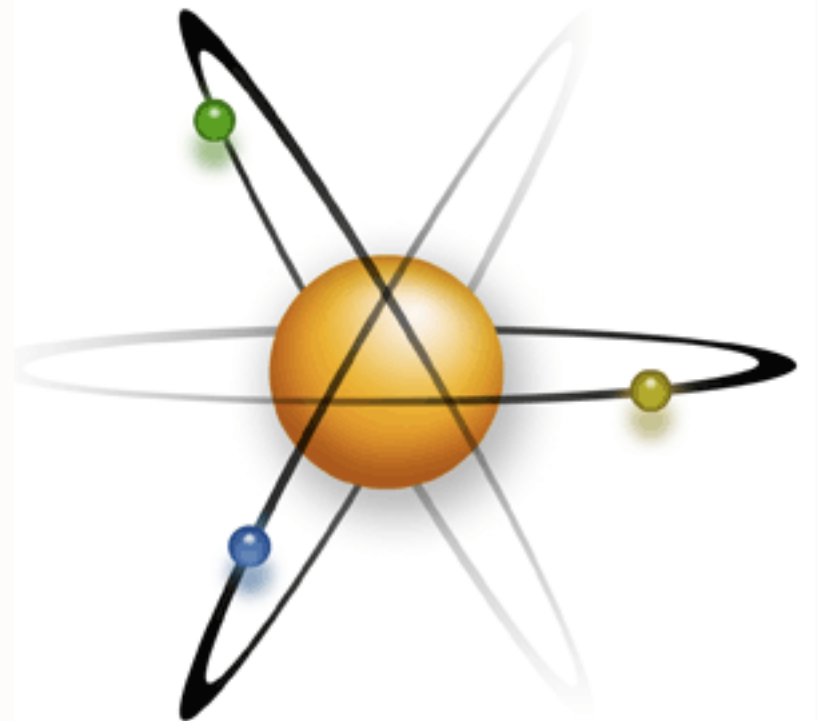


Atoms in Motion



***The remarkable connections between
atomic and hadronic physics***

Stan Brodsky

SLAC National Accelerator Laboratory

Stanford University

University of Rochester

February 8, 2012

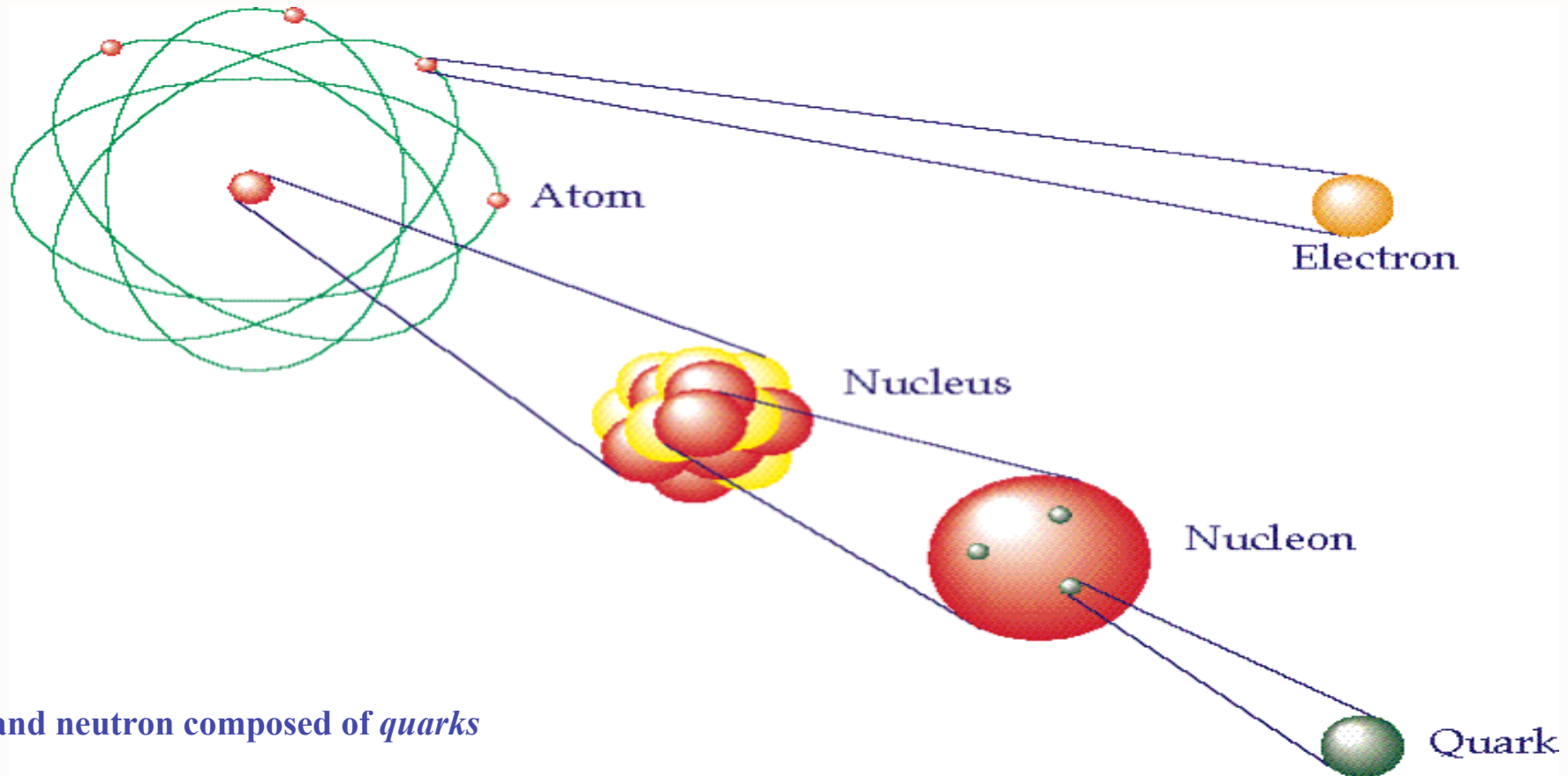


UNIVERSITY of
ROCHESTER

SLAC
NATIONAL ACCELERATOR LABORATORY

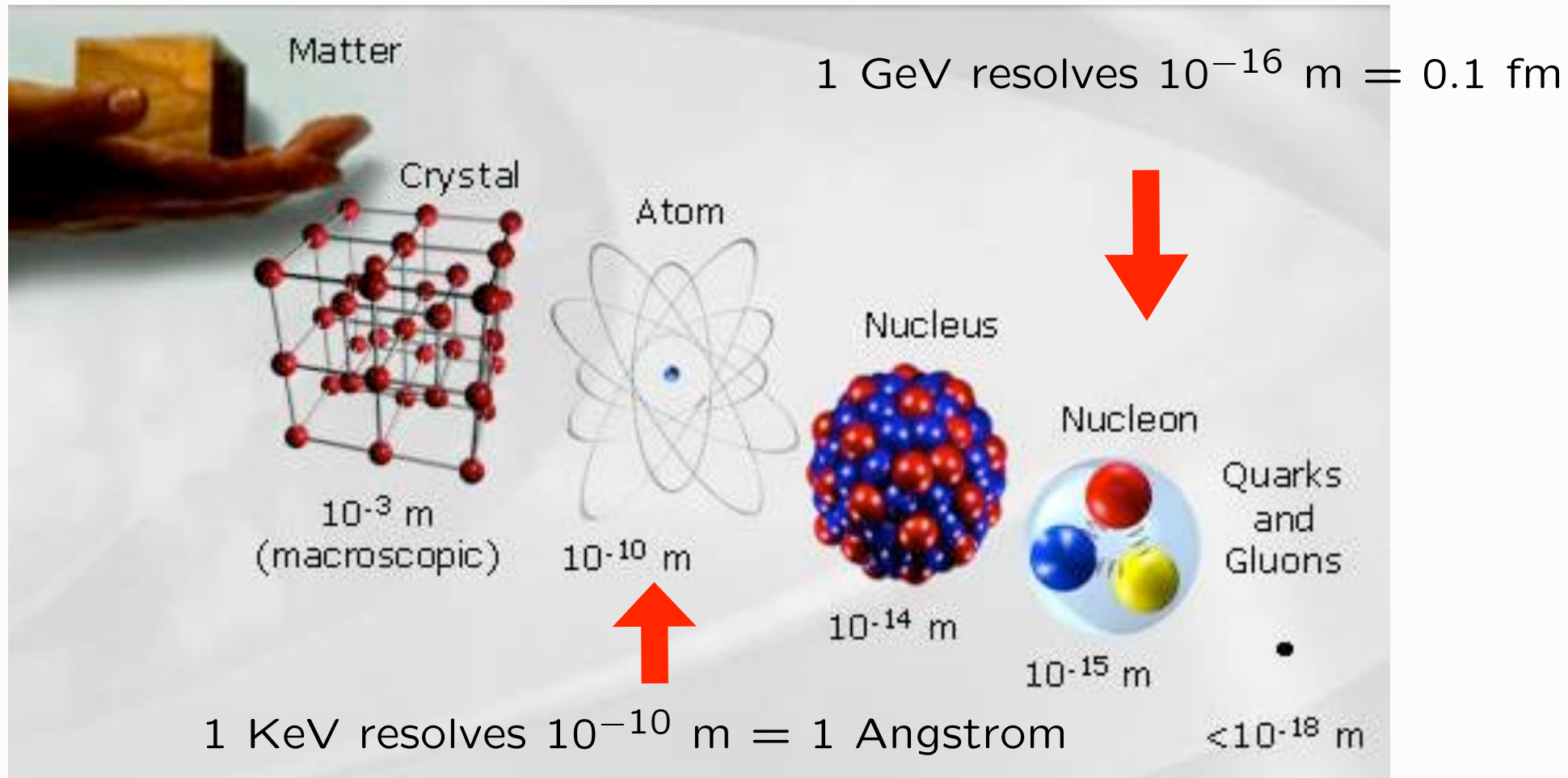
Goal of Science:

To understand the laws of physics and the fundamental composition of matter at the shortest possible distances.



- Proton and neutron composed of *quarks*
- Nuclei composed of protons and neutrons
- Atoms composed of nuclei and electrons ...

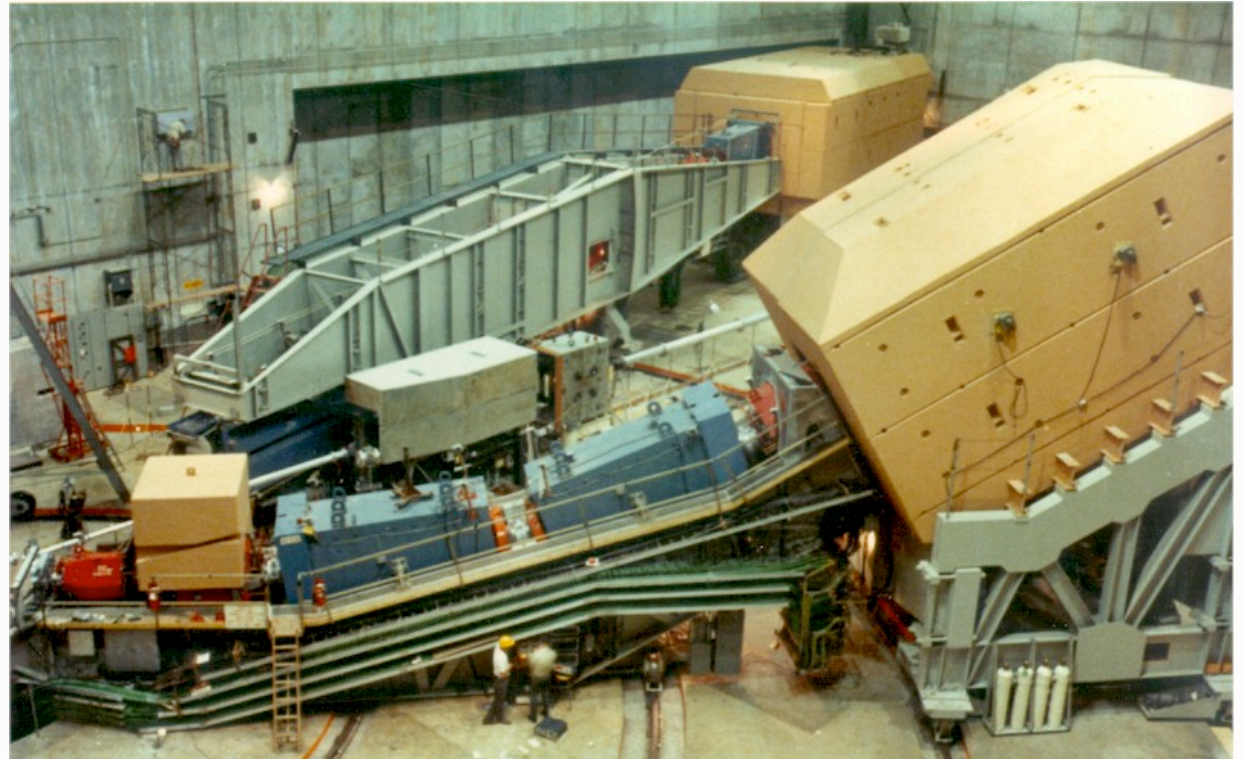
Searching for the Ultimate Constituents



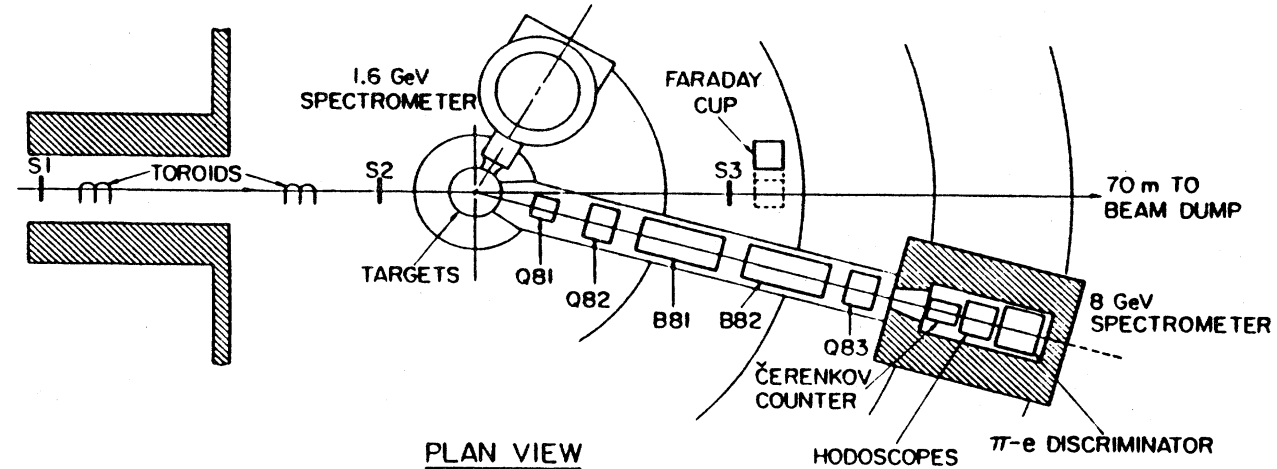
Electrons, Quarks, and Gluons may be truly pointlike!

1 TeV resolves 10^{-19} m = 0.0001 fm

SLAC Two-Mile Linear Accelerator



Pief



Rochester, February 8, 2012

Atoms in Flight

Stan Brodsky

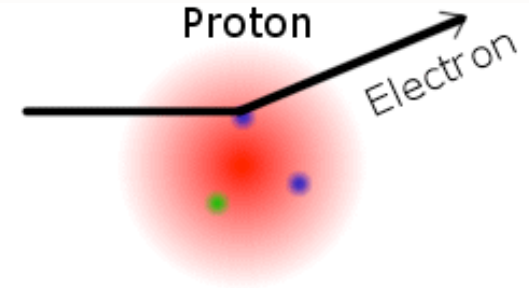
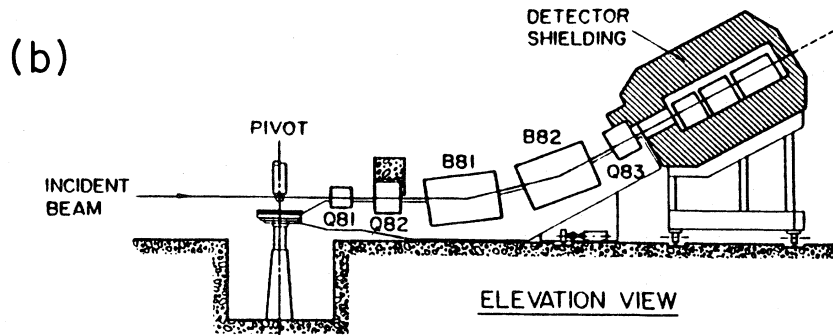
SLAC

1967 SLAC Experiment:

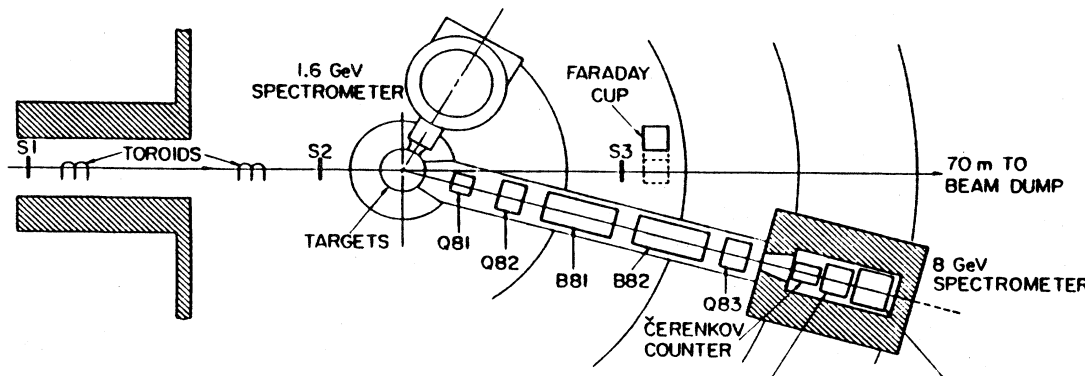
Scatter 20 GeV/c Electrons on protons
in a Hydrogen Target

Discovery of the Quark Structure of Matter

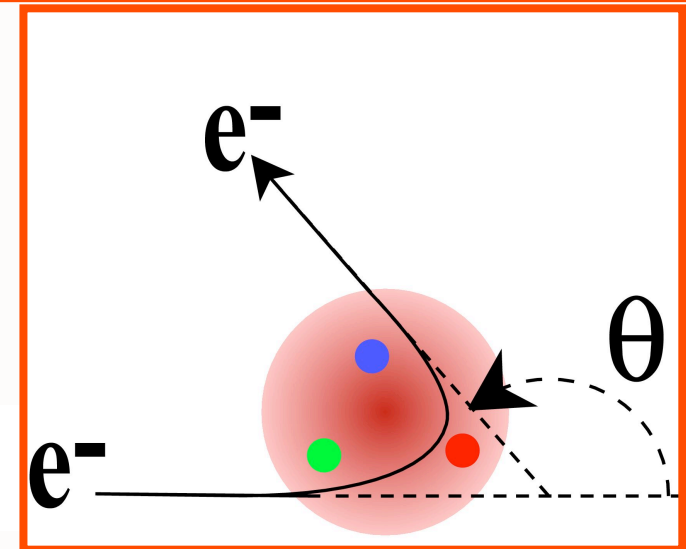
$$ep \rightarrow e'X$$



Discovery of quarks!

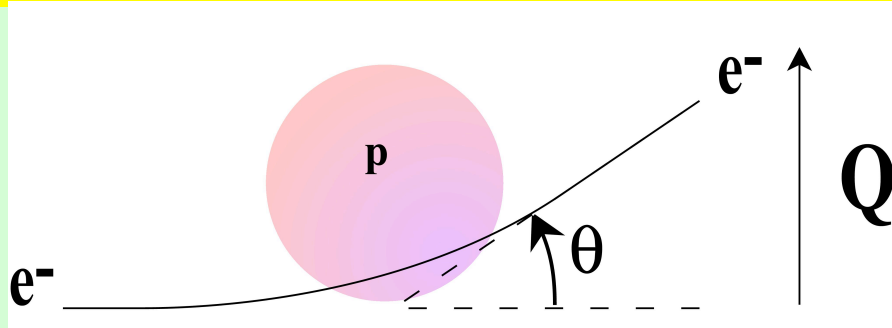


Deep inelastic scattering: Experiments on the proton and the observation of scaling*



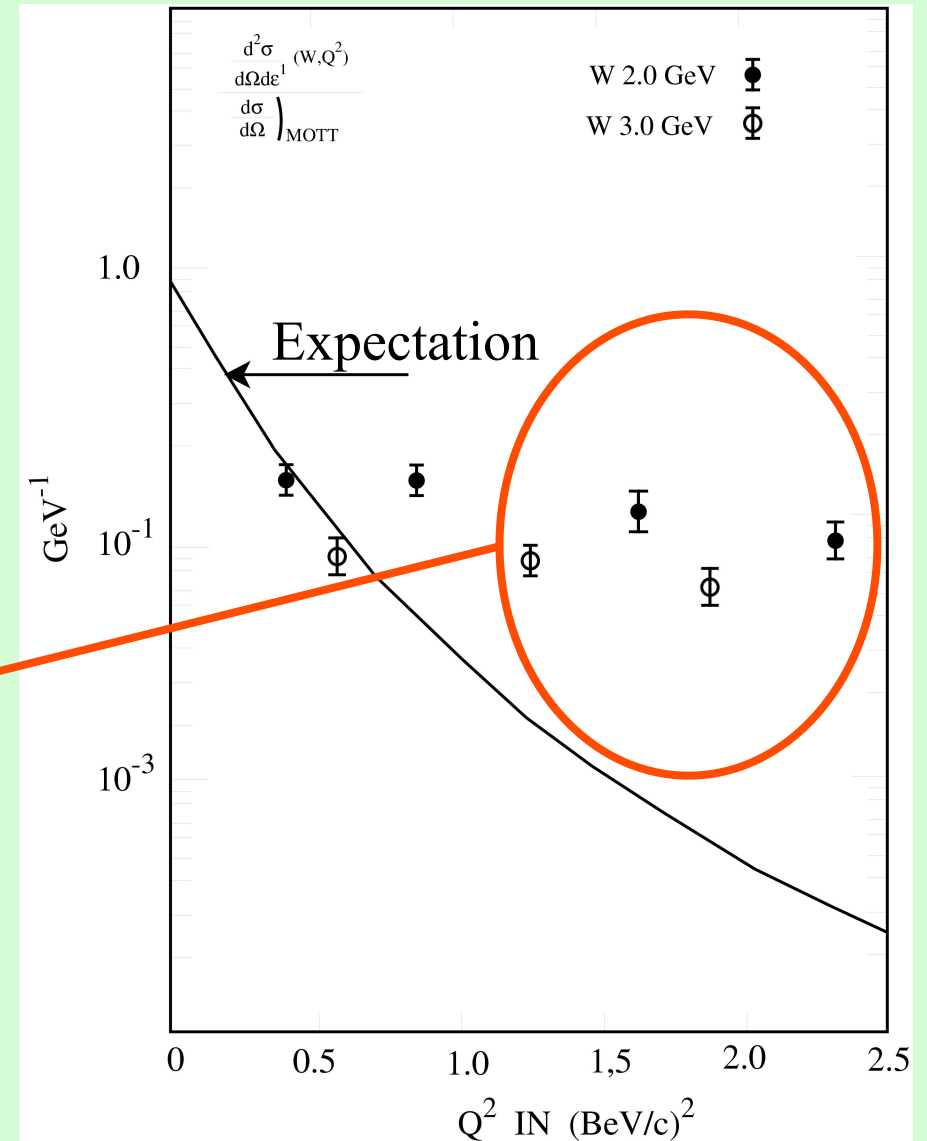
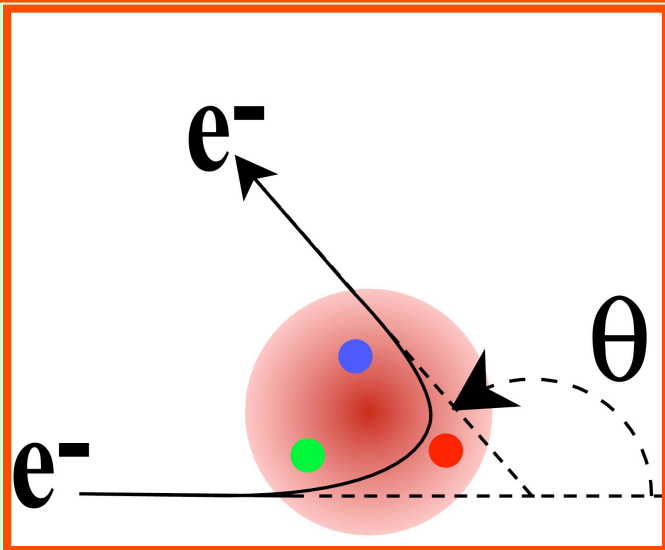
Friedman, Kendall, Taylor: Nobel Prize

Deep inelastic electron-proton scattering

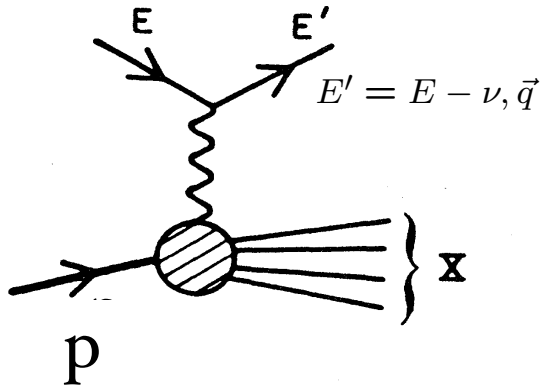


- Rutherford scattering using *very* high-energy electrons striking protons

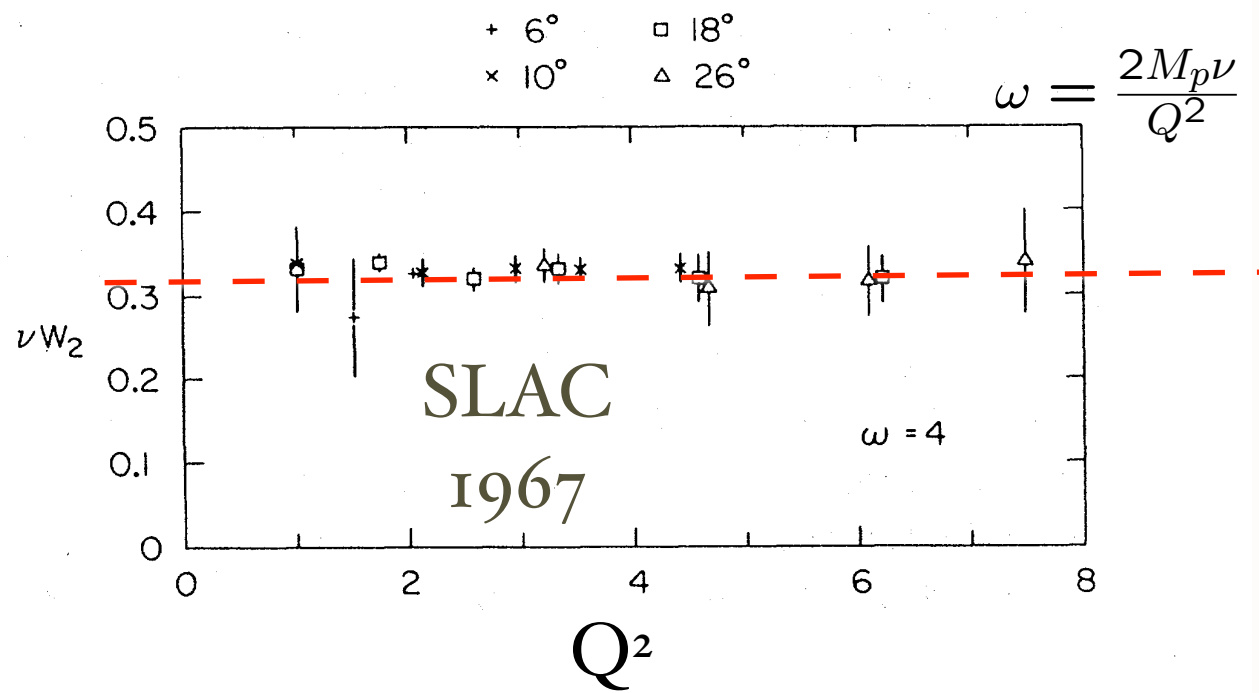
Discovery of quarks!



$$ep \rightarrow e' X$$



$$Q^2 = \vec{q}^2 - \nu^2$$

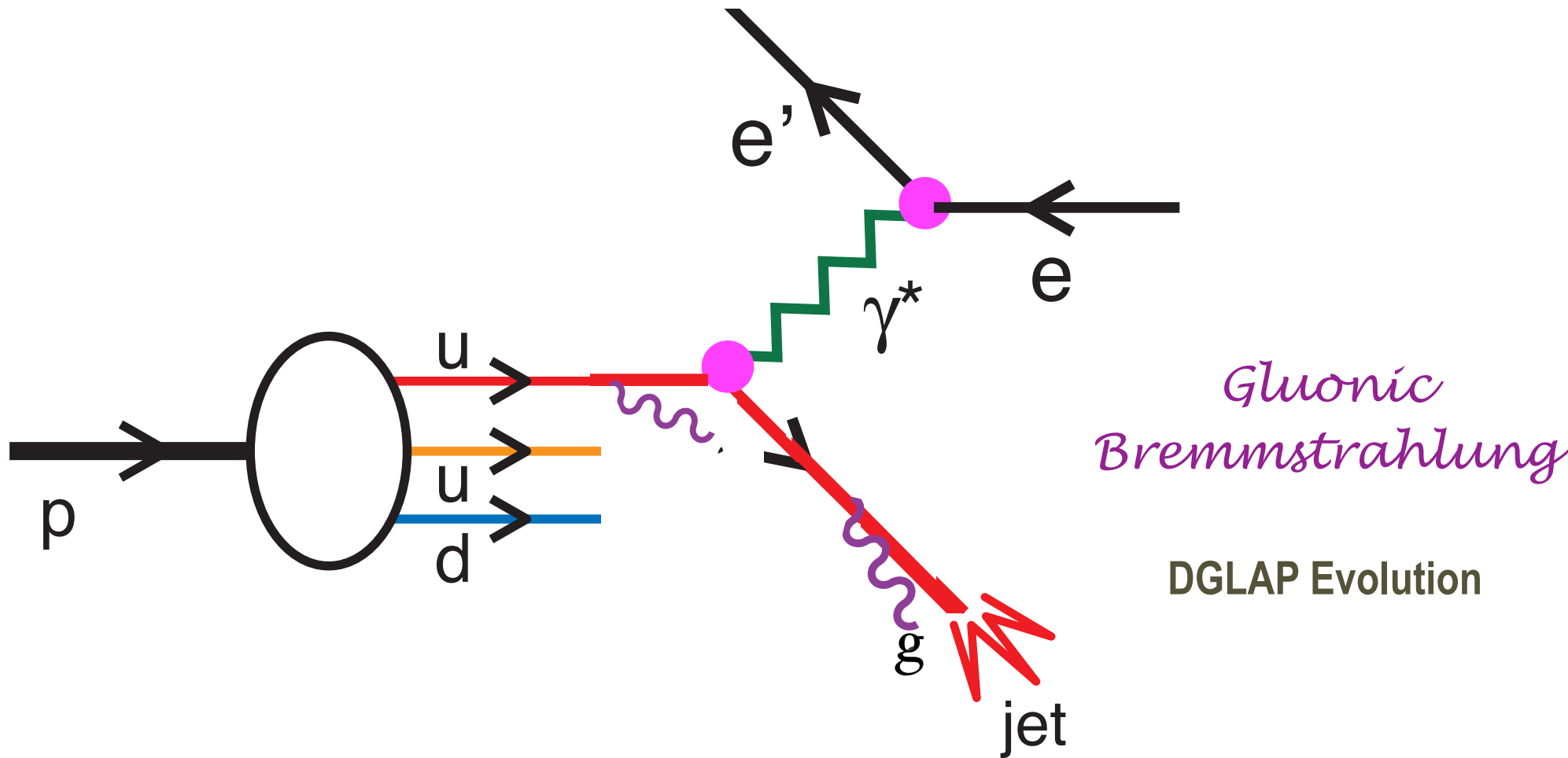


No intrinsic length scale!

Measure rate as a function of energy loss ν and momentum transfer Q
 Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

Discovery of Bjorken Scaling
Electron scatters on point-like quarks!

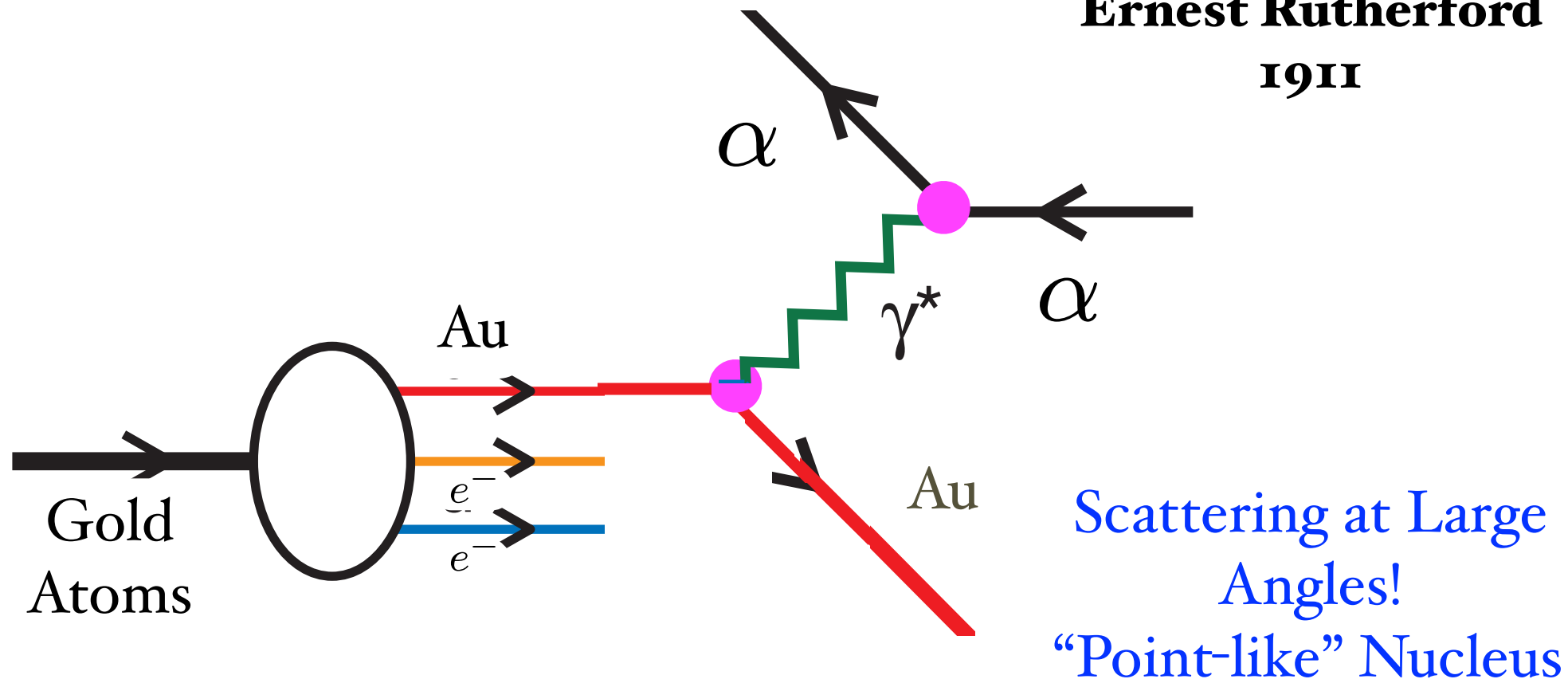
First Evidence for Quark Structure of Matter



Deep Inelastic Electron-Proton Scattering

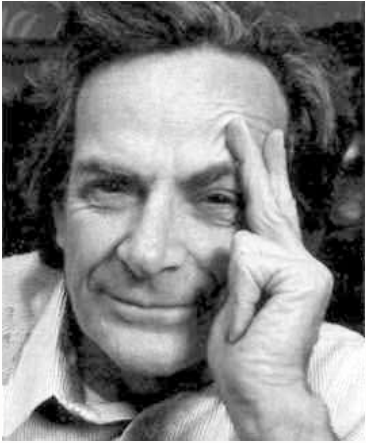
First Evidence for Nuclear Structure of Atoms

Ernest Rutherford
1911



Rutherford Scattering

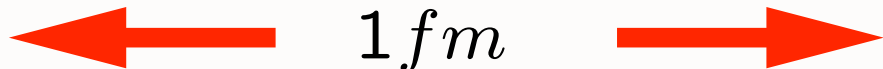
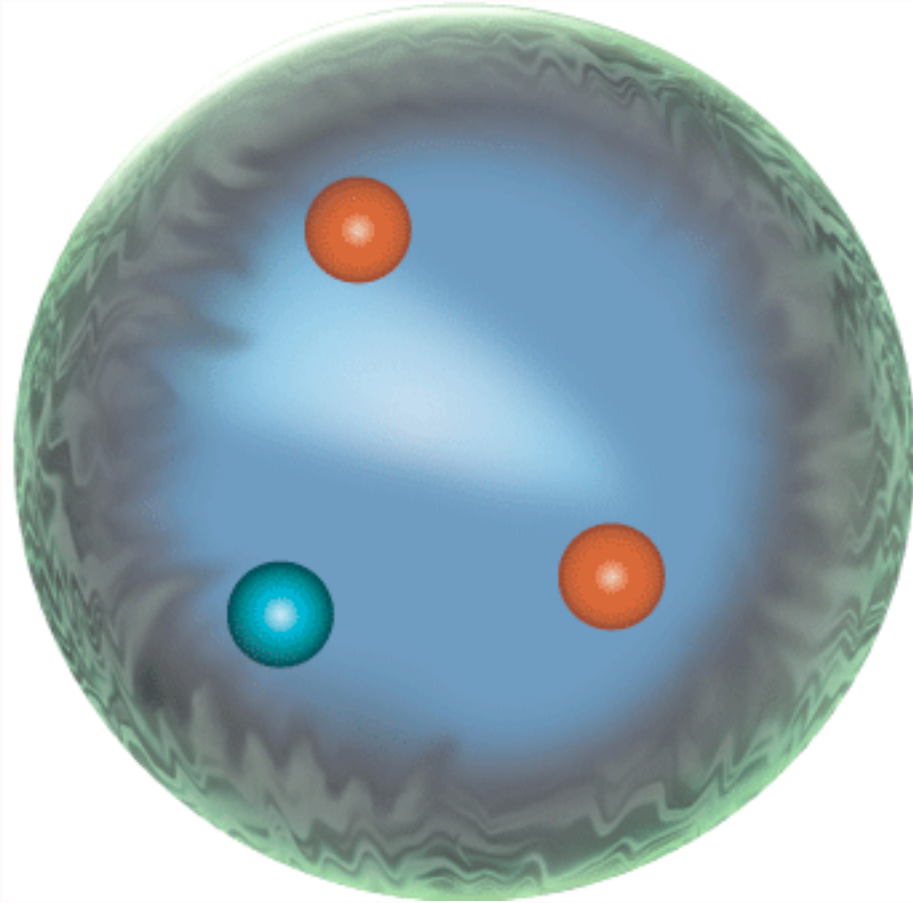
Quarks in the Proton



Feynman & Bjorken:
“Parton” model



$$p = (u u d)$$

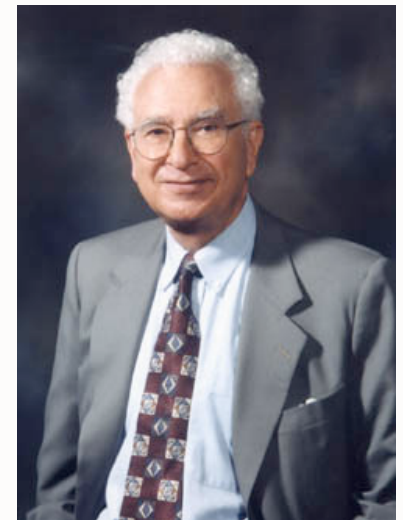


1 fm
 $10^{-15}\text{ m} = 10^{-13}\text{ cm}$

Atoms in Flight



Zweig: “Aces,
Deuces, Treys”



Gell Mann: “Three Quarks for
Mr. Mark”

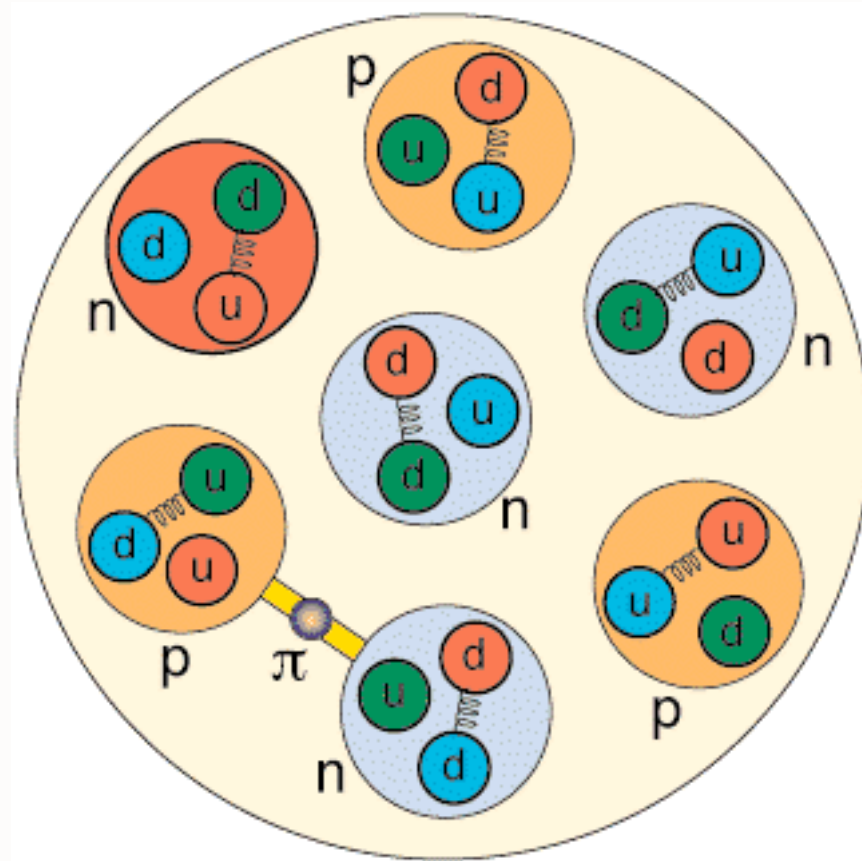
Stan Brodsky **SLAC**

The Quark Structure of the Nucleus

$$e_u = +\frac{2}{3} \quad e_d = -\frac{1}{3}$$

$$p = (uud)$$

$$n = (ddu)$$



$$2e_u + e_d = e_p$$

$$2e_d + e_u = e_n$$

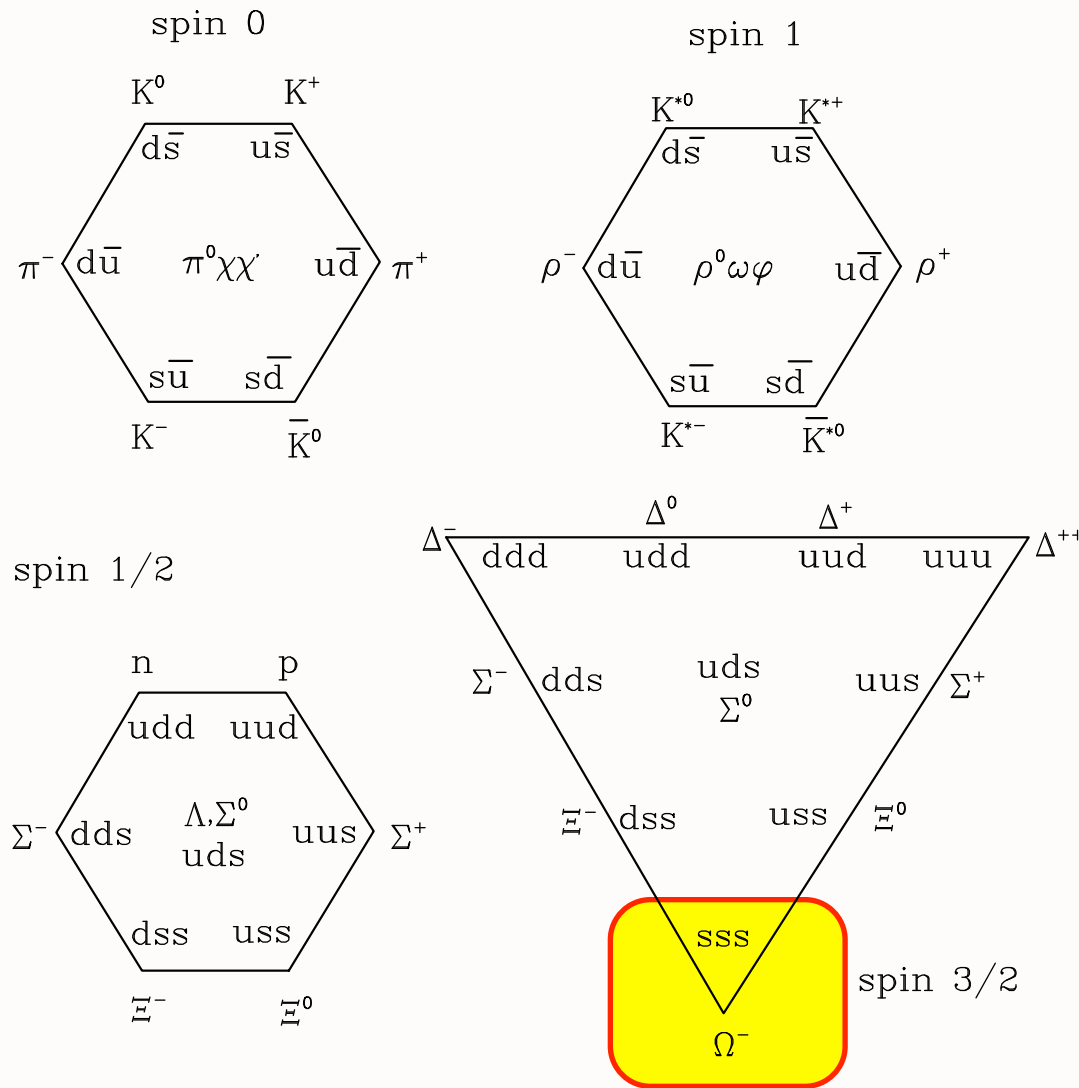
$$2 \times \left(+\frac{2}{3}\right) + 1 \times \left(-\frac{1}{3}\right) = 1$$

$$2 \times \left(-\frac{1}{3}\right) + 1 \times \left(+\frac{2}{3}\right) = 0$$

$SU(N_C), N_C = 3$

The Hadron Spectrum

$SU(3)_{flavor}$



Samios

**Gell Mann,
Zweig**

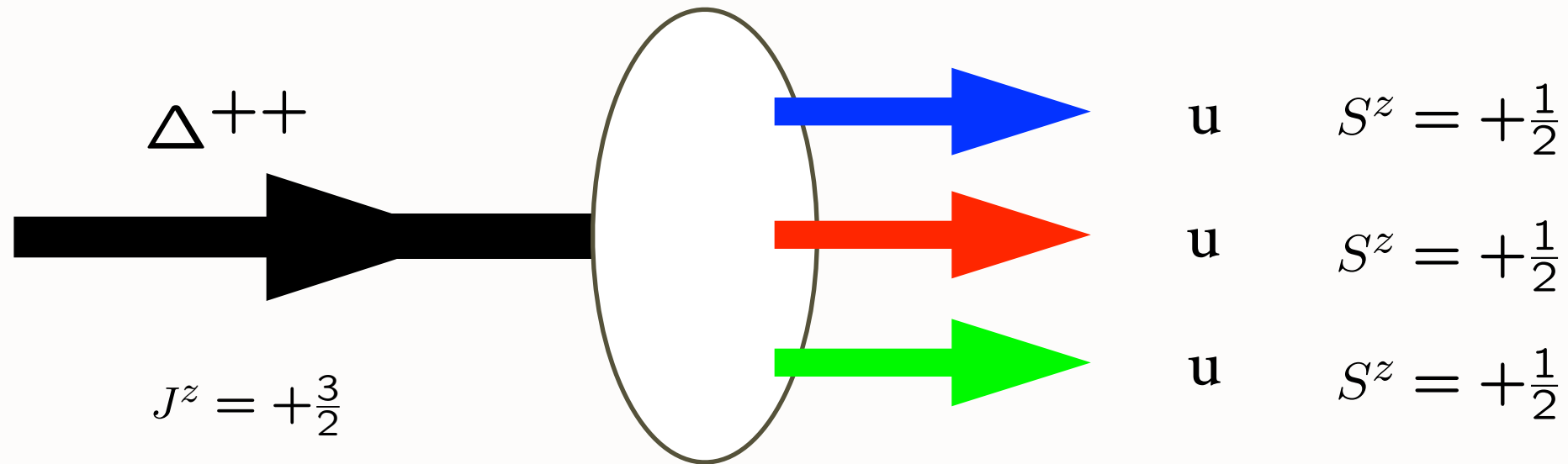
Prediction and Measurement of $\Omega^- = (sss)$

Why are there three colors of quarks?

