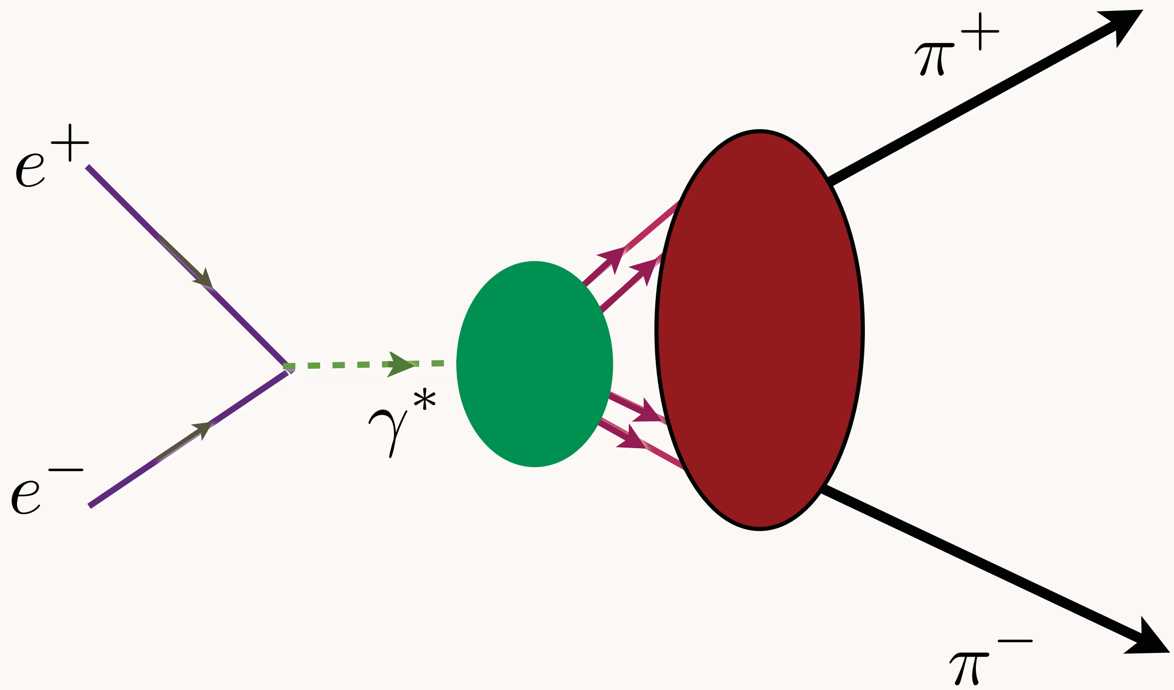
Dressed soft-wall current bring in higher Fock states and more vector meson poles



Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N+z) = (N-1+z)(N-2+z)\ldots(1+z)\Gamma(1+z)$.

- Form factor expressed as $N-1$ product of poles

$$\begin{aligned} F(Q^2) &= \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \\ F(Q^2) &= \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\ &\cdots \\ F(Q^2) &= \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N. \end{aligned}$$

- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \, |\Phi_{\pi^+}(z)|^2 \, ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

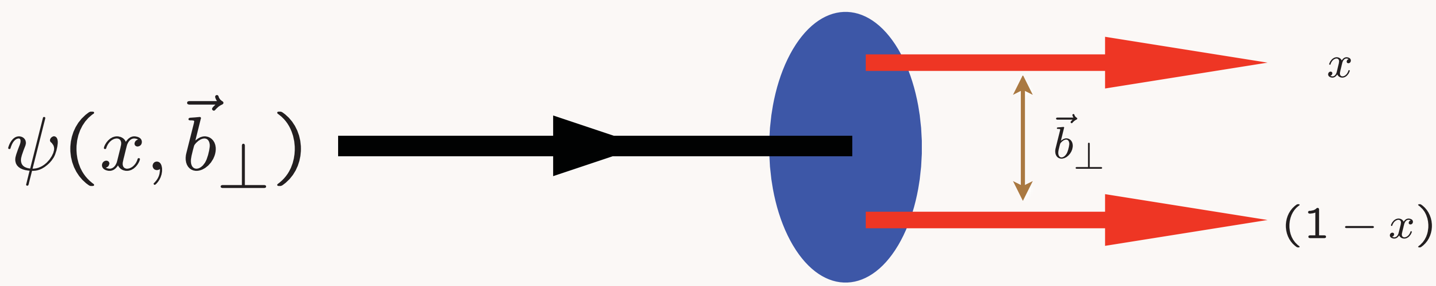
$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) AdS$_5$

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

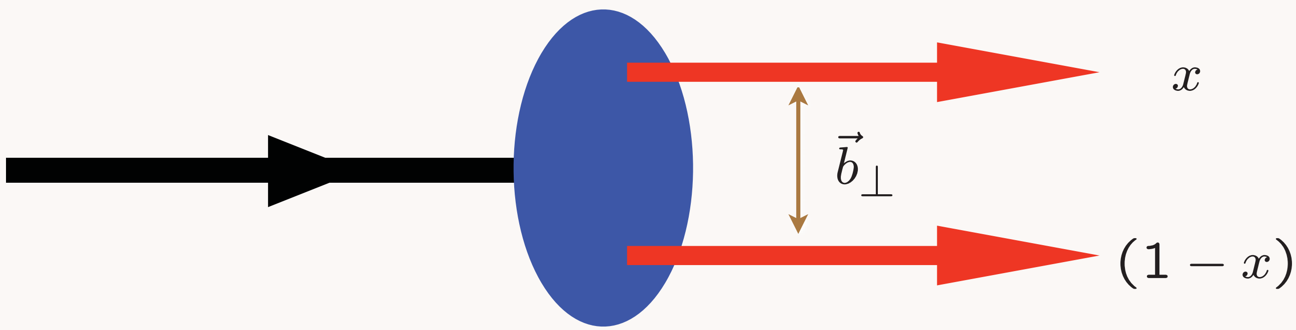
Identical to LF Holography obtained from electromagnetic current

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\begin{aligned}\mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp\, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}\end{aligned}$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp$: $\quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\begin{aligned}\mathcal{M}^2 &= \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)\end{aligned}$$

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \to \lambda x^\mu, \; z \to \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+ dx^- - dx_\perp{}^2$

and

$$\boxed{ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2) \text{ for } x^+ = 0.}$$

Light-Front/AdS_5 Duality

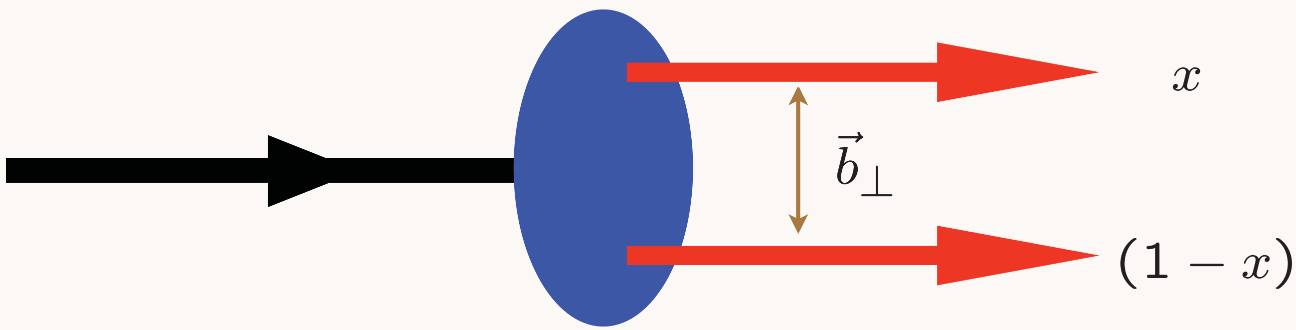
- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

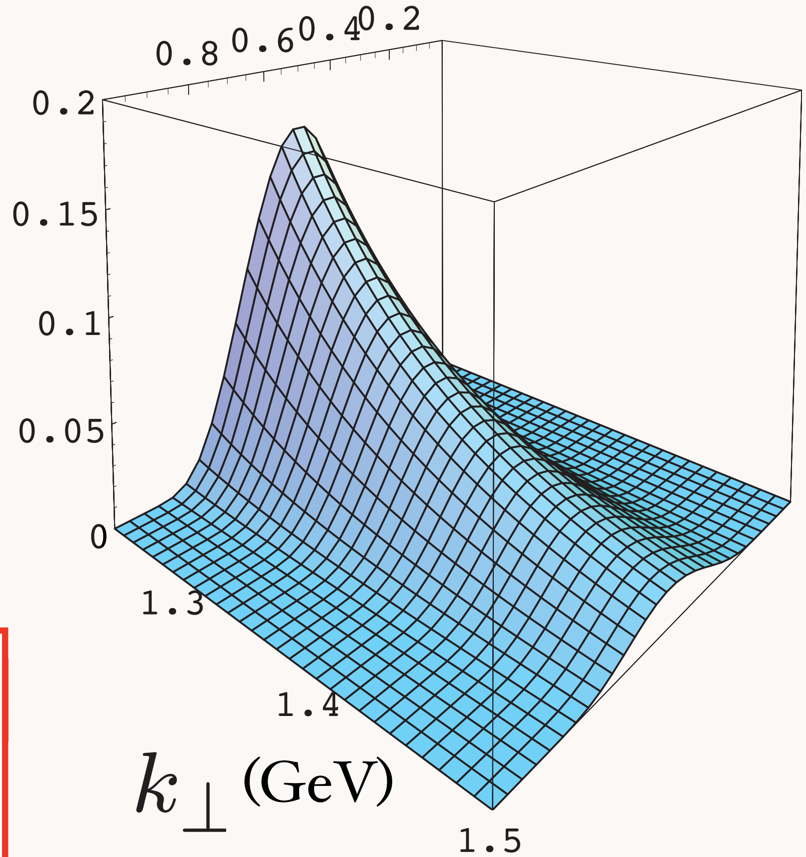
G. de Teramond, sjb

soft wall confining potential:

