Example: Two-parton Pion LFWF

- Hard-Wall Model (P-S)

$$\tilde{\psi}^{HW}_{\bar{q}q/\pi}(x, \mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

- Soft-Wall Model (K-K-S-S)

$$\tilde{\psi}^{SW}_{\bar{q}q/\pi}(x, \mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right)$$

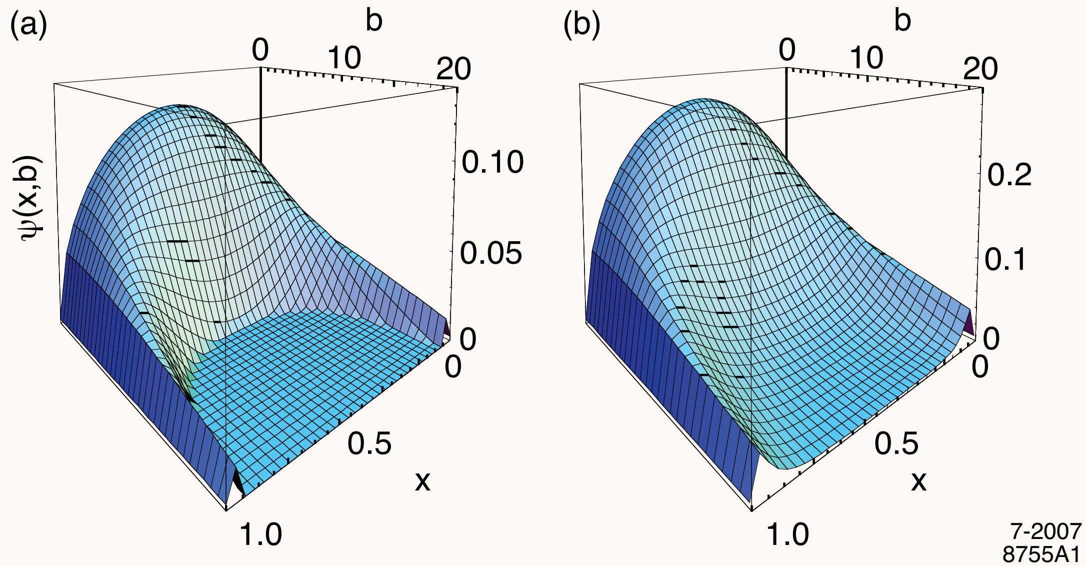


Fig: Ground state pion LFWF in impact space: (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle.$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}} R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

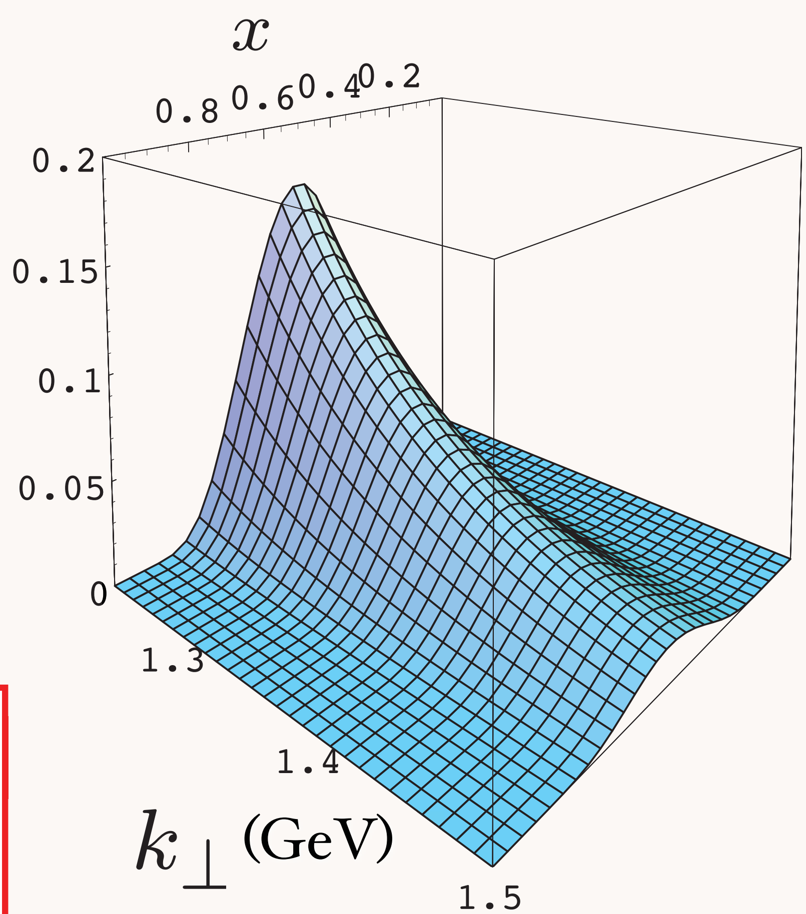
- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{\rm QCD} = 91.7 \,\text{MeV}, \; f_\pi^{SW} = \frac{\sqrt{3}}{8} \kappa = 81.2 \,\text{MeV},$$

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \qquad \phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\left\langle P' \left| \Theta^{++}(0) \right| P \right\rangle = 2 \left(P^+\right)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $S_{\rm QCD} = \int d^4x \sqrt{g}\, \mathcal{L}_{\rm QCD}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta S_{\rm QCD}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \tfrac{1}{2}\overline{\psi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - g^{\mu\nu}\overline{\psi}\left(i\not{D} - m\right)\psi - G^{a\,\mu\lambda} G^{a\,\nu}{}_{\lambda} + \tfrac{1}{4} g^{\mu\nu} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0,\ g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2} \sum_f \overline{\psi}^f(x) \gamma^+ \overleftrightarrow{\partial}^+ \psi^f(x)$$

- Particle number representation

$$\Theta^{++} = \frac{1}{2}\sum_f \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \int \frac{dq'^+ d^2\mathbf{q}'_\perp}{(2\pi)^3} \left(q^+ + q'^+\right) \left\{b^{f\dagger}(q) b^f(q') + d^{f\dagger}(q) d^f(q')\right\}$$

- Gravitational form-factor in momentum space

$$A(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \sum_f x_f \, \psi^*_{n/P'}(x_i, \mathbf{k}'_{\perp i}) \psi_{n/P}(x_i, \mathbf{k}_{\perp i}),$$

where $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + (1 - x_i)\,\mathbf{q}_\perp$ for a struck quark and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\,\mathbf{q}_\perp$ for each spectator

- Gravitational form-factor in impact space

$$A(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_f x_f \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) \left|\tilde\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

Gravitational Form Factor on the LF

$$A_f(q^2) = \int_0^1 x\, dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}_f(x, \vec{\eta}_\perp),$$

where

$$\tilde{\rho}_f(x, \vec{\eta}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{\eta}_\perp \cdot \vec{q}_\perp} \rho_f(x, \vec{q}_\perp)$$

$$= \sum_n \prod_{j=1}^{n-1} \int dx_j\, d^2\vec{b}_{\perp j}\, \delta\Big(1 - x - \sum_{j=1}^{n-1} x_j\Big)$$

$$\times \delta^{(2)}\Big(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp\Big) \left|\tilde{\psi}_n(x_j, \vec{b}_{\perp j})\right|^2.$$

Extra factor of x relati
to charge
form factor

For each quark and
gluon field $x=x_f$

Integrate over angle

$$A_f(q^2) = 2\pi \int_0^1 dx\, (1-x) \int \zeta d\zeta\, J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}_f(x, \zeta)$$

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

The AdS Gravitational Form Factor

Abidin & Carlson

$$ds^2 = \frac{R^2}{z^2}\left((\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - dz^2\right).$$ **linearized metric**

$$h_{zz} = h_{z\mu} = 0$$ **gauge choice**

$$\int d^4x\, dz\, \sqrt{g}\, h^{\ell m}(x,z)\partial_\ell \Phi^*_{P'}(x,z)\partial_m \Phi_P(x,z)$$ **gravitational coupling**

$$z^3\partial_z\left(\frac{1}{z^3}\partial_z h^\nu_\mu\right) - \partial_\rho\partial^\rho h^\nu_\mu = 0.$$ **eqn. of motion from action**

propagation of graviton into AdS from external source

$$h^\nu_\mu(x,z) = \eta^\nu_\mu\, e^{-iq\cdot x} H(q^2,z) \qquad H(q^2=0,z) = H(q^2,z=0) = 1.$$

$$\boxed{H(Q^2,z) = \frac{1}{2}Q^2 z^2 K_2(zQ).}$$ **solution!**

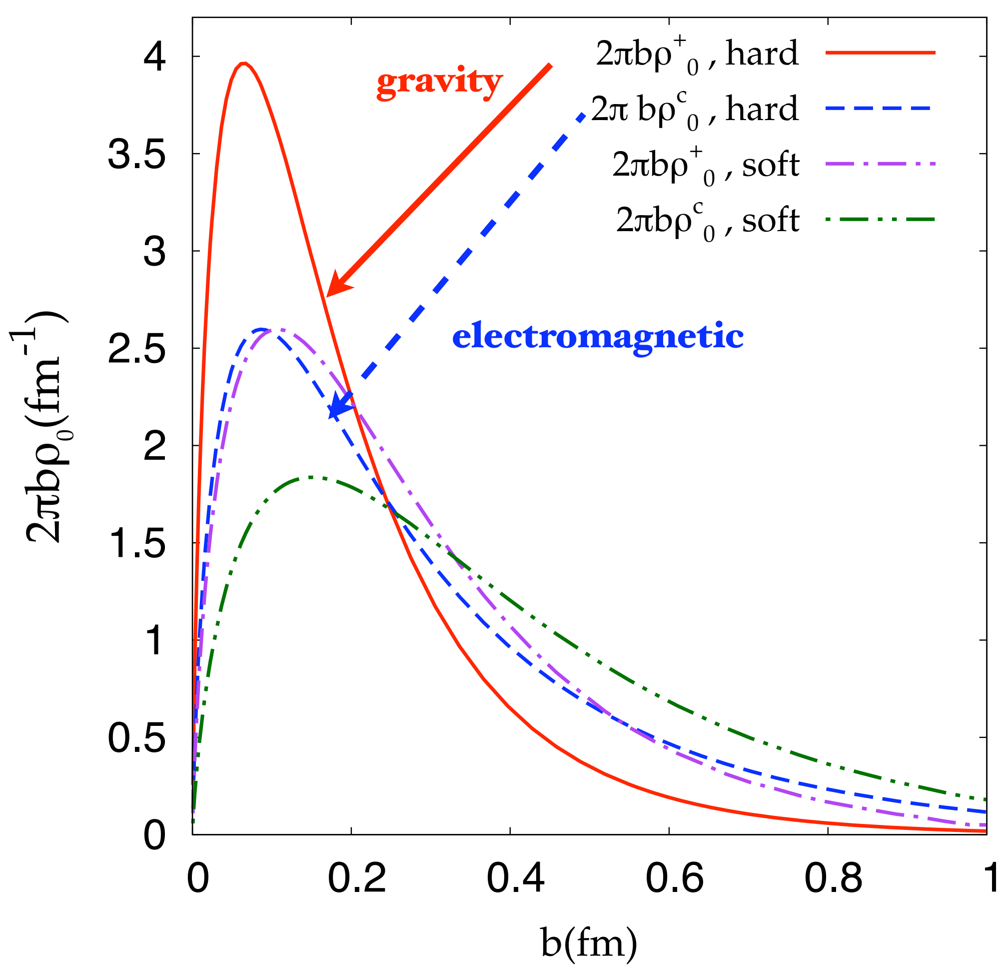
$$\boxed{A(Q^2) = R^3 \int \frac{dz}{z^3}\, \Phi(z) H(Q^2,z)\Phi(z).}$$ **Gravitational Form Factor**