Greenberg

Pauli Exclusion Principle!

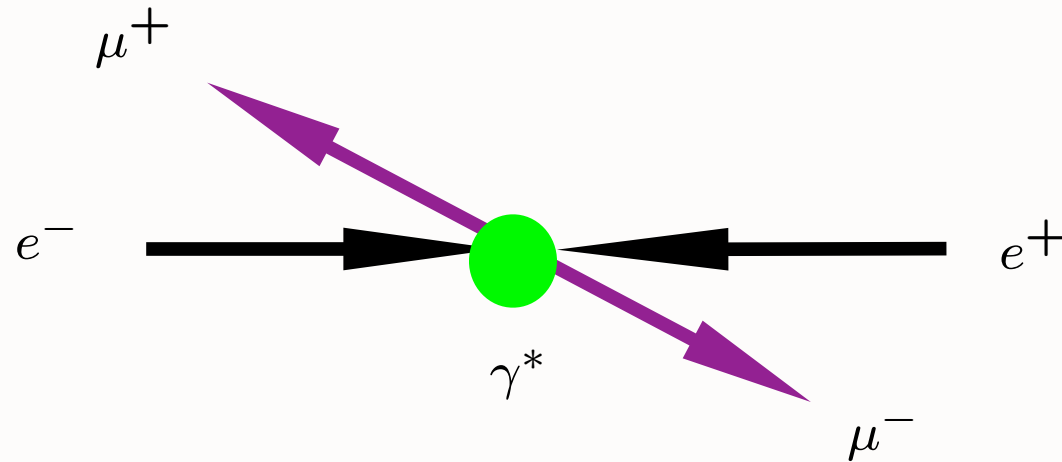
spin-half quarks cannot be in same quantum state !



Three Colors (Parastatistics) Solves Paradox

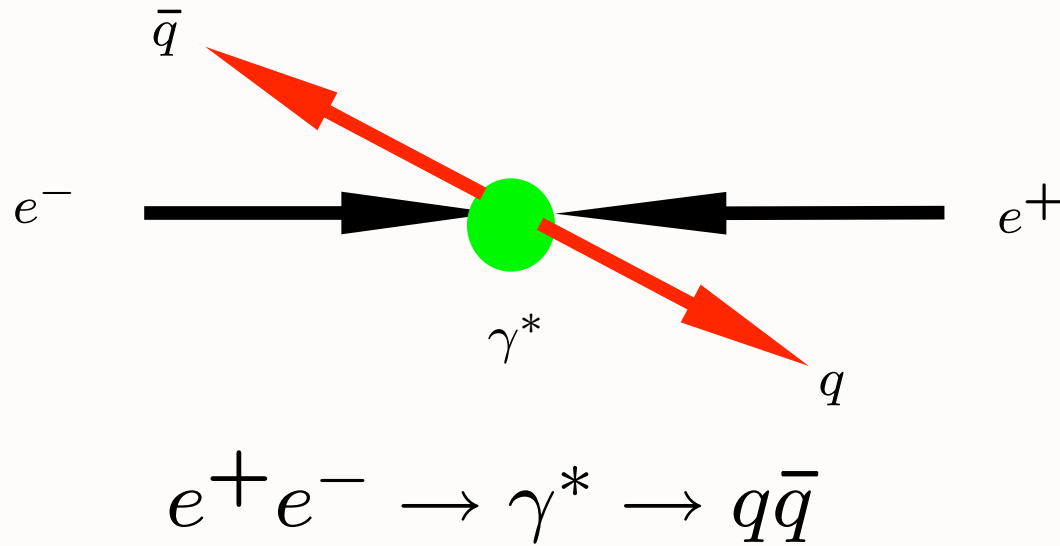
3 Colors Combine : WHITE $SU(N_C), N_C = 3$

Electron-Positron Annihilation



$$e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-$$

Electron-Positron Annihilation



Rate proportional to quark charge squared
and number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

SPEAR Electron-Positron Collider



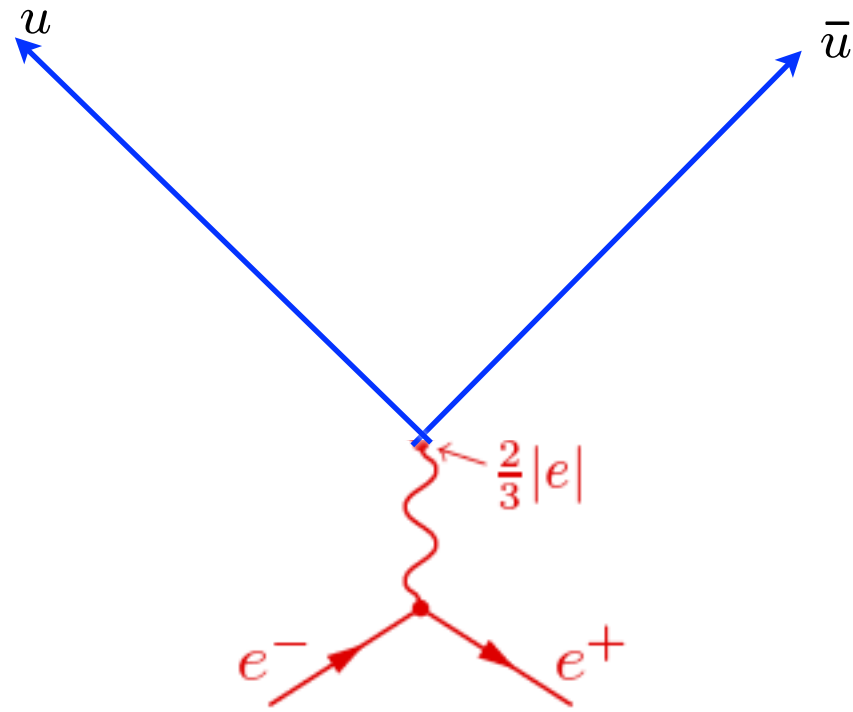
Rochester, February 8, 2012

Atoms in Flight

Stan Brodsky

SLAC

How to Count Quarks



*Color-triplet
quark representation*

For $10 \text{ GeV} < E_{\text{cm}} < 40 \text{ GeV}$,

$$\frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} = 3 \times \left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right]$$

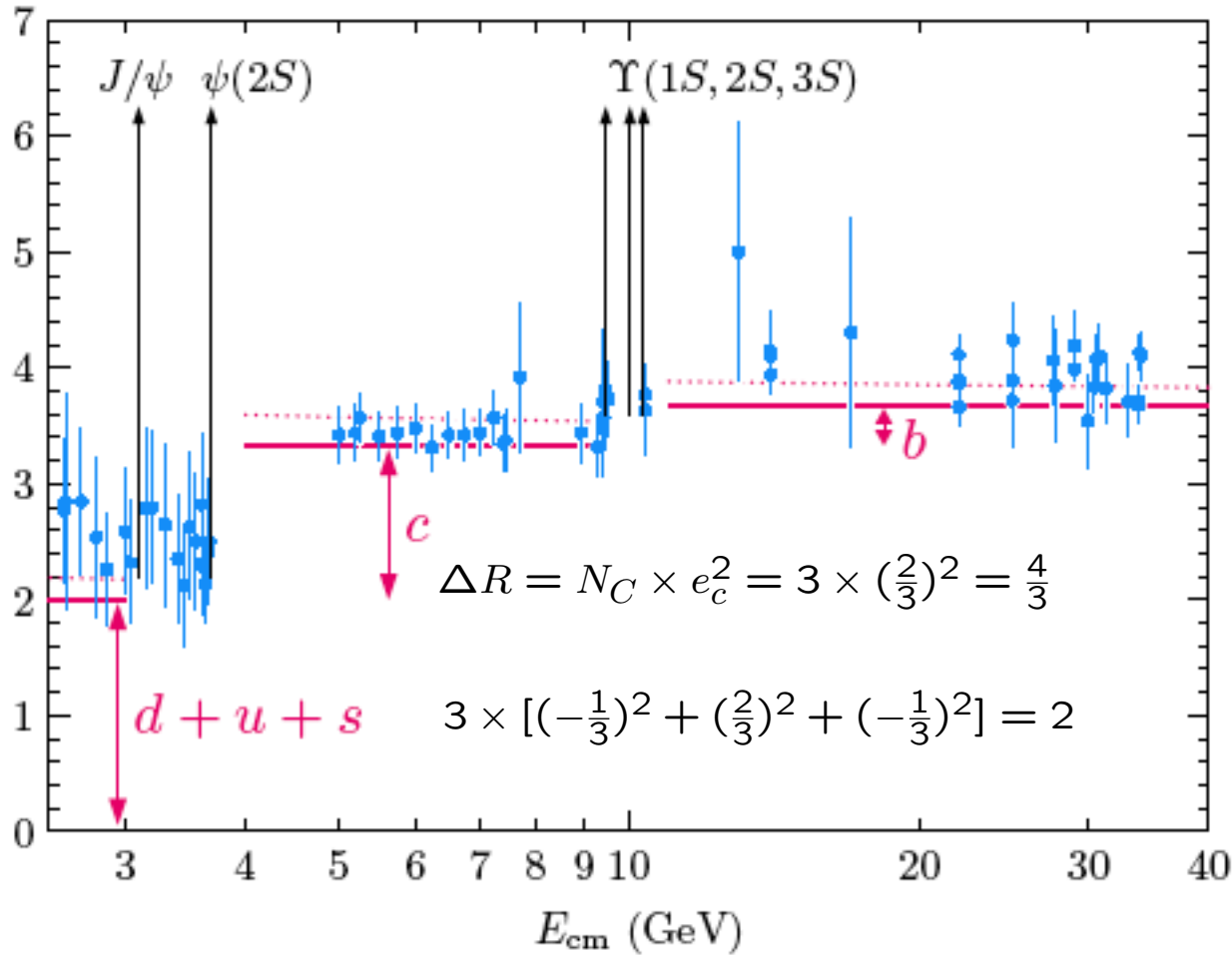
\uparrow colors \uparrow d \uparrow u \uparrow s \uparrow c \uparrow b

$$J/\psi = (c\bar{c})_{1S}$$

How to Count Quarks

$$\Upsilon = (b\bar{b})_{1S}$$

$$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$$



$$3 \times \left(-\frac{1}{3}\right)^2 = \frac{1}{3}$$

$$\Delta R = N_C \times e_c^2 = 3 \times \left(\frac{2}{3}\right)^2 = \frac{4}{3}$$

$$3 \times \left[\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = 2$$

$$N_C = 3$$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

QED Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4} \text{Tr}(F^{\mu\nu} F_{\mu\nu}) + \sum_{\ell=1}^{n_\ell} i \bar{\Psi}_\ell D_\mu \gamma^\mu \Psi_\ell + \sum_{\ell=1}^{n_\ell} m_\ell \bar{\Psi}_\ell \Psi_\ell$$

$$iD^\mu = i\partial^\mu - eA^\mu \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Yang Mills Gauge Principle:
Phase Invariance at Every
Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Landau Pole

QCD Lagrangian

gluon dynamics quark kinetic energy +
quark-gluon dynamics mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

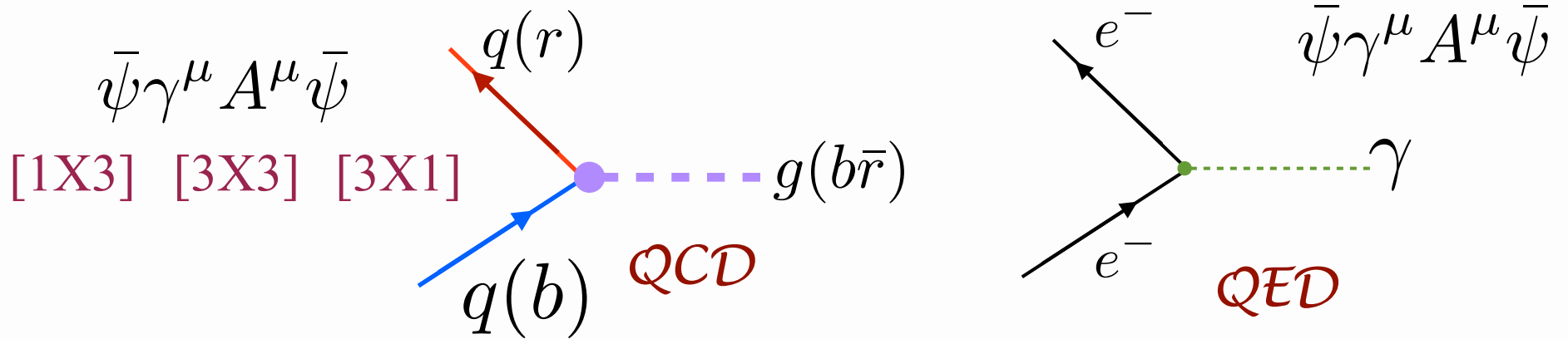
$$iD^\mu = i\partial^\mu - gA^\mu$$

$$G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

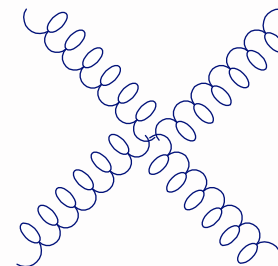
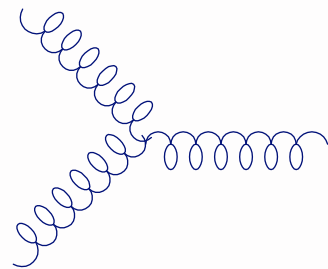
Fundamental Couplings of QCD



$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Gluon vertices



$$G^{\mu\nu} G_{\mu\nu}$$

gluon self couplings

*QED: Underlies Atomic Physics, Molecular Physics,
Chemistry, Electromagnetic Interactions ...*

*QCD: Underlies Hadron Physics, Nuclear Physics,
Strong Interactions, Jets*

Theoretical Tools

- Feynman diagrams and perturbation theory
- Bethe Salpeter Equation, Dyson-Schwinger Equations
- Lattice Gauge Theory,
- Discretized Light-Front Quantization
- AdS/CFT !

In QCD and the Standard Model
the beta function is indeed
negative!

$$\beta(g) = \frac{-g^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{4}{3} \frac{N_F}{2} \right)$$

$$\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} < 0$$

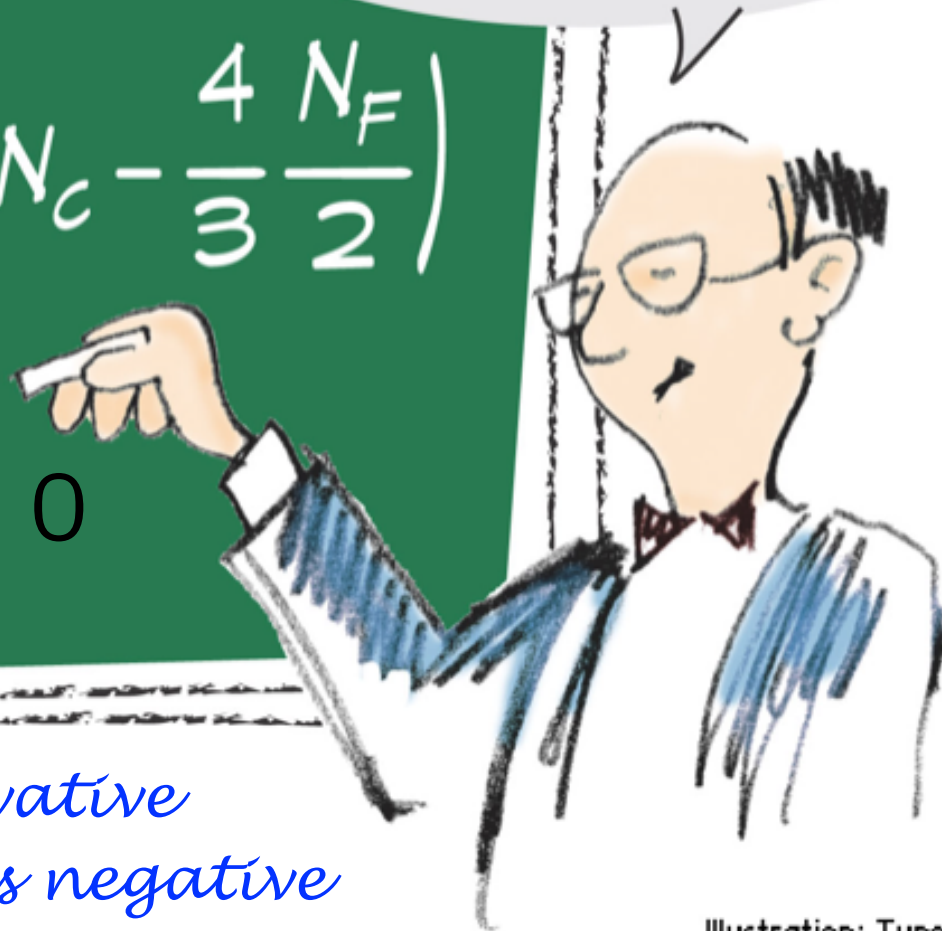


Illustration: Typoform

*logarithmic derivative
of the QCD coupling is negative
Coupling becomes weaker at short
distances or high momentum transfer*

Atoms in Flight

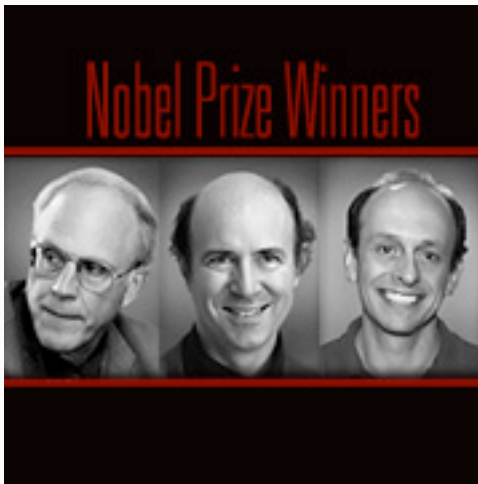
Rochester, February 8, 2012

Stan Brodsky

SLAC

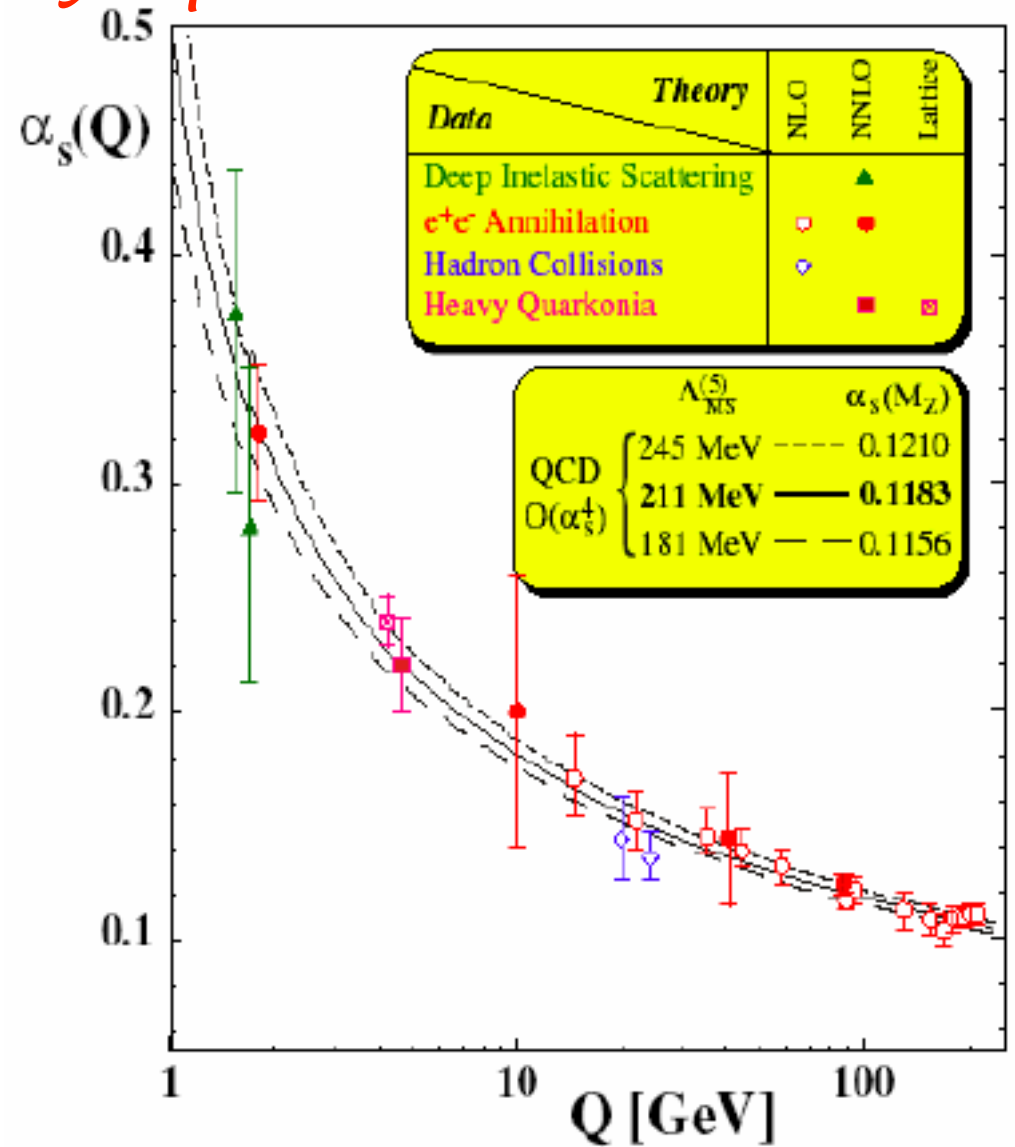
Verification of Asymptotic Freedom

$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$



$$\frac{\sigma(e^+e^- \rightarrow \text{three jets})}{\sigma(e^+e^- \rightarrow \text{two jets})}$$

proportional to $\alpha_s(Q)$

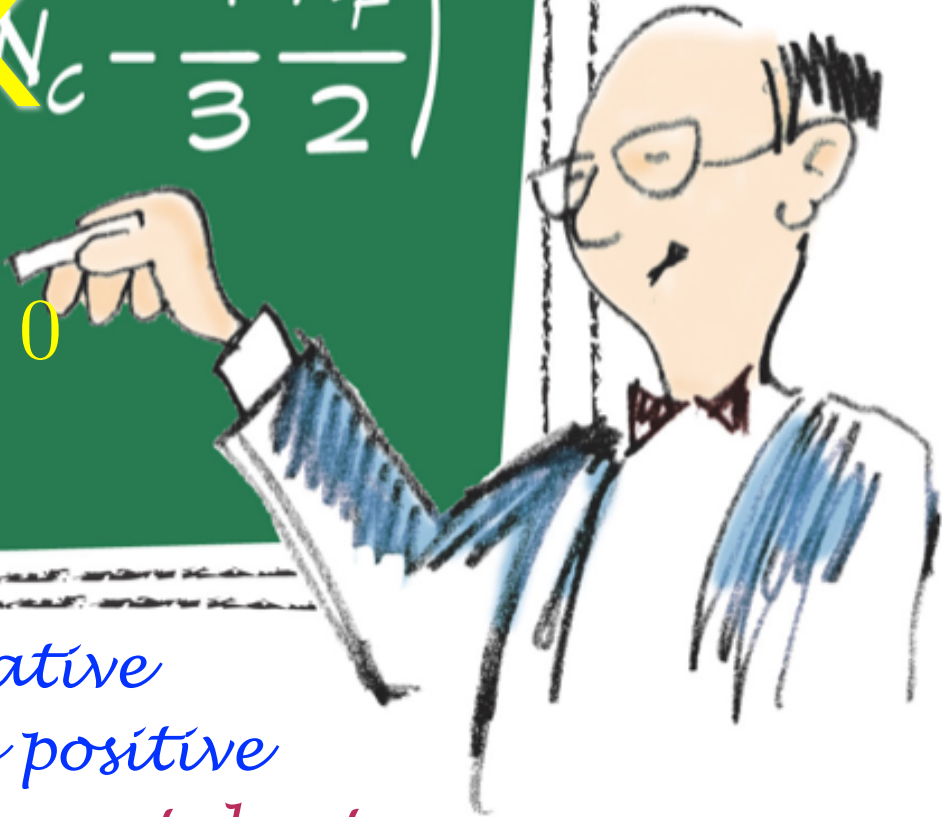


Ratio of rate for $e^+e^- \rightarrow q\bar{q}g$ to $e^+e^- \rightarrow q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$

In QED the β - function
is positive

$$\beta(g) = \frac{-g^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{4}{3} \frac{N_F}{2} \right)$$

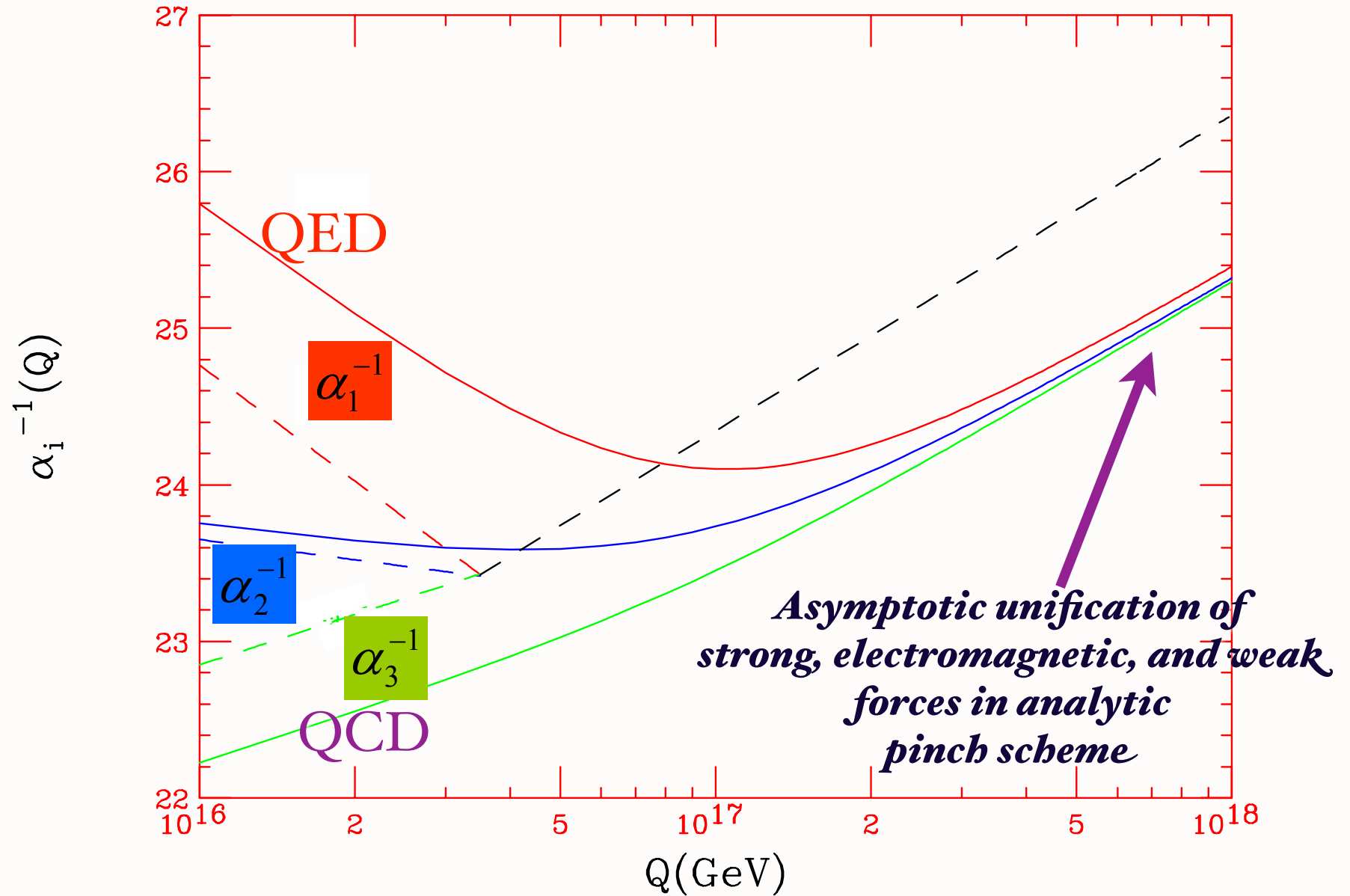
$$\beta = \frac{d\alpha_{QED}(Q^2)}{d \ln Q^2} > 0$$



logarithmic derivative
of the QED coupling is positive
Coupling becomes stronger at short
distances or high momentum transfer

Landau Pole!

Asymptotic Unification




QCD Lagrangian

gluon dynamics quark kinetic energy + quark-gluon dynamics mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$
$$iD^\mu = i\partial^\mu - gA^\mu \qquad [D^\mu, D^\nu] = igG^{\mu\nu}$$

$\lim N_C \rightarrow 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

Analytic limit of QCD: Abelian Gauge Theory

QCD  **QED**

P. Huet, sjb

$\lim N_C \rightarrow 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

QCD \rightarrow Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

*All analyses for Quantum Chromodynamics
must be applicable to Quantum Electrodynamics*

Given the elementary gauge theory interactions, all fundamental processes described in principle!

Example from QED:

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 201(30) \quad \text{QED prediction (Kinoshita, et al.)}$$

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 193(10) \quad \text{Measurement (Dehmelt, et al.)}$$

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 180\ 85 [0.76 \text{ ppt}]$$

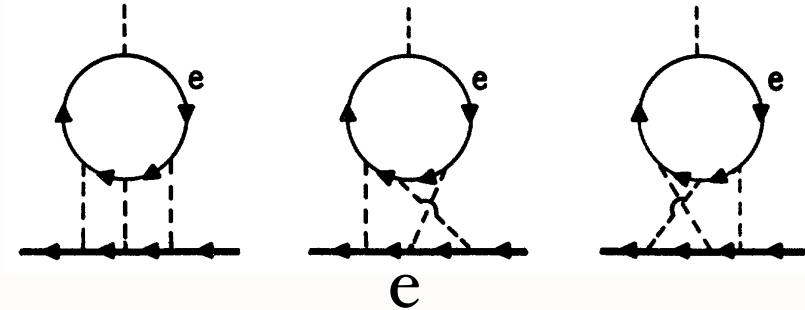
$$\text{Dirac: } g_e \equiv 2 \quad \text{Measurement (Gabrielse, et al.)}$$

QED provides an asymptotic series relating g and α ,

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \dots$$

$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

Light-by-Light Scattering Contribution to C_6



Aldins, Dufner, Kinoshita, sjb

$$\alpha^{-1} = 137.035\,999\,710\,(90)\,(33) [0.66\text{ ppb}][0.24\text{ ppb}],$$

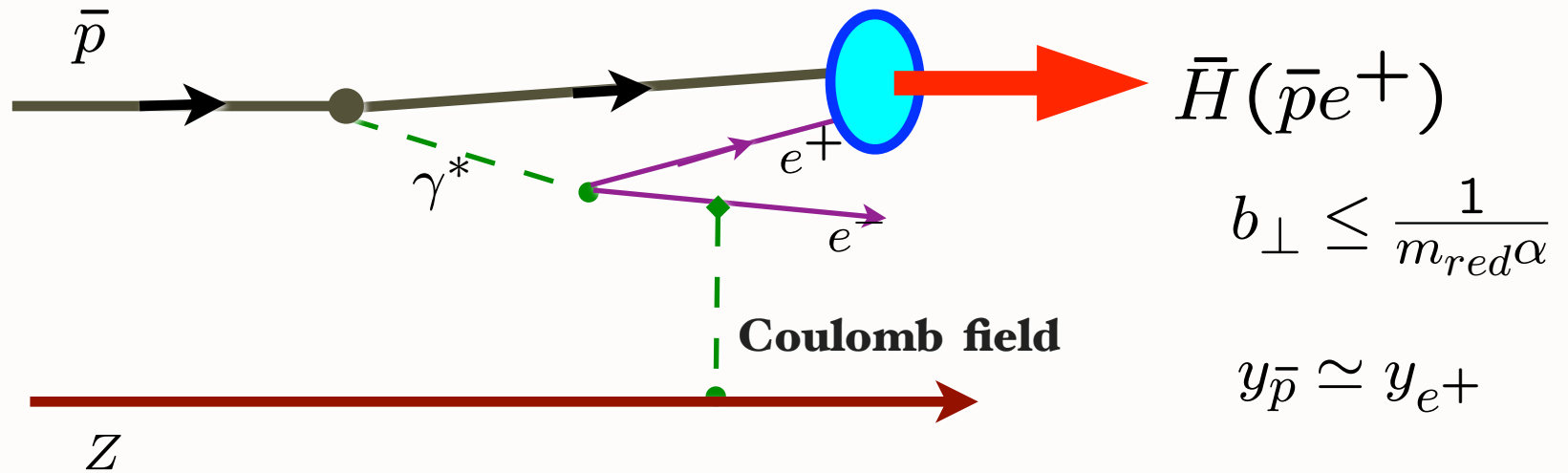
$$= 137.035\,999\,710\,(96) [0.70\text{ ppb}].$$

| G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, Phys. Rev. Lett. **97**, 030802 (2006).

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level



First atoms of antimatter produced at CERN

In September 1995, Prof. Walter Oelert and an international team from Jülich IKP-KFA, Erlangen-Nuernberg University, GSI Darmstadt and Genoa University succeeded for the first time in synthesising atoms of antimatter from their constituent antiparticles. Nine of these atoms were produced in collisions between antiprotons and xenon atoms over a period of three weeks. Each one remained in existence for about forty billionths of a second, travelled at nearly the speed of light over a path of ten metres and then annihilated with ordinary matter. The annihilation produced the signal which showed that the anti-atoms had been created.

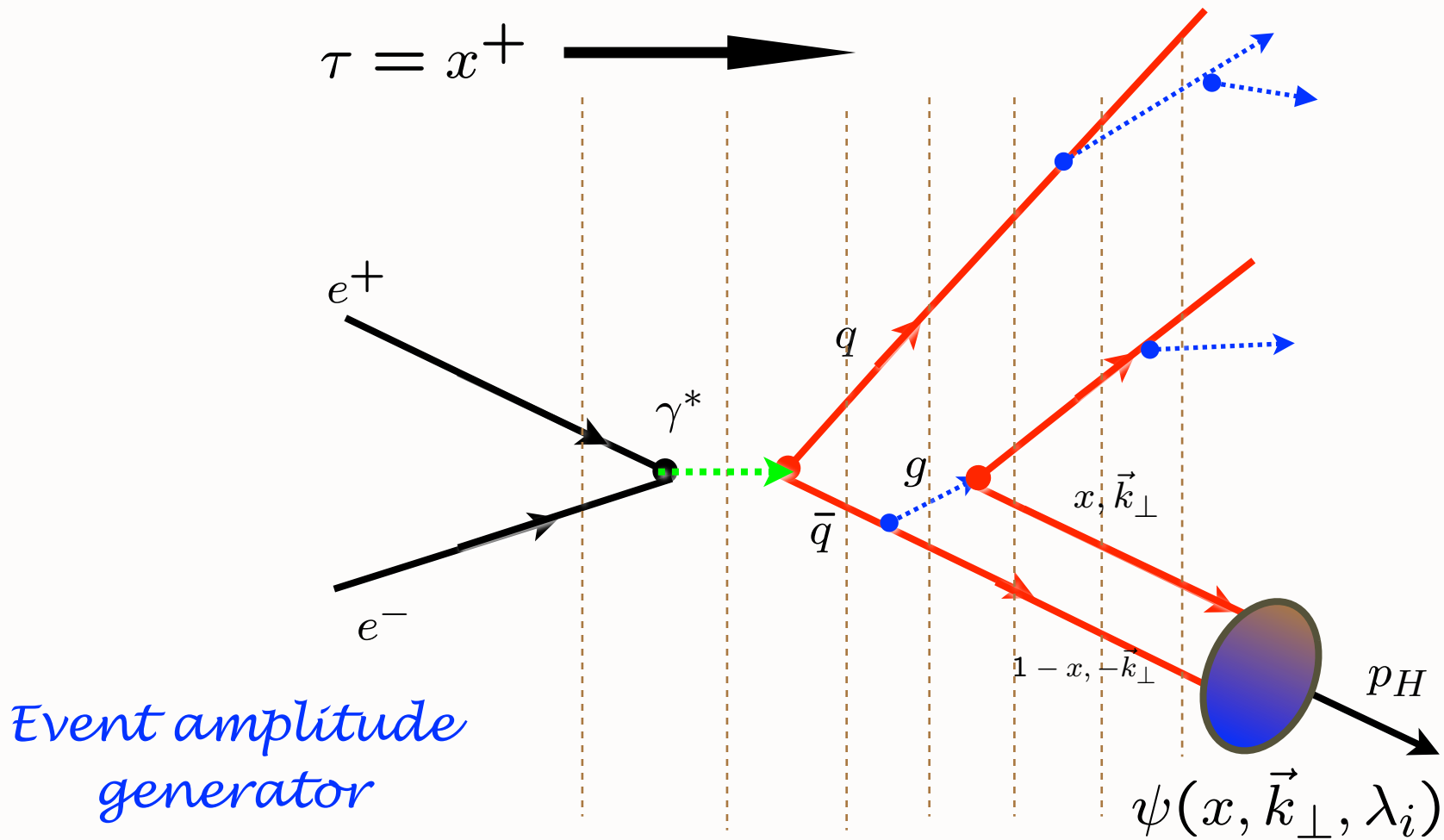
Ordinary atoms consist of a number of electrons in orbit around an atomic nucleus. The hydrogen atom is the simplest atom of all; its nucleus consists of a proton, around which a single electron circulates. The recipe for anti-hydrogen is very simple - take one antiproton, bring up one anti-electron, and put the latter into orbit around the former - but it is very difficult to carry out as antiparticles do not naturally exist on earth. They can only be created in the laboratory. The experimenters whirled previously created antiprotons around the CERN* Low Energy Antiproton Ring (LEAR), passing them through a xenon gas jet each time they went around - about 3 million times each second. (see [scheme](#) of the experiment) Very occasionally, an antiproton converted a small part of its own energy into an electron and an anti-electron, usually called a positron, while passing through a xenon atom. In even rarer cases, the positron's velocity was sufficiently close to the velocity of the antiproton for the two particles to join - creating an atom of anti-hydrogen (see [diagram](#) of the principle) .

Three quarters of our universe is hydrogen and much of what we have learned about it has been found by studying ordinary hydrogen. If the behaviour of anti-hydrogen differed even in the tiniest detail from that of ordinary hydrogen, physicists would have to rethink or abandon many of the established ideas on the symmetry between matter and antimatter. Newton's historic work on gravity was supposedly prompted by watching an apple fall to earth, but would an "anti-apple" fall in the same way? It is believed that antimatter "works" under gravity in the same way as matter, but if nature has chosen otherwise, we must find out how and why.

The next step is to check whether anti hydrogen does indeed "work" just as well as ordinary hydrogen. Comparisons can be made with tremendous accuracy, as high as one part in a million trillion, and even an asymmetry on this tiny scale would have enormous consequences for our understanding of the universe. To check for such an asymmetry would mean holding the anti-atoms still, for seconds, minutes, days or weeks. The techniques needed to store antimatter are under intense development at CERN. New experiments are currently being planned, to capture antimatter in electrical and magnetic bottles or traps allowing for high precision analysis.

The first ever creation of atoms of antimatter at CERN has opened the door to the systematic exploration of the anti world.

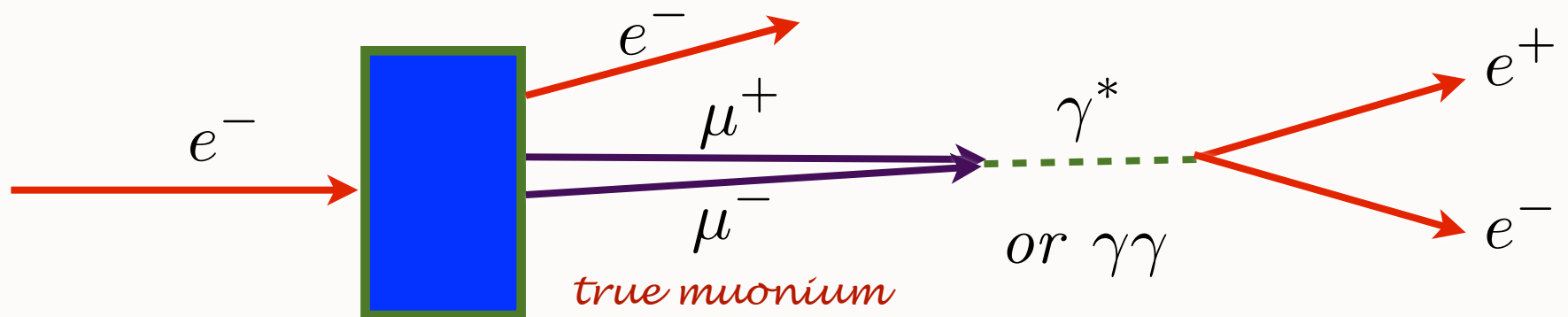
Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

● Production of True Muonium [$\mu^+\mu^-$]

$$eZ \rightarrow eZ[\mu^+\mu^-]_nS \quad q_{min} \simeq \frac{M_{\mu^+\mu^-}^2}{\nu} \sim 10 \text{ MeV}$$



- Produces all Rydberg Levels
- Analytic connection to continuum production -- enhanced by SSS at threshold
- Gap extends in cm multiplied by Lorentz boost
- Excite/De-excite levels with external fields, lasers

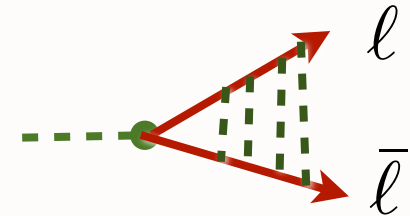
Coulomb Enhancement of Pair Production at Threshold

$$\sigma \rightarrow \sigma S(\beta)$$

$$\beta = \sqrt{1 - \frac{4m_\ell^2}{s}}$$

$$X(\beta) = \frac{\pi\alpha\sqrt{1-\beta^2}}{\beta}$$

$$S(\beta) = \frac{X(\beta)}{1 - e^{-X(\beta)}}$$



Sommerfeld-Schwinger-Sakharov Effect

Bjorken: Analytical Connection to Rydberg Levels below Threshold

$$QCD : \pi\alpha \rightarrow \frac{4}{3}\alpha_s(\beta^2 s)$$

Kühn, Hoang, sjb

Production of True Muonium [$\mu^+\mu^-$]

PRL 102, 213401 (2009)

PHYSICAL REVIEW LETTERS

week ending
29 MAY 2009

Production of the Smallest QED Atom: True Muonium ($\mu^+\mu^-$)

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SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA

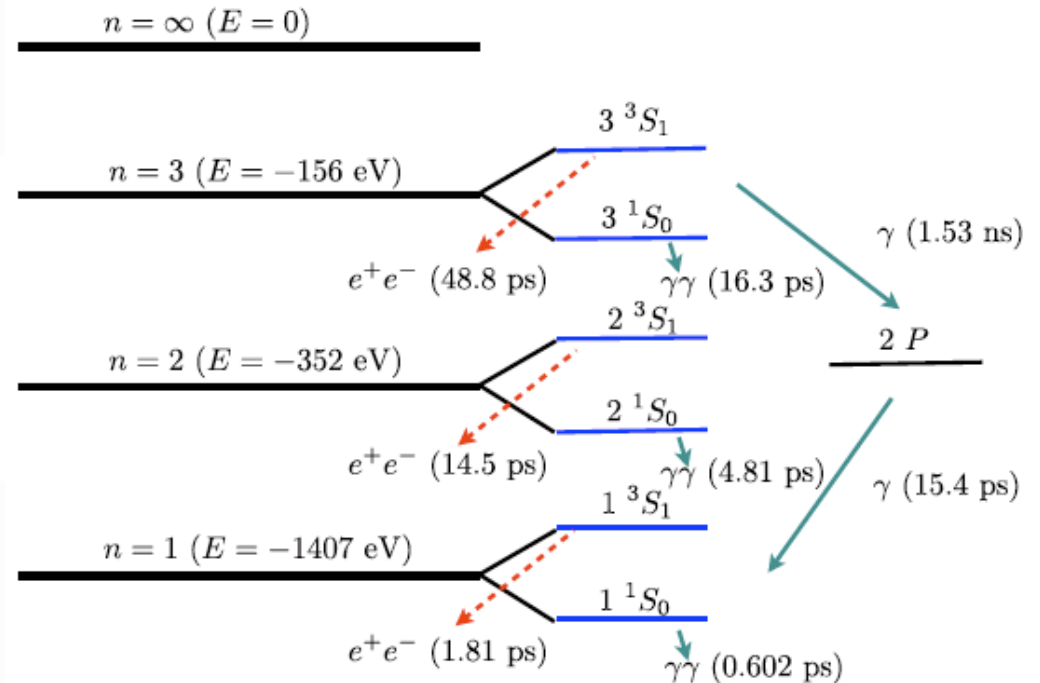
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(Received 22 April 2009; published 26 May 2009)

Rydberg Levels and Decays

$$\begin{aligned} \tau(n^3S_1 \rightarrow e^+e^-) &= \frac{6\hbar n^3}{\alpha^5 mc^2}, & \tau(n^1S_0 \rightarrow \gamma\gamma) &= \frac{2\hbar n^3}{\alpha^5 mc^2}, \\ \tau(2P \rightarrow 1S) &= \left(\frac{3}{2}\right)^8 \frac{2\hbar}{\alpha^5 mc^2}, & \tau(3S \rightarrow 2P) &= \left(\frac{5}{2}\right)^9 \frac{4\hbar}{3\alpha^5 mc^2}, \\ \frac{\tau(n^3S_1 \rightarrow e^+e^-)}{\tau(n^1S_0 \rightarrow \gamma\gamma)} &= 3, & \frac{\tau(2P \rightarrow 1S)}{\tau(n^1S_0 \rightarrow \gamma\gamma)} &= \left(\frac{3}{2}\right)^8 \frac{1}{n^3} = \frac{25.6}{n^3}, \\ \frac{\tau(3S \rightarrow 2P)}{\tau(2P \rightarrow 1S)} &= \left(\frac{5}{3}\right)^9 = 99.2. \end{aligned}$$



Production of bound triplet $\mu^+\mu^-$ system in collisions of electrons with atoms.

