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Quantifying Price Flexibility In Material Procurement as a Real Option

By

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Summary

Title: Quantifying price flexibility in material procurement as a real option

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Abstract

This paper extends theories in finance and economics to compare the cost of a long-term contract with a price cap to that of spot purchases in construction material procurement. In construction, material procurements are usually short-term, project-based, and have a price volatility of up to 30%. These characteristics and the competitive nature of the industry lower the profit margin of general contractors. We have observed that contractors purchase a stable amount of commodity materials such as concrete, structural steel, and lumber throughout the year. For contractors, the price cap reduces the price volatility of materials without their being obliged to a quantity; for suppliers, the contracts give them steady demand and a bigger market share. We evaluate this contract as a real option and solve for the contractor's optimal ordering policy. The challenge is to model price processes when materials are not frequently traded. We model price processes by using as much market information as possible and then evaluate the idiosyncratic uncertainties in a risk-neutral setting. Our methodology does not require market completeness and incorporates some of the latest research in finance such as correlation pricing, option pricing, and zero level pricing, as well as Monte Carlo simulation.

Keywords: material procurement, real option, pricing, ordering policy, finance.

Introduction

Research on the finance side of construction management can provide new ways to

manage cash flow and increase profit margins. Since the seminal work on pricing options by (Black and Scholes, 1973), the financial market of options and other derivatives has been growing exponentially, providing numerous ways for individuals and corporations to hedge risks and improve the performance of their portfolios. An option is valuable because it provides flexibility; it is the right but not the obligation to buy (or sell) an asset under specified terms.¹ Non-paper assets are called real assets and options/flexibilities in acquiring real assets are called real options. A material contract with a price cap is similar to a call option because a buyer pays the cap price when the spot price is larger than the cap. However, the straight application of option pricing to non-financial assets is problematic because most option pricing formulae assume that the options and their underlying assets are frequently traded in efficient and complete markets such as the NYSE. This assumption is needed to construct an artificial replication of the option from the underlying traded asset in order to obtain an unequivocal price for the option. Like other real assets, construction materials are not traded in a national exchange, but there has nevertheless been fruitful research in evaluating real options in oil drilling, software sales and semiconductor facility planning (Benavides, Duley et al., 1998). To evaluate these options, it is important to circumvent the strong assumption of a frictionless and complete market. Our methodology does not require market completeness and incorporates some of the latest research in finance such as the projection pricing, correlation pricing, option pricing, and zero level pricing, as well as Monte Carlo simulation. Our approach is to model the price process of a material by using as much market information as possible and then evaluate the idiosyncratic risks in a risk neutral setting. In appendix II, we show that this is a more accurate model in terms of variance than a simple time series analysis.

We believe that studying real options can bring great value to the construction industry by quantifying the value of managerial flexibility. For example, a contractor solicits take-offs (quotes) from suppliers for his bid preparation but does not place an order until the developer awards the contract. Therefore, his orders are usually short-term, project-based, and subject to fluctuation in price. These characteristics and the competitive nature of the industry lead to an extremely low profit margin for contractors. This profit margin is typically less than 5%, a rate lower than the return on risk free treasury bonds during most of the 1990s. However, through different projects, contractors do purchase a stable amount of commodity materials such as concrete, structural steel, and lumber throughout the year. It is because contractors adjust markup in order to keep the yearly volume relatively stable (Carr, 1982). What if a contractor has the right to place a cap on the price of these commodities? This option would provide buyers with valuable operating flexibility that would allow them to minimize their inventory cost and price volatility. It would also allow

suppliers to secure a share of the market and smooth out production schedule. In this paper, we focus on materials purchased by large door subcontractors who have the volume to realize the value introduced by real options and the power to negotiate complicated long-term contracts with suppliers. We believe options like these have a significant potential in the construction industry and will increase profit margins for many parties along the supply chain by minimizing wastes and increasing efficiency.

Case

W&W is the largest door distributor and subcontractor on the west coast. It buys oriented strand board (OSB) door panels from suppliers like Weyerhaeuser and mills the panels into correct dimensions according to architects' specifications. It then attaches hardware such as knobs, hinges, locks, and stops to the panels and make frames for the finished doors. W&W either sells the doors to other subcontractors as a distributor or installs the doors directly as a subcontractor. The number of doors W&W makes and installs every month is not disclosed but is likely to be in the tens of thousands. For example, a typical Kaiser hospital has 8000 doors and W&W handles most Kaiser hospitals and many Cisco campuses. It turns out that many interior doors in commercial buildings are more or less the same, although they look different. In the old days, doors were made of solid wood, and so a mahogany door had a completely different core from that of a red oak door. Nowadays, solid wood doors are rare because they are very expensive. Instead, particle board or OSB is often used as the core and then a thin layer of laminate is adhered on top of the core to make the door resemble mahogany or red oak. As a result, many doors are only different in dimensions and the paper-thin laminates. If W&W buys the same materials repeatedly for different projects, then a real option embedded contract where the following characteristics hold would be valuable to it:

1. Where the term of the contract is long (nine months in duration, for example) instead of project-based;
2. Where the price of OSB is capped. If the spot price (current market price) of OSB is less than the price cap, W&W pays the spot price. Otherwise, W&W pays the price cap; and,
3. Where the contract terms (such as price) are based on projected order size Q every month but W&W has the option to vary order size month to month.

This contract has a real option because W&W has the option to pay the cap price when the spot price is higher than the cap price. When the spot price is lower than the cap price, W&W pays the spot price. Therefore, it can be seen as a bundle of American call options.

The benefits of such a contract are as follows:

1. W&W can minimize the price volatility of OSB and have a more predictable

cost structure and return on investment. A smoothed-out ROI of 10% per annum for two years is better than 15% for the first year followed by 5% for the second year, especially when the company is publicly traded.

2. W&W can use orders from different projects to diversify and smooth out the order size for each month. Therefore, W&W can make larger commitments and negotiate lower the cap price with suppliers.
3. Flexible order size reduces inventory cost for W&W because there will be less excess material and less buffer inventory needed.
4. W&W can transform cost savings into lower bids to general contractors or developers in order to win more projects.
5. The supplier can increase its market share by signing contracts with multiple buyers like W&W. The supplier can expect W&W always to buy from her because even when the spot price is lower than the cap price, W&W will order from her to avoid loss of goodwill.
6. The supplier can save costs by better forecasting resource allocation, hiring, maintenance, and capacity expansion/contraction.

Formulation and Modeling

We are going to quantify the benefits of the price cap and reduced inventory cost in the following sections. Before we do that, we need to

1. Model the demand dynamics of interior commercial doors that use OSB as core.
2. Model the price dynamics of OSB.
3. Find out the optimal policy for a buyer to exercise the option by minimizing the expected total cost.

A Model for Demand Dynamics

We model the demand for W&W's interior door as a private risk that is independent of the price of OSB. From historical demand data, we model the demand as a geometric Brownian motion (GBM) (see chapter 11 of (Luenberger, 1998) or chapter 2 of (Oksendal, 1998)) because like GBM, demand is never negative, proportional to the length of the period, and has fluctuations that appear to be independent from those of other periods. We denote the demand for time t to be D_t , which has the following forms in continuous time:

$$dD_t = \mu D_t dt + \sigma D_t dB_t \quad \text{and} \quad D_t = D_0 e^{((\mu - 0.5\sigma^2)t + \sigma B_t)}$$

For this particular example, demand is expected to be stationary and lognormally distributed. Therefore, $\mu = 0$ and $dD_t = \sigma D_t dB_t$. In discrete time, demand has mean equal to D_{t-1} and standard deviation equal to $D_{t-1} (e^{\sigma^2} - 1)^{0.5}$ for period t .

A Model for Price Dynamics

In order to evaluate the option objectively, we would like to find the market price for

OSB. According to the famed Capital Asset Pricing Model (CAPM), price of a market asset is determined by its beta:

$$\beta = \frac{\text{cov}(\text{asset}, \text{marketportfolio})}{\sigma_{\text{Marketportfolio}}^2}$$

However, OSB is not traded in any market. It is also difficult to model the price dynamics of OSB (especially the expected growth rate of price) if we do not have sufficient historical data. Nevertheless, we can model the price of OSB using the Correlation Pricing Formula (CPF) (see Appendix I) and the Nested Projection Theorem (Luenberger, 2001). Furthermore, under CPF, the expected growth rate of price of OSB will have a smaller variance than in a simple time series analysis (see Appendix II).

