AN INTERVIEW WITH PAUL BENACERRAF

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EACH YEAR AN interview with a significant modern figure in philosophy is included in The Dualist. This year, Professor Paul Benacerraf graciously agreed to answer questions provided by The Dualist and the Stanford Philosophy Department.

Professor Benacerraf currently teaches in the Philosophy Department at Princeton University. His writings are influential in logic, the philosophy of mathematics, metaphysics, and the philosophy of language. With Hilary Putnam, of Harvard, he is coeditor of *Philosophy of Mathematics: Selected Readings*.

Benacerraf: I thank you and your colleagues for these questions. I'm afraid I shall not do them justice. Still, here goes. I will more or less lump my answers to Crimmins, Zalta, and Zach together, for reasons that should become obvious.

Let me begin with Mark Crimmins' wonderful and provocative proposed line of thought.

Mark Crimmins

I wonder what you think of this line of thought. An ordinary speaker, long having used number-talk, turns to philosophy and encounters the ontological question of whether there ("really") are numbers. Whatever else she thinks about this question, she does not regard her past usage, or the usage of her linguistic peers, as having expressed commitment to a position on the matter (even when they say things like "there are numbers between ten and twenty"). But if the literal meaning (and truth-conditions) of ordinary talk comes from the standard commitments of speakers, it seems to follow that the ontological question is a different question from that of the literal truth of "there are numbers" in English.

I follow it with a quote from myself.

> …To return in closing to our poor abandoned children, I think we must conclude that their education was badly mismanaged – not from the mathematical point of view, since we have concluded that there is no mathematically significant difference between what they were taught and what ordinary

> mortals know, but from the philosophical point of view. They think that numbers are really sets of sets while, if the truth be known, there are no such things as numbers; which is not to say that there are not at least two prime numbers between 15 and 20. ("What Numbers Could Not Be" 294)

If English had a handy and plausible wide-scope negation, I would begin by availing myself of it to say: "No; I haven't stopped beating my wife." But I wouldn't stop there. I would try to explain. I would add, for example, that I have no wife to stop beating….

More seriously, the view Mark is urging on me has considerable appeal. The Carnap of "Empiricism Semantics, and Ontology" pushed that line in some detail, distinguishing what he called "external", or "framework" questions from "internal" questions. The former [e.g. "Are there ("really") numbers?"] should be understood as calling for a practical decision concerning whether we should adopt a (linguistic) framework that speaks of "numbers", the latter ["Are there prime numbers between 12 and 15?"] as answerable in accordance with the rules of the framework once one has adopted it.

> …if the literal meaning (and truth-conditions) of ordinary talk comes from the standard commitments of speakers, it seems to follow that the ontological question is a different question from that of the literal truth of "there are numbers" in English.

I argued in "Mathematical Truth" that there is a strong *prima facie* case for treating pairs of sentences like

1) There are at least three perfect numbers greater than 17.

2) There are at least three large cities older than New York.

as semantically alike, at least to then extent of attributing to their members a shared first-order structure and the parallel truth conditions that such treatment would entail. In so doing, I was implicitly denying what Mark is claiming—namely that the truth conditions of these sentences do not require the existence of the entities they name and over which they quantify. The enigmatic last sentence from "What Numbers Could Not Be" that I quoted above was meant to express my awareness of the tension Mark is calling to our attention. I would like nothing better than an intelligible and viable interpretation of these sentences that did not embroil us in the conundrums that appear to ensue when we treat them as semantically parallel. However, I don't know of any that I find satisfactory. Carnap's does little more than express the wish in more elaborate and (to me) obscure terms. It doesn't constitute a theory of language that one could use to distinguish the contexts where genuine commitments are taken on from those that are merely trivial expressions of something else. Nor does it tell us much about what that something else might be.

