The paradox of emptiness: Much ado about nothing

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Nature's strong messenger – Pion

- 1947 – Pion discovered by Cecil Frank Powell
- Studied tracks made by cosmic rays using photographic emulsion plates
- Despite the fact that Cavendish Lab said method is incapable of "*reliable and reproducible precision measurements*."
- Mass measured in scattering ≈ *250-350 m*$_e$

Nuclear capture of pion

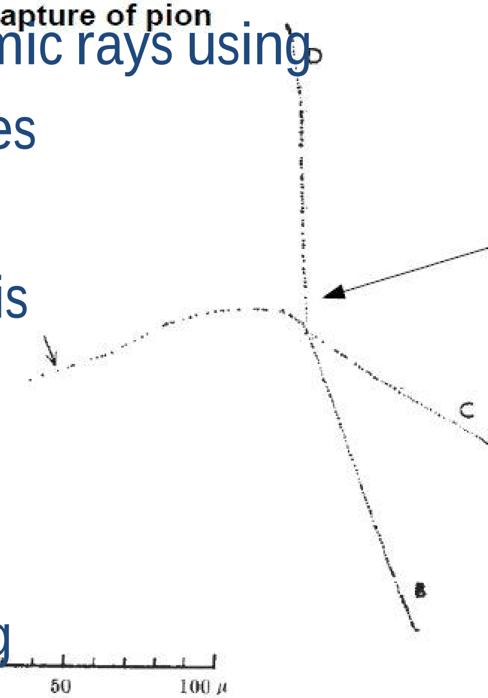
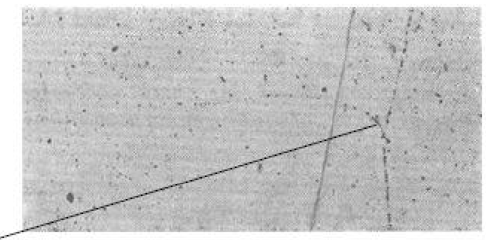


Fig. 1 *b*. TRACE OF COMPLETE STAR ON SCREEN OF PROJECTION MICROSCOPE, SHOWING PROJECTION OF THE TRACKS IN THE PLANE OF THE EMULSION. TRACK *A* CANNOT BE TRACED WITH CERTAINTY BEYOND THE ARROW



g. 1 *a*. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK OF ESON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSION OBJECTIVE. × 500)

- A is the new meson
- B,D,C are likely protons
- Track C goes into the page

Why A is a new meson:
electron: range too large
proton: scattering too large
muon: frequent nuclear interaction

11

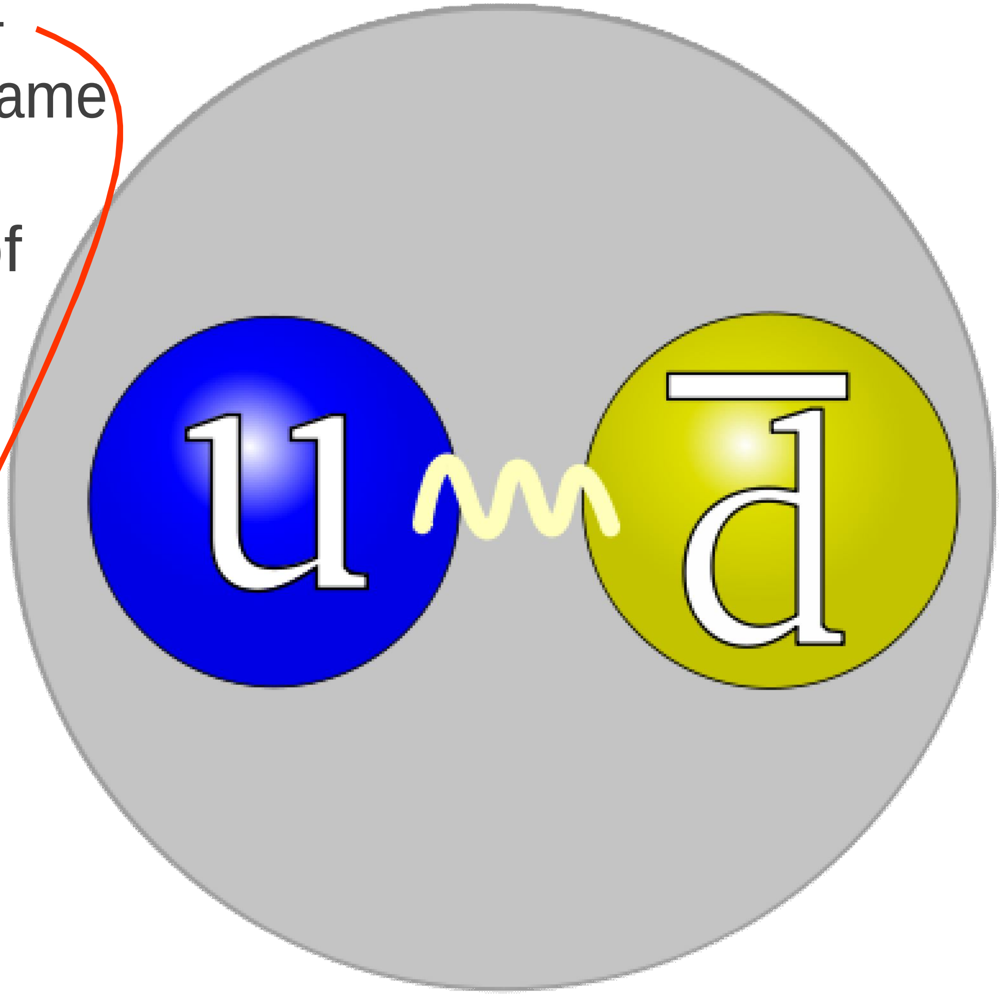
Nature's strong messenger – Pion

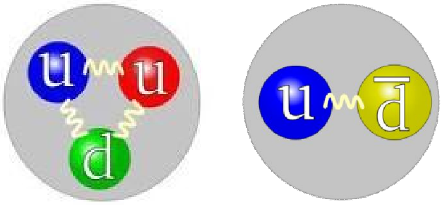
- The beginning of Particle Physics
- Then came
  - Disentanglement of confusion between muon and pion – similar masses
  - Discovery of particles with "strangeness" (e.g., kaon)
- Subsequently, a complete spectrum of mesons and baryons with mass below ≈1 GeV
  - 28 states
- Became clear that pion is "too light"

π	140 MeV
ρ	780 MeV
P	940 MeV

Simple picture - Pion

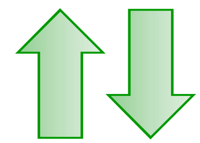
- Gell-Mann and Ne'eman:
  - Eightfold way – a picture based on group theory: *SU(3)*
  - Subsequently, quark model – where the *u*-, *d*-, *s*-quarks became the basis vectors in the fundamental representation of *SU(3)*
- Pion = Two *quantum-mechanical* constituent-quarks - particle+antiparticle - interacting via a *potential*



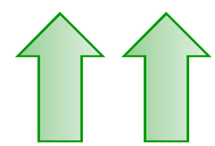


Modern Miracles in Hadron Physics

- proton = three constituent quarks
  - $M_{proton} \approx 1\text{GeV}$
  - Therefore guess $M_{constituent-quark} \approx ⅓ \times \text{GeV} \approx 350\text{MeV}$
- pion = constituent quark + constituent antiquark



  - Guess $M_{pion} \approx ⅔ \times M_{proton} \approx 700\text{MeV}$
- WRONG . . . . $M_{pion} = 140\text{MeV}$
- Rho-meson



  - Also *constituent quark* + *constituent antiquark*
    – just pion with spin of one constituent flipped
  - $M_{rho} \approx 770MeV \approx 2 \times M_{constituent-quark}$

What is “wrong” with the pion?

Dichotomy of the pion

- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless
- However:

  current-algebra (1968)

$$m_\pi^2 \propto m$$

- This is *impossible in quantum mechanics*, for which one always finds:

$$m_{bound-state} \propto m_{constituent}$$

Spontaneous(Dynamical) Chiral Symmetry Breaking

The 2008 Nobel Prize in Physics was divided, one half awarded to Yoichiro Nambu

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Nambu – Jona-Lasinio Model

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345–358

Dynamical Model Of Elementary Particles Based On An Analogy With Superconductivity. II
Y. Nambu, G. Jona-Lasinio, Phys.Rev. 124 (1961) 246-254

- Treats a chirally-invariant four-fermion Lagrangian & solves the gap equation in Hartree-Fock approximation (analogous to rainbow truncation)
- Poss ... ed
- *Esse* ... nd Nam ... & de ... ther ... dege ... Nambu vacua, related by a chiral rotation
- ***Nontrivial Vacuum is "Born"***

The following Lagrangian density will be assumed ($\hbar = c = 1$):

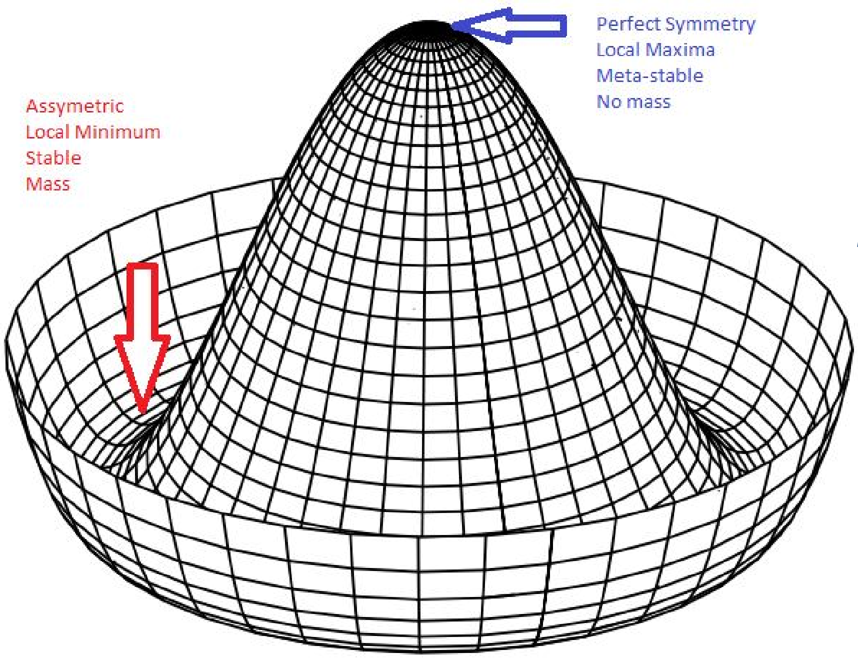
$$L = -\bar{\psi}\gamma_\mu \partial_\mu \psi + g_0[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \qquad (2.6)$$

The coupling parameter g_0 is positive, and has dimensions $[\text{mass}]^{-2}$. The γ_5 invariance property of the interaction is evident from Eq. (2.5). According to the

...rlds described ...3.22) are then

... x, (3.31)

which form another orthogonal set. Since the original total H commutes with X, it will have no matrix elements connecting different "worlds." Moreover, as



Higgs Mechanism

Broken Symmetries and the Masses of Gauge Bosons
P.W. Higgs, Phys. Rev. Lett. 13, 508–509 (1964)

Quotes are in the original

Higgs:

- *Consider the equations [...] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:* (246 GeV!)
- *In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory.*

Gell-Mann – Oakes – Renner Relation

Behavior of current divergences under SU(3) x SU(3).
Murray Gell-Mann, R.J. Oakes , B. Renner
Phys.Rev. 175 (1968) 2195-2199

