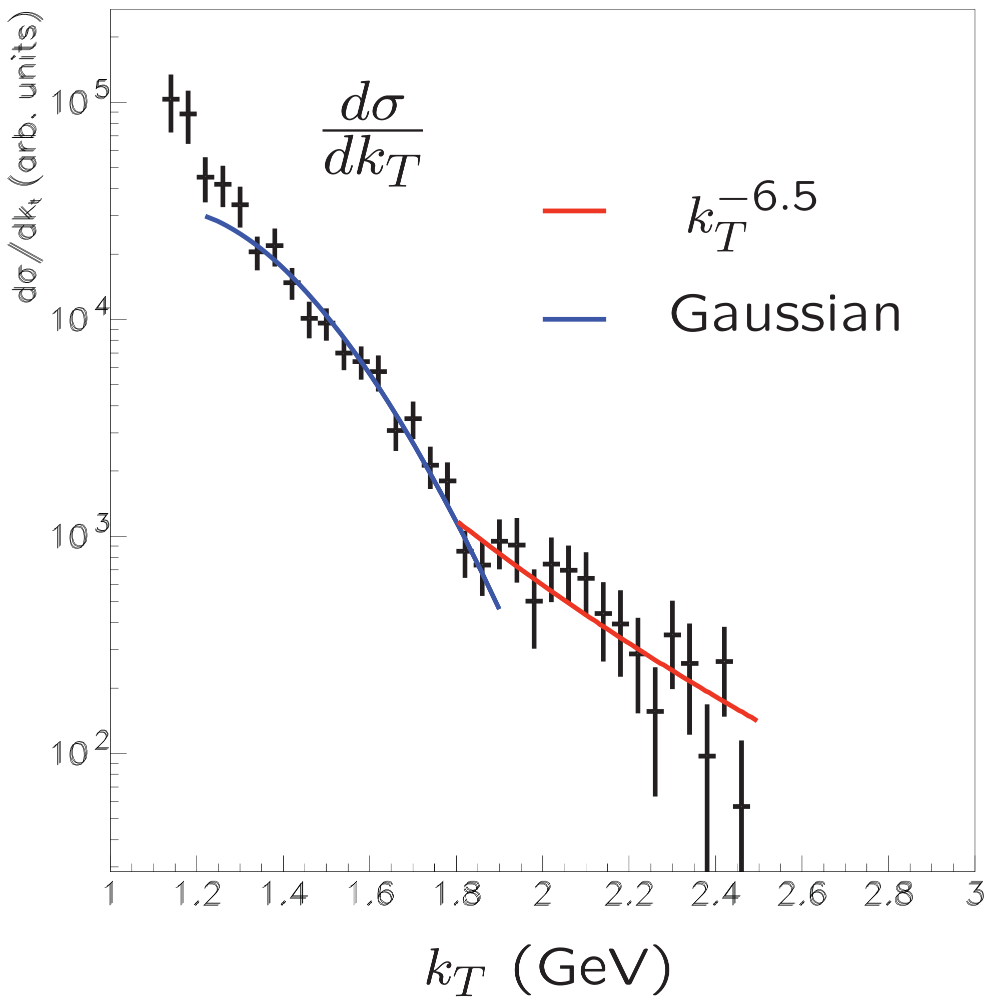
# E791 Diffractive Di-Jet transverse momentum distribution

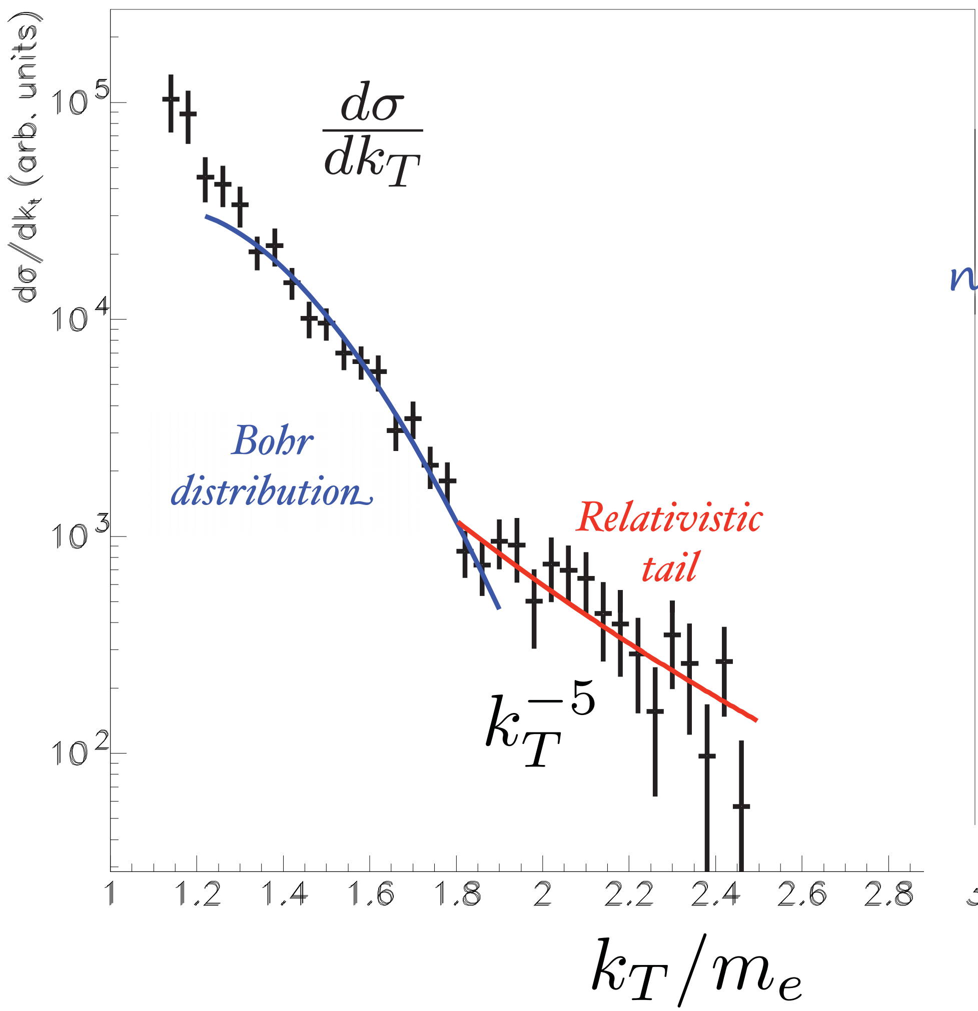



## Two Components

*High Transverse momentum dependence consistent with PQCD, ERBL Evolution* $k_T^{-6.5}$

*Gaussian component similar to AdS/CFT HO LFWF*

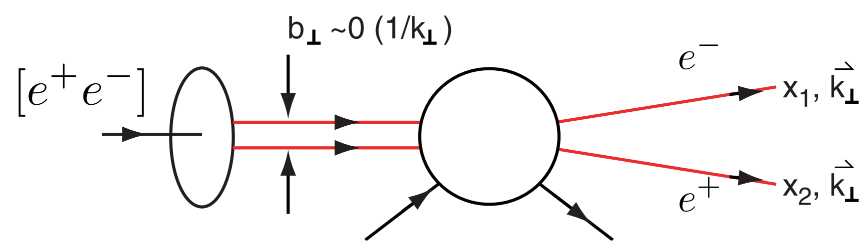
# Simulated diffractive transverse momentum distribution for positronium




## Two Components:

Low momentum dependence from non-relativistic Coulomb interaction

High Transverse momentum dependence from relativistic QED



# Positronium LFWF

$$\left(M^2 - \frac{k_\perp{}^2 + m_1{}^2}{x_1} - \frac{k_\perp{}^2 + m_2{}^2}{x_2}\right)\psi(x_i, k_\perp) = \int_0^1 [dy] \int_0^\infty \frac{d^2 l_\perp}{16\pi^3} \tilde{K}(x_i, k_\perp; y_i, l_\perp; M^2)\psi(y_i, l_\perp)$$

$$\tilde{K} \simeq \frac{-16e^2m^2}{(k_\perp - l_\perp)^2 + (x-y)^2 m^2}$$

## Non-relativistic limit

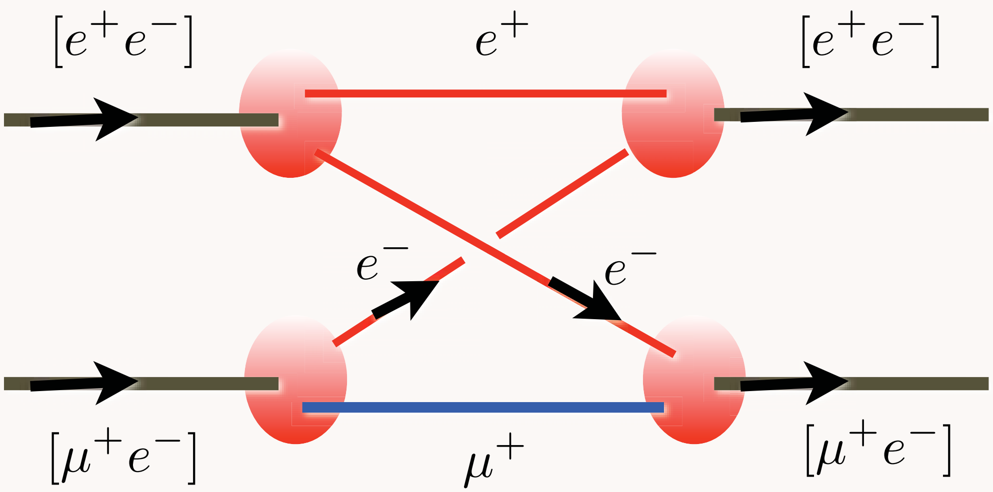
$$\Psi = \left(\frac{m\beta^3}{\pi}\right)^{1/2} \frac{64\pi\beta x_1 x_2}{[k_\perp{}^2 + (x_1 - x_2)^2 m^2 + \beta^2]^2} \times \begin{cases} \frac{u_\uparrow \bar{v}_\downarrow - u_\downarrow \bar{v}_\uparrow}{(2x_1x_2)^{1/2}}, & \text{parapositronium}, \\ \frac{u_\uparrow \bar{v}_\uparrow}{(x_1x_2)^{1/2}}, & \text{orthopositronium}, \\ \cdots, \end{cases}$$

$$\psi_{NR}(x, k_\perp) \sim \frac{1}{k_\perp^4}$$

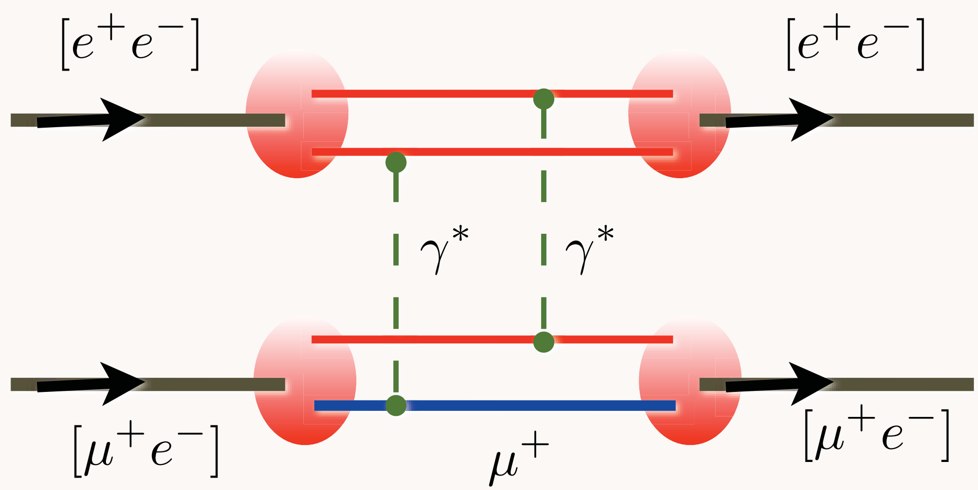
$$\beta = \alpha m_e/2$$

At large $k_\perp^2 >> m_e^2$, $\psi(x, k_\perp) \sim \frac{1}{k_\perp^2}$

CIM: Blankenbecler, Gunion, sjb




Constituent Interchange
Spin exchange in atom-atom scattering



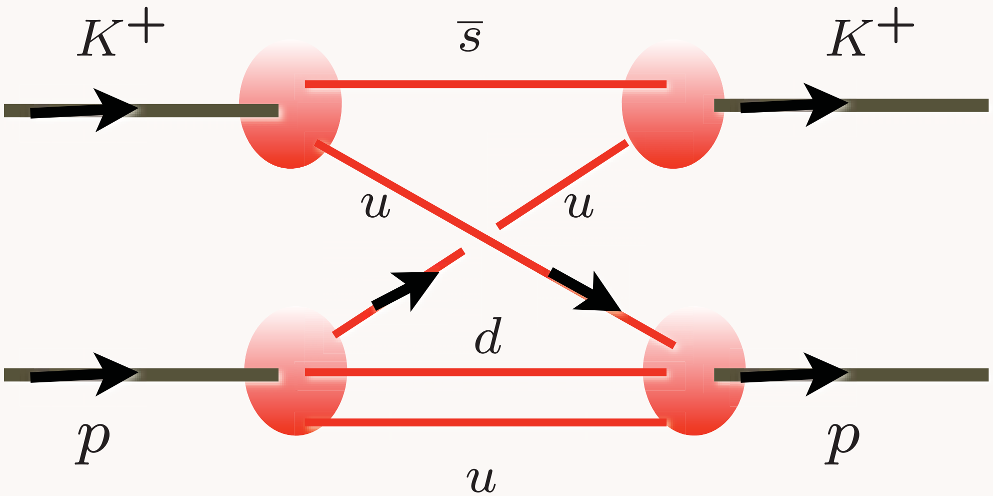

Two-Photon Exchange
(Van der Waal )

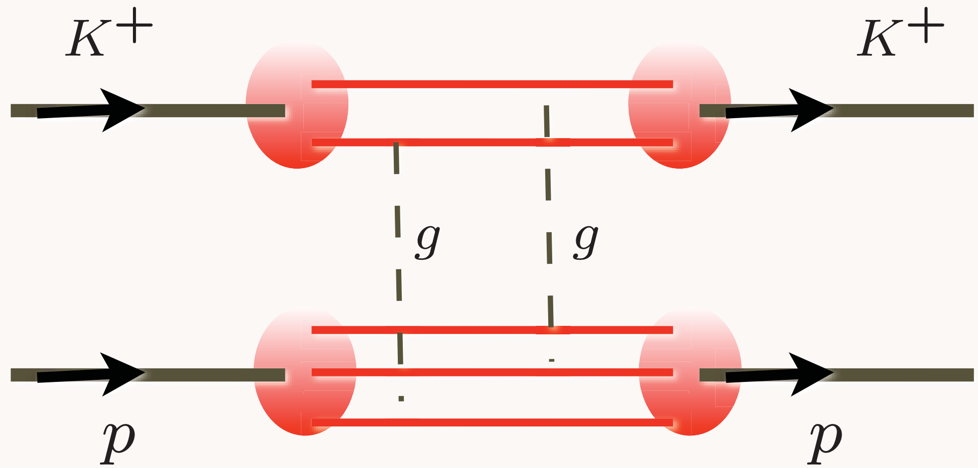
$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

CIM: Blankenbecler, Gunion, sjb




*Quark Interchange*
*(Spin exchange in atom-atom scattering)*

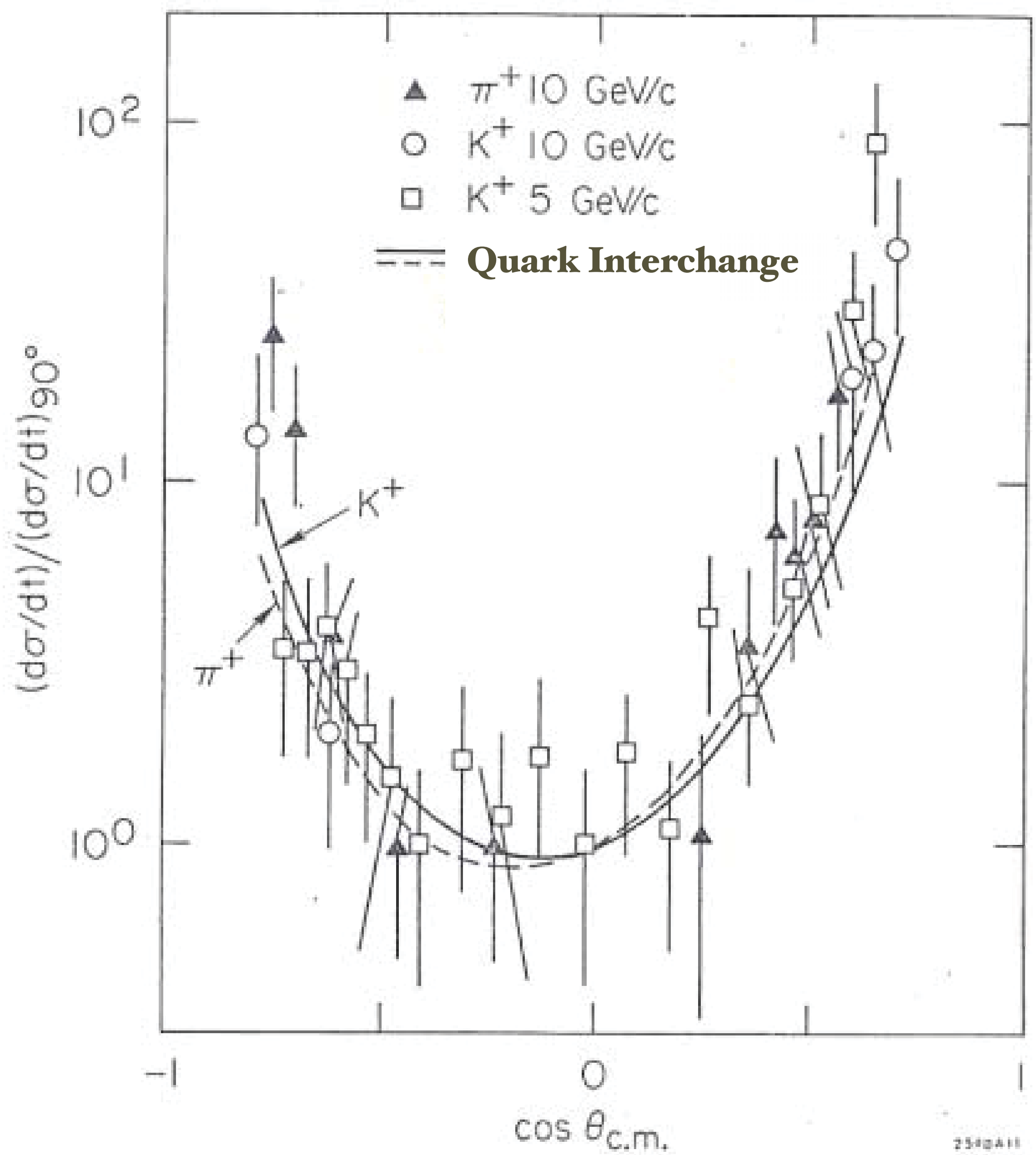
*Gluon Exchange*
*(Van der Waal -- Landshoff)*

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

*MIT Bag Model (de Tar), large $N_C$, ('t Hooft), AdS/CFT all predict dominance of quark interchange:*




*AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions*

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

***Non-linear Regge behavior:***

$$\alpha_R(t) \to -1$$

*Single-spin asymmetries*

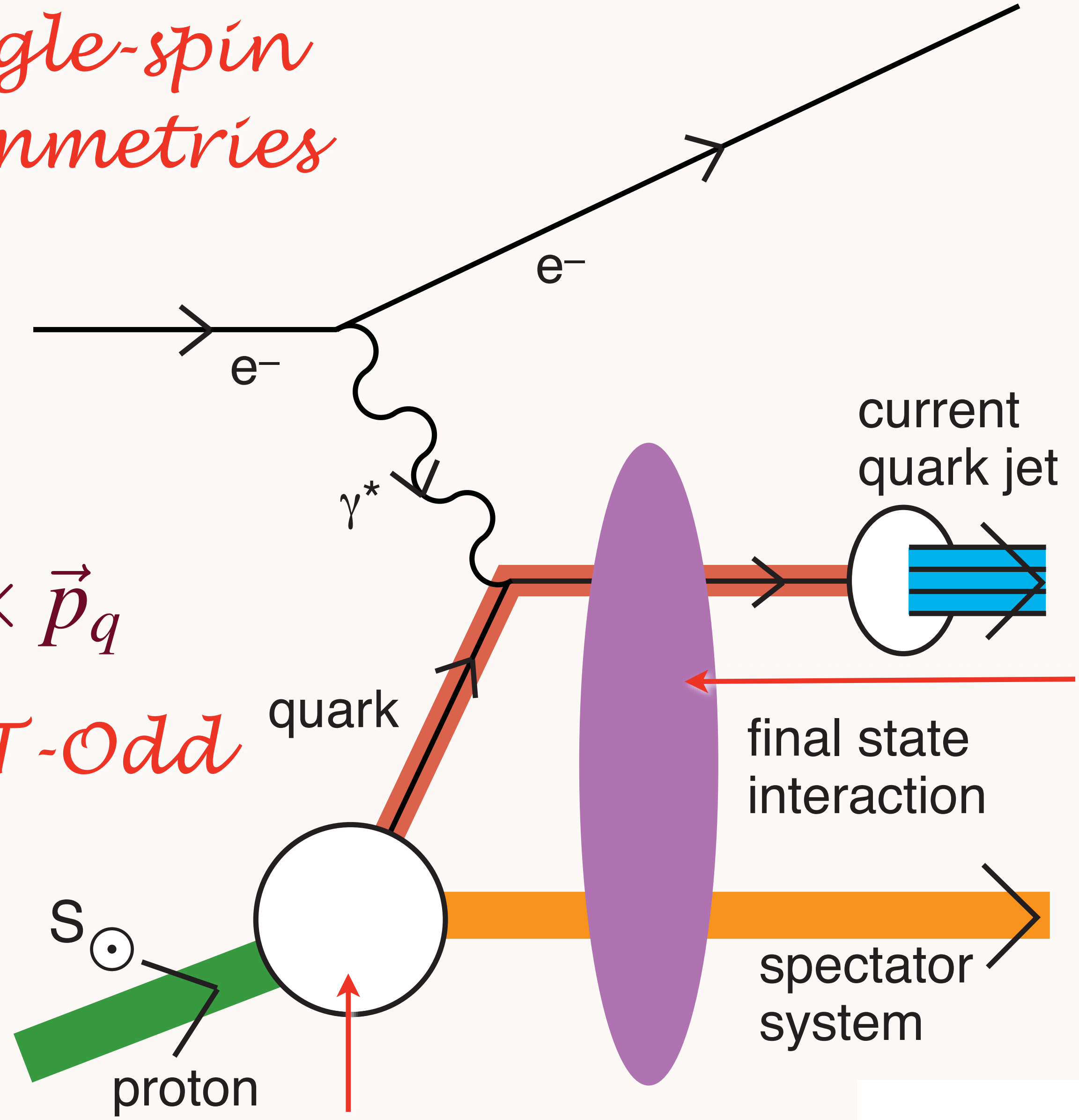
# Leading-Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo-T-Odd*