Holographic result for LFWF identical for electroweak and gravity couplings! Highly nontrivial consistency test

AdS/QCD can predict

- Momentum fractions for each quark flavor and the gluons $$A_f(0) = <x_f>, \sum_f A_f(0) = A(0) = 1$$
- Orbital Angular Momentum for each quark flavor and the gluons $$B_f(0) = <L^3_f>, \sum_f B_f(0) = B(0) = 0$$
- Vanishing Anomalous Gravitomagnetic Moment
- Shape and Asymptotic Behavior of $A_f(Q^2), B_f(Q^2)$

Momentum Density more Compact than Charge Density



Z. Abidin and C. E. Carlson,
"Hadronic Momentum Densities in the Transverse
arXiv:0808.3097 [hep-ph].

Immediate property of LF Holography

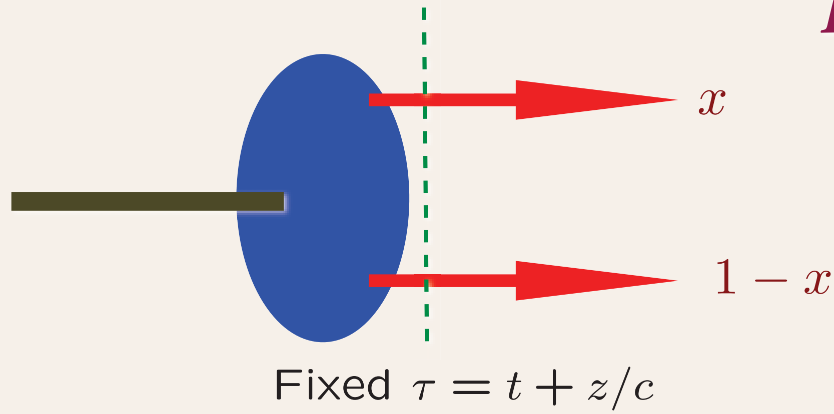
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} F\left(\frac{k_\perp^2}{2\kappa^2 x(1-x)}\right)$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$$k_\perp^2 < Q^2$$

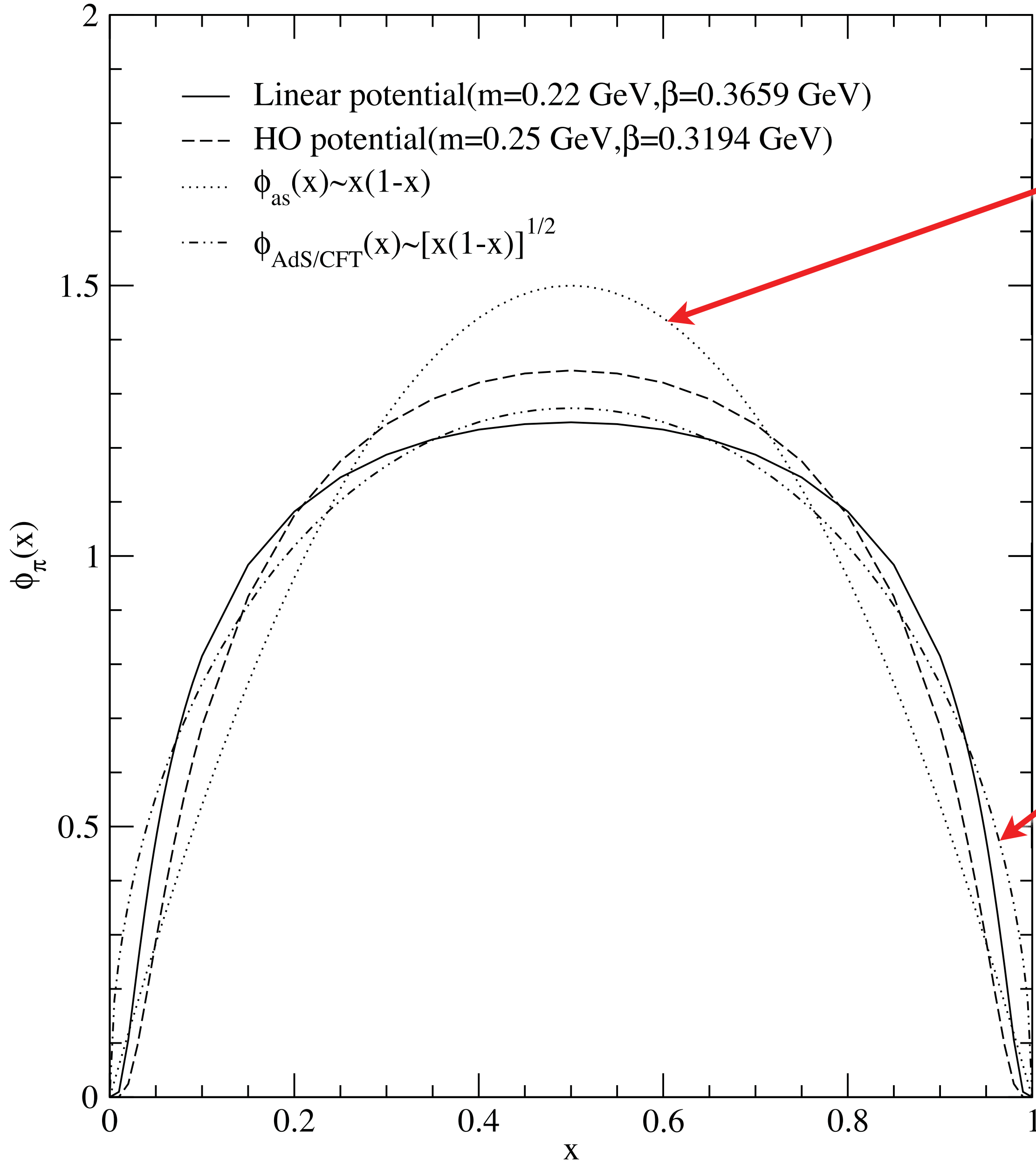
- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$



$$\phi_{asympt} \sim x(1-x)$$

AdS/CFT:

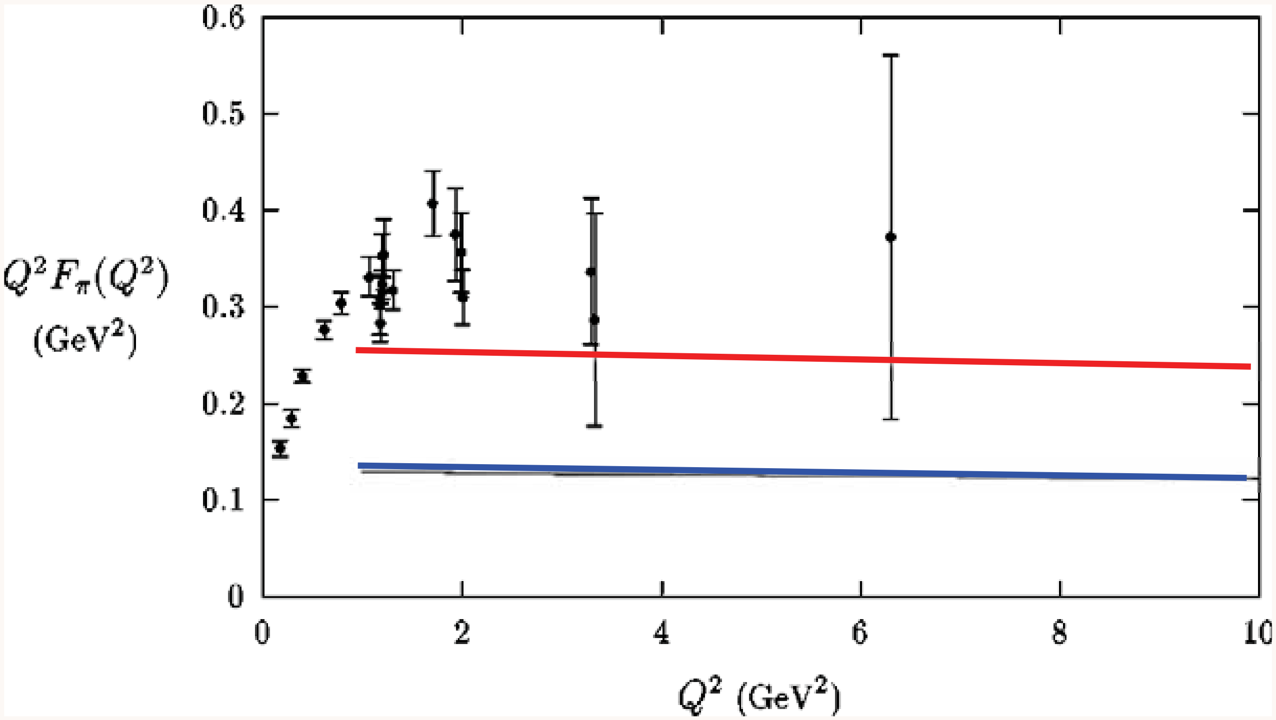
$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\, \phi_\pi(x) \int_0^1 dy\, \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

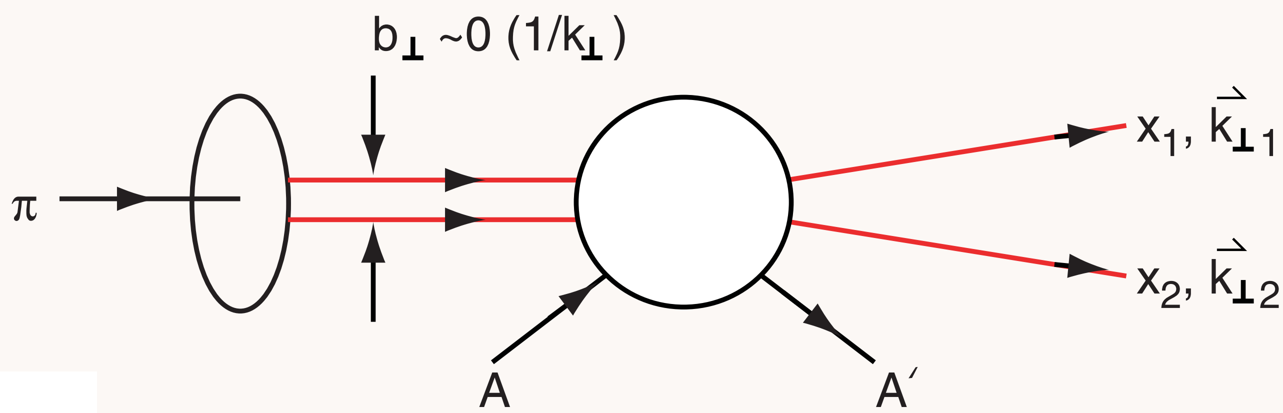
$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