- This paper derives a relation between

  m_π^2 and the expectation-value $<\pi|u_0|\pi>$,

  where u_o is an operator that is linear in the putative Hamiltonian's explicit chiral-symmetry breaking term
  - NB. Quarks were not yet invented, so u_0 was not expressed in terms of quark fields
- PCAC-hypothesis (partial conservation of axial current) is used in the derivation
- Subsequently, the concepts of soft-pion theory
  - Operator expectation values do not change as $t=m_\pi^2 \to t=0$

  to take $<\pi|u_0|\pi> \to <0|u_0|0>$ … *in-pion → in-vacuum*

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Phys.Rev. 175 (1968) 2195-2199

- PCAC hypothesis; viz., pion field dominates the divergence of the axial-vector current

$$\partial_\mu A_\mu \propto \phi_\pi$$

Zhou Guangzhao 周光召
Born 1929 Changsha, Hunan province

- Soft-pion theorem

$$\begin{aligned} & \langle\alpha|\mathcal{O}|\beta\pi(q)\rangle \approx \langle\alpha|[Q_5,\mathcal{O}]|\beta\rangle \\ \Rightarrow\; & \langle 0|\mathcal{O}|0\rangle \approx \langle 0|[Q_5,[Q_5,\mathcal{O}]]|0\rangle \\ \Rightarrow\; & \langle\pi(q)|\mathcal{H}|\pi(q)\rangle \approx \langle 0|[Q_5,[Q_5,\mathcal{H}]]|0\rangle \\ \propto\; & \langle 0|\mathcal{H}_{\rm chiral-symmetry-breaking}|0\rangle \end{aligned}$$

Commutator is chiral rotation
Therefore, isolates explicit chiral-symmetry breaking term in the putative Hamiltonian

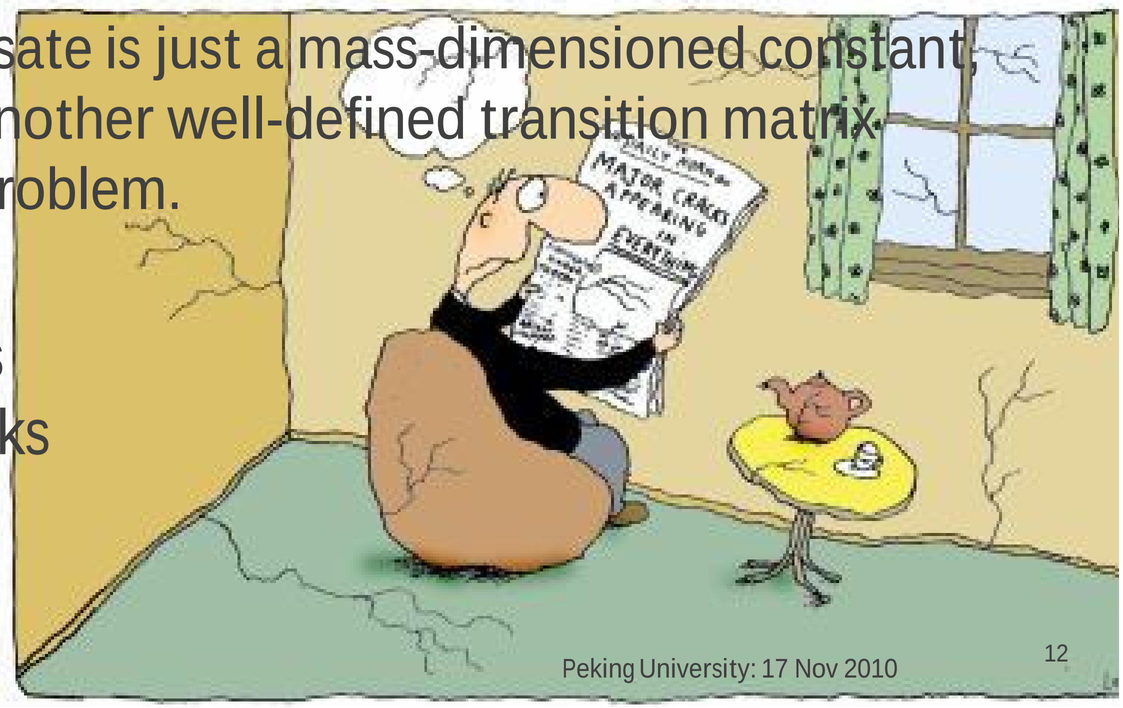
- In QCD, this is $m\bar{q}q$ and one therefore has $m_\pi^2 \propto m\langle 0|\bar{q}q|0\rangle$

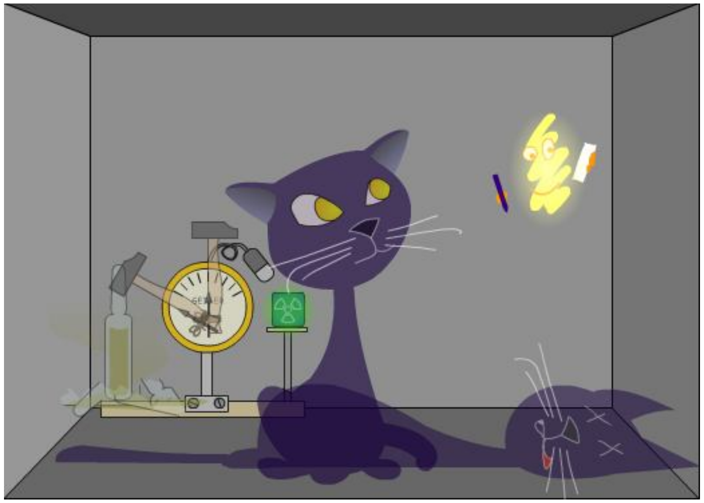
Gell-Mann – Oakes – Renner Relation

$$m_\pi^2 \propto m\langle 0|\bar{q}q|0\rangle$$

$- (0.25\text{GeV})^3$

- Theoretical physics at its best.
- But no one is thinking about how properly to consider or define what will come to be called the *vacuum quark condensate*
  - So long as the condensate is just a mass-dimensioned constant, which approximates another well-defined transition matrix element, there is no problem.
- The problem arises if one over-interprets this number, which textbooks have been doing for a very long time.





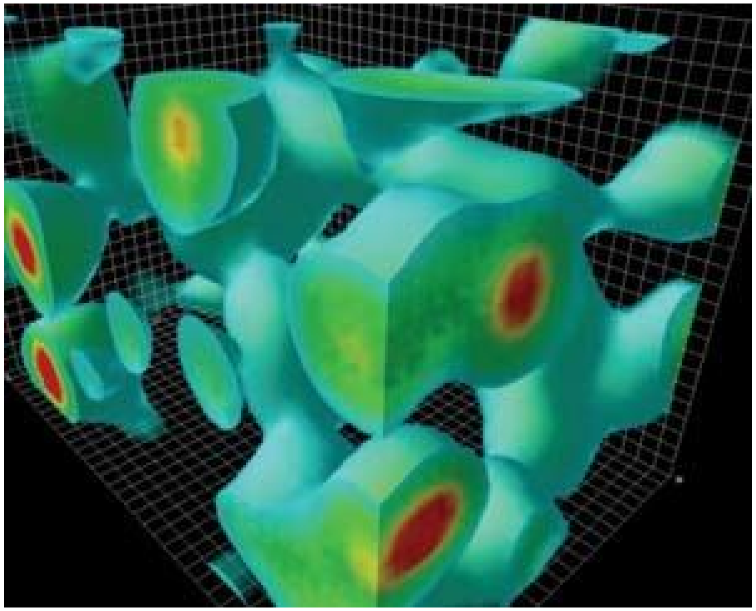
Note of Warning

Chiral Magnetism (or Magnetohadrochironics)
A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration. On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations

- *These authors argue that dynamical chiral-symmetry breaking can be realised as a property of hadrons, instead of via a nontrivial vacuum exterior to the measurable degrees of freedom*

The essential ingredient required for a spontaneous symmetry breakdown in a composite system is the existence of a divergent number of constituents
– DIS provided evidence for divergent sea of low-momentum partons – parton model.



QCD Sum Rules

QCD and Resonance Physics. Sum Rules.
M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov
Nucl.Phys. B147 (1979) 385-447; citations: 3713

- Introduction of the gluon vacuum condensate

$$\frac{\alpha}{\pi}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle = (0.33\,\mathrm{GeV})^4$$

and development of sum rules relating properties of low-lying hadronic states to vacuum condensates

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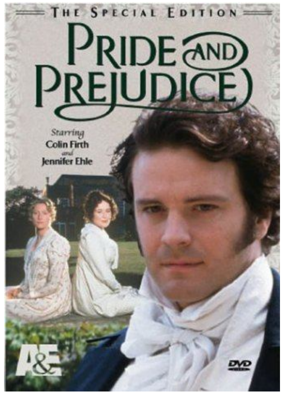
- *At this point (1979), the cat was out of the bag: a physical reality was seriously attributed to a plethora of vacuum condensates*

"quark condensate" 1960-1980

- Instantons in non-perturbative QCD vacuum,
  MA Shifman, AI Vainshtein... - Nuclear Physics B, 1980
- Instanton density in a theory with massless quarks,
  MA Shifman, AI Vainshtein... - Nuclear Physics B, 1980
- Exotic new quarks and dynamical symmetry breaking,
  WJ Marciano - Physical Review D, 1980
- The pion in QCD
  J Finger, JE Mandula... - Physics Letters B, 1980

No references to this phrase before 1980

6290 references to this phrase since 1980

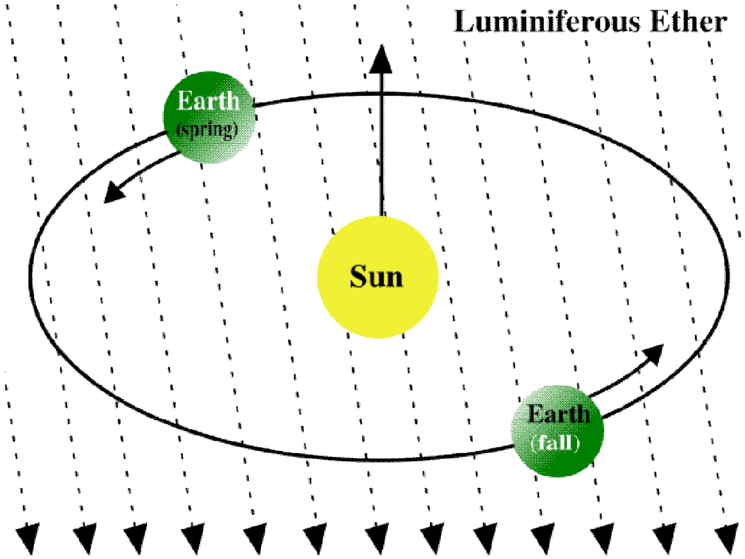


Universal "Truths"

- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."

Precedent?



Precedent-
Luminiferous Aether

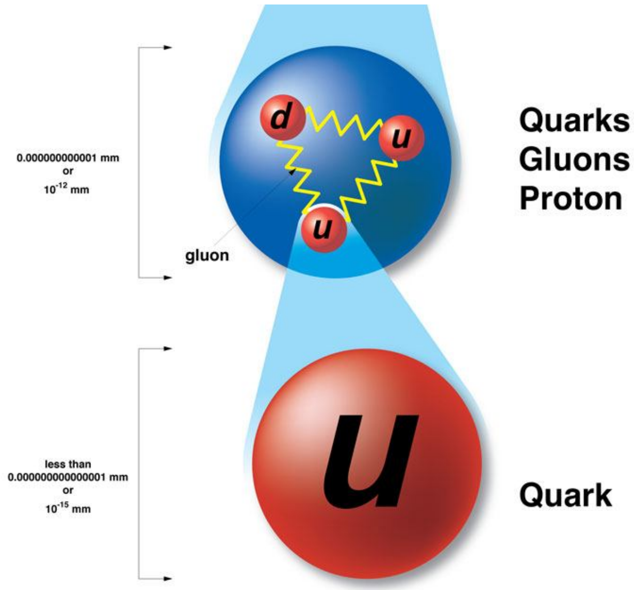
Pre-1887

~~Since the Earth is in motion, the flow of aether across the Earth should produce a detectable "aether wind"~~

- Physics theories of the late 19th century postulated that, just as water waves must have a medium to move across (water), and audible sound waves require a medium to move through (such as air or water), so also light waves require a medium, the "*luminiferous aether*".