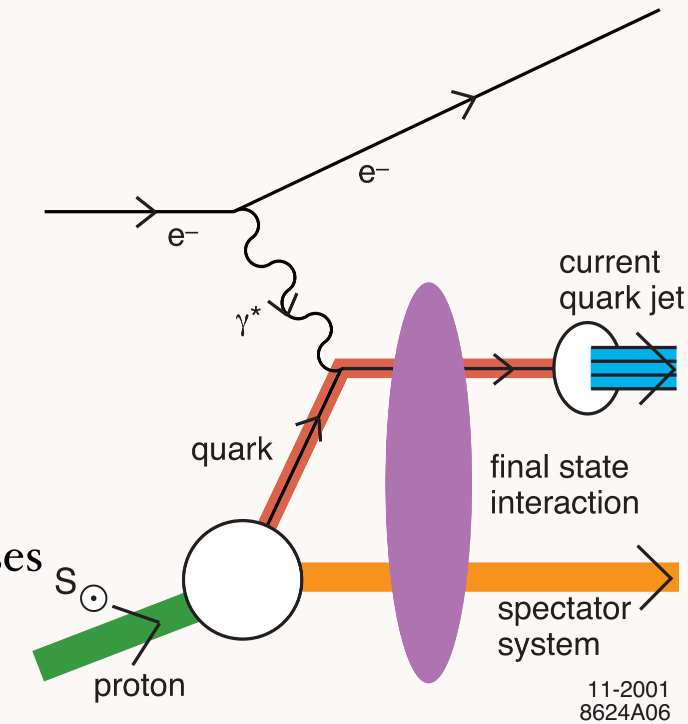

*QCD S- and P-Coulomb Phases --Wilson Line*

*Light-Front Wavefunction S and P- Waves*

Analog of QED FSIs

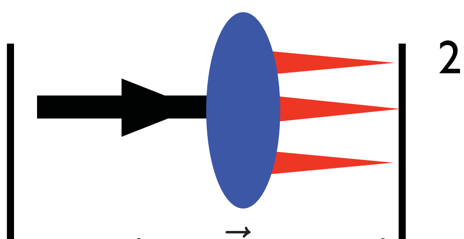
# Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

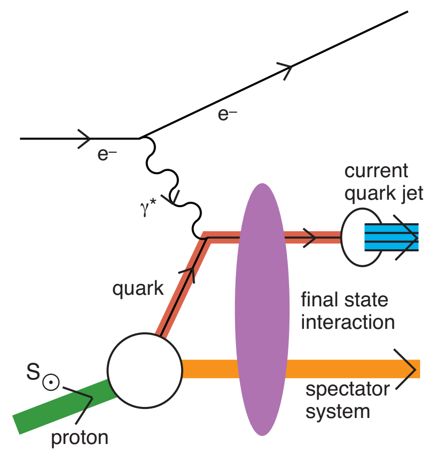



- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

| Static | Dynamic |
|---|---|
| Square of Target LFWFs | Modified by Rescattering: ISI & FSI |
| No Wilson Line | Contains Wilson Line, Phases |
| Probability Distributions | No Probabilistic Interpretation |
| Process-Independent | Process-Dependent - From Collision |
| T-even Observables | T-Odd (Sivers, Boer-Mulders, etc.) |
| No Shadowing, Anti-Shadowing | Shadowing, Anti-Shadowing, Saturation |
| Sum Rules: Momentum and $J^z$ | Sum Rules Not Proven |
| DGLAP Evolution; mod. at large x | DGLAP Evolution |
| No Diffractive DIS | Hard Pomeron and Odderon Diffractive DIS |

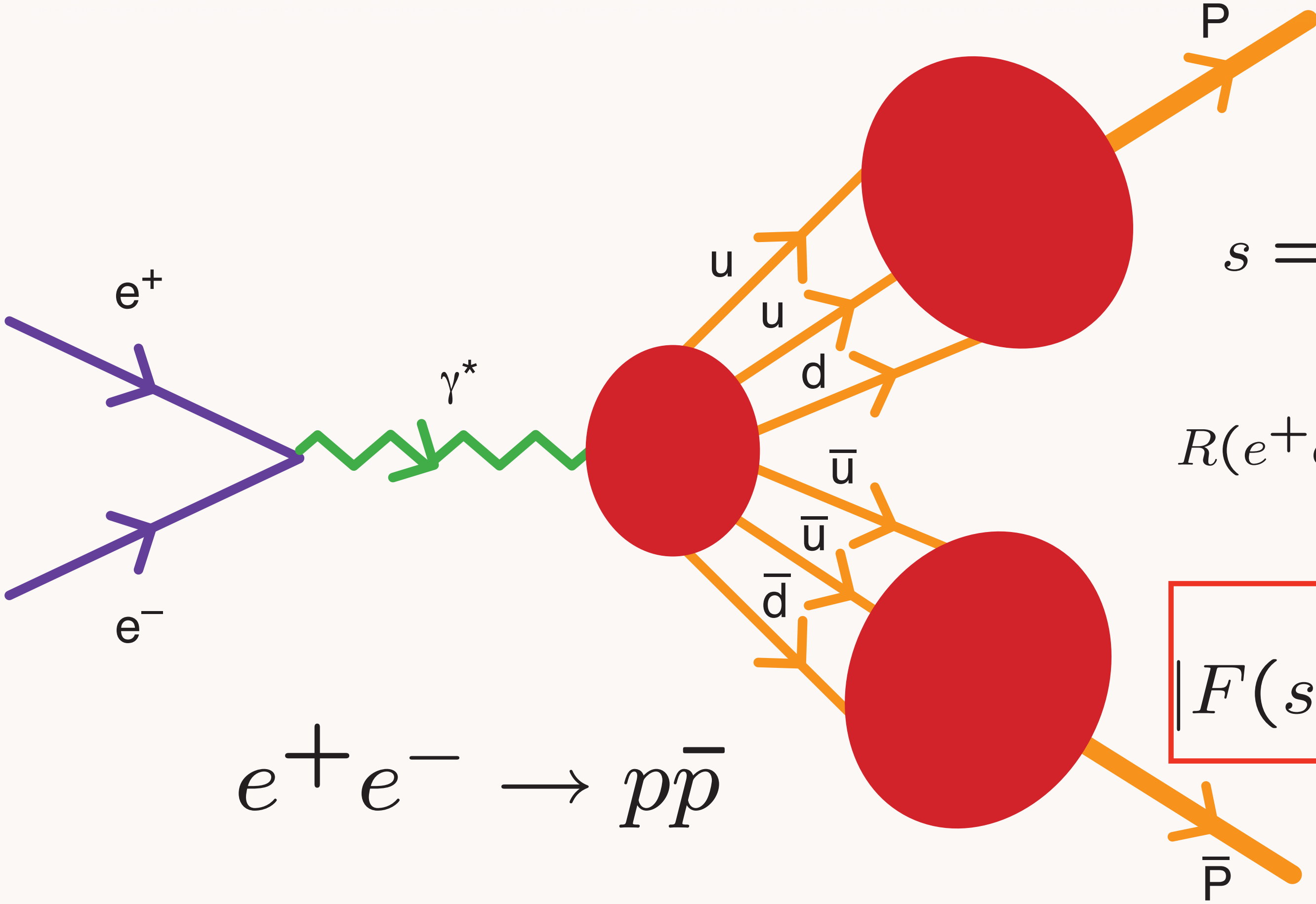



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



# Exclusive Processes

**What if we ask for a specific final state?**




$$e^+e^- \rightarrow p\bar{p}$$

$$s = (E_{e^+} + E_{e^-})^2$$

$$R(e^+e^- \rightarrow H\bar{H}) \propto |F(s)|^2$$

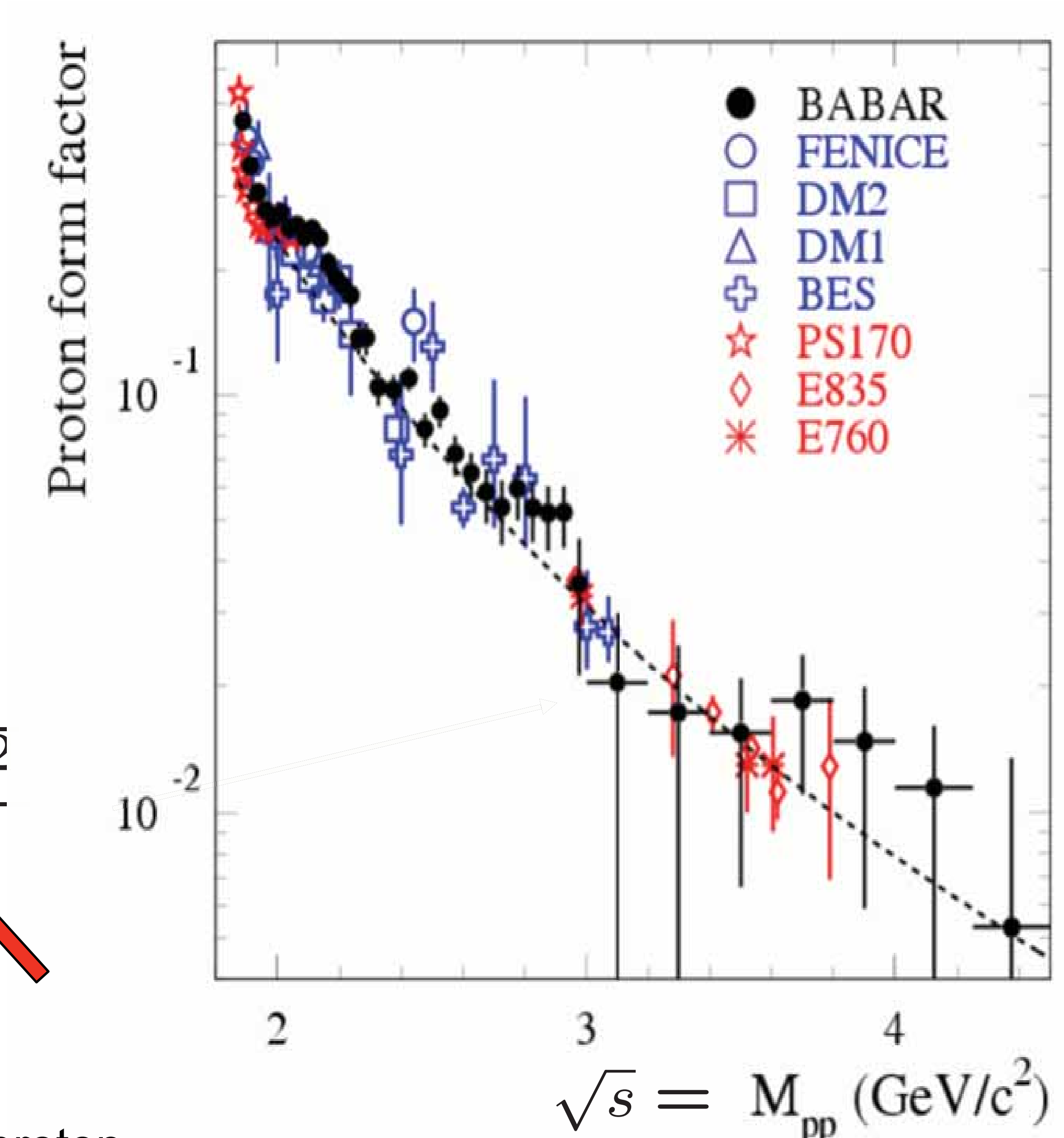
$$|F(s)| \propto [\tfrac{1}{s}]^{n_q - 1}$$

**Probability decreases with number of constituents!**

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2,$$

$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$

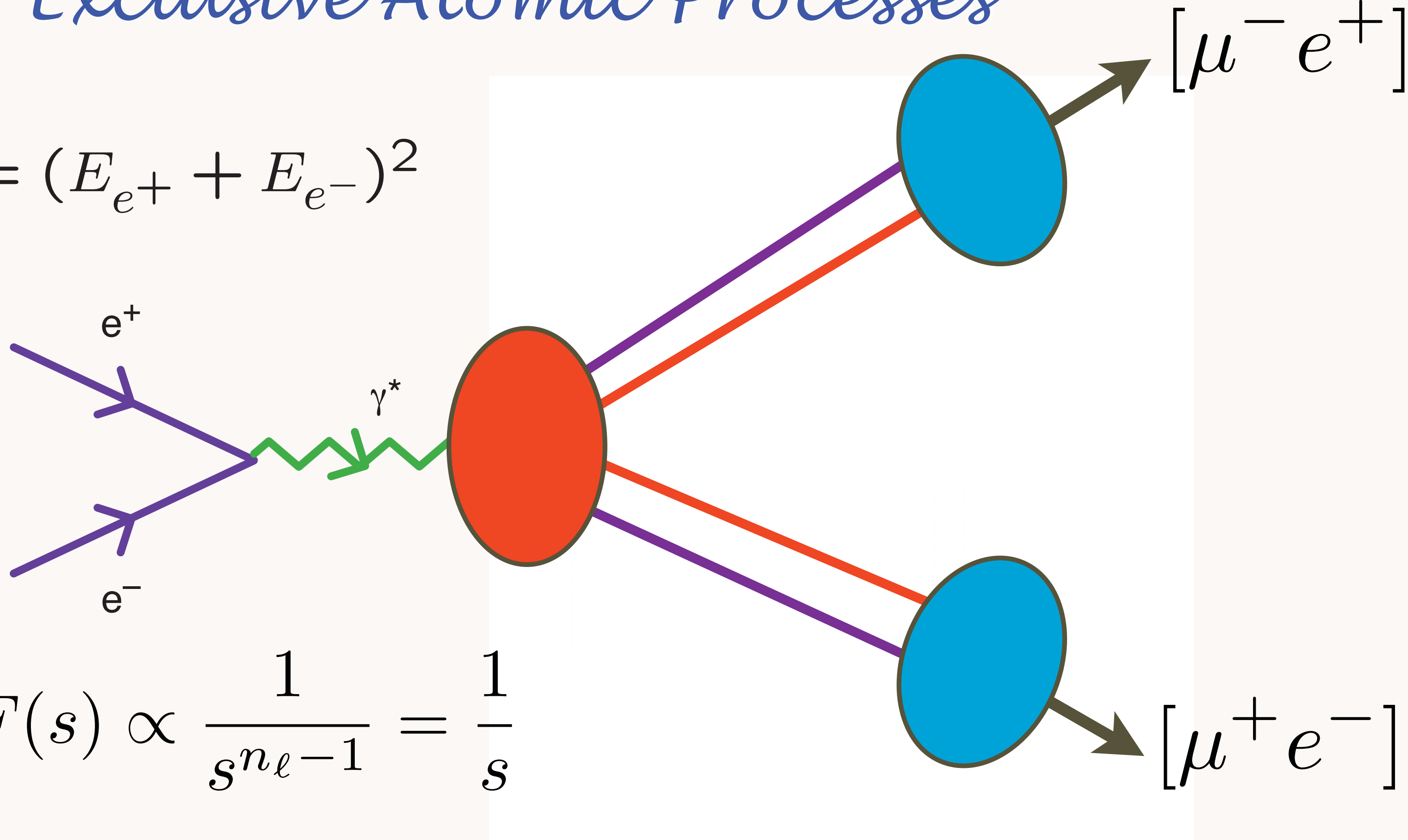
$n_q - 1 = 3 - 1 = 2$




Quark counting for 3 quarks in proton

# Exclusive Atomic Processes

$$s = (E_{e^+} + E_{e^-})^2$$

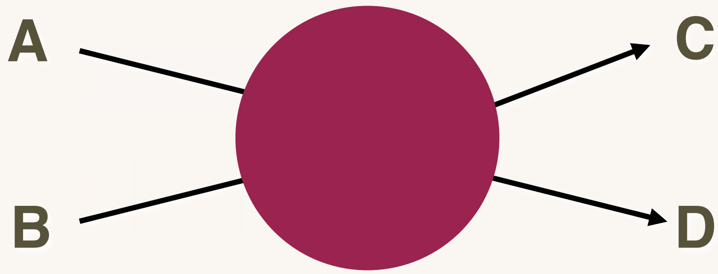



$$F(s) \propto \frac{1}{s^{n_\ell - 1}} = \frac{1}{s}$$

$$R = \frac{\sigma(e^+e^- \to [\mu^+e^-] + [\mu^-e^+])}{\sigma(e^+e^- \to \mu^+\mu^-)} = |F(s)|^2 \propto \frac{1}{s^2}$$

**Probability decreases with number of constituents**

# Constituent Counting Rules




$n_{tot} = n_A + n_B + n_C + n_D$

Fixed $t/s$ or $\cos\theta_{cm}$

$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

**Farrar & sjb;**
**Matveev, Muradyan, Tavkhelidze**

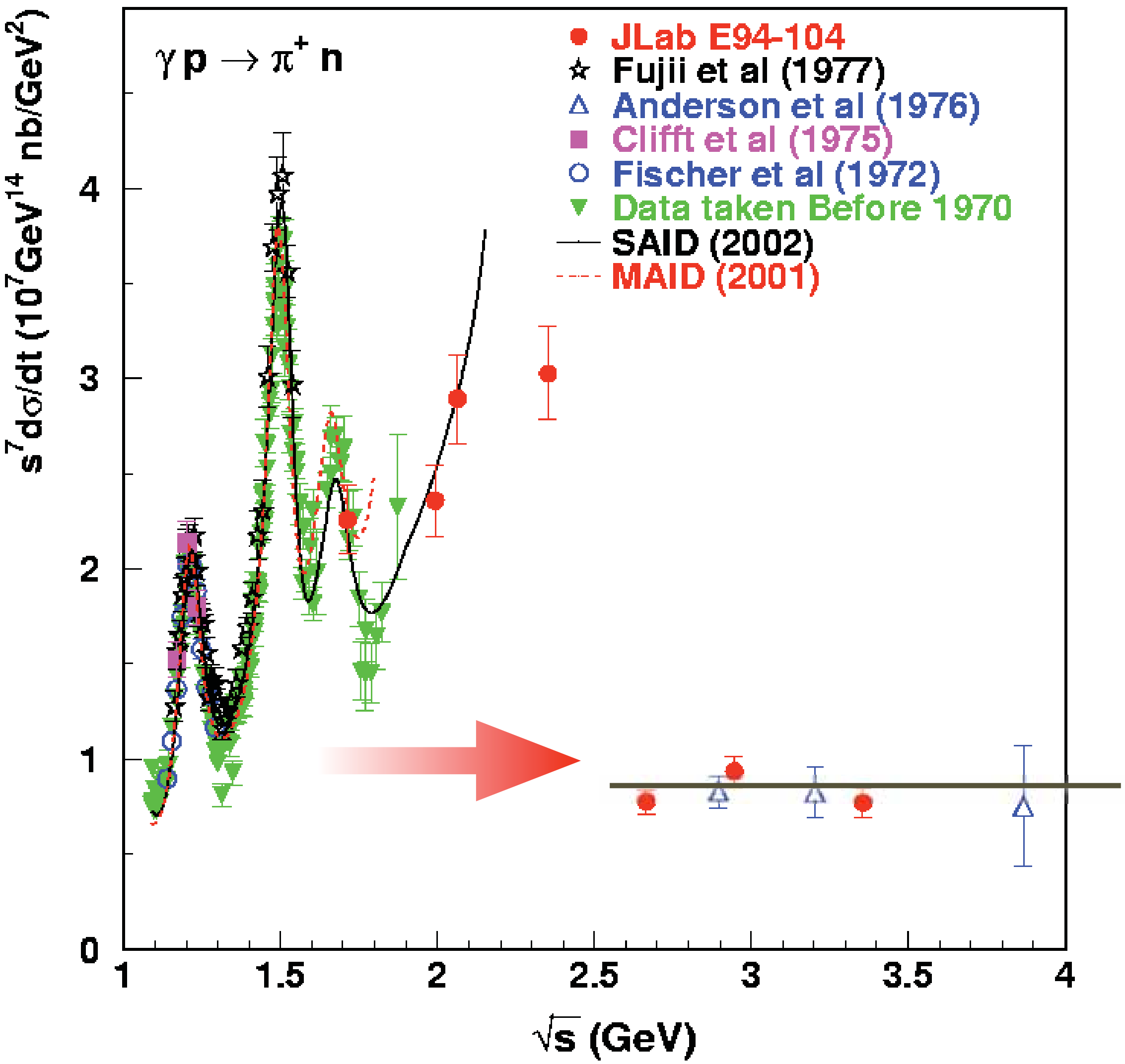
*QED predicts leading-twist scaling behavior of fixed-CM angle exclusive amplitudes*

$$s, -t >> m^2_\ell$$

# Test of Scaling Laws

Constituent counting rules

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719



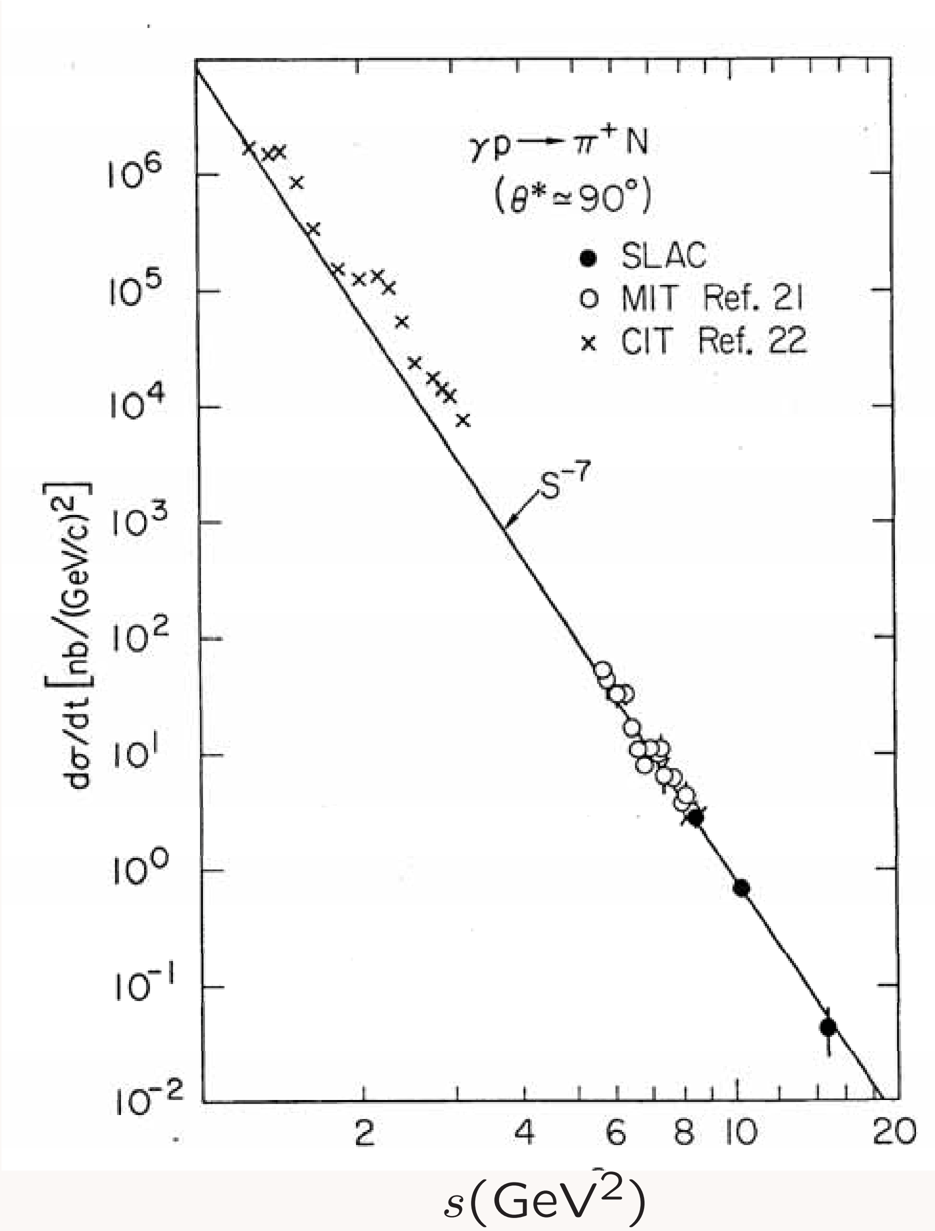

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \rightarrow C+D) = F_{A+B\rightarrow C+D}(\theta_{CM})$$

