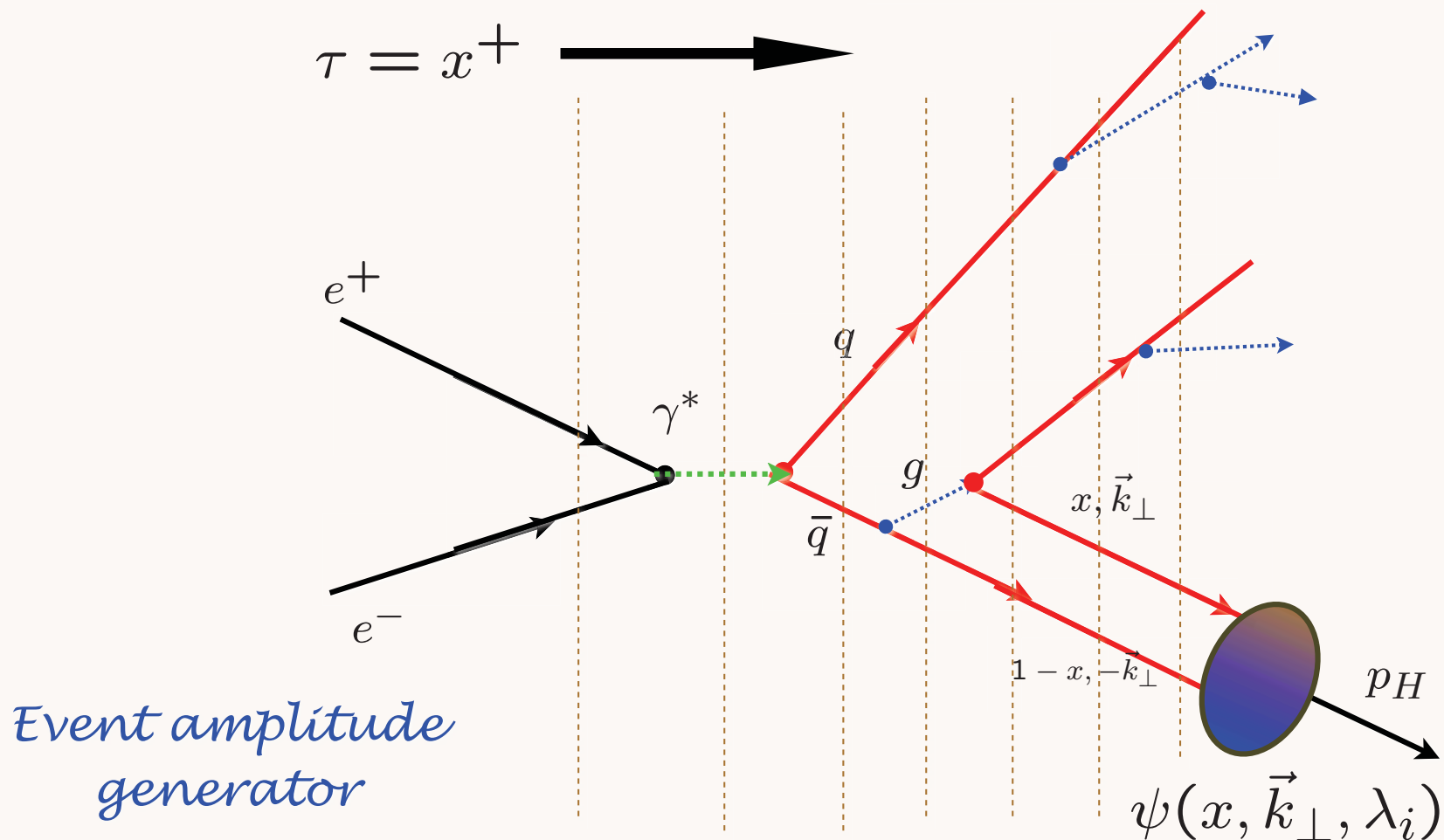
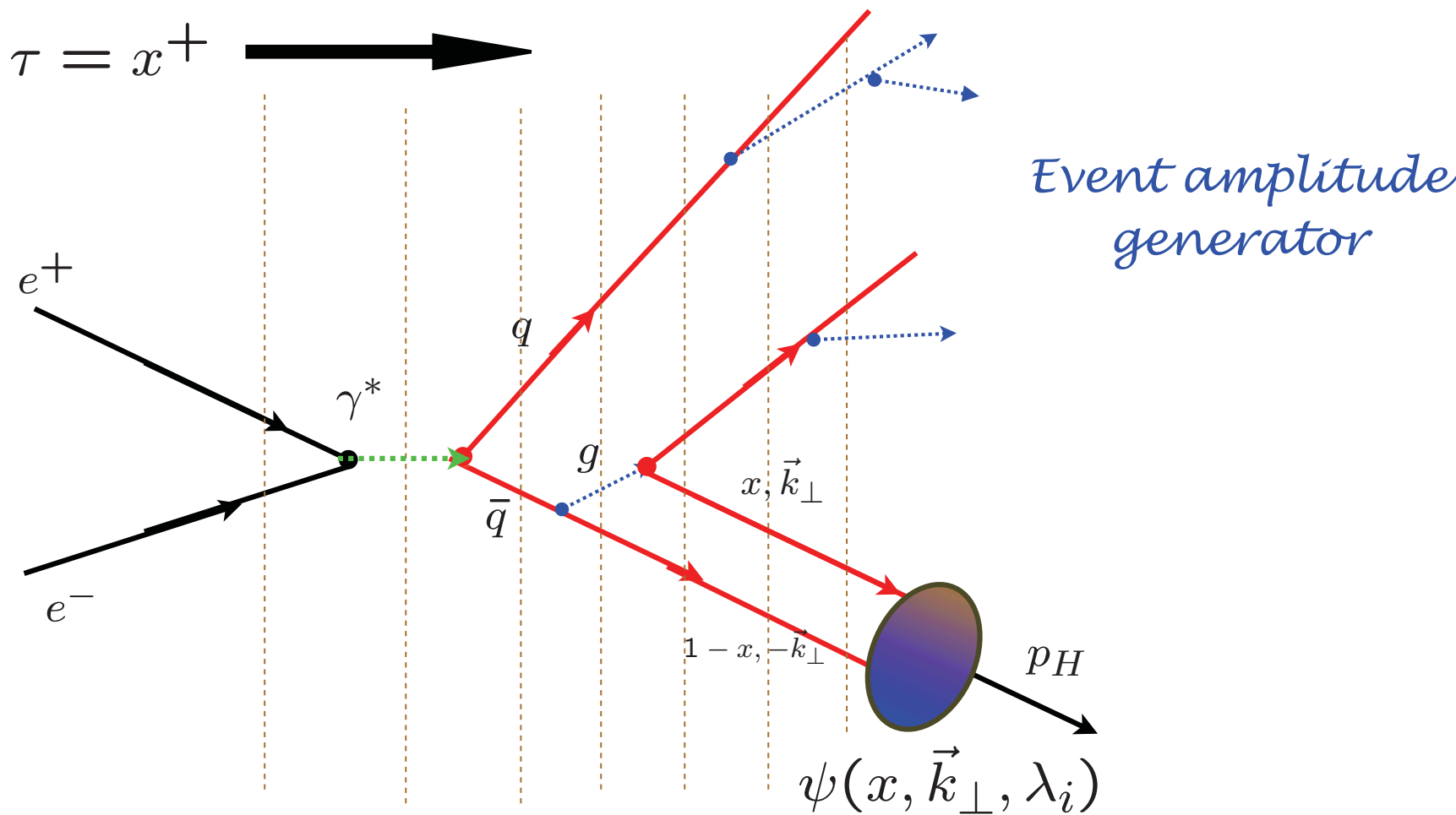


# Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Hadronization at the Amplitude Level



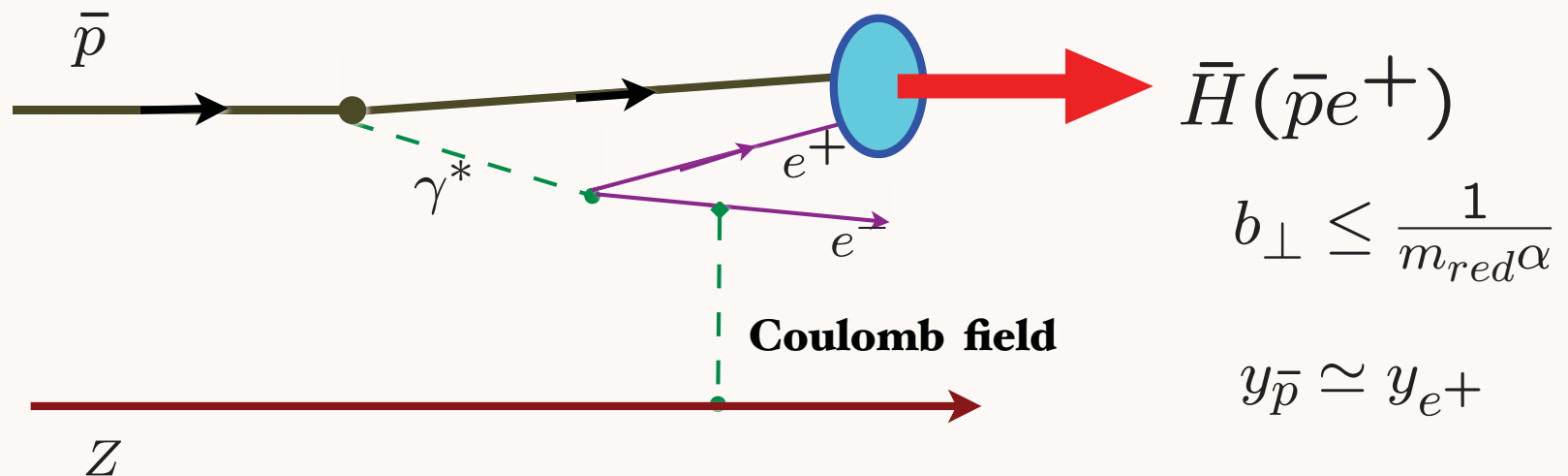
*AdS/QCD*  
*Hard Wall*  
*Confinement:*

Capture if  $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda_{QCD}^2}$   
 i.e.,  
 $\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda_{QCD}^2$

# Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$\bar{H}(\bar{p}e^+)$$

$$b_{\perp} \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

*Coalescence of off-shell co-moving positron and antiproton*

*Wavefunction maximal at small impact separation and equal rapidity*

*“Hadronization” at the Amplitude Level*

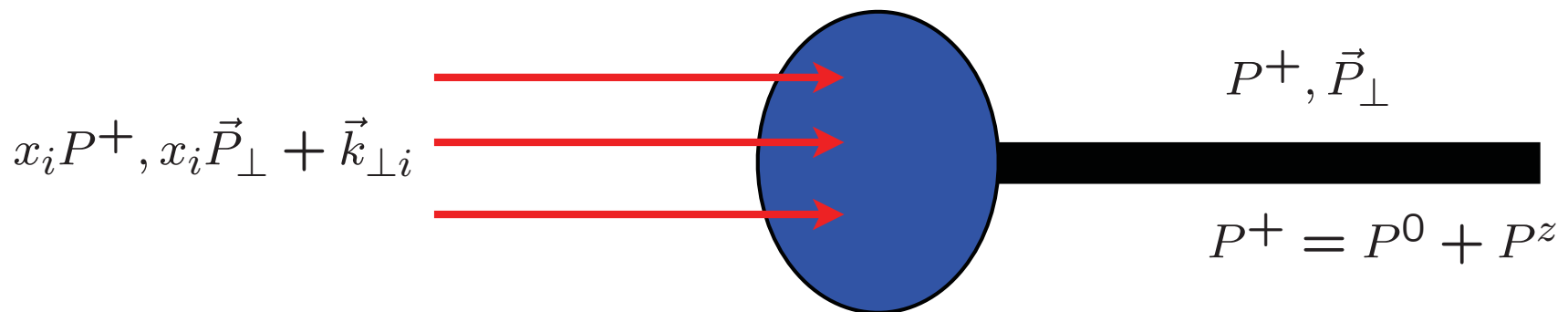
# Features of LF T-Matrix Formalism

## “Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^n \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
- No IR divergences: Maximal gluon and quark wavelength from confinement

