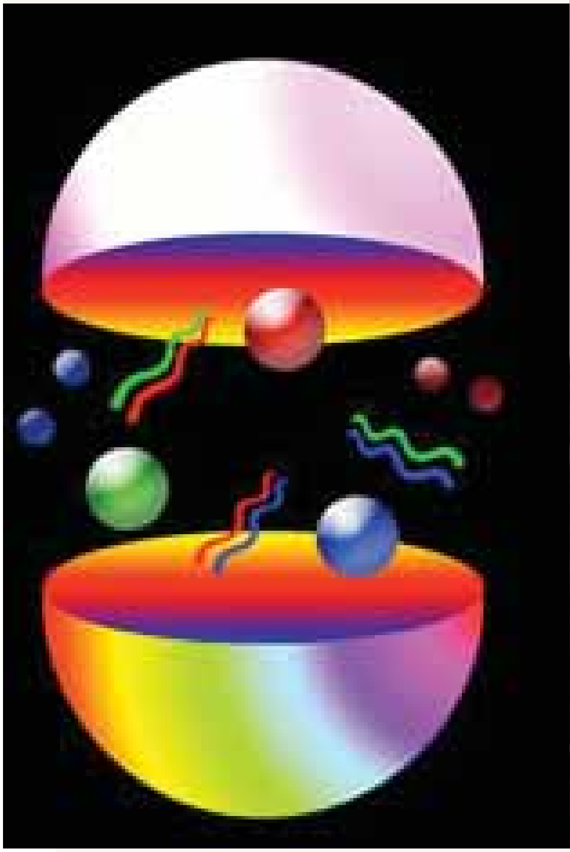
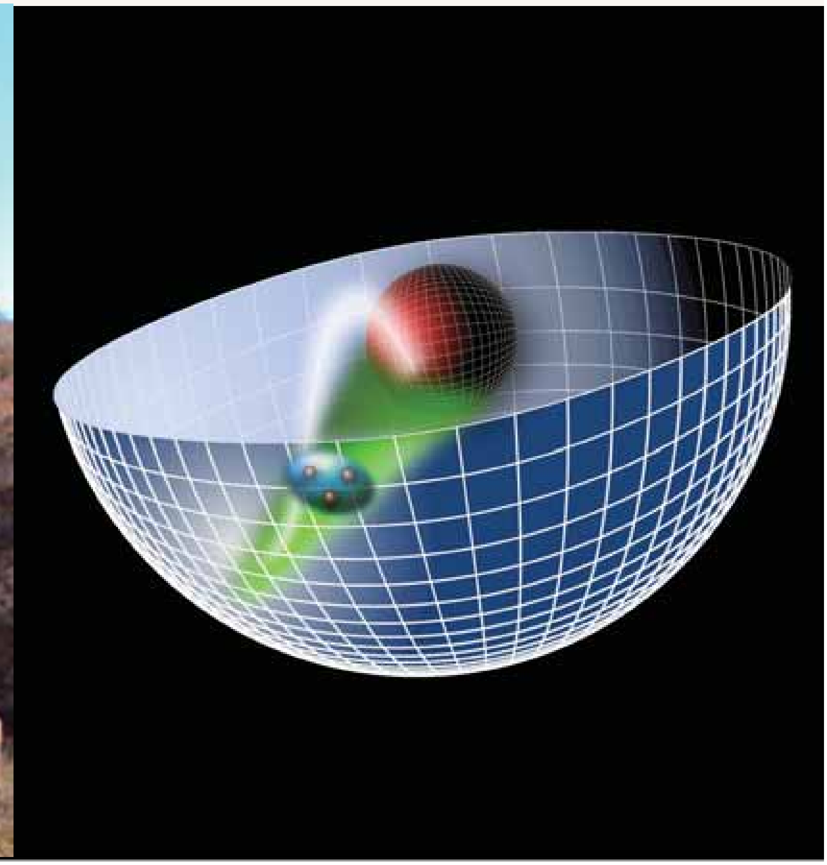
AdS/QCD and Light-Front Holography

A New Approximation to QCD





Stan Brodsky, SLAC National Accelerator Laboratory

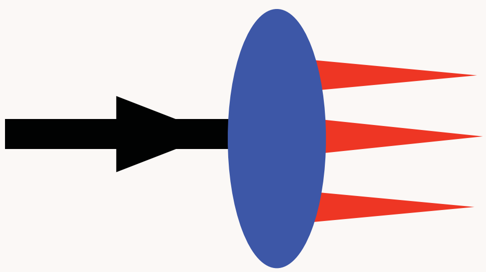
Fifth International Workshop on Quarks and Nuclear Physics (QNP09)

IHEP Beijing, September 21-26 2009

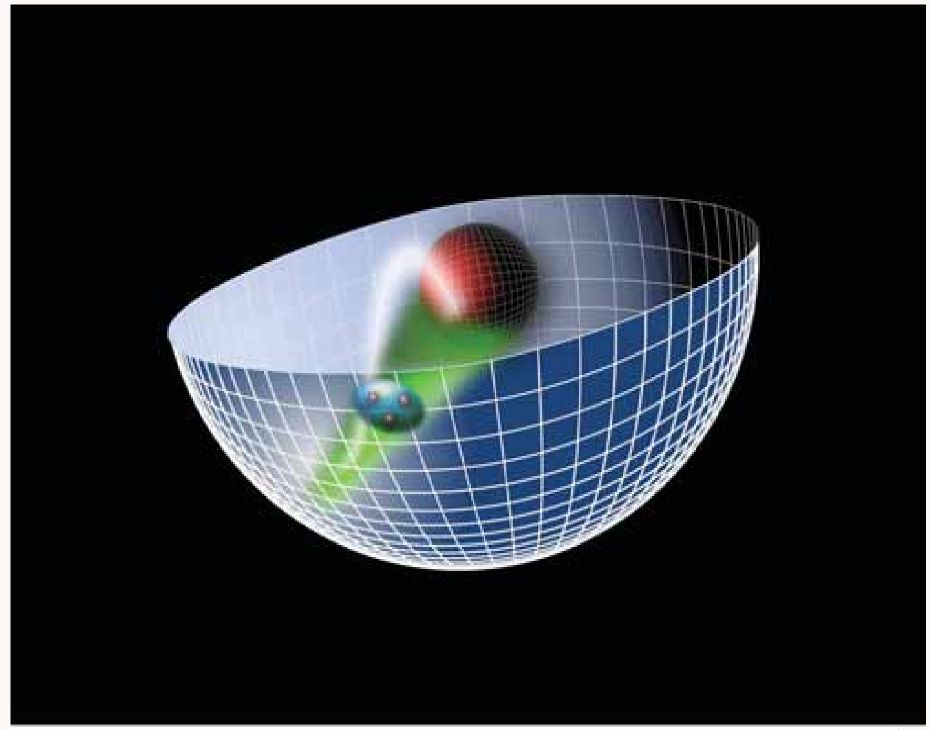
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



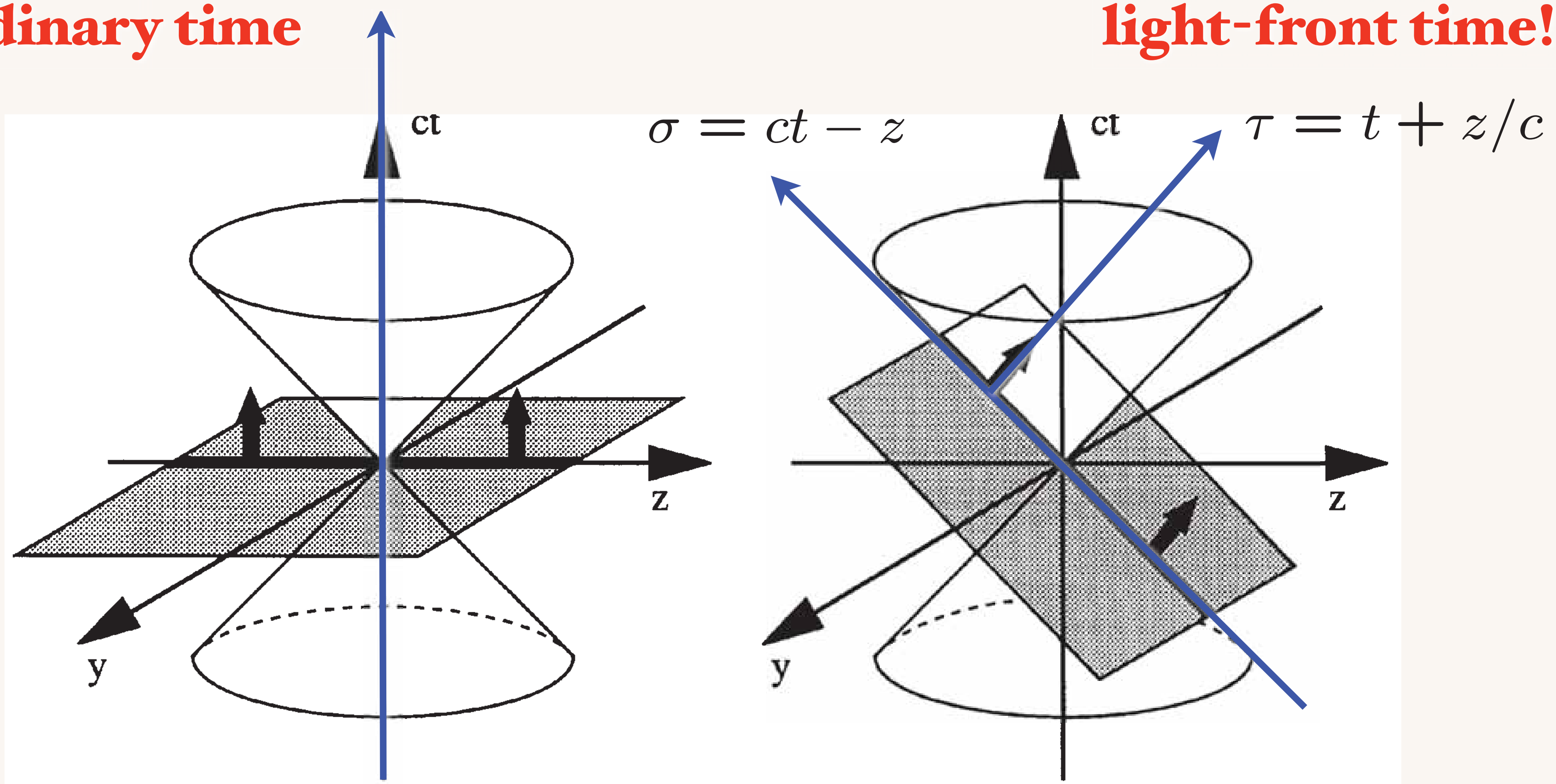
in collaboration with
Guy de Teramond

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same Light Front time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

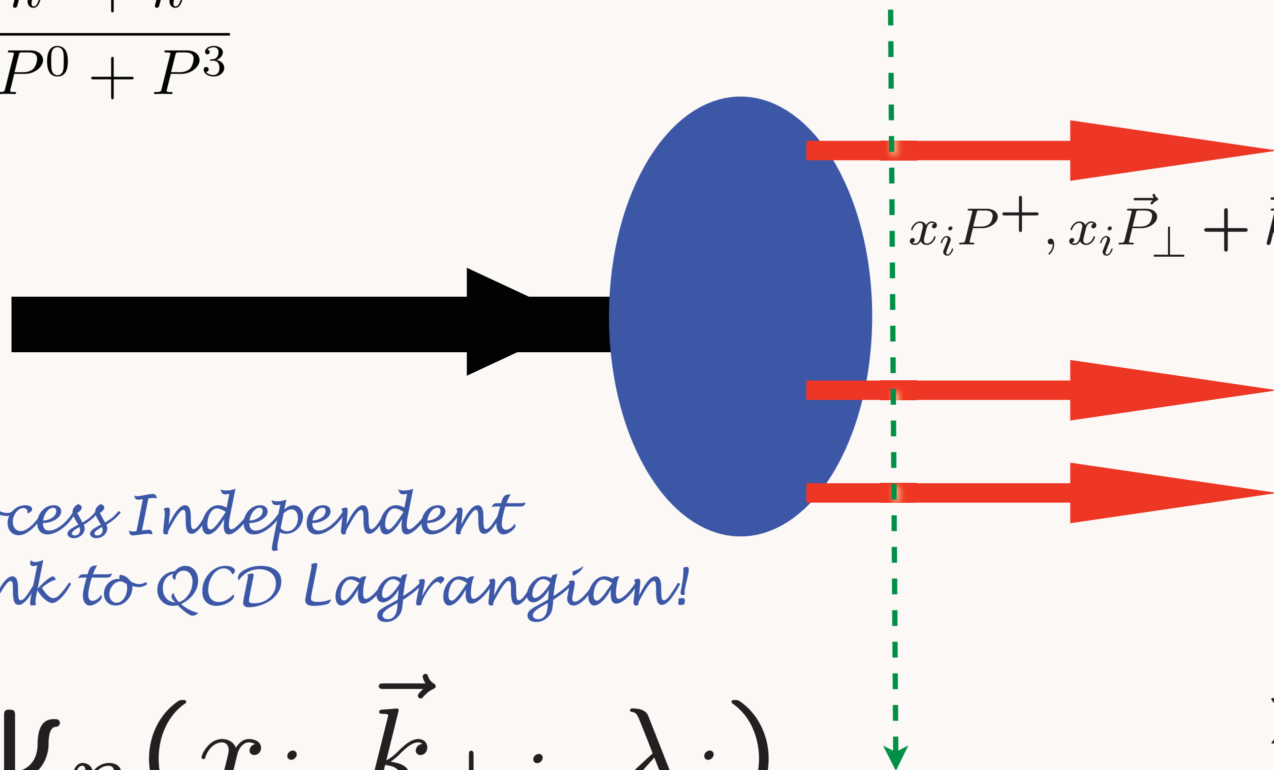
DIS, Form Factors, DVCS, etc. measure proton WF at fixed

$$\tau = t + z/c$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



Process Independent

Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Light-Front QCD

Heisenberg Matrix Formulation

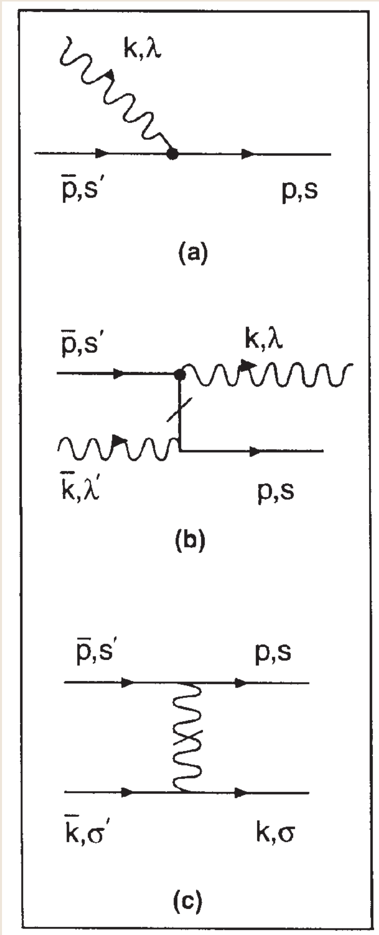
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State!