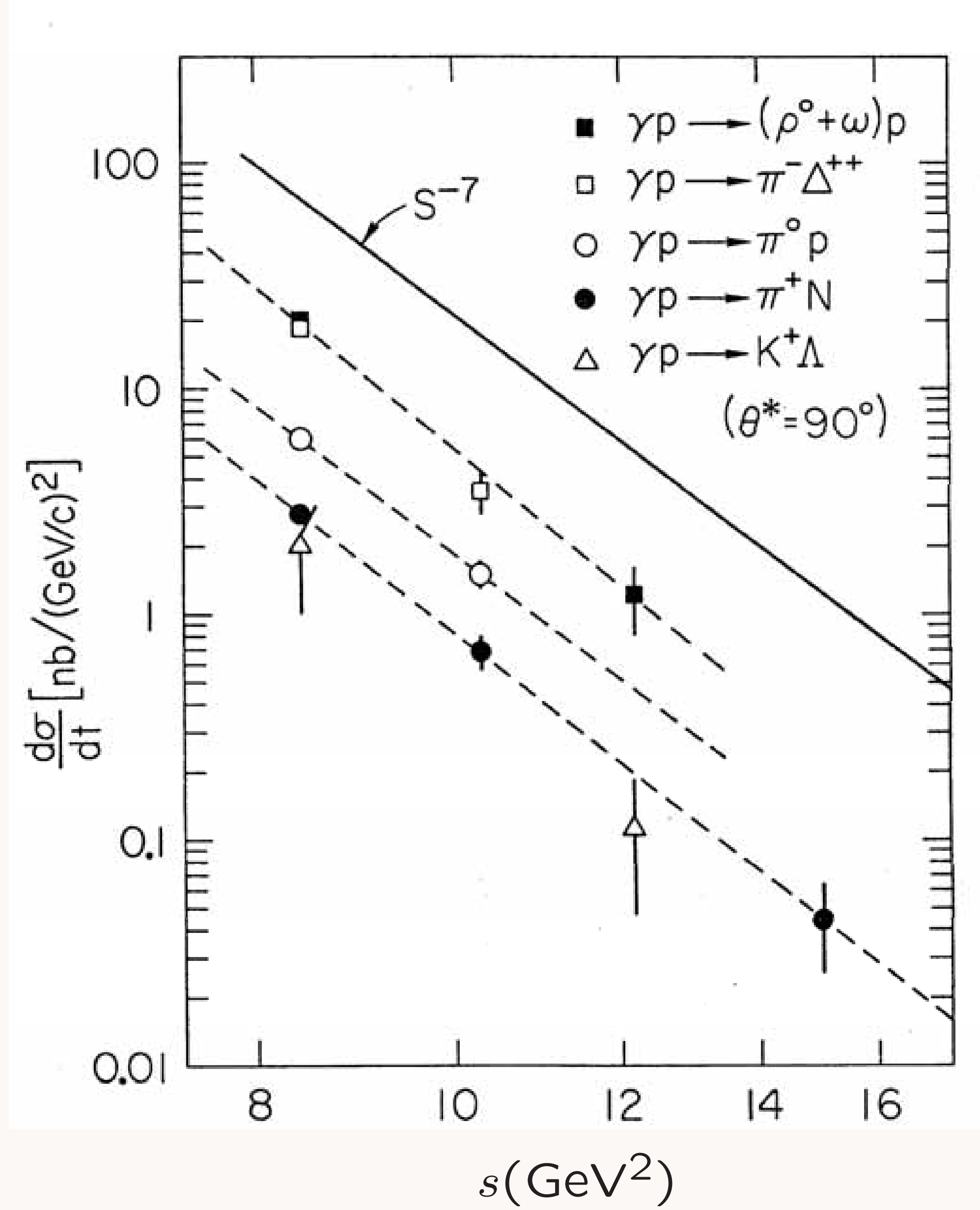
$$s^7\frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

$s^7 d\sigma/dt(\gamma p \rightarrow \pi^+ n) \sim const$
*fixed* $\theta_{CM}$ scaling

Conformal invariance at high momentum transfers!

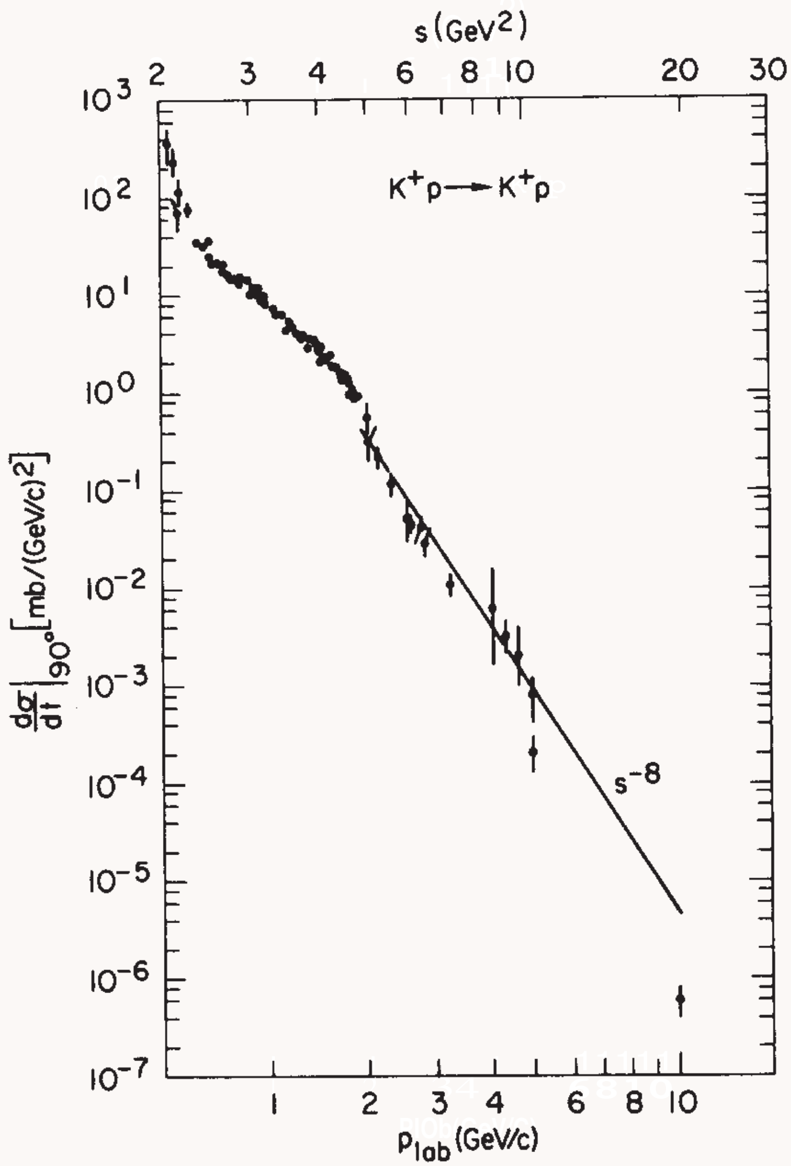



Counting Rules: n=9

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

# Quark-Counting

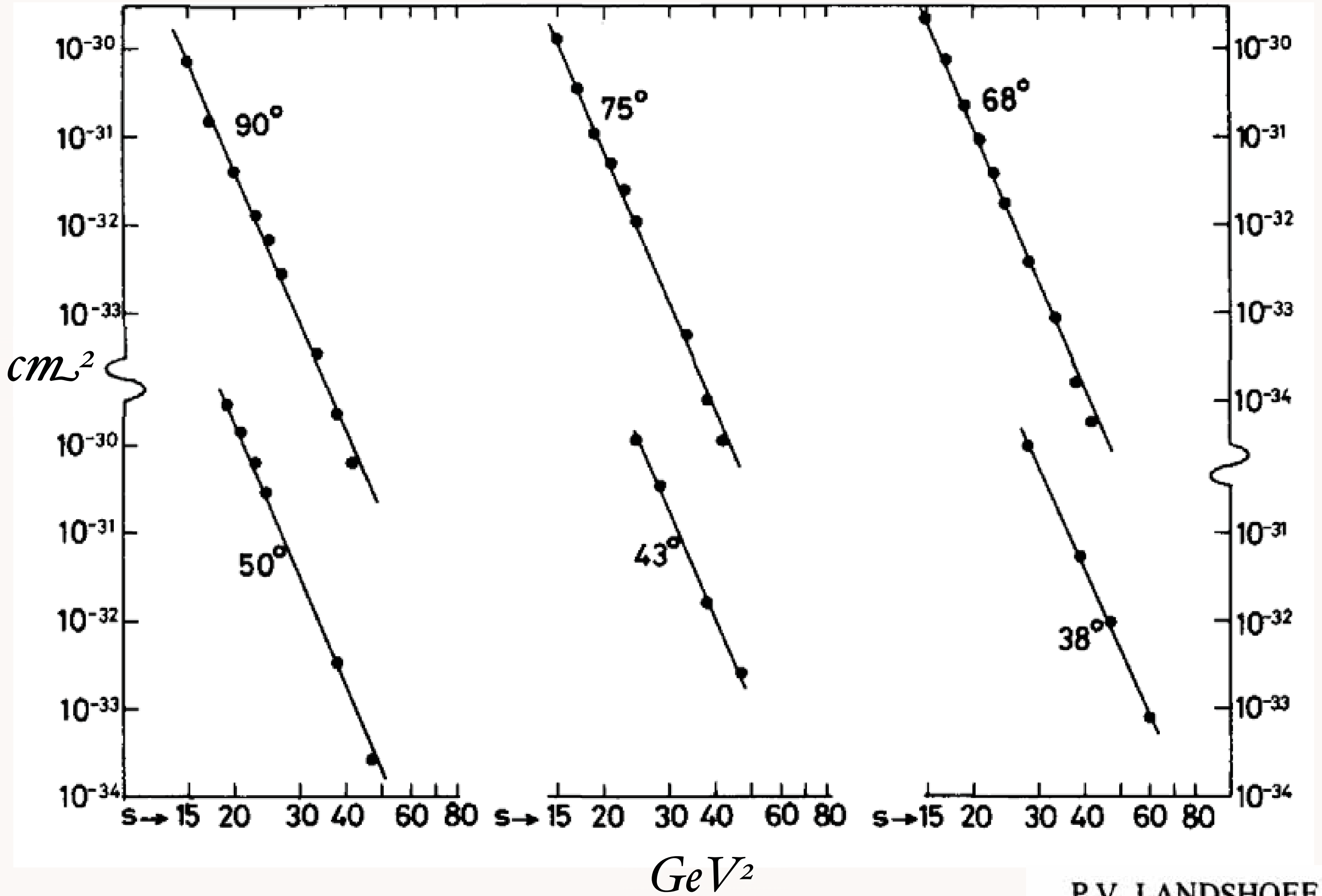



$$\frac{d\sigma}{dt}(K^+p \to K^+p) = \frac{F(\theta_{CM})}{s^8}$$

$$n = 2 \times 3 + 2 \times 2 - 2 = 8$$

## Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$

$n = 4 \times 3 - 2 = 10$



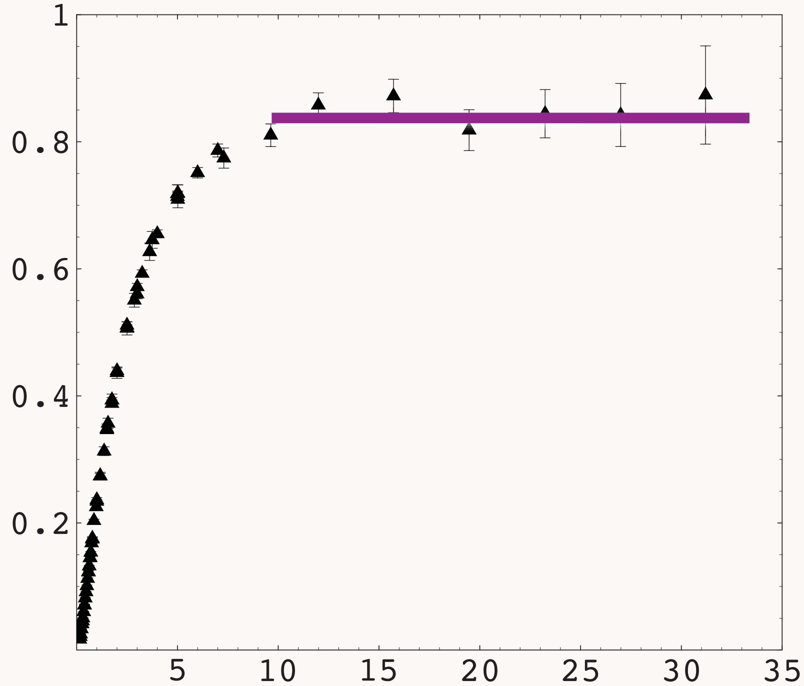

P.V. LANDSHOFF and J.C. POLKINGHORNE

**Best Fit**

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

$Q^4 F_1^p(Q^2)$ [GeV$^4$]




$Q^2$ [GeV$^2$]

$$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$$

*measured in electron-proton elastic scattering*

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

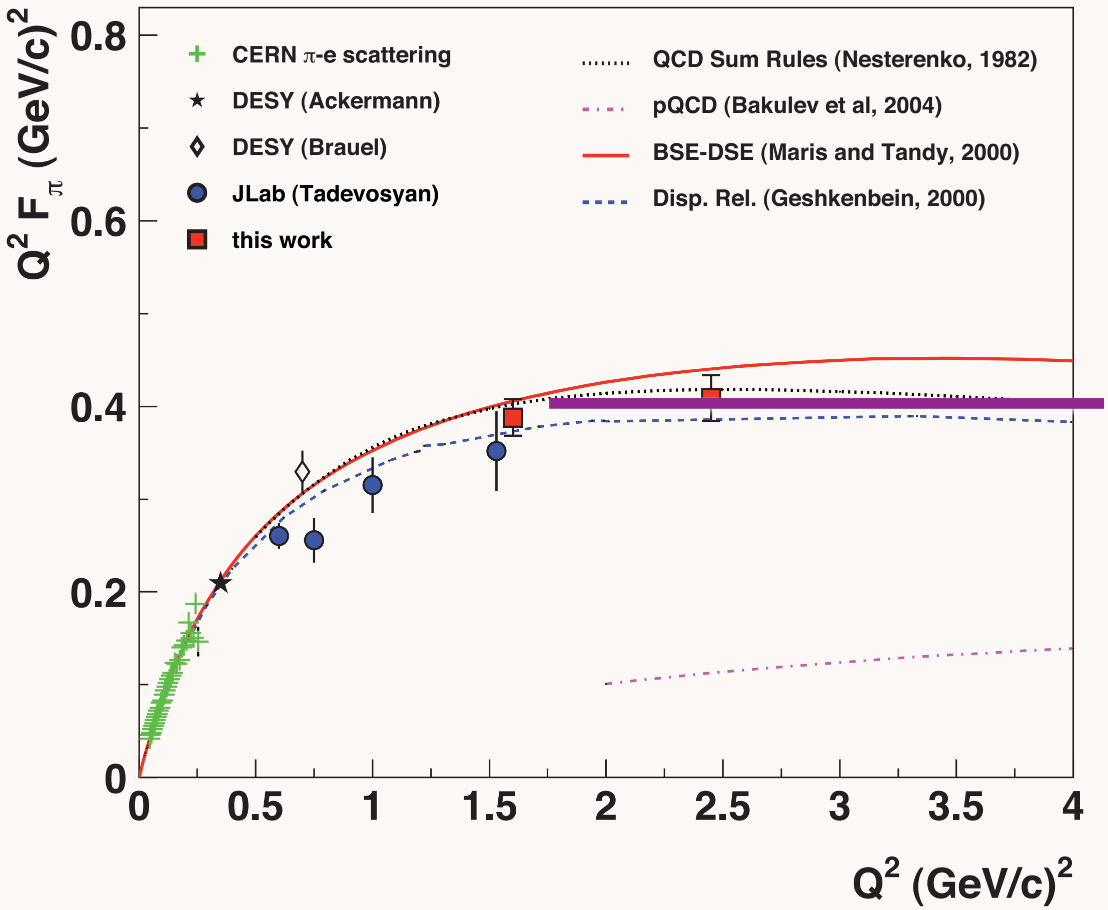
$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

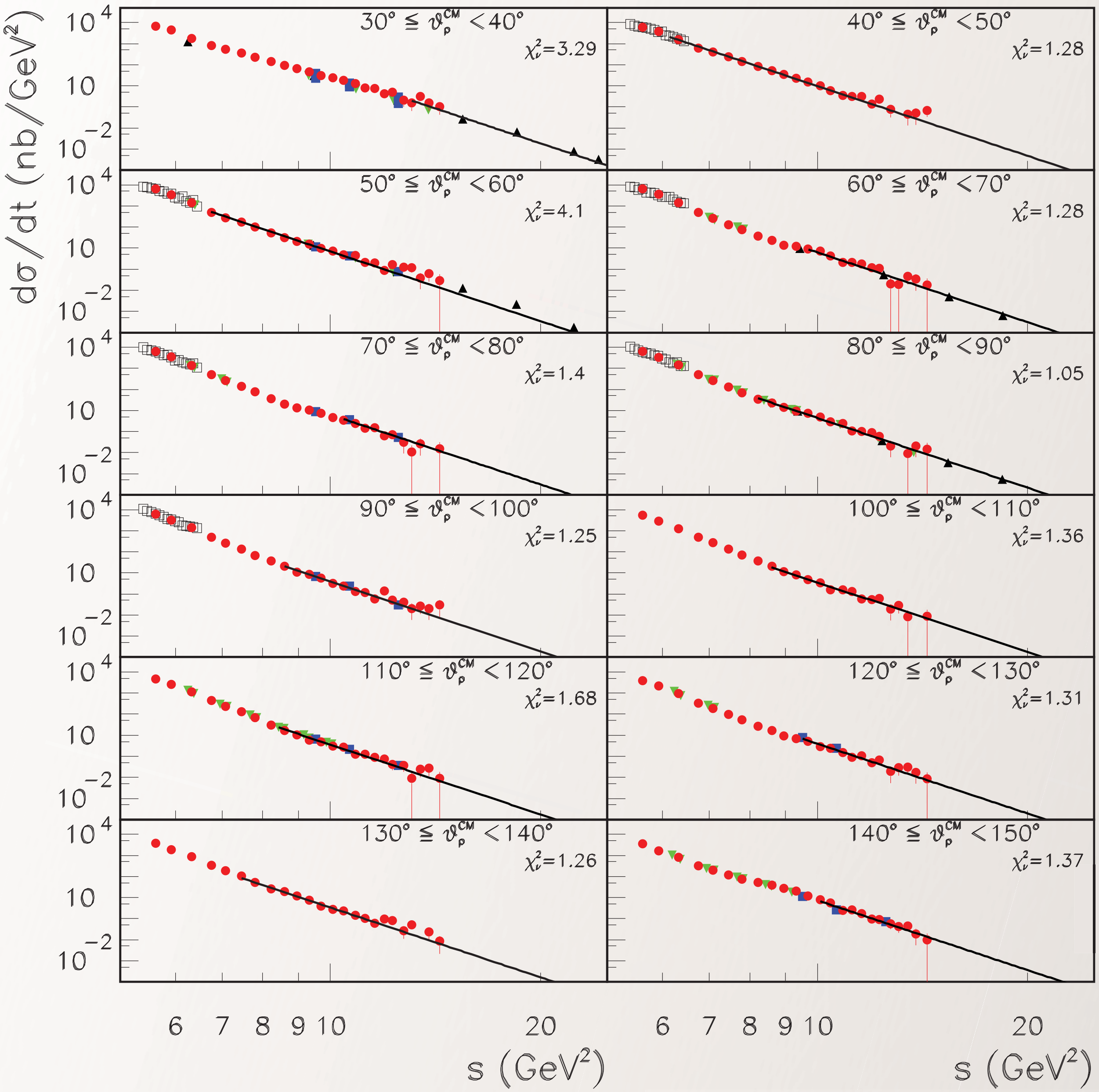
- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Conformal behavior: $Q^2 F_\pi(Q^2) \rightarrow$ const




**Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.**
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

# Deuteron Photodisintegration




J-Lab

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$n_{tot} - 2 =$
$(1 + 6 + 3 + 3) - 2 = 11$

**Reflects conformal invariance**

- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \to np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