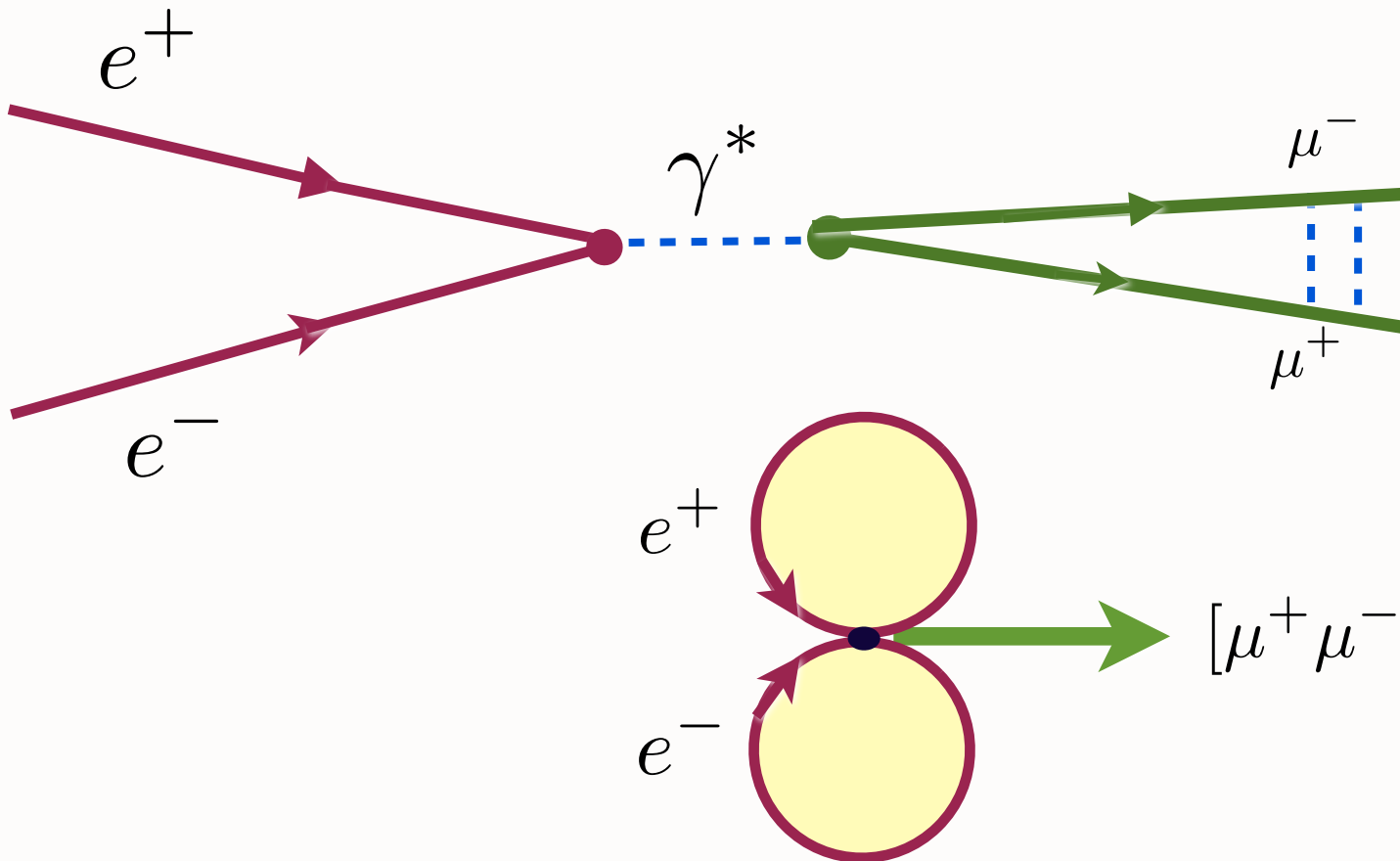
[N. Arteaga-Romero](#), [C. Carimalo](#), ([Paris U., VI-VII](#)) , [V.G. Serbo](#), ([Paris U., VI-VII](#) & [Novosibirsk State U.](#)) . Jan 2000. 10pp.

Published in **Phys.Rev. A62:032501, 2000.**

e-Print: [hep-ph/0001278](#)

Production of True Muonium in an electron-positron collider

Lebed, sjb



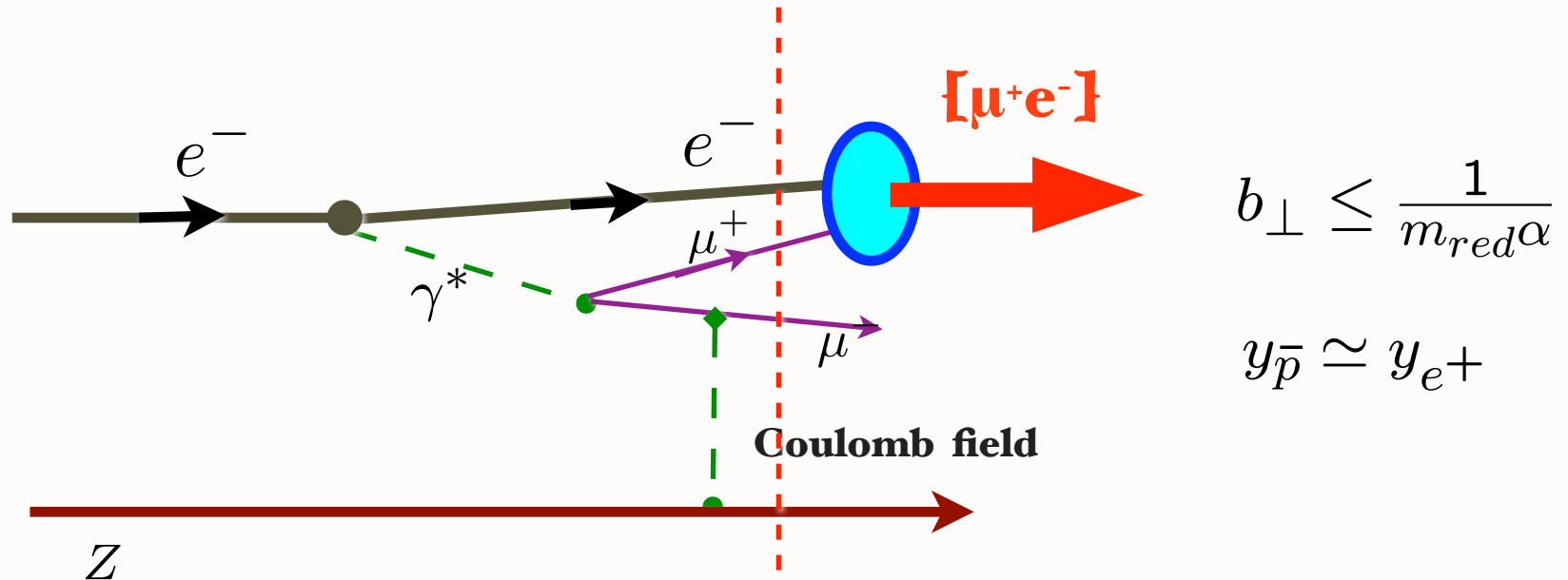
***Electron-Positron Collider:
Bj: FISR (Fool's Intersecting Storage Ring) Frame***

Novel Lepton Physics Studies in electron-nucleus reactions

Use JLab 4 GeV Intense Electron Beam

- **Production of True Muonium [$\mu^+\mu^-$]**
- **Production of Relativistic Muonium [μ^+e^-]**
- **Test All-Orders Bethe-Maximon Formula for Pair Production**
- **Lepton Charge Asymmetry**
- **Test Landau-Pomeranchuk-Migdal (LPM) Effect**

- **Production of Relativistic Muonium $[\mu^+e^-]$**



Coalescence of off-shell co-moving electron and muon

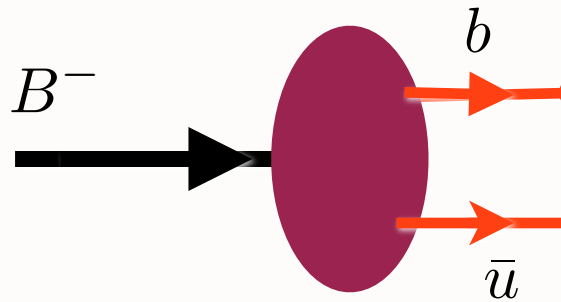
Wavefunction maximal at small impact separation and equal rapidity

“Atom Formation” at the Amplitude Level

Production of Relativistic Muonium $\{\mu^+e^-\}$

- **Never Observed Before?**
- **Measure Lamb Shift of Muonium by Robiscoe Method (Level Crossing by Induced Magnetic Field)**
- **Precision Tests of Time Dilation**
- **Dissociate to muon and electron with foils**
- **Flying Atoms**

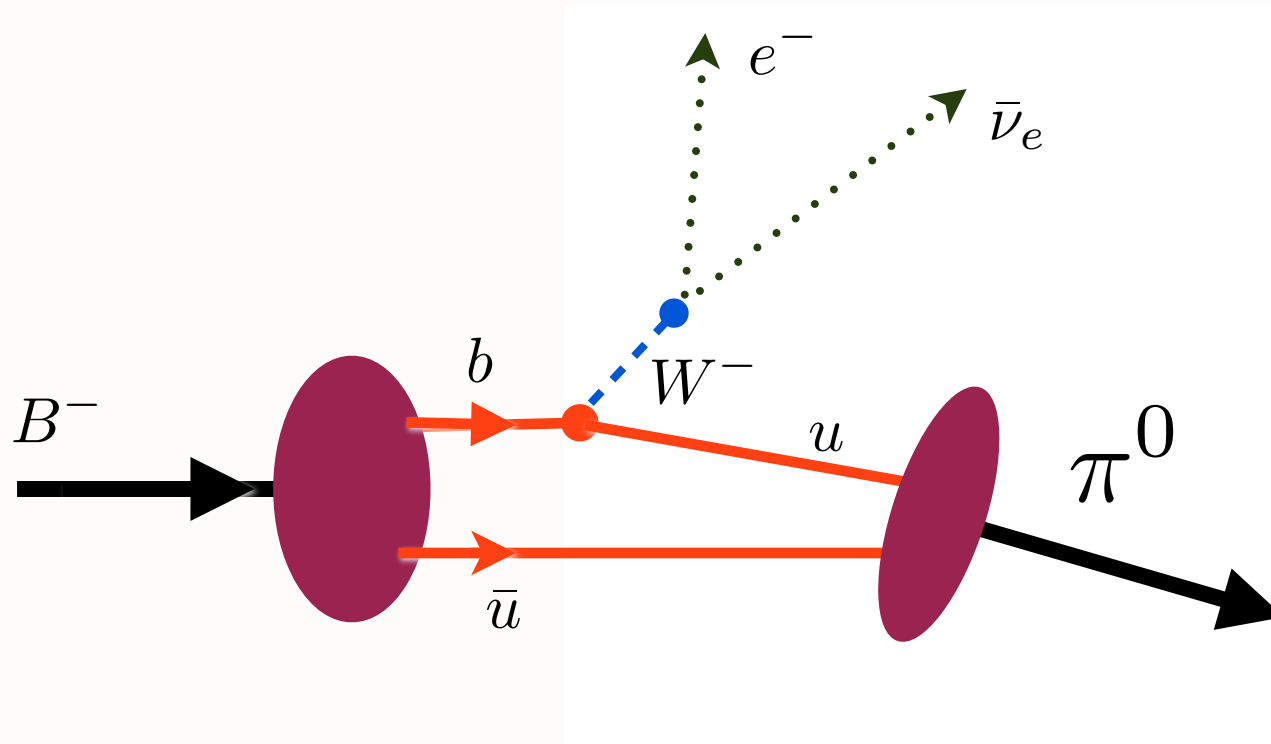
Exclusive B decay



$$\vec{p}_{\pi^0} = \vec{p}_{B^-} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{e^-}$$

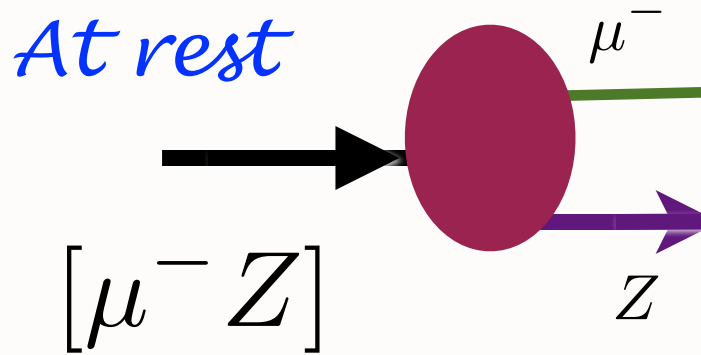
**Decay of the B meson to the pion
plus electron and neutrino**

Exclusive B decay



$$\vec{p}_{\pi^0} = \vec{p}_{B^-} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{e^-}$$

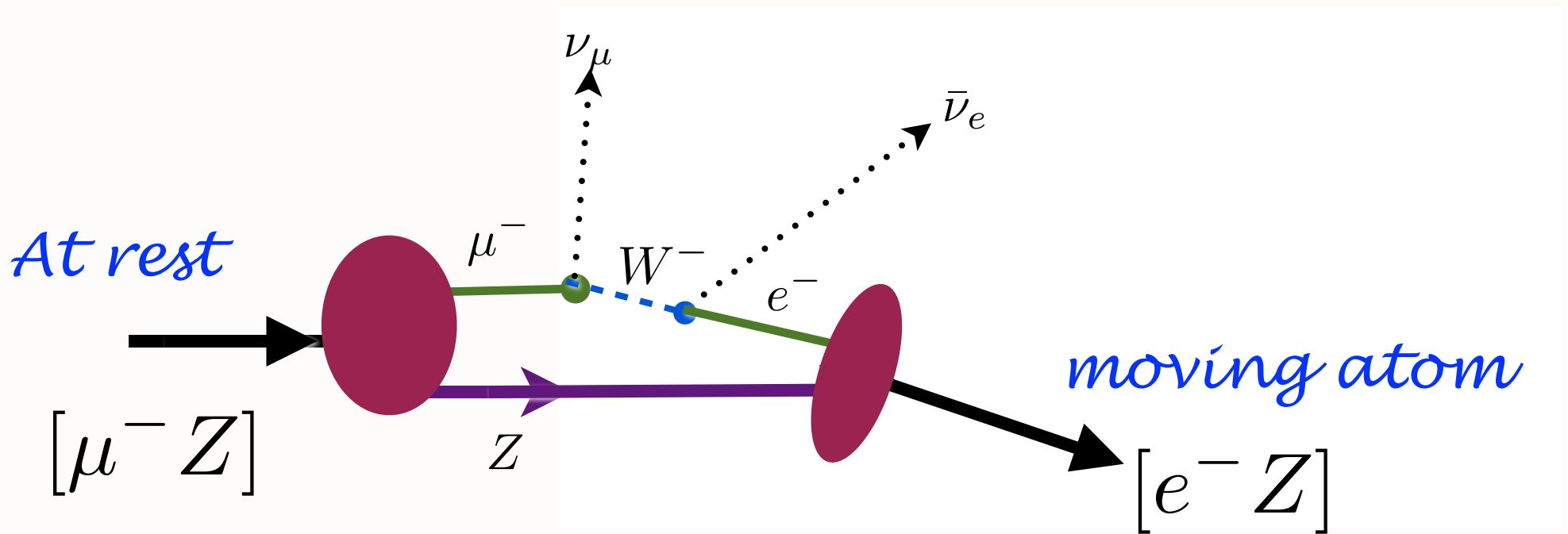
**Decay of the B meson to the pion
plus electron and neutrino**



$$\vec{p}_{[e^- Z]} = \vec{p}_{[\mu^- Z]} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu} = -\vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu}$$

**Decay of a muonic atom to a moving
electronic atom plus two neutrinos**

Measures very high momentum tail of atomic wavefunction



$$\vec{p}_{[e^- Z]} = \vec{p}_{[\mu^- Z]} - \vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu} = -\vec{p}_{\bar{\nu}_e} - \vec{p}_{\nu_\mu}$$

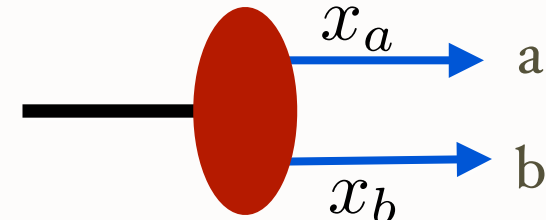
Decay of a muonic atom to a moving electronic atom plus two neutrinos

Measures very high momentum tail of atomic wavefunction

Bethe-Salpeter Equation for Hydrogenic Atoms

$$(p_a - m_a)(p_b - m_b)|N \rangle = G|N \rangle$$

$$p_a^\mu + p_b^\mu = P^\mu = (E_N, \vec{P})$$



an eigenvalue problem for $P^0 = E_N = \sqrt{M_N^2 + \vec{P}^2}$

$$(i\partial_a - m_a)(i\partial_b - m_b)\chi_N(x_a, x_b) = (G\chi_N)(x_a, x_b)$$

In momentum space: $P = p_a + p_b$ $p = \tau_b p_a - \tau_a p_b$

$$\begin{aligned} & [\gamma^{(a)} \cdot (\tau_a P + p) - m_a][\gamma^{(b)} \cdot (\tau_b P - p) - m_b]\Psi_N(p, P) \\ &= \int d^4 p' G(p, p'; P)\Psi_N(p', P) \end{aligned}$$

$$\tau_a = \frac{m_a}{m_a + m_b}$$

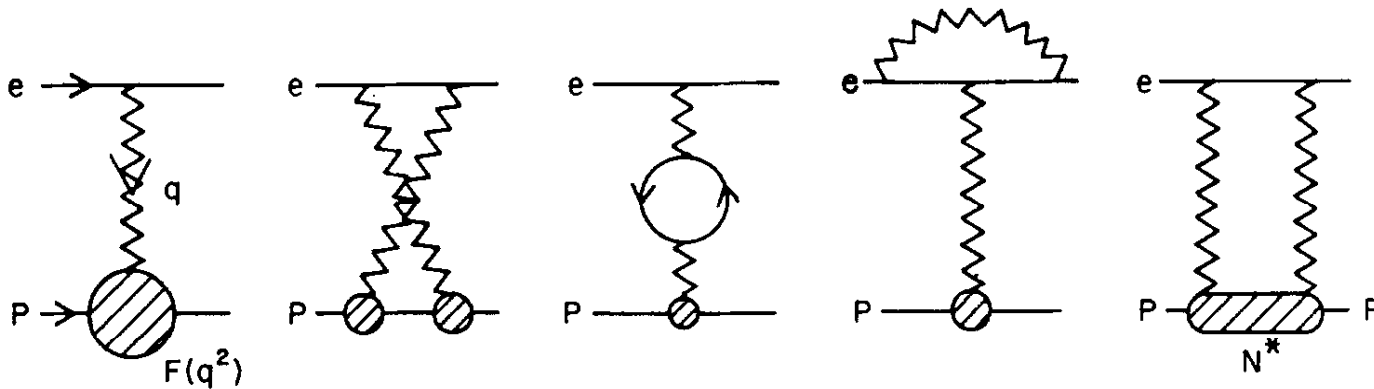
$$\tau_b = \frac{m_b}{m_a + m_b}$$

Bethe-Salpeter Theory of Hydrogenic Atoms

Bethe-Salpeter Equation

$$(\not{p}_e - m_e)(\not{p}_p - m_p) \chi = G \chi$$

$$G = G_{1\gamma} + G_{\text{CROSSED}} + G_{\text{VAC.POL.}} + G_{\text{SELF ENERGY}} + G_{\text{NUC-POL}} + \dots$$



$$G_{1\gamma} = G_{\text{COULOMB}} + G_{\text{TRANSVERSE}}$$

$$-\epsilon_\mu \frac{1}{q^2} \epsilon^\mu = \epsilon_0 \frac{1}{q^2} \epsilon_0 + \sum_{\substack{\text{TRAN} \\ i=1,2}} \epsilon_i \frac{1}{q^2} \epsilon_i$$

G_{COULOMB} → Schrödinger equation, proton finite size correction

+ G_{TRANS} → reduced mass corrections, HFS splittings

+ $G_{\text{CROSSED}}^{(\text{all})}$ → Dirac equation, relativistic reduced mass correction

+ $G_{\text{VAC-POL}} + G_{\text{SELF ENERGY}}$ → Lamb shift, radiative corrections to HFS

+ $G_{\text{NUC-POL}}$ → correction to HFS

Features of Bethe-Salpeter Equation

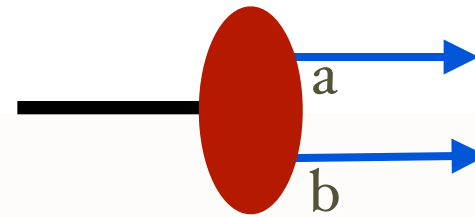
- **Exact Bound-State Formalism for QED if one includes all 2PI kernels**
- **Eigenvalues give complete spectrum, bound state and continuum**
- **Relativistic, Frame Independent**
- **Feynman virtualities:** $p_i^2 \neq m_i^2$
- **Reduces to Dirac Coulomb Equation if one includes all crossed graph 2PI kernels**
- **Matrix Elements of electromagnetic current from sum of all 2PI contributions**
- **Normalization of Bethe-Salpeter Wavefunctions also requires sum of all 2PI kernels**
- **n-body formulation difficult**
- **No cluster decomposition theorem**

Solution to Salpeter Equation in CM frame

Total spin S ,
Projection $S^z = M$

$$\varphi_{\mathcal{M}}(\mathbf{x}_a, \mathbf{x}_b, X^0)_{SM}$$

$$= \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a^0 + m_a}{2p_a^0} \frac{p_b^0 + m_b}{2p_b^0} \right)^{1/2} \left(\frac{1}{2m_a + k_a} \begin{pmatrix} 1 \\ \boldsymbol{\sigma}_a \cdot \mathbf{p} \end{pmatrix} \right) \otimes \left(\frac{1}{2m_b + k_b} \begin{pmatrix} 1 \\ -\boldsymbol{\sigma}_b \cdot \mathbf{p} \end{pmatrix} \right) \\ \times \phi_{\mathcal{M}}(\mathbf{p}) \chi_{SM} e^{i\mathbf{p} \cdot \mathbf{x} - i\mathcal{M}X^0}$$



$$k_{a,b} \equiv -\tau_{b,a}(U + W)$$

$$\int d^3x_a d^3x_b \varphi_{\mathcal{M}}(\mathbf{x}_a, \mathbf{x}_b)^\dagger (\Lambda_{++} - \Lambda_{--}) \varphi_{\mathcal{M}}(\mathbf{x}_a, \mathbf{x}_b) = 1$$

$$\int d^3p |\phi_{\mathcal{M}}(\mathbf{p})|^2 = 1.$$

$$\chi_{\mathcal{M}}^{\alpha\beta}(x_a, x_b)_{SM} = \langle 0 | T(\psi_a^\alpha(x_a) \psi_b^\beta(x_b)) | \mathbf{0} \mathcal{M} SM \rangle$$

Lorentz Boost

$$\Phi_{\mathcal{M}}(x_a, x_b)_{SM} = \langle 0 | T(\psi(x_a)\psi_b(x_b)) | \vec{P} = \vec{0}, \mathcal{M}, S, M \rangle$$

$$\Phi_{E, \vec{P}}(x'_a, x'_b)_{SM} = S_a(\Lambda) S_b(\Lambda) \Phi_{\mathcal{M}}(x_a, x_b)_{SM}$$

$$S_a(\Lambda) = \sqrt{\frac{E + \mathcal{M}}{2\mathcal{M}}} \left(1 + \frac{\vec{\alpha}_a \cdot \vec{P}}{\mathcal{M} + E} \right)$$

$$S_a(\Lambda) u(0) = u(p) = \sqrt{\frac{p^0 + m}{2m}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{p}}{p^0 + m} \end{pmatrix} \chi$$

Single particle wave-packet

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}} u(p) \phi(p) e^{-ip \cdot x}$$

$$u(p) = \sqrt{\frac{p^0 + m}{2m}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{p^0 + m} \end{pmatrix} \chi.$$

Guess wavefunction for moving bound state

$$\varphi_{EP}(\mathbf{x}_a, \mathbf{x}_b, X^0)_{SM}$$

$$= \frac{E + \mathcal{M}}{2\mathcal{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a^0 + m_a}{2p_a^0} \frac{p_b^0 + m_b}{2p_b^0} \right)^{1/2}$$

$$\times \begin{pmatrix} 1 \\ \boldsymbol{\sigma}_a \cdot \left(\frac{\mathbf{P}}{\mathcal{M} + E} + \frac{\mathbf{p}}{2m_a + k_a} \right) \end{pmatrix} \otimes \begin{pmatrix} 1 \\ \boldsymbol{\sigma}_b \cdot \left(\frac{\mathbf{P}}{\mathcal{M} + E} - \frac{\mathbf{p}}{2m_b + k_b} \right) \end{pmatrix}$$

$$\times \phi_{\mathcal{M}}(\mathbf{p}) \chi_{SM} \exp[i\mathbf{p} \cdot \tilde{\mathbf{x}} + i\mathbf{P} \cdot \mathbf{X}] \exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma - 1) \hat{\mathbf{V}} \hat{\mathbf{V}} \cdot \mathbf{x} \quad ; \quad p_{a,b}^0 = \sqrt{\mathbf{p}^2 + m_{a,b}^2}, \quad k_{a,b} \equiv -\tau_{b,a}(U + W).$$

Single particle wave-packet

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}} u(p) \phi(p) e^{-ip \cdot x}$$

Primack, sjb

$$u(p) = \sqrt{\frac{p^0 + m}{2m}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{p^0 + m} \end{pmatrix} \chi.$$

Correct wavefunction for moving bound state

Not product of independent boosts!!

$$\varphi_{EP}(\mathbf{x}_a, \mathbf{x}_b, X^0)_{SM}$$

$$= \frac{E + \mathcal{M}}{2\mathcal{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a^0 + m_a}{2p_a^0} \frac{p_b^0 + m_b}{2p_b^0} \right)^{1/2}$$

$$\times \begin{pmatrix} 1 + \frac{\boldsymbol{\sigma}_a \cdot \mathbf{P}}{\mathcal{M} + E} \frac{\boldsymbol{\sigma}_a \cdot \mathbf{p}}{2m_a + k_a} \\ \boldsymbol{\sigma}_a \cdot \left(\frac{\mathbf{P}}{\mathcal{M} + E} + \frac{\mathbf{p}}{2m_a + k_a} \right) \end{pmatrix} \otimes \begin{pmatrix} 1 - \frac{\boldsymbol{\sigma}_b \cdot \mathbf{P}}{\mathcal{M} + E} \frac{\boldsymbol{\sigma}_b \cdot \mathbf{p}}{2m_b + k_b} \\ \boldsymbol{\sigma}_b \cdot \left(\frac{\mathbf{P}}{\mathcal{M} + E} - \frac{\mathbf{p}}{2m_b + k_b} \right) \end{pmatrix}$$

$$\times \phi_{\mathcal{M}}(\mathbf{p}) \chi_{SM} \exp[ip \cdot \tilde{\mathbf{x}} + i\mathbf{P} \cdot \mathbf{X}] \exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma - 1) \hat{\mathbf{V}} \hat{\mathbf{V}} \cdot \mathbf{x} \quad ; \quad p_{a,b}^0 = \sqrt{\mathbf{p}^2 + m_{a,b}^2}, \quad k_{a,b} \equiv -\tau_{b,a}(U + W).$$

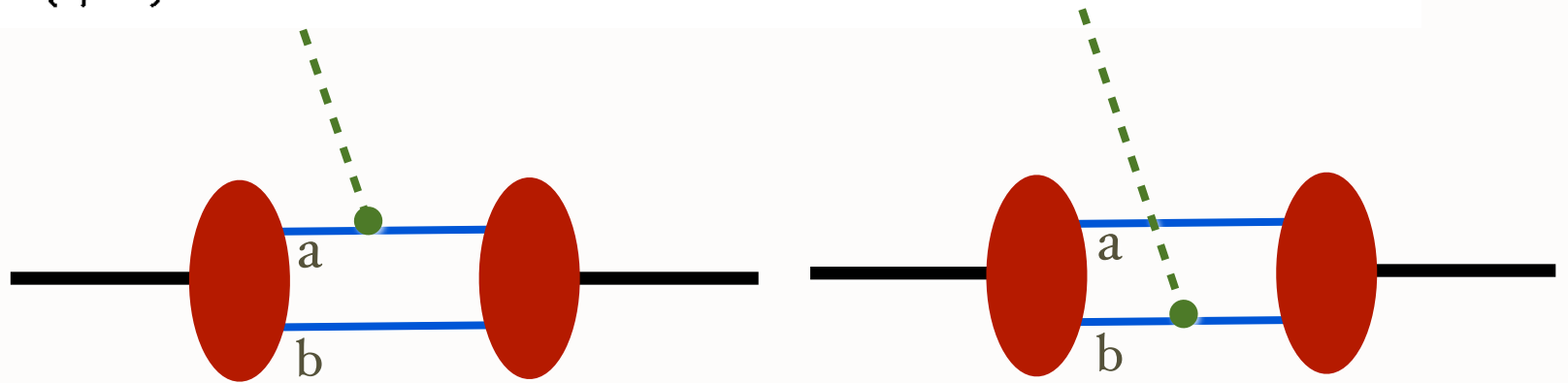
Correct reduction of electromagnetic interaction
in nonrelativistic limit

$$H_{\text{NR}}^{\text{em}} = \sum_{s=a,b} \left[\frac{-\mathbf{p}_s \cdot e_s \mathbf{A}_s}{m_s} + \frac{e_s^2 \mathbf{A}_s^2}{2m_s} + e_s A_s^0 - \mu_s \boldsymbol{\sigma}_s \cdot \mathbf{B}_s \right. \\ \left. - \left(2\mu_s - \frac{e_s}{2m_s} \right) \boldsymbol{\sigma}_s \cdot \mathbf{E}_s \times \frac{(\mathbf{p}_s - e_s \mathbf{A}_s)}{2m_s} \right]$$

Correction to
Foldy-Wouthyusen!
↓

$$+ \frac{1}{4M_T} \left(\frac{\boldsymbol{\sigma}_a}{m_a} - \frac{\boldsymbol{\sigma}_b}{m_b} \right) \cdot (e_b \mathbf{E}_b \times (\mathbf{p}_a - e_a \mathbf{A}_a) - e_a \mathbf{E}_a \times (\mathbf{p}_b - e_b \mathbf{A}_b))$$

$$+ 0(1/m^3).$$



Bound state of two spin-1/2 particles

Boost of a Composite System

- *Boost is not product of independent boosts of constituents since constituents are already moving*
- *Only known at weak binding*
- *Dirac: Boosts are dynamical*
- *Correct form needed to prove Low Energy Theorem for Compton scattering and Drell-Hearn Gerasimov Sum Rule*

Drell Hearn Gerasimov Sum Rule

$$\int_{\omega_{\text{th}}}^{\infty} \frac{\sigma_P(\omega) - \sigma_A(\omega)}{\omega} d\omega = 8\pi^2 \left(\mu - \frac{Z_T e}{2\mathcal{M}} \right)^2$$

anomalous magnetic
moment squared

Proof

Optical Theorem from Unitarity

Forward spin-flip amplitude given by LET

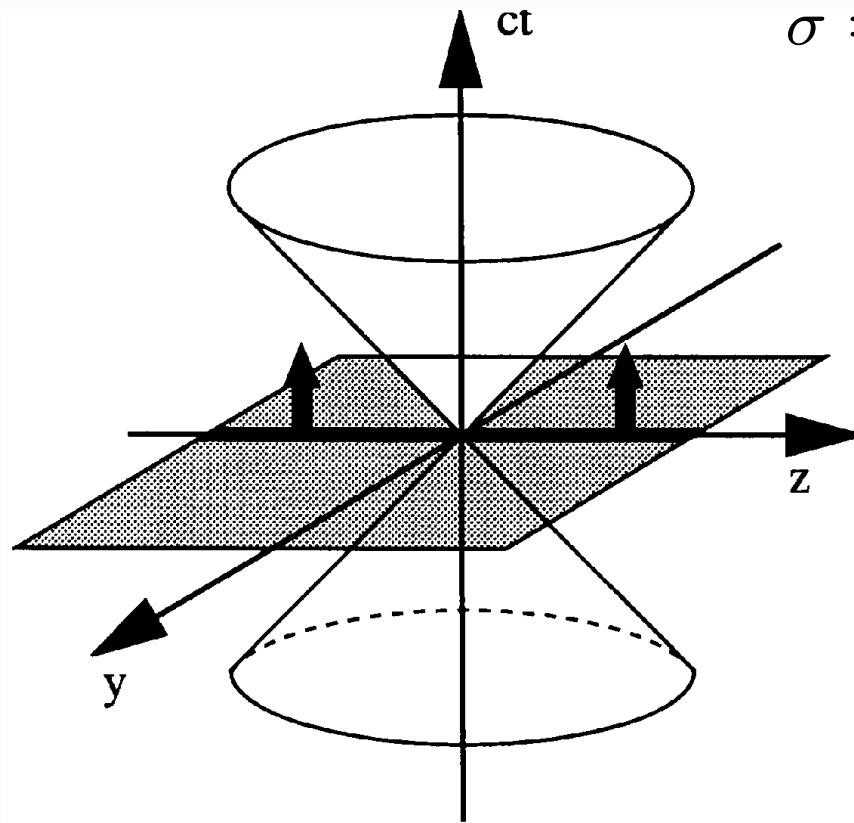
Unsubtracted dispersion relation

$$M_{fi} = \frac{1}{2\omega} (2\pi)^3 \delta^3(P_f - P_i) \left[\frac{Z_T^2 e^2}{\mathcal{M}} \hat{\mathbf{e}}' \cdot \hat{\mathbf{e}} \delta_{fi} + 2i\omega \left(\mu - \frac{Z_T e}{2\mathcal{M}} \right)^2 \boldsymbol{\sigma}_{fi} \cdot \hat{\mathbf{e}}' \times \hat{\mathbf{e}} + O(\omega^2) \right]$$

$$M^{\uparrow \rightarrow \downarrow}(\theta = 0)$$

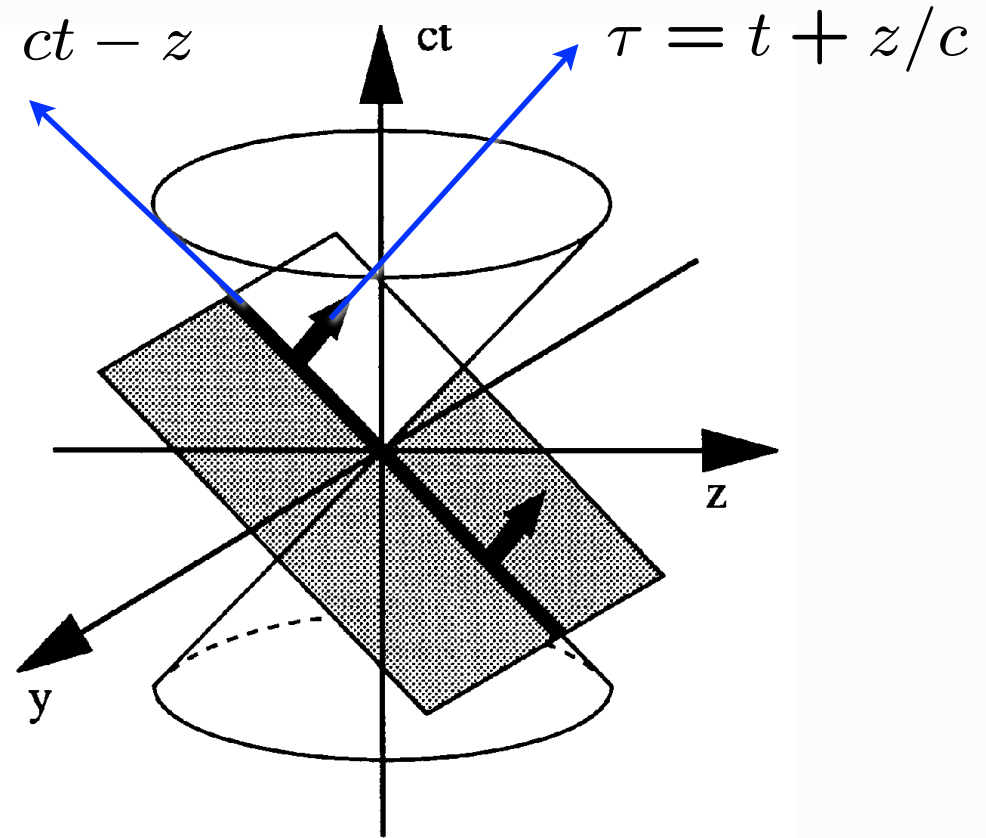
Dirac's Amazing Idea: The "Front Form"

Evolve in
"light-front" time



Instant Form

$$\sigma = ct - z$$



Front Form

*Each element of
flash photograph
illuminated
at same LF time*

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenstate -- independent of τ

Causally-Connected Domains



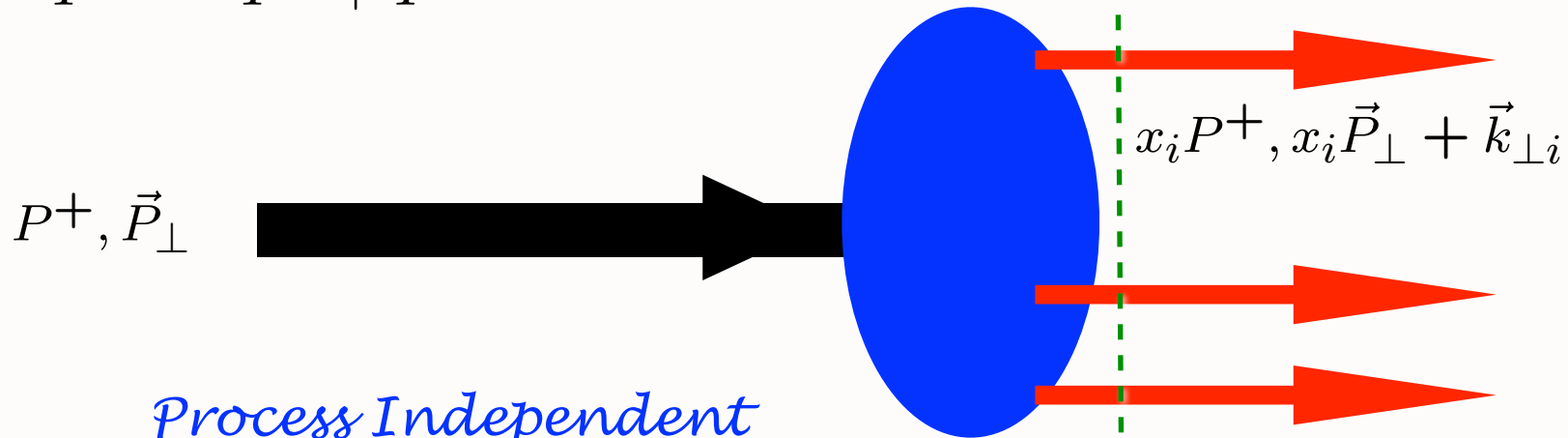
*'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing*

Montgomery

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



*Process Independent
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock
State!

LF Spin Sum Rule

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

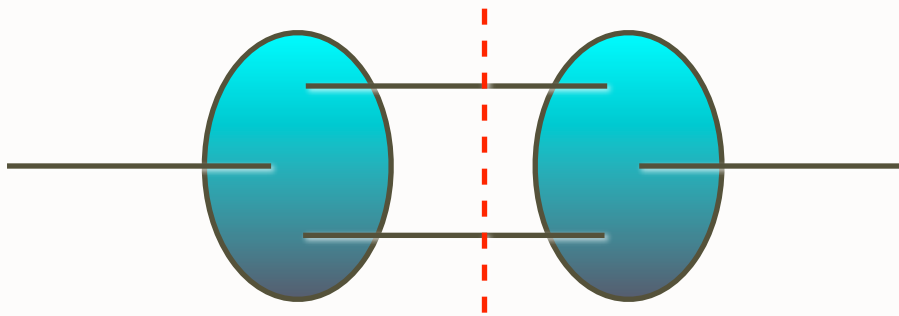
n-1 orbital angular momenta

Orbital angular momentum is a property of Light-Front Wavefunctions

Nonzero Anomalous Moment --> Nonzero orbital angular momentum

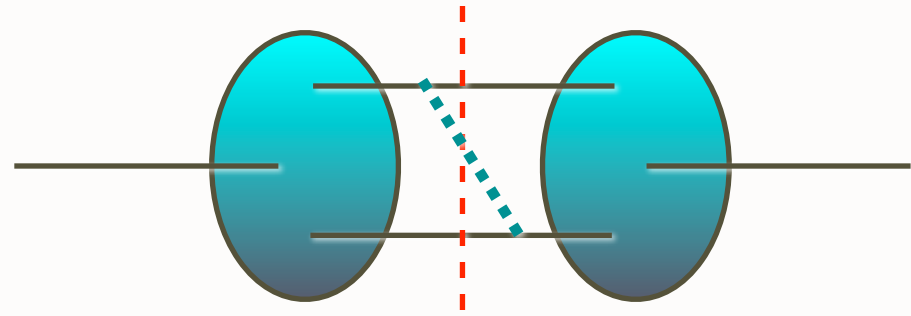
Quantum Mechanics: Uncertainty in p , x , spin

Relativistic Quantum Field Theory:
Uncertainty in particle number n



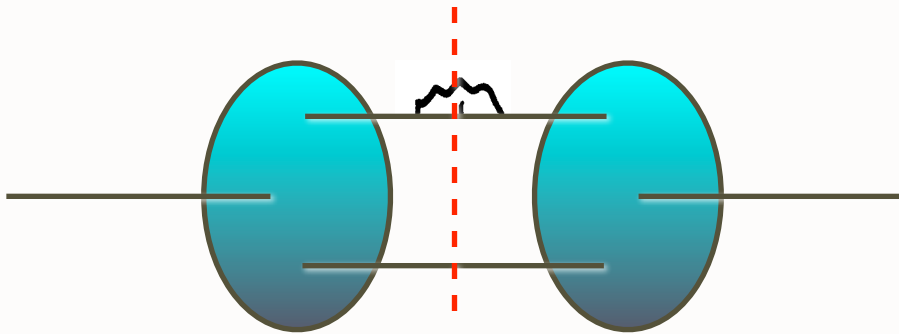
Positronium $n=2$

$$e^+e^-$$



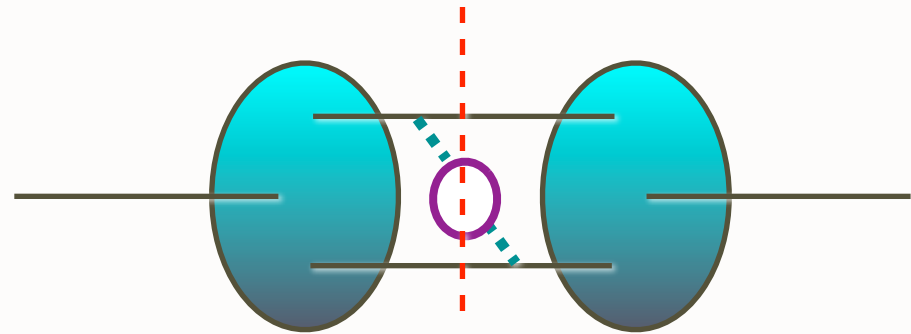
Hyperfine splitting $n=3$

$$e^+e^-\gamma$$



Lamb Shift $n=3$

$$e^+e^-\gamma$$



Vacuum Polarization $n=4$

$$e^+e^-e^+e^-$$

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with n=3, 4, ... constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

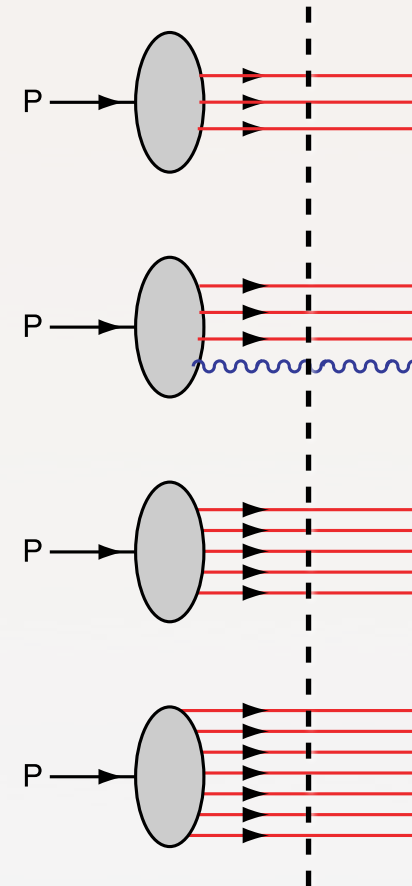
$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks

$c(x), b(x)$ at high x

$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$



Fixed LF time

Light-Front QED

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

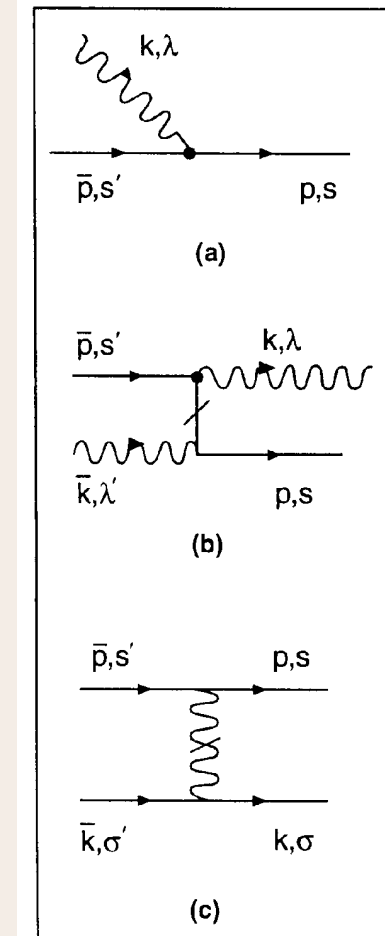
$$L^{QED} \rightarrow H_{LF}^{QED}$$

$$H_{LF}^{QED} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QED} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

Eigenvalues and Eigensolutions give Positronium Spectrum and Light-Front wavefunctions