CPF states that an asset W , which is not frequently traded, should be priced by a market asset which is most correlated with it. It turns out that the payoff of the market asset is the orthogonal projection of the payoff of W , W_f , onto the market space and CPF has the same form as the CAPM formula. The only difference between CPF and CAPM is that the market asset in CPF does not need to be the market portfolio and can be made up by any linear combination of existing market assets (stocks, bonds, derivatives, etc). The form of CPF is:

$$P_W = \frac{1}{R} \left[\bar{W} - \beta_{W_f, Y} * (\bar{Y} - P_Y * R) \right]$$

The Nested Projection Theorem states that if K and M are subspaces of a Hilbert space and M is a subspace of K , the projections of W_f onto M and K , W_M and W_K respectively, have the following relationship: $W_M = (W_K)_M$ ³. The elements of the vector, W_f , in the Hilbert space are the payoff of W in different states one period later, each with a different probability of happening.

OSB is made of generic lumber strands bonded together by resin. Both lumber and resin are commodities that have much broader applications and market than doors. Although, lumber and resin may not be traded in an efficient market, regional price indices for lumber and resin exist in some parts of the country. We are therefore able to compute the beta of the price of lumber with respect to P (P can be either the market portfolio or an efficient portfolio in the market space), $\beta_{L,P}$, as

well as the beta of the price of resin with respect to P , $\beta_{S,P}$ ⁴. If we find a linear combination of lumber and resin, $a*L + b*S=T$, which has the biggest correlation with the price of OSB⁵ (a and b probably being the proportions of lumber and resin in OSB), we can find the beta of T with respect to P by,

$$\text{cov}(T, P) = \text{cov}(a * L + b * S, P) = a * \text{cov}(L, P) + b * \text{cov}(S, P)$$

$$\beta_{T,P} = \frac{\text{cov}(T,P)}{\sigma_P^2} = \frac{a * \text{cov}(L,P) + b * \text{cov}(S,P)}{\sigma_P^2} = a * \beta_{L,P} + b * \beta_{S,P}$$

Suppose L and S are not in the subspace spanned by market assets but are most correlated to market assets Lm and Sm respectively. From Appendix I, the prices of Lm and Sm are:

$$P_{Lm} = \frac{1}{R} [\bar{l}m - \beta_{Lm,P} * (\bar{p} - P_P * R)]; \quad P_{Sm} = \frac{1}{R} [\bar{l}s - \beta_{Sm,P} * (\bar{p} - P_P * R)]$$

$$\therefore P_L = \frac{1}{R} [\bar{l} - \beta_{L,Lm} * (\bar{l}m - P_{Lm} * R)] = \frac{1}{R} [\bar{l} - \beta_{L,Lm} * (\beta_{Lm,P} * (\bar{p} - P_P * R))]$$

Since $\text{cov}(P_L, P_P) = \beta_{L,Lm} * \text{cov}(P_{Lm}, P_P) \Rightarrow \beta_{L,P} = \beta_{L,Lm} * \beta_{Lm,P}$, we have

$$P_L = \frac{1}{R} [\bar{l} - \beta_{L,P} * (\bar{p} - P_P * R)]. \quad \text{Similarly, } P_S = \frac{1}{R} [\bar{s} - \beta_{S,P} * (\bar{p} - P_P * R)]$$

If we add L and S to the market space, from CPT,

$$P_{OSB} = \frac{1}{R} [\overline{osb} - \beta_{OSB,T} * (\bar{t} - P_T * R)] \quad <1>$$

where $\bar{t} = a * \bar{l} + b * \bar{s}$

By linear pricing, $P_T = a * P_L + b * P_S$

$$\therefore \bar{t} - P_T * R = a * \beta_{L,P} * (\bar{p} - P_P * R) + b * \beta_{S,P} * (\bar{p} - P_P * R)$$

$$= (a * \beta_{L,P} + b * \beta_{S,P}) * (\bar{p} - P_P * R)$$

Putting back into equation <1> for P_{OSB}, we have,

$$P_{OSB} = \frac{1}{R} [\overline{osb} - \beta_{OSB,T} * (a * \beta_{L,P} + b * \beta_{S,P}) * (\bar{p} - P_P * R)]$$

$$P_{OSB} = \frac{1}{R} [\overline{osb} - \beta_{OSB,T} * \beta_{T,P} * (\bar{p} - P_P * R)] \quad <2>$$

Equation <2> is consistent with the Nested Projection Theorem in that we find the price of OSB by first projecting the random vector of OSB onto the plane LS spanned by vectors of lumber (L) and resin (S). The resulting vector T on LS is a linear combination of L and S and correlates most closely with OSB among vectors on LS as well as the market space. We then further project T onto the space M spanned by market assets and the resulting vector Tm is a linear combination of projections of L on M, Lm, and S on M, Sm. The price of Tm is found by linear pricing and its beta is also a linear combination of betas of L and S. As a result, the price of OSB by projection and correlation pricing is really the price of Tm in the market space plus the expected non-market payoffs discounted by the risk free rate. Expressing the last equation in terms of growth rates, we have⁶,

$$R_f = \frac{1}{P_{OSB}} \left[\overline{OSB} - \frac{P_{OSB} * P_T}{P_T^2} * \beta_{R_{OSB}, R_T} * \frac{P_T * P_P}{P_P^2} * \beta_{R_T, R_P} * P_P * (\overline{R_P} - R_f) \right]$$

$$\therefore \overline{R_{OSB}} = R_f + \beta_{R_{OSB}, R_T} * \beta_{R_T, R_P} * (\overline{R_P} - R_f) \quad <3>$$

Equation <3> has the similar form to the CAPM formula. If we collect data on β_{R_{OSB}, R_T} , β_{R_T, R_P} , $\overline{R_P}$, and R_f , we can find $\overline{R_{OSB}}$, the expected growth rate of price of OSB. The CPF gives us the foundation to work with a risk neutral martingale measure rather than real probabilities because the price of OSB is determined by its replicable projection onto the expanded albeit incomplete market space. Therefore, we can discount cash flows according to a risk-free interest rate in a risk neutral world.⁷ Otherwise, we would have to find the risk-adjusted discount rate for OSB. Due to storage cost, in the risk neutral world, $E[R_{OSB}] = F_L / P_L$ where F_L and P_L are the forward and the spot prices of lumber respectively.⁸

An Optimal Exercising Policy

We now solve for the optimal policy for W&W to order OSB because the value of the contract to W&W depends on how much she orders. We first construct a simple model to describe the fulfillment of demand, handling of inventory and backorder, and various cost factors with the following conditions:

1. The contract is for nine periods. Each period (Δt) is one month.
2. The demand in each month is D_t and comes at the beginning of each month. It is independent from month to month and is log-normally distributed as described above.
3. The cost of each OSB door panel is $\$c_t$ and is governed by the price dynamics described above.
4. The cap price for each door panel is $\$c$ with option.
5. The discount factor is constant at $1/R$ per month.
6. The order size for each month is y_t , which is determined by the optimal policy.
7. The initial inventory of doors for each month is x_t and unfulfilled demand is back ordered. Therefore, $x_{t+1} = y_t + x_t - D_t$ (see Figure 1).
8. The storage cost is $\$h$ /panel/month, and the shortage cost is $\$p$ /panel/month.⁹ The shortage cost should be understood as either the loss of goodwill to W&W's customers or as a discount given to a customer to ensure her patience. The contract involves a volume requirement and will therefore result in a loss of goodwill if the buyer orders less than expected. The supplier expects the buyer to order on average Q door panels every month for a total of $9Q$ over nine months. If the buyer orders less than $9Q$, the supplier will be disappointed. The loss and gain of goodwill is modeled as $\$w * (9Q - \sum_{t=1}^9 y_t)^+$ where $\$w$ is the cost/loss of

goodwill quantified in monetary terms and is discounted in the same way as other costs are. We use goodwill because monetary penalty for not making an order quota is hard to implement in a long-term business relationship as every buyer finds it objectionable and supplier does not want to force the buyer to pay.

9. The order and delivery times are at the beginning of each month.
10. The inventory at the end of the 9th month can be salvaged for no more than the cap price.

We assume that W&W has the ability to carry a small inventory and that the terms of the contract (length, price cap, expected monthly order size, and so on) are the result of a negotiation between W&W and the supplier. The cost of goodwill is an estimate made by W&W.