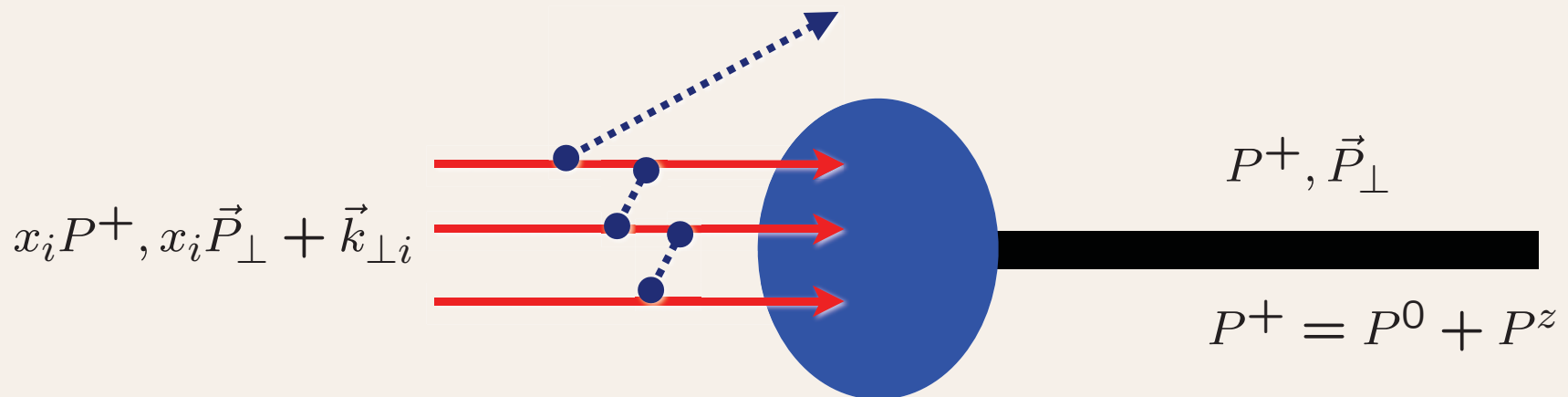


# Features of LF T-Matrix Formalism

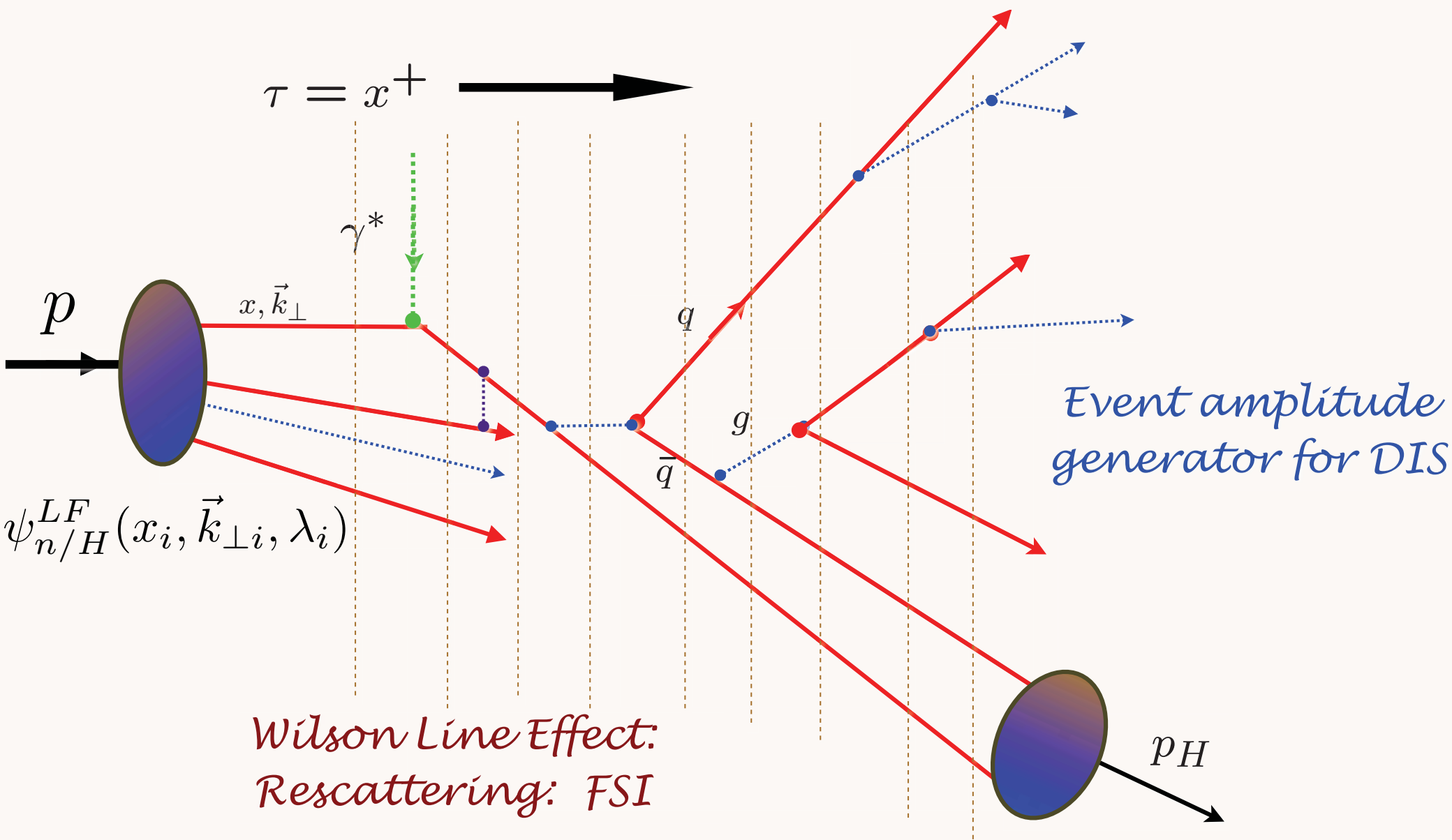
## “Event Amplitude Generator”

If  $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$  use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange
- Factorization Scale for structure functions and fragmentation functions set:  $\mu_{fact} = \Lambda_{QCD}$



# Hadronization at the Amplitude Level



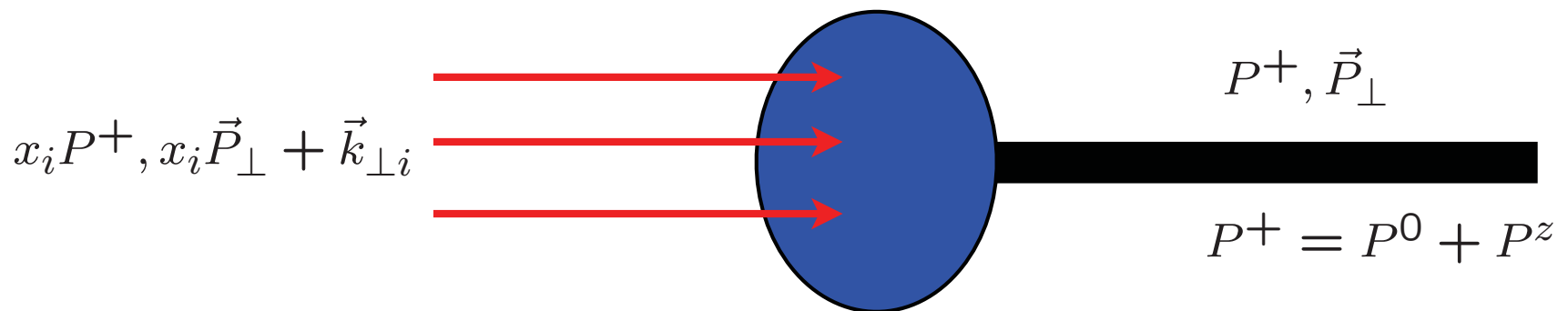
# *Features of LF T-Matrix Formalism*

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of  $P^+$  and  $P^z$
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$  conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using “alternating denominator” method
- Multiple renormalization scales (BLM)

# Features of LF T-Matrix Formalism

## “Event Amplitude Generator”

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large  $z$
- hadron helicity conservation if hadron LFWF has  $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

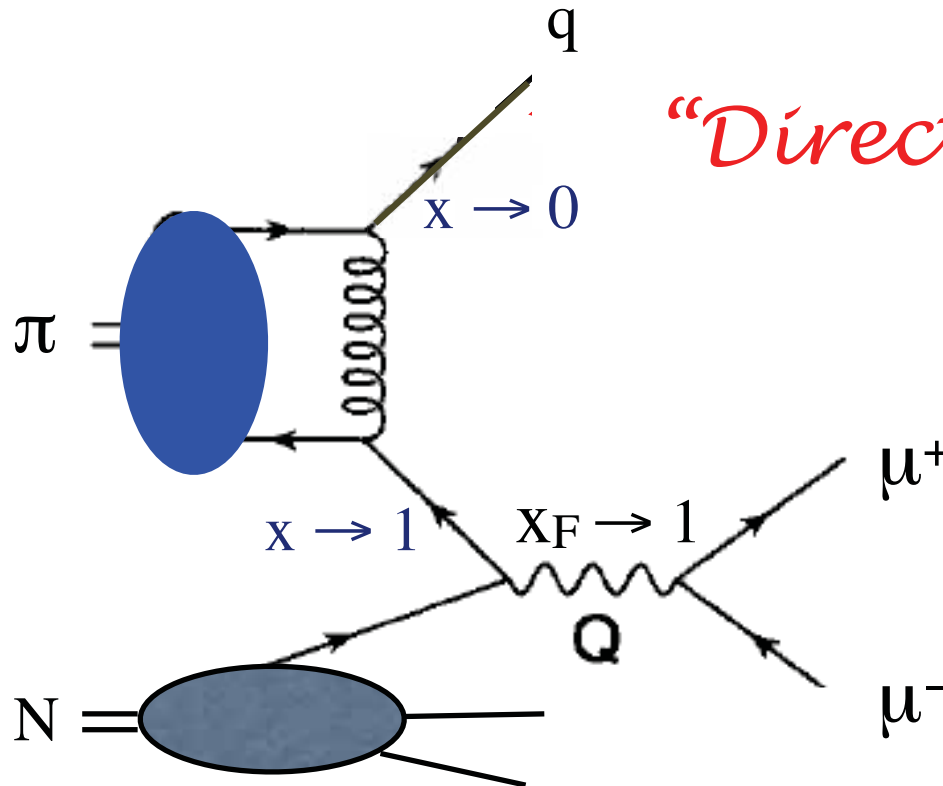




$$\pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F$$

In the limit where  $(1-x_F)Q^2$  is fixed as  $Q^2 \rightarrow \infty$

Entire pion wf  
contributes to  
hard process



*“Direct” Subprocess*

Virtual photon is  
longitudinally  
polarized

Berger, sjb  
Khoze, Brandenburg, Muller, sjb  
Hoyer Vanttinen

$$\pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

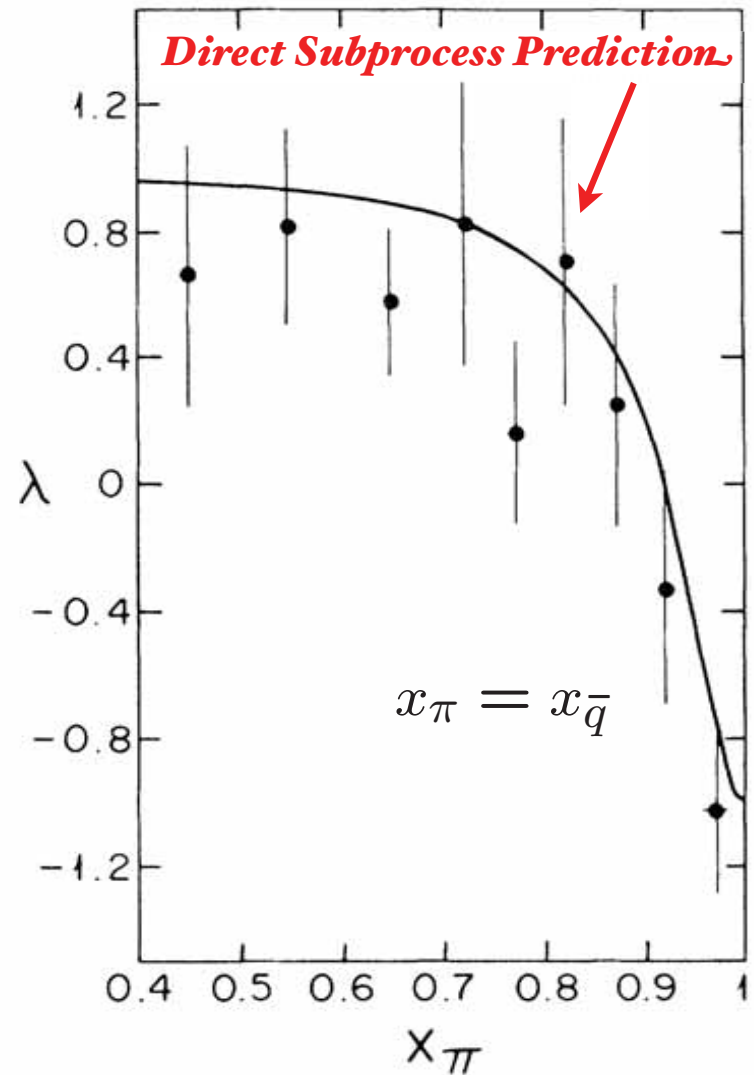
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[ (1-x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2$$