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling
$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \qquad \phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2 \, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0 \left[b \, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

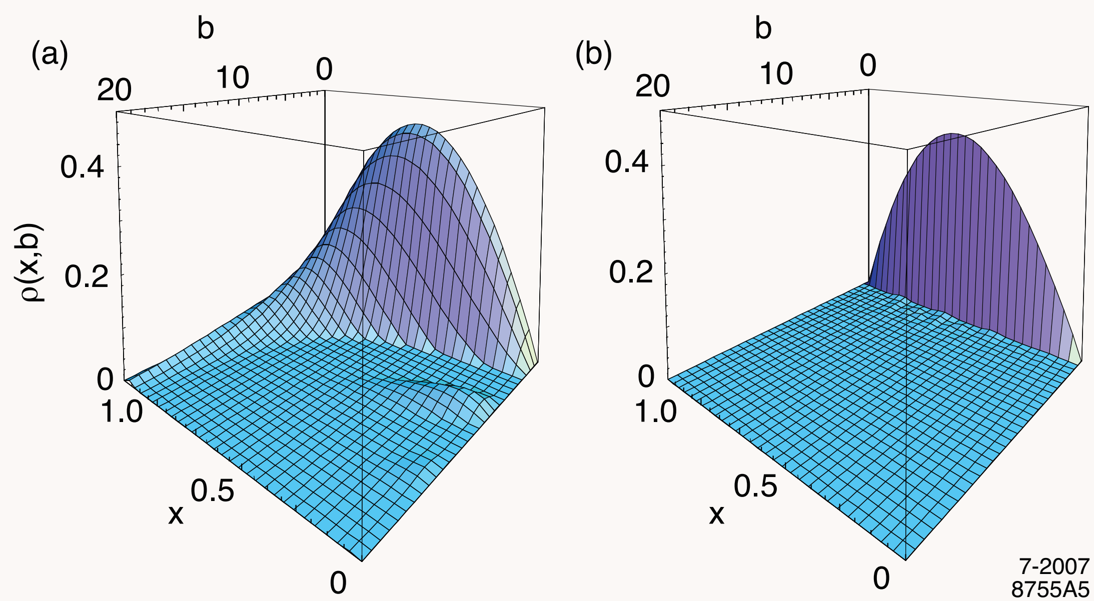


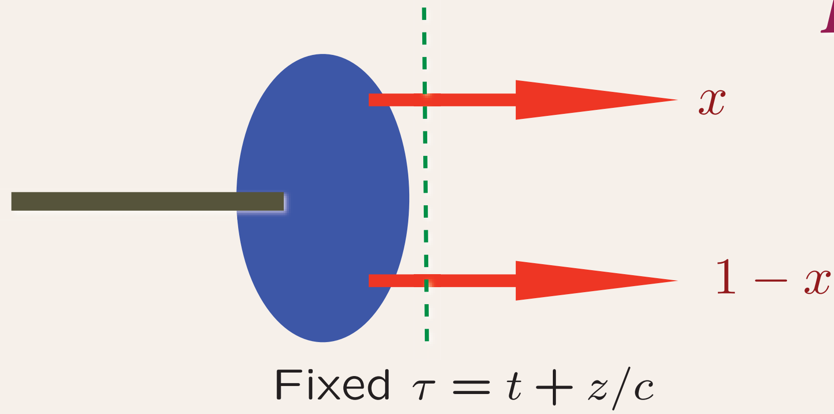
Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$$k_\perp^2 < Q^2$$

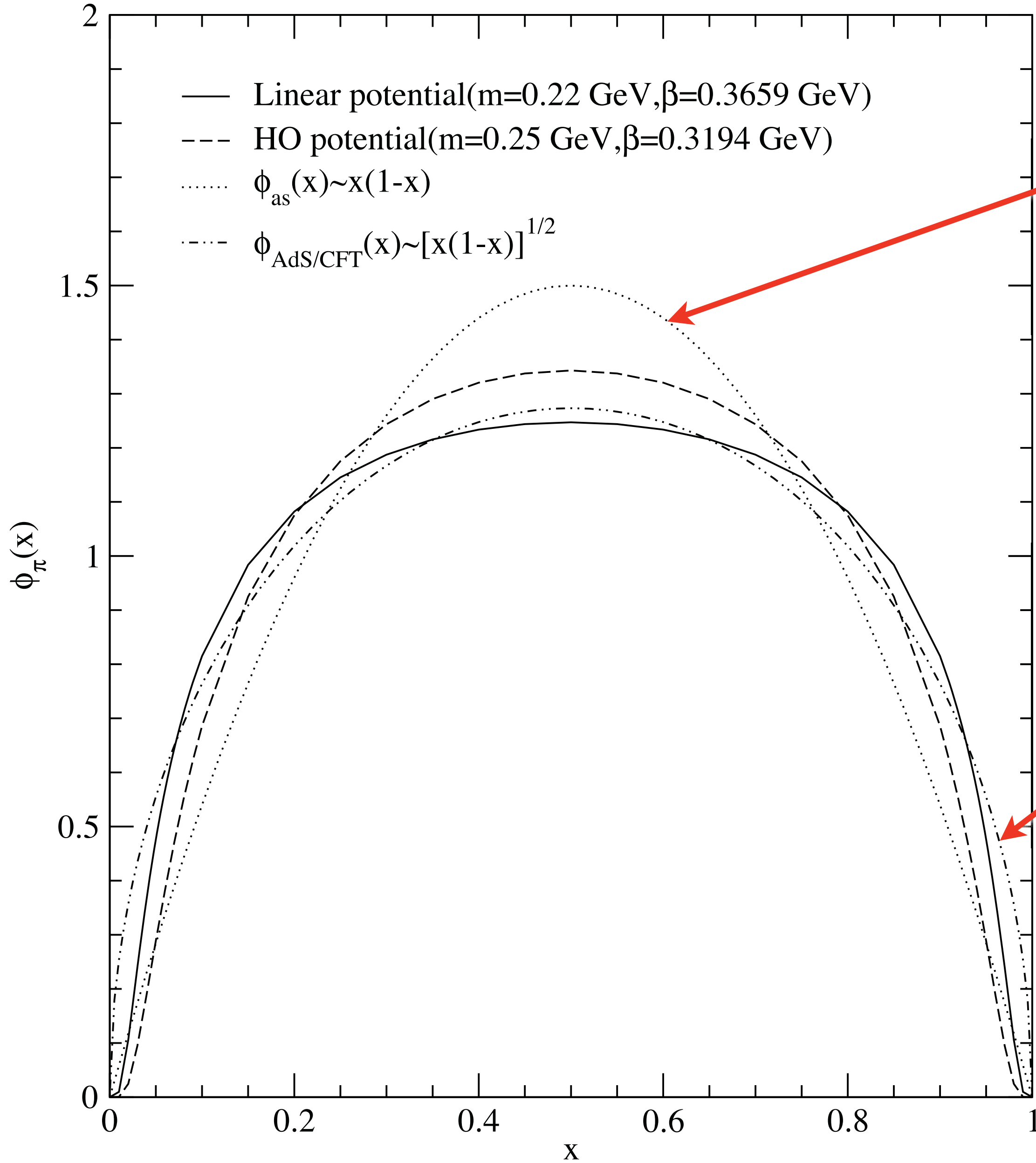
- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$



$$\phi_{asympt} \sim x(1-x)$$

AdS/CFT:

