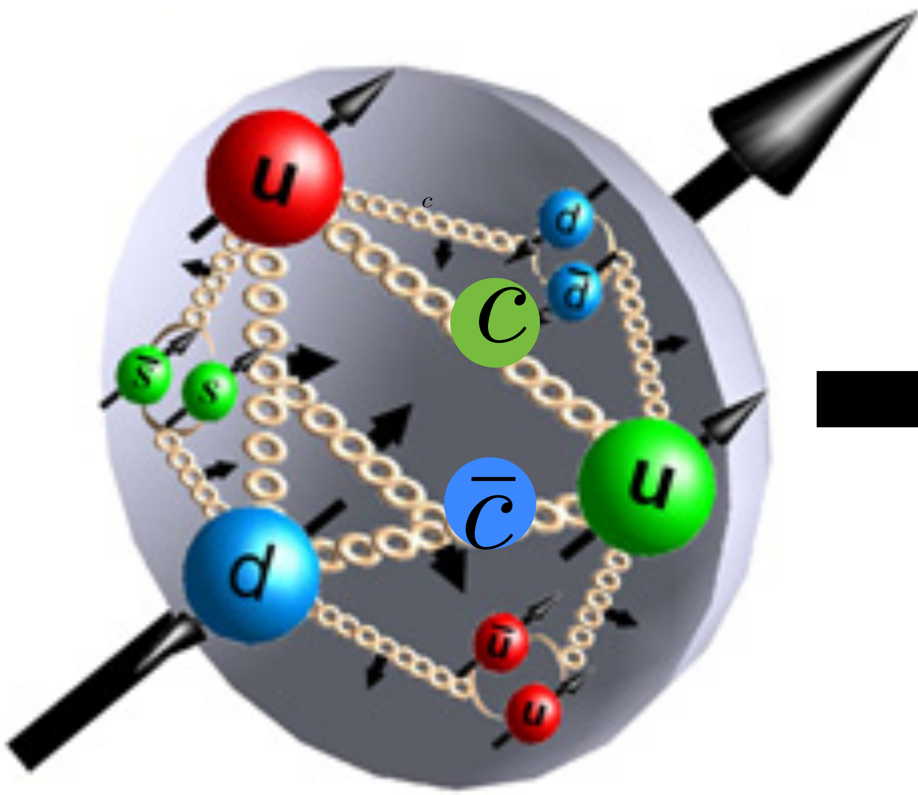
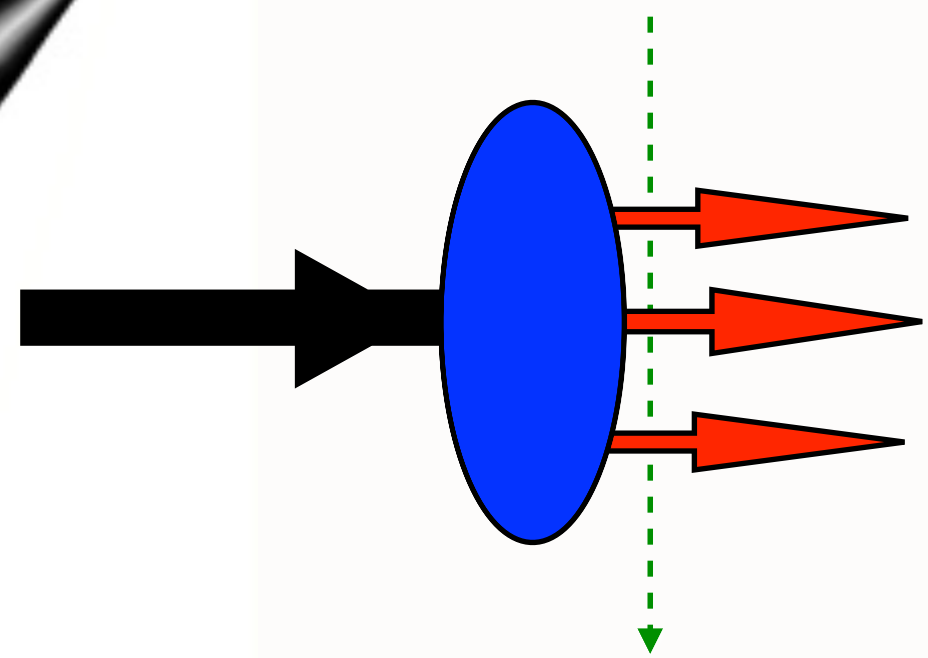
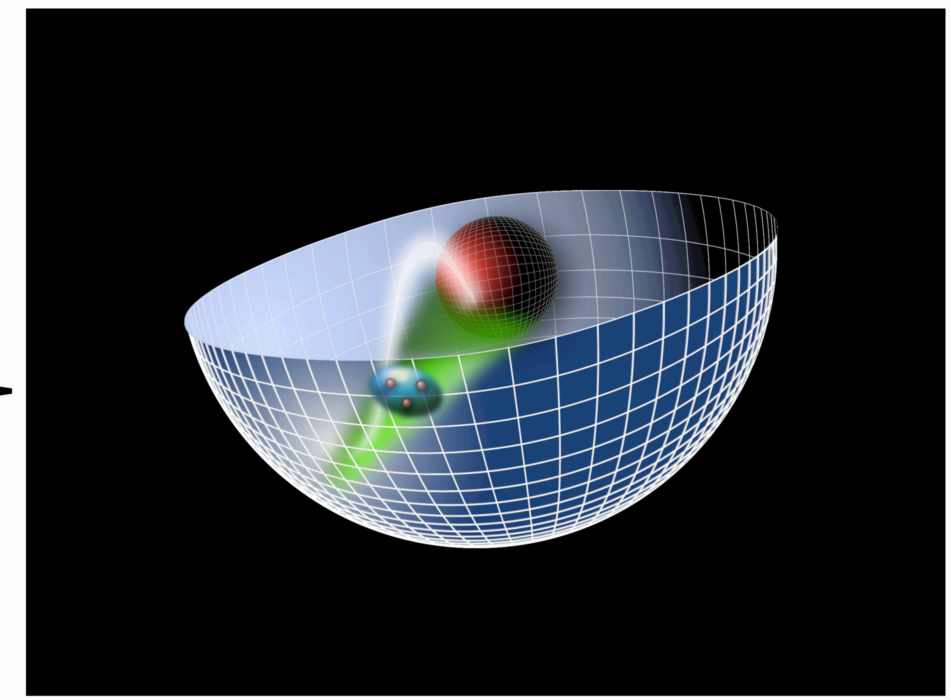
AdS/QCD and Light-Front Holography

Guy de Teramond
Hans Günter Dosch







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

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May 21, 2013
Skiathos, Greece

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

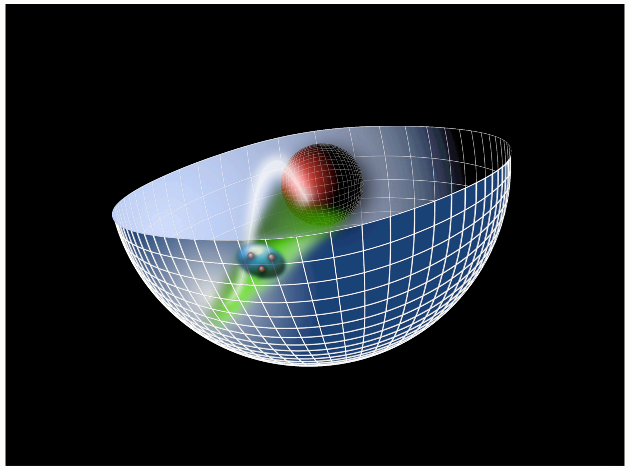
How does color confinement arise?

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Unique potential!

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

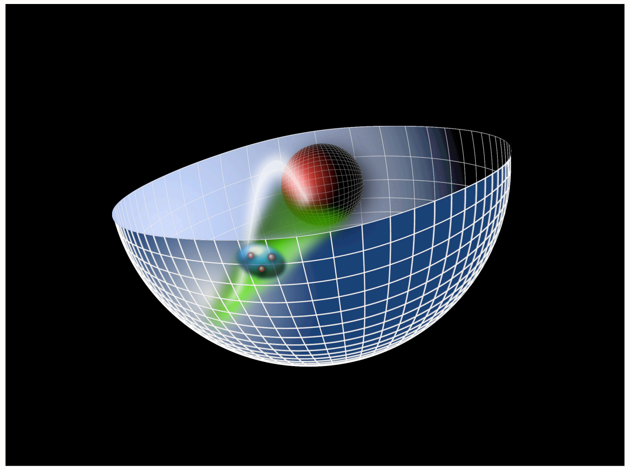
Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal
Symmetry
of the action

Light-Front Schrödinger Equation

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\psi(\zeta)=\mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

$$\zeta^2=x(1-x)\mathbf{b}^2_\perp .$$

Confinement scale: $\kappa \simeq 0.5 \; GeV$

$1/\kappa \simeq 0.4 \; fm$

Unique Confinement Potential!

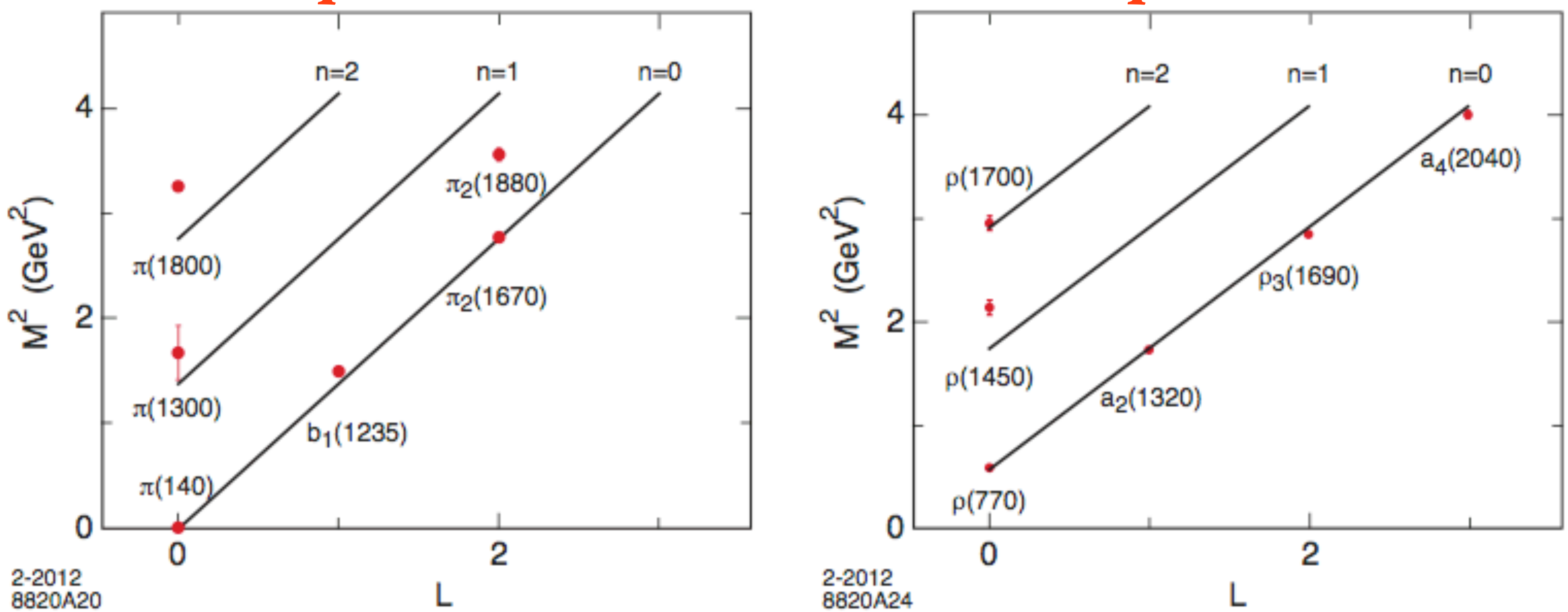
Conformal Symmetry of the action

- $J = L + S$, $I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***



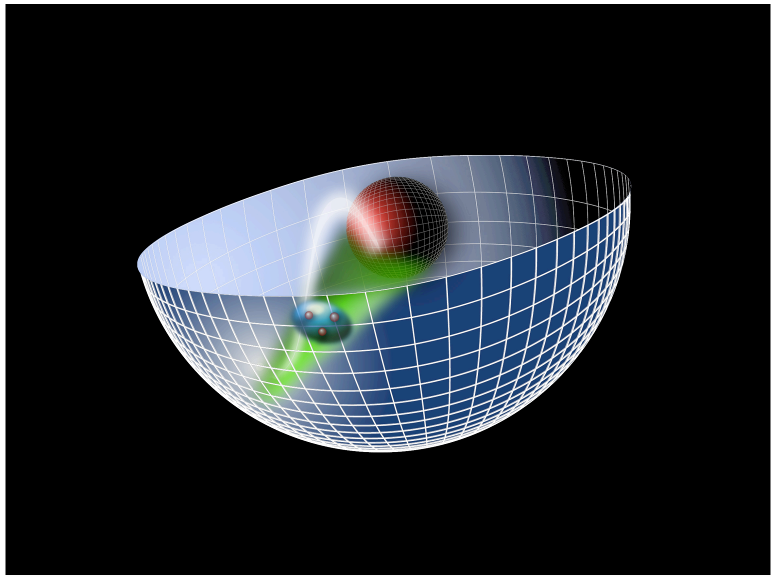
I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

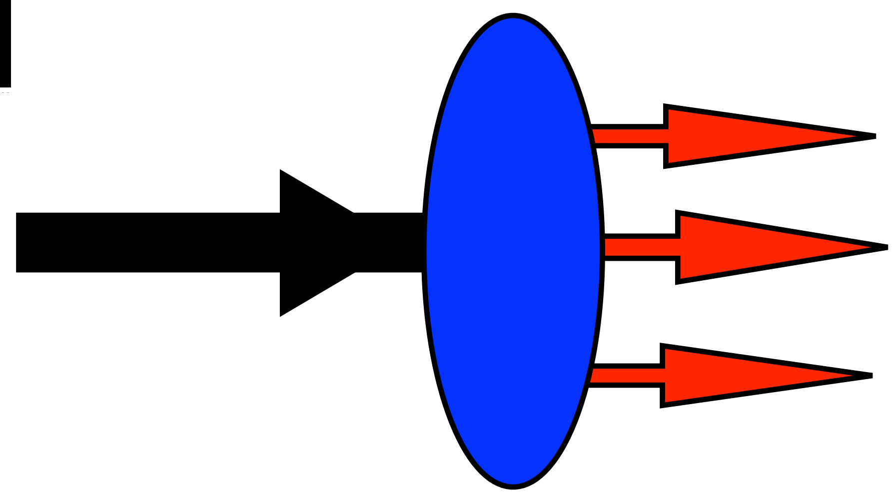
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

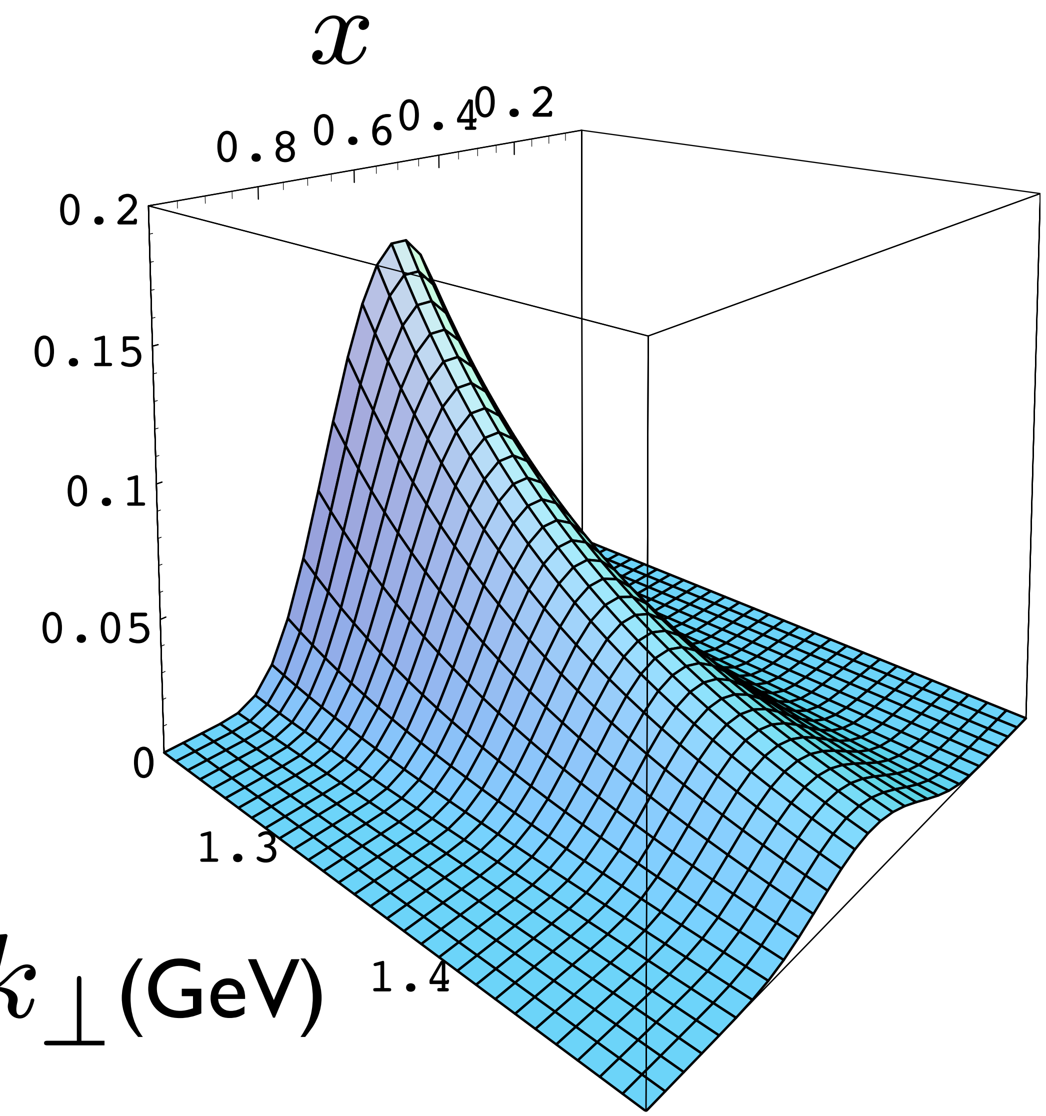
$\phi(z)$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

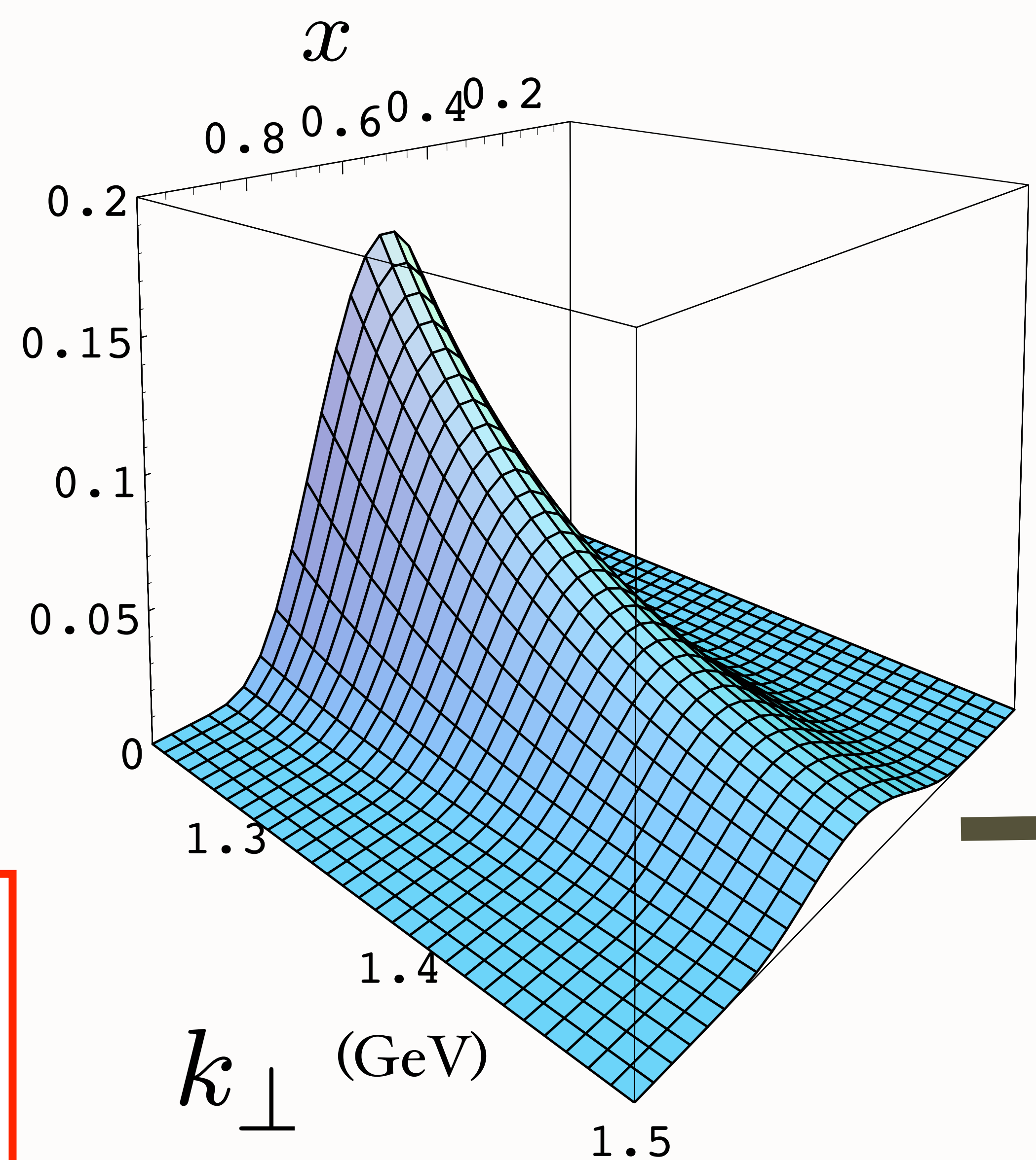


- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

Prediction from AdS/QCD: Meson LFWF

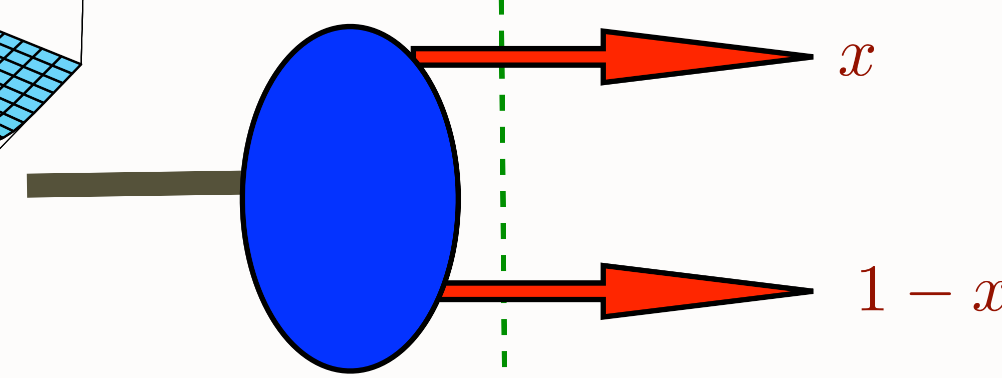
$$\psi_M(x, k_\perp^2)$$



$k_\perp$

de Teramond, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Provides Connection of Confinement to TMDs

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\phi(x, \zeta) = \mathcal{N} \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right),$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

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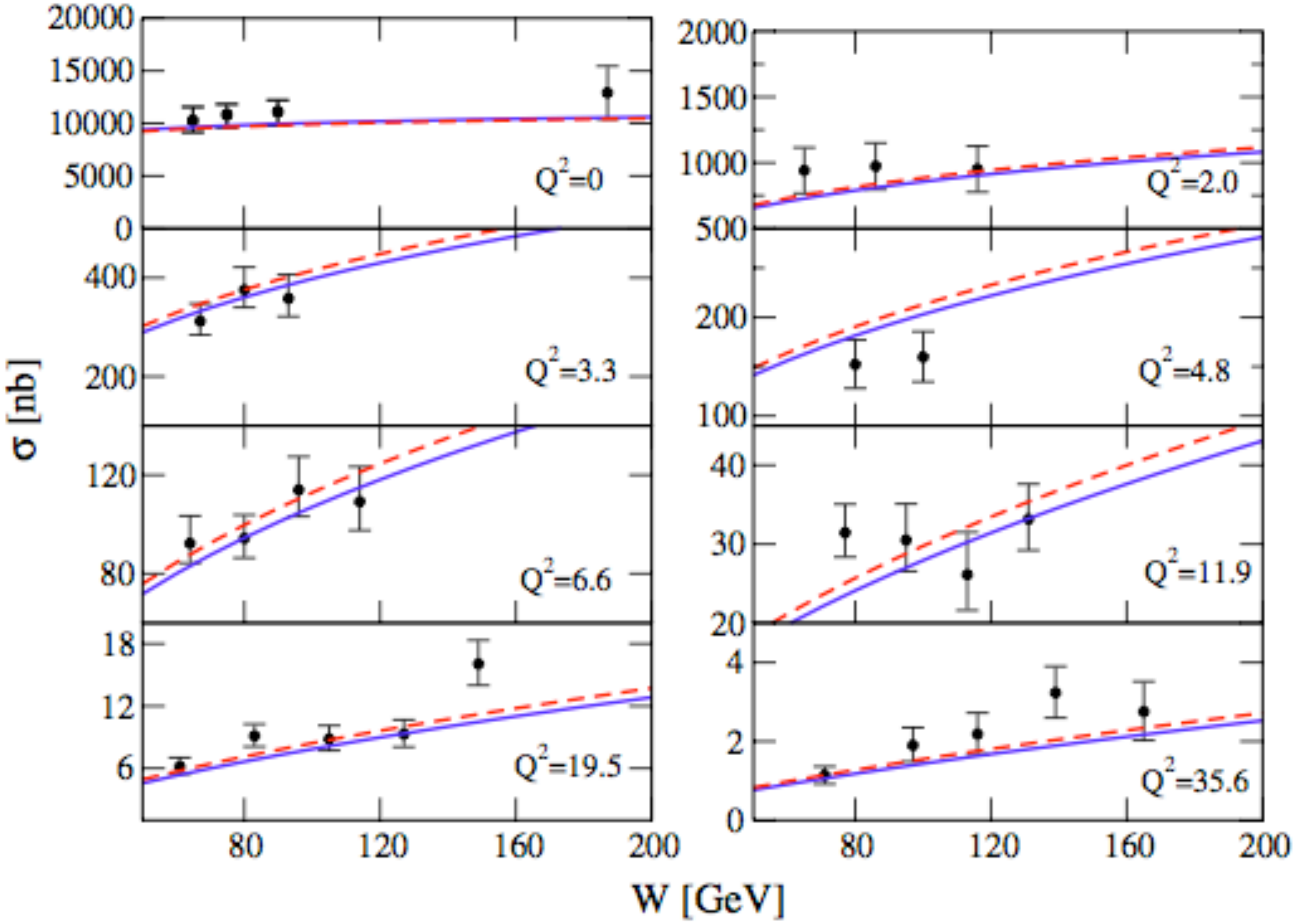
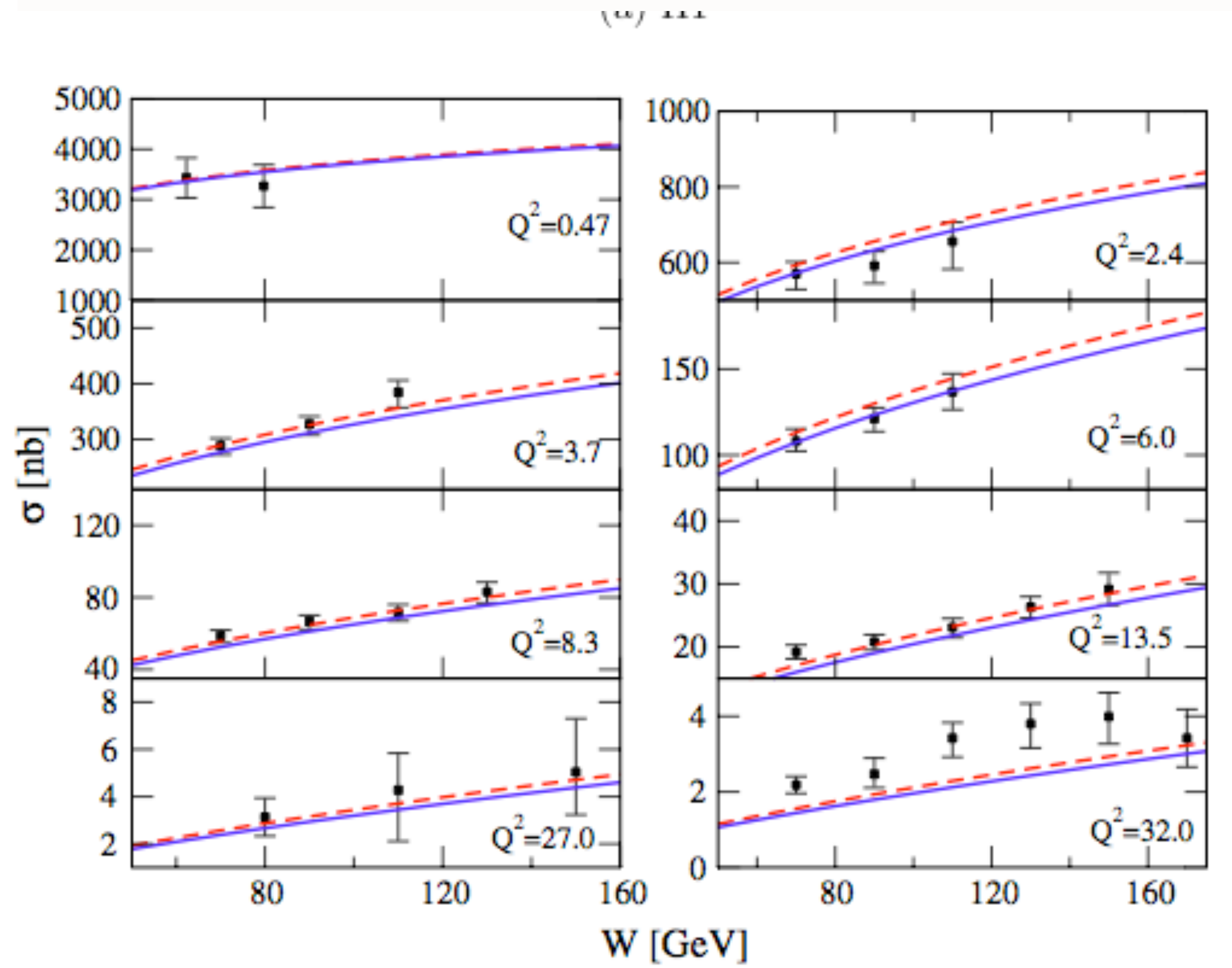
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



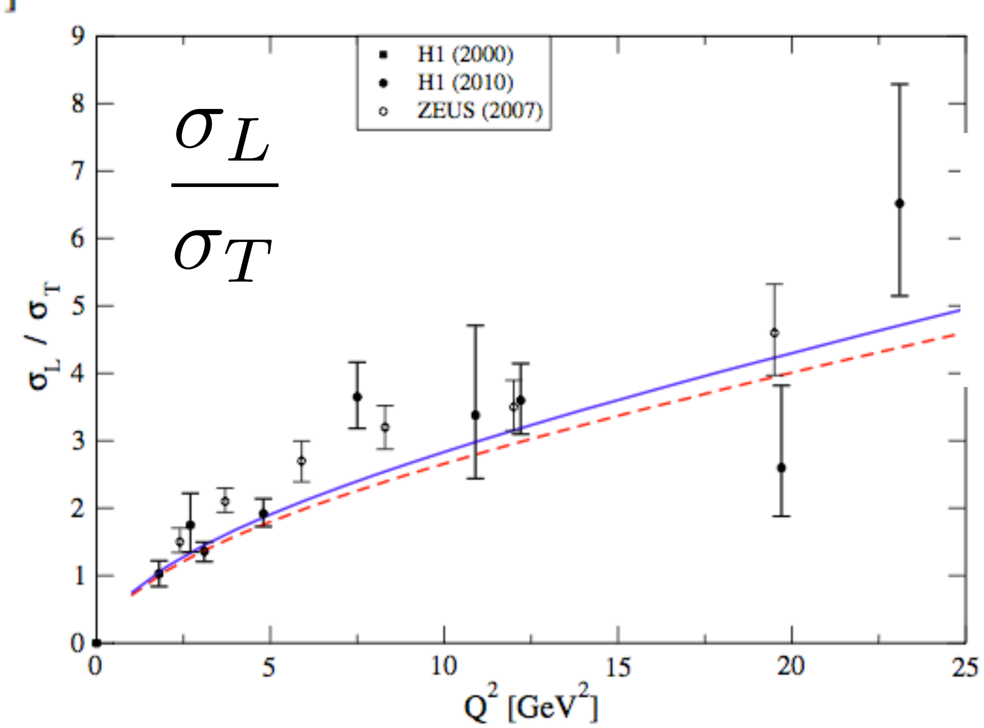
(a) H



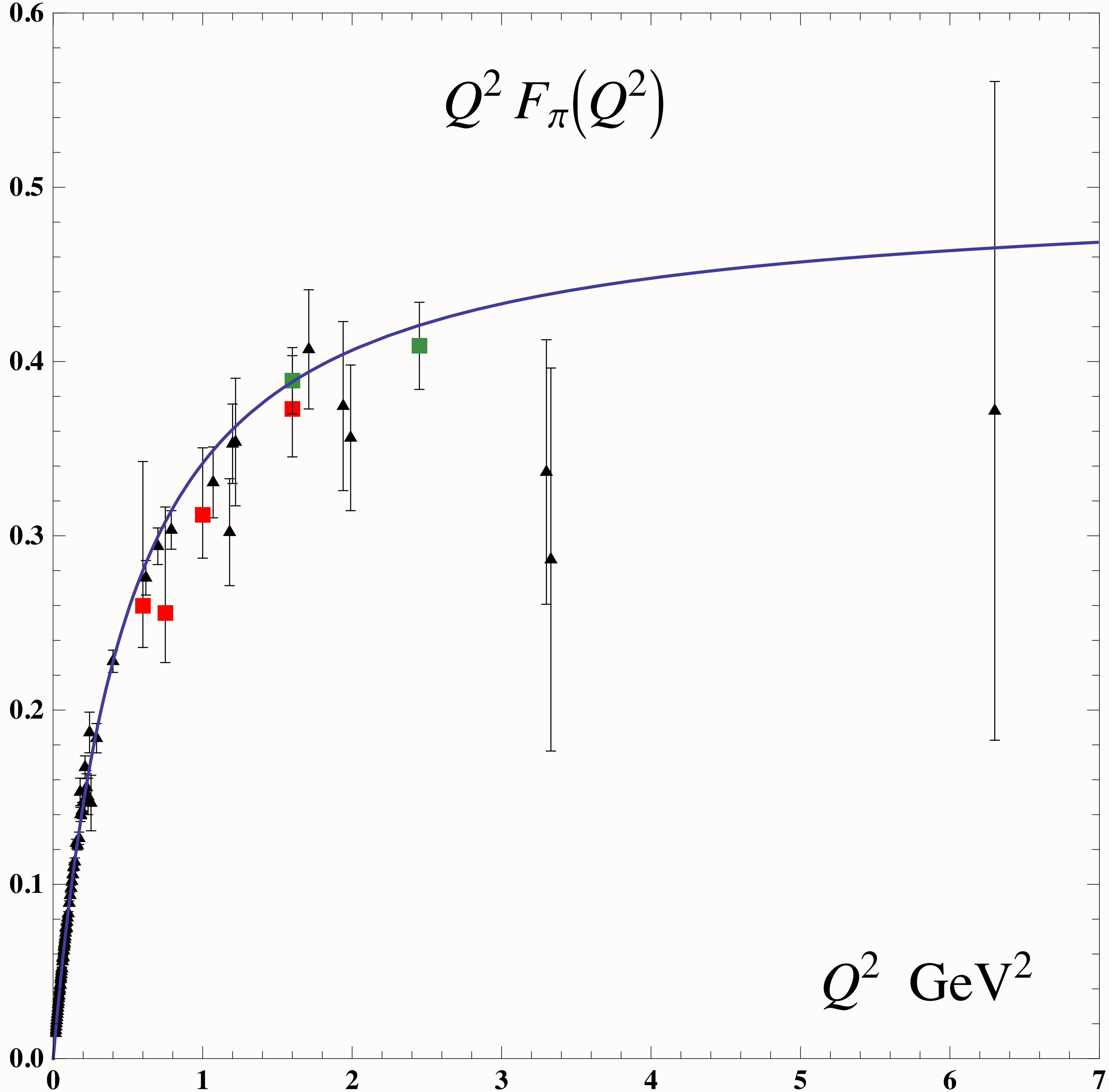
(b) ZEUS

J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Quantization

- LF coordinates

$x^+ = x^0 + x^3$	light-front time	$P^+ = P^0 + P^3$	longitudinal momentum
$x^- = x^0 - x^3$	longitudinal space variable	$P^- = P^0 - P^3$	light-front Hamiltonian
$\mathbf{x}_\perp = (x^1, x^2)$	transverse space variable	$\mathbf{P}_\perp = (P^1, P^2)$	transverse momentum

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamilnotian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i \frac{\partial}{\partial x^+} |\psi(P)\rangle = P^- |\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} |\psi(P)\rangle$$

where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

- Construct LF Lorentz invariant Hamiltonian $P^2 = P^- P^+ - \mathbf{P}_\perp^2$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

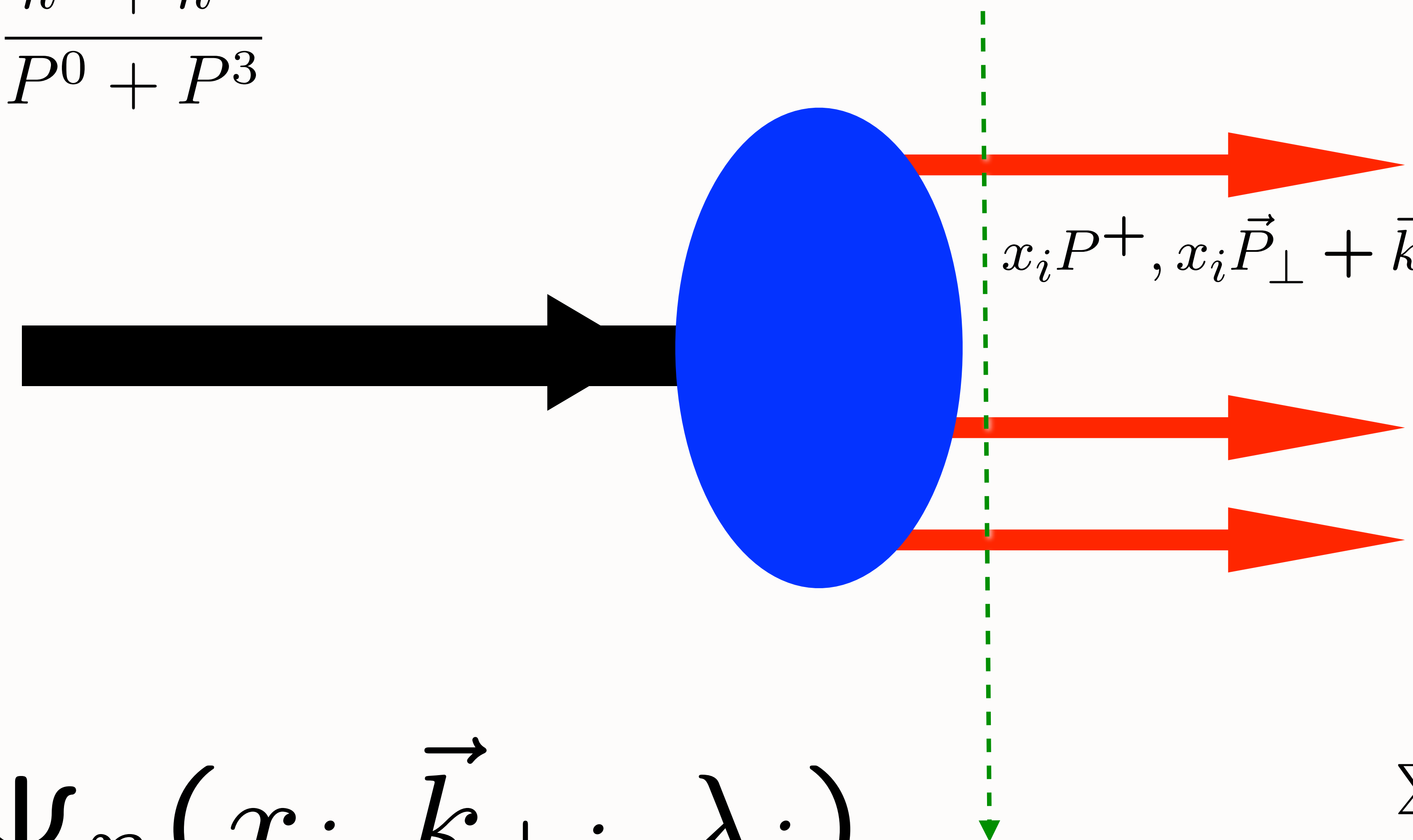
- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows unambiguous definition of partonic content of a hadron

Causal, Frame-independent, Simple Vacuum,
Current Matrix Elements are overlap of LFWFS

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

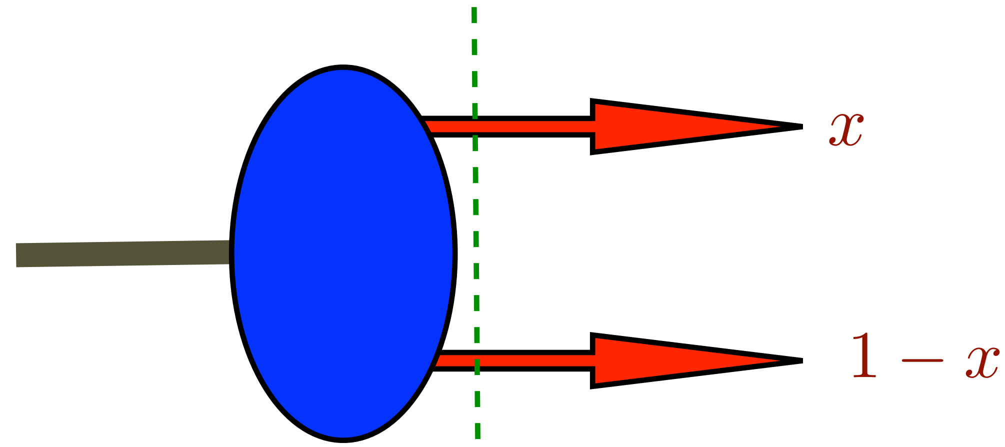
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Bethe-Salpeter WF integrated over k^-

