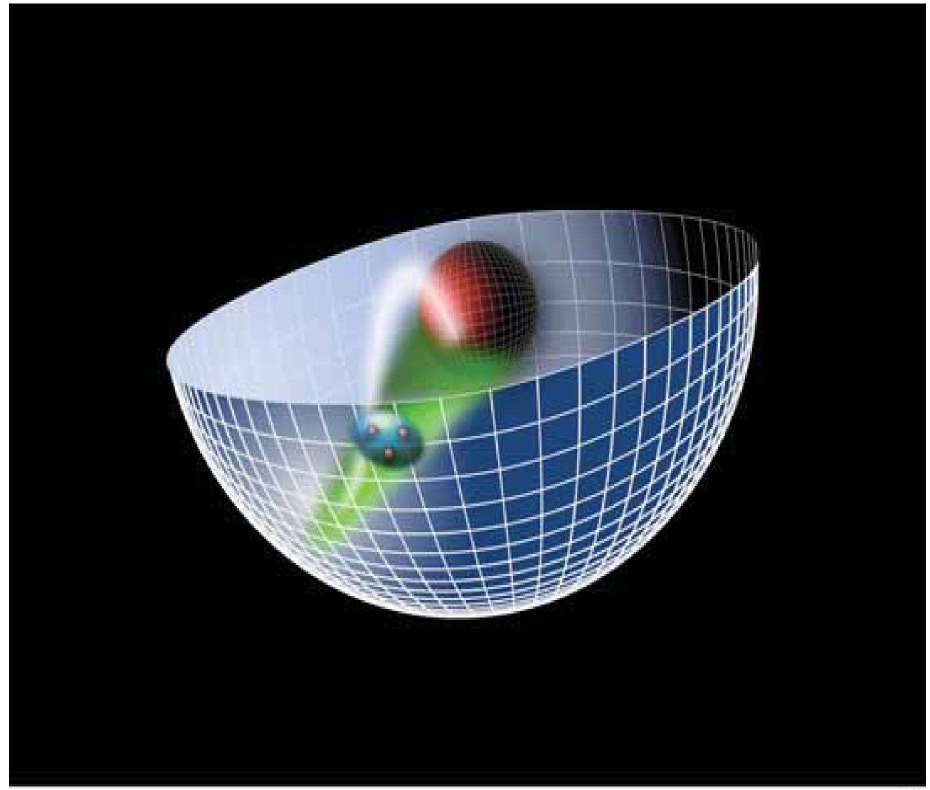
Light-Front Holography and Non-Perturbative QCD



Stan Brodsky, SLAC

LC2009

Light-Cone 2009: Relativistic Hadronic and Particle Physics

July 8-13, 2009

Instituto Tecnológico de Aeronáutica (ITA)

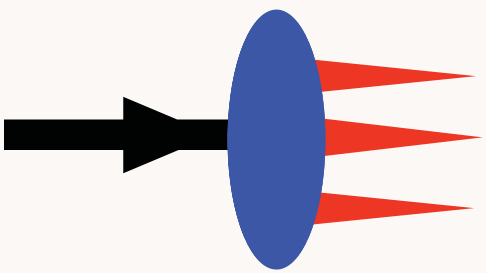
Comando-Geral de Tecnologia Aeroespacial (CTA)

São José dos Campos, Brazil

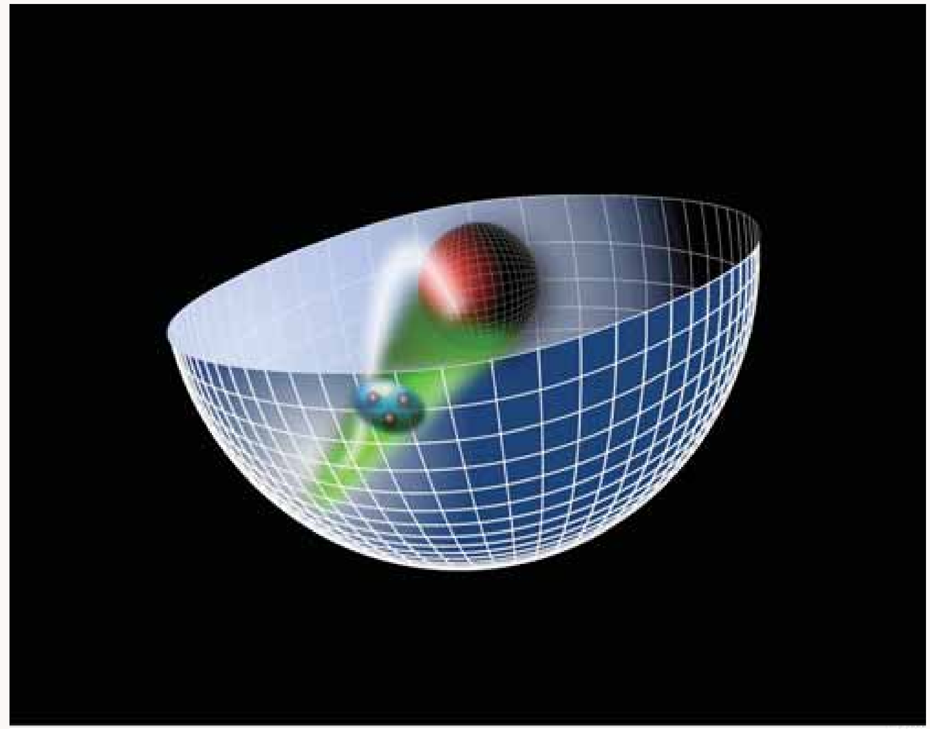
Light-Front Holography and Non-Perturbative QCD

Main Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

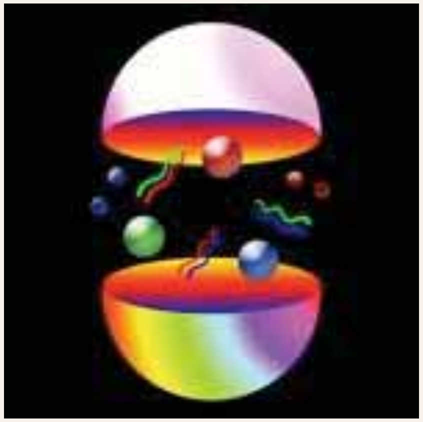


in collaboration with
Guy de Teramond

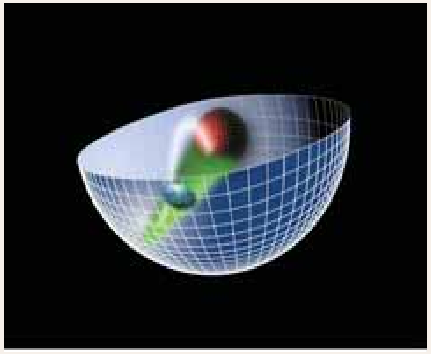
AdS/QCD: New interpretation of chiral symmetry and QCD vacuum structure

with Robert Shrock

- Quarks and Gluons:
  Fundamental constituents of hadrons and nuclei

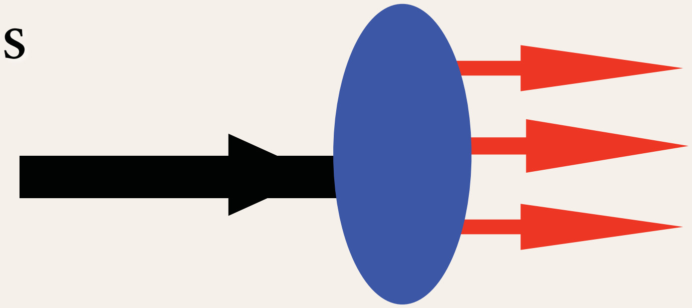


- *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: *AdS/QCD*



- *Light-Front Holography*
- *Hadronization at the Amplitude Level*
- *Light Front Wavefunctions:* analogous to the Schrodinger wavefunctions of atomic physics

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

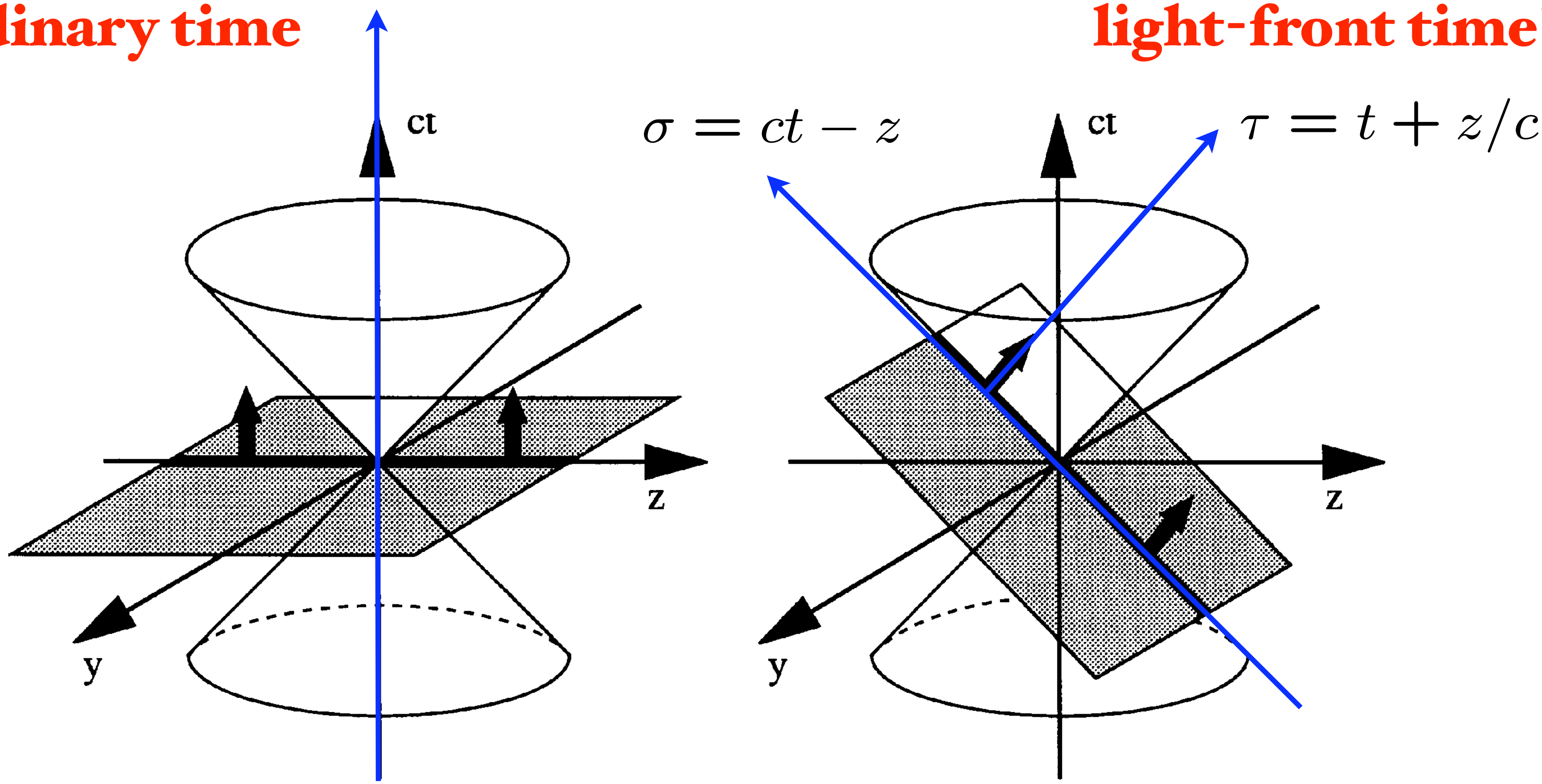


P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form

Front Form

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

Causally-Connected Domains

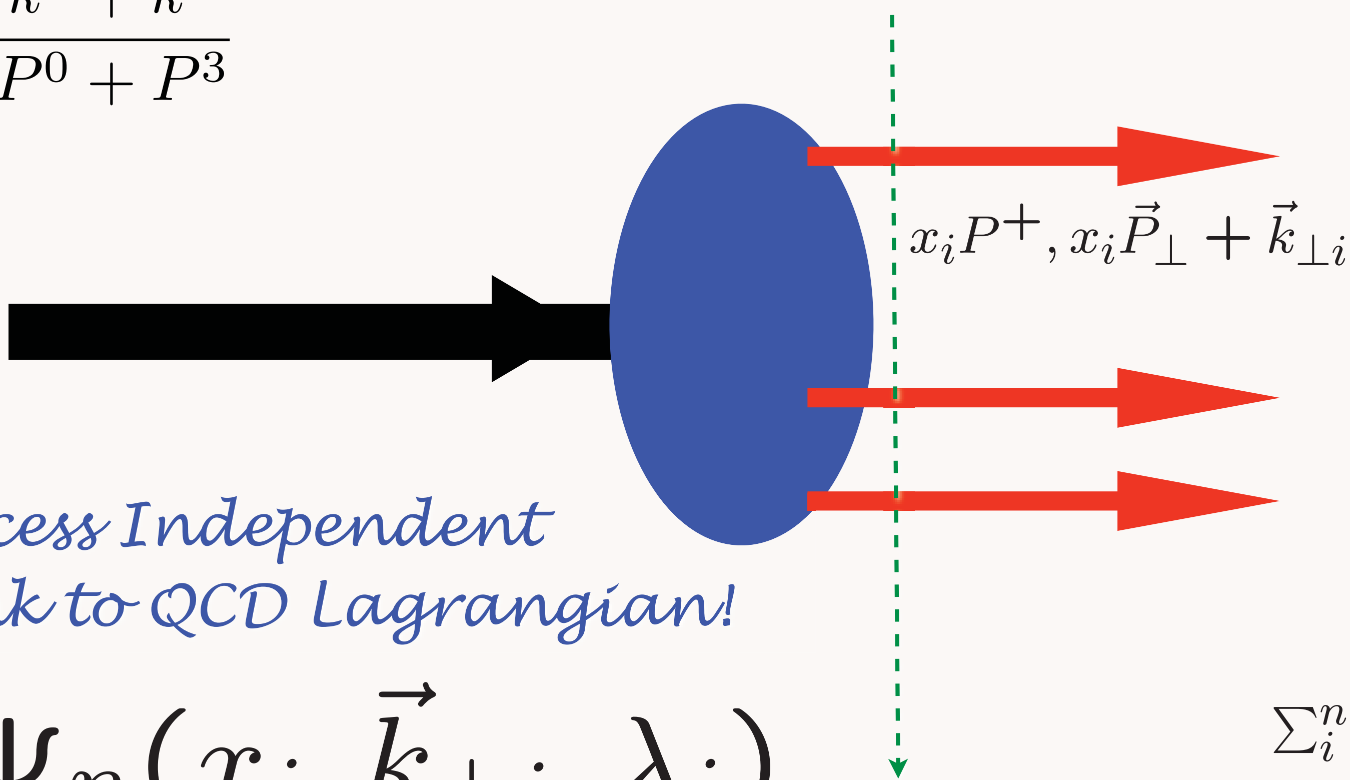
HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



Process Independent

Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front QCD

Heisenberg Matrix Formulation

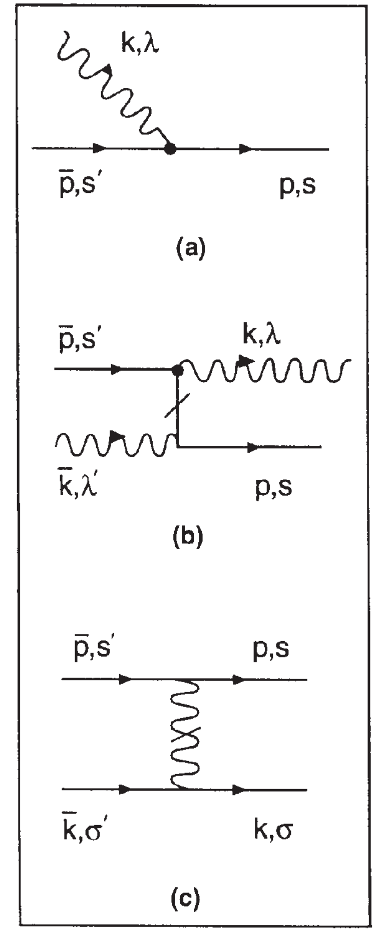
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