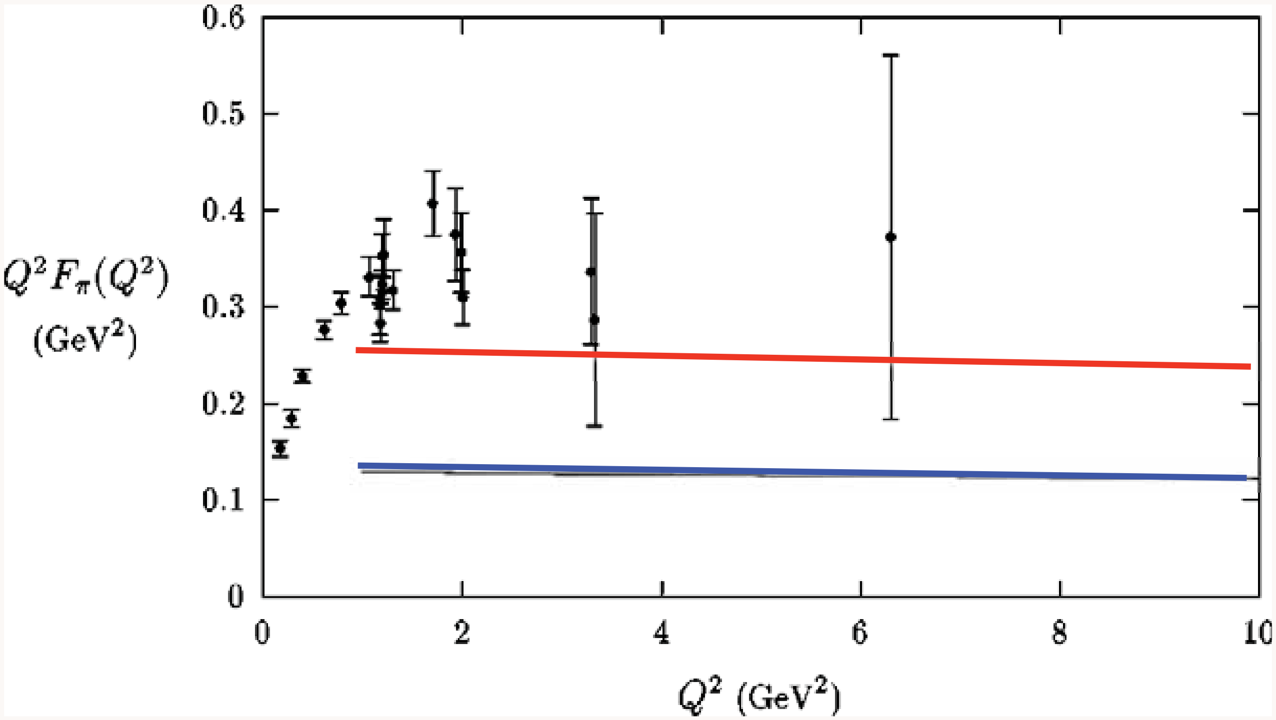
$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD leading twist predictio
$F_\pi(Q^2)$ by factor 16/9

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\, \phi_\pi(x) \int_0^1 dy\, \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \ \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

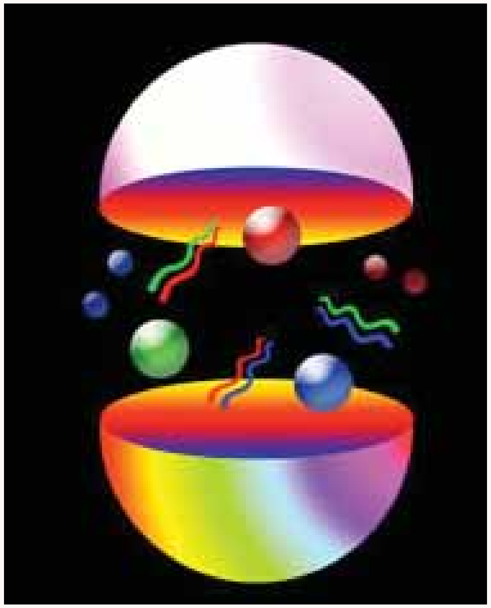
Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

See also T. Sakai and S. Sugimoto

Baryons in Ads/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \,\sqrt{g}\,\overline{\Psi}(x,z)\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right]\Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

  Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

  given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

  with a scale independent mass ratio.

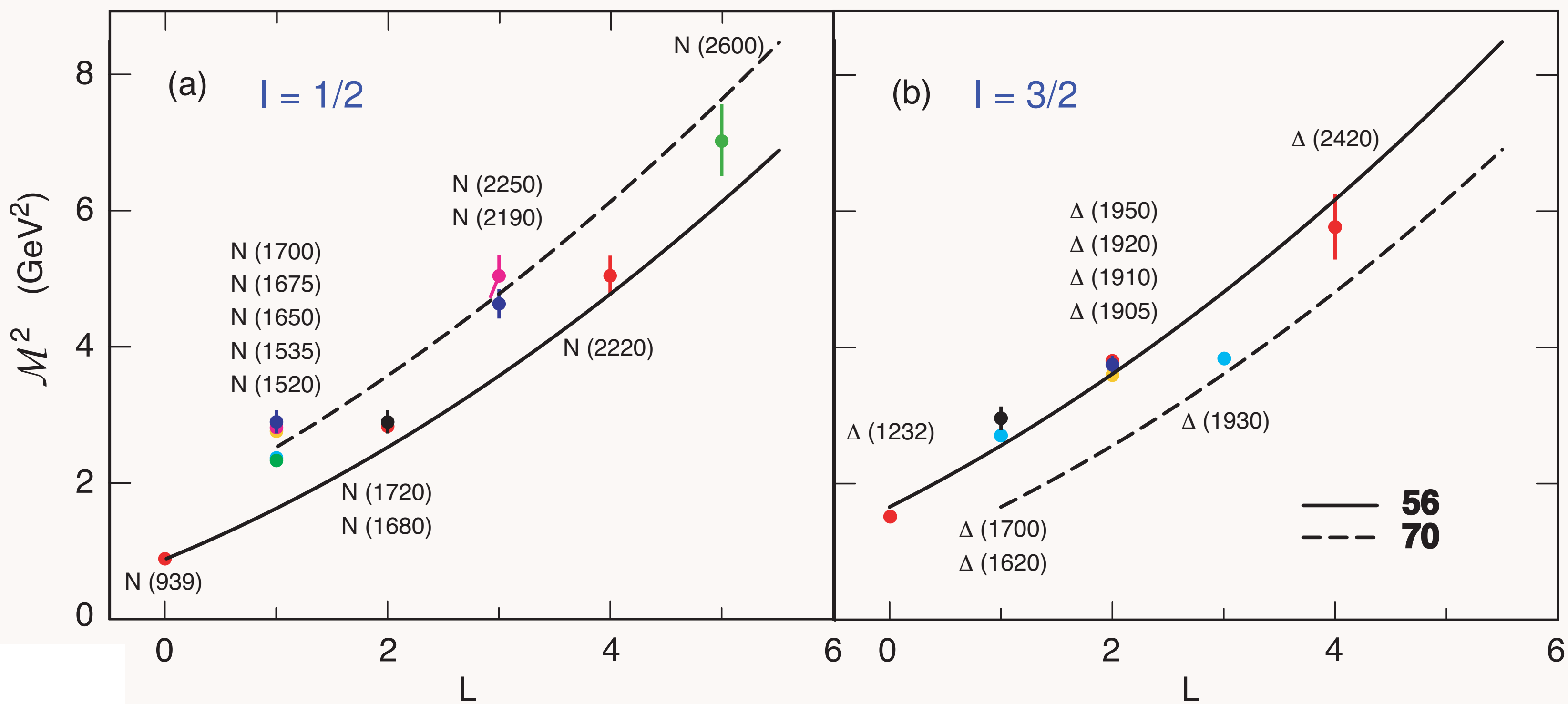


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^{+}(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^{+}(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^{-}(1535)$ $N\frac{3}{2}^{-}(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^{-}(1650)$ $N\frac{3}{2}^{-}(1700)$ $N\frac{5}{2}^{-}(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^{-}(1620)$ $\Delta\frac{3}{2}^{-}(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^{+}(1720)$ $N\frac{5}{2}^{+}(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^{+}(1910)$ $\Delta\frac{3}{2}^{+}(1920)$ $\Delta\frac{5}{2}^{+}(1905)$ $\Delta\frac{7}{2}^{+}(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}$
	$\frac{3}{2}$	3	$N\frac{3}{2}^{-}$ $N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}(2190)$ $N\frac{9}{2}^{-}(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^{-}(1930)$ $\Delta\frac{7}{2}^{-}$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^{+}$ $N\frac{9}{2}^{+}(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^{+}$ $\Delta\frac{7}{2}^{+}$ $\Delta\frac{9}{2}^{+}$ $\Delta\frac{11}{2}^{+}(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^{-}$ $N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}$ $N\frac{13}{2}^{-}$

Linear Holographic Confinement

- Compare with usual Dirac equation in AdS space in presence of a potential $V(z)$ $\quad\left(x^\ell = (x^\mu, z)\right)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + V(z)\right]\Psi(x^\ell) = 0.$$

- We consider the linear confining potential $\quad V(z) = \kappa^2 z.$

- Upon substitution $\Psi(x,z) = e^{-iP\cdot x} z^2 \psi(z),\ z \to \zeta$ we find

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\,\psi(\zeta)$$

with

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right), \quad \mu R = \nu + \frac{1}{2},$$

our previous result.

- Soft-wall model for baryons corresponds to a linear confining potential in the LF transverse variable ζ!