Apparently unassailable logic

- Until, of course, "… *the most famous failed experiment to date.*" *On the Relative Motion of the Earth and the Luminiferous Ether*
Michelson, Albert Abraham & Morley, Edward Williams
American Journal of Science 34 (1887) 333–345.



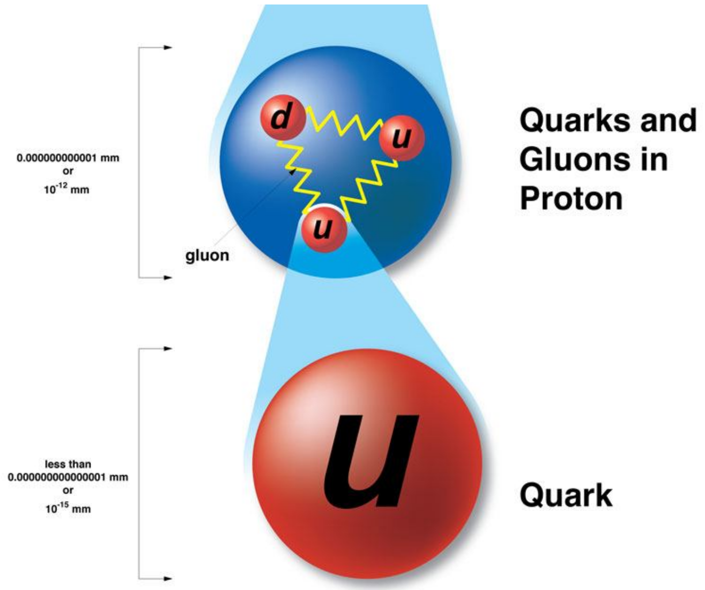
$$\langle 0 | \bar{q}q | 0 \rangle$$

QCD

1973-1974

- How should one approach this problem, understand it, within Quantum ChromoDynamics?

1) Are the quark and gluon “condensates” theoretically well-defined?
2) Is there a physical meaning to this quantity or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?

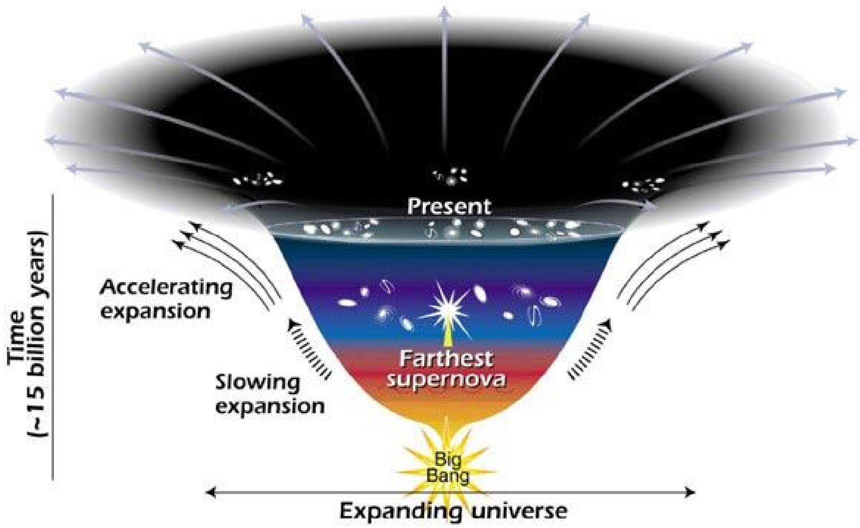


$$\langle 0 | \bar{q}q | 0 \rangle$$

QCD

1973-1974

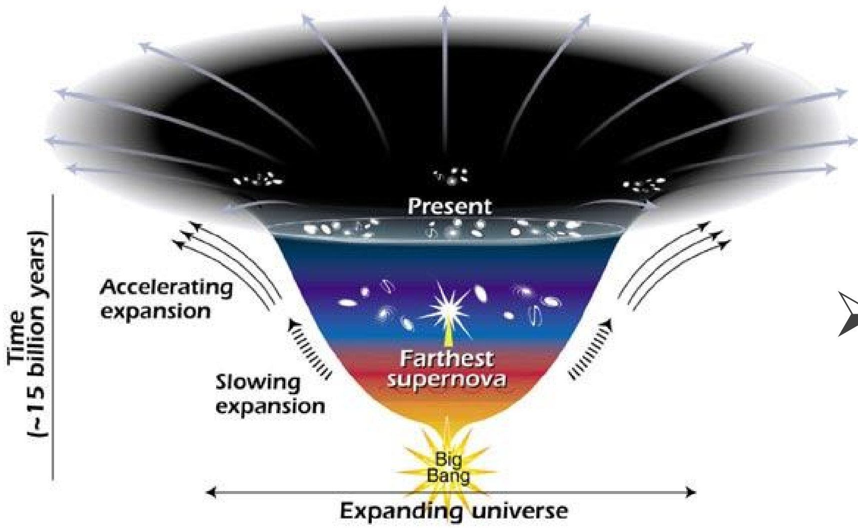
Why does it matter?



"Dark Energy"

- Two pieces of evidence for an accelerating universe
  1) Observations of type Ia supernovae
     → the rate of expansion of the Universe is growing
  2) Measurements of the composition of the Universe point to a missing energy component with negative pressure: CMB anisotropy measurements indicate that the Universe is at $\Omega_0 = 1 \pm 0.04$.
     In a flat Universe, the matter density and energy density must sum to the critical density. However, matter only contributes about ⅓ of the critical density, $\Omega_M = 0.33 \pm 0.04$.
     Thus, two thirds of the critical density is missing.

"Dark Energy"

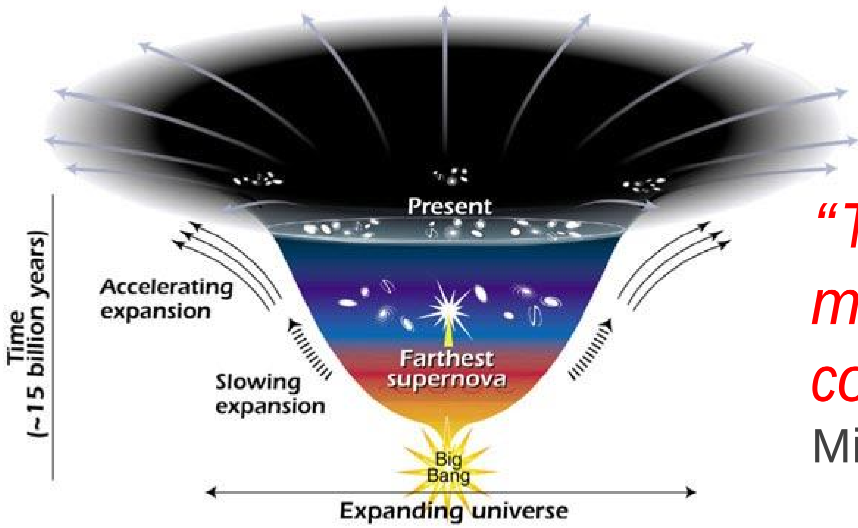


- In order to have escaped detection, the missing energy must be smoothly distributed.

- In order not to interfere with the formation of structure (by inhibiting the growth of density perturbations) the energy density in this component must change more slowly than matter (so that it was subdominant in the past).
- Accelerated expansion can be accommodated in General Relativity through the Cosmological Constant, *Λ*.
  - Contemporary cosmological observations mean:

$$\rho_\Lambda^{obs} = \frac{\Lambda}{8\pi G} \le (10^{-12} GeV)^4$$

  - Einstein introduced the repulsive effect of the cosmological constant in order to balance the attractive gravity of matter so that a static universe was possible. He promptly discarded it after the discovery of the expansion of the Universe.



"Dark Energy"

"The advent of quantum field theory made consideration of the cosmological constant obligatory not optional."
Michael Turner, "Dark Energy and the New Cosmology"

- The only possible covariant form for the energy of the (quantum) vacuum; viz.,

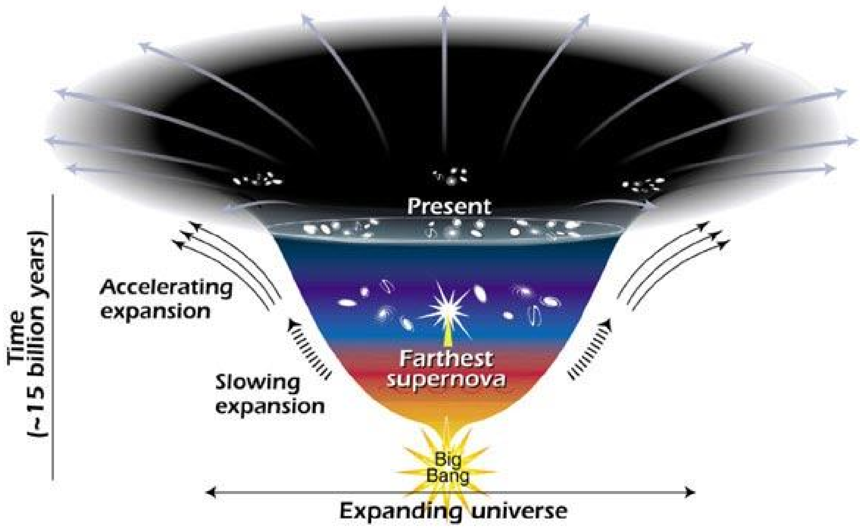
$$T^{\mu\nu}_{\rm VAC} = \rho_{\rm VAC}\, g^{\mu\nu}$$

is mathematically equivalent to the cosmological constant.

"It is a perfect fluid and precisely spatially uniform"

"Vacuum energy is almost the perfect candidate for dark energy."

"Dark Energy"



$$T^{\mu\nu}_{\rm VAC} = \rho_{\rm VAC}\, g^{\mu\nu}$$

Enormous and even greater contribution from Higgs VEV!

- *QCD vacuum contribution*
  - If chiral symmetry breaking is expressed in a nonzero expectation value of the quark bilinear, then the energy difference between the symmetric and broken phases is of order

$$M_{QCD} \approx 0.3 \text{ GeV}$$

Mass-scale generated by spacetime-independent condensate

  - One obtains therefrom:

$$\rho_\Lambda^{QCD} = 10^{46}\, \rho_\Lambda^{obs}$$

"The biggest embarrassment in theoretical physics."

$$\langle 0 | \bar{q}q | 0 \rangle$$

QCD

1973-1974

Are the condensates real?

- Is there a physical meaning to the vacuum quark condensate (and others)?
- Or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?

S. Weinberg, Physica 96A (1979)
Elements of truth in this perspective

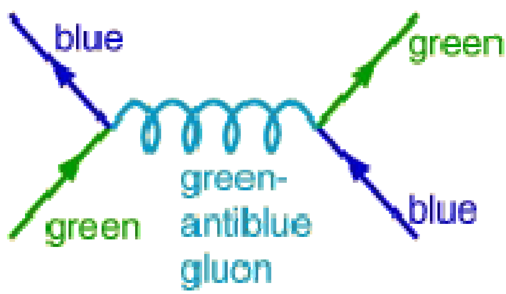
What is measurable?