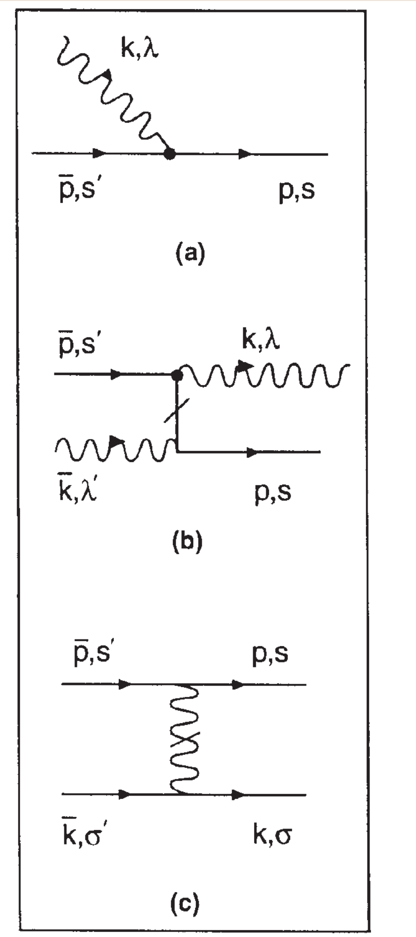
Light-Front QCD

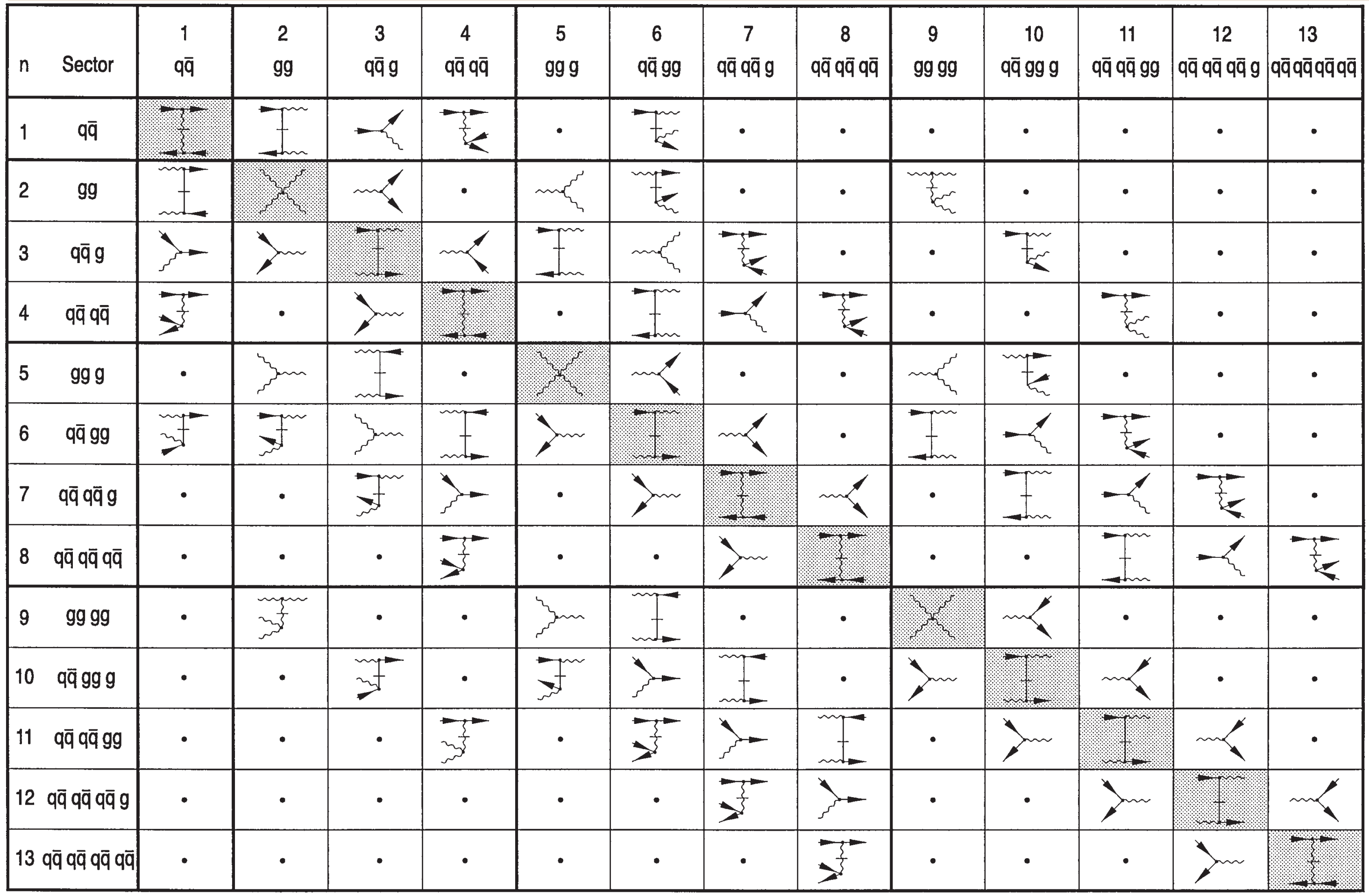
Heisenberg Matrix Formulation

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

H.C. Pauli & sjb

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

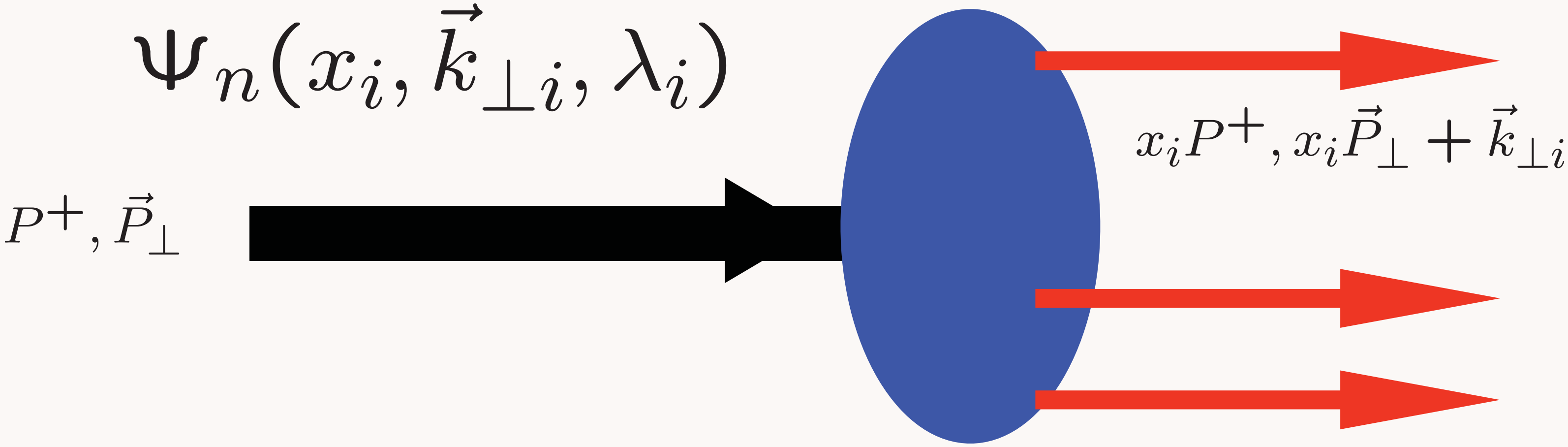
DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

$\sum_i^n x_i = 1$ $\quad$ $\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_{i=1}^n k_i^+ = \sum_{i=1}^n x_i \vec{P}^+ = \vec{P}^+$ $\quad$ $\sum_{i=1}^n (x_i \vec{P}_\perp + \vec{k}_{\perp i}) = \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$$\vec{\ell}_j \equiv (\vec{k}_\perp \times \vec{b}_\perp)_j = (\vec{k}_\perp \times \frac{i\partial}{\partial \vec{k}_\perp})_j$$

n-1 Intrinsic Orbital Angular Momenta
Frame Independent

$j = 1, 2, \cdots (n-1)$

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State!

LF Spin Sum Rule

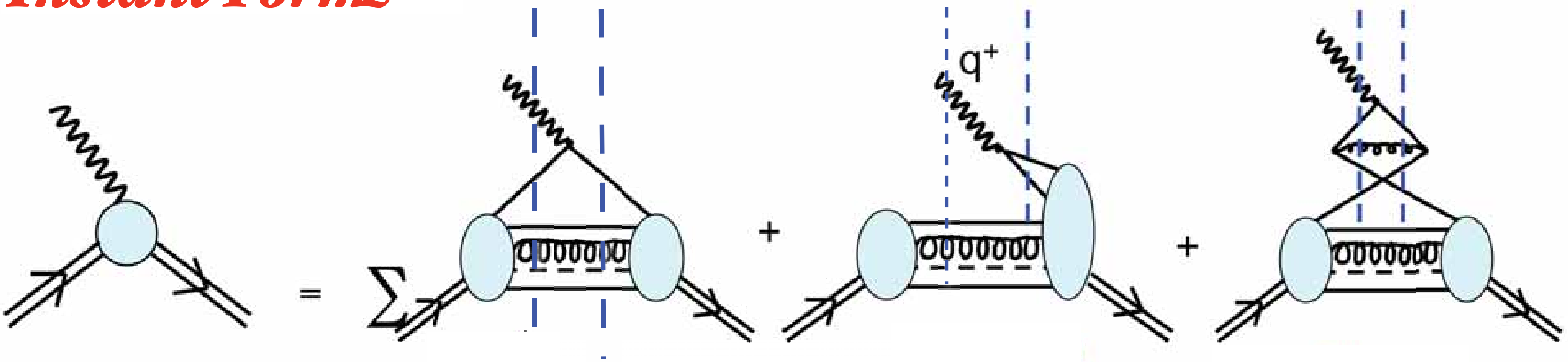
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Calculation of Form Factors in Equal-Time Theory

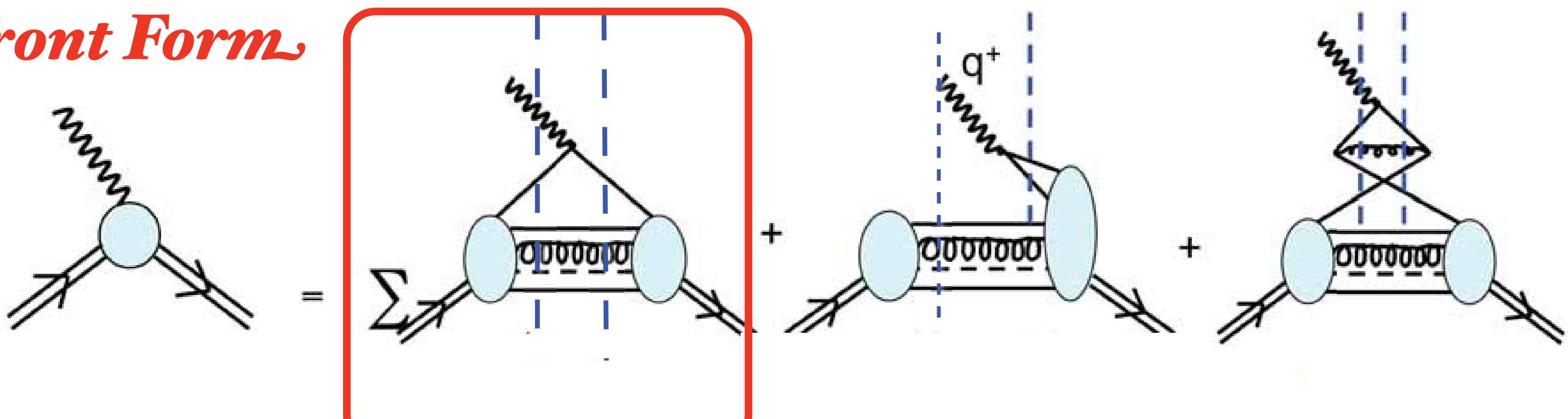
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



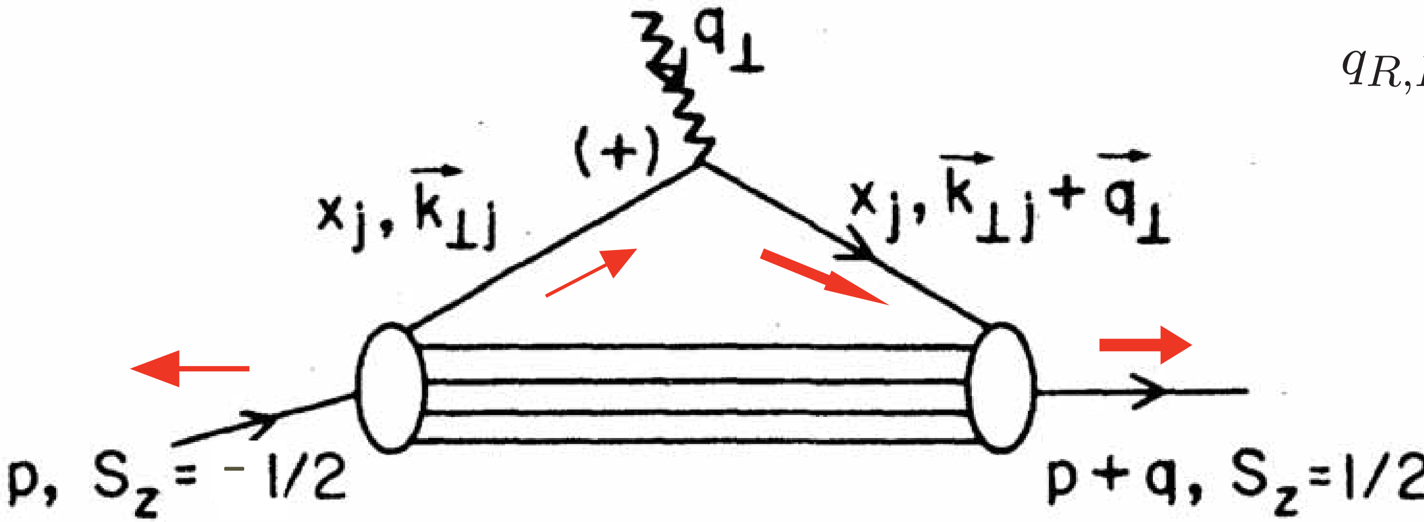
Absent for $q^+ = 0$ **zero !!**

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

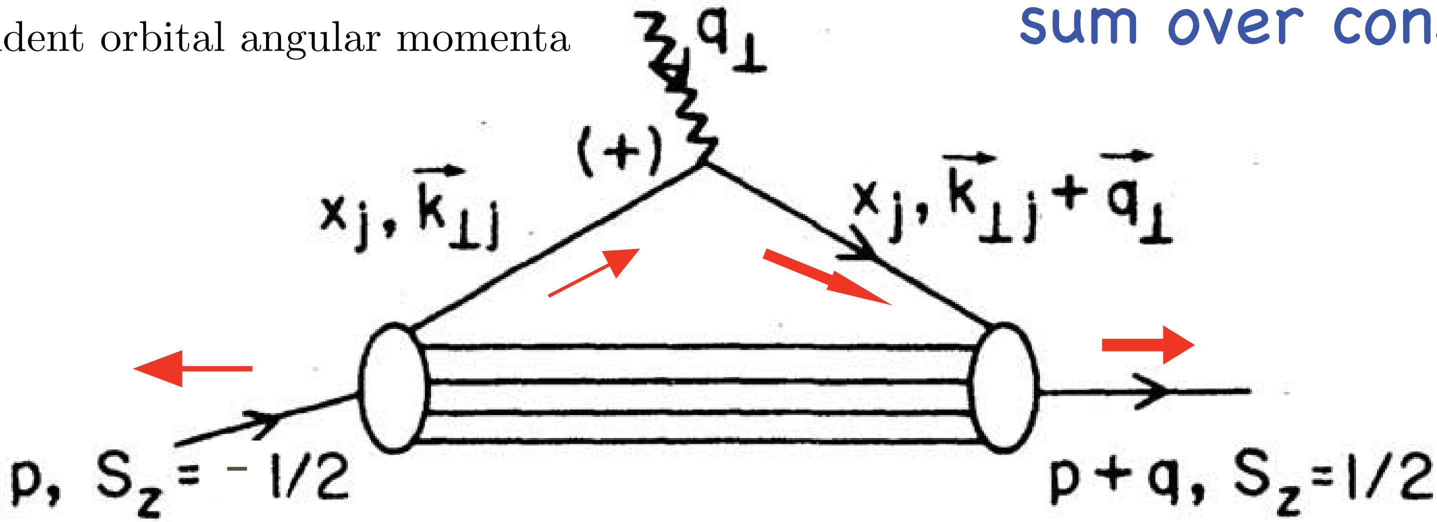
Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

$\sum_{i=1}^{n} L_i = 0$
$n-1$ independent orbital angular momenta

sum over constituents



Hwang, Schmidt, sjb;
Holstein et al

$B(0) = 0$

Each Fock State

Special Features of LF Spin

- LF Helicity and chirality refer to z direction, not the particle's 3-momentum p!!
- LF spinors are eigenstates of $S^z = \pm\frac{1}{2}$
- Gluon polarization vectors are eigenstates of $S^z = \pm 1$

$$\epsilon^\mu = (\epsilon^+, \epsilon^-, \vec{\epsilon}_\perp) = (0, 2\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp)$$

$$\vec{\epsilon}^{\,\pm}_\perp = \mp\frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \qquad k^\mu \epsilon_\mu = 0 \qquad \epsilon^+ = 0$$

Light-Cone Gauge

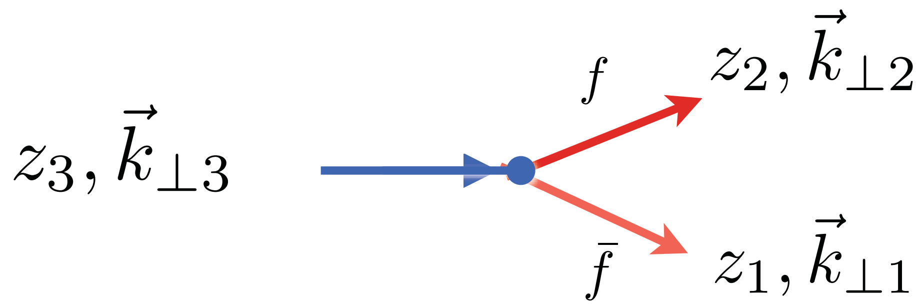
Light-Cone Spinors $S^z = \pm 1/2$ G. P. Lepage and sjb

$$\left.\begin{matrix} u_\uparrow(p) \\ u_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} (p^+ + \beta m + \alpha_\perp \cdot \mathrm{p}_\perp) \times \left\{\begin{matrix} \chi(\uparrow) \\ \chi(\downarrow) \end{matrix}\right. ,$$