*Dramatic change in  
angular distribution at  
large  $x_F$*

**Example of a higher-twist  
direct subprocess**



Chicago-Princeton  
Collaboration

Phys.Rev.Lett.55:2649,1985

# *Crucial Test of Leading -Twist QCD: Scaling at fixed $x_T$*

$$x_T = \frac{2p_T}{\sqrt{s}}$$

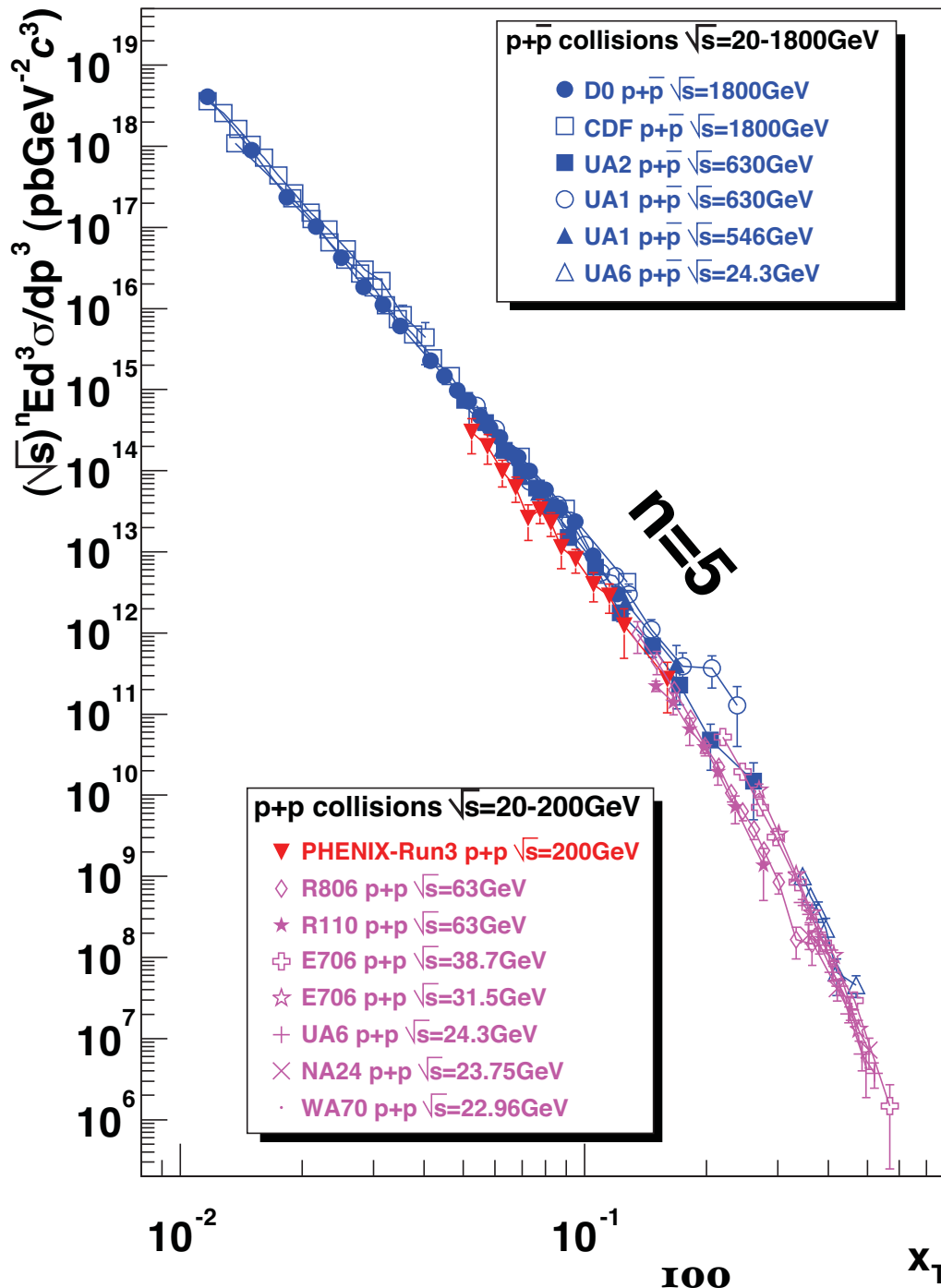
$$E \frac{d\sigma}{d^3p}(pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

***Parton model:  $n_{eff} = 4$***

***As fundamental as Bjorken scaling in DIS***

**Conformal scaling:  $n_{eff} = 2 n_{active} - 4$**

$$\sqrt{s}^n E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) \text{ at fixed } x_T$$