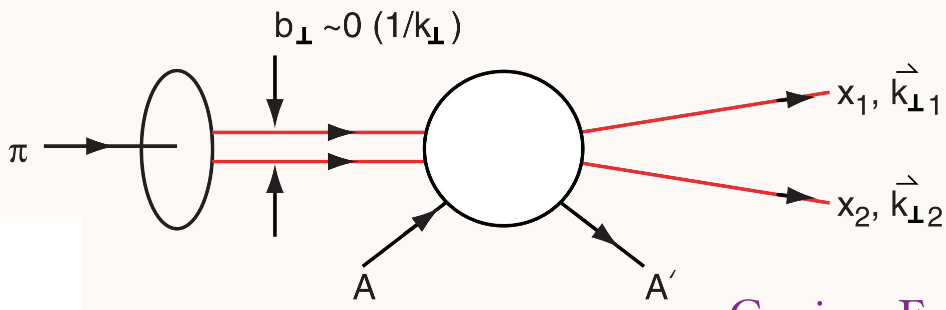


$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

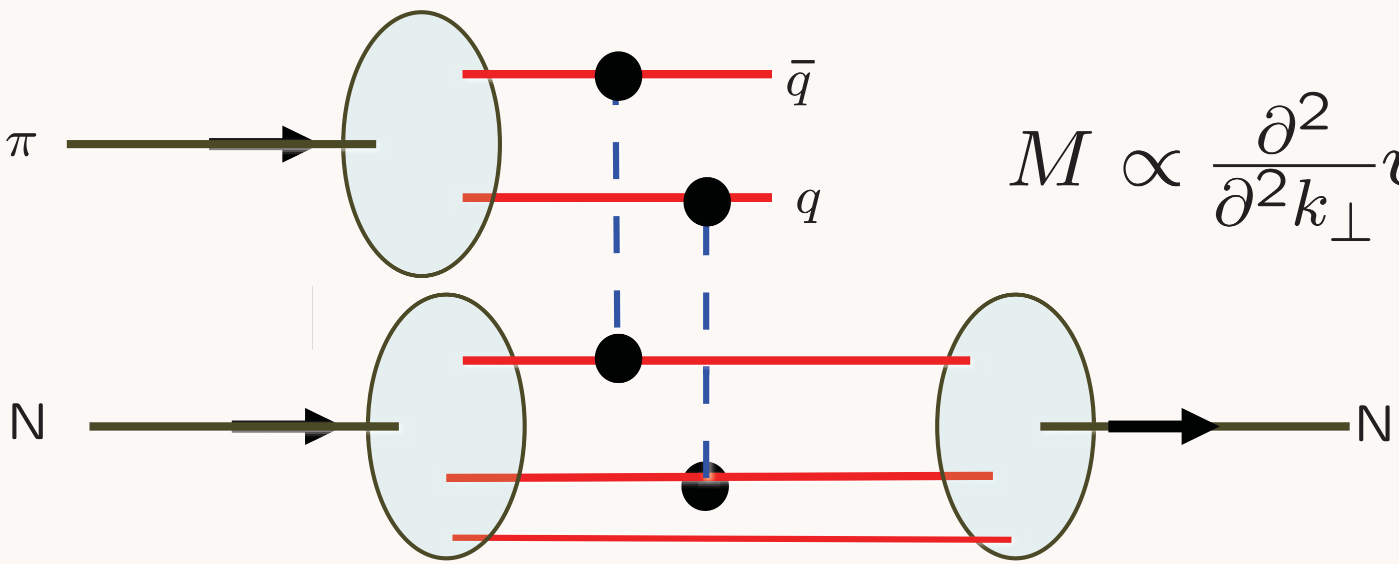
Minimal momentum transfer to nucleus
Nucleus left Intact!

E791 FNAL Diffractive DiJet



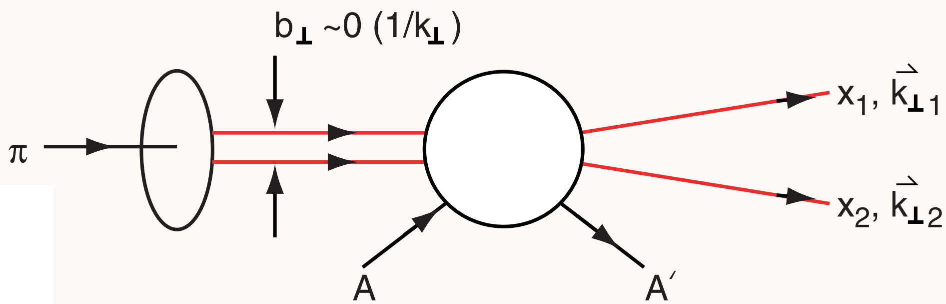
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

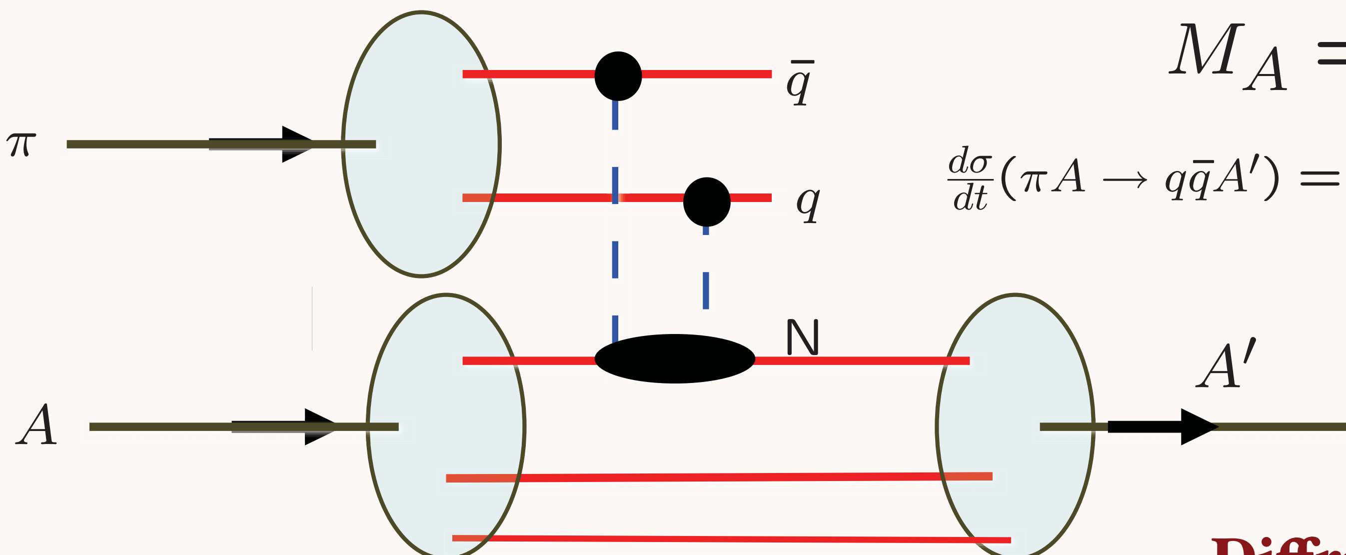
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb

A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

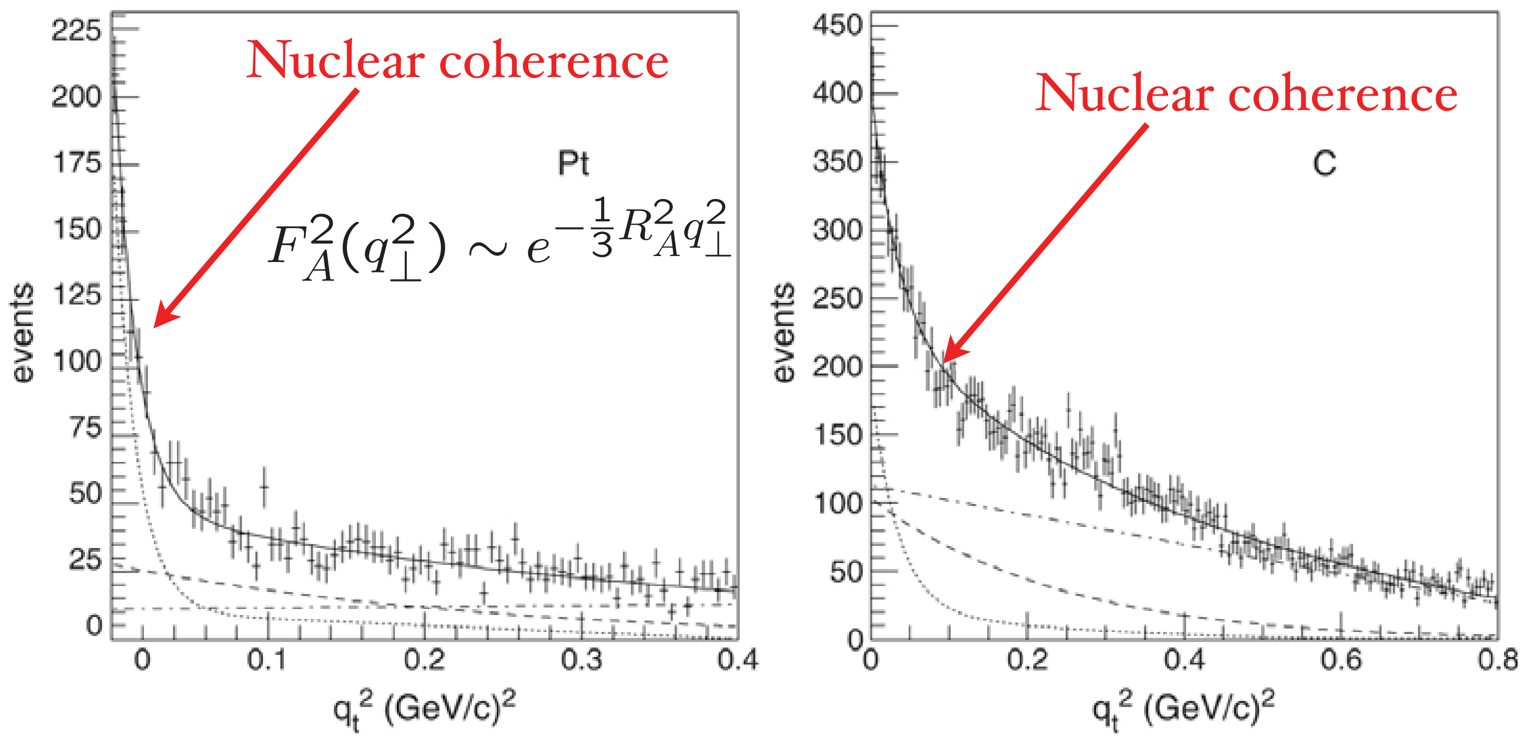
- Fully coherent interactions between pion and nucleons.