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

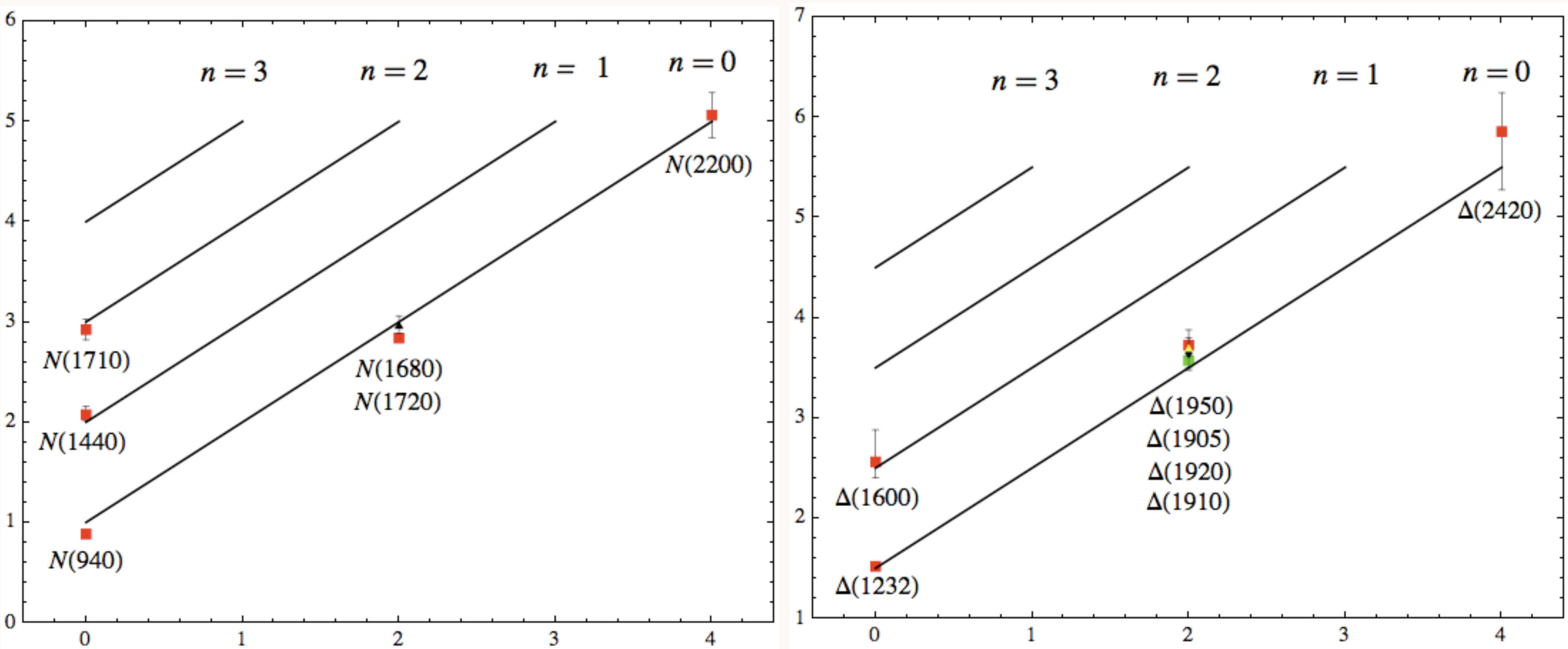
$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$\mathcal{M}^2$



L

Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

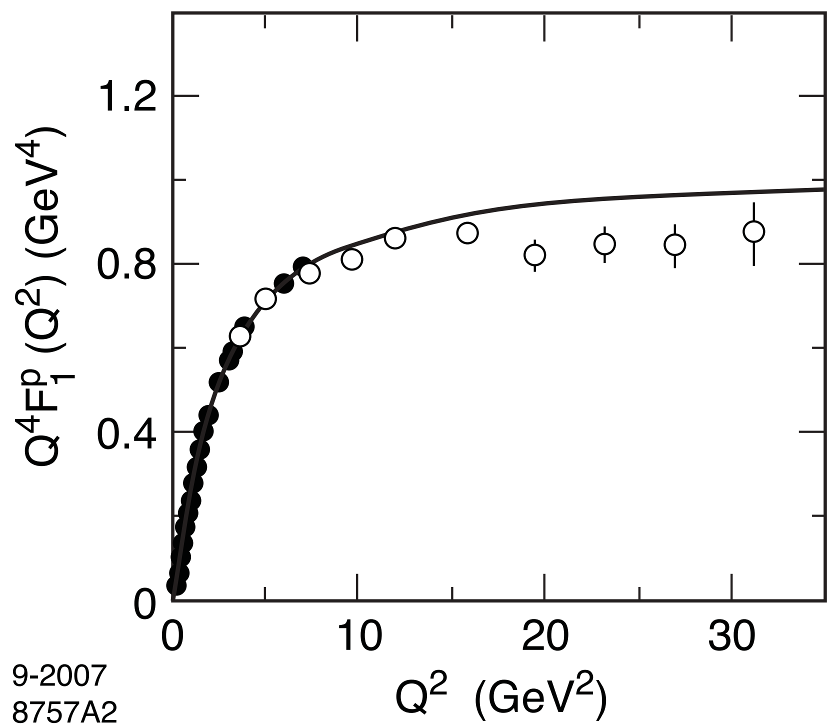
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

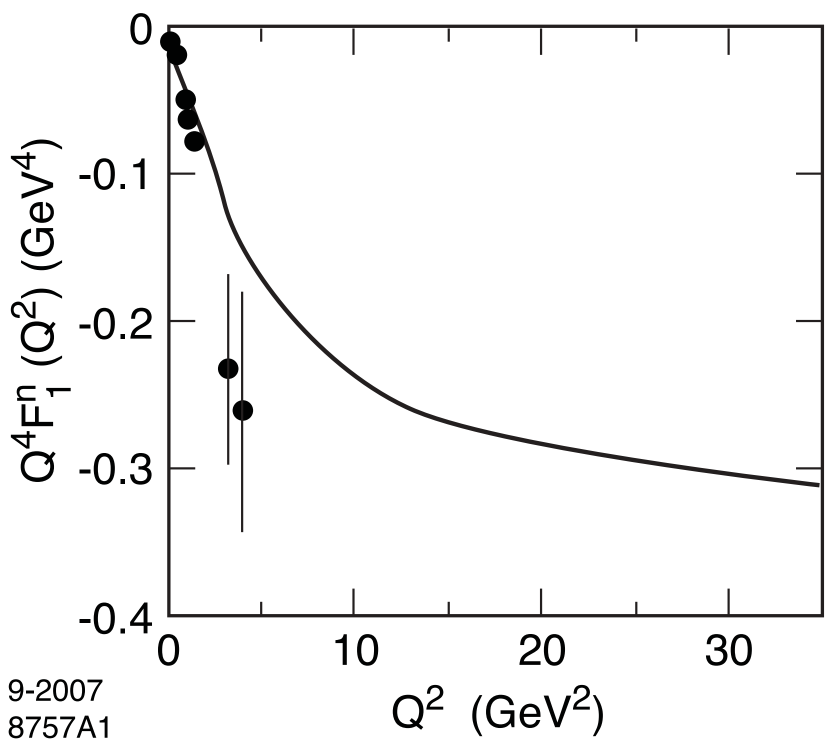
  where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

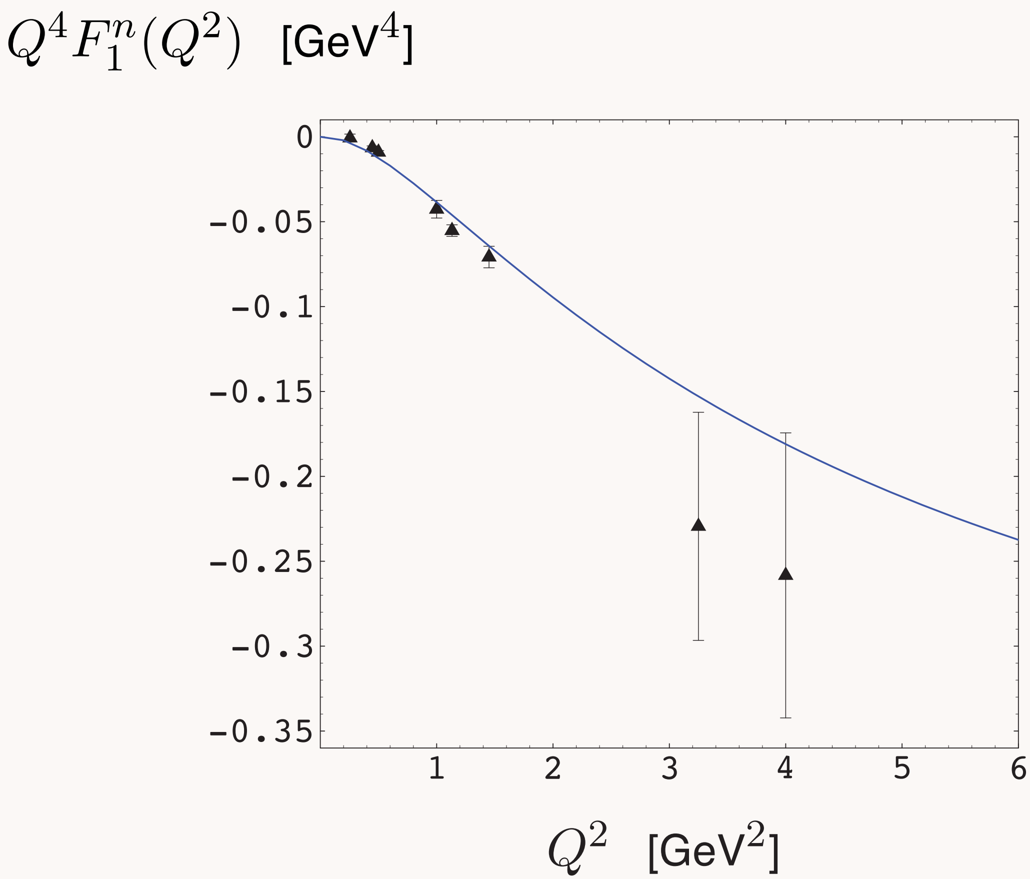
- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron} \;\; \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement

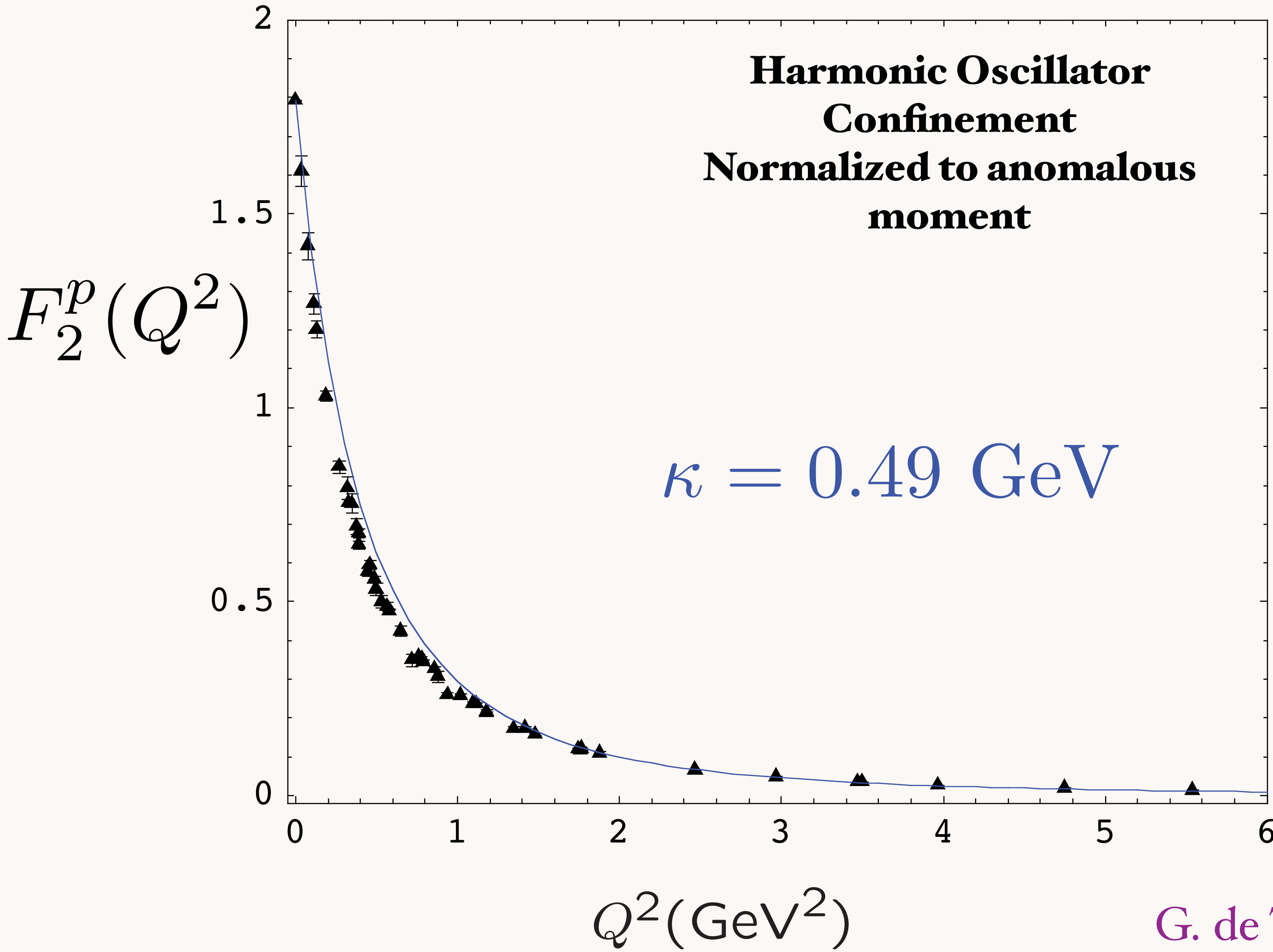


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

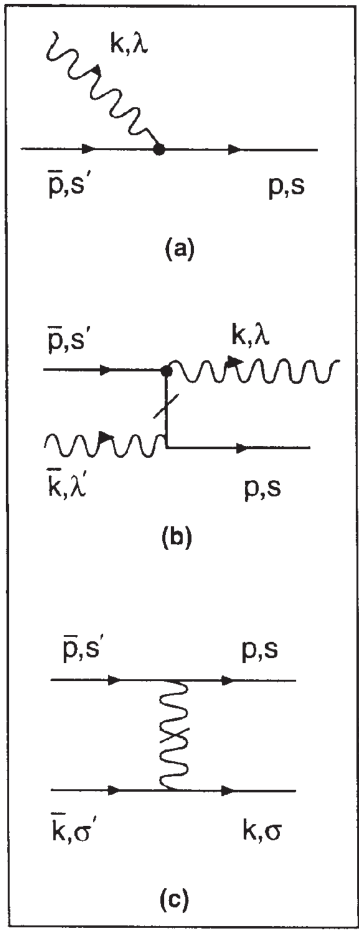
From overlap of $L = 1$ and $L = 0$ LFWFs

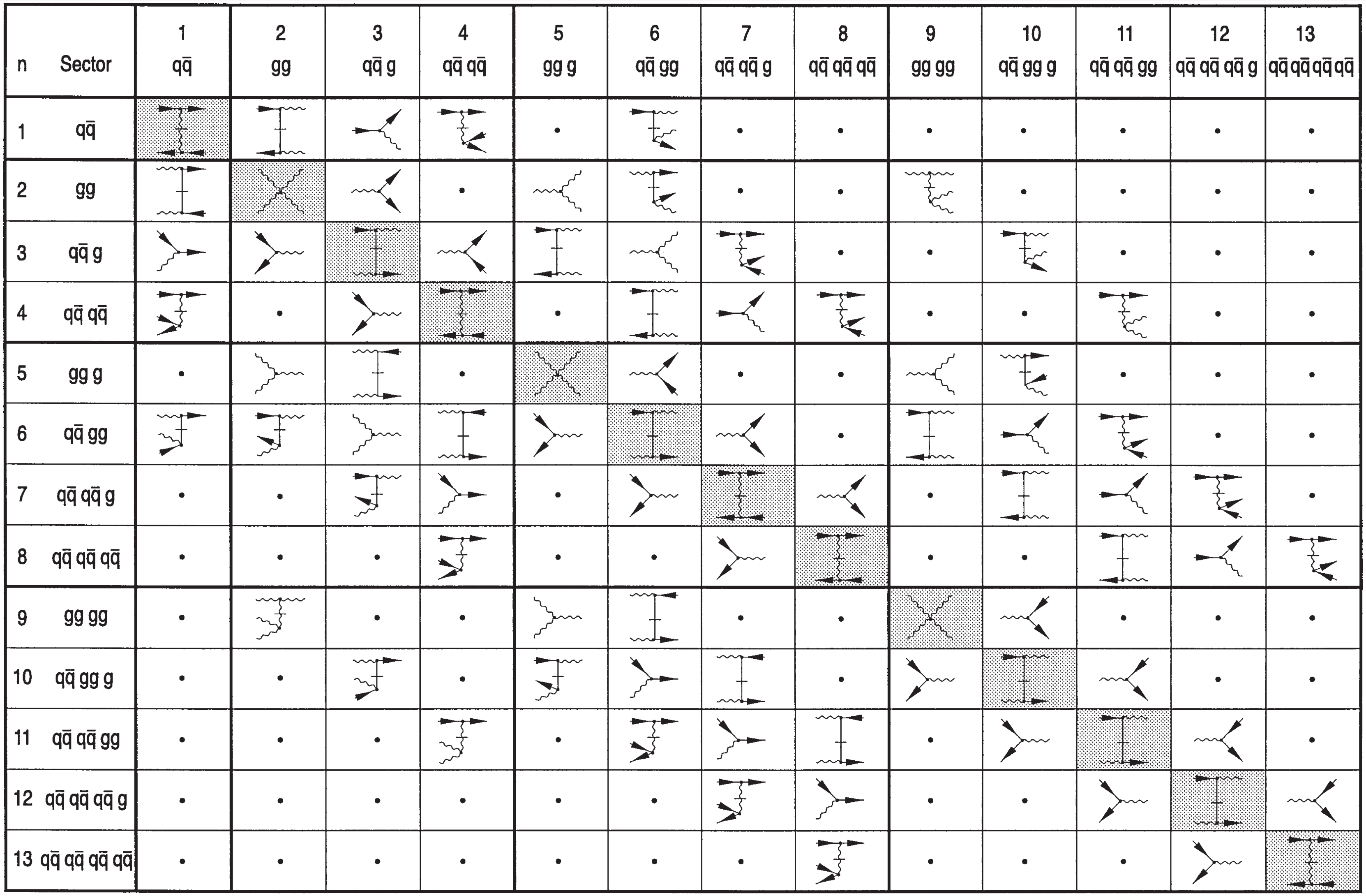


G. de Teramond, sjb

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\ |\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

