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Probabilistic Consideration Method for weight/score-based decisions in systems engineering-related applications

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Abstract

Systems engineering (SE) uses a number of tools to structure project, product, and service development in an attempt to maximize workflow efficiencies and the satisfaction of project stakeholders and end users. Many of these tools—e.g., Quality Function Deployment (QFD) matrices—assign numerical weights and scores to SE considerations in order to organize and prioritize characteristics that are used to drive project decisions. Unfortunately, these weights and scores are susceptible to numerical variances due to human bias and other uncertainties. As a result, there is concern with regard to the robustness of these tools toward such variances and how this may impact major project decisions. To address this we introduce a probability-based method that uses computer simulations to evaluate the influence of numerical variance in weight/score-based SE tool implementations. This method can be used to develop confidence in weight/score-based decisions, determine if SE implementations should be re-evaluated, identify trends related to probabilistic variance, and set tolerances for such variances. The method can be expanded for use in other numerical based decision making frameworks that are susceptible to probabilistic variances.

Keywords-score-based decisions; systems engineering; sensitivity analysis; Quality Function Deployment; probabilistic modeling; computer simulation

1. Introduction

Systems engineering (SE) is a methodological approach to developing and realizing products, processes, and services in a manner that optimizes critical considerations in the corresponding project and system lifecycles. This is done by constructing frameworks that maximize project workflow efficiencies, system functionalities, and operational reliability while appropriately considering all project stakeholders, workflow tasks, technological/knowledge capabilities, etc. Elements of SE are implemented in various project phases or realms of considerations. For example, SE processes can be used in project ideation to identify potential stakeholders and their values or goals in order to determine how to prioritize value considerations to maximize stakeholder fulfillment. In design engineering, SE is used to guide design choices, activities, and/or analyses that can help manage time/monetary budgets, select system concepts, architectures, or refine functionalities, etc. Altogether, SE tools and constructs create a path for a succession of practices that attempt to yield the best possible services or systems (Weiss, 2013).

Many of the tools used in SE applications are quantitative constructs that capture qualitative considerations through the use of numerical weights and scores. Examples of these include Quality Function Deployment (QFD), Kepner-Tregoe (Kepner, 1973), and Pugh matrices (Weiss, 2013); QFD in particular is extensively used in SE applications to compare and prioritize stakeholder values and requirements. A concern however, is that the numbers used in QFD and other tools are determined by humans, via surveys, intuition, etc., and are susceptible to uncertainty that can spur numerical variances in weights/scores and as a result impact the verity of the SE application.

This paper expands work related to model uncertainty in weight/score-based tools (Yuventi, 2013a) in order to introduce a method that provides insight into the potential influence of uncertainty in implementations of these tools. This method uses probabilistic computer simulations to analyze a given implementation and investigate how results may be affected by the presence of numerical variances. It can be used to validate SE-based project decisions, provide grounds to re-evaluate tool-implementations, identify probabilistic relationships, etc. More details into this method will be provided in subsequent sections of this paper. However, we begin this discussion with an overview of SE process flows, considerations, and decision tools in order to develop a base of understanding for this work and its motivations.

2. Value-driven systems engineering

A system integrates interrelated elements to provide utility that is greater than the sum of the utilities of each individual element (Weiss, 2013). The term is often used to describe complex products such as automobiles, airplanes, and integrated electronics that involve multi-disciplinary development teams and technologies. However, it can also extend to "systems-of-systems", such as transportation and energy-grid infrastructures, or single-discipline product/service projects developed through fragmented supply chain organizational models. SE attempts to reduce the complexities involved in realizing a system into a logical collection of rules that introduce common understandings to steer projects. This can be done in various ways; however, Figure 1 illustrates an example process flow for SE. In a value-driven SE approach, identifying the projects stakeholders—i.e., those involved in all phases of the project from ideation to operation—and their values with regard to that project, is one of the more important activities. This knowledge can be used to determine system requirements, functionalities, and architectures (e.g., in the case of innovative consumer products) or prioritize design activities and select system concepts or themes (in the case of standardized products or services). Also, another important, but sometimes overlooked, aspects of SE is the necessity for iteration and communication in each stage and between stages and teams. Such activity ensures consistency in the themes used to drive decisions and allows for adjustments to tools and frameworks if unpredicted challenges were to arise.