Light-Front QCD

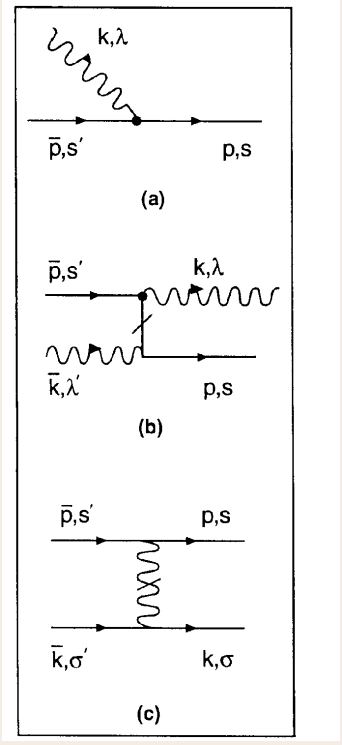
Heisenberg Matrix Formulation

$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization

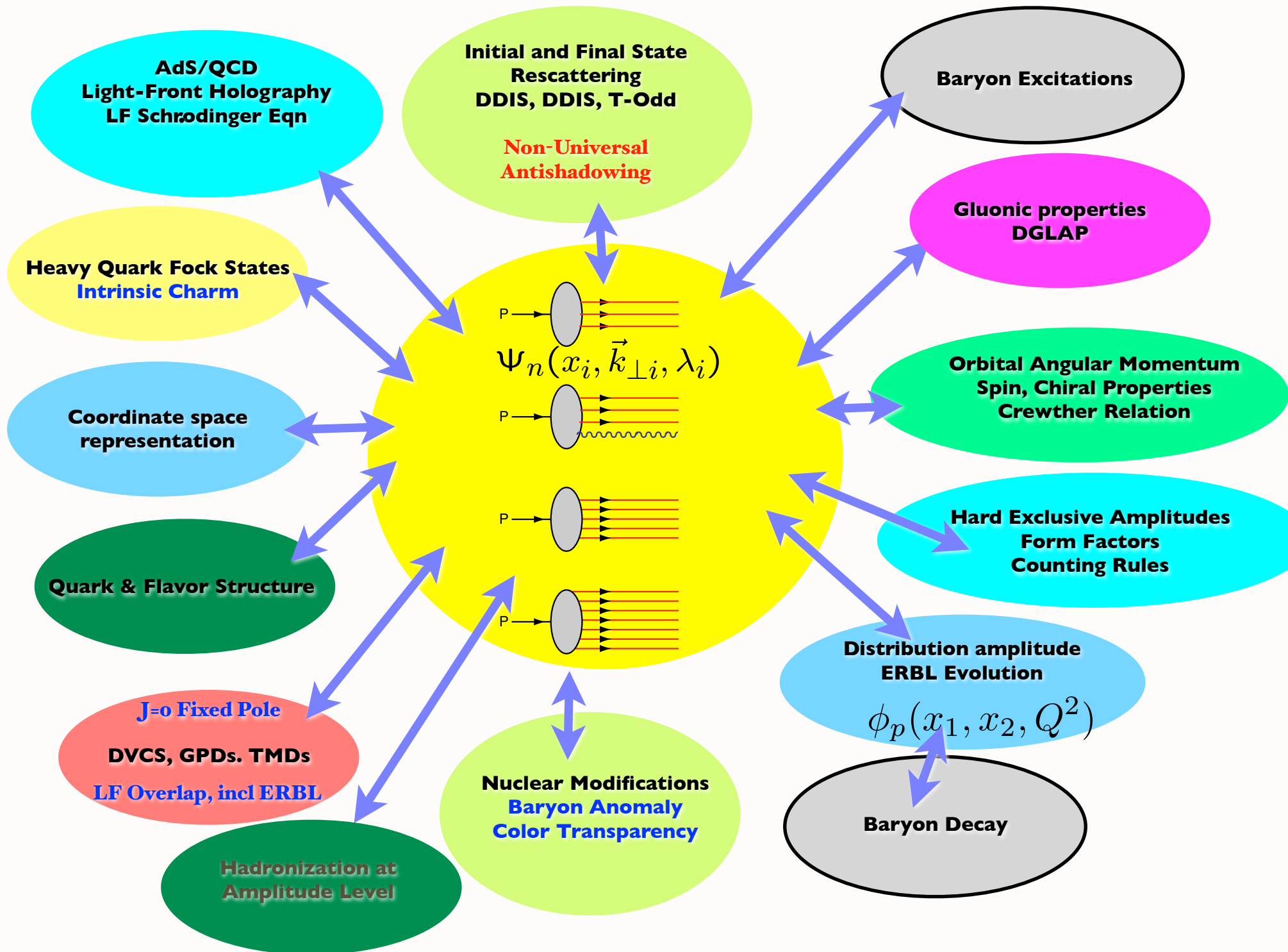
| n | Sector | 1 q \bar{q} | 2 gg | 3 q \bar{q} g | 4 q \bar{q} q \bar{q} | 5 gg g | 6 q \bar{q} gg | 7 q \bar{q} q \bar{q} g | 8 q \bar{q} q \bar{q} q \bar{q} | 9 gg gg | 10 q \bar{q} gg g | 11 q \bar{q} q \bar{q} gg | 12 q \bar{q} q \bar{q} q \bar{q} g | 13 q \bar{q} q \bar{q} q \bar{q} q \bar{q} |
|----|---|------------------|---------|--------------------|------------------------------|-----------|---------------------|--------------------------------|--|------------|------------------------|----------------------------------|---|---|
| 1 | q \bar{q} | | | | | . | | . | . | . | . | . | . | . |
| 2 | gg | | | | . | | | . | . | | . | . | . | . |
| 3 | q \bar{q} g | | | | | | | | . | . | | . | . | . |
| 4 | q \bar{q} q \bar{q} | | . | | | . | | | | . | . | | . | . |
| 5 | gg g | . | | | . | | | . | . | | | . | . | . |
| 6 | q \bar{q} gg | | | | | | | . | . | | | | . | . |
| 7 | q \bar{q} q \bar{q} g | . | . | | | . | | | | . | | | | . |
| 8 | q \bar{q} q \bar{q} q \bar{q} | . | . | . | | . | . | | | . | . | | | |
| 9 | gg gg | . | | . | . | | . | . | . | | | . | . | . |
| 10 | q \bar{q} gg g | . | . | | . | | | | . | | | | . | . |
| 11 | q \bar{q} q \bar{q} gg | . | . | . | | . | | | | . | | | | . |
| 12 | q \bar{q} q \bar{q} q \bar{q} g | . | . | . | . | . | | | . | . | | | | |
| 13 | q \bar{q} q \bar{q} q \bar{q} q \bar{q} | . | . | . | . | . | . | | | . | . | | | |



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Hans Christian Pauli & sjb

QCD and the LF Hadron Wavefunctions



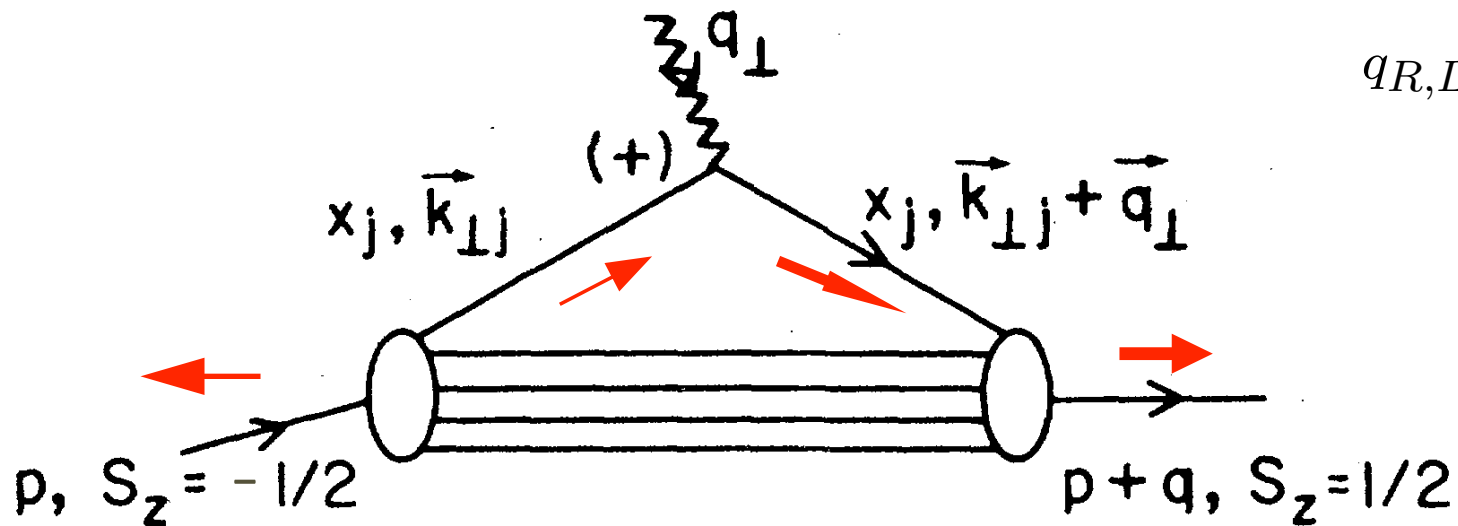
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$



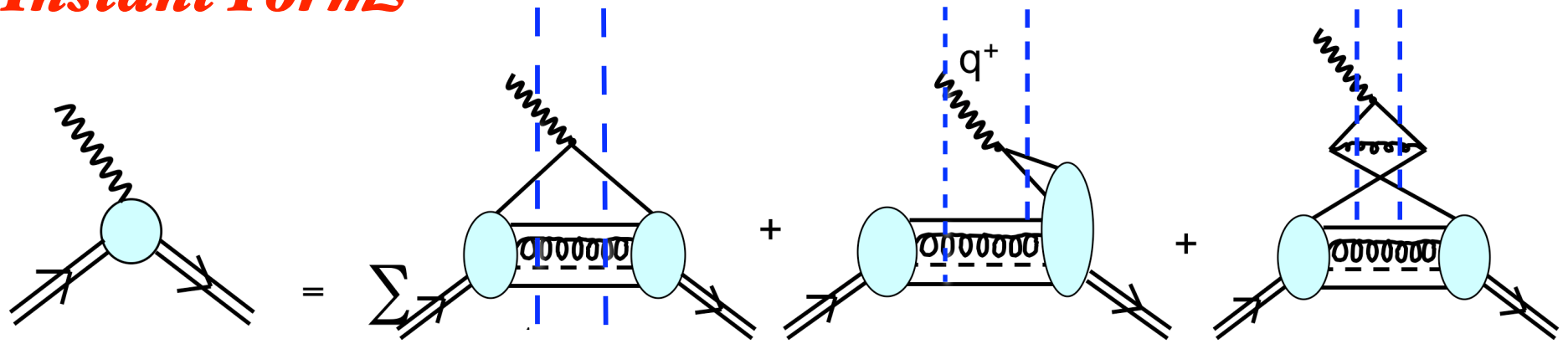
$$q_{R,L} = q^x \pm iq^y$$

Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

*Same matrix elements appear in Sivers effect
 -- connection to quark anomalous moments*

Calculation of Form Factors in Equal-Time Theory

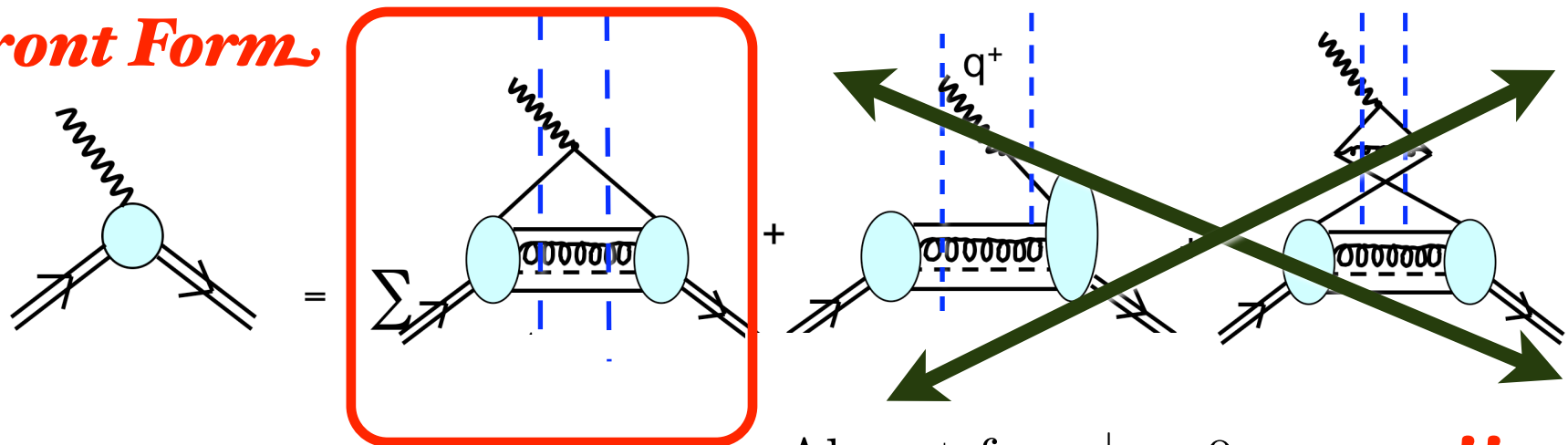
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

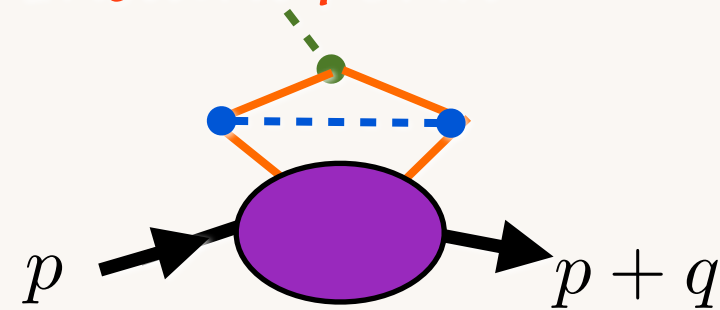
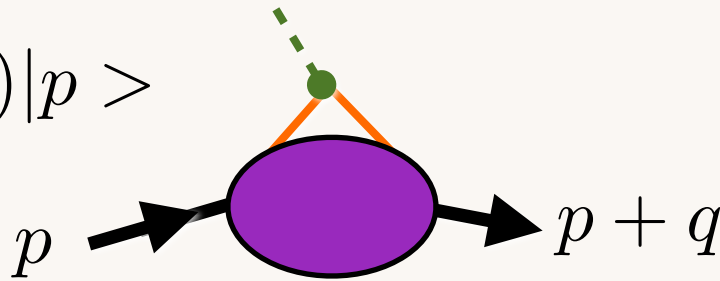
Front Form



Absent for $q^+ = 0$ **zero !!**

Calculation of proton form factor in Instant Form

$$\langle p + q | J^\mu(0) | p \rangle$$

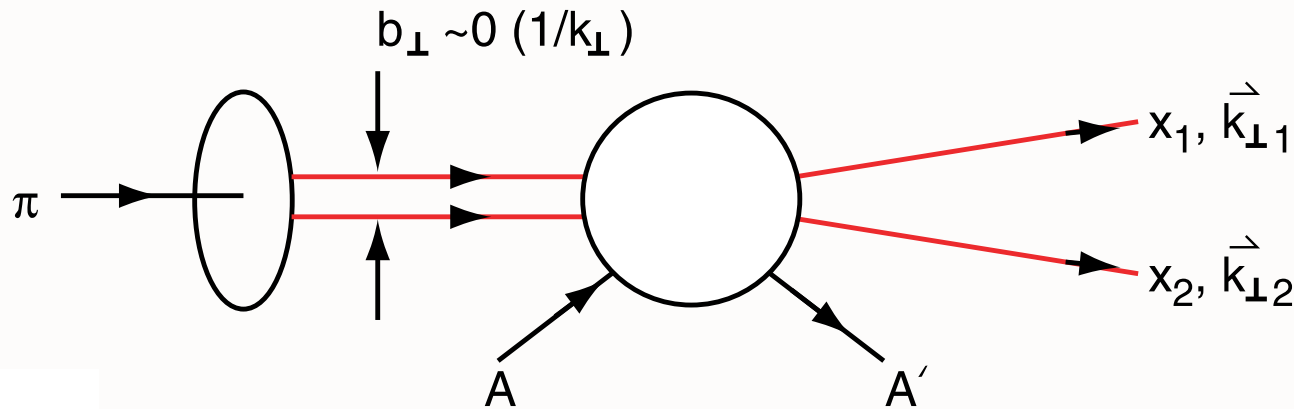


- **Need to boost proton wavefunction from p to $p+q$: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!!**
- **Wavefunction insufficient to compute matrix elements**
- **Each time-ordered contribution is frame-dependent**
- **States built on normal-ordered acausal vacuum**
- **Divide by disconnected vacuum diagrams**
- **Light-Front vacuum trivial! No conflict with cosmology**

Cosmological constant 10^{120} too large from QED?

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

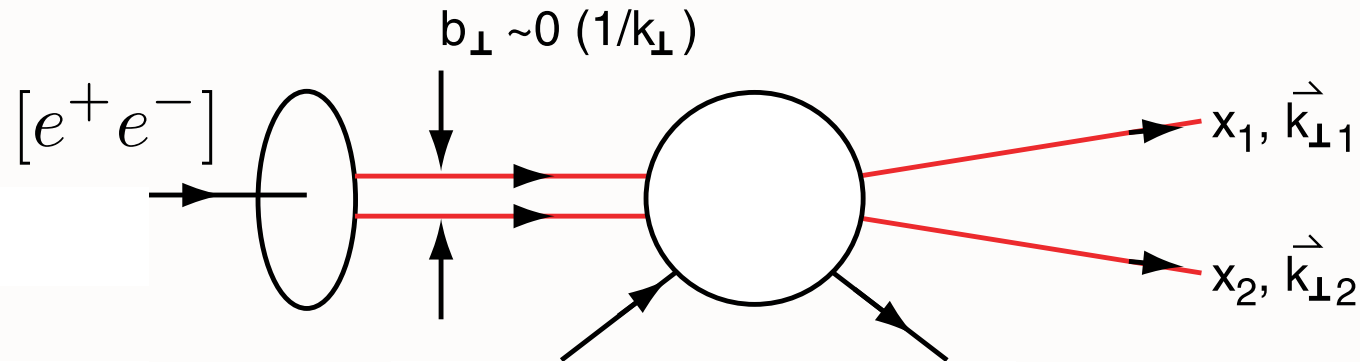


$$M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp})$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!

Diffractional Dissociation of Atoms

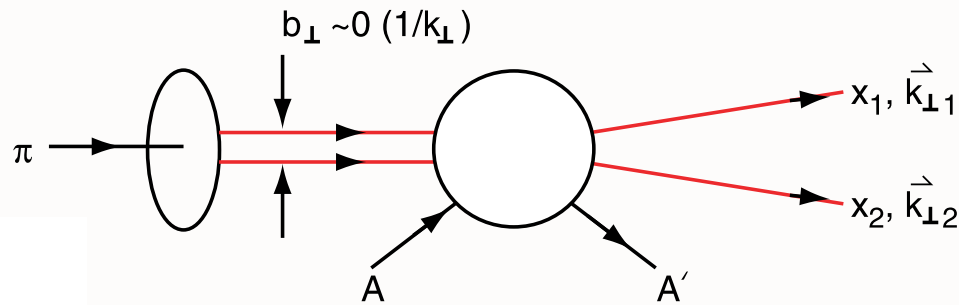


$$M \propto \frac{\partial}{\partial \vec{k}_{\perp}} \psi_{e^+ e^-} (x, \vec{k}_{\perp})$$

Measure Light-Front Wavefunction of Positronium
and Other Atoms

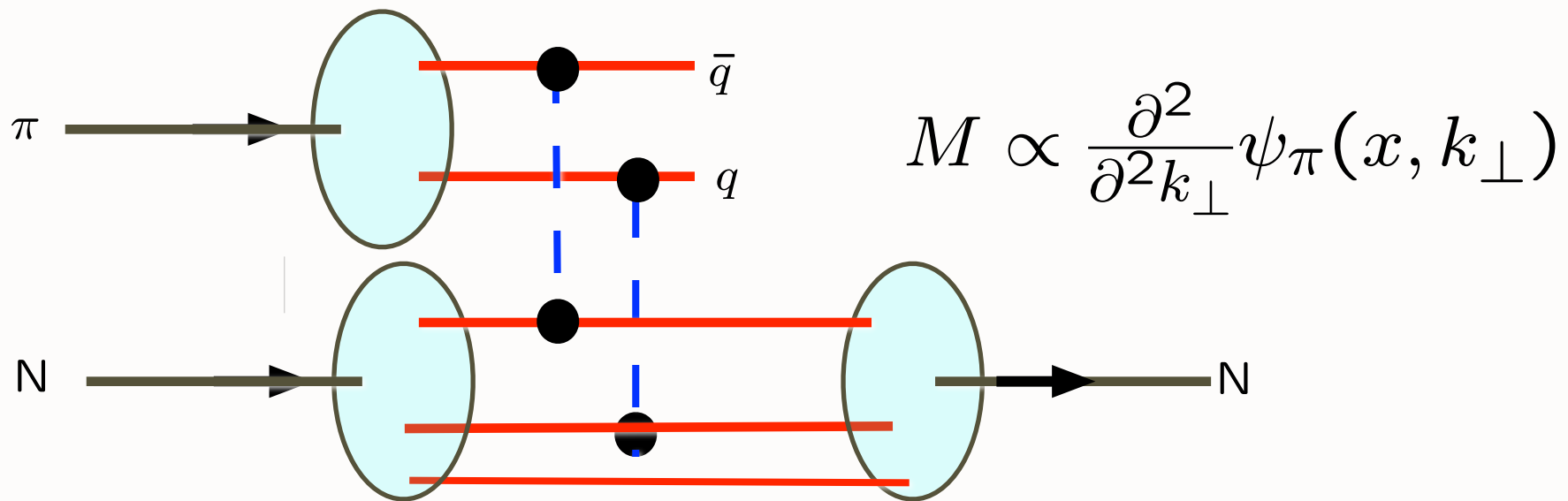
Minimal momentum transfer to Target
Target left Intact!

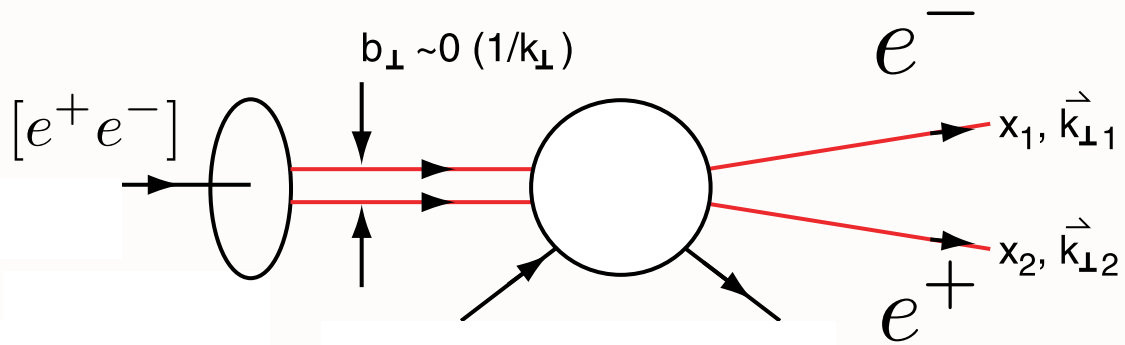
E791 FNAL Diffractive DiJet



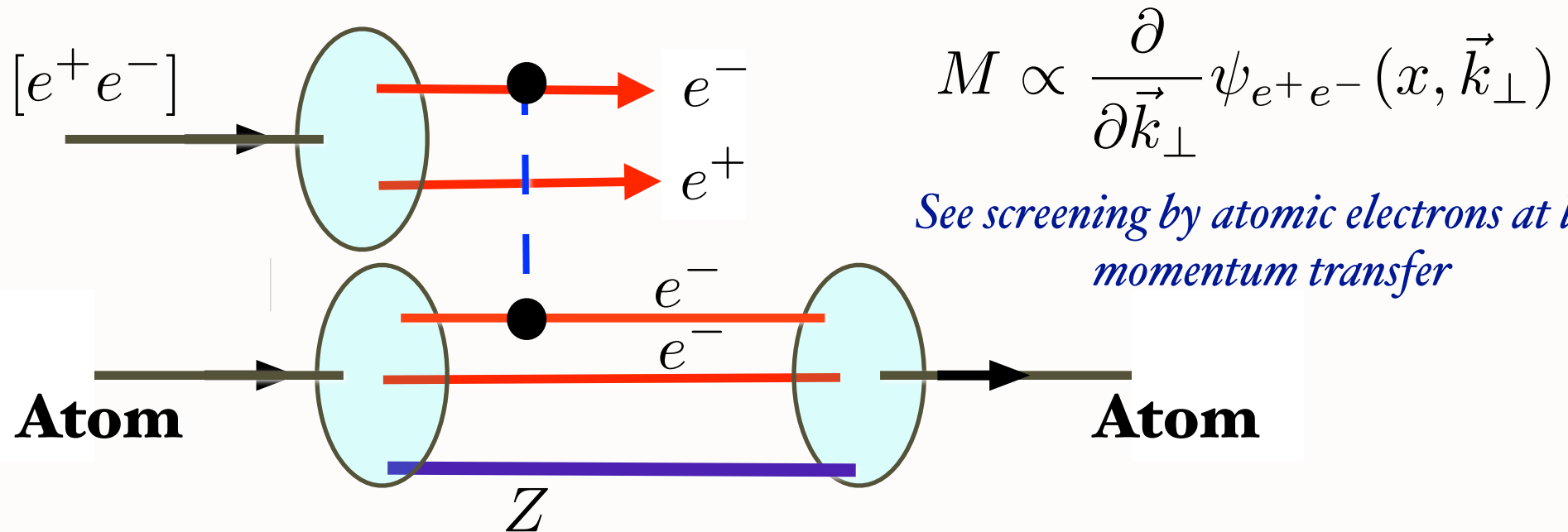
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction

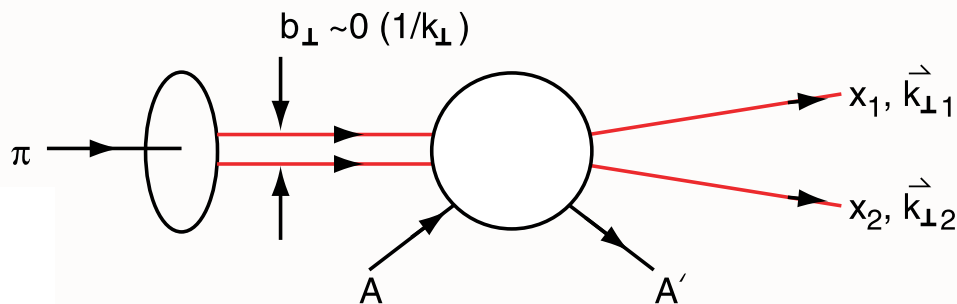




*Coulomb-Photon exchange
measures the derivative of the positronium
light-front wavefunction*



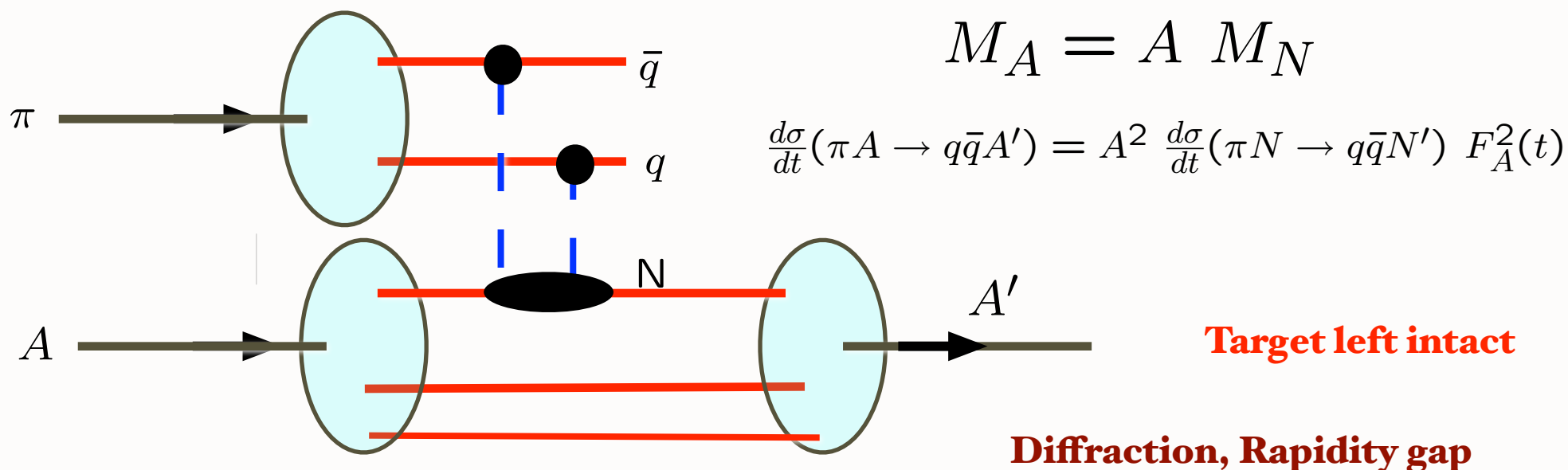
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller
Strikman

*Small color-dipole moment pion not absorbed;
interacts with each nucleon coherently*

QCD COLOR Transparency

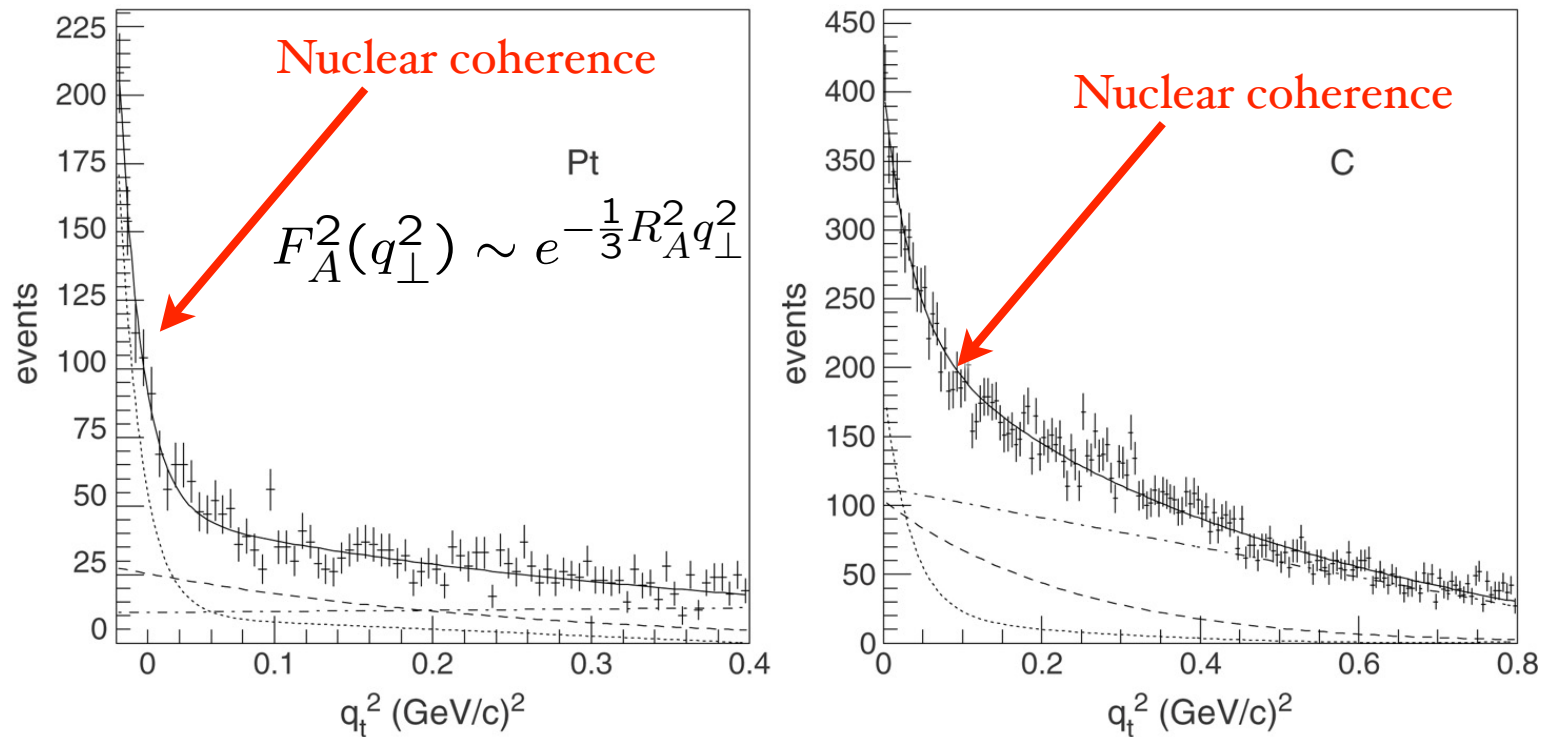


- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$M(A) = A \cdot M(N)$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Measure pion LFWF in diffractive dijet production

Confirmation of color transparency

A-Dependence results: $\sigma \propto A^\alpha$

| <u>k_t range (GeV/c)</u> | <u>α</u> | <u>α (CT)</u> |
|---------------------------------------|----------------------------|---------------------------------|
| $1.25 < k_t < 1.5$ | $1.64 +0.06 -0.12$ | 1.25 |
| $1.5 < k_t < 2.0$ | 1.52 ± 0.12 | 1.45 |
| $2.0 < k_t < 2.5$ | 1.55 ± 0.16 | 1.60 |

Ashery E79I

α (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out !

Factor of 7

Color Transparency

Bertsch, Gunion, Goldhaber, sjb

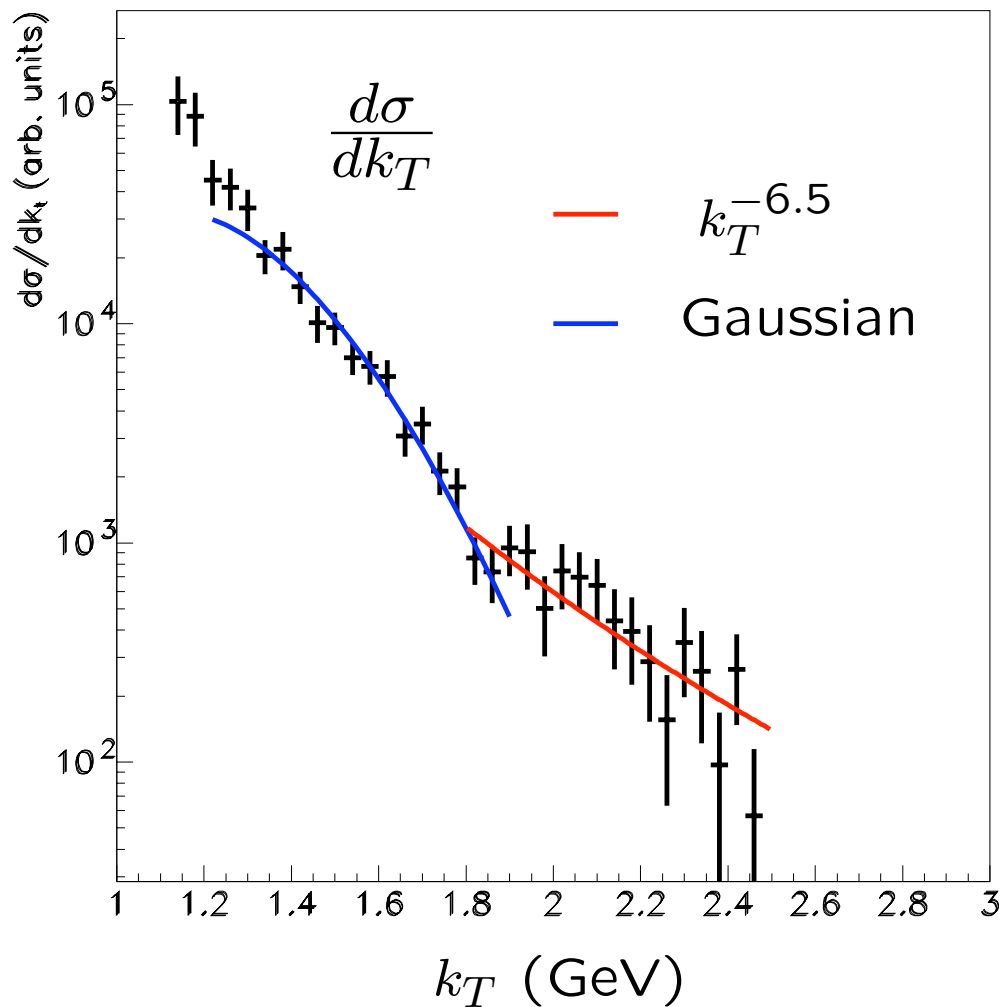
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Atomic Transparency

- Fundamental test of gauge theory in atomic physics
- Small electric dipole moments interact weakly in target
- Complete coherence at high energies -- crystals!

E791 Diffractive Di-Jet transverse momentum distribution



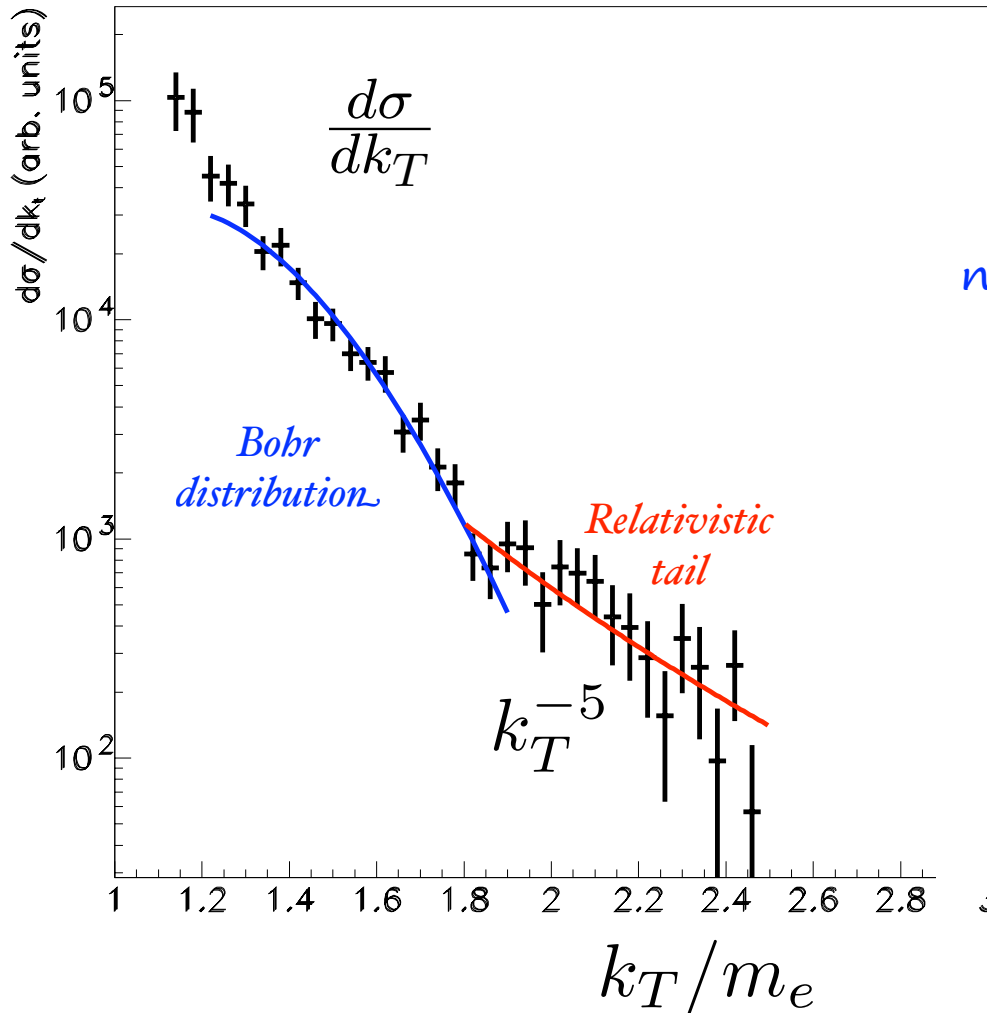
Two Components

High Transverse momentum dependence consistent with PQCD, ERBL Evolution

$$k_T^{-6.5}$$

Gaussian component similar to AdS/CFT HO LFWF

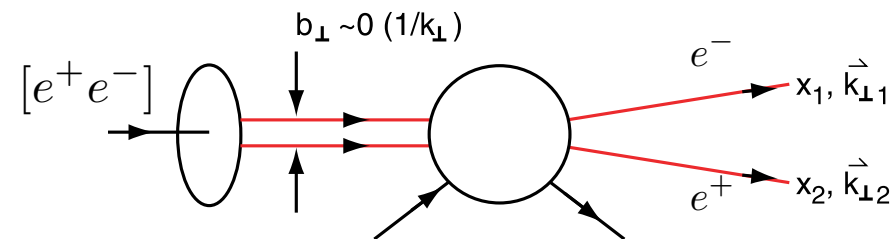
Simulated diffractive transverse momentum distribution for positronium



Two Components:

Low momentum dependence from non-relativistic Coulomb interaction

High Transverse momentum dependence from relativistic QED



$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

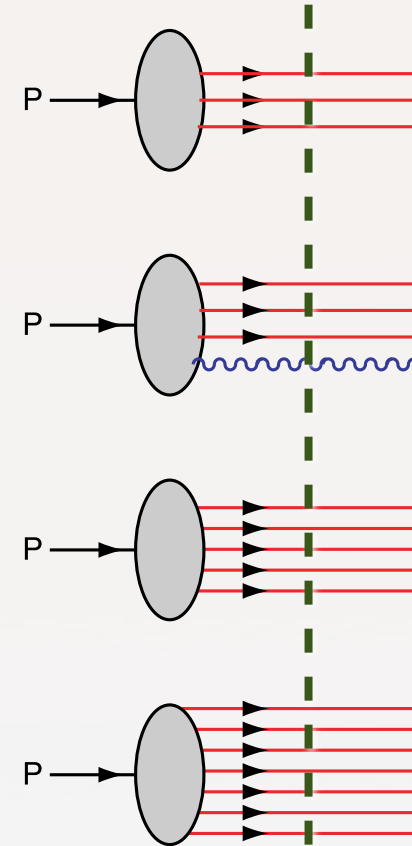
$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks
 $c(x), b(x)$ at high x !

$\bar{s}(x) \neq s(x)$
 $\bar{u}(x) \neq \bar{d}(x)$



Fixed LF time
Coupled. infinite set

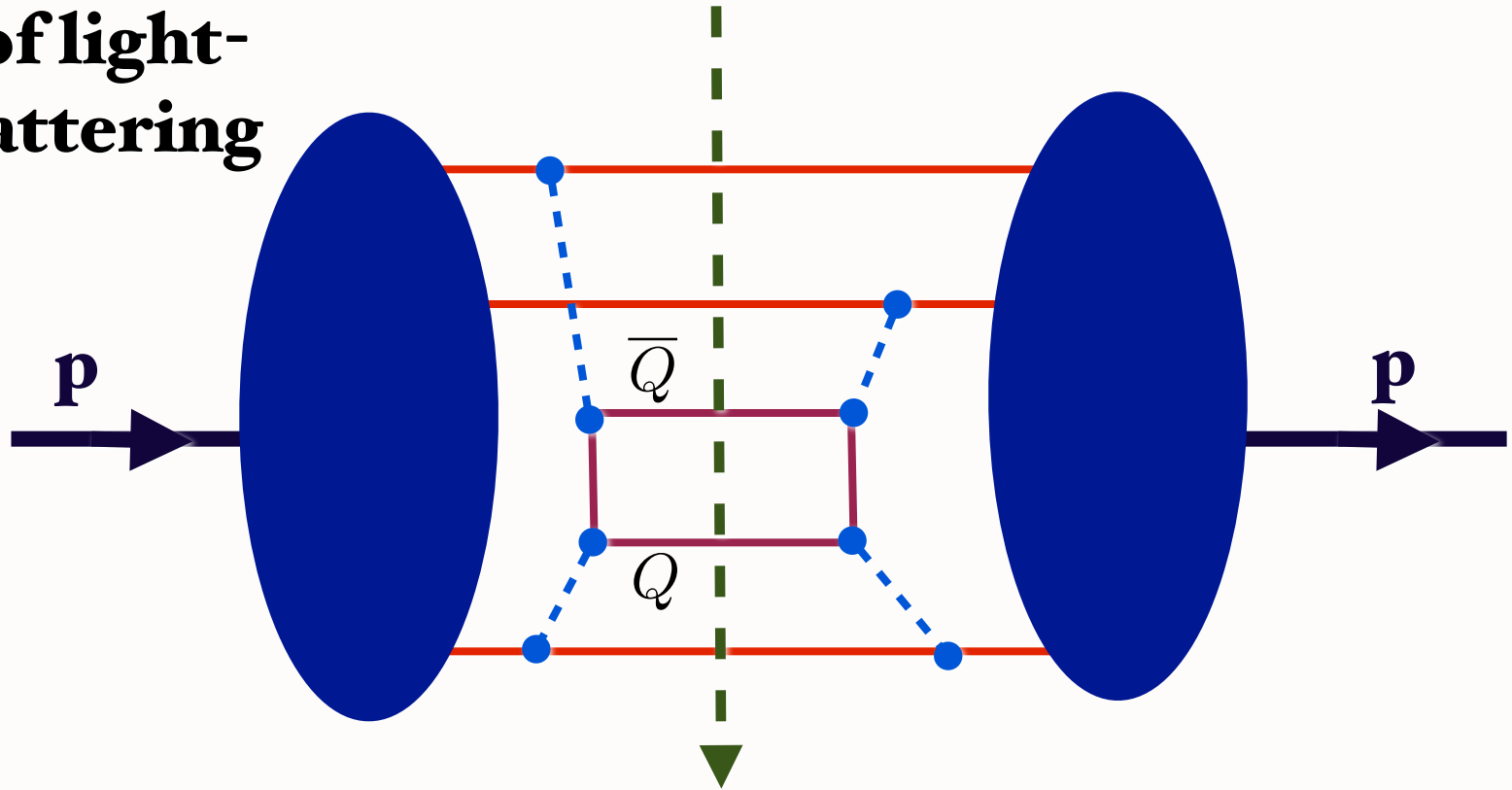
Mueller: gluon Fock states BFKL

Deuteron: Hidden Color

Proton Self Energy

QCD predicts Intrinsic Heavy Quarks!