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\; \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

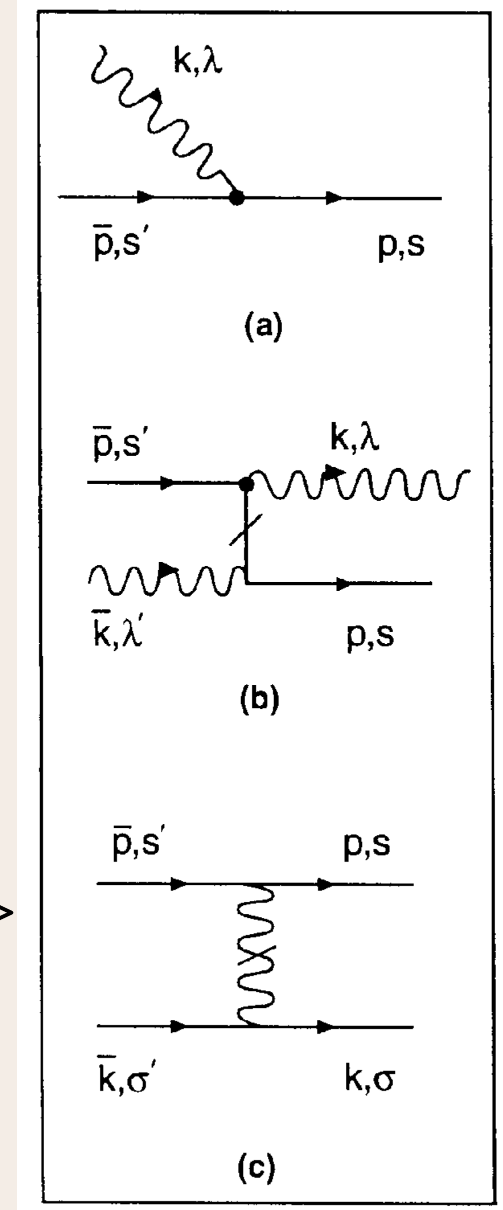
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

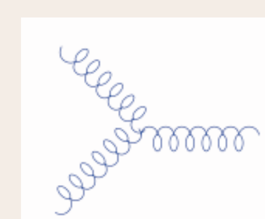
$$H^{QCD}_{LF} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$

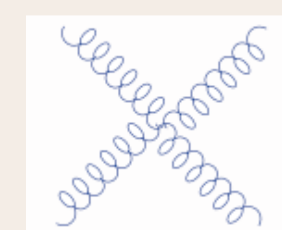
$$|p, J_z >= \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i >$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





H^{int}_{LF}

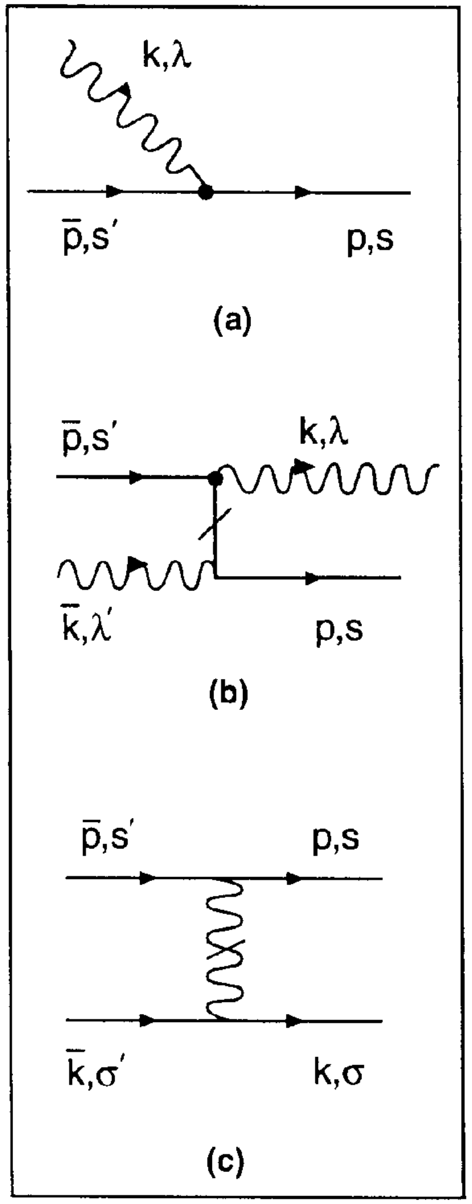
Light-Front QCD

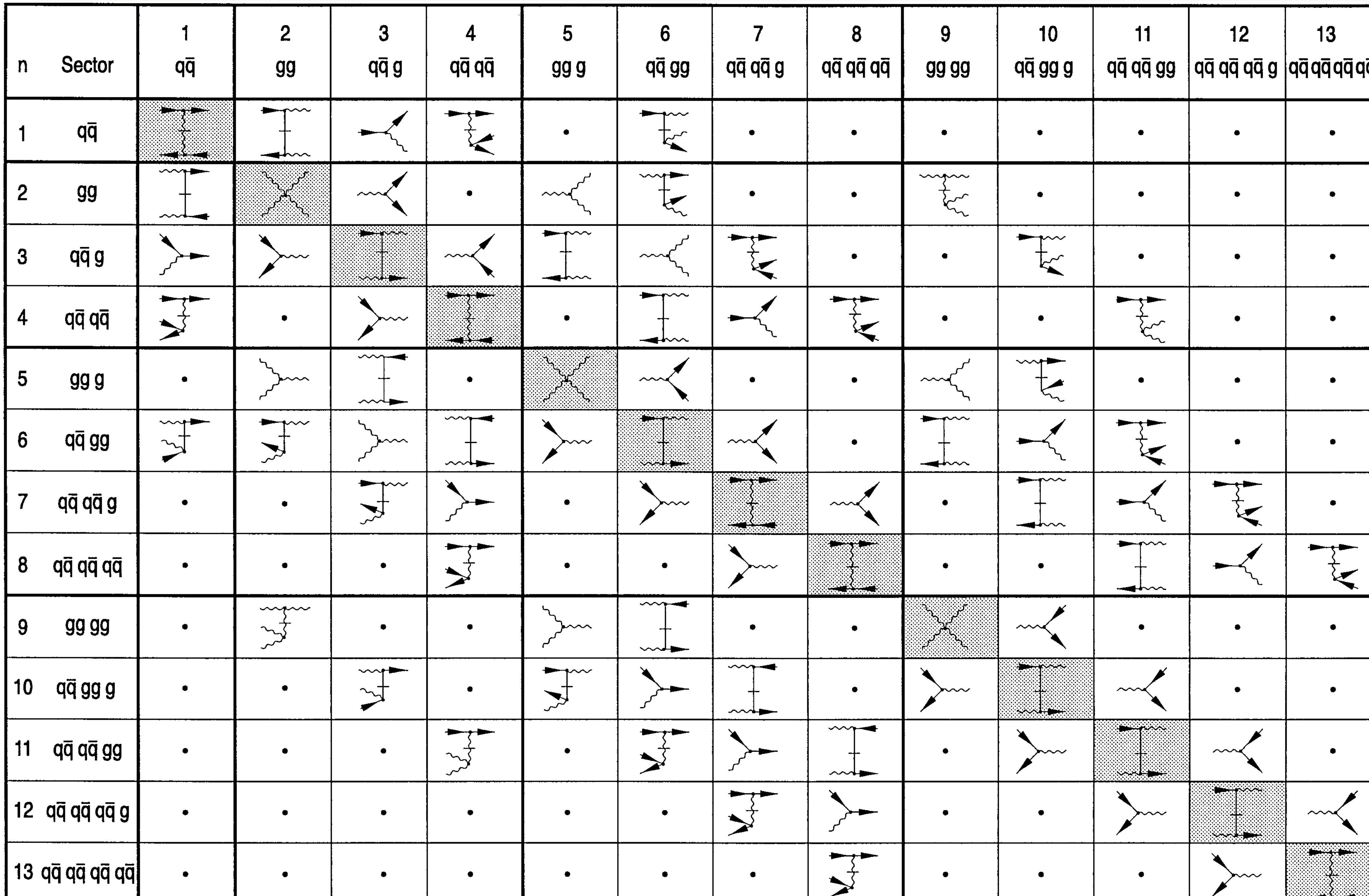
Heisenberg Equation

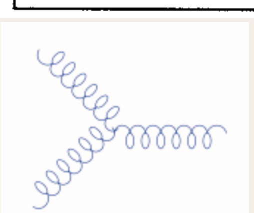
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

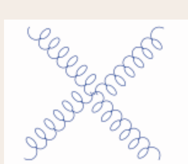
DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb









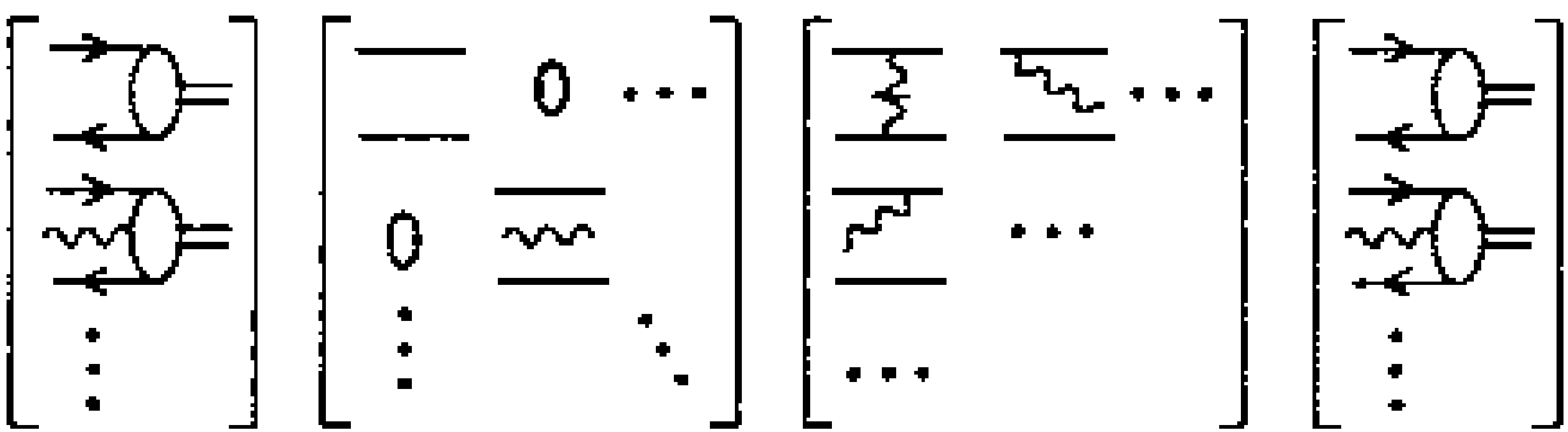
Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}| V |q\bar{q}\rangle & \langle q\bar{q}| V |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g| V |q\bar{q}\rangle & \langle q\bar{q}g| V |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

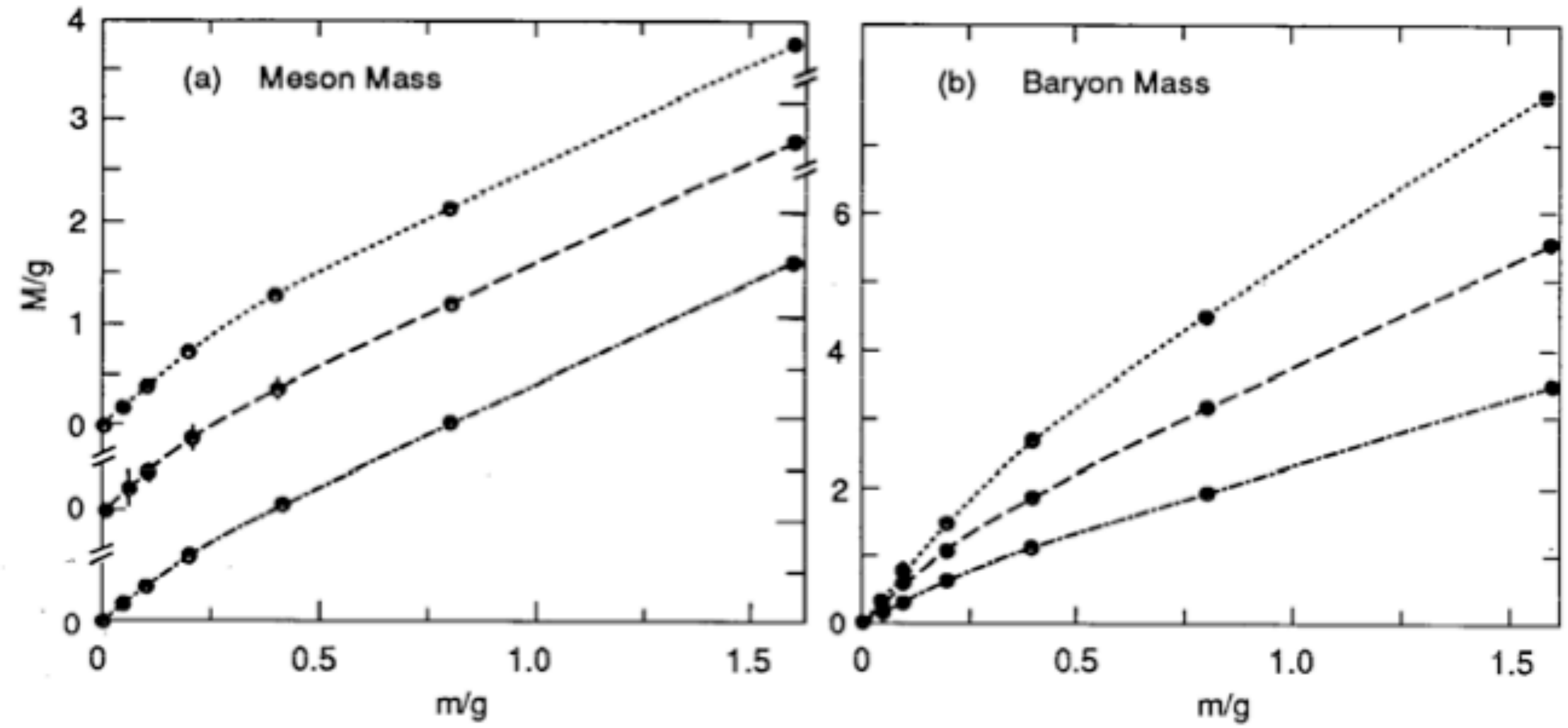
$A^+ = 0$



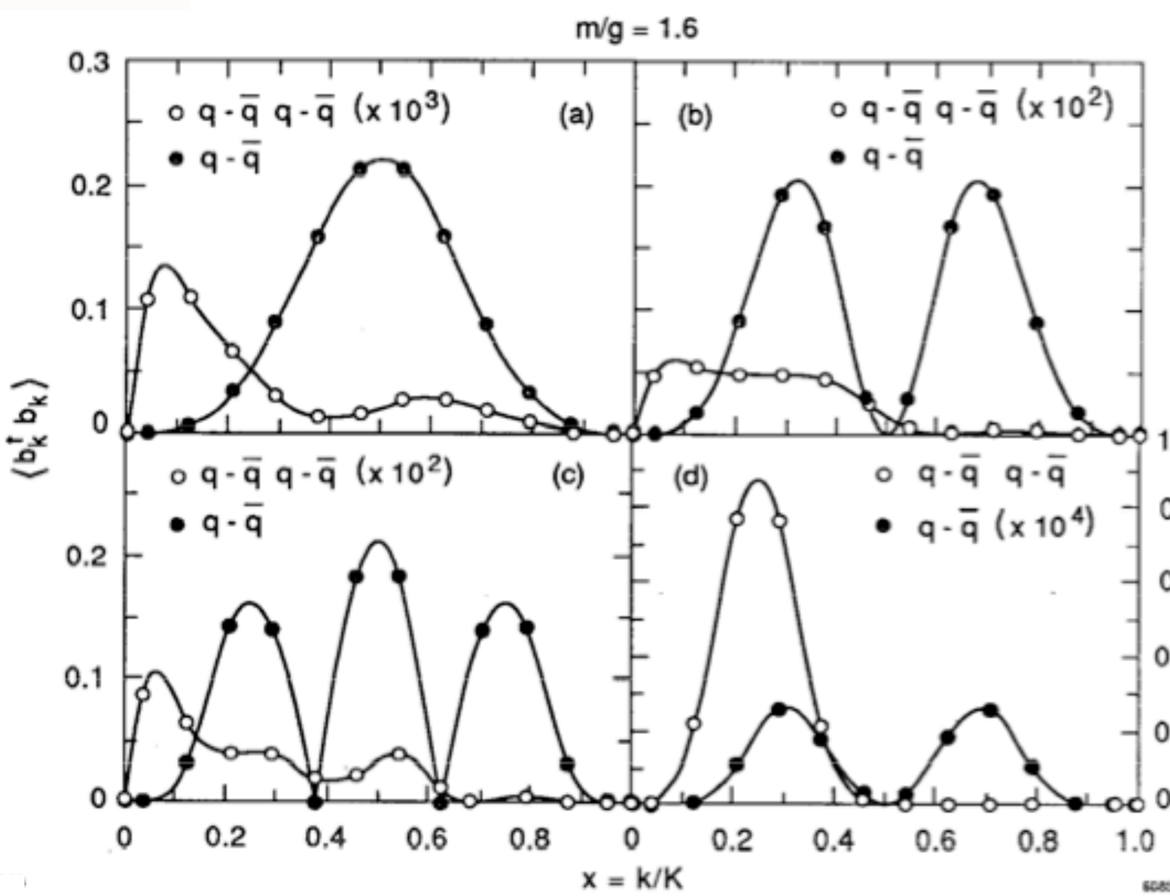
Minkowski space; frame-independent; no fermion doubling; no ghosts

- Light-Front Vacuum = vacuum of free Hamiltonian!

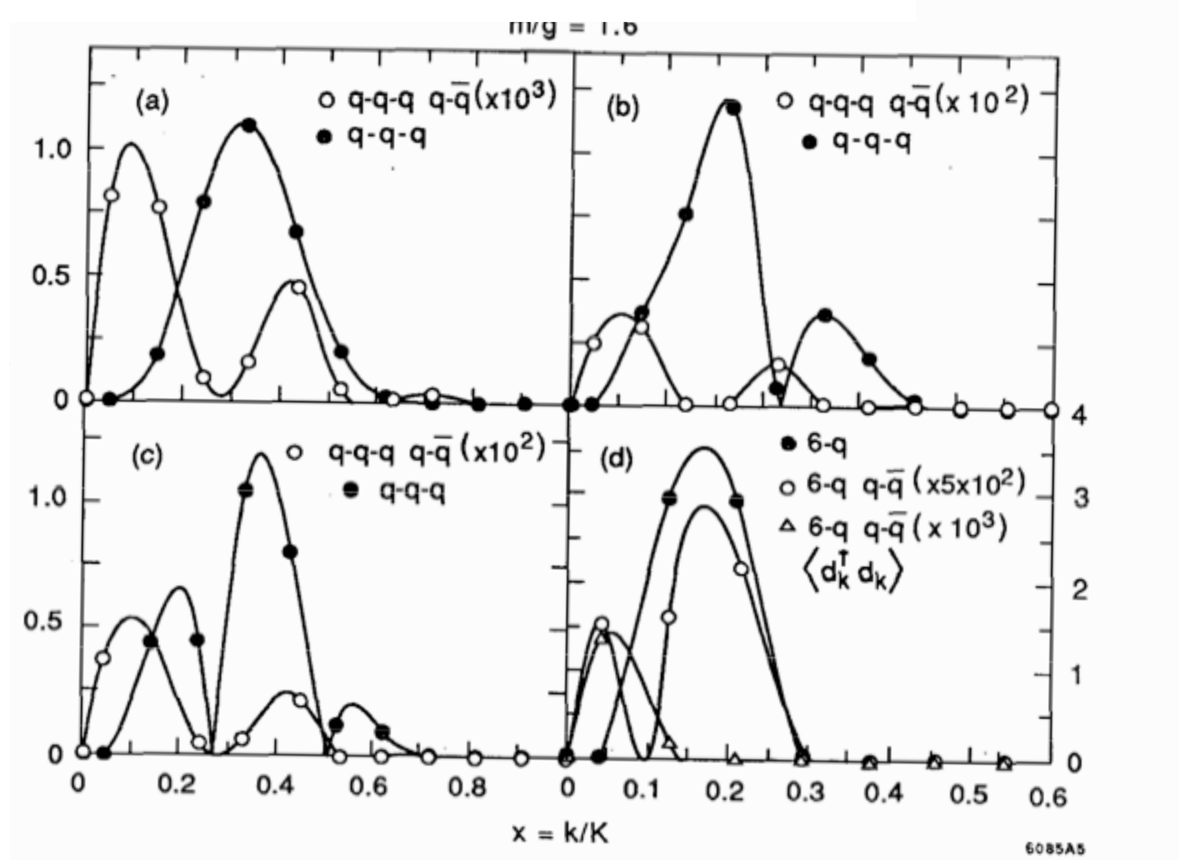
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.

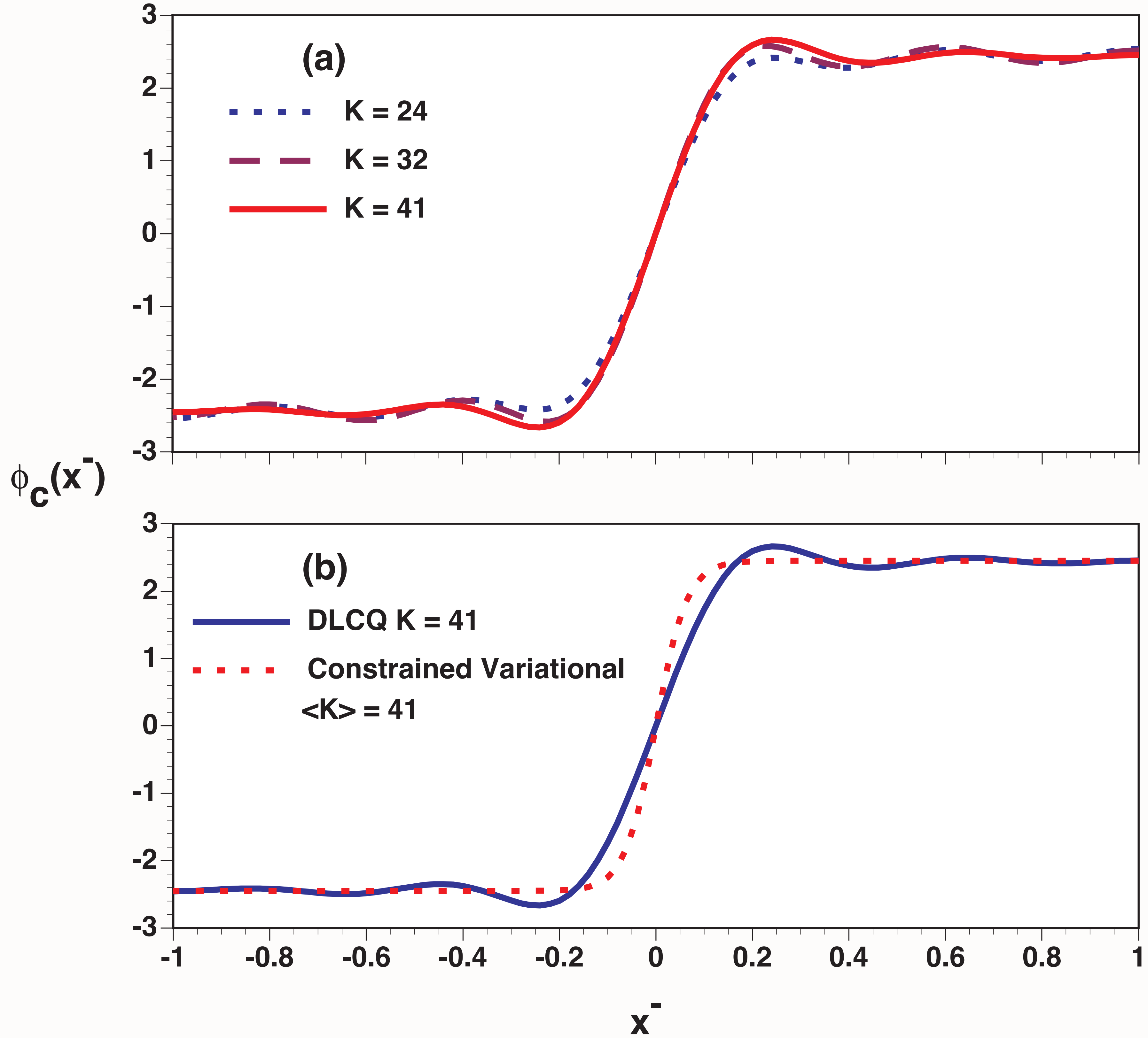


a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

Kinks in Discrete Light-Cone Quantization

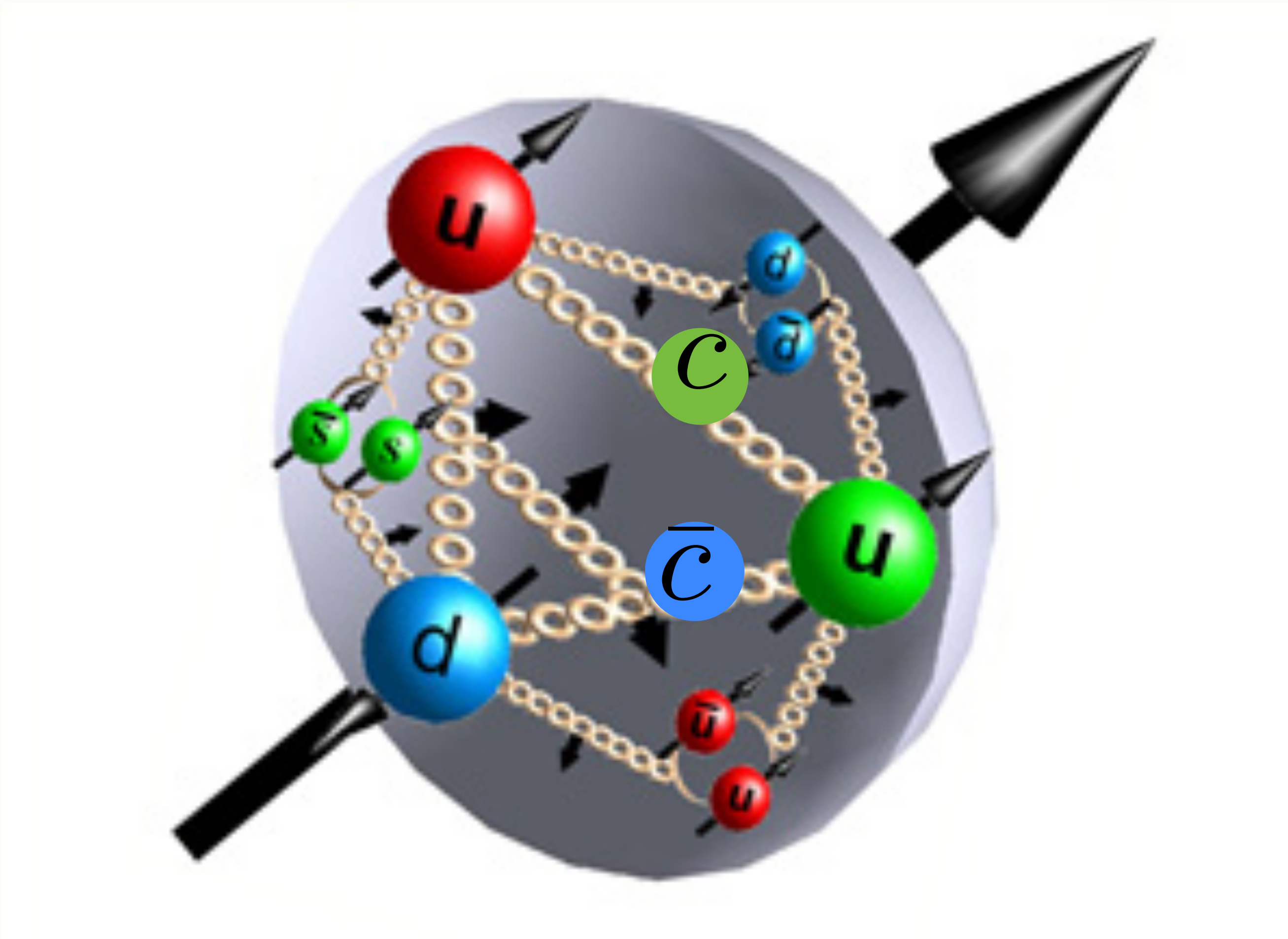
Chakrabarti, Harindranath, Martinovic, Vary



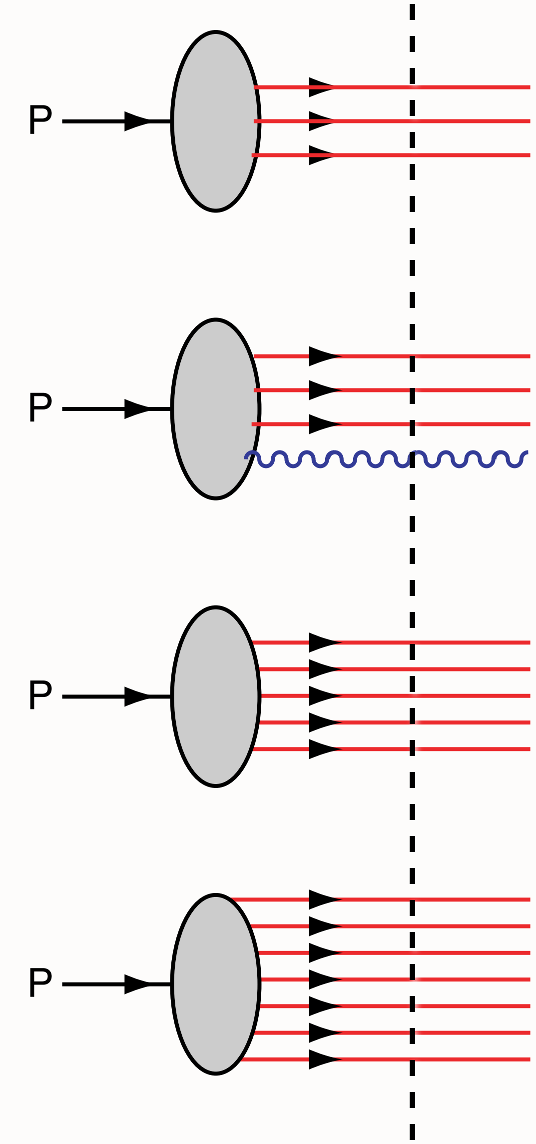
$$
\begin{aligned}
H &= \frac{1}{2}\int d^3x \overline{\widetilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\widetilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2\int d^3x \mathbf{Tr}\left[\widetilde{A}^\mu, \widetilde{A}^\nu\right]\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2\int d^3x \overline{\widetilde{\psi}}\gamma^+ T^a \widetilde{\psi}\frac{1}{(\mathrm{i}\partial^+)^2}\overline{\widetilde{\psi}}\gamma^+ T^a\widetilde{\psi} \\
&- g^2\int d^3x \overline{\widetilde{\psi}}\gamma^+\left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right)\widetilde{\psi} \\
&+ g^2\int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+\widetilde{A}^\kappa, \widetilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2\int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\widetilde{\not{A}}\widetilde{\psi} \\
&+ g\int d^3x \overline{\widetilde{\psi}}\widetilde{\not{A}}\widetilde{\psi} \\
&+ 2g\int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu\widetilde{A}^\nu\left[\widetilde{A}_\mu, \widetilde{A}_\nu\right]\right)
\end{aligned}
$$

Wavefunction at fixed LF time: Arbitrarily Off-Shell in Invariant Mass

Eigenstate: all Fock states contribute



Higher Fock States of the Proton



Fixed LF time

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

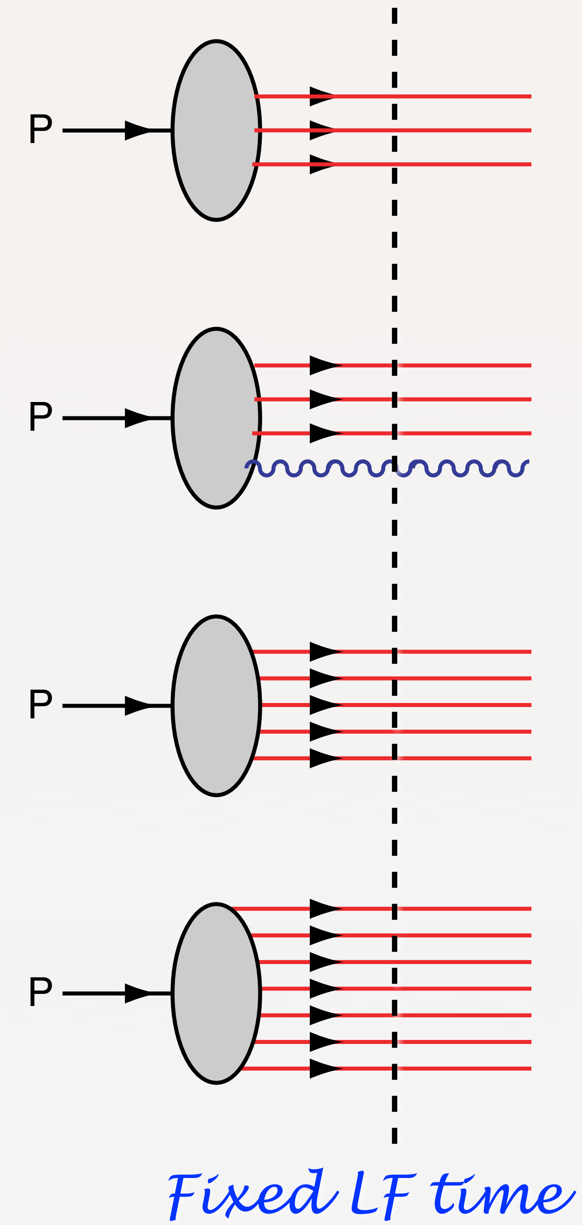
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



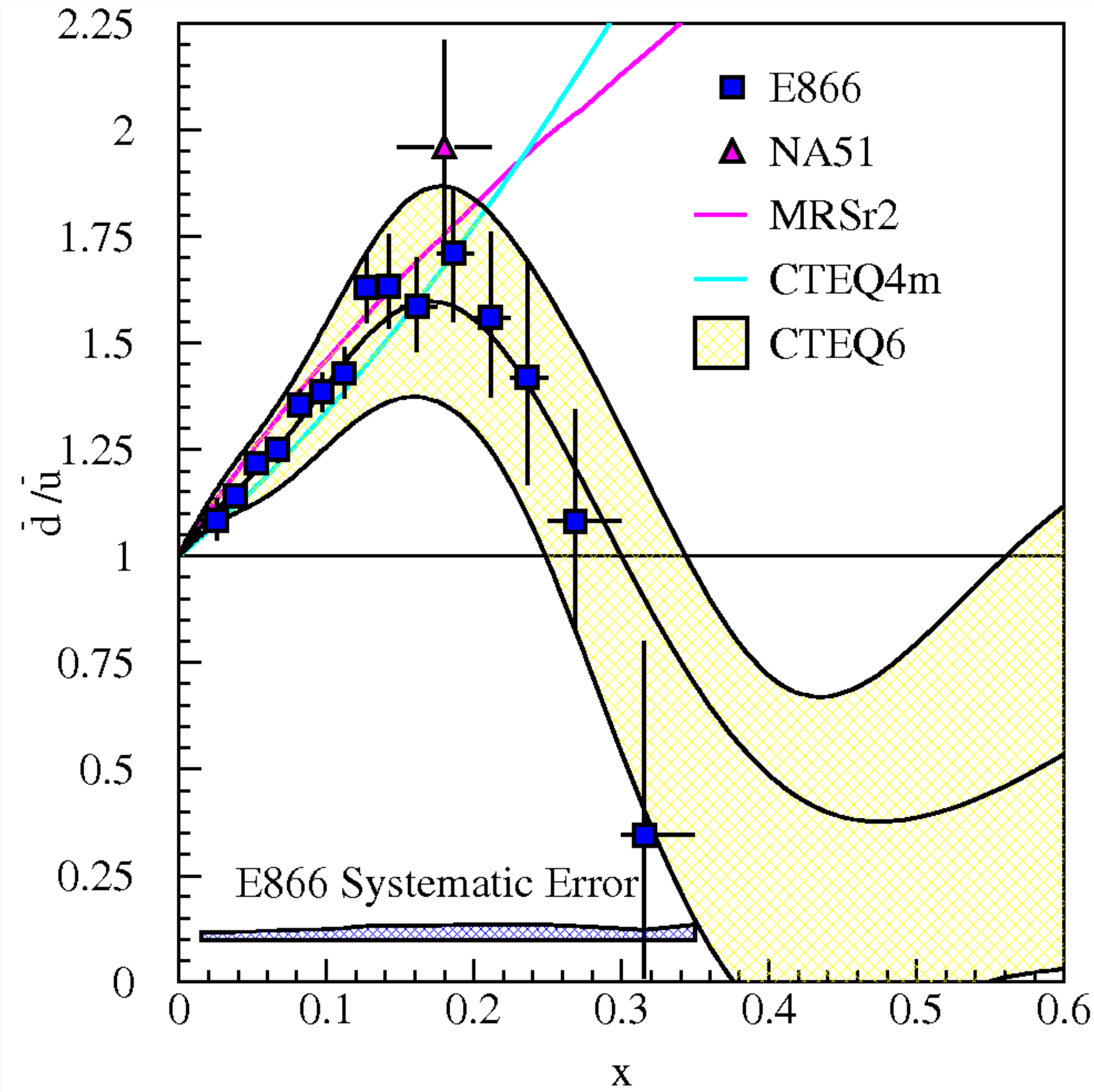
Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states BFKL Pomeron

Hidden Color

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$



- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

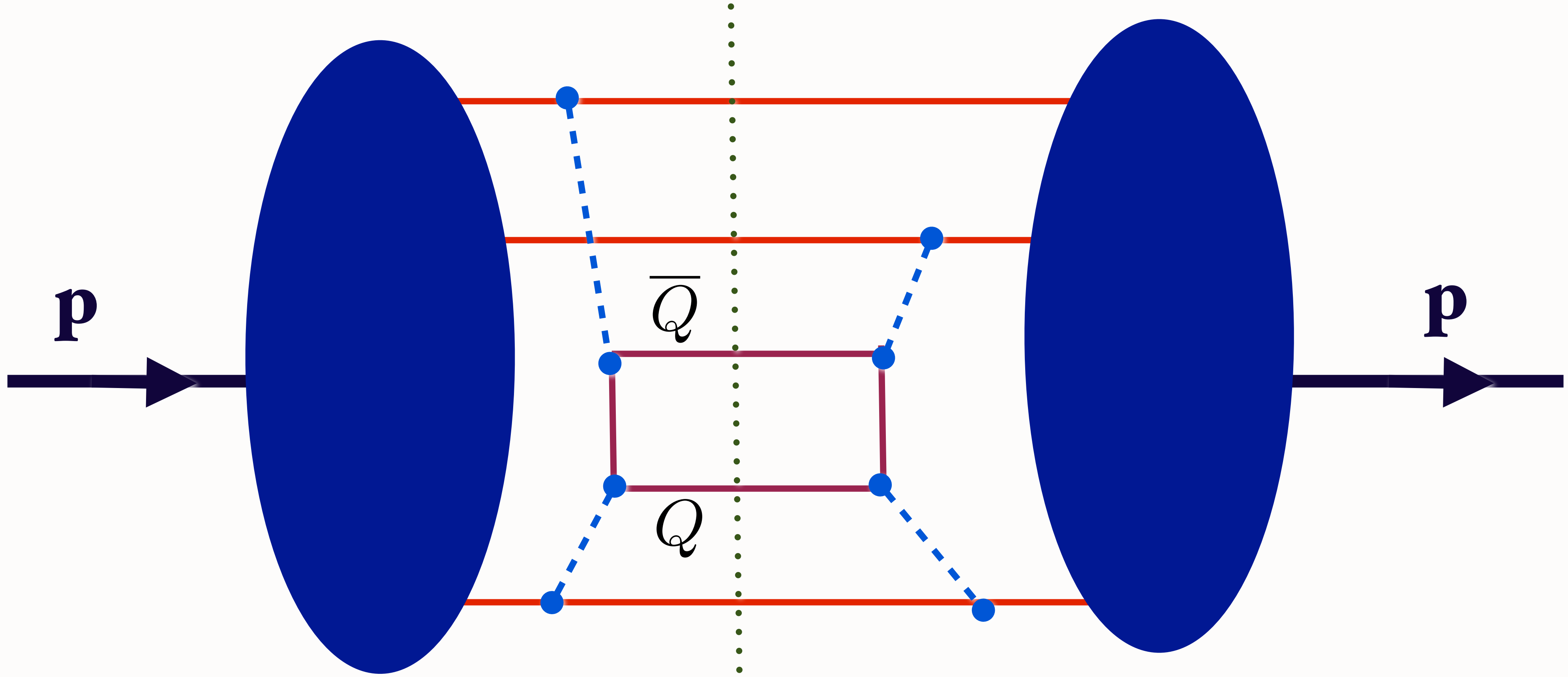
$$s(x) \neq \bar{s}(x)$$

Intrinsic glue, sea, heavy quarks

Fixed LF time

Proton Self Energy
Intrinsic Heavy Quarks

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$



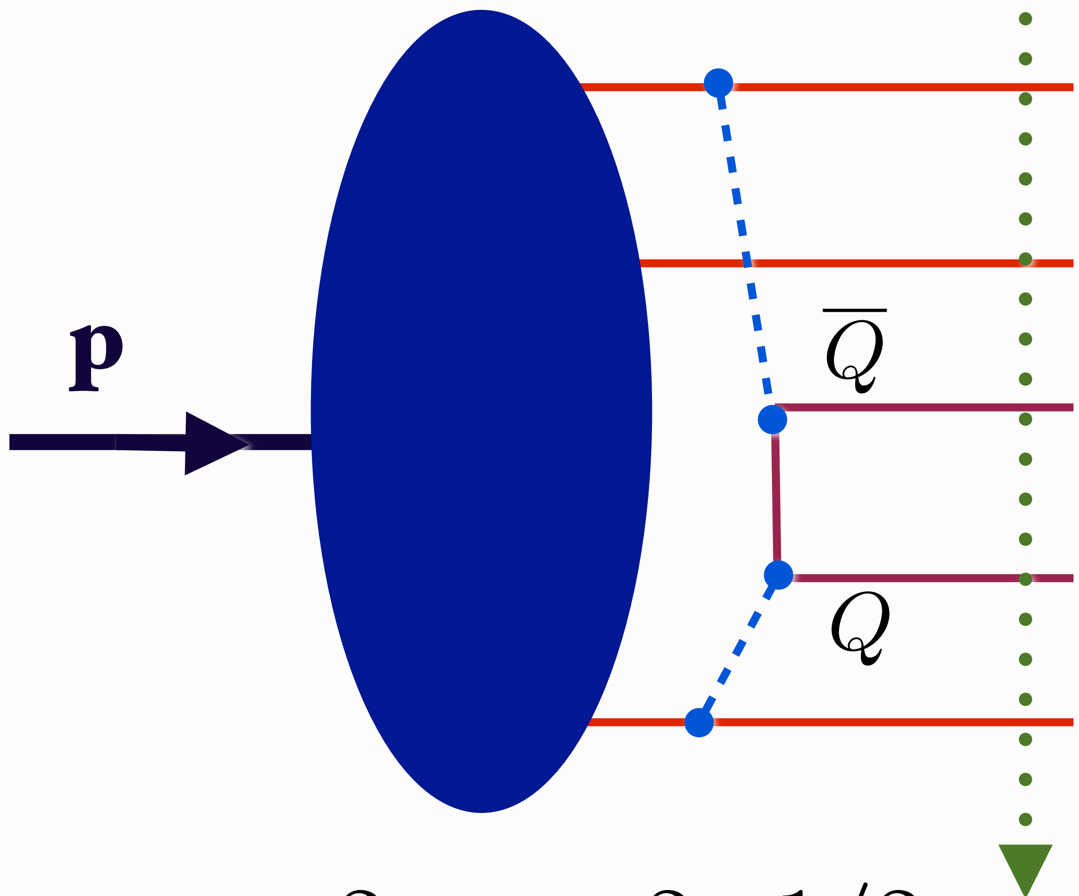
Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

Fixed LF time

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x!