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says [redacted] in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

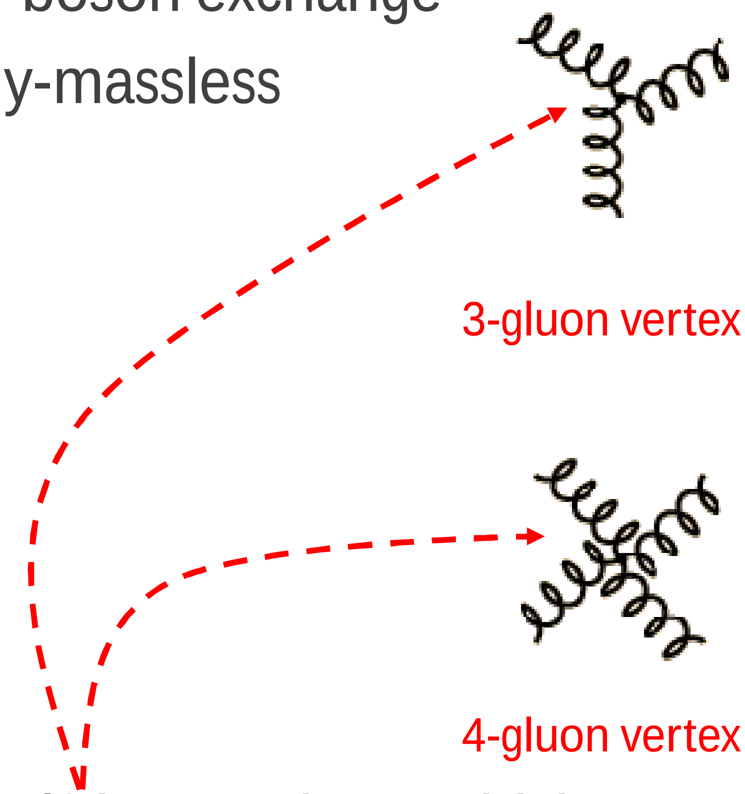
What is QCD?

Relativistic Quantum Gauge Theory:

- Interactions mediated by vector boson exchange
- Vector bosons are perturbatively-massless



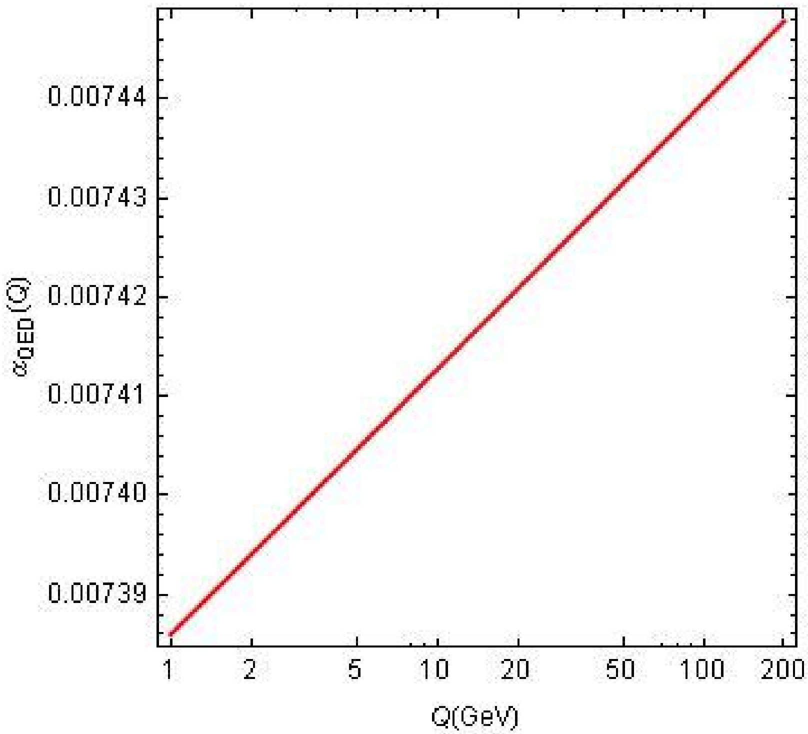
Feynman diagram for an interaction between quarks generated by a gluon.

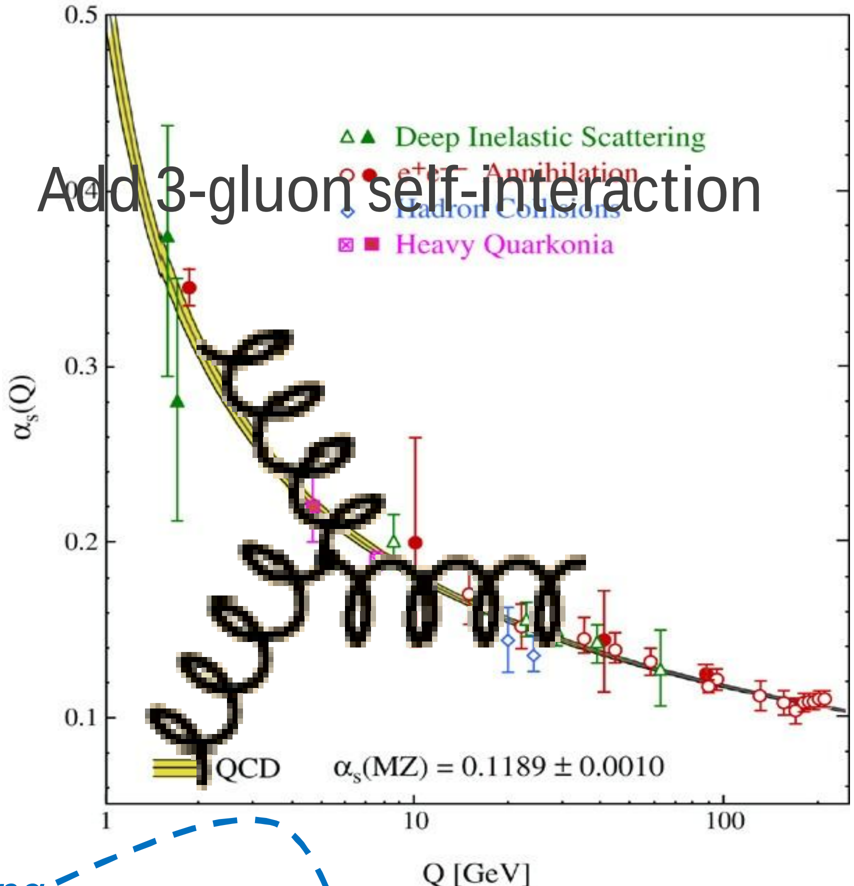


- Similar interaction in QED
- Special feature of QCD – gluon self-interactions, which completely change the character of the theory

QED cf. QCD?

✓ 2004 Nobel Prize in Physics : Gross, Politzer and Wilczek



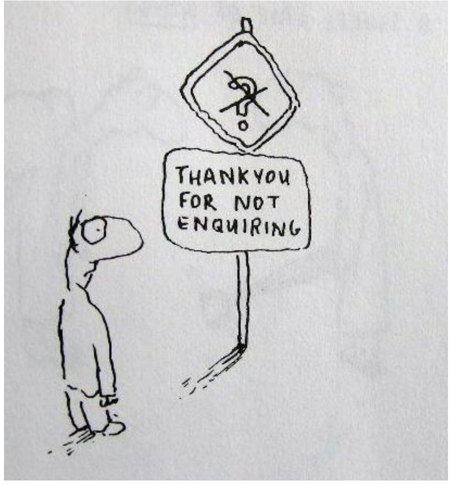


$$\alpha_{QED}(Q) = \frac{\alpha}{1 - \frac{2\alpha}{3\pi} \ln \frac{Q}{m_e}}$$

gluon antiscreening

fermion screening

$$\alpha_{QCD}(Q) = \frac{6\pi}{(33 - 2N_f) \ln \frac{Q}{\Lambda}}$$



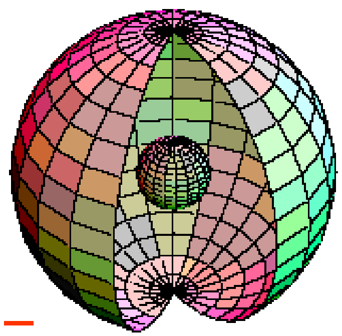
QCD's Challenges

Understand emergent phenomena

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses; e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^P=+$ and $J^P=-$ *(parity partners)*
- Neither of these phenomena is apparent in QCD's Lagrangian
  - ***Yet*** they are the dominant determining characteristics of real-world QCD.
- QCD
  - – Complex behaviour arises from apparently simple rules.

Why don't we just stop talking & solve the problem?

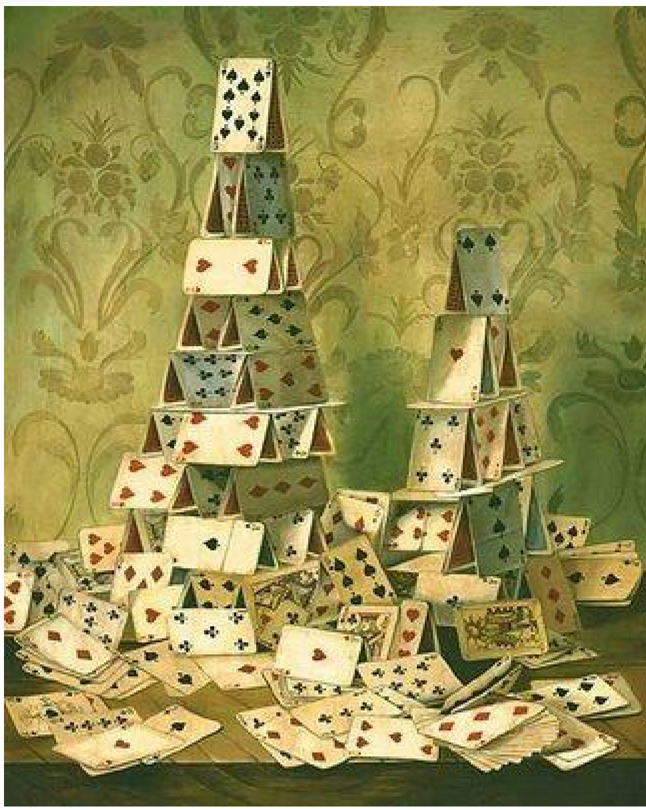
- Emergent phenomena can't be studied using perturbation theory
  - *So what? Same is true of bound-state problems in quantum mechanics!*
- Differences:
  - Here relativistic effects are crucial – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential –
    Unknown throughout > 98% of the pion's/proton's volume!



- Understanding requires *ab initio* nonperturbative solution of fully-fledged interacting relativistic quantum field theory

How can we tackle the SM's Strongly-interacting piece?

The Traditional Approach

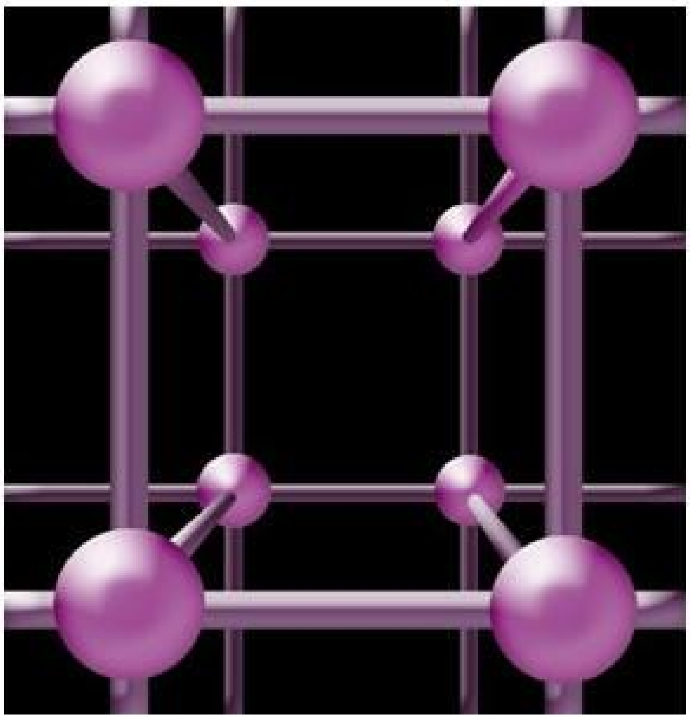


– Modelling

– has its problems.