Insertion of light-by-light scattering



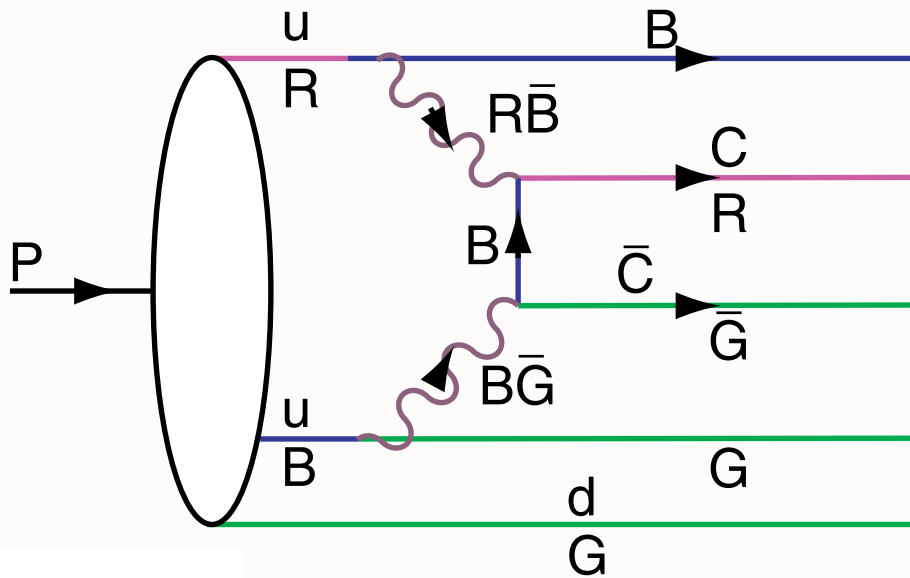
$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

**Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.**

BHPS: Hoyer, Peterson, Sakai, sjb



$|uudc\bar{c}\rangle$ Fluctuation in Proton

QCD: Probability $\sim \frac{\Lambda_{QCD}^2}{M_Q^2}$

$|e^+e^-\ell^+\ell^-\rangle$ Fluctuation in Positronium

QED: Probability $\sim \frac{(m_e\alpha)^4}{M_\ell^4}$

OPE derivation - M.Polyakov et al.

$$\langle p | \frac{G_{\mu\nu}^3}{m_Q^2} | p \rangle \text{ vs. } \langle p | \frac{F_{\mu\nu}^4}{m_\ell^4} | p \rangle$$

$c\bar{c}$ in Color Octet

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

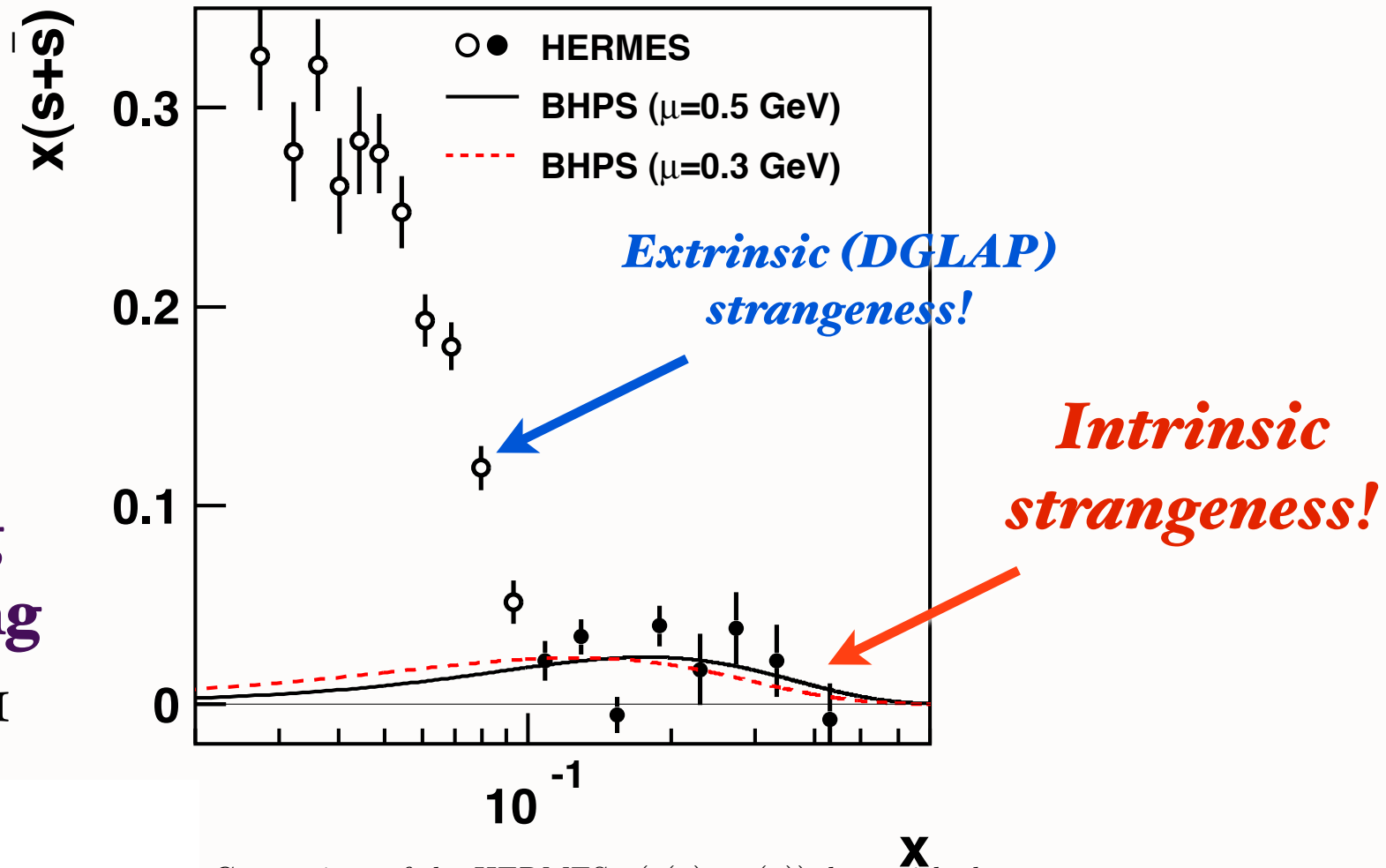
High x charm!

Charm at Threshold

Action Principle: Minimum KE, maximal potential

HERMES: Two components to $s(x, Q^2)$!

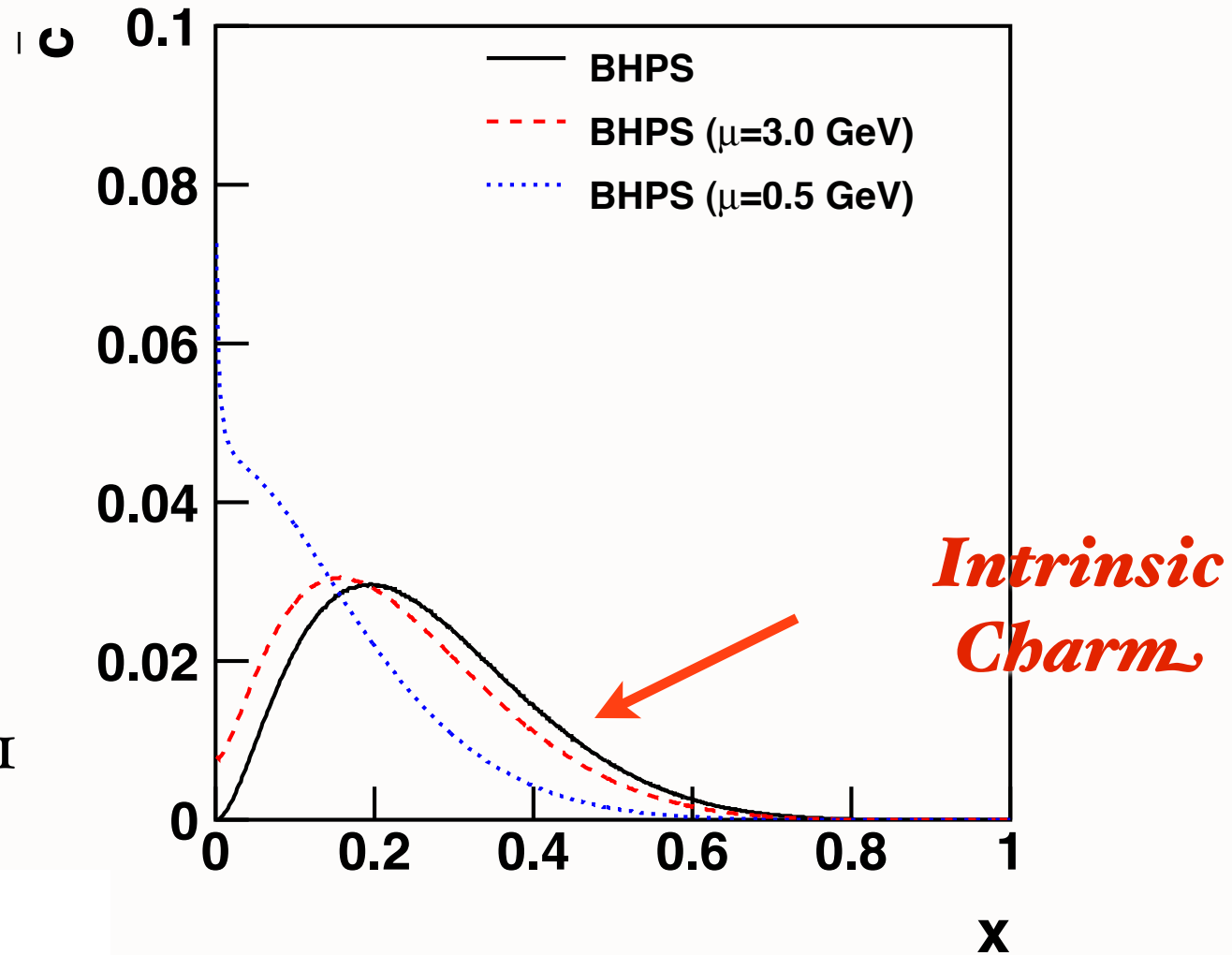
W. C. Chang
and J.-C. Peng
arXiv:1105.2381



Comparison of the HERMES $x(s(x) + \bar{s}(x))$ data with the calculations based on the BHPs model. The solid and dashed curves are obtained by evolving the BHPs result to $Q^2 = 2.5$ GeV² using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$

Scale intrinsic strangeness by $\frac{1}{m_Q^2}$



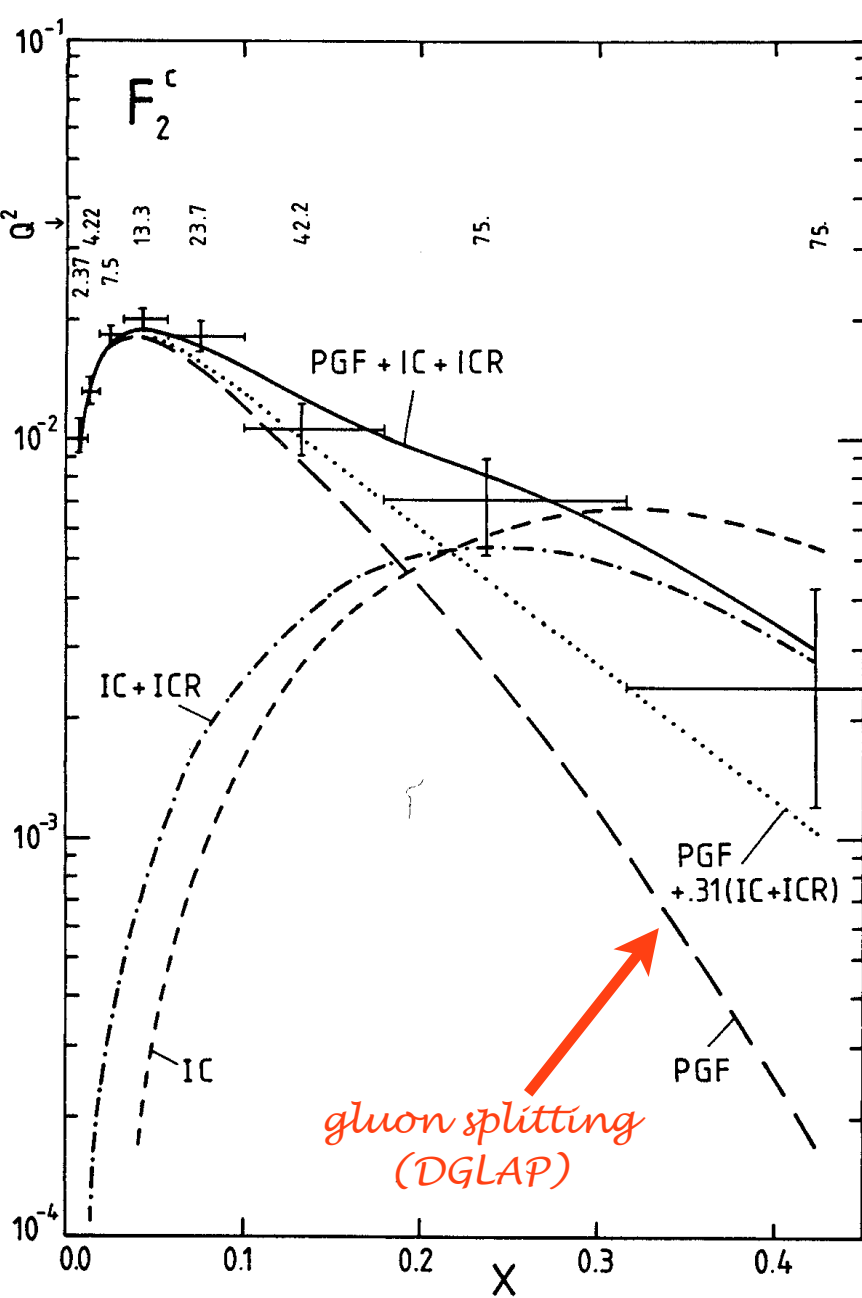
W. C. Chang
J.-C. Peng

arXiv:1105.2381

Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75 \text{ GeV}^2$ using $\mu = 3.0 \text{ GeV}$, and $\mu = 0.5 \text{ GeV}$, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

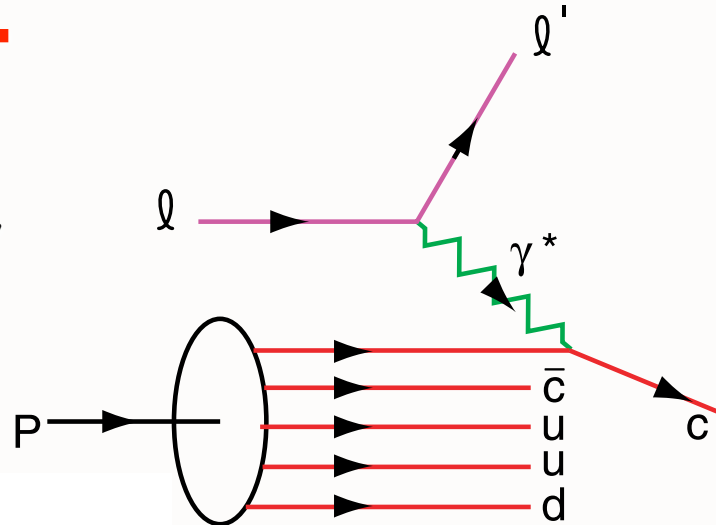
Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



First Evidence for Intrinsic Charm

factor of 30!

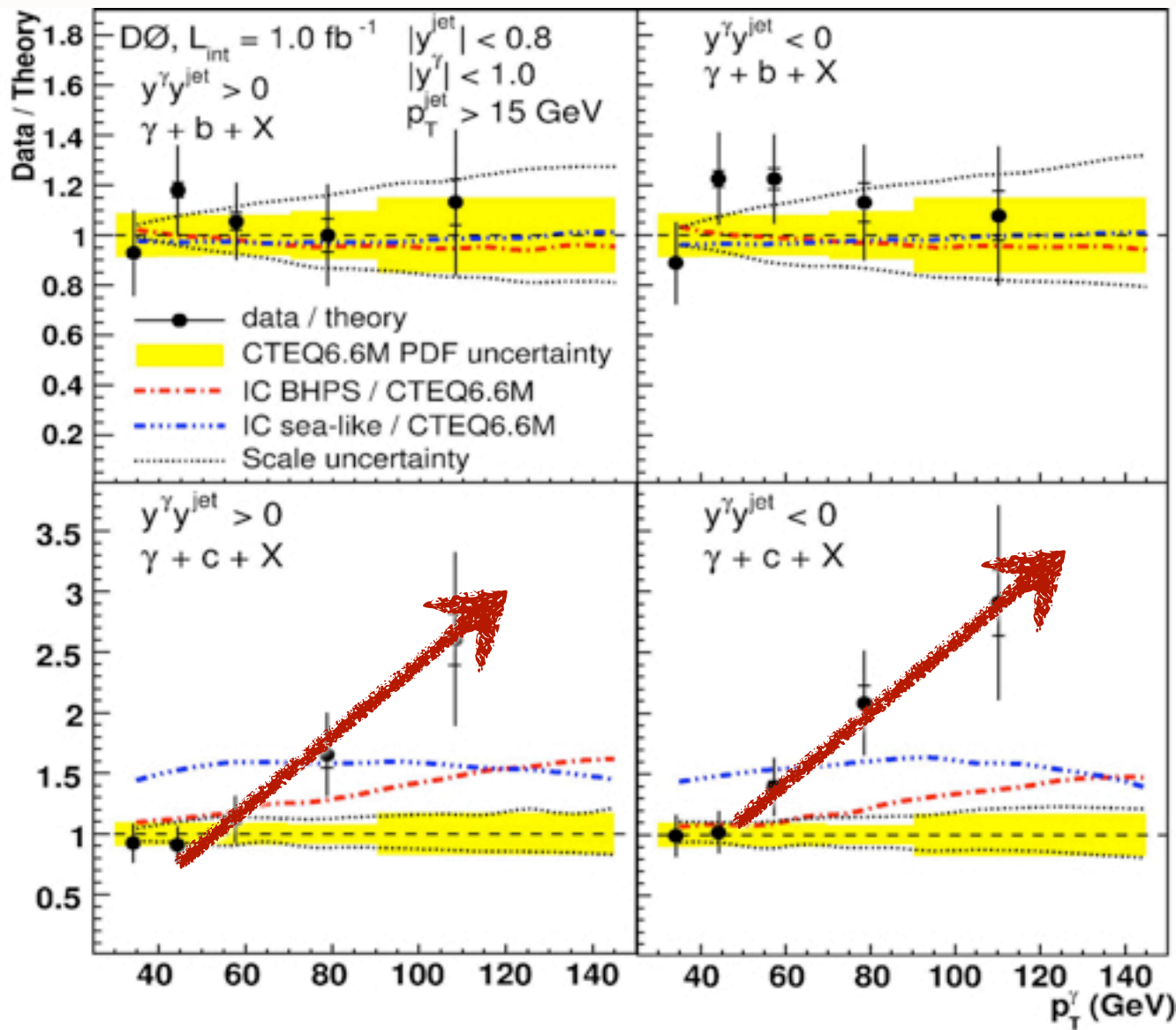


DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections
in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV



$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

Ratio
insensitive to
gluon PDF,
scales

Signal for
significant IC
at $x > 0.1$?

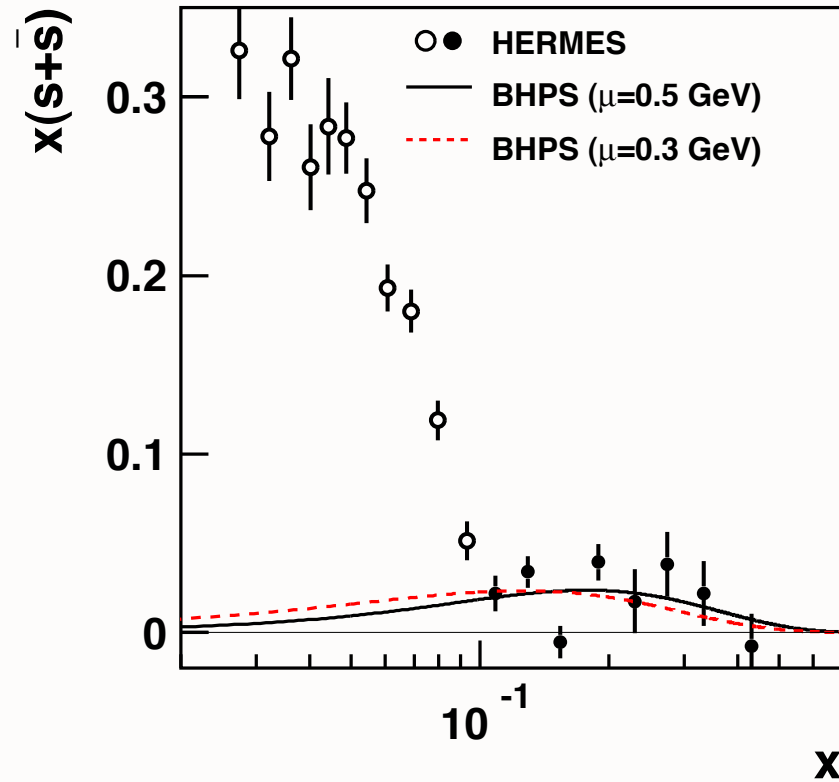
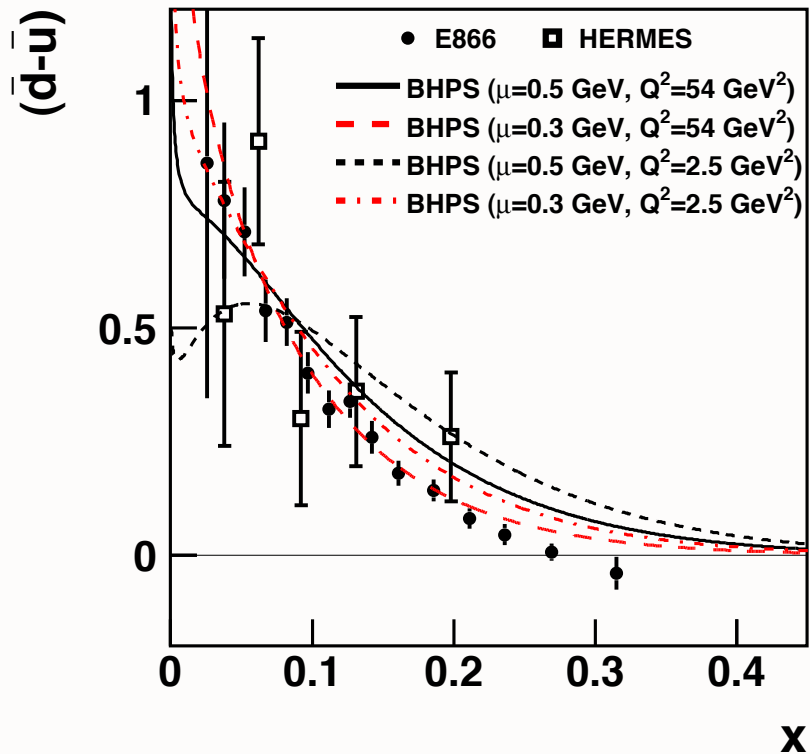
DGLAP evolution issues?

Extraction of Various Five-Quark Components of the Nucleons

Wen-Chen Chang^a, Jen-Chieh Peng^{a,b}

^a*Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*

^b*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*



Intrinsic Heavy-Quark Fock States

- **Rigorous** prediction of QCD, OPE

- Color-Octet Color-Octet Fock State!

- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$

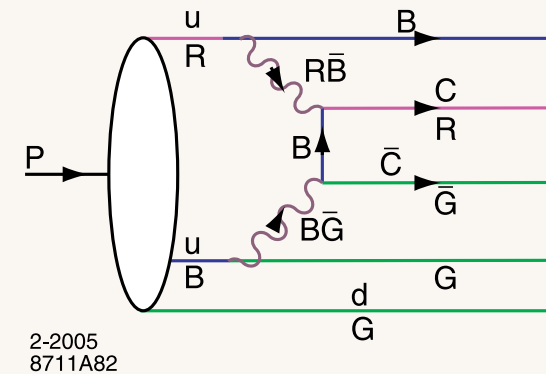
- Large Effect at high x!

- **Greatly increases kinematics of colliders such as Higgs production**
(*Kopeliovich, Schmidt, Soffer, sjb*)

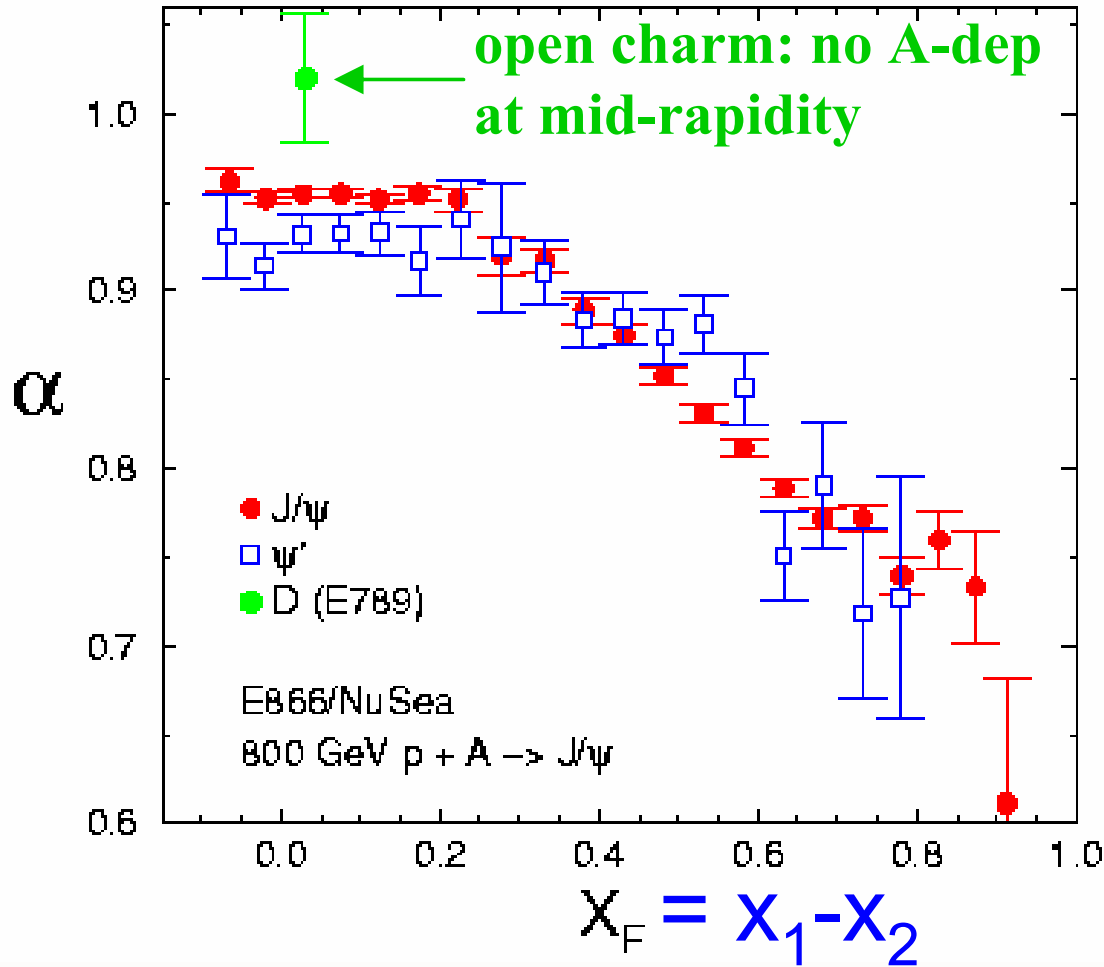
- Severely underestimated in conventional parameterizations of heavy quark distributions (*Pumplin, Tung*)

- Slow evolution compared to extrinsic quarks from gluon splitting!

- Many empirical tests



800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$
 PRL 84, 3256 (2000); PRL 72, 2542 (1994)



$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$$

Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization!

Violation of factorization in charm hadroproduction.

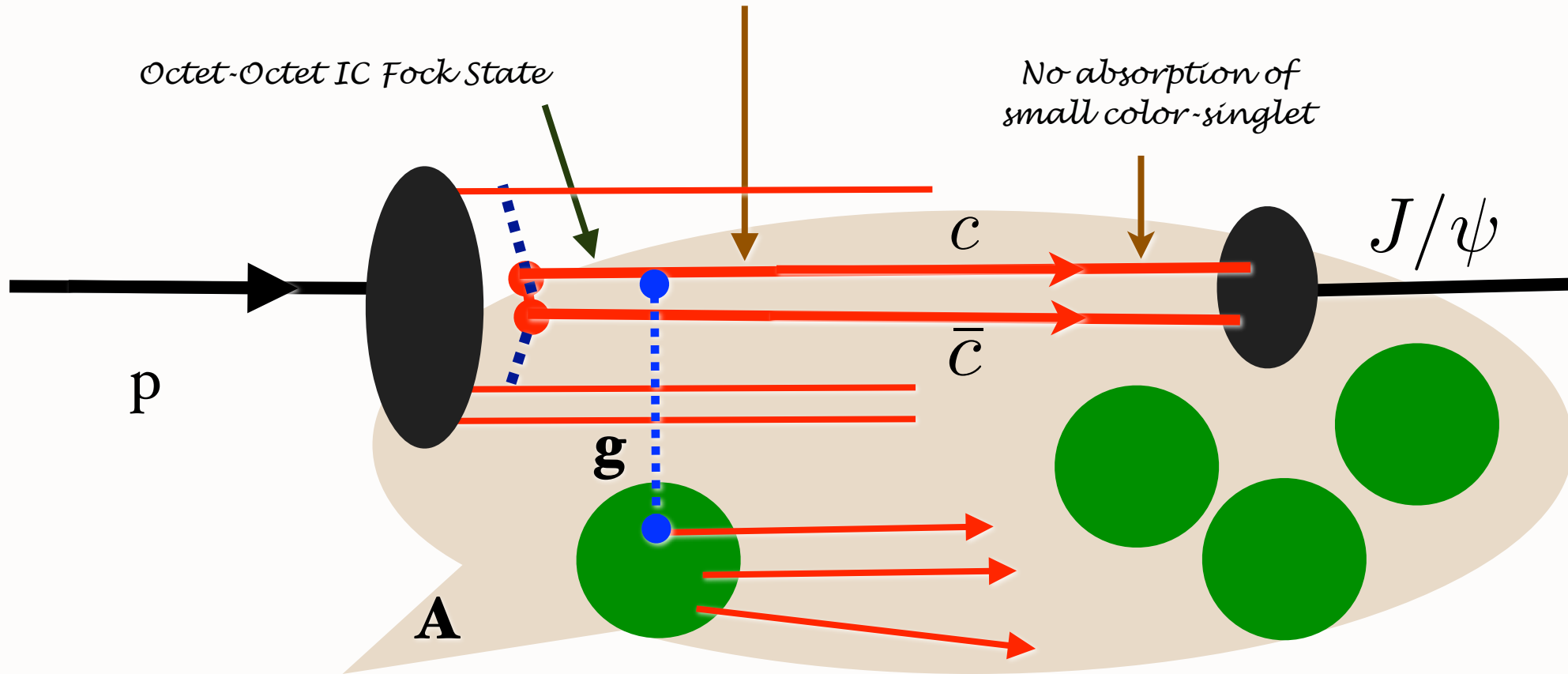
[P. Hoyer](#), [M. Vanttinen](#) ([Helsinki U.](#)), [U. Sukhatme](#) ([Illinois U., Chicago](#)) . HU-TFT-90-14, May 1990. 7pp.

Published in Phys.Lett.B246:217-220,1990

IC Explains large excess of quarkonia at large x_F , A-dependence

*Color-Opaque IC Fock state
interacts on nuclear front surface*

Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$

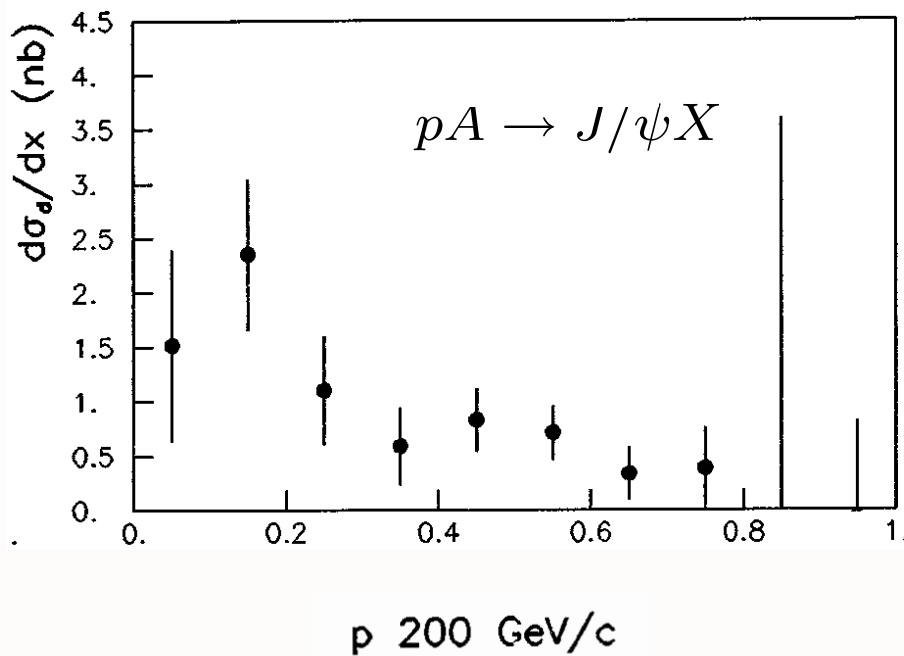
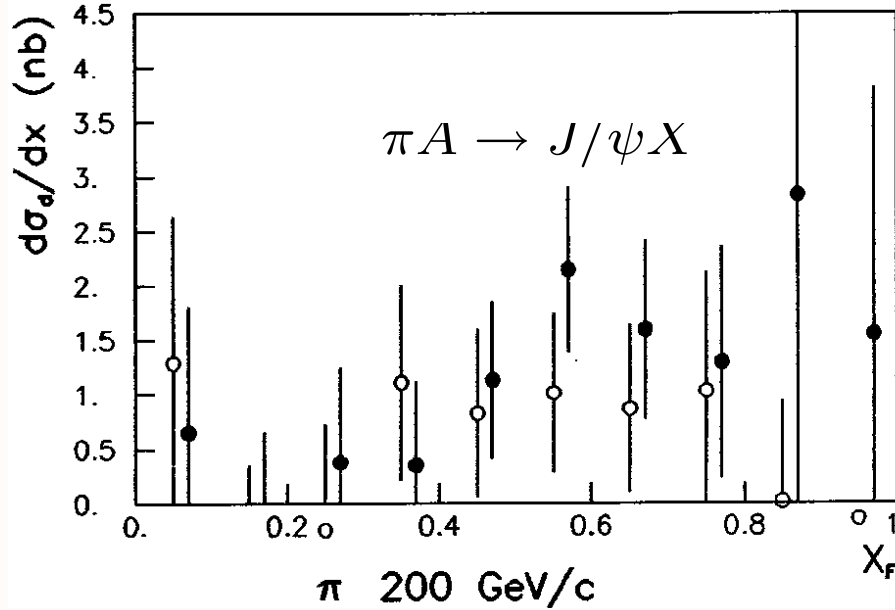
J. Badier et al, NA3

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_2}{dx_F}$$

$A^{2/3}$ component

High x_F :

*Consistent with
color-octet intrinsic
charm*

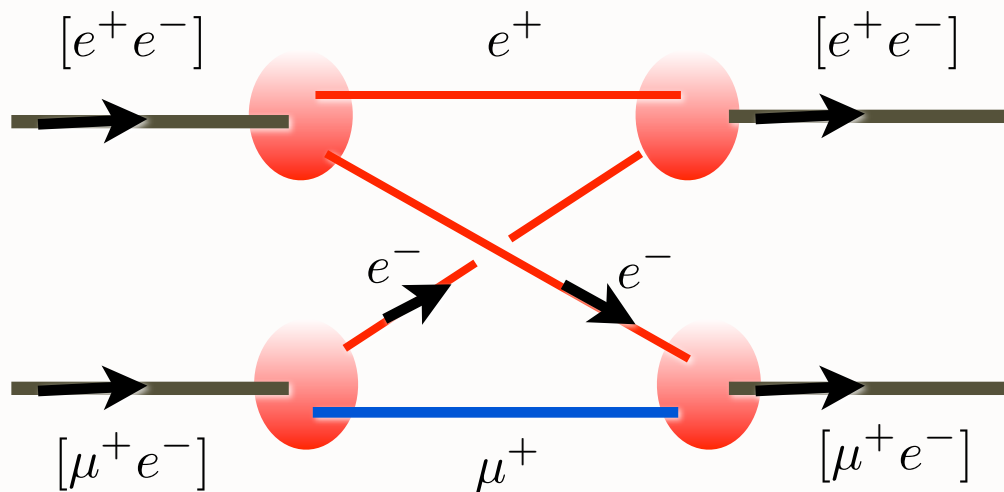


**Excess beyond conventional gluon-splitting
PQCD subprocesses**

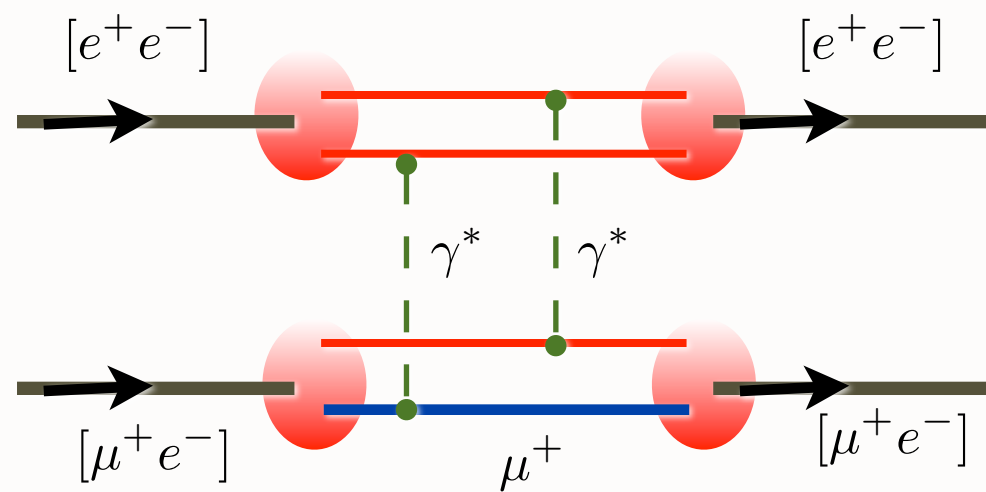
Why is IQ Important for Flavor Physics?

- **New perspective on fundamental nonperturbative hadron structure**
- **Charm structure function at high x**
- **Dominates high x_F charm and charmonium production**
- **Hadroproduction of new heavy quark states such as ccu, ccd, bcc, bbb, at high x_F**
- **Intrinsic charm -- long distance contribution to penguin mechanisms for weak decay** *Gardner, sjb*
- **$J/\psi \rightarrow \rho\pi$ puzzle explained** *Karliner, sjb*
- **Novel Nuclear Effects from color structure of IC, Heavy Ion Collisions**
- **New mechanisms for high x_F Higgs hadroproduction**
- **Dynamics of b production: LHCb** *New Multi-lepton Signals*
- **Fixed target program at LHC: produce bbb states**

Blankenbecler, Gunion, sjb



*Constituent Interchange
Spin exchange in atom-
atom scattering*

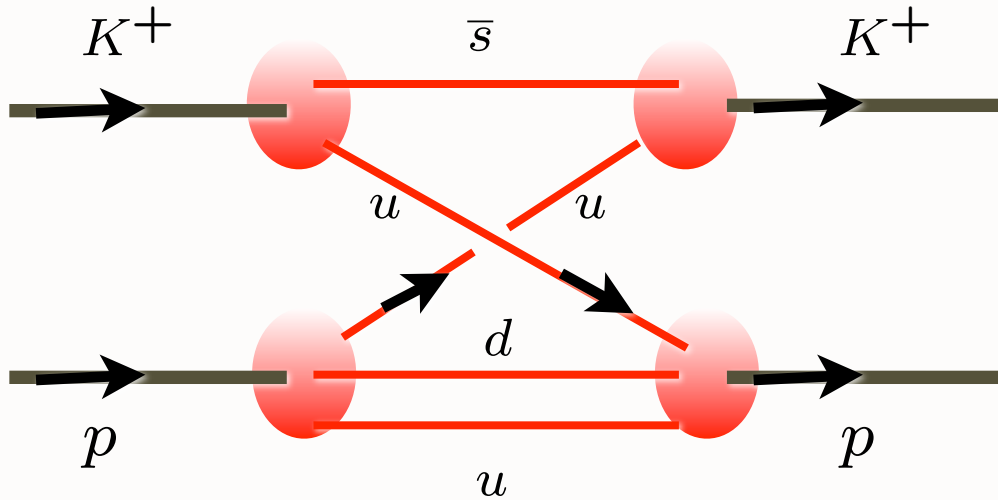


*Two-Photon Exchange
(Van der Waal)*

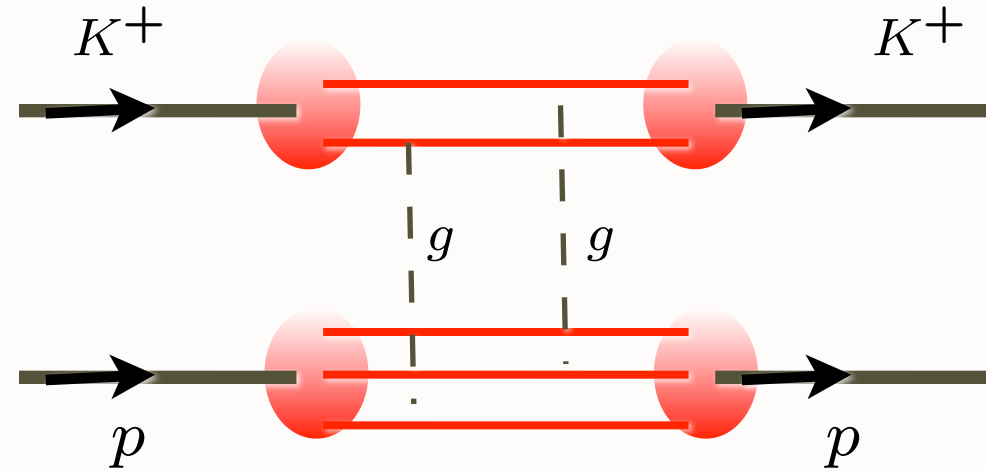
$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s, t)_{\text{gluonexchange}} \propto sF(t)$$



*Quark Interchange
(Analog of Spin exchange
in atom-atom scattering)*



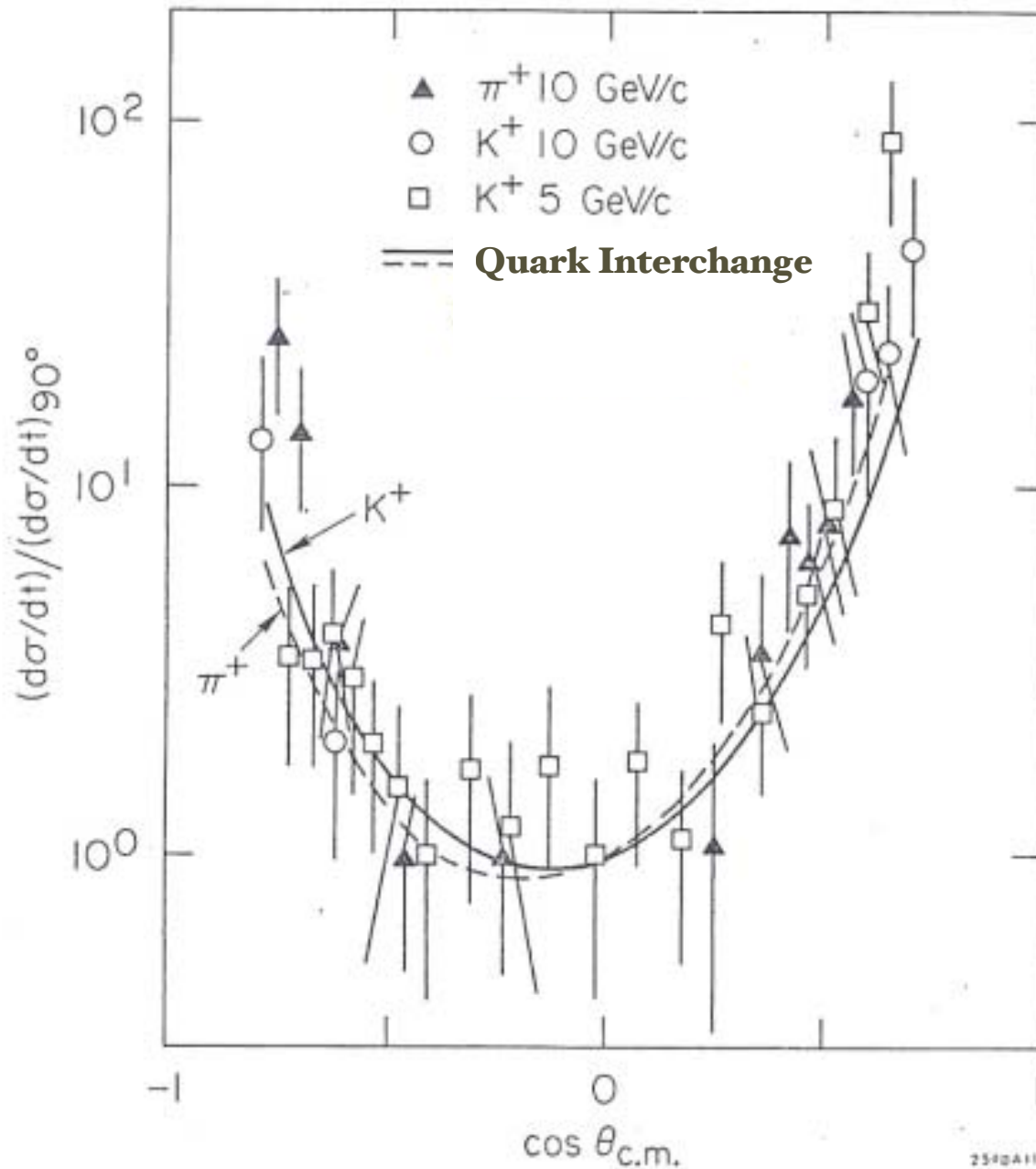
*Gluon Exchange
(Van der Waal --
Landshoff)*

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s, t)_{\text{gluonexchange}} \propto sF(t)$$

*MIT Bag Model (de Tar), large N_c , ('t Hooft), AdS/CFT
all predict dominance of quark interchange:*