- Emerging Di-Jets do not interact with nucleus.

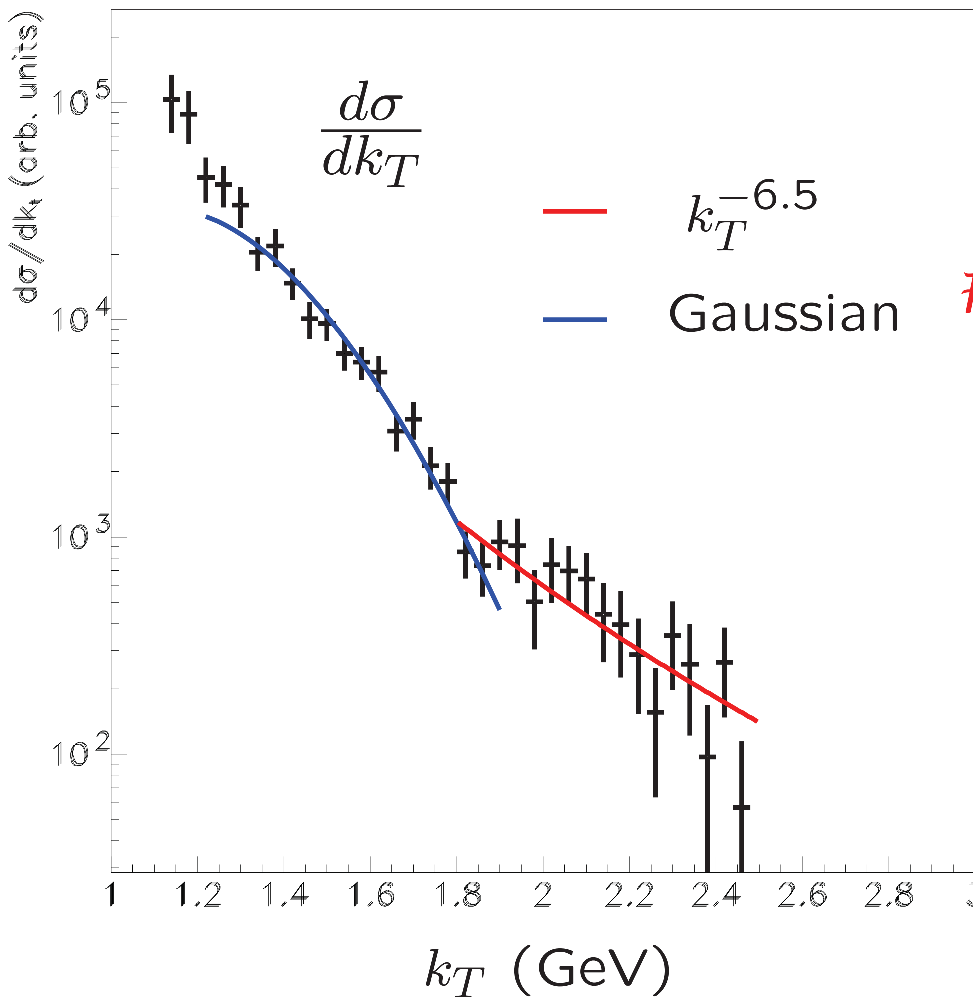
$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\underline{\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0}$$

$$\underline{\sigma \propto A^{4/3}}$$



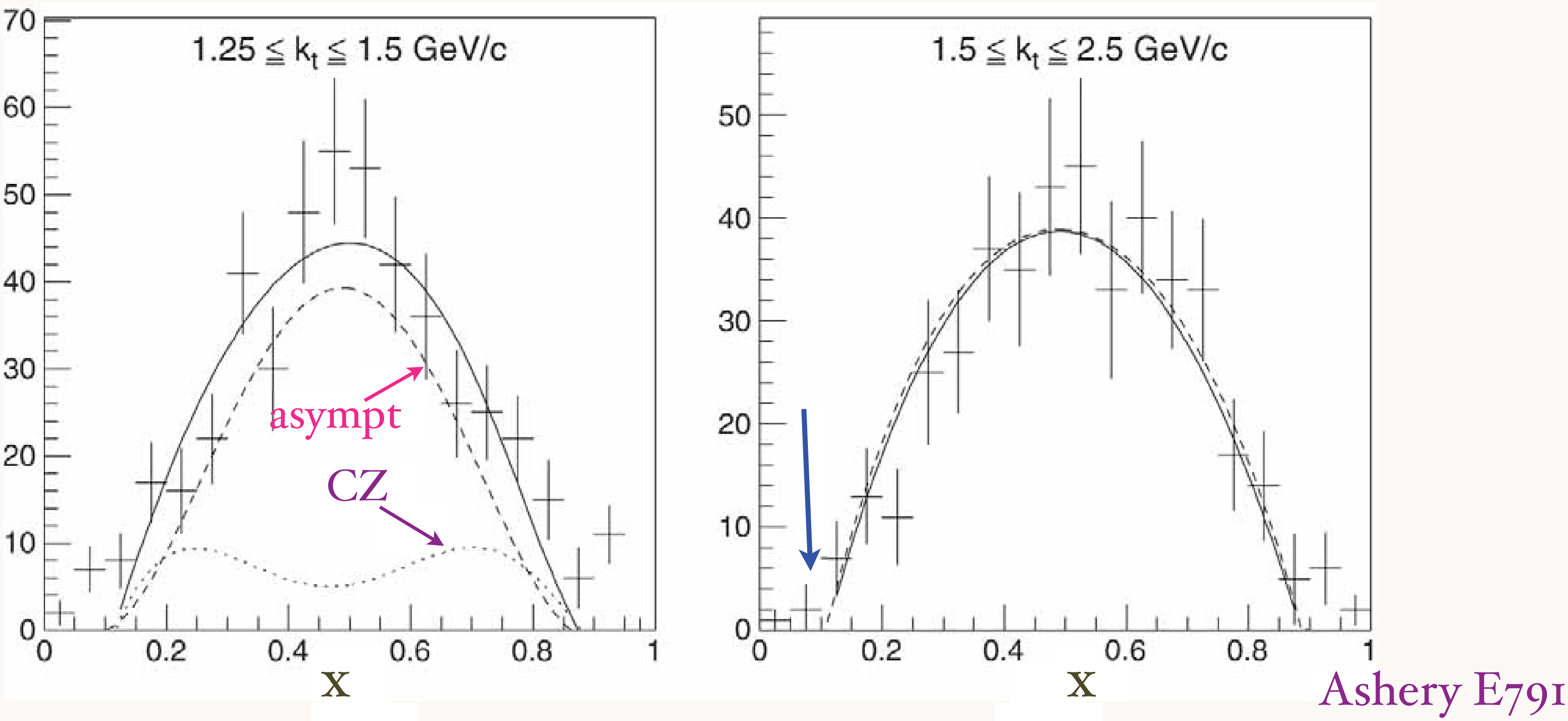
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with $k_T^{-6.5}$ *PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF

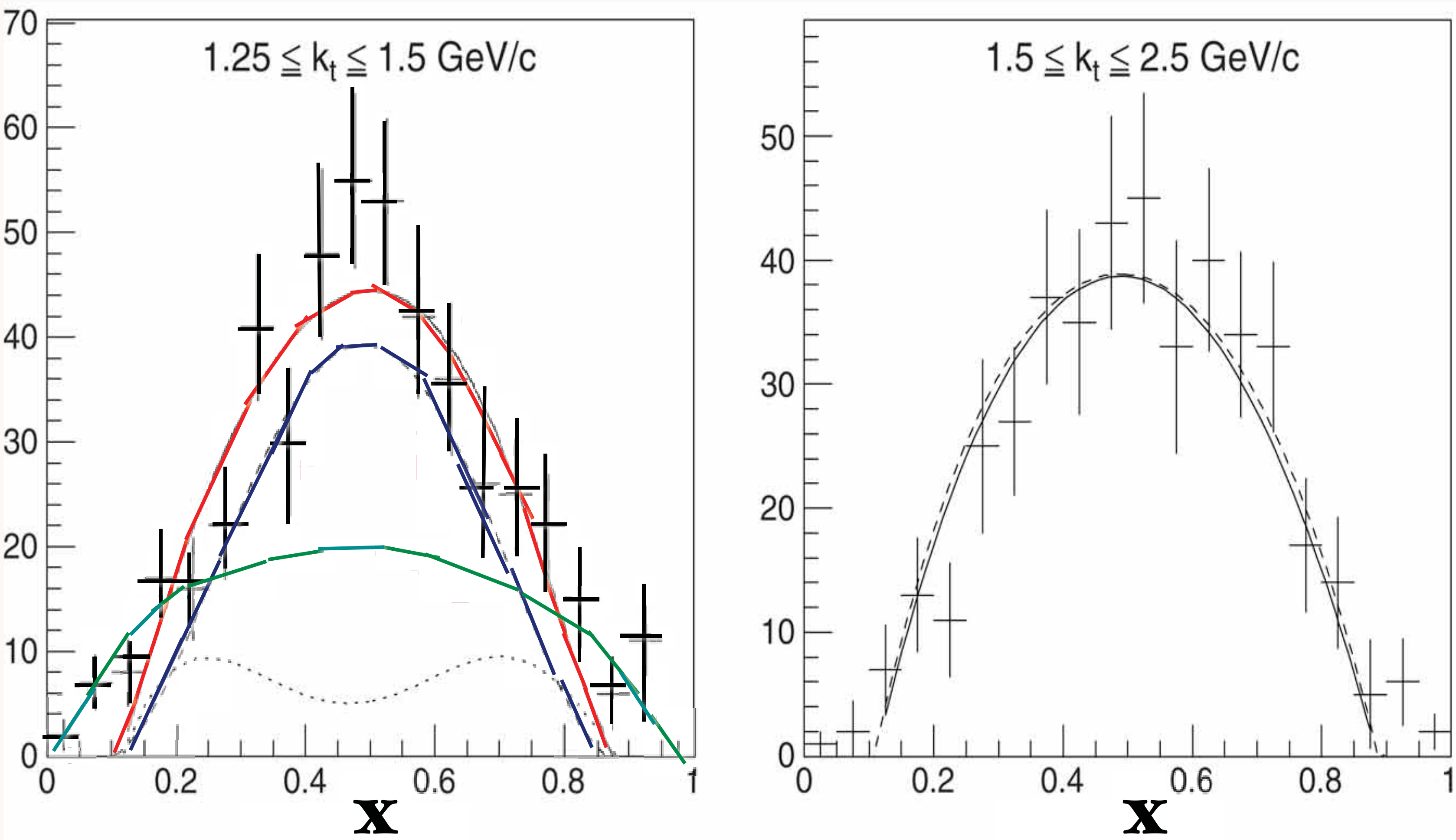


Narrowing of x distribution at higher jet transverse momentum

x: distribution of diffractive dijets from the platinum target for $1.25 \le k_t \le 1.5$ GeV/c (left) and for $1.5 \le k_t \le 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components: Nonperturbative (AdS/CFT) and Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$