We now solve for the optimal policy for W&W to order OSB because the value of the contract to W&W depends on how much she orders. With option, the expected cost at the beginning of the ninth month is:

$$E[Cost_9] = E \left[\begin{array}{l} c'_9 * y_9 + (p+h) * (y_9 + x_9 - D_9)^+ - p * (y_9 + x_9 - D_9) \\ + w * \left(9Q - \left(x_9 - x_1 + y_9 + \sum_{t=1}^8 D_t \right) \right)^+ - \frac{c'_{10}}{R} * (y_9 + x_9 - D_9)^+ \\ + \frac{c_{10}}{R} * (D_9 - y_9 - x_9)^+ \end{array} \right] \quad <4>$$

In <4>, the first term in the bracket is the purchase cost and the second and the third terms are the storage and the shortage costs. The fourth term is the goodwill cost and the fifth term is the salvage value of the ending inventory. The last term is the backorder cost in case not all demand can be fulfilled in the ninth month. Since we have the option to buy the panel at \$c with the option, $c'_i = \min(c, c_i)$.

Since $E[Cost_9]$ is a convex function, finding the first order condition is sufficient.

Minimizing <4> with respect to y_9 by differentiation gives¹⁰:

$$\underset{y_9}{Min}(E[Cost_9]) \Rightarrow \frac{dE[Cost_9]}{dy_9} = E \left[\begin{array}{l} c'_9 + (p+h) * I(y_9 > (D_9 - x_9)) - p \\ - w * I(y_9 < L) - \frac{c'_{10}}{R} * I(y_9 > (D_9 - x_9)) \\ - \frac{c_{10}}{R} * I(y_9 \leq (D_9 - x_9)) \end{array} \right] = 0$$

$$\text{where } L = 9Q - \left(x_9 - x_1 + \sum_{t=1}^8 y_8 \right)$$

$$\therefore c'_9 + (p+h) * P(D_9 < (y_9 + x_9)) - p - w * I(y_9 < L) - \frac{E[c'_{10}]}{R} * P(D_9 < (y_9 + x_9))$$

$$-\frac{E[c_{10}]}{R} * (1 - P(D_9 < (y_9 + x_9))) = 0$$

$$\therefore P(D_9 < (y_9^* + x_9)) = \frac{p + \frac{E[c'_{10}]}{R} + w^* I(y_9 < L) - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}} \quad <5>$$

$I(y_9 > (D_9 - x_9))$ is the indicator variable that is one when $y_9 > (D_9 - x_9)$ and zero

otherwise. Then the derivative of $(y_9 > (D_9 - x_9))^+$ with respect to y_9 is equal to

$I(y_9 > (D_9 - x_9))$ except at $y_9 = D_9 - x_9$, which occurs with probability zero. Finally,

$E[I(y_9 > (D_9 - x_9))] = P(D_9 < (y_9 + x_9))$. Since $E[c'_{10}] \leq E[c_{10}]$, <4> is convex

and y_9^* is the order size that minimizes the expected cost. In words, W&W should

order y_9^* doors such that the probability that the demand does not exceed $y_9^* + x_9$ is

equal to the right hand side of <5>.

We will analyze <5> by setting cases:

Case 1: $L \leq 0$

$$\Rightarrow P(D_9 < (y_9^* + x_9)) = \frac{p + \frac{E[c'_{10}]}{R} - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}}$$

$$\text{Case 2: } \left\{ \begin{array}{l} L = M > 0, y_9 = M^+, \text{ and } P(D_9 < (M^+ + x_9)) \leq \frac{p + \frac{E[c'_{10}]}{R} - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}} \end{array} \right.$$

$$\Rightarrow y_9^* > L = M$$

$$\text{Case 3: } \left\{ \begin{array}{l} L = M > 0, y_9 = M^-, \text{ and } P(D_9 < (M^- + x_9)) \geq \frac{p + \frac{E[c'_{10}]}{R} + w^* I(y_9 < L) - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}} \end{array} \right.$$

$$\Rightarrow y_9^* < L = M$$

$$\text{Case 4: } \left\{ \begin{array}{l} L = M > 0, y_9 = M^+, P(D_9 < (M^+ + x_9)) > \frac{p + \frac{E[c'_{10}]}{R} - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}} \\ \text{but } P(D_9 < (M^- + x_9)) < \frac{p + \frac{E[c'_{10}]}{R} + w * I(y_9 < L) - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}} \end{array} \right.$$

$$\Rightarrow y_9^* = L = M$$

At the beginning of the eighth month, the expected cost is:

$$E[Cost_8] = E \left[\begin{array}{l} c'_8 * y_8 + (p+h) * (y_8 - D_8 + x_8)^+ - p * (y_8 - D_8 + x_8) \\ + \frac{E[Cost_9(y_9^*(y_8))]}{R} \end{array} \right]$$

Since $E[Cost_8]$ is a convex function, finding the first order condition is sufficient.

Minimizing with respect to y_8 by differentiation gives:

$$\text{Min}_{y_8} (E[Cost_8]) \Rightarrow \frac{\delta E[Cost_8]}{\delta y_8} = E \left[c'_8 + (p+h) * I(y_8 > (D_8 - x_8)) - p - \frac{c'_9}{R} \right] = 0$$

because <4> and cases 1 to 4 implies that y_8 increases one to one with x_9 , and

$x_9 + y_9$ is independent of y_8 . For example in case 2,

$$\text{suppose } P(D_9 < (M^+ + x_9)) = \frac{p + \frac{E[c'_{10}]}{R} - c'_9}{p + h - \frac{E[c'_{10}]}{R} + \frac{E[c_{10}]}{R}},$$

x_9 increases by one $\Rightarrow L$ decreases by one $\Rightarrow y_9 = M^+$ decreases by one.

$$\therefore P(D_8 < (y_8^* + x_8)) = \frac{p + (E[c'_9])/R - c'_8}{p + h}$$

$$\text{For other months, } P(D_t < (y_t^* + x_t)) = \frac{p + (E[c'_{t+1}])/R - c'_t}{p + h} \text{ for } t = 1 \dots 8$$

Similarly, the ordering policy without the price cap is:

$$P(y_i^* + x_i > D_i) = \frac{p + (E[c_{t+1}]) / R - c_t}{p + h} \text{ for } t = 1 \dots 9$$

To find out the value of the contract, we use Monte Carlo simulation to generate 100,000 sample pairs of total costs (with and without price cap) over nine months.¹¹ We assume that the demand for OSB core doors has two properties. First, it is independent of the price movements. Second, it is a private uncertainty and the OSB doors business only makes up a small portion of the total W&W business (which encompasses, amongst other things, other types of doors, exterior GFRC and glass walls, and windows.). The second property allows us to use zero level pricing (see chapter 16 of (Luenberger, 1998)) and the risk neutral probabilities for OSB demand is the same as the real probabilities. Aside from the demand distribution and the shortage, goodwill, and holding costs (which are specific to W&W), we estimate the σ 's, risk free interest rate, and other parameters using market data (see Appendix III). The value of the option is then estimated by the mean of percentage cost savings:¹²

$$= \frac{E[TC_{NO}] - E[TC_O]}{E[TC_{NO}]} * 100\% .^{13}$$

Sensitivity Analysis

We perform a sensitivity analysis for two reasons. First, we would like to find out how the expected costs savings would change as the parameters vary because we estimate the parameters from market data, which may not be abundant (see Appendix III) or the most closely correlated ones. Second, we would like to identify the sensitive parameters—those that affect the value of the contract more than others. The two most sensitive parameters are the cap price and the volatility of the price of OSB. We tabulate the percentage of savings over a range of cap prices and a range of the volatility of the price of OSB (see Figure 2). Not surprisingly, the value of option increases with the volatility of the price of OSB and decreases with the cap price. Furthermore, the savings are quite significant when compared with the current profit margin of contractors. For example, if volatility is 0.1 and the cap price is \$105 (that is, higher than the initial OSB price and about the same as the expected OSB price at the end of nine months), the cost savings will be 13.16%. If material costs making up half of the bid, a 13.16% increase in margin becomes a 6.58% increase in margin for the whole project. For many contractors, this could mean increasing the margin for projects by 100%. On the other hand, suppliers, who absorb the price volatility risk, may want to share the cost savings with contractors in the form of contract fees.

Conclusion

We have shown a method to quantify the cost savings of a long-term material contract with a price cap over spot purchases. The contract has a real option because it provides the flexibility to buy at the cap price when the price is high and the option to buy at the spot price when the price is low. We evaluate the contract using correlation pricing and Monte Carlo simulation. The correlation pricing formula allows us to calculate the price of OSB more accurately when the historical pricing data of OSB is limited. We avoid the assumption that the market is complete and we did not find the value of the option by replicating the material contract. Instead, we price OSB by pricing something that closely correlates with it. The process is similar to pricing one's home by checking out the sale prices of one's neighbors' homes. We believe the value of the option is larger than what we have presented here as we have only quantified two of the six benefits listed above. In the future, we would like to take account of the fact that some contractors, unlike W&W, are unwilling to take any inventory. We aim to construct a model in which contractors estimate their need K in advance and can later order any quantity less than K while suppliers use lumber futures and call options to create "stack and roll" minimum variance hedge positions to minimize price risk. Furthermore, we will investigate how we can design the contracts so that contractors do not have the incentive to overestimate the demand. Finally, we would also like to quantify how such a long-term contract with a price cap can increase a company's market share as well as the impact on cost savings when the demand and the price of OSB are correlated.