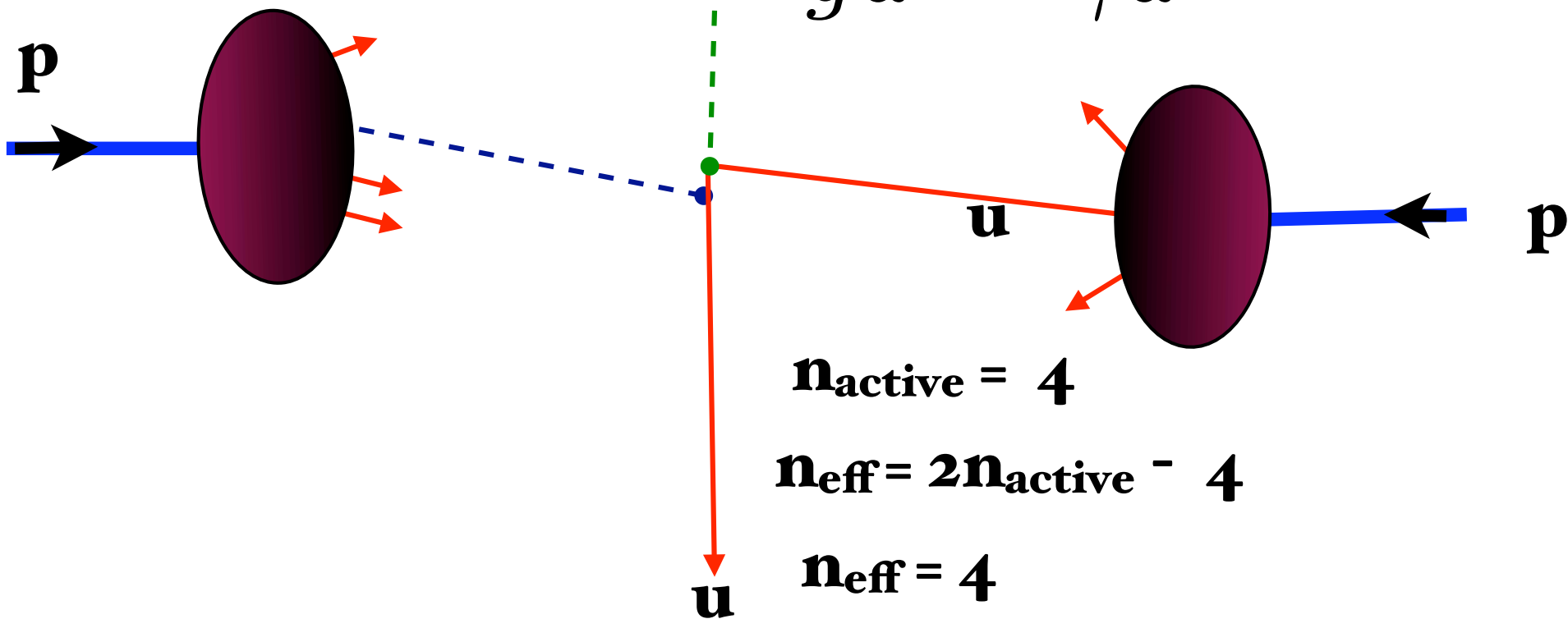


**Scaling of direct  
photon production  
consistent with  
PQCD**

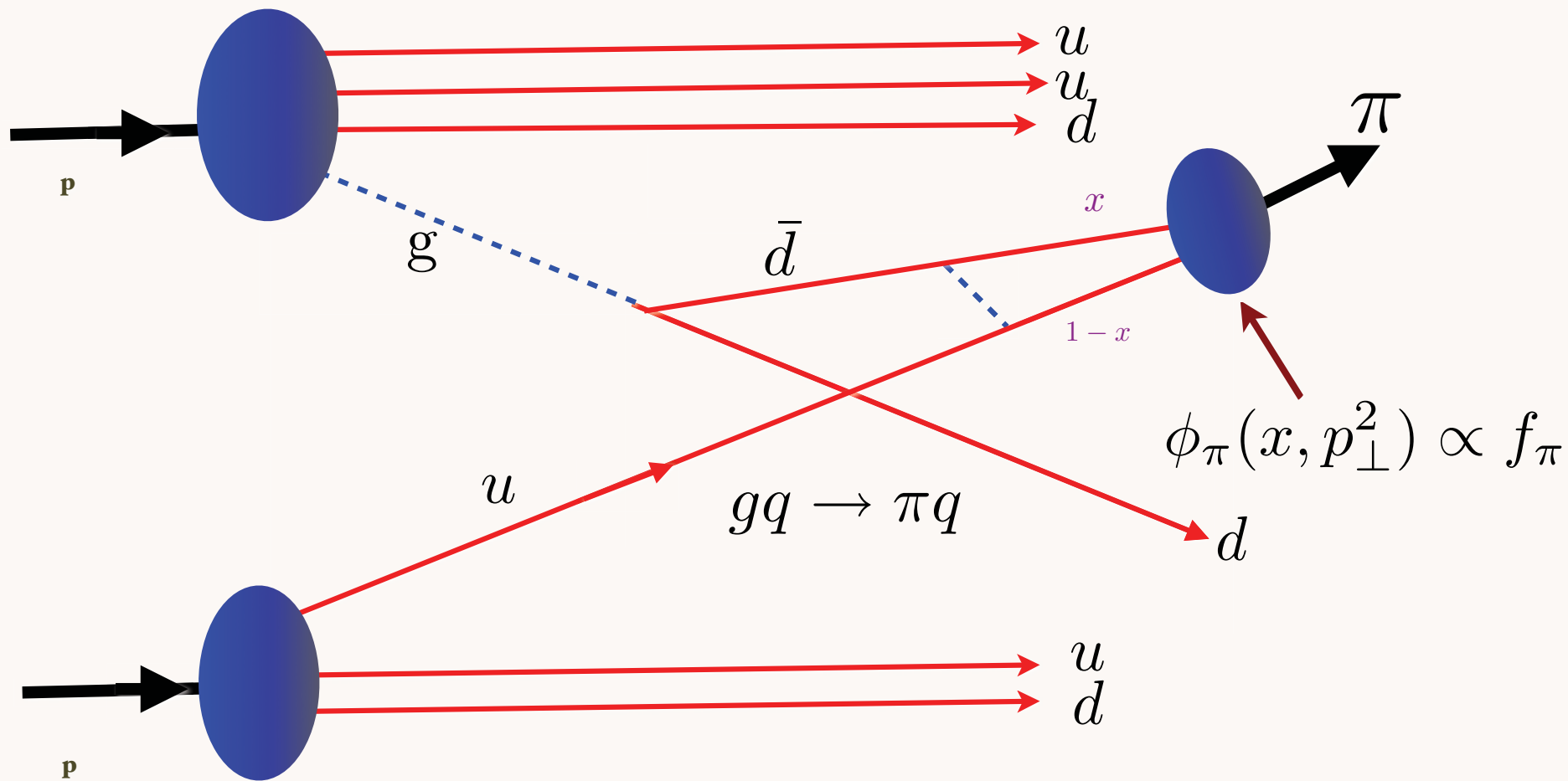
$$pp \rightarrow \gamma X$$

$$E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \rightarrow \gamma u$$



*Higher-Twist Contribution to Hadron Production*

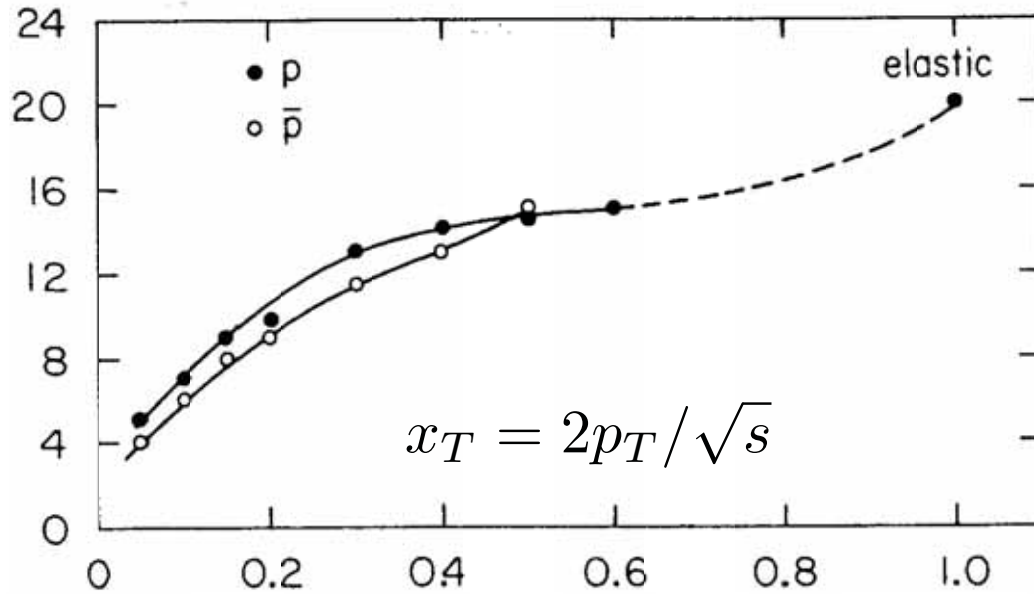


$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

*No Fragmentation Function*

$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{eff}}}$$

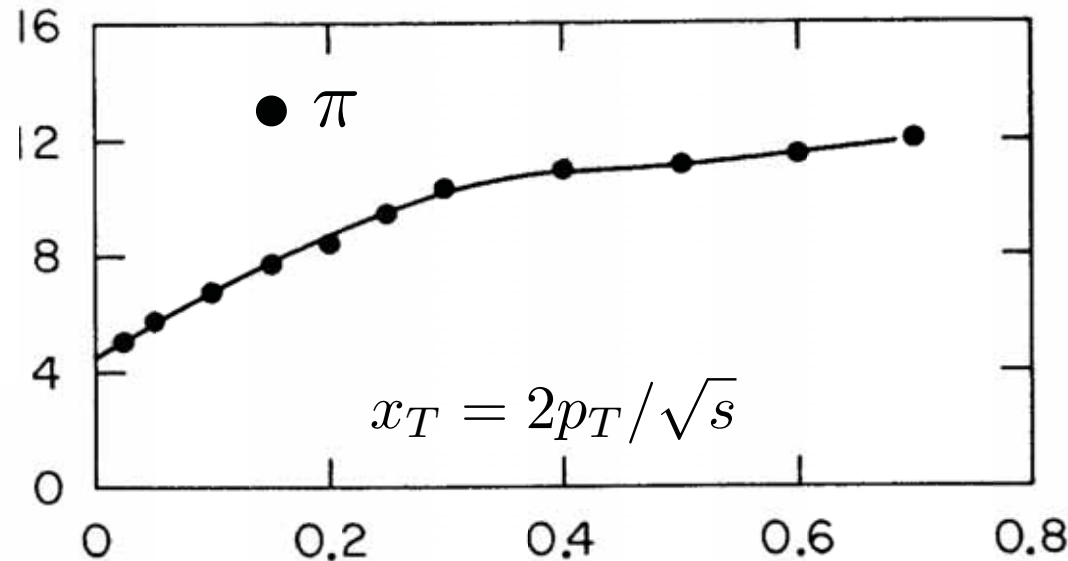
$n_{eff}$



*Clear evidence for higher-twist contributions*

**Fermilab, ISR data**

$n_{eff}$

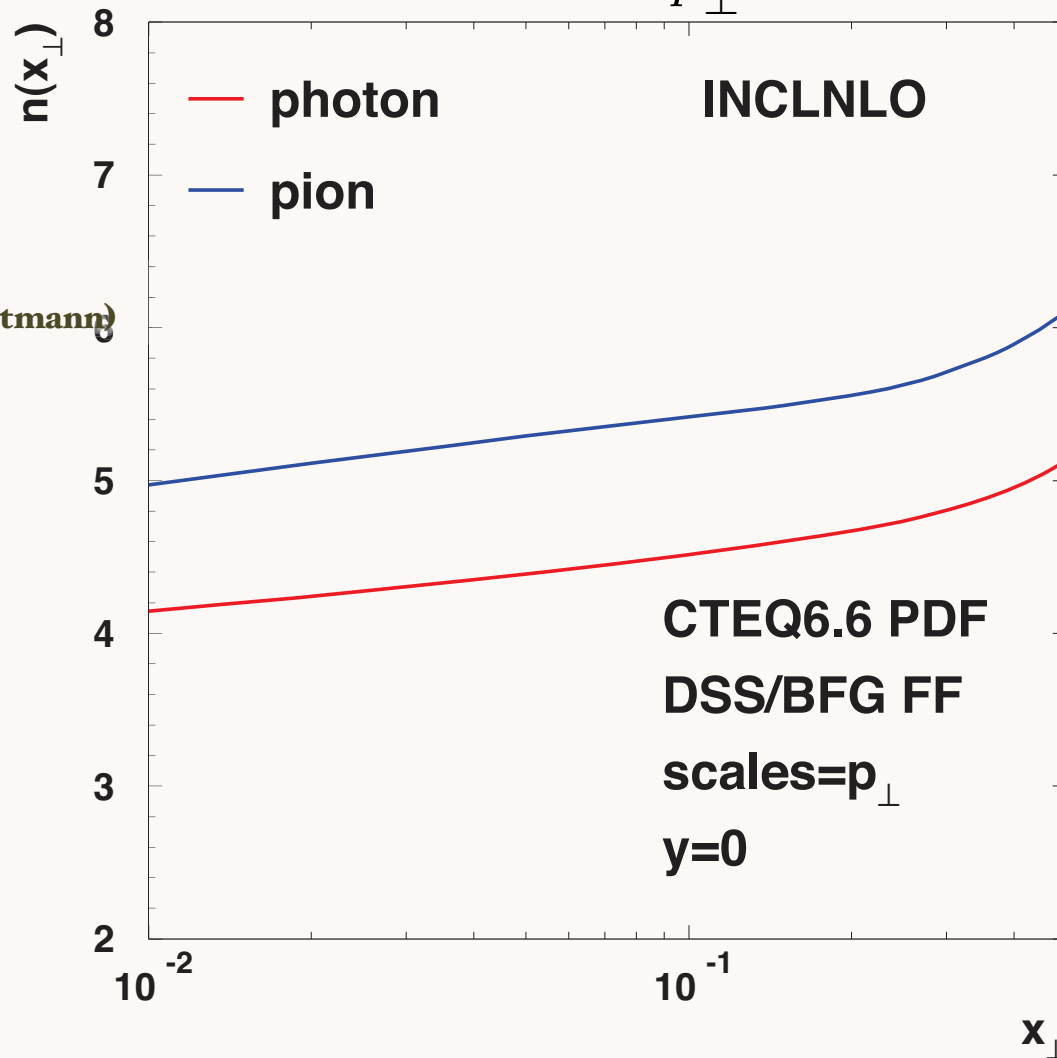


*Continuous Rise of  $n_{eff}$*

**Stan Brodsky  
SLAC**

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_{\perp}, y)}{p_{\perp}^{n(x_{\perp})}}$$



$$pp \rightarrow \pi X$$

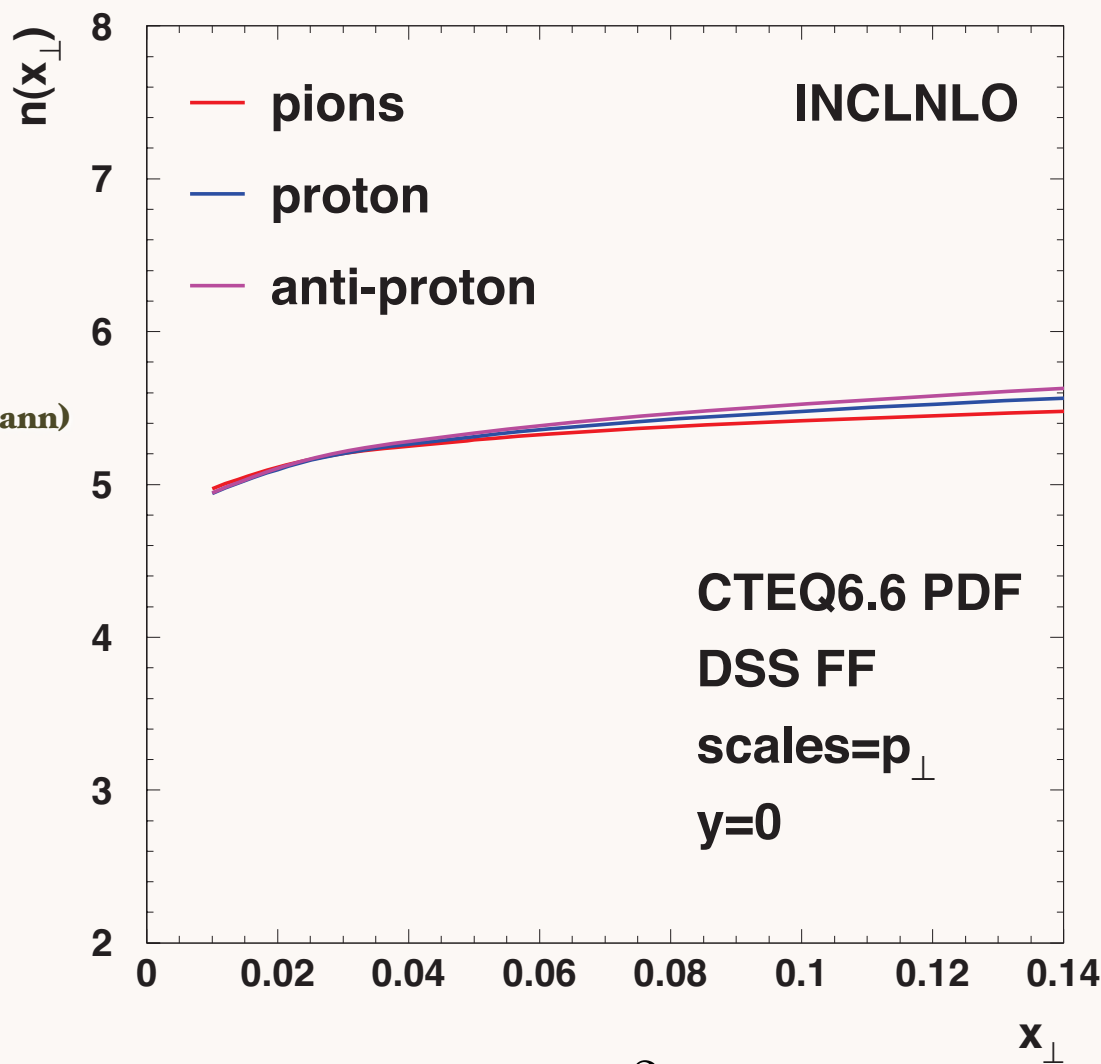
$$pp \rightarrow \gamma X$$

$$5 < p_{\perp} < 20 \text{ GeV}$$

$$70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV}$$



$$5 < p_{\perp} < 20 \text{ GeV} \quad 70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV}$$



$$pp \rightarrow \pi X$$



$$pp \rightarrow p X$$



$$pp \rightarrow \bar{p} X$$



$$\frac{d\sigma}{d^3p/E} = \frac{F(x_{\perp}, y)}{p_{\perp}^{n(x_{\perp})}}$$

Arleo, Aurenche

$$x_{\perp} = \frac{2p_{\perp}}{\sqrt{s}}$$

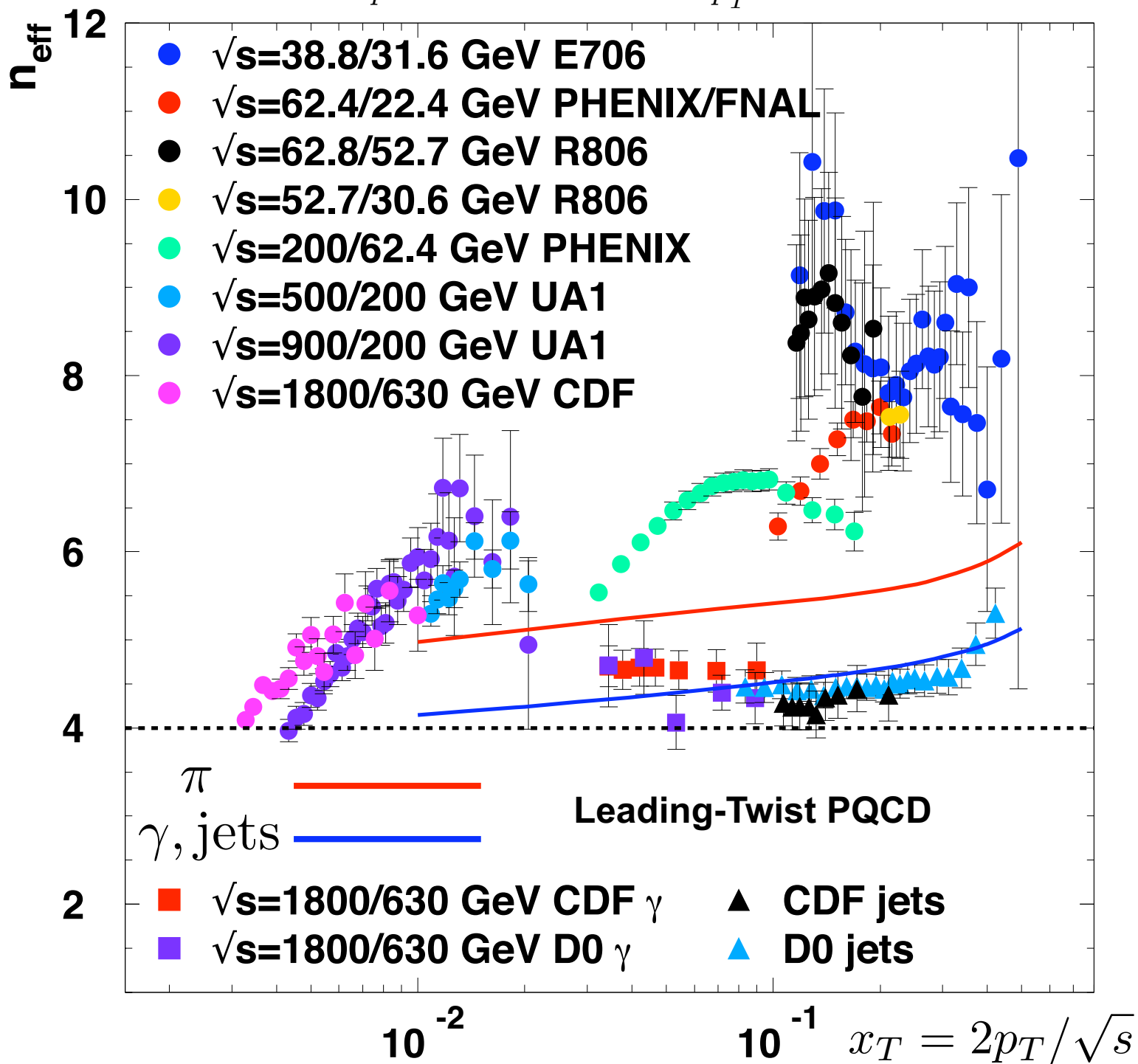
**JTI Workshop ANL**  
**April 16, 2009**