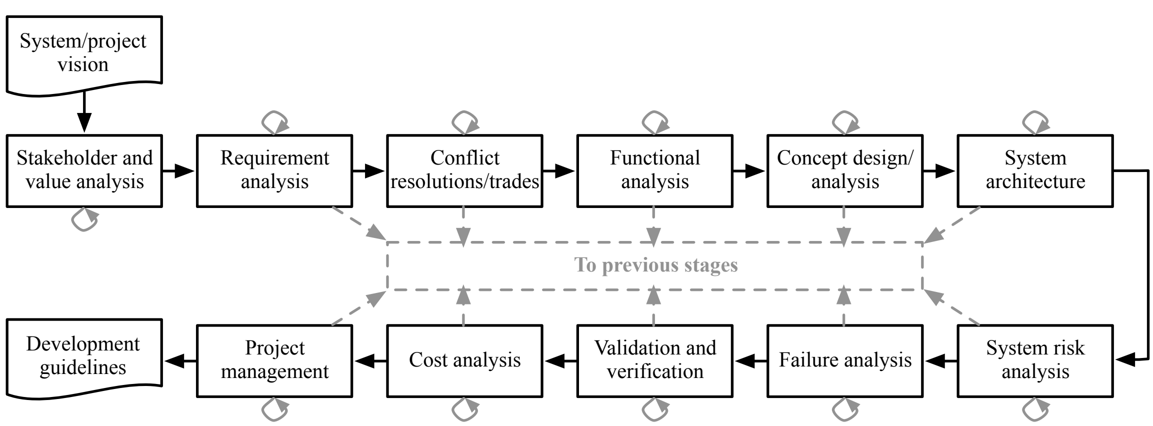

Figure 1. Example stages in a value-driven Systems Engineering application

2.1 Weight/score-based decision tools: QFD

As mentioned, many SE tools, such as QFD matrices, use numerical values to 'weigh' and 'score' considerations or elements, such as stakeholders, stakeholder-values, requirements, technical characteristics, etc. In QFD matrices, each i^{th} input element is assigned a weight $W_{in,i}$ determined manually or calculated from a previous QFD matrix or similar tool. These weights represent the importance of each element to the SE process relative to other input elements; with higher weights representing greater importance. The input elements are used as references to score a collection of other elements, referred to in this discussion as the "scored elements" (e.g., stakeholders are input elements when determining stakeholder-value scores). Each j^{th} scored element is assigned a numerical score $S_{i,j}$ in reference to each i^{th} input element. These scores represent the relationship between each scored element and each input element in terms of correlations or importance, with higher scores representing greater positive correlation or more importance. $W_{in,i}$ and $S_{i,j}$ are used to determine the weighted sum $Sum_{weighted,j}$ for each scored element; where k represents the number of input elements, then

$$Sum_{weighted,j} = \sum_{i=1}^{k} W_{in,i} \cdot S_{i,j}. \tag{1}$$

$Sum_{weighted,j}$ is used to determine relative weights for these elements $W_{out,j}$; e.g.

$$W_{out,j} = 10 \cdot \frac{Sum_{weighted,i}}{\max\left(Sum_{weighted,all\ j}\right)}. \tag{2}$$

It is also useful to determine the un-weighted sum scored elements, $Sum_{raw,j}$, where

$$Sum_{raw,j} = \sum_{i=1}^{k} S_{i,j}. \tag{3}$$

It is common for the scored elements to have different $Sum_{raw,j}$ in a given implementation. When certain elements have significantly higher $Sum_{raw,j}$ than others this suggests that there is a bias toward this scored element in reference to the input elements (e.g., stakeholder-bias toward a particular value) and further helps in decision-making, conflict-resolutions and trades.

$W_{out,j}$ are the outputs of the QFD and can be used to prioritize the scored-elements, make decisions, and/or provide inputs to another QFD. Quite often QFD matrices are used in a succession of stages, wherein the outputs of one stage form the inputs of another, as illustrated in Figure 2. QFD matrices are often referred to—or are constituents of—"Houses of Quality" (HOQ) (Hauser, 1998), where the roofs of the HOQ capture the relationships, or conflicts between the scored elements, as illustrated in Figure 2. These relationships in the HOQ roofs are often qualitative and do not fall within the scope of this discussion. As a result, although conflict resolution is an important part of the SE process, these roofs are not illustrated in the other QFD matrices presented in this paper and conflicts are not discussed in much detail. However, it is important to note that prioritizations derived from $W_{out,j}$ play an important role in determining how to manage conflict resolutions.

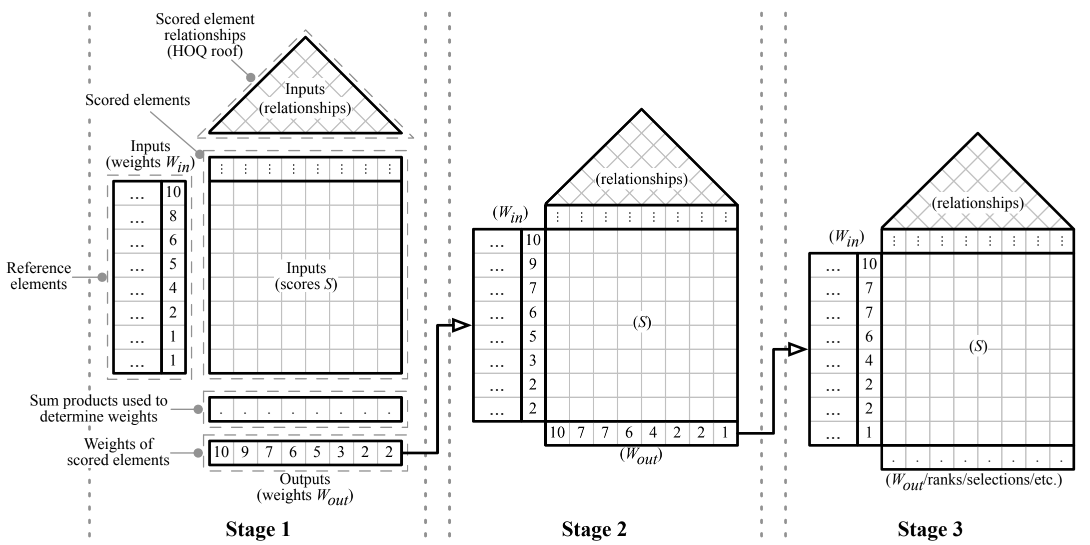

Figure 2. Successive stages in a QFD or other weigh/score-based tool implementation

Various standards exist for determining the useable set of numbers for weights and scores. For example, weights can be limited between 0 and 10, as in the case represented in (2), where 10 represents highest importance, 1 represents low importance, and 0 representing no importance (and hence will likely be ignored). In practice a variety of scoring methods are used, including: 0-10 integer scales, similar to that described for weights, and more discrete methods, for example, having only 3 usable scores to capture (1) weak, (2) medium, and (3) strong scored element-to-input element relationships. In these discrete systems, weak-medium-strong scores may be assigned numerical values such as 1-3-5, 1-3-9, 1-5-9, 1-7-9, etc. (Park, 1998) with a score of 0 assigned when there is absolutely no consideration or relevance. Although there is no scientific basis for using one scoring method over another (Cohen, 1995), the results for each scoring system are likely to differ. In addition, different scoring systems may be affected differently by uncertainty as a result of human bias toward what the numerical magnitudes represent.