- $n_{tot} = 1 + 6 + 3 + 3 = 13$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$

at high $p_T$

# Primary Evidence for Quarks

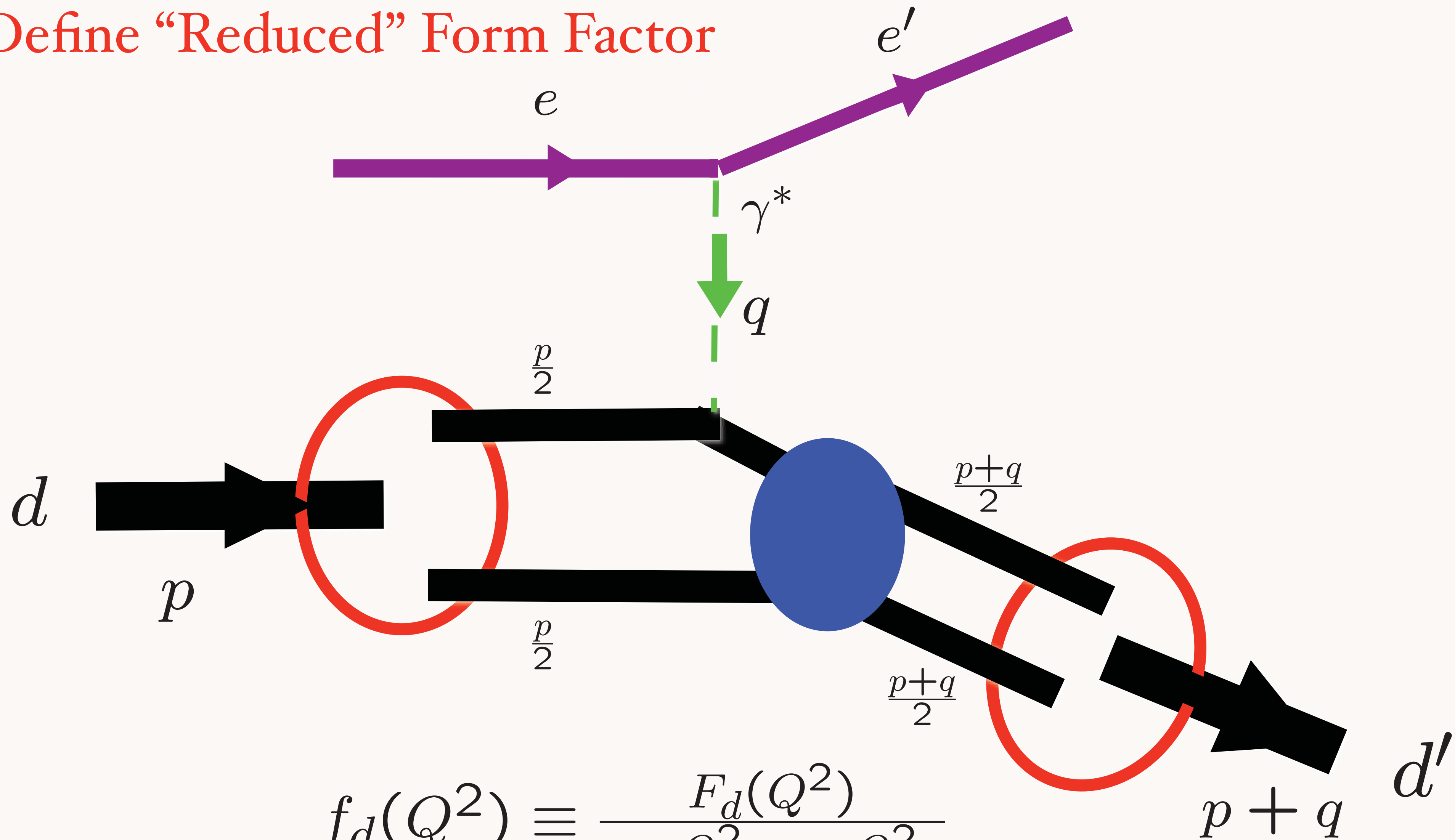
- Electron-Proton Inelastic Scattering: $ep \to e'X$
  Electron scatters on pointlike constituents with fractional charge; final-state jets

- Electron-Positron Annihilation: $e^+e^- \to X$
  Production of pointlike pairs with fractional charges and 3 colors; quark, antiquark, gluon jets

- Exclusive hard scattering reactions: $pp \to pp, \ \gamma p \to \pi^+ n, \ ep \to ep$
  probability that hadron stays intact counts number of its pointlike constituents:

**Quark Counting Rules**

# Define "Reduced" Form Factor




$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

*Elastic electron-deuteron scattering*

# QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)} \,.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

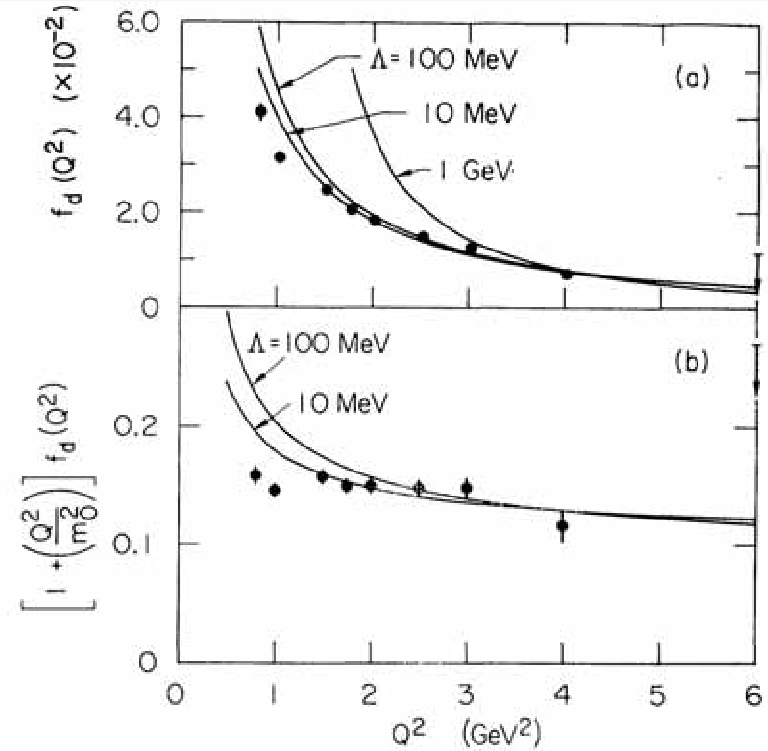
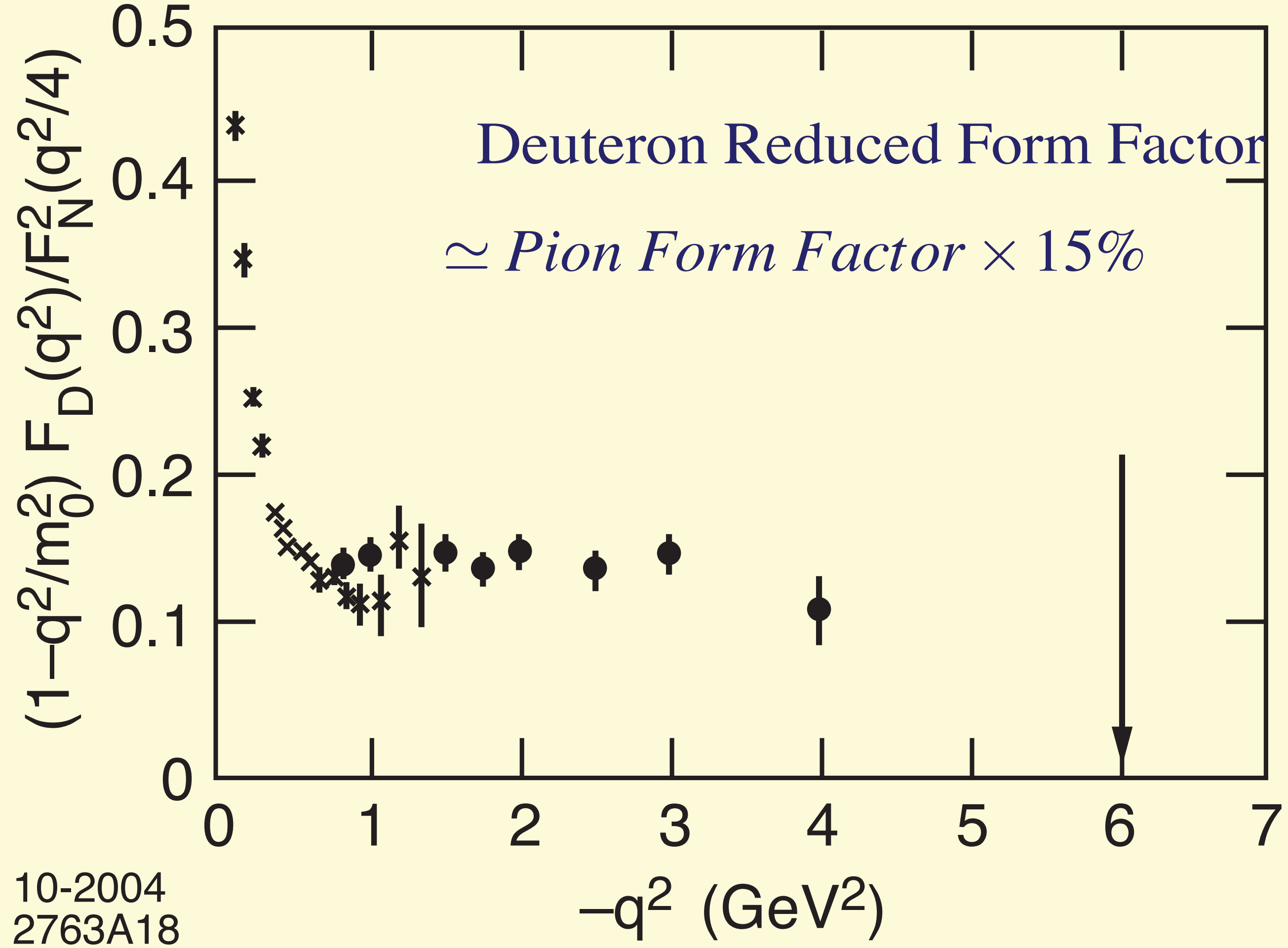



FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71 \text{ GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4$ GeV$^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28$ GeV$^2$ is used (Ref. 8).



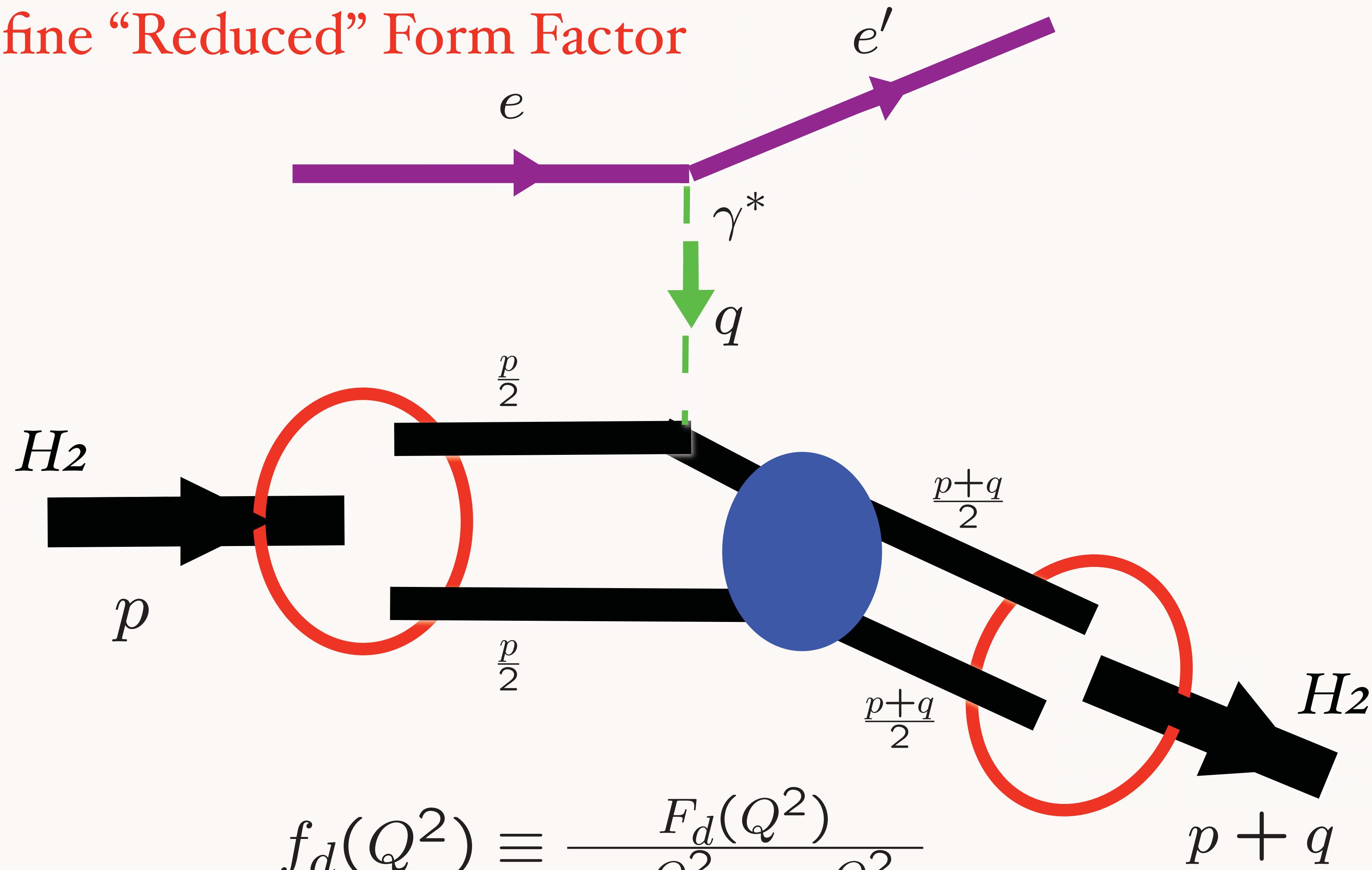

- 15% Hidden Color in the Deuteron

# Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high $Q^2$

# Define "Reduced" Form Factor




$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

*Elastic electron-molecule scattering!*

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

# Quark Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
- $F_H(Q) \propto \frac{1}{(Q^2)^{n-1}}$ n = # elementary constituents

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

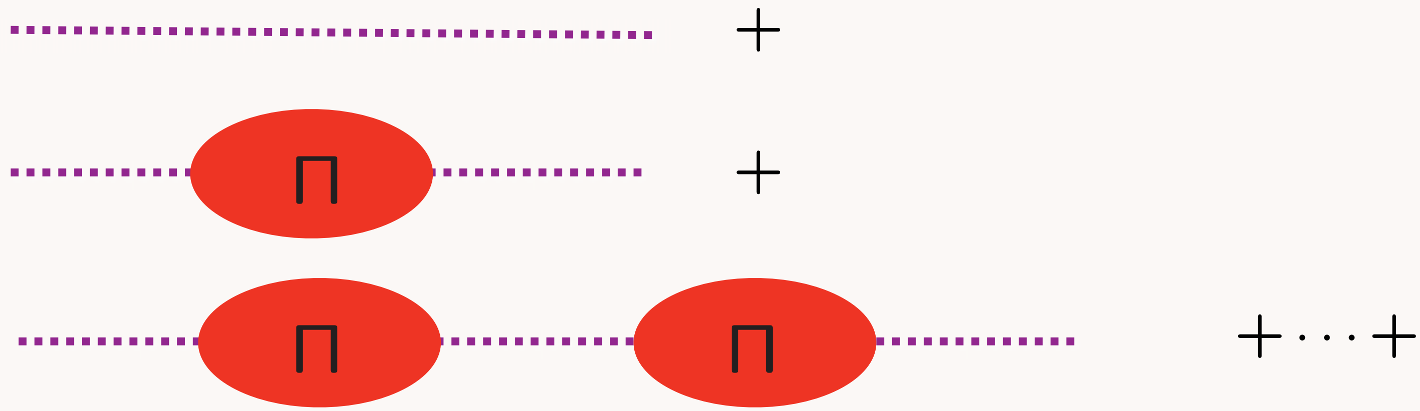

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

**Gell Mann-Low Effective Charge**

# QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

*All-orders lepton loop corrections to dressed photon propagator*

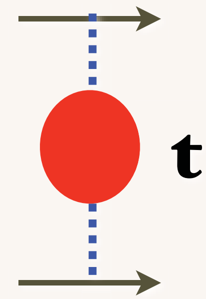



$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

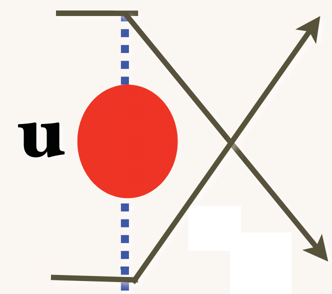
**Initial scale $t_0$ is arbitrary -- Variation gives RGE Equations**
**Physical renormalization scale t not arbitrary**

# *Electron-Electron Scattering in QED*

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

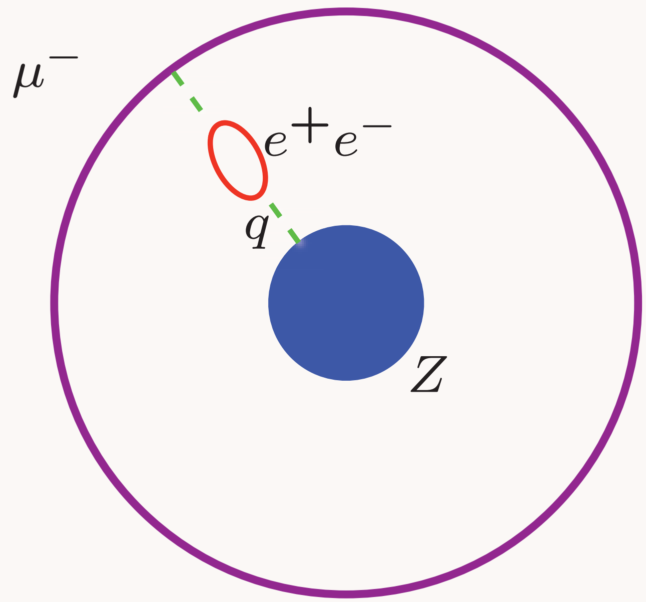



- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**




- **If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- *No renormalization scale ambiguity!*
- *Two separate physical scales.*

# Scale Setting in QED: Muonic Atoms




$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

**Scale is unique: Tested to ppm**

Gyulassy: Higher Order VP verified to 0.1% precision in $\mu$ Pb

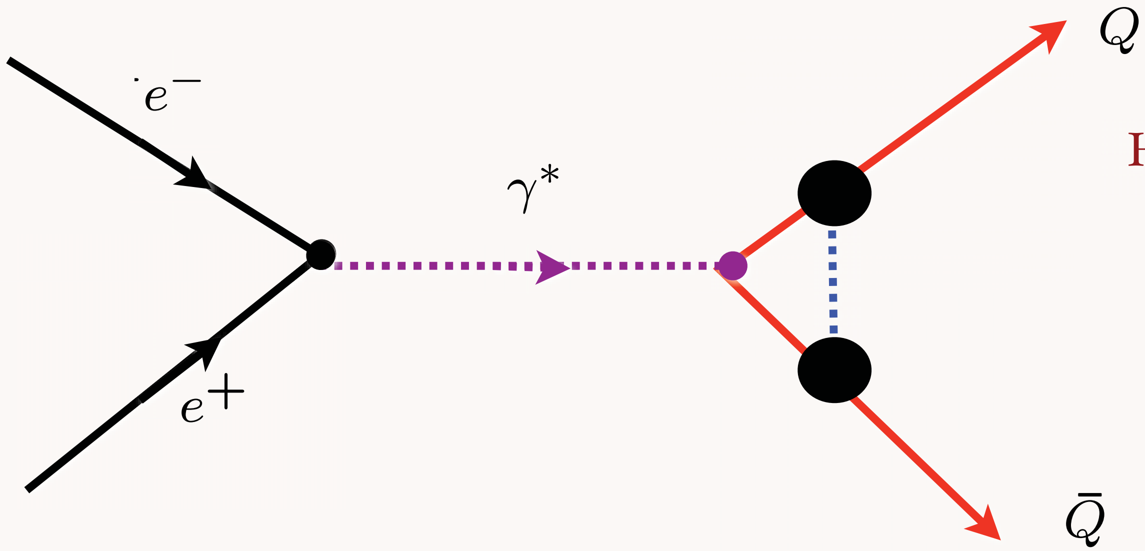
Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F\alpha_s, n_\ell = n_F/C_F$$

QCD $\rightarrow$ Abelian Gauge Theory

*Analytic Feature of SU(Nc) Gauge Theory*

*Scale-Setting procedure for QCD must be applicable to QED*




Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(s\beta^2)}{4\beta}]$$

Angular distributions of massive quarks close to threshold.

# Example of Multiple BLM Scales

**Need QCD coupling at small scales at low relative velocity β**

# The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis $r, \theta, \phi$

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

*Need a First Approximation to QCD*

***Comparable in simplicity to Schrödinger Theory in Atomic Physics***

**Relativistic, Frame-Independent, Color-Confining**

# Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$
$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp\, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

**Change variables** $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$
$$+ \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$
$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

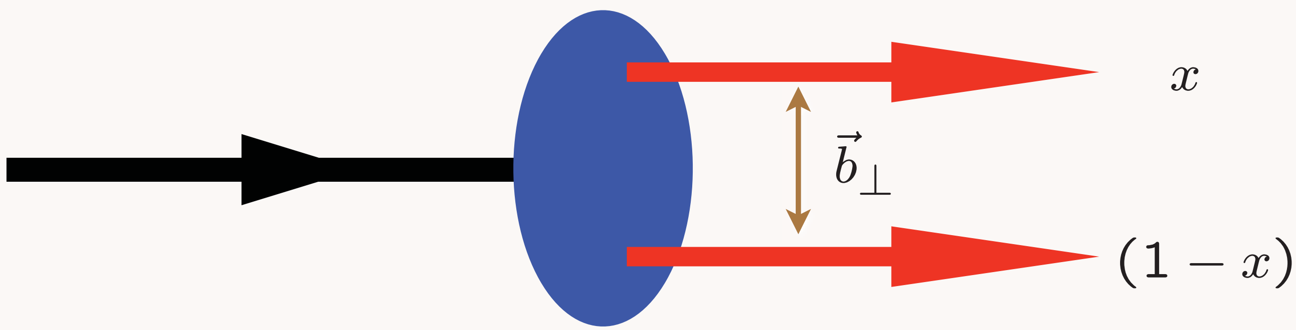
# Light-Front Holography:
# Map AdS/CFT to 3+1 LF Theory

*Relativistic LF radial equation!*

*Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



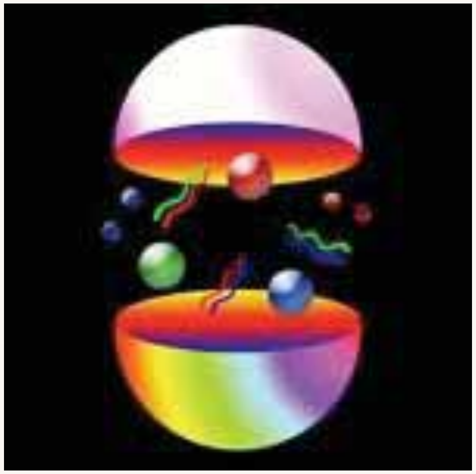

$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

*soft wall*
*confining potential:*

# The World of Quarks and Gluons:

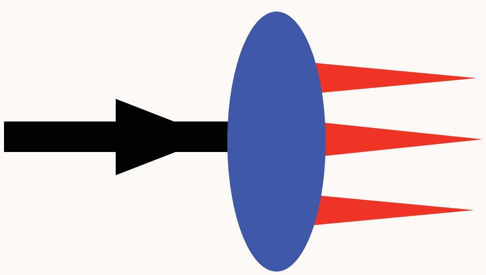
- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: Holography: AdS/CFT
- Need to understand QCD at the Amplitude Level: Hadron wavefunctions!