How can we tackle the SM's Strongly-interacting piece?

Lattice-QCD

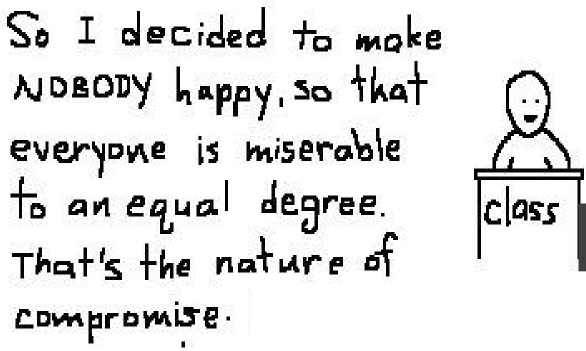


– Spacetime becomes an hypercubic lattice
– Computational challenge, many millions of degrees of freedom

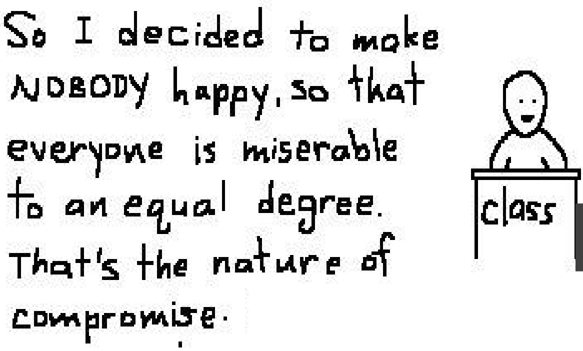
How can we tackle the SM's Strongly-interacting piece?

Lattice-QCD

– Spacetime becomes an hypercubic lattice
– Computational challenge, many millions of degrees of freedom
– *Approximately 500 people worldwide & 20-30 people per collaboration.*

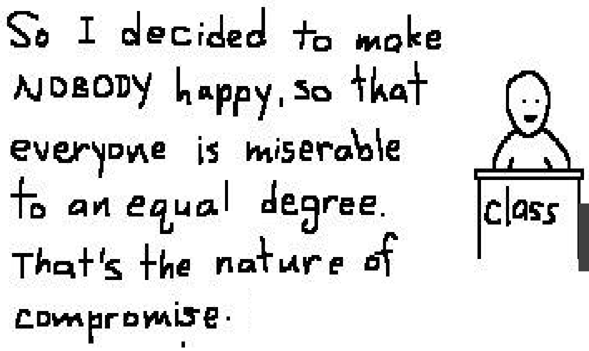


A Compromise? Dyson-Schwinger Equations



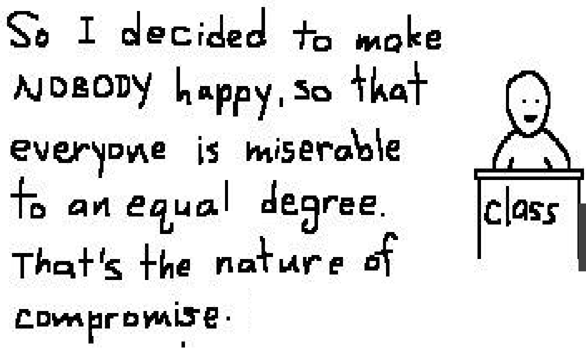
A Compromise? Dyson-Schwinger Equations

- 1994 . . . "*As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . . .*"



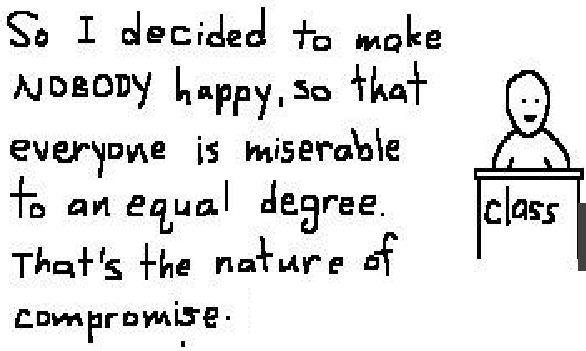
A Compromise? Dyson-Schwinger Equations

- 1994 . . . "*However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF (VA) and RHIC (NY), there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .* "



A Compromise? Dyson-Schwinger Equations

- 1994 . . . “*Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD.*”



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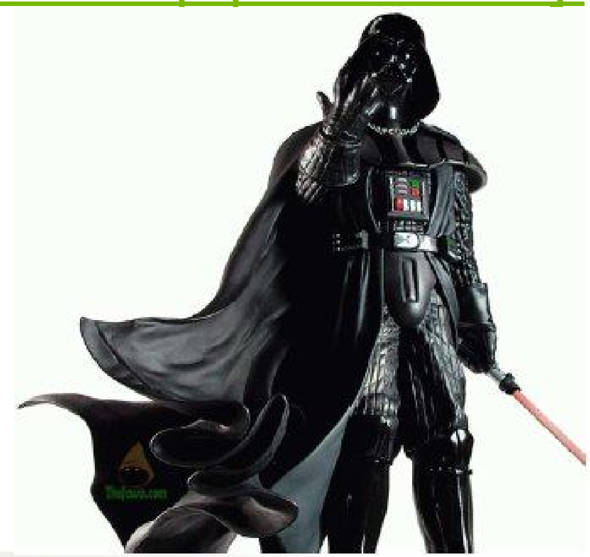
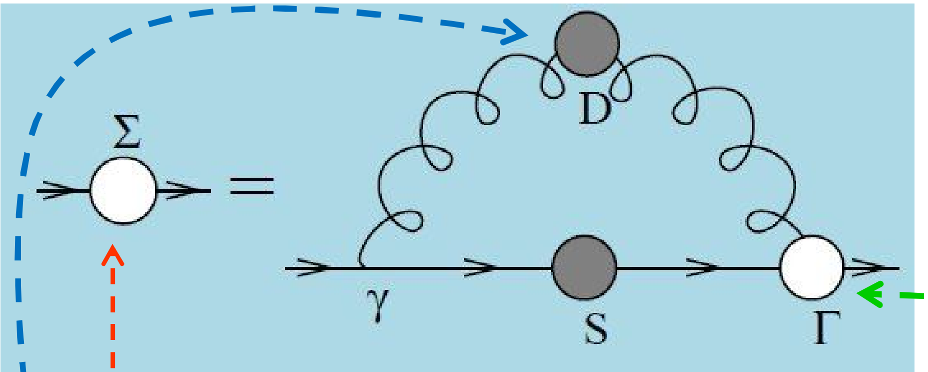
- C. D. Roberts and ~~A. G. Williams~~, "Dyson-Schwinger equations and their application to hadronic physics,"
  Prog. Part. Nucl. Phys. **33**, 477 (1994) [arXiv:hep-ph/9403224].
  (473 citations)





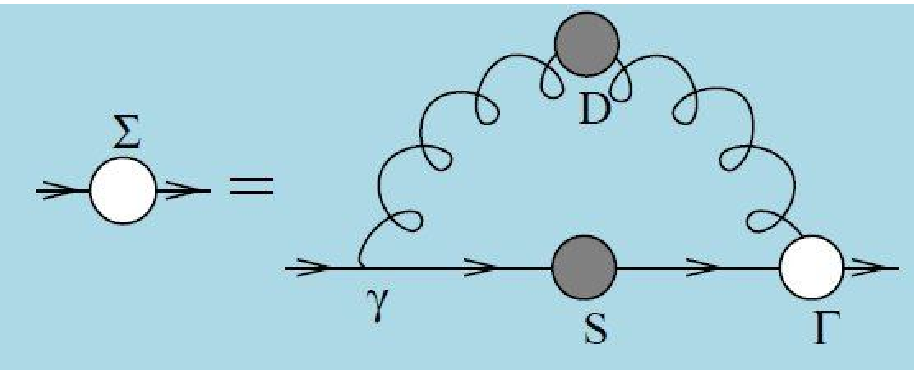
A Compromise? DSEs

- Dyson (1949) & Schwinger (1951) . . . One can derive a system of coupled integral equations relating all the Green functions for a theory, one to another.

  Gap equation:
  - fermion self energy
  - gauge-boson propagator
  - fermion-gauge-boson vertex

$$S(p) = \frac{1}{i\gamma \cdot p + \Sigma(p)}$$

- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.

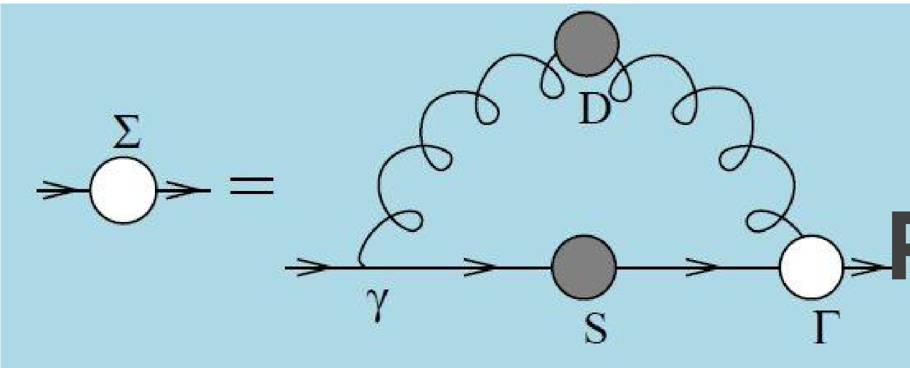


Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  . . . Materially Reduces Model-Dependence
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    – Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    – Coloured objects not detected, not detectable?

- In doing this, arrive at understanding of long-range behaviour of strong running-coupling
- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling



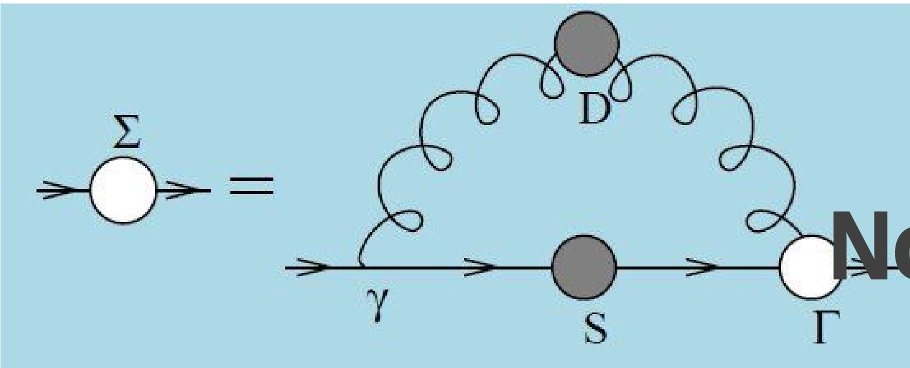
Mass from Nothing?! Perturbation Theory

- QCD is asymptotically-free (2004 Nobel Prize)
  - Chiral-limit is well-defined;
    i.e., one can truly speak of a massless quark.
  - NB. This is nonperturbatively *impossible* in QED.

- Dressed-quark propagator: $S(p) = \dfrac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$
- Weak coupling expansion of gap equation yields every diagram in perturbation theory
- In perturbation theory:
  If *m=0*, then *M(p²)=0*
  Start with no mass,
  Always have no mass.