Ashery
E791

Possibly two components:

Perturbative (ERBL) + Nonperturbative (AdS/CFT)

$$\phi(x) = A_{\rm pert}(k_\perp^2)x(1-x) + B_{\rm nonpert}(k_\perp^2)\sqrt{x(1-x)}$$

Narrowing of x distribution at high jet transverse momentum

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2 \, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0 \left[b \, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

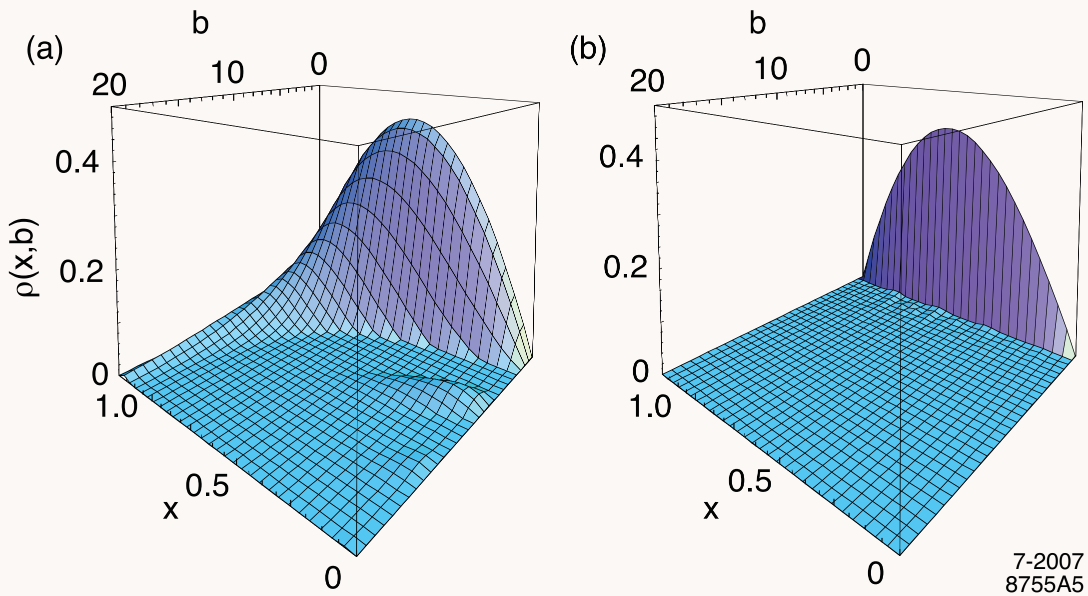


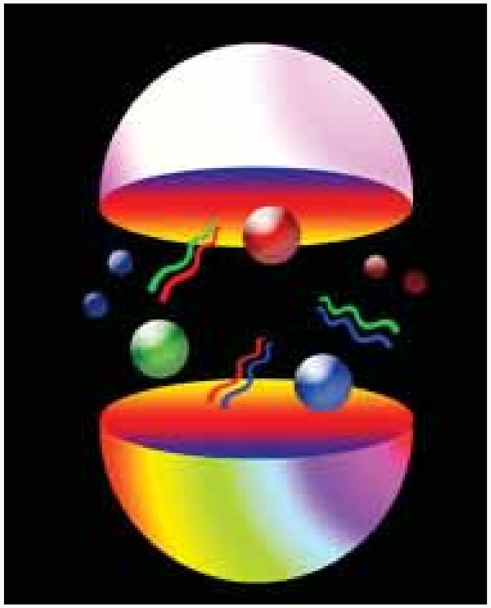
Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

See also T. Sakai and S. Sugimoto

Baryons in AdS/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g}\, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right] \Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

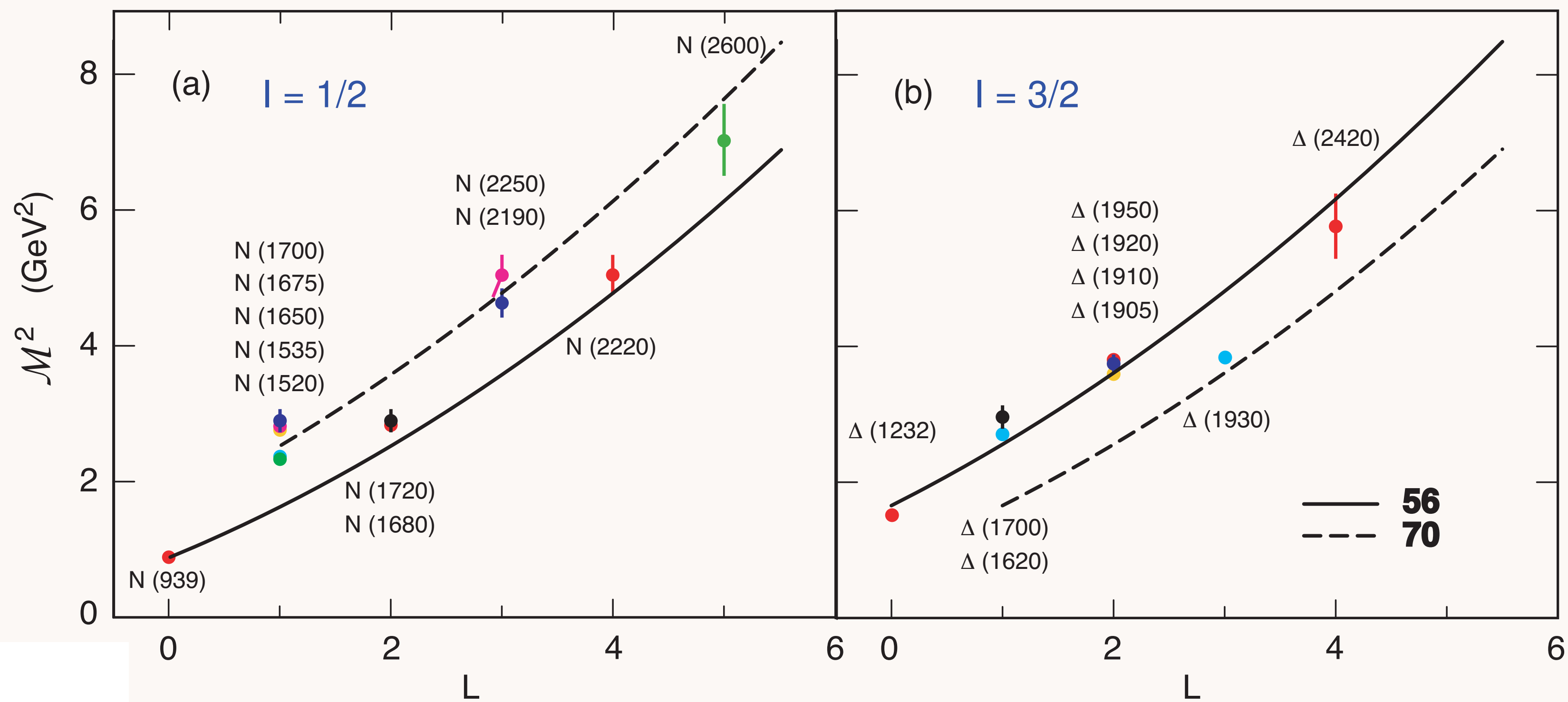


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

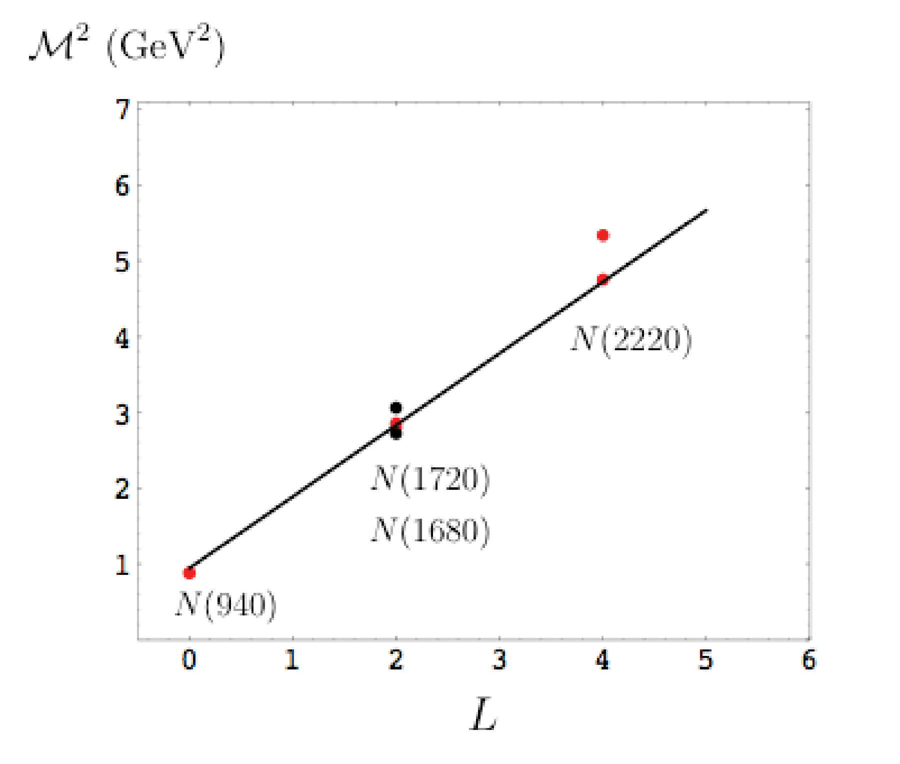
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