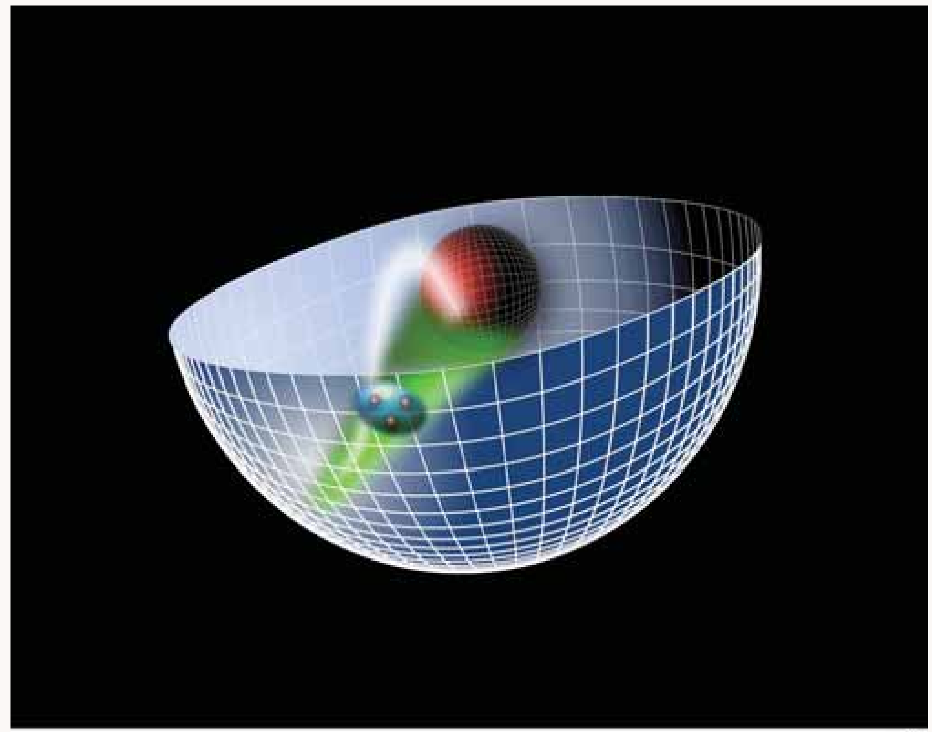
# Light-Front Holography and Non-Perturbative QCD

**Goal:**
**Use AdS/QCD duality to construct a first approximation to QCD**

*Hadron Spectrum*
*Light-Front Wavefunctions,*
*Form Factors, DVCS, etc*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



**in collaboration with**
**Guy de Teramond**

# *Goal:*

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

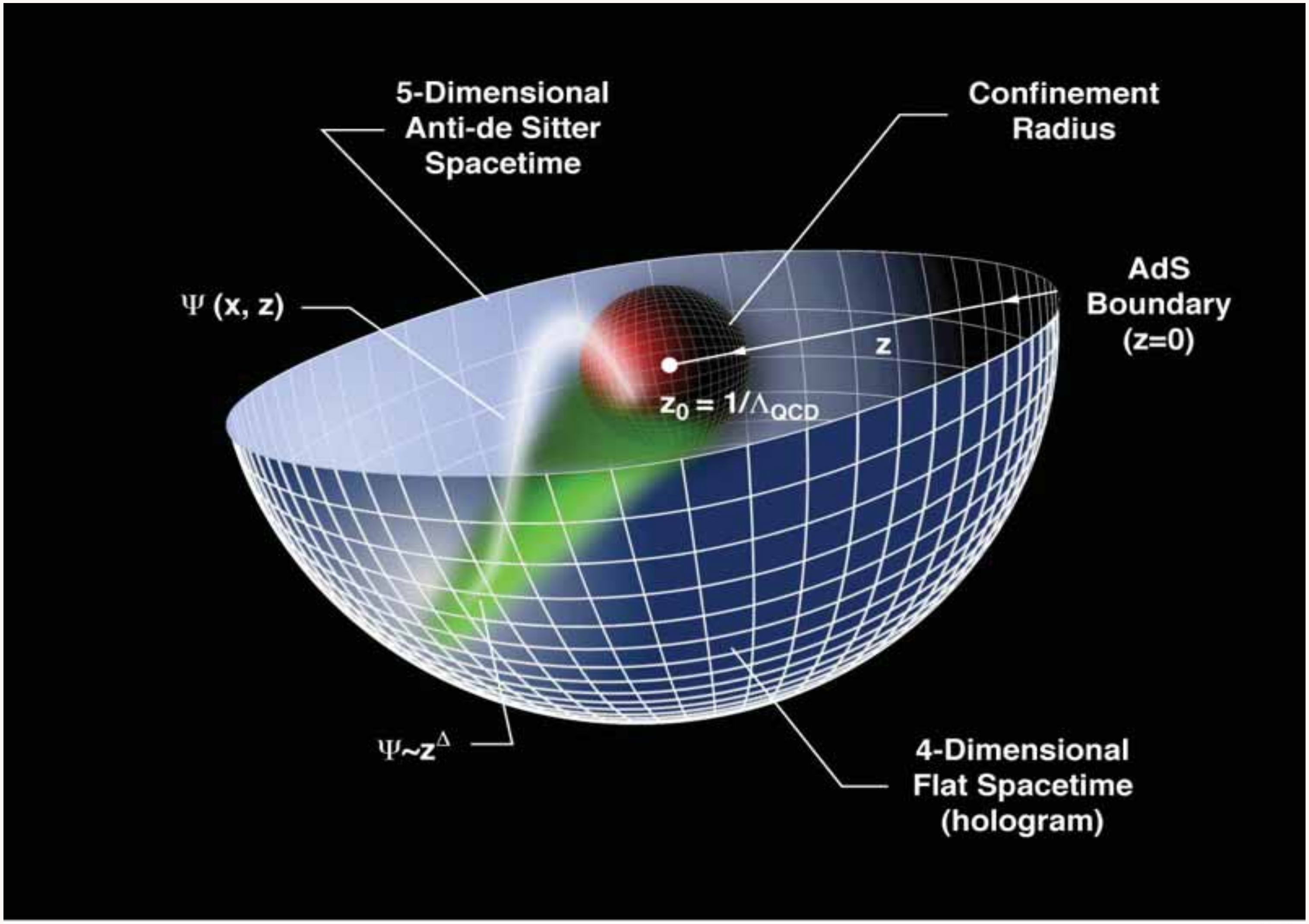
Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

**SO(4,2) has a mathematical representation on AdS5**

# Applications of AdS/CFT to QCD




*Changes in physical length scale mapped to evolution in the 5th dimension z*

**in collaboration with Guy de Teramond**

# AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

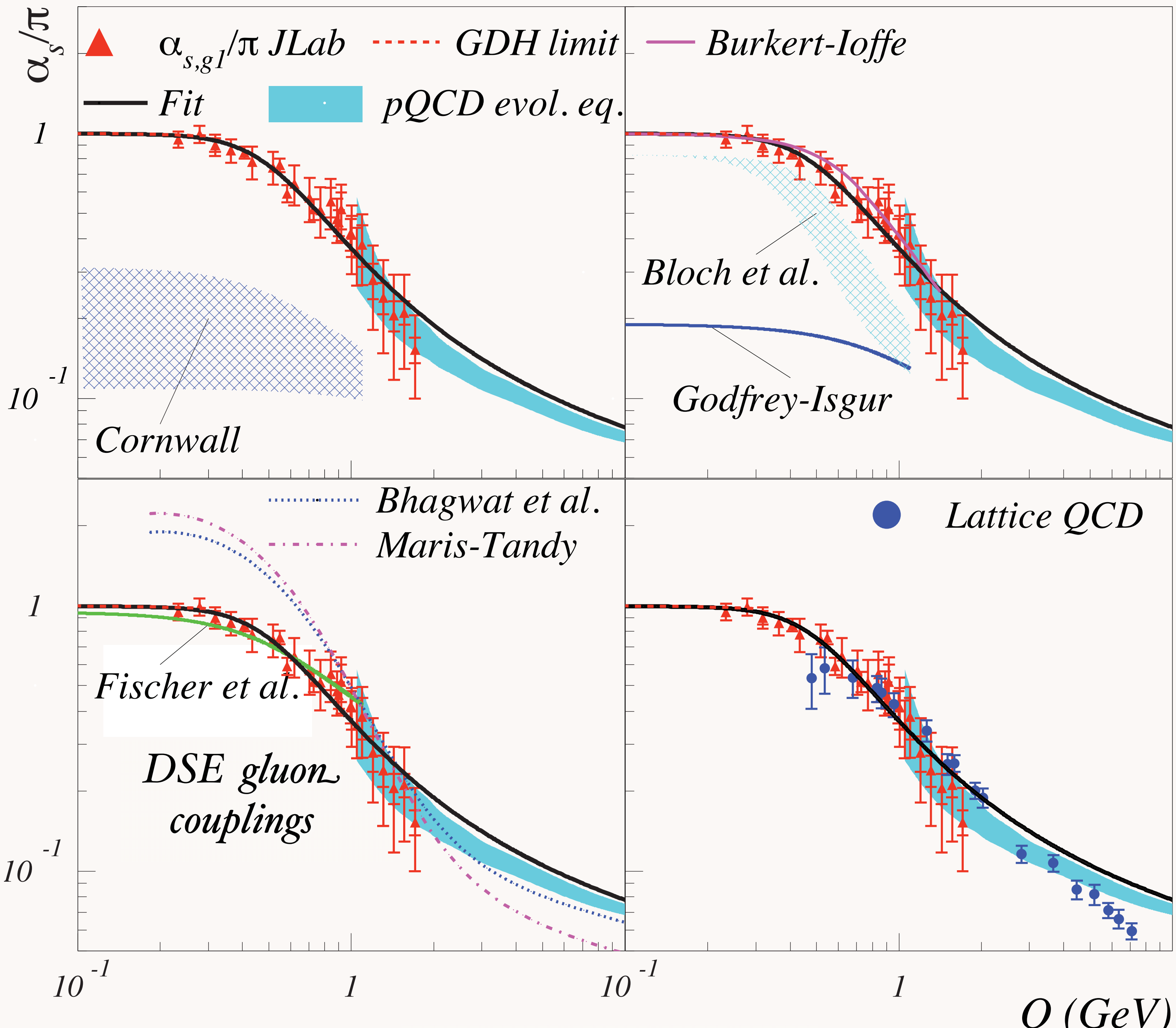
Map $AdS_5$ X $S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

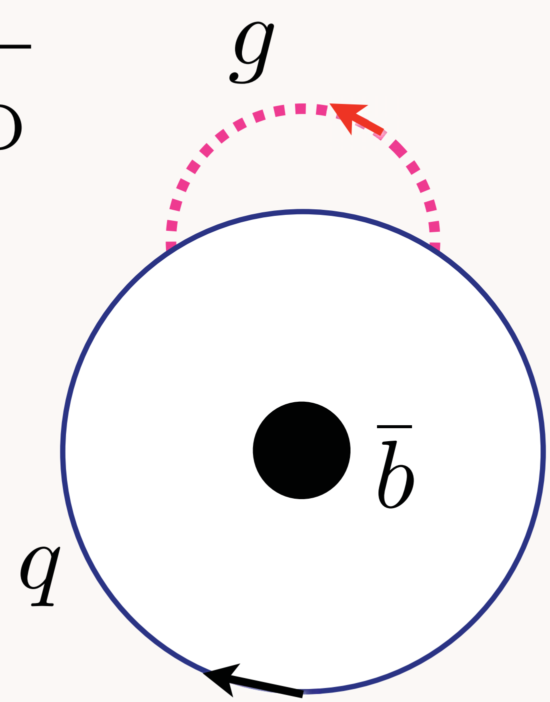
# Deur, Korsch, et al.



# Lesson from QED and Lamb Shift:

*maximum wavelength of bound quarks and gluons*

$$k > \frac{1}{\Lambda_{QCD}} \qquad \lambda < \Lambda_{QCD}$$




**B-Meson**

*gluon and quark propagators cutoff in IR*
*because of color confinement*

**Shrock, sjb**

# Maximal Wavelength of Confined Fields

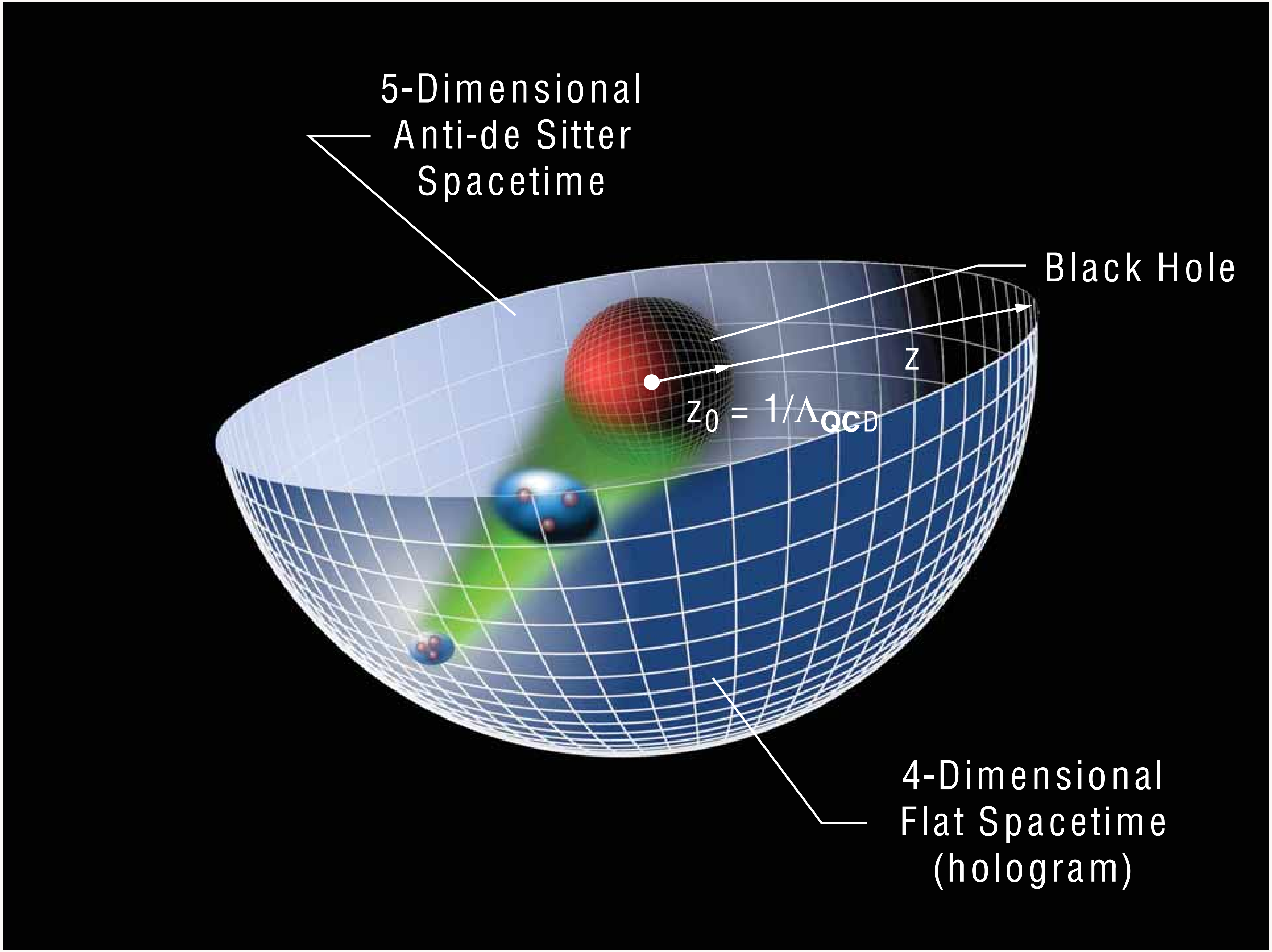
- **Colored fields confined to finite domain** $(x-y)^2 < \Lambda^{-2}_{QCD}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

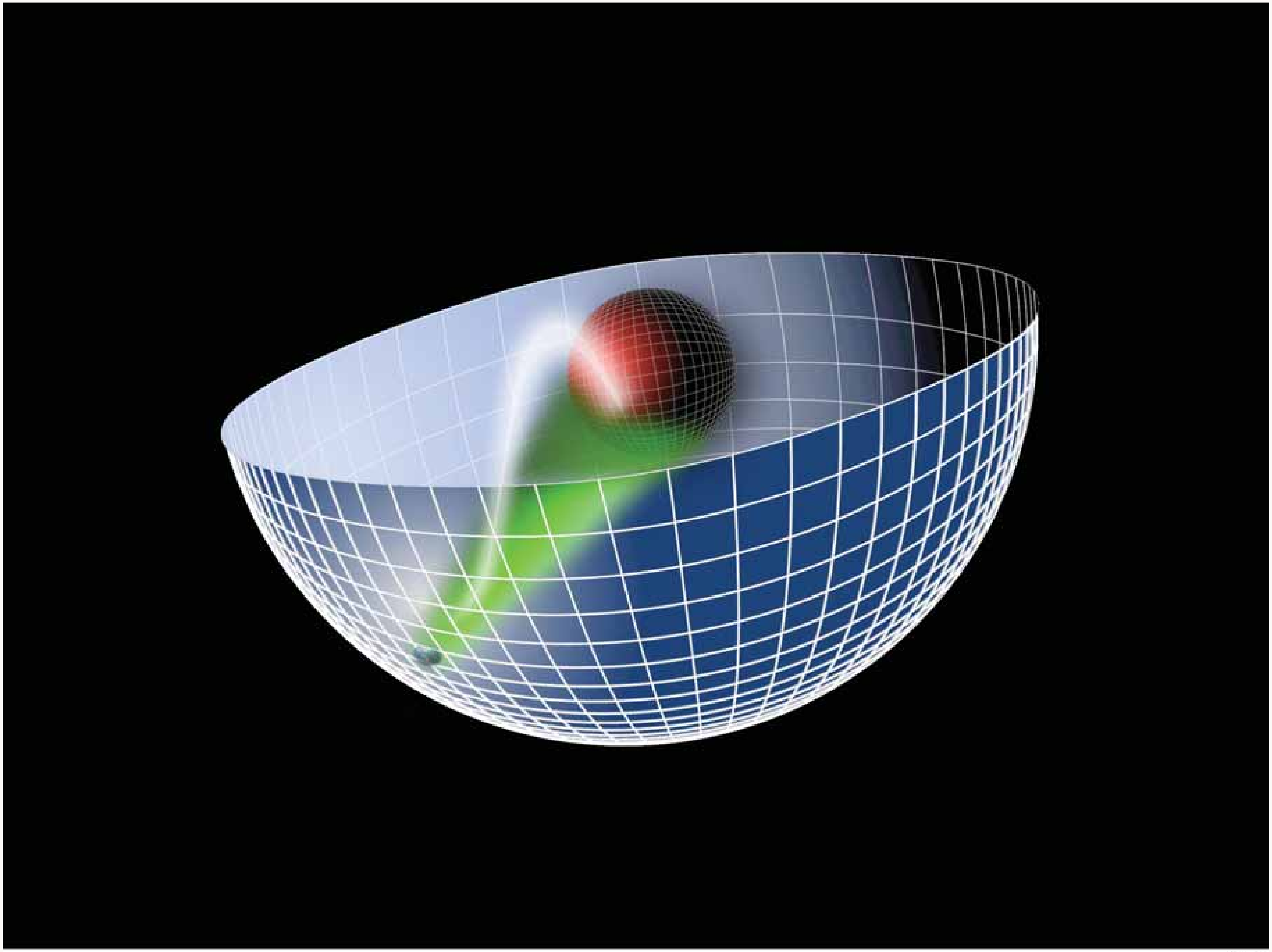
Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