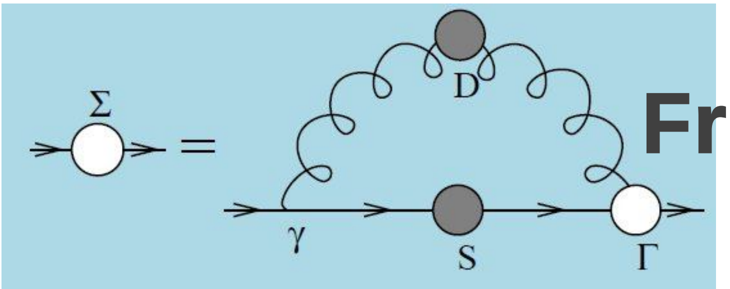
$$M(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{\Lambda^2}\right] + \ldots\right)$$

0



Mass from Nothing?! Nonperturbative DSEs

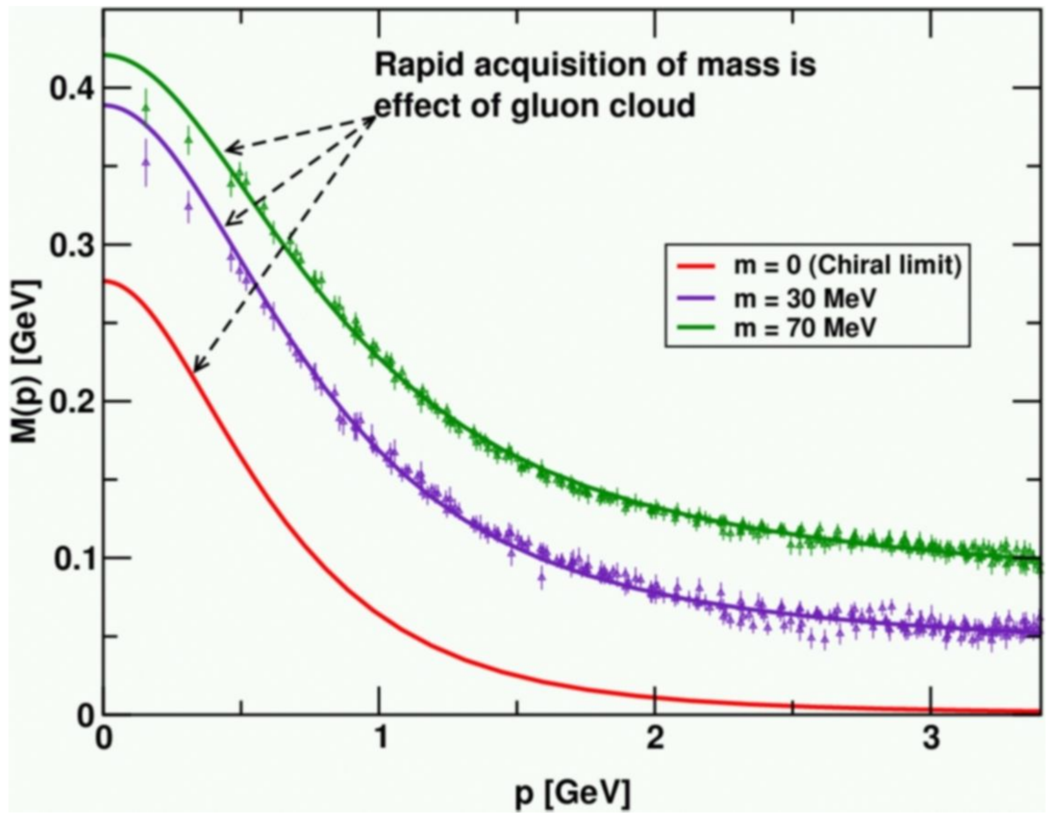
- Gap equation is a nonlinear integral equation
  - Modern computers enable it to be solved, self-consistently, with ease
- In the last ten years, we have learnt a great deal about the nature of its kernel
  - What do the self-consistent, nonperturbative solutions tell us?

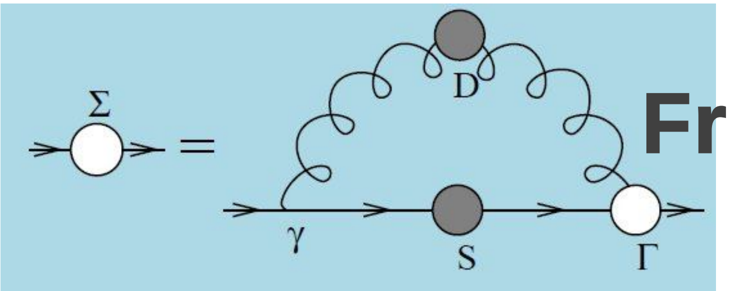


Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.





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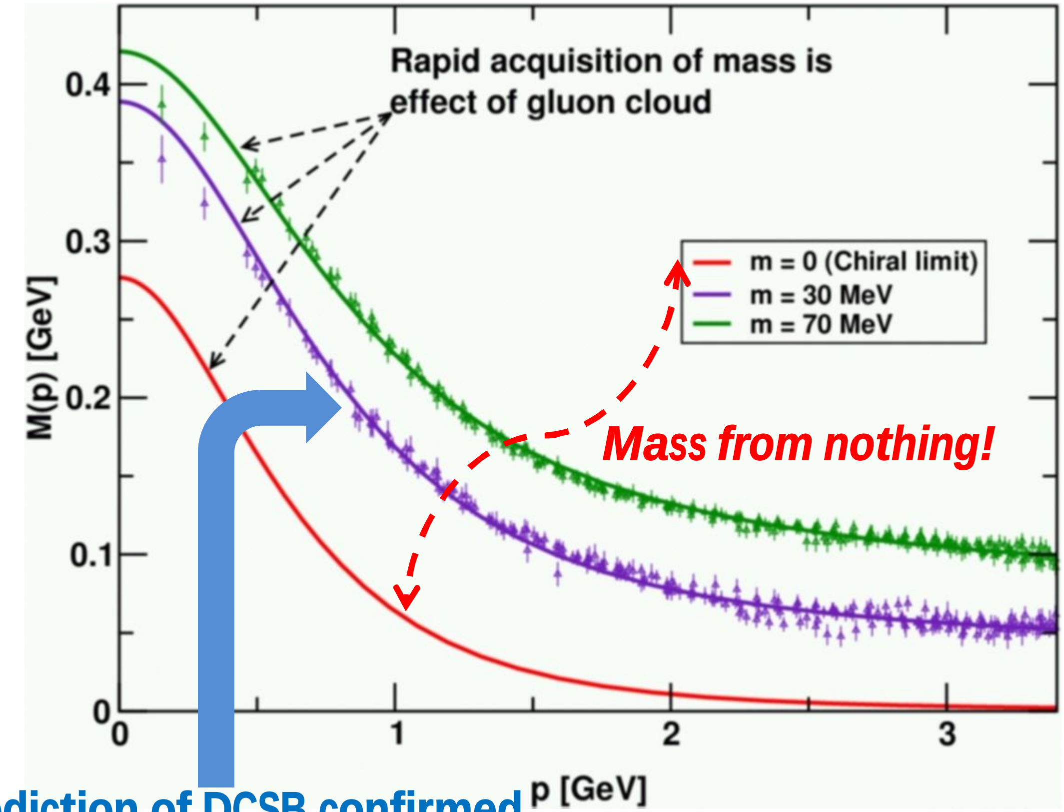
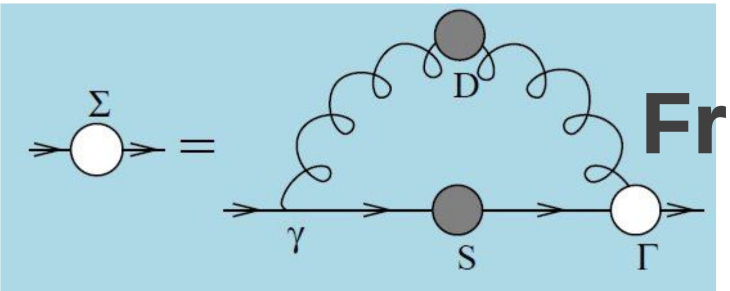


DSE prediction of DCSB confirmed



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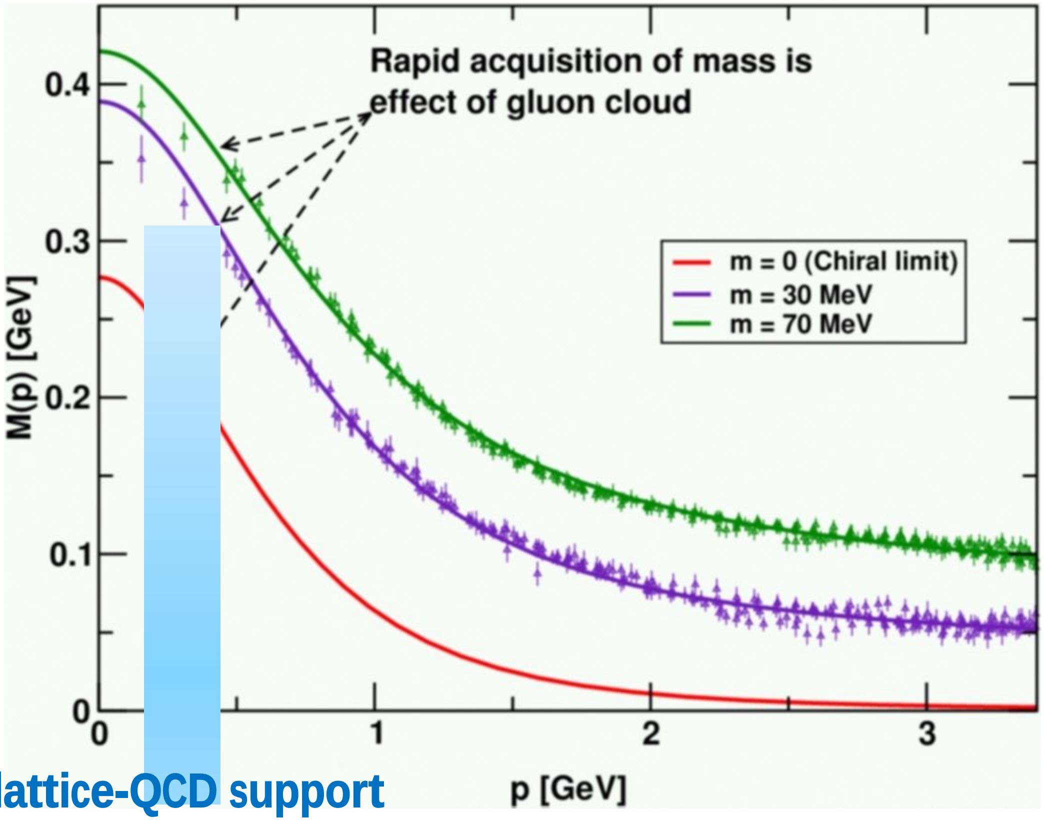
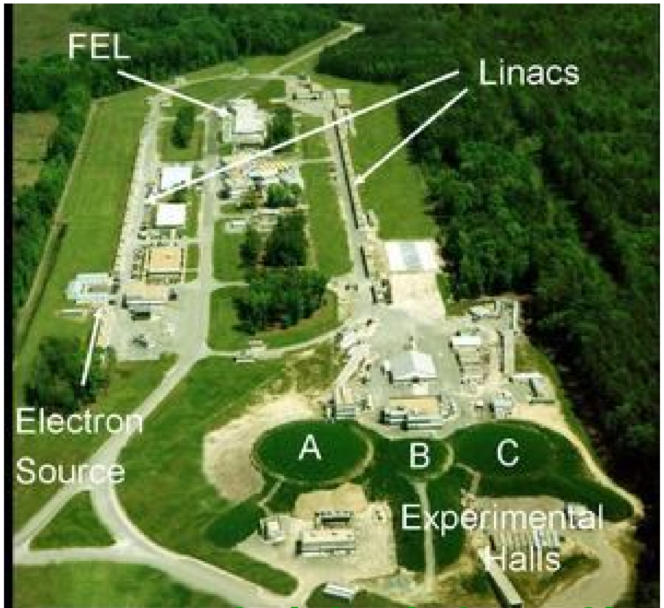
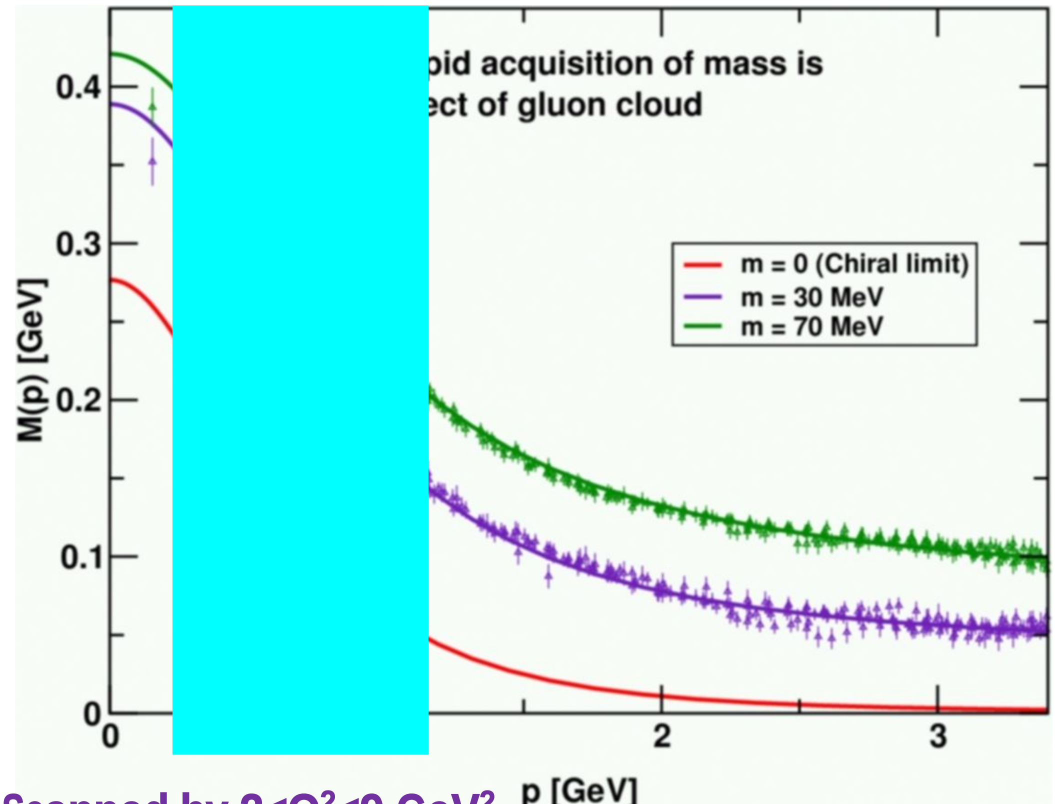