Minimal off-shellness

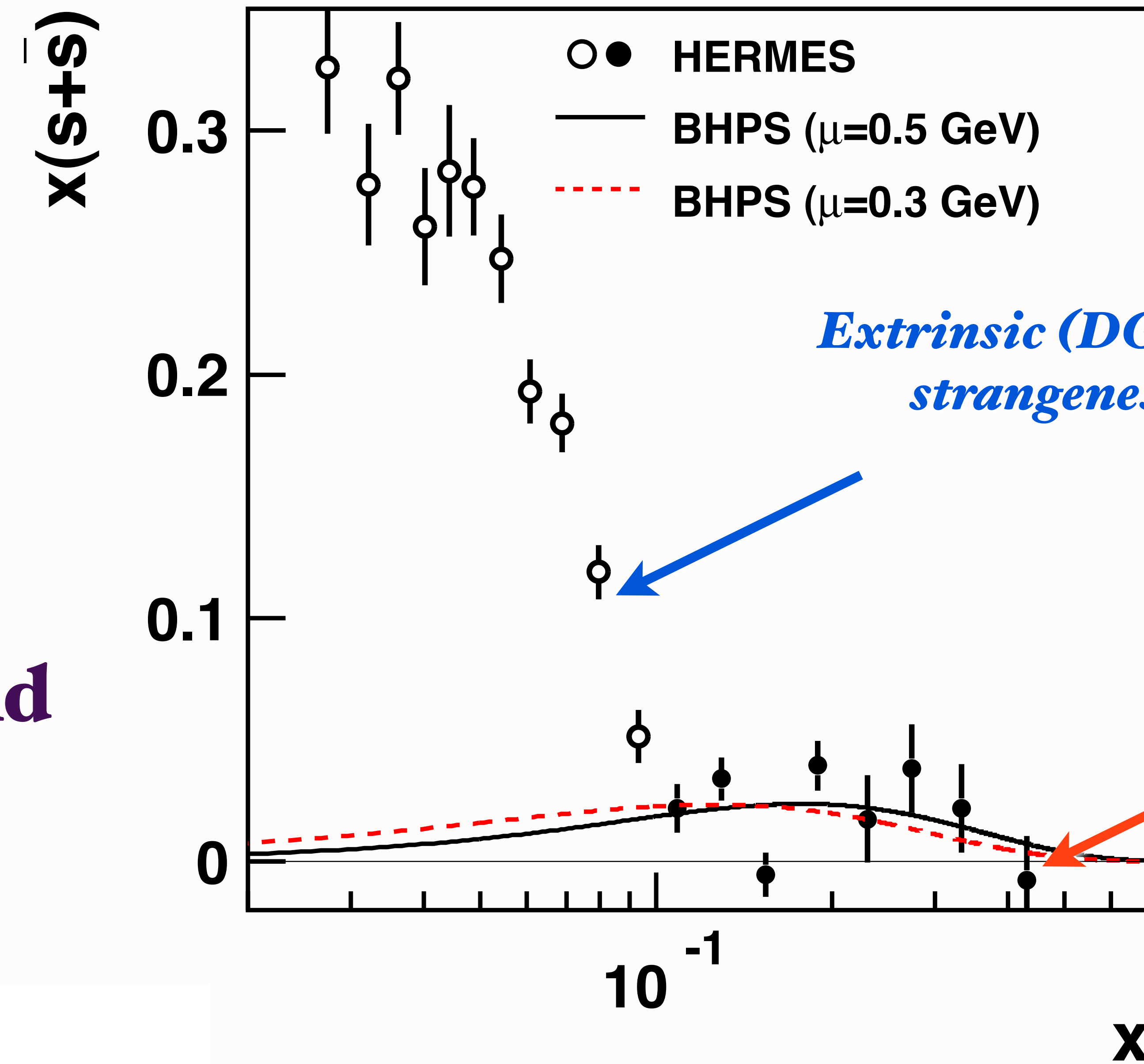
$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

HERMES: Two components to s(x,Q²)!

BHPS: Hoyer, Sakai, Peterson, sjb



Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

W. C. Chang and J.-C. Peng

arXiv:1105.2381

Intrinsic strangeness!

Consistent with intrinsic charm data

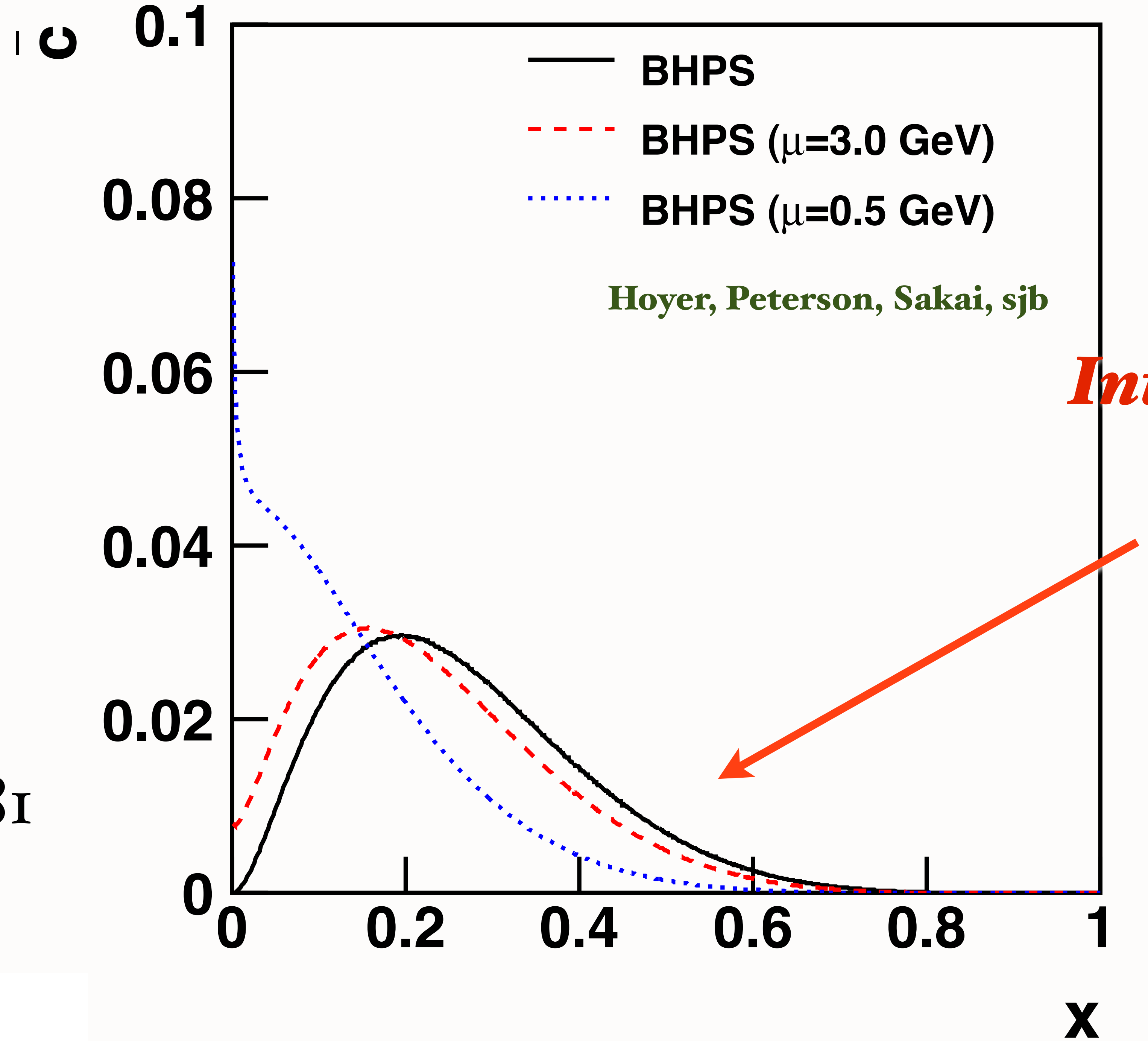
QCD: $\frac{1}{M_Q^2}$ scaling

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

QCD ($1/m_Q^2$) scaling: predict IC !

W. C. Chang and J.-C. Peng

arXiv:1105.2381



Intrinsic Charm

Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV^2 using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

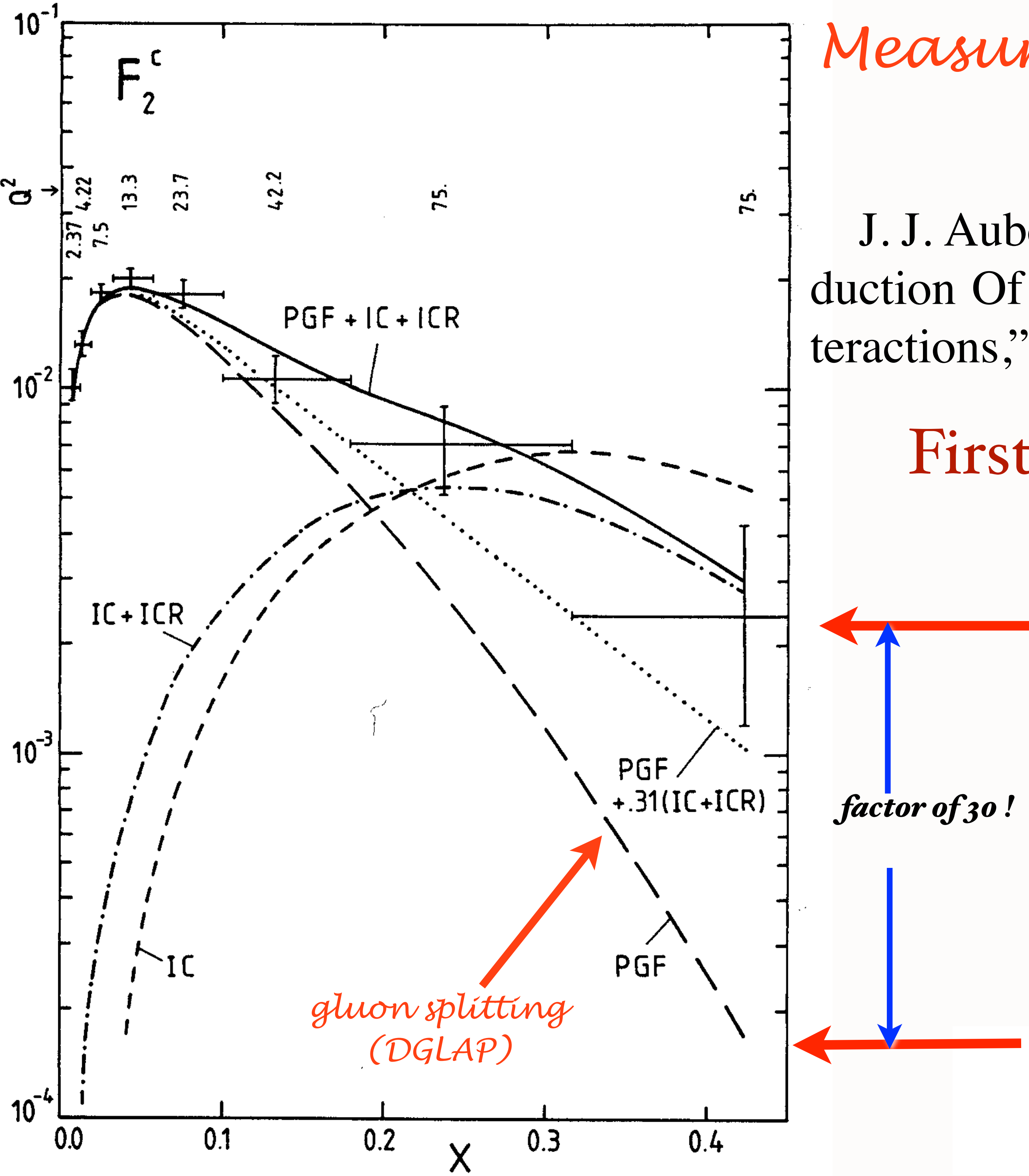
Consistent with EMC

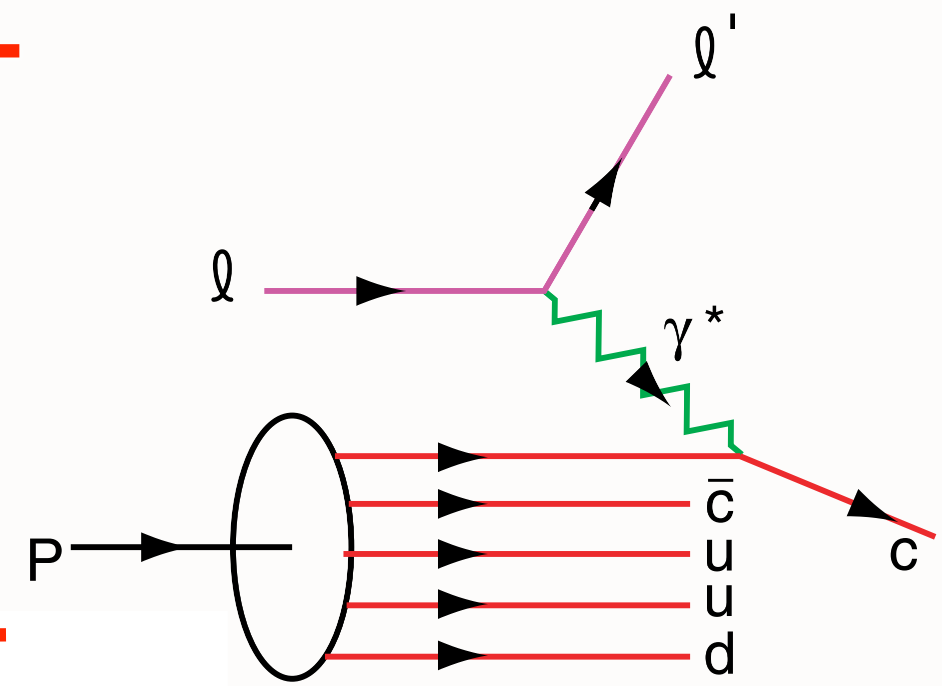
Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb





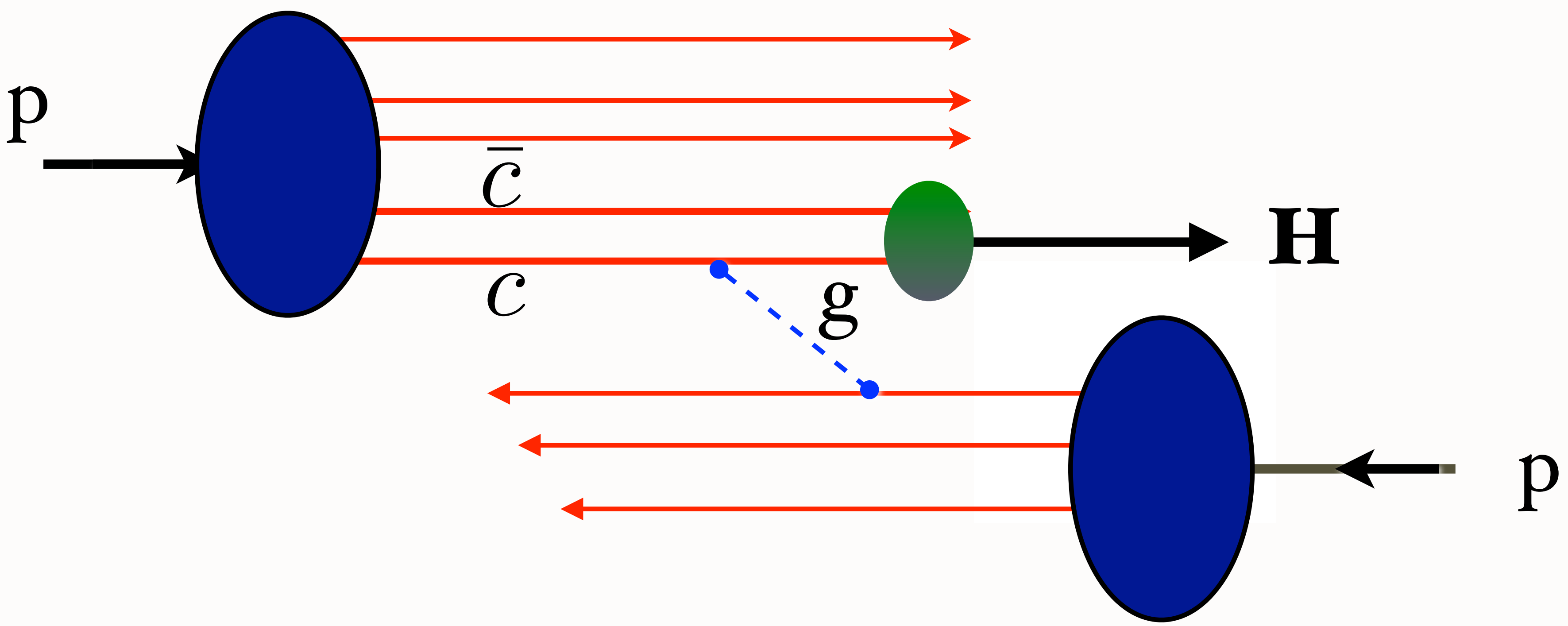
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x,Q^2) = c(x,Q^2)_{\text{extrinsic}} + c(x,Q^2)_{\text{intrinsic}}$$

Goldhaber, Kopeliovich, Schmidt, sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



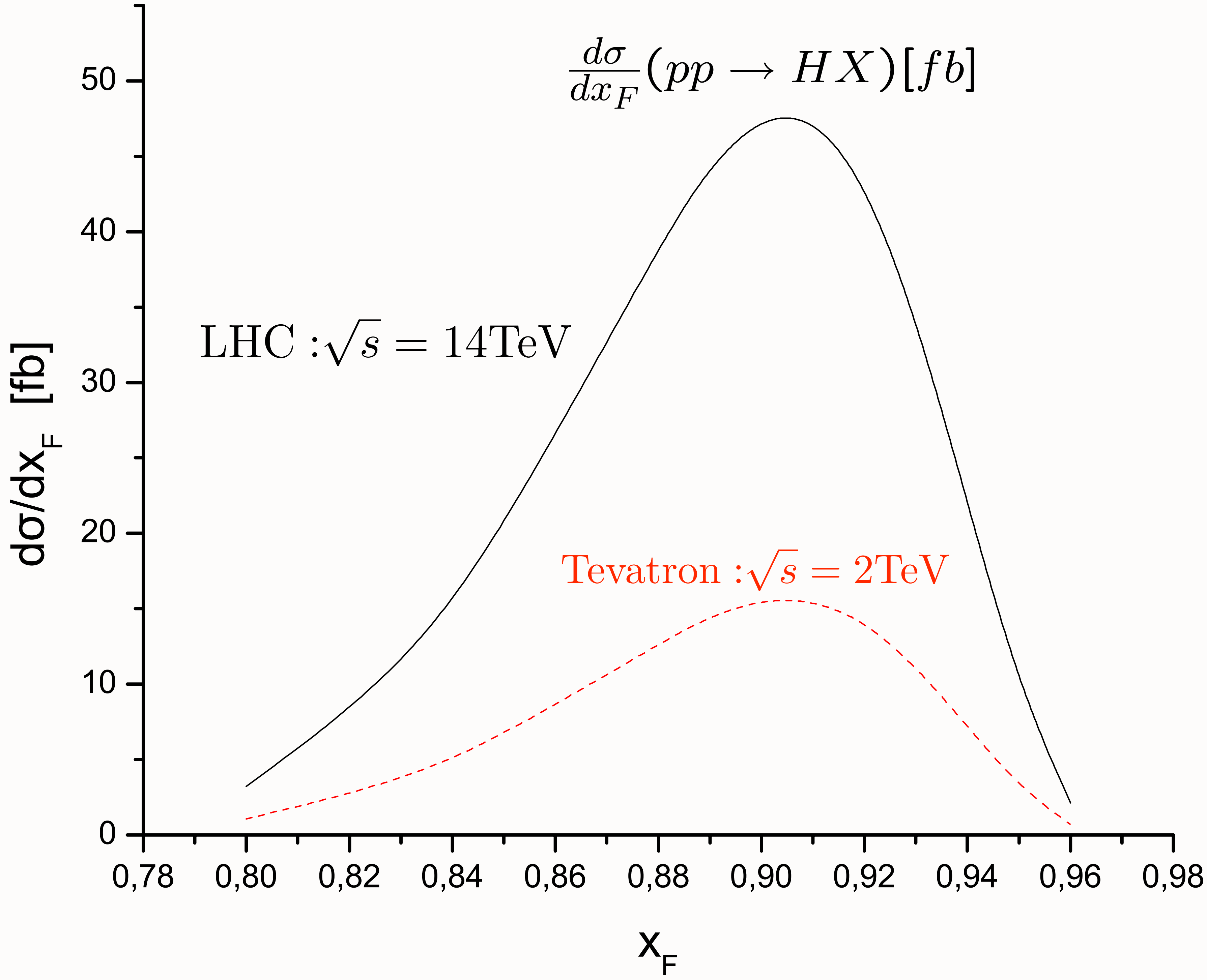
$$pp \to H X$$

Also: intrinsic bottom, top

Higgs can have 80% of Proton Momentum!

New production mechanism for Higgs

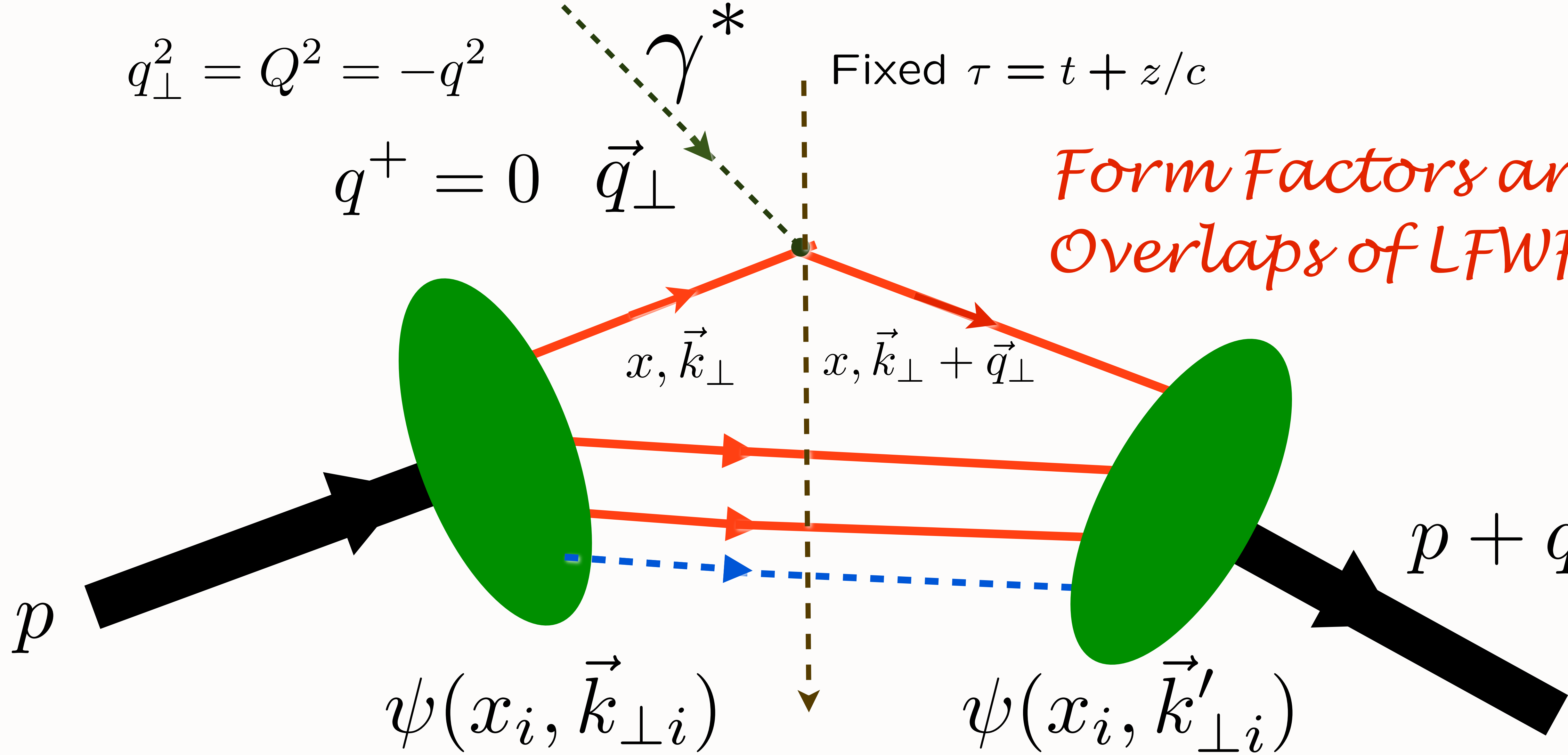
Intrinsic Bottom Contribution to Inclusive Higgs Production



Goldhaber, Kopeliovich, Schmidt, sjb

$$< p+q|j^+(0)|p >= 2p^+ F(q^2)$$

Interaction picture

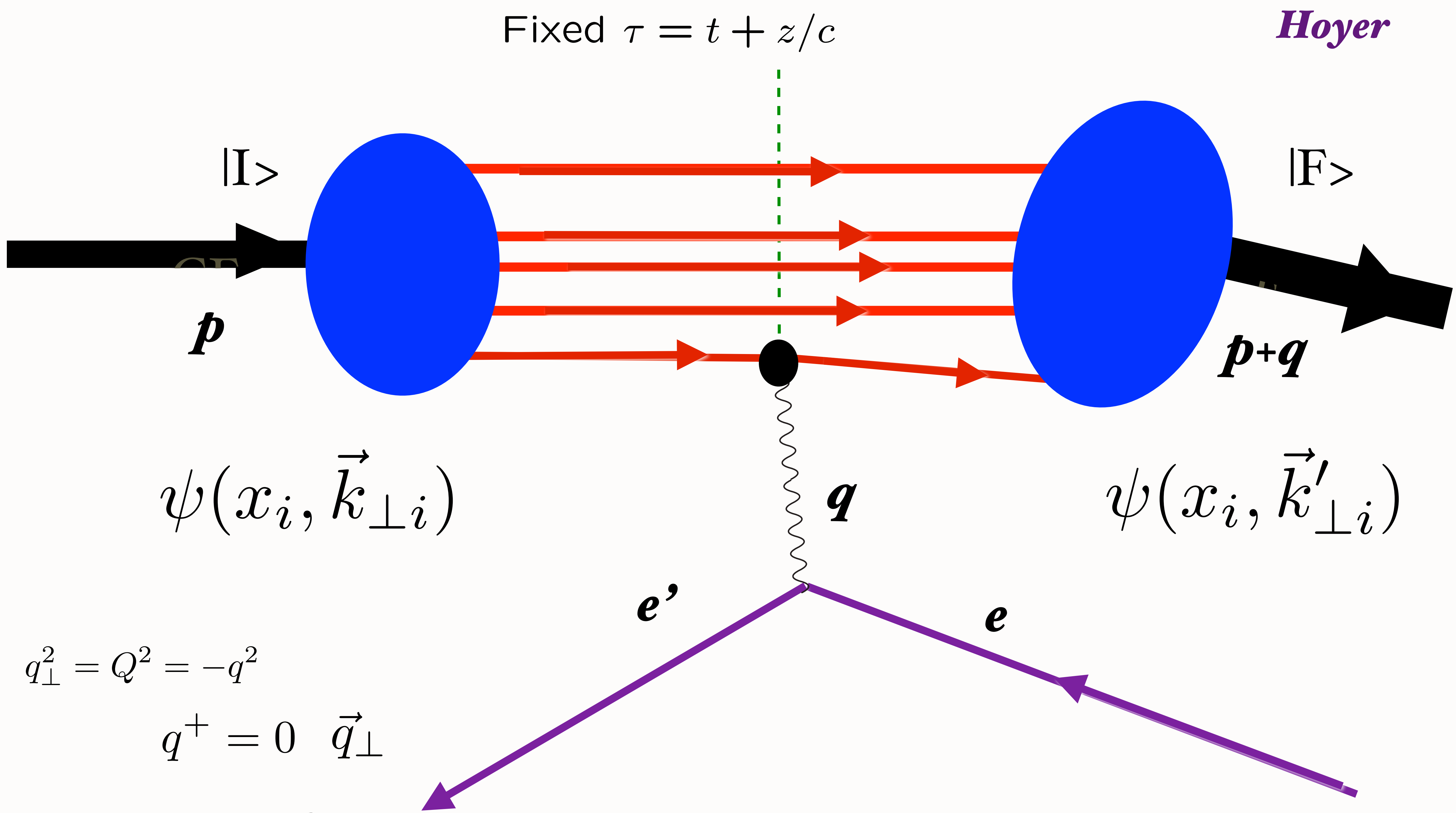


struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Light-Front Wavefunctions and Electron-Proton Collisions

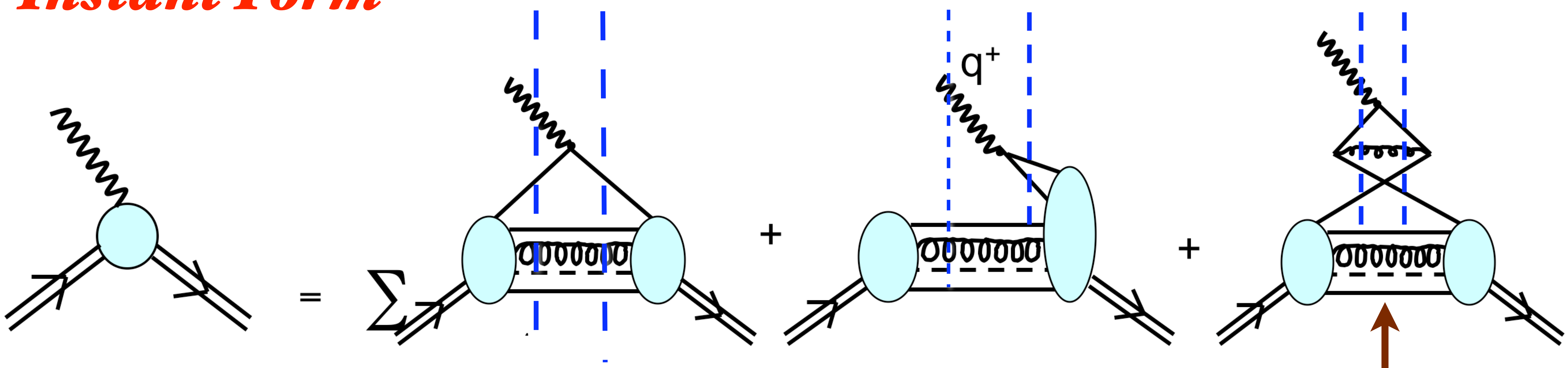


All final states |F> in electroproduction produced
Diagonal n to n overlap of LFWFs

|F> = resonances, multiparticle states.
Confinement: Only Color Singlets!

Calculation of Form Factors in Equal-Time Theory

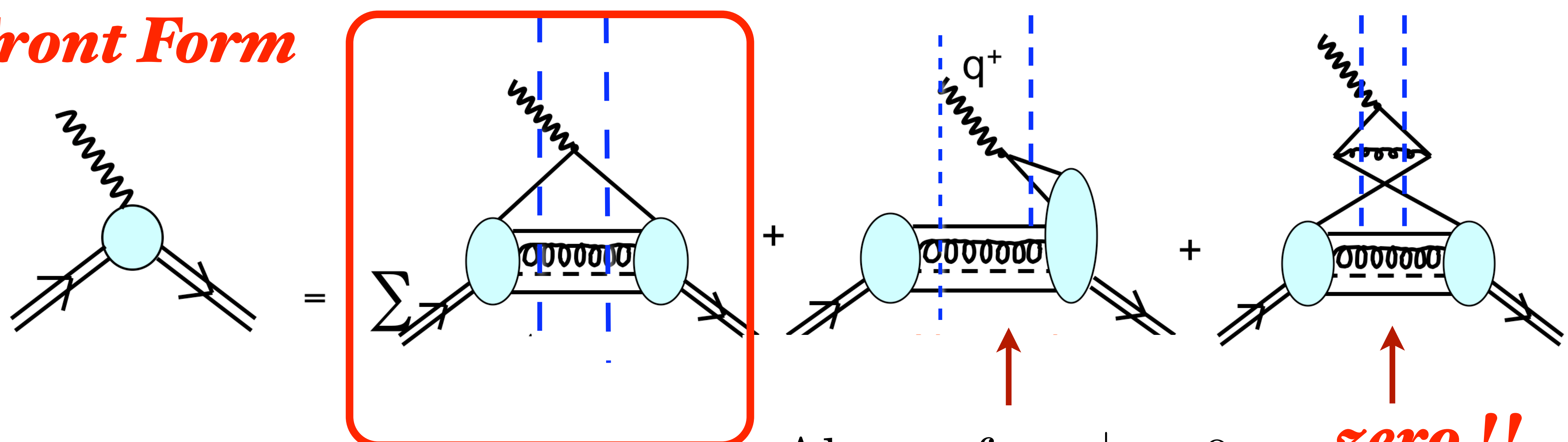
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Exact Answer!

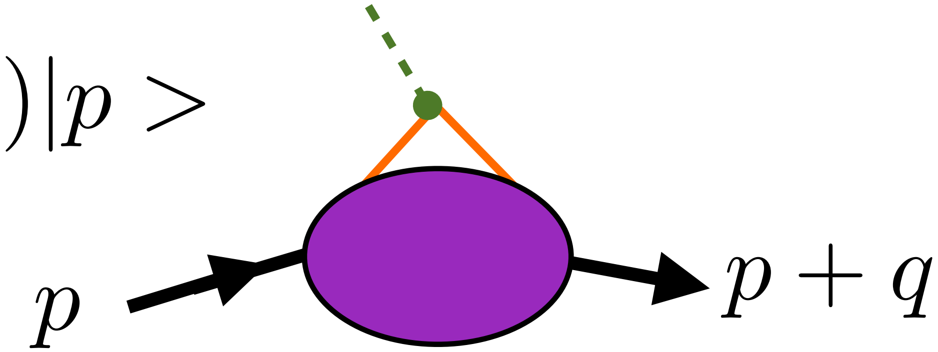
Absent for $q^+ = 0$

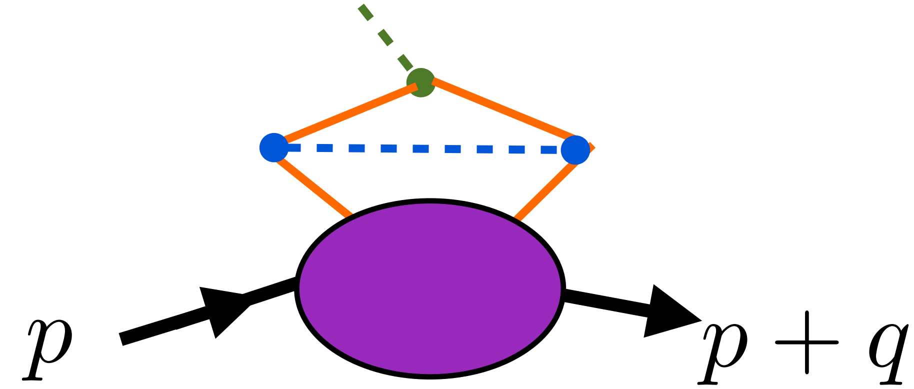
zero !!