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \rightarrow -1$$

Single-spin asymmetries

**Leading-Twist
Sivers Effect**

*Hwang,
Schmidt, sjb*

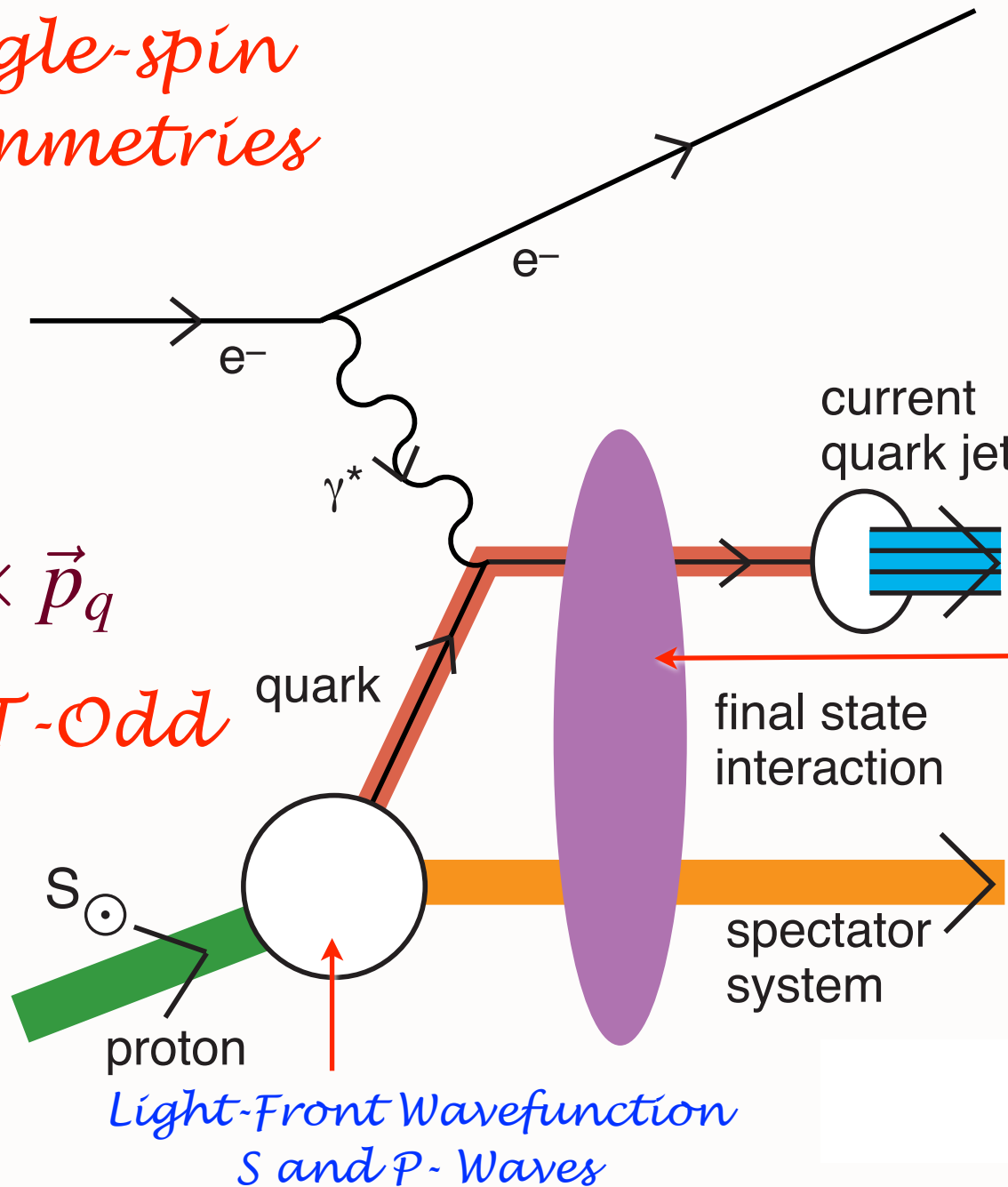
*Collins, Burkardt,
Ji, Yuan*

*QCD S- and P-
Coulomb Phases
--Wilson Line*

**Analog of QED
Coulomb Phases**

Pseudo-T-Odd

$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

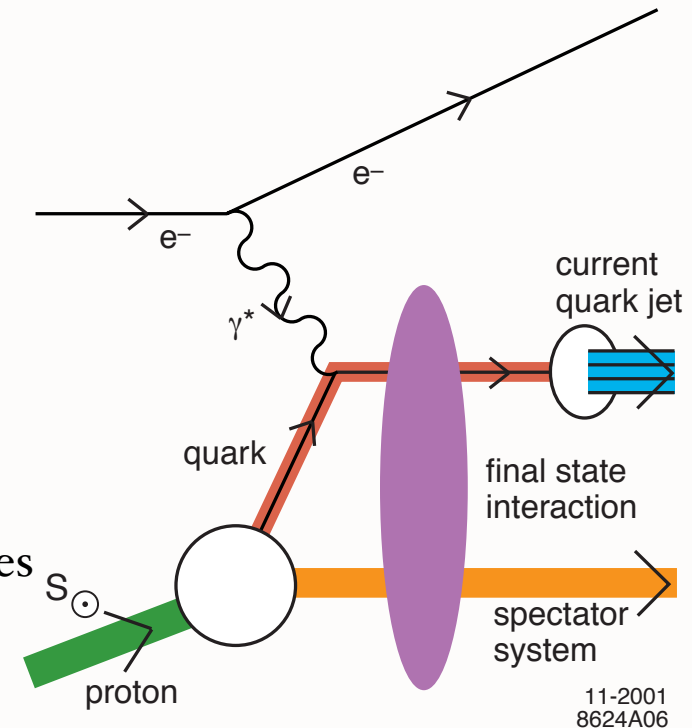


*Light-Front Wavefunction
S and P-Waves*

Final-State Interactions Produce Pseudo-T-Odd (Sivers Effect)

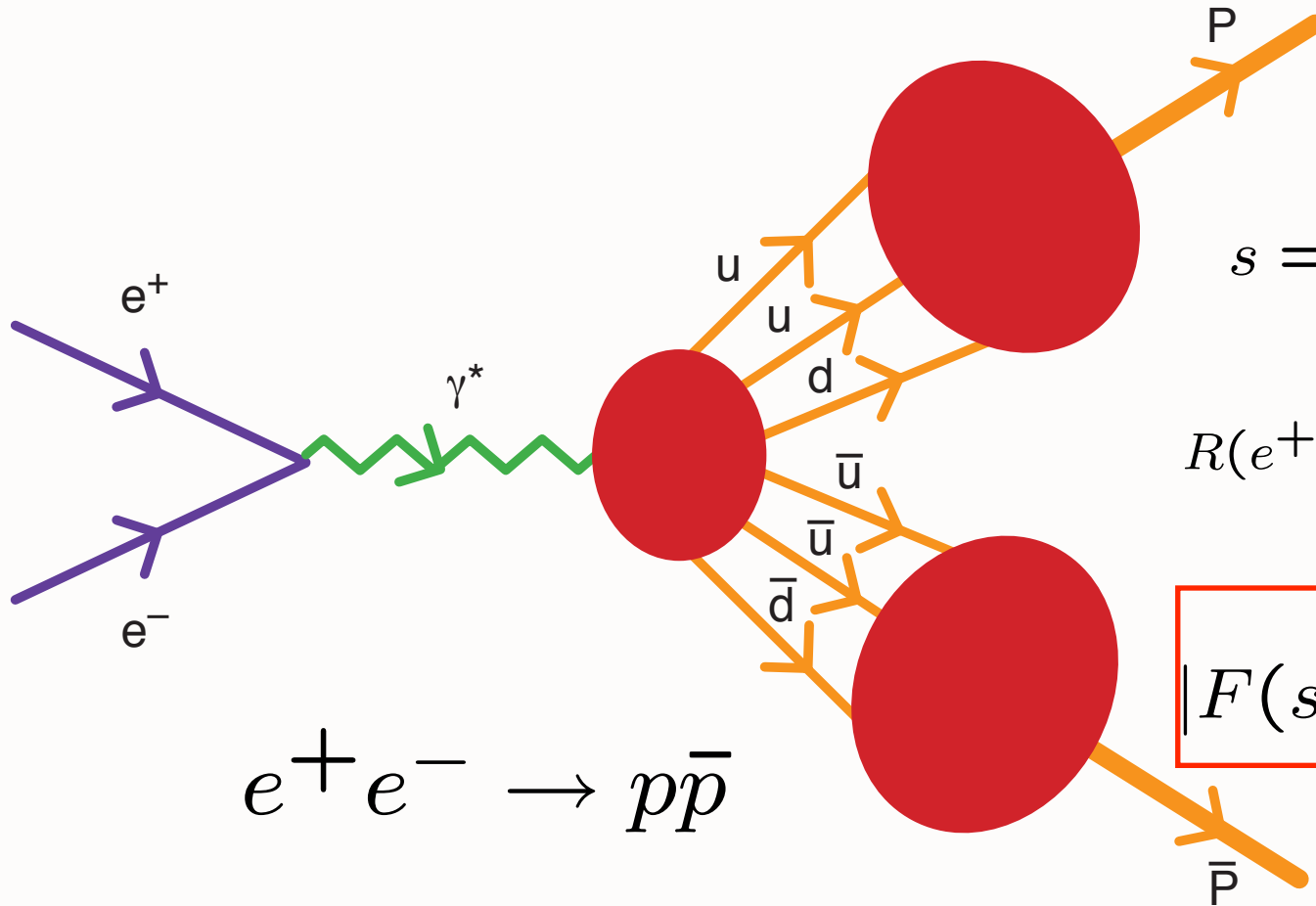
- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

$$\mathbf{i} \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$



Exclusive Processes

What if we ask for a specific final state?



$$e^+ e^- \rightarrow p \bar{p}$$

$$s = (E_{e^+} + E_{e^-})^2$$

$$R(e^+ e^- \rightarrow H \bar{H}) \propto |F(s)|^2$$

$$|F(s)| \propto \left[\frac{1}{s}\right]^{n_q - 1}$$

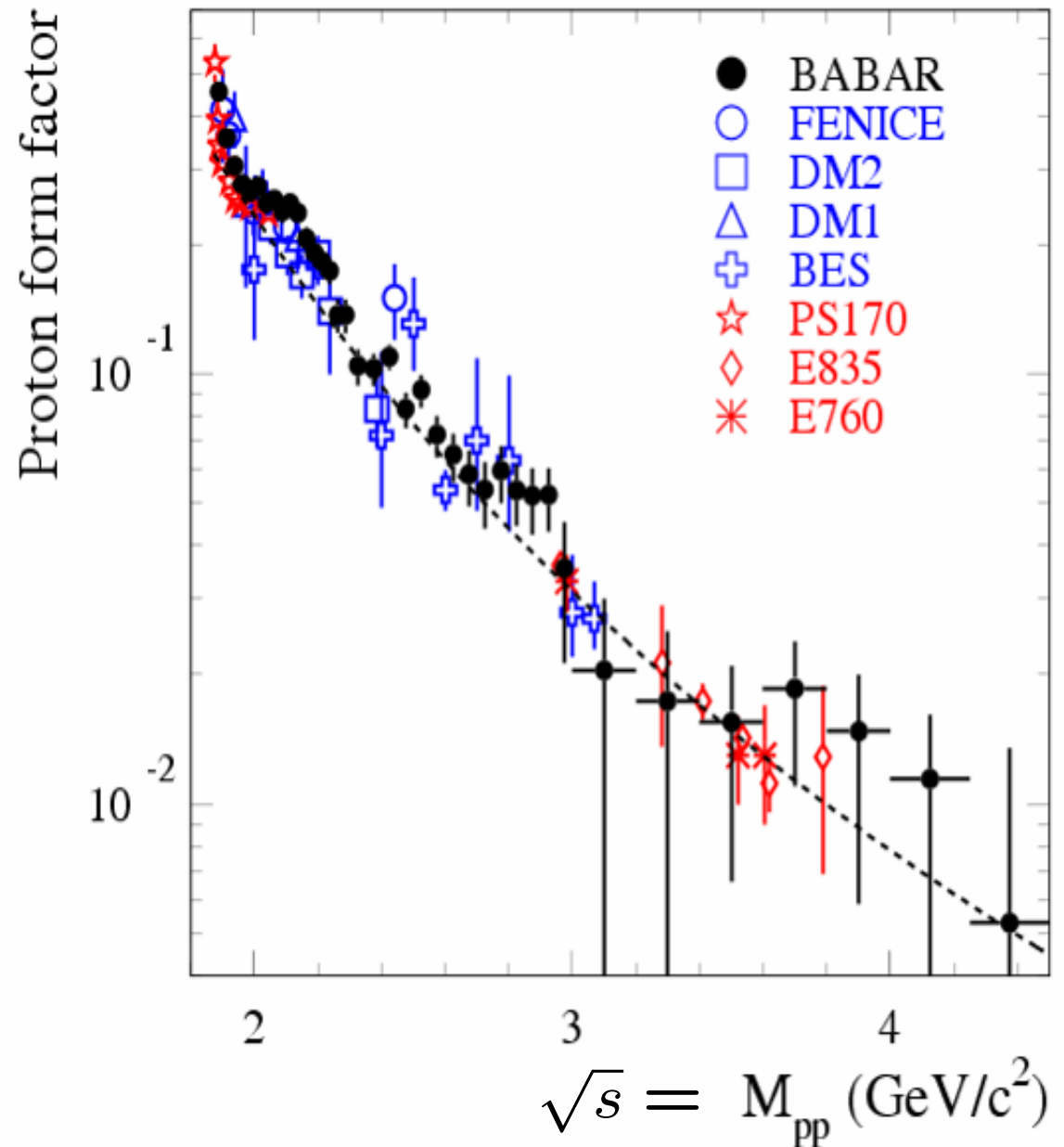
Probability decreases with number of constituents!

Timelike Proton Form Factor

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2} |F|^2,$$

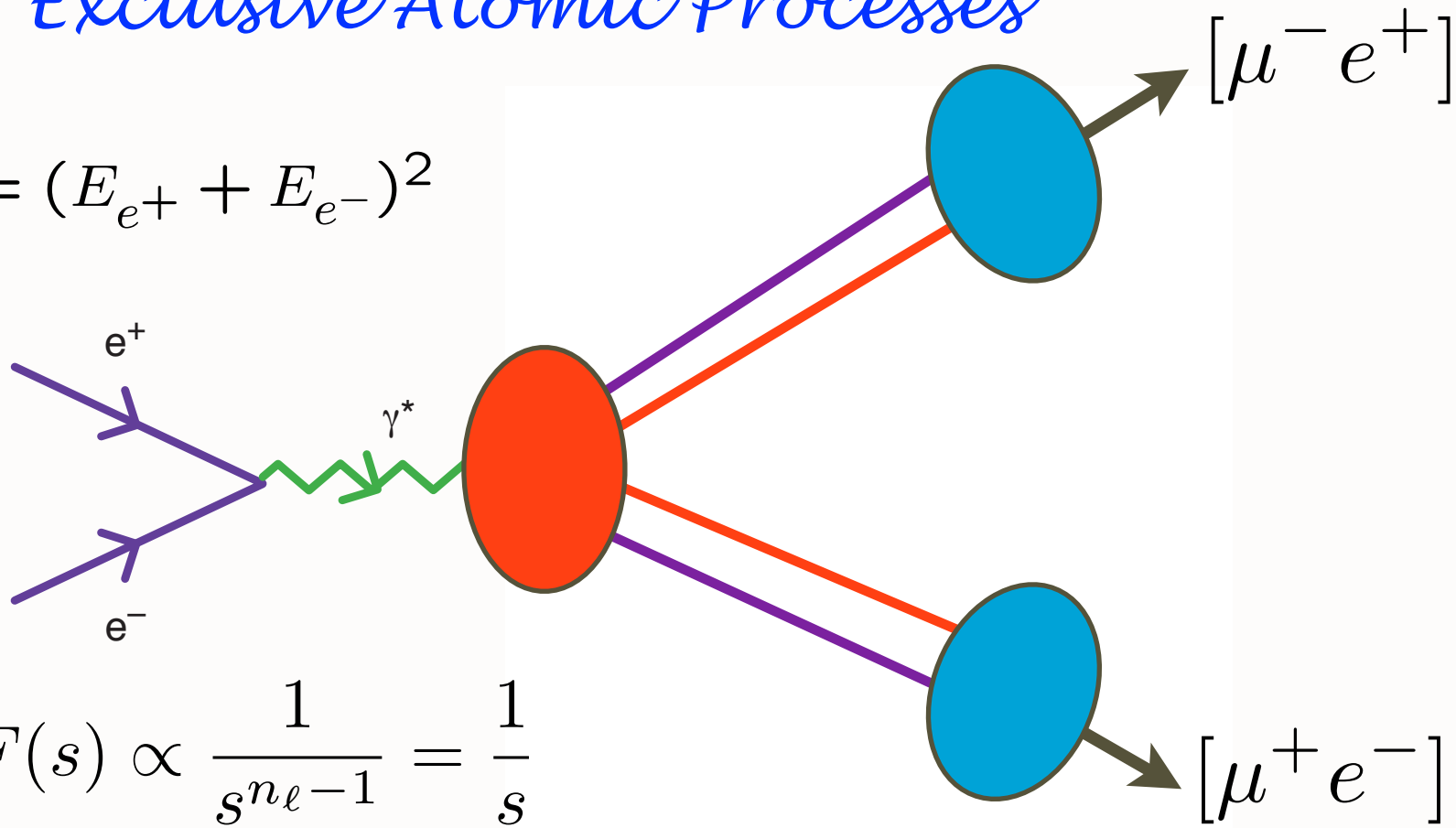
$$F(s) \propto \frac{\log^{-2} \frac{s}{\Lambda^2}}{s^2}$$

$$n_q - 1 = 3 - 1 = 2$$



Exclusive Atomic Processes

$$s = (E_{e^+} + E_{e^-})^2$$

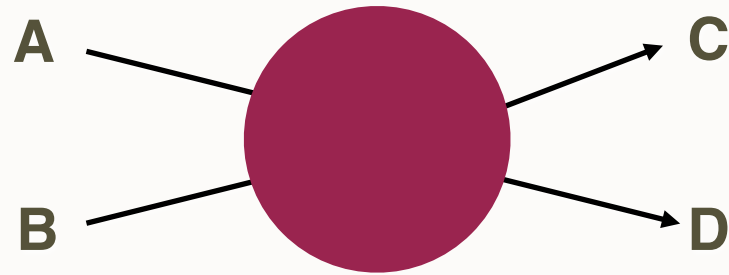


$$F(s) \propto \frac{1}{s^{n_\ell - 1}} = \frac{1}{s}$$

$$R = \frac{\sigma(e^+e^- \rightarrow [\mu^+e^-] + [\mu^-e^+])}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = |F(s)|^2 \propto \frac{1}{s^2}$$

Probability decreases with number of constituents

Constituent Counting Rules



$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos \theta_{cm}$

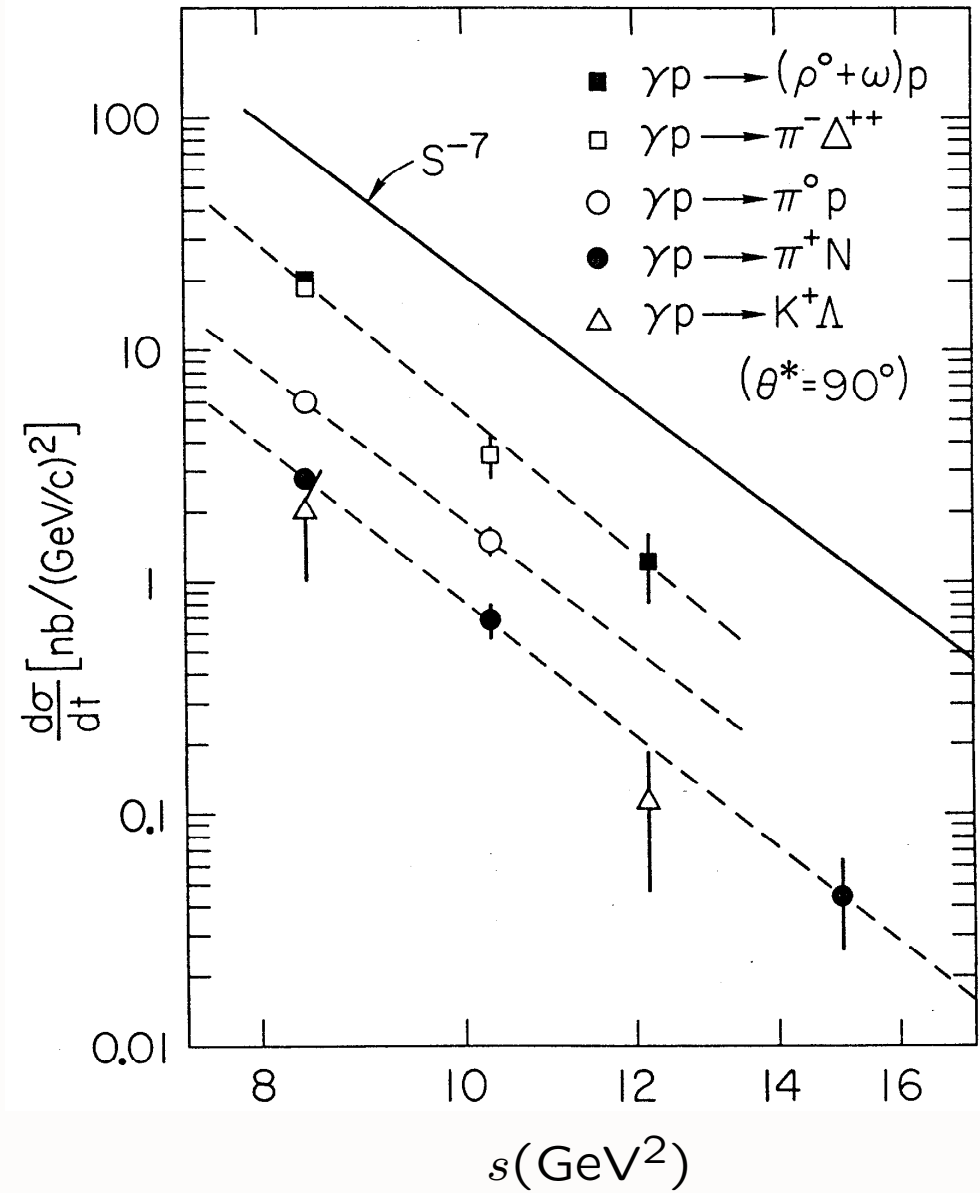
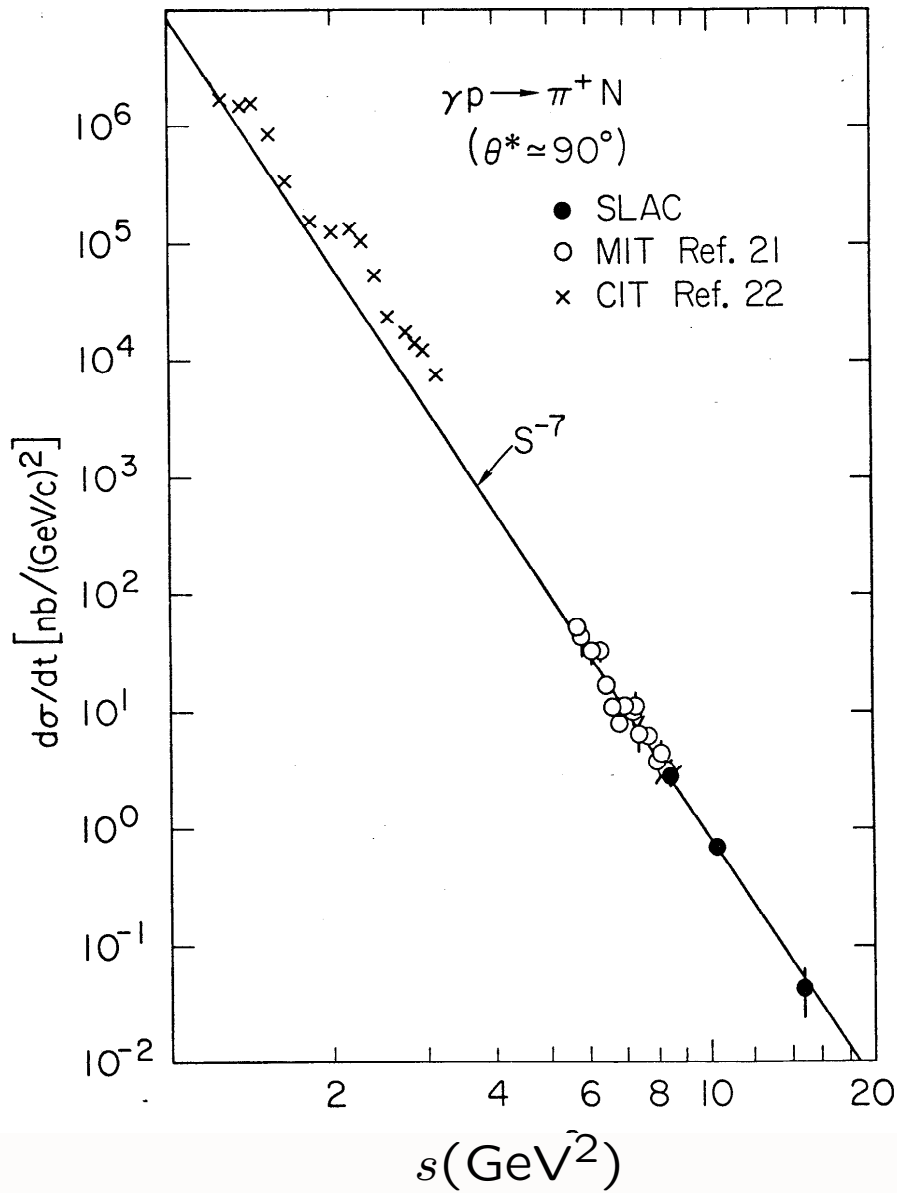
$$\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s^{[n_{tot}-2]}} \quad s = E_{cm}^2$$

$$F_H(Q^2) \sim \left[\frac{1}{Q^2}\right]^{n_H-1}$$

Farrar & sjb;
Matveev, Muradyan, Tavkhelidze

QED and QCD predicts leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

$$s, -t \gg m_\ell^2$$

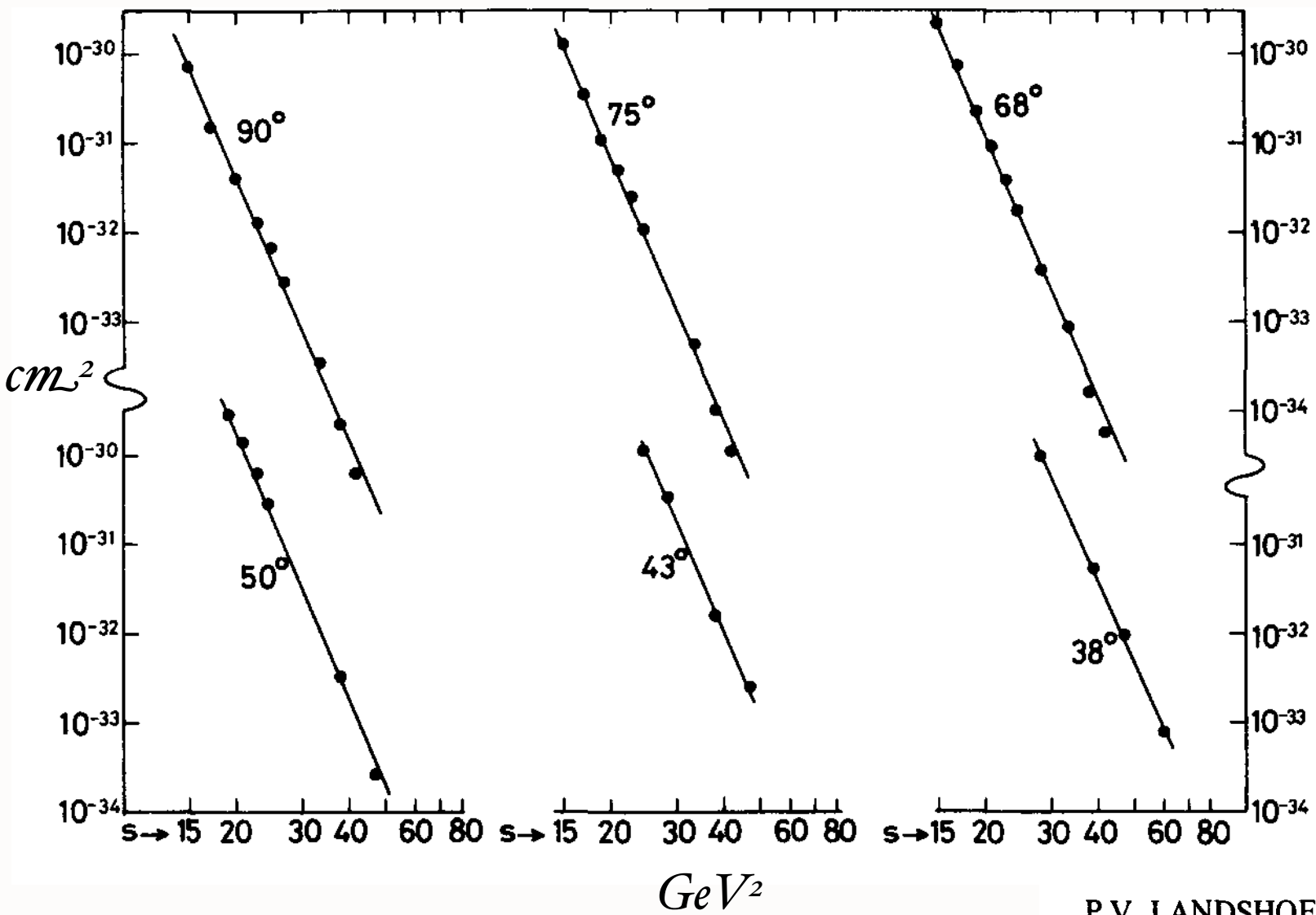


Counting Rules: $n=9$

$$\frac{d\sigma}{dt} (\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}}$

$n = 4 \times 3 - 2 = 10$

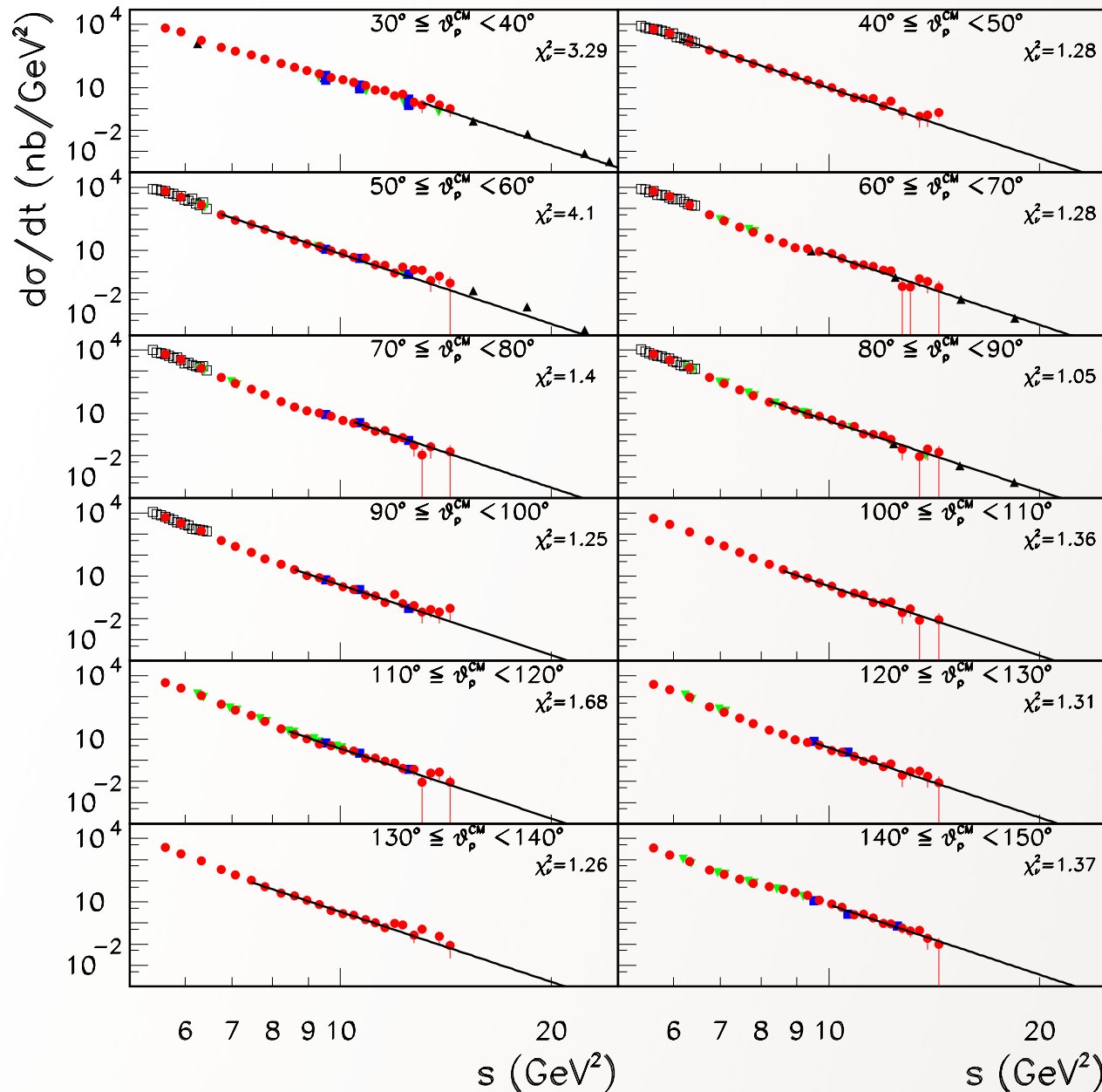


Best Fit
 $n = 9.7 \pm 0.5$
 Reflects underlying conformal scale-free interactions

P.V. LANDSHOFF and J.C. POLKINGHORNE

Deuteron Photodisintegration

J-Lab



PQCD and AdS/CFT:

$$s^{n_{tot}-2} \frac{d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B \rightarrow C+D}(\theta_{CM})$$

$$s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM})$$

$$n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11$$

Reflects conformal invariance

- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \rightarrow np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

- $n_{tot} = 1 + 6 + 3 + 3 = 13$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

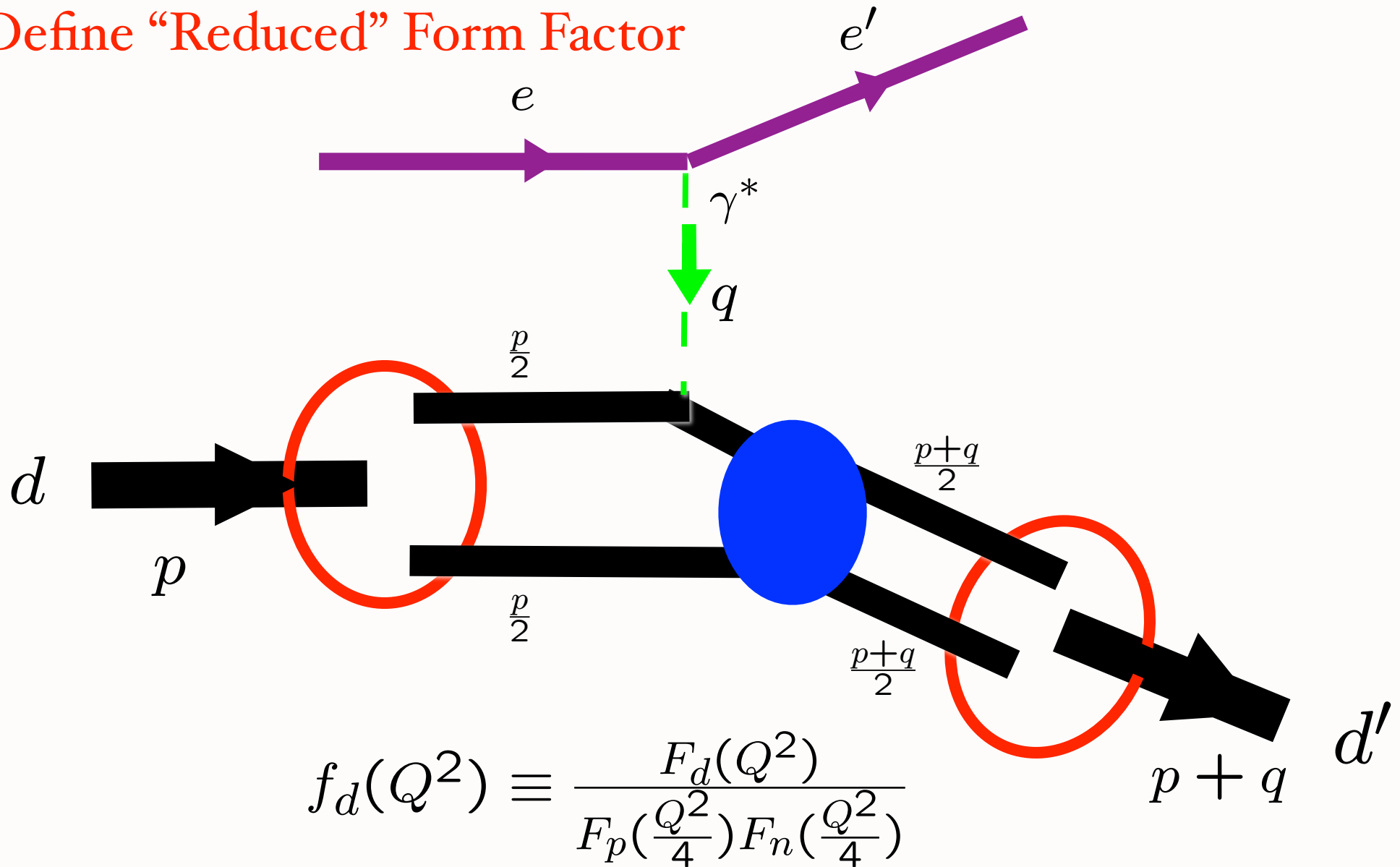
Hidden color: $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$
at high p_T

Primary Evidence for Quarks

- Electron-Proton Inelastic Scattering: $ep \rightarrow e'X$
Electron scatters on pointlike constituents with fractional charge; final-state jets
- Electron-Positron Annihilation: $e^+e^- \rightarrow X$
Production of pointlike pairs with fractional charges and 3 colors; quark, antiquark, gluon jets
- Exclusive hard scattering reactions: $pp \rightarrow pp, \gamma p \rightarrow \pi^+ n, ep \rightarrow ep$
probability that hadron stays intact counts number of its pointlike constituents:

Quark Counting Rules

Define "Reduced" Form Factor



Elastic electron-deuteron scattering

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n^d - \gamma_m^d} \left[1 + \mathcal{O} \left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-(2/5) C_F/\beta}$$

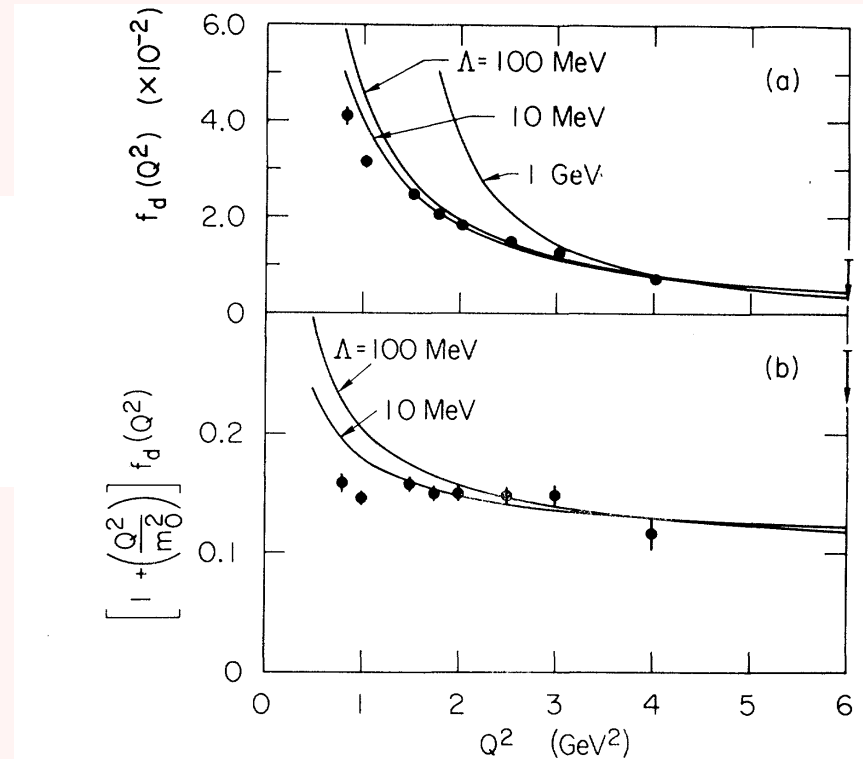
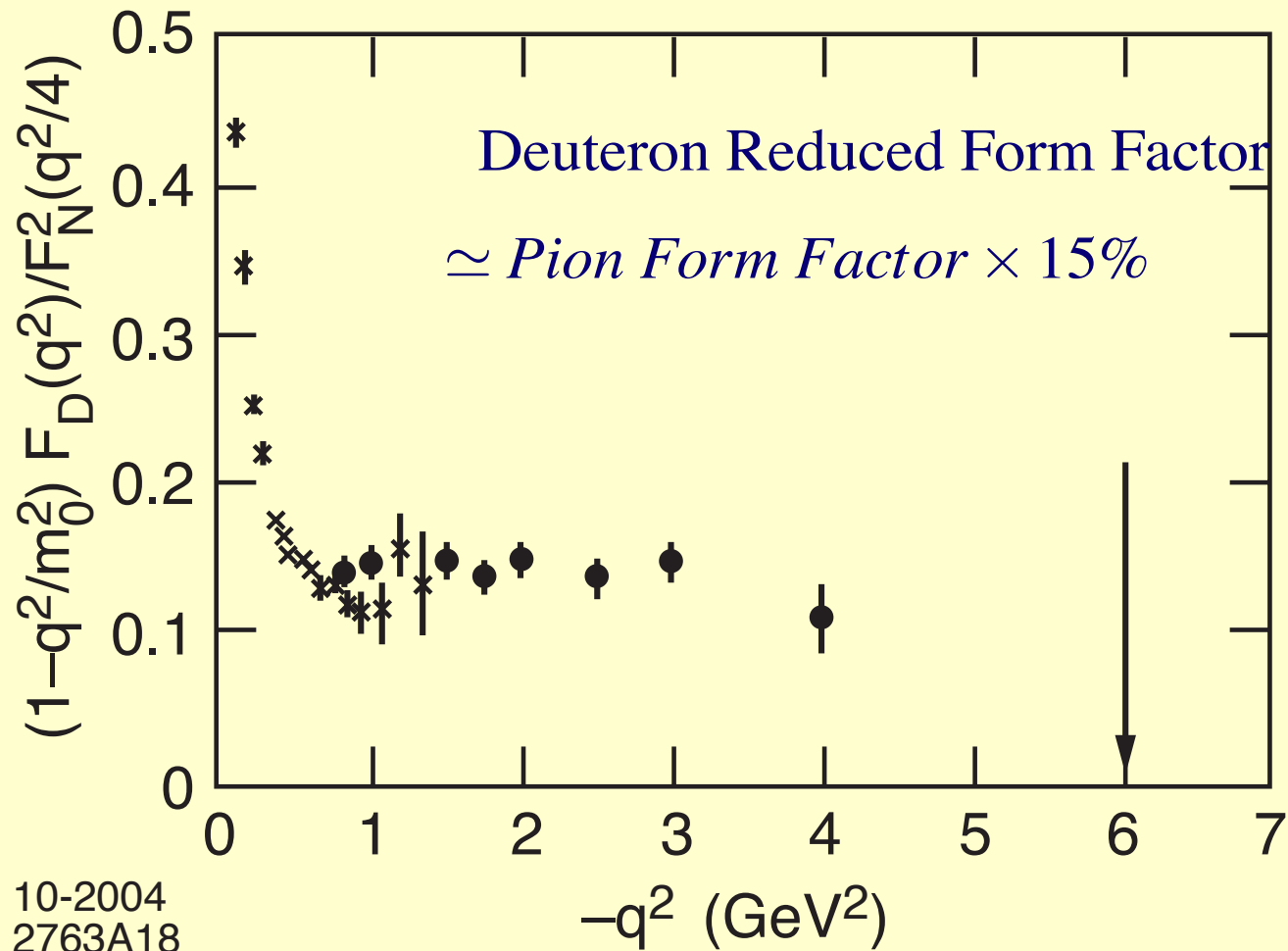


FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2) [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71 \text{ GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4 \text{ GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28 \text{ GeV}^2$ is used (Ref. 8).



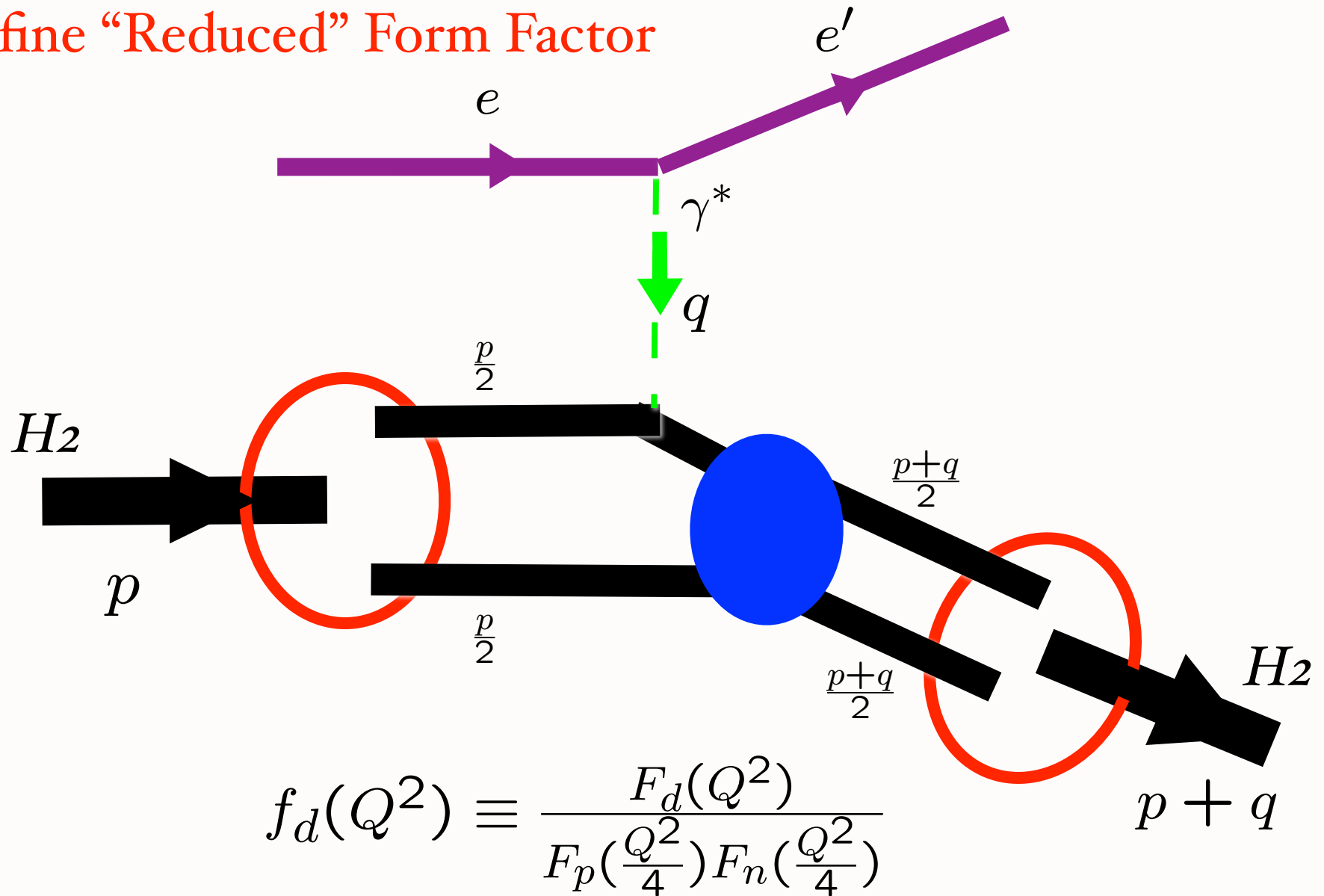
- 15% Hidden Color in the Deuteron

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is $|n\ p\rangle$
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- **Predict** $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$ at high Q^2

Define "Reduced" Form Factor



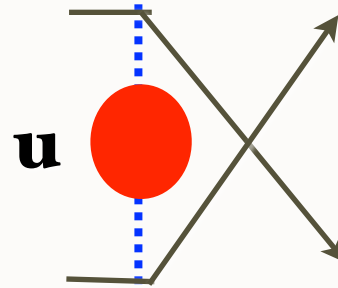
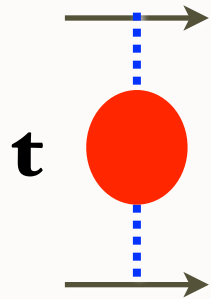
Elastic electron-molecule scattering!

Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
- $F_H(Q) \propto \frac{1}{(Q^2)^{n-1}}$ **n = # elementary constituents**

Electron-Electron Scattering in QED

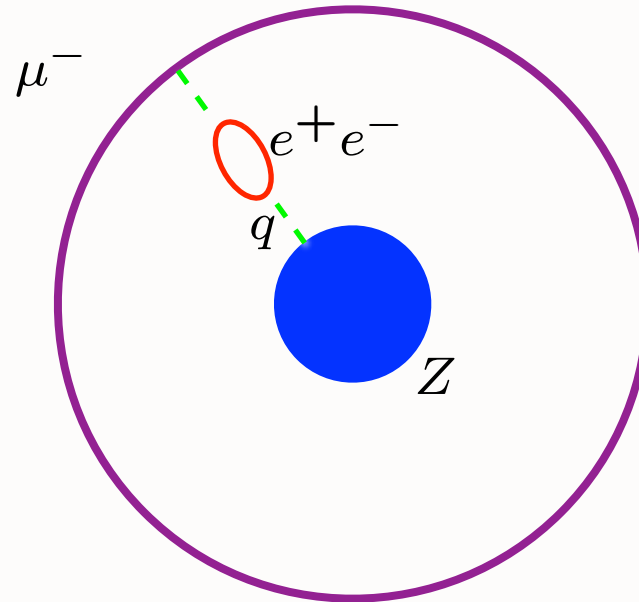
$$\mathcal{M}_{ee \rightarrow ee}(++; ++)=\frac{8\pi s}{t}\alpha(t)+\frac{8\pi s}{u}\alpha(u)$$



$$\alpha(t)=\frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge

Scale Setting in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

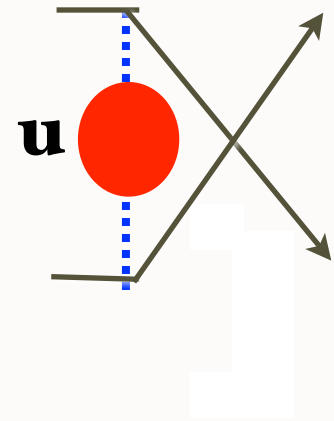
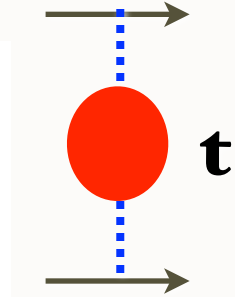
$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to
0.1% precision in μ Pb

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$



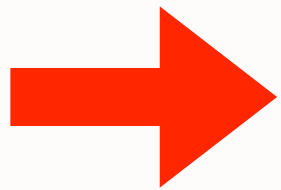
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**
- **If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- *No renormalization scale ambiguity!*
- *Two separate physical scales.*

$\lim N_C \rightarrow 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

QCD \rightarrow Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

*Scale-Setting procedure for QCD
must be applicable to QED*



Principle of Maximum Conformality
Scheme-independent!

Di Giustino, Wu, sjb

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

↑
**Scale-Free
 Conformal Series**

↑
**Running Coupling
 Effects**

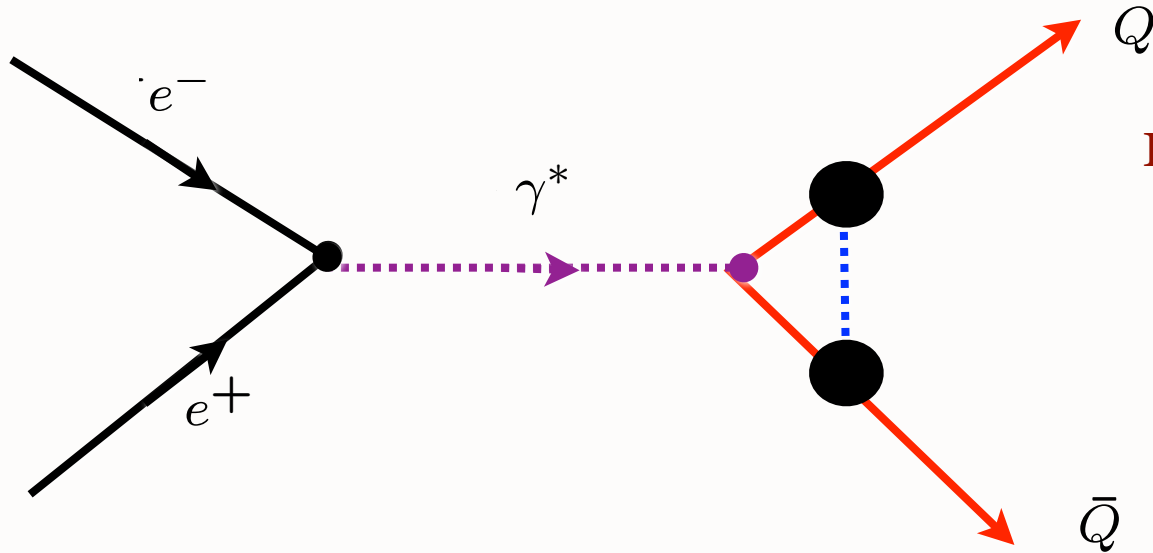
↑
**Higher Twist from
 Hadron Dynamics**

↑
**Intrinsic Heavy
 Quarks**

↑
**Light by Light
 Loops**

BLM/PMC: Absorb β -terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$



Hoang, Kuhn, Teubner, sjb

Di Giustino, Wu, sjb

$$F_1 + F_2 = \left[1 - 2 \frac{\alpha_s (s e^{3/4} / 4)}{\pi} \right] \times \left[1 + \frac{\pi \alpha_s (s \beta^2)}{4 \beta} \right]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple PMC Scales

Need QCD coupling at small scales at low relative velocity β

Features of PMC/BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- **“Principle of Maximum Conformality”** Di Giustino, Wu, sjb
- **All terms associated with nonzero beta function summed into running coupling**
- **Standard procedure in QED**
- **Resulting series identical to conformal series**
- **Renormalon $n!$ growth of PQCD coefficients from beta function eliminated!**
- *Scheme Independent !!!*
- **In general, BLM/PMC scales depend on all invariants**
- **Single Effective PMC scale at NLO**

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

Need a First Approximation to QCD

*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

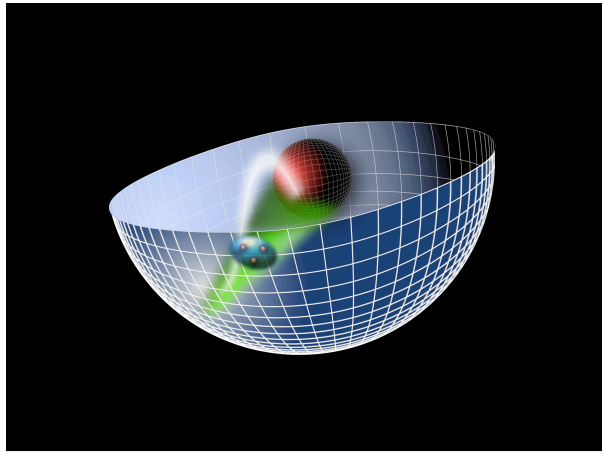
Relativistic, Frame-Independent, Color-Confining

Goal: an analytic first approximation to QCD

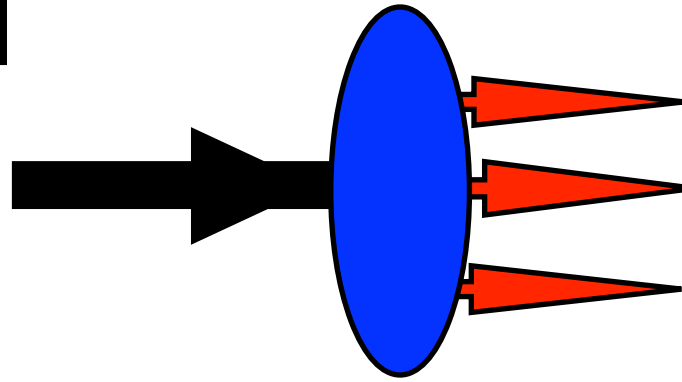
- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

de Teramond, sjb

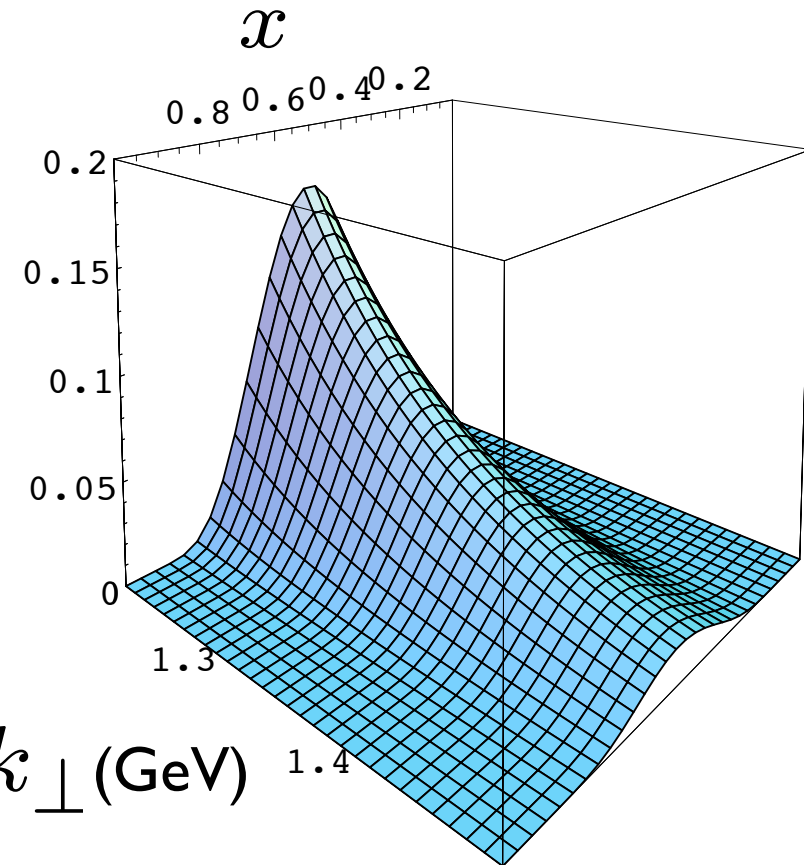
$$\phi(z)$$



- *Light-Front Holography*



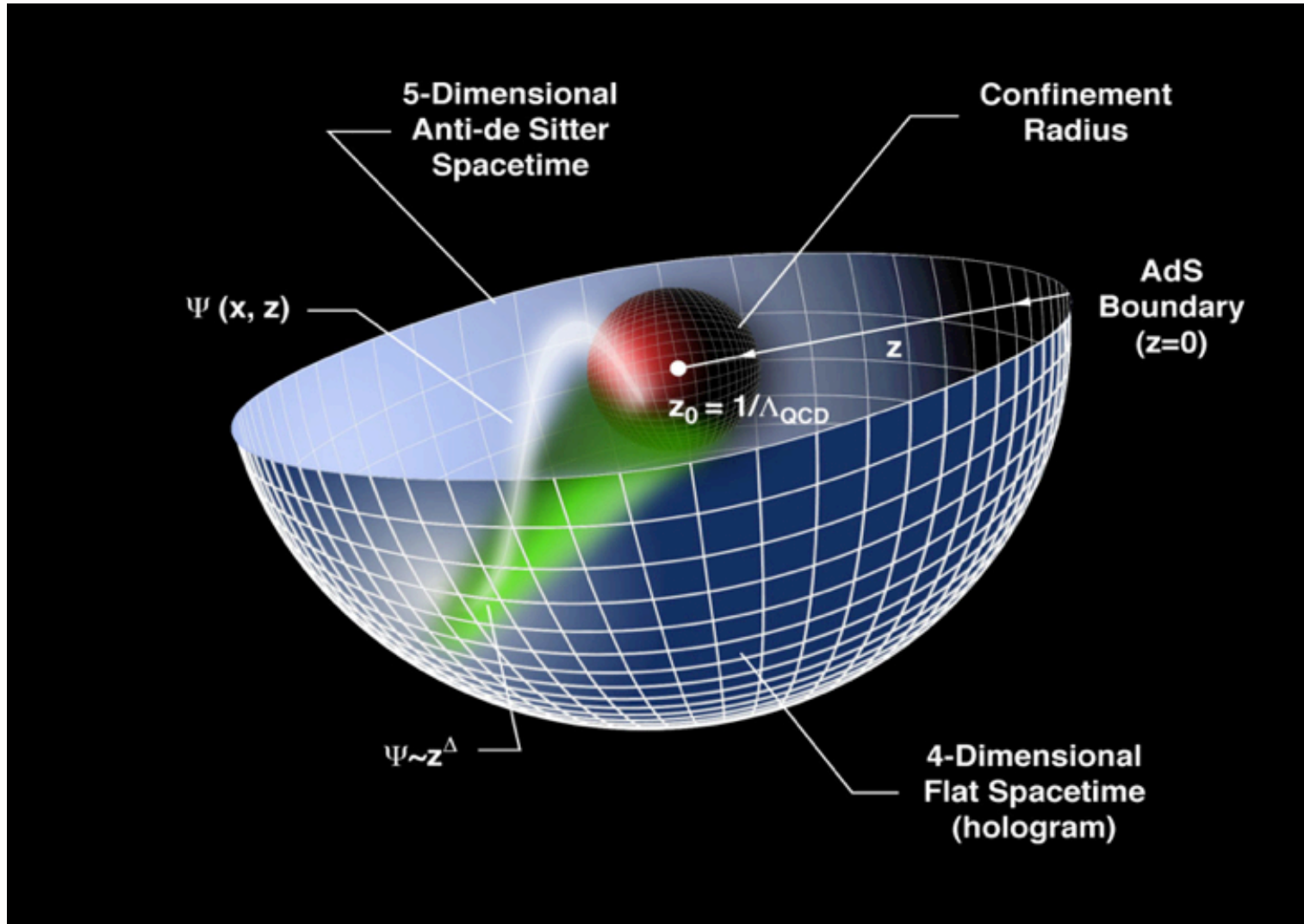
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

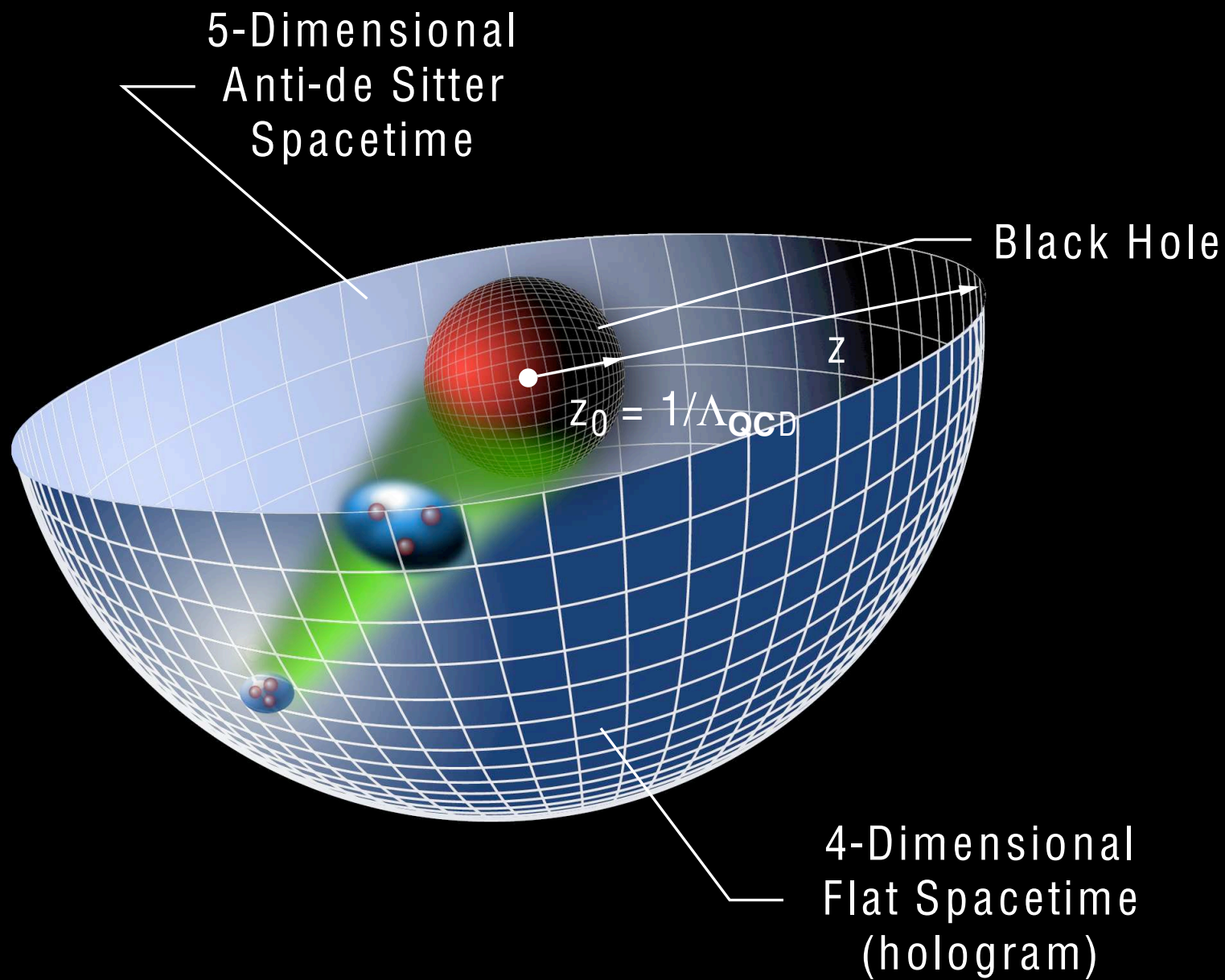
Schrödinger Wavefunctions
of Hadron Physics

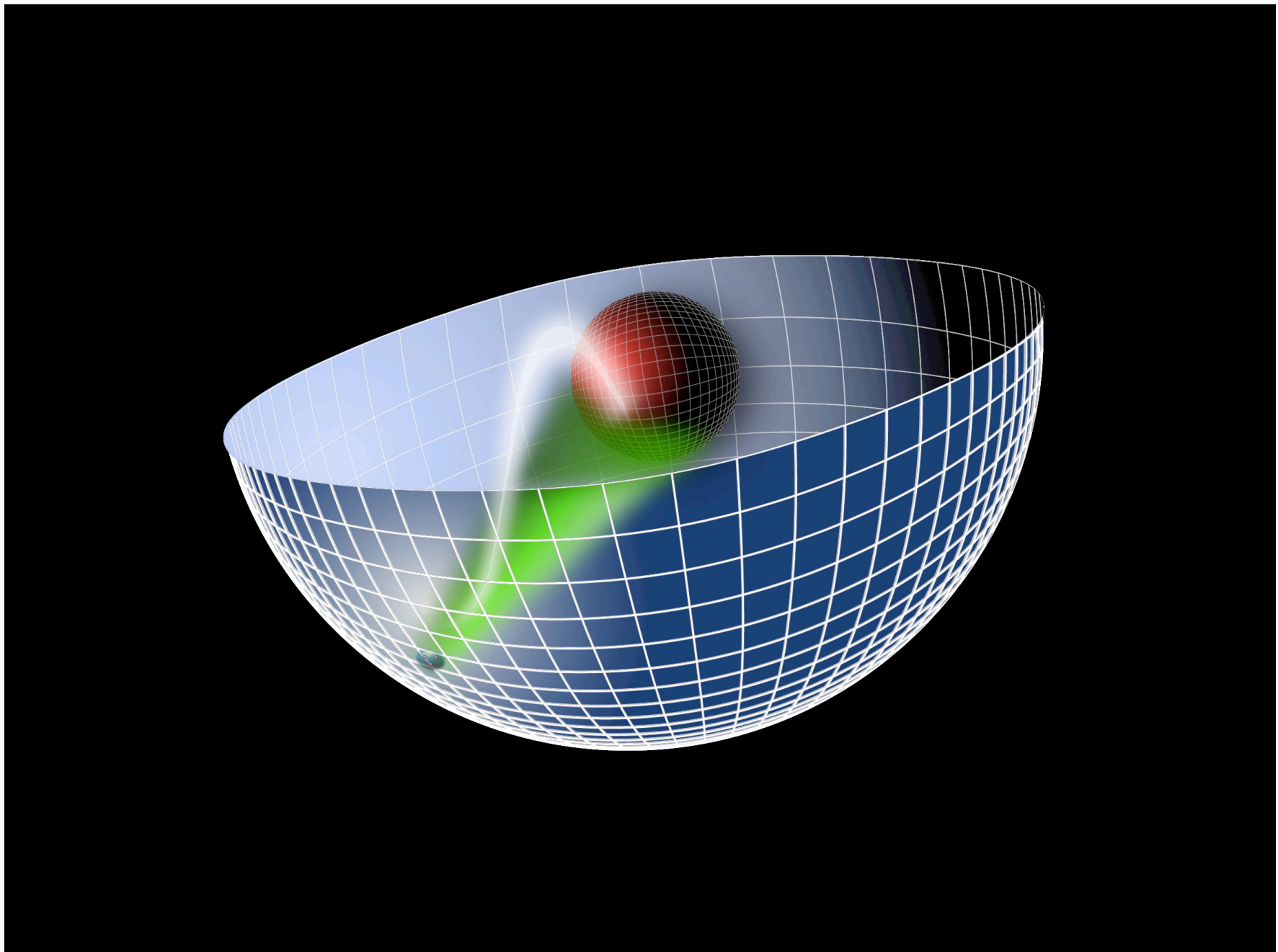
Applications of AdS/CFT to QCD

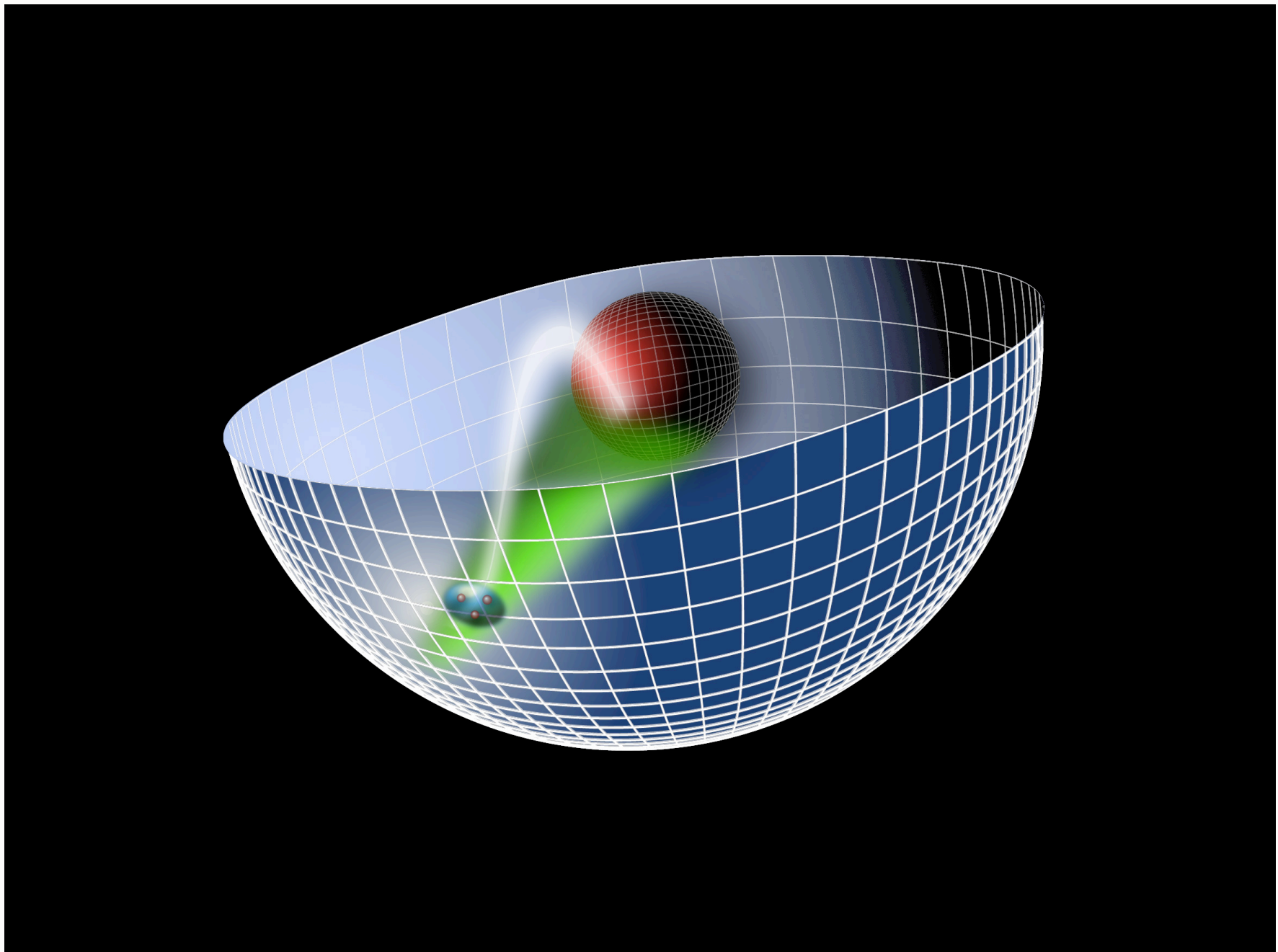


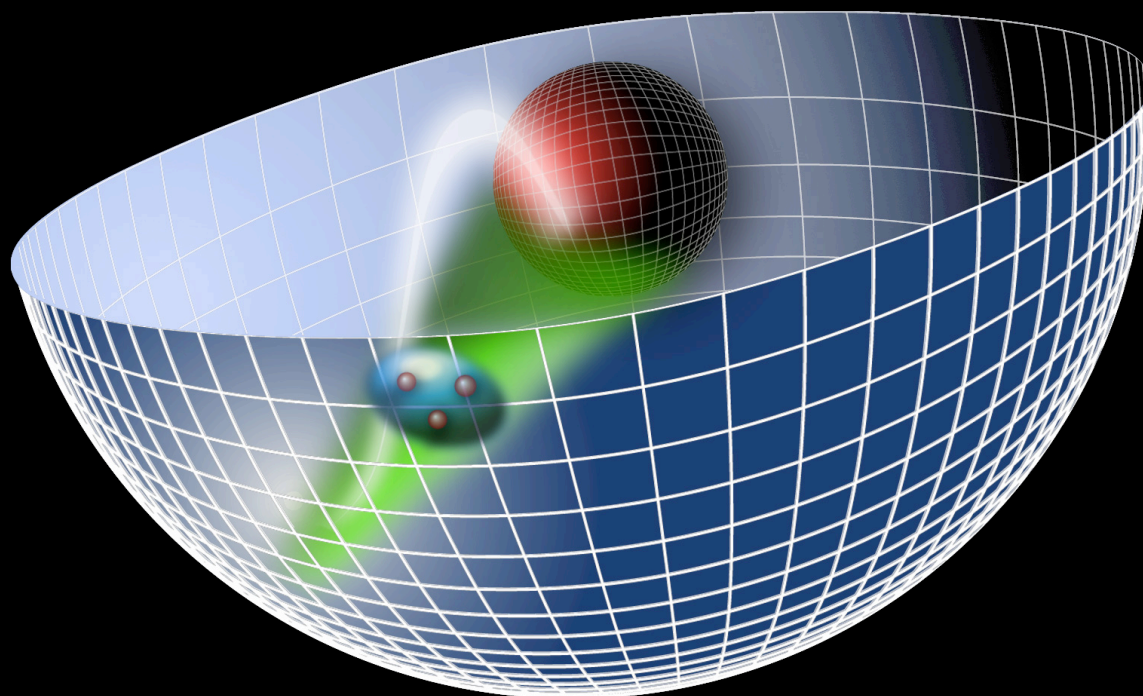
Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond







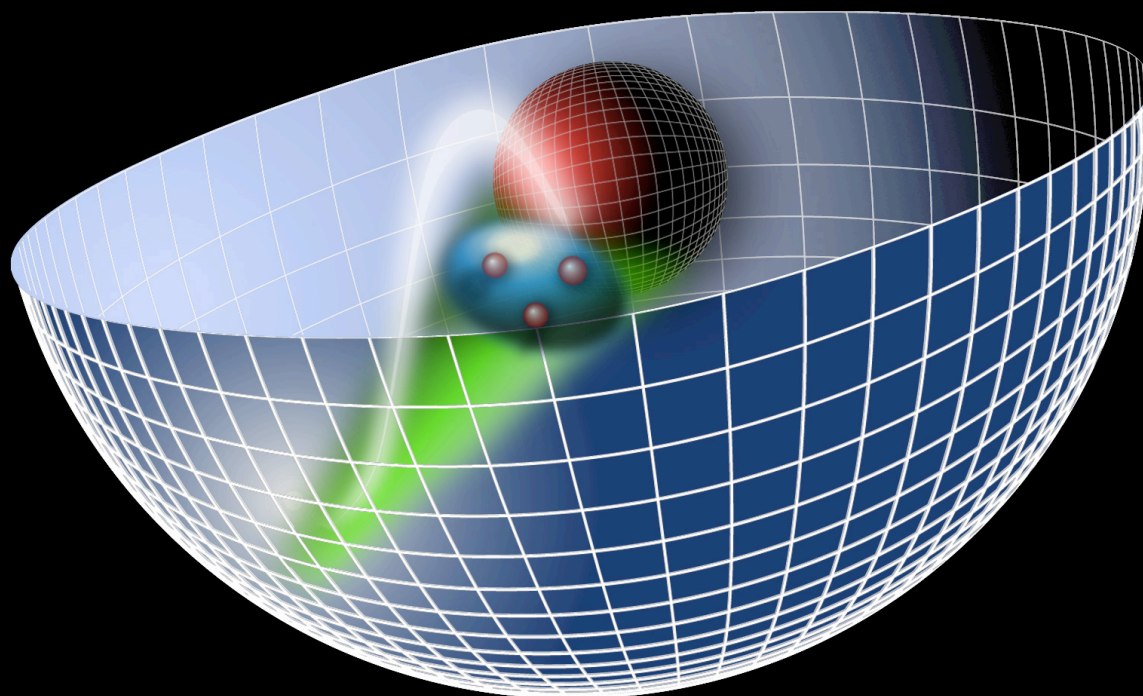


Rochester, February 8, 2012

Atoms in Flight

Stan Brodsky

SLAC



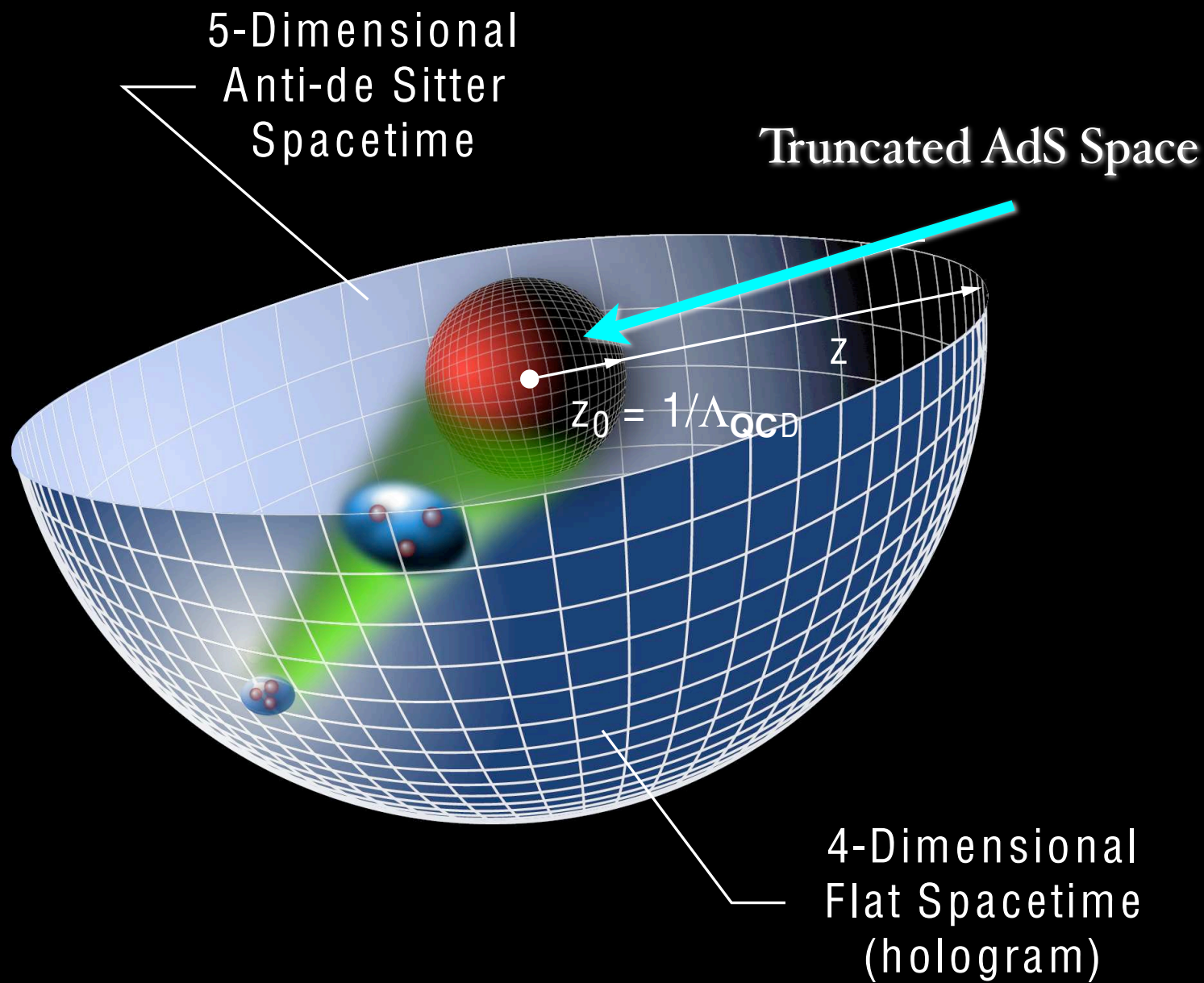
Rochester, February 8, 2012

Atoms in Flight

Stan Brodsky

SLAC


128



Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

Soft-Wall Model

$$S = \int d^4x dz \sqrt{g} e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action
Dilaton-Modified AdS₅*

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

Quark separation increases with L

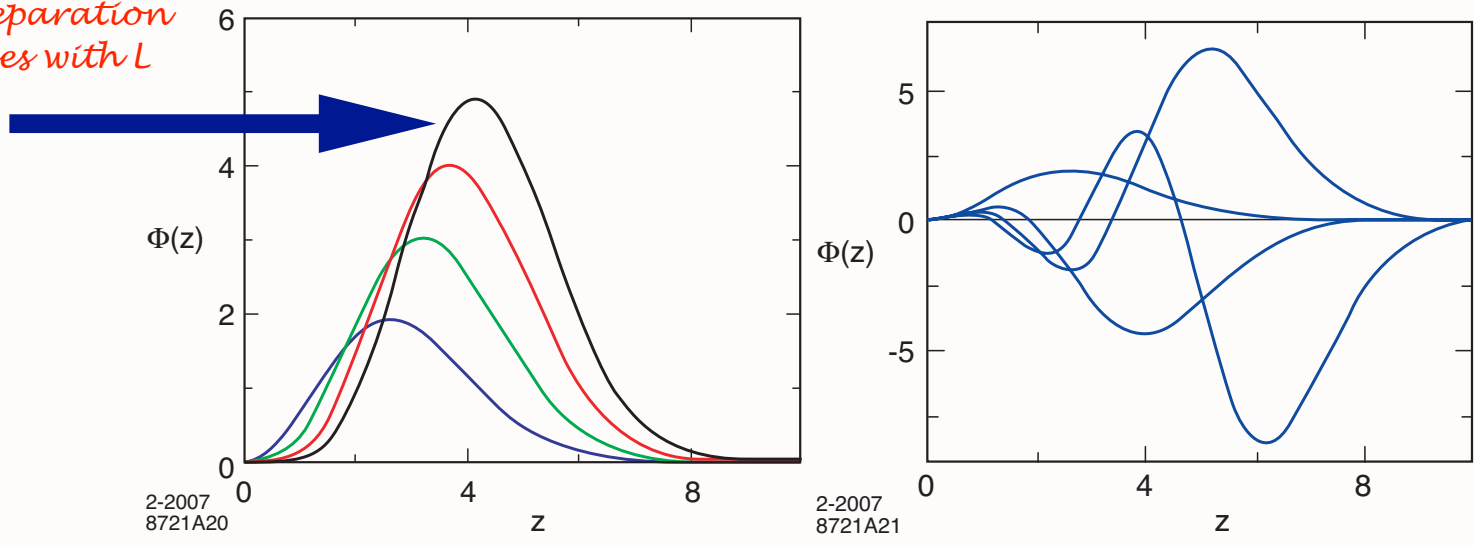
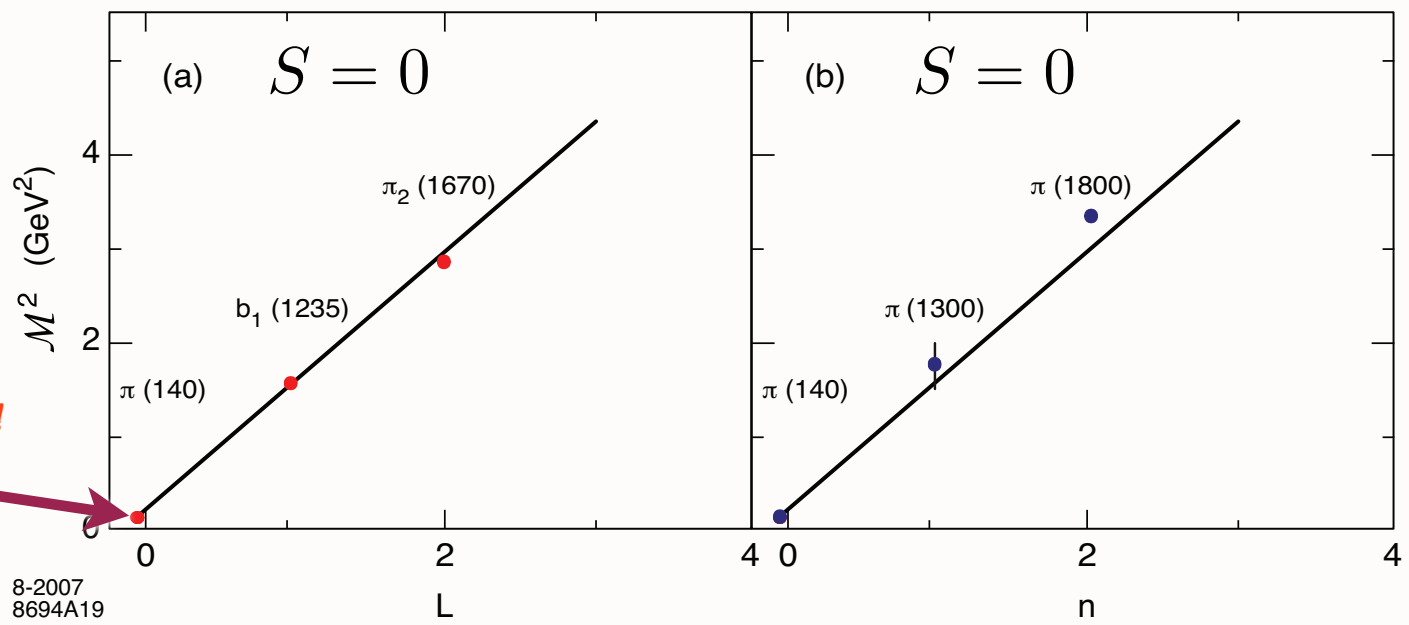


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

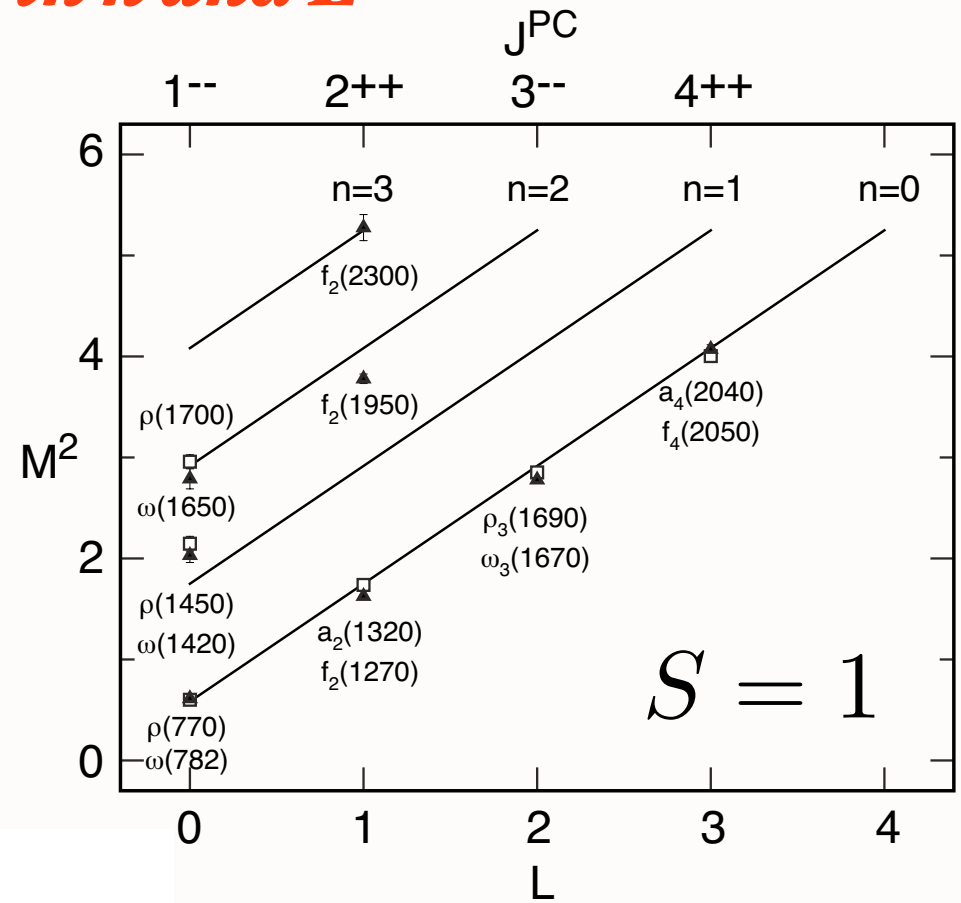
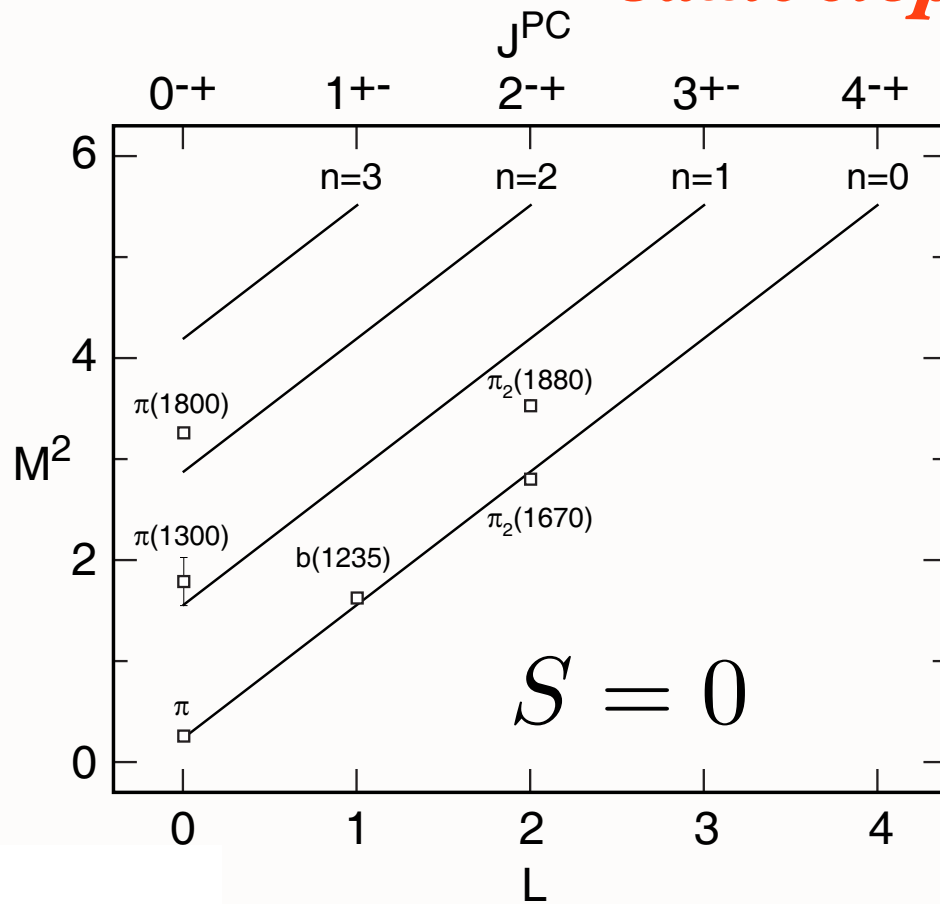
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Bosonic Modes and Meson Spectrum

$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \rightarrow 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
 $4\kappa^2$ for $\Delta L = 1$
 $2\kappa^2$ for $\Delta S = 1$

Same slope in n and L



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I=1$ ρ -meson and $I=0$ ω -meson families ($\kappa = 0.54$ GeV)

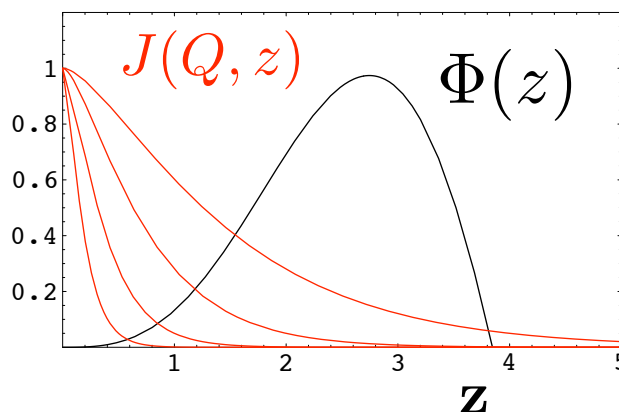
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