Hint of lattice-QCD support for DSE prediction of violation of reflection positivity



12GeV
The Future of JLab

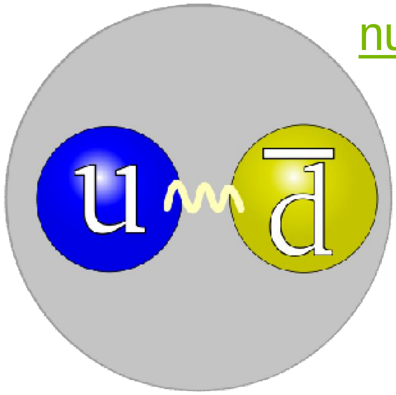
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Numerical simulations of lattice QCD (data, at two different bare masses) **have confirmed model predictions** (solid curves) **that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates**. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.

Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.

P. Maris, C.D. Roberts & P.C. Tandy
nucl-th/9707003



Dichotomy of the pion

- Building on the concepts and theory that produces the features that have been described, one can derive numerous exact results in QCD.
- One of them explains the peculiar nature of the pion's mass; i.e., it's relationship to the Lagrangian current-quark mass *m(ς)*:

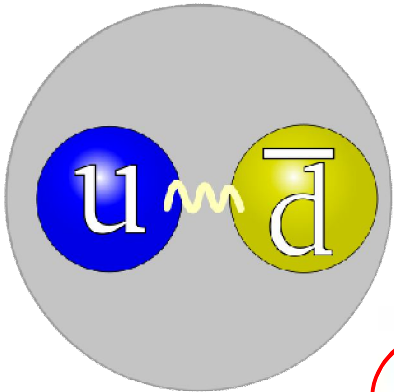
$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

$$i f_\pi P_\mu = \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle$$
$$= Z_2(\zeta, \Lambda)\, \mathrm{tr}_{\mathrm{CD}} \int^\Lambda \frac{d^4 q}{(2\pi)^4} i\gamma_5 \gamma_\mu S(q_+) \Gamma_\pi(q; P) S(q_-), \tag{5}$$

$$i \rho_\pi = -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$
$$= Z_4(\zeta, \Lambda)\, \mathrm{tr}_{\mathrm{CD}} \int^\Lambda \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+) \Gamma_\pi(q; P) S(q_-). \tag{6}$$

This is an almost-familiar relation , a peculiar, non-quantum-mechanical Identity – looks like the GMOR

What are the constants of proportionality, physically?



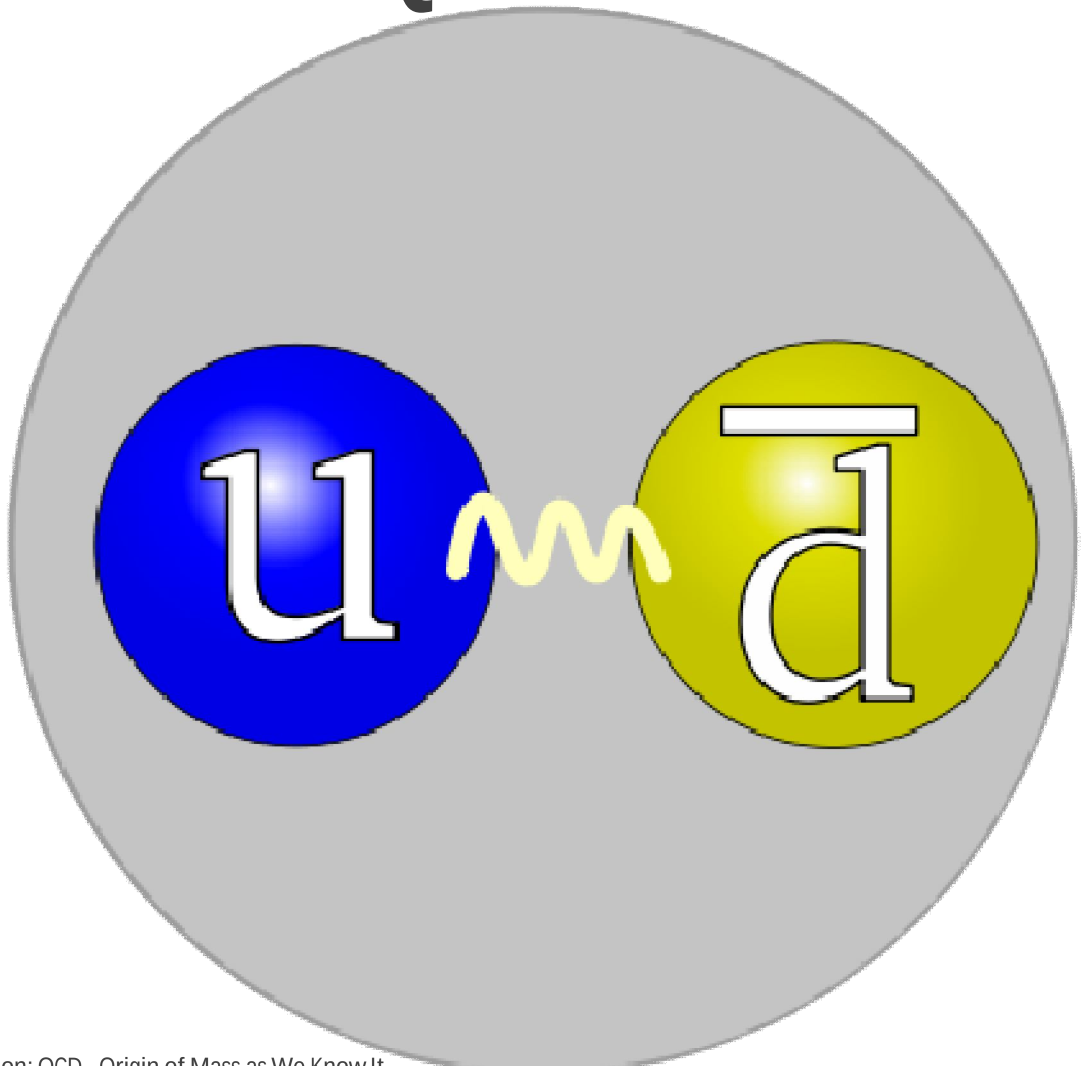
Gell-Mann – Oakes – Renner Relation (1968)

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

- Pion's leptonic decay constant, mass-dimensioned observable which describes rate of process $\pi^+ \to \mu^+ \nu$
- *Vacuum quark condensate*

How is this expression modified and interpreted in a theory with confinement?