**AdS/QCD and LF Holography**

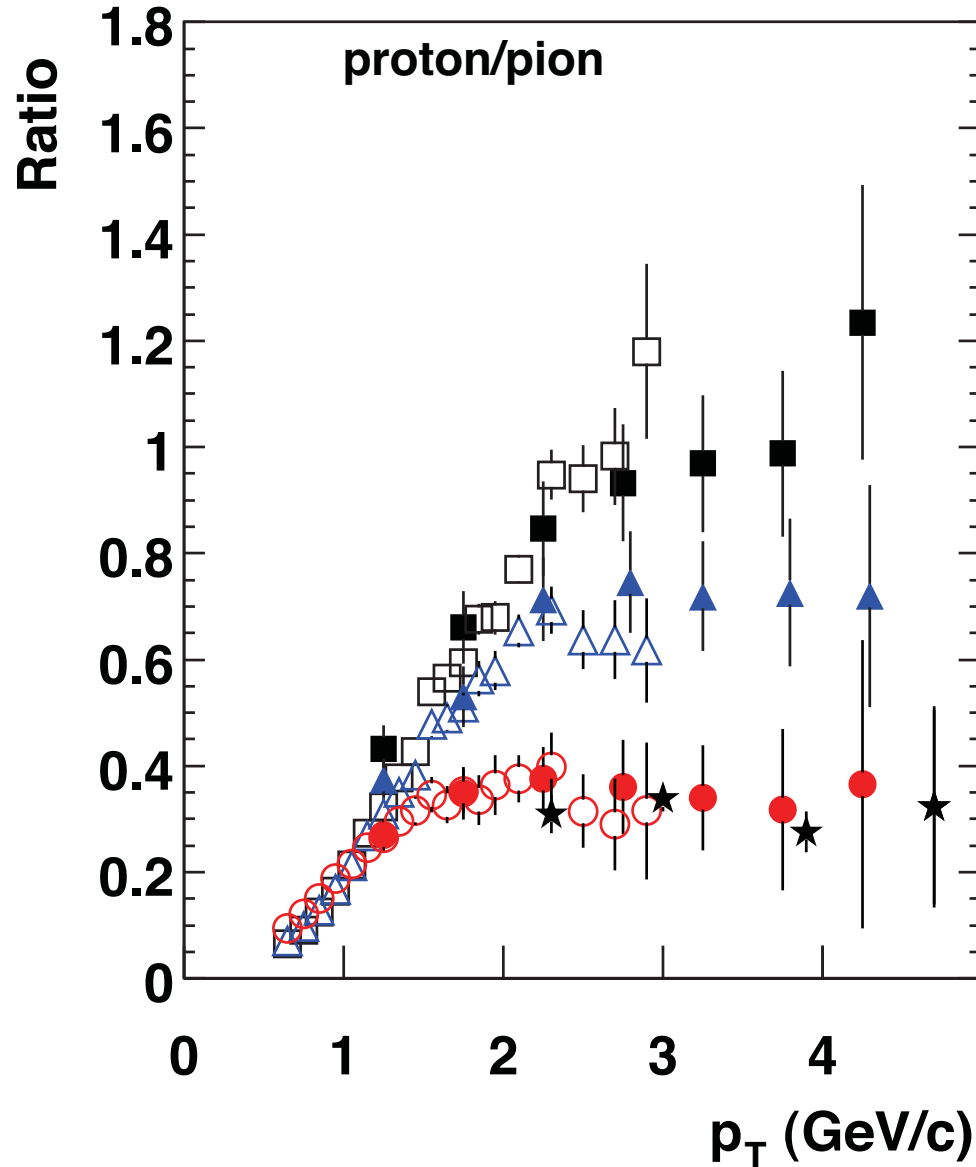
**105**

**Stan Brodsky**  
**SLAC**

$$E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\text{eff}}}}$$



*Baryon Anomaly: Particle ratio changes with centrality!*



*Protons less absorbed  
in nuclear collisions than pions*

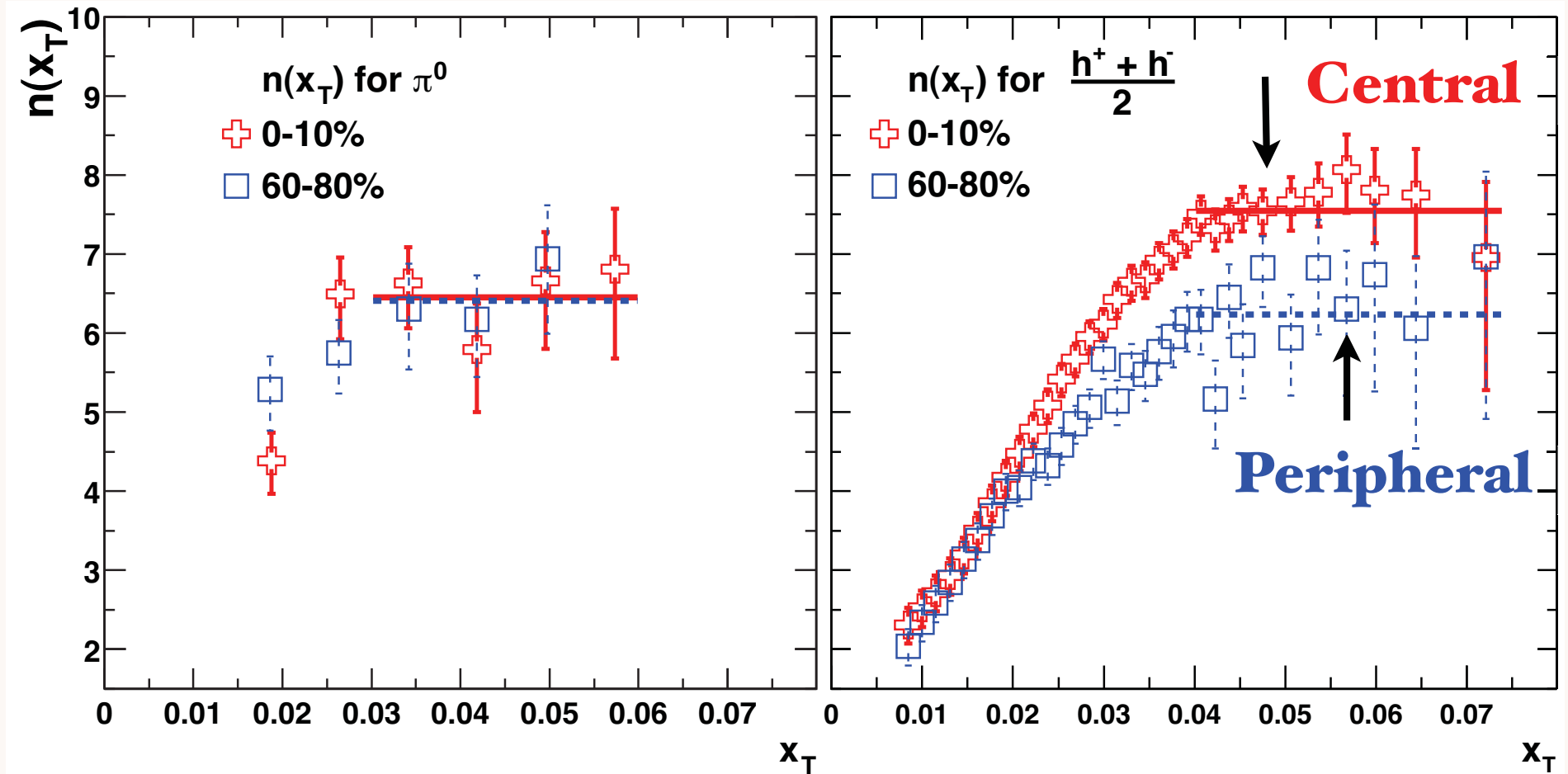
← **Central**

- ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ● Au+Au 60-92%
- ★ p+p,  $\sqrt{s} = 53$  GeV, ISR
- e<sup>+</sup>e<sup>-</sup>, gluon jets, DELPHI
- ..... e<sup>+</sup>e<sup>-</sup>, quark jets, DELPHI

← **Peripheral**

Sickles, sjb

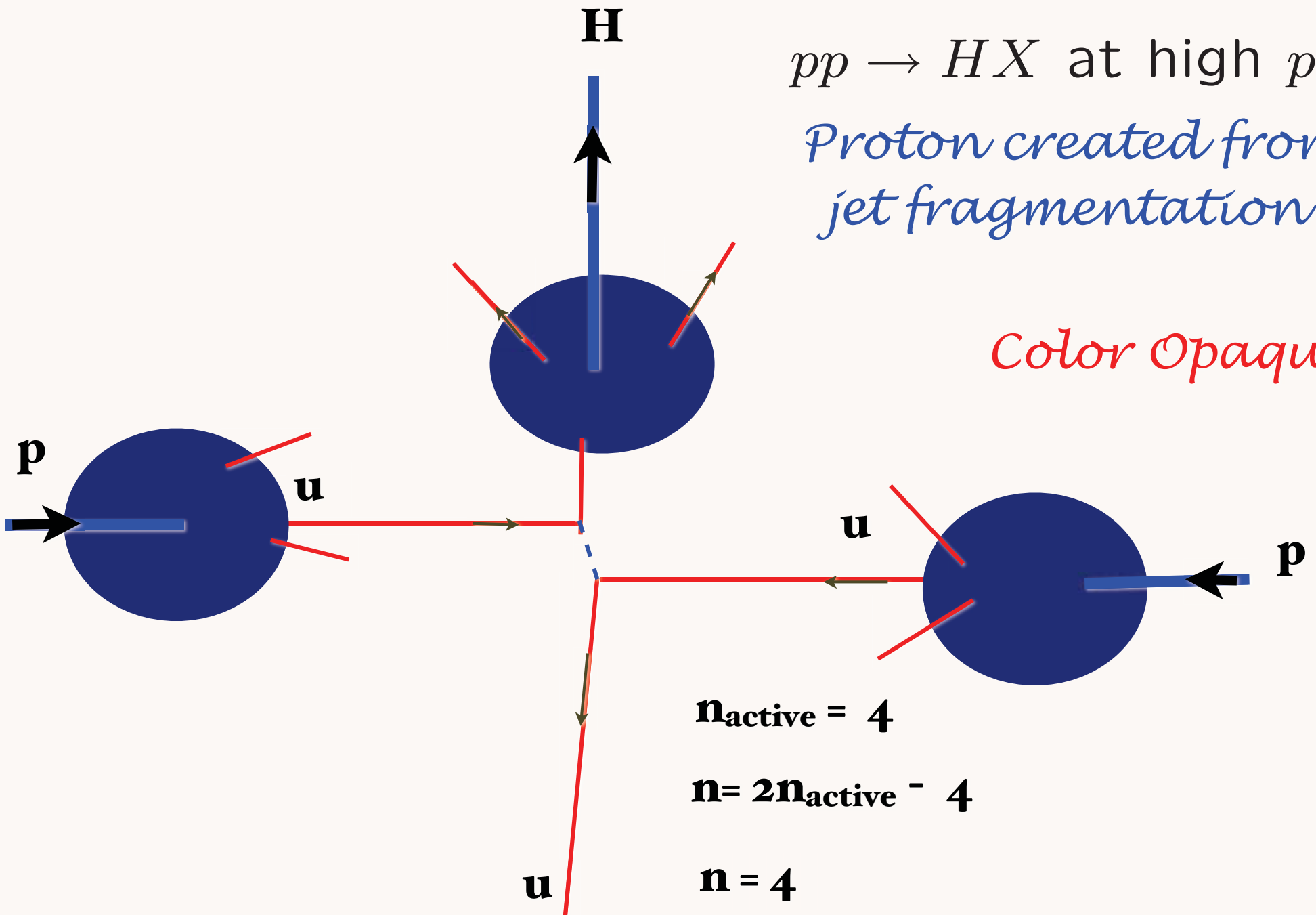
$$\sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV}$$



*Proton power changes with centrality !*

$pp \rightarrow HX$  at high  $p_T$   
*Proton created from  
jet fragmentation*

*Color Opaque*



$$\mathbf{n}_{\text{active}} = 4$$

$$\mathbf{n} = 2\mathbf{n}_{\text{active}} - 4$$

$$\mathbf{n} = 4$$

*Baryon can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

$$b_{\perp} \simeq 1/p_T$$

**p**

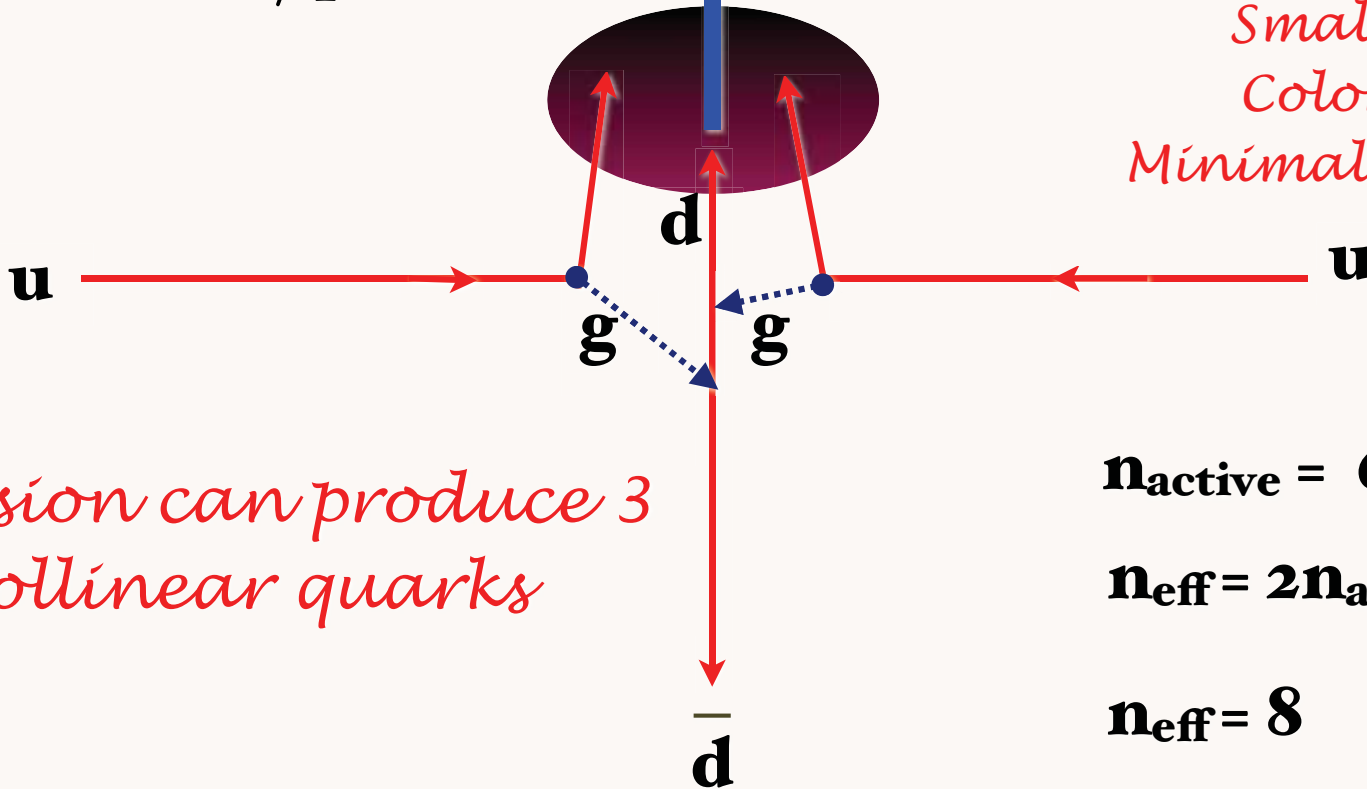
$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