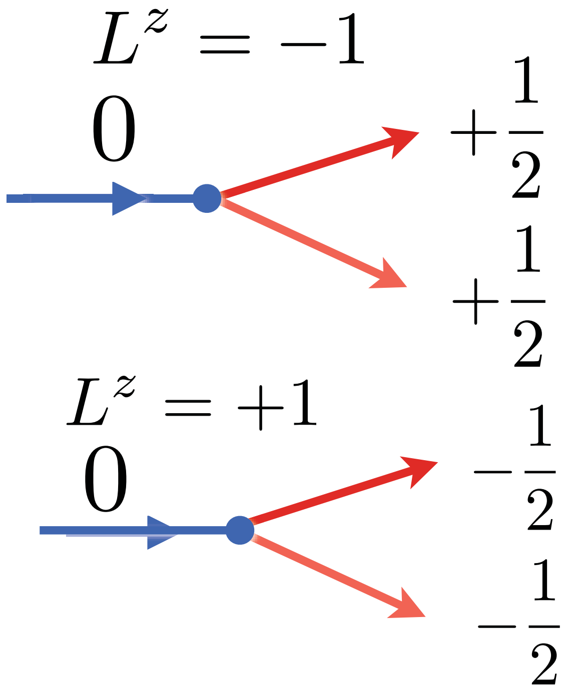
$$\left.\begin{matrix} v_\uparrow(p) \\ v_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} (p^+ - \beta m + \vec{\alpha}_\perp \cdot \vec{\mathrm{p}}_\perp) \times \left\{\begin{matrix} \chi(\downarrow) \\ \chi(\uparrow) \end{matrix}\right.$$

$$\chi(\uparrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},$$

Angular Momentum on the Light-Front



P-Wave Decay
Spin-0 coupling
to fermion pair



spinor overlap

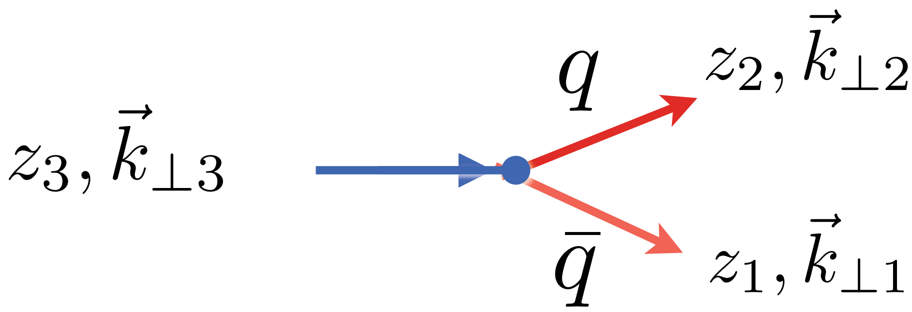
$$< ij >=< i - |j+ >= -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

$$[ij] =< i^+ |j- >= \sqrt{2 z_i z_j} \vec{\epsilon}^{\,(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

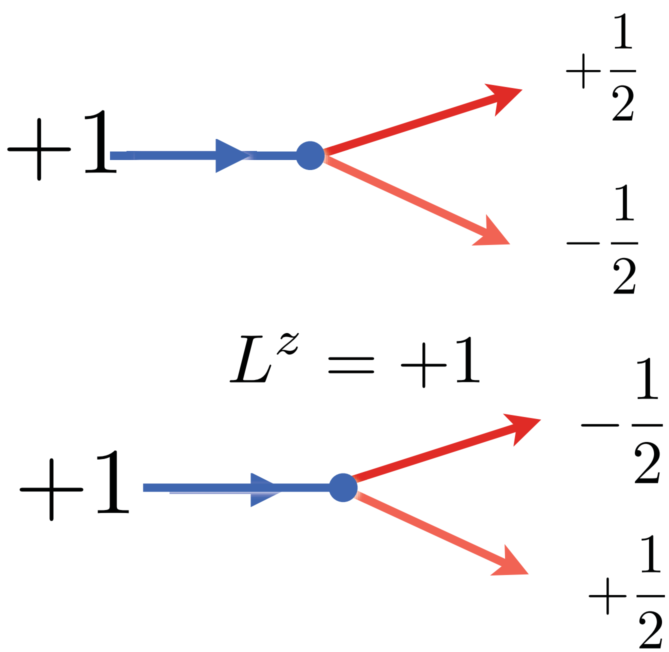
$$< ij > [ij] = z_i z_j \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)^2 = \mathcal{M}^2_{ij}$$

Identity

Angular Momentum on the Light-Front



Spin-1 coupling
to massless fermion pair



$$\vec{\epsilon}_{\perp}^{(+)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_{\perp}^{(-)} \cdot \frac{\vec{k}_{\perp 3}}{z_3},$$

P-Wave Decay

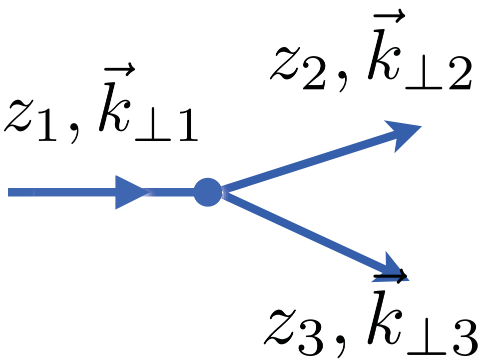
$$\vec{\epsilon}_{\perp}^{(-)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_{\perp}^{(+)} \cdot \frac{\vec{k}_{\perp 3}}{z_3}$$

Compare CM distribution $1 + \cos^2 \theta_{CM}$

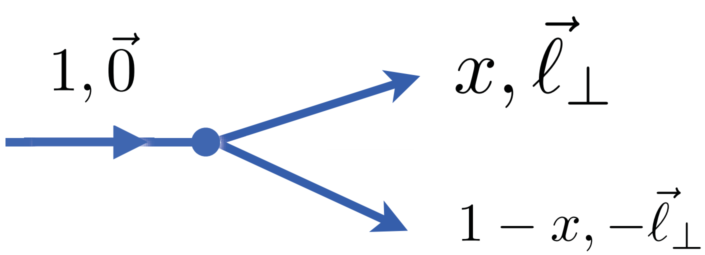
Looks like S and D-Wave Decay

Angular Momentum on the Light-Front

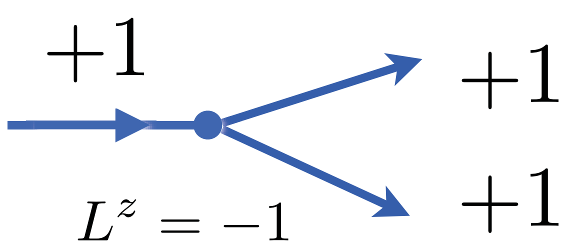
Triple-Gluon Coupling



$$g z_1 \vec{\epsilon}_{\perp}^{\,+} \cdot \vec{v}_{23} = g z_1 \vec{\epsilon}_{\perp}^{\,+} \cdot \left(\frac{\vec{k}_{\perp 2}}{z_2} - \frac{\vec{k}_{\perp 3}}{z_3} \right)$$



$$g z_1 \vec{\epsilon}_{\perp}^{\,+} \cdot \vec{v}_{23} = g \vec{\epsilon}_{\perp}^{\,+} \cdot \frac{\vec{\ell}_{\perp}}{x(1-x)}$$

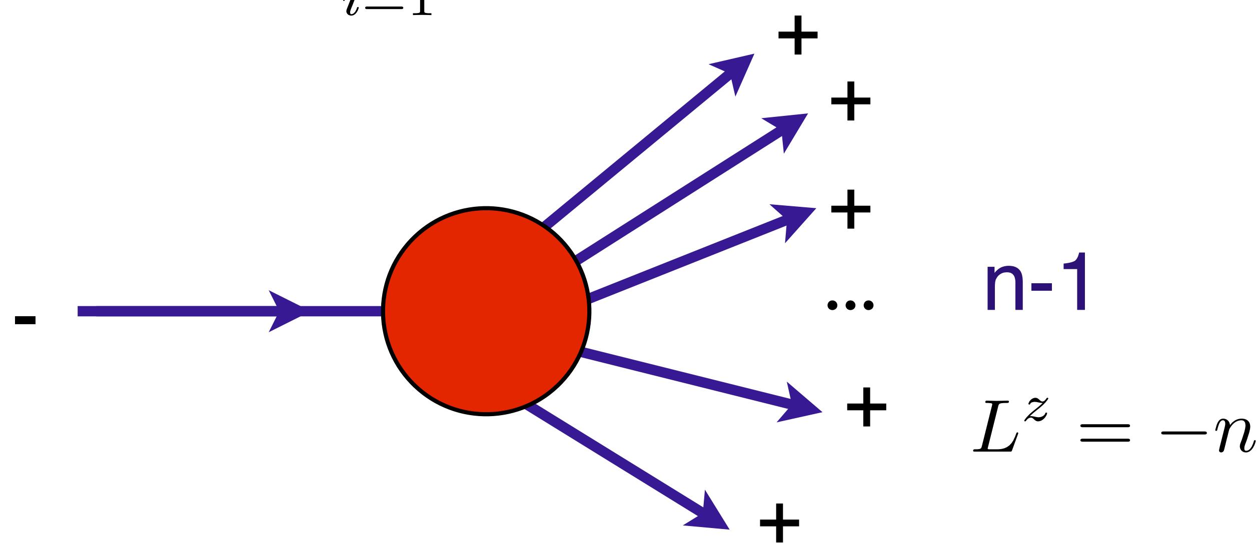


$$< ij >= -\sqrt{2 z_i z_j} \vec{\epsilon}_{\perp}^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j} \right)$$

G. de Teramond and sjb

$$M(-1 \to +1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-1) + L^z$$



Tree Graphs

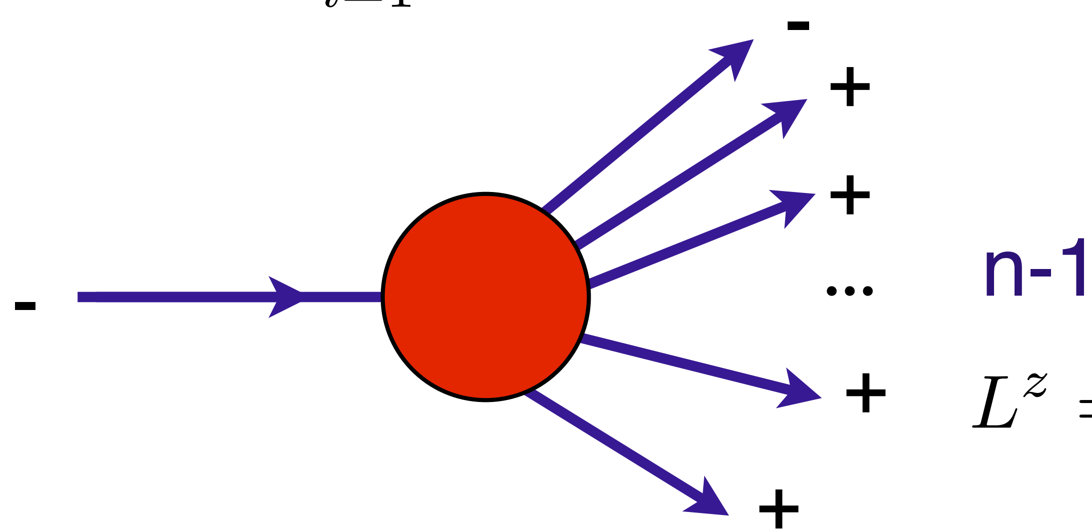
Vanishes Because Maximum $|L^z| = n-2$

Renormalizability

G. de Teramond and sjb

$$M(-1 \to -1 + 1 + 1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-2) + L^z$$



$$L^z = -(n-1)$$

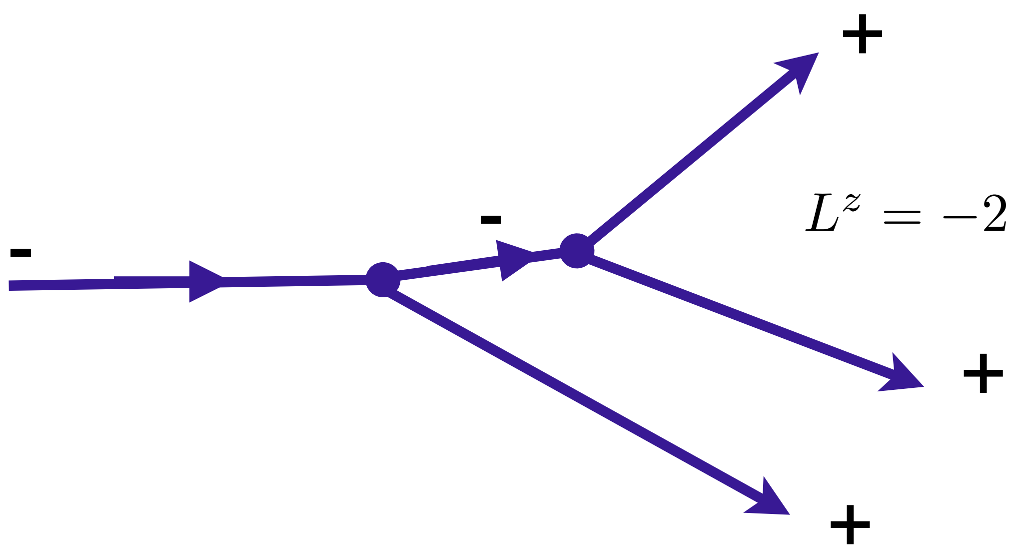
Vanishes Because Maximum $|L^z| = n - 2$

Tree Graphs

Light Front Helicity Explains MHV rules

n=4 example

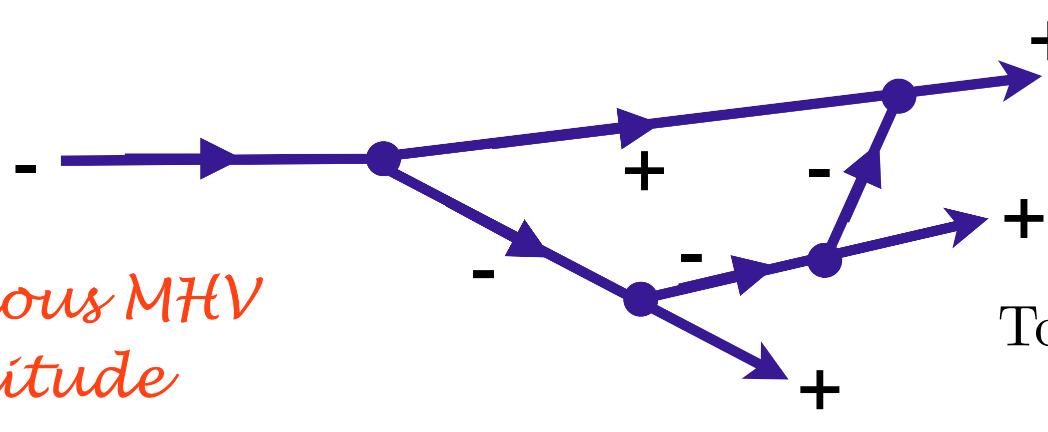
Tree Graph $M(-1 \to +1+1+1) \propto g^2 = 0$



$L^z = -2$ vertex not allowed

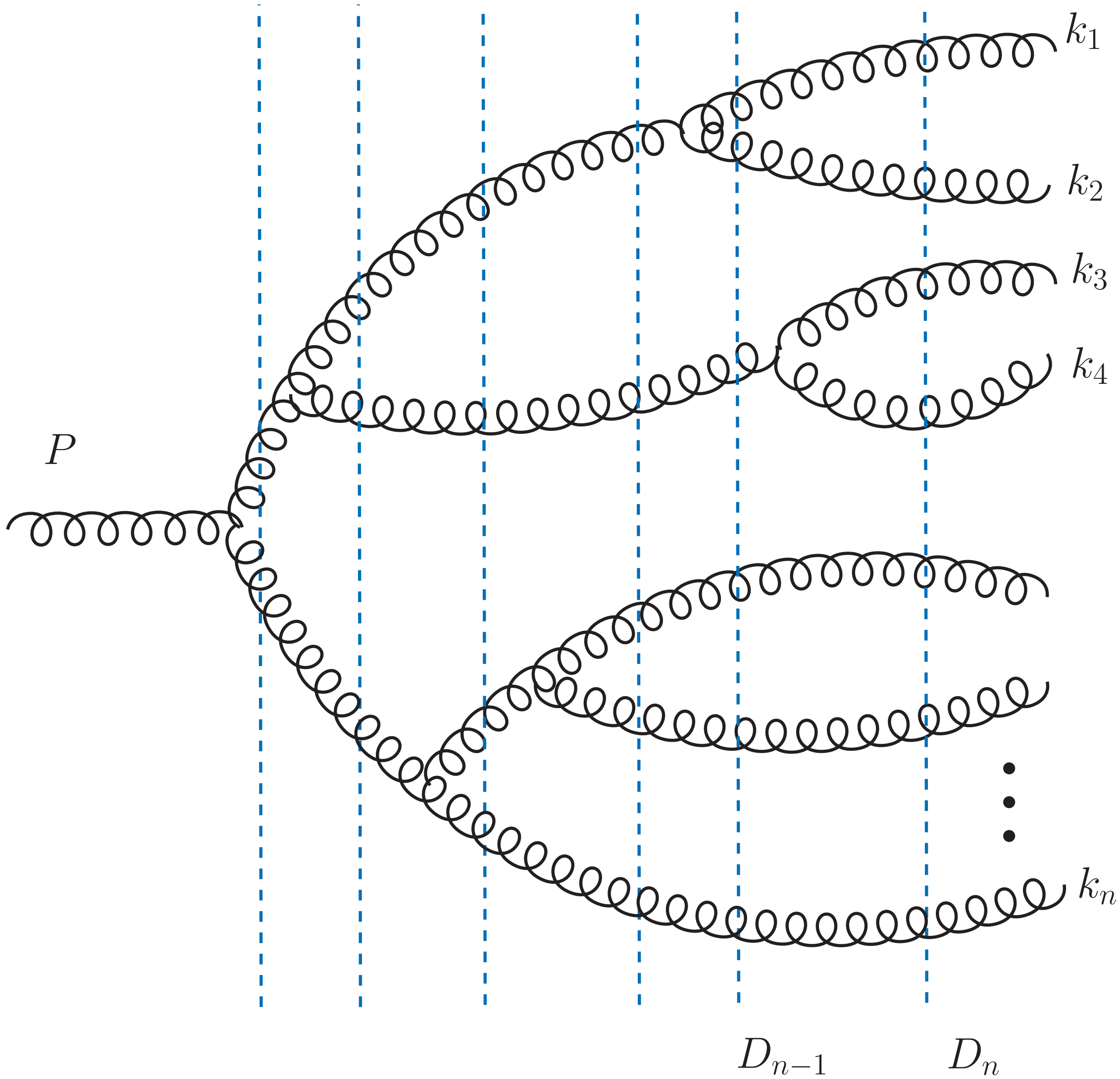
However:

Loop Graph $M(-1 \to +1+1+1) \propto g^4 \neq 0$



Anomalous MHV Amplitude

Total $L^z = -4$ allowed

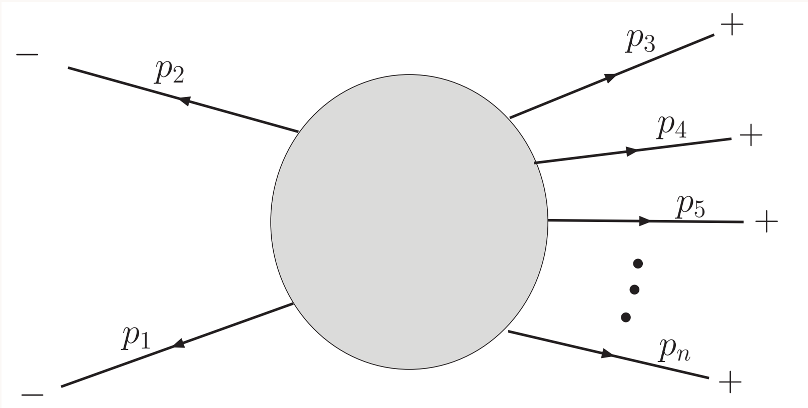


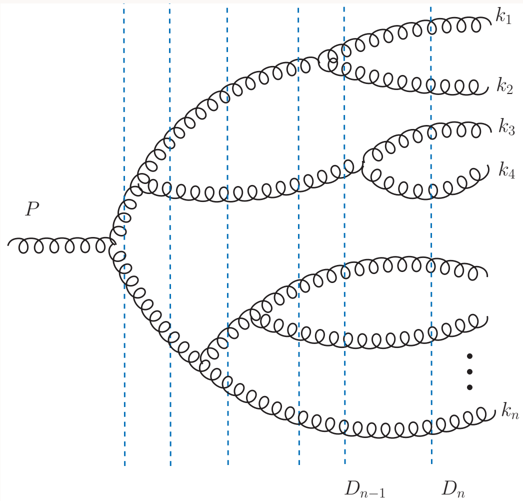
Exact kinematics in the small x evolution of the color dipole and gluon cascade.