Nature of the Pion: QCD's Goldstone Mode

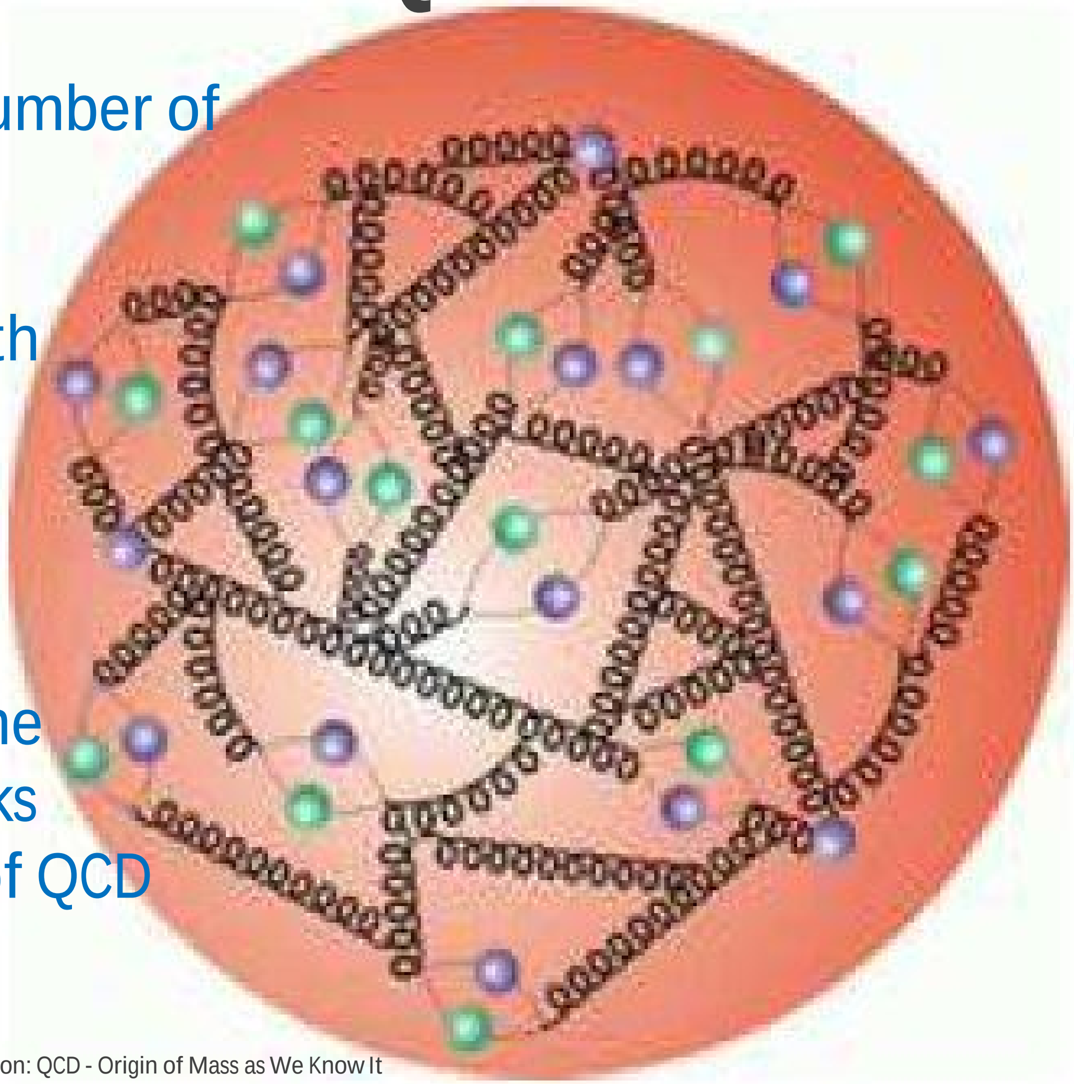


Nature of the Pion: QCD's Goldstone Mode

2 →
many or infinitely many

Nature and number of constituents depends on the wavelength of the probe

Constituent-quarks are replaced by the dressed-quarks and –gluons of QCD



Maris, Roberts and Tandy
nucl-th/9707003

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

Exact in Chiral QCD

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \end{aligned}$$

Miracle: two body problem solved, almost completely, once solution of one body problem is known

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts
nucl-th/9708029

- Pseudoscalar projection of pion's Bethe-Salpeter wave-function onto the origin in configuration space

  – or the pseudoscalar pion-to-vacuum matrix element

$$i\rho_\pi = -\langle 0|\bar{q} i\gamma_5 q|\pi\rangle$$
$$= Z_4(\zeta,\Lambda)\, \mathrm{tr}_{\rm CD} \int^\Lambda \frac{d^4q}{(2\pi)^4} \gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-)$$

- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
  - For arbitrary current-quark masses
  - For any pseudoscalar meson

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

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  - – or the pseudoscalar pion-to-vacuum matrix element
  - – or the pion's leptonic decay constant

$$\begin{aligned} i f_\pi P_\mu &= \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle \\ &= Z_2(\zeta, \Lambda)\, \mathrm{tr}_{\mathrm{CD}} \int^\Lambda \frac{d^4 q}{(2\pi)^4} i \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi(q;P) S(q_-) \end{aligned}$$

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Maris & Roberts
nucl-th/9708029

- Define $$-\langle \bar{q} q \rangle_\zeta^\pi \equiv - f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$$
- Then, owing to the pion Goldberger-Treiman relations one derives, in the chiral limit

Chiral limit

$$\kappa_\pi(0;\zeta) = - < \bar{q} q >_\zeta^0$$

- Namely, the so-called *vacuum quark condensate* is the *chiral-limit value* of the *in-pion condensate*
- The *in-pion condensate* is the *only well-defined function of current-quark mass* in QCD that is *smoothly connected to the vacuum quark condensate*.

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$

There is *only one* condensate

Langeld, Roberts et al.
nucl-th/0301024

I. Casher Banks formula:

Density of eigenvalues of Dirac operator

$$-\langle 0|\bar{q}q|0\rangle = 2m \int_0^\infty d\lambda \, \frac{\rho(\lambda)}{\lambda^2 + m^2}$$

II. Constant in the Operator Product Expansion:

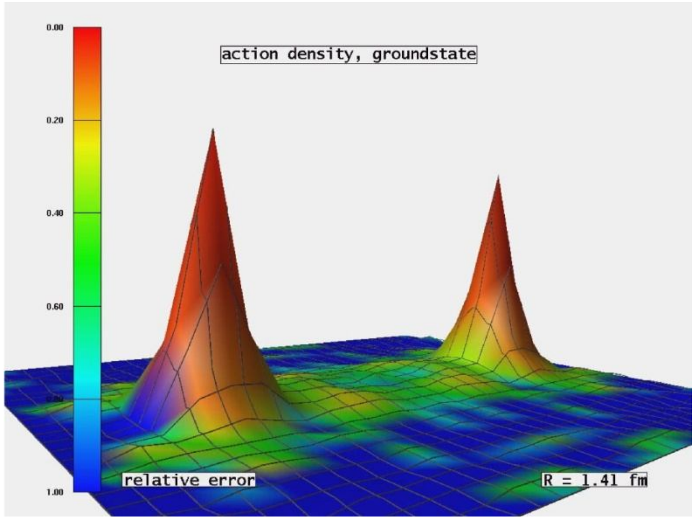
$$M(p^2) \overset{\text{large}-p^2}{=} \frac{2\pi^2\gamma_m}{3} \frac{\left(-\langle\bar{q}q\rangle^0\right)}{p^2\left(\frac{1}{2}\ln\left[\frac{p^2}{\Lambda_{\rm QCD}^2}\right]\right)^{1-\gamma_m}}$$

III. Trace of the dressed-quark propagator:

$$\tilde{\sigma}(m) := N_c \, \mathrm{tr}_D \int_p^\Lambda \tilde{S}_m(p)$$

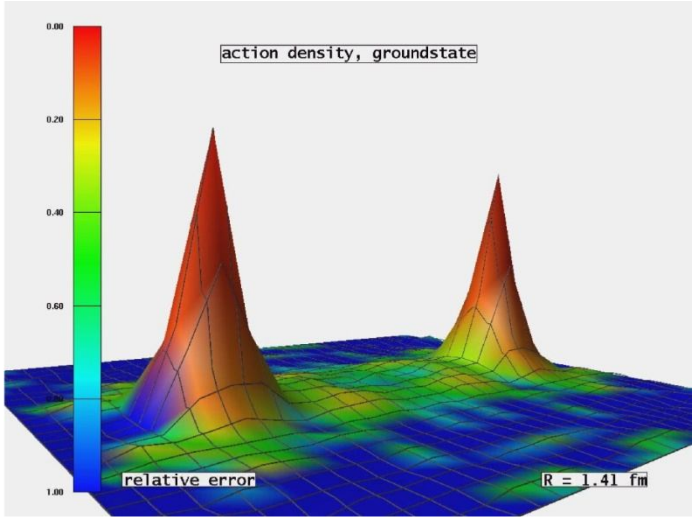
$m \to 0$

No matter how one chooses to calculate it, one is always calculating the same thing.



Confinement

- Notion of a "vacuum" state is a quantum mechanics idea, grounded in second-quantisation of quantum field theory
  - There is no nonperturbative definition of a quark or gluon creation operator , hence one cannot define a second-quantised vacuum within which they can condense.
- Quark-hadron duality
  - All observable consequences of QCD can, in principle, be computed using a hadronic basis. ("hadron" means any one of the states or resonances in the complete spectrum of color-singlet bound-states generated by the theory)

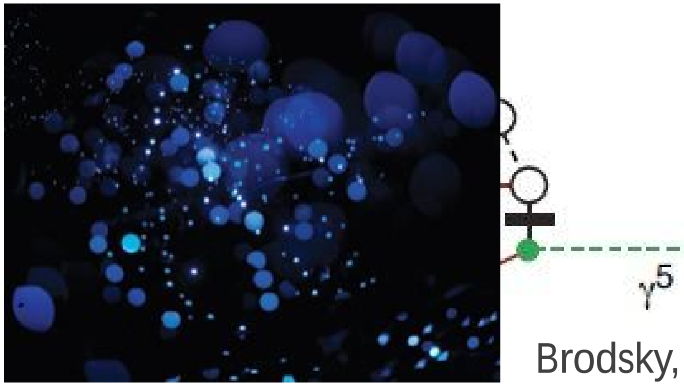


Confinement

- Generates nonperturbatively an infrared mass-scale

  $M_q(0) \approx 0.4$ GeV $\approx M_g(0)$

  - Mass-scale provides an infrared cutoff within an hadron, such that the role played by constituent field modes with $p^2 < m_{ir}^2$ is exponentially suppressed.
  - Define $\lambda := 1/\sqrt{p^2}$, then an equivalent statement is that modes with $\lambda > m_{ir}$ play no part in defining the bound-state's properties.
  - There are no long wavelength modes within an hadron, hence, the quarks and gluon can't "leak out" and condense elsewhere.



Paradigm shift: In-Hadron Condensates

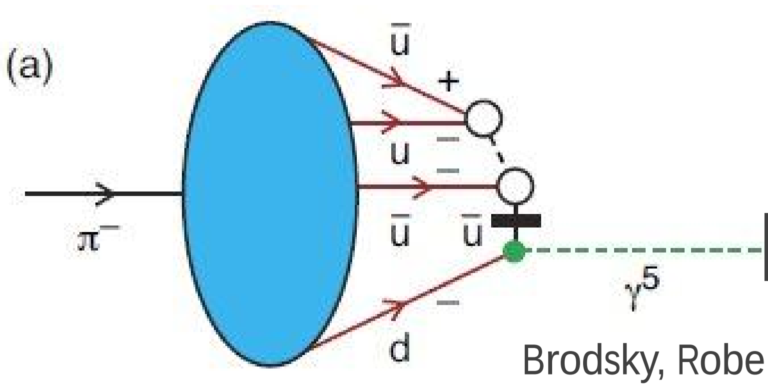
Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, arXiv:0905.1151 [hep-th], to appear in PNAS

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **GMOR cf.**

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

QCD

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$



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  - **No qualitative difference between f_π and ρ_π**

$$i f_\pi P_\mu = \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle = Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \quad (5)$$

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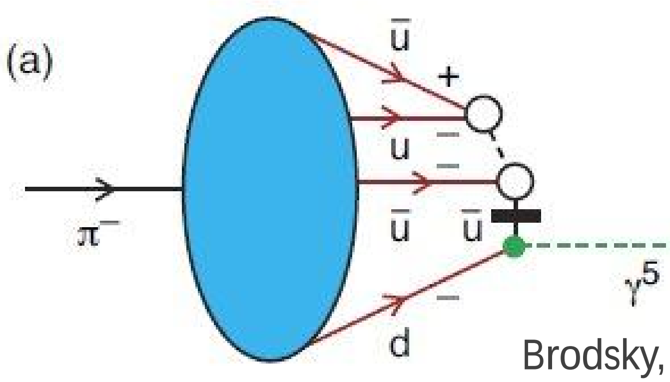

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  - **No qualitative difference between f_π and ρ_π**
  - **And**

Chiral limit

$$\kappa_\pi(0;\zeta) = -<\bar{q}q>_\zeta^0$$

$$-\langle \bar{q}q\rangle_\zeta^\pi \equiv -f_\pi \langle 0|\bar{q}\gamma_5 q|\pi\rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$$

Paradigm shift: In-Hadron Condensates

"Void that is truly empty
solves dark energy puzzle"
Rachel Courtland, New Scientist 4th Sept. 2010

"EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe."

$$\Omega_{QCD-condensates} = 8\pi G_N \frac{\Lambda_{QCD}^4}{3H_0^2} \simeq 10^{45}$$

Cosmological Constant:

- ✓ **Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}**
- ✓ **Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model**