- Baryon: twist-dimension $3+L \quad (\nu = L+1)$

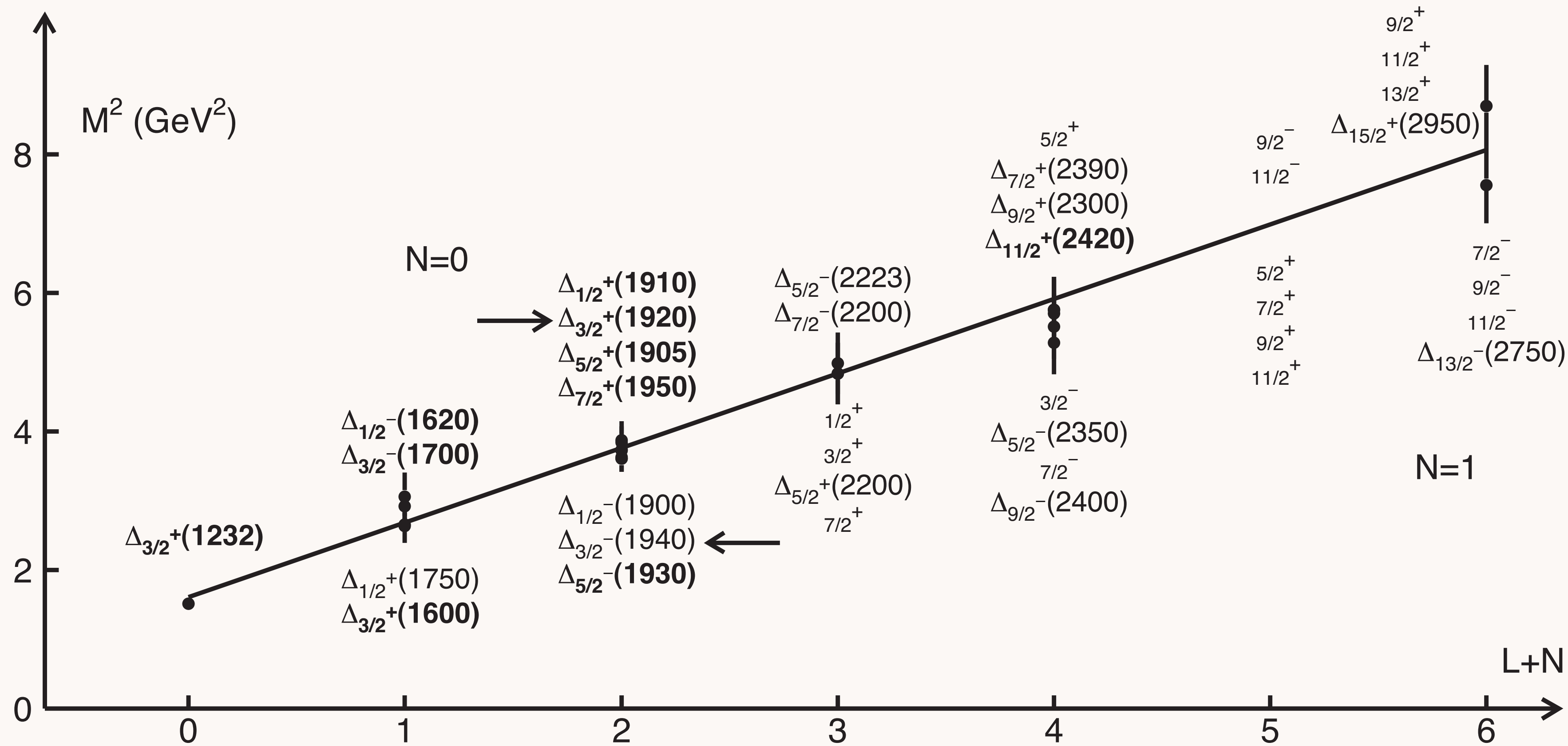
$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

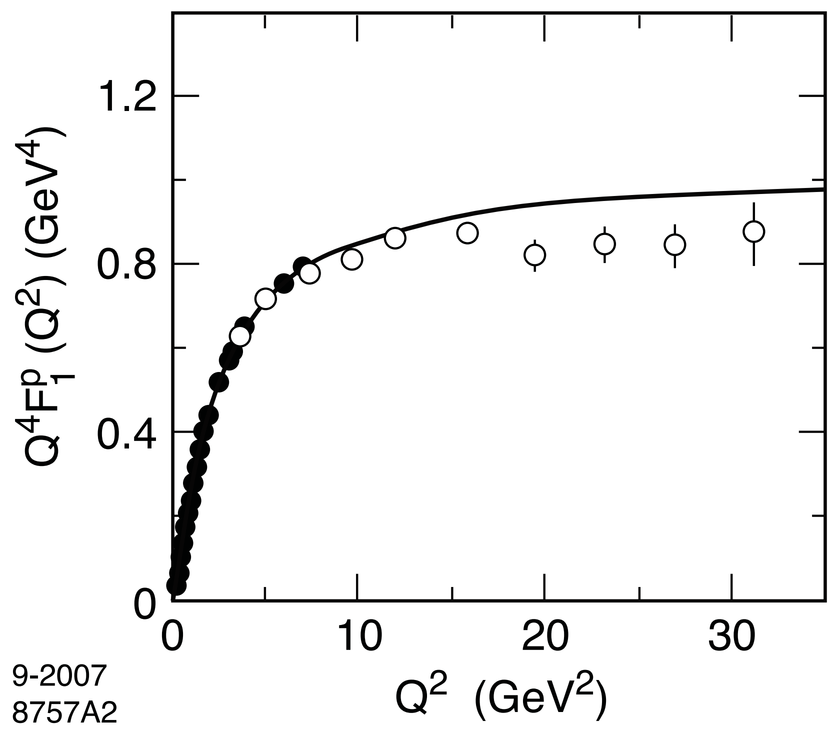
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

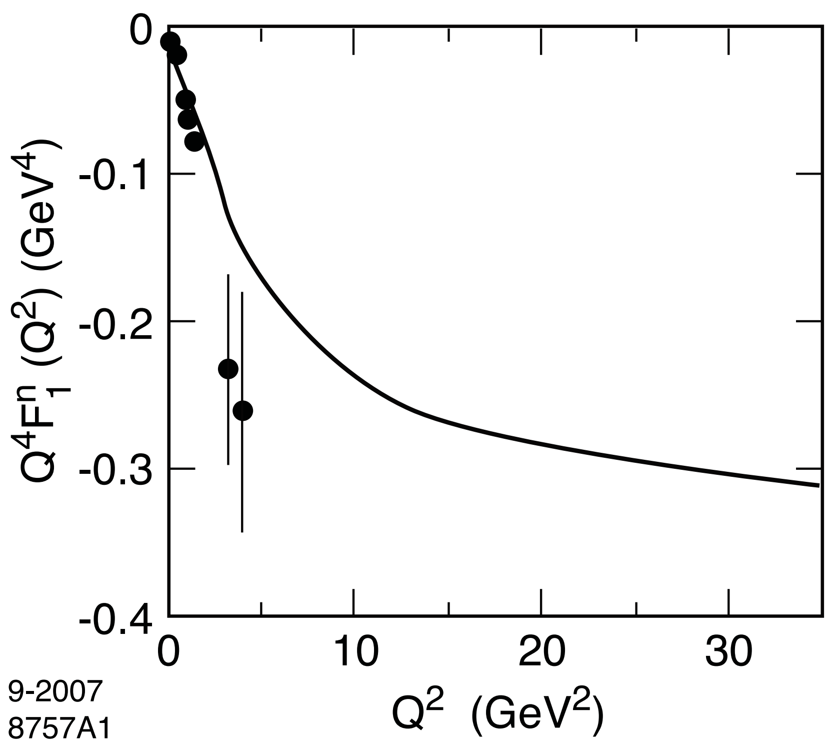
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

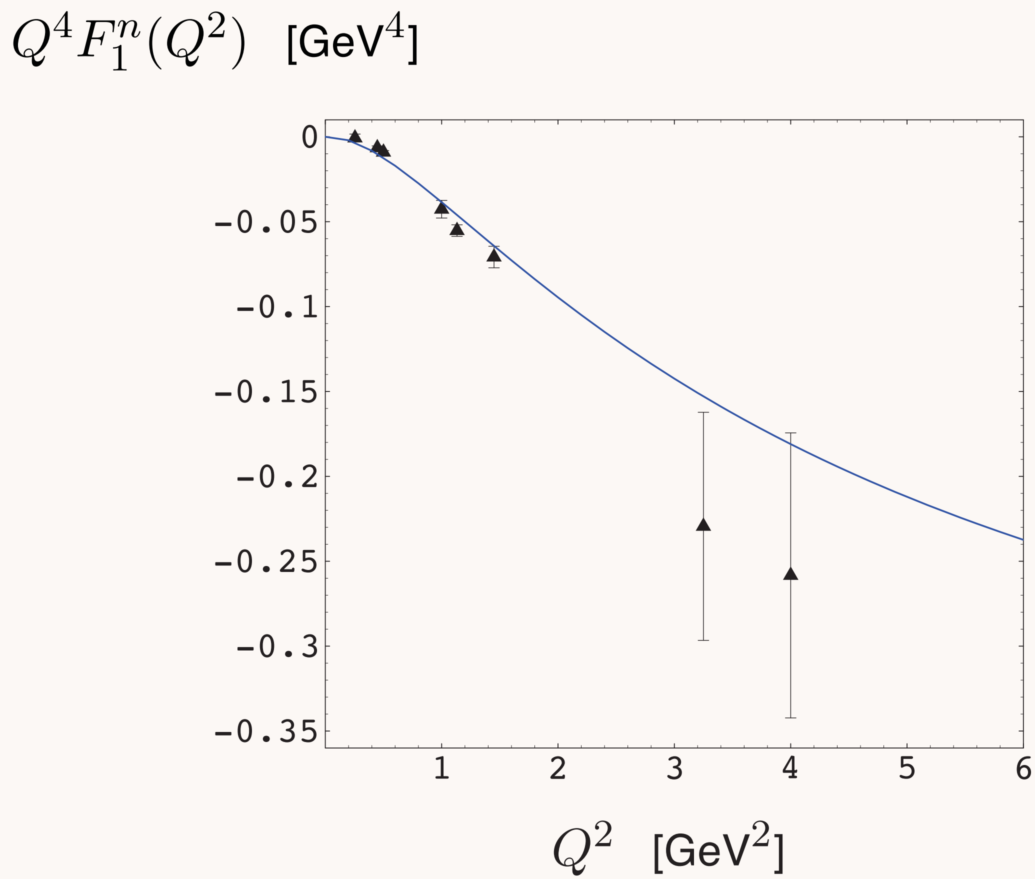
- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement

