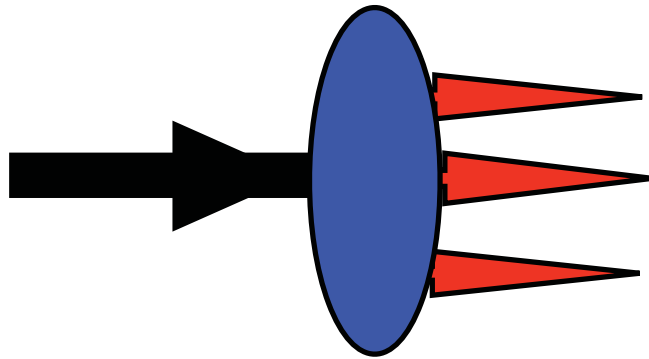
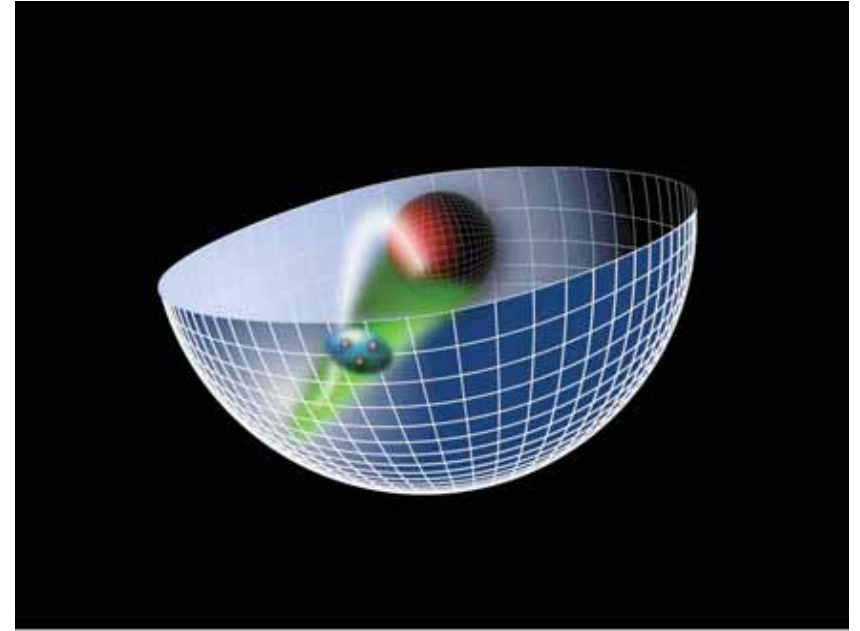


New Insights into Hadron Physics from AdS/QCD and Light-Front Holography



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond



The Abdus Salam
International Centre for Theoretical Physics

May 27, 2010



Stan Brodsky

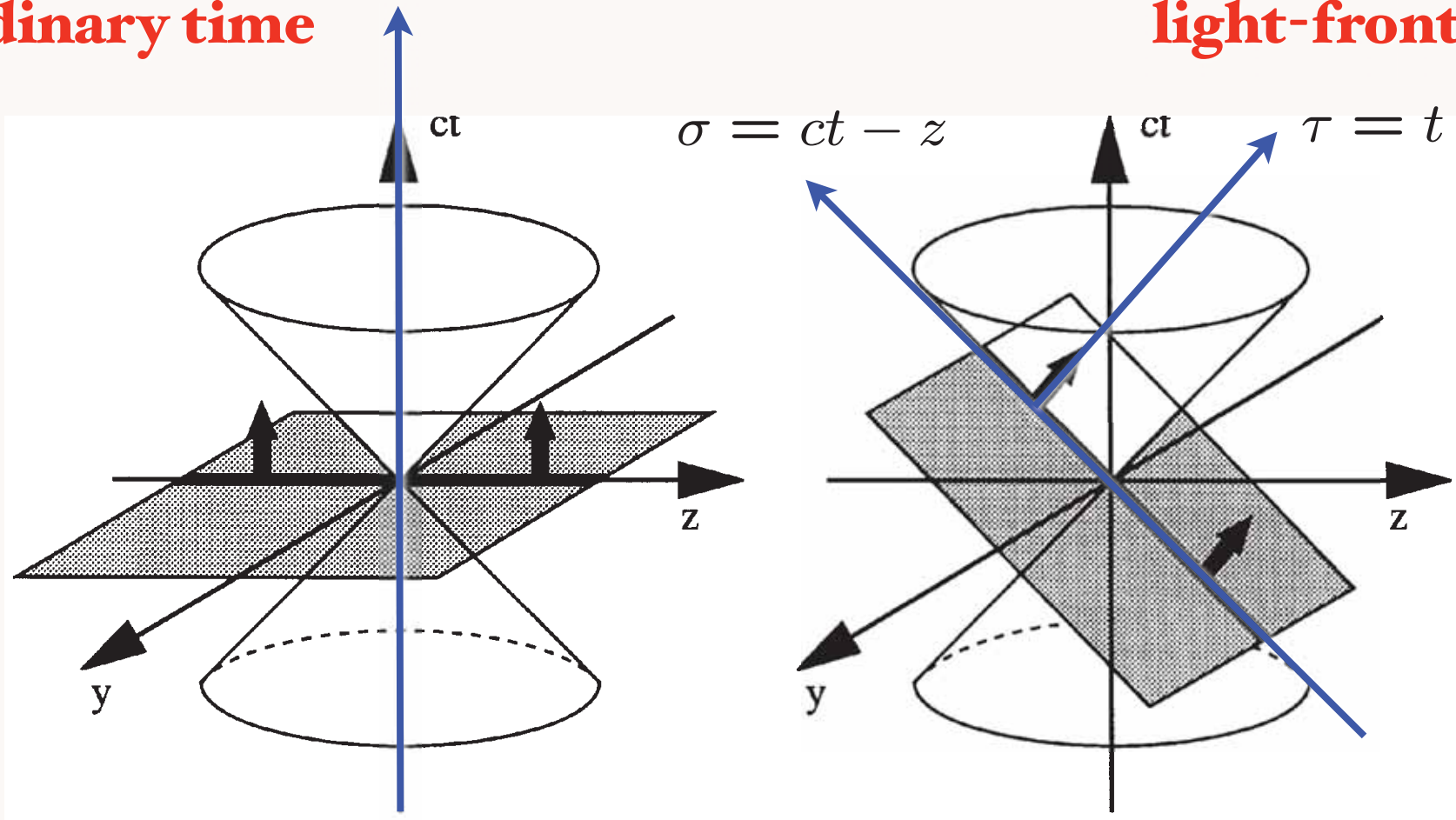


& CP³-Origins

Dirac's Amazing Idea: The Front Form

**Evolve in
ordinary time**

**Evolve in
light-front time!**



Instant Form

Front Form

*Each element of
flash photograph
illuminated
at same Light Front
time*

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

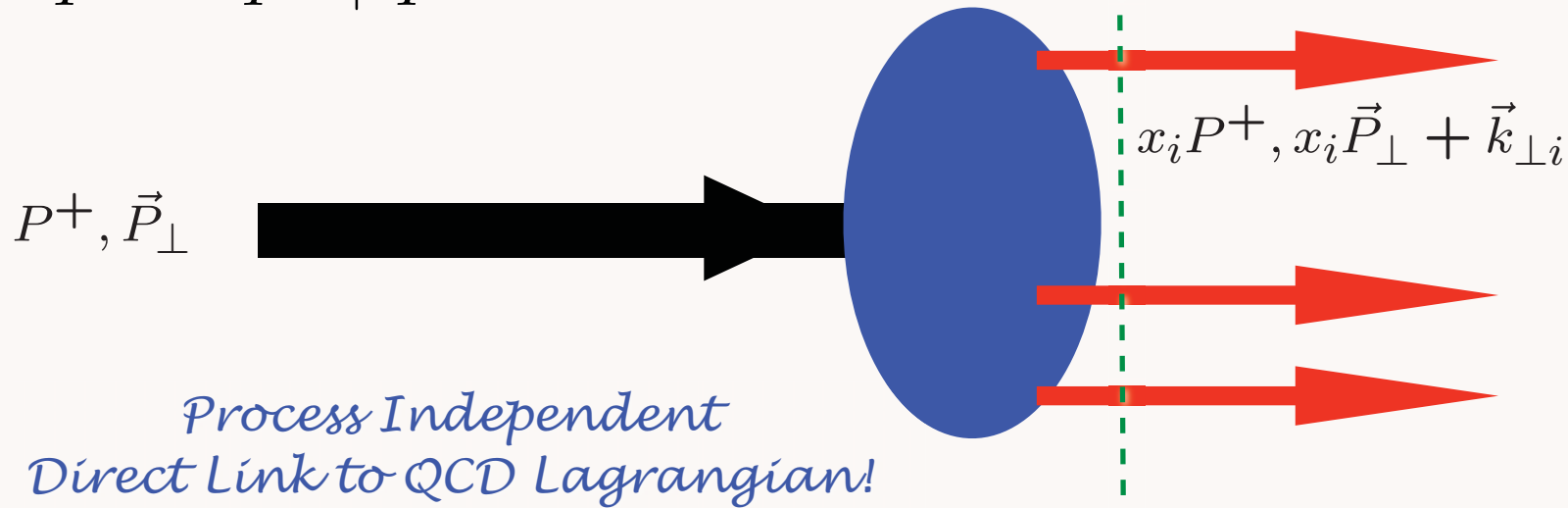
Causal, Trivial Vacuum



Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

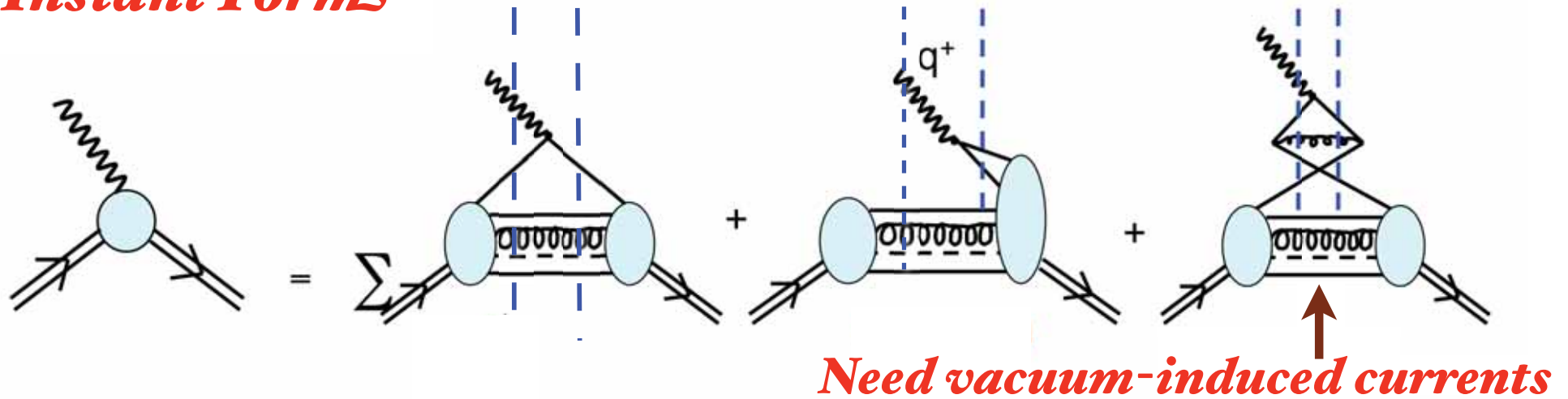
$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

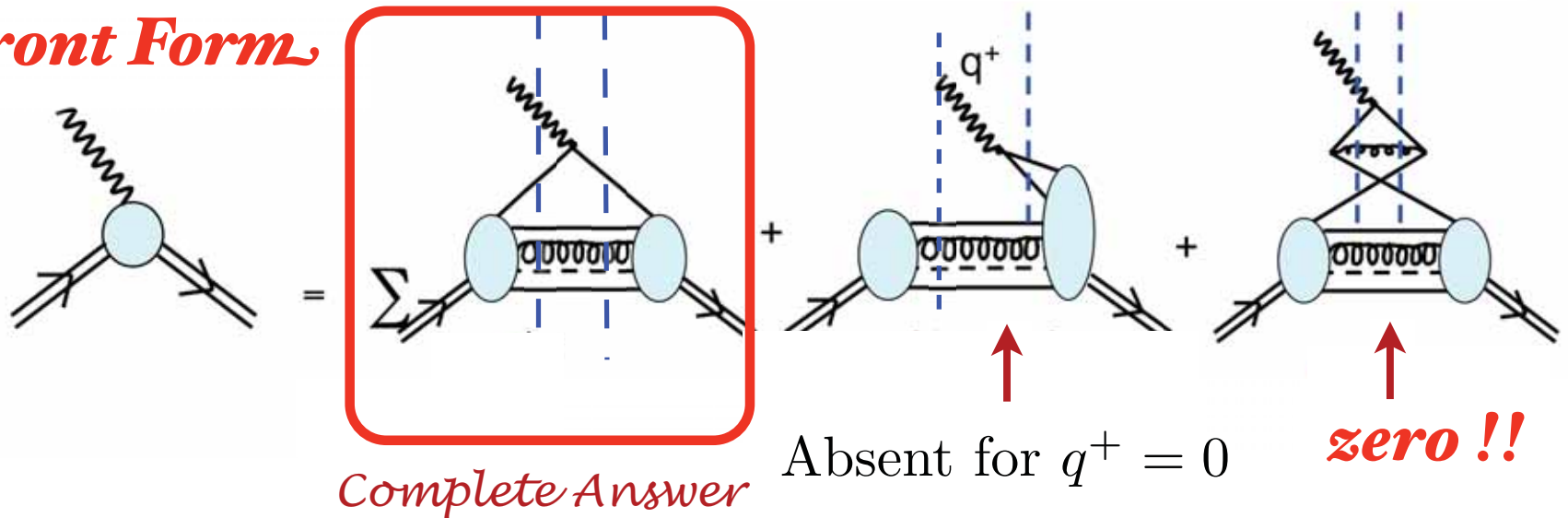
Calculation of Form Factors in Equal-Time Theory

Instant Form



Calculation of Form Factors in Light-Front Theory

Front Form



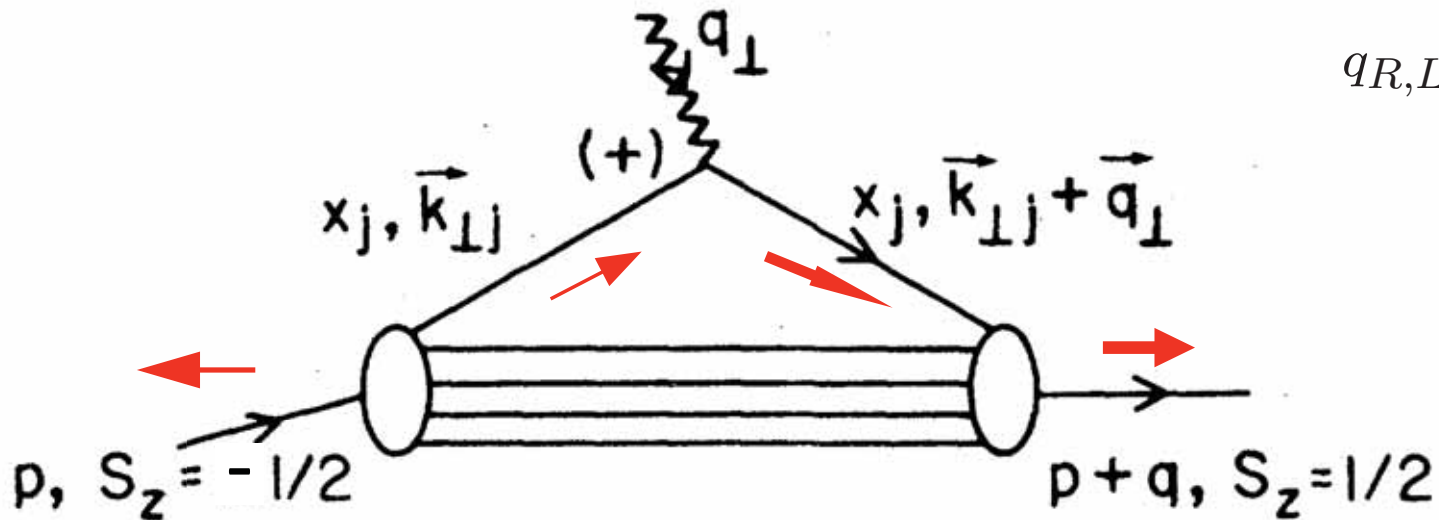
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