LF Spin Sum Rule

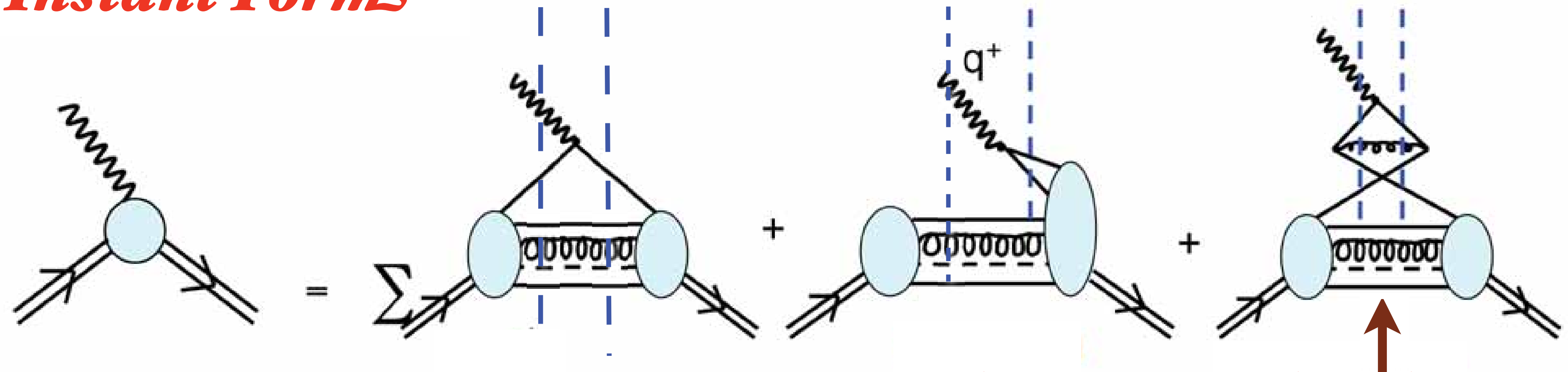
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment --> Nonzero orbital angular momentum!

Calculation of Form Factors in Equal-Time Theory

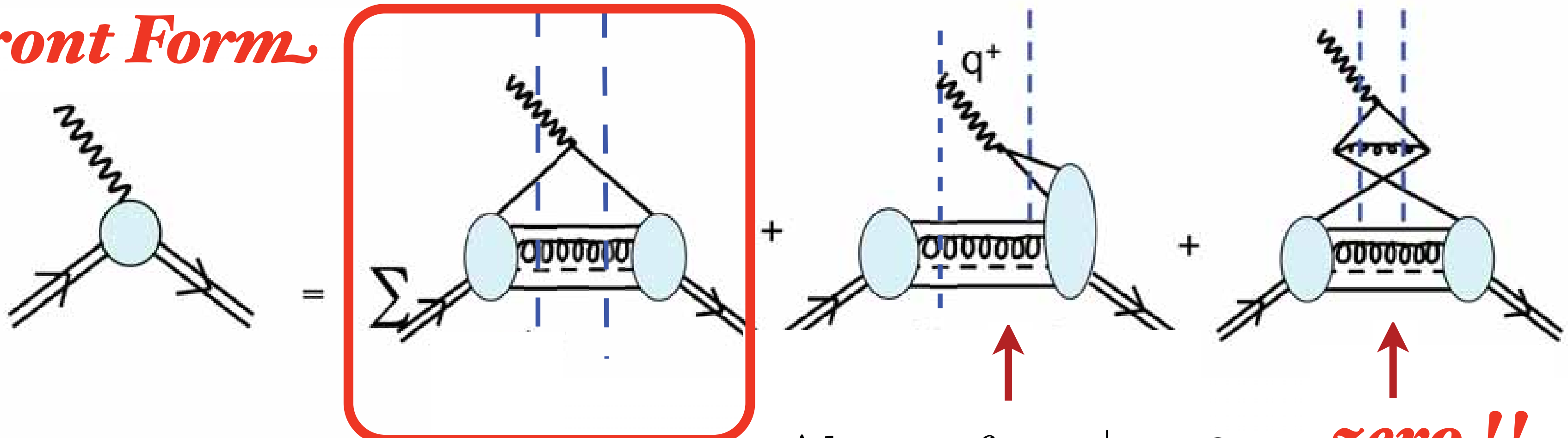
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

Absent for $q^+ = 0$

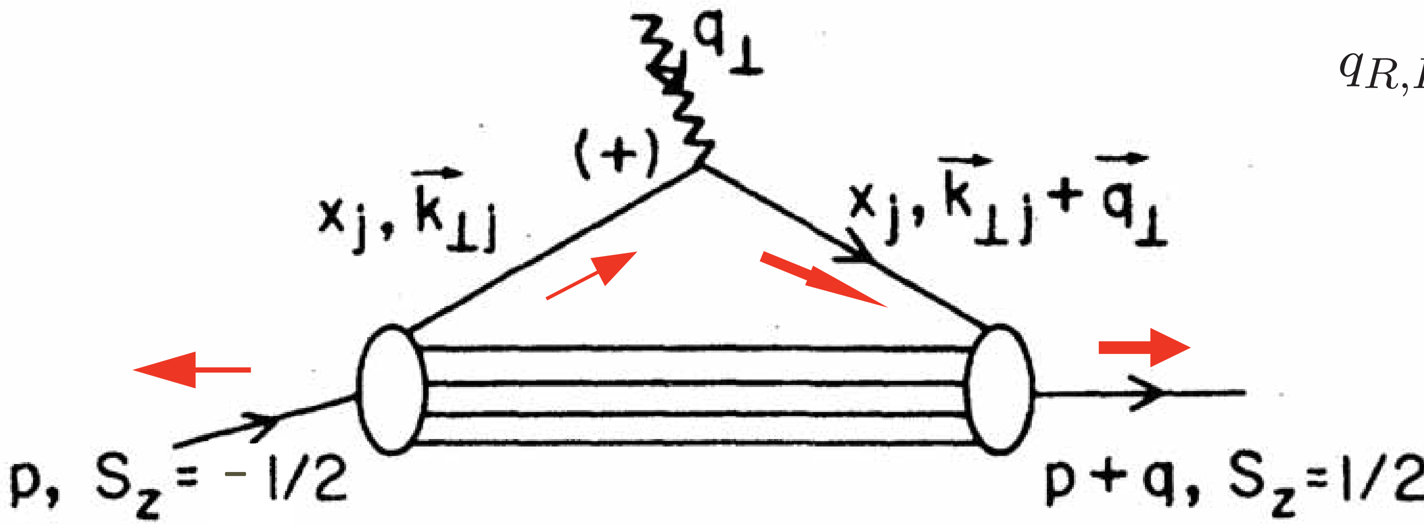
zero !!

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

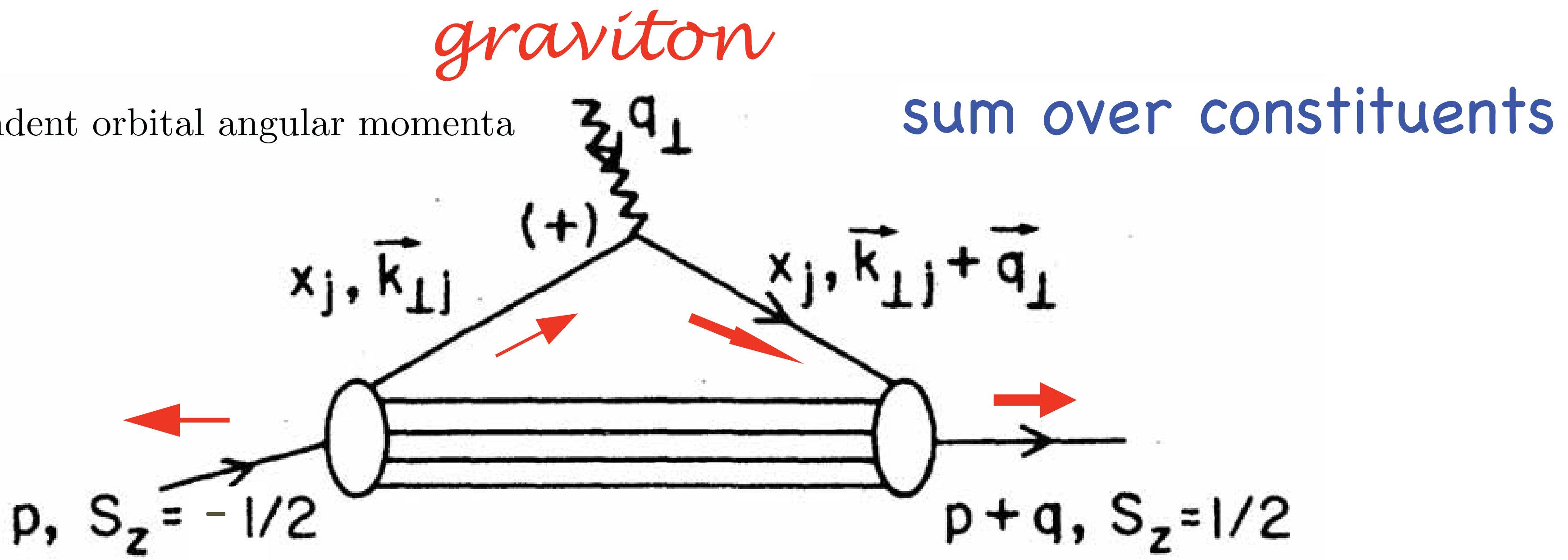
Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

$\sum_{i=1}^{n} L_i = 0$
$n-1$ independent orbital angular momenta



Hwang, Schmidt, sjb;
Holstein et al

B(0) = 0

Each Fock State

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

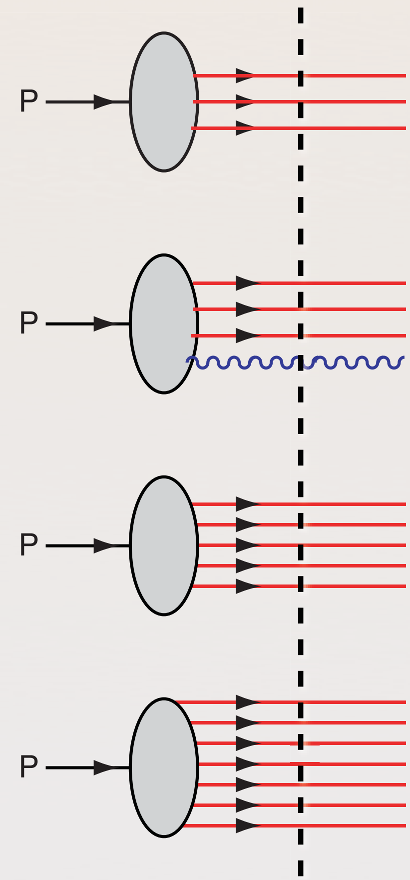
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

$c(x), b(x)$ at high x

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x - x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1 - \zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1 - x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi=t=0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^{1} dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int_{-1}^{1} dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl,Hwang, sjb

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

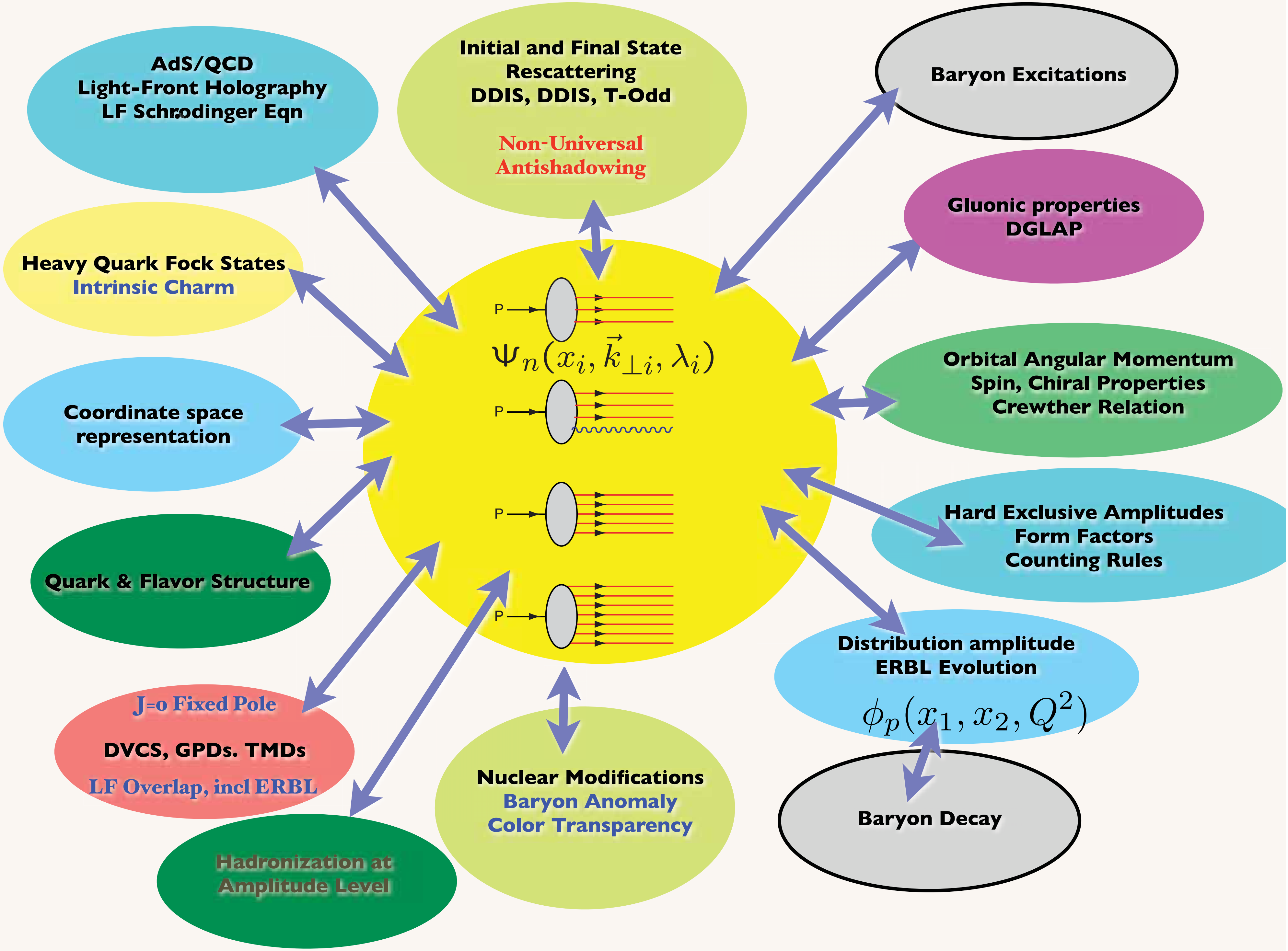
X. Ji, Phy.Rev.Lett.78,610(1997)