Appendix I—Projection Pricing and the Correlation Pricing Formula

This is a derivation of the Correlation Pricing Formula (CPF) through a simple one period example in a general n -dimension Hilbert space. It is analogous to Luenberger's (Luenberger, 2001) in the L^2 space of random variables but is easier to visualize and interpret. It also states the CPF in terms of rates of return, which obliterates the need to determine the expected values of assets. In the vector space, a vector represents the uncertain value of an asset one period from now. The k -tuple of a k -dimensional vector ($k \leq n$) corresponds to the k possible states of the value of the asset one period from now. Each state has a probability of occurring and the probabilities of the states sum to one. The states belong to two main groups, the market states and the private states. Market states form the subspace of the n -dimension space and are spanned by assets traded in financial markets. Private states are states specific to some non-traded assets and are independent of the market states. All future values mean the values one period from now and are denoted by the subscript f , while the present values are denoted by the subscript p . For example, suppose W_f , the future value of a lumber mill W , depends on the price of lumber and

the cost of its maintenance. Lumber is a traded commodity and therefore a market asset. Suppose there are three possible future market states with probabilities 0.3, 0.3, and 0.4 respectively¹⁴. The portion of the future value of the mill related to the price of lumber will be x_1 , x_2 , or x_3 depending on which of the market states will happen. On the other hand, the cost of maintenance will be in one of two different states with probabilities 0.4 and 0.6 respectively. The portion of the future value of the mill related to maintenance will be z_1 or z_2 , depending on which of the private states will occur. Assuming that the two portions are additive and that the market states and private states are independent of each other, there are six possible values for W_f (see Figure 3). On the other hand, suppose there are three market assets, two stocks, Y_1 and Y_2 , and one risk free asset, R . Y_{1f} , Y_{2f} , and R_f form the basis of the three dimensional market space one period from now (see their representation in the six-dimensional vector space in Figure 4). Any market asset can be represented as a linear combination of the basis assets, Y_{1f} , Y_{2f} , and R_f such as $a*Y_{1f}+b*Y_{2f}+c*R_f$. Its price will be the same linear combination of the prices of the basis assets and its future value will be the linear combination of Y_{1f} , Y_{2f} , and R_f .

We define the weighted inner product of vectors W and Y to be

$$(W | Y) = \sum_{i=1}^n p_i \cdot w_i \cdot y_i = \|w\| \|y\| \cos \Theta, \text{ where } \|W\| = \sqrt{(W | W)}, \Theta \text{ is the angle between}$$

W and Y , y_i is the future value of Y in state i , and p_i is the probability of realizing state i . Therefore, the projection of vector W on vector Y is $\left(\frac{(W | Y)}{\|Y\|^2}\right)Y$. Two vectors are orthogonal to each other if their weighted inner product is equal to zero.

We define

$$Y'_{jf} = \begin{bmatrix} y_{j1} - \overline{y_j} \\ y_{j1} - \overline{y_j} \\ y_{j2} - \overline{y_j} \\ y_{j2} - \overline{y_j} \\ y_{j3} - \overline{y_j} \\ y_{j3} - \overline{y_j} \end{bmatrix} \text{ where } \overline{y_j} = \sum_{i=1}^3 q_i * y_{ji}, \text{ the expected value of } Y_j \text{ one period from}$$

now.

It is easy to verify that $(Y'_{jf} | R_f) = 0$. Let R_f , Y_{1f} , and Y_{2f} span a space M and let Y'_{1f} and Y'_{2f} span a space M' that is orthogonal to R_f . Therefore, the projection of W_f onto M can be expressed as the sum of the projection of W_f onto M' and the projection of W_f onto R_f .

If W is only a small portion of our asset portfolio, we can price the private

portion of W_f in a risk-neutral way regardless of our utility function—it is equal to $\bar{z} = v_1 * z_1 + v_2 * z_2$. On the other hand, we can find the linear combination of market assets which replicates and therefore has the same price as the market portion of W_f . We will show that the sum of the prices of the private and market portions is equal to that of the projection of W_f onto M , which can be split into two additive parts.

$$\text{The projection of } W_f \text{ onto } M = P(W_f, M) = \frac{(W_f | R_f)R_f}{\|R_f\|^2} + (W_f | Y')Y'$$

$$\text{where } Y' = \begin{bmatrix} y_a \\ y_a \\ y_b \\ y_b \\ y_c \\ y_c \end{bmatrix} \text{ is a linear combination of } Y'_{1f} \text{ and } Y'_{2f} \text{ and is proportional to}$$

the projection of W_f onto M' . Also, $\bar{y}' = \sum_{i=1}^6 p_i * y_i' = 0$ and we scale Y' so that

$\|Y'\| = 1$. By the dual projection theorem, Y' is also the unique unit vector in M'

that maximize $\text{cov}(W_f, Y')$.

$$\begin{aligned} (W_f | R_f) &= 0.12 * (x_1 + z_1) * R + 0.18 * (x_1 + z_2) * R + 0.12 * (x_2 + z_1) * R \\ &+ 0.18 * (x_2 + z_2) * R + 0.16 * (x_3 + z_1) * R + 0.24 * (x_3 + z_2) * R \\ &= \sum_{i=1}^3 q_i * x_i * R + \sum_{i=1}^2 v_i * z_i * R = \bar{x} * R + \bar{z} * R = (\bar{x} + \bar{z}) * R \end{aligned}$$

$$\begin{aligned} (W_f | Y') &= 0.12 * (x_1 + z_1) * Y_a + 0.18 * (x_1 + z_2) * Y_a + 0.12 * (x_2 + z_1) * Y_b \\ &+ 0.18 * (x_2 + z_2) * Y_b + 0.16 * (x_3 + z_1) * Y_c + 0.24 * (x_3 + z_2) * Y_c \\ &= (X | Y') + \bar{z} * \bar{y}' = (X | Y') = \text{cov}(X, Y') = \text{cov}(W_f, Y') \end{aligned}$$

$$\therefore \frac{(W_f | R_f)R_f}{\|R_f\|^2} + (W_f | Y')Y' = (\bar{x} + \bar{z}) \frac{R_f}{\|R_f\|} + \text{cov}(X, Y')Y'$$

From the above, we can see that the length of the projection of the private part onto R_f and M' is \bar{z} and 0 respectively; and that the length of the projection of the market part onto R_f and M' is \bar{x} and $\text{cov}(X, Y')$ respectively.

If $Y' = a * Y_1' + b * Y_2'$,

$$\text{cov}(X, Y') = \text{cov}(W_f, Y') = \text{cov}(W_f, a * Y_1 + b * Y_2) = \text{cov}(W_f, Y)$$

Furthermore,

$$\|Y'\|^2 = (Y_j - \bar{y}_j | Y_j - \bar{y}_j) = \sum_{i=1}^6 p_i * (y_{ji} - \bar{y}_j)^2 = \bar{y}_j^2 - (\bar{y}_j)^2 = \text{var}(Y_j)$$

$$\text{Define } \beta_{W_f, Y} = \frac{\text{cov}(W_f, Y)}{\text{Var}(Y)}$$

$$\therefore \frac{(W_f | R_f) R_f}{\|R_f\|^2} + (W_f | Y') Y' = (\bar{x} + \bar{z}) \frac{R_f}{\|R_f\|} + \beta_{W_f, Y} [a * (Y_1 - \bar{y}_1) + b * (Y_2 - \bar{y}_2)]$$

The price of R , Y_1 , and Y_2 are 1, P_1 , and P_2 respectively. As a result, the price of

$$P(W_f, M) = \frac{(\bar{x} + \bar{z})}{R} + \beta_{W_f, Y} \left[a * P_1 + b * P_2 - \frac{(a * \bar{y}_1 + b * \bar{y}_2)}{R} \right]$$

Let $Y = a * Y_1 + b * Y_2$ and $\bar{y} = a * \bar{y}_1 + b * \bar{y}_2$. Since $\bar{x} + \bar{z} = \sum_{i=1}^6 w_i * p_i = \bar{w}$, price of

$$P(W_f, M) = \frac{1}{R} \left[\bar{w} - \beta_{W_f, Y} [\bar{y} - P_Y * R] \right]$$