Leszek Motyka (Hamburg U. & Jagiellonian U.) , Anna M. Stasto (Penn State U. & RIKEN BNL & Cracow, INP) . Jan 2009. 37pp.

e-Print: **arXiv:0901.4949** [hep-ph]

LF Derivation of Parke-Taylor





$$m(1^-,2^-,3^+,\dots,n^+) = ig^{n-2}\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n-2\, n-1\rangle\langle n-1\, n\rangle\langle n1\rangle},$$

$$m(\pm,\pm,\dots,\pm) = m(\mp,\pm,\pm,\dots,\pm) = 0\,.$$

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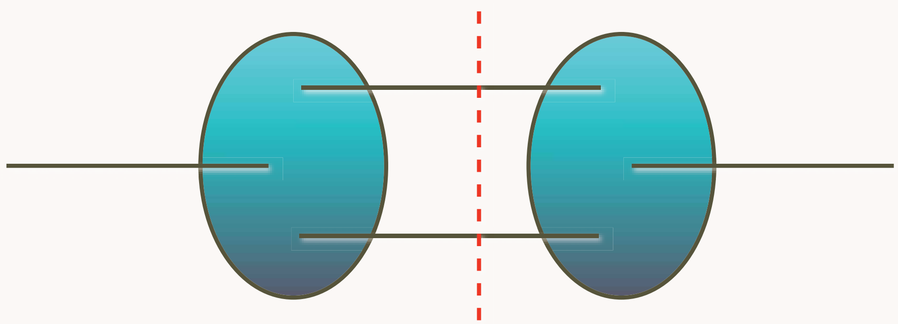
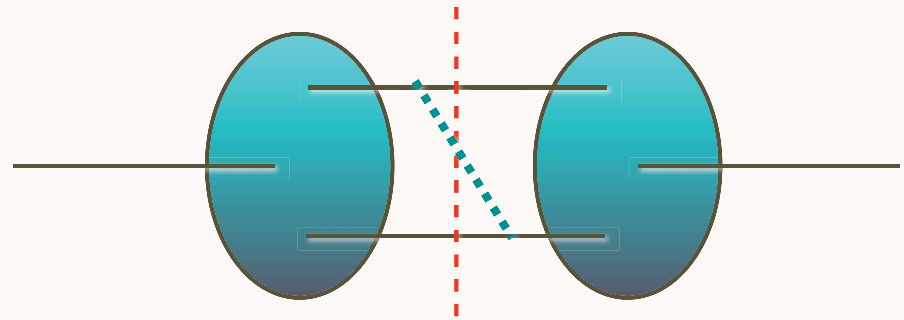
Quantum Mechanics: Uncertainty in p, x, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



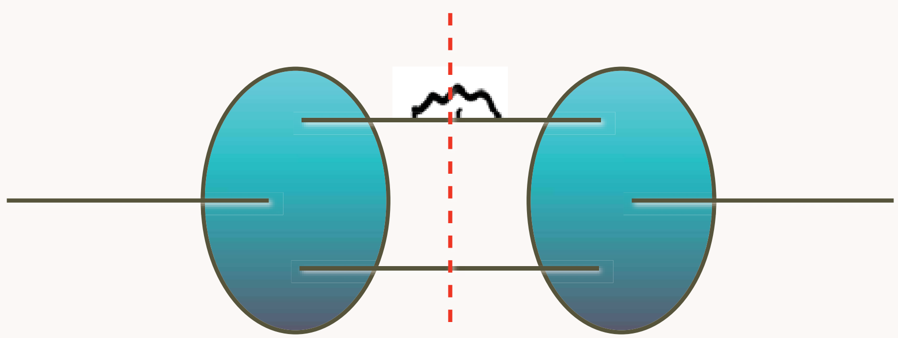
Positronium n=2

e^+e^-



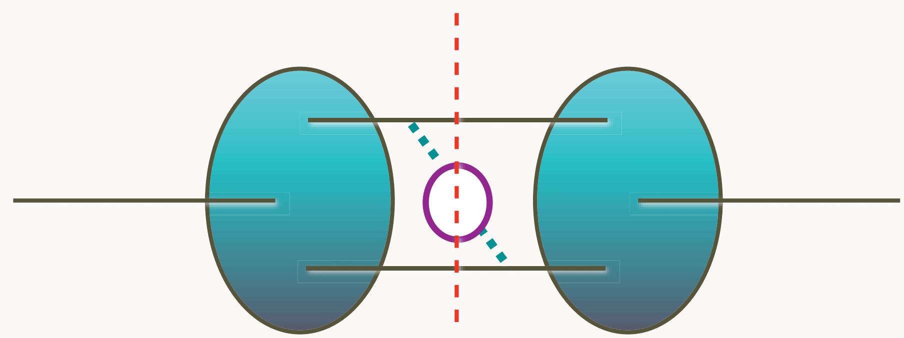
Hyperfine splitting n=3

$e^+e^-\gamma$



Lamb Shift n=3

$e^+e^-\gamma$



Vacuum Polarization n=4

$e^+e^-e^+e^-$

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

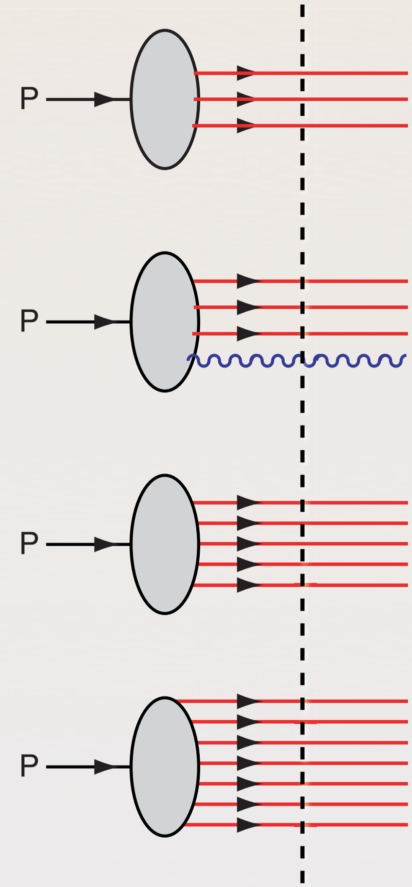
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0+k_i^z}{P^0+P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

$c(x), b(x)$ at high x

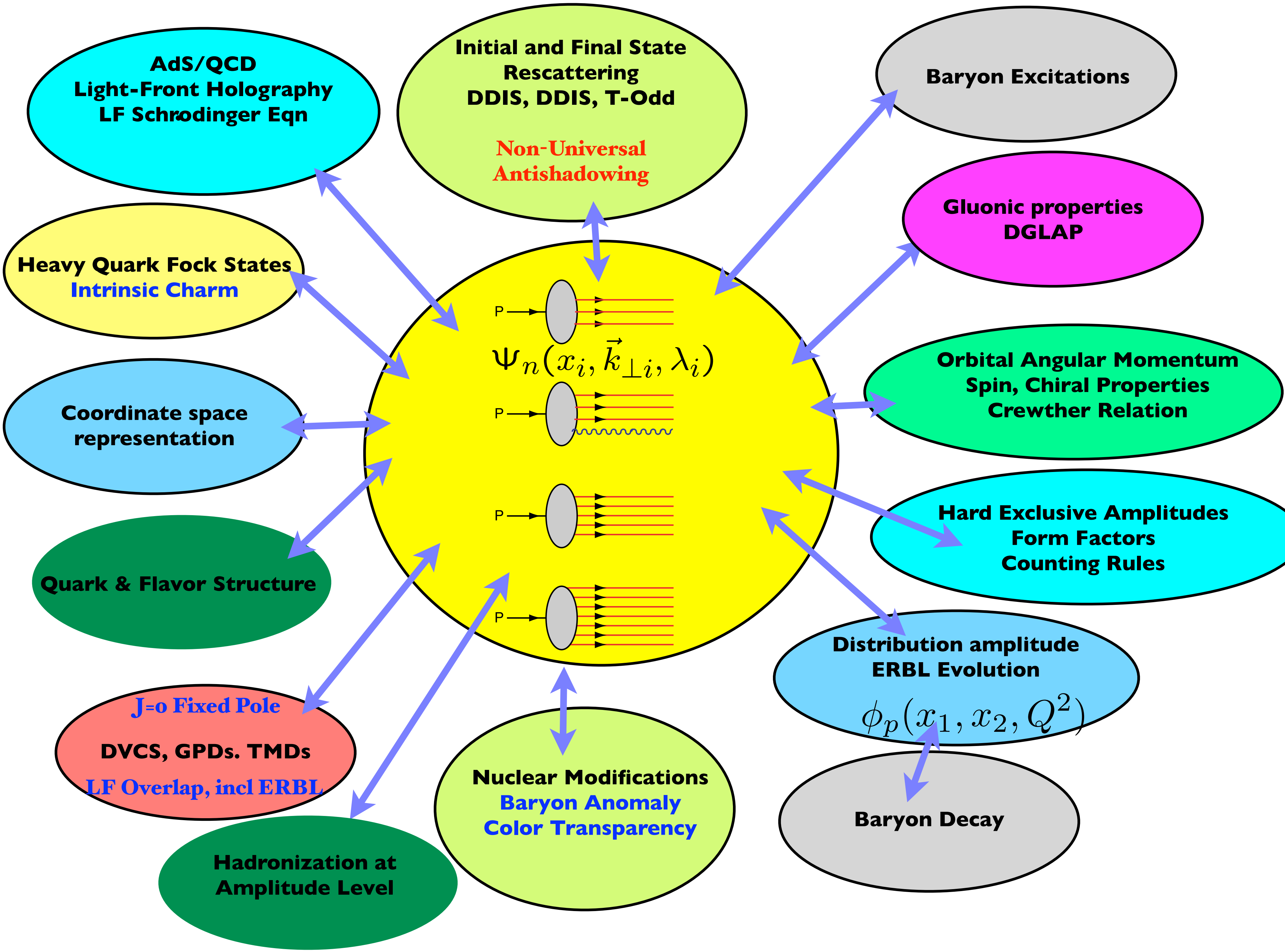
$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Light-Front QCD Features and Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

QCD and the LF Hadron Wavefunctions

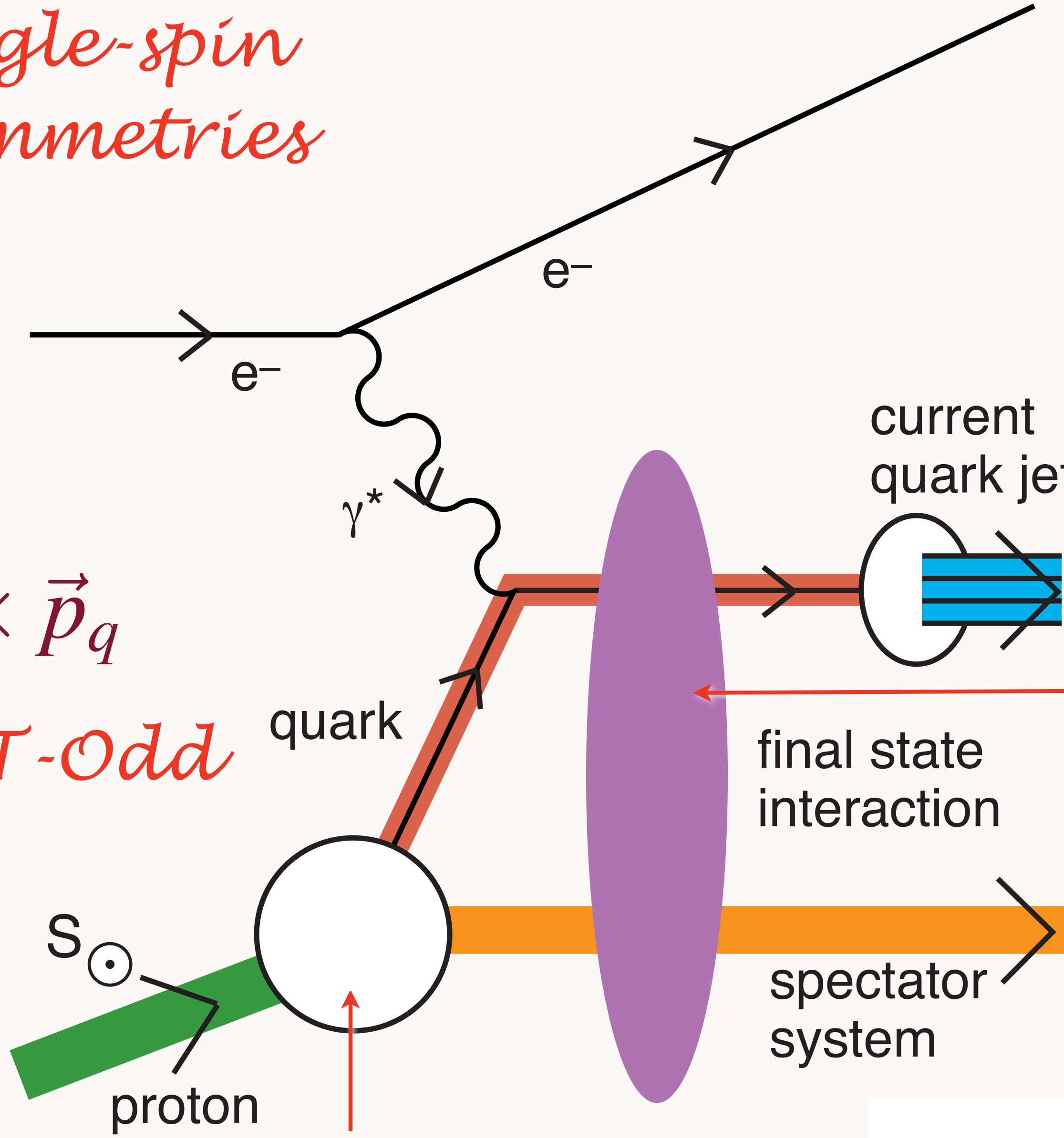


Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan



$$i\,\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd

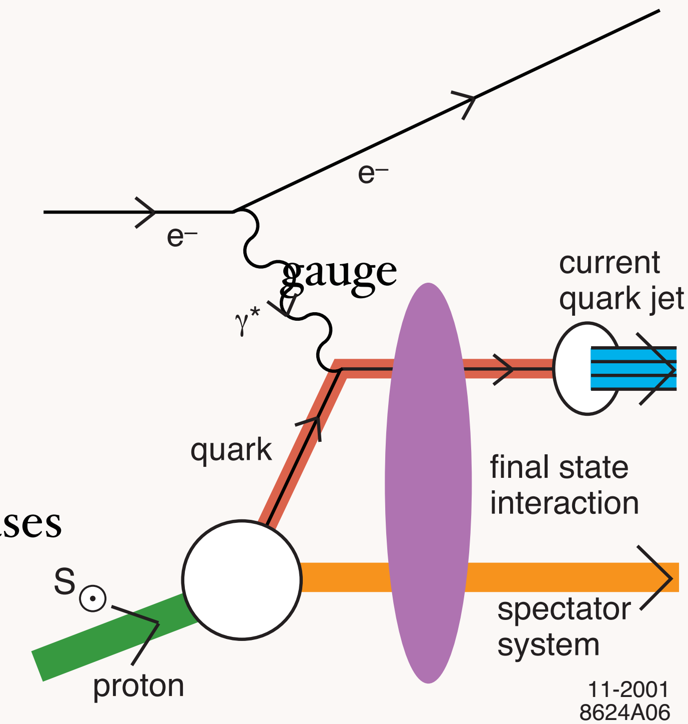
QCD S- and P- Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!



Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t) \right] = F_1(t) \quad \text{Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t) \right] = F_2(t) \quad \text{Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

Verified using LFWFs

Diehl, Hwang, sjb

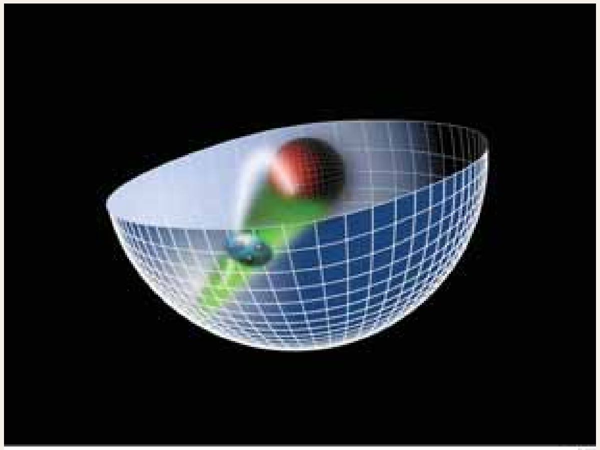
$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Quark angular momentum (Ji's sum rule)

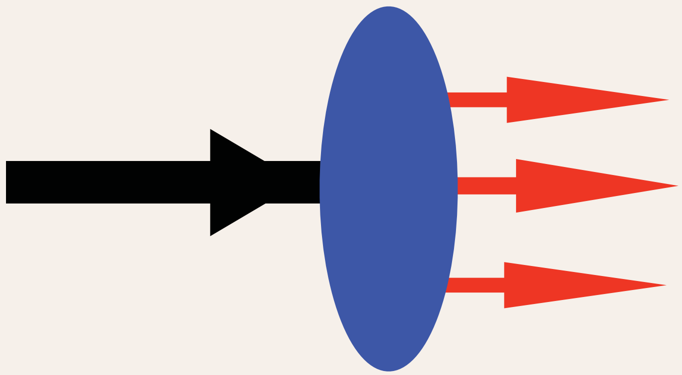
$$J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0) \right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

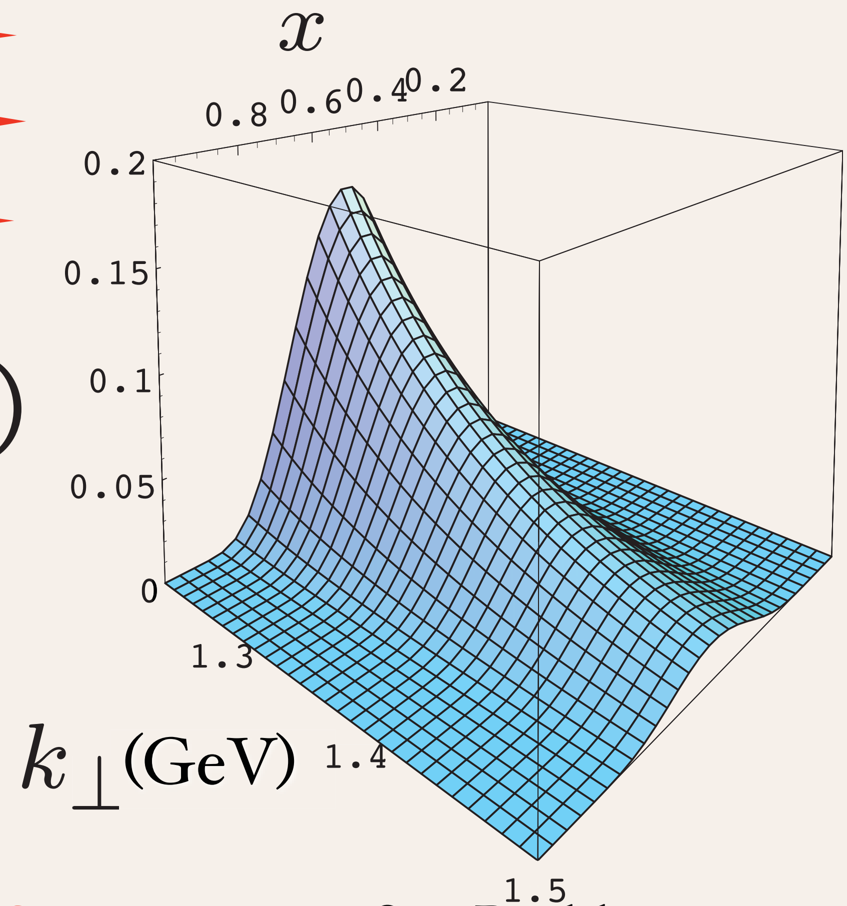
$\phi(z)$



- *Light-Front Holography*



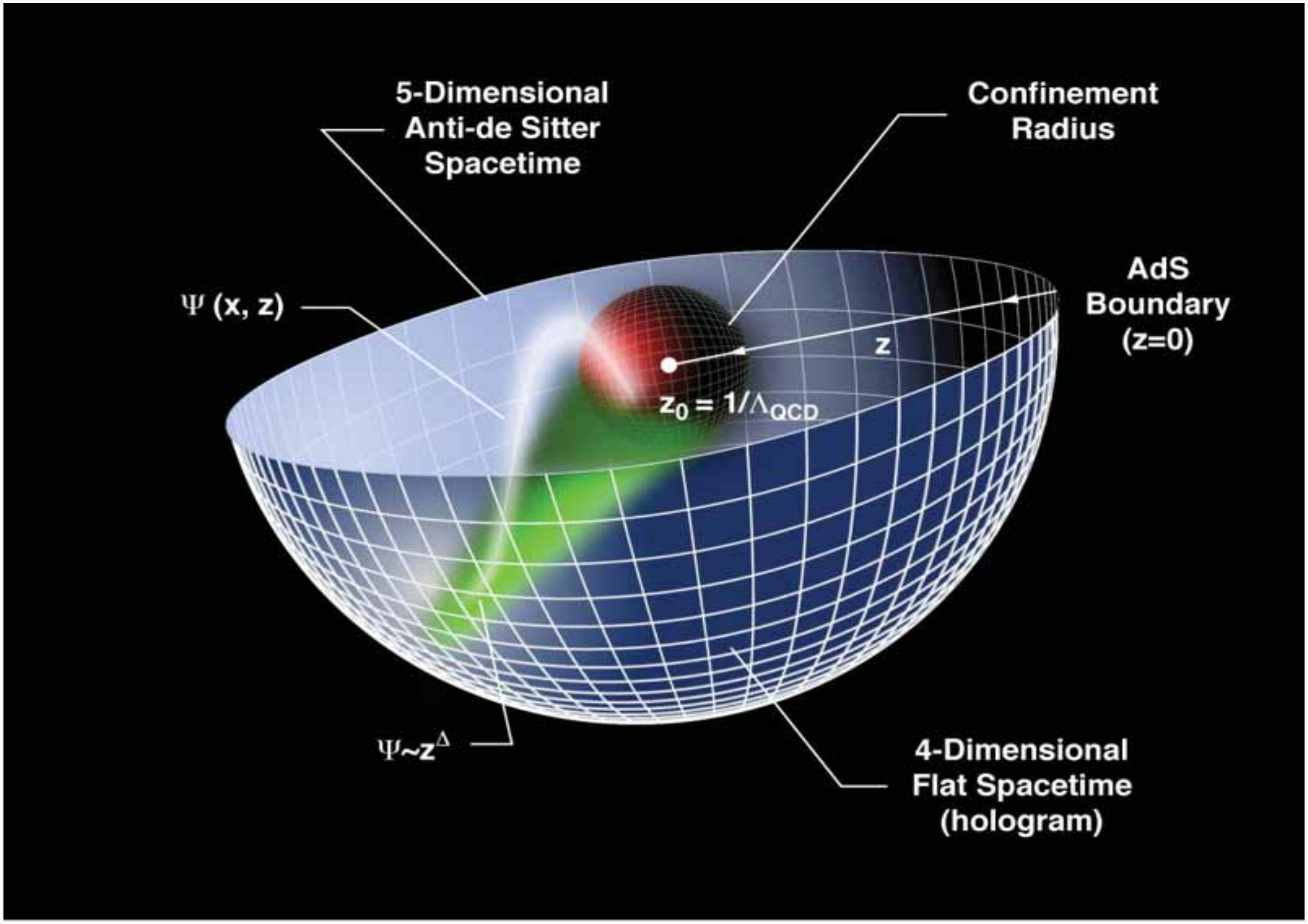
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

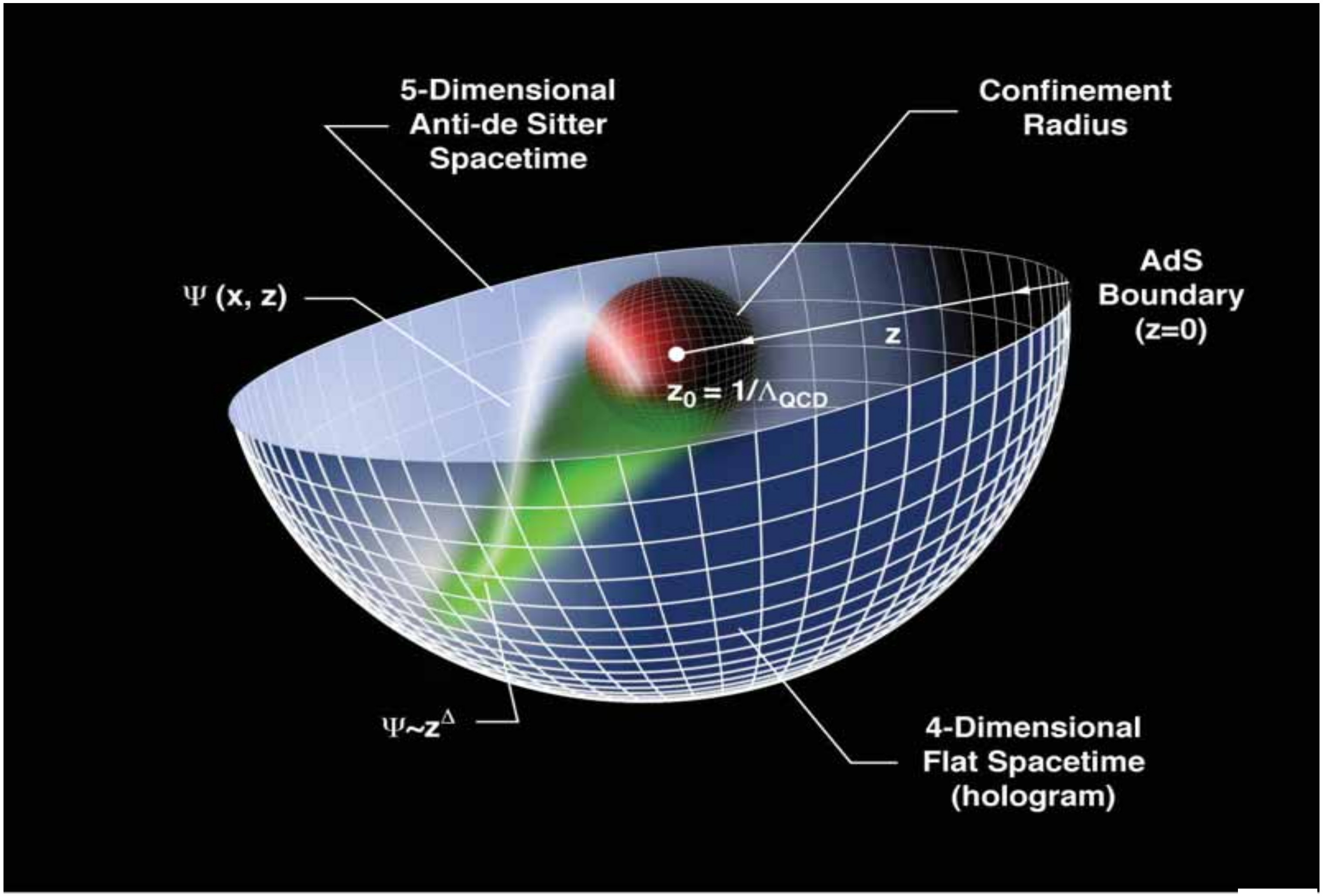
Applications of AdS/CFT to QCD



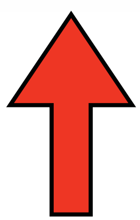
Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

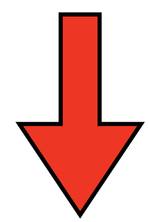
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z



String Theory



Bottom-Up

Top-Down

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

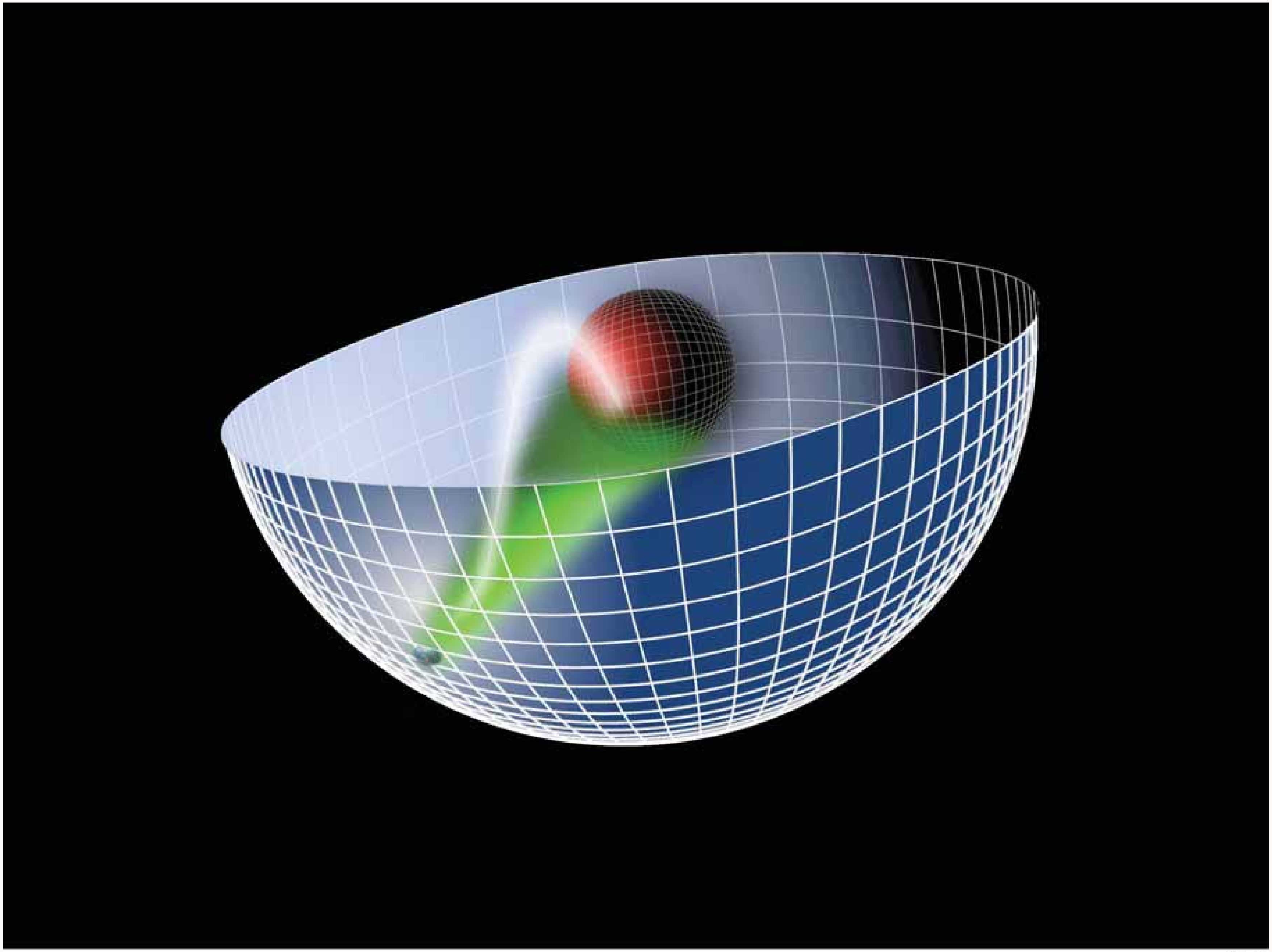
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

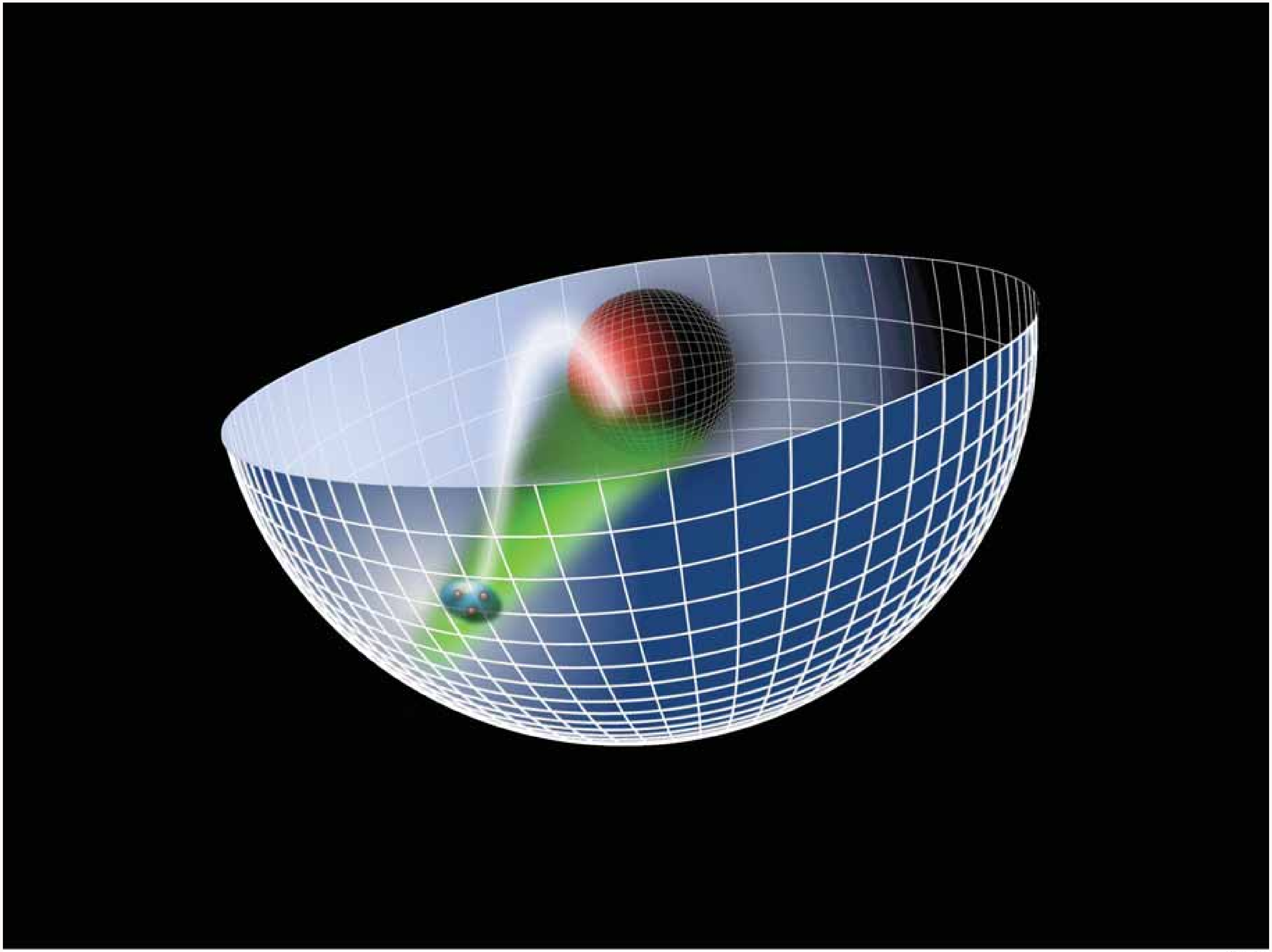
- Different values of z correspond to different scales at which the hadron is examined.