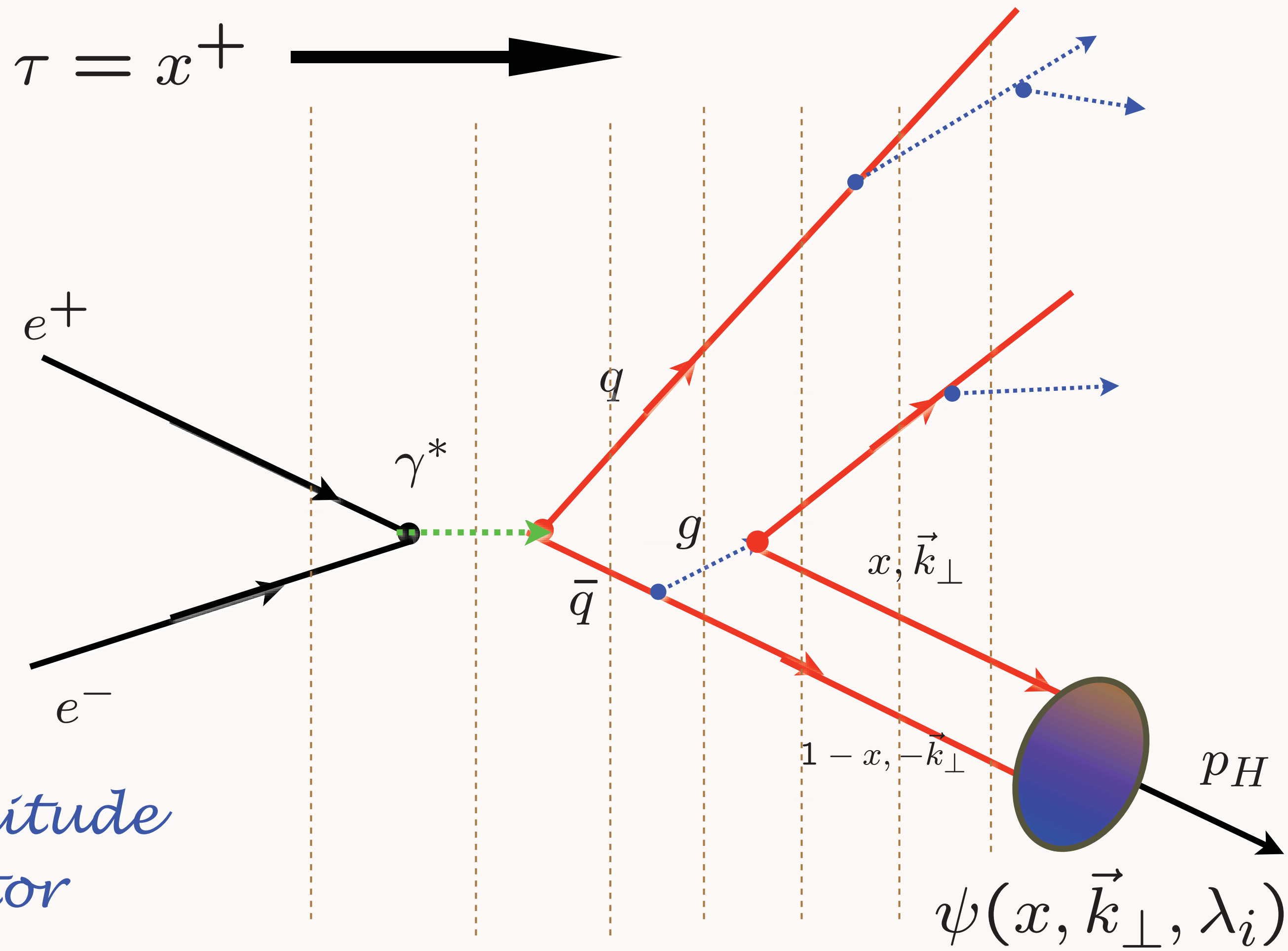
Pauli, Hornbostel, Hiller, McCartor, sjb

- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

Hadronization at the Amplitude Level

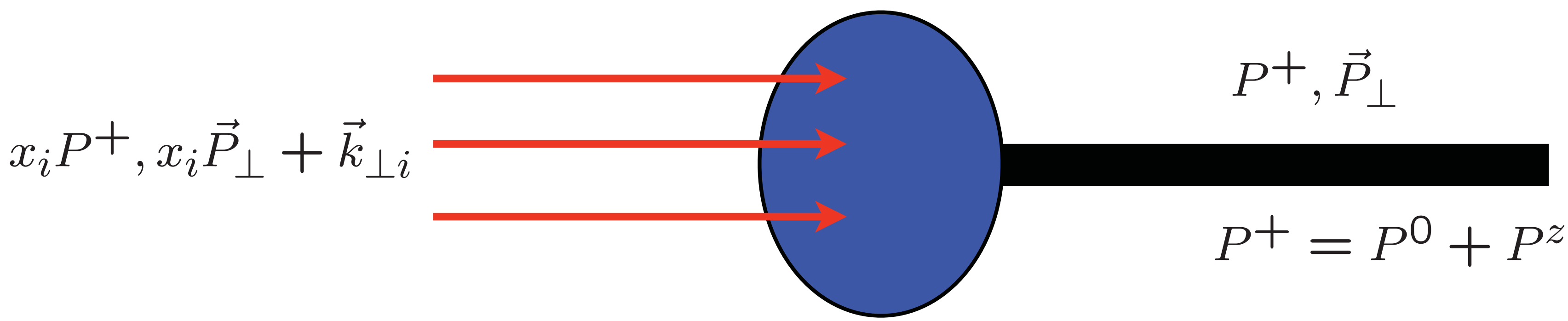


Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

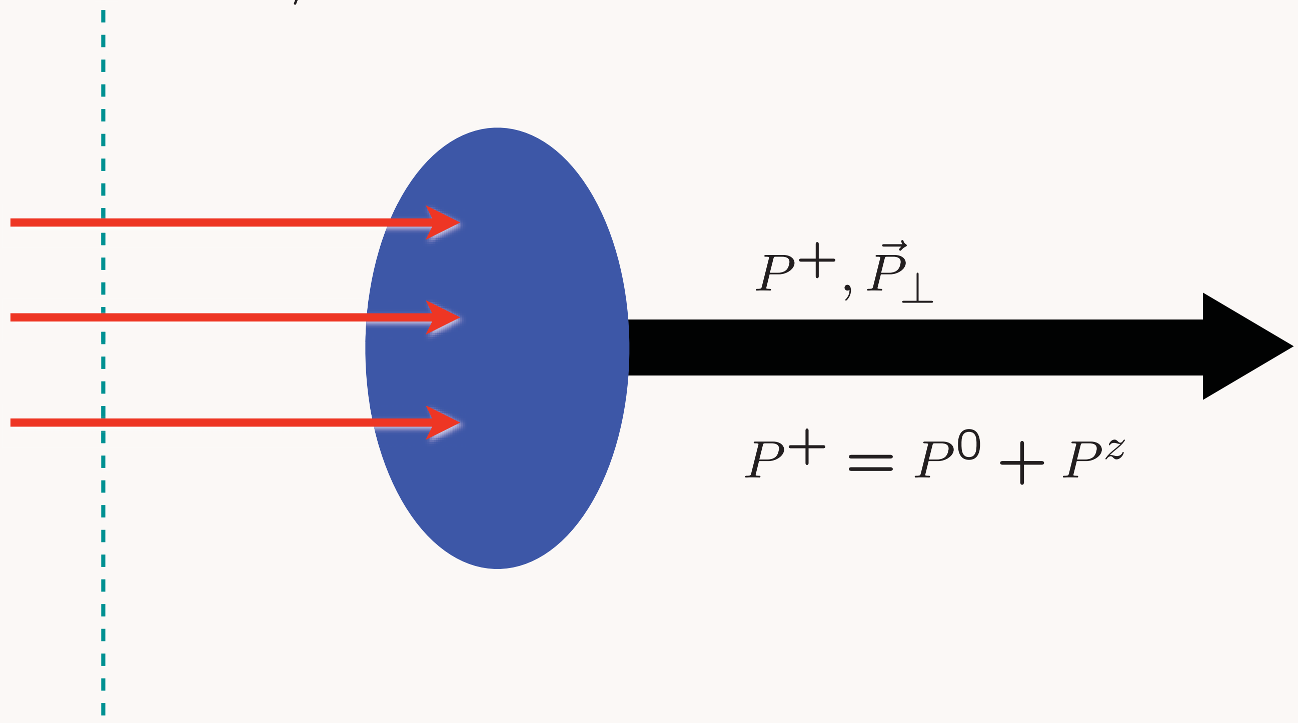
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