Bjorken  
Blankenbecler, Gunion, sjb  
Berger, sjb  
Hoyer, et al: Semi-Exclusive

Sickles, sjb

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



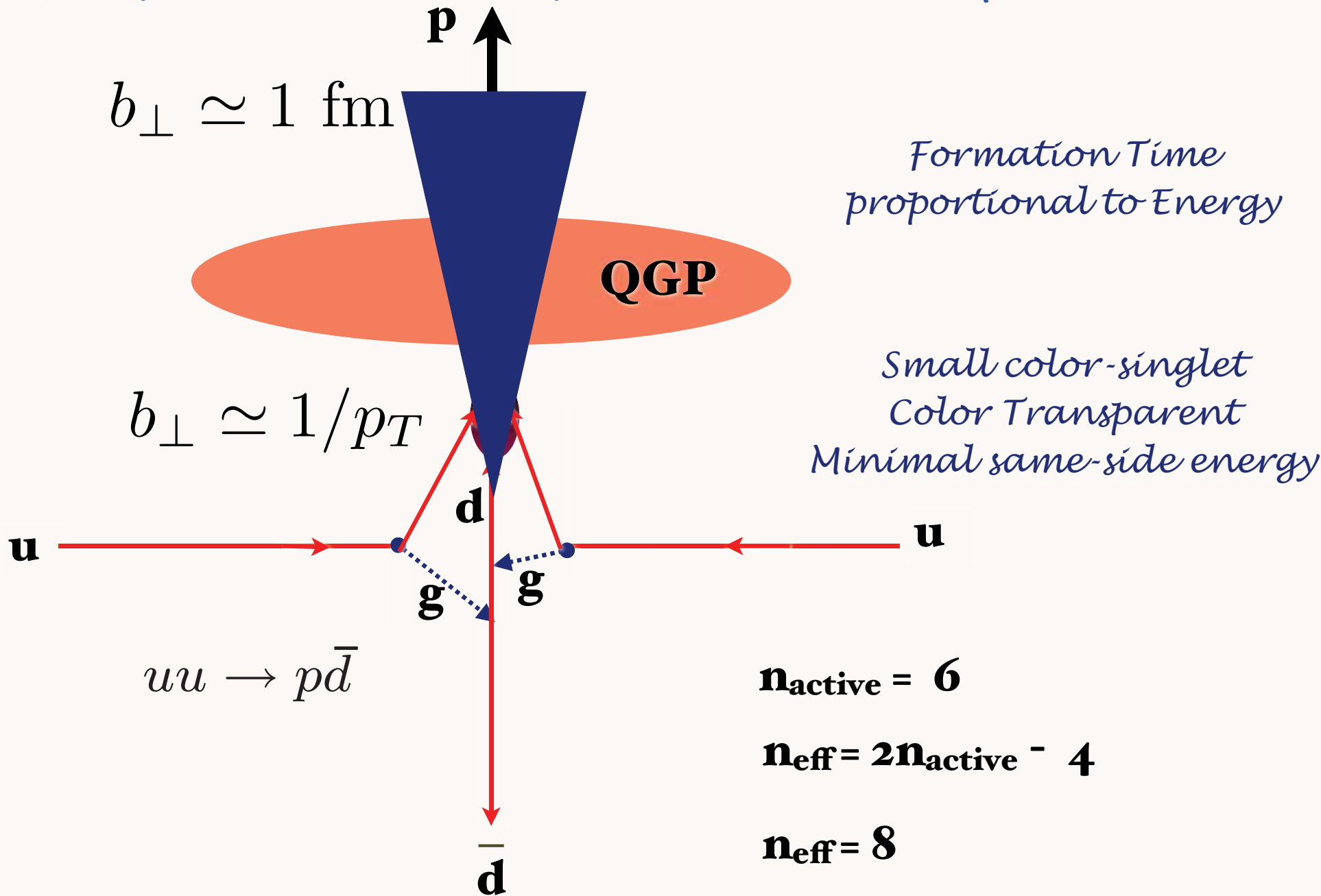
*Collision can produce 3  
collinear quarks*

$$\mathbf{n}_{\text{active}} = 6$$

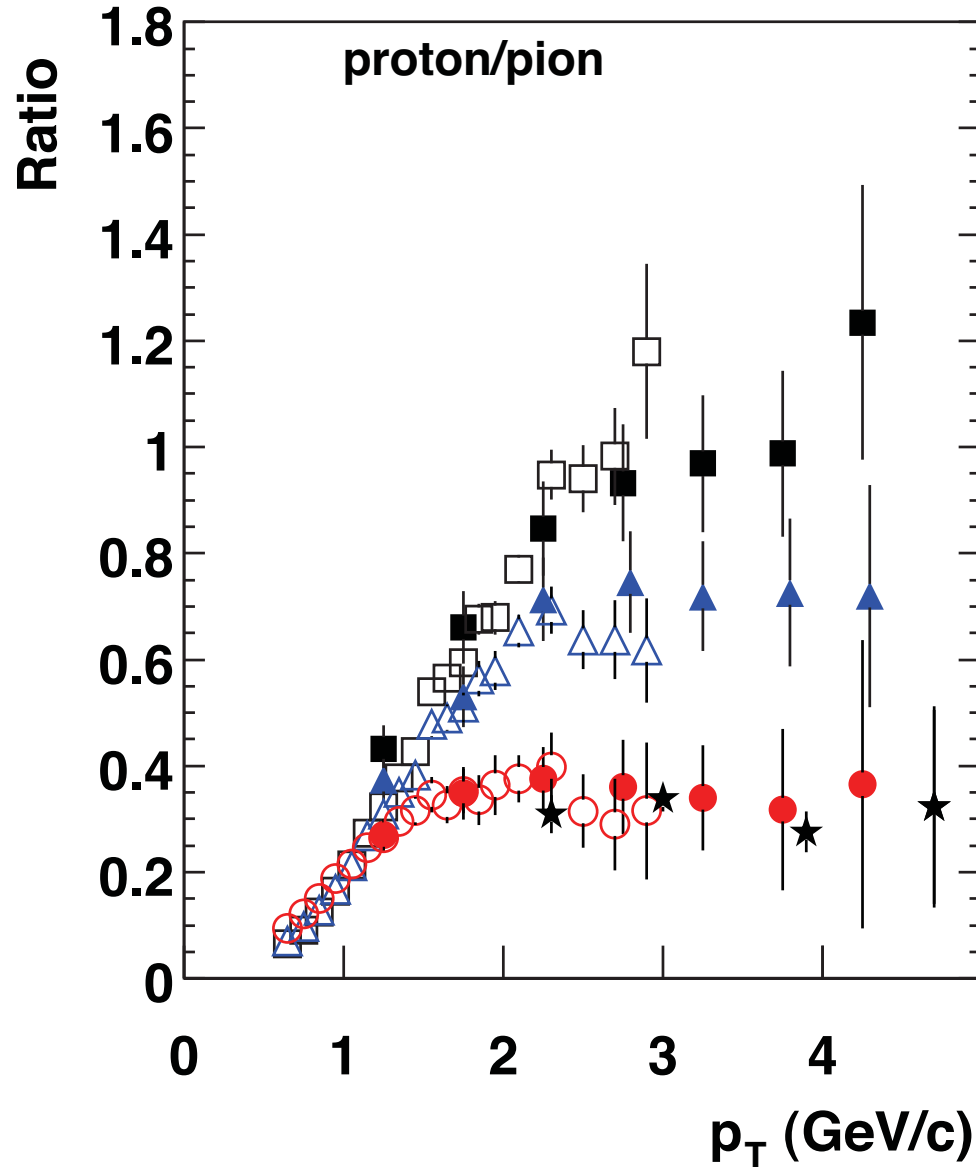
$$\mathbf{n}_{\text{eff}} = 2\mathbf{n}_{\text{active}} - 4$$

$$\mathbf{n}_{\text{eff}} = 8$$

*Baryon made directly within hard subprocess*



*Particle ratio changes with centrality!*



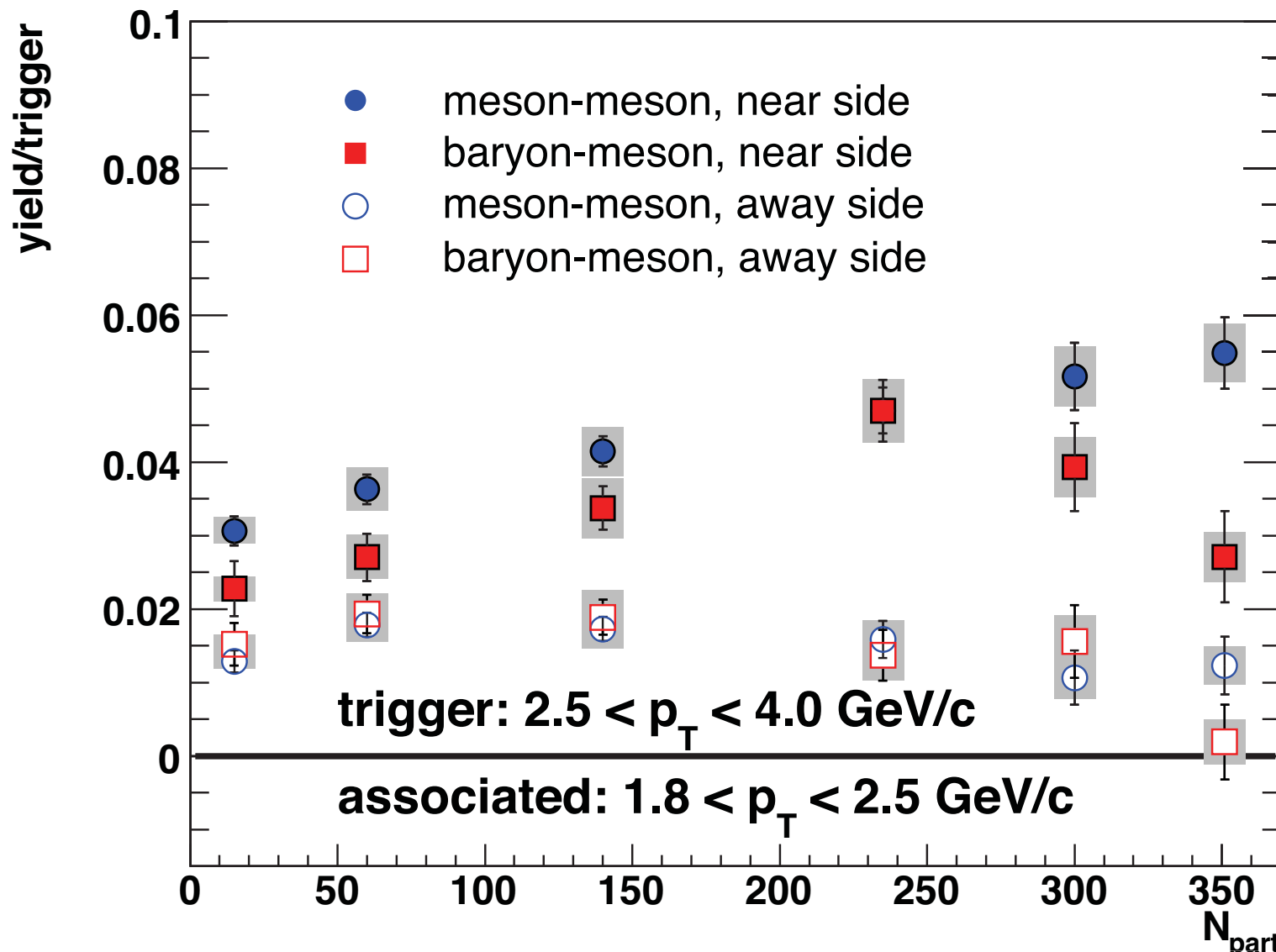
*Protons less absorbed  
in nuclear collisions than pions  
because of dominant  
color transparent higher twist process*

← **Central**

- ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ● Au+Au 60-92%
- ★ p+p,  $\sqrt{s} = 53$  GeV, ISR
- e<sup>+</sup>e<sup>-</sup>, gluon jets, DELPHI
- ..... e<sup>+</sup>e<sup>-</sup>, quark jets, DELPHI