2.2 Scoring uncertainties

The data used to determine these weights and scores for SE tools are often limited, inaccurate, or vague (Xie, 2003), or are susceptible to mishandling due to human error, bias, etc. Matters are exacerbated for newer projects since there are either fewer credible data sources or limited experience among the investigators (Kim, 2007). In addition, small uncertainties can propagate through multi-stage tool implementations to significantly impact final results. Researchers identify four dominant sources of uncertainty in QFD and other similar tools (Kim, 2007):

1. *Fuzziness*: lack of precision or consistency in implementation,
2. *Fluctuation*: change in the relationships between SE elements over time,
3. *Heterogeneity*: differing analysis perspective or bias, and
4. *Incompleteness*: limited information.

Uncertainties influence variances in manually derived weights/scores, i.e., these numbers may differ from their 'true values' with some probabilistic distribution. Research has been done to understand and reduce the extent of these uncertainties, especially with regard to fuzziness sources (e.g., Chan, 1999). Some researchers have also proposed adjustments to QFD frameworks to account for uncertainty using probabilistic modeling (e.g., Kim, 2007). However, it is impossible to completely remove the threat of uncertainty and, as a result, it is important to develop a general understanding of the robustness of SE tools to these uncertainties and/or the potential impact of these uncertainties on project decisions.

3. The Probabilistic Consideration Method

Researchers have investigated the susceptibility of standard QFD matrices to produce misleading outputs as a result of numerical variances (e.g., Ghiya, 1999; Yuventi, 2013a). This paper builds on these discussions and investigations to create a method—i.e., the Probabilistic Consideration Method (PCM)—that quantifies the potential impact of numerical variances in weight/score-based tools. Figure 3 illustrates a PCM pseudo-code algorithm used to analyze a QFD implementation in which a selection or decision is made in the final stage (i.e., the final stage is similar to a Kepner-Tregoe decision matrix). Assuming that there are x number of QFD stages, $S_{i,j}$ in each stage are treated as random numbers with mean $\mu_{i,j}$ equal to the original $S_{i,j}$ and standard deviation $\sigma_{i,j}$ that represents the estimated or evaluated numerical variance of $S_{i,j}$ due to uncertainty. The same is done for manually determined $W_{in,i}$ (which are typically constrained to the first stage). The random numbers are generated based on a Probability Density Function (PDF) that is believed to best capture the probabilistic distribution of scores, e.g., Gaussian Distribution, Beta Distribution, etc. A different PDF can be used in successive applications of PCM as more insight is developed with regard to the nature of the numerical variances in a given tool implementation. The generated numbers are constrained to the useable range of scores, e.g., between 0 and 10 for 0-10 scoring, and rounded to the closest useable score, e.g., 1.65 is rounded to 3 in 1-3-5 scoring.