So as you might suspect, I am not altogether happy with Mark's presupposed premise that the speaker's view about whether she was committed by her usage to the view that there "really" are numbers should prevail here over the considered and reasoned theoretical pronouncements of linguists, or whoever is in charge of thinking about these matters [this is the wife I haven't stopped beating]. In my opinion, the best candidate theory that has emerged in the intervening years is some form of fictionalism. Do I find that totally satisfying? No, I do not. For many reasons, but space does not permit their enumeration, which would, in any case, have little more than autobiographical interest. Let me just say, then that the challenge to fictionalism that I find most compelling derives from an article by Eugene Wigner ("The Unreasonable Effectiveness of Mathematics") in which he simply expresses wonderment at the many ways in which mathematics turns out to be useful—the ubiquitous π and e are examples he cites with effect. Unless we also made up he world ….

So, in a word, I am not persuaded.

Edward Zalta

Could you please briefly sketch your current view about the relationship between the meaning of mathematical sentences and our knowledge of mathematics?

Ed Zalta asks a good question too. I have no answer. I simply do not have any view that I find satisfactory. Perhaps the Neo-Fregeans [Wright *et al*.] are on to something [Of course they are, that's a fatuous way to put it.]. But even if their view can be made out, the gap between finding a plausible argument that arithmetical truths are indeed analytic and explaining how we know them would still remain—analyticity as an *epistemic* category has never fully recovered from Quine's attacks; but that connection would have to be made if we were to have an account that linked the semantics to the epistemology.

Richard Zach

Your papers "What numbers could not be" (1965) and "Mathematical Truth" (1973) have significantly shaped the debate in the philosophy of mathematics for the last quarter century by setting out important questions that need to be answered by a satisfactory account of mathematical truth and knowledge. Among the many rival *accounts proposed to address your questions, do you think a clear winner has emerged? What do you see as the most impotant open questions in the philosophy of mathematics?*

I indicated above which approaches to the foundations of arithmetic I find most promising; since I don't find either of them wholly persuasive, there is, in my opinion, no clear winner. As for which open questions in the philosophy of mathematics I see as the most important, I would list, in no particular order, the ones mentioned above plus a host of more particular ones about the foundations of set theory. Here I have in mind all the questions regarding constructivism or finitism that have occupied mathematicians and philosophers ever since they first arose. I would like to understand better what it is to postulate some things. When are we entitled to do so? How does postulation relate to existence? A related set of questions has to do with our grasp of mathematical concepts and the relation between the concepts and their linguistic expression in some formalism. This last is a much-vexed question in all the debates over Skolemite interpretations of languages. I have taken loud, confident, but not very enlightening realist positions on these matters. My true view is that the anti-realist *arguments* I attack are no good. But I am a long way from having a view of my own that I find satisfactory in the sense that it meets the challenge of precisely what the boundaries are of the concepts that I claim are expressed [e.g. non-denumerable set], and how our own abilities are related to the possession of those concepts—what about us makes it so that we possess these concepts? These are questions that worried Gödel throughout his philosophical career and on which he took some rather fantastic stands (which I discussed in a recent talk). They puzzle me too; but I think that at bottom, they are but a species of the generic questions about the nature of mathematical knowledge, and especially, what is it about us in virtue of which we may possess some particular bit of it.

Grigori Mints

Do you think logic is still important for philosophy?

I do think that logic is important for philosophy. First of all, it is part of philosophy. Moreover, as (broadly speaking) the normative theory of logical consequence, it underlies rational thought and its understanding is important for anyone who is interested in *that* subject. It's hard going at times, especially in its nether regions when it is indistinguishable from some branch of mathematics. But that doesn't make it any the less philosophy for all that. It can be taken in small or large doses, and can be beneficial in either. What size dose is appropriate depends on the interests and capacities of the patient.

The Dualist

In papers published in the 1990's you revised some of your assertions from "Mathematical Truth" and "What Numbers Could Not Be." Have your views changed further over the last few years?