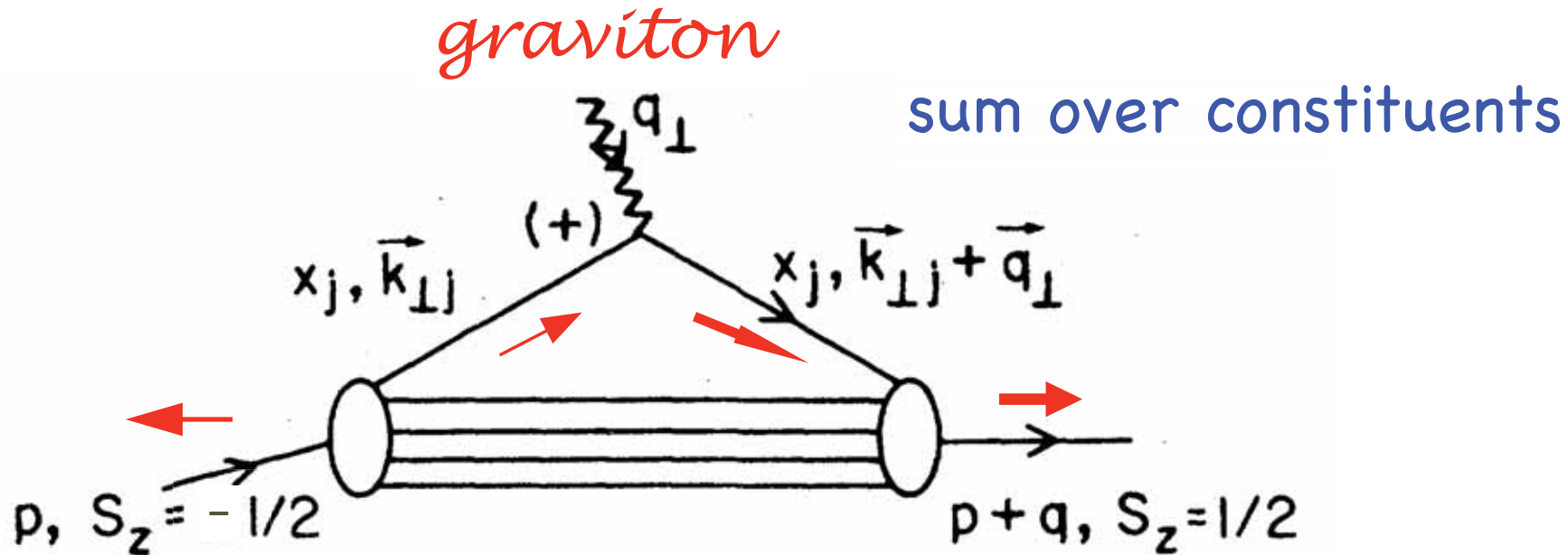


Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum*

Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem



**Hwang, Schmidt, sjb;
Holstein et al**

$$B(0) = 0$$

Each Fock State

Angular Momentum on the Light-Front

Jaffe definition

LC gauge

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

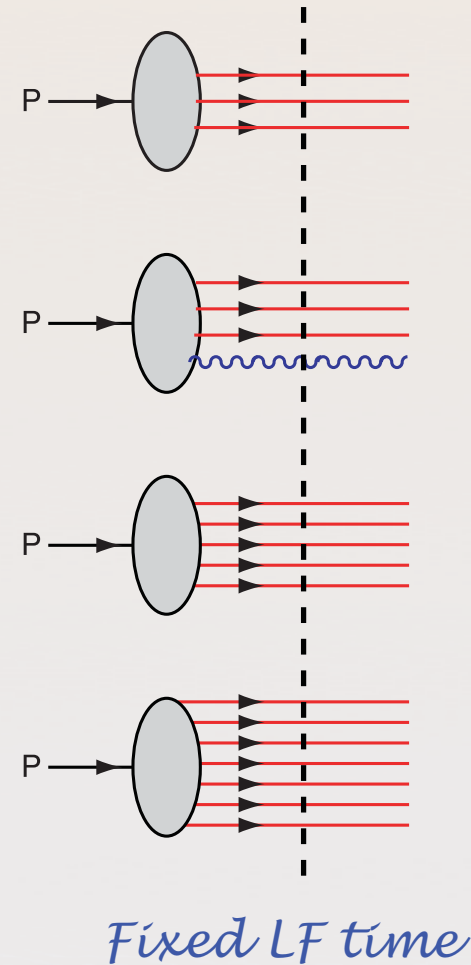
are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks
 $c(x), b(x)$ at high x !

$$\bar{s}(x) \neq s(x)$$

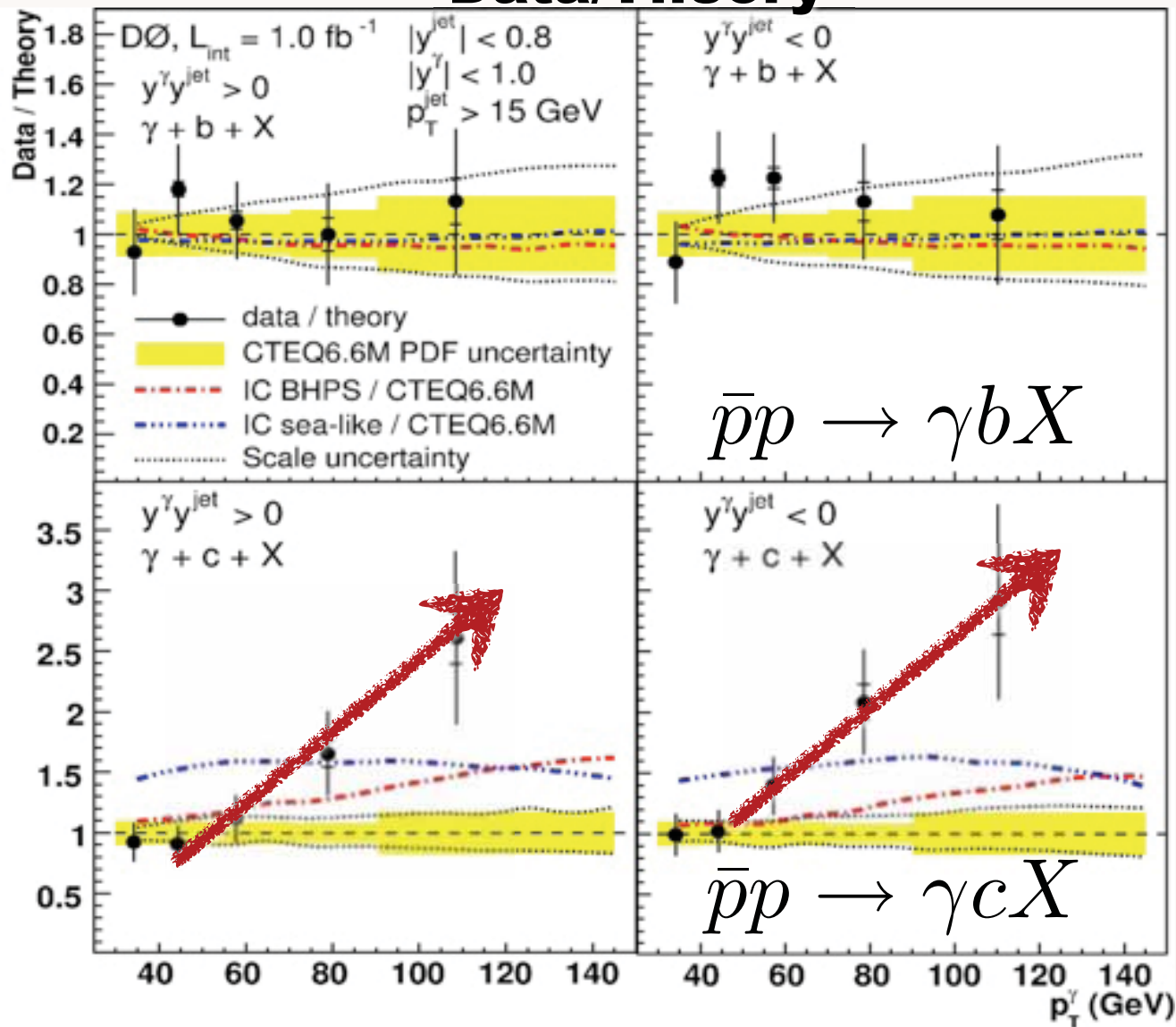
$$\bar{u}(x) \neq \bar{d}(x)$$



Mueller: gluon Fock states \rightarrow BFKL Pomeron *Hidden Color*

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections
in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

Data/Theory



$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

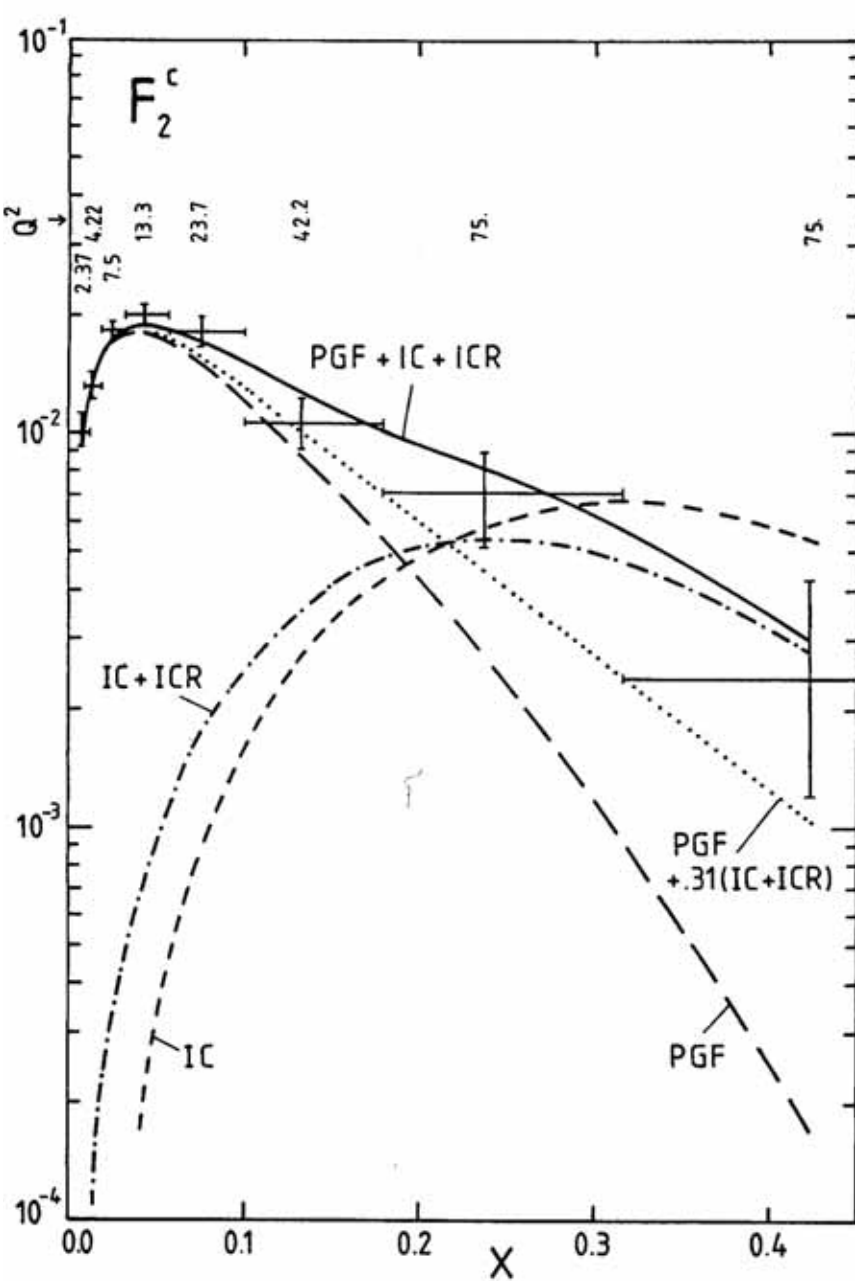
Ratio
insensitive to
gluon PDF,
scales

Signal for
significant IC
at $x > 0.1$

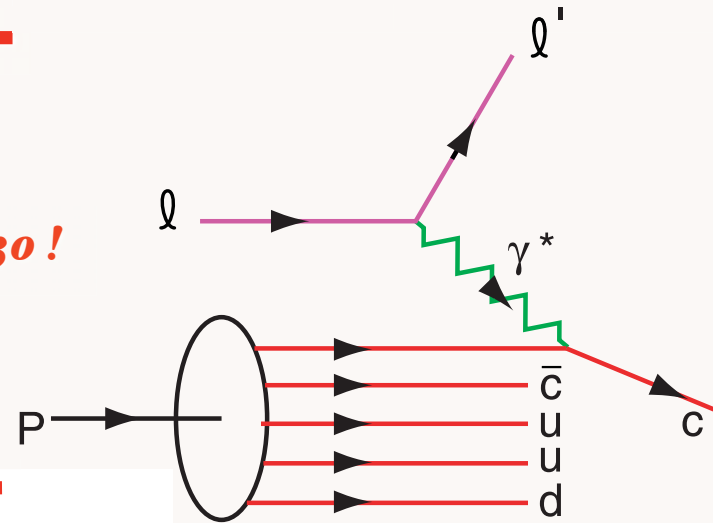
Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm
Never been checked!

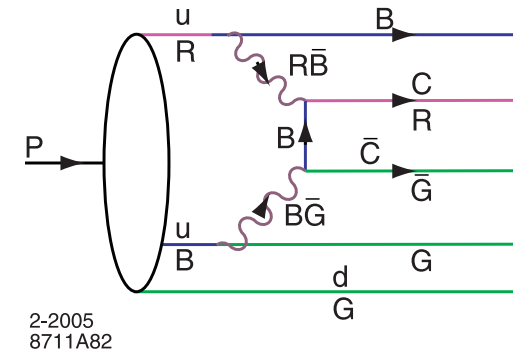


←
factor of 30!
←



DGLAP / Photon-Gluon Fusion: factor of 30 too small

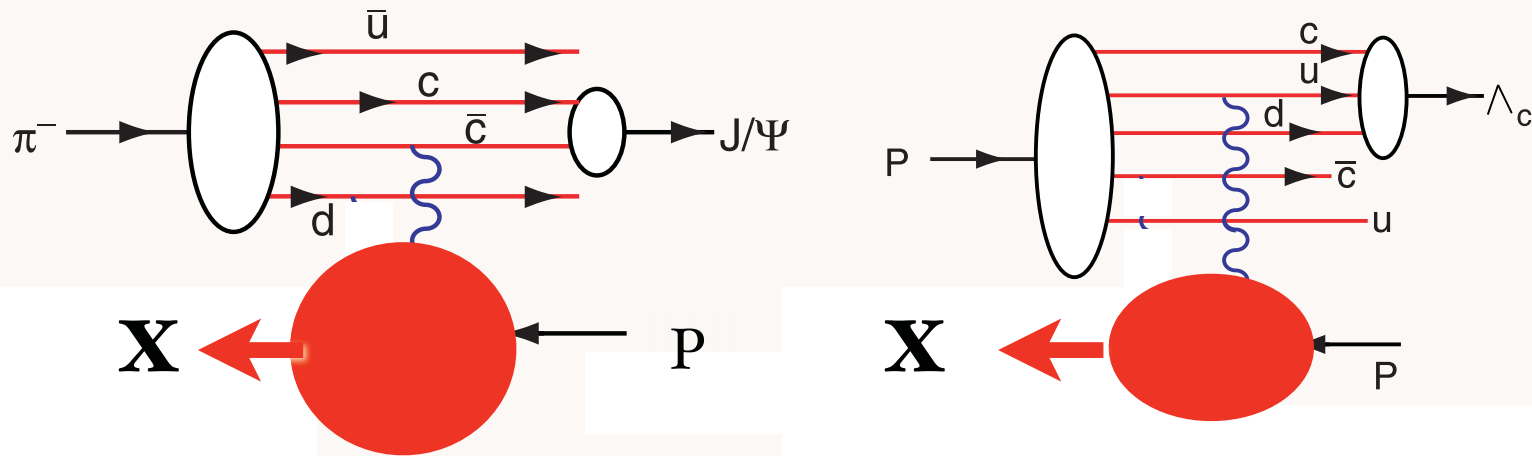
Intrinsic Heavy-Quark Fock States



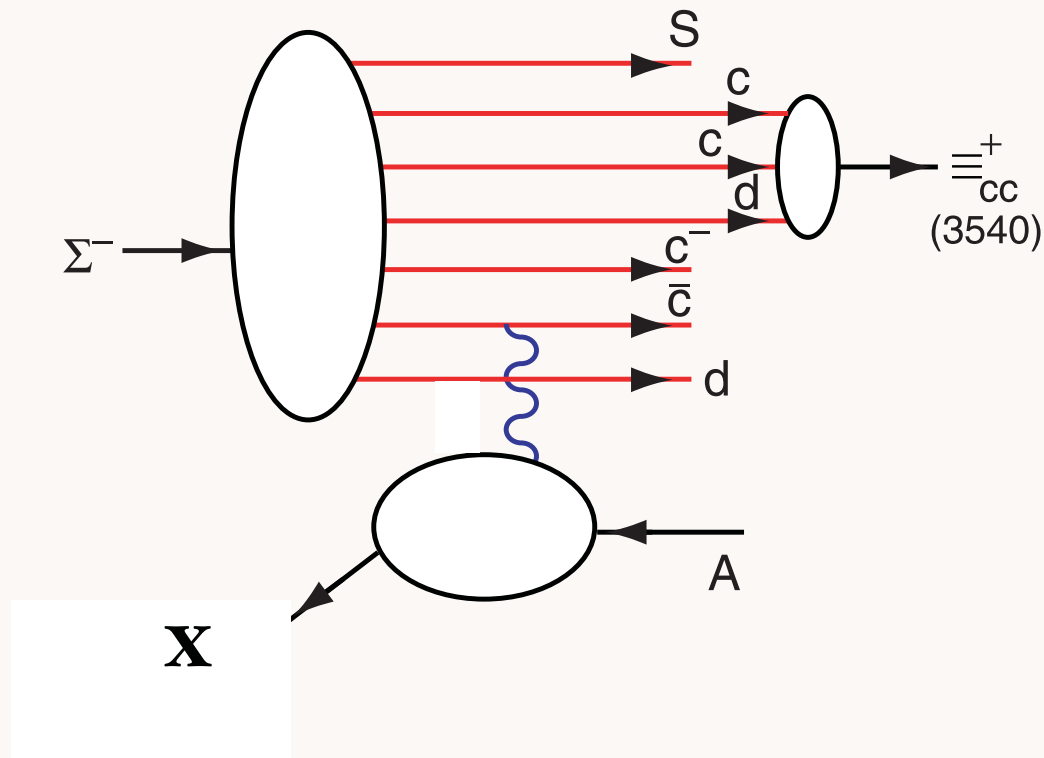
- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

- EMC data: $c(x, Q^2) > 30 \times \text{DGLAP}$
 $Q^2 = 75 \text{ GeV}^2, x = 0.42$
- High x_F $pp \rightarrow J/\psi X$
- High x_F $pp \rightarrow J/\psi J/\psi X$
- High x_F $pp \rightarrow \Lambda_c X$ ISR
- High x_F $pp \rightarrow \Lambda_b X$ ISR
- High x_F $pp \rightarrow \Xi(ccd) X$ (SELEX)

Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ , Λ_c and other Charm Hadrons at High x_F