The standard deviation can be the same for all scores or for a subset of scores based on an understanding of who determines the original scores and how or why they did so. For example, if the same person determined all of the scores in a given stage then it may be more likely that all $\sigma_{i,j}$ will be the same in that stage as opposed to if different people scored subsets of that stage. PCM uses Monte Carlo simulations wherein the QFD is recalculated and evaluated for (n) simulation runs. The scores and calculated weights in each run should differ based on the probabilistic considerations, scoring system, etc. (Figure 4 conveys these influences for a given numerical set). Also, using more Monte Carlo runs increases the potency and accuracy of the analysis (i.e., preferably n should be greater than 1,000). Statistics are collected based on the weights and scores for each stage and/or, this case, the selection percentage of each of the selectable elements in the final QFD stage.

```
for n Monte Carlo runs:
    for each of the x QFD stages:
        for each i^th input element:
            if W_in,i is manually derived (typical only first stage):
                μ_i←original W_in,i; σ_i←evaluated deviation for W_in,i
                W_in,i←random number given a PDF, μ_i, and σ_i
                round W_in,i to the nearest useable weight
            for each j^th scored element:
                μ_i,j←original S_i,j; σ_i,j←evaluated deviation for S_i,j
                S_i,j←random number given a PDF, μ_i,j, and σ_i,j
                round S_i,j to the nearest usable score
        for each j^th scored element:
            calculate Sum_raw,j, Sum_weighted,j and W_out,j
            gather statistics on weights, ranks, etc.
        if in selection stage (e.g., stage x):
            select scored element with highest Sum_weighted,j or W_out,j
            gather statistics on selection rates (etc.) for element
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Figure 3. Example pseudo-code for Probabilistic Consideration Method

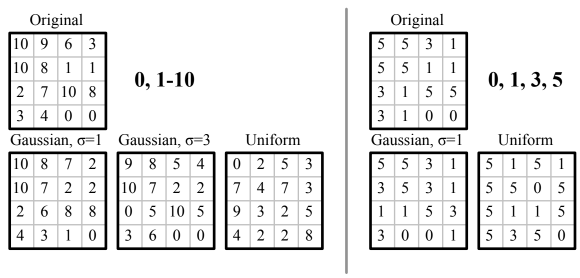

Figure 4. Impact of scoring method and σ on number generation

3.1 Example QFD implementation: Photovoltaic construction

To demonstrate PCM we use a value-driven QFD implementation wherein a system concept is selected for a large-scale photovoltaic (PV) construction project. This example was chosen in support of propositions that SE should be applied in more construction initiatives to improve the quality of the constructed system and productivity in project workflows (Yuventi, 2013b).

Large-scale PV systems are construction initiatives that are designed and installed by electrical contractors on behalf of a project developer. The purpose of the facilities is to produce large amounts of electricity and, are therefore normally commissioned and operated by power utility companies to increase their generation capacity. In the case of typical utility-run systems, a developer envisions the project, works with a landowner to purchase or lease land area for installation, aligns with a utility that will take over the system after construction, works with an engineer to develop system concepts, and secures funding from lenders/investors. Afterward, the developer hires contractors—primarily an electrical contractor but sometimes other subcontractors are needed—to design and construct the system based on the selected concept(s). These contractors often work with component suppliers and labor unions when installing the systems. After the construction is complete, the utility company takes over operation of the system. The end consumer is anyone who purchases the electricity—for direct consumption or transmission—from the utility. Regulatory bodies, such as local governments, oversee engineering, construction, and operation efforts to ensure that the system meets applicable building codes, such as the National Electrical Code (NEC), safety standards, etc.

A few metrics are commonly used to evaluate large-scale PV projects. The most common are: project costs, prior to utility handoff; maximum power/energy generation capacity; and the cost-to-performance ratio ($/W), which relates the construction costs to the estimated maximum power output. However, other considerations may include: the cost of energy as seen by the consumer over the system lifecycle, i.e., the levelised energy costs (LEC); the operational reliability of the system; operator safety; building code compliance; environmental implications, such as the recyclability the land and components; project completion time; etc. These metrics can also be used to capture the stakeholder-values. For example, the lenders/investors may only care about maximizing their financial return and therefore want to minimize project costs. Utility companies care about lifetime energy costs and want to minimize LEC. The land/site host may care about the potential damage that the system can have on the future value of their land and therefore may be interested in maximizing environmental sustainability metrics.

The identified stakeholder-values can be used to prioritize design choices or engineering activities in the engineering phase. For example, higher efficiency PV modules can be selected to increase power/energy generation; special analyses—that are not covered in the building codes—could be performed to optimize for operator safety; etc. If there is a fixed set of system concepts to choose from, these design choice priorities can be used to determine which concept to select based on all of the SE considerations. An example of this entire determination is illustrated in the three-stage QFD shown in Figure 5, done from the perspective of the project developer. Here PV_1, PV_2, and PV_3 are the three available concepts to select from, where PV_1 is the 'cheap' concept, allowing for the smallest $/W, project time, and project costs; PV_2 is the 'performance' concept, providing the highest power/energy, reliability and lowest LEC, and PV_3 is the 'conscientious' concept, being the most environmentally friendly, maximizing operator safety and code compliance.