Therefore, the Correlation Pricing Formula for W_f is

$$P_W = \frac{1}{R} \left[\bar{w} - \beta_{W_f, Y} [\bar{y} - P_Y * R] \right]. \text{ In other words, the formula posits that the price of}$$

W_f is equal to the expected payoff of the private uncertainty plus the price of Y , which is the market asset most correlated with W_f .

$$\text{Let } \bar{R}_Y = \frac{\bar{y}}{P_Y}, \bar{R}_W = \frac{\bar{w}}{P_W}, R_Y = \frac{Y}{P_Y}, R_W = \frac{W_f}{P_W}. \text{ Therefore, } \beta_{W_f, Y} = \frac{P_w * \text{cov}(R_W, R_Y) * P_Y}{P_Y^2 * \sigma_{R_Y}^2}$$

Rearranging the CPF, we have,

$$\frac{\bar{w}}{P_W} = R + \frac{\beta_{W_f, Y} [\bar{y} - P_Y * R]}{P_W} = R + \frac{P_w * \text{cov}(R_W, R_Y) * P_Y}{P_W * P_Y^2 * \sigma_{R_Y}^2} [\bar{R}_Y * P_Y - P_Y * R]$$

$$\therefore \bar{R}_W = R + \frac{\text{cov}(R_W, R_Y)}{\sigma_{R_Y}^2} [\bar{R}_Y - R] = R + \beta_{R_W, R_Y} [\bar{R}_Y - R]$$

The benefit of expressing CPF in terms of rates of return is that we do not have to define the market and private states nor calculate \bar{w} . The CPF determines \bar{R}_W from

the interest rates, correlation between W and Y , and \bar{R}_y . On the other hand, if W lies in the market space and we have an optimal portfolio P^{15} , we have the CAPM:

$\bar{R}_W = R + \beta_{R_W, R_P} [\bar{R}_P - R]$ (see page 178 of (Luenberger, 1998) for proof). Therefore,

$$P_W = \frac{1}{R} \left[\bar{W} - \beta_{W_f, P} [\bar{P} - P_P * R] \right]$$

Appendix II---Variance Reduction

The CPF is a more accurate model in terms of variance than a simple time series analysis. One of the problems of pricing a non-traded asset is the scarcity of past price data. This makes the forecasting of the growth rates of prices difficult. The problem is exacerbated when the length of the time period is short or when trying to estimate the expected growth rate of a price. Suppose r_i is the growth rate of price per month and the growth rates are identically distributed and independent of each other from month to month. Then the yearly growth rate r_{yr} is given by:

$$1 + r_{yr} = (1 + r_1) * (1 + r_2) * (1 + r_3) \cdots (1 + r_{12})$$

$$\approx 1 + r_1 + r_2 + r_3 + \cdots + r_{12}$$

$$\therefore r_{yr} = \sum_{i=1}^{12} r_i$$

$$E[r_{yr}] = \sum_{i=1}^{12} E[r_i] = 12E[r_i] = 12\bar{r}$$

$$\sigma_{r_{yr}}^2 = E \left[\sum_{i=1}^{12} r_i - 12\bar{r} \right]^2 = E \left[\sum_{i=1}^{12} (r_i - \bar{r}) \right]^2 = E \left[\sum_{i=1}^{12} (r_i - \bar{r})^2 \right] = \sum_{i=1}^{12} E (r_i - \bar{r})^2$$

$$= \sum_{i=1}^{12} \sigma_{r_i}^2 = 12\sigma_{r_i}^2$$

$$\therefore \sigma_{r_i} = \frac{\sigma_{r_{yr}}}{\sqrt{12}}$$

This shows that the standard deviation of the growth rate decreases slower than the growth rate itself as the length of the time period shortens. This implies that we need to collect more data to estimate the monthly growth rate than for a yearly growth rate. Similar problem arises when we try to estimate the expected growth rate as shown below.

$$\bar{r}_i = E[r_i] = \frac{1}{n} \sum_{i=1}^n E[r_i] = \frac{1}{n} \sum_{i=1}^n \bar{r}_i$$

$$\sigma_{r_i}^2 = E \left[\frac{1}{n} \sum_{i=1}^n r_i - \bar{r} \right]^2 = \frac{1}{n^2} E \left[\sum_{i=1}^n r_i - n \bar{r} \right]^2 = \frac{1}{n^2} E \left[\sum_{i=1}^n (r_i - \bar{r}) \right]^2 = \frac{1}{n^2} \sum_{i=1}^n \sigma_{r_i}^2 = \frac{\sigma_{r_i}^2}{n}$$

$$\therefore \sigma_{\bar{r}} = \frac{\sigma_{r_i}}{\sqrt{n}}$$

The CPF says the expected growth rate of the price of a non-traded asset W is given by $\bar{R}_W = R + \beta_{R_W, R_P} [\bar{R}_P - R]$. As with a linear regression model, we can forecast the future expected growth rate of the price using the following equation:

$$\bar{R}_W = R + \hat{\beta}_{R_W, R_P} [\bar{R}_P - R]$$

where $\hat{\beta}$ is an estimator of β . Therefore, $\text{var}(\bar{R}_W) = \text{Var}(\hat{\beta}_{R_W, R_P} \bar{R}_P) + R^2 \text{var}(\hat{\beta}_{R_W, R_P})$

In a linear regression model, let $Y_i = \alpha + \beta X_i + u_i$ where u_i is the iid error term

with $E[u_i] = 0$, $\text{Var}(u_i) = \sigma_{u_i}^2$ and $\text{cov}(u_i, x_i) = 0$.

$$\begin{aligned} \hat{\beta}_{Y, X} &= \frac{\text{cov}(X, Y)}{\text{var}(X)} = \frac{\frac{1}{n} \sum_{i=1}^n X_i Y_i - \frac{1}{n^2} \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{\frac{1}{n} \sum_{i=1}^n X_i^2 - \frac{1}{n^2} \left(\sum_{i=1}^n X_i \right)^2} \\ &= \frac{\sum_{i=1}^n n^2 X_i Y_i - n \sum_{i=1}^n X_i \sum_{i=1}^n Y_i - n \sum_{i=1}^n X_i \sum_{i=1}^n Y_i + n \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n^2 \sum_{i=1}^n X_i^2 - 2n \sum_{i=1}^n X_i \sum_{i=1}^n X_i + n \sum_{i=1}^n X_i \sum_{i=1}^n X_i} \\ &= \frac{\sum_{i=1}^n \left[X_i Y_i - \frac{1}{n} \left(\sum_{i=1}^n X_i \right) Y_i - X_i \left(\frac{\sum_{i=1}^n Y_i}{n} \right) + \frac{1}{n} \left(\sum_{i=1}^n X_i \right) \left(\frac{\sum_{i=1}^n Y_i}{n} \right) \right]}{\sum_{i=1}^n \left[X_i^2 - 2X_i \left(\frac{\sum_{i=1}^n X_i}{n} \right) + \left(\frac{1}{n} \left(\sum_{i=1}^n X_i \right) \right)^2 \right]} \\ &= \frac{\sum_{i=1}^n [(X_i - \bar{X})(Y_i - \bar{Y})]}{\sum_{i=1}^n (X_i - \bar{X})^2} = \frac{\sum_{i=1}^n [(X_i - \bar{X})Y_i]}{\sum_{i=1}^n (X_i - \bar{X})^2} - \frac{\bar{Y} \sum_{i=1}^n (X_i - \bar{X})}{\sum_{i=1}^n (X_i - \bar{X})^2} \\ &= \frac{\sum_{i=1}^n [(X_i - \bar{X})Y_i]}{\sum_{i=1}^n (X_i - \bar{X})^2} \quad \text{since } \sum_{i=1}^n (X_i - \bar{X}) = 0 \end{aligned}$$