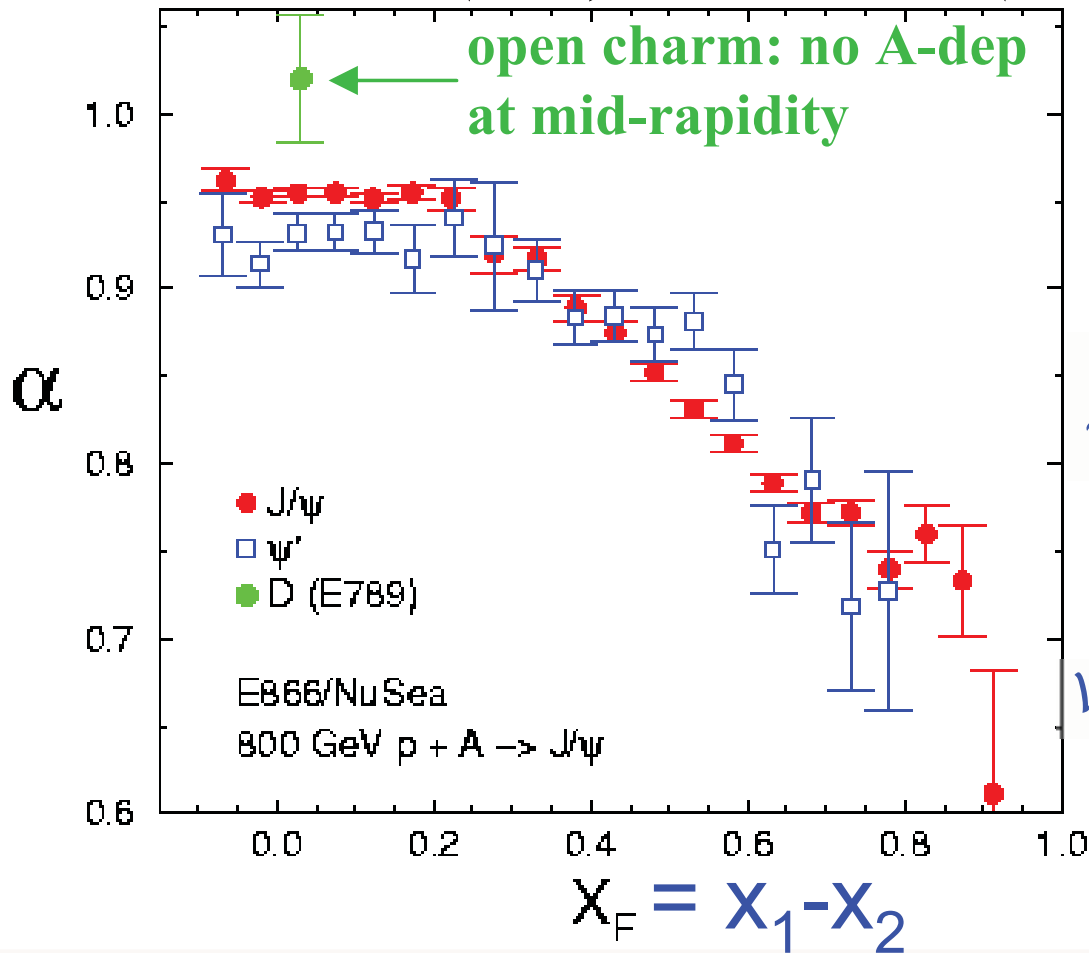


Production of a Double-Charm Baryon

SELEX high x_F $\langle x_F \rangle = 0.33$

800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$
 PRL 84, 3256 (2000); PRL 72, 2542 (1994)

$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$$



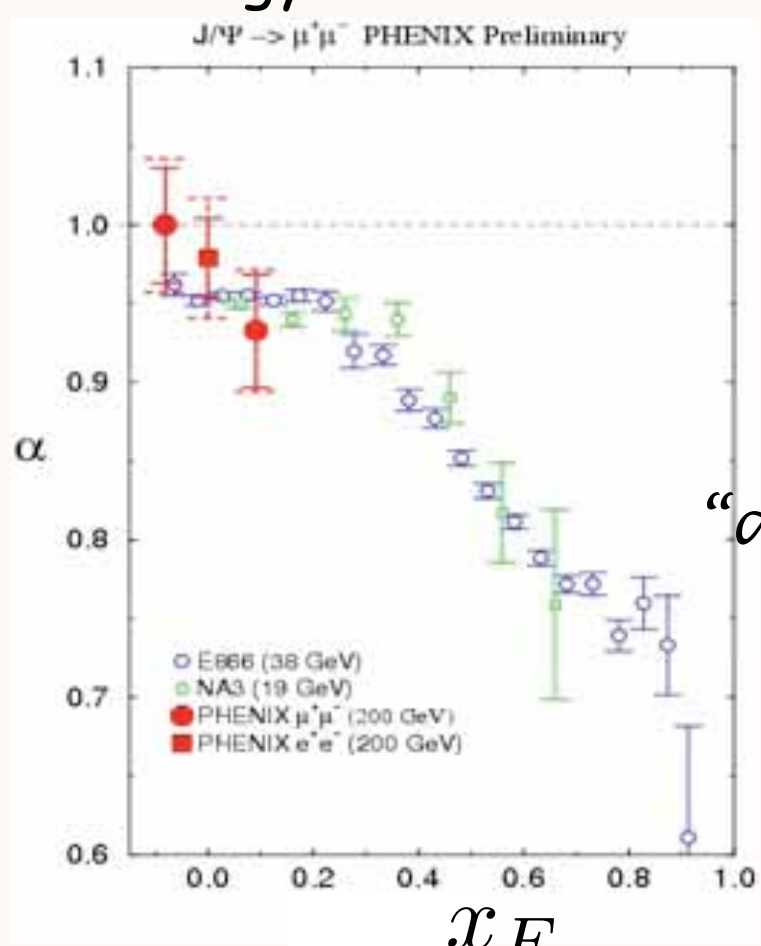
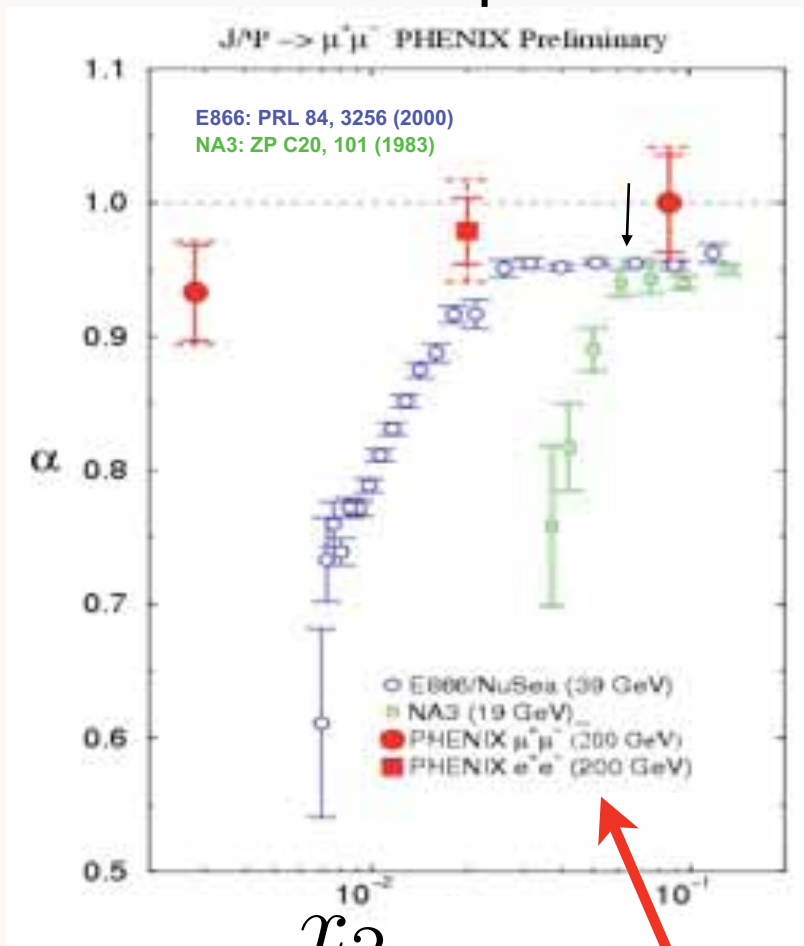
Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization!

Violation of factorization in charm hadroproduction.

[P. Hoyer](#), [M. Vanttinen \(Helsinki U.\)](#), [U. Sukhatme \(Illinois U., Chicago\)](#). HU-TFT-90-14, May 1990. 7pp.
 Published in Phys.Lett.B246:217-220,1990

PHENIX compared to lower energy measurements



Huge "absorption" effect



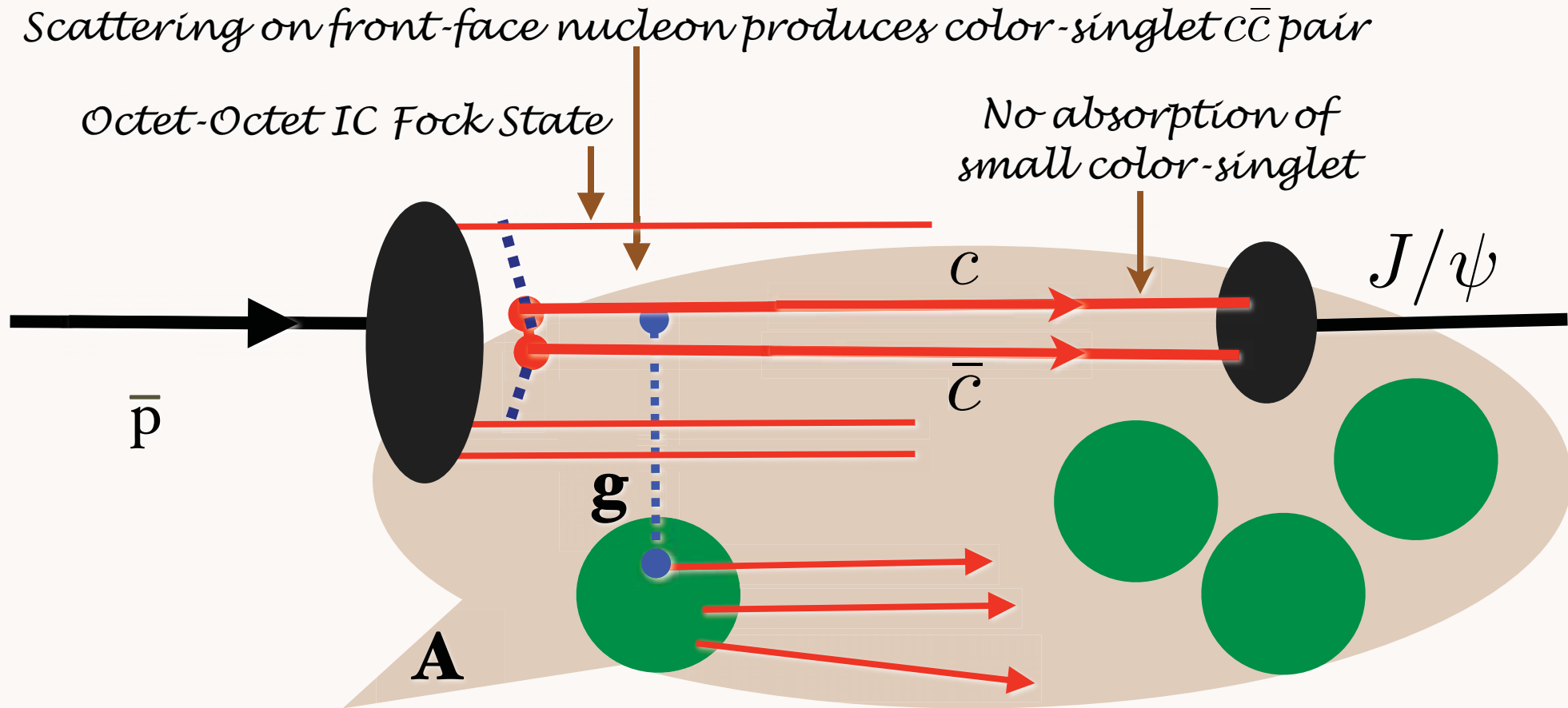
Klein, Vogt, PRL 91:142301, 2003
 Kopeliovich, NP A696:669, 2001

Violates PQCD factorization!

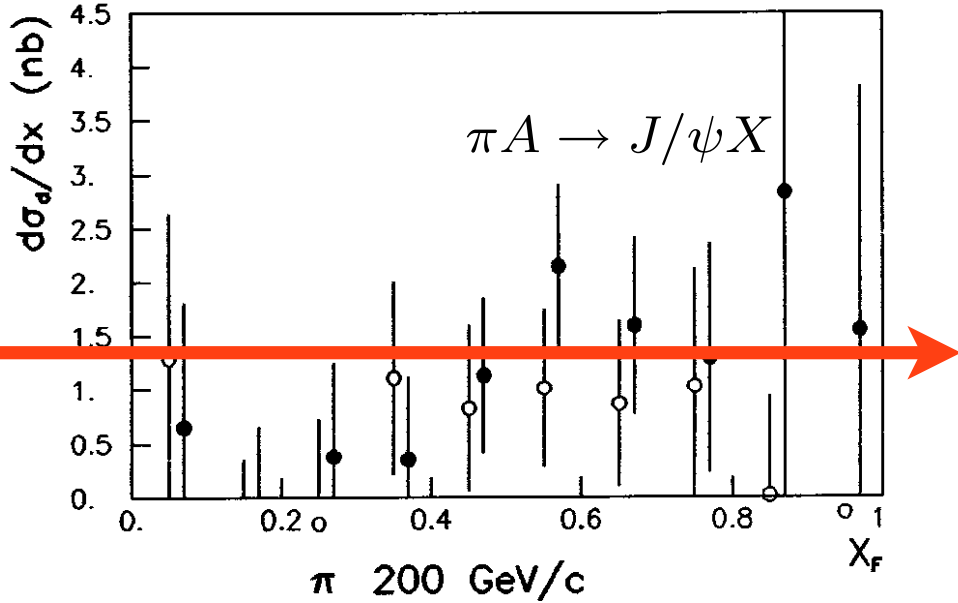
$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

*Color-Opaque IC Fock state
interacts on nuclear front surface*

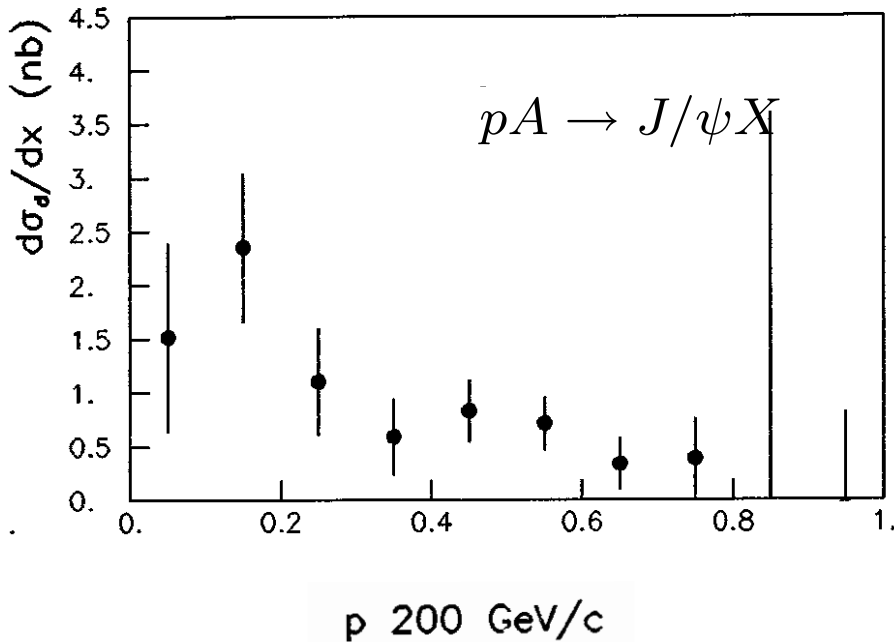


$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$



$A^{2/3}$ component

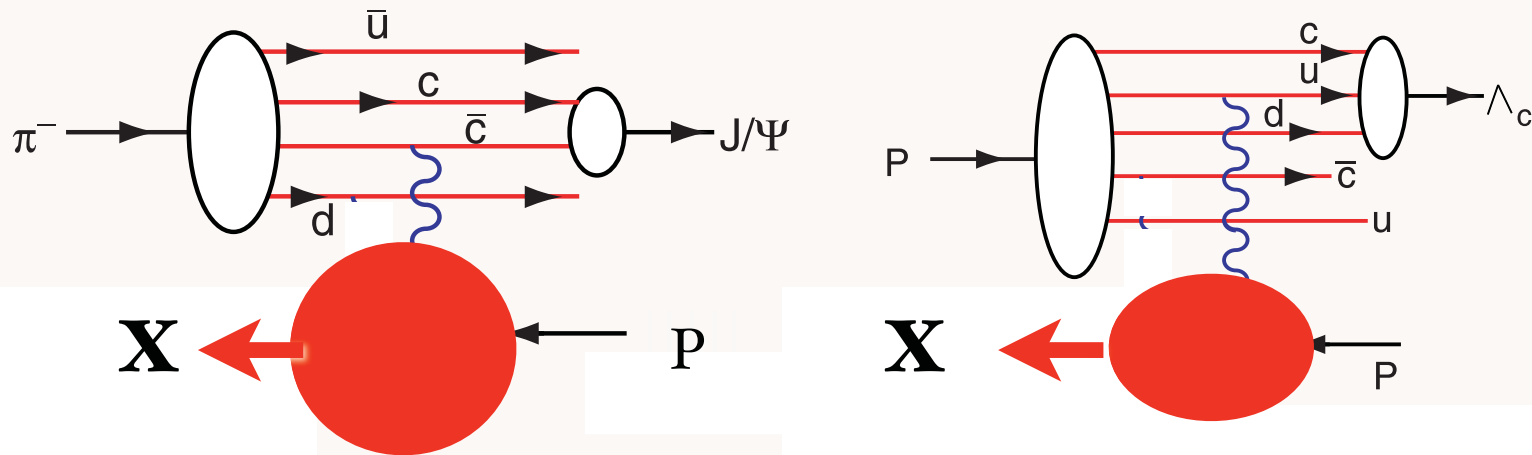
J. Badier et al, NA3



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

Excess beyond conventional PQCD subprocesses

Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ , Λ_c and other Charm Hadrons at High x_F