No vacuum graphs

Calculation of proton form factor in Instant Form

$< p + q | J^\mu(0) | p >$





- **Need to boost proton wavefunction from p to p +q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**
- **Instant form: Violates causality**

Exact LF Formula for Pauli Form Factor

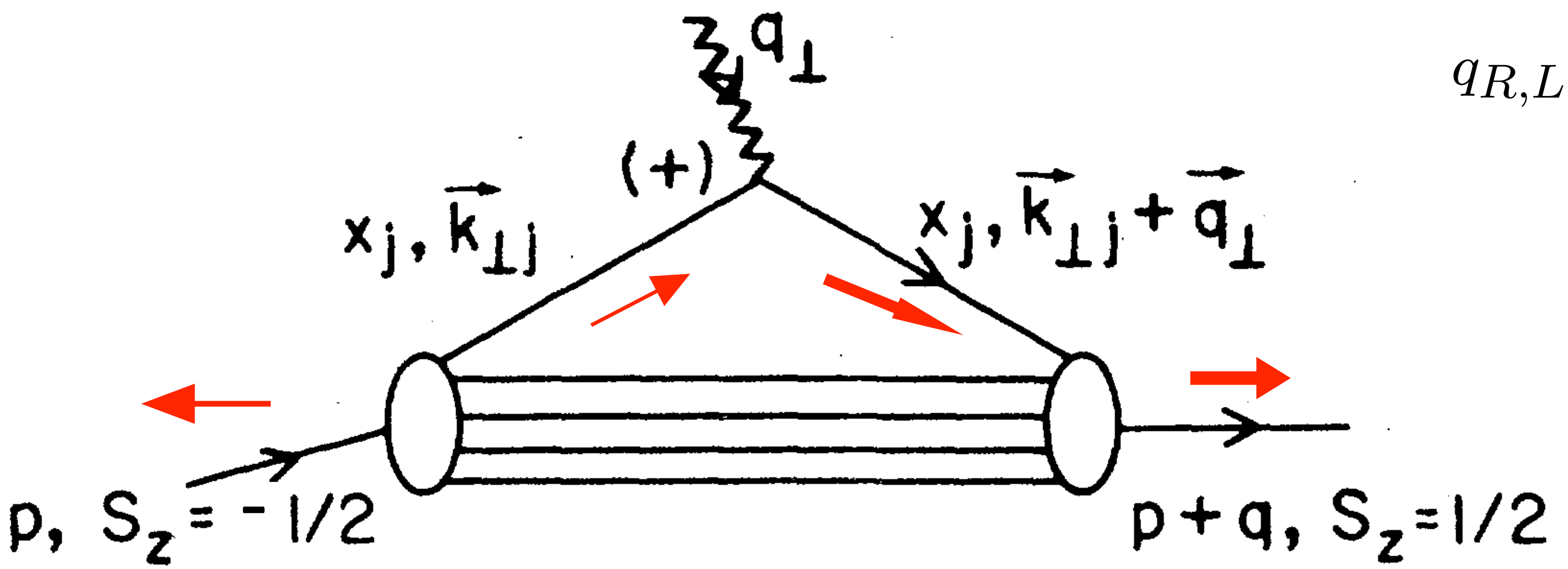
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

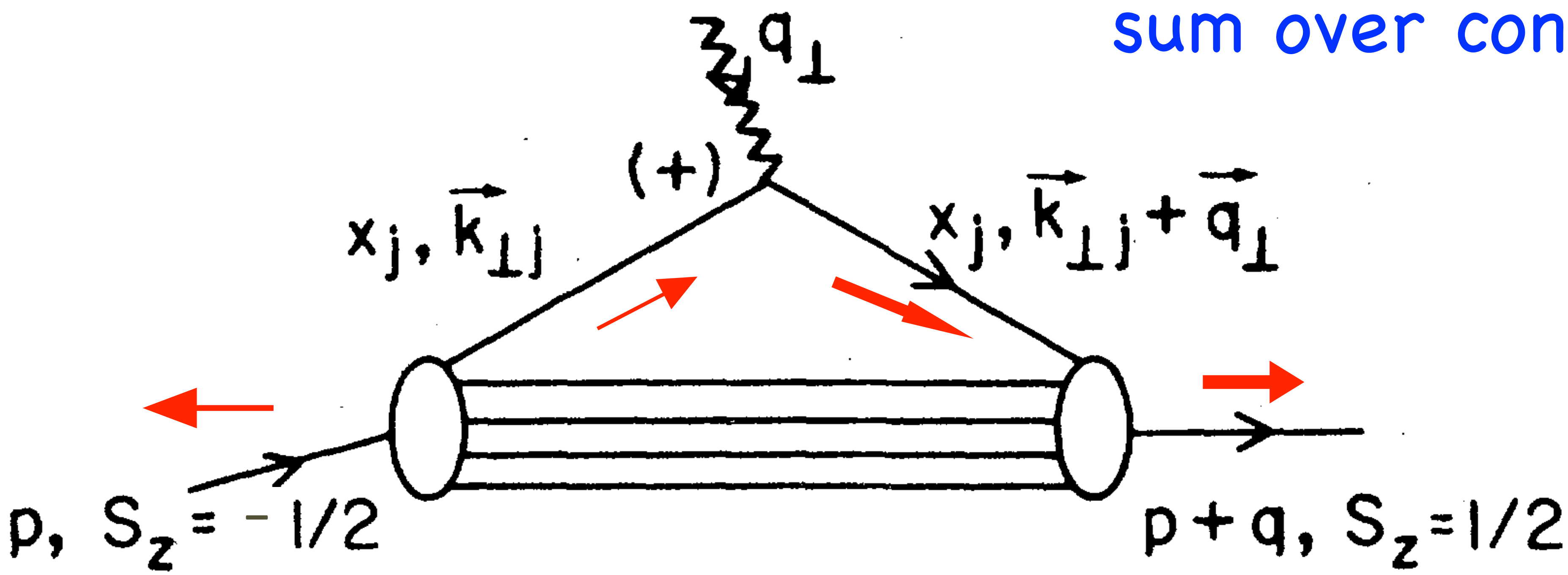


Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*



sum over constituents

Hwang, Schmidt, sjb; Holstein et al

$B(0) = 0$

Each Fock State

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock-State by Fock-State
Every Vertex

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Parke-Taylor Amplitudes **Stasto**

Nonzero Anomalous Moment <--> Nonzero orbital angular momentum
Drell, sjb

Advantages of the Dirac's Front Form for Hadron Physics

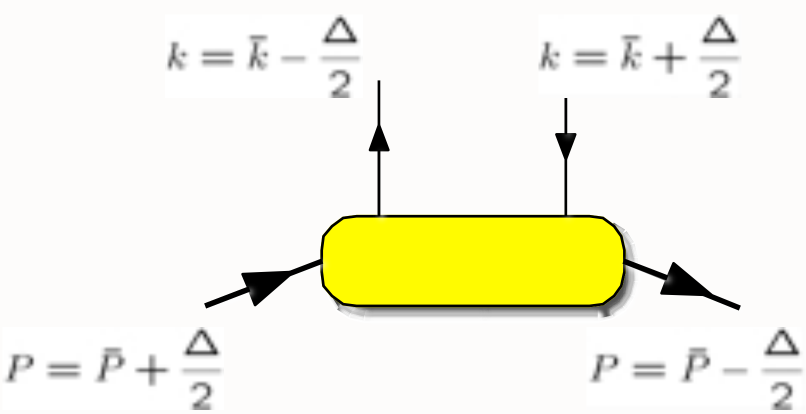
- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts
- No dependence on observer's frame
- Dual to AdS/QCD
- LF Vacuum trivial -- no condensates
- Implications for Cosmological Constant

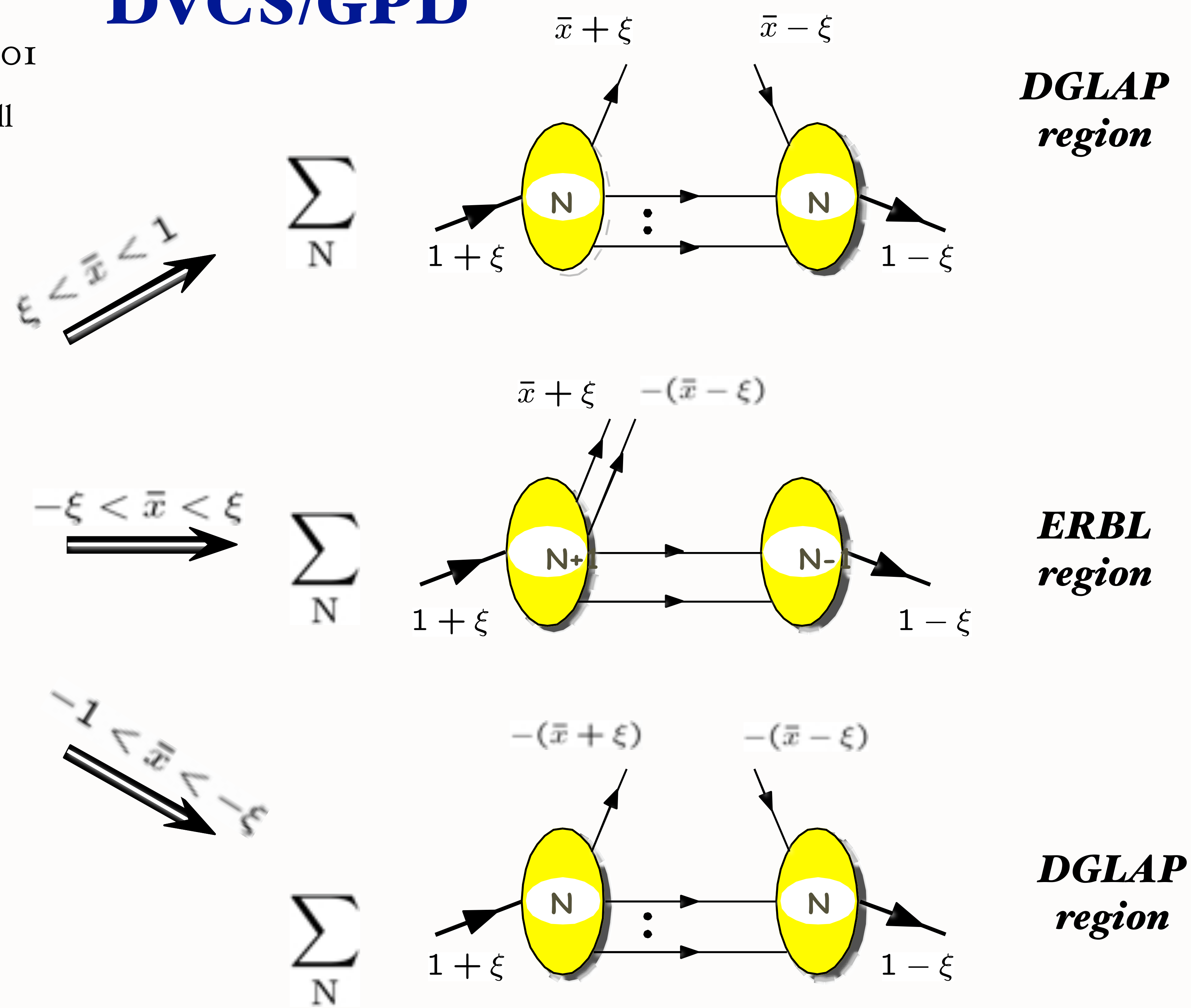
Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

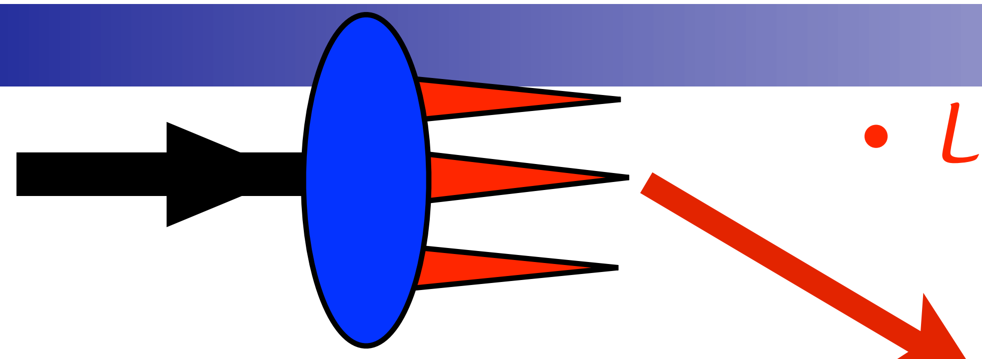
See also: Diehl, Feldmann, Jakob, Kroll





Bakker & Ji
Lorce

- *Light Front Wavefunctions:*



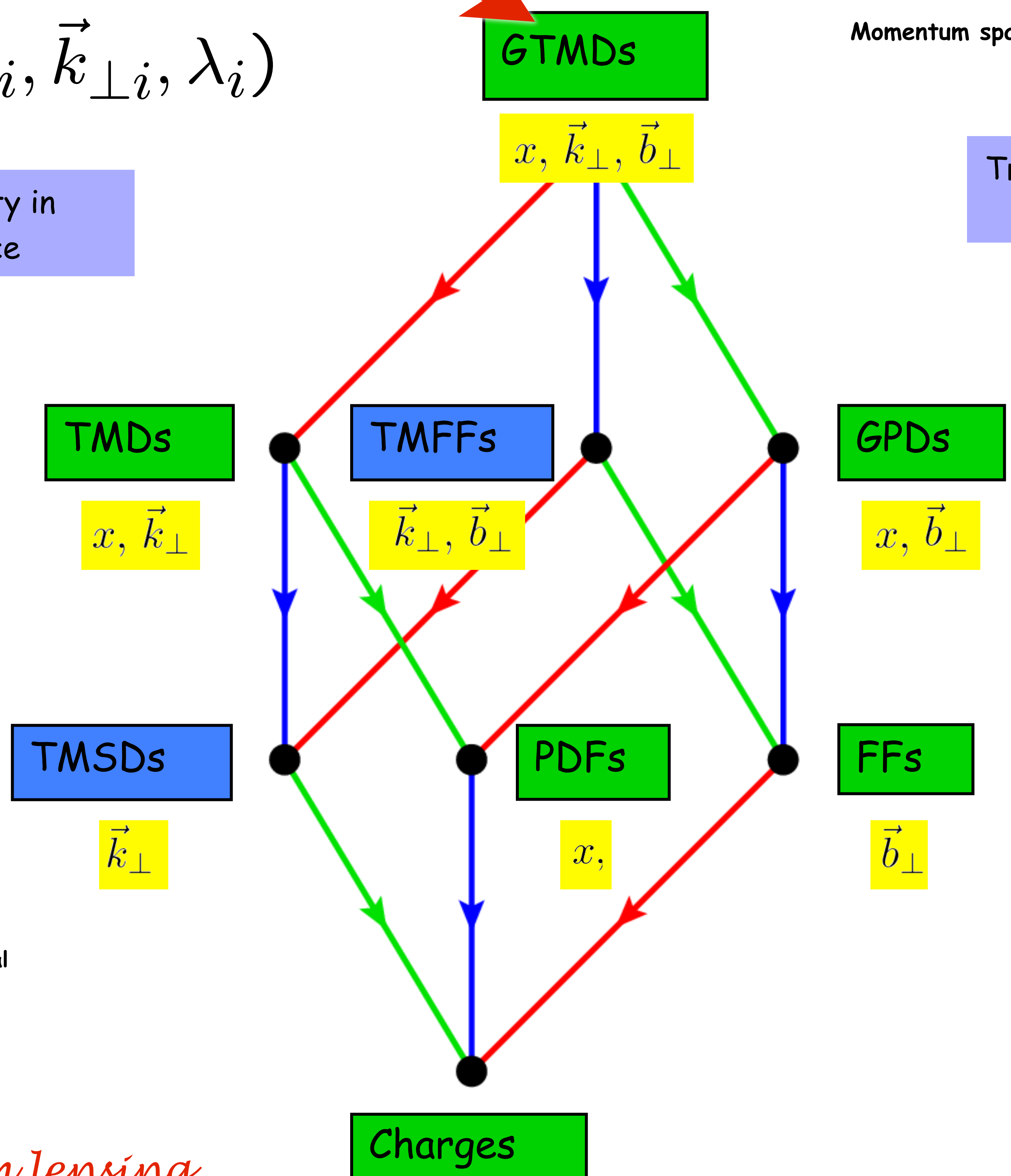
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

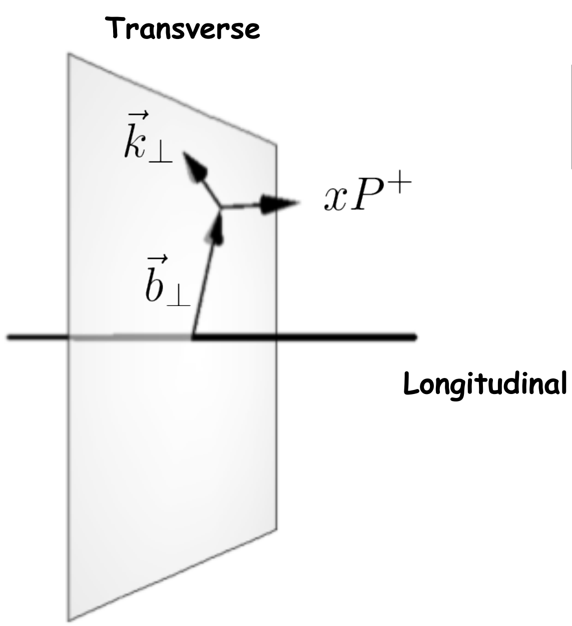
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

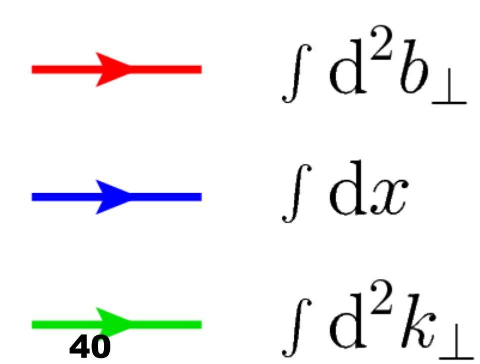
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



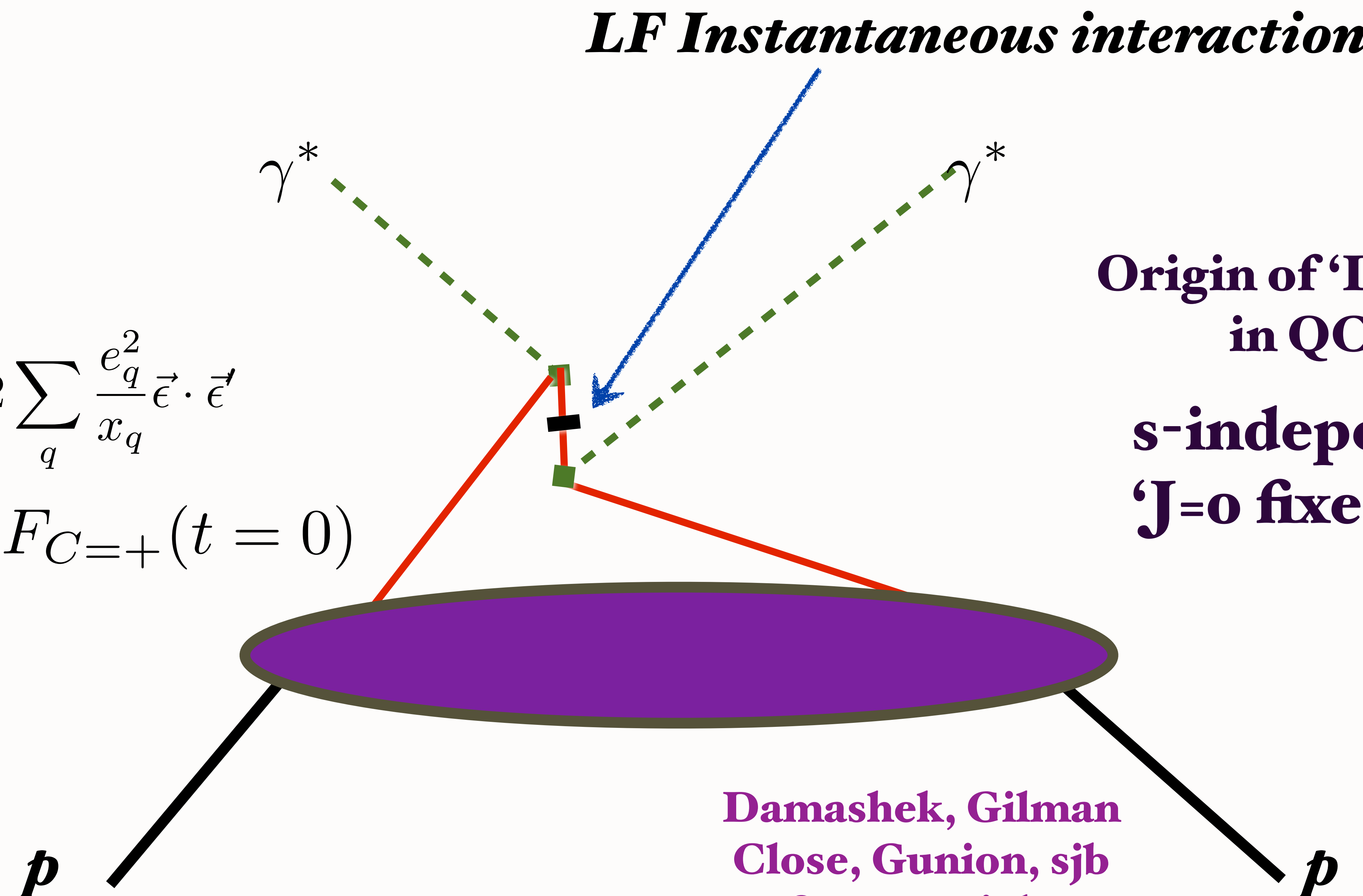
Lorce, Pasquini





Sivers, T-odd from lensing

Leading-Twist Contribution to Real Part of DVCS



$$T = -2 \sum_q \frac{e_q^2}{x_q} \vec{\epsilon} \cdot \vec{\epsilon}'$$

$$T \propto s^0 F_{C=+}(t=0)$$

Origin of 'D-Term' in QCD

s-independent 'J=0 fixed pole'

Damashek, Gilman
Close, Gunion, sjb
Szczepaniak, Llanes Estrada, sjb

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

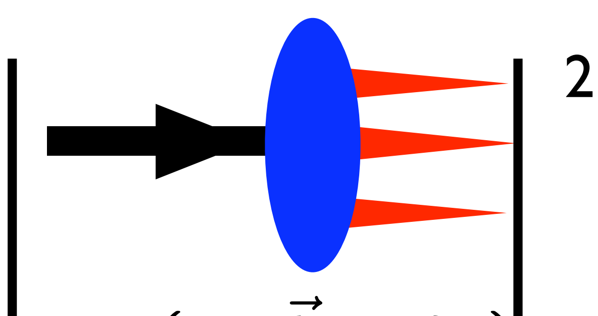
Hwang, Schmidt, sjb,

Mulders, Boer

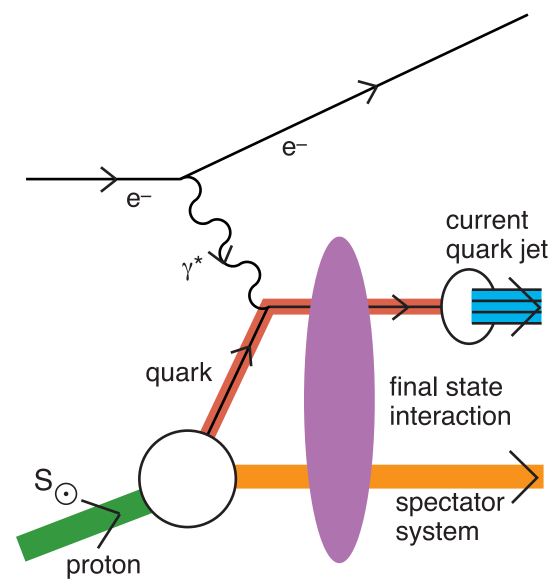
Qiu, Sterman

Collins, Qiu

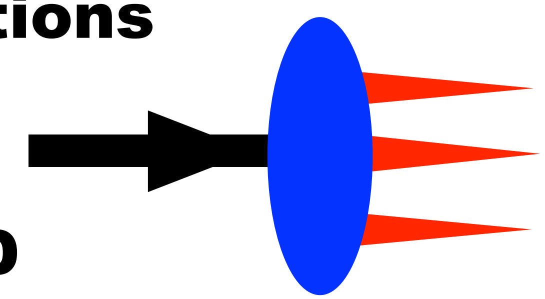
Pasquini, Xiao, Yuan, sjb



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



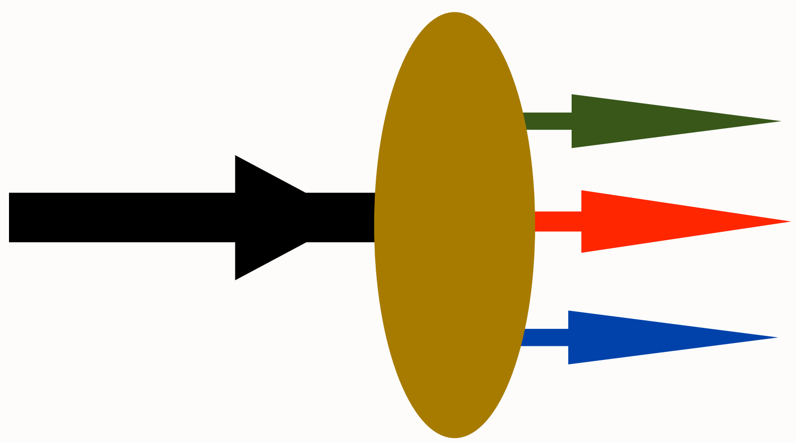
- LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian
- Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors
- Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs
- Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!
- Hadron Physics without LFWFs is like Biology without DNA!

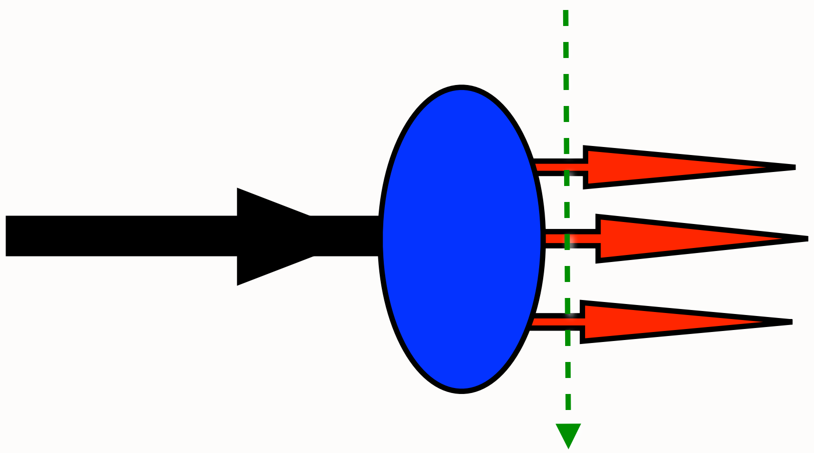
- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

$$H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x,\vec{k}_\perp) = M^2\ \psi_{LF}(x,\vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\,\zeta^2} + U(\zeta,S,L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis ζ, ϕ

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Light-Front Schrödinger Equation

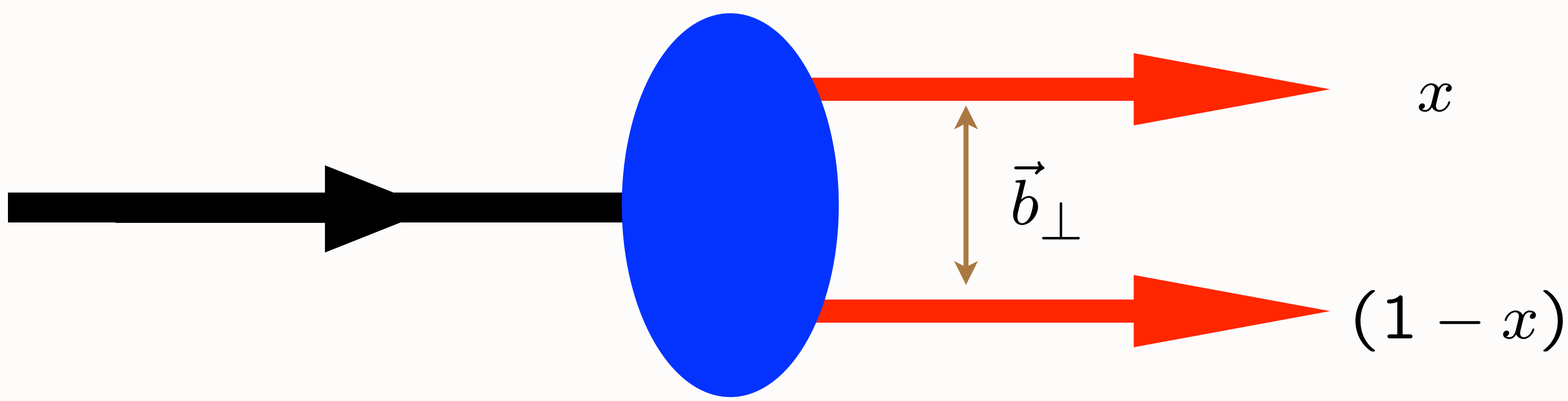
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



AdS/QCD:

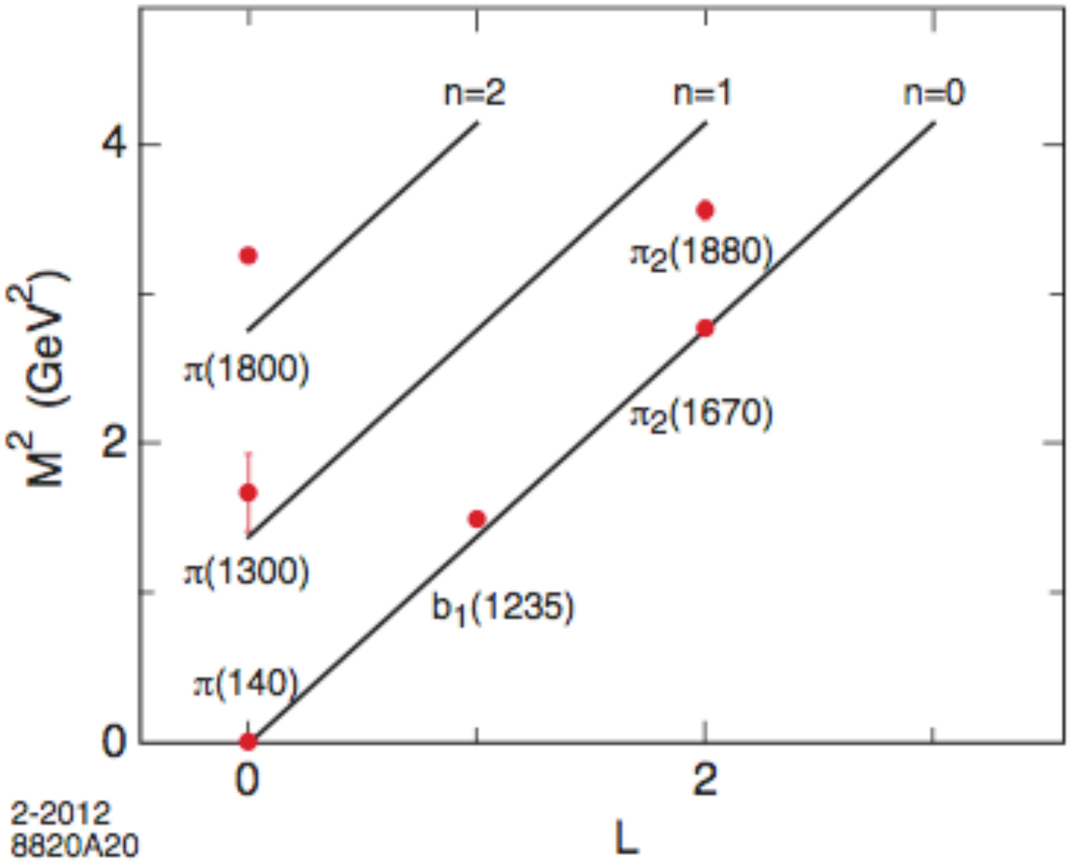
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

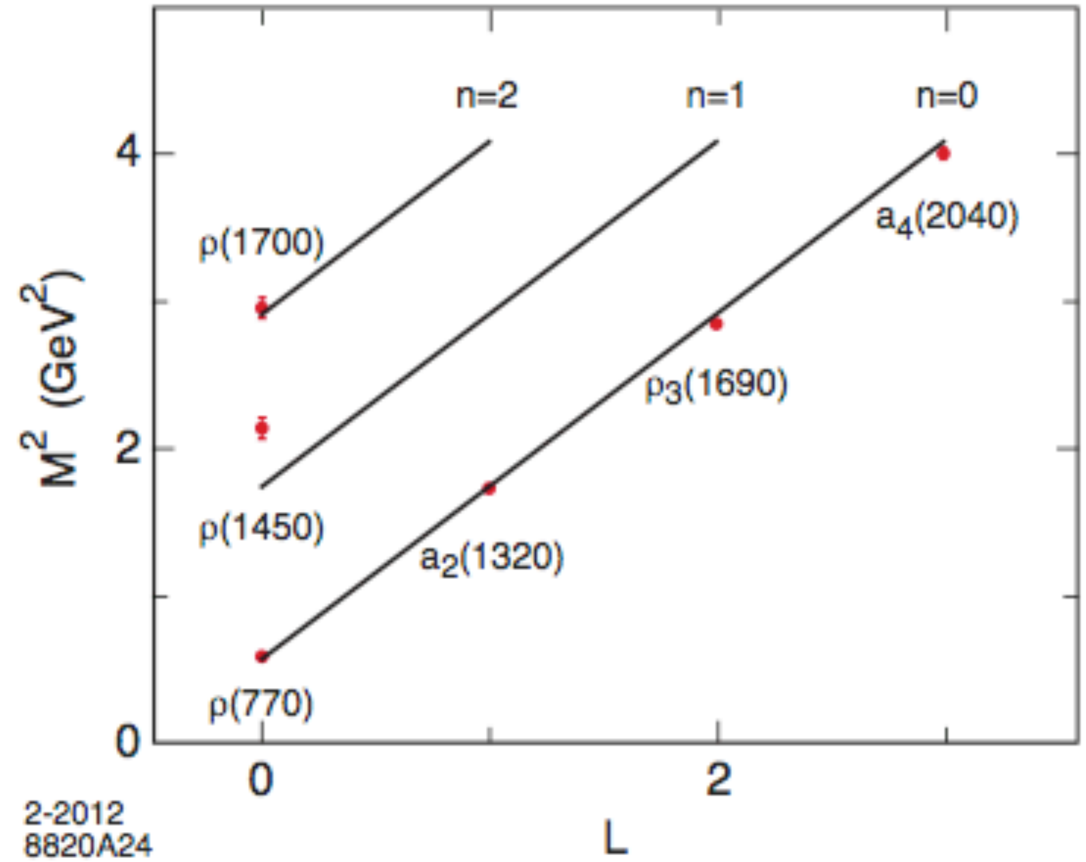
- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Same slope in n and L! ***Massless pion in Chiral Limit!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

Light-Front Schrödinger Equation

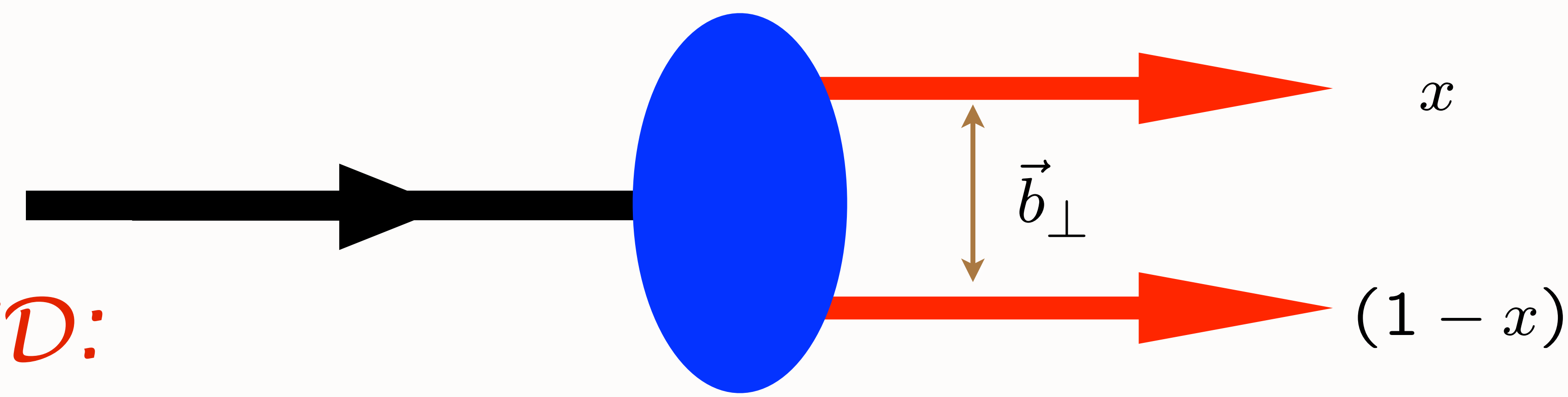
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

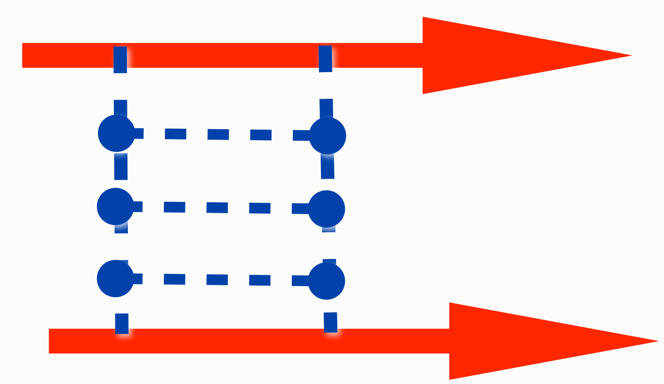
$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



AdS/QCD:

U is the exact QCD potential
Conjecture: 'H'-diagrams generate



$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

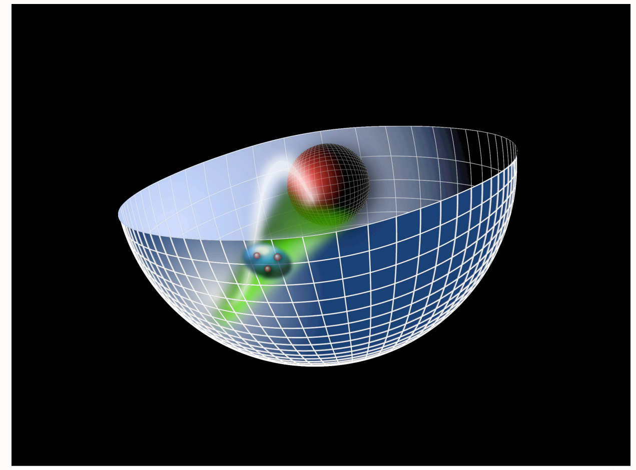
Remarkable Features of Light-Front Schrödinger Equation

- *Relativistic, frame-independent*
- *QCD scale appears spontaneously - unique LF potential*
- *Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter*
- *Zero-mass pion for zero mass quarks!*
- *Regge slope same for n and L -- not usual HO*
- *Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry*
- *Phenomenology: LFWFs, Form factors, electroproduction*
- *Extension to heavy quarks*

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

de Teramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

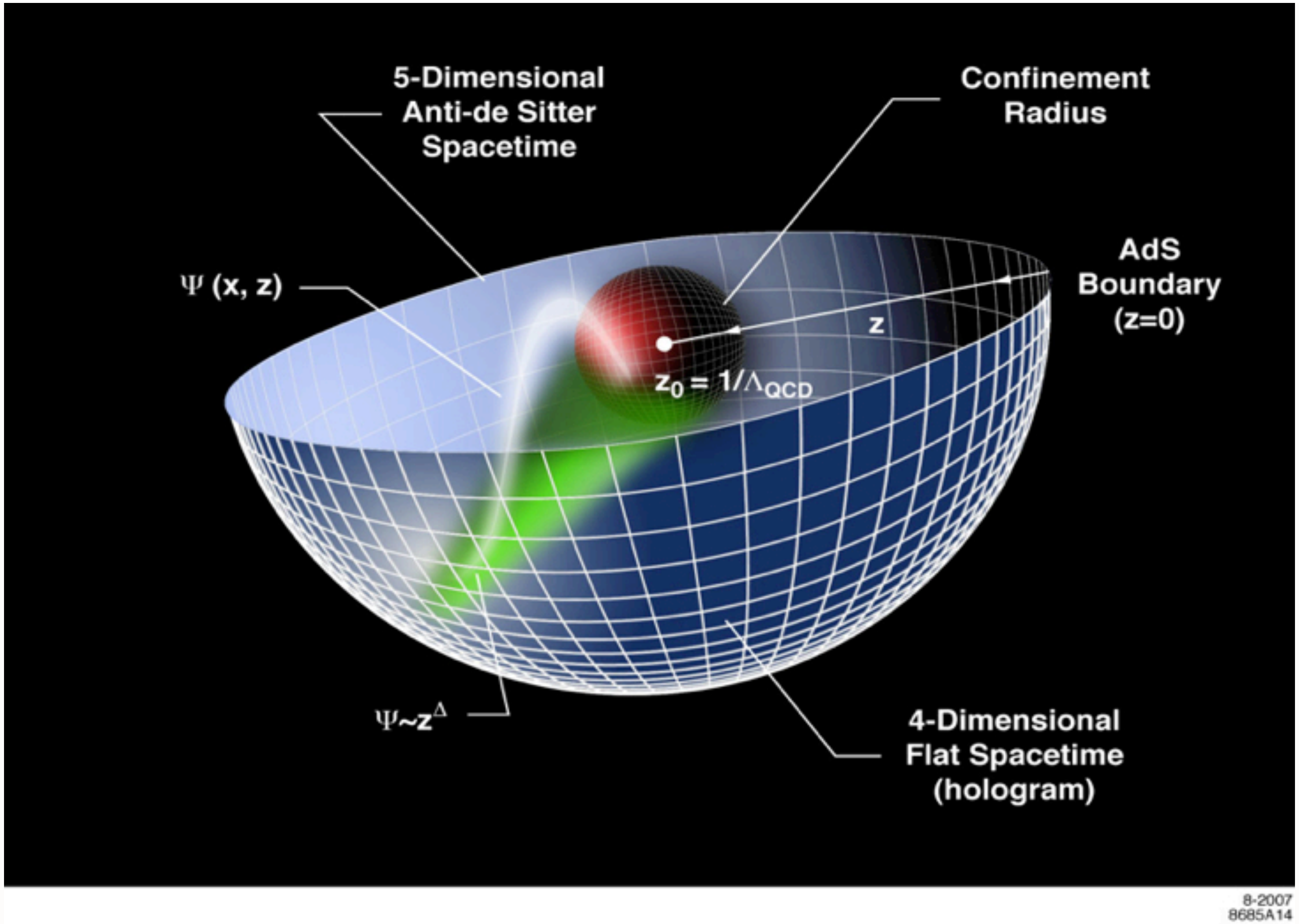
Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$$1/\kappa \simeq 0.4\ fm$$



Changes in physical length scale mapped to evolution in the 5th dimension z

Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$ ← invariant measure

$x^\mu \rightarrow \lambda x^\mu, \; z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2}\int d^{d+1}x\sqrt{g}\,\tfrac{1}{2}\left[g^{\ell m}\partial_\ell\Phi\partial_m\Phi - \mu^2\Phi^2\right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^\ell}\left(\sqrt{g}\,g^{\ell m}\frac{\partial}{\partial x^m}\Phi\right) + \mu^2\Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2\partial_z^2 - (d-1)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2R^2}\,\right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

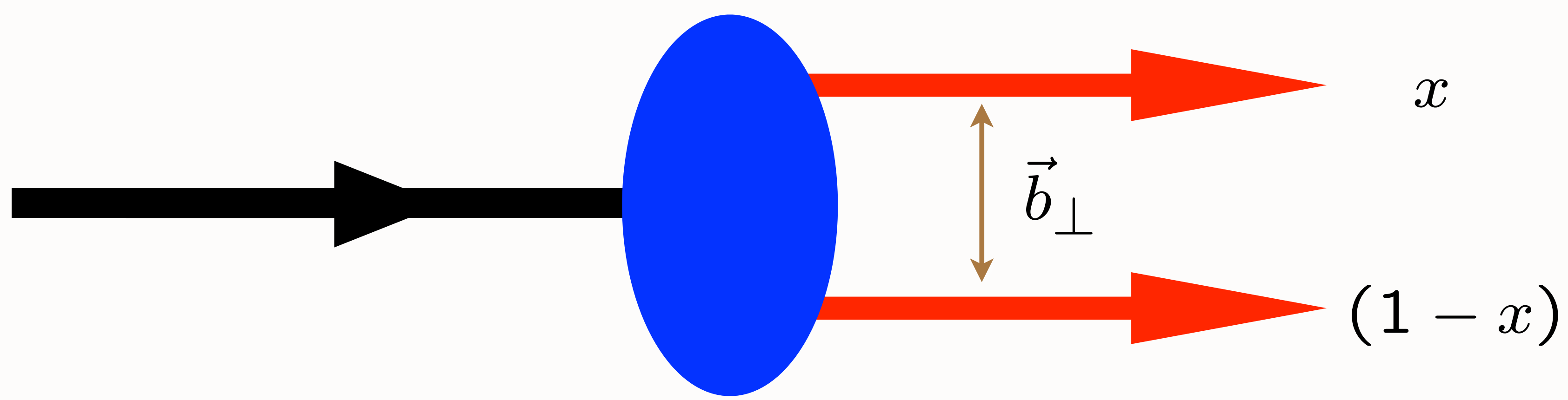
$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$

de Teramond, sjb

LF(3+1) ↔ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

G. de Teramond and sjb, PRL 102 081601 (2009)

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

- $z \to \zeta$ $\quad \phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

$(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

$(\mu R)^2 = -(2-J)^2 + L^2$

- $(\mu R)^2 \geq -4$ $\quad L^2 \geq 0$
- τ $\quad \hat{\Phi}_J \quad \tau = 2 + L$

Higher Spin Wave Equations in AdS Space and LF Holographic Mapping

H. G. Dosch, G. de Teramond, sjb PRD 87 (2013)

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Integer spin-J fields in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{eff} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{MM'} D_M \Phi^*_{N_1 \ldots N_J} \, D_{M'} \Phi_{N_1' \ldots N_J'} - \mu_{eff}^2(z) \, \Phi^*_{N_1 \ldots N_J} \, \Phi_{N_1' \ldots N_J'} \Big)$$

where D_M is the covariant derivative which includes affine connection

- The z-dependent effective AdS mass $\mu_{eff}(z)$ can absorb the contribution from different contractions in the action and is *a priori* unknown

- Effective mass $\mu_{eff}(z)$ allows a separation of kinematical and dynamical effects and is determined by precise mapping to light-front physics

- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

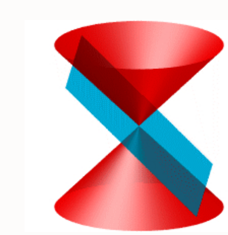
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$



with $(\mu R)^2 = -(2-J)^2 + L^2$

Light-Front Schrödinger Equation

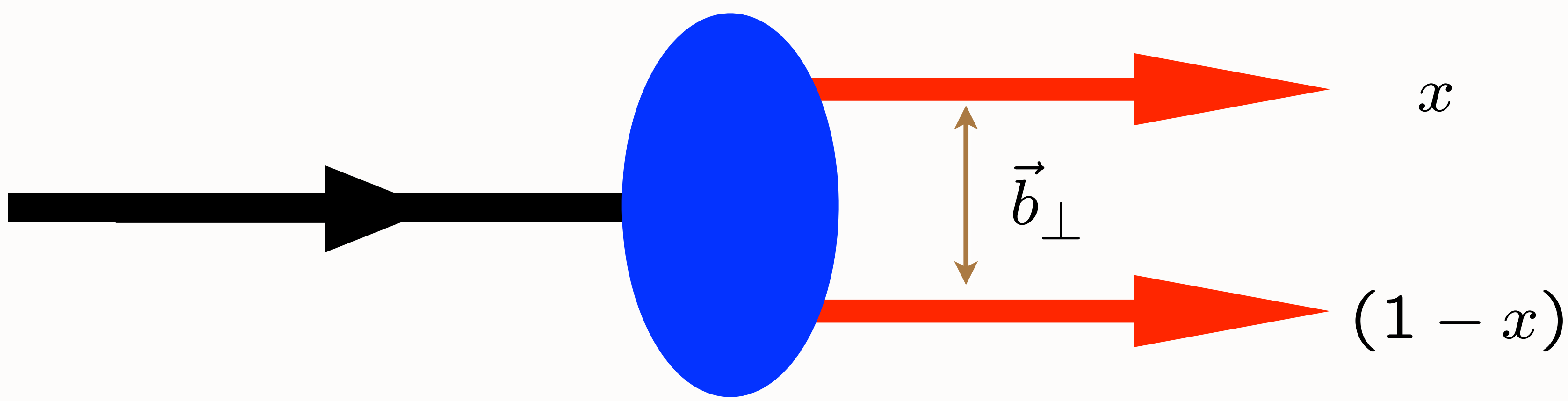
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$



AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Meson Spectrum in Soft Wall Model

- Dilaton profile $\varphi(z) = +\kappa^2 z^2$ $\quad z \to \zeta$
- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

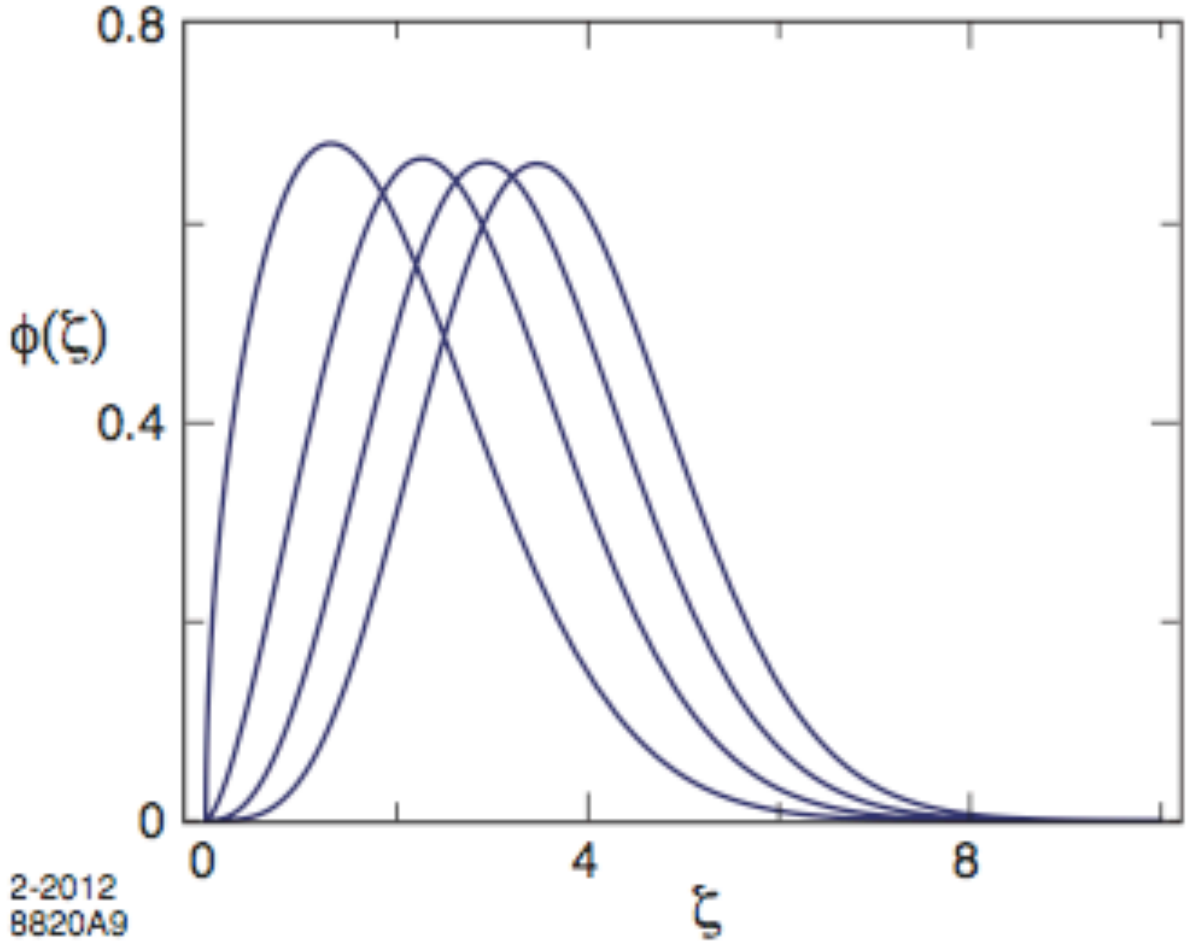
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

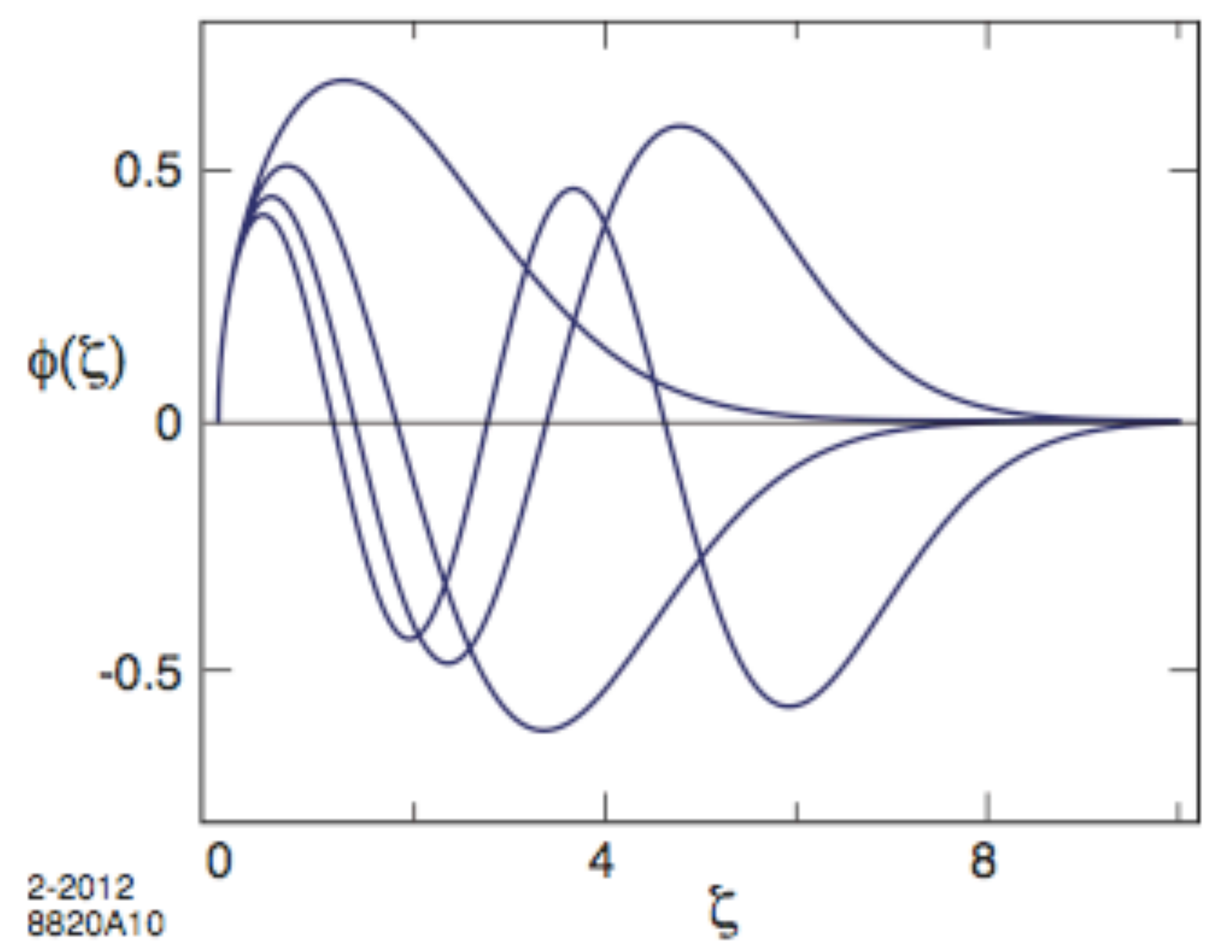
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, sjb





LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes

Quark separation increases with L

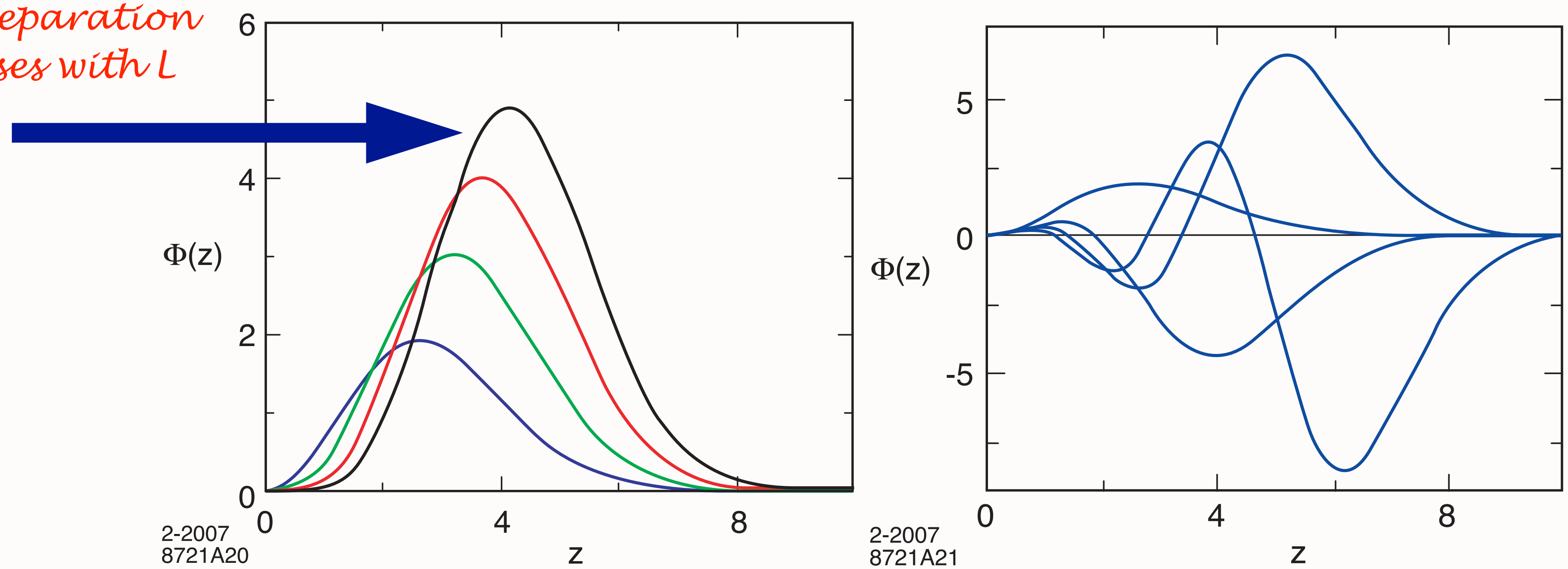
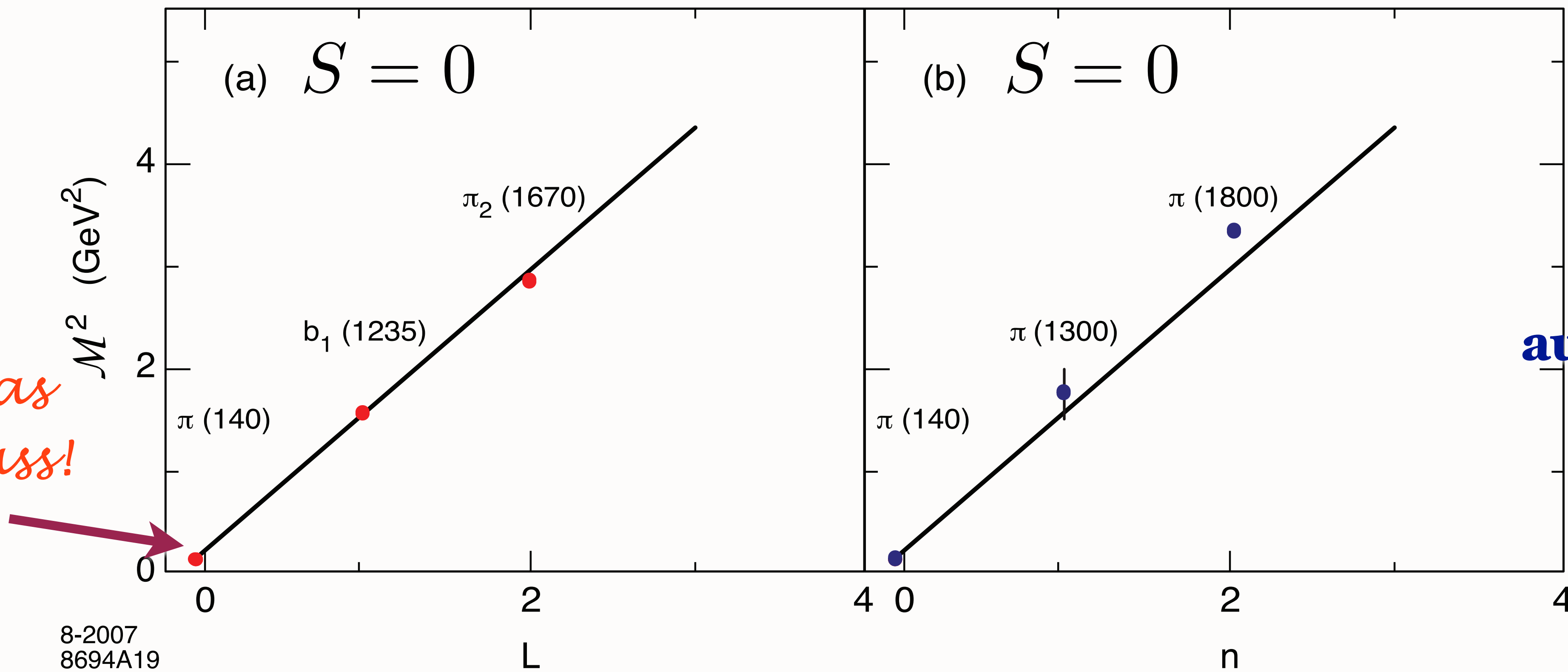


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Pion has zero mass!

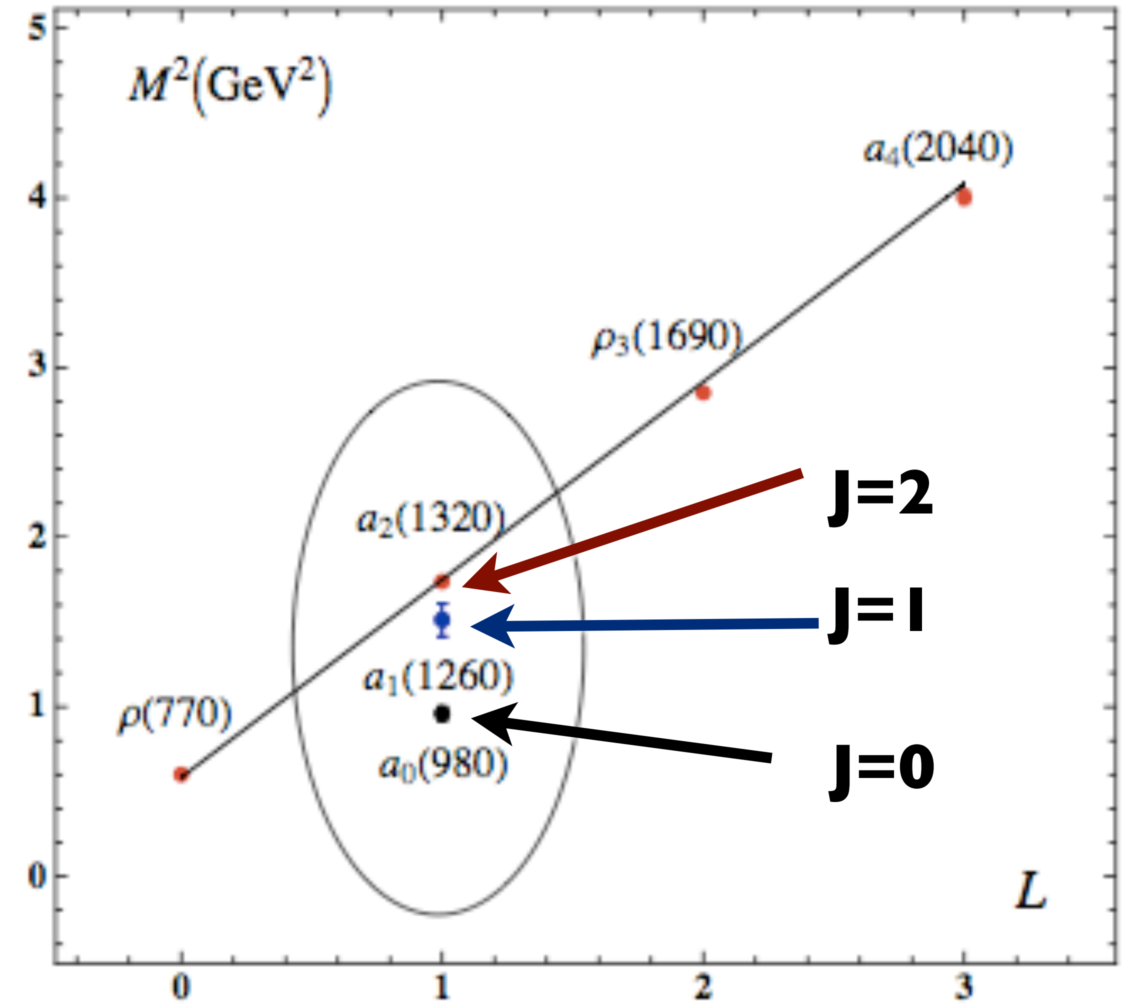
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$$\lambda = \kappa^2$$

$$\mathcal{M}^2_{n,J,L} = 4\lambda \left(n + \frac{J+L}{2}\right)$$

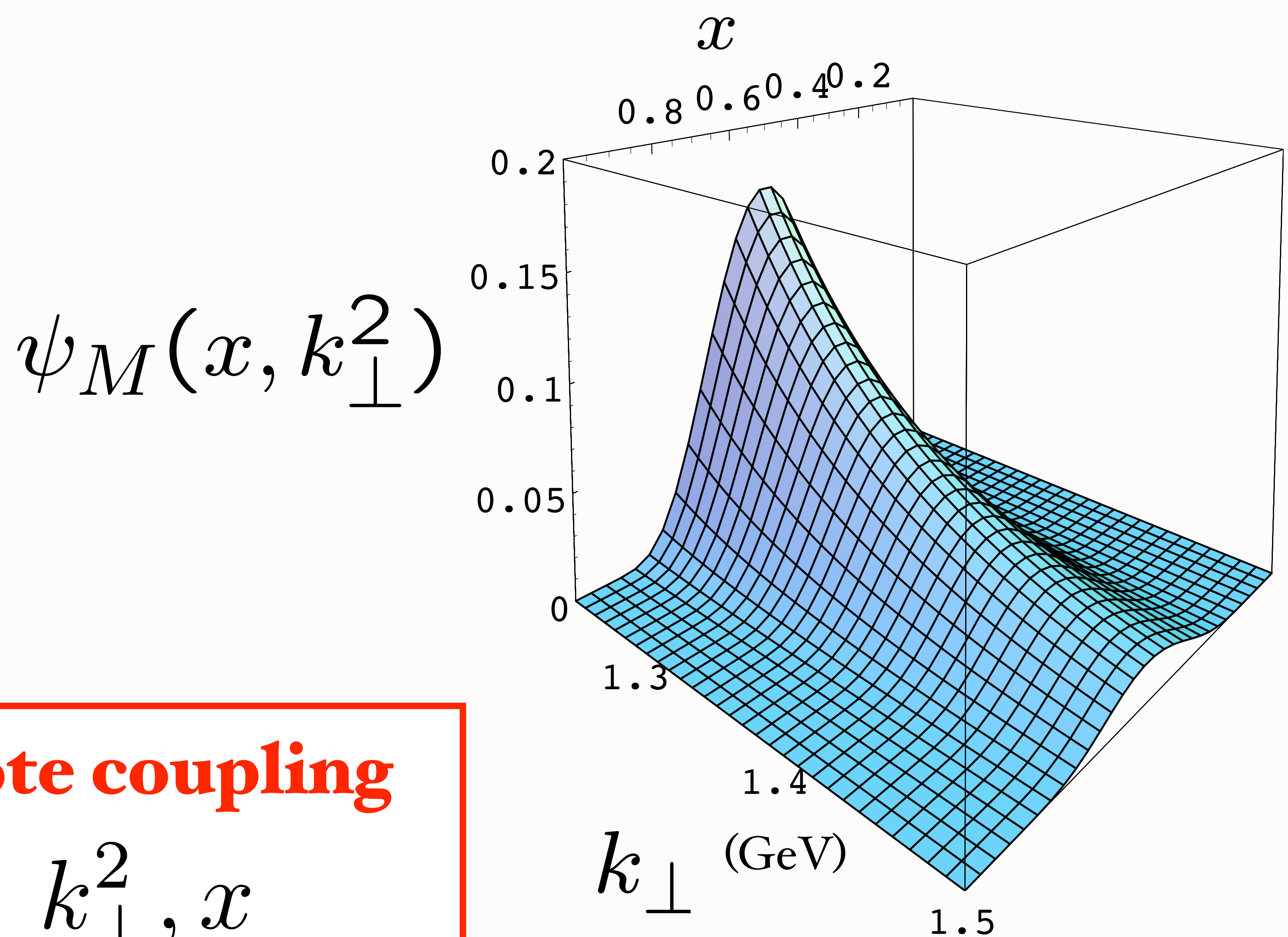


- Triplet splitting for the $L = 1$, $J = 0, 1, 2$ vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

- Systematics of light meson spectra – orbital and radial excitations as well as important $J - L$ splitting, well described by light-front harmonic confinement model
- Linear Regge trajectories, a massless pion and relation between the ρ and a_1 mass $M_{a_1}/M_\rho = \sqrt{2}$ usually obtained from Weinberg sum rules [Weinberg (1967)]

Prediction from AdS/CFT: Meson LFWF



de Teramond, sjb

"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

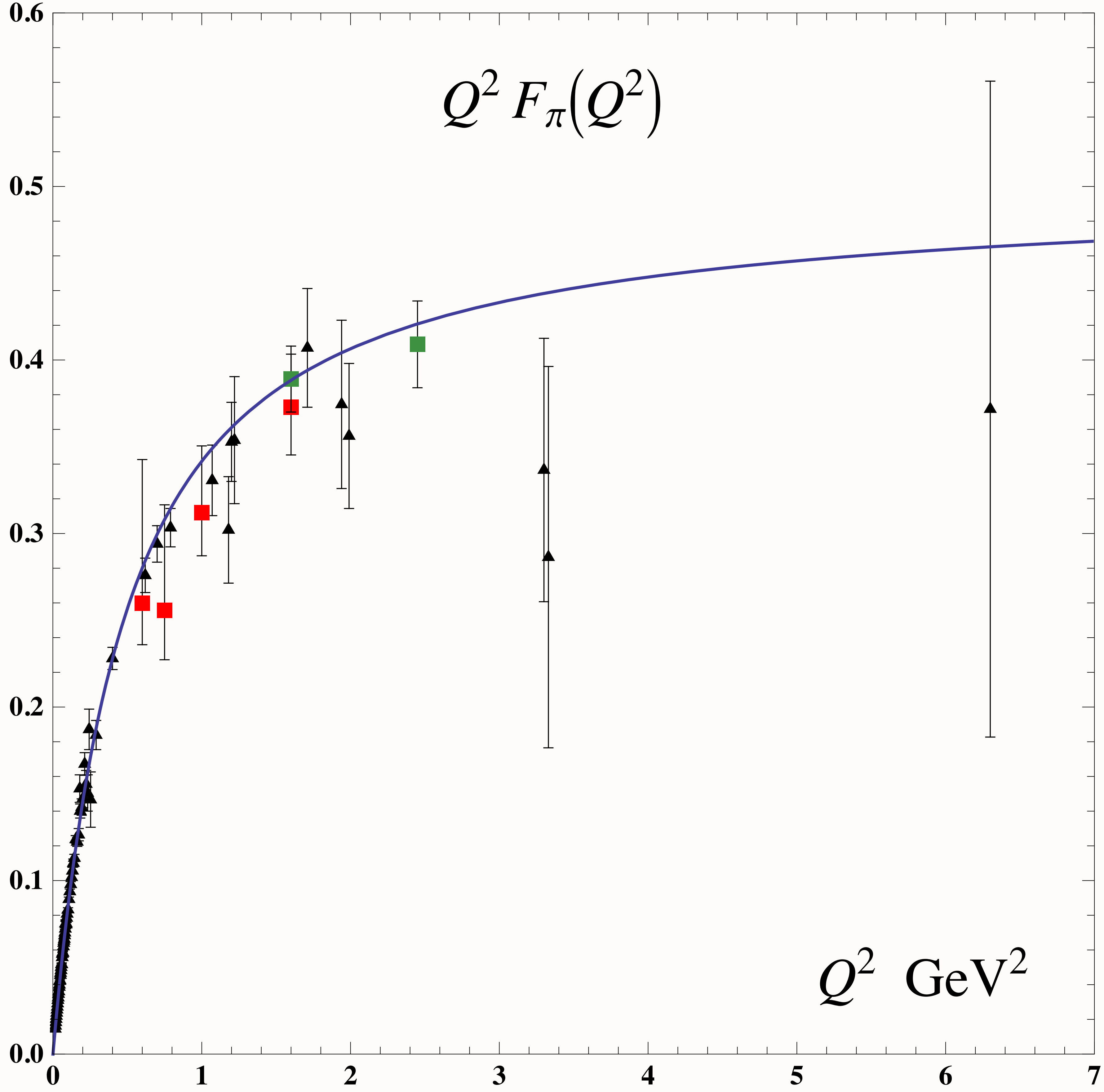
Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Provides Connection of Confinement to TMDs


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

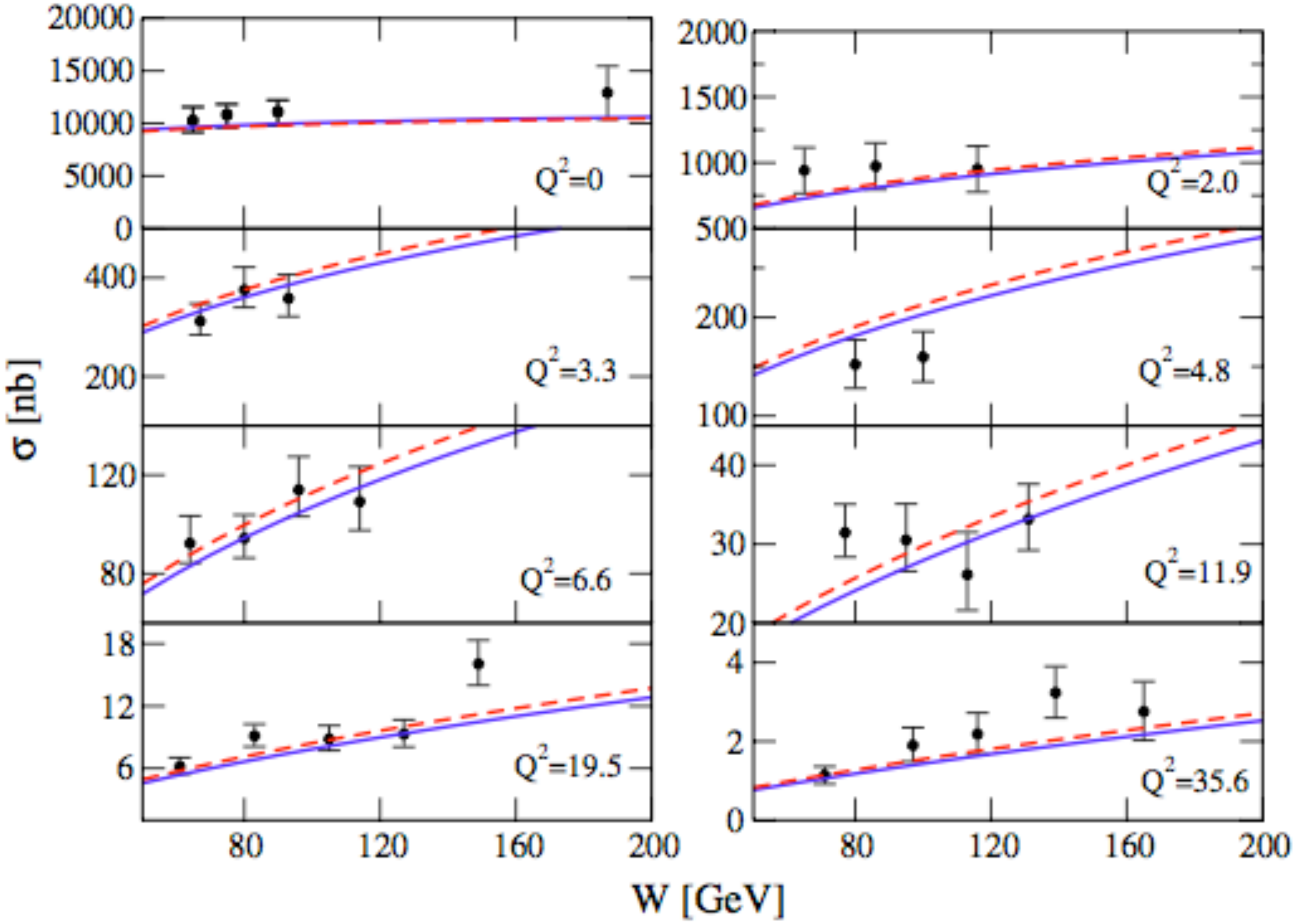
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\phi(x,\zeta) = \mathcal{N}\frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right),$$

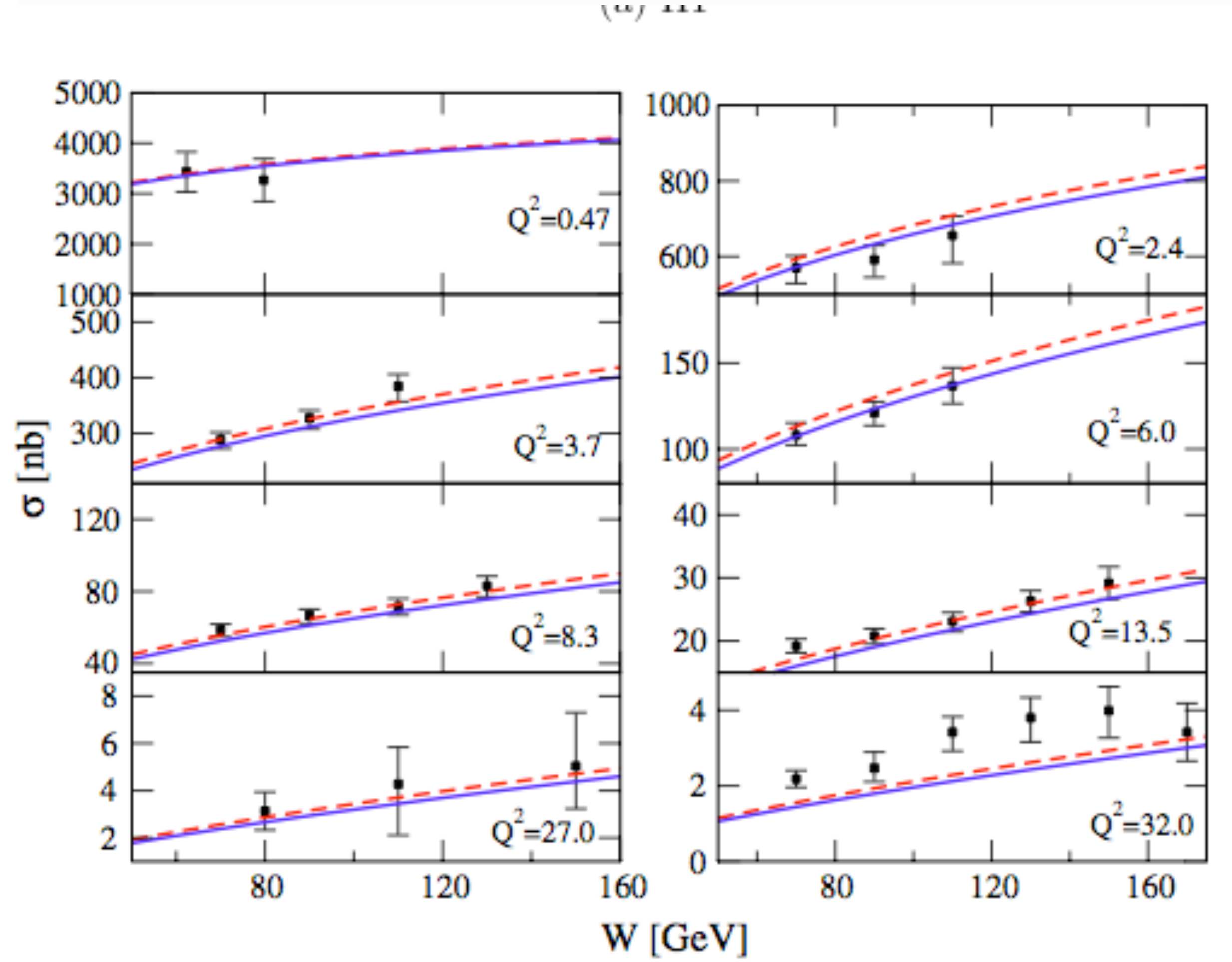
$$\tilde{\phi}(x,k) \propto \frac{1}{\sqrt{x(1-x)}}\exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



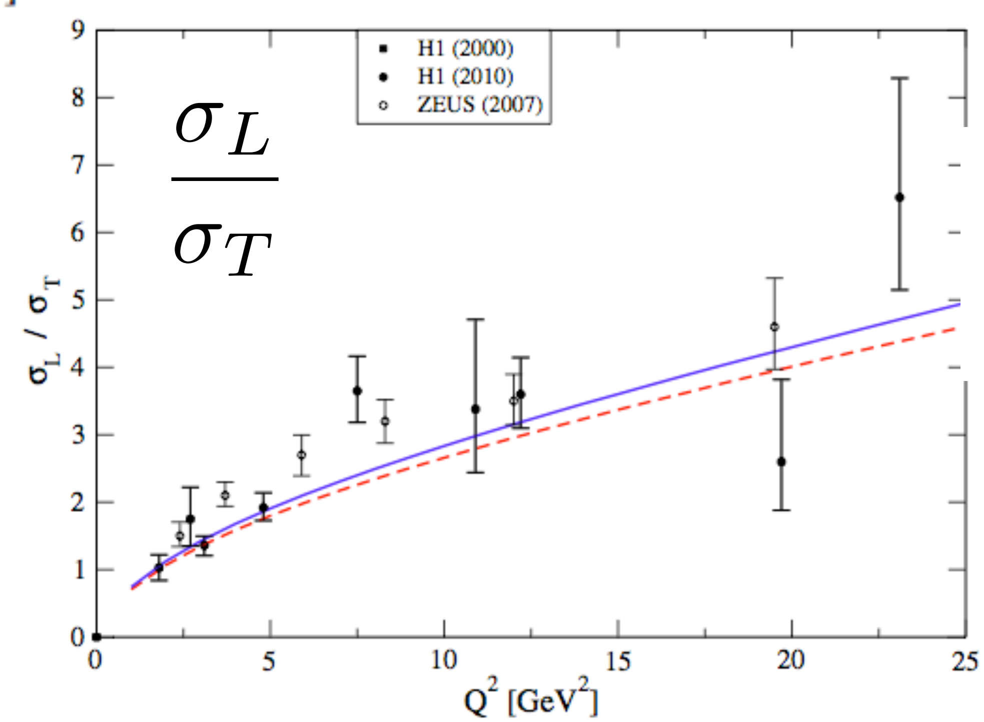
(a) H



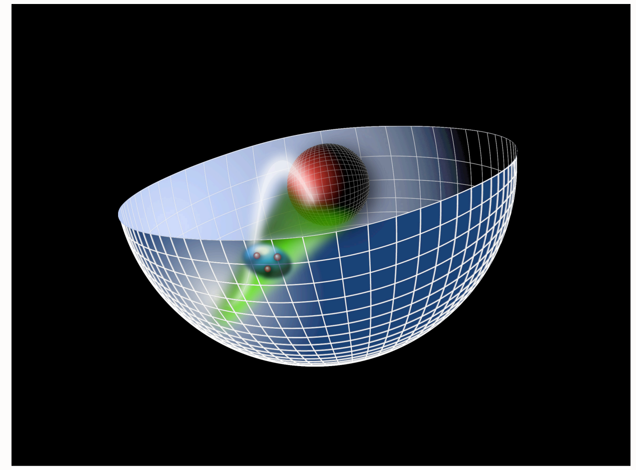
(b) ZEUS

J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$1/\kappa \simeq 0.4\ fm$

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Uniqueness

de Teramond, Dosch, sjb

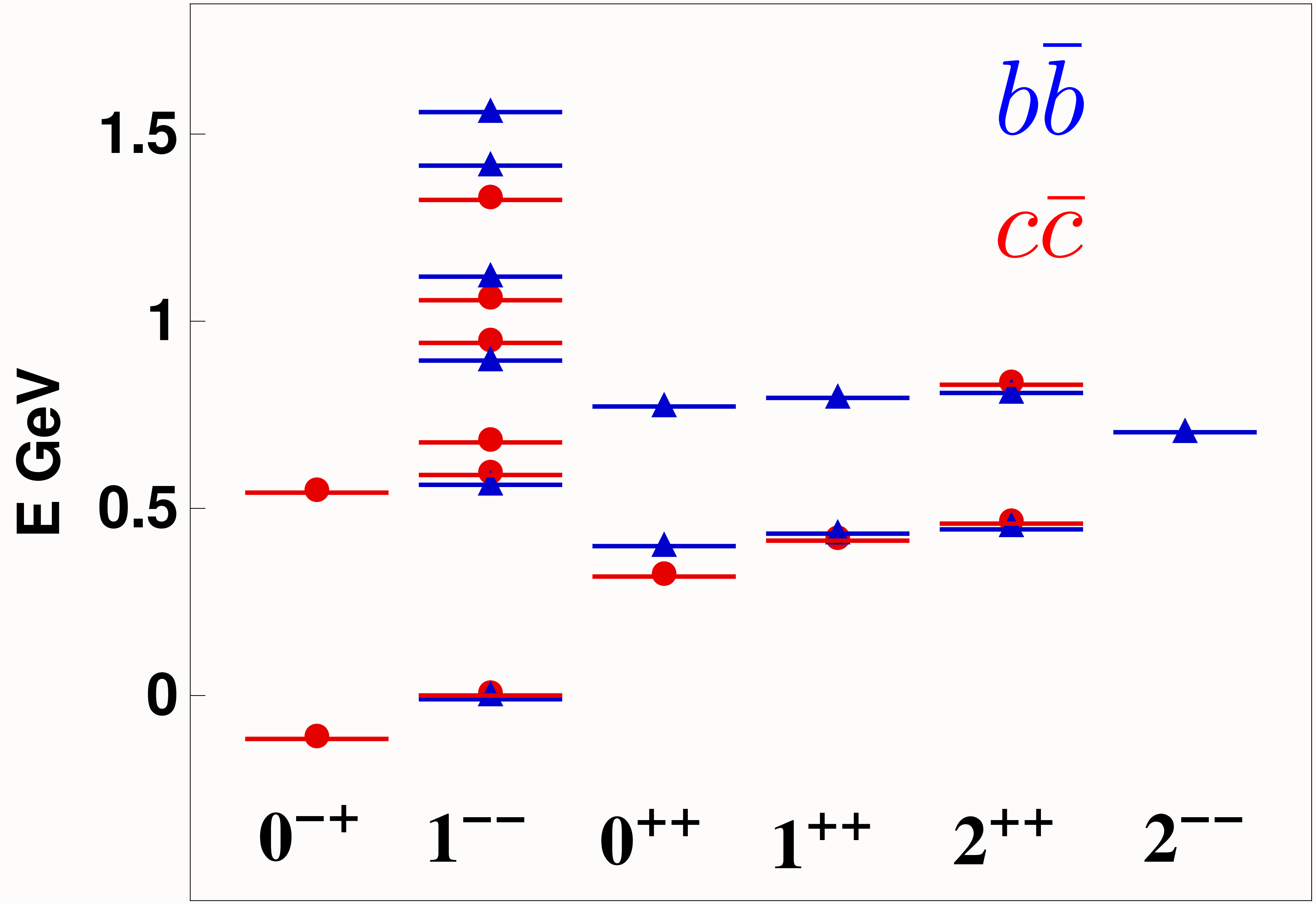
- ζ^2 confinement potential and dilaton profile unique!
- Linear Regge trajectories in n and L: same slope!
- Massless pion in chiral limit!   No vacuum condensate!
- Derive from conformal invariance: conformally invariant action for massless quarks despite mass scale
- Same principle, equation of motion as de Alfaro, Fubini, Furlan
- <u>Conformal Invariance in Quantum Mechanics</u> Nuovo Cim. A34 (1976) 569

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

Quigg and Rosner (1979:

Excitation energies of quarkonia appear to be flavor-independent



logarithmic potential?