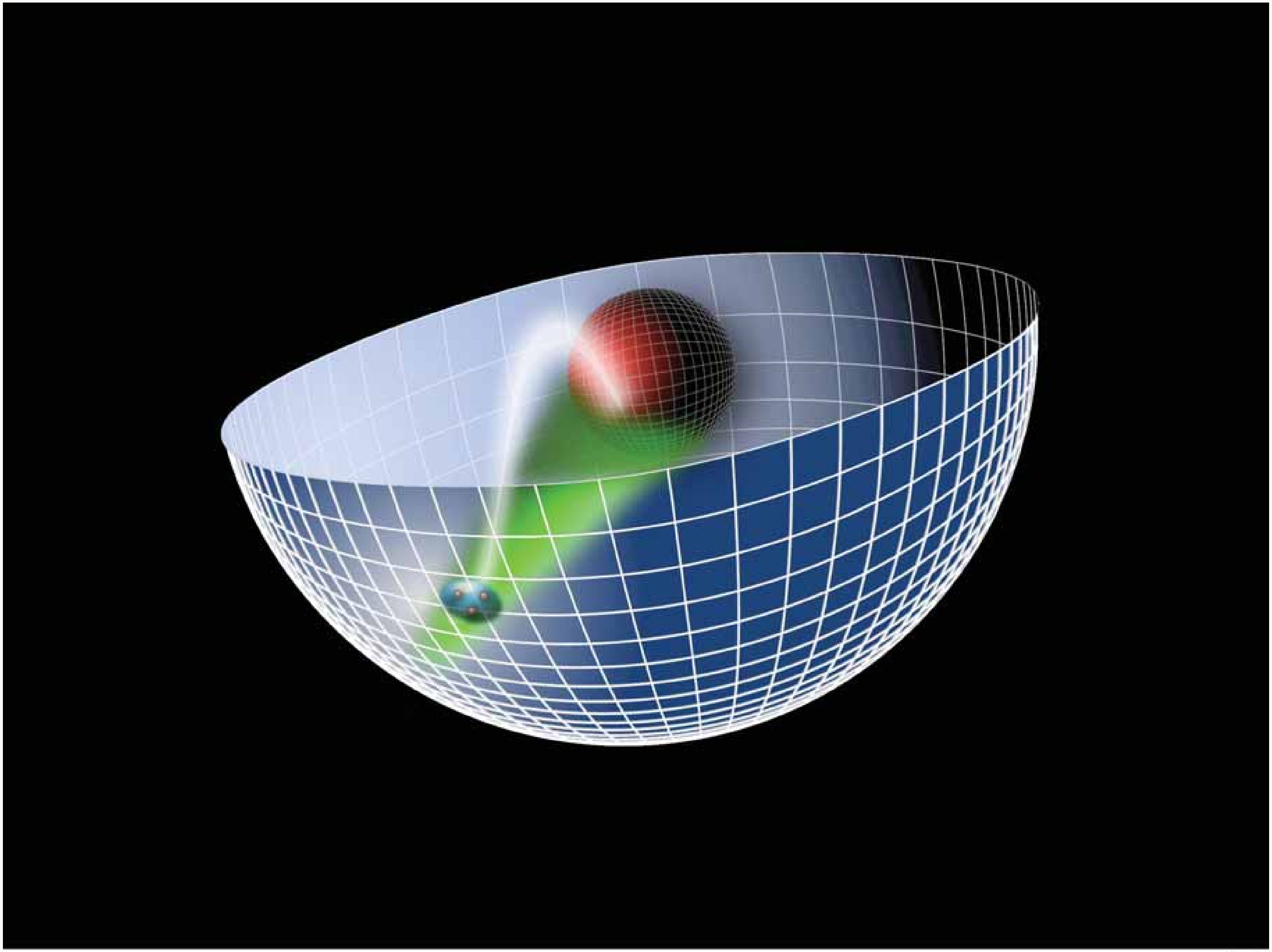
**Shrock, sjb**

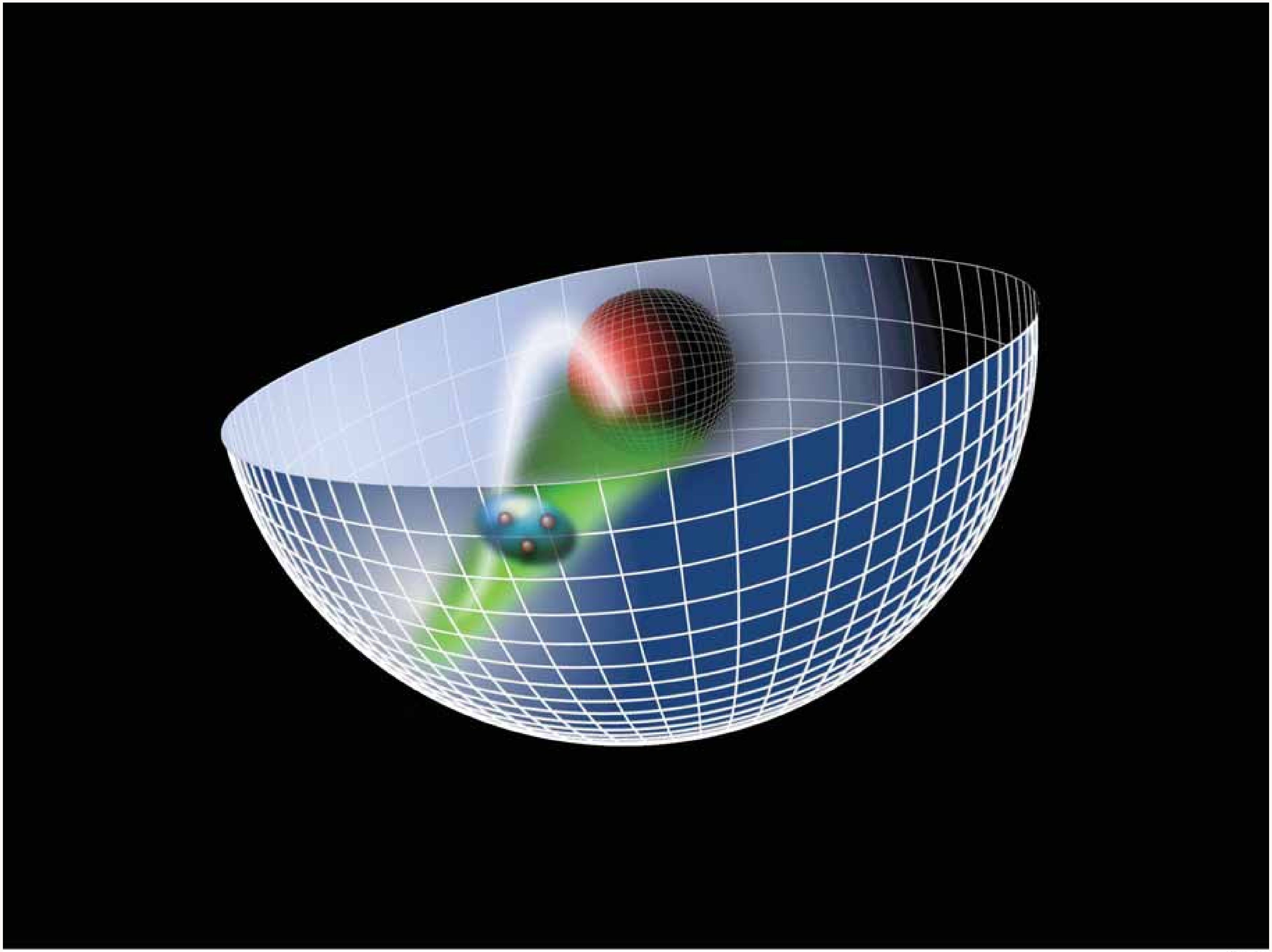
J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

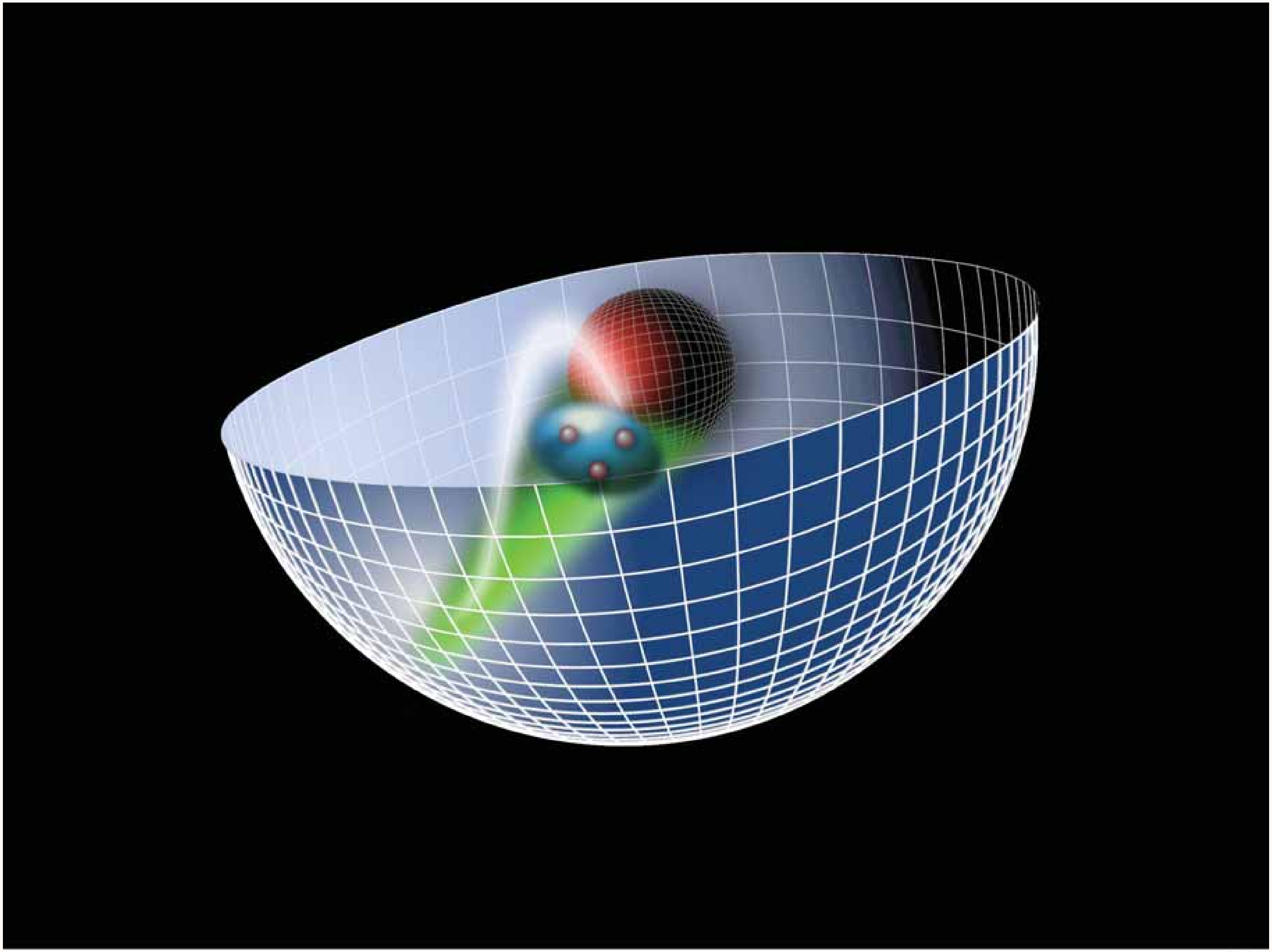
***A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).***

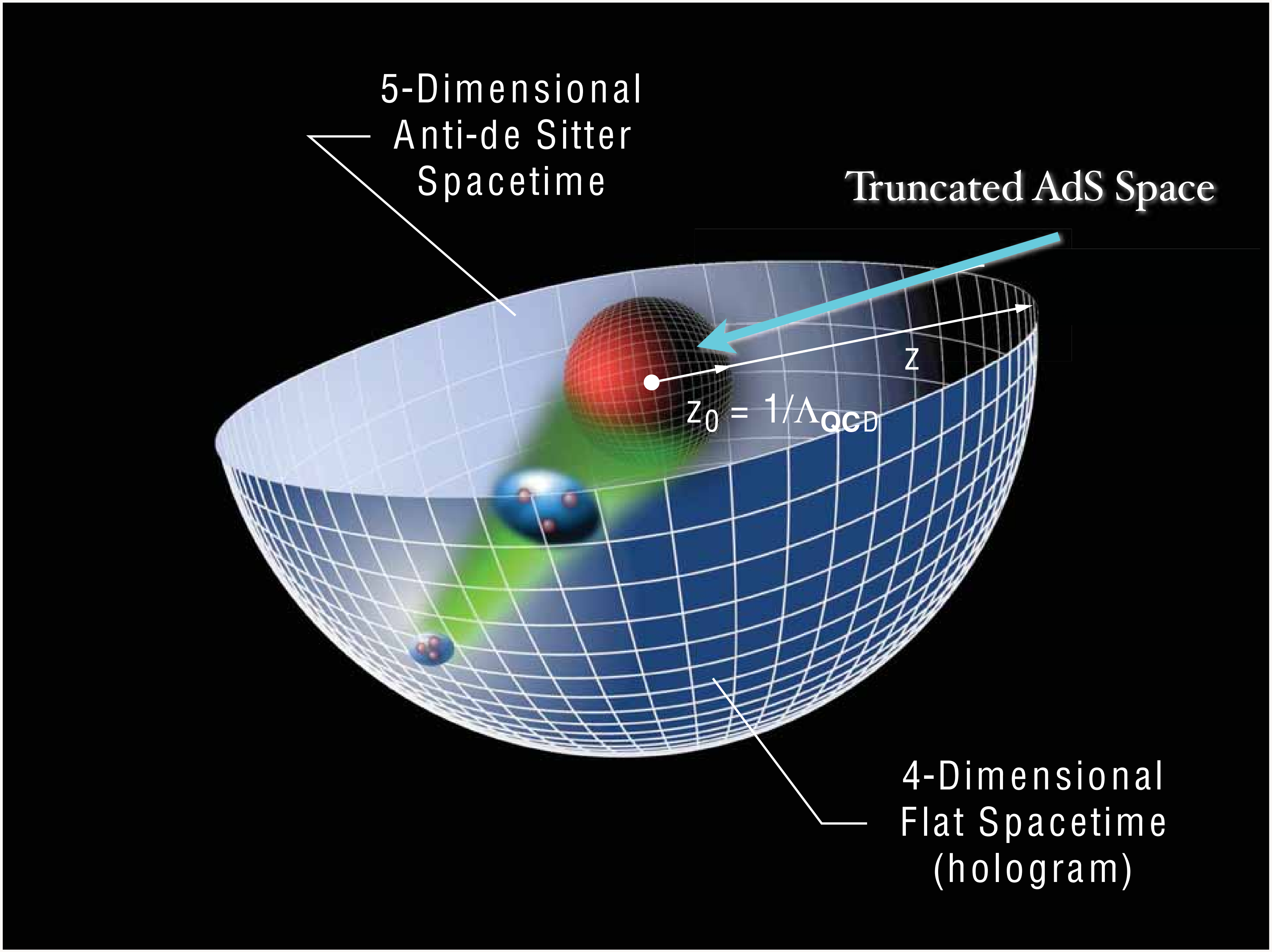

5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)










5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

# Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial "classical" approximation to QCD: Remarkable agreement with light hadron spectroscopy

Guy de Teramond, sjb

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{LF}_{QCD}$ ; variational methods

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

# AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

$$\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$$

Same slope in n and L

Derived from variation of Action Dilaton-Modified $AdS_5$

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Quark separation increases with L

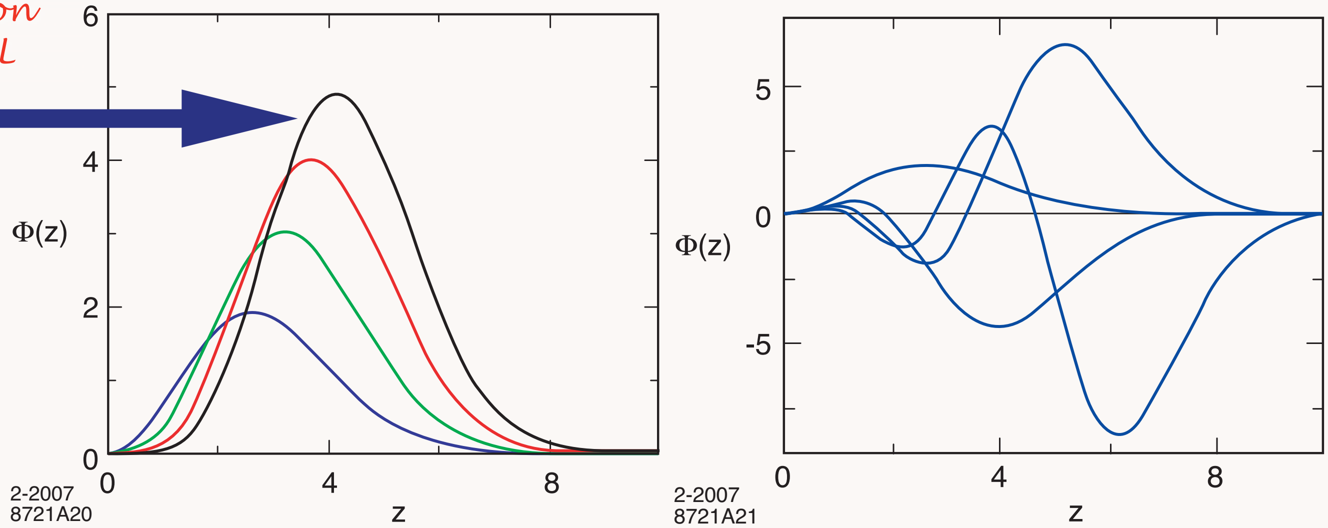
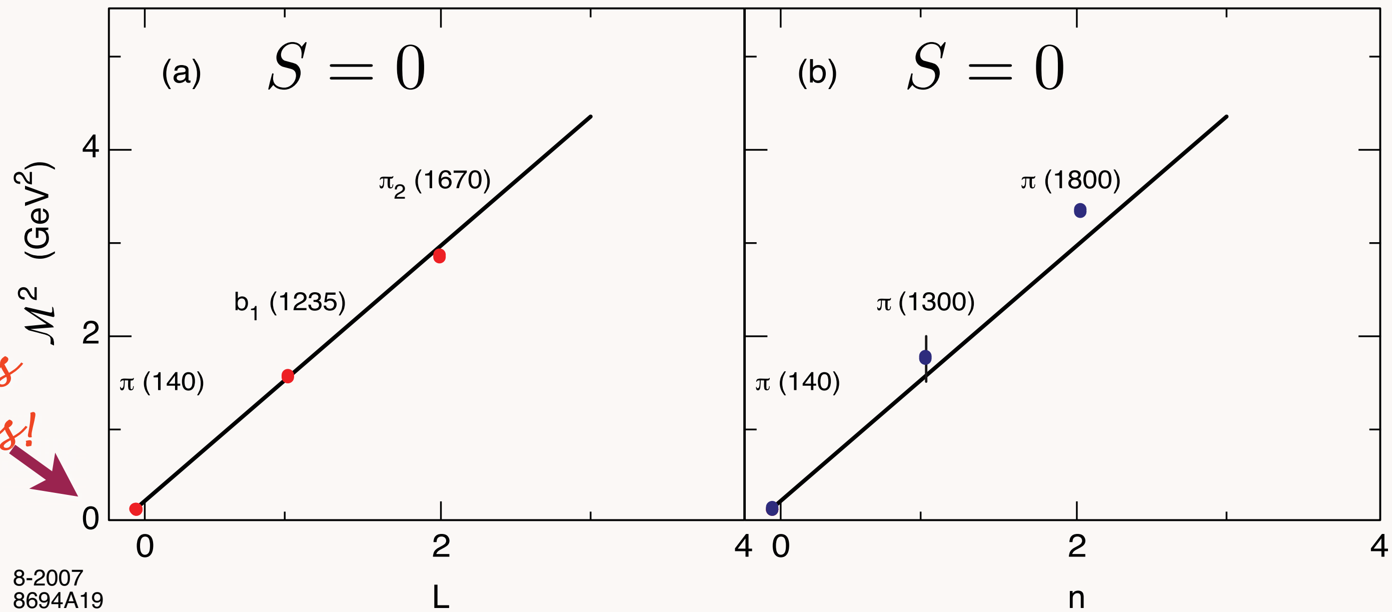



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

# Soft Wall Model



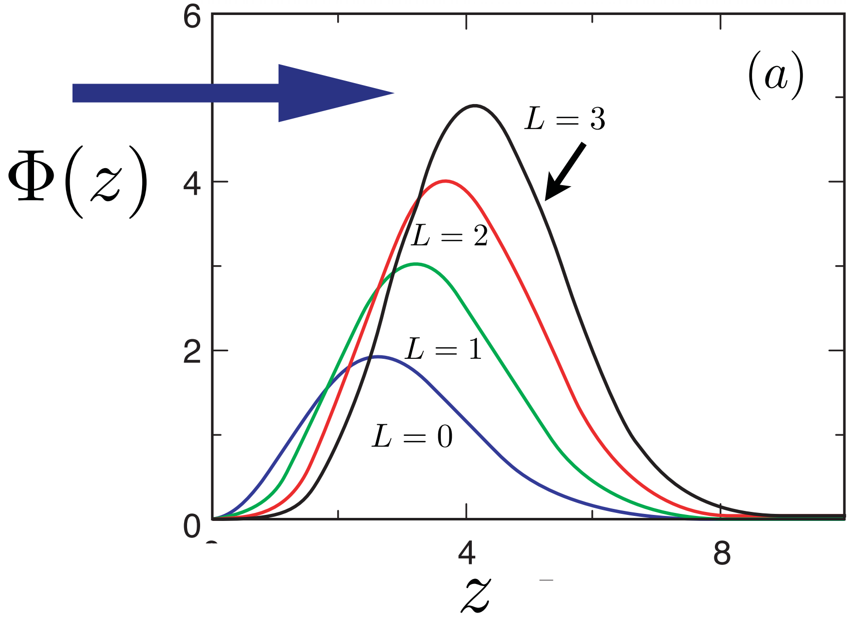

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

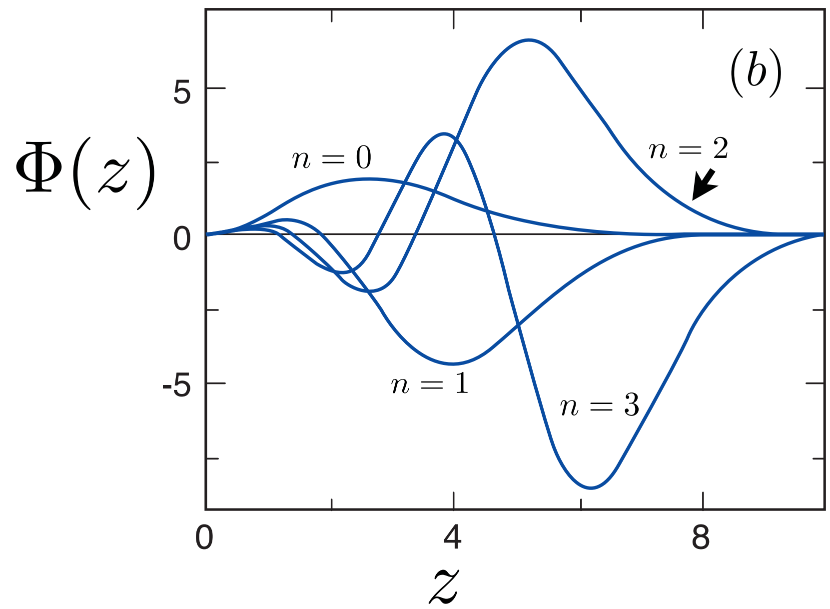
Pion has zero mass!