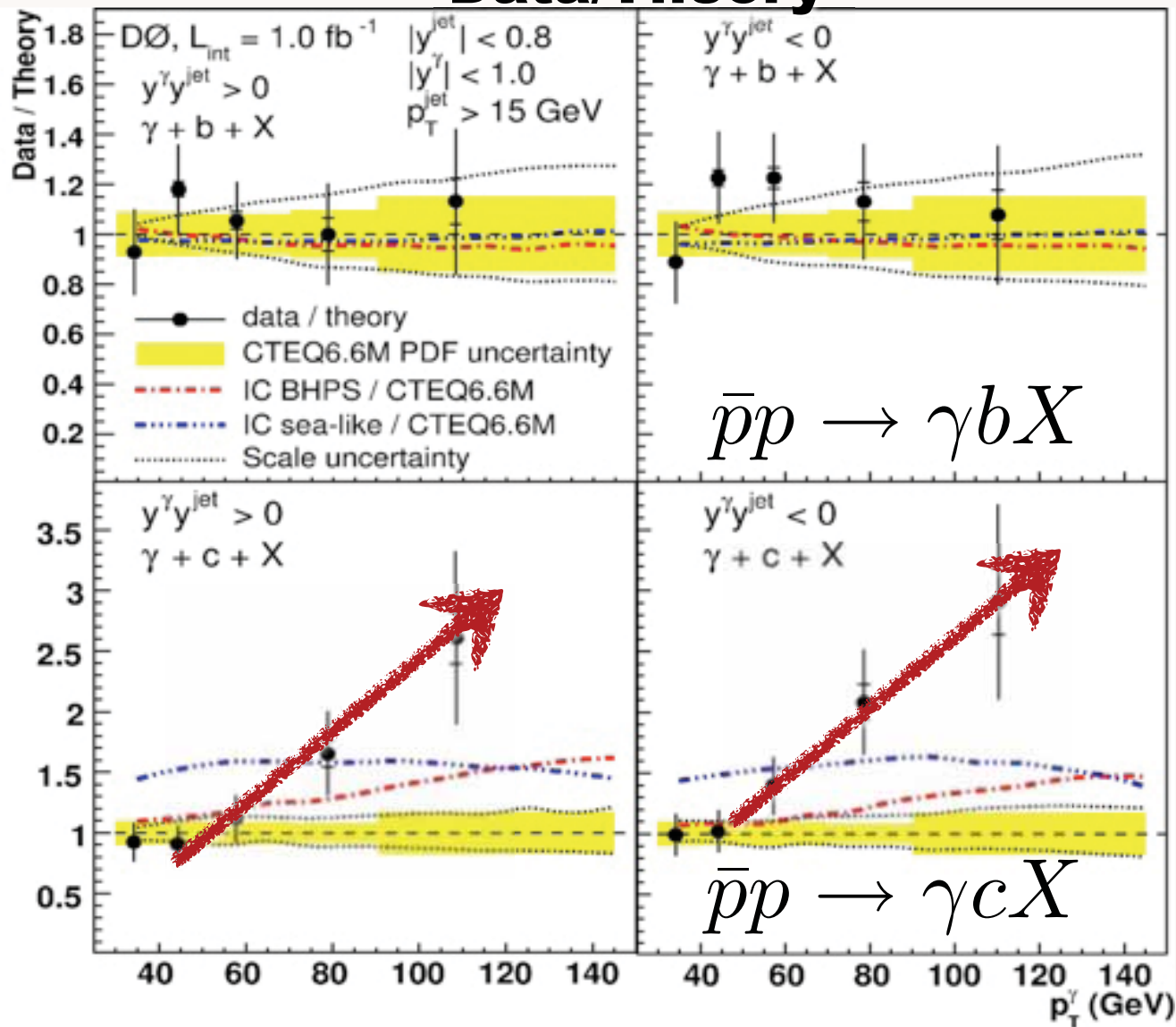
- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \rightarrow J/\psi X$ (Mueller, Gunion, Tang, SJB)
- Color Octet IC Explains $A^{2/3}$ behavior at high x_F (NA3, Fermilab) *Color Opacity* (Kopeliovitch, Schmidt, Soffer, SJB)
- IC Explains $J/\psi \rightarrow \rho\pi$ puzzle (Karliner, SJB)
- IC leads to new effects in B decay (Gardner, SJB)

Higgs production at $x_F = 0.8$!

Goldhaber, Kopeliovich,
Schmidt, Soffer, sjb

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections
in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

Data/Theory

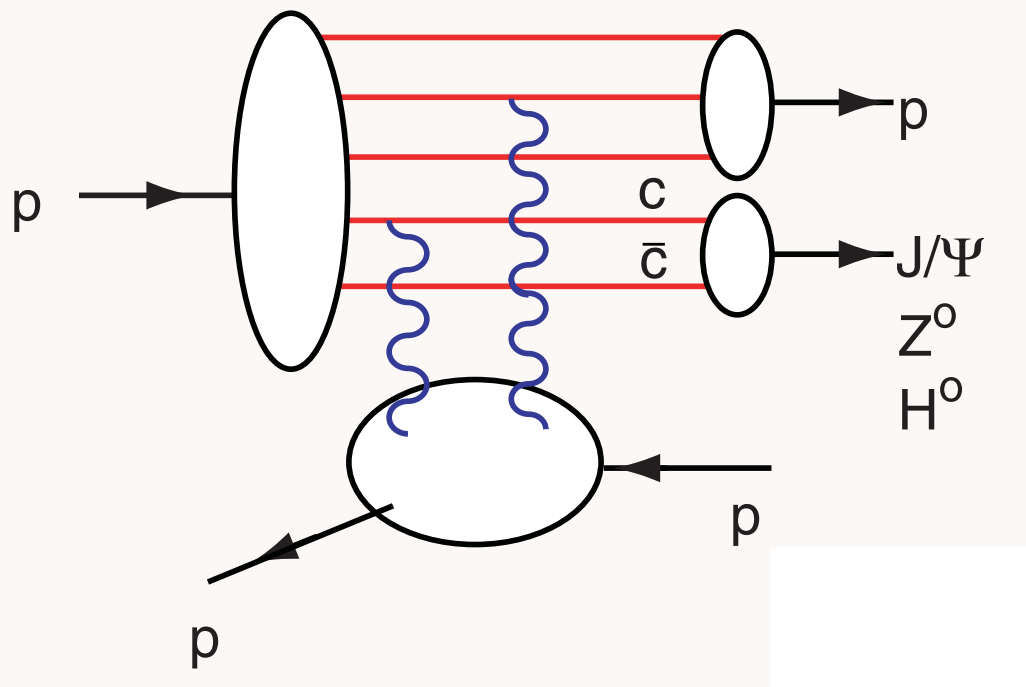


$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

Ratio
insensitive to
gluon PDF,
scales

Signal for
significant IC
at $x > 0.1$

Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$p p \rightarrow J/\psi p p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

*Exclusive Diffractive
High- x_F Higgs Production!*

Kopeliovich, Schmidt,
Soffer, sjb

Intrinsic $c\bar{c}$ pair formed in color octet 8_C in proton wavefunction Large Color Dipole

Collision produces color-singlet J/ψ through color exchange

RHIC Experiment

Hadron Distribution Amplitudes

$$\phi_M(x, Q) = \int^Q d^2\vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

$\sum_i x_i = 1$

Lepage, sjb
 $k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE,

- Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

Light-Front formalism links dynamics to spectroscopy

Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

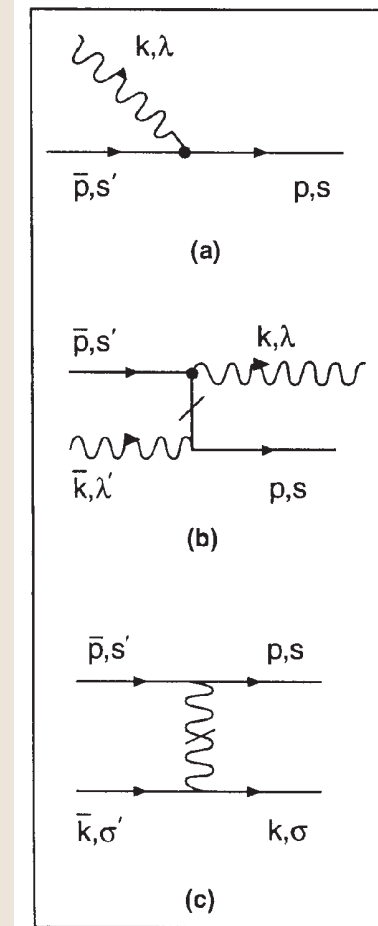
Heisenberg Matrix Formulation

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

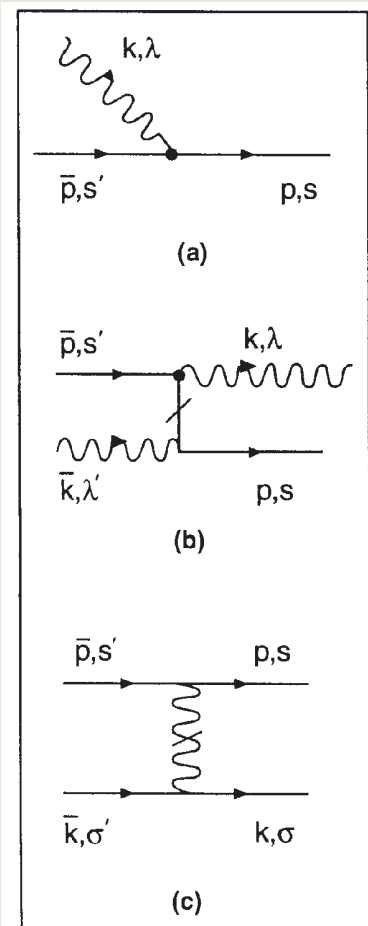
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions



Heisenberg Matrix Formulation

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

Discretized Light-Cone Quantization



n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	gg g
6	q \bar{q} gg								.				.	.
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gg gg
10	q \bar{q} gg g
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g				
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}			

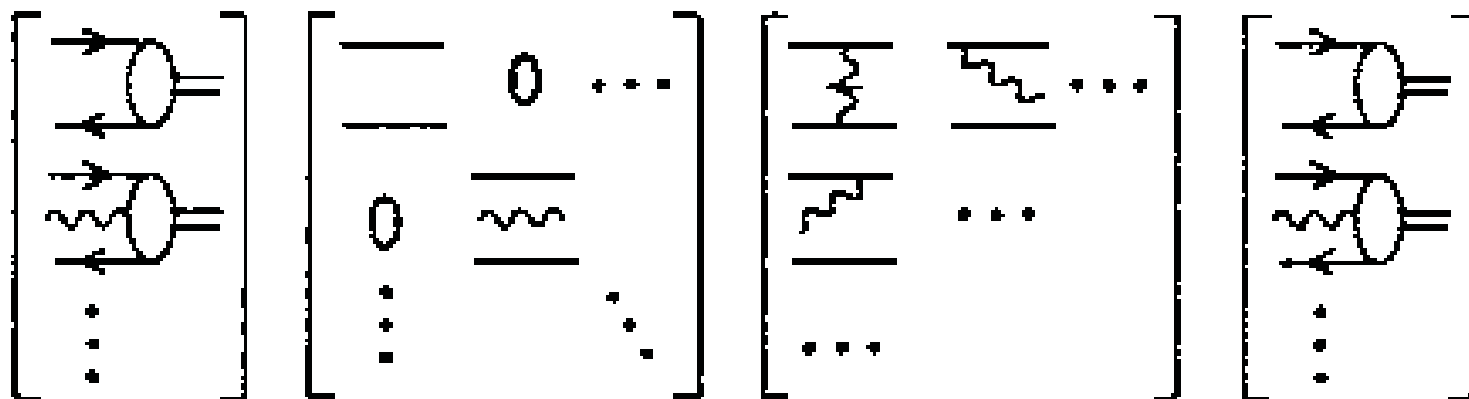
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^- . Discrete k^+ ; frame-independent truncation

LIGHT-FRONT SCHRÖDINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$$A^+ = 0$$

G.P. Lepage, sjb

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

de Teramond, Deur, Shrock, Roberts, Tandy

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi(x, k_{\perp}) \quad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^{μ}

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

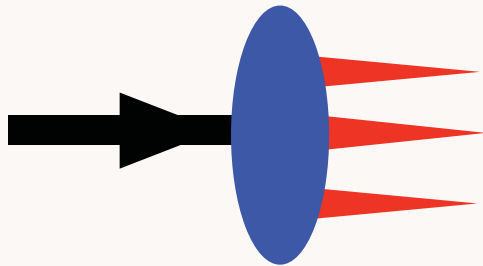
Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Holography and Non-Perturbative QCD

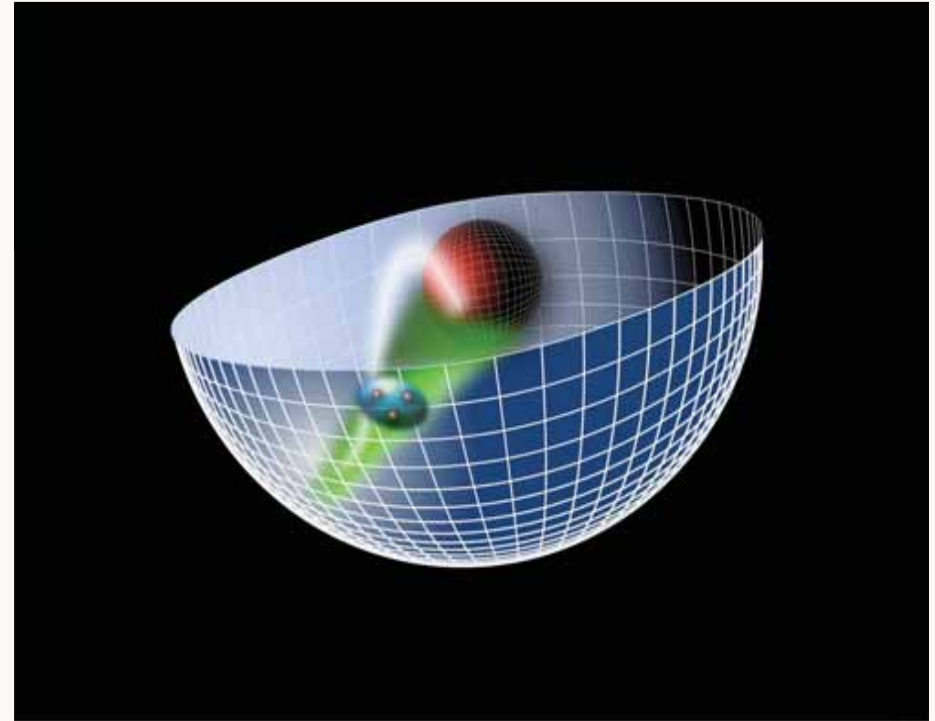
Goal:

**Use AdS/QCD duality to construct
a first approximation to QCD**

*Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR*



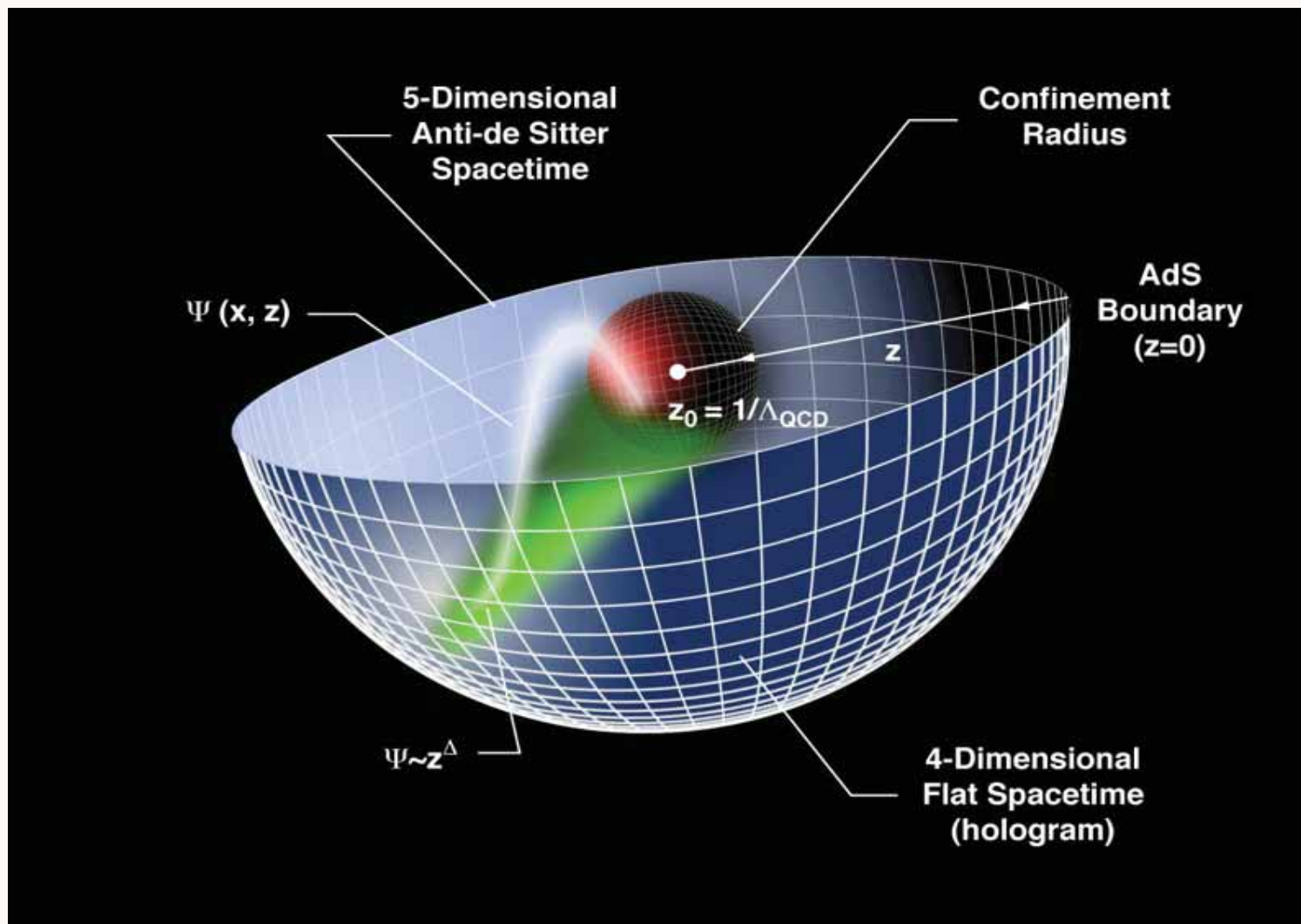
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