Light Front Definitions Light-Cone Gauge

Light-Front QCD Features and Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

QCD and the LF Hadron Wavefunctions

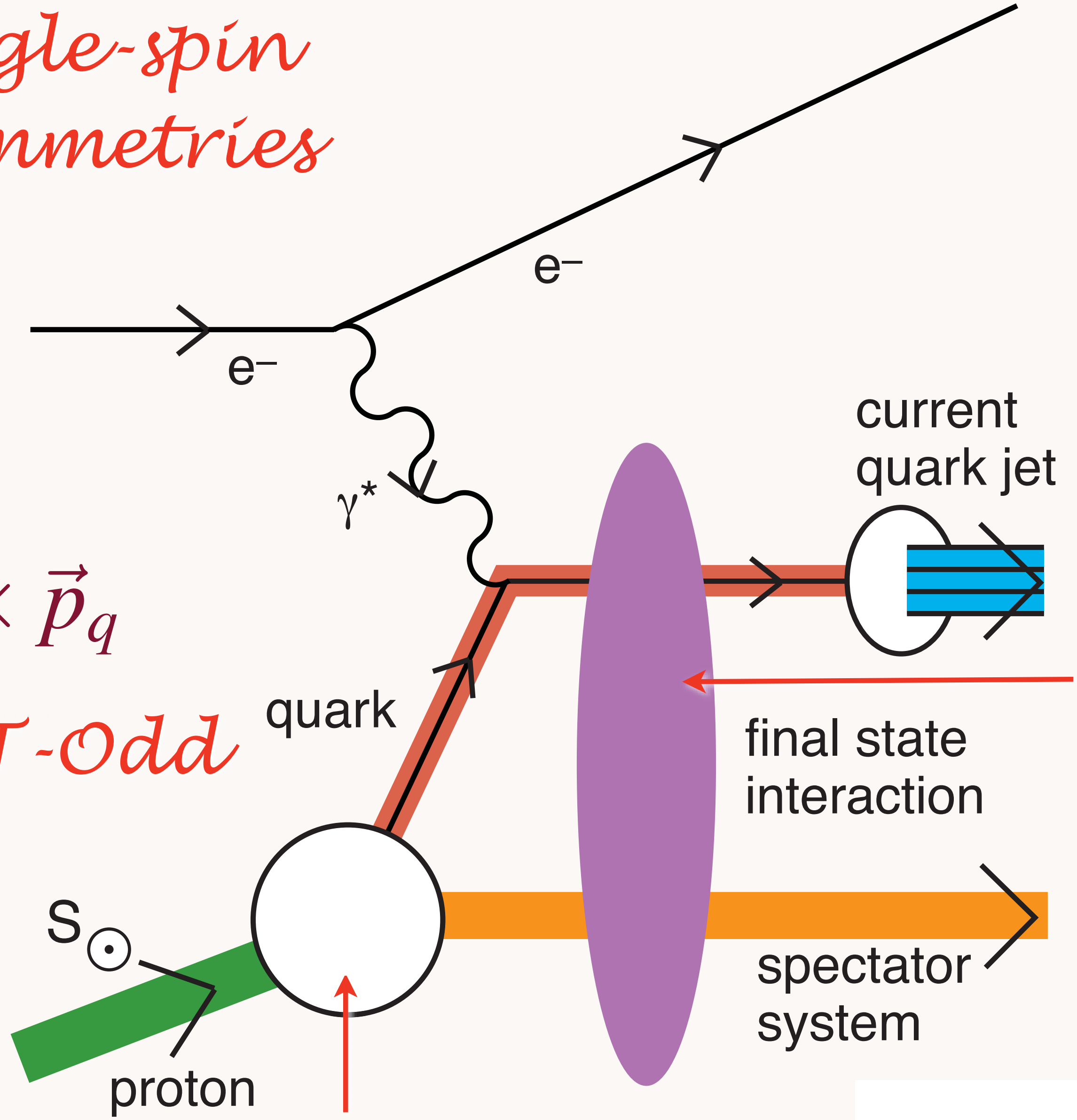


Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan



$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

QCD S- and P- Coulomb Phases --Wilson Line

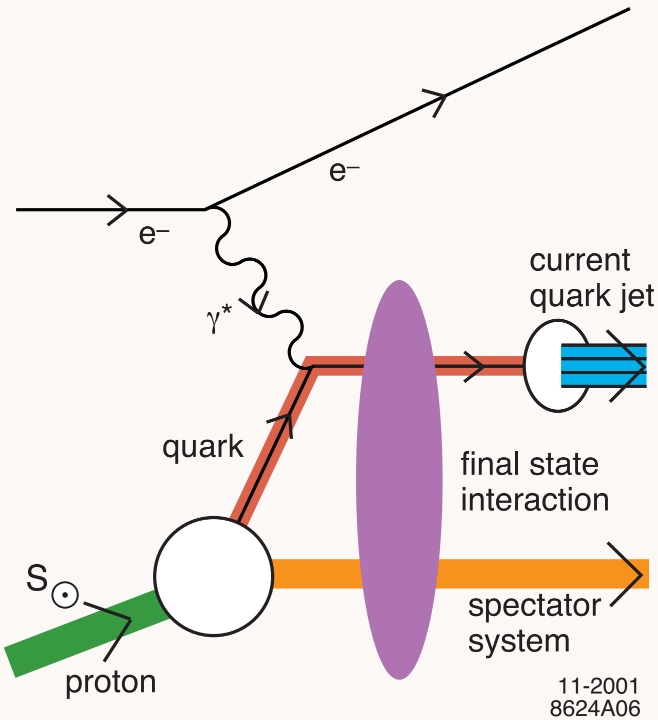
Light-Front Wavefunction S and P- Waves

Violates pQCD Factorization!

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

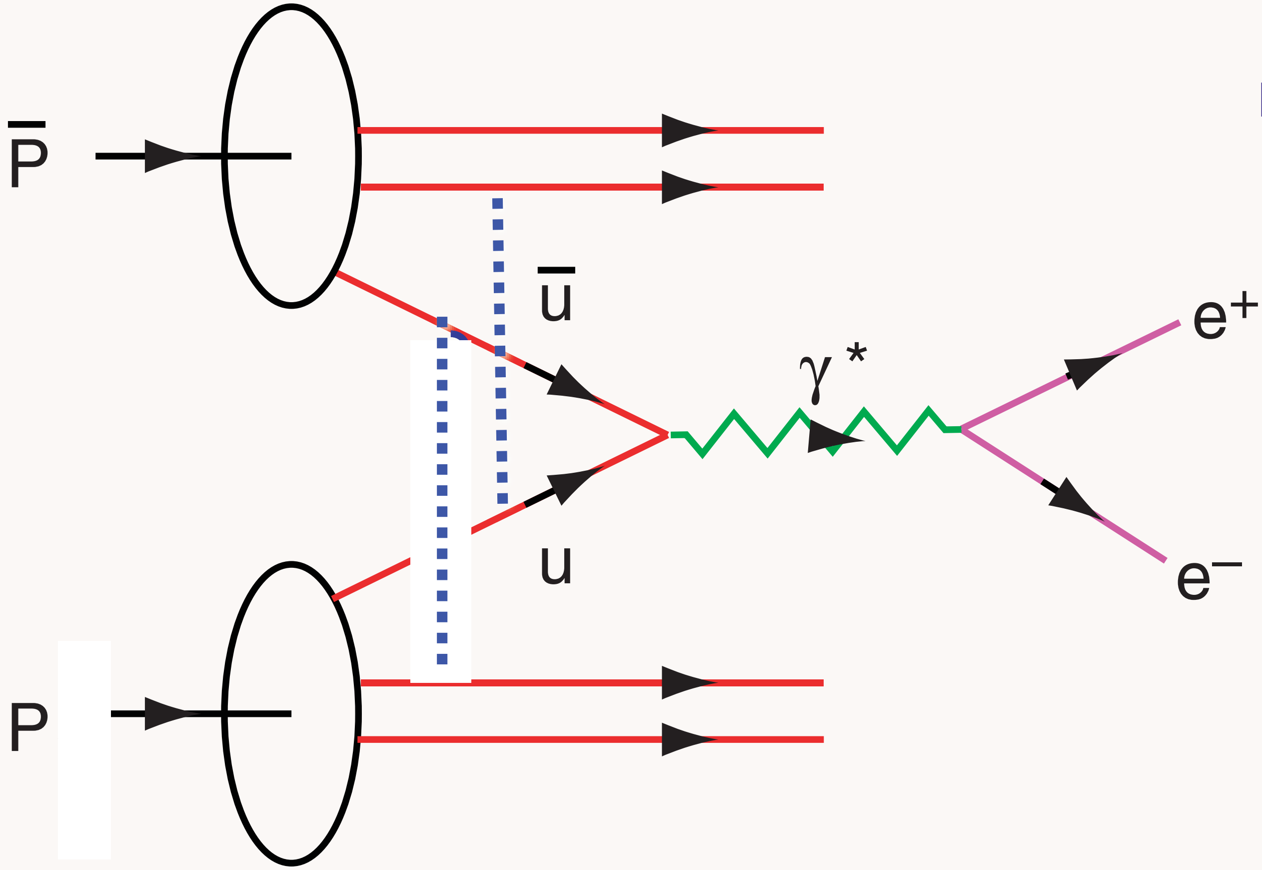
$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;**
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**



Boer, Hwang, sjb

F. Yuan et al.

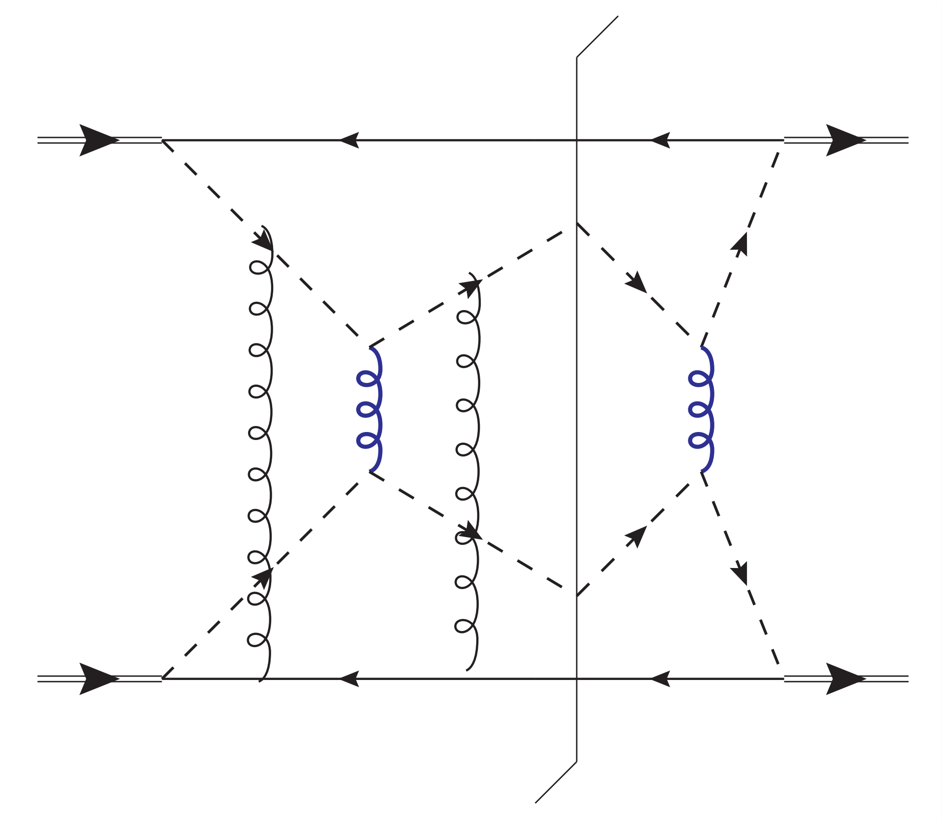


DY $\cos 2\phi$ correlation at leading twist from double ISI

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

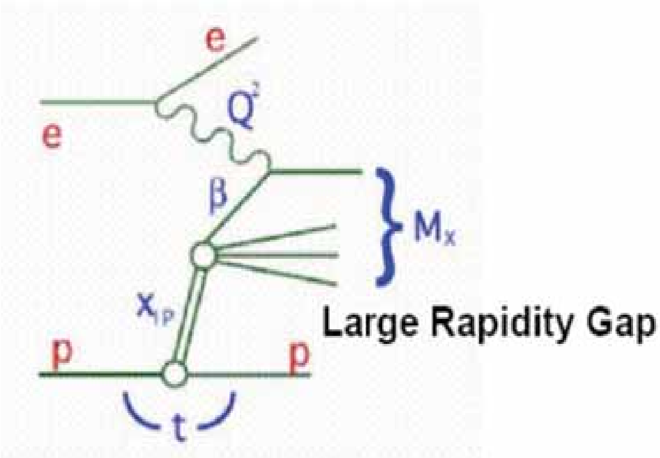
John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