Stakeholder-values

Stakeholders	W_{in}	Min. project costs	Min. project time	Max. power/energy	Max. reliability	Min. LEC	Min. $/W	Max. safety	Code compliance	Recycling/Environ.
Developer	10	10	9	6	3	5	7	1	0	1
Lenders/investors	8	10	8	1	1	1	1	0	0	0
Utility company	7	2	7	10	8	8	3	6	2	0
Land/site host	5	3	4	0	0	0	0	2	0	10
Electrical contractor	5	7	7	7	5	4	10	3	8	0
Regulators	4	0	0	1	3	0	0	9	10	5
Other contractors	3	10	7	2	5	0	4	3	7	0
Energy consumers	3	0	0	3	9	10	0	1	0	1
Component suppliers	1	10	2	0	5	3	3	5	5	4
Sum_{raw}		52	44	30	39	31	28	30	32	21
$Sum_{weighted}$		284	281	192	178	167	164	130	120	87
W_{out}		10.0	9.89	6.76	6.27	5.88	5.77	4.58	4.23	3.06

Design choices/activities

Stakeholder-values	W_{in}	NEC-min wiring	Wiring/layout opt.	Module binning	High eff. modules	Inexpensive modules	Non-code analyses	Solar tracking	High quality BOS	Sustainability studies
Min. project costs	10.0	7	5	1	1	10	0	1	1	0
Min. project time	9.89	10	1	1	0	0	1	1	1	1
Max. power/energy	6.76	3	9	10	10	1	3	9	5	0
Max. reliability	6.27	1	6	9	9	1	5	5	5	5
Min. LEC	5.88	3	8	8	8	5	0	2	2	0
Min. $/W	5.77	9	7	7	5	10	5	1	1	0
Max. safety	4.58	3	1	0	0	1	10	2	5	7
Code compliance	4.23	8	0	0	0	0	5	0	0	5
Recycling/Environ.	3.06	0	0	0	0	0	3	0	0	10
Sum_{raw}		44	37	36	33	28	32	21	20	28
$Sum_{weighted}$		313	250	231	210	205	166	139	125	125
W_{out}		10.0	8.01	7.40	6.71	6.55	5.32	4.44	4.01	4.00

Concepts

Design choices	W_{in}	PV_1	PV_2	PV_3
NEC-min wiring	10.0	10	2	4
Wiring/layout opt.	8.01	5	7	2
Module binning	7.40	2	6	1
High eff. modules	6.71	1	10	3
Inexpensive modules	6.55	10	1	1
Non-code analyses	5.32	5	1	8
Solar tracking	4.44	1	5	3
High quality BOS	4.01	1	3	4
Sustainability studies	4.00	1	1	10
Sum_{raw}		36	36	36
$Sum_{weighted}$		266	238	202
Selection		✓		

Figure 5. Three-stage QFD implementation used to select a PV system concept

Since this QFD is done from the developer's perspective it makes sense that the developer will have the highest W_{in} in the first stage of the QFD. Lenders/investors and the utility company are assigned high weights because the developer needs to have strong relationships with these parties in order to realize the project. On the other hand, it is unlikely that the developer will deal directly with the energy consumers or the component suppliers, so these stakeholders have low weights. A 0-10 integer system was used for all scores and for manually derived stakeholder weights in Figure 5; however, other scoring systems could have been used without changing the framework. Also, this QFD implementation could be expanded into more stages to capture more considerations, or into fewer stages to more directly relate stakeholders to the concept selections (single-stage and two-stage variants for this QFD are presented in the Appendix). It is important to note that all of the SE elements, weights, and scores shown Figure 5 may not represent all of the considerations in every project. That is, there may be more—or different—stakeholder-values, etc.; however, this is just an example used to illustrate the use of PCM. Also, although conflict resolution is beyond the scope of this discussion, various conflicts may exist between the stakeholder-values and the design choices. For example, maximizing power/energy may mean using more efficient PV modules and conducting more engineering analyses and hence likely increasing project time and costs; using NEC-minimum wiring minimizes costs but negatively impacts power/energy output and system reliability (Yuventi, 2013b); etc.

3.2 PCM analyses and results

PCM was used to analyze how concept selection will change if numerical variance was introduced into the QFD presented in Figure 5. This analysis was also done for weak-medium-strong—i.e., 1-3-5, 1-3-9, 1-5-9, and 1-7-9—scoring systems by converting the 0-10 scores using the logic captured in Table 1. To investigate probabilistic relationships, the analyses were done under three sets of conditions:

1. Assuming the numerical variances follow a Gaussian distribution, the QFD was analyzed using a boundary-constrained Gaussian PDF in the PCM, that is, the probabilities of numbers above or below that of the original QFD are approximately equal (for mid boundary scores), as illustrated in Figure 6. Allowing all $\sigma_{i,j}$ to be the same were the same, this analysis was done for $\sigma_{i,j}$=1, 2, 3, 4, and 5 (where $\mu_{i,j}\pm\sigma_{i,j}$ accounts for 68.2% of the expected scores). This simulates the case in which the same person or mindset is applied for all of the scoring.

2. Again the QFD was analyzed assuming a Gaussian distribution, however now each $\sigma_{i,j}$ was different and represented by a uniformly distributed random number between 0 and 5. This case assumes that the general consensus leads toward the original scores but there is inconsistency in the scoring mindsets or personnel.
3. Assuming that there is no general consensus toward any particular score, then the QFD was analyzed assuming all of the scores are uniformly distributed over the range of useable scores. This case assumes that there is no structure or training toward the scoring determinations.