**in collaboration with
Guy de Teramond and Alexandre Deur**

Central problem for strongly-coupled gauge theories

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Conformal Theories are invariant under the Poincare and conformal transformations with

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$

the generators of $SO(4,2)$

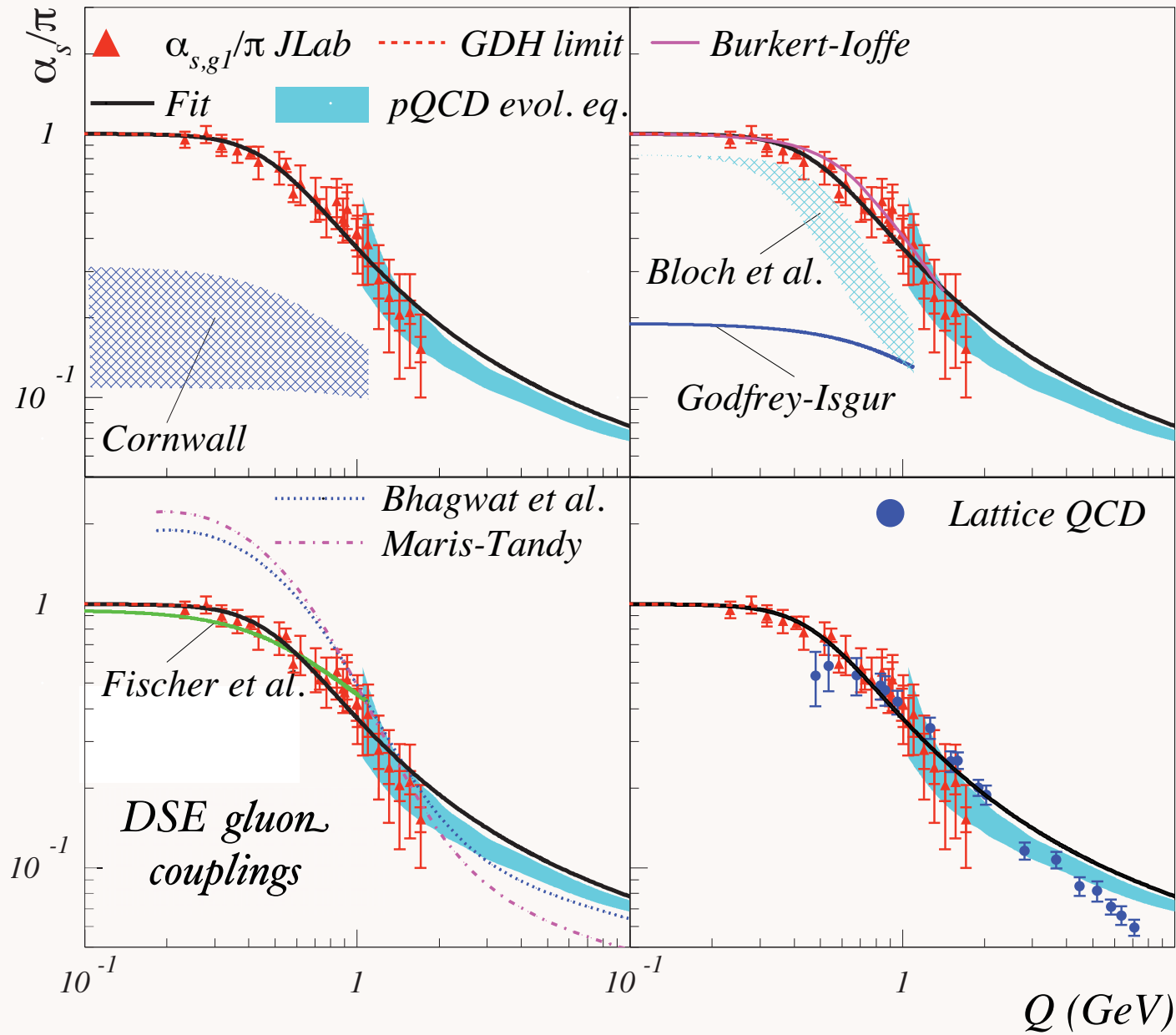
$SO(4,2)$ has a mathematical representation on AdS_5

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S^5$ to conformal $N=4$ SUSY

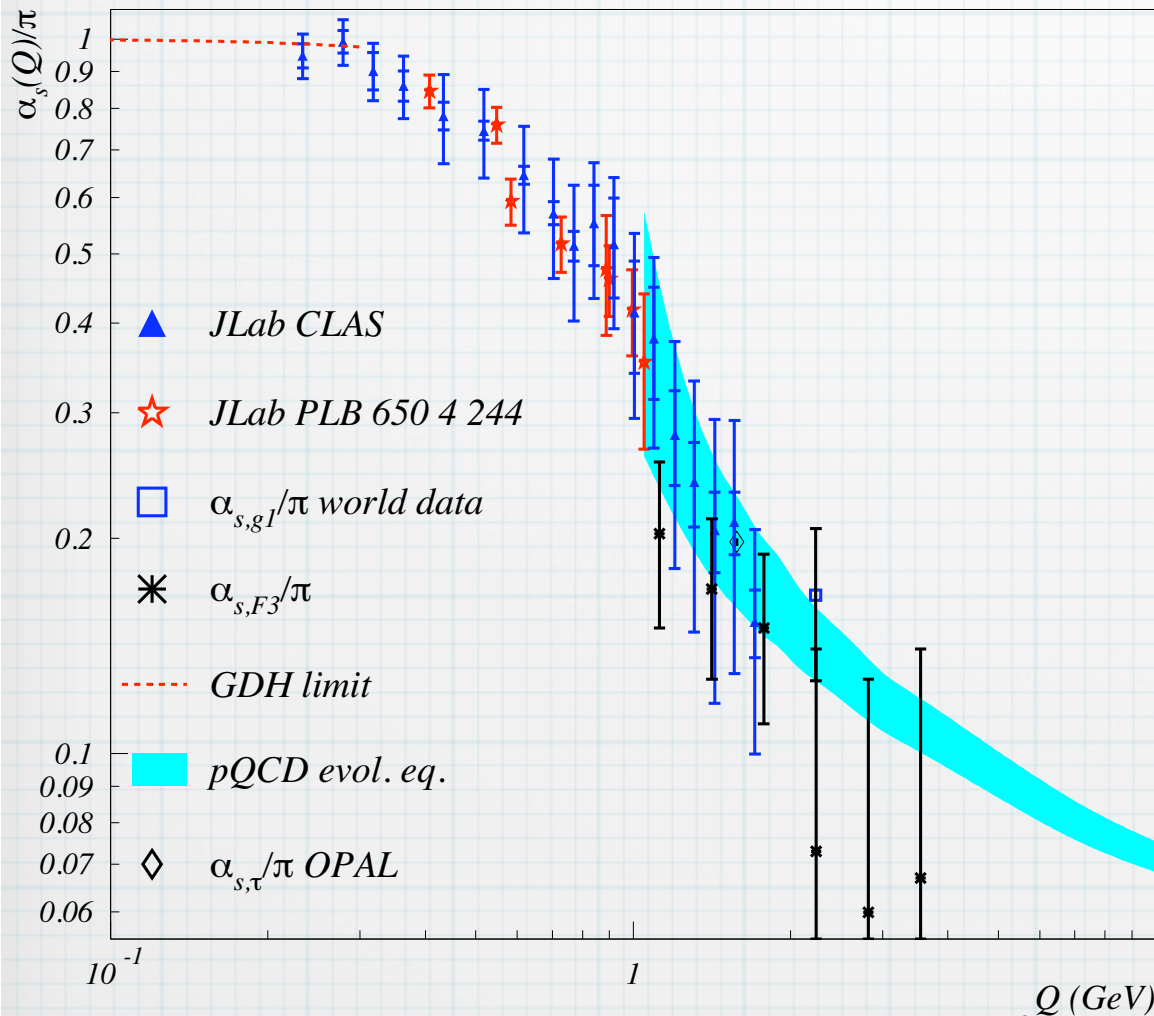
- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window**: $\alpha_s(Q^2) \simeq \text{const}$ at small Q^2
- **Use mathematical mapping of the conformal group $SO(4,2)$ to AdS_5 space**



Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left(g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi} \right)$$



g_1 = spin dependent structure function

Recent JLab data from E01(2008), CLAS, and Hall A

α_s runs only modestly at small Q^2

Gribov

Fig. from 08034119, Duer et al.

Maximal Wavelength of Confined Fields

$$(x - y)^2 < \Lambda_{QCD}^{-2}$$

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

Shrock, sjb

*Quark and Gluon vacuum polarization insertions
decouple: IR fixed Point*

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

**Trieste ICTP
May 27, 2010**


AdS/QCD and Hadronic Physics

**Stan Brodsky
SLAC-CP3**

Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

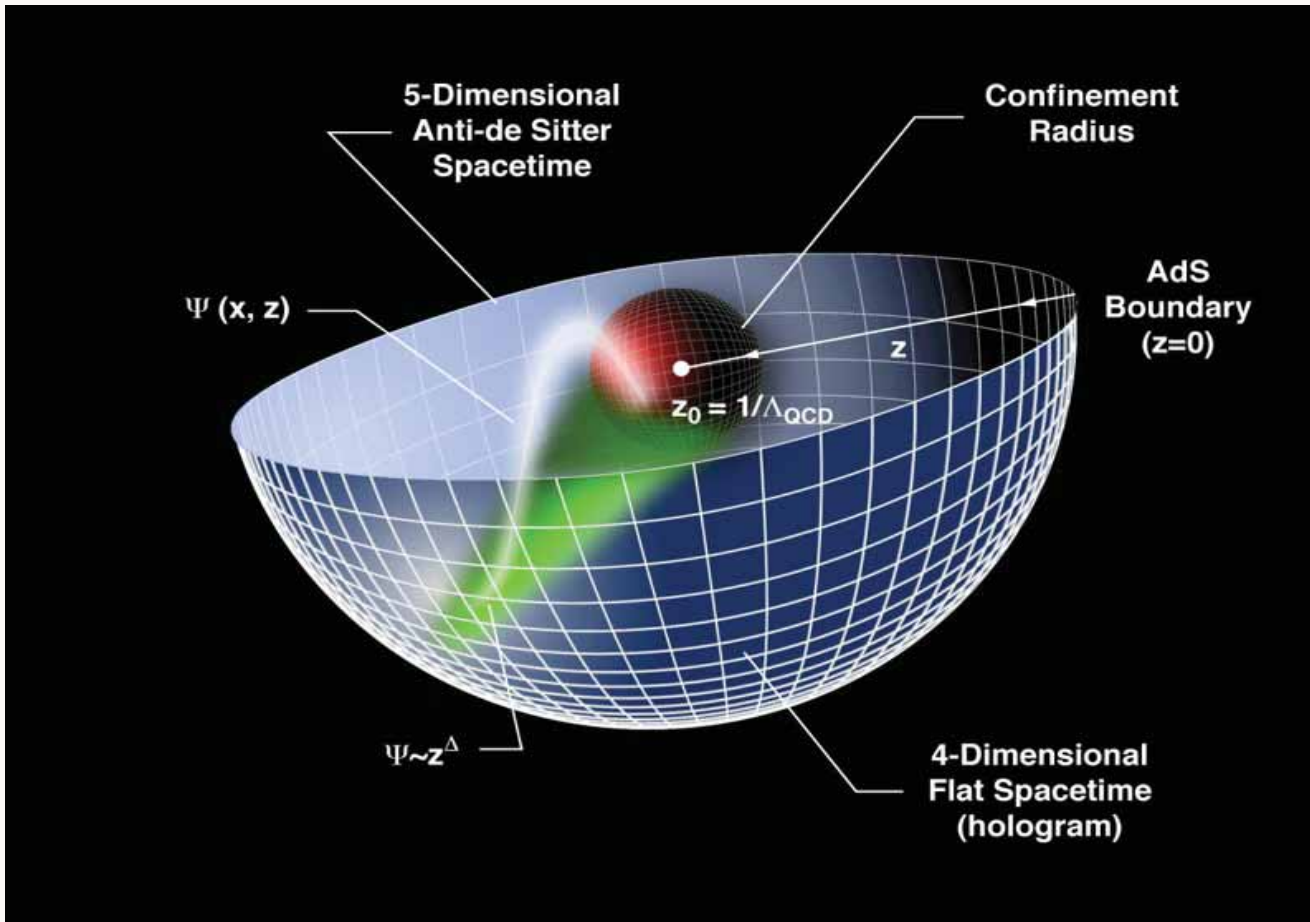
$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



8-2007
8685A14

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) [Polchinski and Strassler \(2001\)](#).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) [Karch, Katz, Son and Stephanov \(2006\)](#).

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.
- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates, $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

*AdS Schrodinger Equation for bound state
of two scalar constituents:*

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

**L: light-front orbital angular
momentum**

Derived from variation of Action in AdS₅

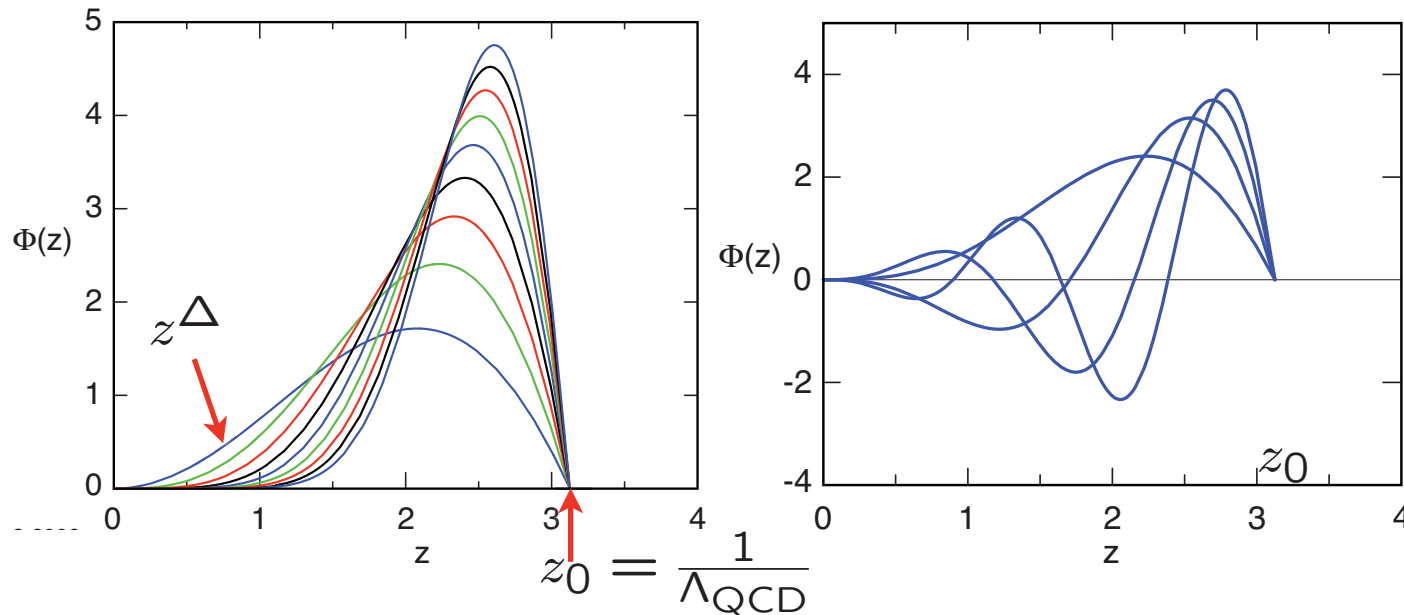
Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

*Match fall-off at small z to conformal twist-dimension,
at short distances*

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1 \dots \ell_m\}} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4- d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

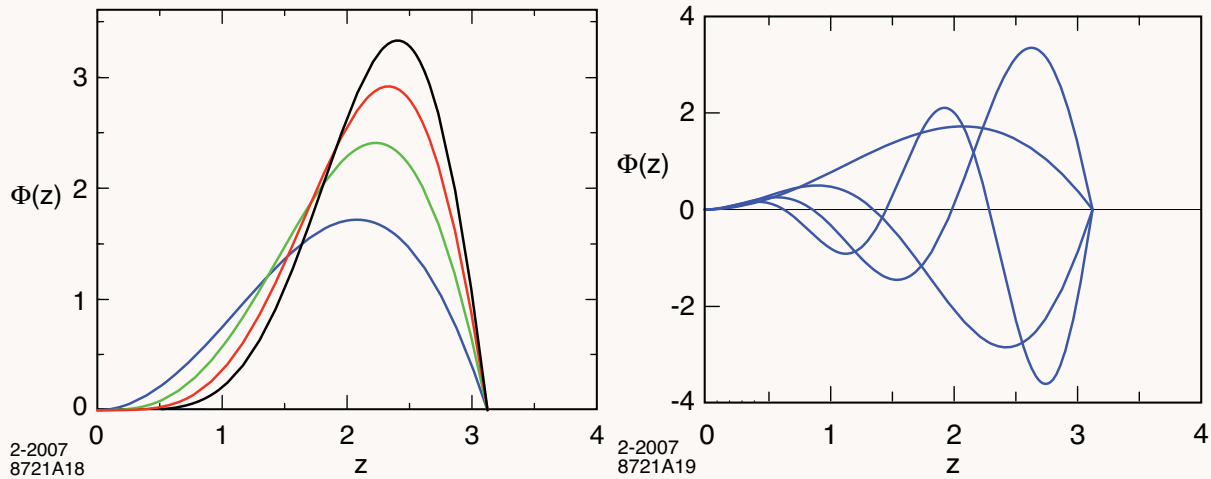


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$.