Heavy-Quark Systems and Conformal Invariance

H.G. Dosch, G. de Teramond, sjb

- Structure of excitations for heavy quark bound states is largely independent of the reduced mass
- Quigg and Rosner (1979) "For what form of the quark-antiquark potential is the level spacing independent of the reduced mass ?" : $V(r) = C \ln(r/r_0)$
- New perspective from non-relativistic realization of the dAFF construction!
- Consider the operator

$$G_{NR} = a\, H_t + b\, D + c\, K,$$

- The corresponding dAFF NR Hamiltonian is

$$H_{NR} = \tfrac{1}{2}\left(\dot{q}^2 + \frac{g}{q^2} + \frac{4\,ac - b^2}{4} q^2\right),$$

$$q \to \sqrt{m} r, \quad \dot{q} \to \frac{1}{\sqrt{m}} i \frac{d}{dr} \qquad [q(t), \dot{q}(t)] = i$$

H.G. Dosch, G. de Teramond, sjb (in progress)

- NR dAFF Schrödinger representation

$$q \to \sqrt{m}\, r, \qquad \dot{q} \to -\frac{1}{\sqrt{m}} i \frac{d}{dr}$$

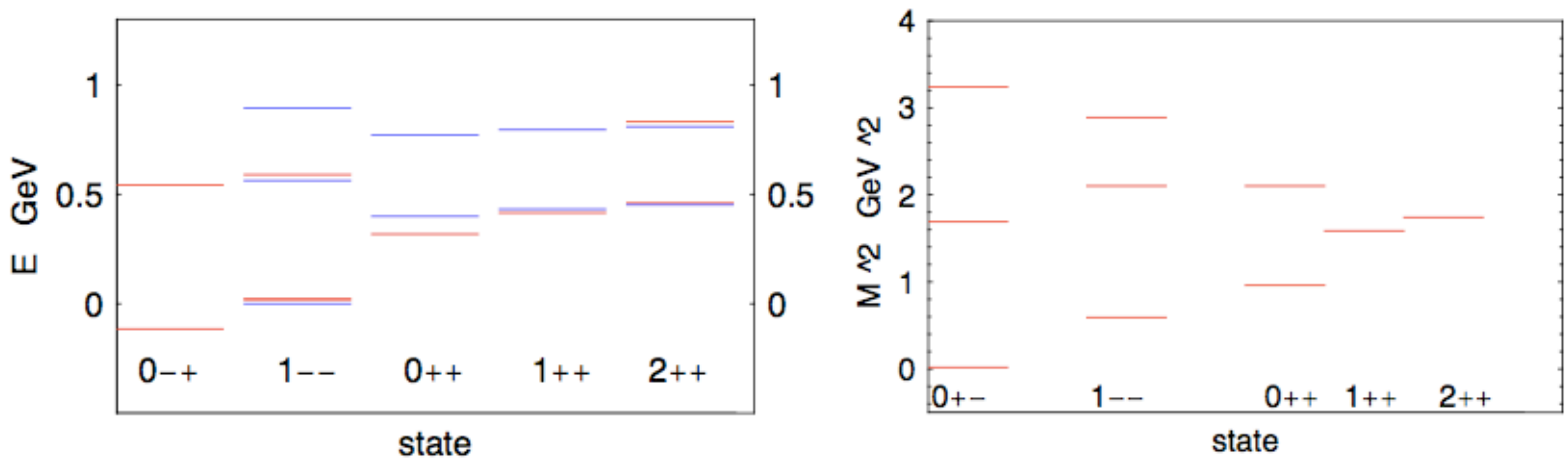
compatible with canonical commutation relations $[q(t), \dot{q}(t)] = i$

- Find

$$H_{NR} = -\frac{1}{2m}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{2mr^2} + \frac{1}{2} m w^2 r^2$$

where $g = \ell(\ell+1)$ and $\omega^2 = 4ac - b^2/4$ $\qquad \omega = \Lambda = 0.28\ GeV$

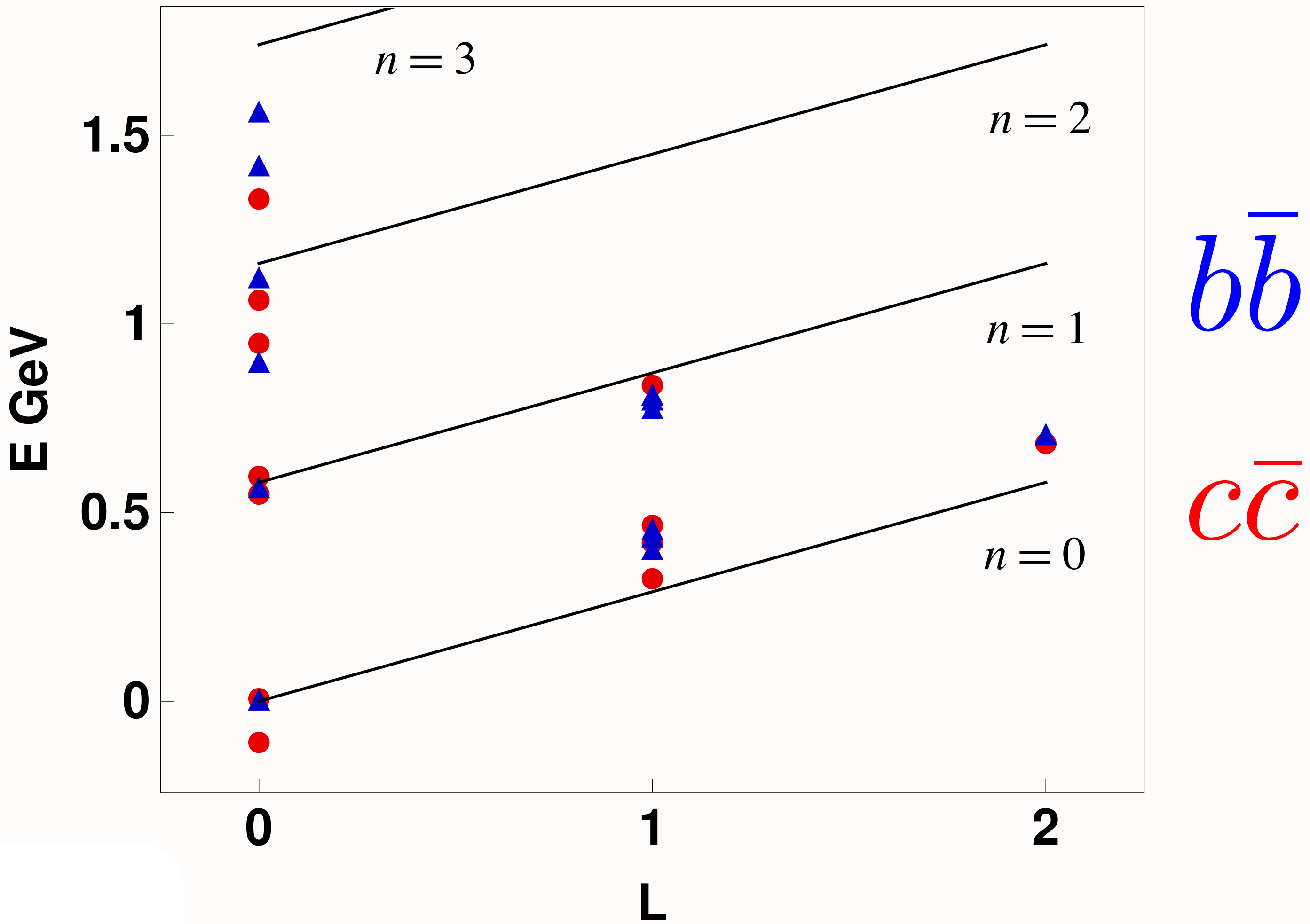
- NR Hamiltonian has reduced mass m but level spacing is independent of m as suggested by data



Excitation energy E of $c\bar{c}$ (red) and $b\bar{b}$ (blue) $\qquad$ Squared mass M^2 of light $I = 1$ mesons

H.G. Dosch, G. de Teramond, sjb (in progress)

$$H = -\frac{1}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2}\right) + \frac{1}{2}m\,\Lambda^2 r^2,$$



$$E_{n\ell} = \left(2\,n + \ell + \frac{3}{2}\right)\Lambda$$

$$\Lambda = 0.28\ GeV$$

Flavor independent

Universal confinement time!

Excitation energies of $c\bar{c}$ (red boxes) and $b\bar{b}$ (blue diamonds) with different values of angular momentum ℓ. Only well established states below open flavour threshold are shown.

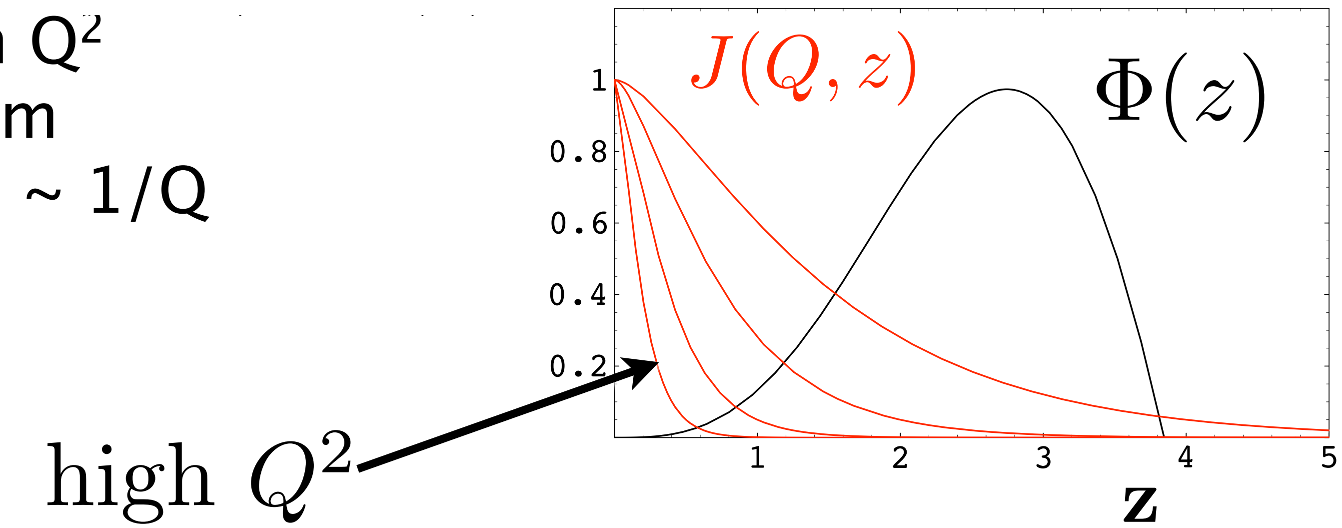
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q²
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

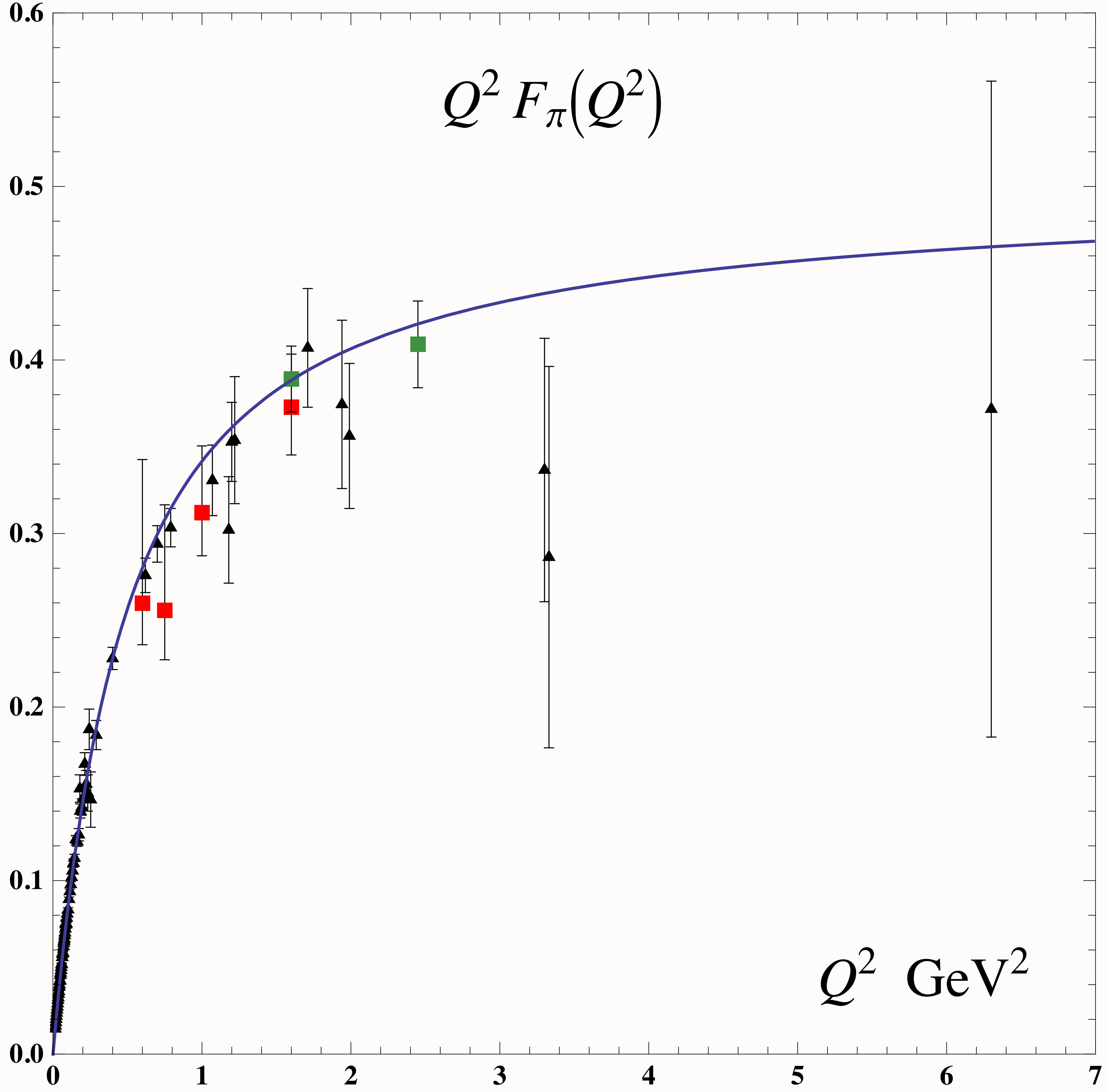
- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

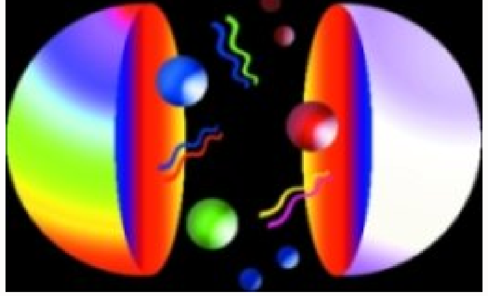


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g}\, e^{\varphi(z)} \left(i \overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi} \Psi - \mu \overline{\Psi} \Psi \right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x} \Psi(z)$

$$\left[i \left(z \eta^{\ell m} \Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $\left(\nu = \mu R - \frac{1}{2},\ \nu = L + 1\right)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2 (n + L + 1)$$

positive parity

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2 (N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\, \Lambda_{\rm QCD}$

Light-Front Mapping

- A physical baryon satisfies the Rarita-Schwinger equation for spinors in physical space-time

$$(i\gamma^\mu \partial_\mu - M)\, u_{\nu_1 \cdots \nu_T}(P) = 0, \qquad \gamma^\nu u_{\nu\nu_2\cdots\nu_T}(P) = 0.$$

- Upon substitution in AdS wave equation for spin J ($u^\pm$ chiral spinors)

$$\Psi^\pm_{\nu_1\cdots\nu_T}(x,z) = e^{iP\cdot x}\left(\frac{R}{z}\right)^{T-d/2} \psi^\pm_T(z)\, u^\pm_{\nu_1\cdots\nu_T}(P),$$

and $z \to \zeta$ find LFWE

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - V(\zeta)\psi_- &= M\psi_+, \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - V(\zeta)\psi_+ &= M\psi_- \end{aligned}$$

provided that $|\mu R| = \nu + \frac{1}{2}$ and $\psi^\pm_T = \psi_\pm$ with effective LF potential

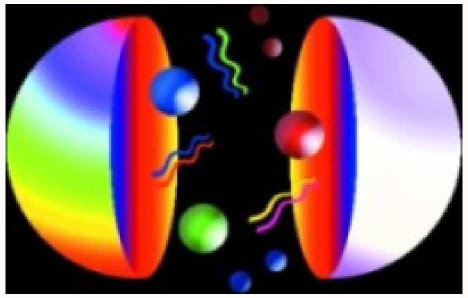
$$V(\zeta) = \frac{R}{\zeta}\rho(\zeta),$$

a J-independent potential – No spin-orbit coupling !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

PDG 2012

Baryon Spectrum

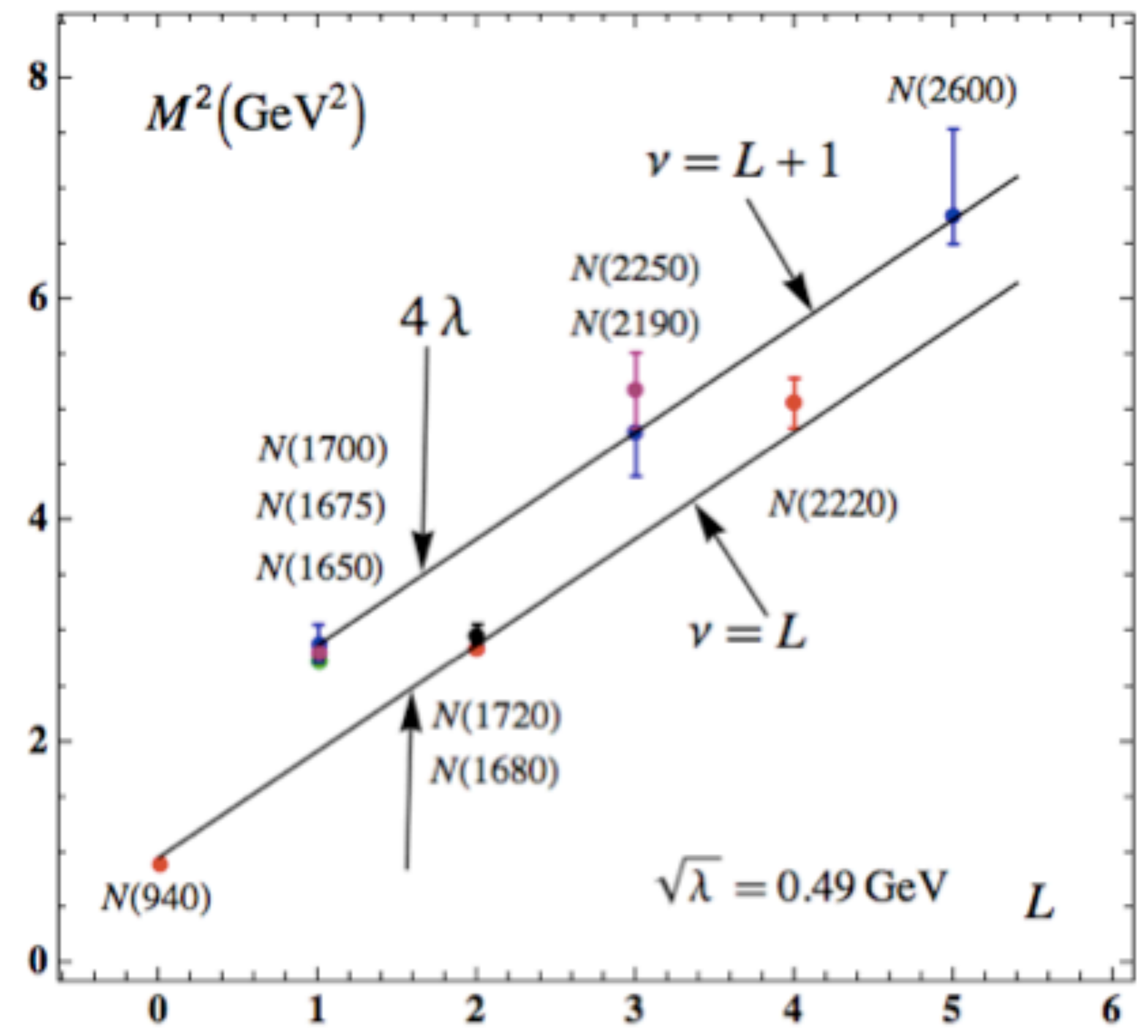
- Choose linear potential $V = \lambda\,\zeta, \quad \lambda > 0$
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^\nu(\lambda\zeta^2), \qquad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

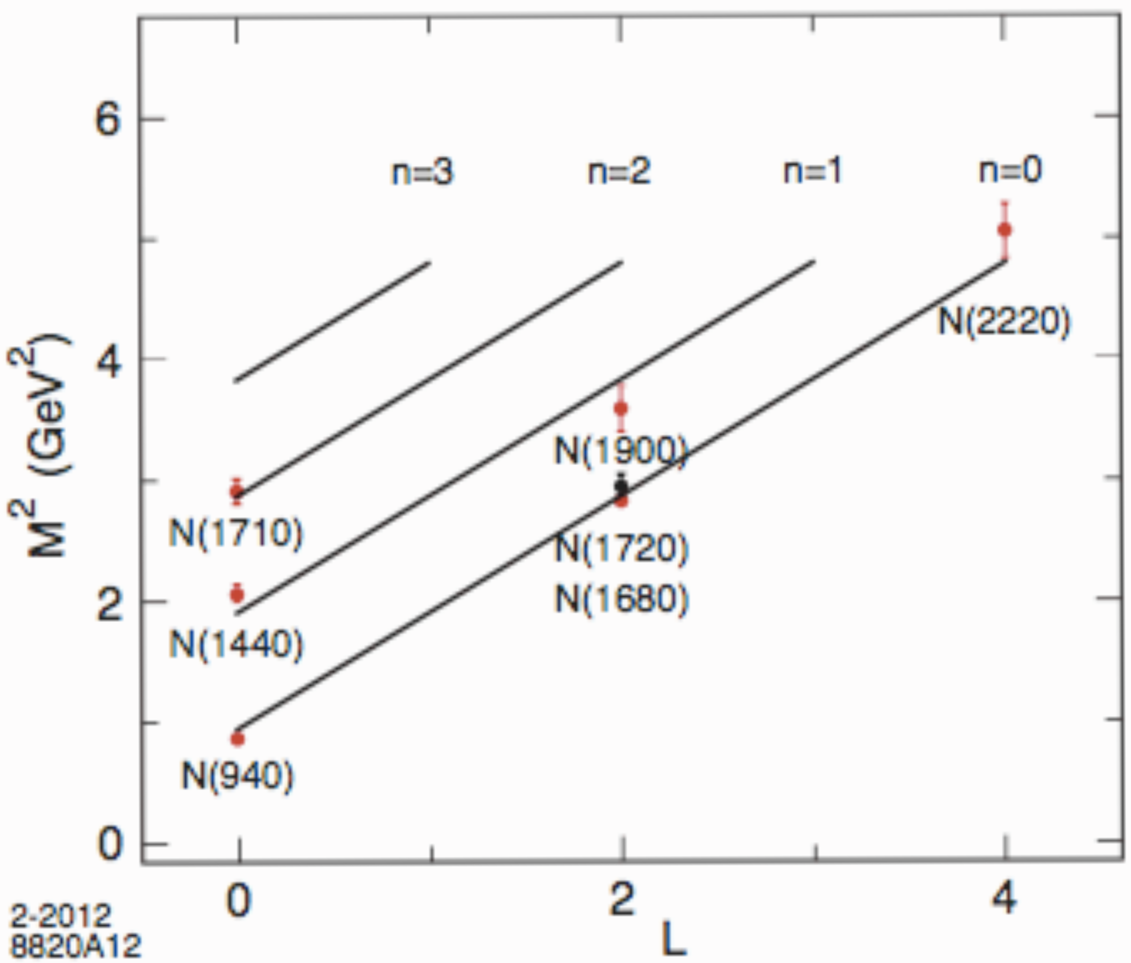
- Gap scale 4λ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families !
- For nucleons $\nu^+_{1/2} = L, \quad \nu^-_{3/2} = L + 1,$ where L is the relative LF angular momentum between the active quark and spectator cluster
- For $\lambda < 0$ no solution possible

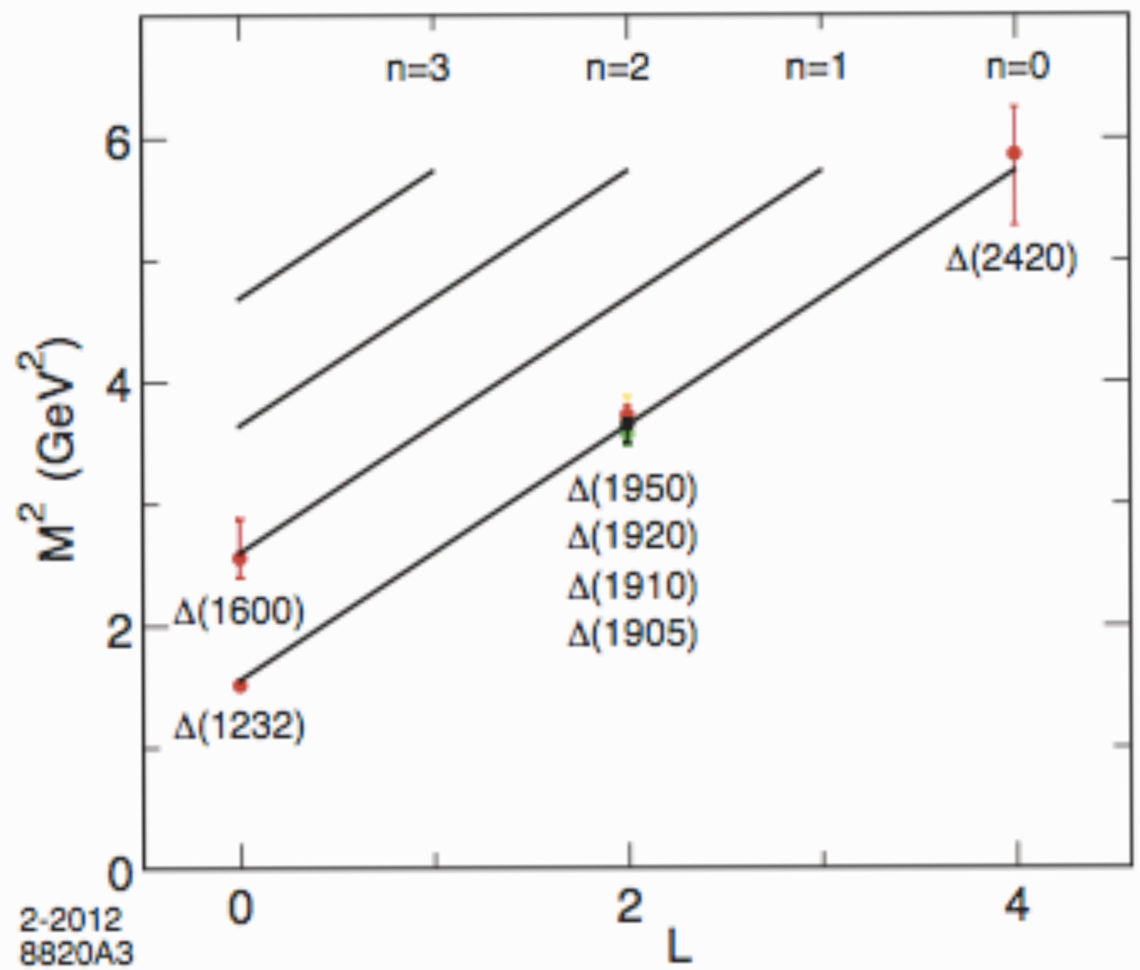


Identify L with ν

- Phenomenological identification to describe the full baryon spectrum: plus and negative sectors have internal spin $S = \frac{1}{2}$ and $S = \frac{3}{2}$

$$\nu^+_{1/2} = L, \qquad \nu^+_{3/2} = L + 1/2$$
$$\nu^-_{1/2} = L + 1/2, \qquad \nu^-_{3/2} = L + 1$$





Example: Orbital and radial excitations for positive parity N and Δ baryon families ($\sqrt{\lambda} \simeq 0.5$ GeV)

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

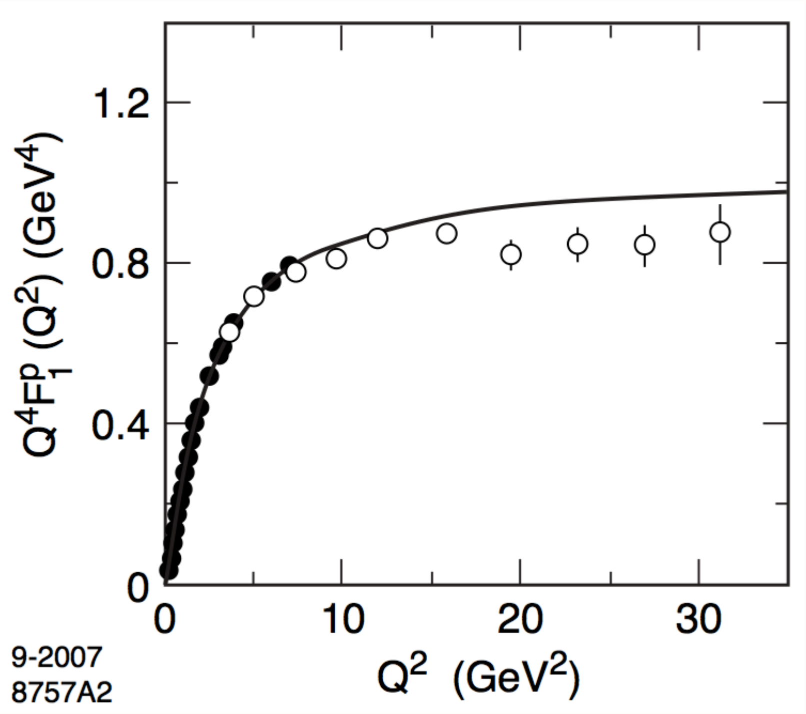
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

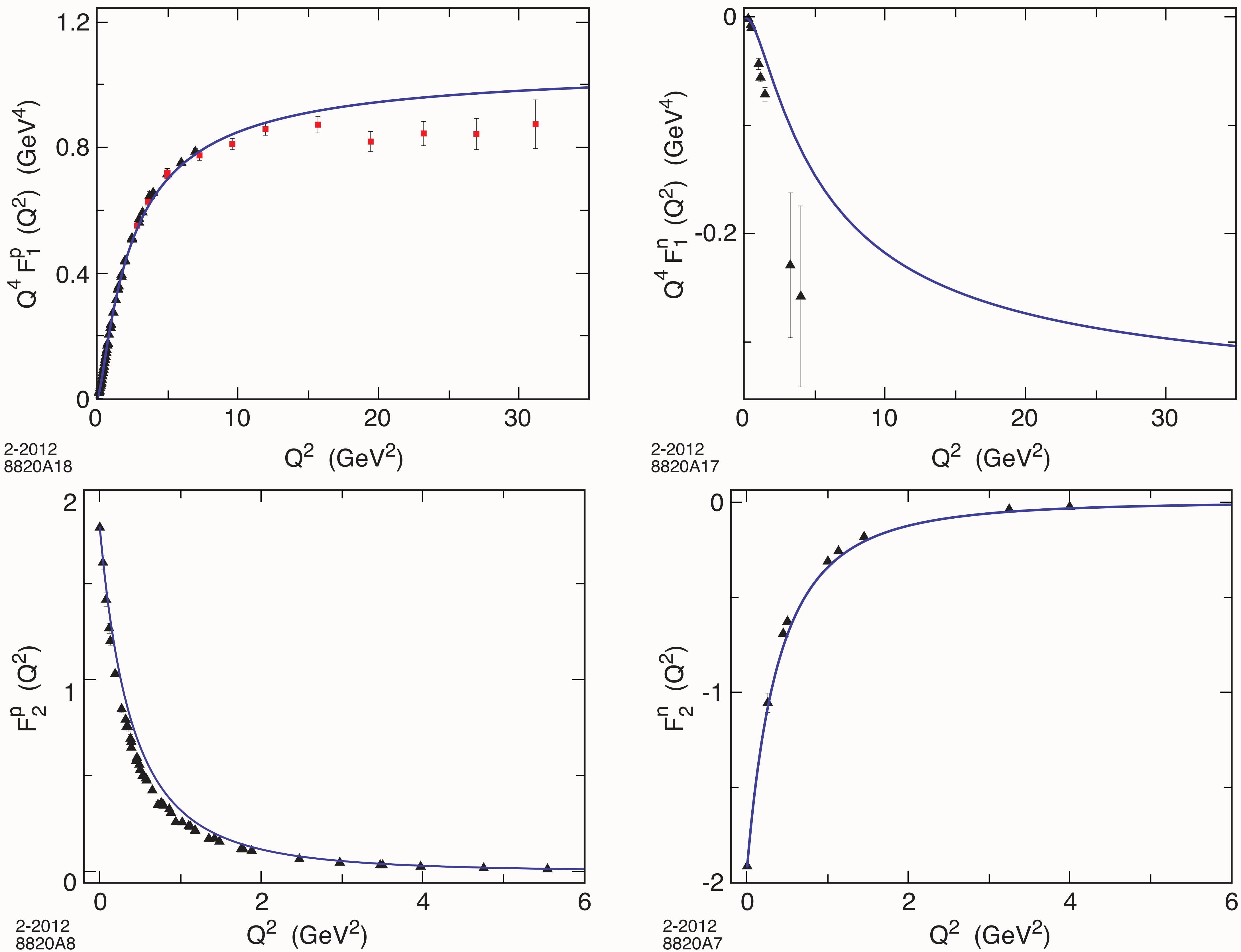
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$



Using $SU(6)$ flavor symmetry and normalization to static quantities



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\big(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\big)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

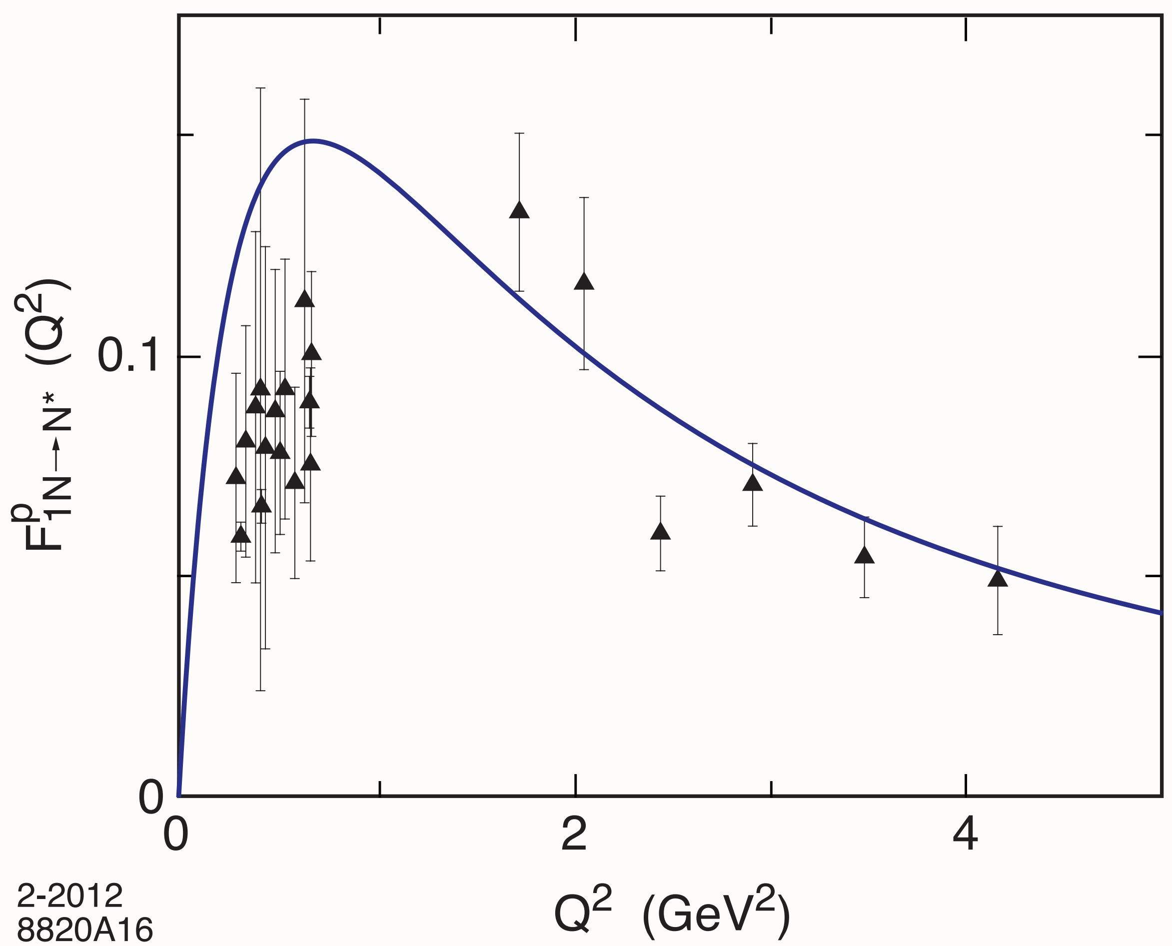
with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

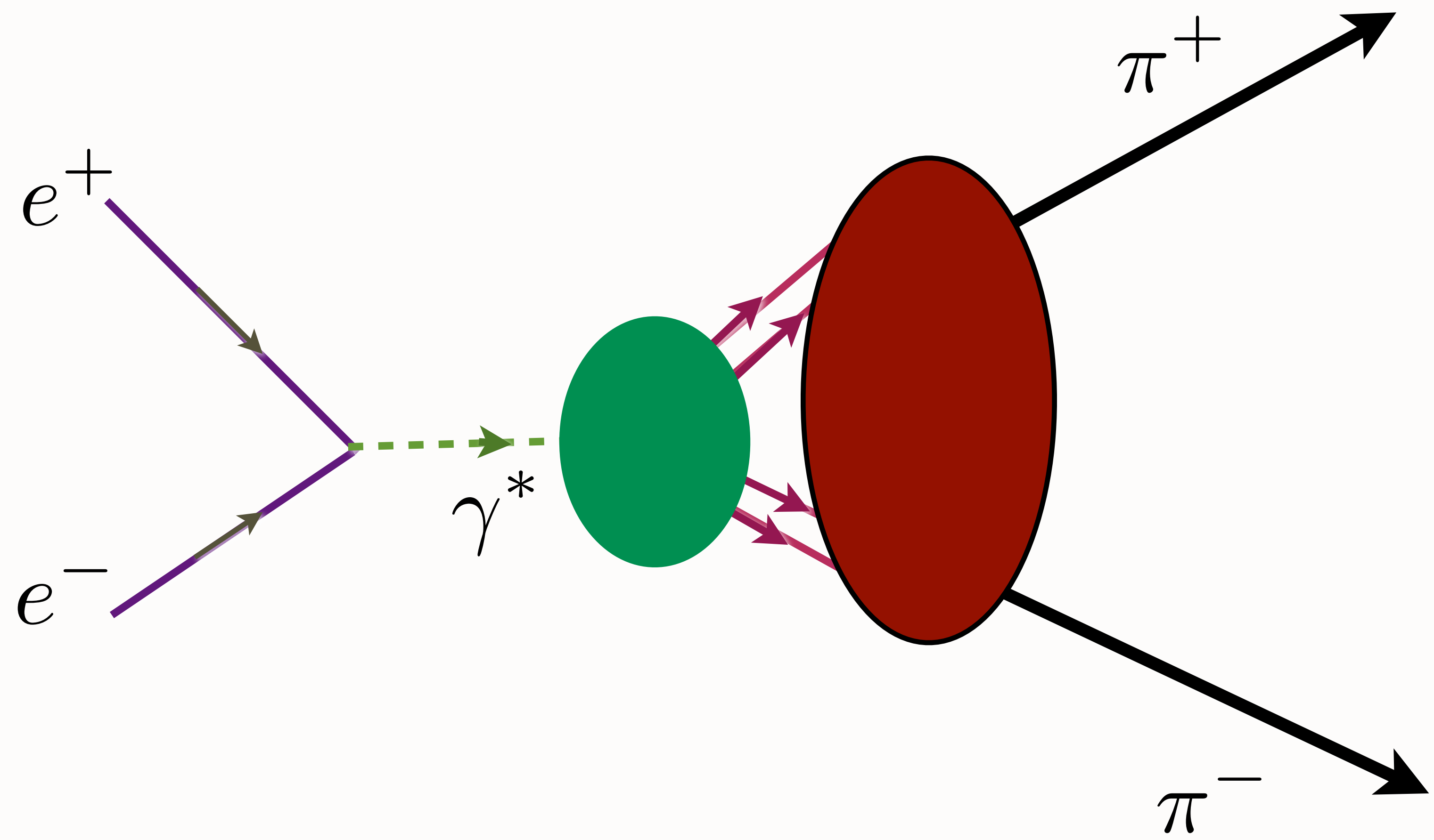
Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Dressed soft-wall current brings in higher Fock states and more vector meson poles