$$\begin{aligned}
\widehat{\beta}_{Y,X} - \beta_{Y,X} &= \frac{\sum_{i=1}^n [(X_i - \bar{X})(\alpha + \beta_{Y,X}X_i + u_i)]}{\sum_{i=1}^n (X_i - \bar{X})^2} - \beta_{Y,X} \\
\widehat{\beta}_{Y,X} - \beta_{Y,X} &= \frac{\sum_{i=1}^n ((X_i - \bar{X})\beta_{Y,X}X_i + (X_i - \bar{X})u_i) - \beta_{Y,X} \sum_{i=1}^n (X_i - \bar{X})^2}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
&= \frac{\beta_{Y,X} \sum_{i=1}^n ((X_i - \bar{X})X_i - (X_i - \bar{X})^2) + \sum_{i=1}^n (X_i - \bar{X})u_i}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
&= \frac{\beta_{Y,X} \sum_{i=1}^n (X_i^2 - \bar{X}X_i - (X_i^2 - 2X_i\bar{X} + \bar{X}^2)) + \sum_{i=1}^n (X_i - \bar{X})u_i}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
&= \frac{\beta_{Y,X} \sum_{i=1}^n (\bar{X}X_i - \bar{X}^2) + \sum_{i=1}^n (X_i - \bar{X})u_i}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
&= \frac{\beta_{Y,X} \bar{X} \sum_{i=1}^n X_i - n\beta_{Y,X} \bar{X}^2 + \sum_{i=1}^n (X_i - \bar{X})u_i}{\sum_{i=1}^n (X_i - \bar{X})^2} = \frac{\sum_{i=1}^n (X_i - \bar{X})u_i}{\sum_{i=1}^n (X_i - \bar{X})^2} \\
&= \frac{\sum_{i=1}^n (X_i^* * u_i)}{\sum_{i=1}^n X_i^{*2}} \text{ if we let } X_i - \bar{X} = X_i^*.
\end{aligned}$$

$$\therefore \text{var}(\widehat{\beta}_{Y,X}) = E[\widehat{\beta}_{Y,X} - \beta_{Y,X}]^2 = \frac{E\left[\sum_{i=1}^n (X_i^* * u_i)\right]^2}{\left(\sum_{i=1}^n X_i^{*2}\right)^2} = \frac{E\left[\sum_{i=1}^n (X_i^{*2} * u_i^2)\right]}{\left(\sum_{i=1}^n X_i^{*2}\right)^2}$$

$$= \frac{\sum_{i=1}^n X_i^{*2} [Eu_i^2]}{\left(\sum_{i=1}^n X_i^{*2}\right)^2} = \frac{\sigma_{u_i}^2 \sum_{i=1}^n X_i^{*2}}{\left(\sum_{i=1}^n X_i^{*2}\right)^2} = \frac{\sigma_{u_i}^2}{\left(\sum_{i=1}^n X_i^{*2}\right)}$$

$$\therefore \text{Var}(\hat{\beta}_{R_W, R_P}) = \frac{\sigma_{u_i}^2}{\left(\sum_{i=1}^n (R_{P_i} - \overline{R_P})^2 \right)}$$

Then we calculate $\text{Var}(\hat{\beta}_{R_W, R_P} \overline{R_P})$.

If X and Y are independent,

$$\begin{aligned} \text{Var}(XY) &= E[(XY)^2] - (E[XY])^2 = E[X^2Y^2] - (E[X]^2 E[Y]^2) \\ &= E[X^2]E[Y^2] - \overline{X}^2 \overline{Y}^2 = [\sigma_X^2 + \overline{X}^2][\sigma_Y^2 + \overline{Y}^2] - \overline{X}^2 \overline{Y}^2 \\ &= \sigma_X^2 \sigma_Y^2 + \sigma_X^2 \overline{Y}^2 + \sigma_Y^2 \overline{X}^2 \end{aligned}$$

We assume that we can observe R_p , the growth rate of a market asset, much more often than the price of OSB. Suppose that we have N data points for R_p , but only n data points for OSB, where $N \gg n$. We can separate the data for R_p into two sets, a smaller set (n) for calculating $\hat{\beta}_{R_W, R_P}$ and $\sigma_{\hat{\beta}}^2$ and a larger set ($N-n$) for calculating

$\overline{R_P}$ and $\sigma_{R_P}^2$. Then $\overline{R_P}$ and $\sigma_{R_P}^2$ are independent, and we have,

$$\begin{aligned} \text{Var}(\hat{\beta}_{R_W, R_P} \overline{R_P}) &= \sigma_{R_P}^2 (\hat{\beta}_{R_W, R_P})^2 + \sigma_{\hat{\beta}}^2 (\overline{R_P})^2 + \sigma_{R_P}^2 \sigma_{\hat{\beta}}^2 \\ \therefore \sigma_{R_W}^2 &= \sigma_{R_P}^2 (\hat{\beta}_{R_W, R_P})^2 + \sigma_{\hat{\beta}}^2 \left((\overline{R_P})^2 + \sigma_{R_P}^2 + R^2 \right) \\ &= \sigma_{R_P}^2 (\hat{\beta}_{R_W, R_P})^2 + \frac{\sigma_{u_i}^2 \left((\overline{R_P})^2 + \sigma_{R_P}^2 + R^2 \right)}{\left(\sum_{i=1}^{N-n} (R_{P_i} - \overline{R_P})^2 \right)} \\ &= \left[\frac{1}{(N-n)(N-n-1)} \sum_{i=1}^{N-n} (R_{P_i} - \overline{R_P})^2 \right] \left[\frac{\sum_{i=1}^n [(R_{W_i} - \overline{R_{W_i}}) R_{P_i}]}{\sum_{i=1}^n (R_{P_i} - \overline{R_P})^2} \right]^2 \\ &\quad + \frac{1}{\left(\sum_{i=1}^{N-n} (R_{P_i} - \overline{R_P})^2 \right)} \left(\frac{1}{(n-1)} \sum_{i=1}^n \left(R_{W_i} - \left(R + \widehat{\beta}_{R_W, R_P} [\overline{R_P} - R] \right) \right)^2 \right) \\ &\quad * \left(\left(\frac{1}{N-n} \sum_{i=1}^{N-n} R_{P_i} \right)^2 + R^2 \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\left(\sum_{i=1}^{N-n} (R_{P_i} - \overline{R_{P_i}})^2 \right)} \left(\frac{1}{(n-1)} \sum_{i=1}^n \left(R_{W_i} - \left(R + \widehat{\beta}_{R_W, R_P} [\overline{R_{P_i}} - R] \right) \right)^2 \right) \\
& * \left[\frac{1}{(N-n)(N-n-1)} \sum_{i=1}^{N-n} (R_{P_i} - \overline{R_{P_i}})^2 \right] \\
& = \frac{\sigma_{R_P}^2 \left(\widehat{\beta}_{R_W, R_P} \right)^2}{(N-n)} + \frac{\sigma_{u_i}^2 \left((\overline{R_P})^2 + R^2 \right)}{(N-n)\sigma_{R_P}^2} + \frac{\sigma_{u_i}^2}{(N-n)(N-n-1)} = V
\end{aligned}$$

When $N \gg n$, $V \leq \frac{1}{n(n-1)} \sum_{i=1}^n (R_{W_i} - \overline{R_{W_i}})^2 = \frac{\sigma_{R_W}^2}{n}$ under most situations.¹⁶

Therefore, we can use the fact that there are many more data points for R_P than R_W to reduce the variance when we are estimating $\overline{R_W}$.

Appendix III—Market Data

This section presents some market data to show that the parameters we have used in the Monte Carlo simulation are reasonable.

1. The historical annual return of the Dow Jones Industrial Average (INDU) is 12%. If we use INDU to approximate the efficient market asset P, the monthly return of P is 1%.
2. Oriented strand boards are made of strands of southern pine (or poplar or aspen) bonded together by resin (phenol formaldehyde or isocyanate) under intense pressure and heat. Each panel is 3" thick, 12'x12' in area, and weighs 44lbs. Resin comes in liquid form in which 50% of the weight is water and costs about 20 cents per pound in early April, 2002. After processing, all the water is vaporized and resin makes up about 3.5%-5% of the weight of an OSB. Therefore, the cost of resin is about \$4.9 per thousand square feet of OSB if we assume resin makes up 4% of the weight of an OSB. On April 5, 2002, the price index of OSB in New York is \$182.5 per thousand square board feet. As a result, the cost of resin makes up 3.44% of the cost of good sold of OSB if we assume that the gross margin of the seller is 22%.¹⁷ We interviewed OSB manufacturers and they estimated labor and lumber each makes up 35% of the total cost.¹⁸ As a result, the ratio of the cost of lumber to the cost of resin is ten to one and we use $T=10*L+S$ as the product that correlates most closely with OSB.¹⁹
3. After studying the yields of US Treasury bonds (see Table 1), we use 2% as

the annual return of the risk free asset during the next nine months. The monthly return is therefore 0.167%.

4. On April 5, 2002, the price of the lumber future expiring on May 15, 2002 closes at \$299.5 on the Chicago Mercantile Exchange. On the other hand, the spot price of lumber closes at \$289.5 on April 5, 2002 in New York.