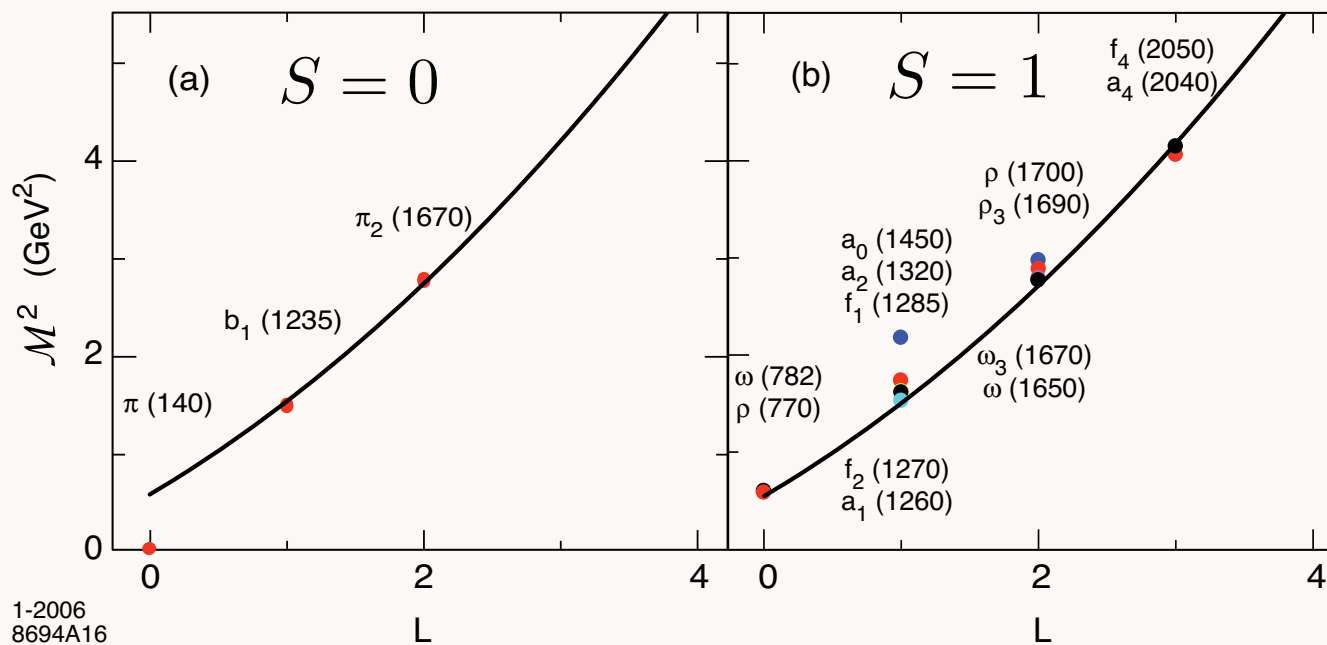


Fig: Light meson and vector meson orbital spectrum $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$

Soft-Wall Model

$$S = \int d^4x dz \sqrt{g} e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action
Dilaton-Modified AdS₅*

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

Quark separation increases with L

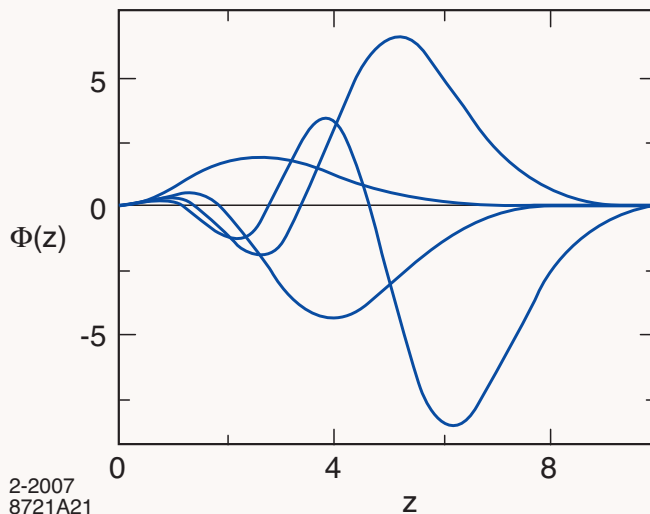
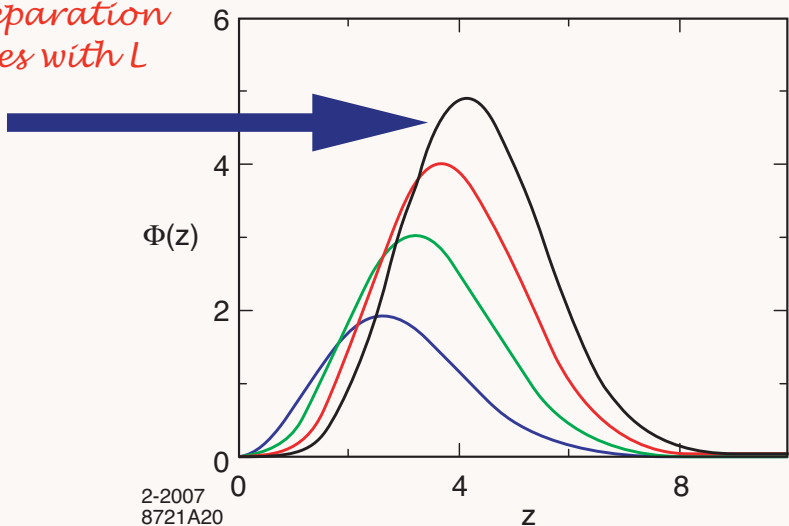
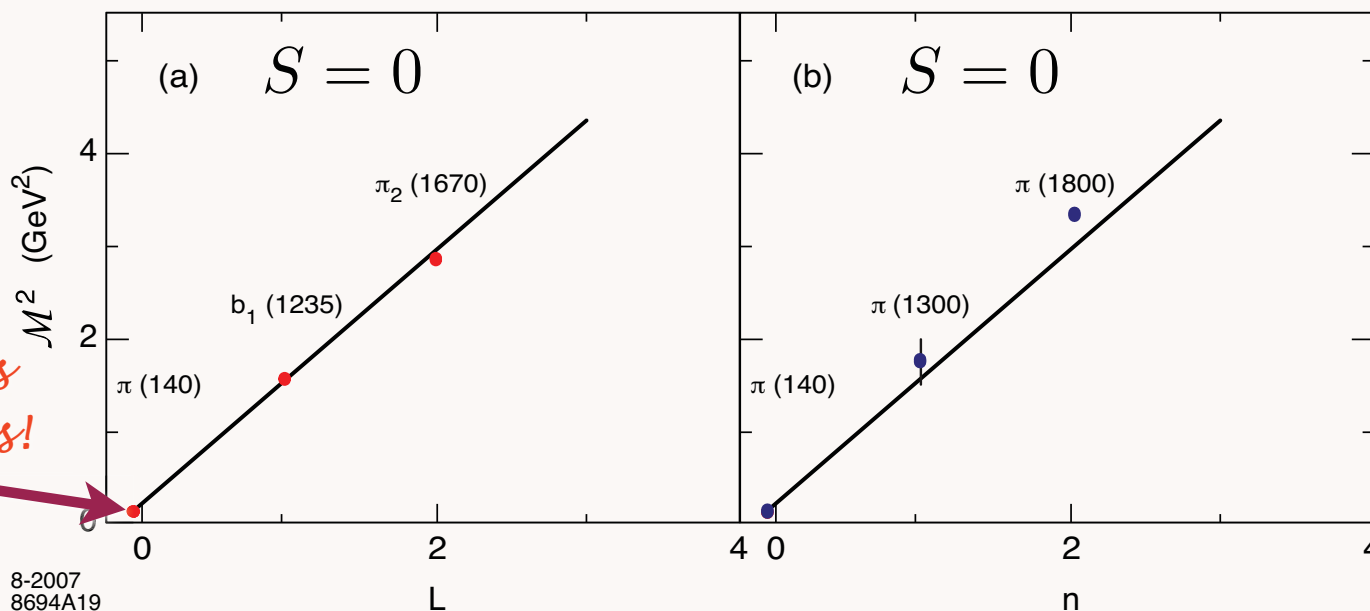


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model

Pion mass automatically zero!

$$m_q = 0$$



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion has zero mass!

Higher-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

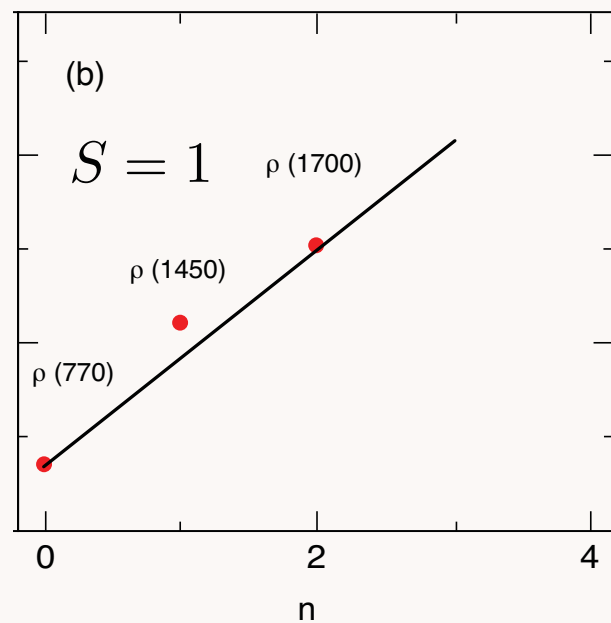
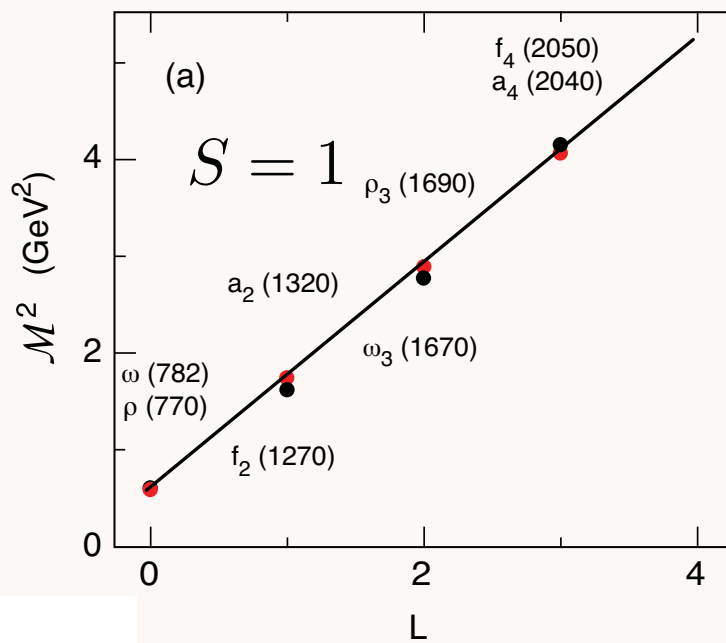
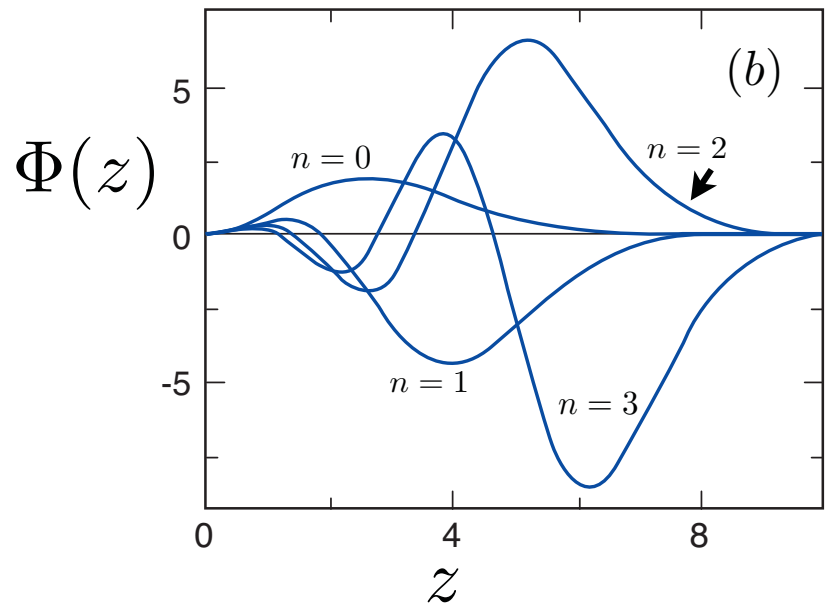
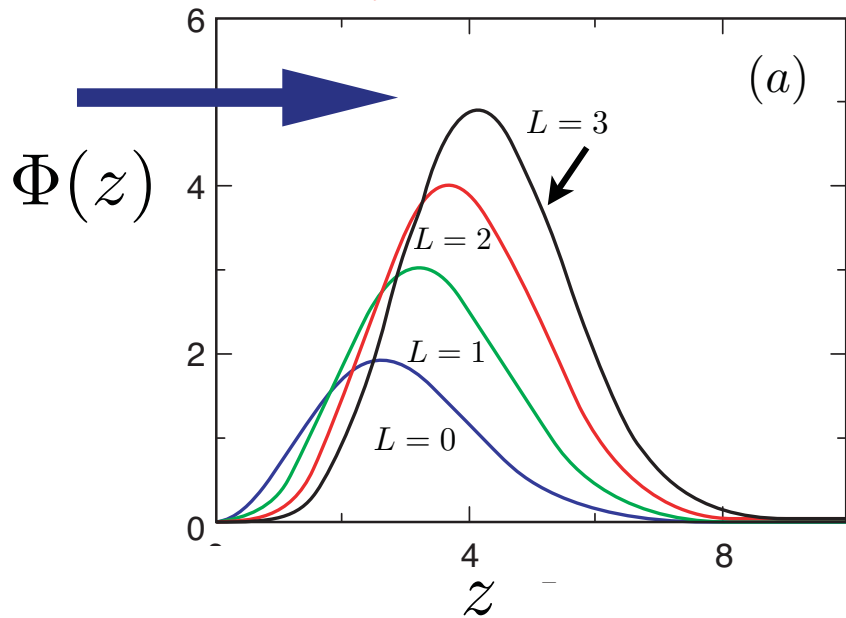
we find the LF wave equation

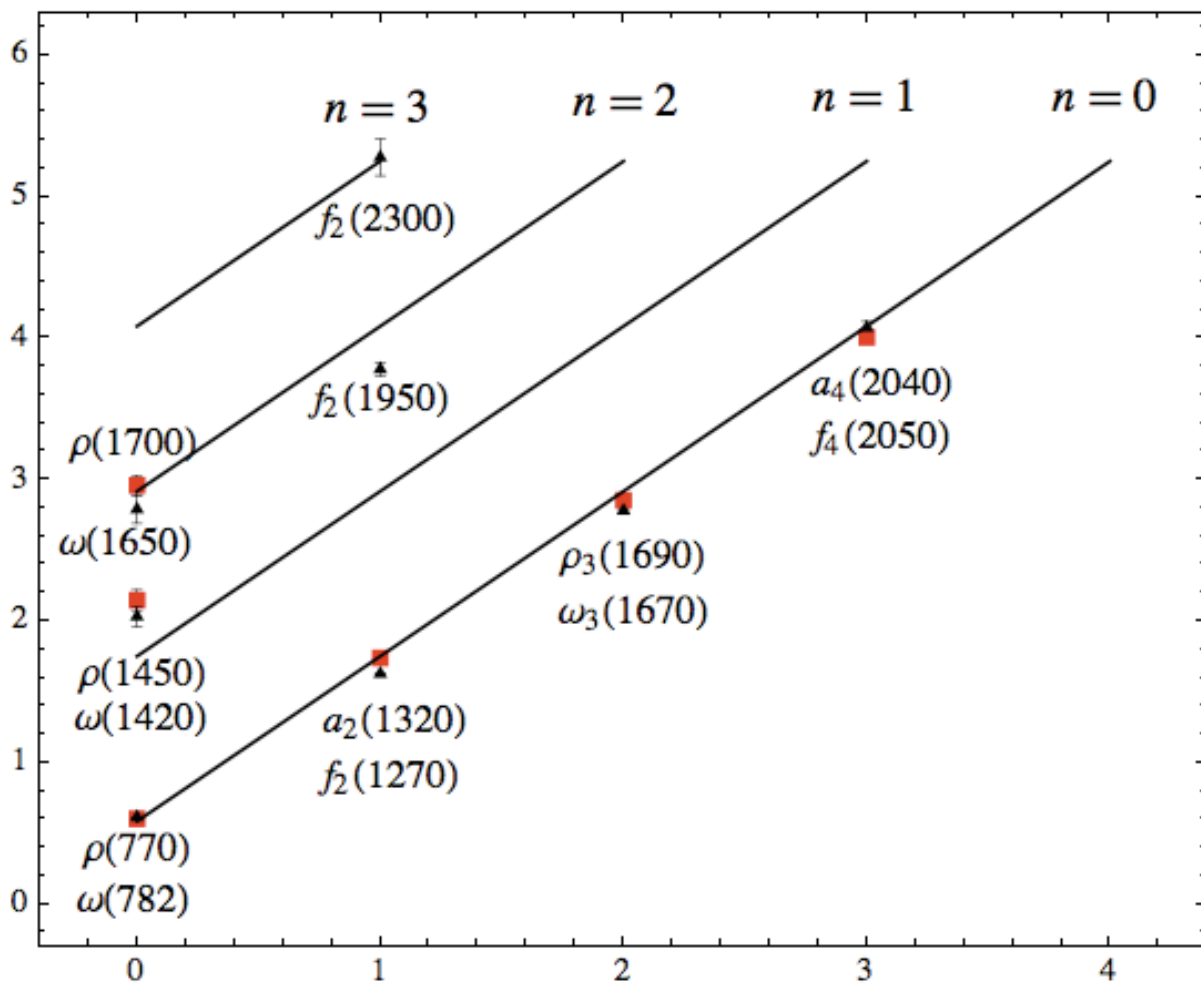
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