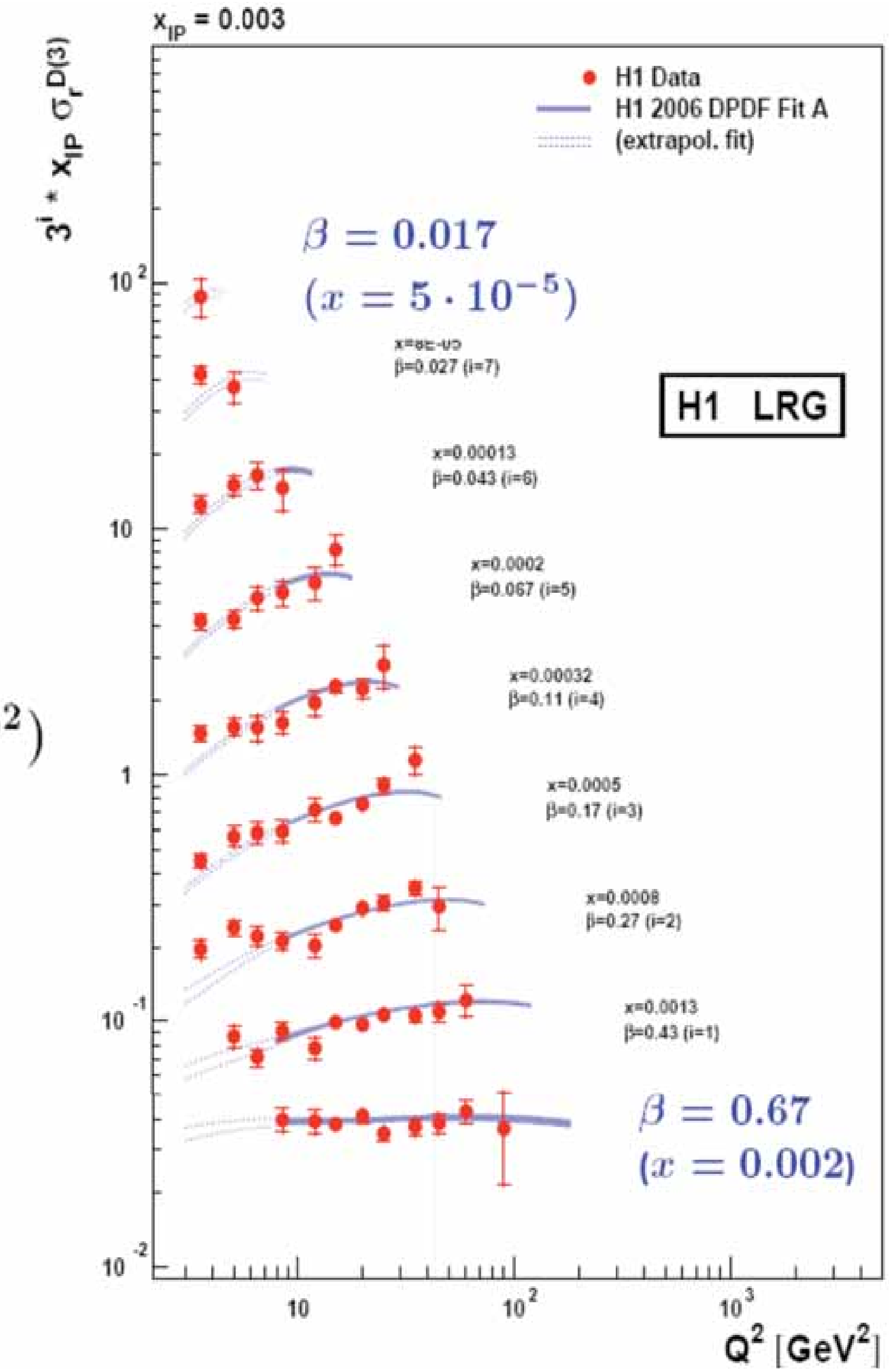
e-Print: **arXiv:0705.2141** [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Diffractive Structure Function F_2^D





Diffractive inclusive cross section

$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\, d\beta\, dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$$

$$F_2^D(x_{I\!P}, \beta, Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

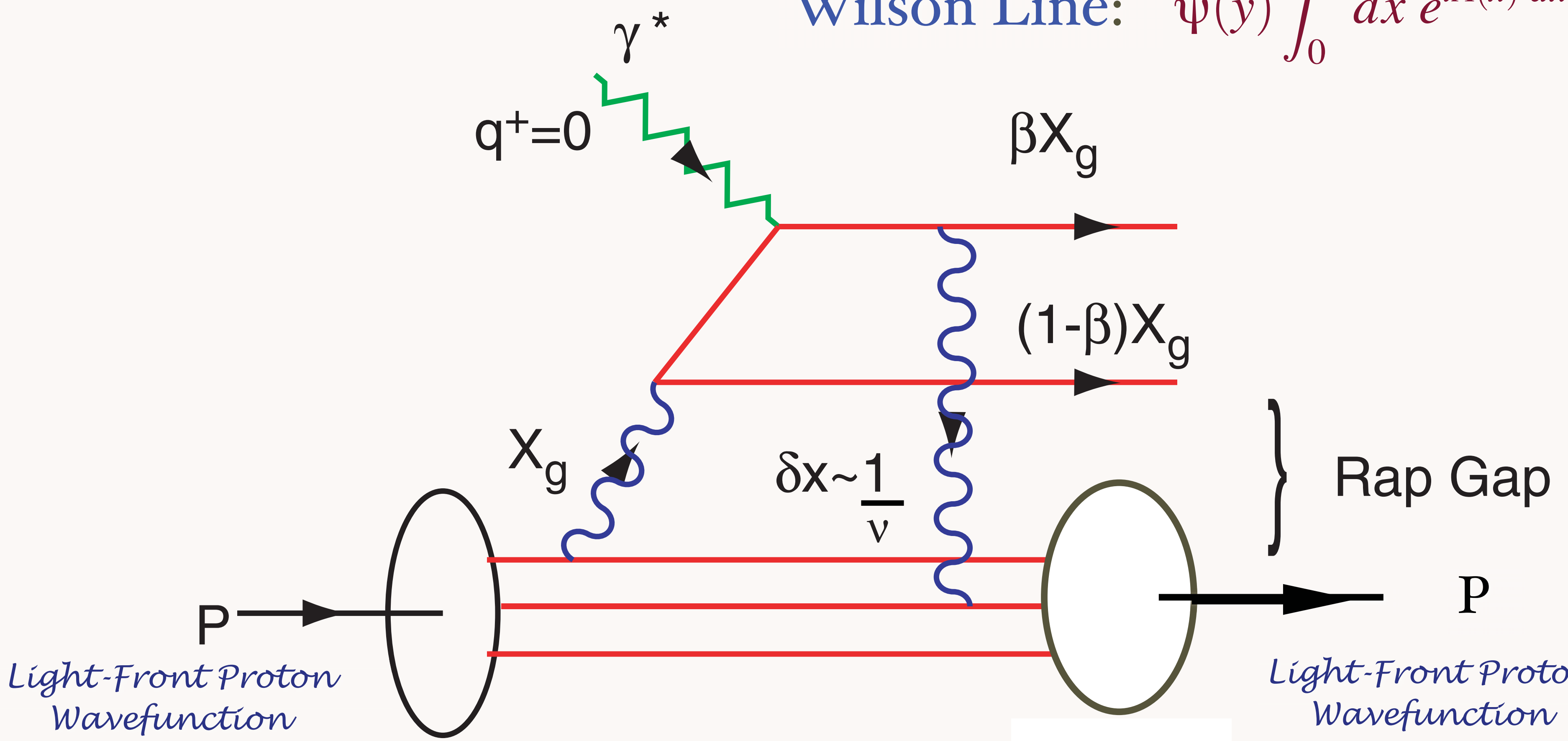
Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

Precise measurements *sys 5%, stat 5–20 %*

Hoyer, Marchal, Peigne, Sannino, sjb

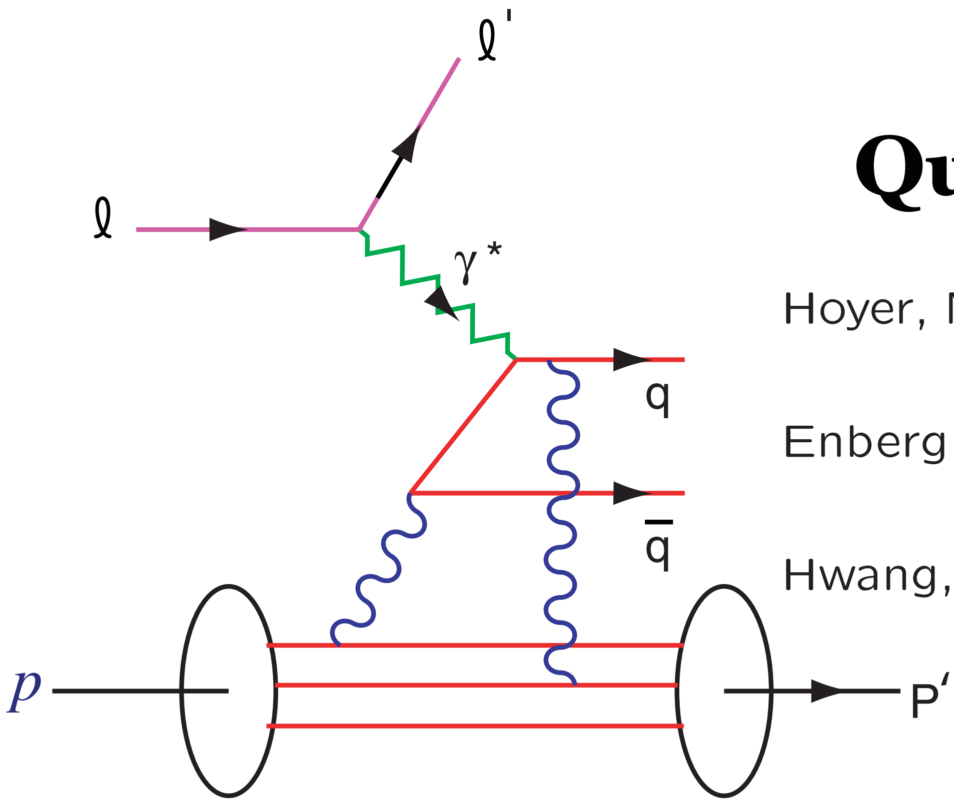
QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



Reproduces lab-frame color dipole approach

Final-State QCD Interaction Produces Diffractive DIS



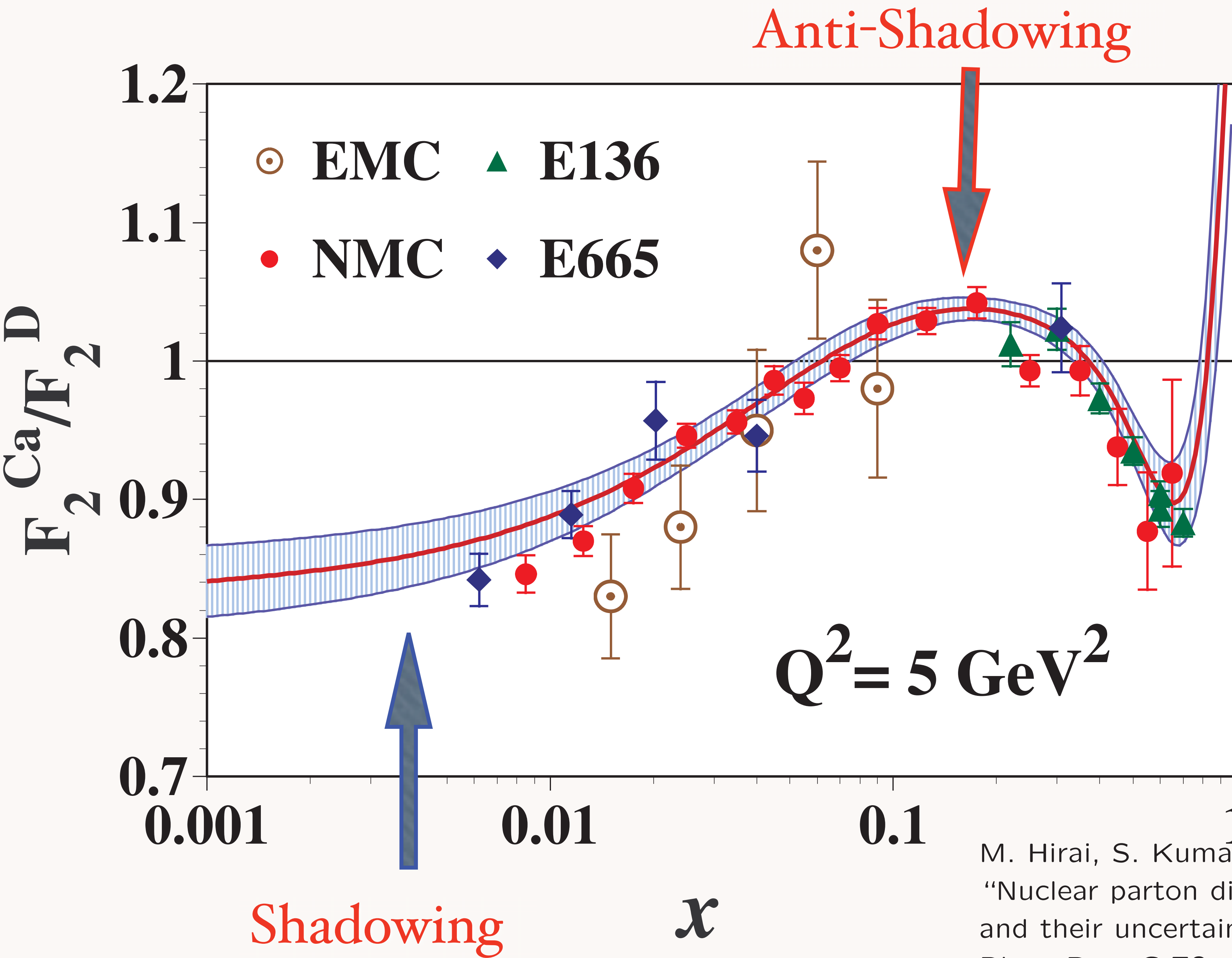
Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