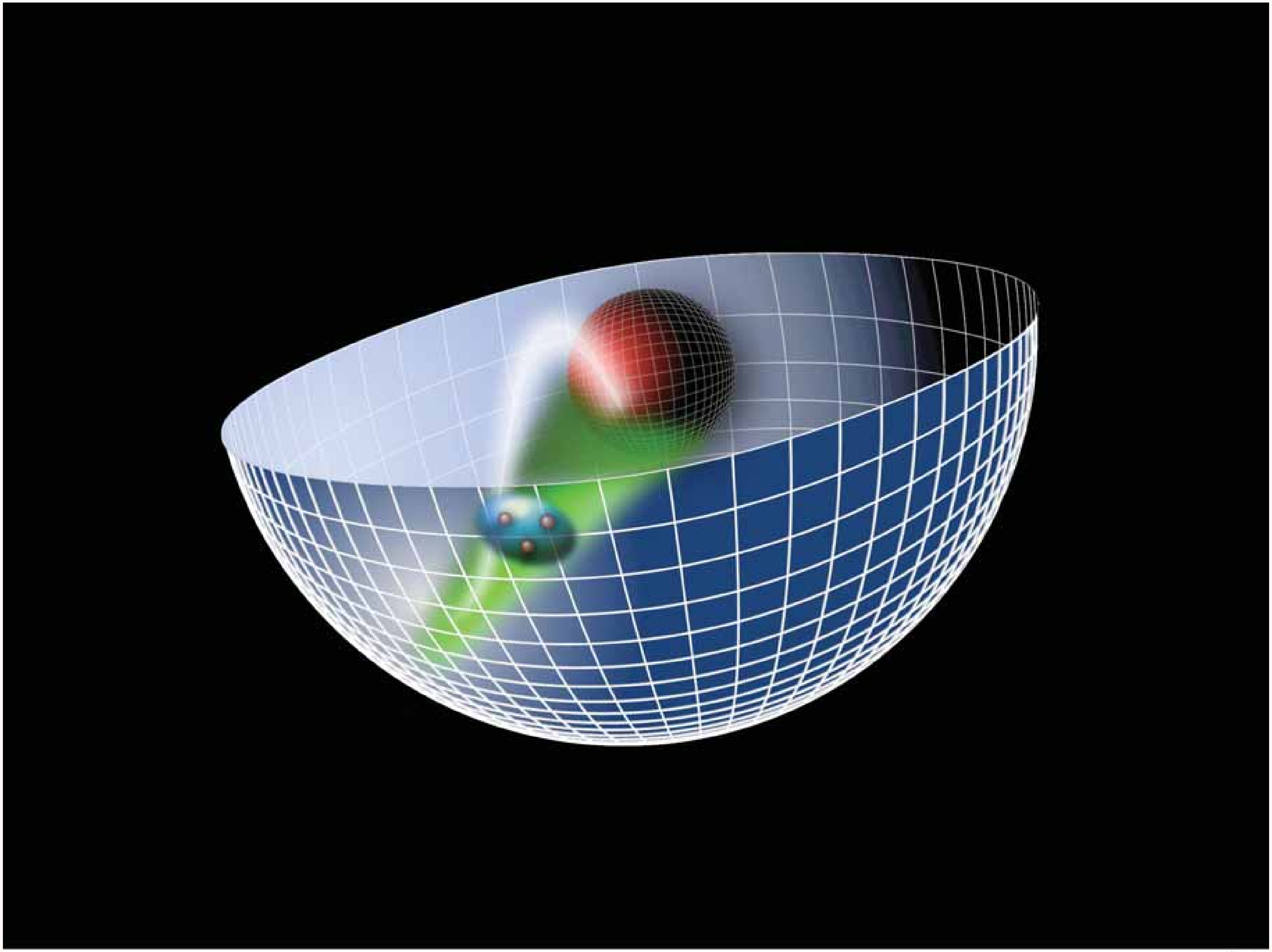
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

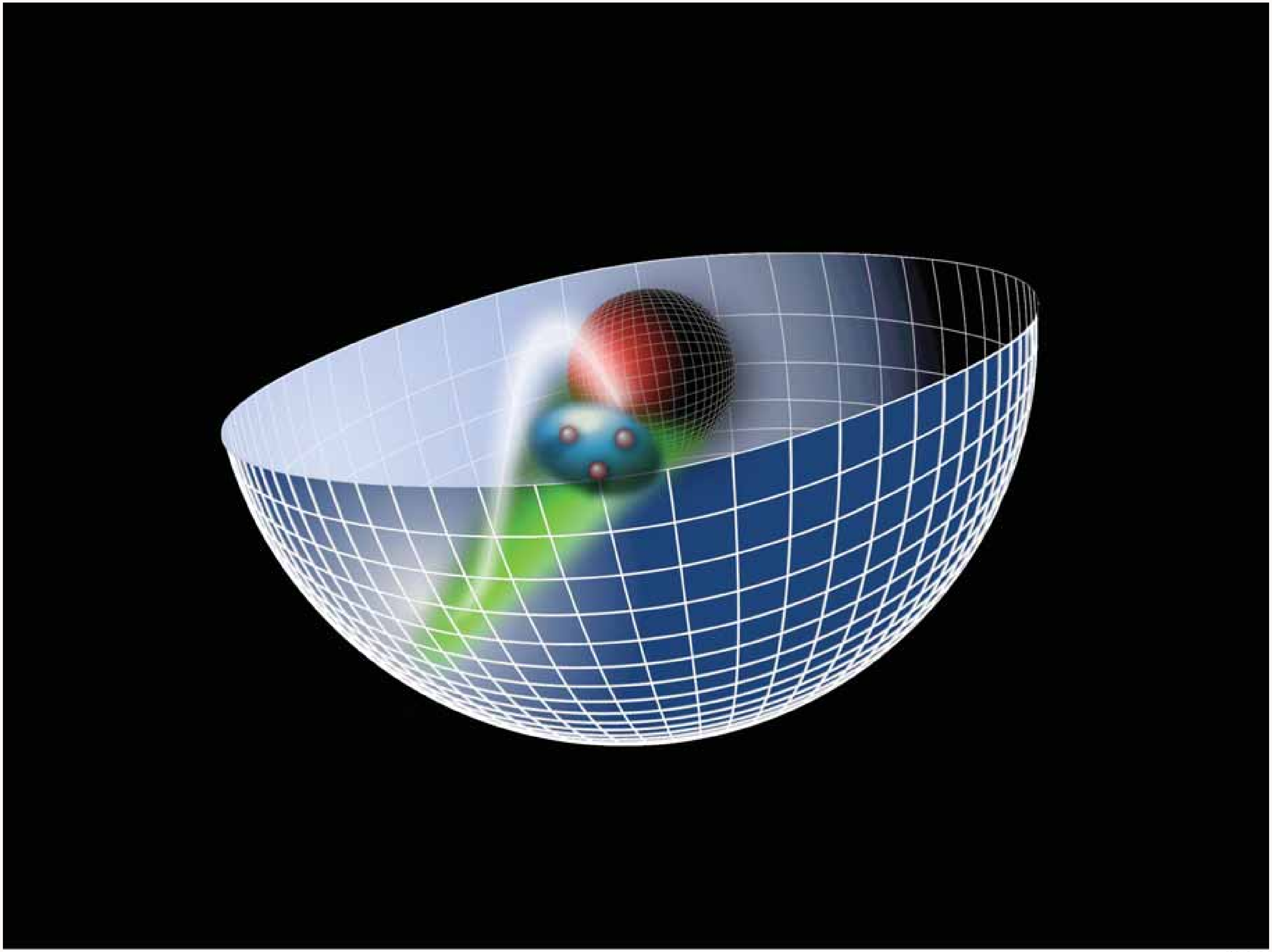
$x^2 = x_\mu x^\mu$: invariant separation between quarks

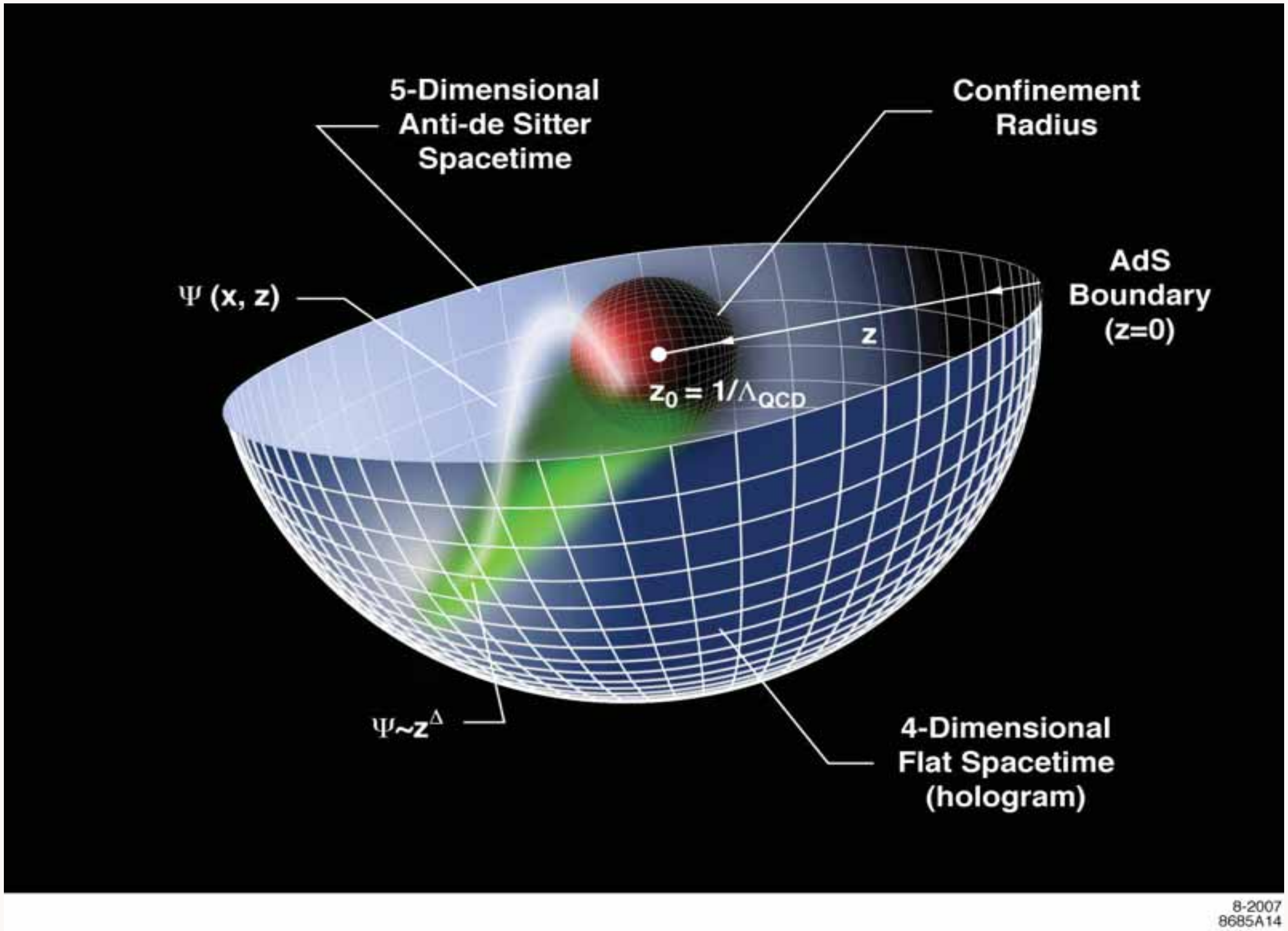
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.











- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

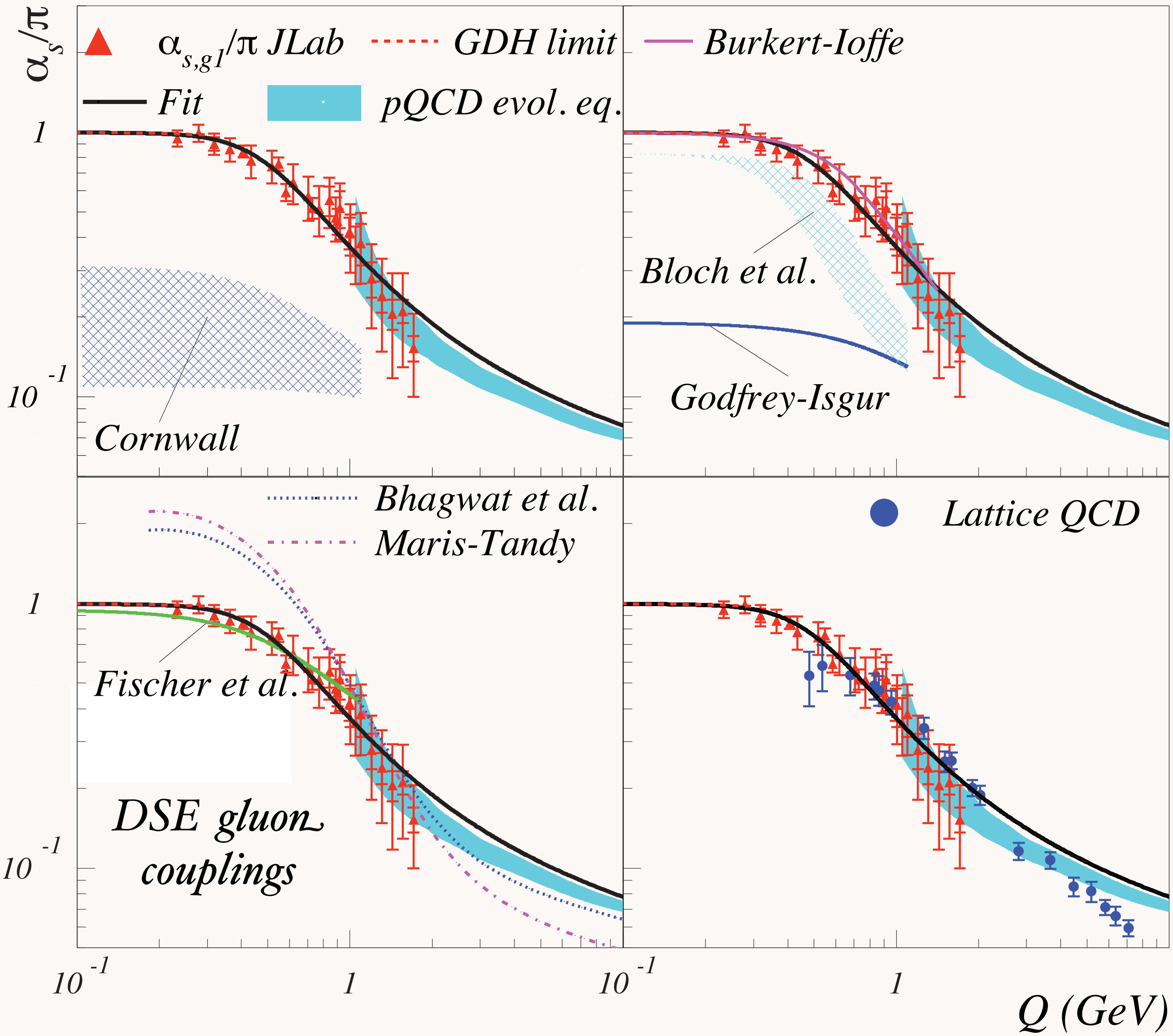
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map AdS_5 X S_5 to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Deur, Korsch, et al.



IR Conformal Window for QCD?