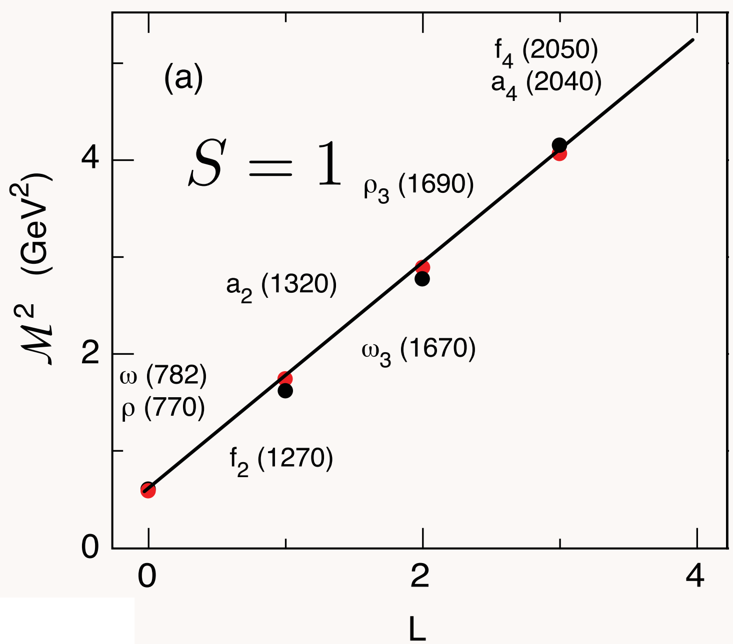
**Pion mass automatically zero!**

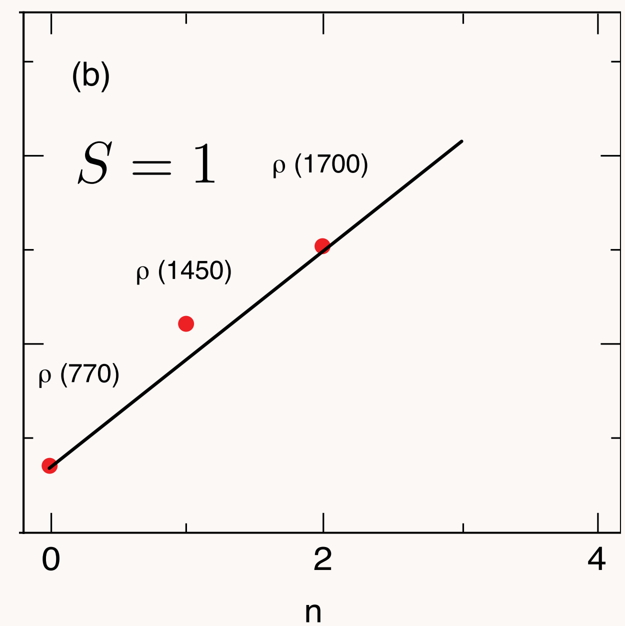
$$m_q = 0$$

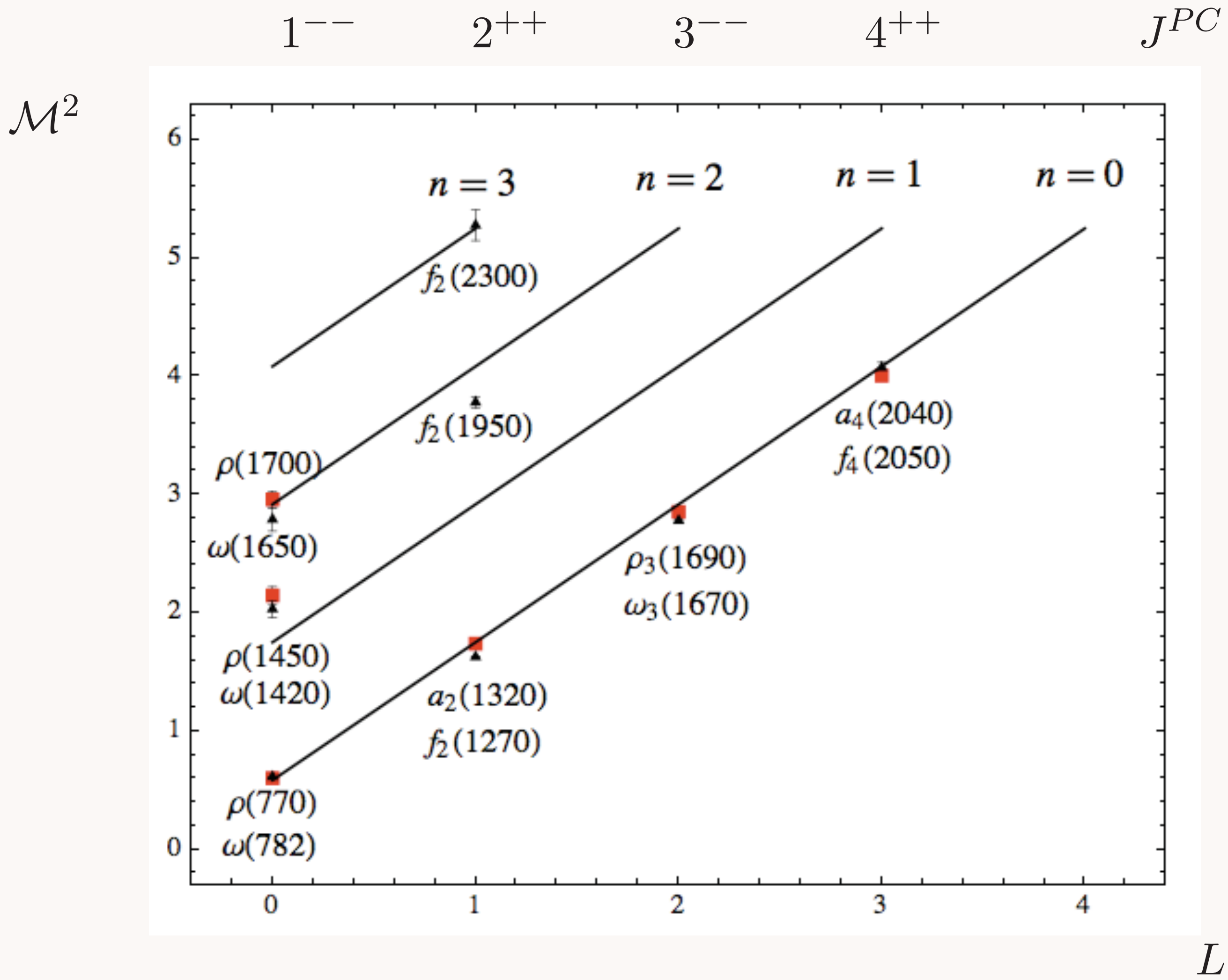
Quark separation increases with L



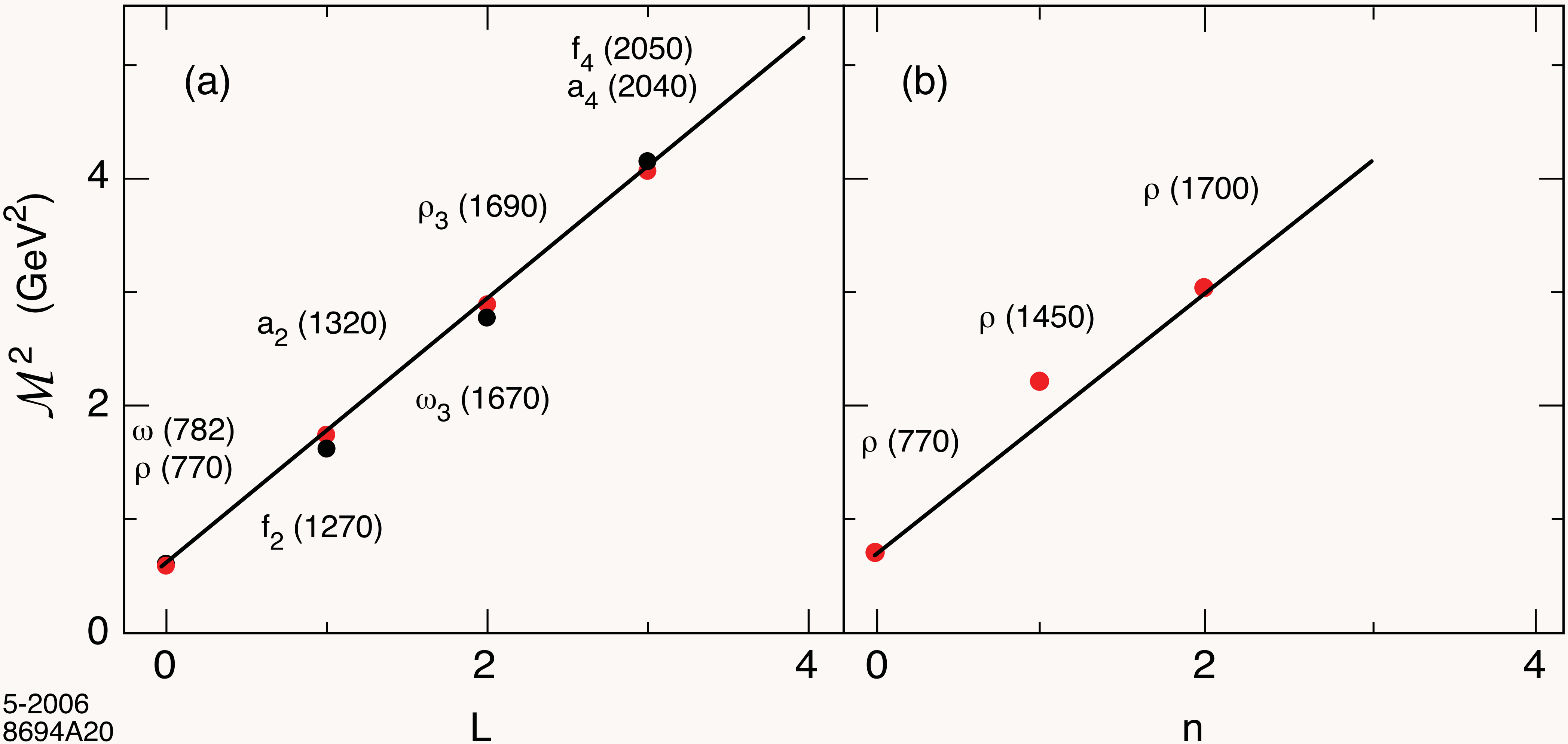






Parent and daughter Regge trajectories for the $I = 1$ $\rho$-meson family (red)
and the $I = 0$ $\omega$-meson family (black) for $\kappa = 0.54$ GeV

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



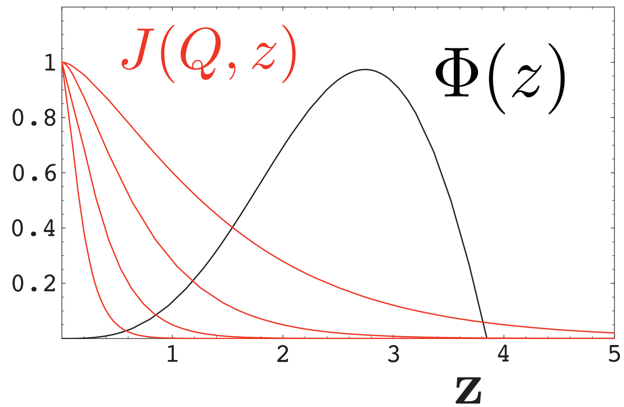
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High $Q^2$
from
small z ~ 1/Q




**Polchinski, Strassler**
**de Teramond, sjb**

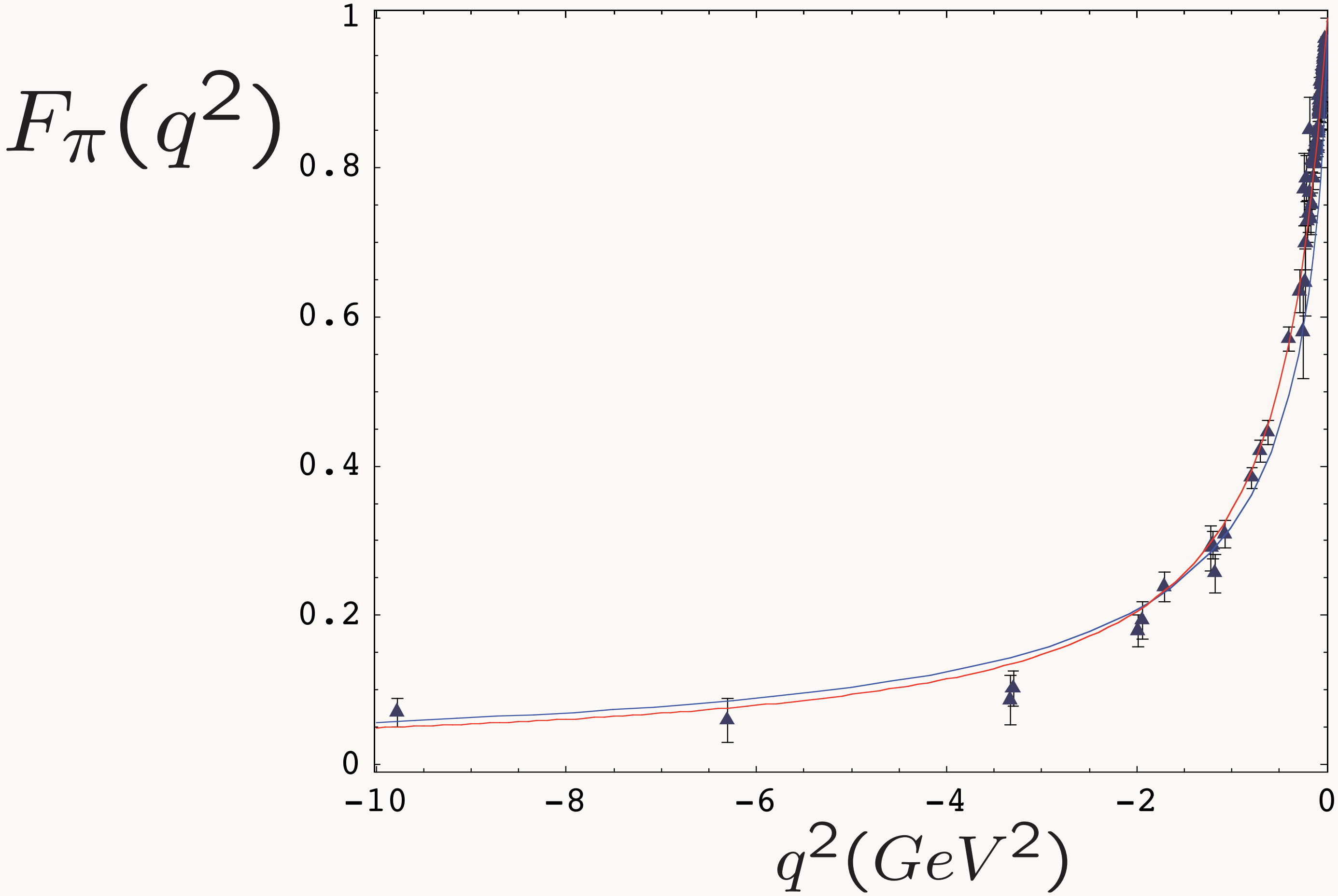
Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

**Dimensional Quark Counting Rules:**
**General result from**
**AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

# Spacelike pion form factor from AdS/CFT




**Data Compilation**
**Baldini, Kloe and Volmer**

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

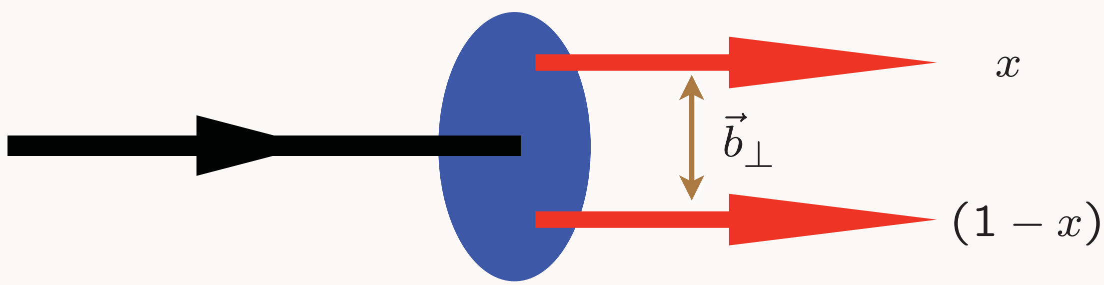
**de Teramond, sjb**
**See also: Radyushkin**

## LF(3+1)   AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

$\psi(x, \vec{b}_\perp)$



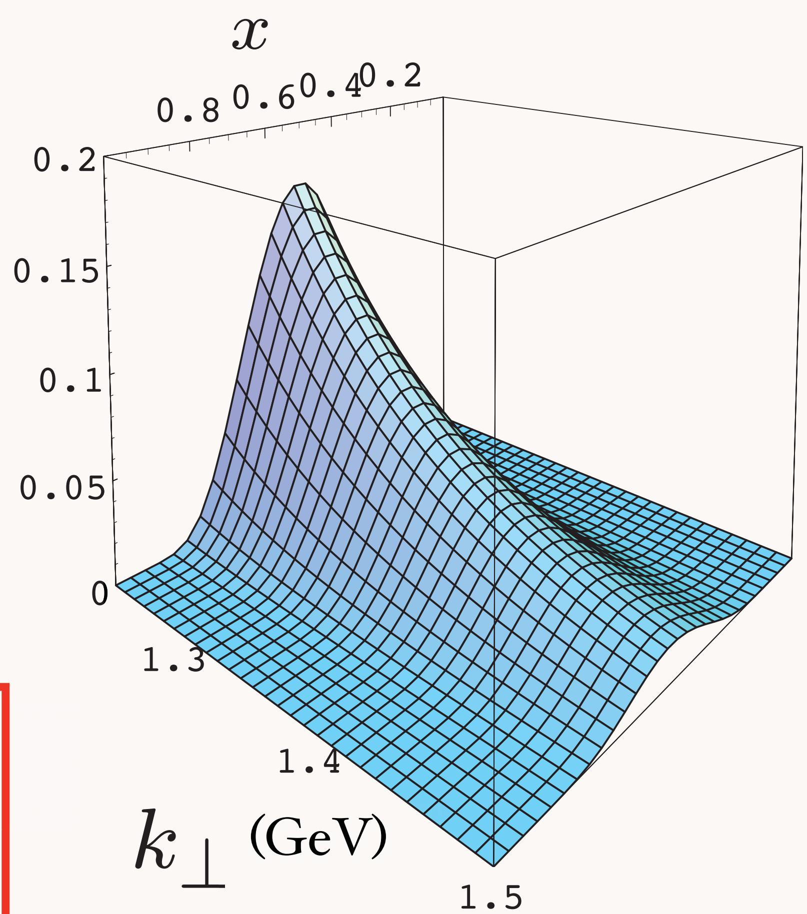

$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\phi(\zeta)$$

*Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

# Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$




**"Soft Wall" model**

$\kappa = 0.375$ GeV

massless quarks

**Note coupling**

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

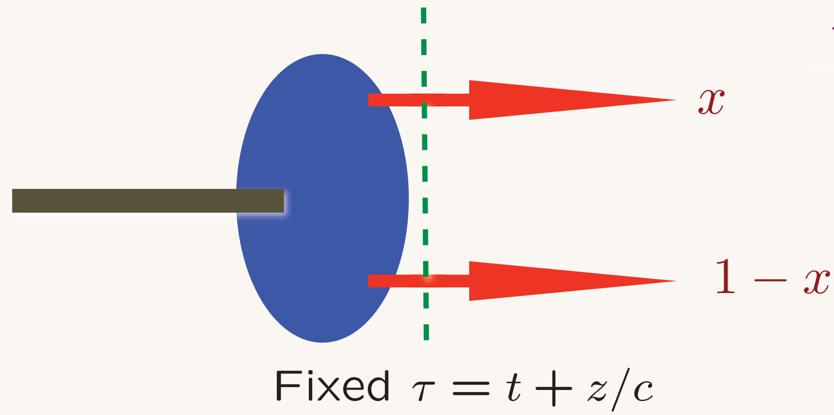
*Connection of Confinement to TMDs*

# Hadron Distribution Amplitudes

*Lepage, sjb*

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$




$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

*Lepage, sjb*
*Efremov, Radyushkin*
*Sachrajda, Frishman Lepage, sjb*
*Braun, Gardi*

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

# Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

# AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momenta
- Hadron Spectra, Regge Trajectories

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model) *Baryons*

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator $\Pi$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

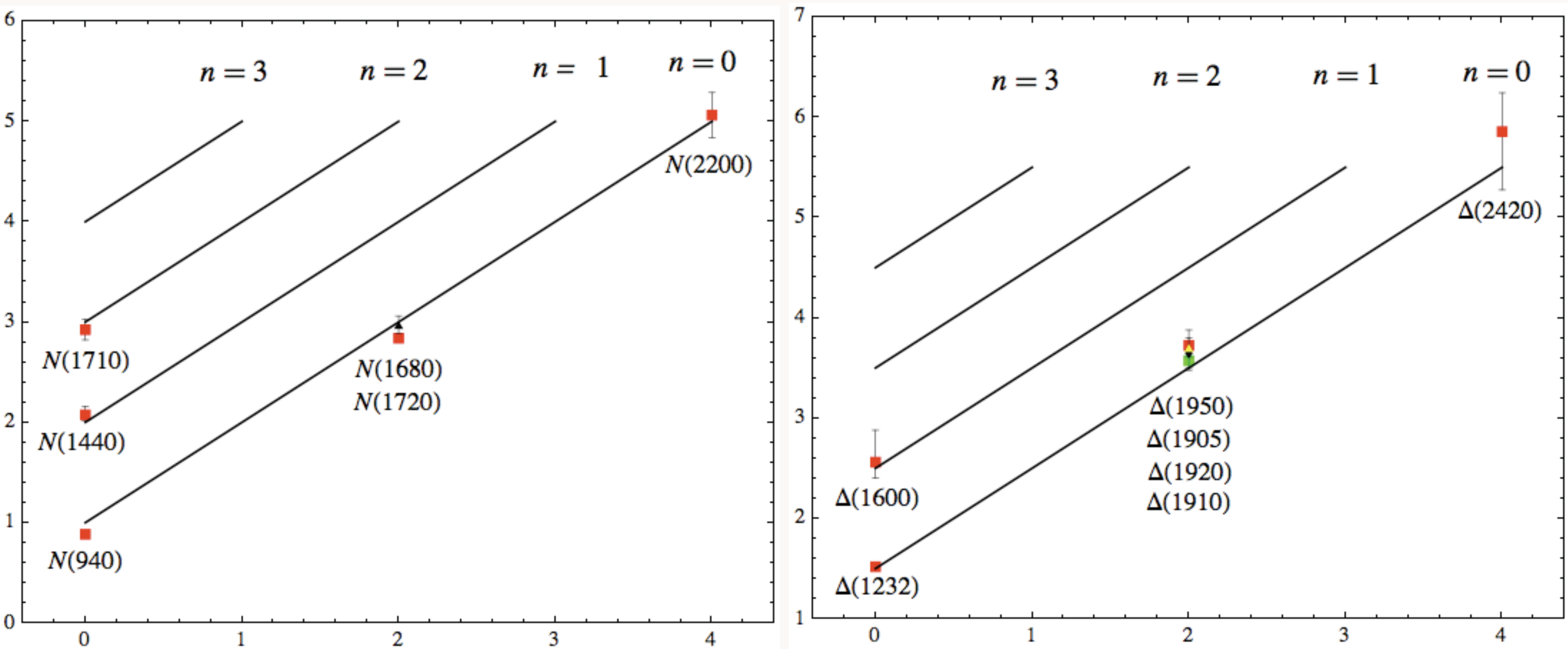
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