Therefore, we use the ratio of the price of the lumber future to the spot price of lumber to calculate the expected return of OSB in the risk neutral world:

$$\left(\frac{\$299.5}{\$289.5} - 1 \right) \left(\frac{30 \text{ days}}{40 \text{ days}} \right) + 1 = 1.026$$

5. Volatility is the annualized (260 weekdays) standard deviation of the day-to-day logarithmic price changes. After the market close on April 5, 2002, the volatilities calculated using the closing prices of the last 100 days are shown in Table 2.²⁰ Therefore, the volatility of OSB per month is:

$$0.395 / \sqrt{12} = 0.114.$$

6. The annualized standard deviation of the returns of the prices of LUMBOSB1, LUMBSP24, LBK2, INDU, and BFK2 are listed in Table 3. We calculate them using the closing prices between Oct. 19, 2001 and April 16, 2002.

The correlation matrix of the returns of the prices of LUMBOSB1, LBK2, INDU, and BFK2 and the corresponding t-statistics are shown in Table 4 and Table 5. We calculate the matrices using the closing prices between Oct. 19, 2001 and April 16, 2002. Finally, if we use benzene, which is a principal component of many resins, to approximate phenol formaldehyde and LBK2

to approximate lumber, we can calculate β_{R_{OSB}, R_p} in the following way:

Let the projection of W_f , $P(W_f, M)$, be OSB_M

$$OSB_M - \overline{OSB} \frac{R_f}{\|R_f\|} = \beta_{OSB, T} * (T - \bar{T})$$

$$\left(OSB_M - \overline{OSB} \frac{R_f}{\|R_f\|} \middle| P \right) = \beta_{OSB, T} * \left((T|P) - (\bar{T}|P) \right)$$

$$E[OSB_M * P] - E[OSB]E[P] = \beta_{OSB, T} * E[T * P] - E[T]E[P]$$

$$\text{But } E[OSB_M * P] = E[OSB * P]$$

$$\therefore \text{cov}(OSB, P) = \beta_{OSB, T} * \text{cov}(T, P)$$

$$\frac{\text{cov}(OSB, P)}{\sigma_P^2} = \frac{\beta_{OSB, T} * \text{cov}(T, P)}{\sigma_P^2}$$

$$\therefore \beta_{OSB, P} = \beta_{OSB, T} * \beta_{T, P}$$

$$\Rightarrow \frac{P_{OSB} * P_P}{P_P^2} * \beta_{R_{OSB}, R_P} = \frac{P_{OSB} * P_T}{P_T^2} * \beta_{R_{OSB}, R_T} * \frac{P_T * P_P}{P_P^2} * \beta_{R_T, R_P}$$

$$\Rightarrow \beta_{R_{OSB}, R_P} = \beta_{R_{OSB}, R_T} * \beta_{R_T, R_P}$$

$$\begin{aligned} \text{where } \beta_{R_T, R_P} &= 0.91 * \beta_{R_{LBK2}, R_P} + 0.09 * \beta_{R_S, R_P} \\ &= \frac{0.91 * 0.700 * 0.287}{0.173} + \frac{0.09 * 0.653 * 0.449}{0.173} = 1.209 \end{aligned}$$

$$\begin{aligned} \text{and } \beta_{R_{OSB}, R_T} &= 0.91 * \beta_{R_{OSB}, R_{LBK2}} + 0.09 * \beta_{R_{OSB}, R_S} \\ &= \frac{0.91 * 0.878 * 0.287 * 0.392}{\sigma_{R_T}^2} + \frac{0.09 * 0.758 * 0.392 * 0.449}{\sigma_{R_T}^2} \end{aligned}$$

$$\sigma_{R_T}^2 = (0.91 * 0.287)^2 + (0.09 * 0.449)^2 = 0.0698$$

$$\therefore \beta_{R_{OSB}, R_T} = 1.460 \text{ and } \beta_{R_{OSB}, R_P} = 1.460 * 1.209 = 1.765$$

Figures

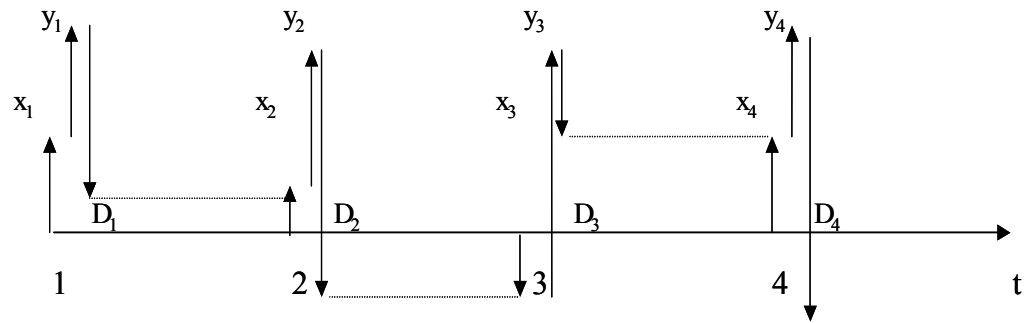


Figure 1. Relationship among x_t , y_t , and D_t

Price Cap	Volatility of the price of OSB										
	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15
85	25.91%	26.24%	26.60%	26.96%	27.32%	27.67%	28.00%	28.33%	28.64%	28.95%	29.24%
90	21.61%	21.97%	22.36%	22.76%	23.17%	23.58%	23.98%	24.38%	24.77%	25.14%	25.50%
95	17.36%	17.78%	18.25%	18.74%	19.24%	19.75%	20.24%	20.73%	21.20%	21.66%	22.10%
100	13.27%	13.80%	14.38%	14.98%	15.59%	16.19%	16.77%	17.34%	17.89%	18.42%	18.93%
105	9.74%	10.40%	11.09%	11.79%	12.48%	13.16%	13.82%	14.46%	15.08%	15.67%	16.24%
110	6.86%	7.57%	8.31%	9.04%	9.77%	10.49%	11.18%	11.85%	12.50%	13.12%	13.72%
115	4.77%	5.49%	6.23%	6.98%	7.72%	8.44%	9.15%	9.83%	10.50%	11.14%	11.76%
120	3.19%	3.87%	4.58%	5.29%	6.00%	6.70%	7.39%	8.07%	8.72%	9.36%	9.97%
125	2.08%	2.69%	3.34%	4.00%	4.67%	5.33%	6.00%	6.64%	7.27%	7.89%	8.49%
130	1.31%	1.86%	2.44%	3.04%	3.66%	4.28%	4.90%	5.52%	6.13%	6.73%	7.32%

Figure 2. Sensitivity of expected cost saving in percentage.