The PCM analyses were performed using 10,000 Monte Carlo runs in which all of the scores and the stakeholder weights were treated as random numbers. A large number of run was used to ensure that the histogram of the Monte Carlo randomizations matched well with the evaluated PDF, i.e., more runs increase the accuracy of the PCM analyses. Table 2 captures the results of this analysis, illustrating how the selection rates—in percentages—changed for each concept given different probabilistic conditions. These results indicate that for the Gaussian distribution, wherein there is are defined structures or frameworks for the QFD, that the concept chosen in the original analysis, i.e., PV_1, will be chosen the majority of the time and hence is the dominant concept. This is consistent for all of the evaluated $\sigma_{i,j}$ and across the various scoring systems, although the relationship between $\sigma_{i,j}$ and selection rates differed for each system. This was expected, especially for the 1-3-5 system since the values of $\sigma_{i,j}$ used will cover a larger set of useable scores than with the other systems. It is also intuitive that the selection percentages for each concept will change with respect to $\sigma_{i,j}$ in each scoring system and that the dominance of a concept will decrease. As $\sigma_{i,j}$ increases, the distribution resembles a uniform distribution, as illustrated in Figure 6. (The PDF in this PCM analysis has small spikes at the minimum and maximum values to capture the probability of values outside of the useable range.) Similar to the results of the uniform distribution, significantly large $\sigma_{i,j}$ represent a controlled case, wherein the selection rates of each concept should approach the same value (33.3% for this example) as n→∞, if the QFD framework is set up correctly and if all manually derived weights and scores are randomized. The results for $\sigma_{i,j}$=0 are also a controlled case. These represent the output of the original QFD with no numerical variance. If the concept selected in the original QFD is not selected 100% of the time then this means that PCM was set up incorrectly. Interesting results were be seen when $\sigma_{i,j}$ is a random number, represented by σ=rand in Table 2. These results show that even with this additional variability, PV_1 is still the most selected design for all of the scoring systems. This suggests that the original QFD implementation and the QFD framework in general can be very robust toward numerical variance.

Table 1. 0-10 to weak-medium-strong score conversion

0-10 score	**0**	**0<$S_{i,j}$≤3**	**3<$S_{i,j}$≤7**	**7<$S_{i,j}$≤10**
New relationship	0	Weak	Medium	Strong

Table 2. PCM results: concept selection rates for various probabilistic conditions

		0-10 scoring			1-3-5 scoring			1-3-9 scoring		
	PDF	**PV_1**	**PV_2**	**PV_3**	**PV_1**	**PV_2**	**PV_3**	**PV_1**	**PV_2**	**PV_3**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-	100.00%	-	-
Gaussian	$\sigma_{i,j}$=1	77.12%	21.27%	1.61%	54.73%	24.79%	20.48%	97.76%	1.73%	0.51%
Gaussian	$\sigma_{i,j}$=2	55.10%	32.19%	12.71%	42.05%	28.09%	29.86%	75.14%	14.21%	10.65%
Gaussian	$\sigma_{i,j}$=3	45.81%	33.66%	20.53%	38.34%	29.68%	31.98%	54.33%	21.35%	24.32%
Gaussian	$\sigma_{i,j}$=4	39.87%	35.09%	25.04%	37.33%	30.07%	32.60%	45.42%	25.18%	29.40%
Gaussian	$\sigma_{i,j}$=5	38.19%	32.87%	28.94%	34.67%	31.42%	33.91%	42.79%	25.50%	31.71%
Gaussian	$\sigma_{i,j}$=rand	54.18%	30.56%	15.26%	43.62%	26.83%	29.55%	66.12%	16.49%	17.39%
	Uniform	33.27%	33.23%	33.50%	32.86%	33.48%	33.66%	32.86%	34.41%	32.73%

		1-5-9 scoring			1-7-9 scoring		
	PDF	**PV_1**	**PV_2**	**PV_3**	**PV_1**	**PV_2**	**PV_3**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-
Gaussian	$\sigma_{i,j}$=1	90.46%	7.69%	1.85%	80.59%	18.40%	1.01%
Gaussian	$\sigma_{i,j}$=2	56.74%	24.90%	18.36%	52.82%	31.22%	15.96%
Gaussian	$\sigma_{i,j}$=3	47.21%	26.05%	26.74%	44.00%	31.12%	24.88%
Gaussian	$\sigma_{i,j}$=4	41.67%	28.67%	29.66%	38.62%	31.23%	30.15%
Gaussian	$\sigma_{i,j}$=5	39.69%	28.61%	31.70%	36.78%	31.40%	31.82%
Gaussian	$\sigma_{i,j}$=rand	56.57%	23.23%	20.20%	50.66%	29.77%	19.57%
	Uniform	33.62%	32.84%	33.54%	33.49%	33.21%	33.30%

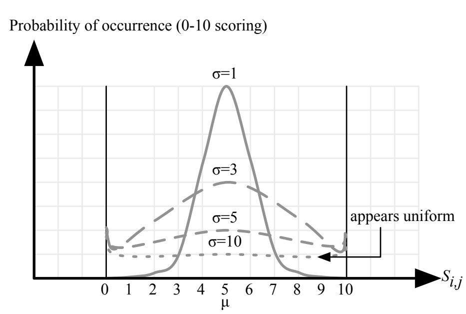

Figure 6. Gaussian PDF as σ increases

There may be more confidence toward the preciseness of certain QFD scores or stages than others in most SE applications. For example, there may be less uncertainty in the relationships between design choices and system concepts in comparison to the stakeholder and stakeholder-value relationships, since the former may be easier to derive from best-practices, experience, technical considerations, etc. Also, stakeholder weights may be well established, based on contract stipulations, project/organizational networks, etc. Table 3 illustrates PCM results of such limiting cases for the 0-10 scoring system, analyzed using 10,000 Monte Carlo runs. In these simulations, the 'fixed' weights/scores had standard deviation, i.e., they remained as the original values in Figure 5. Comparing these results to that shown in Table 2 indicates that numerical variation in the concept scores has a much larger impact on the concept selection rates than variations in the stakeholder weights. In fact, using fixed stakeholder weights did not change the result much in comparison to Table 2. Therefore this suggests that—in this case—it is more important to reduce uncertainty in the concept score determinations than the stakeholder weighting in order to increase confidence in the QFD results. Table 3 also indicates that the uniform results approach 33.3% for each concept. This is because the concepts had equal Sum_{raw} in the original QFD.