← **Peripheral**

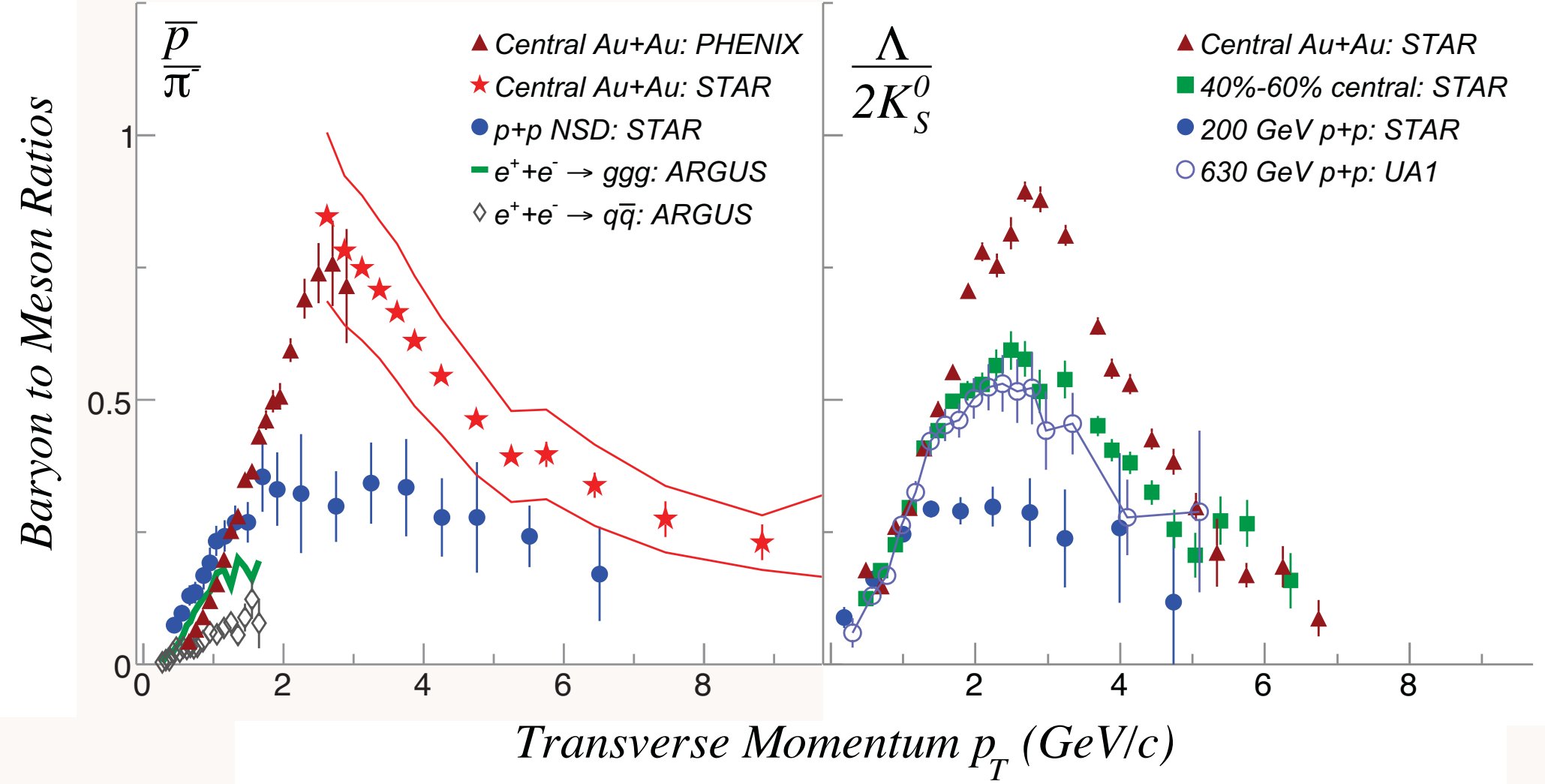




*proton trigger:  
# same-side  
particles  
decreases with  
centrality*

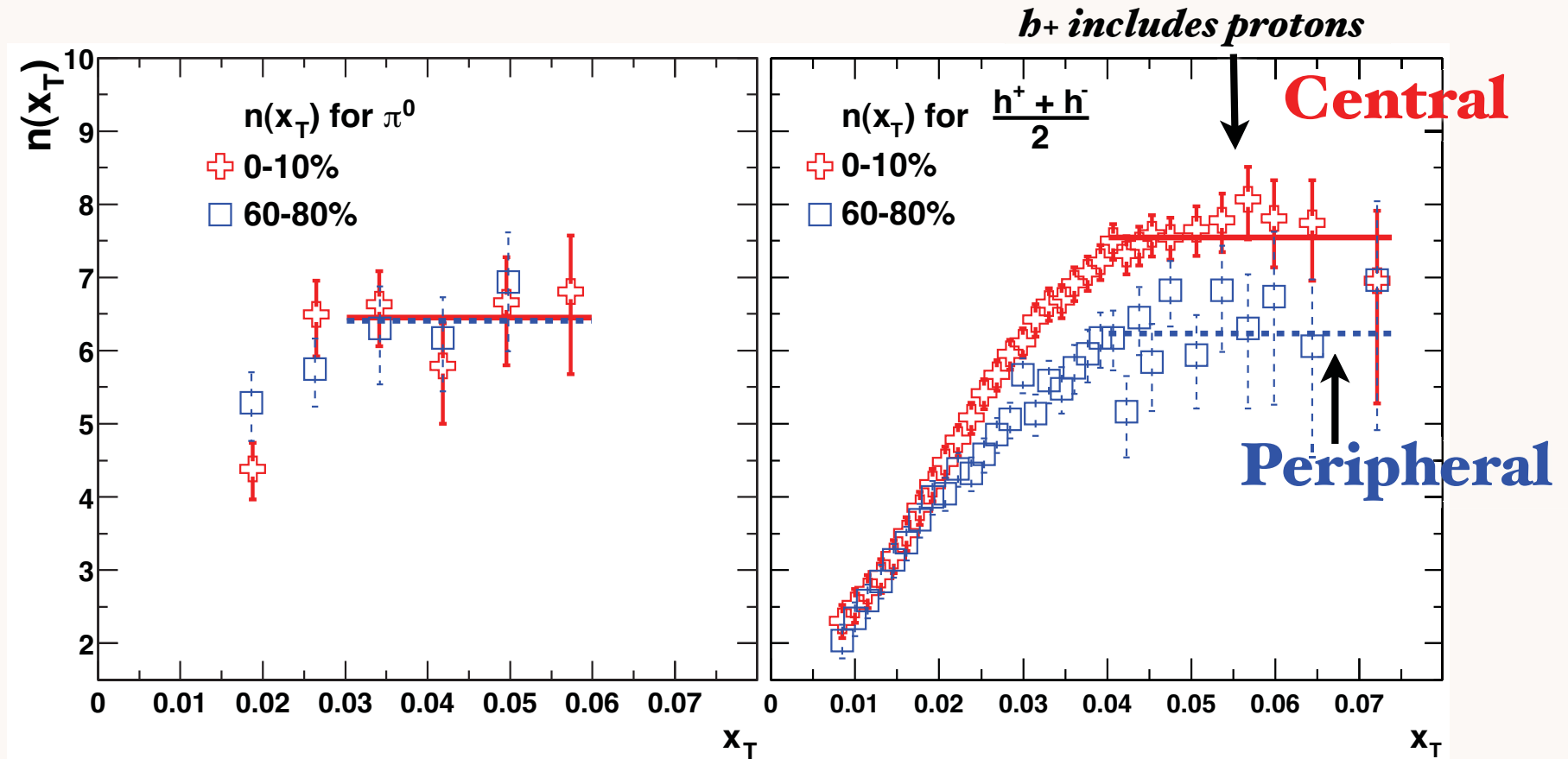


**Proton production more dominated by  
color-transparent direct high- $n_{eff}$  subprocesses**



Power-law exponent  $n(x_T)$  for  $\pi^0$  and  $h$  spectra in central and peripheral Au+Au collisions at  $\sqrt{s_{NN}} = 130$  and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev. C* **69**, 034910 (2004) [nucl-ex/0308006].



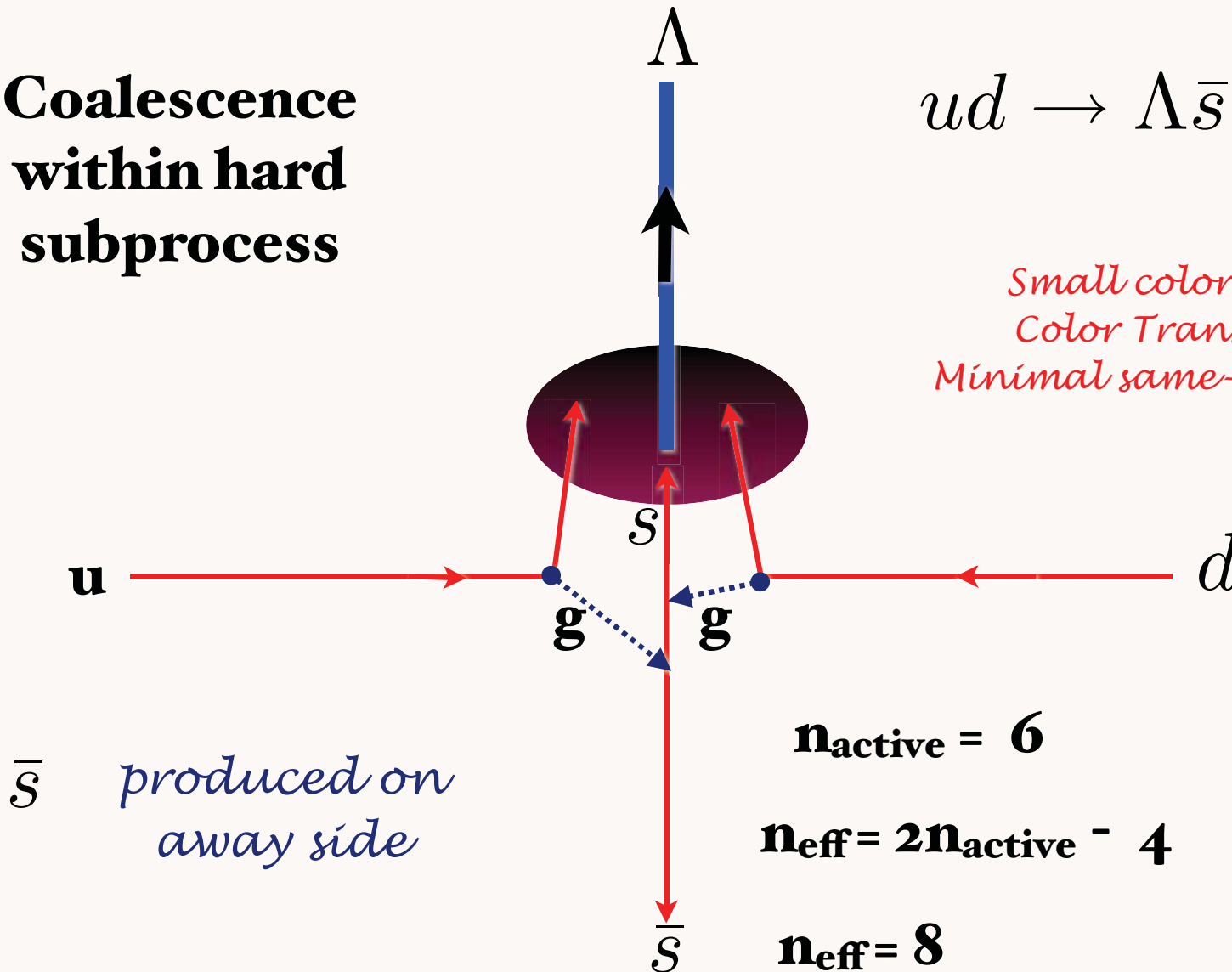
*Proton production dominated by color-transparent direct high  $n_{eff}$  subprocesses*

*Lambda can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

$$ud \rightarrow \Lambda \bar{s}$$

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

Sickles, sjb

# *Baryon Anomaly: Evidence for Direct, Higher-Twist Subprocesses*

- Explains anomalous power behavior at fixed  $x_T$
- Protons more likely to come from direct higher-twist subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Proton power  $n_{\text{eff}}$  increases with centrality since leading twist contribution absorbed
- Fewer same-side hadrons for proton trigger at high centrality
- Exclusive-inclusive connection at  $x_T = 1$

# Chiral Symmetry Breaking in AdS/QCD

We consider the action of the  $X$  field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left( g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right), \quad (1)$$

with equations of motion

$$z^3 \partial_z \left( \frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left( \frac{\mu_X R}{z} \right)^2 X = 0. \quad (2)$$

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$\left[ z^2 \partial_z^2 - 3z \partial_z + 3 \right] X(z) = 0. \quad (3)$$

for  $(\mu_X R)^2 = -3$ , which corresponds to scaling dimension  $\Delta_X = 3$ . The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \quad (4)$$

where  $A$  and  $B$  are determined by the boundary conditions.

**Ehrlich, Katz, Son, Stephanov**  
**Babington, Erdmenger, Evans, Kirsch, Guralnik, Thelfall**  
de Teramond, Shrock, sjb  
(preliminary)

$$A \propto m_q \qquad B \propto \langle \bar{\psi} \psi \rangle$$

*Expectation value taken inside hadron*

In presence of quark masses the Holographic LF wave equation is ( $\zeta = z$ )

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2} \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (1)$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \quad (2)$$

The parameter  $a$  is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,} \quad (3)$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle, \quad (4)$$

where we have used the sum over fractional longitudinal momentum  $\sum_i x_i = 1$ .

*Mass shift from dynamics inside hadronic boundary*

# Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted  $z^2$  distribution, not constant condensate**

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \phi^2(z) z^2$$

- **$z^2$  weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD supports confined condensate picture**

de Teramond, Shrock, sjb



# *Quark and Gluon condensates reside within hadrons, not vacuum*

Casher and Susskind

Roberts et al.

Shrock and sjb

- **Bound-State Dyson-Schwinger Equations**

Roberts et al.