Quark separation increases with L



1^{--} 2^{++} 3^{--} 4^{++} J^{PC} \mathcal{M}^2  L

Parent and daughter Regge trajectories for the $I = 1$ ρ -meson family (red)
and the $I = 0$ ω -meson family (black) for $\kappa = 0.54$ GeV

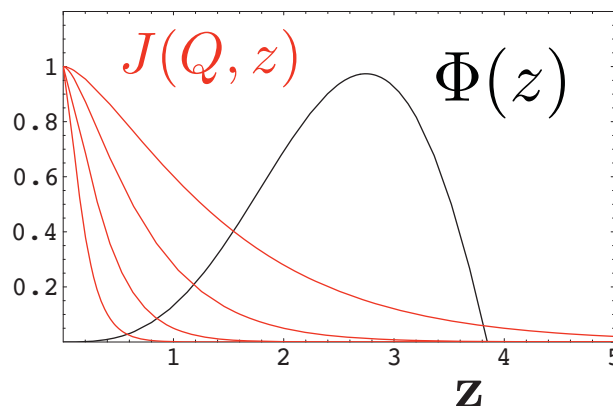
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



**Polchinski, Strassler
de Teramond, sjb**

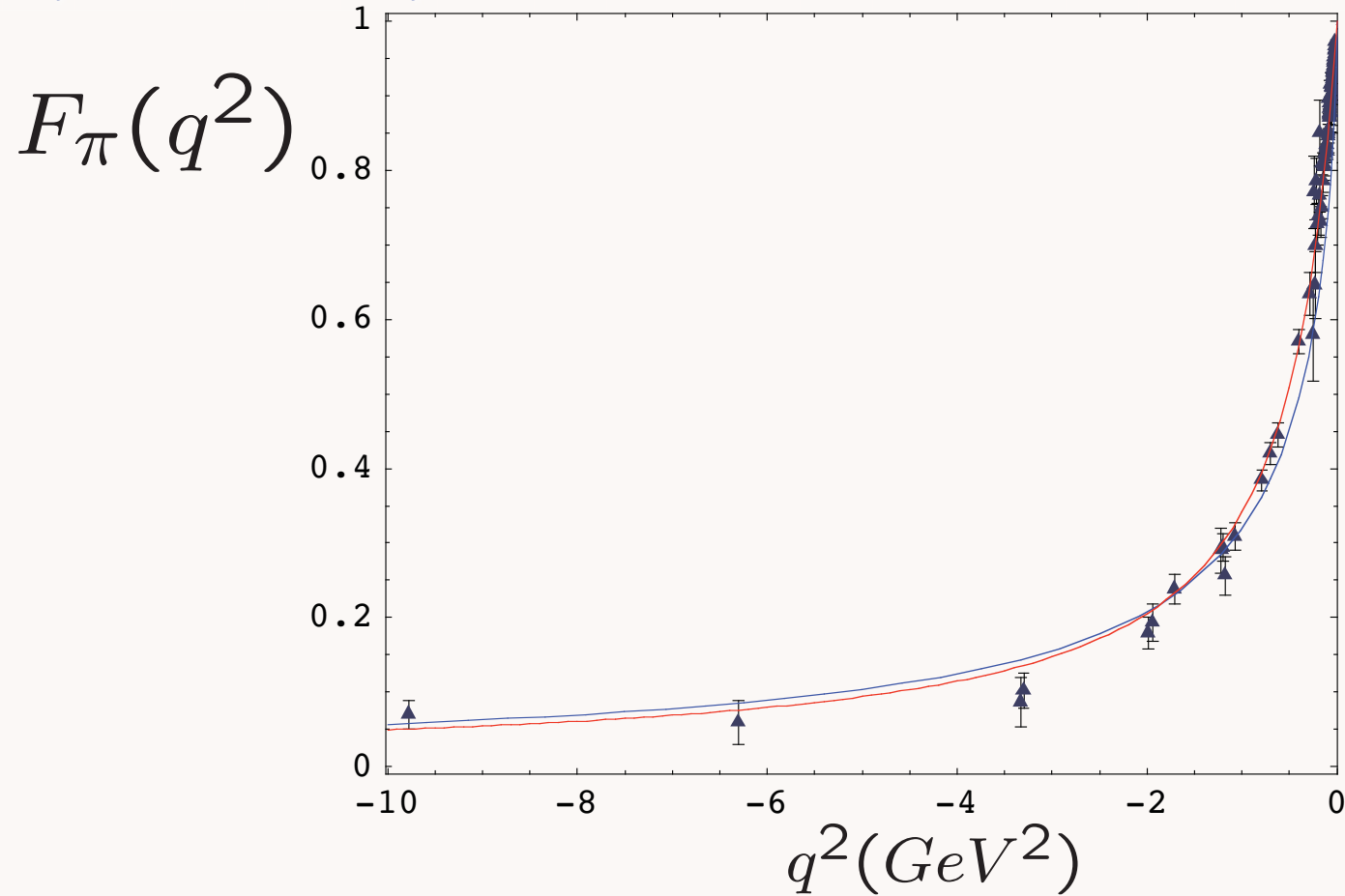
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

**Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space \vec{b}_\perp

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$):

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

Identical to LF Holography obtained from electromagnetic current

$LF(3+1)$

AdS_5

$$\psi(x, \vec{b}_\perp)$$

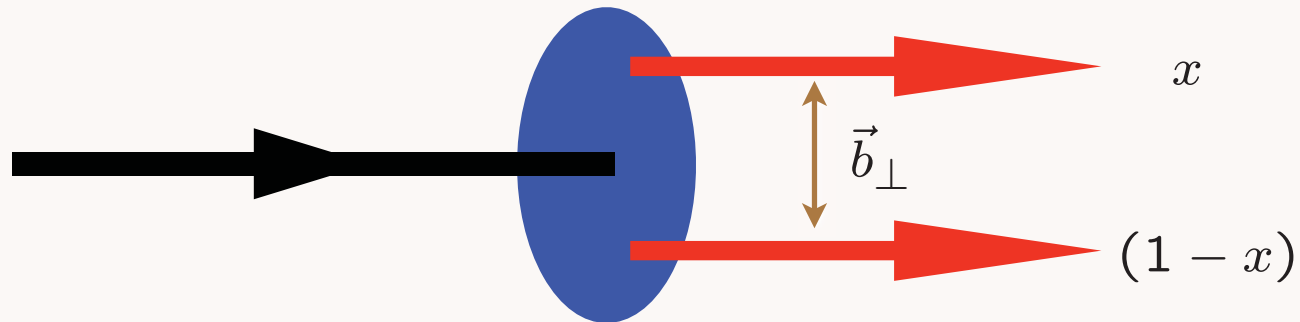


$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$



$$z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

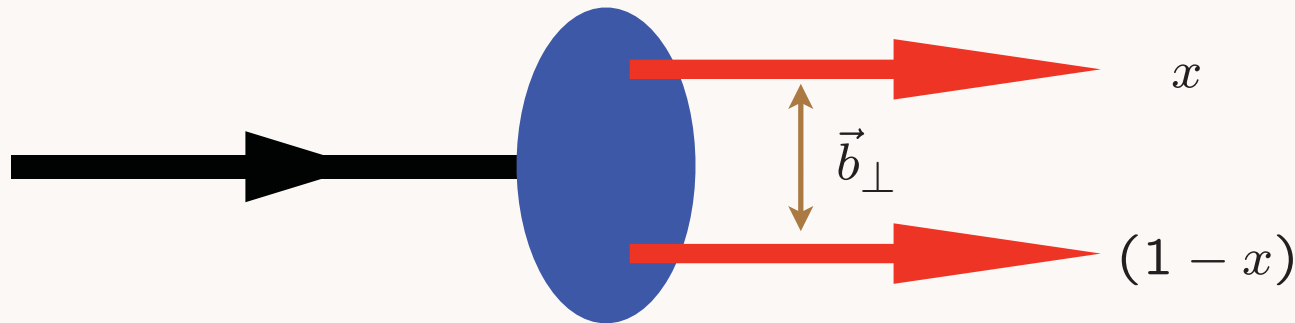
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*soft wall
confining potential*

G. de Teramond, sjb

- Propagation of external current inside AdS space described by the AdS wave equation

$$[z^2 \partial_z^2 - z(1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

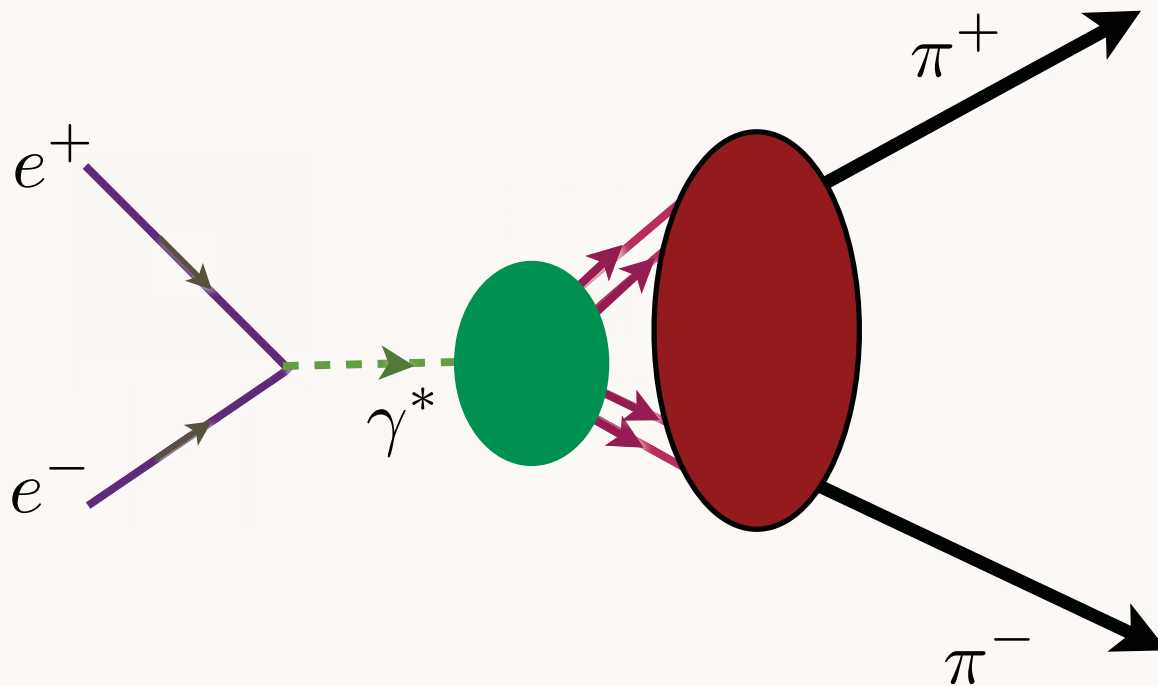
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Soft Wall
Model*

Dressed soft-wall current bring in higher Fock states and more vector meson poles



Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

...

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \quad N,$$

Positive Dilaton Background $\exp(+\kappa^2 z^2)$ $\mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right)$

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}$$

$$Q^2 \rightarrow \infty$$

Constituent Counting

AdS/CFT now extensive field---apologies for all omitted references
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: "fancy"
Start from string theory, develop QCD analogs
on lower dimensional branes

Sakai & Sugimoto

"Bottom-up"
Anticipate what 5D Lagrangian must be (guess),
directly involving desired ρ , π , a_1 , ... fields and
connect to matching QCD structures

Erlich et al.
Da Rold & Pomarol

EM form factors in "bottom-up" approach

Brodsky & de Teramond
Radyushkin & Grigoryan

Gravitational form factors in bottom-up approach

Zainul Abidin & me

Soft-wall

Karch, Katz, Son, and Stephanov
Batell, Gherghetta, and Sword

LF(3+1)

AdS₅

$$\psi(x, \vec{b}_\perp)$$



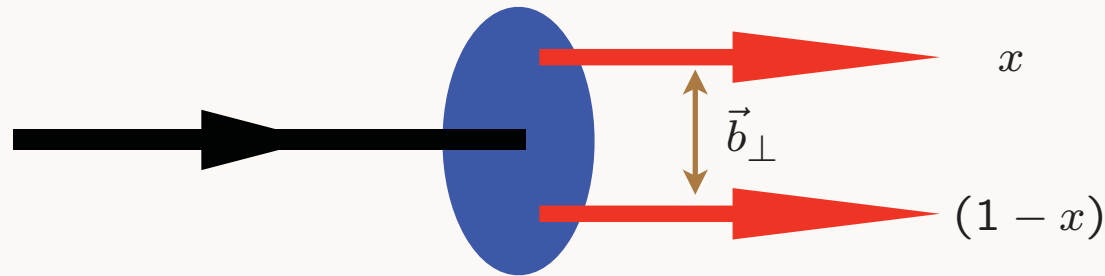
$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$



$$z$$

$$\psi(x, \vec{b}_\perp)$$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

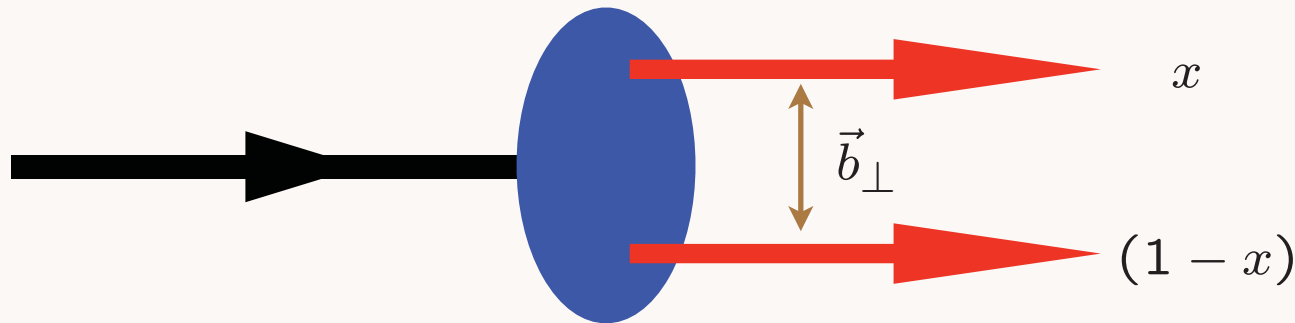
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Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

G. de Teramond, sjb

*soft wall
confining potential:*

Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions.} \end{aligned}$$

**Change
variables**

$$(\vec{\zeta}, \varphi), \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

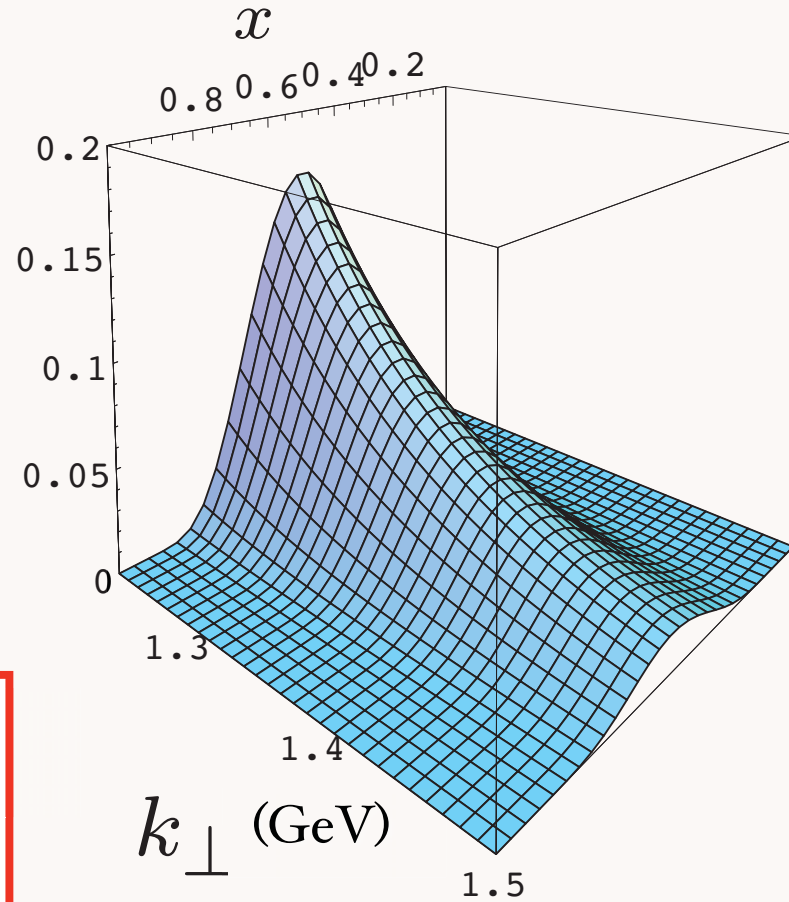
Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

**“Soft Wall”
model**

$\kappa = 0.375$ GeV
massless quarks

$$\psi_M(x, k_{\perp}^2)$$



Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20$$

$$\phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25$$

$$\phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

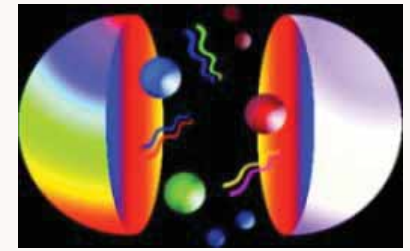
$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD
GdT and Brodsky: hep-th/0409074, hep-th/0501022.