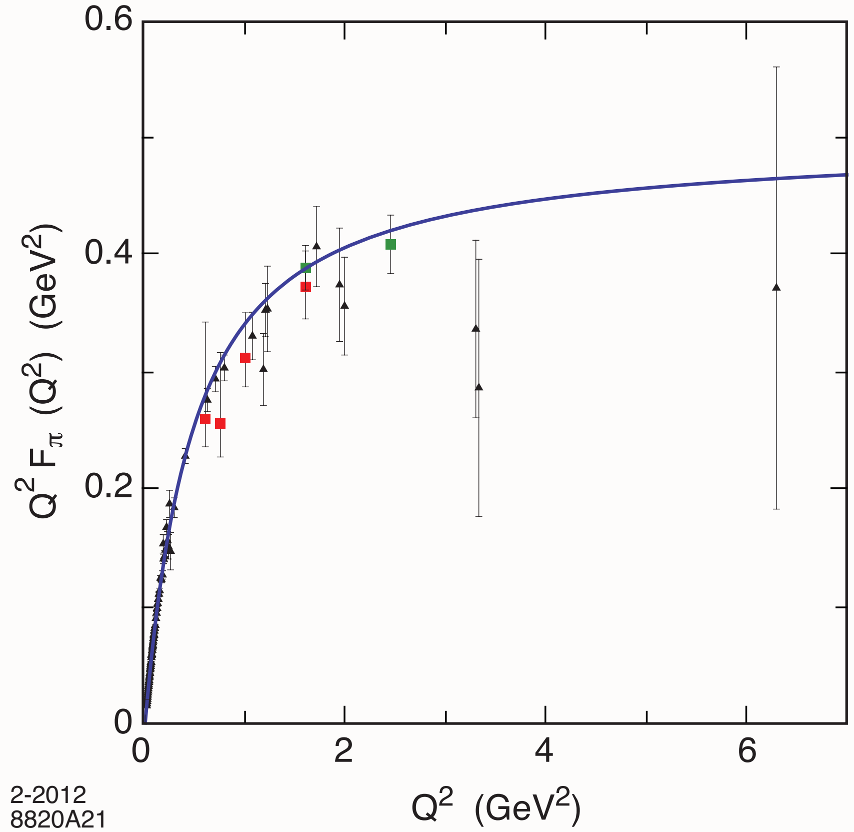


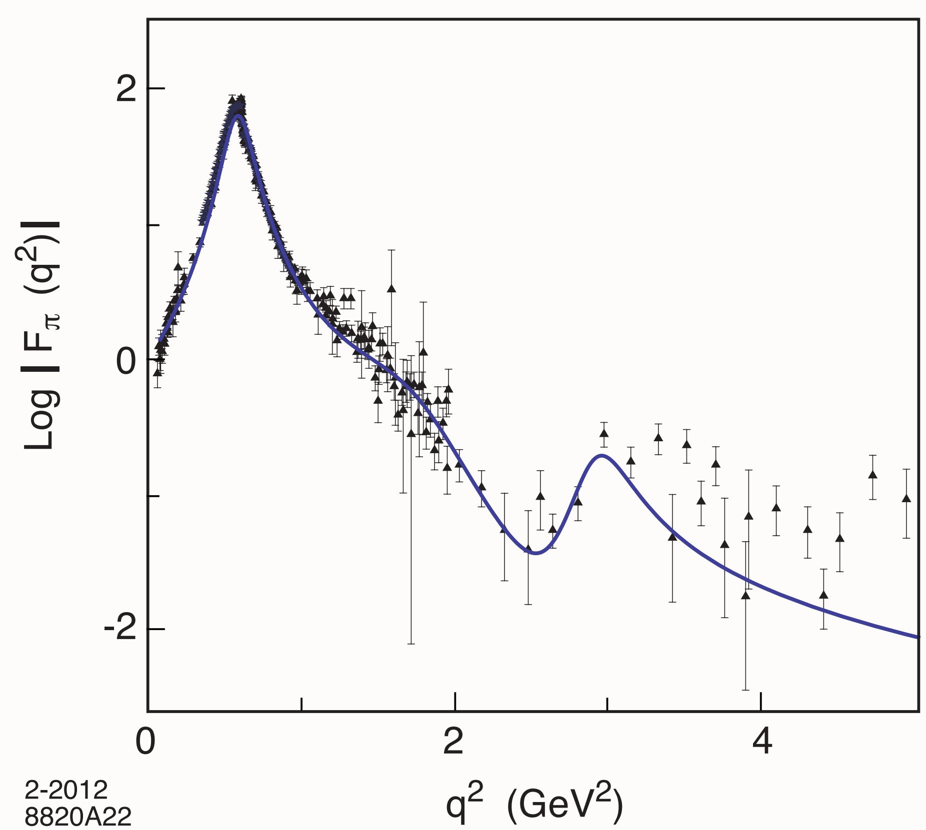
Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \to qq$, $q\bar{q} \to q\bar{q}$ but also to $q \to qq\bar{q}$ and $\bar{q} \to \bar{q}q\bar{q}$
- Higher Fock states can have any number of extra $q\bar{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

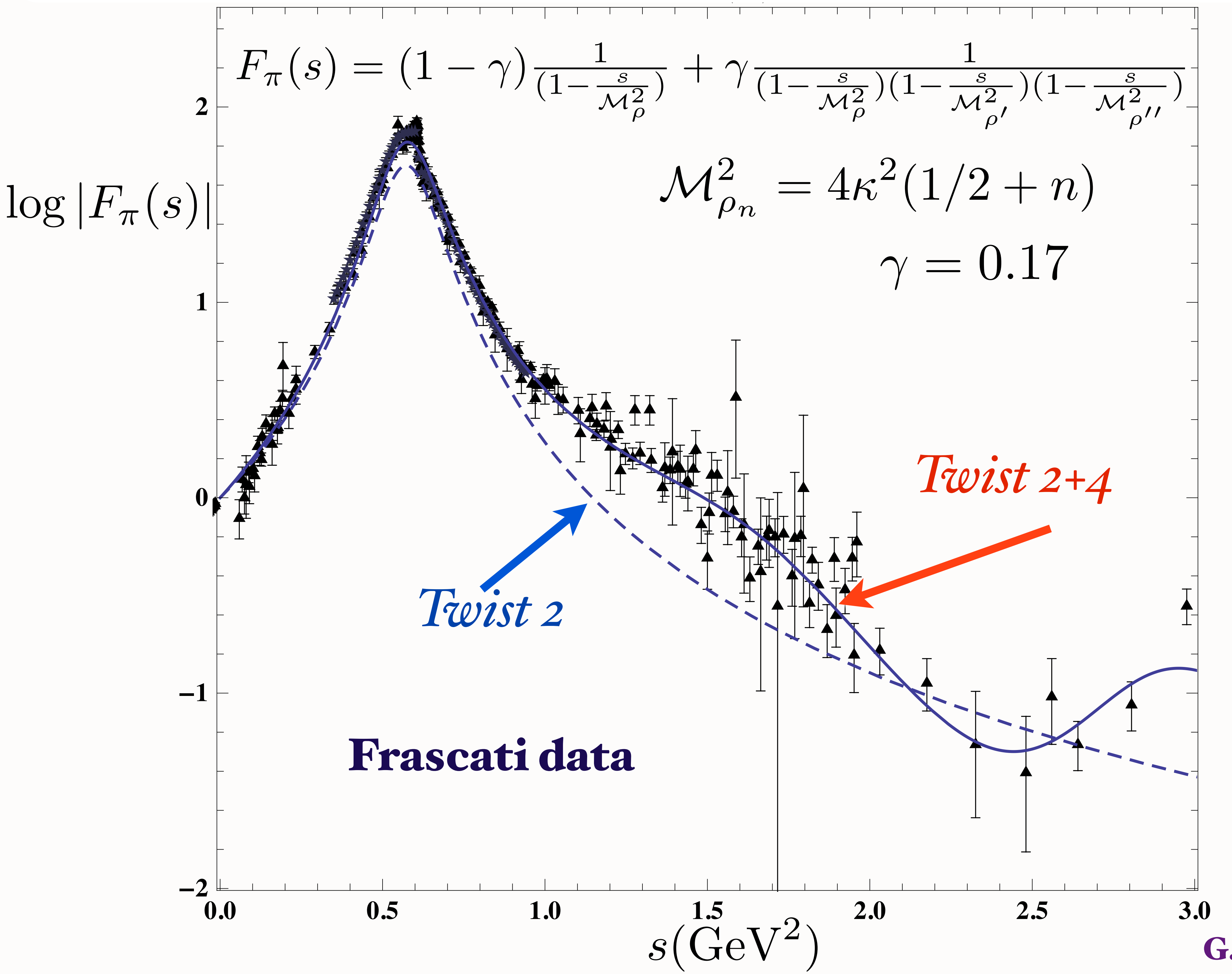
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \to q^2 + \sqrt{2}i\mathcal{M}\Gamma$ $\quad (P_{q\bar{q}q\bar{q}}$ = 13 %)





Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



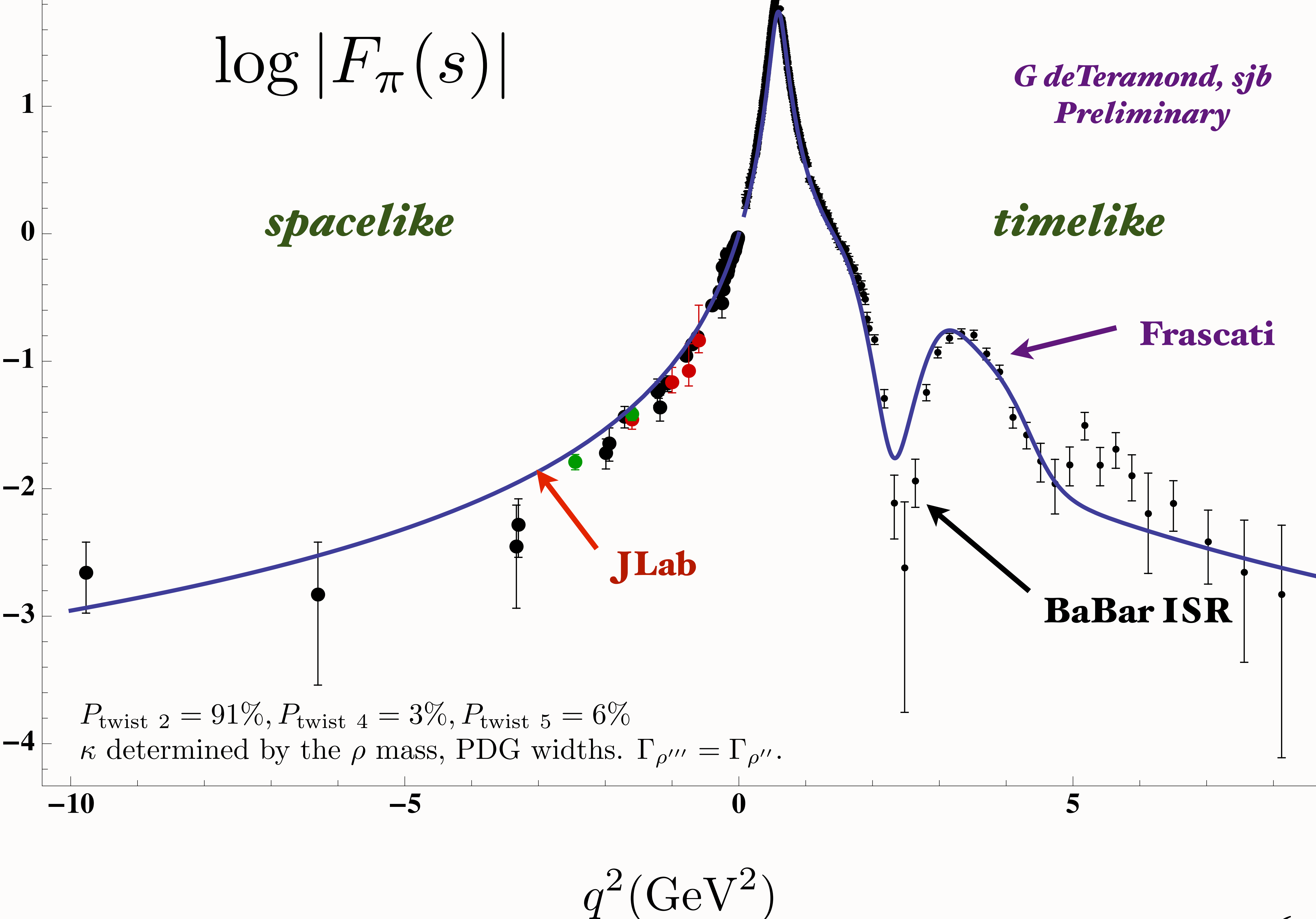
Prescription for Timelike poles :

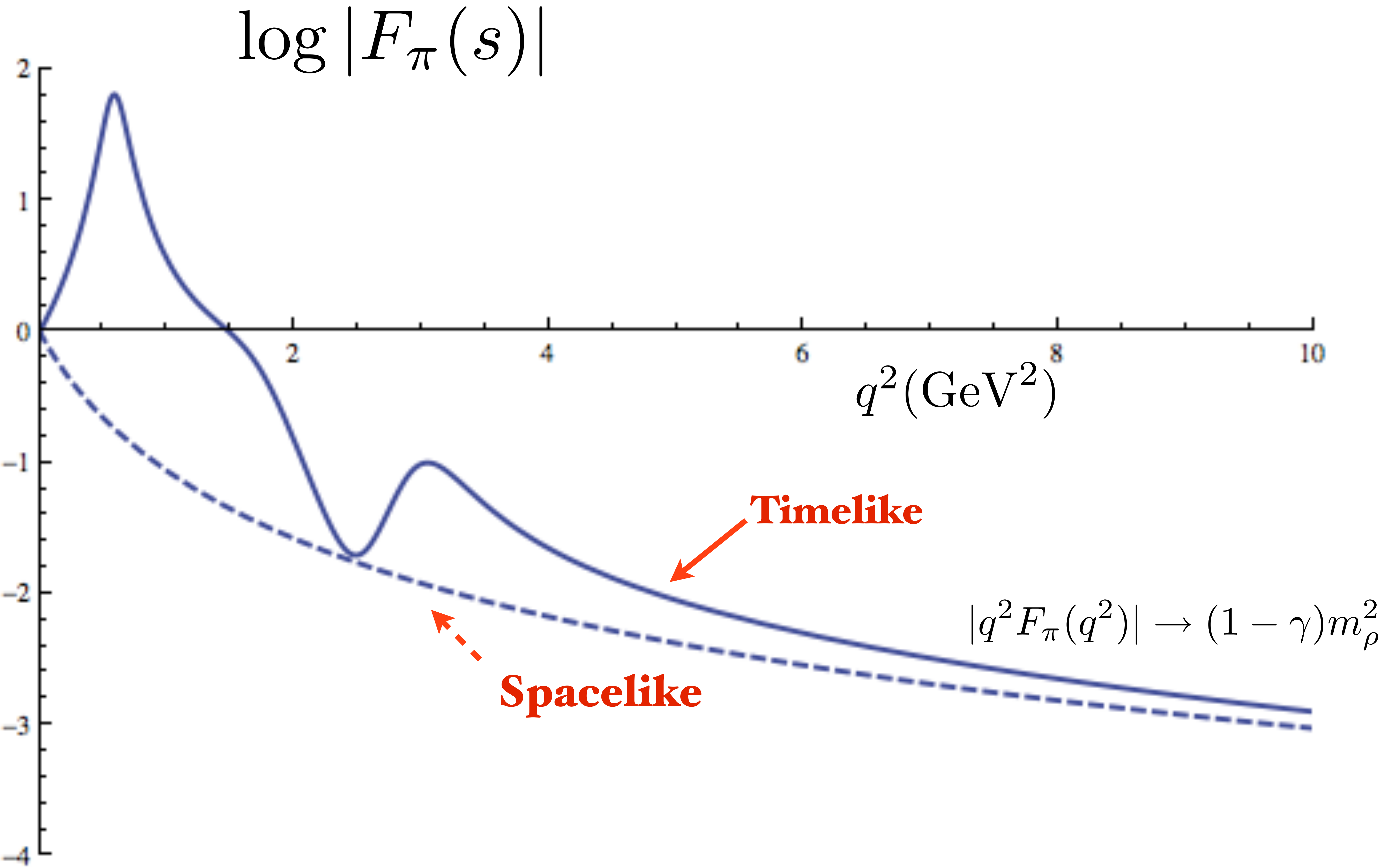
$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography





$\log|F_\pi(s)|$
2
1
0
−1
−2
−3
−4
2
4
6
8
10
$q^2(\mathrm{GeV}^2)$
Timelike
Spacelike
$|q^2 F_\pi(q^2)| \to (1-\gamma)m_\rho^2$

Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

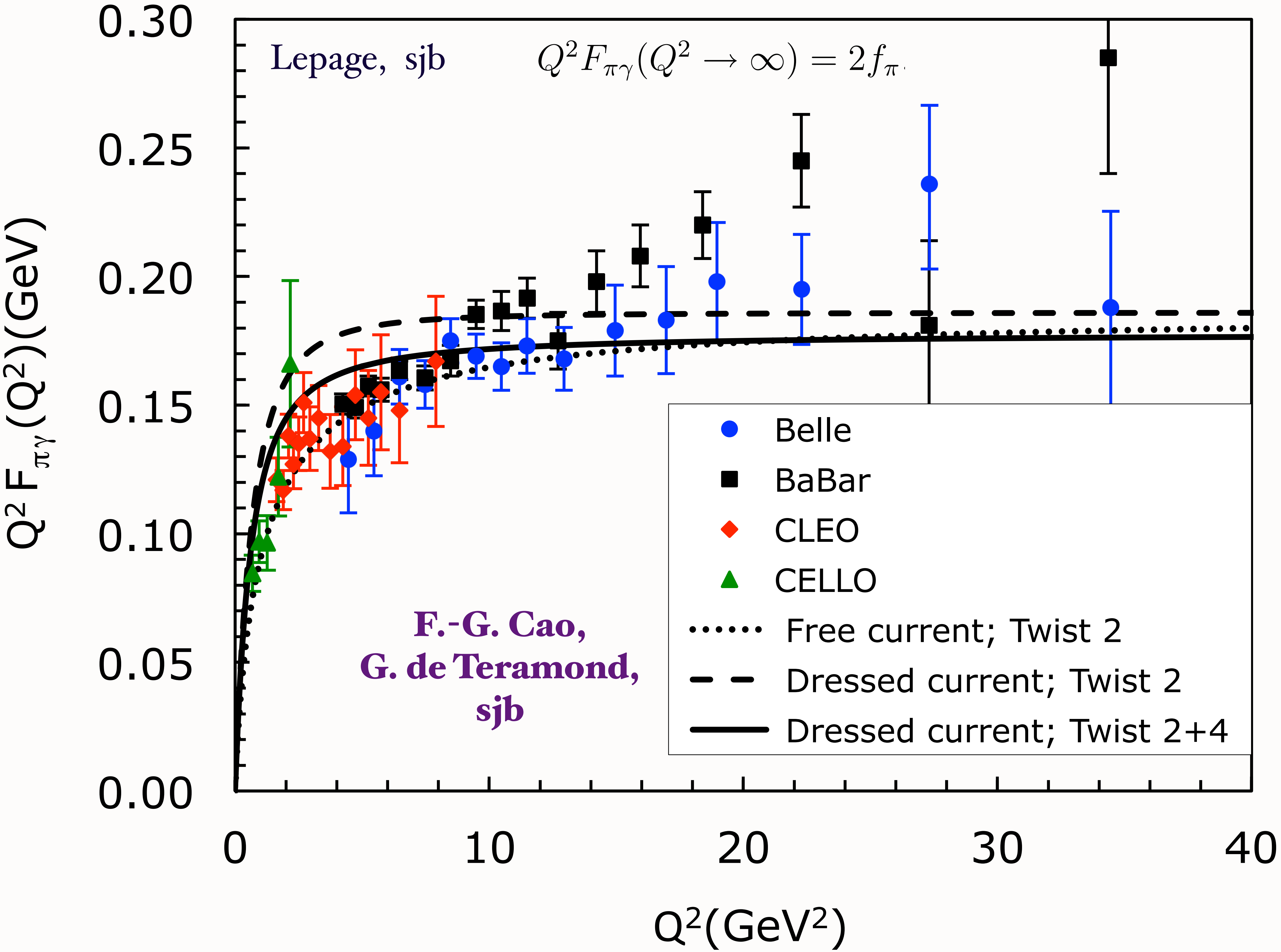
- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2 P_{q\overline{q}}} \, \kappa \, z^2 e^{-\kappa^2 z^2/2}, \qquad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\overline{q}}$
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\overline{q}}} \, \kappa/\sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\overline{q}} Q^2 (1-x)/4\pi^2 f_\pi^2 \, x}\right]$$

normalized to the asymptotic DA $[P_{q\overline{q}} = 1 \rightarrow$ Musatov and Radyushkin (1997)]

- Large Q^2 TFF is identical to first principles asymptotic QCD result $\quad Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

Photon-to-pion transition form factor



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

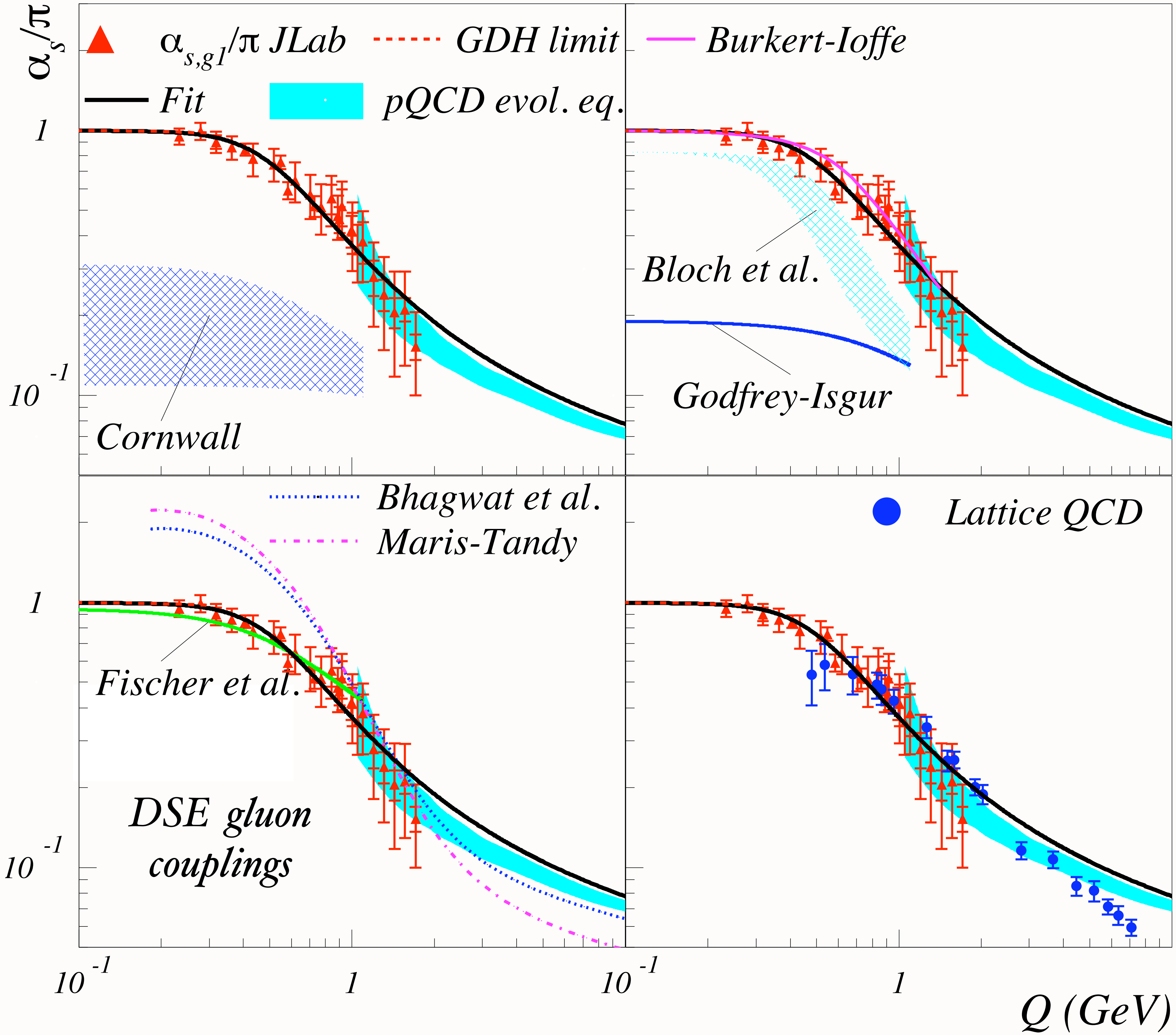
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

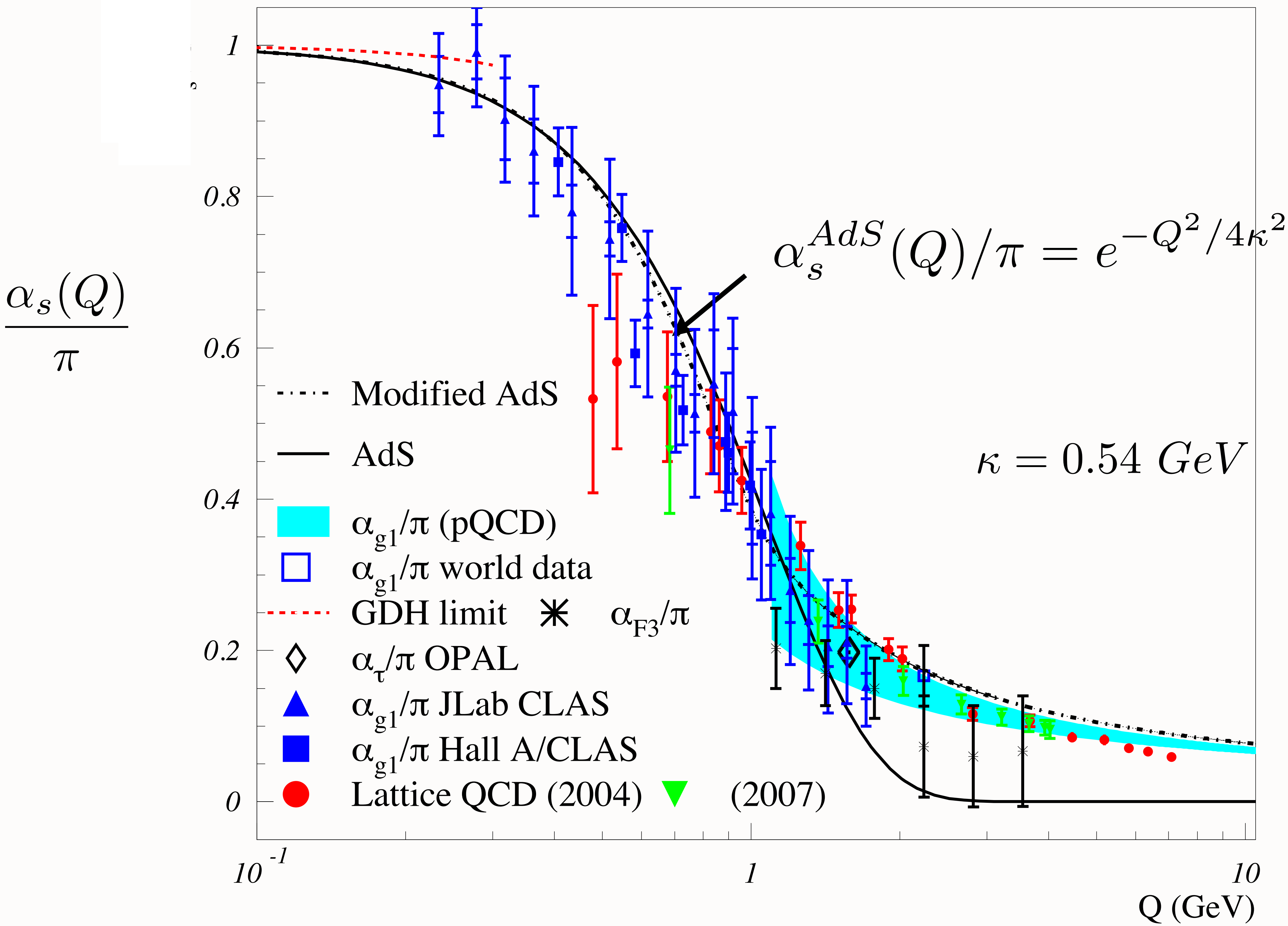
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Deur, Korsch, et al.



Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Gell-Mann Oakes Renner Formula in QCD

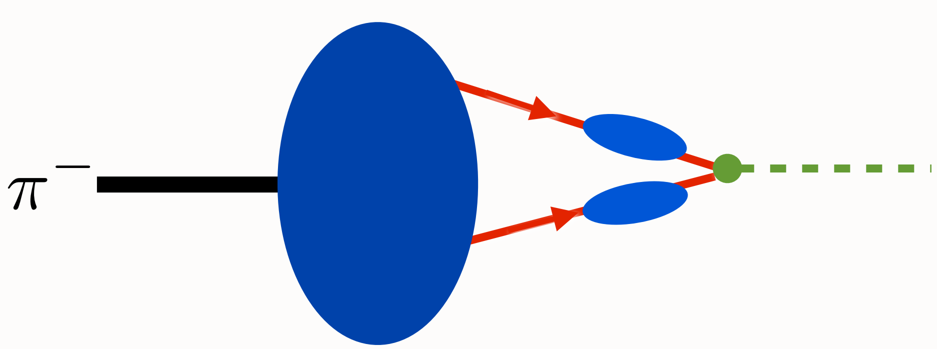
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is normal pion decay matrix element



$$< 0|\bar{q}\gamma_5 q|\pi >$$

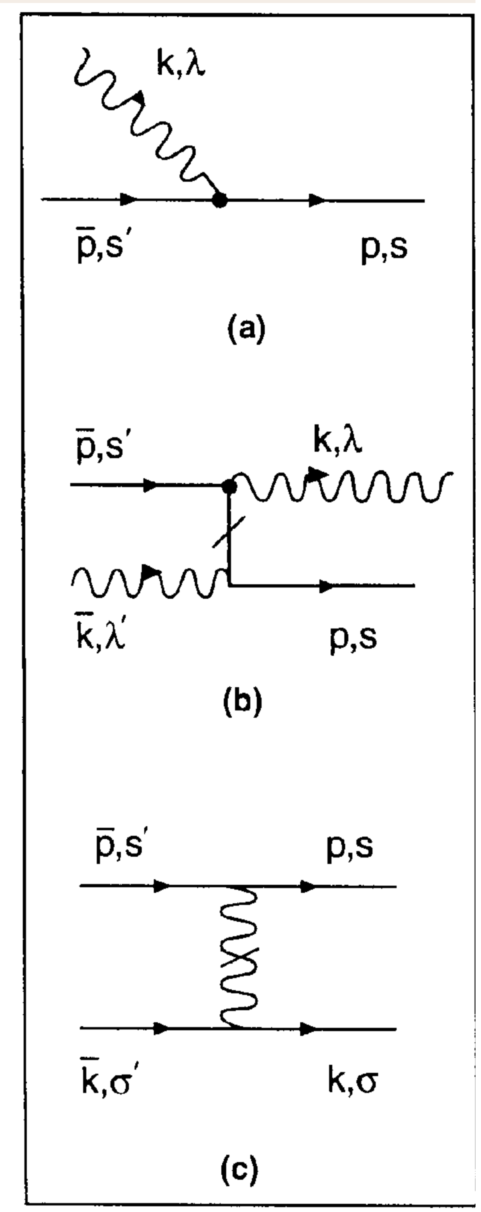
Maris, Roberts, Tandy

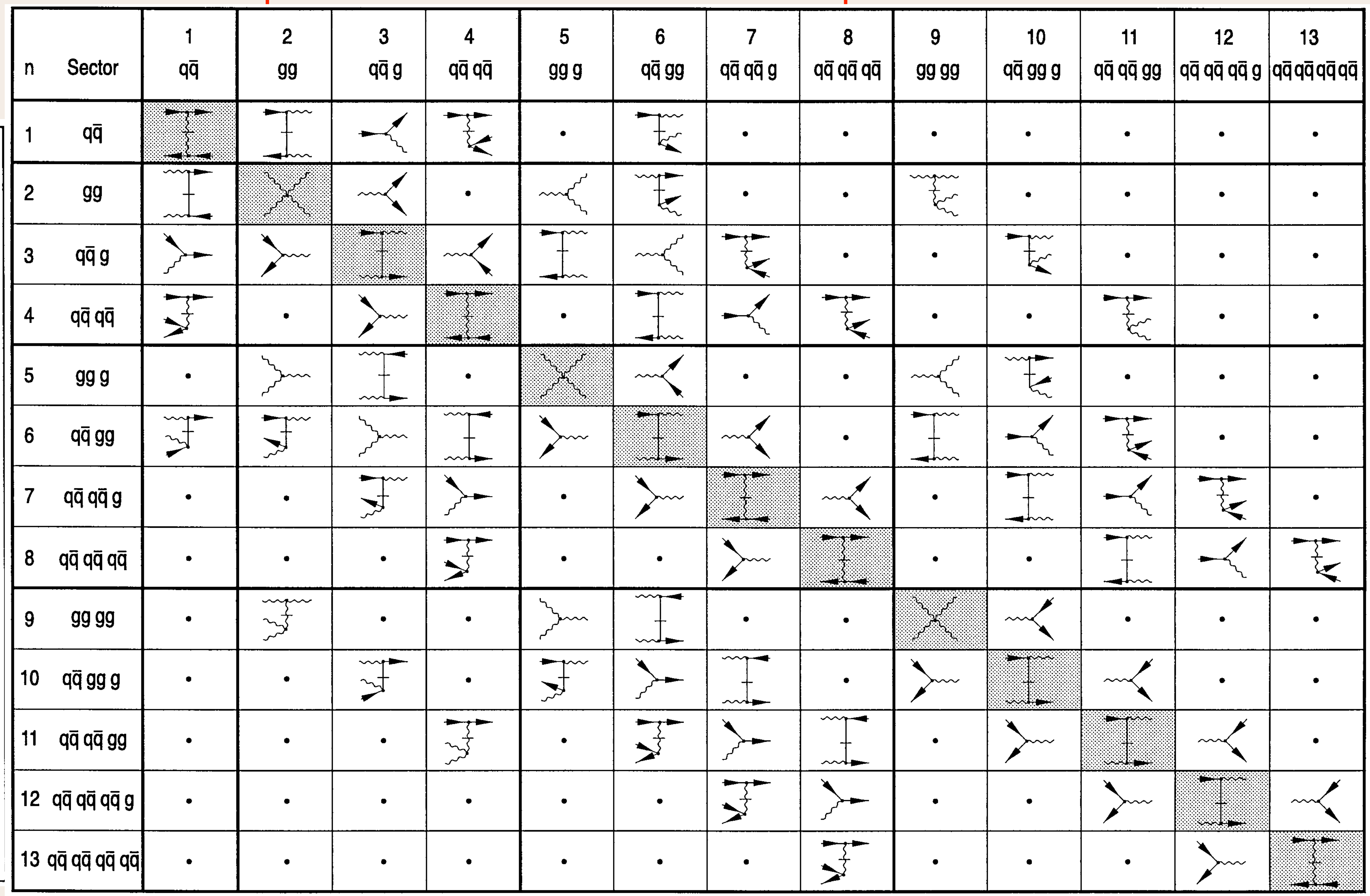
AdS/QCD and Light-Front Holography

- AdS/QCD: Incorporates scale transformations characteristic of QCD with a single scale -- RGE
- Light-Front Holography; unique connection of AdS_5 to Front-Form
- Profound connection between gravity in 5th dimension and physical 3+1 space time at fixed LF time τ
- Gives unique interpretation of z in AdS to physical variable ζ in 3+1 space-time

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\;|\Psi_h\rangle$$





BLFQ: Use AdS/QCD basis functions!

Basis Light-Front Quantization Approach to Quantum Field Theory

Use AdS/QCD orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

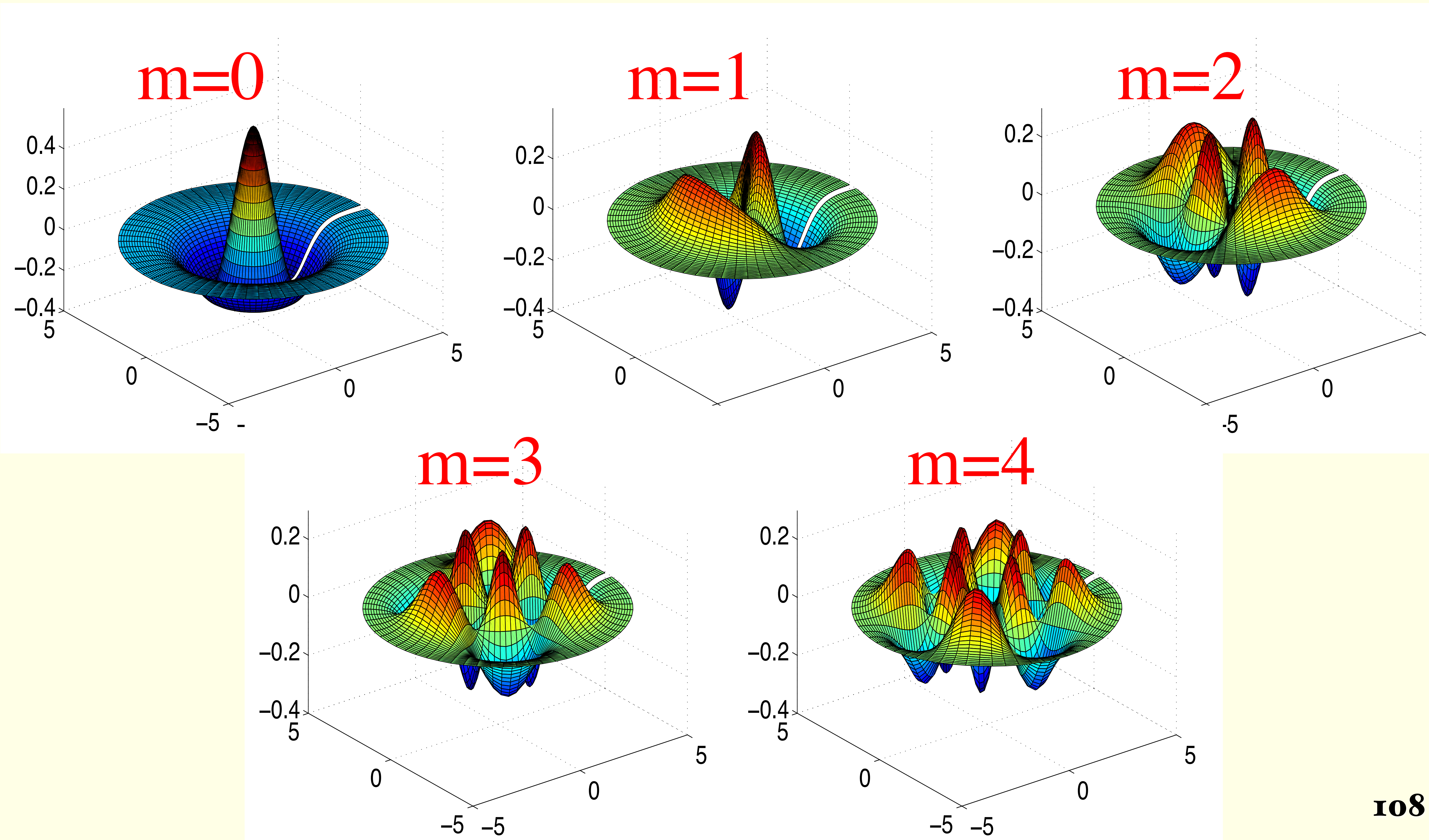
BLFQ

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**

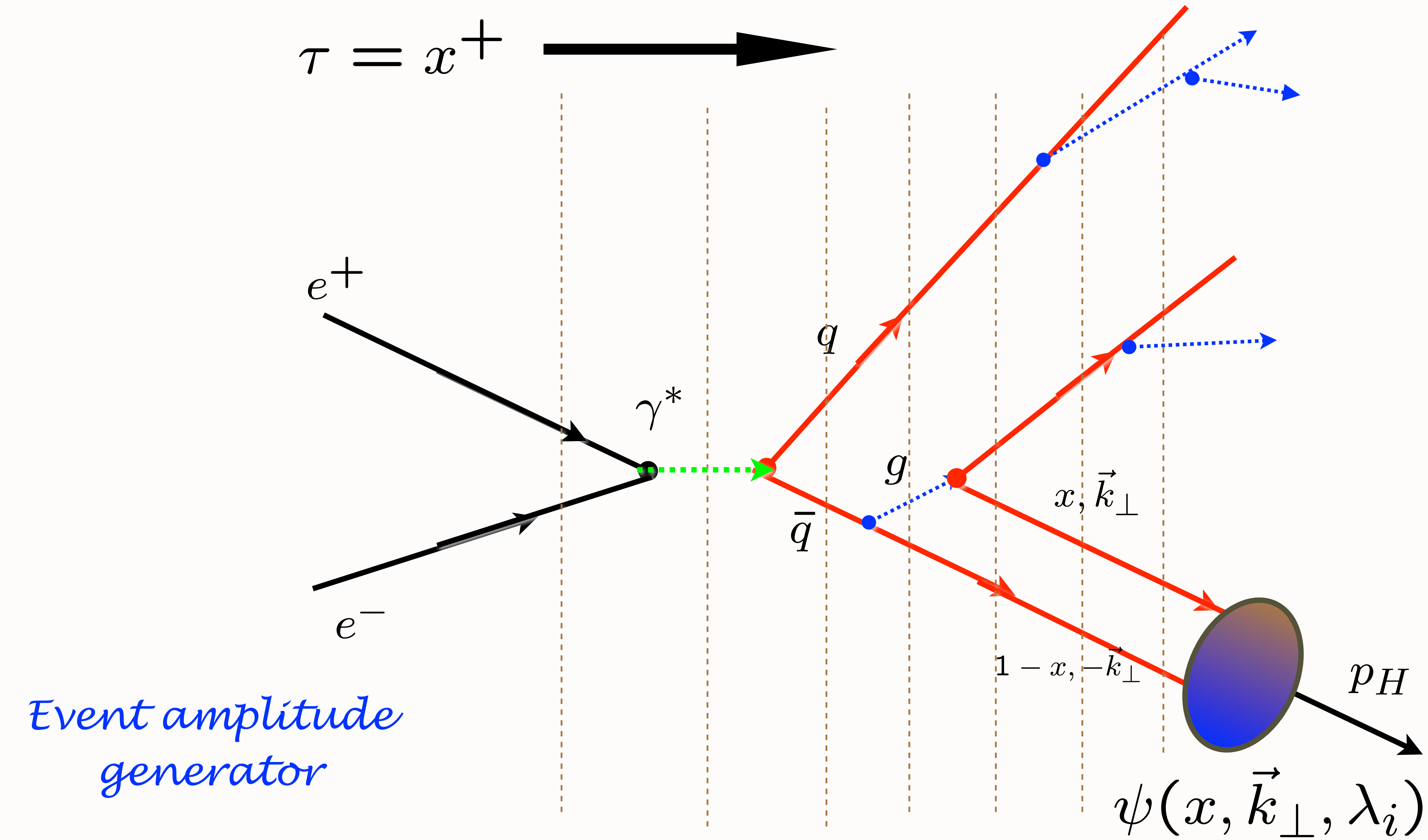
- Xingbo Zhao
- Anton Ilderton,
- Heli Honkanen
- Pieter Maris,
- James Vary
- Stan Brodsky

Set of transverse 2D HO modes for $n = 1$

J.P. Vary, H. Honkanen, Jun Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, PRC