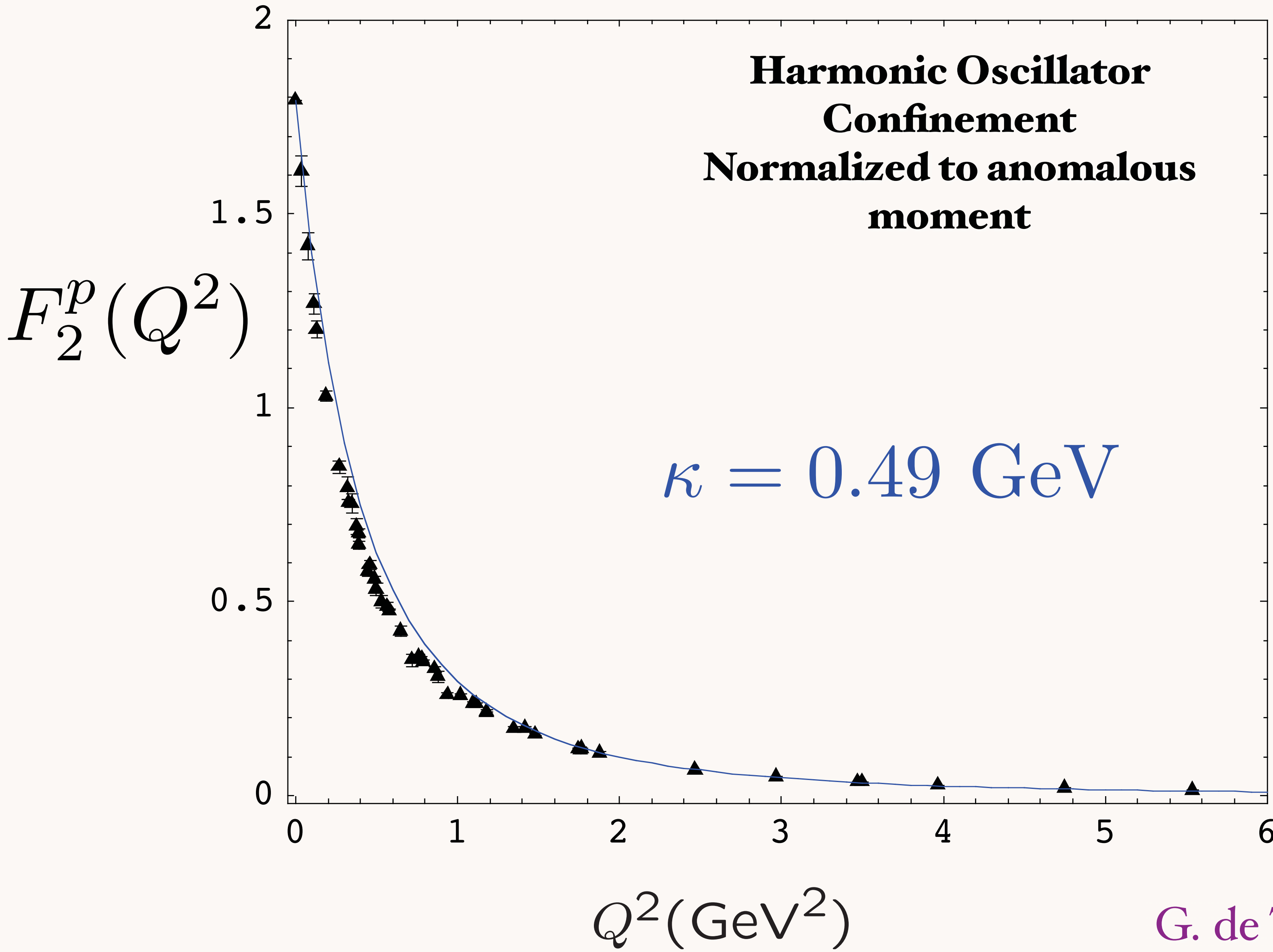


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



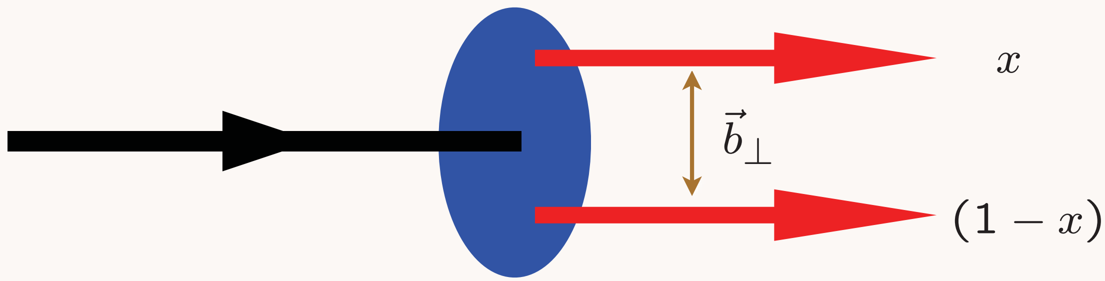
G. de Teramond, sjb

LF(3+1) $\qquad\qquad$ AdS_5

$$\psi(x,\vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \quad \longleftrightarrow \quad z$$

$$\psi(x,\vec{b}_\perp)$$

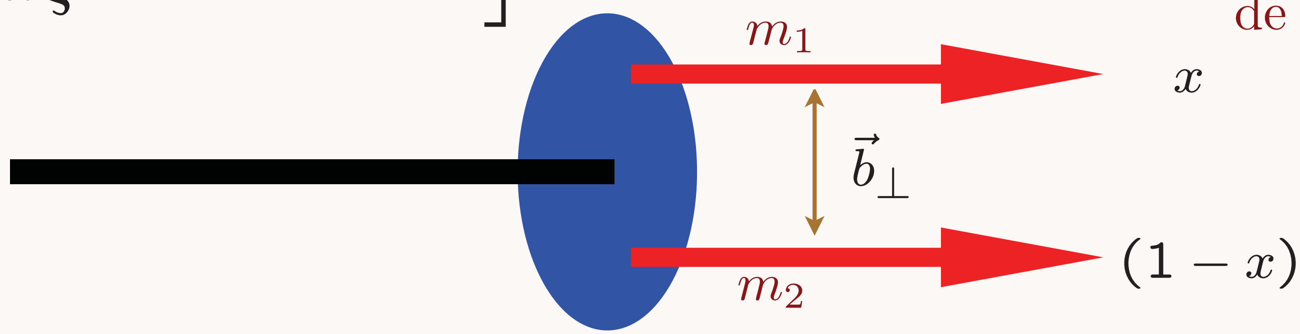


$$\psi(x,\vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

ground state LFWF

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

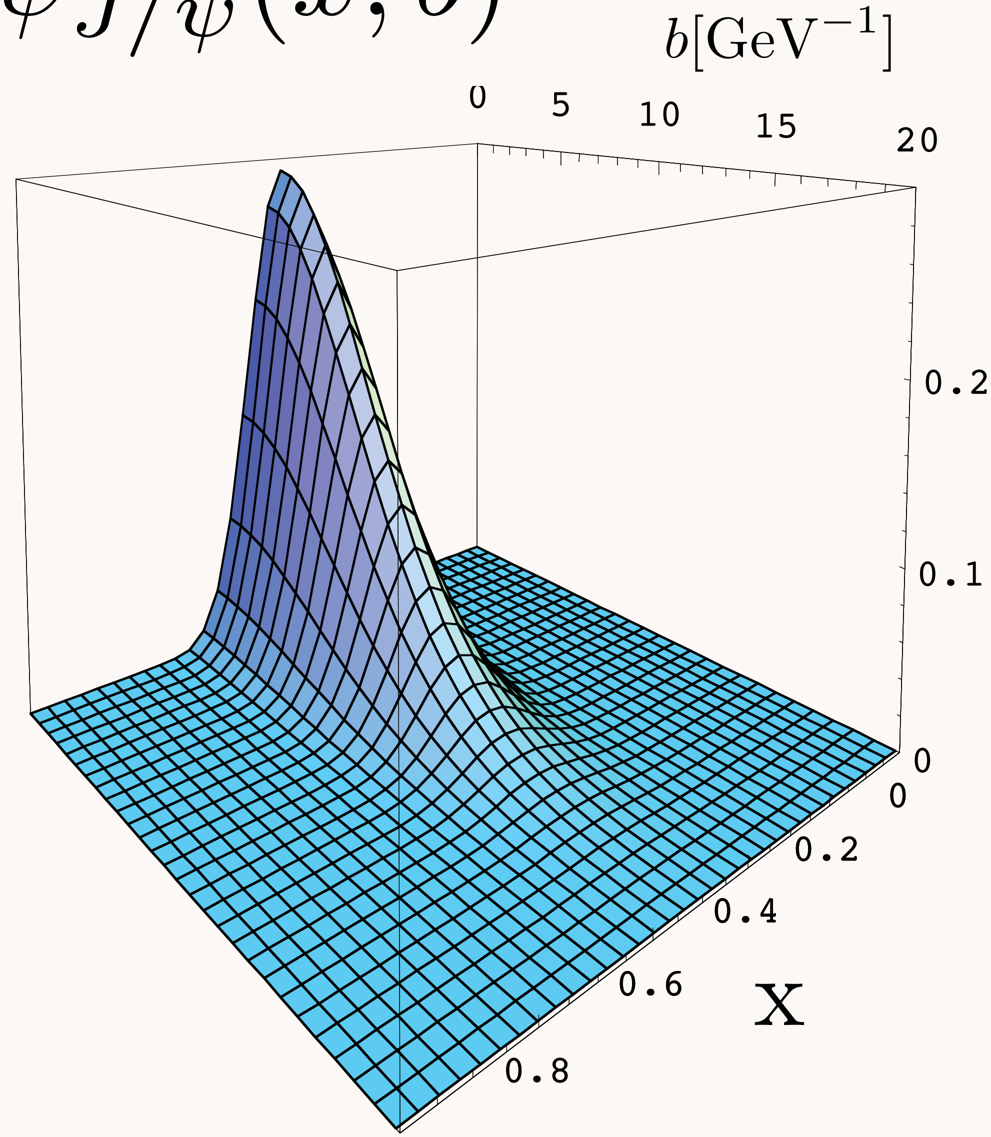
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