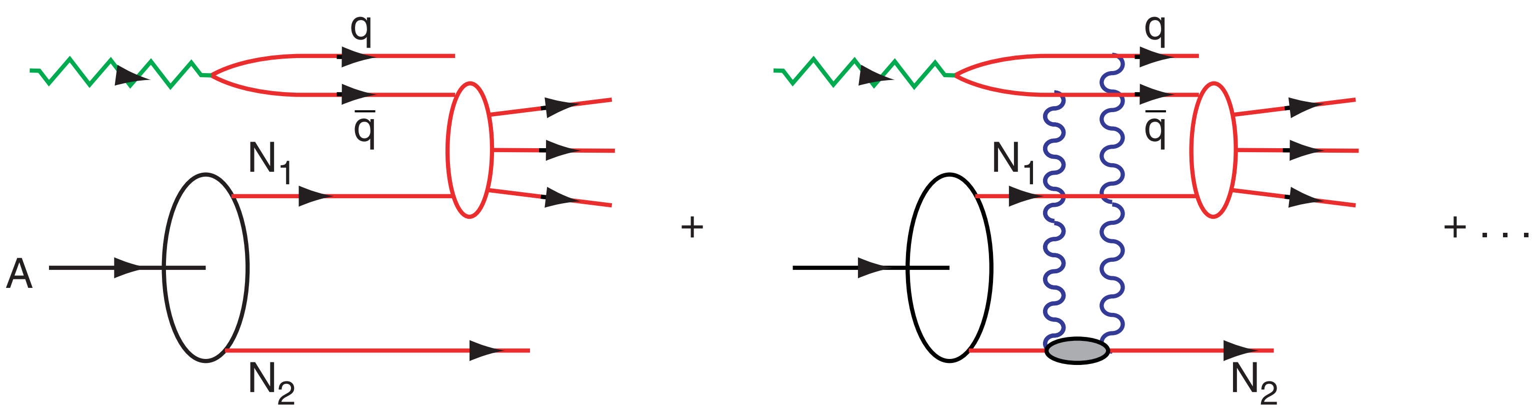


M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

Stodolsky
Pumplin, sjb
Gribov

Nuclear Shadowing in QCD

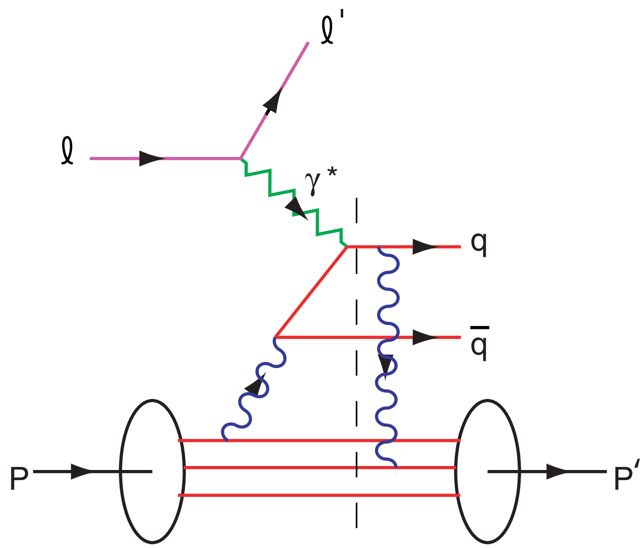
Destructive Interference



Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus



Shadowing depends on leading-twist DDIS

Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

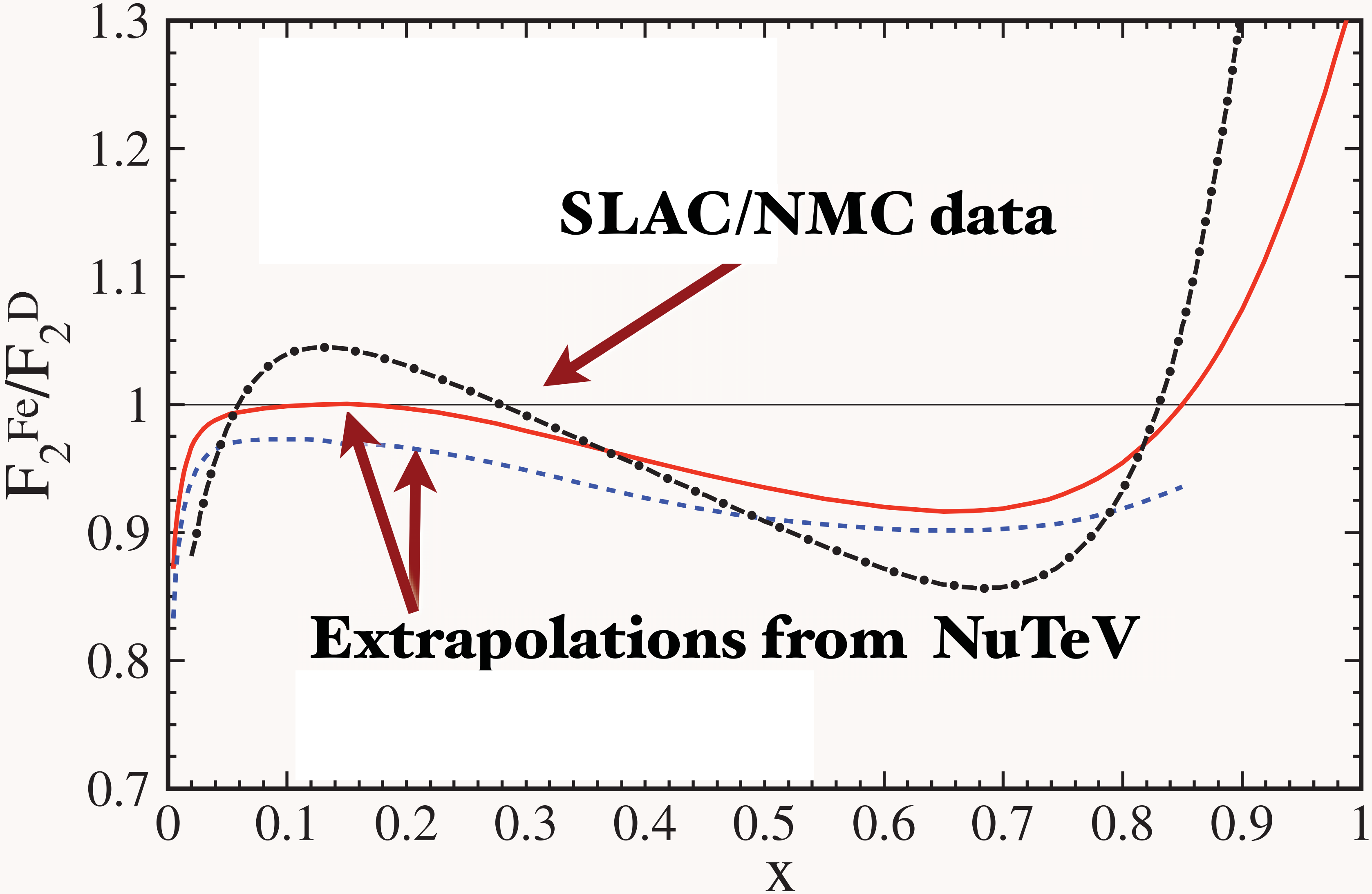
Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

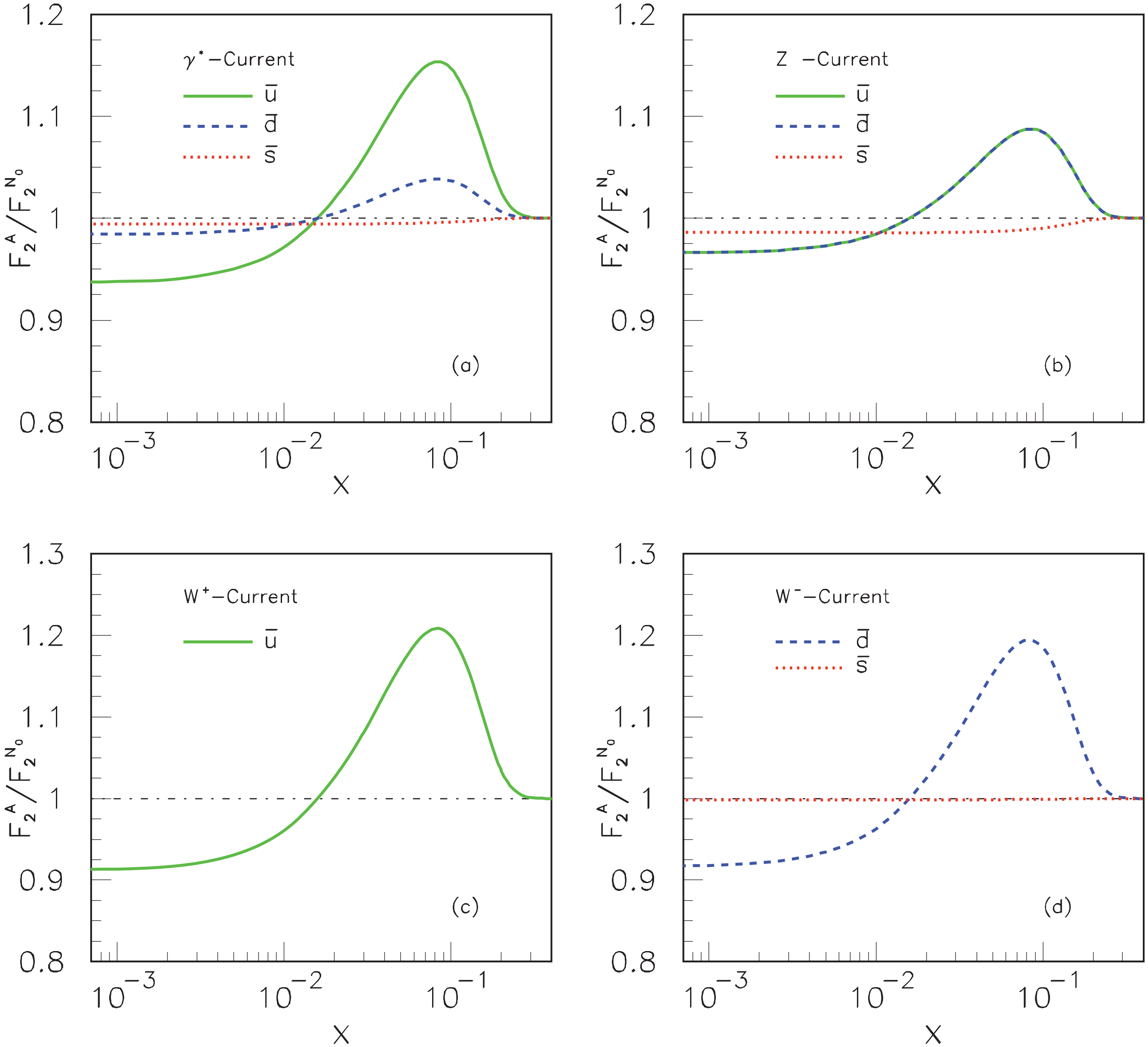
Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb

$$Q^2 = 5 \text{ GeV}^2$$



Scheinbein, Yu, Keppel, Morfin, Olness, Owens

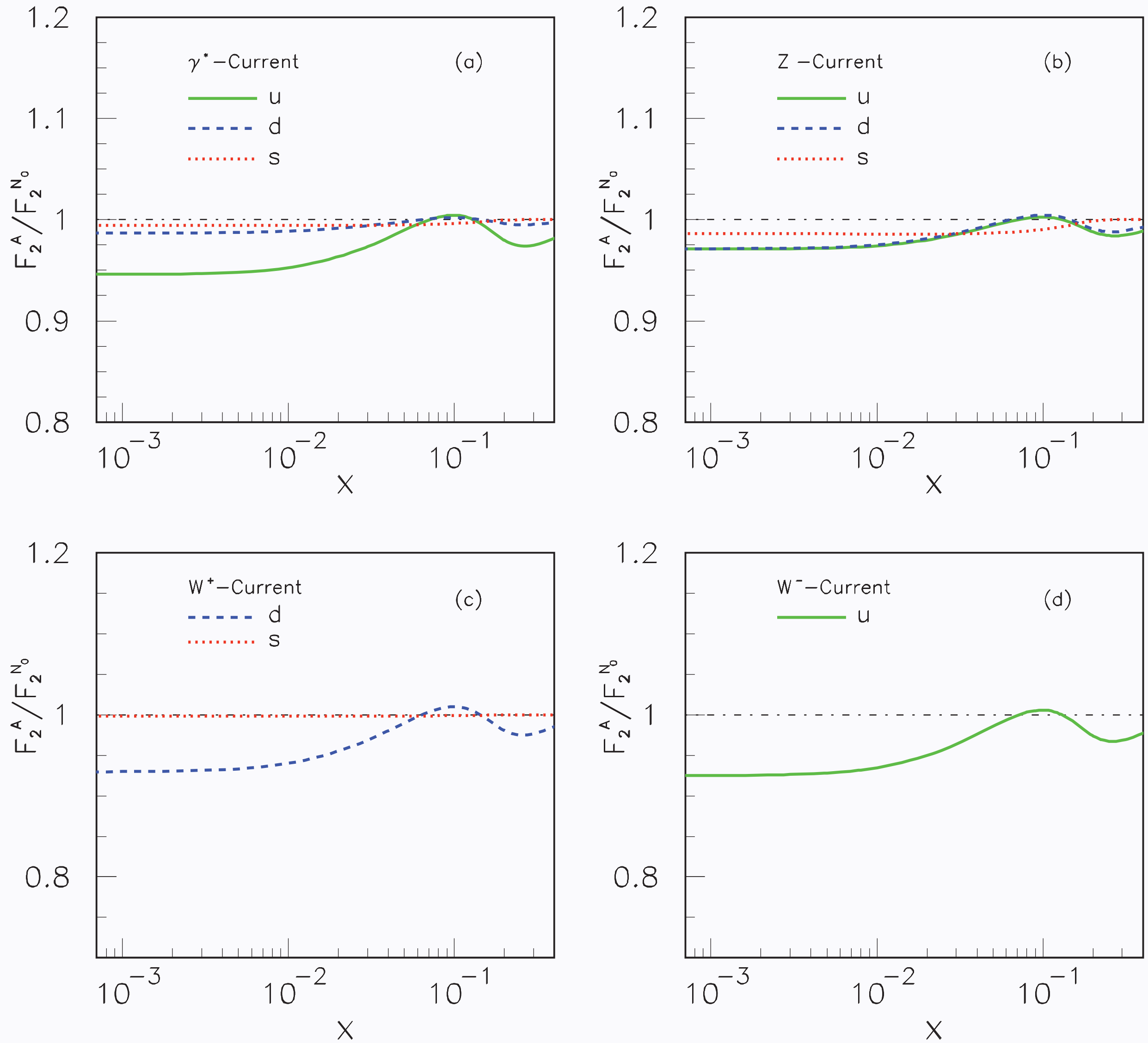


Schmidt, Yang; sjb

Reggeon Contribution to DDIS Constructive Interference!