Table 3. PCM results: concept selection rates for limiting conditions (0-10 scoring)

		Fixed stakeholder weights And variable concept scores			Fixed concept scores and Variable stakeholder weights			Fixed stakeholder weights And fixed concept scores		
	PDF	**PV_1**	**PV_2**	**PV_3**	**PV_1**	**PV_2**	**PV_3**	**PV_1**	**PV_2**	**PV_3**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-	100.00%	-	-
	$\sigma_{i,j}$=1	77.97%	20.46%	1.57%	98.99%	1.01%	0.00%	99.42%	0.58%	0.00%
	$\sigma_{i,j}$=2	55.19%	32.84%	11.97%	87.31%	12.58%	0.11%	88.92%	11.08%	0.00%
	$\sigma_{i,j}$=3	45.63%	34.89%	19.48%	75.70%	22.22%	2.08%	77.19%	21.51%	1.30%
	$\sigma_{i,j}$=4	40.94%	34.20%	24.86%	66.41%	26.76%	6.83%	66.99%	28.06%	4.95%
	$\sigma_{i,j}$=5	37.77%	34.78%	27.45%	58.53%	30.21%	11.26%	60.33%	30.47%	9.20%
	$\sigma_{i,j}$=rand	54.29%	30.33%	15.38%	81.40%	16.58%	2.02%	82.52%	15.82%	1.66%
	Uniform	33.92%	33.49%	32.59%	34.19%	33.75%	32.06%	34.44%	33.85%	31.71%

4. Insights derivable from PCM analyses

The example PCM analyses and results presented in this paper convey a relationship between probabilistic uncertainty and the results of a given QFD implementation. A similar analysis could have been performed using other probabilistic distributions or on other weight/score-based decision-making tools that are used in SE and other processes. That is, PCM can be adjusted for use on Kepner-Tregoe and Pugh matrices, numerical decision-trees, multi-criteria decision analysis, etc. These implementations do not have to be as detailed and those presented in this paper, e.g., PCM could only be performed using a single standard deviation and/or PDF. However, evaluation over a range of probabilistic considerations provides insight into the robustness of the evaluated tool implementation and possible probability-based trends. For example, if D_{sum} and D_{sel} respectively represents the difference between $Sum_{weighted,j}$ and selection rates of the selected concept and the second ranked concept in the original QFD. Then, based on the Gaussian distribution analysis and results shown in Table 2, where $\sigma_{i,j}$ is the same for all scores and $\sigma_{i,j}>0$,

$$\frac{D_{sum}}{50 \cdot \sigma_{i,j}} \geq D_{sel} \tag{4}$$

approximately captures the relationship between D_{sum}, D_{sel}, and $\sigma_{i,j}$ for all of the scoring systems. This formulaic relationship differs slightly for PCM analyses done on the single-stage and two-stage QFD variants shown in the Appendix, and will likely differ for other QFD implementations. Possibly, theoretical evaluations can be performed to develop generalized equations using other parameters and PDFs. PCM facilitates this exploration and provides practitioners with an idea of how to manage weigh/score-based tool implementations when considering uncertainties. In addition, PCM can be useful in assisting the following SE application considerations:

1. Training scorers to use mindsets, sources, and systems to reduce potential numerical variance to a tolerated amount, and
2. Determine whether the weight/score-based tool should be re-evaluated.

With respect to both of these considerations, it is common in practice to iteratively expand the breadth of QFD analyses throughout a SE initiative as more information is gathered or unplanned decisions have to be made. Therefore, PCM can be used to structure these iterations to improve the final results. PCM can be used to identify when/if an implementation should be re-evaluated based on defining an acceptable selection rate for the 'dominant' concept or 'clear winner' (e.g., 60% selection rate or higher) for a given set of probabilistic considerations. If there is/are no clear winner(s) then the re-evaluation can involve appropriately rescoring or reweighting QFD elements; considering additional stakeholders, values, or concepts; adding more stages to the QFD implementation; or any combination of these activities.

5. Conclusion

The unavoidable threat of uncertainty that affects inputs to weight/score-based decision-making tools creates concern as to how these uncertainties may impact the verity of results produced by these tools. This is especially a concern for SE applications wherein weight/score-based tools, such as QFD matrices, are used extensively and iteratively. The numerical variances in weights and scores that are introduced due to uncertainties, such as human-bias and lack of education, can propagate through stages and iterations to significantly impact critical project decisions. In this paper, we introduced an algorithm that we called the Probabilistic Consideration Method or PCM that analyzes an implementation of a weight/score-based tool to evaluate the resilience of the implementation in the presence of probabilistic numerical variances. PCM was used to analyze a QFD implementation that selects system concepts for a large-scale PV construction project illustrating that the original implementation was quite robust when factoring potential Gaussian distributions of weights and scores. Such PCM analyses could be used to identify probability-based trends, set tolerances for levels of numerical variance, provide grounds to further justify project decisions, or be used as a reason to re-evaluate tool implementations. These takeaways can be used to train scorers and refine successive implementations of these tools in SE and other processes. PCM can also be implemented using different PDFs and adjusted for use on other numerical-based decision making tools, such as Pugh matrices, numerical decision-trees, etc.