Light-Front Wavefunctions

Fixed $\tau = t + z/c$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+, \vec{P}_\perp$

$\sum_i^n x_i = 1$

$P^+ = P^0 + P^z$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

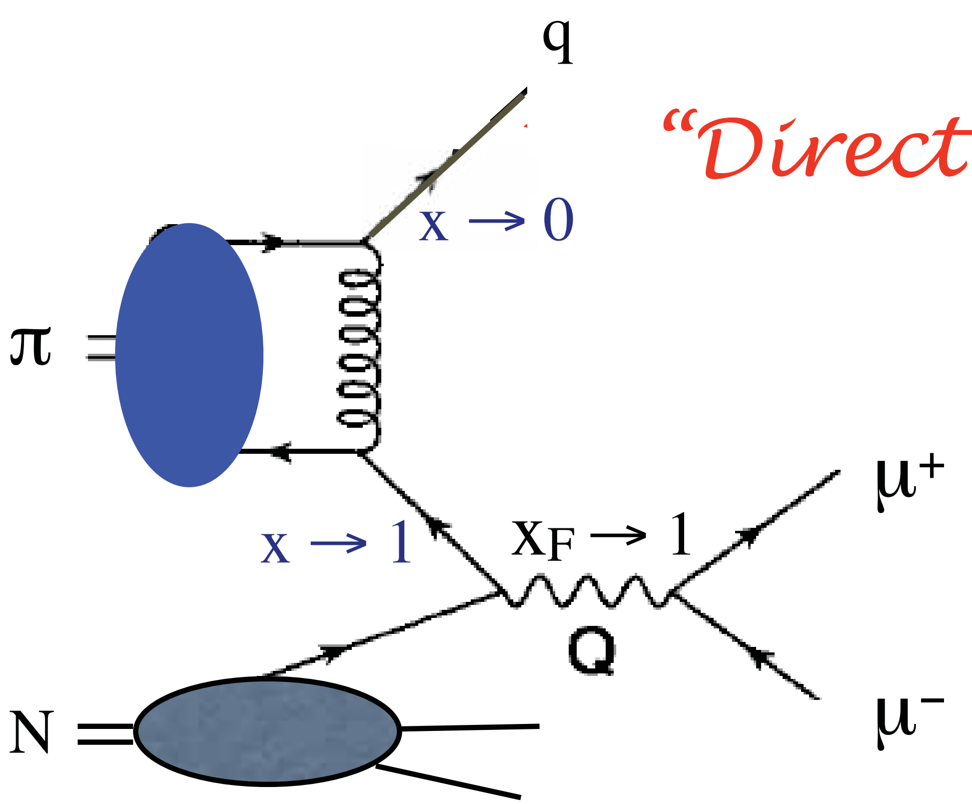
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S. Spectrum is independent of S
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

Entire pion wf contributes to hard process



"Direct" Subprocess

Virtual photon is longitudinally polarized

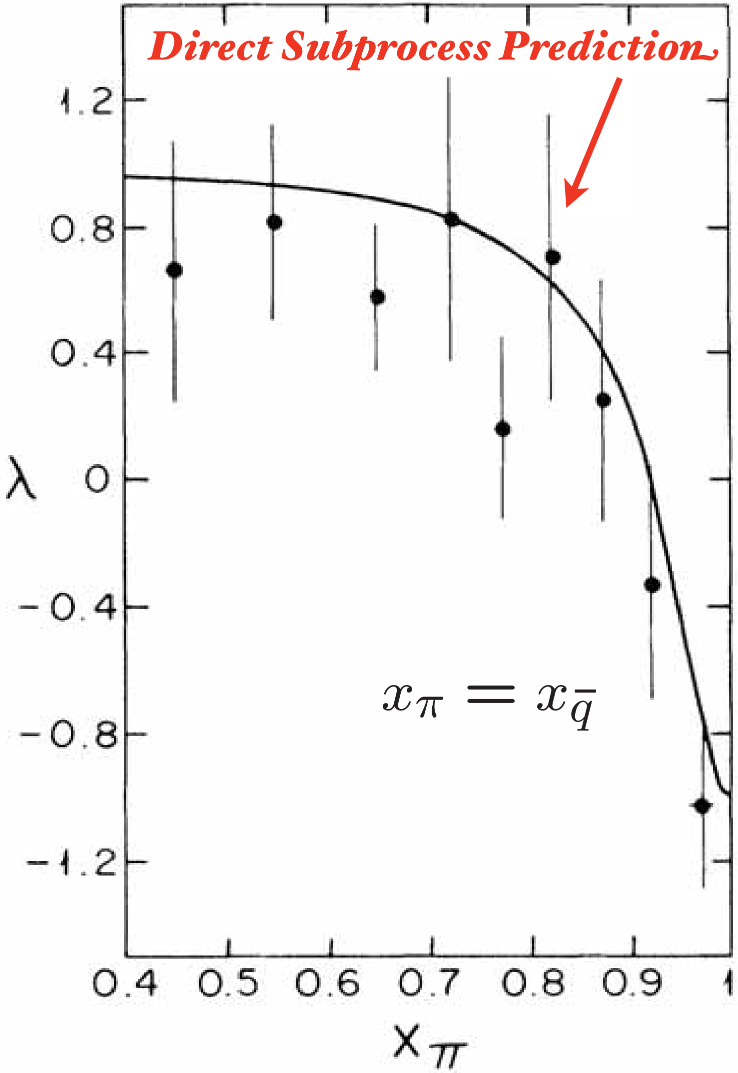
Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen

$\pi^- N \rightarrow \mu^+ \mu^- X$ at 80 GeV/c

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi .$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \mathrm{GeV}^2/c^2$$



Dramatic change in angular distribution at large x_F

Example of a higher-twist direct subprocess

Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

$$x_T = \frac{2p_T}{\sqrt{s}}$$

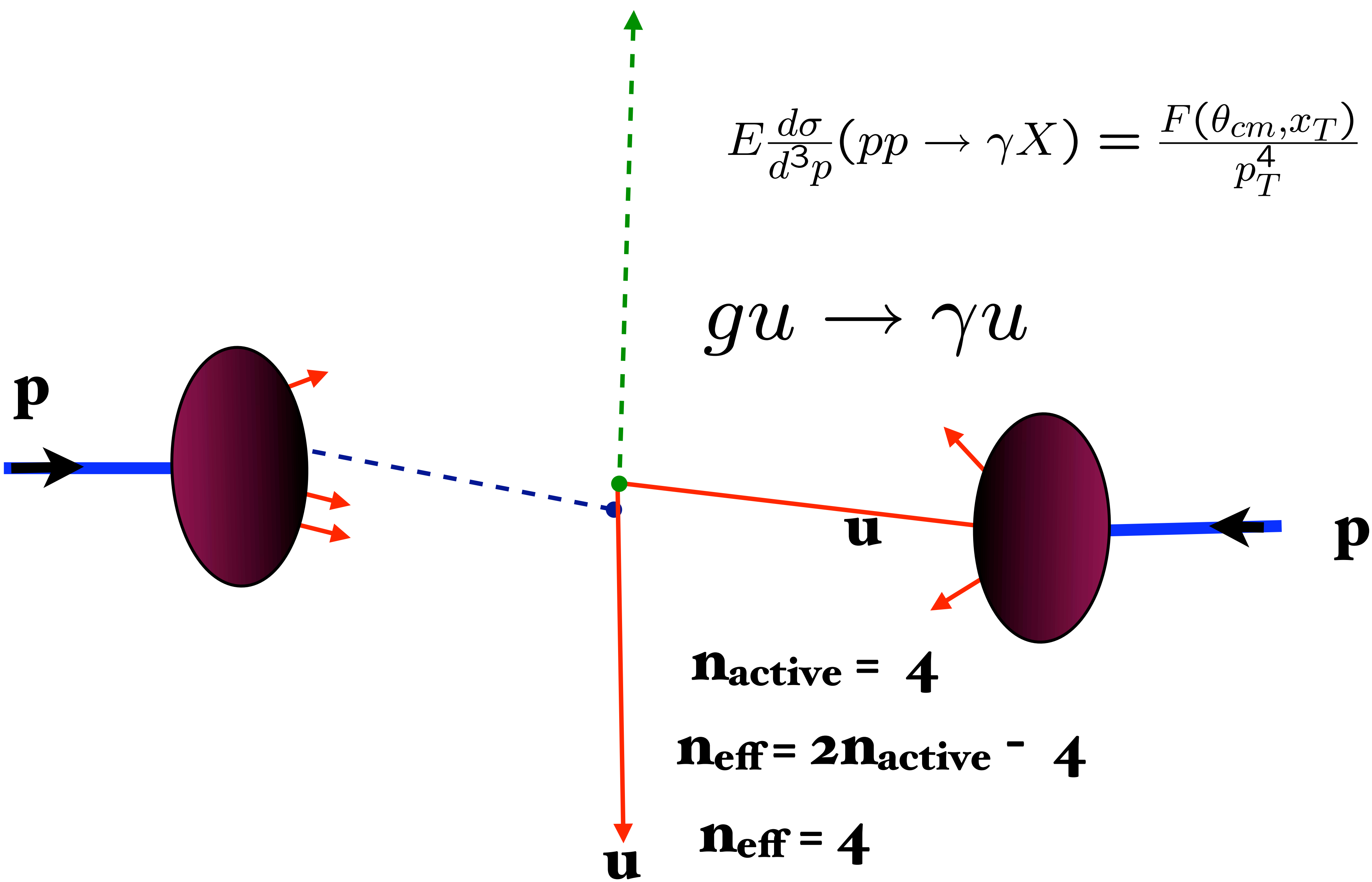
$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