Nuclear Antishadowing not universal !

Shadowing and Antishadowing of DIS Structure Functions

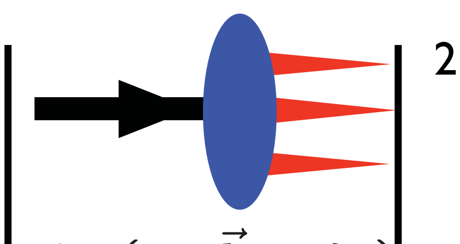


S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

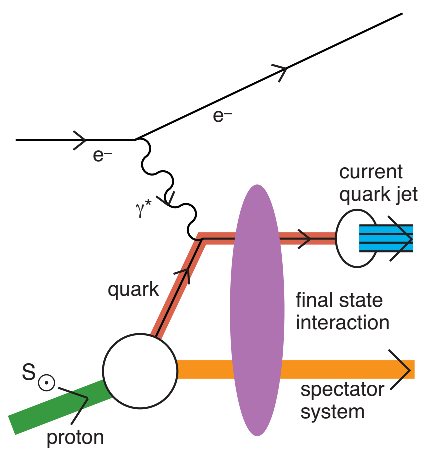
Modifies NuTeV extraction of $\sin^2\theta_W$

Test in flavor-tagged lepton-nucleus collisions

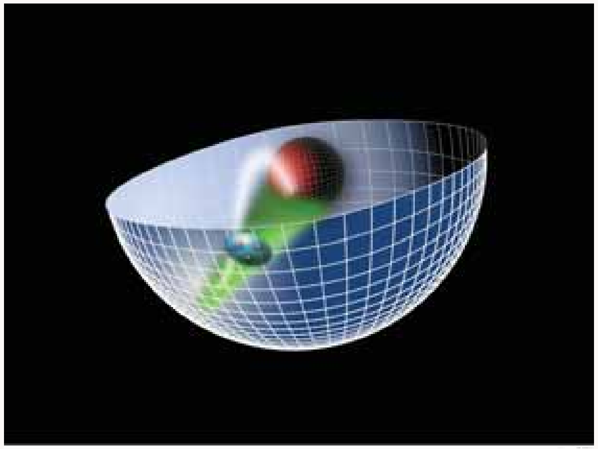
Static	Dynamic
• Square of Target LFWFs	Modified by Rescattering: ISI & FSI
• No Wilson Line	Contains Wilson Line, Phases
• Probability Distributions	No Probabilistic Interpretation
• Process-Independent	Process-Dependent - From Collision
• T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
• No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
• Sum Rules: Momentum and J^z	Sum Rules Not Proven
• DGLAP Evolution; mod. at large x	DGLAP Evolution
• No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

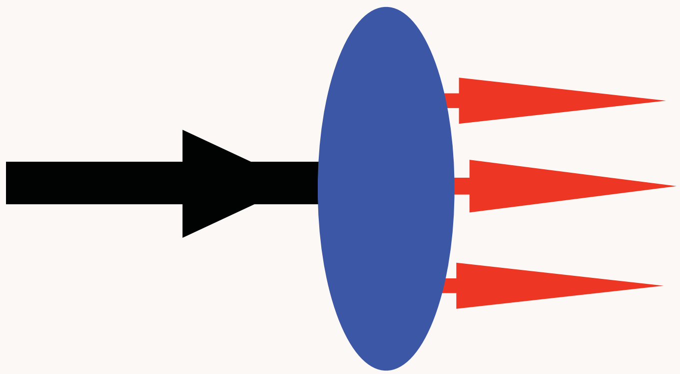


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

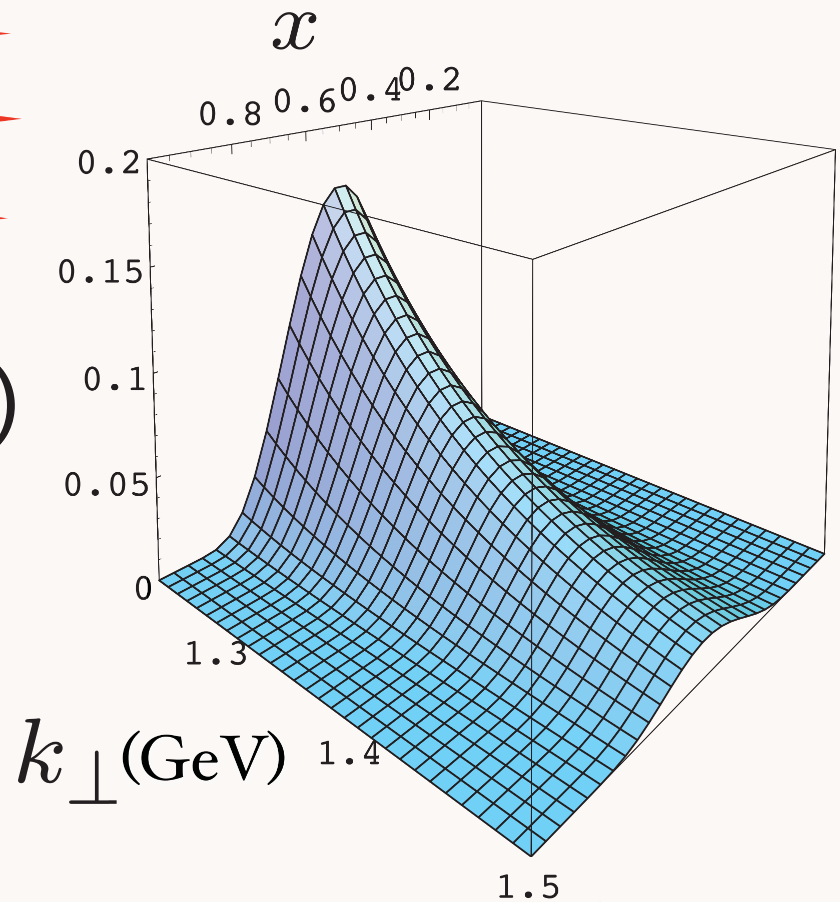


$\phi(z)$





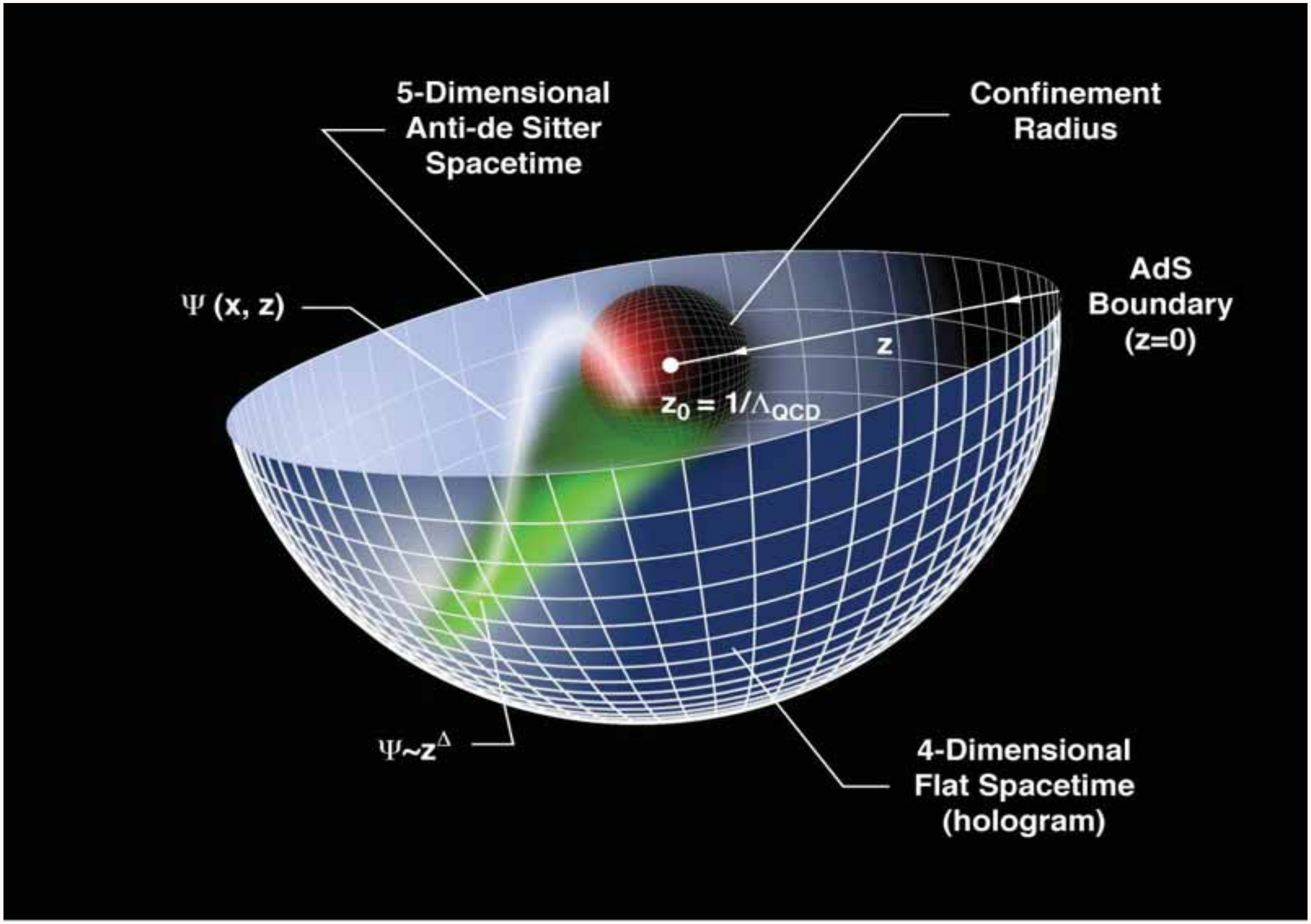
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

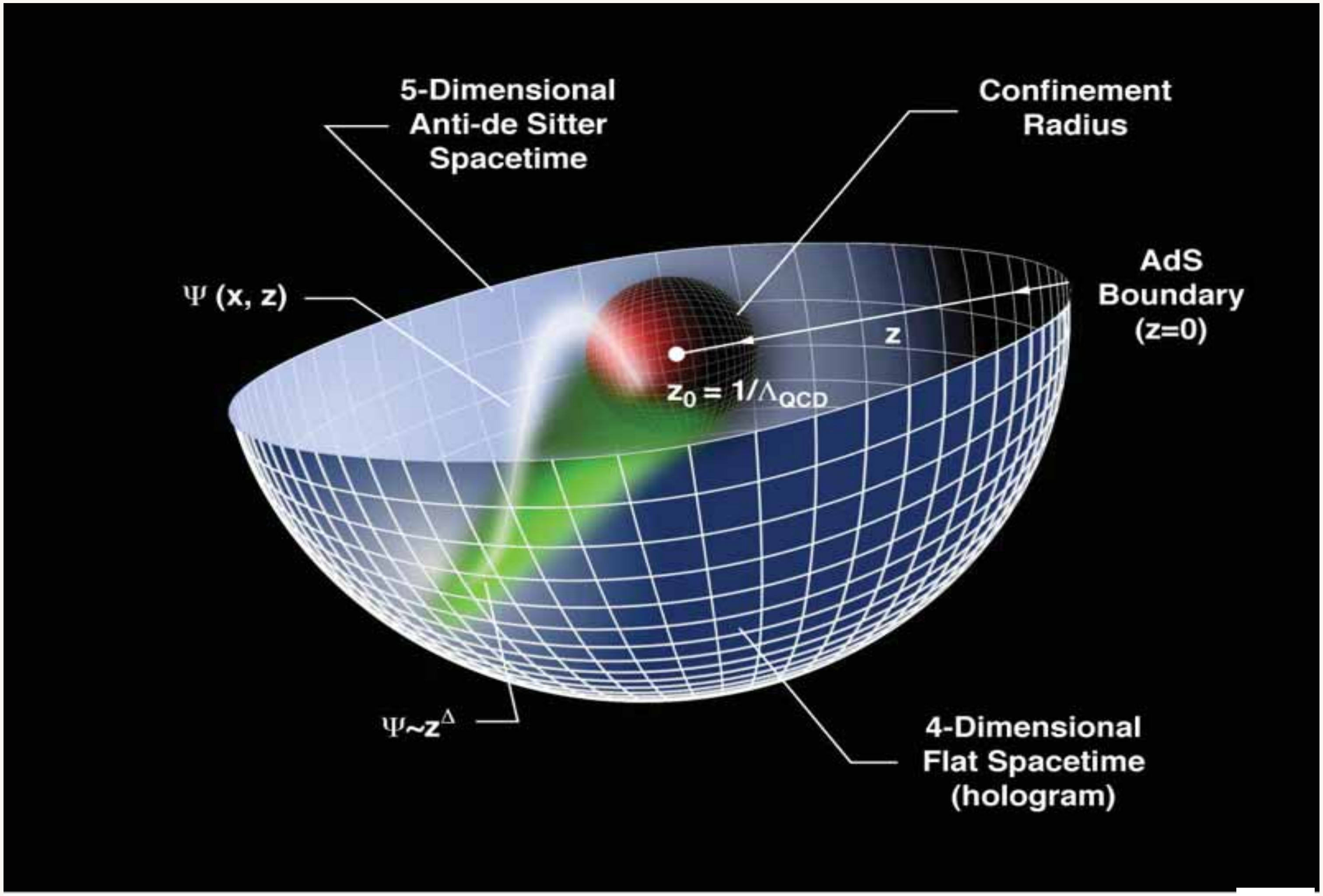
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

String Theory

Bottom-Up

Top-Down

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathrm{M}^{\mu\nu}, \mathrm{P}^{\mu}, \mathrm{D}, \mathrm{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \rightarrow \lambda x^\mu, \; z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z.