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Appendix

Figure A.1 illustrates single-stage and two-stage variants of the QFD implementation analyzed in the paper. In the single-stage implementation, the system concept is selected directly from the stakeholders, wherein the scores represent how much a particular stakeholder aligns with the corresponding concept. In the two-stage variant, the design-choice scoring stage was removed from the three-stage QFD shown in Figure 5 to directly relate the stakeholder-values to the system concepts. The PCM analysis conducted on the three-stage QFD implementation was also done on the single-stage and two-stage variants, using the same scoring systems, to investigate/illustrate how the impact of variance changes with the number of stages. Table A.1 and Table A.2 capture a subset of these results, showing that trends that are similar to the results obtained for the three-stage QFD. Table A.1 illustrates the influence of the various scoring systems when all manual weights and scores are varied and Table A.2 illustrates the results of limiting cases for the 0-10 scoring system to for respective comparisons with Table 2 and Table 3.

Single-stage QFD

		Concepts		
Stakeholders	W_{in}	PV_1	PV_2	PV_3
Developer	10	10	5	3
Lenders/investors	8	10	2	1
Utility company	7	1	10	8
Land/site host	5	1	1	10
Electrical contractor	5	9	7	5
Regulators	4	1	2	10
Other contractors	3	7	4	3
Energy consumers	3	1	10	1
Component suppliers	1	5	4	4
Sum_{raw}		45	45	45
$Sum_{weighted}$		270	230	225
Selection		✓		

Two-stage QFD

		Stakeholder-values								
Stakeholders	W_{in}	Min. project costs	Min. project time	Max. power/energy	Max. reliability	Min. LEC	Min. $/W	Max. safety	Code compliance	Recycling/Environ.
Developer	10	10	9	6	3	5	7	1	0	1
Lenders/investors	8	10	8	1	1	1	1	0	0	0
Utility company	7	2	7	10	8	8	3	6	2	0
Land/site host	5	3	4	0	0	0	0	2	0	10
Electrical contractor	5	7	7	7	5	4	10	3	8	0
Regulators	4	0	0	1	3	0	0	9	10	5
Other contractors	3	10	7	2	5	0	4	3	7	0
Energy consumers	3	0	0	3	9	10	0	1	0	1
Component suppliers	1	10	2	0	5	3	3	5	5	4
Sum_{raw}		52	44	30	39	31	28	30	32	21
$Sum_{weighted}$		284	281	192	178	167	164	130	120	87
W_{out}		10.0	9.89	6.76	6.27	5.88	5.77	4.58	4.23	3.06

⇨

		Concepts		
Stakeholder-values	W_{in}	PV_1	PV_2	PV_3
Min. project costs	10.0	10	1	1
Min. project time	9.89	8	1	2
Max. power/energy	6.76	5	10	2
Max. reliability	6.27	1	8	5
Min. LEC	5.88	2	9	1
Min. $/W	5.77	9	6	1
Max. safety	4.58	1	1	9
Code compliance	4.23	3	3	9
Recycling/Environ.	3.06	1	1	10
Sum_{raw}		40	40	40
$Sum_{weighted}$		303	246	196
Selection		✓		

Figure A.1. Single-stage and two-stage variants of the QFD implementation

Table A-1. PCM results for the single-stage and two-stage QFD variants

		Single-stage QFD								
		0-10 scoring			1-3-5 scoring			1-3-9 scoring		
	PDF	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-	100.00%	-	-
	$\sigma_{i,j}$=1	86.17%	7.23%	6.60%	67.68%	18.51%	13.81%	97.30%	0.05%	2.65%
	$\sigma_{i,j}$=2	63.11%	18.69%	18.20%	48.74%	27.53%	23.73%	77.16%	5.12%	17.72%
	$\sigma_{i,j}$=3	51.37%	24.76%	23.87%	41.74%	29.21%	29.05%	59.91%	14.50%	25.59%
	$\sigma_{i,j}$=4	43.99%	28.78%	27.23%	40.02%	30.81%	29.17%	50.25%	20.11%	29.64%
	$\sigma_{i,j}$=5	39.93%	30.19%	29.88%	37.39%	31.76%	30.85%	45.45%	23.64%	30.91%
	$\sigma_{i,j}$=rand	59.07%	20.42%	20.51%	50.74%	26.01%	23.25%	68.47%	10.72%	20.81%
	Uniform	33.65%	32.87%	33.48%	34.06%	32.67%	33.27%	33.41%	33.10%	33.49%

		Two-stage QFD								
		0-10 scoring			1-3-5 scoring			1-3-9 scoring		
	PDF	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-	100.00%	-	-
	$\sigma_{i,j}$=1	96.15%	3.80%	0.05%	69.15%	24.30%	6.55%	98.07%	1.88%	0.05%
	$\sigma_{i,j}$=2	77.20%	18.37%	4.43%	48.22%	31.62%	20.16%	78.36%	16.90%	4.74%
	$\sigma_{i,j}$=3	60.38%	26.51%	13.11%	39.46%	32.93%	27.61%	58.24%	26.75%	15.01%
	$\sigma_{i,j}$=4	49.67%	29.89%	20.44%	37.75%	32.49%	29.76%	48.31%	29.58%	22.11%
	$\sigma_{i,j}$=5	44.29%	31.25%	24.46%	36.12%	32.78%	31.10%	41.71%	31.51%	26.78%
	$\sigma_{i,j}$=rand	69.65%	20.41%	9.94%	48.53%	30.49%	20.98%	67.63%	21.59%	10.78%
	Uniform	32.98%	33.25%	33.77%	33.61%	33.27%	33.12%	33.22%	33.16%	33.62%

Table A.2. Single-stage and two-stage results for concept selection rates for limiting conditions

		Single-stage QFD		
		Fixed concept scores and Variable stakeholder weights		
	PDF	**PV₁**	**PV₂**	**PV₃**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-
	$\sigma_{i,j}$=1	98.00%	0.81%	1.19%
	$\sigma_{i,j}$=2	81.48%	8.38%	10.14%
	$\sigma_{i,j}$=3	67.30%	14.51%	18.19%
	$\sigma_{i,j}$=4	58.59%	19.07%	22.34%
	$\sigma_{i,j}$=5	54.22%	20.78%	25.00%
	$\sigma_{i,j}$=rand	77.23%	9.93%	12.84%
	Uniform	35.80%	29.91%	34.29%

		Two-stage QFD								
		Fixed stakeholder weights And variable concept scores			Fixed concept scores and Variable stakeholder weights			Fixed stakeholder weights And fixed concept scores		
	PDF	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**	**PV₁**	**PV₂**	**PV₃**
Gaussian	$\sigma_{i,j}$=0	100.00%	-	-	100.00%	-	-	100.00%	-	-
	$\sigma_{i,j}$=1	96.88%	3.11%	0.01%	100.00%	-	-	100.00%	-	-
	$\sigma_{i,j}$=2	79.57%	17.07%	3.36%	98.79%	1.18%	0.03%	99.91%	0.09%	-
	$\sigma_{i,j}$=3	64.37%	25.25%	10.38%	92.21%	6.49%	1.30%	97.82%	2.13%	0.05%
	$\sigma_{i,j}$=4	53.16%	29.80%	17.04%	82.44%	13.04%	4.52%	92.36%	6.79%	0.85%
	$\sigma_{i,j}$=5	46.91%	31.19%	21.90%	74.17%	16.81%	9.02%	85.53%	11.33%	3.14%
	$\sigma_{i,j}$=rand	72.17%	19.58%	8.25%	92.88%	5.40%	1.72%	96.55%	3.02%	0.43%
	Uniform	33.96%	33.17%	32.87%	33.05%	30.59%	36.36%	33.21%	31.02%	35.77%

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