As for whether my views have changed in the years since "Mathematical Truth" and "What Numbers Could Not Be", I can only say this. Neither of those papers has as its central focus the expression of certain positive views (although some are invariably attributed to me on the basis of things I say there); they were written to discuss certain arguments and positions that I found puzzling, even baffling—reductionism in arithmetic, the seeming incompatibility of views of mathematics stimulated by semantics on the one hand, and epistemology on the other… These and others of my papers are less meant to advance positive views of my own than to discuss, or debunk, if you prefer, some arguments that have some prominence in the literature and that I find troubling for one reason or another. I still find these puzzling; so those views haven't changed. Nor have I found adequate positive replacements that resolve the issues I raise to my own satisfaction.

The Dualist

"What numbers could not be" is often taken as a starting point for structuralist philosophies of mathematics. Do you think your article provides a good argument for the structuralist viewpoint? Do you find the structuralist viewpoint "promising"?

You ask about structuralism and my own attitude to it. I admit to having flirted with structuralism in some form in "What Numbers Could Not Be". But I couldn't make it work to resolve some of the issues raised there—I found structures are every bit as indeterminate as the objects they were meant to replace. The intuition that structural properties (rather than definite objects) were what mattered struck me then, and strikes me now, as a possibly promising line of thought. But to make it do the needed work one needs a plausible (and epistemically transparent) theory of properties and relations. As they say, "reduces to previous case".

The Dualist

If you were to edit a third edition of "Philosophy of Mathematics:

Selected Readings" what new papers would you add?

I pass on the third edition of *Philosophy of Mathematics: Selected Readings*. There's a ton of good material out there and the selection would be difficult indeed. I think now that a third edition would be in two volumes. I still believe in reading the early stuff, but a companion volume with more recent discussions would, I think, be a valuable addition, pedagogically.

The Dualist

Can you describe some of your current and future projects? In what ways do you hope your work might set the agenda for future philosophy of mathematics?

My most recent musings have been on the issues I mentioned a bit earlier on, regarding our grasp of mathematical concepts. I taught a seminar this past Spring on Gödel's philosophical views, as expressed in his published and posthumous papers, as well as in a few conversations I was privileged to have with him. I'm not sure if that will come to anything of more than anecdotal interest, but I will soon write up a talk I gave this summer on the matter under the title "Gödel's Monadology". I am also writing a small monograph of which "What Mathematical Truth Could Not Be – I" is the first installment and "What Mathematical Truth Could Not Be – II" the second. These are meant to review the issues in "Mathematical Truth" and "What Numbers Could Not Be" in the light of the intervening discussion. I do have some further things to say, but they are too rough right now even to attempt to put them down here [I tried and only managed to delay this response unpardonably].

As to whether I hope that my work would set the agenda for future philosophy of mathematics, I had no such hope 35 years ago and to the extent that it has, have been mostly very surprised, but also very embarrassed. I have no such hope now. At best, I hope that students of the subject find in it something to help them think through the thorny issues that constitute this wonderful subject.

The Dualist

While undergraduates may have experience with natural science, English, Mathematics in high school, undergraduates rarely enter college with exposure to analytic philosophy. In your experience, what has been the best way for teachers to introduce students to analytic philosophy?

In my opinion, the best way to introduce students to philosophy, analytic or otherwise, is to find some issues in which they are likely to be interested and to expose them to ways of thinking about these issues, setting a standard of argument and exposition as you go along that draws the students into a discussion in whose development they feel they have a stake. This can be done either with contemporary issues or with classical texts; but it has to be done in such a way as to engage their active commitment and participation. I don't mean that there has to be discussion—not everyone is comfortable with that, especially at first. But there has to be some active participation of some sort—and some writing.

The Dualist

Do you have advice for students considering a career in philosophy?

Advice for students who are considering a career in philosophy? Be prepared to suffer; and be prepared to conclude after some attempts that it isn't for you. You can "do" philosophy wherever you are and whatever else you do. But a career in philosophy is very demanding, with rewards that aren't always evident. It's a truism that the job market is tough and competition awful. I'm assuming everyone knows that. My point concerns the benefits and costs of *a career* in philosophy, if you succeed in overcoming these initial hazards. It is a subject that doesn't repay effort with trappings of success—there are always far more reasons to be dissatisfied with what you may have achieved than to glory in it. You have to be prepared for that.

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