Hadronization at the Amplitude Level

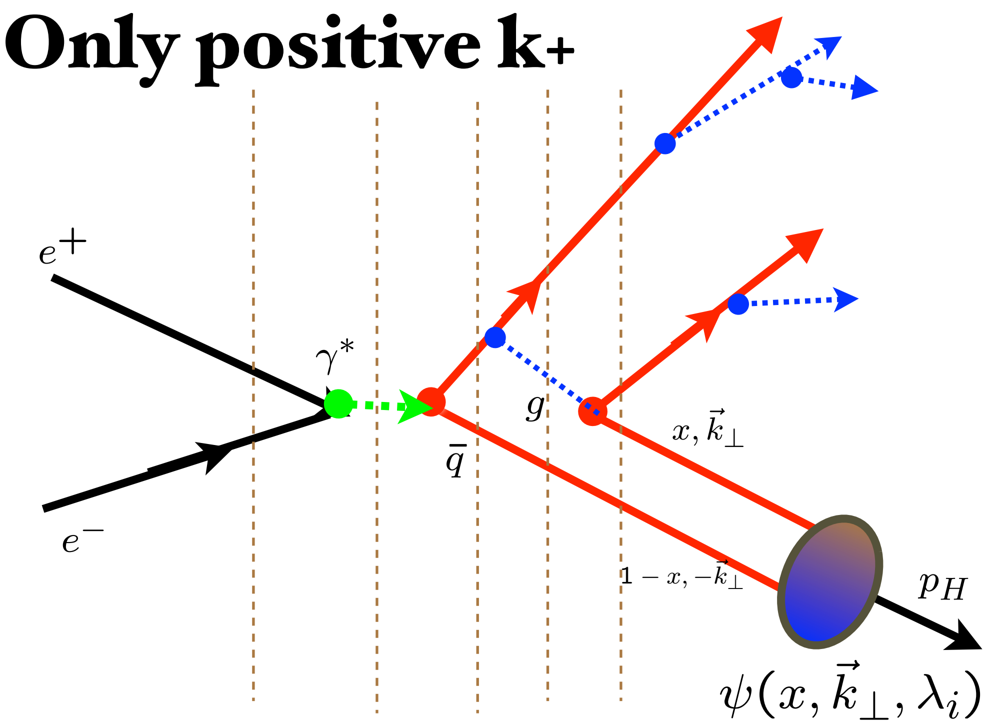


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Off -Shell T-Matrix

Event amplitude generator

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** **Chueng Ji, sjb**
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**

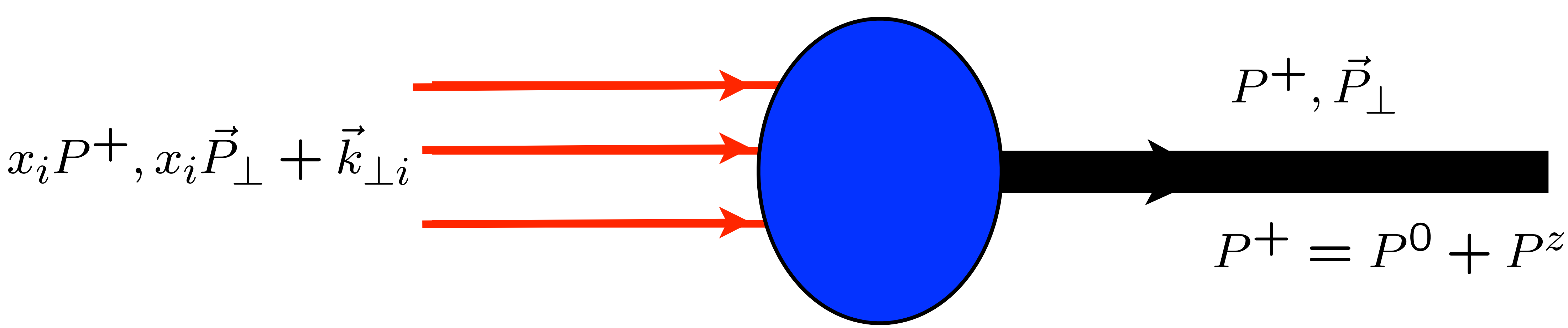


Roskies, Suaya, sjb

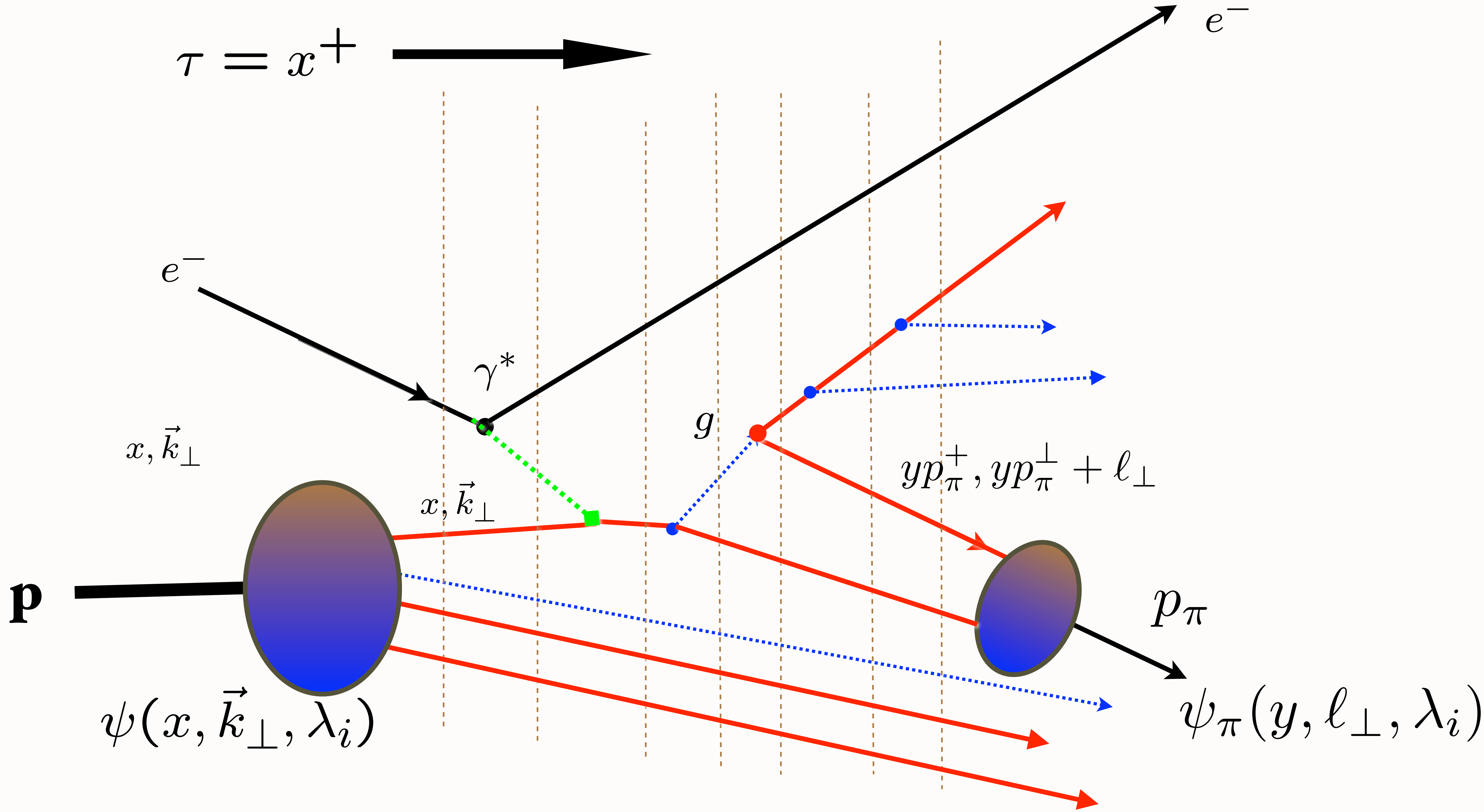
Features of LF T-Matrix Formalism

"Event Amplitude Generator"

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

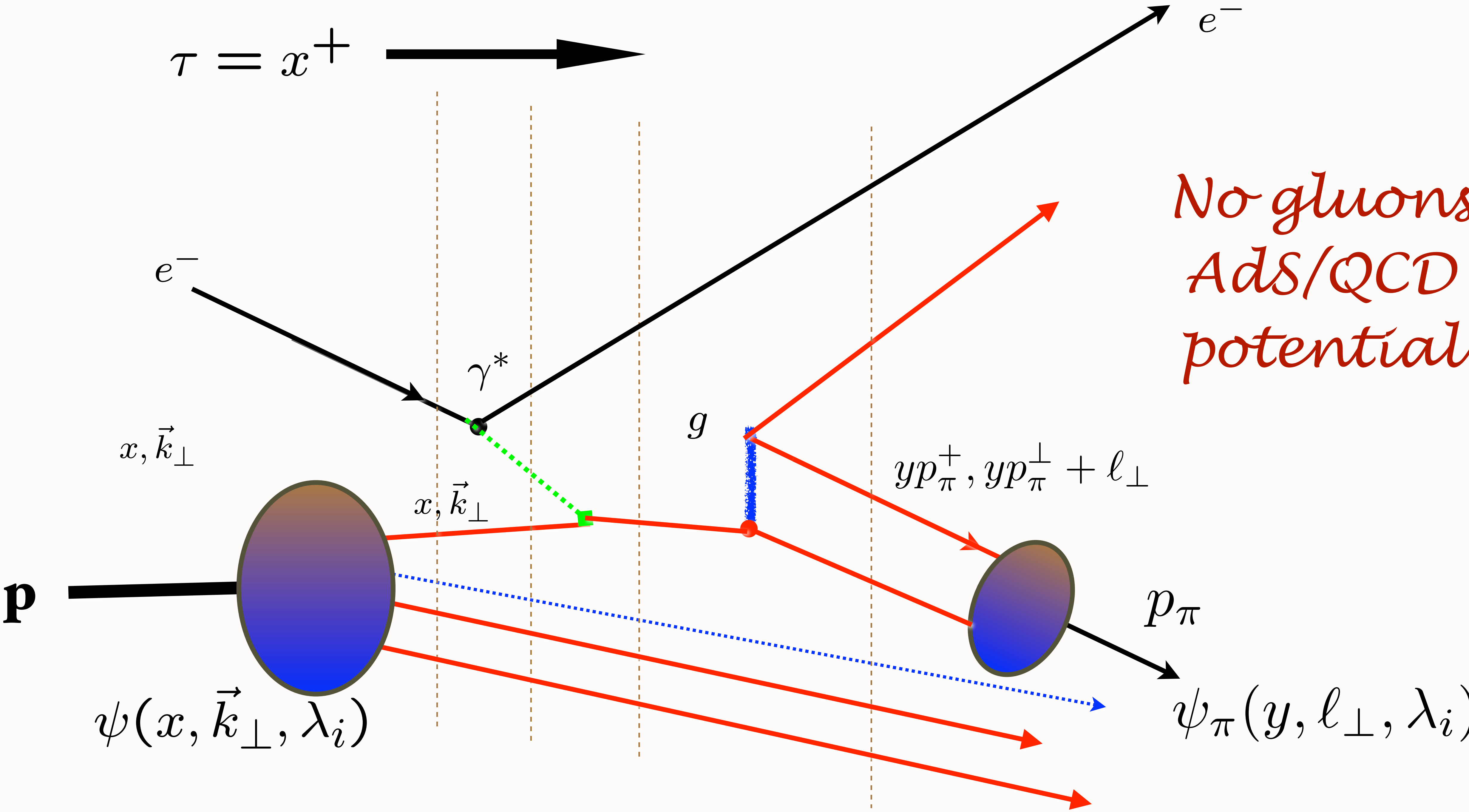


Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



No gluons
AdS/QCD
potential

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Only Hadrons can Appear!

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

Light by Light Loops

BLM/PMC: Absorb β-terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

↓

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

↓

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

$through\ the\ PMC - BLM\ correspondence\ principle$

↓

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\} - terms$

↓

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

Same as QED Scale Setting

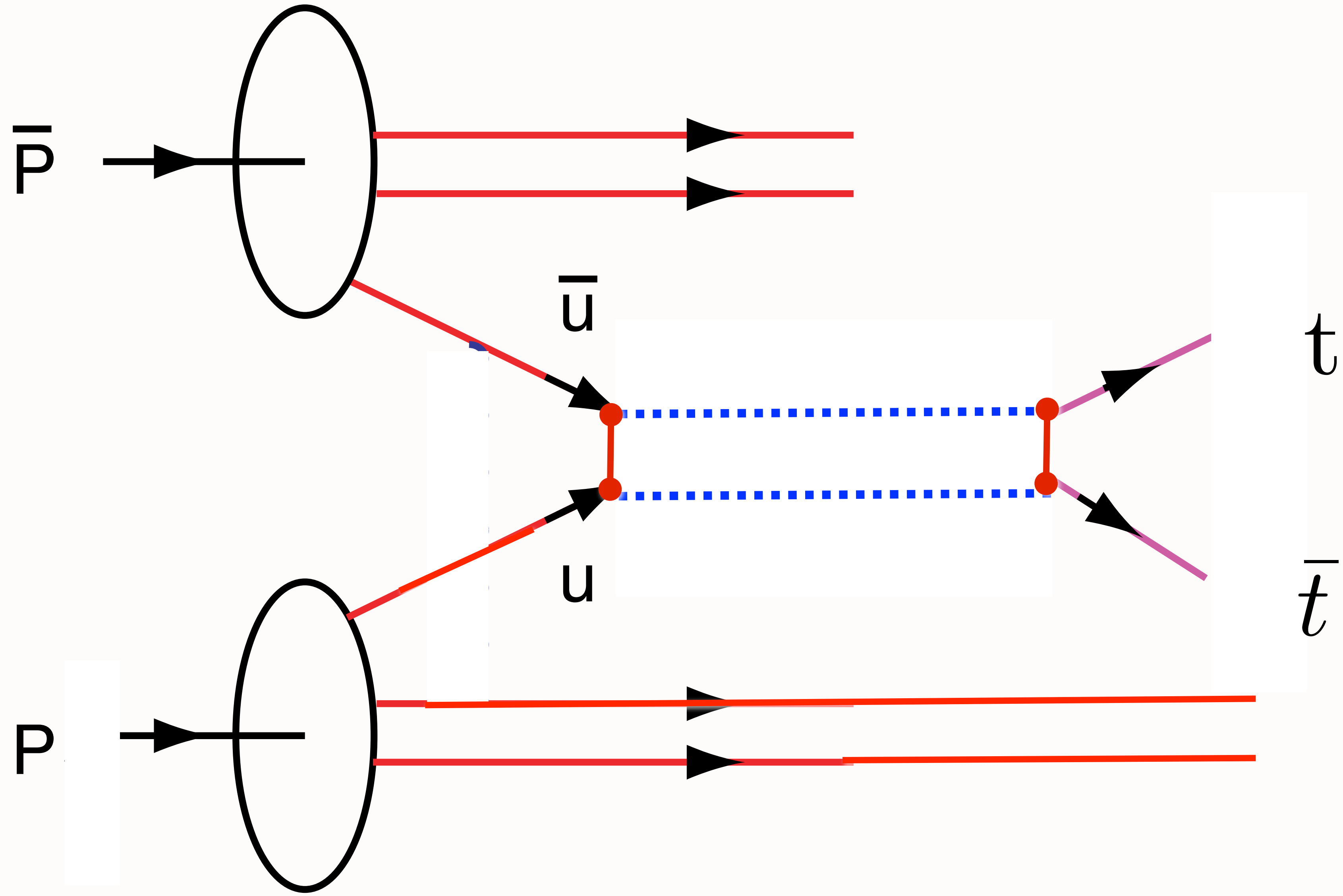
Apply to Evolution kernels, hard subprocesses

Eliminates unnecessary systematic uncertainty

δ-scheme automatically identifies QCD β function terms

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

Contributes to the $\bar{p}p \to \bar{t}tX$ asymmetry at the Tevatron

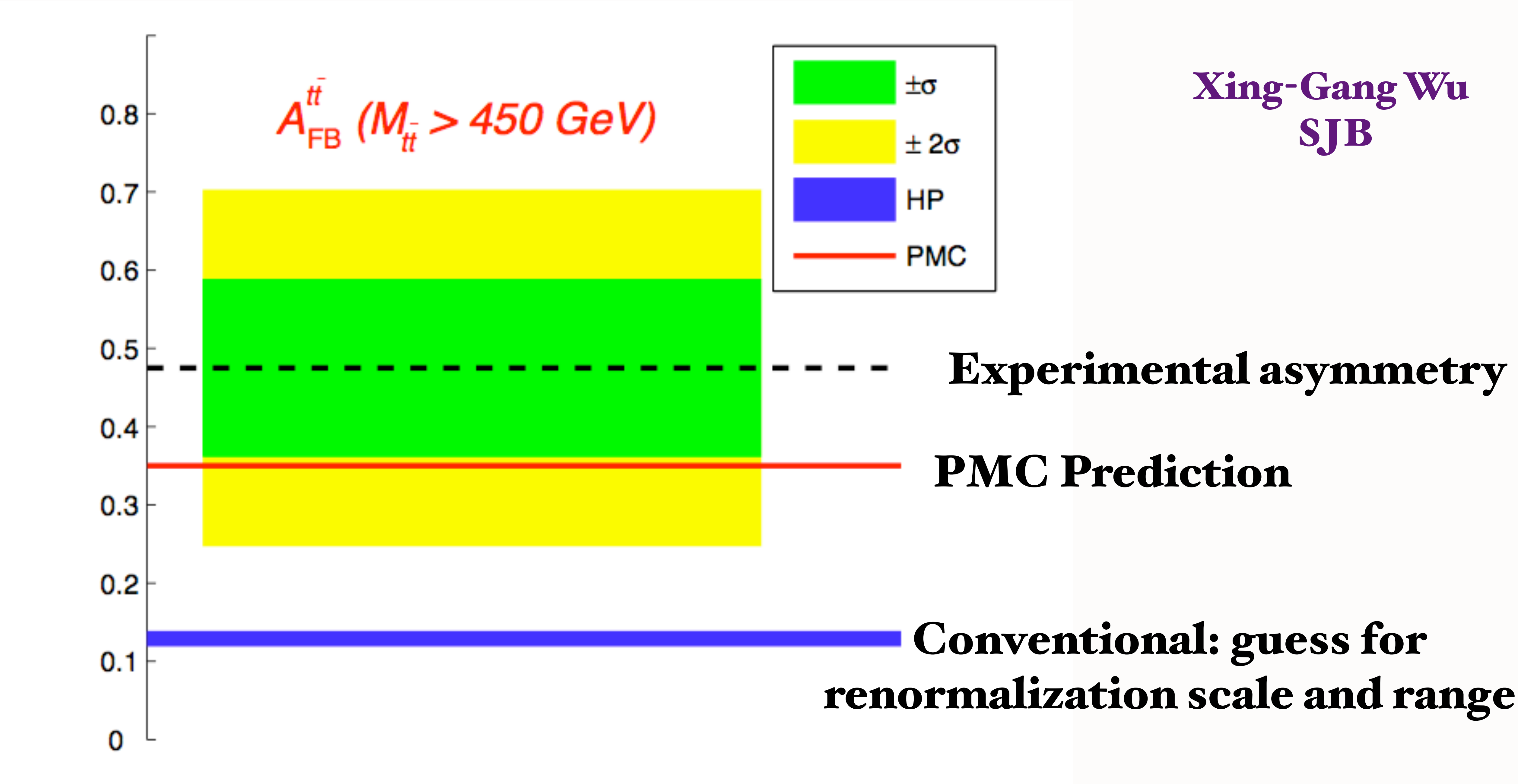


Interferes with Born term.

Small value of renormalization scale increases asymmetry

Xing-Gang Wu, sjb

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the 'Principle of Maximum Conformality' (PMC)



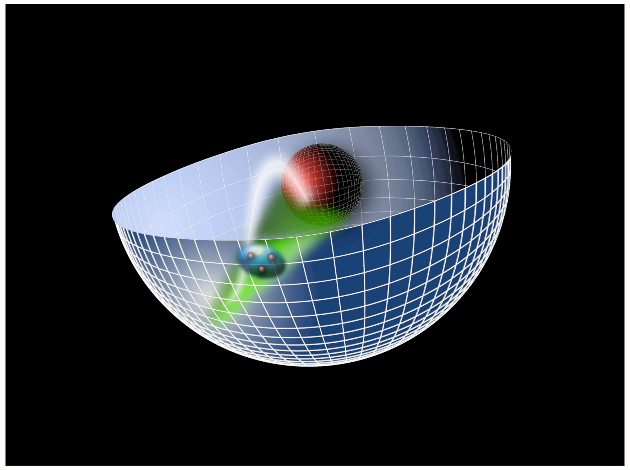
Xing-Gang Wu
SJB

$t\bar{t}$ asymmetry predicted by pQCD NNLO within $1\ \sigma$ of CDF/D0 measurements using PMC/BLM scale setting

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$$1/\kappa \simeq 0.4\ fm$$

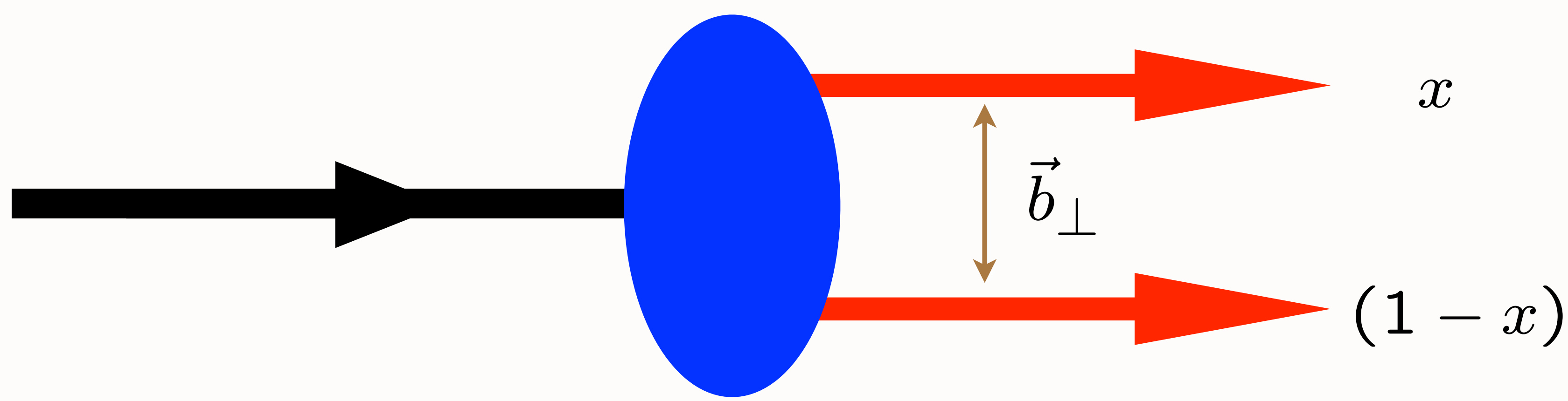
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holography

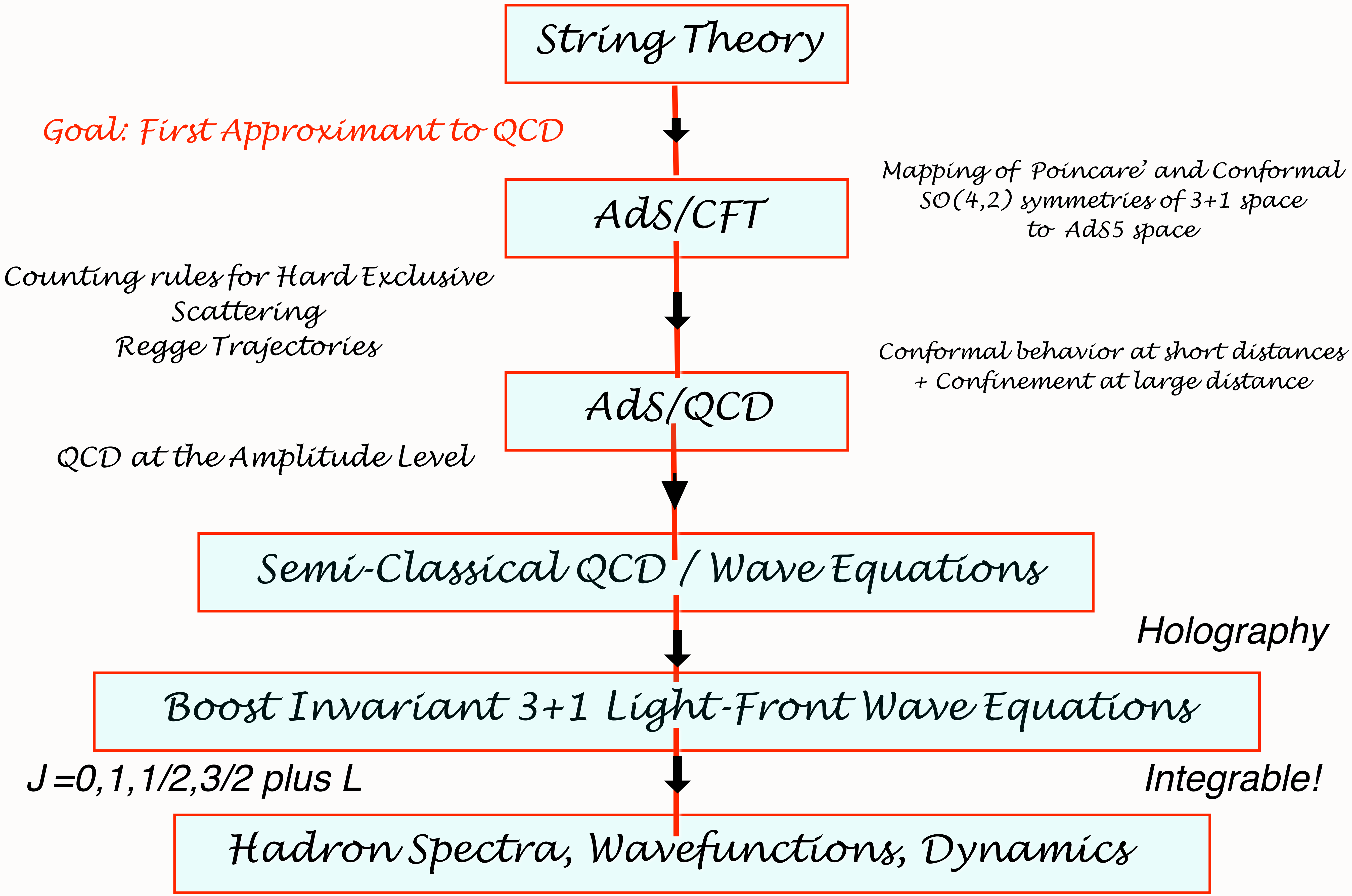
$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors



String Theory
Goal: First Approximant to QCD
AdS/CFT
Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Counting rules for Hard Exclusive Scattering
Regge Trajectories
AdS/QCD
Conformal behavior at short distances + Confinement at large distance
QCD at the Amplitude Level
Semi-Classical QCD / Wave Equations
Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics

Light-Front Schrödinger Equation

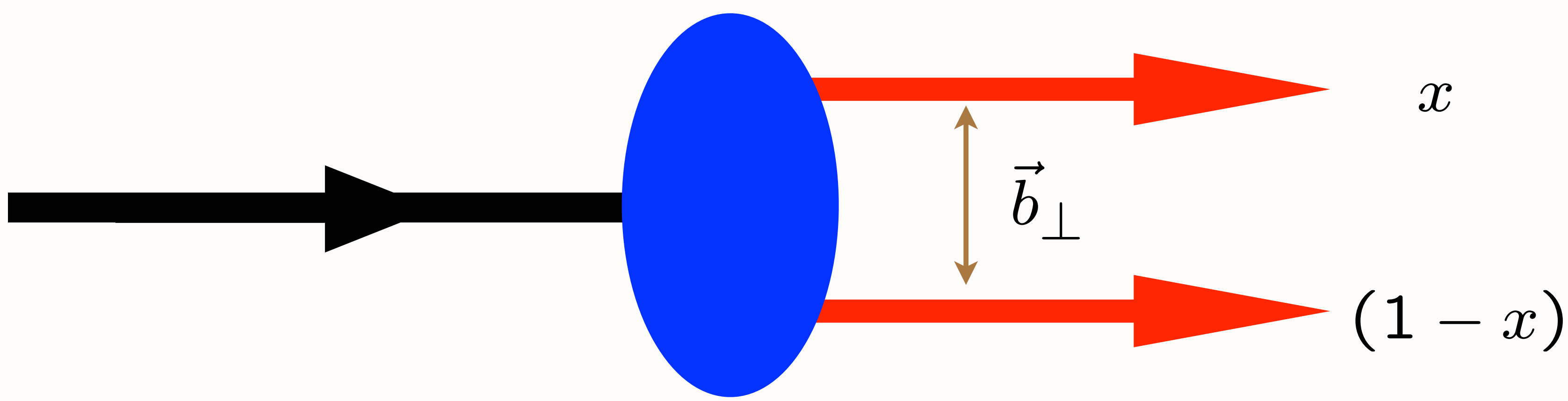
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

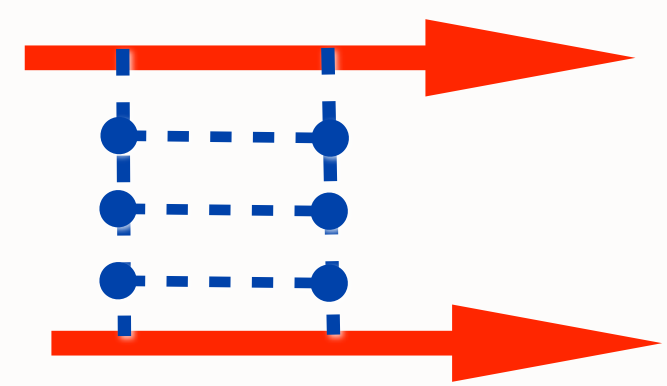
$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2, J, L, M^2)]\Psi_{J,L}(\zeta^2) = M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

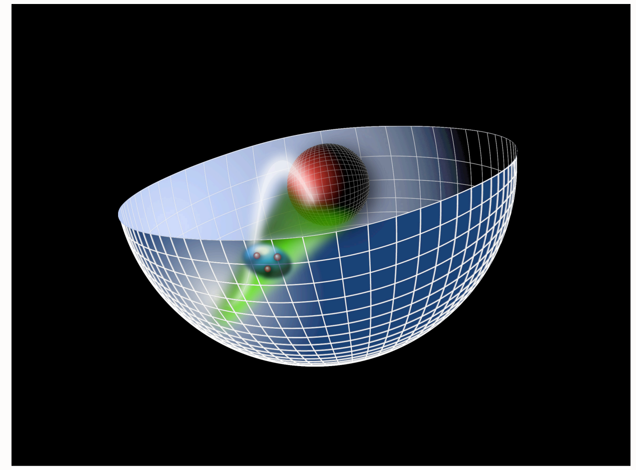


U is the exact QCD potential
Conjecture: 'H'-diagrams generate

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$



AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\psi(\zeta)=\mathcal{M}^2\psi(\zeta)$$

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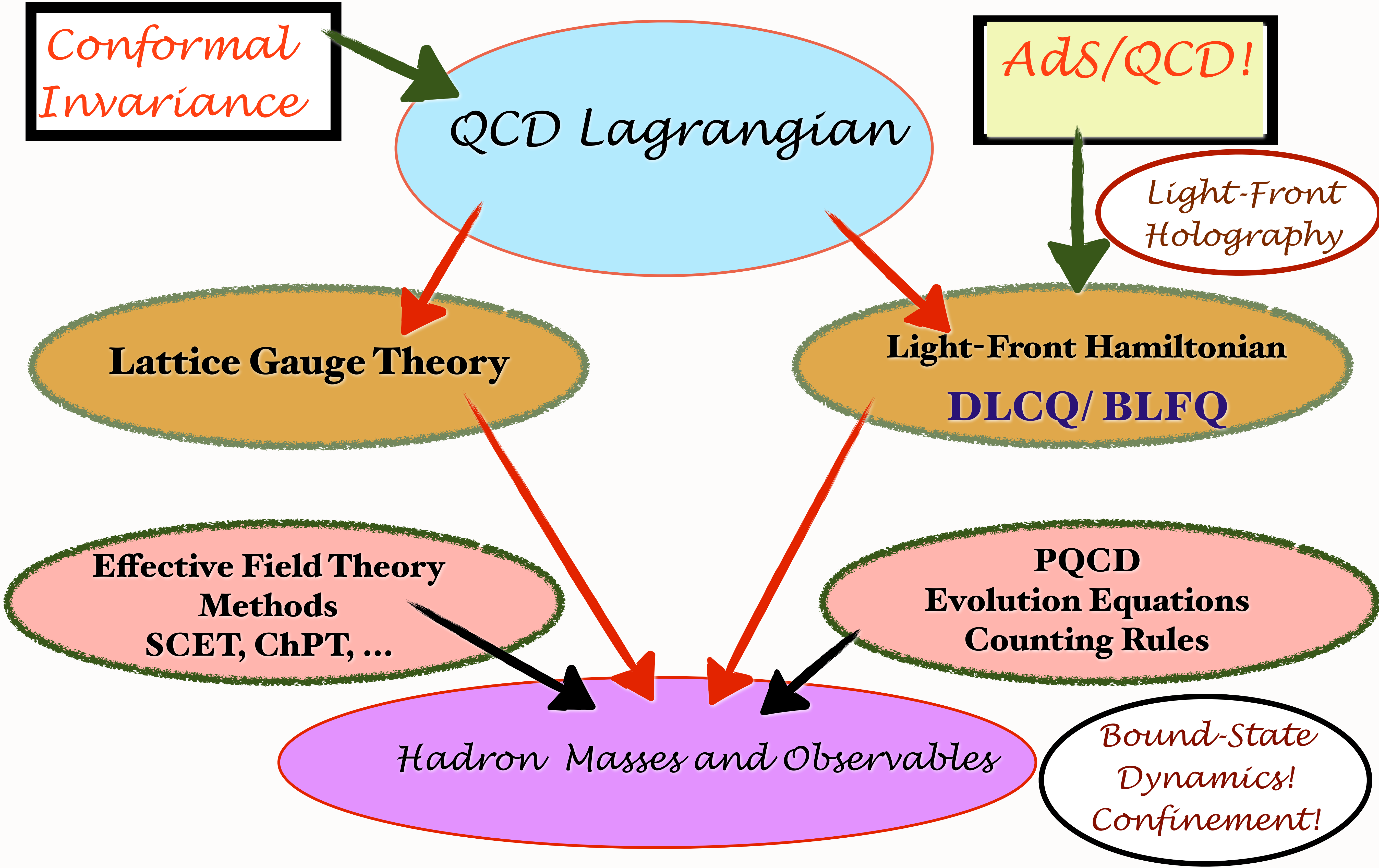
Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5 \; GeV$

$1/\kappa \simeq 0.4 \;\; fm$

Predict Hadron Properties from First Principles!



Basis Light-Front Quantization Approach to Quantum Field Theory

Use AdS/QCD orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

BLFQ

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**

- Xingbo Zhao
- Anton Ilderton,
- Heli Honkanen
- Pieter Maris,
- James Vary
- Stan Brodsky

Solving nonperturbative QCD using the Front Form

- **Heisenberg: Diagonalize the QCD LF Hamiltonian** *Hornbostel, Pauli, sjb*
- **DLCQ: Complete solutions QCD(1+1): any number of colors, flavors, quark masses**
- **AdS/QCD and Light-Front Holography: Soft-Wall Model predicts light-quark spectrum and dynamics** *de Teramond, sjb*
- **BFLQ: Use AdS/QCD orthonormal basis functions** *Vary, Maris. et al*
- **RGPEP: Systematically reduce off-diagonal elements; RG equations which evolve LFQCD in scale** *Glazek*
- **Reduce QCD to equation for LF valence state with effective potential** *Pauli*
- **Reduce QCD to one dimensional LF Schrödinger Equation in radial coordinate conjugate to the invariant mass.** *de Teramond, sjb*
- **Lippmann-Schwinger expansion in $\Delta U = U_{QCD} - U_{AdS}$** *Hiller-sjb*
- **Cluster expansion methods** *Hiller-Chabysheva*

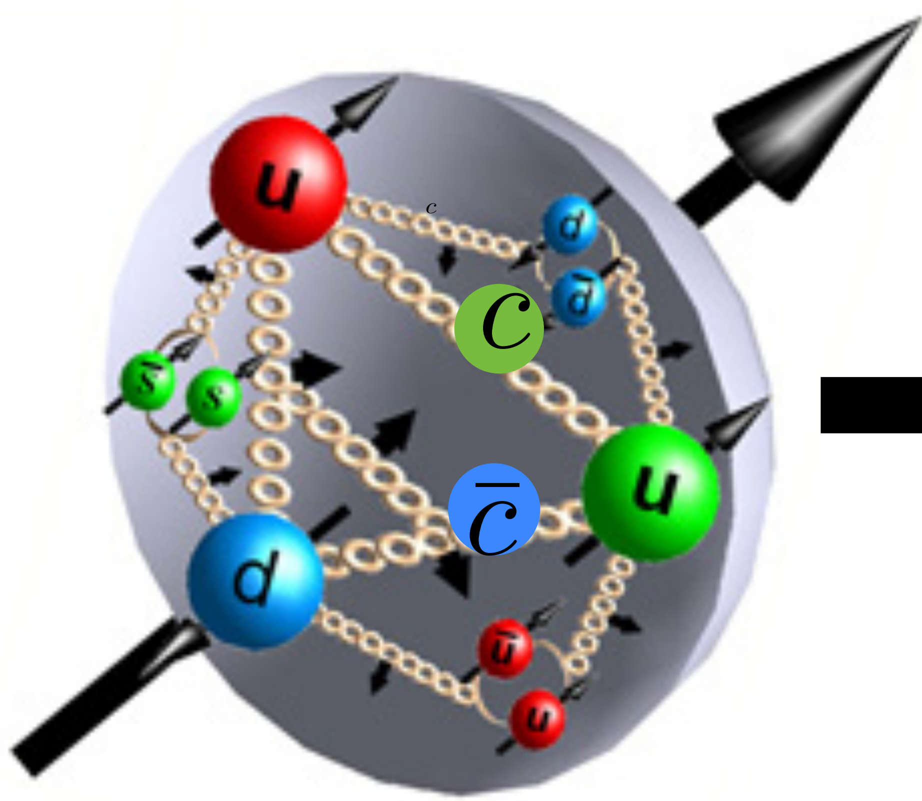
Features of AdS/QCD LF Holography

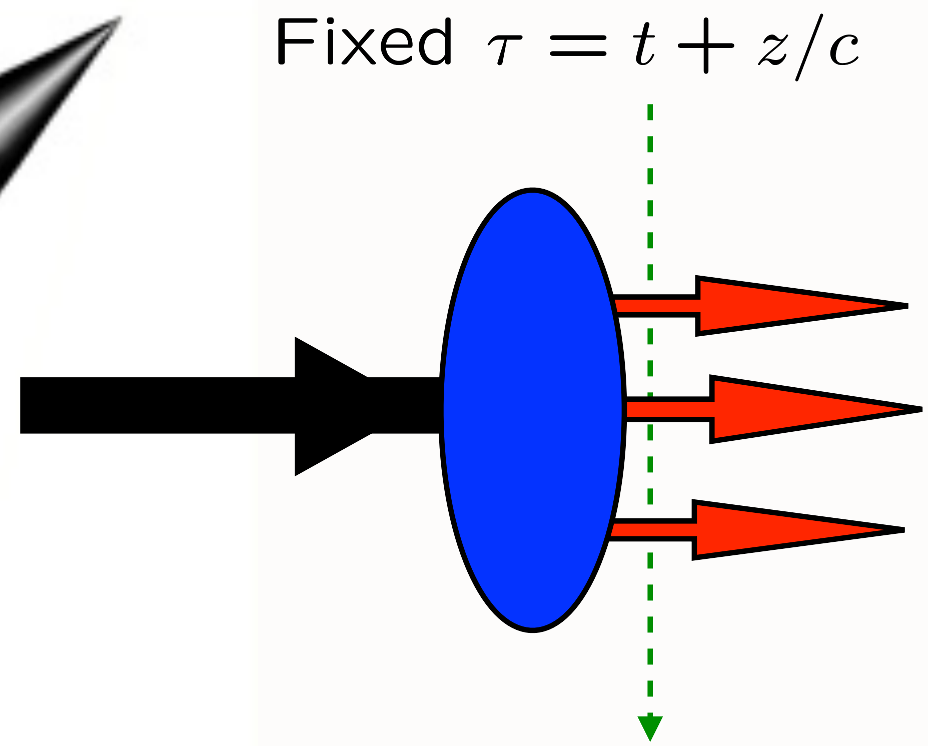
- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on q +(qq) -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

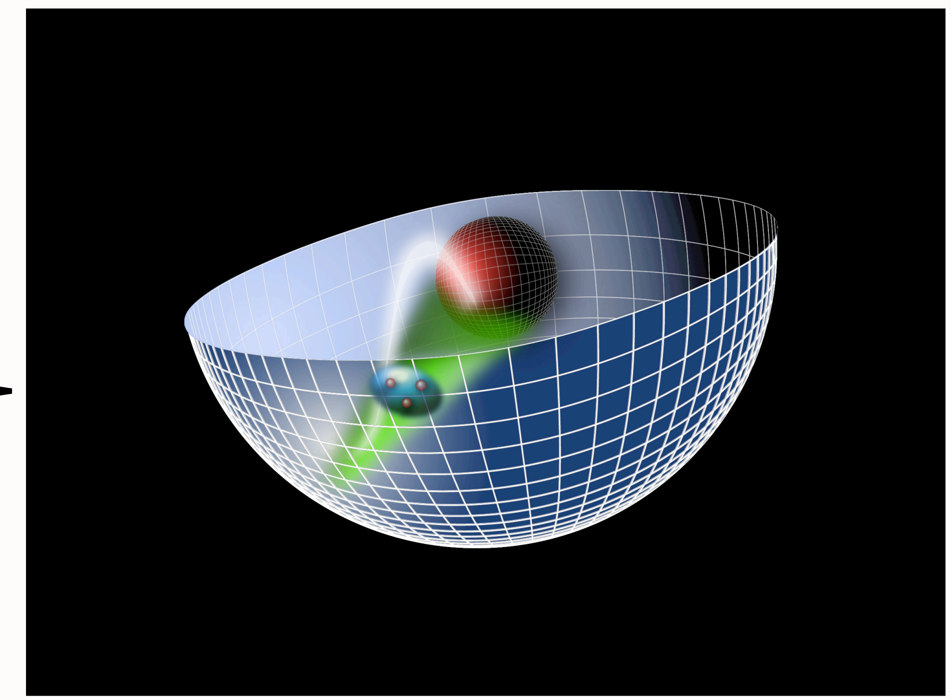
New Directions

- **Hadronization at the Amplitude Level**
- **LF Confinement potential and LFWFs predicted**
- **Eliminate Factorization Scale: Fracture function determines off-shellness**
- **Eliminate Renormalization Scale Ambiguity: Principle of Maximal Conformality (PMC)**
- **Exclusive Channels: PQCD Gluon exchange versus Soft Interactions**
- **Different mechanisms at x \to 1 and high k_\perp**
- **Massive quark spectroscopy**
- **Sublimated Gluons: Gluons appear at high virtuality**
- **Hidden Color of Nuclear Wavefunctions**
- **Duality: connection to DIS at high x**

AdS/QCD and Light-Front Holography







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LC 2013
May 21, 2013
Skiathos, Greece