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

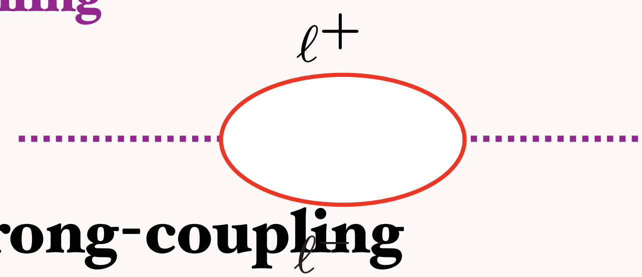
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

Shrock, de Teramond, sjb

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Serber-Uehling

$$\Pi(Q^2) \to \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

L: orbital angular momentum

Derived from variation of Action in AdS_5

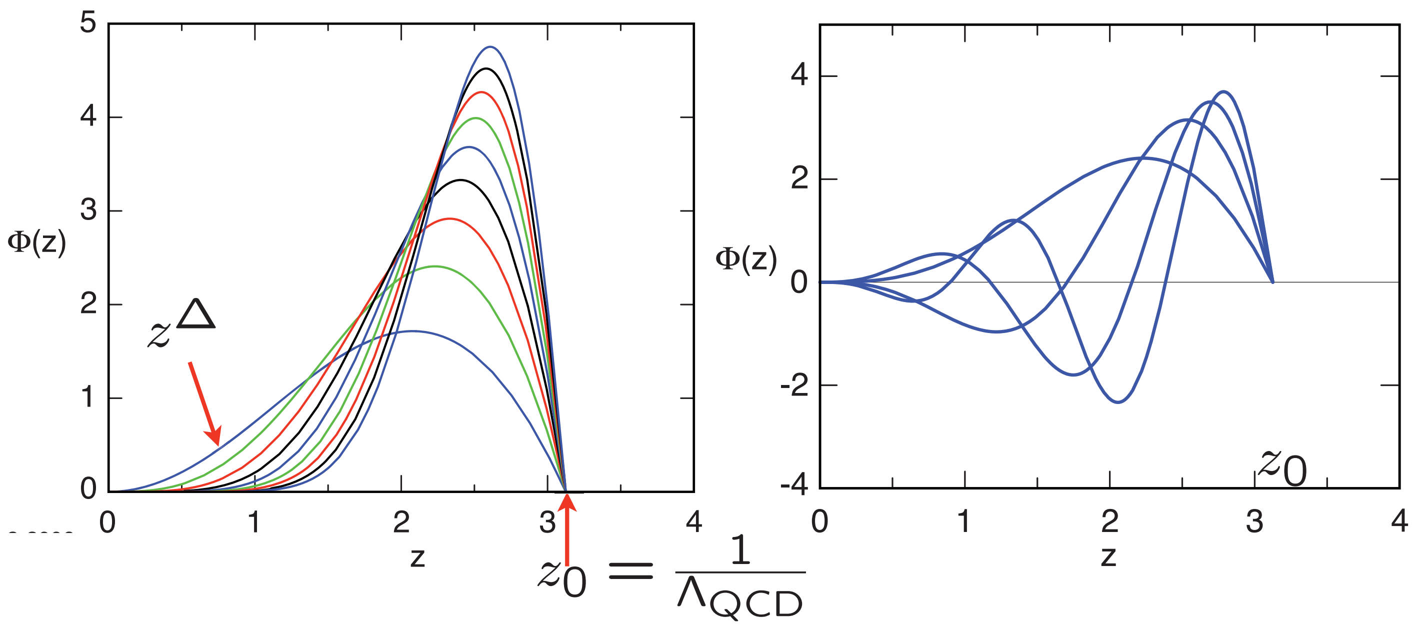
Hard wall model: truncated space

$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_c}) = 0.$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

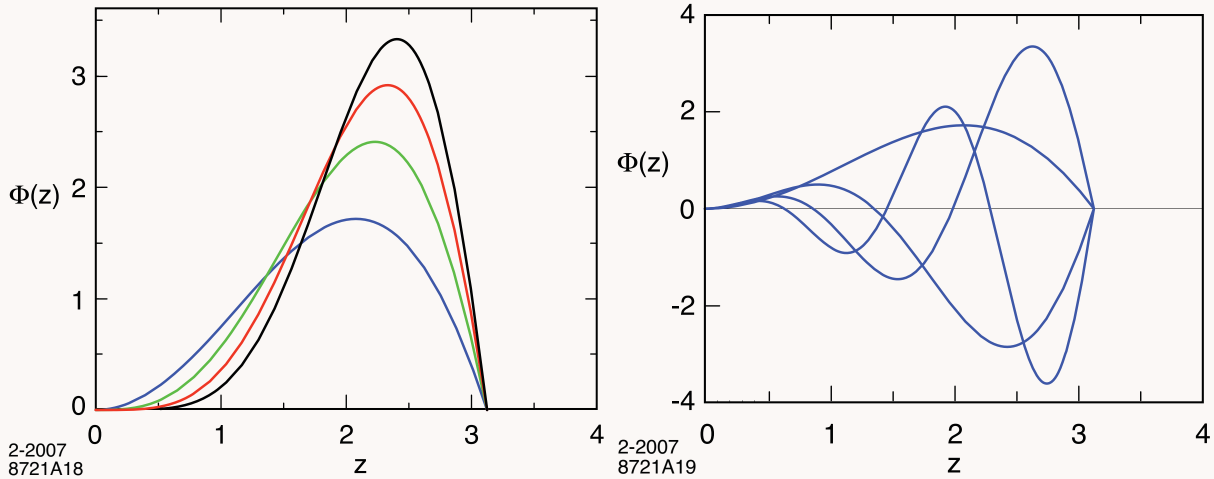


Fig: Orbital and radial AdS modes in the hard wall model for Λ_{QCD} = 0.32 GeV .

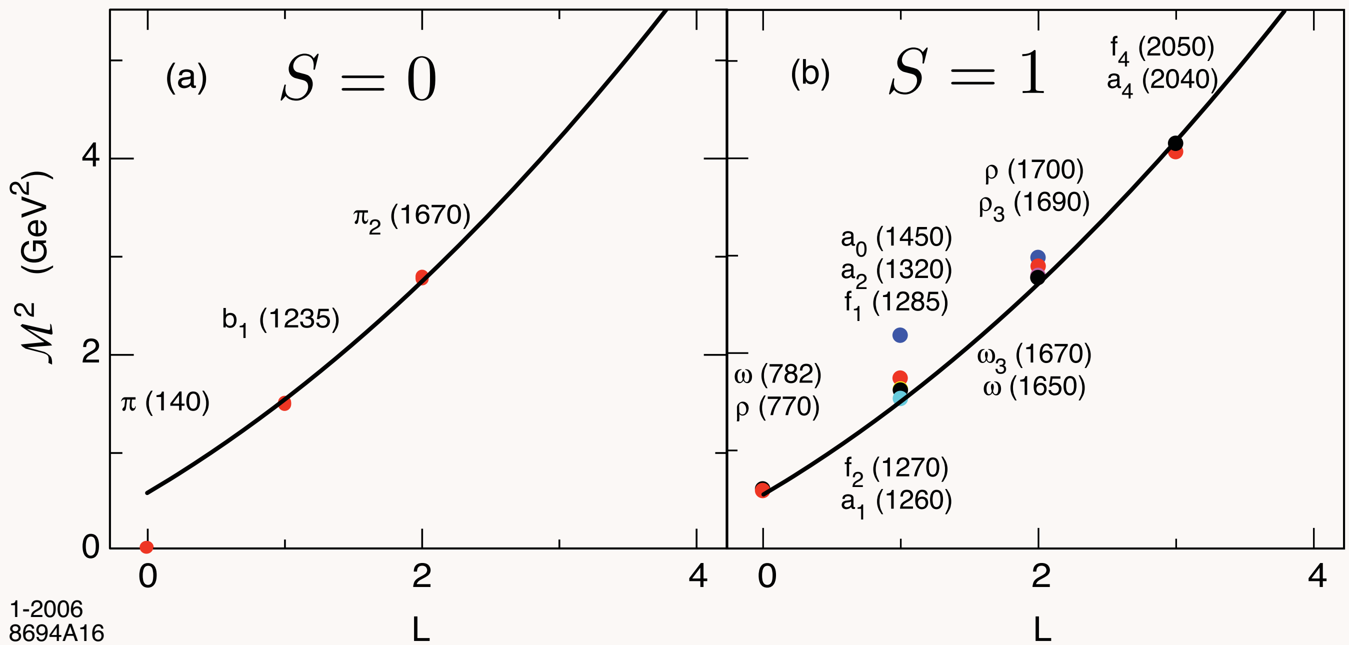


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32\ \mathrm{GeV}$

Higher Spin Bosonic Modes HW

- Each hadronic state of integer spin $S \leq 2$ is dual to a normalizable string mode

$$\Phi(x,z)_{\mu_1\mu_2\cdots\mu_S} = \epsilon_{\mu_1\mu_2\cdots\mu_S}\, e^{-iP\cdot x}\, \Phi_S(z).$$

with four-momentum P_μ and spin polarization indices along the 3+1 physical coordinates.

- Wave equation for spin S-mode W. S. l'Yi, Phys. Lett. B **448**, 218 (1999)

$$\left[z^2\partial_z^2 - (d+1-2S)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_S(z) = 0,$$

- Solution

$$\widetilde{\Phi}(z)_S = \left(\frac{z}{R}\right)^S \Phi(z)_S = C e^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}(z\mathcal{M})\,\epsilon(P)_{\mu_1\mu_2\cdots\mu_S},$$

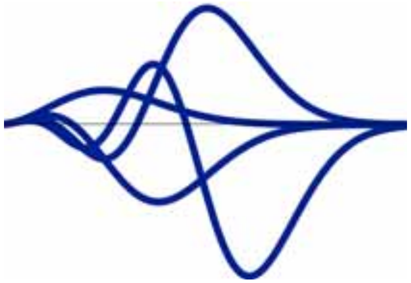
- We can identify the conformal dimension:

$$\Delta = \frac{1}{2}\left(d + \sqrt{(d-2S)^2 + 4\mu^2 R^2}\right).$$

- Normalization:

$$R^{d-2S-1}\int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^{d-2S-1}}\,\Phi_S^2(z) = 1.$$

Soft-Wall Model



- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff wich depends on the profile of a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(d - 1 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$. See also [Metsaev (2002), Andreev (2006)]

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$ a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2}-\frac{1-4L^2}{4z^2}+U(z)\right]\phi(z)=\mathcal{M}^2\phi(z)$$

$$U(z)=\kappa^4 z^2+2\kappa^2(L+S-1)$$

Derived from variation of Action
Dilaton-Modified AdS_5

$$e^{\Phi(z)}=e^{+\kappa^2 z^2}$$

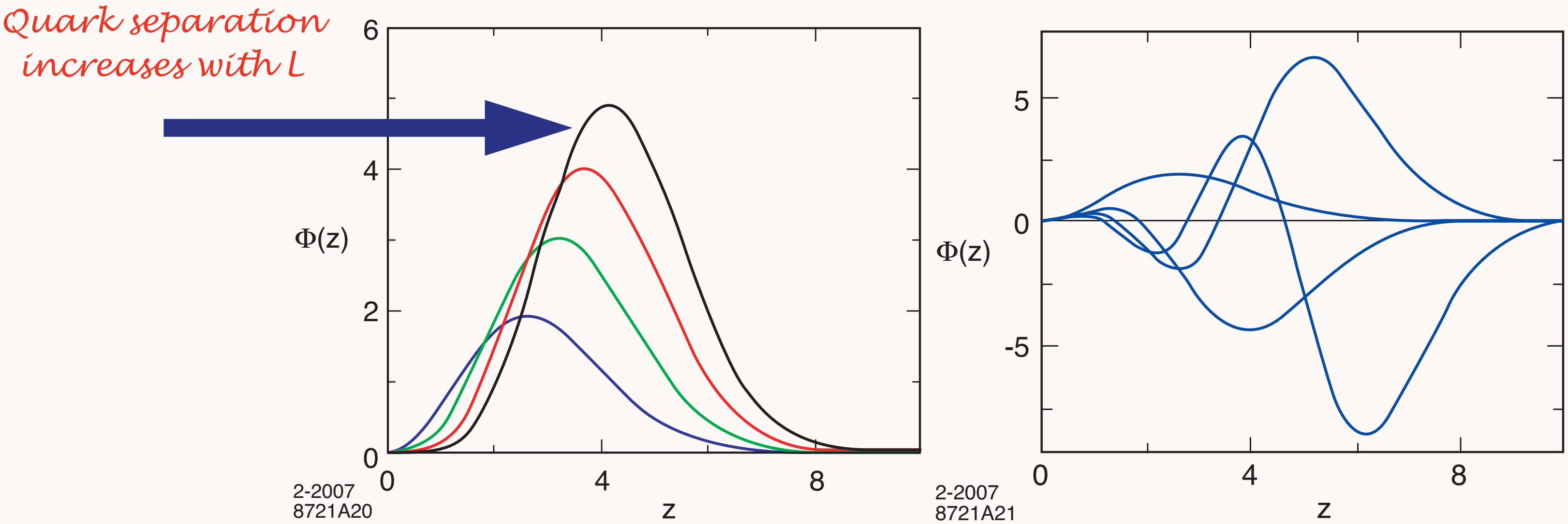
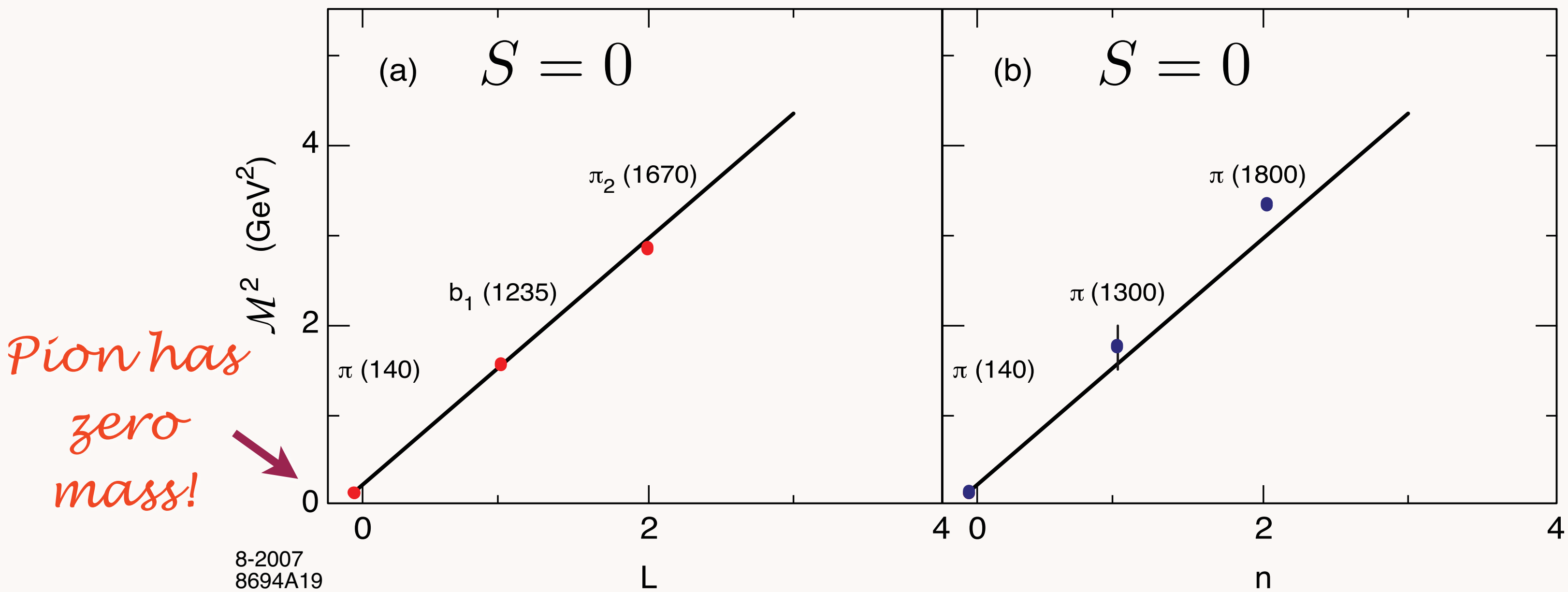


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

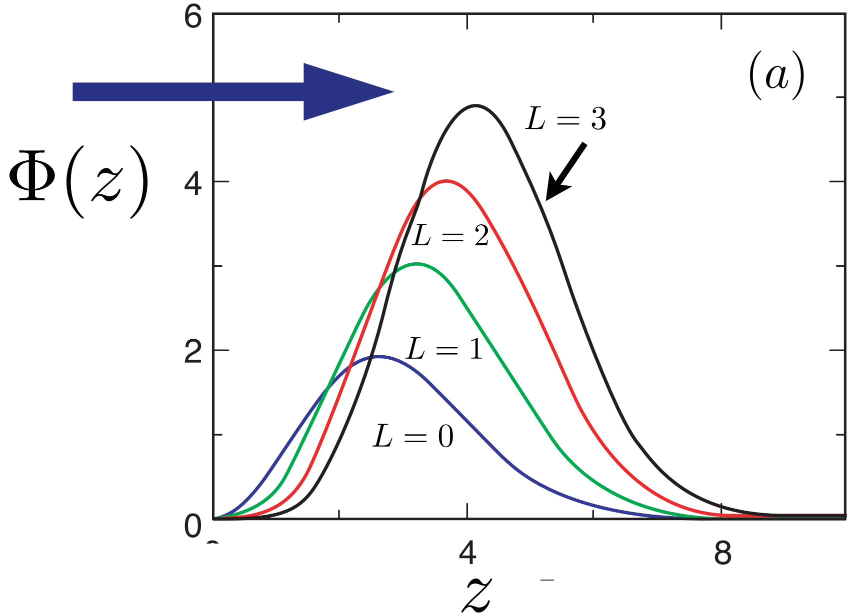


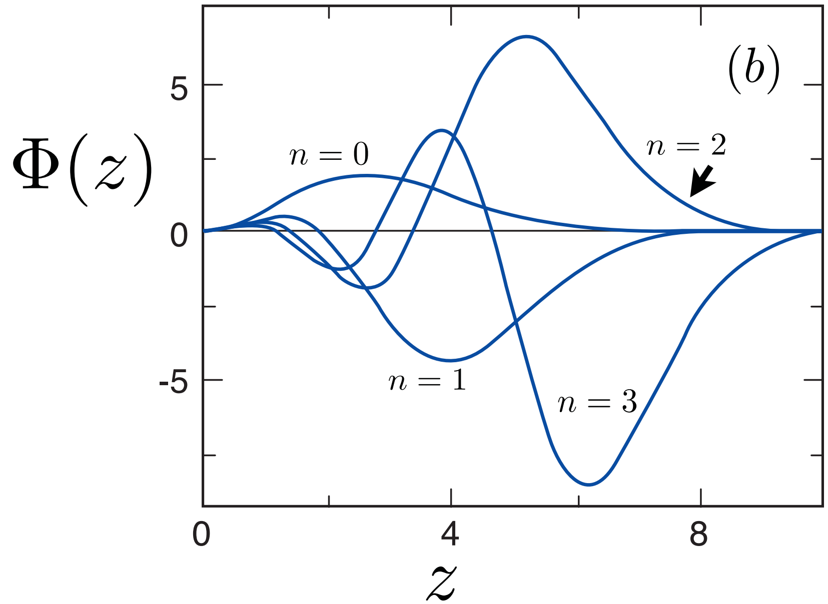
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

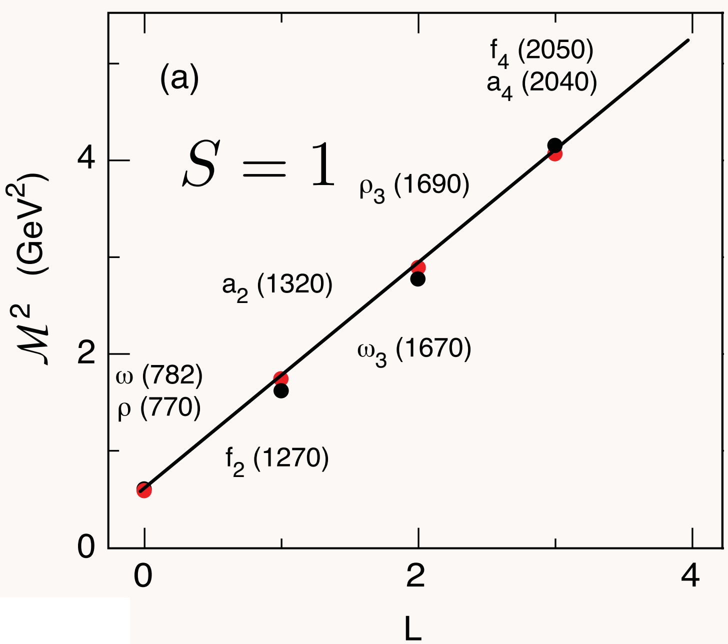
Pion mass automatically zero!