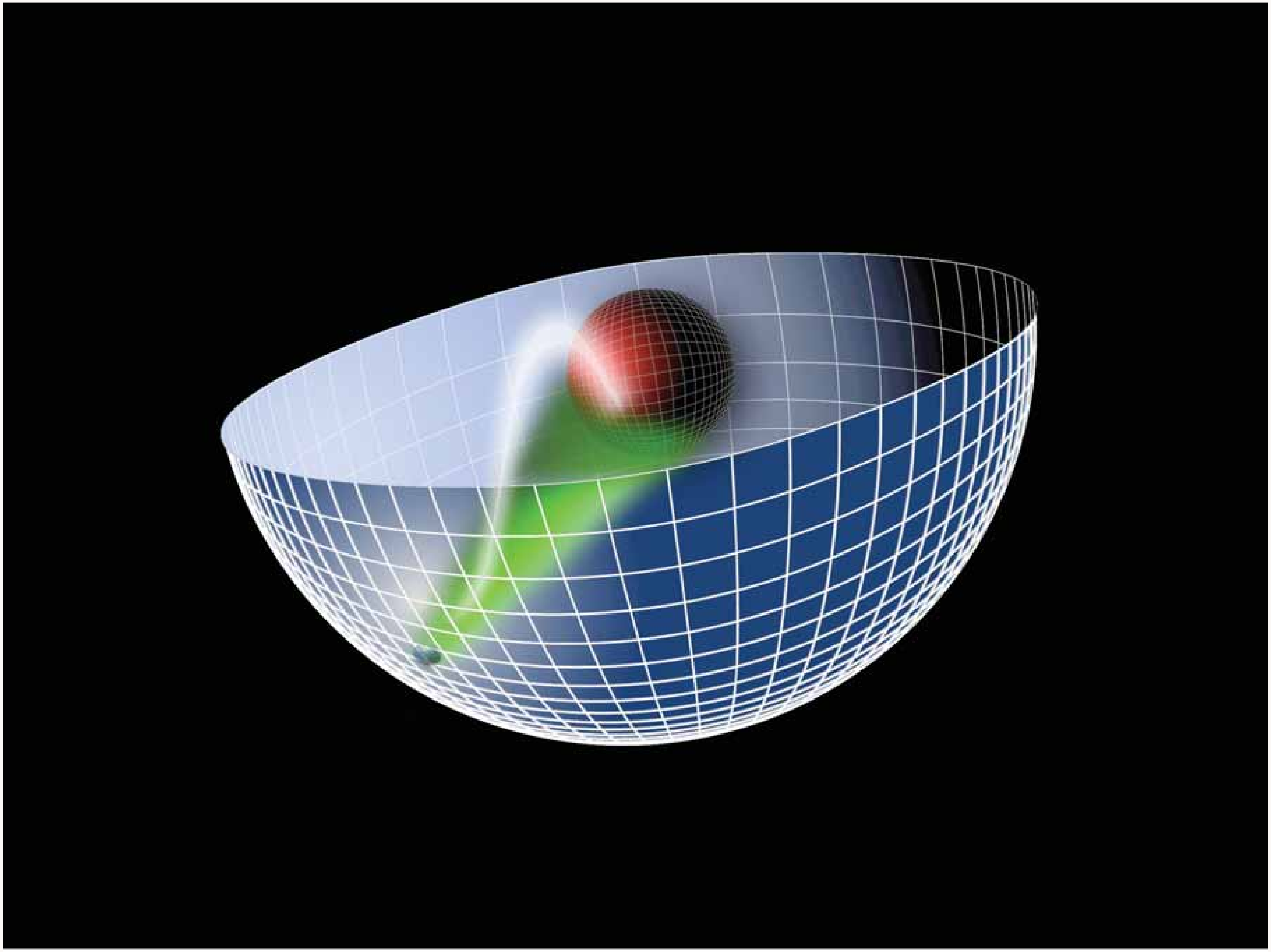
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

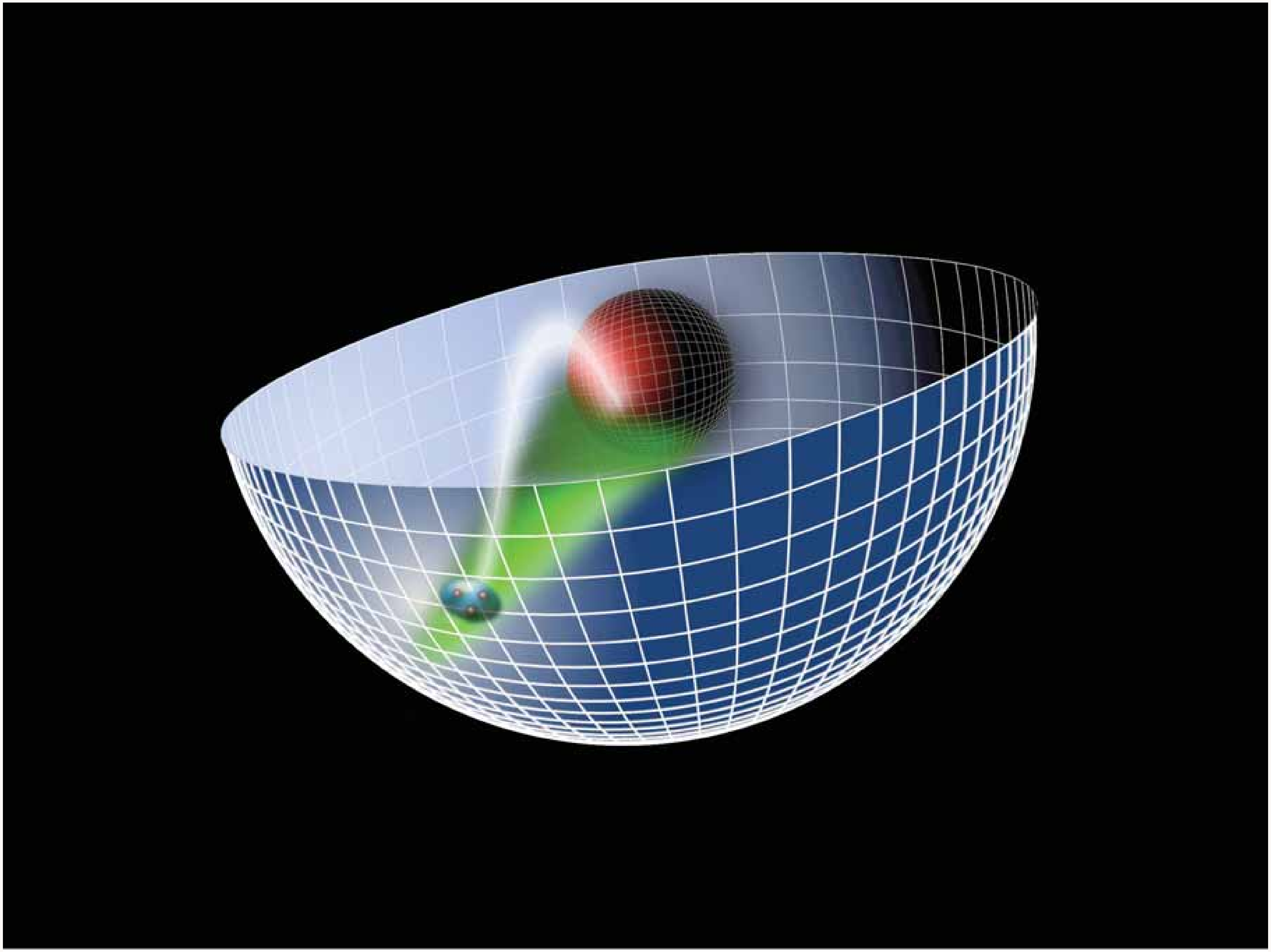
- Different values of z correspond to different scales at which the hadron is examined.

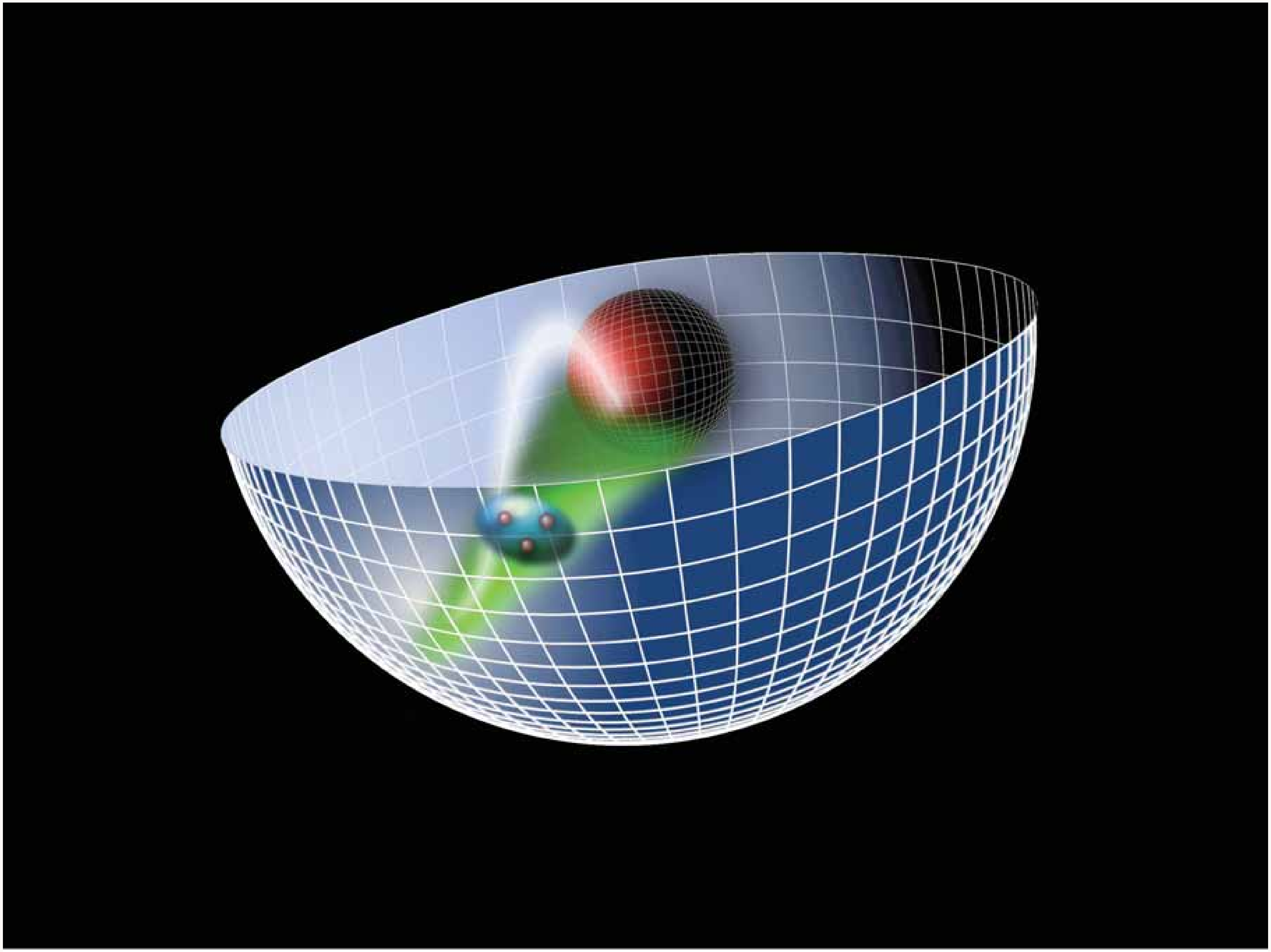
$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

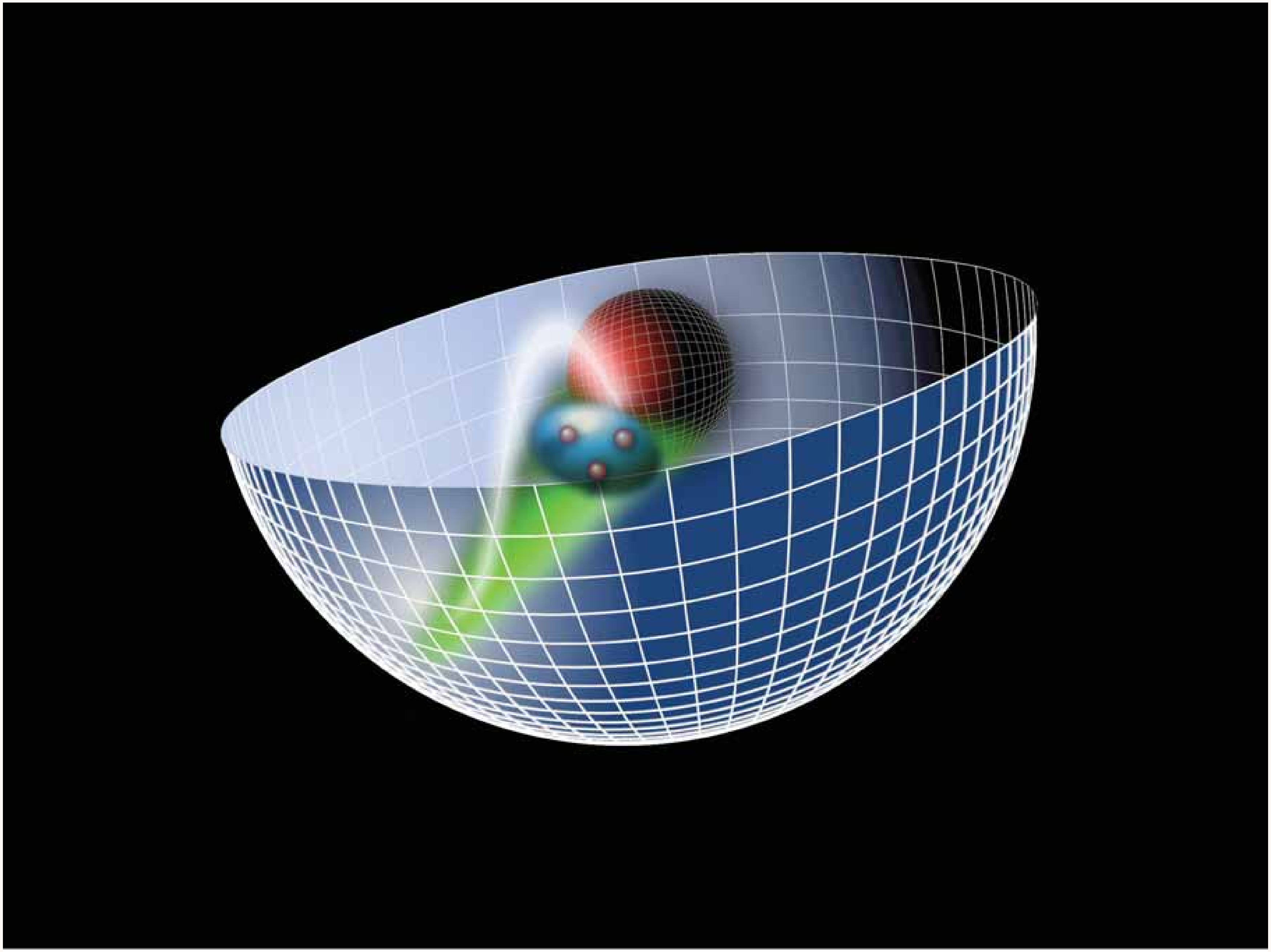
$x^2 = x_\mu x^\mu$: invariant separation between quarks