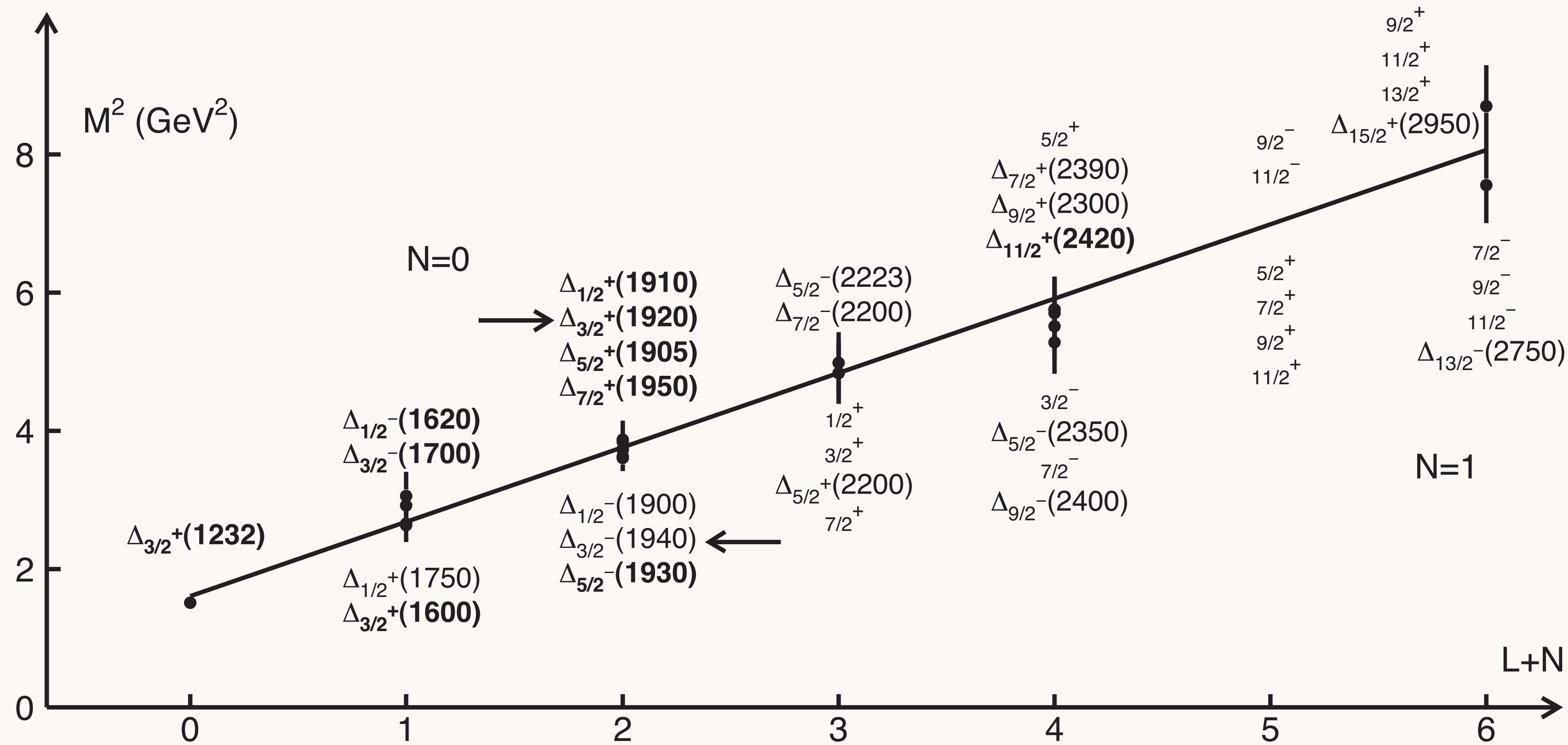
$2\kappa^2$ for $\Delta S = 1$

$\mathcal{M}^2$




$L$

Parent and daughter **56** Regge trajectories for the $N$ and $\Delta$ baryon families for $\kappa = 0.5$ GeV




E. Klempt *et al.*: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

| $SU(6)$ | $S$ | $L$ | Baryon State |
|---|---|---|---|
| **56** | $\frac{1}{2}$ | 0 | $N\frac{1}{2}^+(939)$ |
| | $\frac{3}{2}$ | 0 | $\Delta\frac{3}{2}^+(1232)$ |
| **70** | $\frac{1}{2}$ | 1 | $N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$ |
| | $\frac{3}{2}$ | 1 | $N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$ |
| | $\frac{1}{2}$ | 1 | $\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$ |
| **56** | $\frac{1}{2}$ | 2 | $N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$ |
| | $\frac{3}{2}$ | 2 | $\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$ |
| **70** | $\frac{1}{2}$ | 3 | $N\frac{5}{2}^-$ $N\frac{7}{2}^-$ |
| | $\frac{3}{2}$ | 3 | $N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$ |
| | $\frac{1}{2}$ | 3 | $\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$ |
| **56** | $\frac{1}{2}$ | 4 | $N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$ |
| | $\frac{3}{2}$ | 4 | $\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$ |
| **70** | $\frac{1}{2}$ | 5 | $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ |
| | $\frac{3}{2}$ | 5 | $N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$ |

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

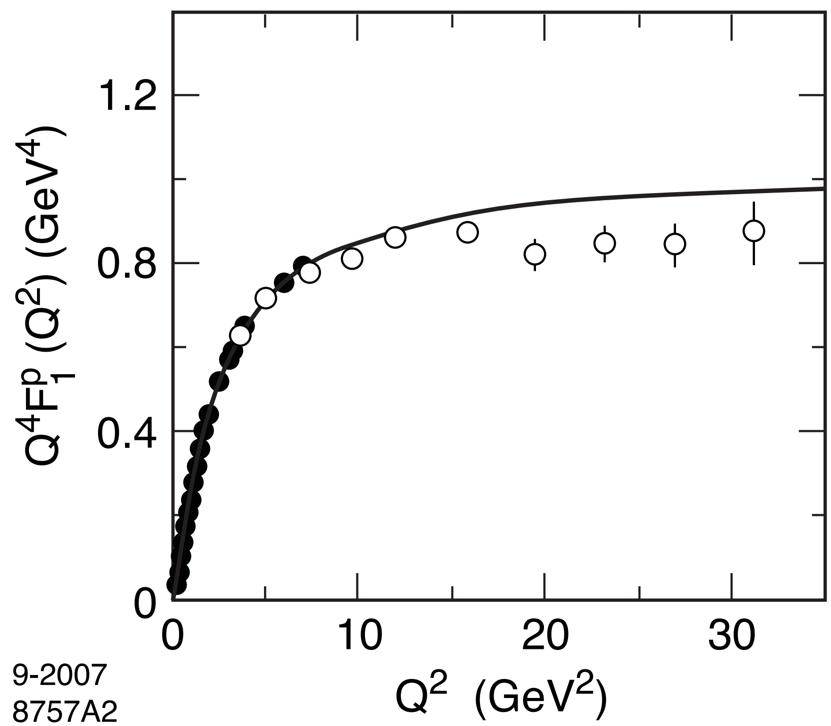
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

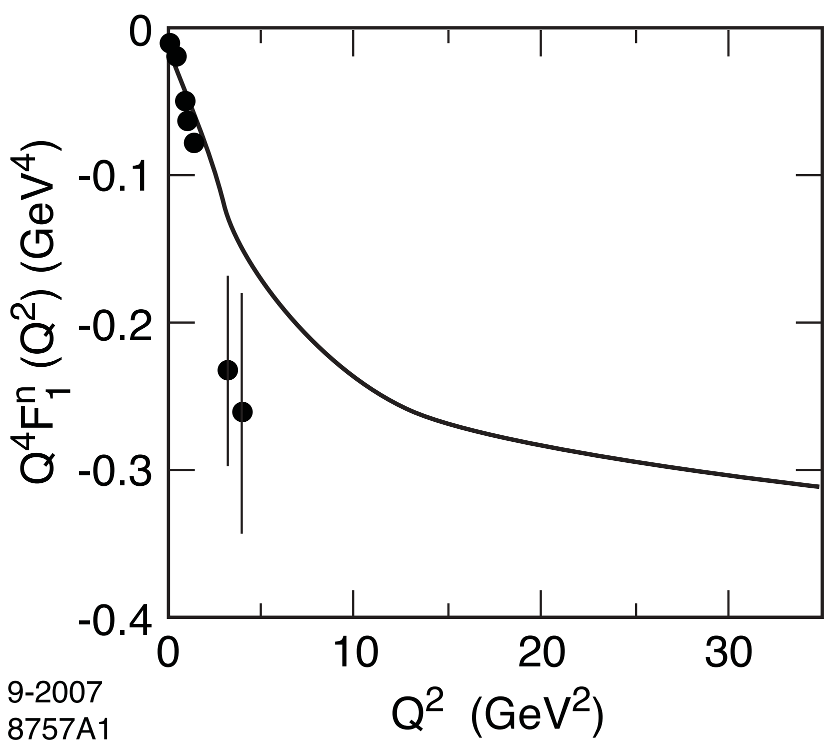
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$




SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large $Q^2$: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron} \;\; \tau = 3}$

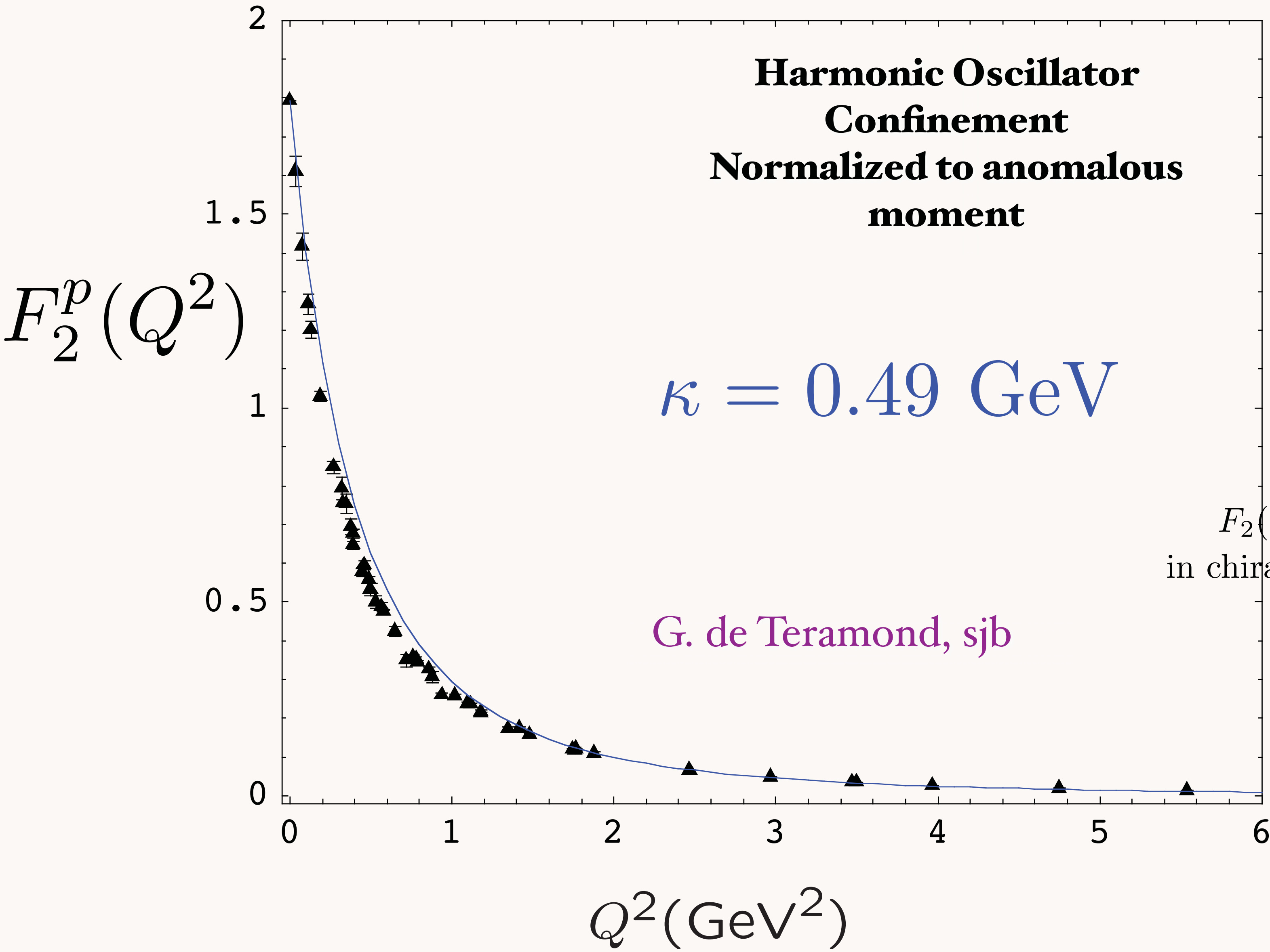



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

# Spacelike Pauli Form Factor

*Preliminary*

From overlap of $L = 1$ and $L = 0$ LFWFs




*AdS/QCD No chiral divergence!*

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$
in chiral perturbation theory

# Non-Perturbative Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

**Five dimensional action in presence of dilaton background**

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g}\; e^{\phi(z)} \frac{1}{g_5^2} G^2 \quad \text{where } \sqrt{g} = \left(\frac{R}{z}\right)^5 \text{ and } \phi(z) = +\kappa^2 z^2$$

**Define an effective coupling** $g_5(z)$

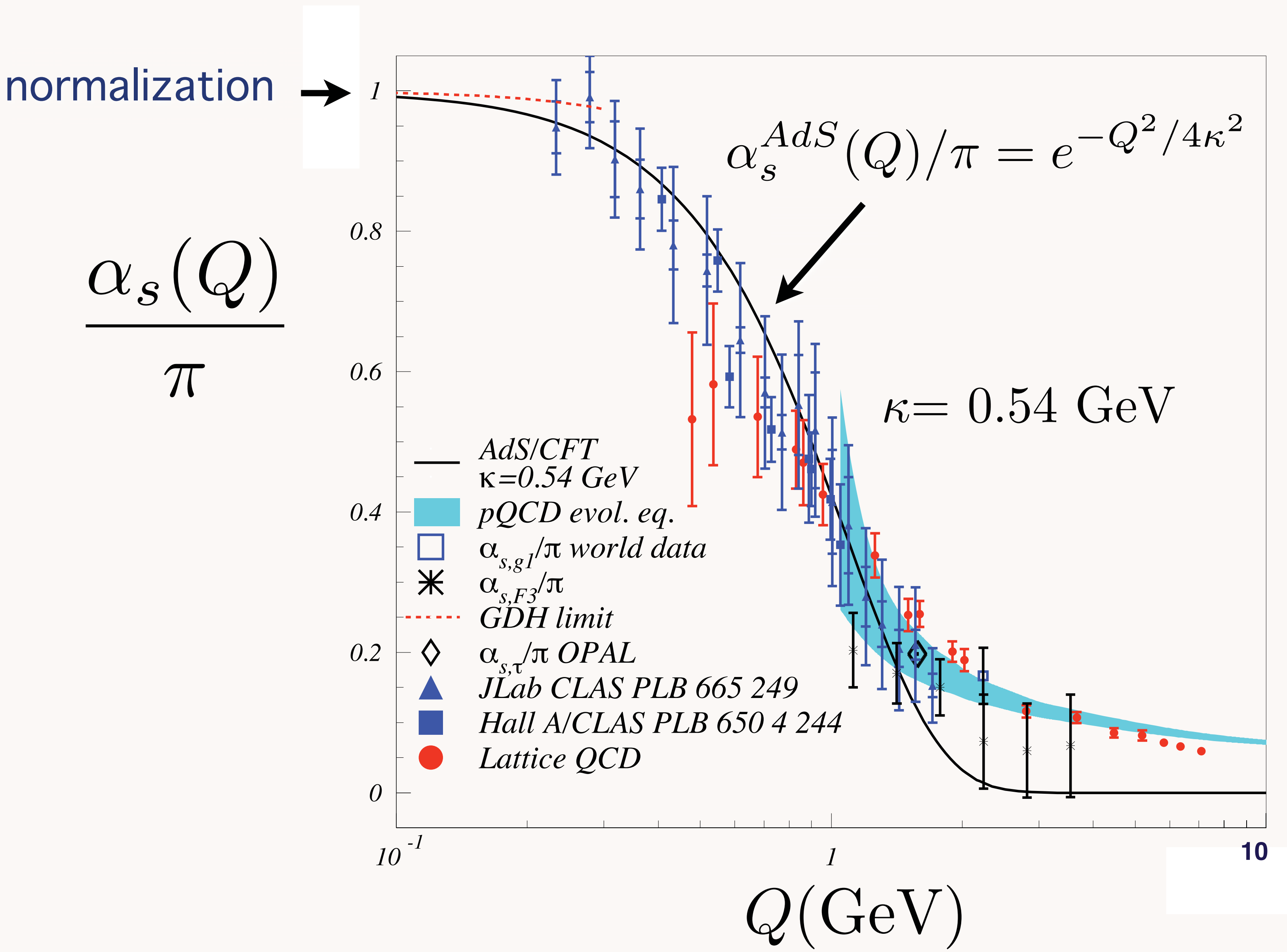
$$S = -\frac{1}{4}\int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2$$

$$\text{Thus } \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

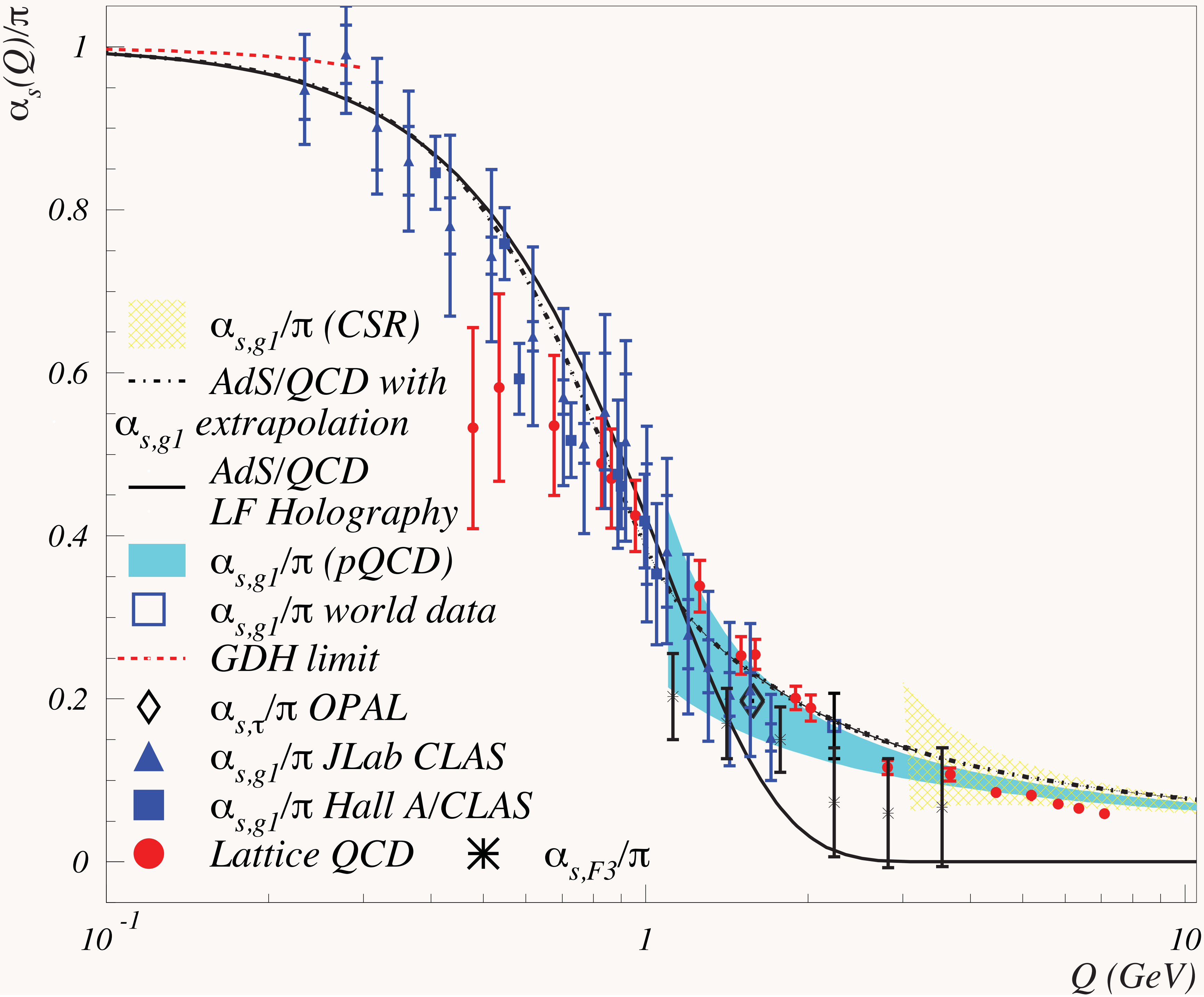
**Light-Front Holography:** $z \to \zeta = b_\perp \sqrt{x(1-x)}$

$$\alpha_s(q^2) \propto \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \quad \text{where } \alpha_s(\zeta) = e^{-\kappa^2 \zeta^2} \alpha_s(0)$$

# Running Coupling from AdS/QCD




Deur, de Teramond, sjb



αs(Q)/π
1
0.8
0.6
0.4
0.2
0
10-1
1
10
Q (GeV)
αs,g1/π (CSR)
AdS/QCD with
αs,g1 extrapolation
AdS/QCD
LF Holography
αs,g1/π (pQCD)
αs,g1/π world data
GDH limit
αs,τ/π OPAL
αs,g1/π JLab CLAS
αs,g1/π Hall A/CLAS
Lattice QCD
αs,F3/π

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓

Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

J=0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

# New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

# Lesson from QED and Lamb Shift:

## *Consequences of Maximum Quark and Gluon Wavelength*

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling

$$\alpha_s(Q^2) \text{ finite}, \beta \to 0 \text{ at small } Q^2$$

- Bound state quark and gluon Dyson-Schwinger Equation

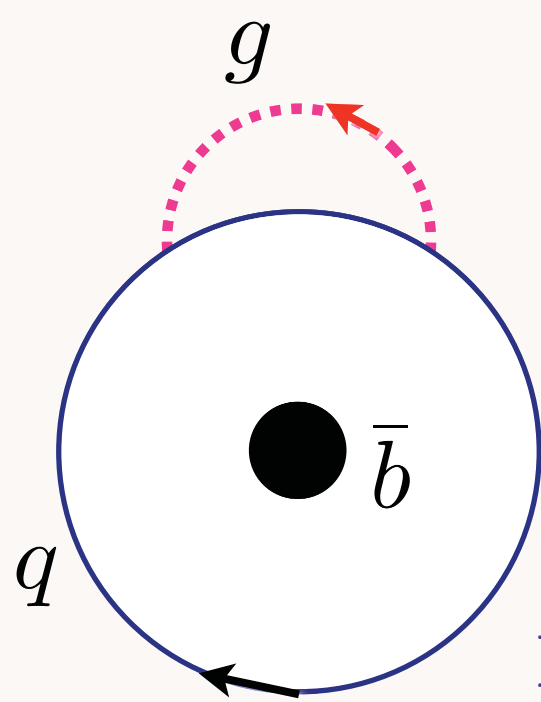
**Roberts et al.**

**Casher, Susskind**

- Quark and Gluon Condensates exist within hadrons

**Shrock, sjb**

*Use Dyson-Schwinger Equation for bound-state quark propagator:*




**B-Meson**

**Shrock, sjb**

**Roberts, Tandy Maris**

**Alkofer**

$$< \bar{b}|\bar{q}q|\bar{b} > \text{ not } < 0|\bar{q}q|0 >$$

# "One of the gravest puzzles of theoretical physics"

**DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA*
*Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*
*zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

*QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum*

**Shrock, sjb**

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, ...)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

**Shrock and sjb**

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.

- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, QGP, CGC, ...

*Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities.* —*Mark Twain*

# Many Analogs: QED/QCD

- Diffractive Dissociation of Atoms/Hadrons
- Atomic/Color Transparency
- Light-Front Wavefunctions
- Atomic Alchemy/B decay
- Atom Formation/Hadronization
- Spontaneous pair production/ Confinement
- Intrinsic heavy leptons/Intrinsic Charm
- True Muonium/Quarkonium

SCIENCE VOL 265 15 SEPTEMBER 1995

# A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

Frank and Ernest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