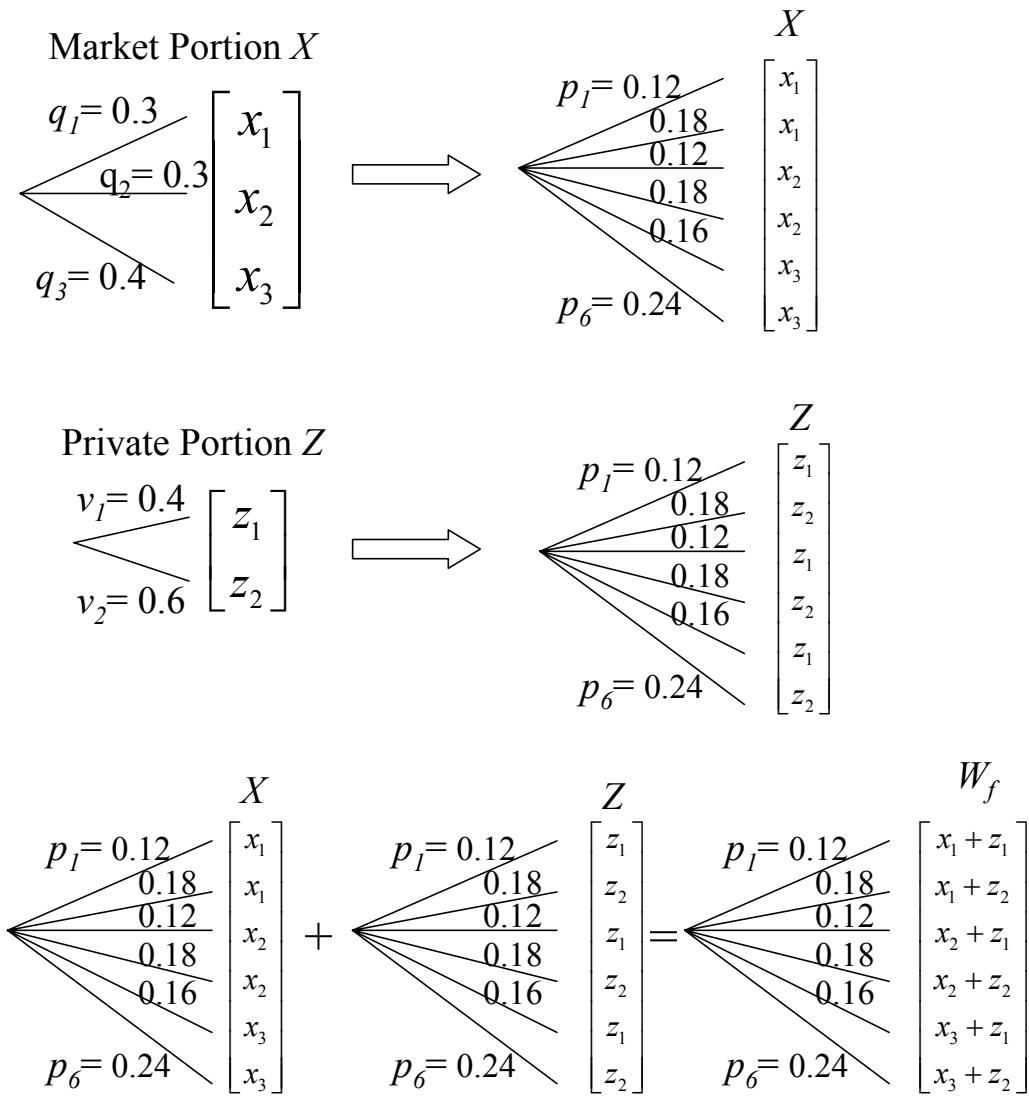


Figure 3. X_f , Z_f and W_f in two and six dimensional vector space.

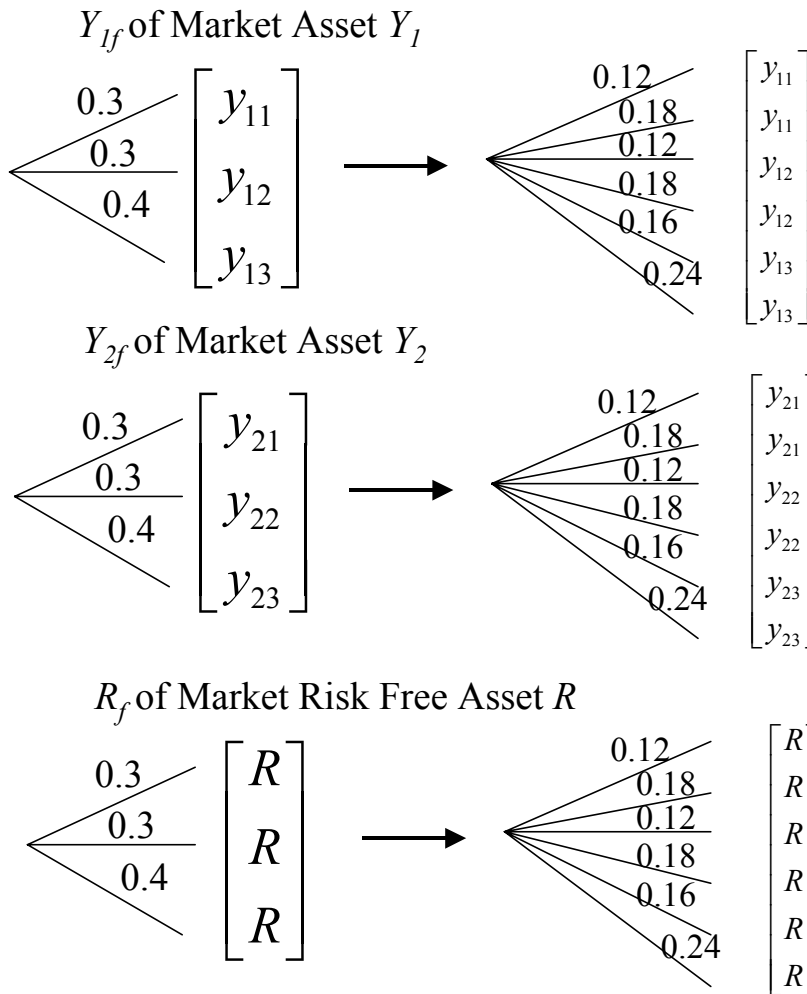


Figure 4. Y_{1f} , Y_{2f} , and R_f in three and six dimensional vector space.

Maturity	Three months	Six months	Two years	Five years	Ten years	Thirty years
Yield	1.64	1.85	3.38	4.52	5.18	5.65

Table 1. Maturities and yields of treasury bills, notes, and bonds

<i>Market Asset</i>	<i>Annualized Volatility</i>
OSB price index in NY (Bloomberg ticker: LUMBOSB1)	0.395
Lumber future expiring on 5/15/02 and traded in Chicago Mercantile Exchange (Bloomberg ticker: LBK2)	0.294
Dow Jones Industrial Average (Bloomberg ticker: INDU)	0.166
Benzene future expiring on 5/16/02 and traded in Chicago Mercantile Exchange (Bloomberg ticker: BFK2)	0.437

Table 2. Annualized volatilities of market assets.

<i>Market Asset</i>	<i>Annualized σ_R</i>
Spot OSB traded in NY (Bloomberg ticker: LUMBOSB1)	0.392
Lumber future expiring on 5/15/02 and traded in Chicago Mercantile Exchange (Bloomberg ticker: LBK2)	0.287
Dow Jones Industrial Average (Bloomberg ticker: INDU)	0.173
Benzene future expiring on 5/16/02 and traded in Chicago Mercantile Exchange (Bloomberg ticker: BFK2)	0.449

Table 3. Annualized standard deviations of the prices of market assets

	LUMBOSB1	INDU	LBK2	BFK2
LUMBOSB1	1	0.636	0.878	0.758
INDU	0.636	1	0.700	0.653
LBK2	0.878	0.700	1	0.697
BFK2	0.758	0.653	0.697	1

Table 4. Correlation matrix of market assets.

	LUMBOSB1	INDU	LBK2	BFK2
LUMBOSB1		8.99	20.00	12.67
INDU	8.99		10.68	9.39
LBK2	20.00	10.68		10.61
BFK2	12.67	9.39	10.61	

Table 5. t-statistics of the correlation matrix of market assets.

End of Figures

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Endnotes

- ¹ A right to buy is called a call option and a right to sell is called a put option.
- ² In geometric Brownian motion, μ is called the drift and σ is called the volatility.
- ³ Hilbert space is a complete inner product vector space.
- ⁴ We estimate $\beta_{L,P}$ and $\beta_{S,P}$ by past prices of lumber, resin, and P.
- ⁵ It is reasonable to assume that a linear combination of lumber and resin has the biggest correlation with the OSB among all market assets.
- ⁶ We estimate $\beta_{OSB,T}$ and $\beta_{T,P}$ by past prices of OSB, lumber, resin, and P.
- ⁷ See chapter 9 of (Luenberger, 1998) for the prerequisite of the existence and the uniqueness of risk neutral probabilities/measures.
- ⁸ We use the prices of lumber because we assume that lumber and OSB have the same storage costs.
- ⁹ Since W&W has limited storage space, we assume that she has a higher average holding cost than that implied by the forward prices.
- ¹⁰ We minimize the expected cost because we assume W&W is risk neutral.
- ¹¹ See (Boyle, 1995) for an introduction of Monte Carlo simulation of GBM in equivalent martingale or risk neutral measure.
- ¹² The percentages are independent of the mean of demand and the initial price of OSB at the beginning of the contract.
- ¹³ $E[TC_0]$ and $E[TC_{NO}]$ are the estimators of the expected total costs over nine months with and

without the price cap respectively.

¹⁴ In this simple example, the market space has three dimensions. In the general case, the market space can have as many dimensions as there are linearly independent assets that are traded in the market. Similarly, there can be many private states.

¹⁵ Optimal in terms of maximizing utility or rate of return given a certain level of volatility

¹⁶ $\hat{\beta}_{R_W, R_P}, \overline{R_P}$, and R are less than two in most cases. $\sigma_{R_P}^2, \sigma_{u_i}^2$, and $\sigma_{R_W}^2$ are comparable in magnitude.

¹⁷ Weyerhaeuser and Georgia Pacific, both sellers of OSB, have gross margins of 21.46% and 22.54% respectively in the past 12 months.

¹⁸ Total cost includes sales and market as well as general and administrative.

¹⁹ We use the assumption that operating margin is 5% (the same as that of Weyerhaeuser).

²⁰ The daily volumes of LUMBOSB1 and BFK2 are not available. Therefore, we have no idea of the their liquidity. The volume and open interest of LBK2 on April 16, 2002 are 737 and 1189 respectively. When compared with the volume (2383) and open interest (8375) of the May cotton future traded on the Chicago Mercantile Exchange on April 17, 2002, LBK2 has limited liquidity.