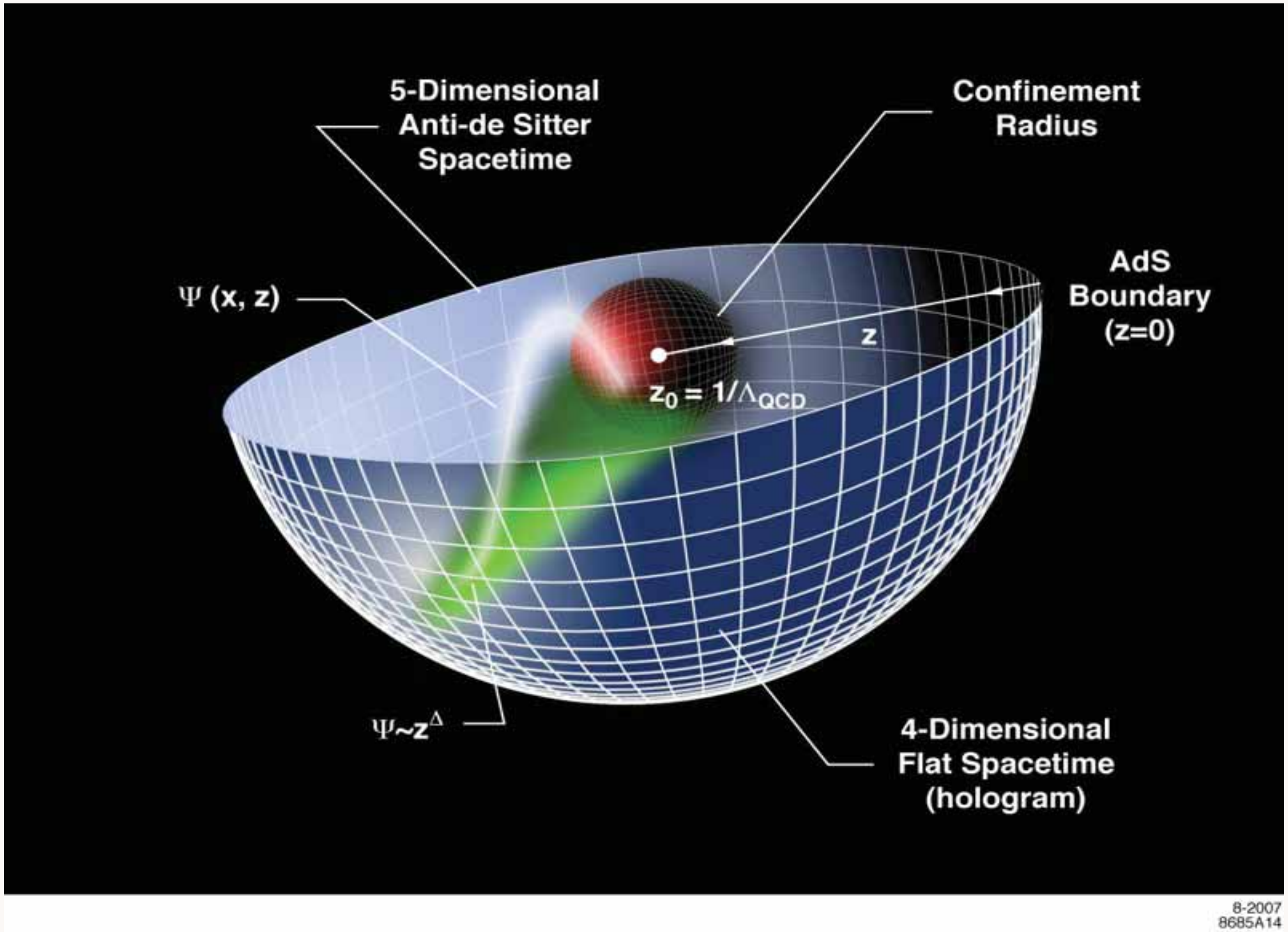
- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.









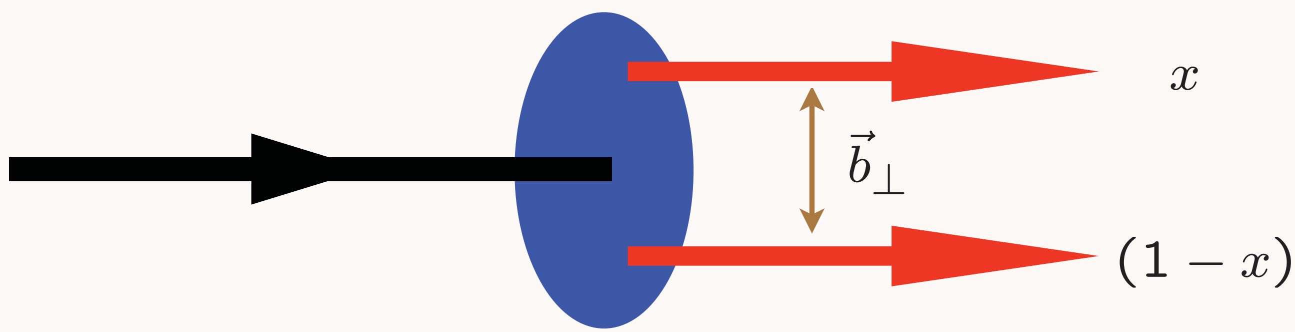


- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

LF(3+1) $\quad\quad$ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

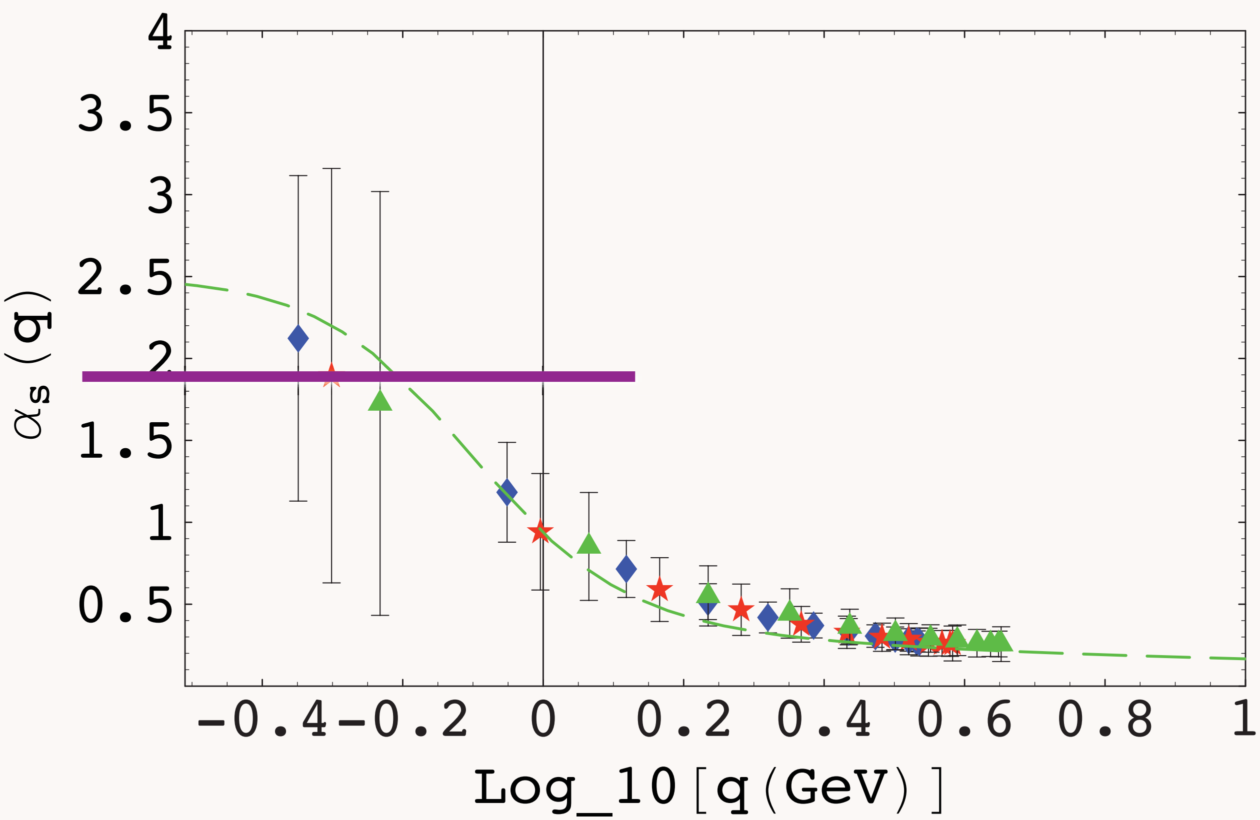
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

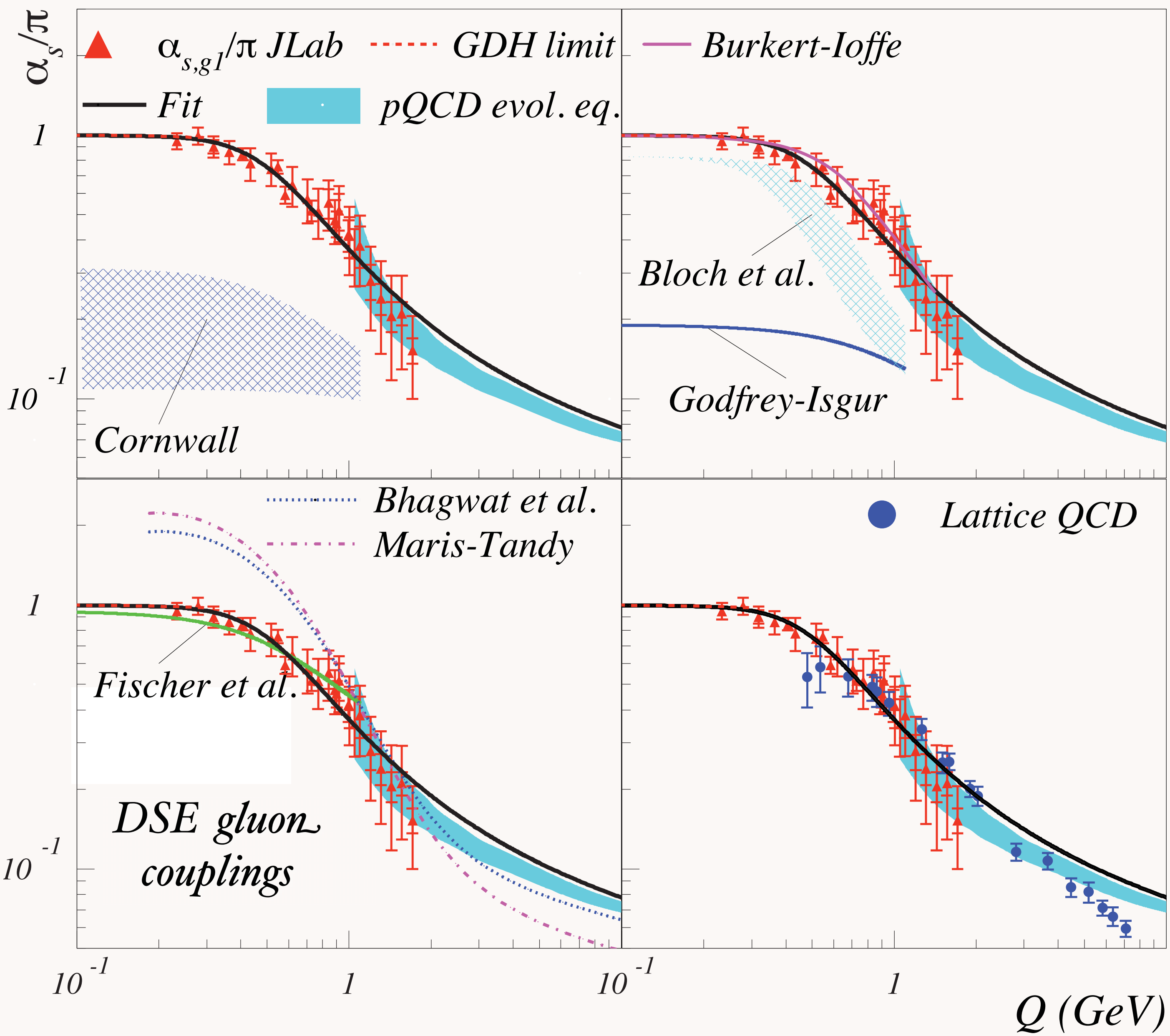
Conformal QCD Window in Exclusive Processes

- Does α_s develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .

- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).



Deur, Korsch, et al.



Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}\left[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}\right]$$