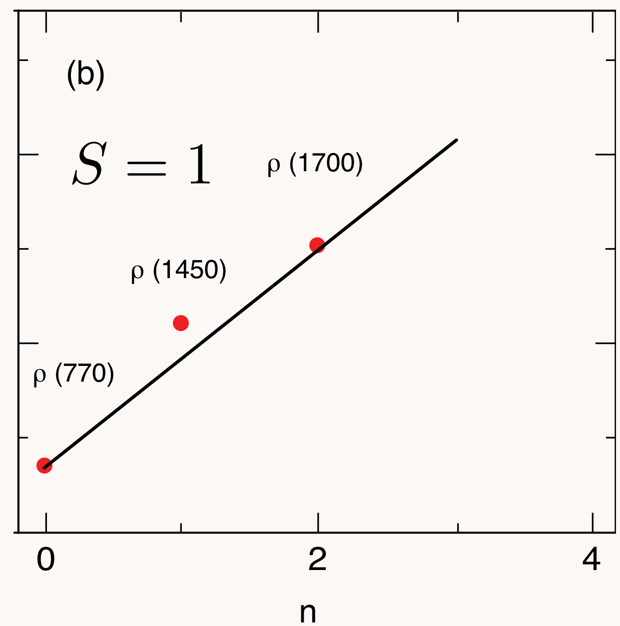
$$m_q = 0$$

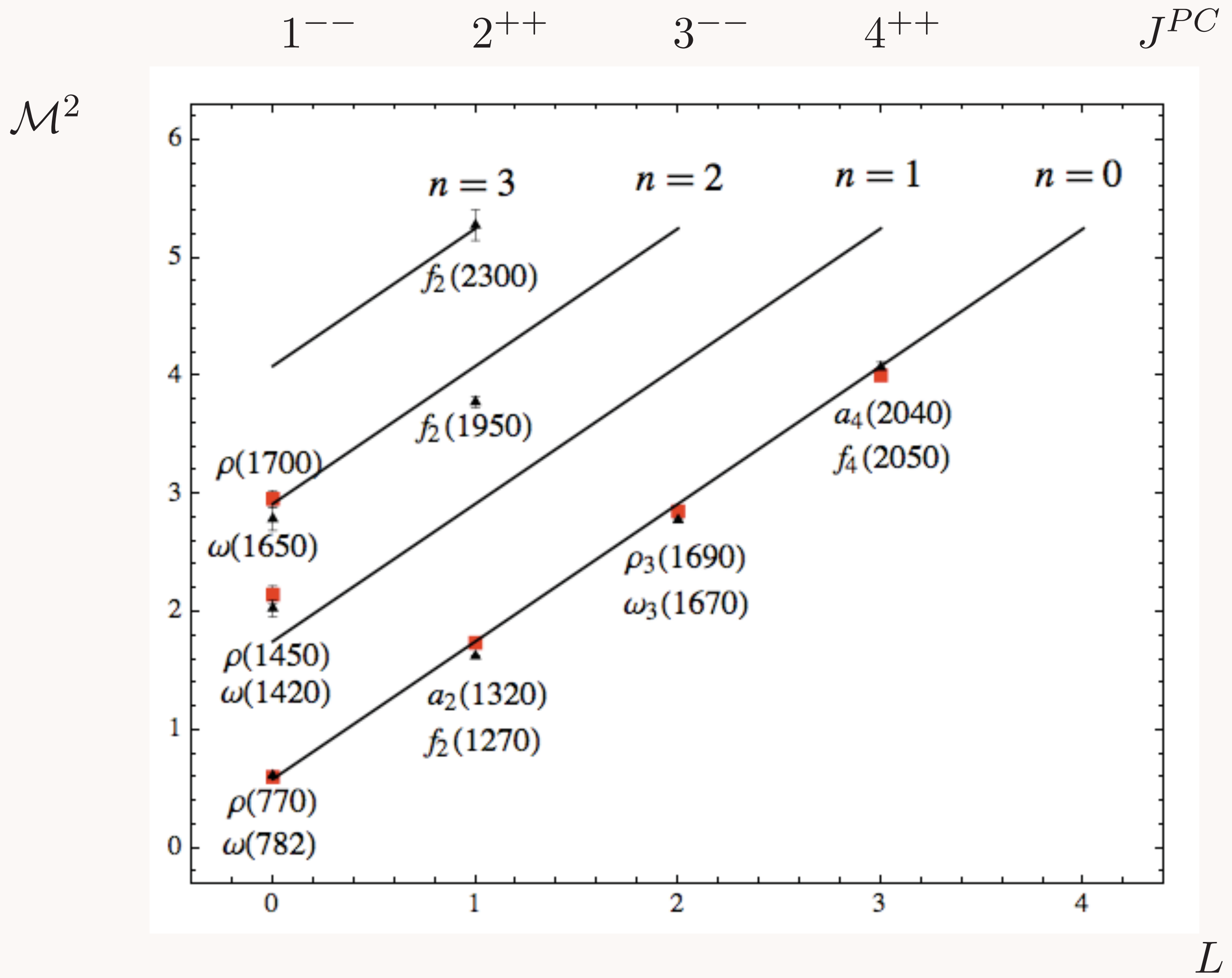
Quark separation increases with L











Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red) and the $I = 0$ ω-meson family (black) for $\kappa = 0.54$ GeV

Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \; x^\ell = (x^\mu, z)$

$$i g_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0.$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z \, \partial_z - z^2 Q^2\right] J(Q,z) = 0,$$

subject to boundary conditions $J(Q=0,z) = J(Q, z=0) = 1.$

- Solution

$$J(Q,z) = zQK_1(zQ).$$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z), \quad \Phi(z) \to z^\Delta, \;\; z \to 0.$$

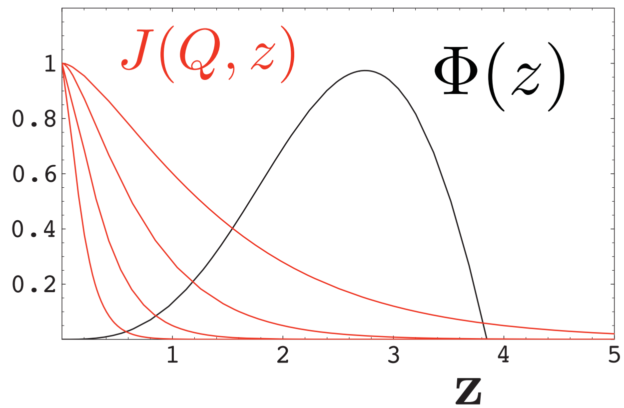
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

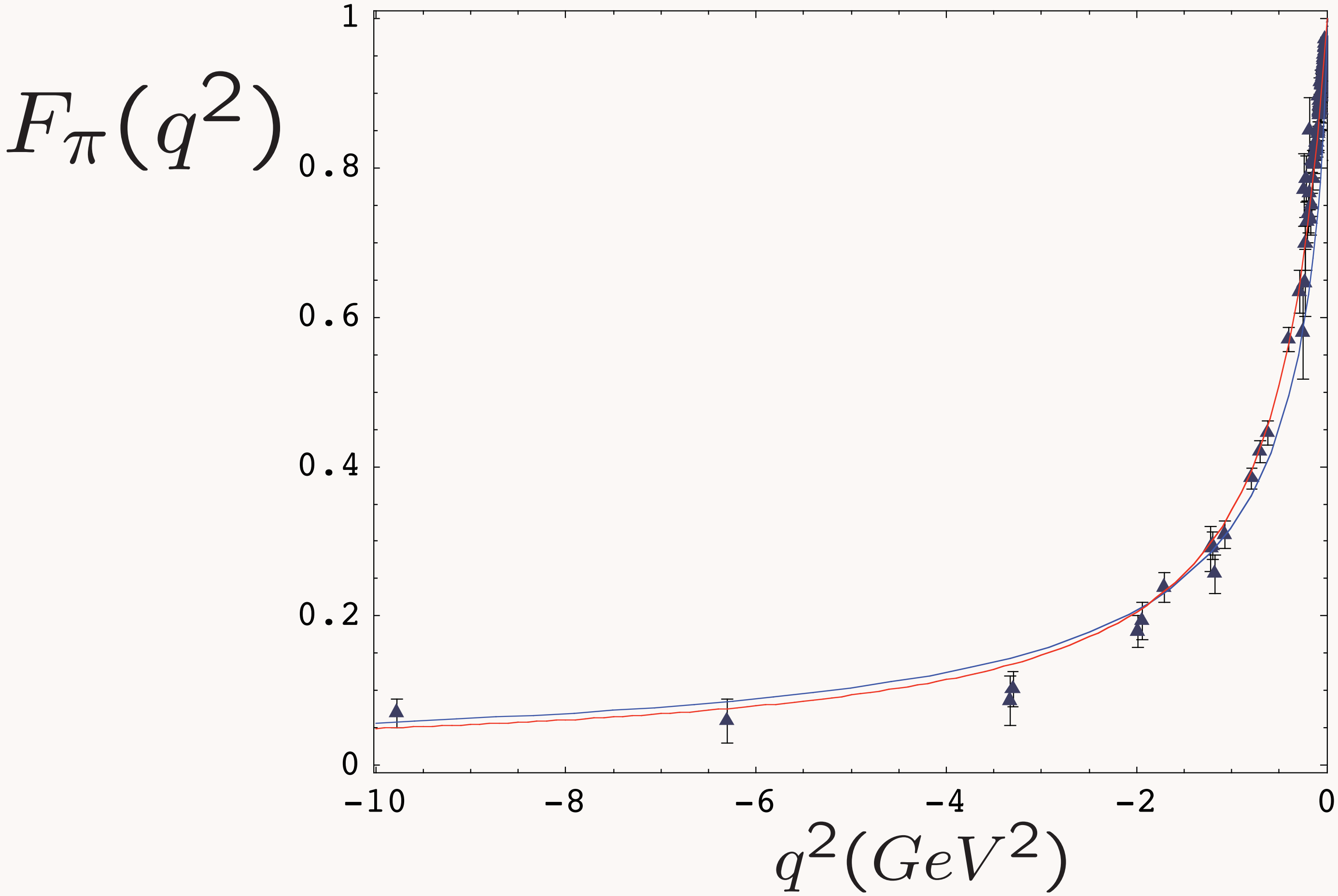
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

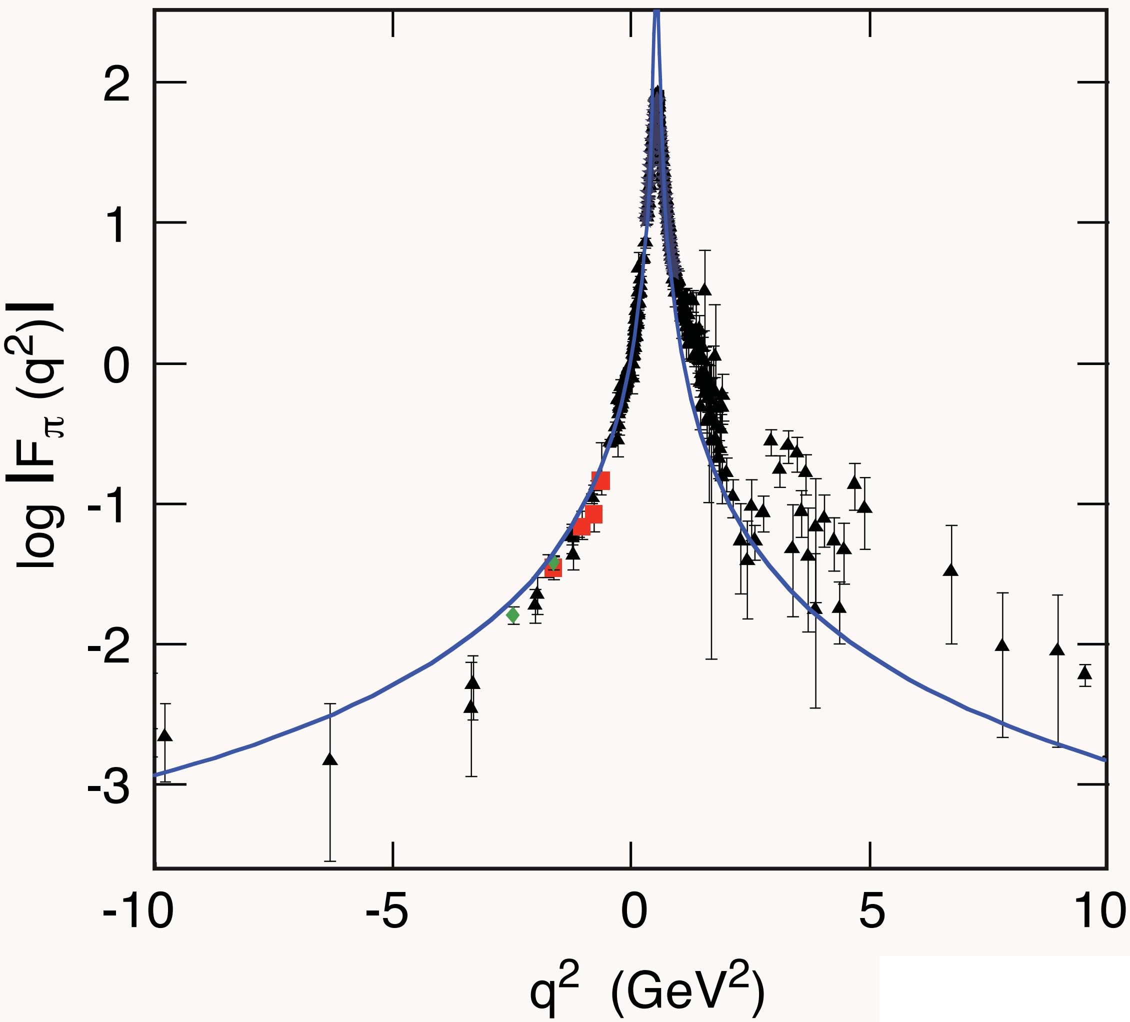
One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

- Analytical continuation to time-like region $q^2 \to -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

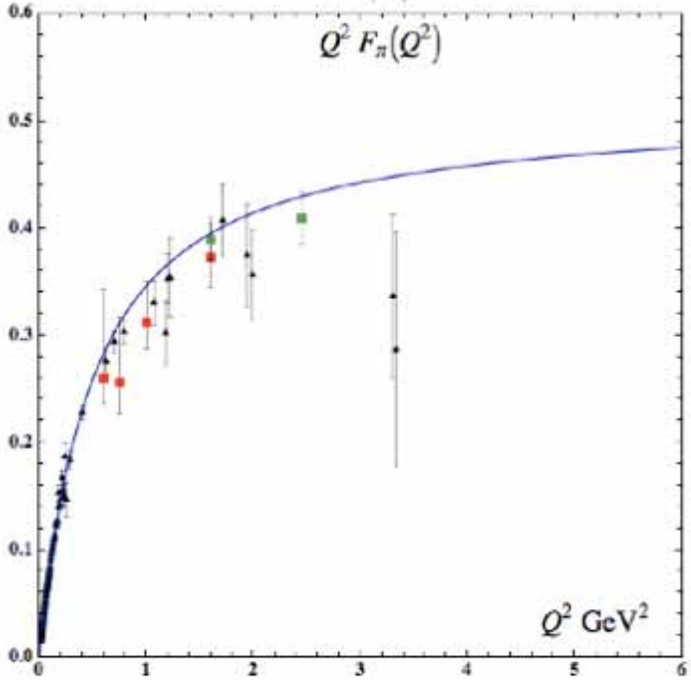
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

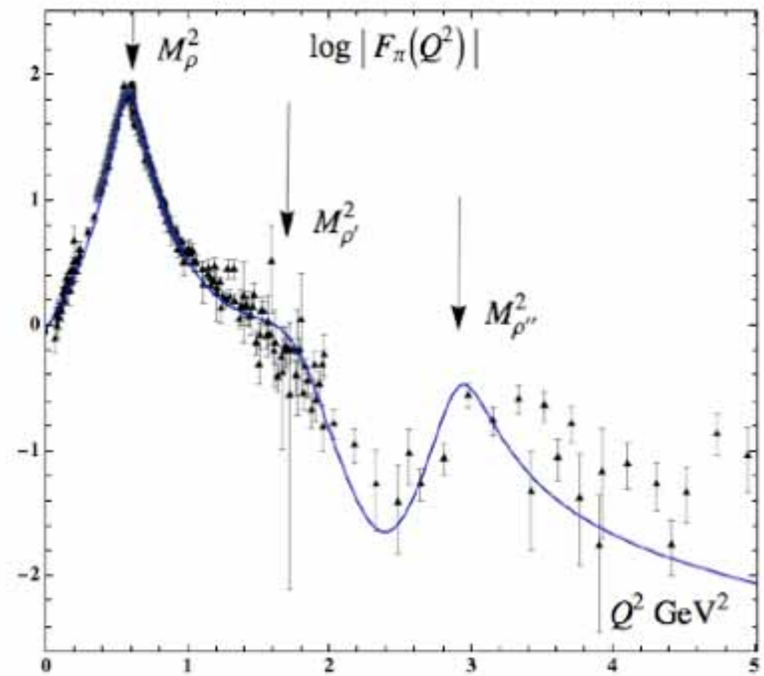


Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Space- and Time Like Pion Form-Factor (HFS)   PRELIMINARY





$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle$$

$$\mathcal{M}^2 \to 4\kappa^2(n+1/2)$$

$\kappa = 0.54$ GeV

$$\Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300 \text{ MeV}$$

$P_{q\bar{q}q\bar{q}} = 13$ %