- **LF vacuum trivial up to  $k^+ = 0$  zero modes**

- **Analogous to finite size superconductor**

- **Implications for cosmological constant --  
Eliminates 45 orders of magnitude conflict**

Shrock and sjb

Pion mass and decay constant.

[Pieter Maris](#), [Craig D. Roberts](#) ([Argonne, PHY](#)), [Peter C. Tandy](#) ([Kent State U.](#)) . ANL-PHY-8753-TH-97, KSUCNR-103-97, Jul 1997. 12pp.

Published in **Phys.Lett.B420:267-273,1998**.

e-Print: [nucl-th/9707003](#)

Pi- and K meson Bethe-Salpeter amplitudes.

[Pieter Maris](#), [Craig D. Roberts](#) ([Argonne, PHY](#)) . ANL-PHY-8788-TH-97, Aug 1997. 34pp.

Published in **Phys.Rev.C56:3369-3383,1997**.

e-Print: [nucl-th/9708029](#)

Concerning the quark condensate.

[K. Langfeld](#) ([Tubingen U.](#)), [H. Markum](#) ([Vienna, Tech. U.](#)), [R. Pullirsch](#) ([Regensburg U.](#)), [C.D. Roberts](#) ([Argonne, PHY](#) & [Rostock U.](#)), [S.M. Schmidt](#) ([Tubingen U.](#) & [HGF, Bonn](#)) . ANL-PHY-10460-TH-2002, MPG-VT-UR-239-02, Jan 2003. 7pp.

Published in **Phys.Rev.C67:065206,2003**.

e-Print: [nucl-th/0301024](#)

“*In-Meson Condensate*”

$$- \langle \bar{q}q \rangle_{\zeta}^{\pi} = f_{\pi} \langle 0 | \bar{q} \gamma_5 q | \pi \rangle .$$

Valid even for  $m_q \rightarrow 0$

$f_{\pi}$  nonzero

# QCD Symmetries

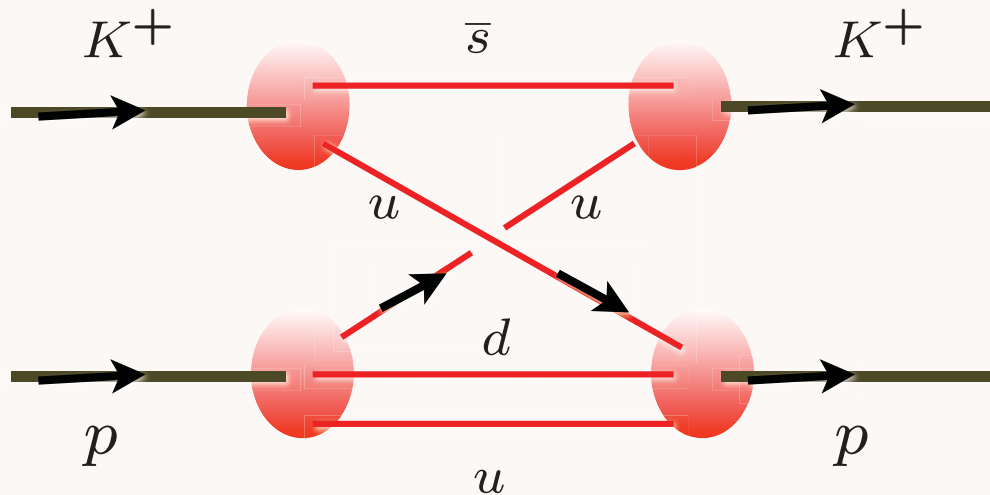
- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Provides Conformal Template
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict

# *Hadron Dynamics at the Amplitude Level*

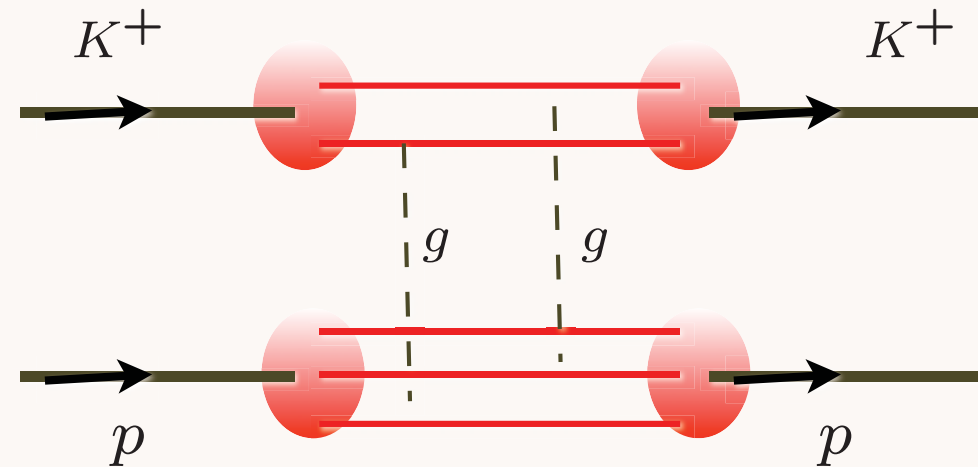
- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect
- Higher Fock States give GMOR Relations, Chiral Symmetry Breaking

# *New Perspectives for QCD from AdS/CFT*

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support  $0 < x < 1$ .
- Quark Interchange dominant force at short distances



*Quark Interchange  
(Spin exchange in atom-atom scattering)*



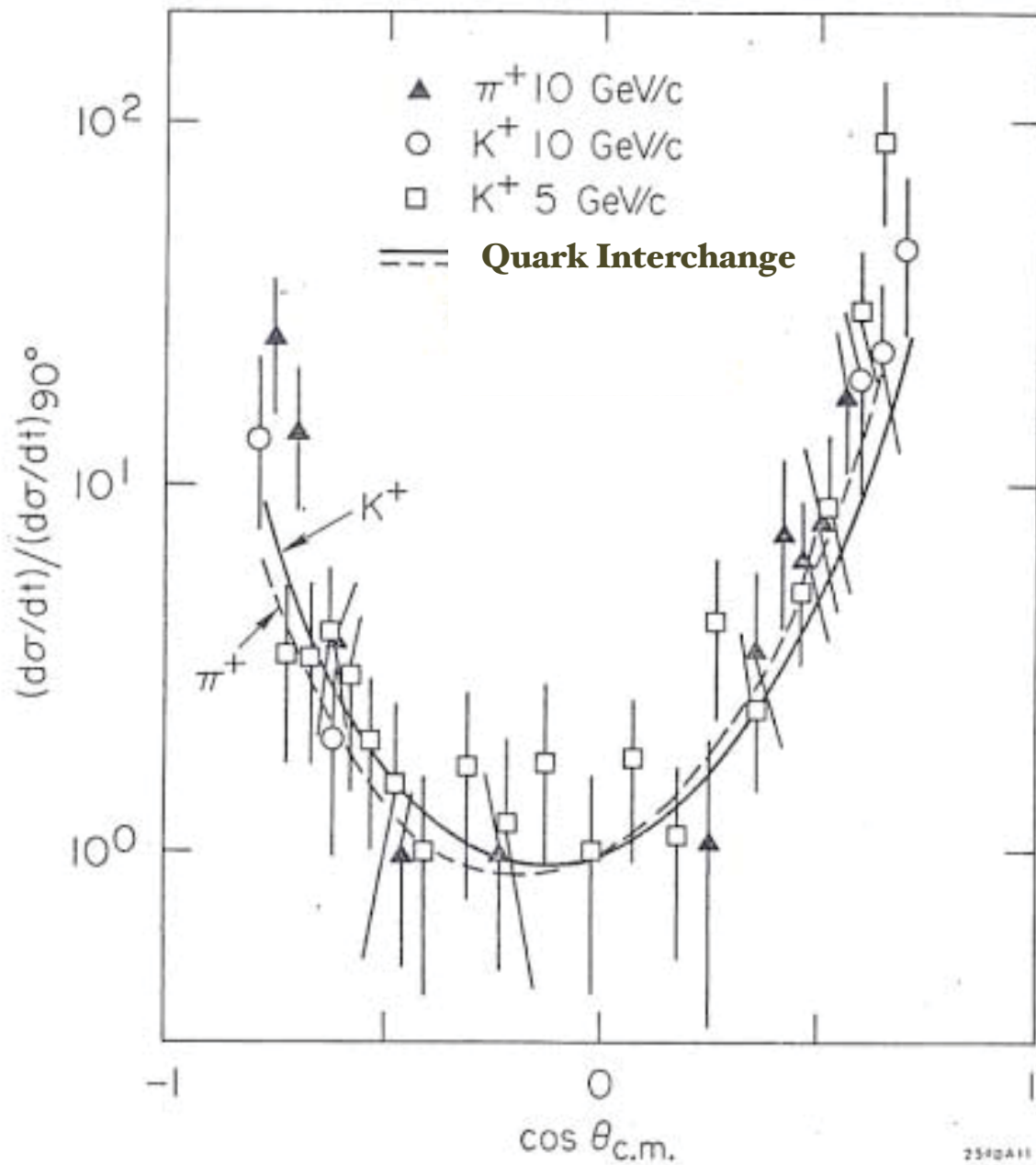
*Gluon Exchange  
(Van der Waal -- Landshoff)*

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s, t)_{\text{gluonexchange}} \propto sF(t)$$

*MIT Bag Model (de Tar), large \$N\_c\$, ('t Hooft), AdS/CFT  
all predict dominance of quark interchange:*



*AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions*

$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

**Non-linear Regge behavior:**

$$\alpha_R(t) \rightarrow -1$$

# Why is quark-interchange dominant over gluon exchange?

Example:  $M(K^+_p \rightarrow K^+_p) \propto \frac{1}{ut^2}$

Exchange of common  $u$  quark

$$M_{QIM} = \int d^2k_\perp dx \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$$

Holographic model (Classical level):

Hadrons enter 5th dimension of  $AdS_5$

Quarks travel freely within cavity as long as separation  $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.



## Comparison of Exclusive Reactions at Large $t$

B. R. Baller,<sup>(a)</sup> G. C. Blazey,<sup>(b)</sup> H. Courant, K. J. Heller, S. Heppelmann,<sup>(c)</sup> M. L. Marshak,  
E. A. Peterson, M. A. Shupe, and D. S. Wahl<sup>(d)</sup>

*University of Minnesota, Minneapolis, Minnesota 55455*

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi

*Brookhaven National Laboratory, Upton, New York 11973*

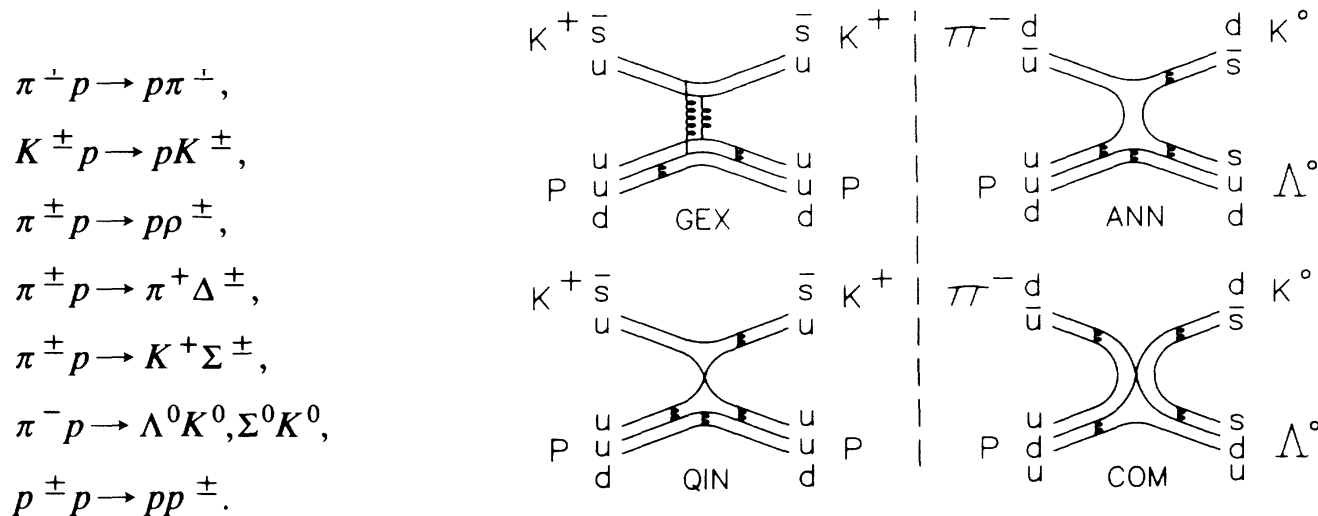
and

S. Gushue<sup>(e)</sup> and J. J. Russell

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Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.:  $\pi^\pm p \rightarrow p\pi^\pm, p\rho^\pm, \pi^+\Delta^\pm, K^+\Sigma^\pm, (\Lambda^0/\Sigma^0)K^0, K^\pm p \rightarrow pK^\pm; p^\pm p \rightarrow pp^\pm$ . By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.



# Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion ( $m_q = 0$ )
- Regge Trajectories: universal slope in  $n$  and  $L$
- Valid for all integer  $J$  &  $S$ . Spectrum is independent of  $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large  $N_c$  limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize  $H_{LF}$  on AdS basis

String Theory



AdS/CFT

Mapping of Poincare' and Conformal  $SO(4,2)$  symmetries of 3+1 space to AdS5 space



AdS/QCD

Conformal behavior at short distances + Confinement at large distance

*Goal: First Approximant to QCD*  
Counting rules for Hard Exclusive Scattering  
Regge Trajectories  
QCD at the Amplitude Level



Semi-Classical QCD / Wave Equations

Holography



Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$  plus  $L$

Integrable!



Hadron Spectra, Wavefunctions, Dynamics