Polchinski, Strassler
de Teramond, sjb

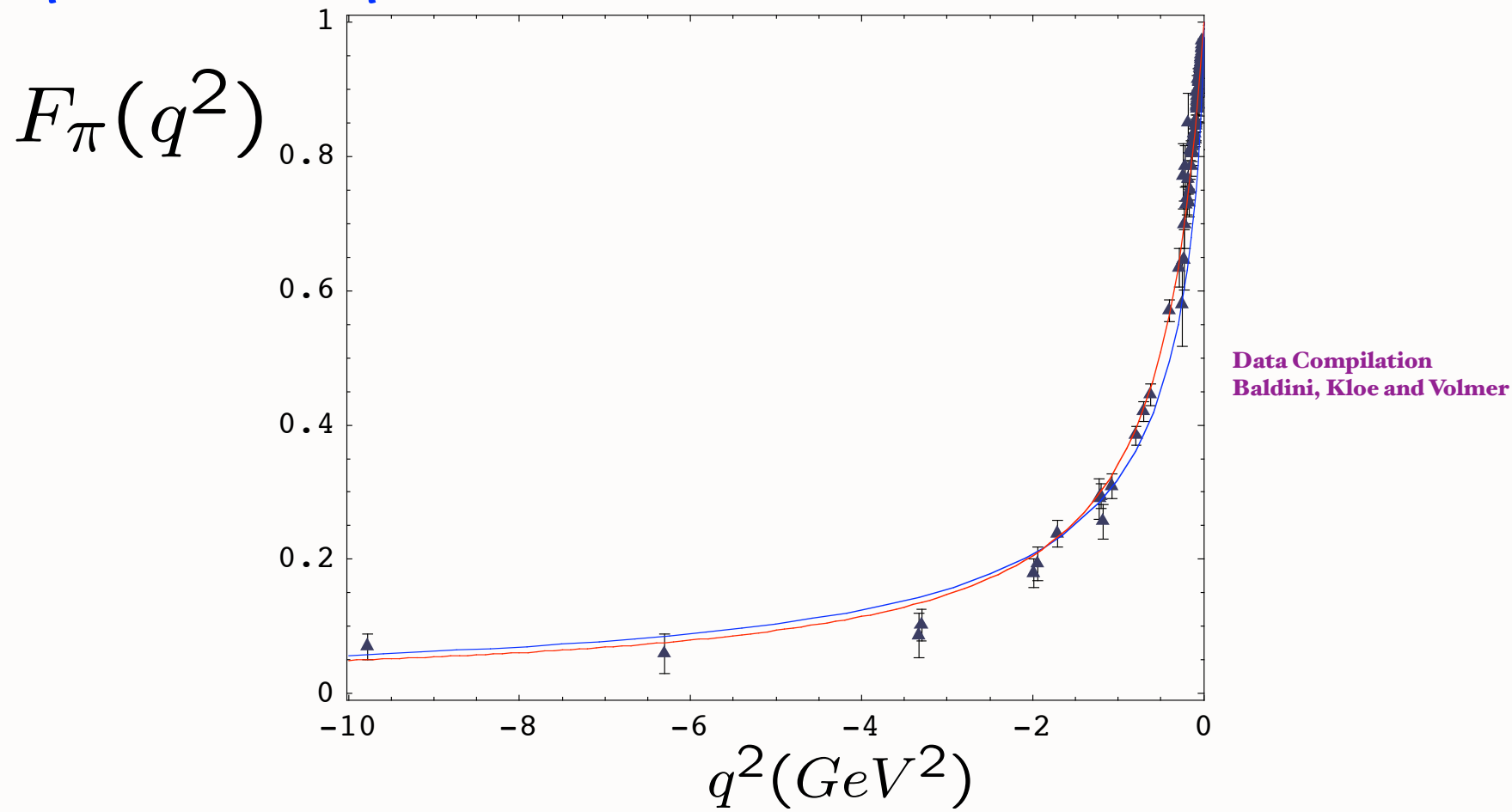
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

de Teramond, sjb
See also: Radyushkin

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space \vec{b}_\perp

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$):

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

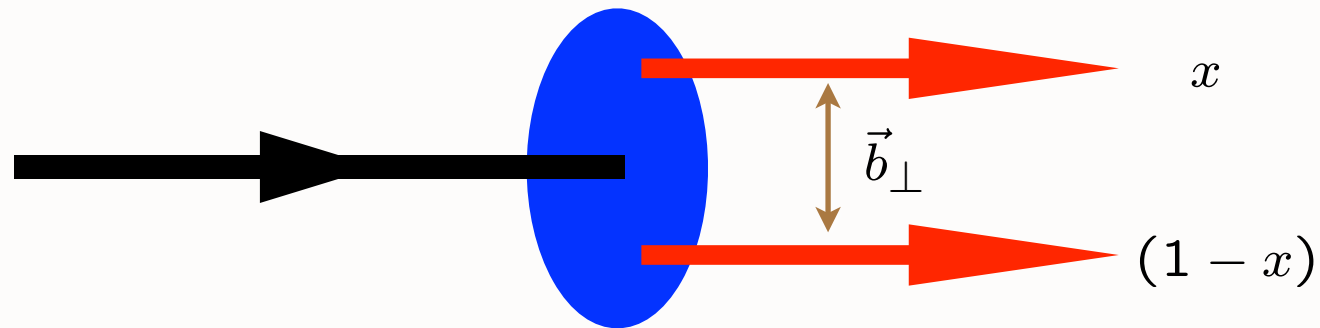
- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

$LF(3+1) \longleftrightarrow AdS_5$
 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$
 $\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$


$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

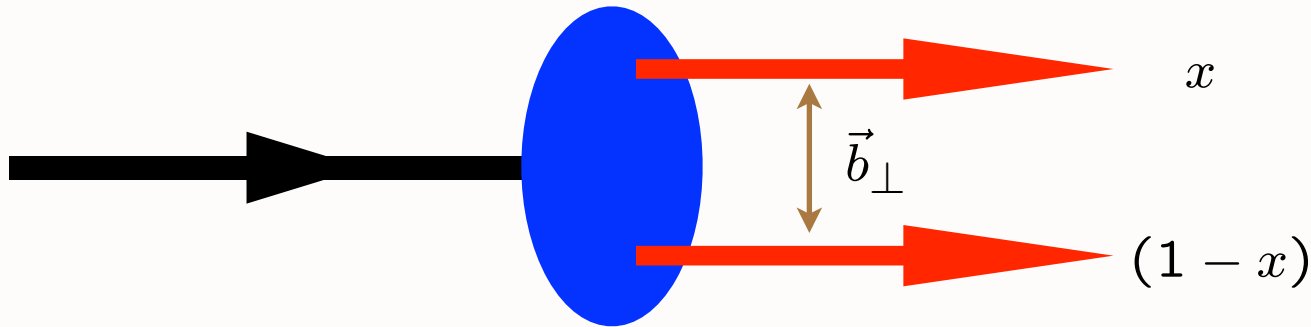
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(z) = \kappa^4 z^2 + 2\kappa^2(L + S - 1)$$

*soft wall
confining potential:*

G. de Teramond, sjb

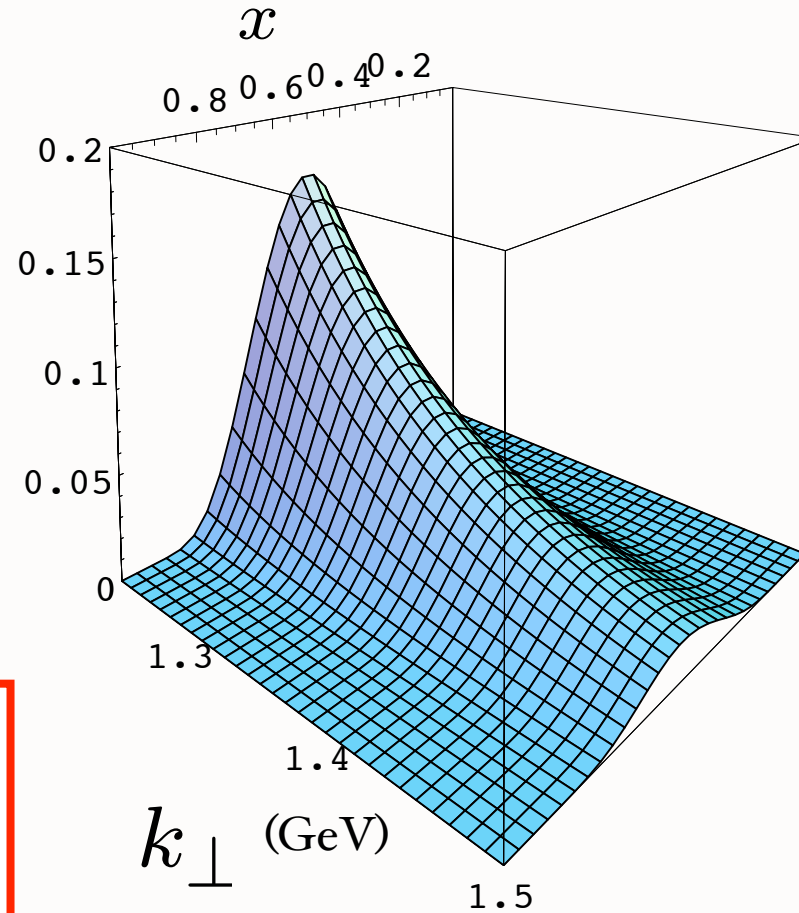
Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

**“Soft Wall”
model**

$\kappa = 0.375 \text{ GeV}$
massless quarks

$$\psi_M(x, k_{\perp}^2)$$



Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

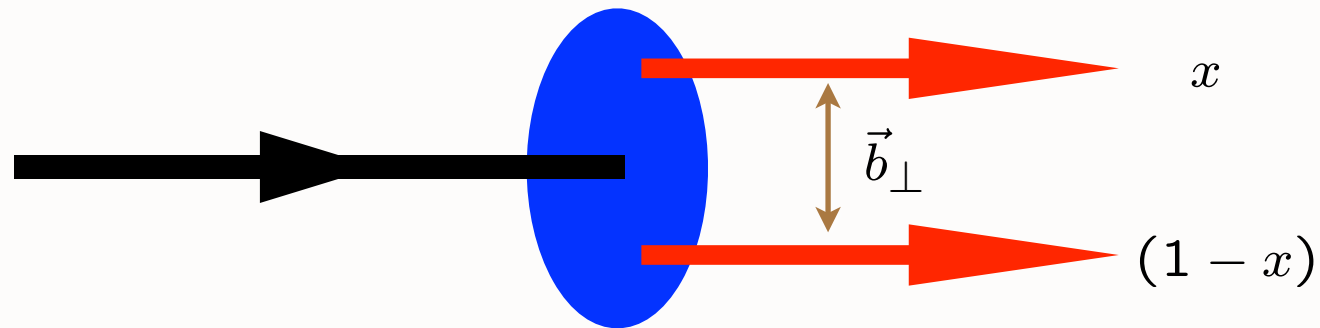
Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$
 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$
 $\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$


$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

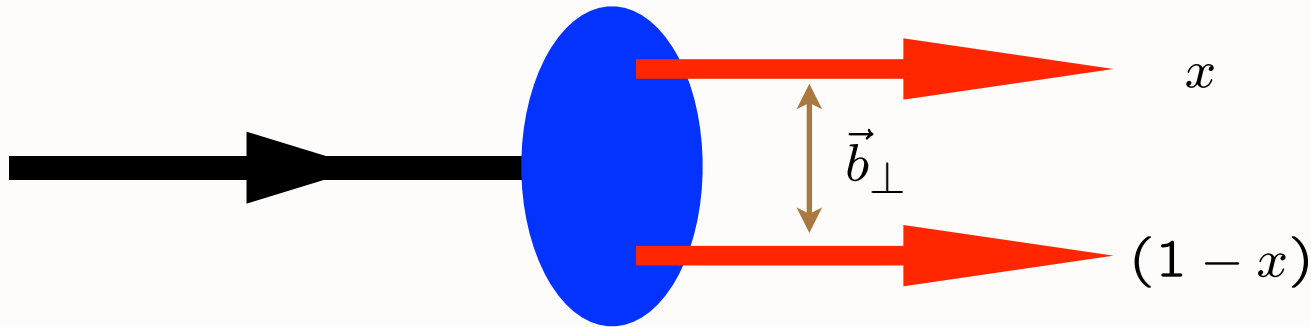
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*soft wall
confining potential:*

G. de Teramond, sjb

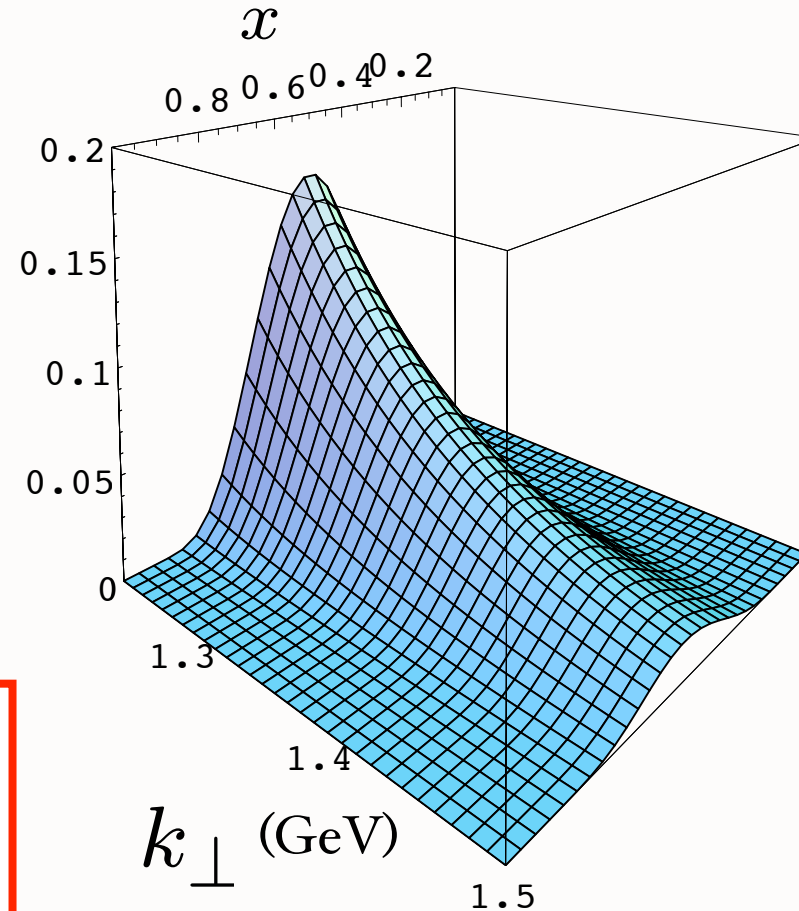
Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

**“Soft Wall”
model**

$\kappa = 0.375$ GeV
massless quarks

$$\psi_M(x, k_{\perp}^2)$$



Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

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Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

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$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

de Teramond, sjb

Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions.} \end{aligned}$$

**Change
variables**

$$(\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

Light-Front Holography

AdS Space matches 3+1 spacetime at fixed Light-Front Time!

- *Matching of AdS and LF Expressions for EM and Gravitational Form Factors*
- *Overlap of LFWFs Only – No Vacuum Currents so cannot match to Instant-Time formula*
- *Matches Equations of LF Hamiltonian Theory*
- *Matches LF Kinetic Energy*
- *Angular Momentum Matches to AdS Mass*

Baryons in AdS/QCD

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint Π^\dagger , with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

- Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

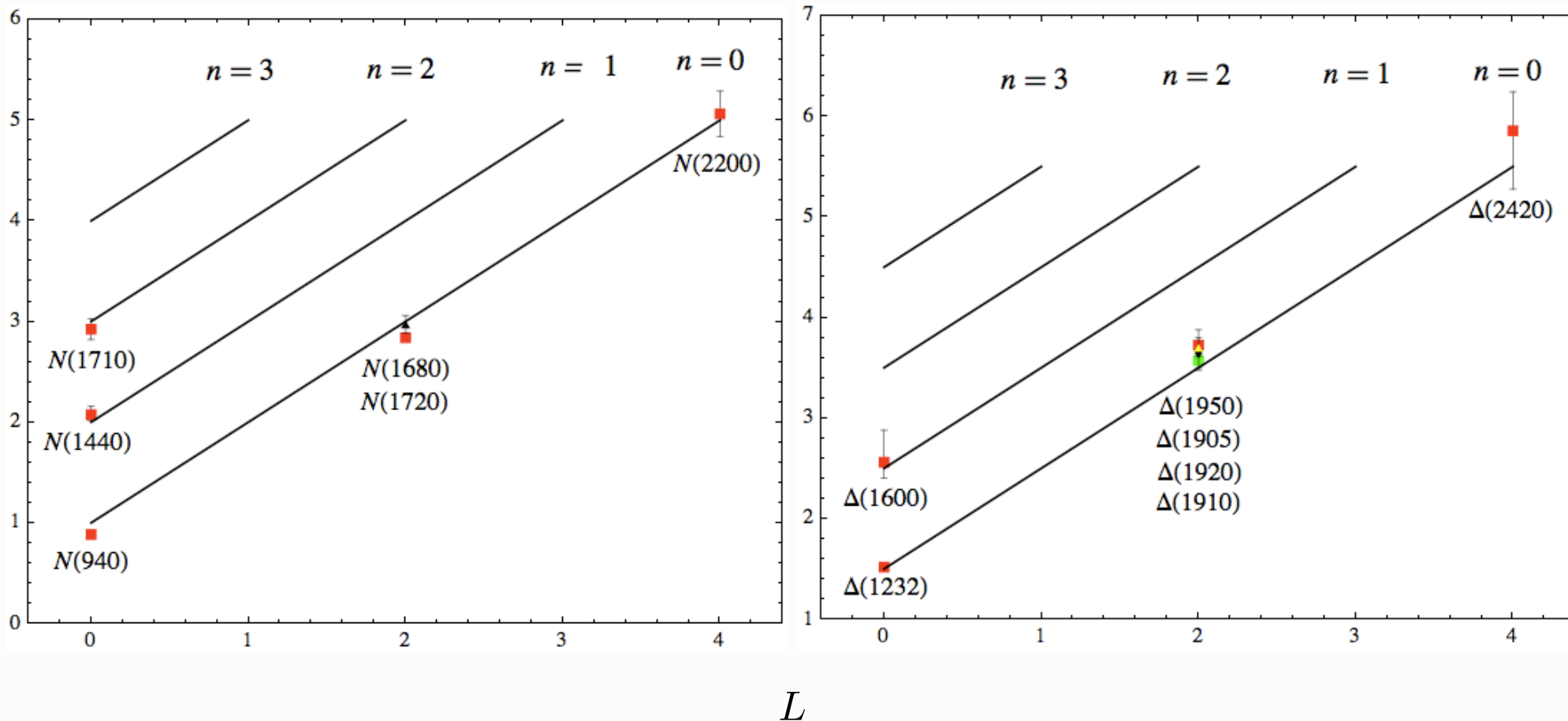
$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$

Same multiplicity of states for mesons and baryons!

$$\mathcal{M}^2$$



Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Chiral Features of Soft-Wall AdS/ QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.** *Proton spin carried by quark angular momentum!*
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
 $J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

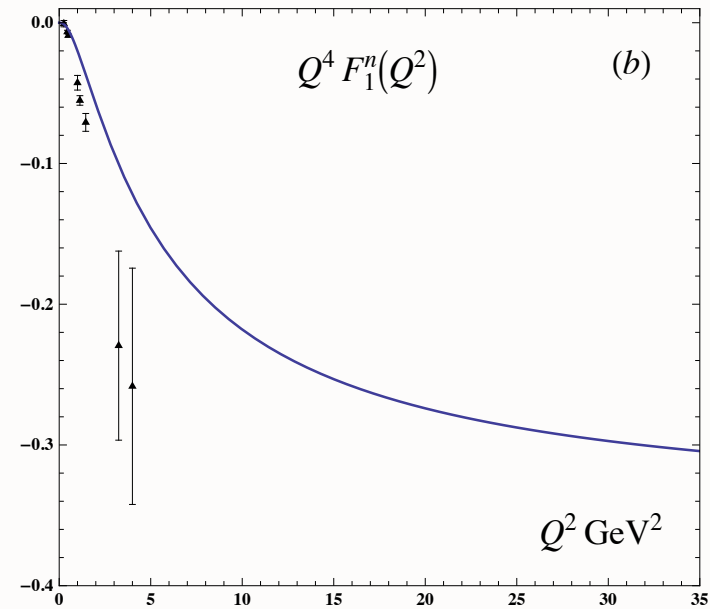
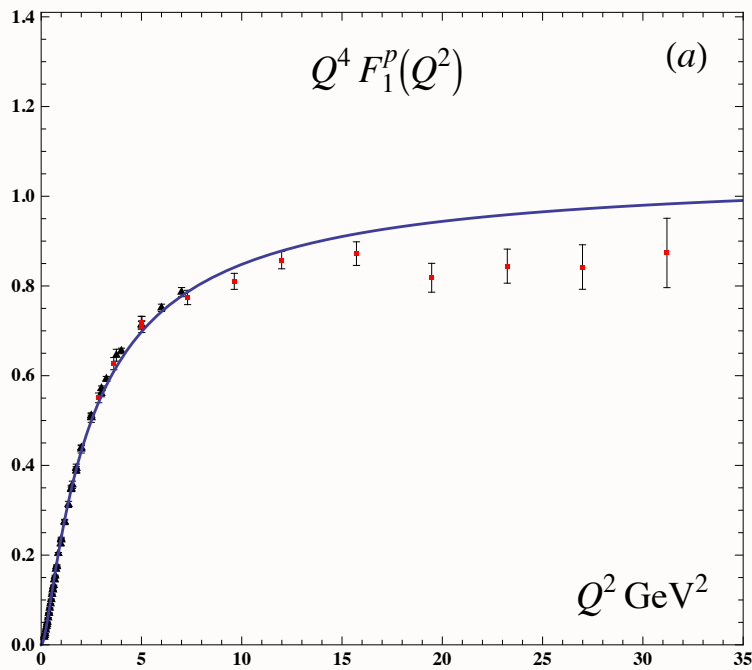
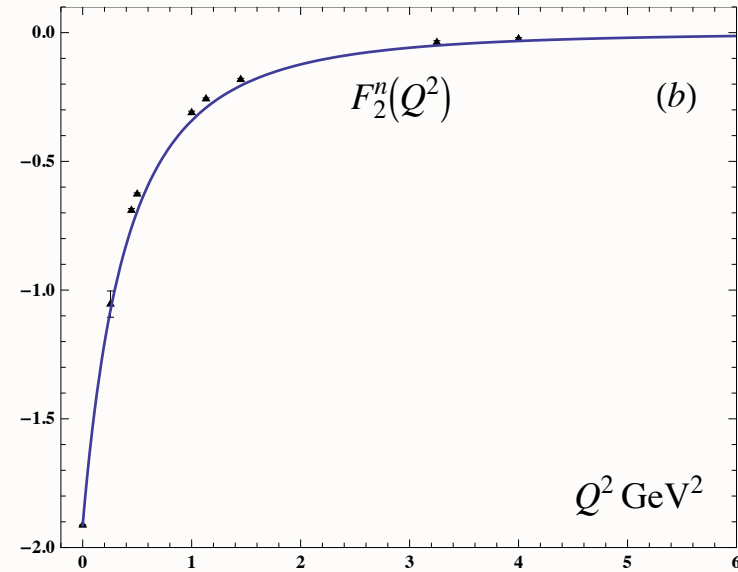
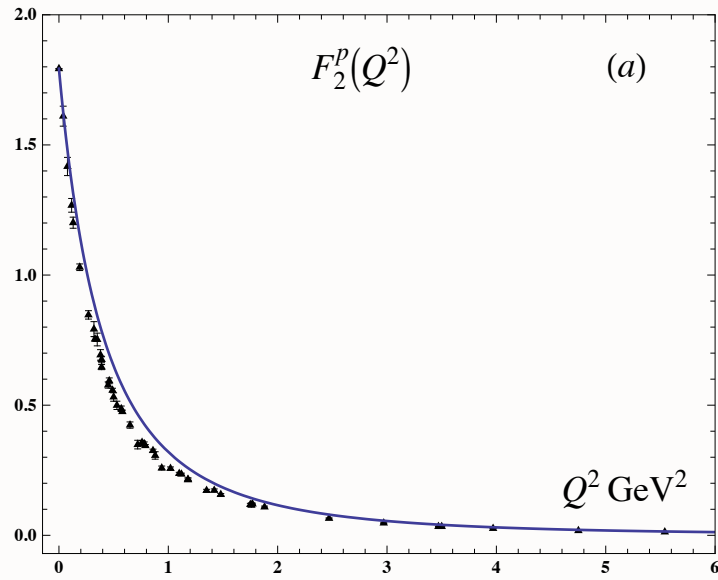
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

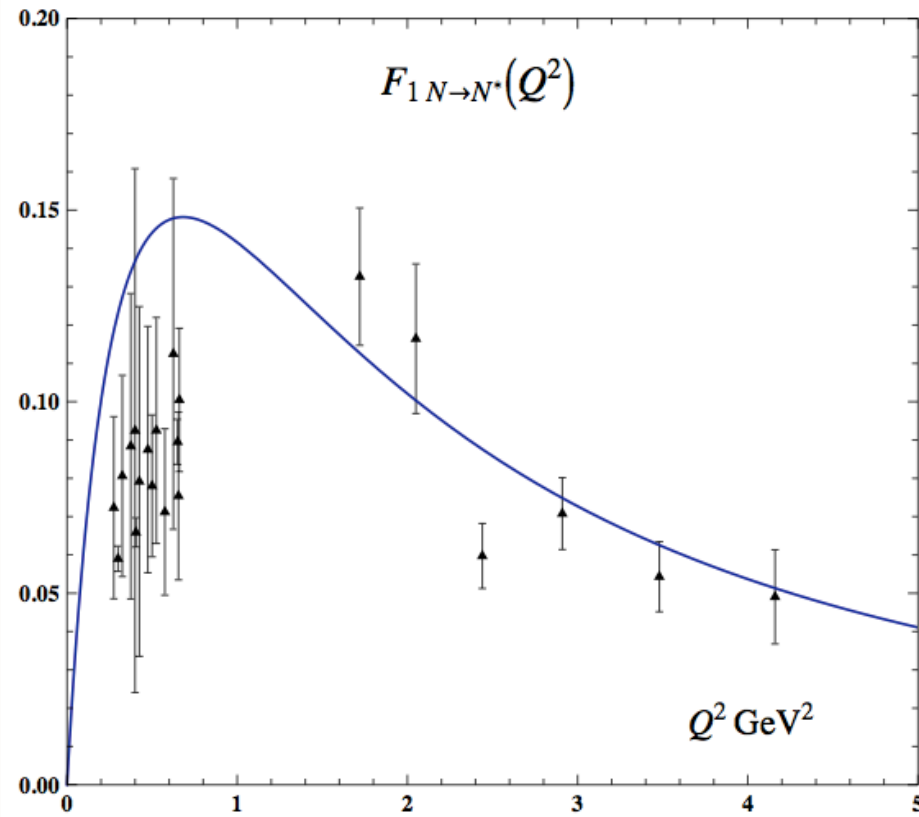
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

Proton and neutron form factors from AdS/QCD -- one parameter



$$N(940) \rightarrow N^*(1440): \Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$$



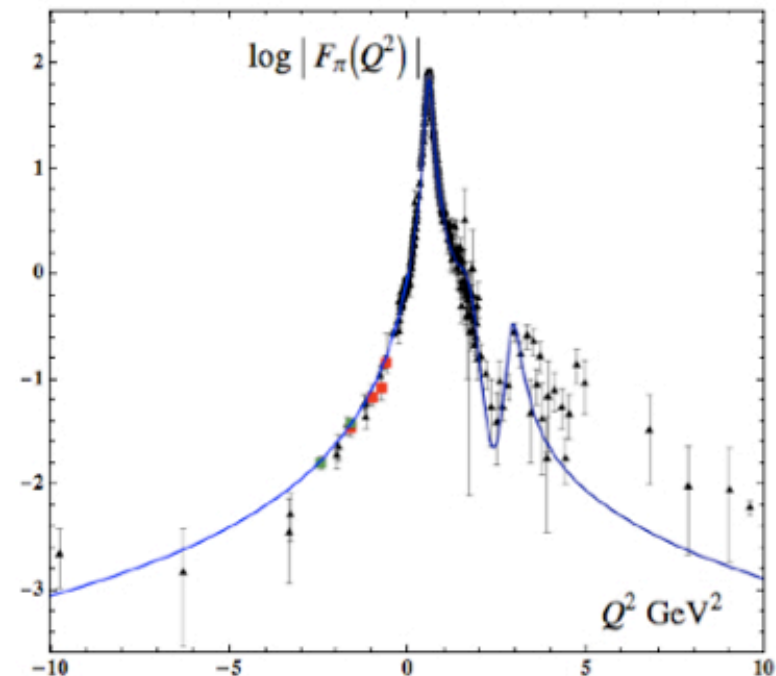
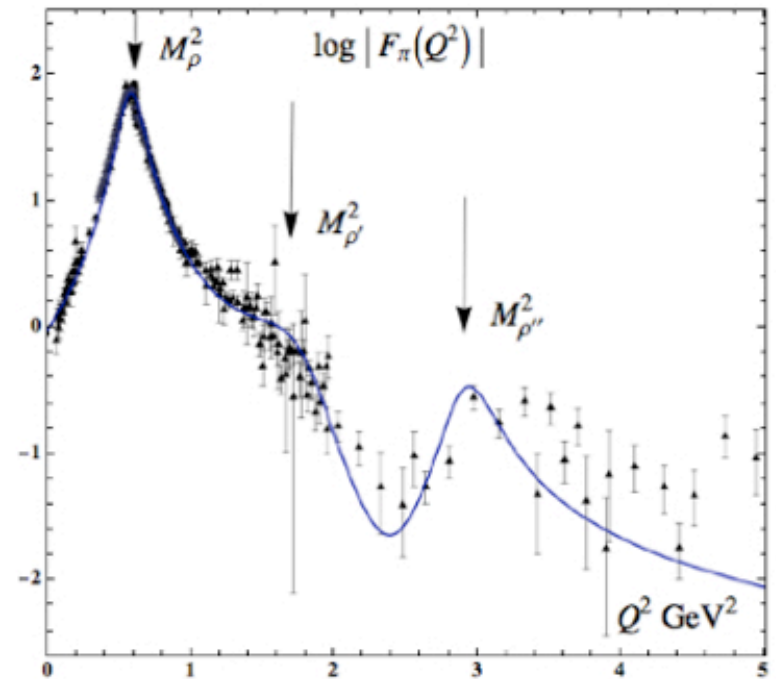
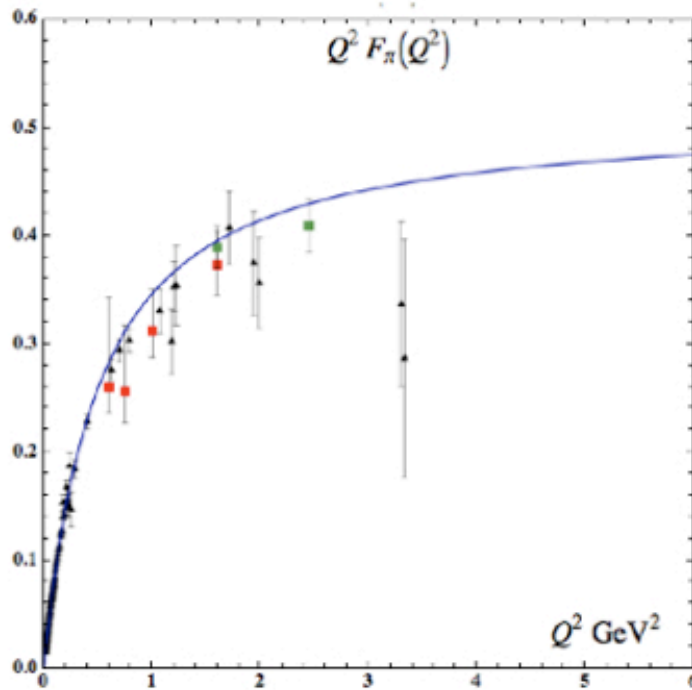
Data from I. Aznauryan, *et al.* CLAS (2009)

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{M_\rho^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)}$$

with $M_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

Space- and Time Like Pion Form-Factor (HFS)

PRELIMINARY



$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle$$

$$\mathcal{M}^2 \rightarrow 4\kappa^2(n + 1/2)$$

$$\kappa = 0.54 \text{ GeV}$$

$$\Gamma_\rho = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 13\%$$

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

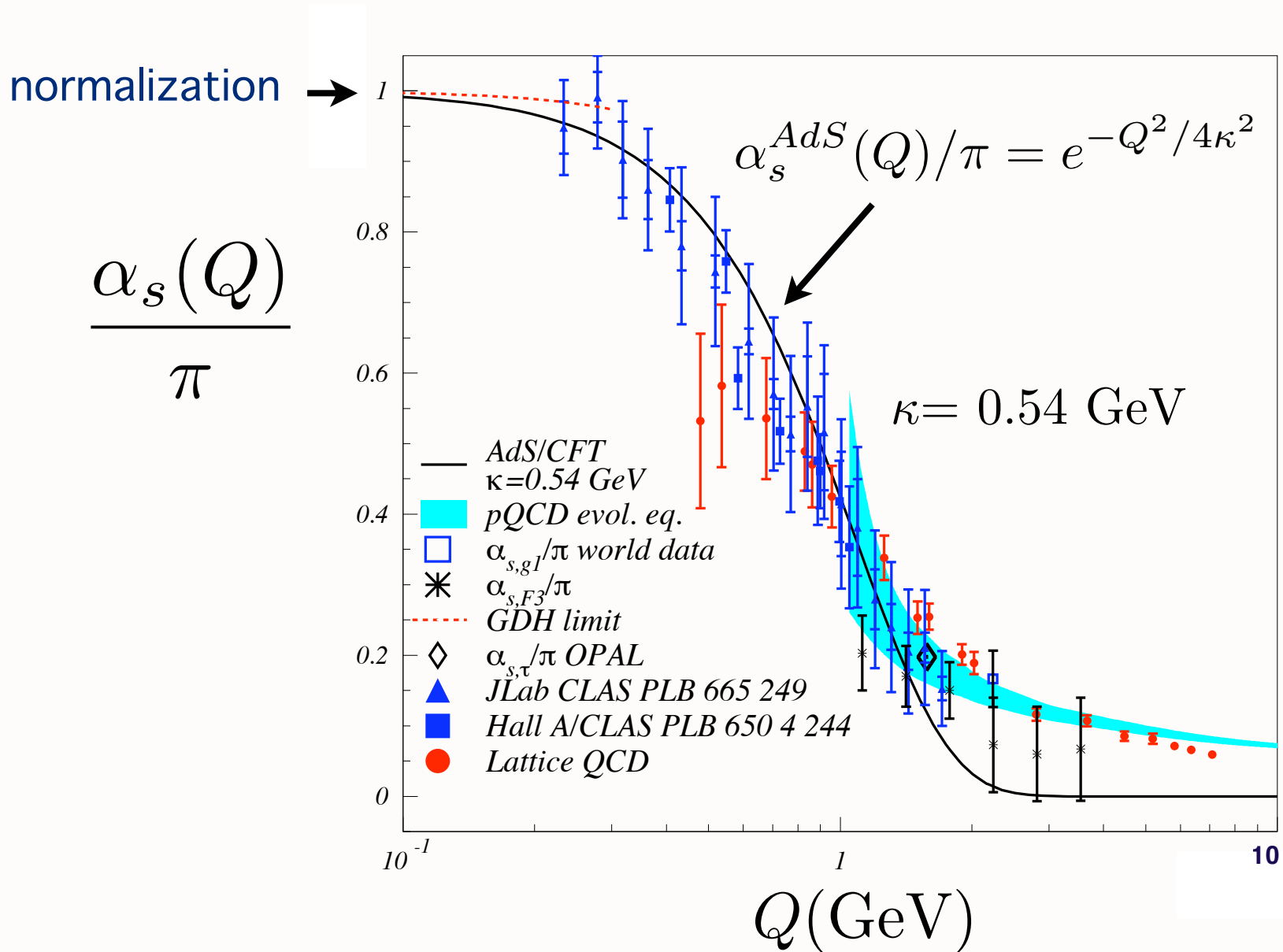
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

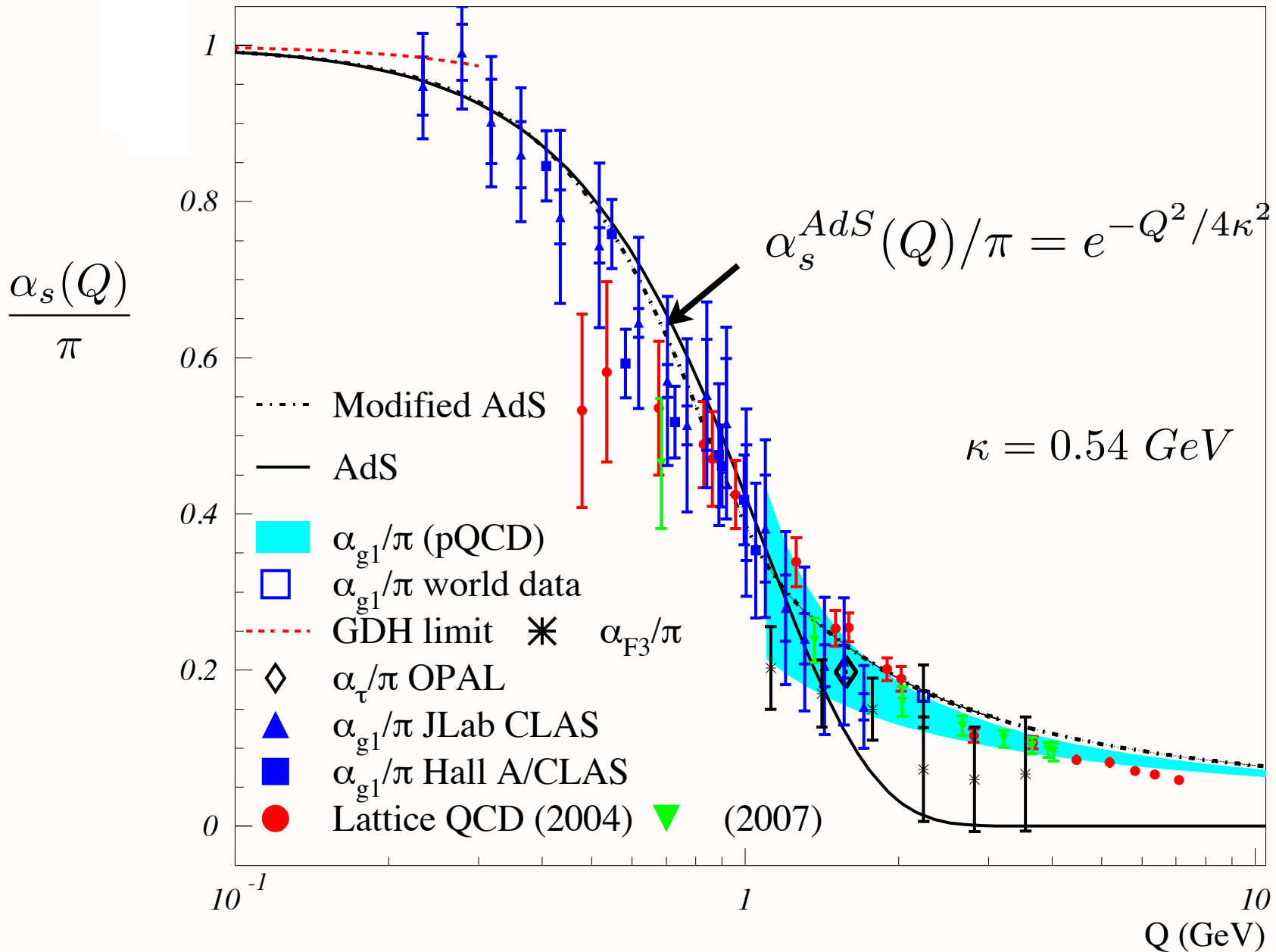
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from AdS/QCD



Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS₅**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite $N_c = 3$: Baryons built on 3 quarks -- Large N_c limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

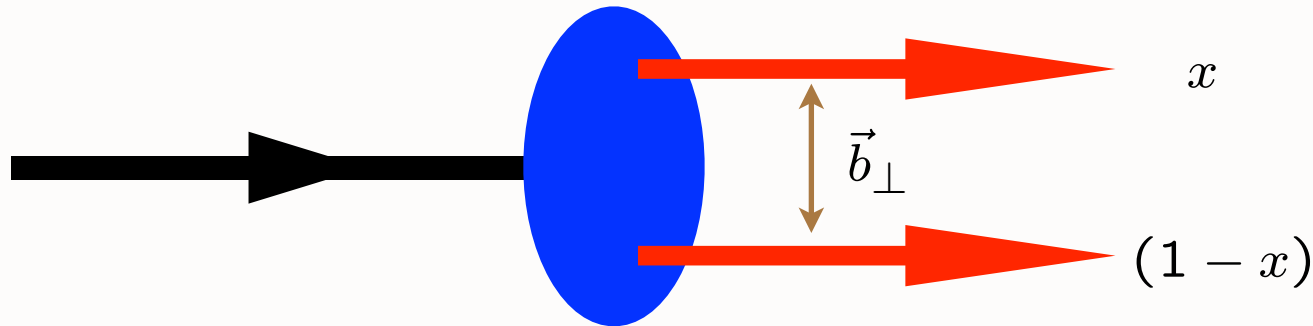
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



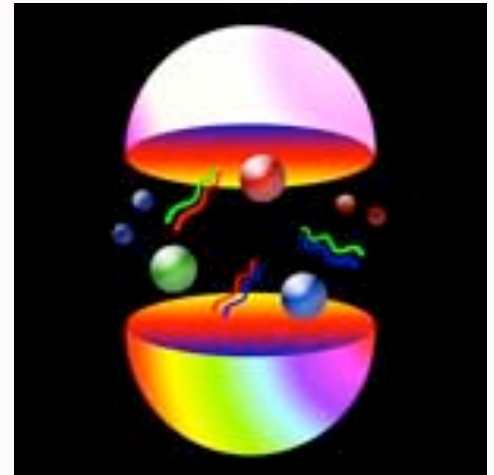
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*soft wall
confining potential:*

G. de Teramond, sjb

The World of Quarks and Gluons:

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: Holography: AdS/CFT
- Need to understand QCD at the Amplitude Level: Hadron wavefunctions!



String Theory

AdS/CFT

Mapping of Poincare' and Conformal $SO(4,2)$ symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

AdS/QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

Conformal behavior at short distances + Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$ plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

*Use AdS/CFT orthonormal LFWFs
as a basis for diagonalizing
the QCD LF Hamiltonian*

- Good initial approximant
- Better than plane wave basis Pauli, Hornbostel, Hiller,
McCartor, sjb
- DLCQ discretization -- highly successful I+I
- Use independent HO LFWFs, remove CM motion Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations

*“One of the gravest puzzles of
theoretical physics”*

**DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

*QCD Problem Solved if Quark and Gluon condensates reside
within hadrons, not LF vacuum*

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind

Roberts et al.

Shrock and sjb

- **Bound-State Dyson-Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant --
Eliminates 45 orders of magnitude conflict**

Roberts et al.

Shrock and sjb

Gell-Mann Oakes Renner Formula in QCD

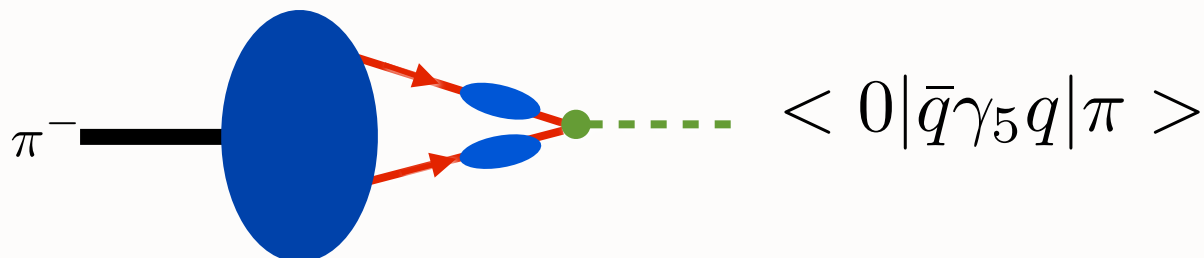
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} \langle 0 | \bar{q}q | 0 \rangle$$

**current algebra:
effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} \langle 0 | i\bar{q}\gamma_5 q | \pi \rangle$$

**QCD: composite pion
Bethe-Salpeter Eq.**

vacuum condensate actually is an “in-hadron condensate”



Maris, Roberts, Tandy

PHYSICAL REVIEW C **82**, 022201(R) (2010)**New perspectives on the quark condensate**Stanley J. Brodsky,^{1,2} Craig D. Roberts,^{3,4} Robert Shrock,⁵ and Peter C. Tandy⁶¹*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*²*Centre for Particle Physics Phenomenology: CP³-Origins, University of Southern Denmark, Odense 5230 M, Denmark*³*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*⁴*Department of Physics, Peking University, Beijing 100871, China*⁵*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*⁶*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

Light-Front vacuum: trivial, causal, frame-independent

Rochester, February 8, 2012

Atoms in Flight

Stan Brodsky

SLAC

Summary on QCD 'Condensates'

- Condensates do not exist as space-time-independent phenomena
- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: “In-Hadron Condensates”
- Find: $\frac{\langle 0|\bar{q}q|0 \rangle}{f_\pi} \rightarrow - \langle 0|i\bar{q}\gamma_5 q|\pi \rangle = \rho_\pi$
 $\langle 0|\bar{q}i\gamma_5 q|\pi \rangle$ similar to $\langle 0|\bar{q}\gamma^\mu\gamma_5 q|\pi \rangle$
- Zero contribution to cosmological constant! Included in hadron mass
- Q_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states
- Light-Front Vacuum: Causal, trivial, no normal ordering needed

Many Analogs: QED/QCD

- **Diffraction Dissociation of Atoms/Hadrons**
- **Atomic/Color Transparency**
- **Light-Front Wavefunctions**
- **Atomic Alchemy/B decay**
- **Atom Formation/Hadronization**
- **Spontaneous pair production/ Confinement**
- **Intrinsic heavy leptons/Intrinsic Charm**
- **True Muonium/Quarkonium**
- **Scale Setting, Counting Rules**

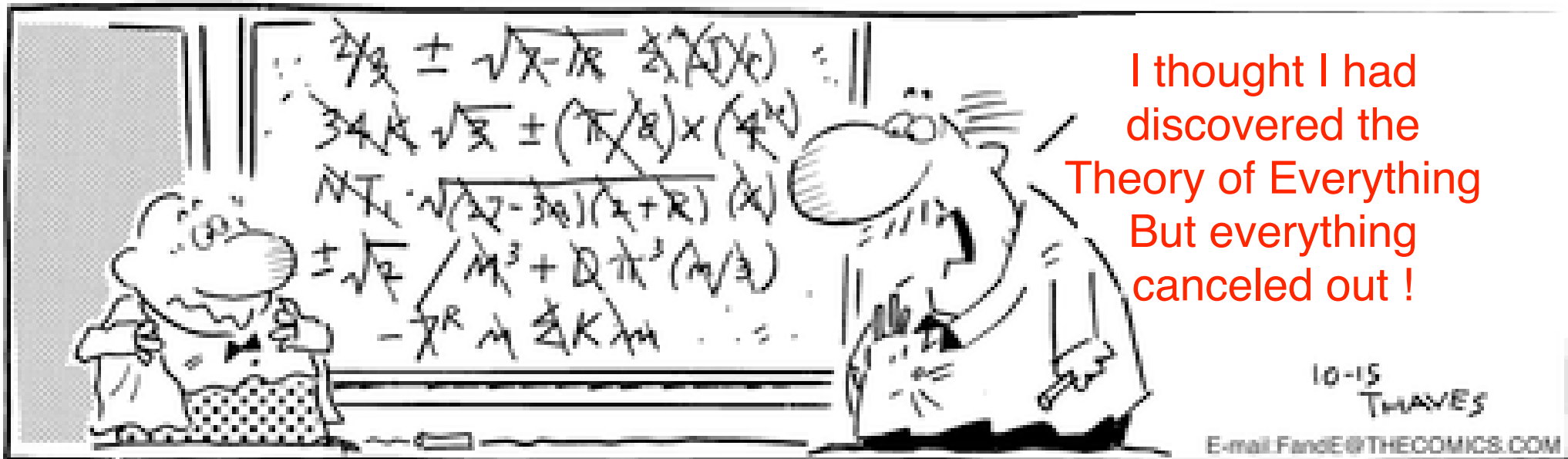
- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.
- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, quark-gluon plasma, ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities. *—Mark Twain*

A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

Frank and Ernest



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