From Nick Evans

Baryons in AdS/CFT

- Action for massive fermionic modes on AdS₅:

$$S[\bar{\Psi}, \Psi] = \int d^4x dz \sqrt{g} \bar{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion: $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution ($\mu R = \nu + 1/2$)

$$\Psi(z) = C z^{5/2} [J_\nu(z\mathcal{M})u_+ + J_{\nu+1}(z\mathcal{M})u_-]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm(z = 1/\Lambda_{\text{QCD}}) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}$$

with scale independent mass ratio

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin- $\frac{1}{2}$ modes $\psi(\zeta)$ and spin- $\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are **two-component spinor** solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5\beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L$ ($\nu = L + 1$)

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_-].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,$$

given by

$$\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},$$

with a scale independent mass ratio.

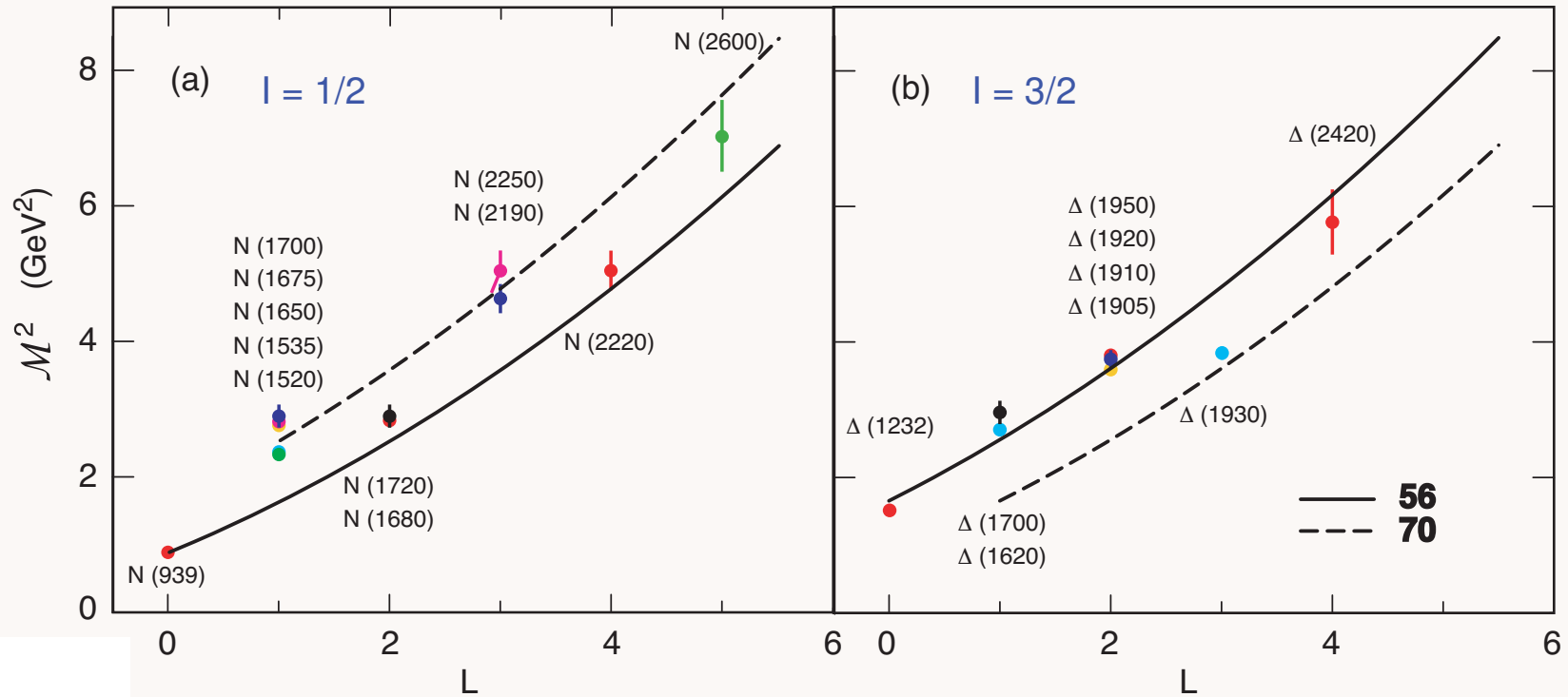


Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The 56 trajectory corresponds to L even $P = +$ states, and the 70 to L odd $P = -$ states.

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint Π^\dagger , with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned}$$

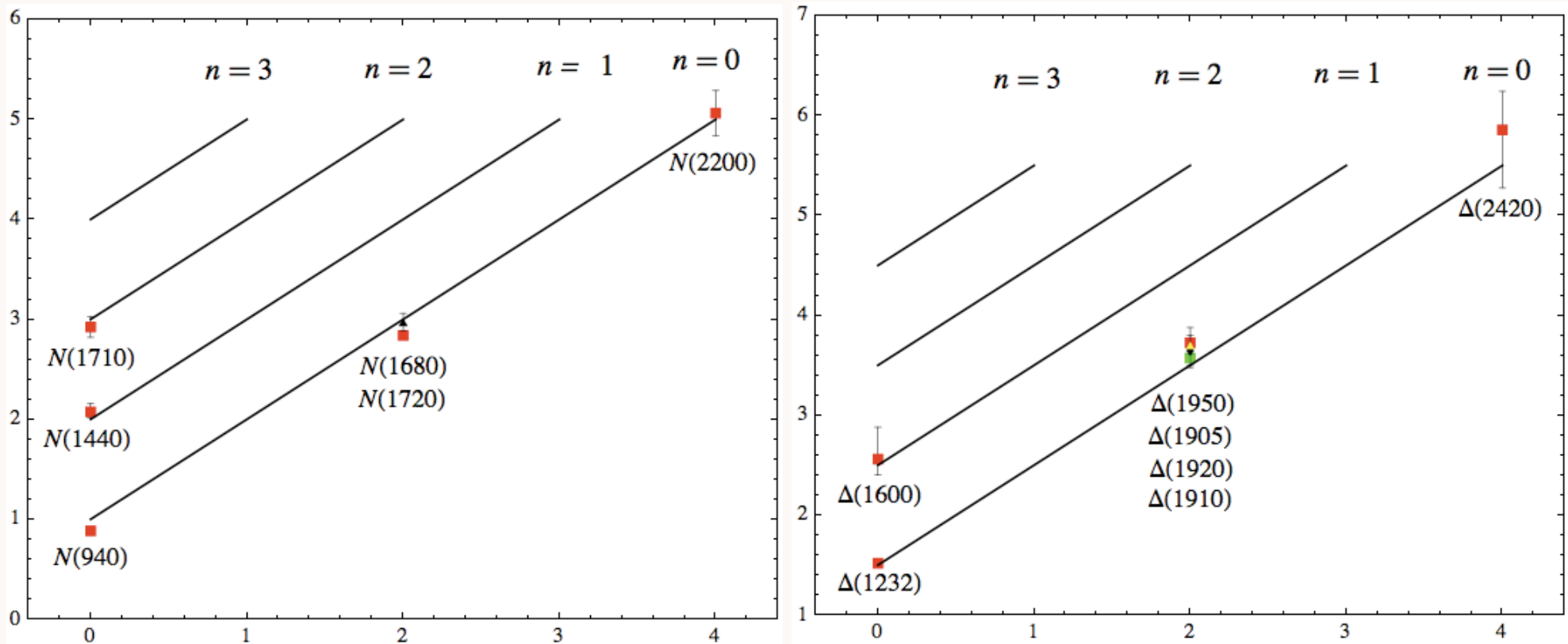
- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

- Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

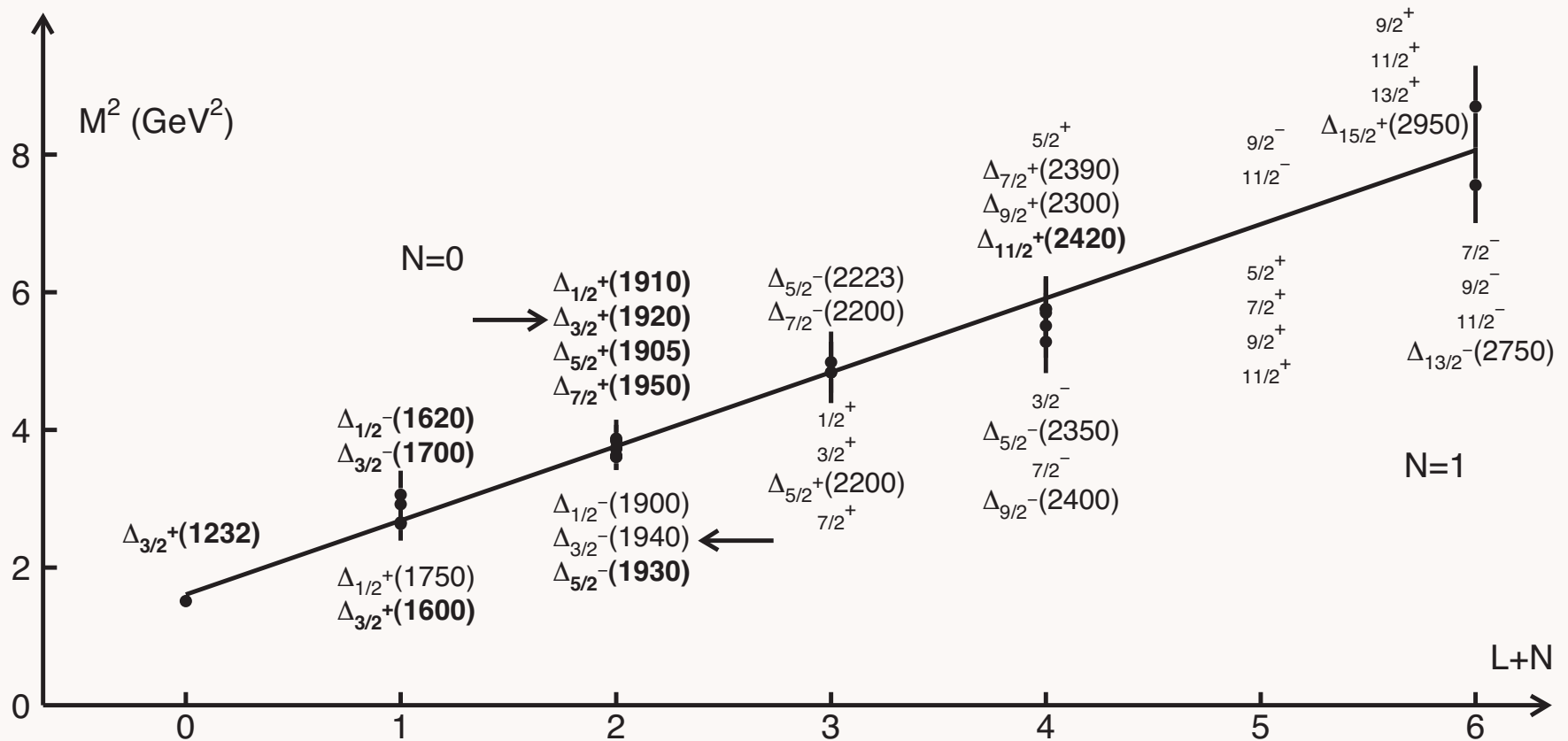
$4\kappa^2$ for $\Delta n = 1$
 $4\kappa^2$ for $\Delta L = 1$
 $2\kappa^2$ for $\Delta S = 1$

$$\mathcal{M}^2$$



$$L$$

Parent and daughter 56 Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.

H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

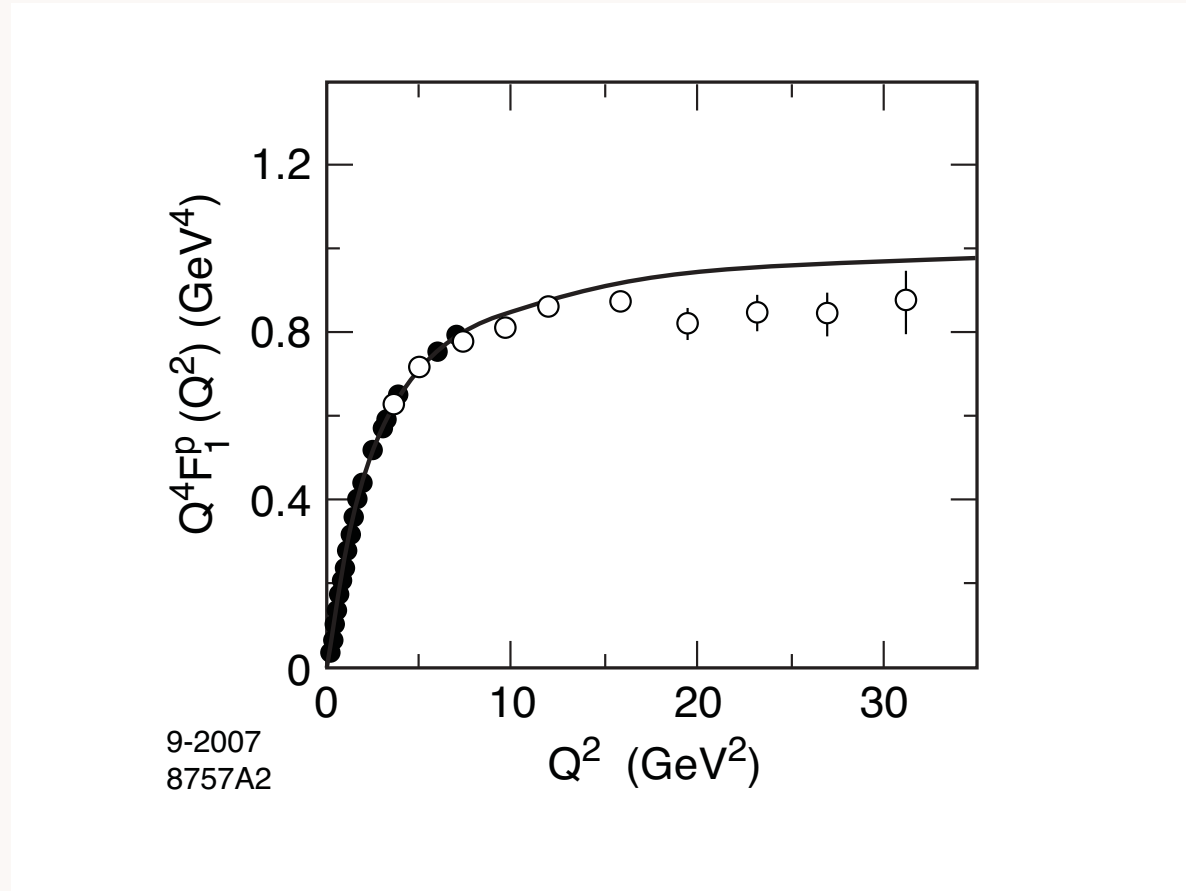
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

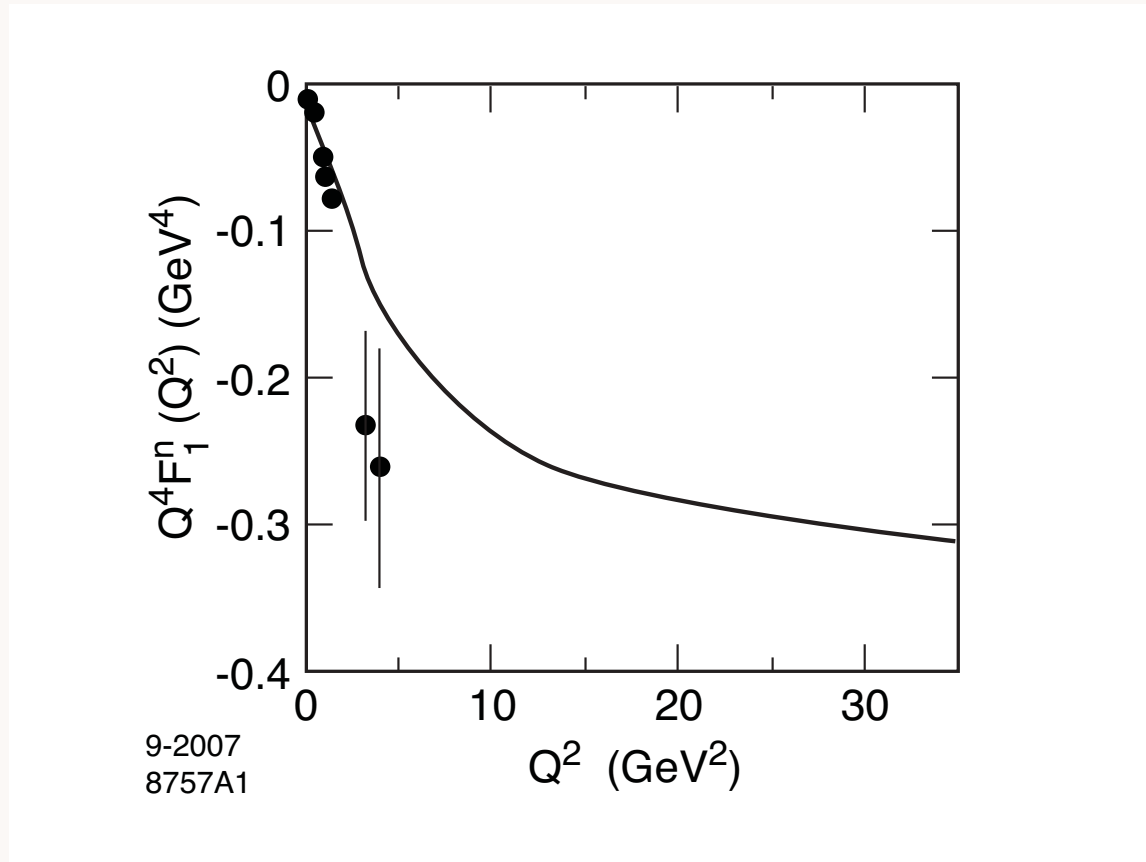
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

- Scaling behavior for large Q^2 : $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ Proton $\tau = 3$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2 : $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ Neutron $\tau = 3$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).