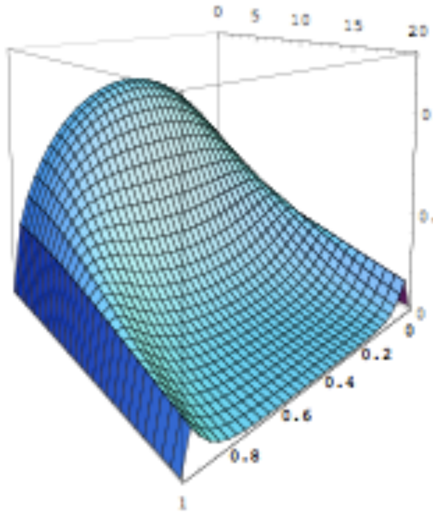
$$\psi_{J/\psi}(x, b)$$

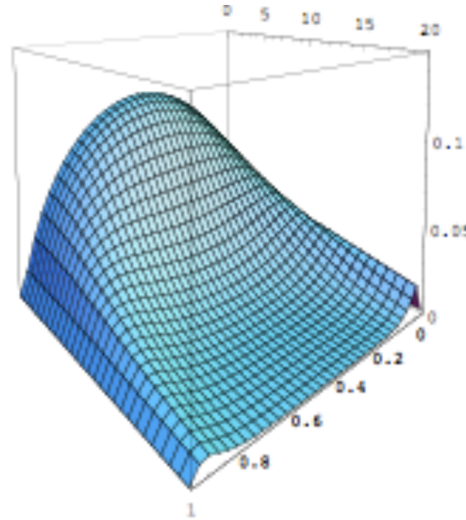


$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



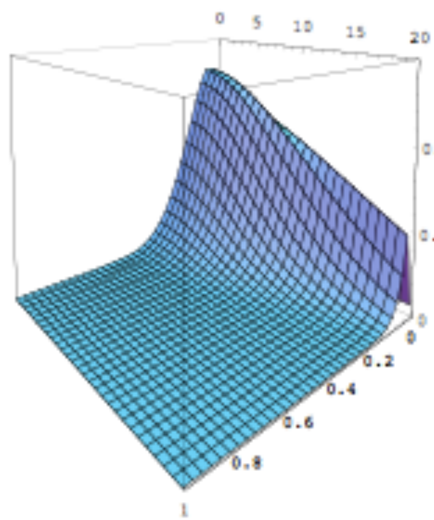


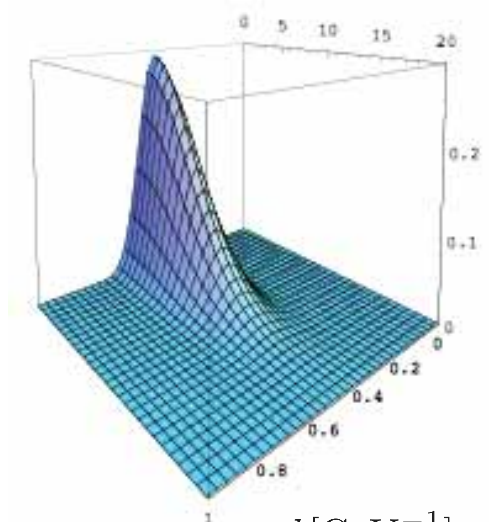
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

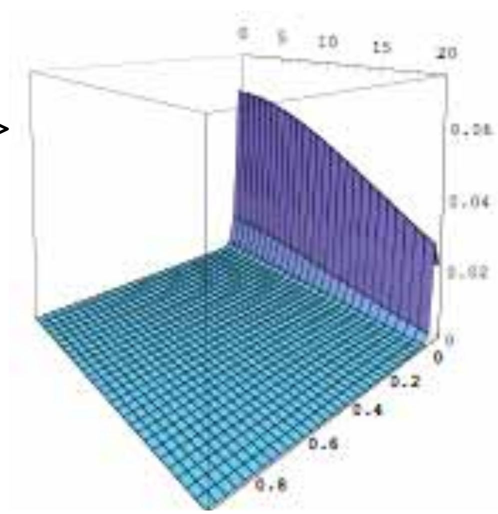


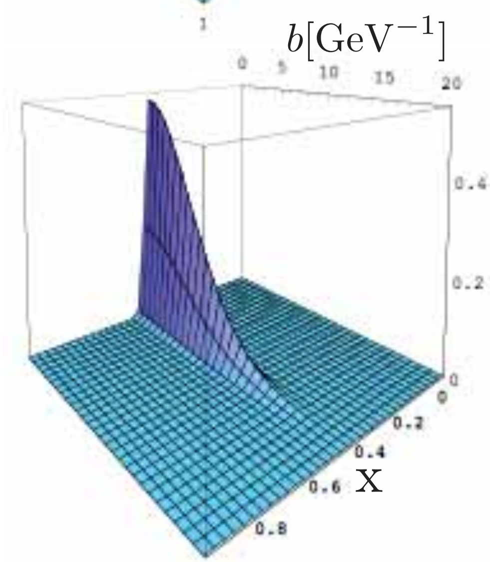


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





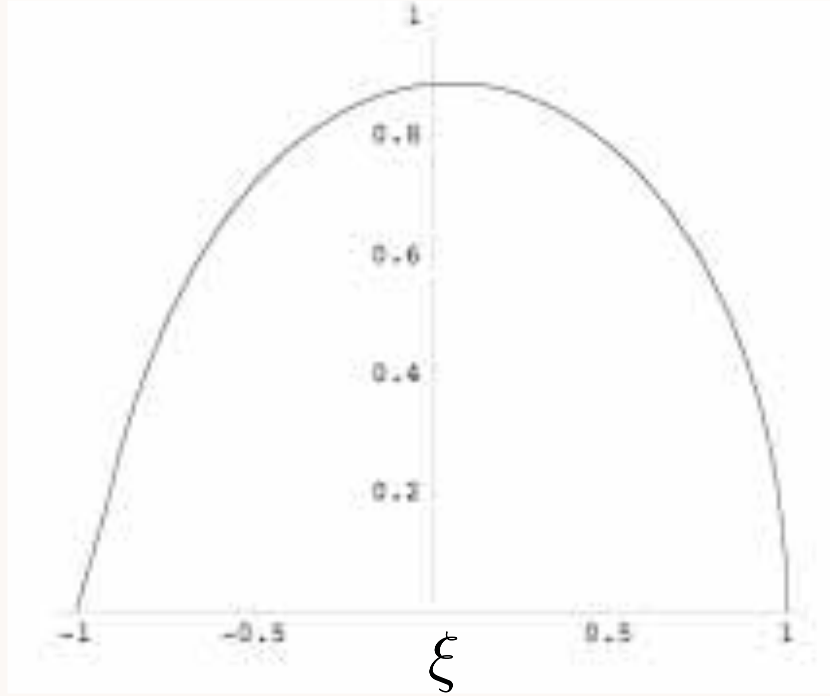
$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

First Moment of Kaon Distribution Amplitude

$$< \xi >= \int_{-1}^{1} d\xi \; \xi \; \phi(\xi)$$

$$\xi = 1 - 2x$$



$$< \xi >_K = 0.04 \pm 0.02 \qquad \kappa = 375 \; MeV$$

Range from $m_s = 65 \pm 25$ MeV (PDG)

$$< \xi >_K = 0.029 \;\; \pm 0.002$$ Donnellan et al.

$$< \xi >_K = 0.0272 \pm 0.0005$$ Braun et al.

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

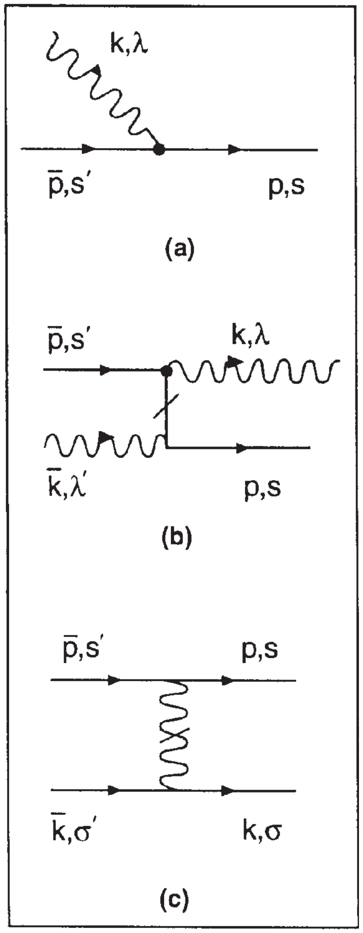
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

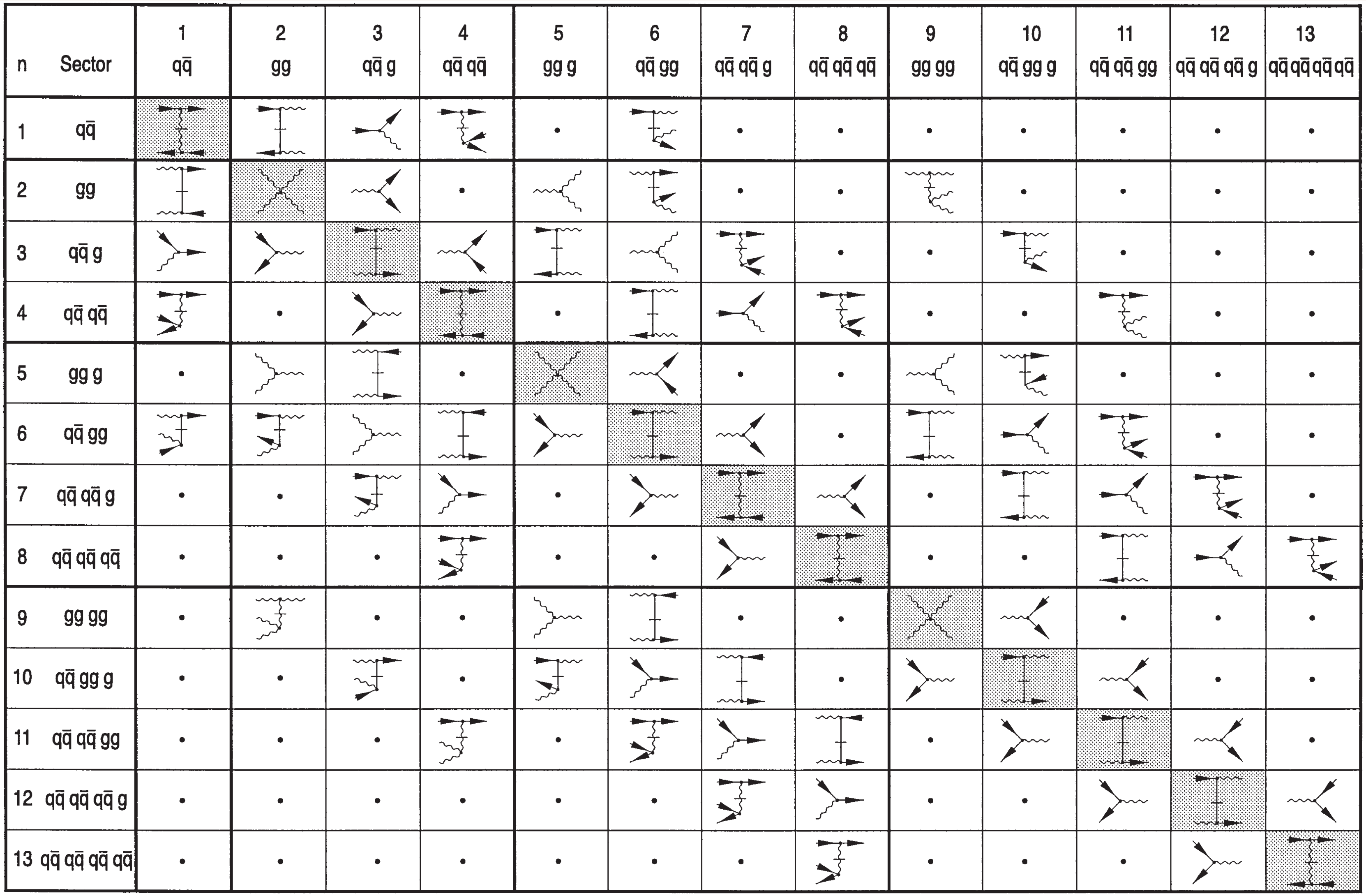
Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

Light-Front QCD
Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





Use AdS/QCD basis functions