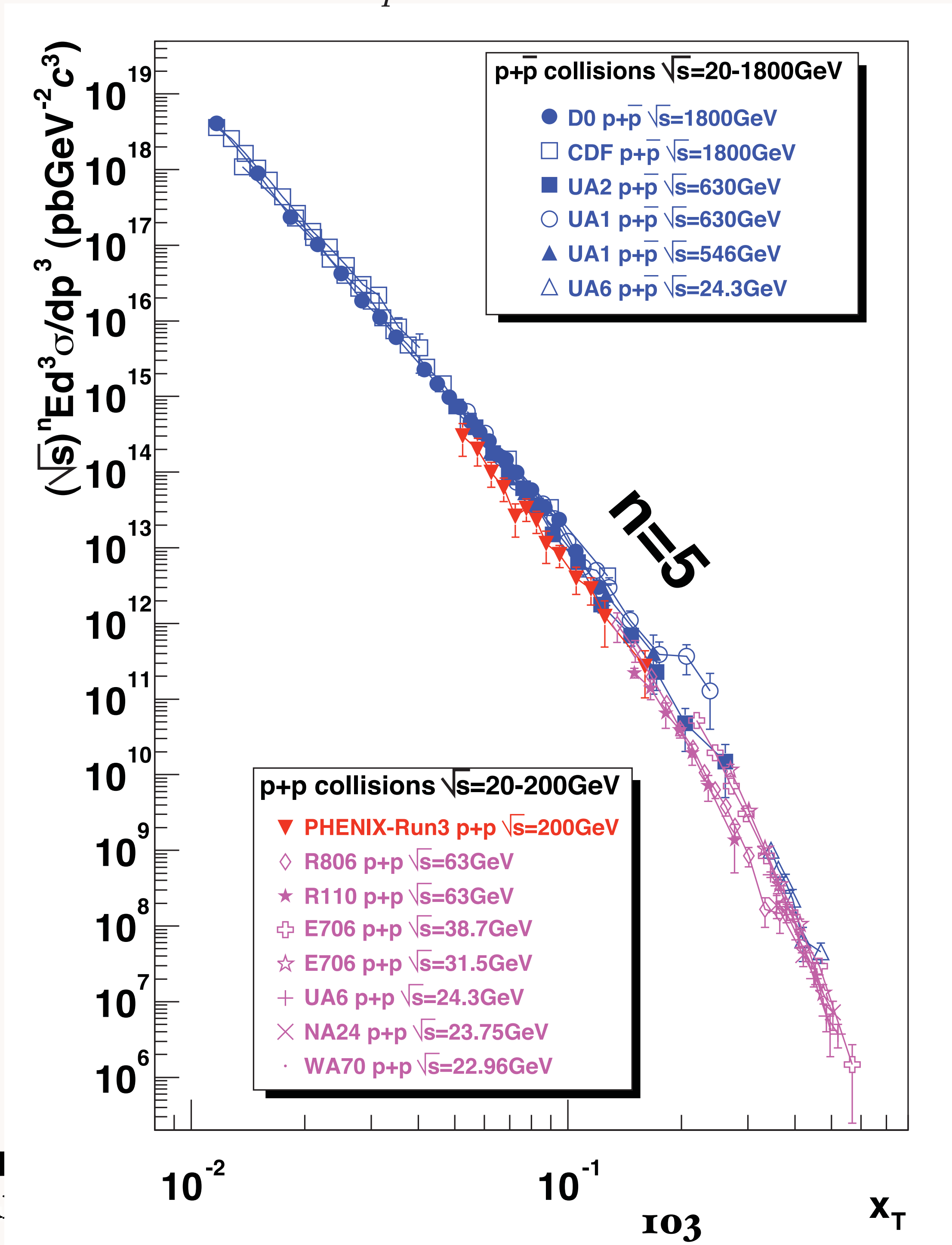
Conformal scaling: $n_{eff} = 2\, n_{active} - 4$

$$pp \longrightarrow \gamma X$$



$$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) \text{ at fixed } x_T$$

Tannenbaum

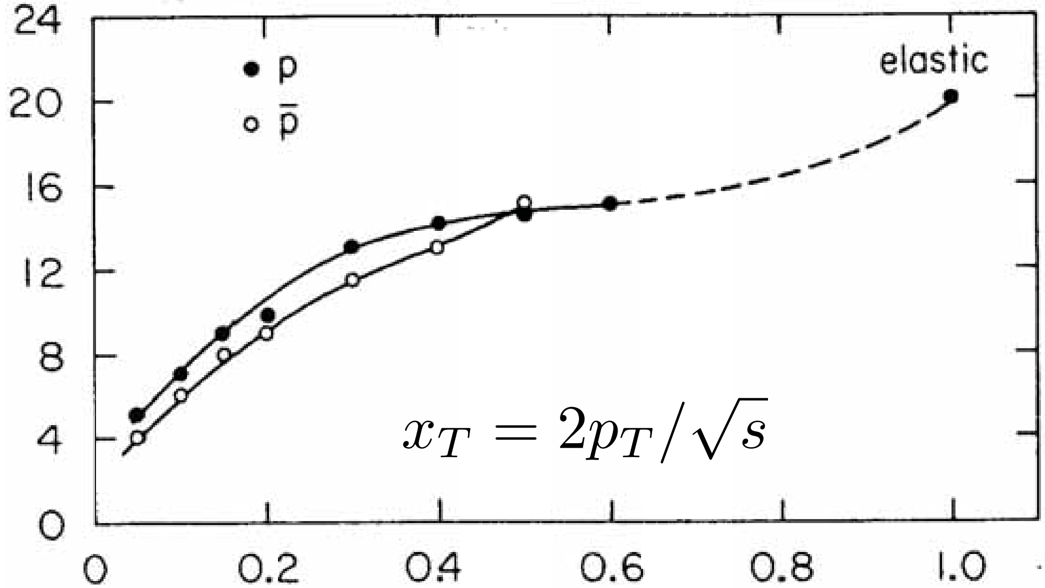


Scaling of direct photon production consistent with PQCD

Stan Brodsky
SLAC

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^{n_{eff}}}$$

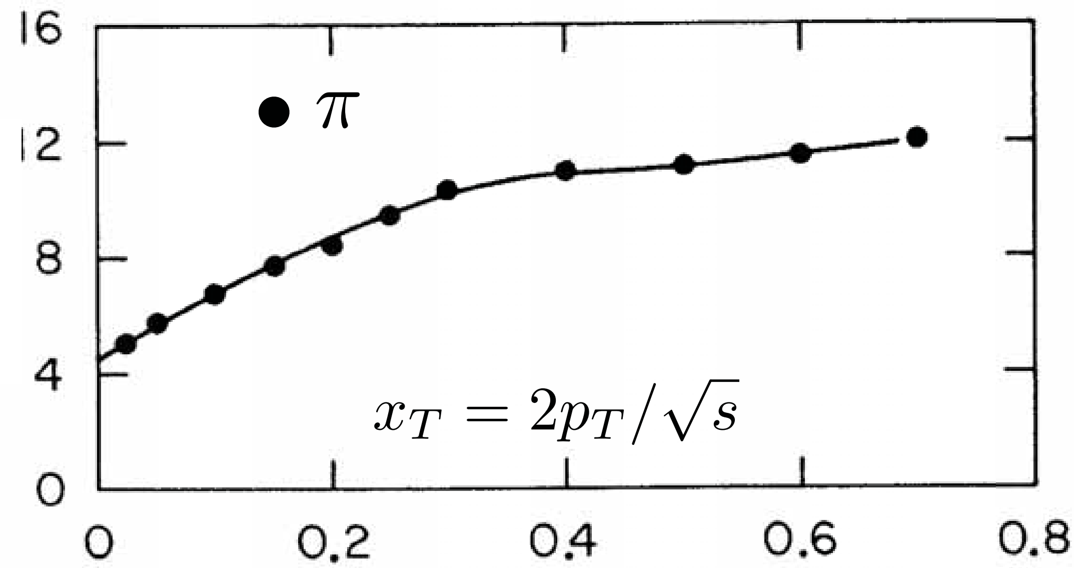
$n_{\rm eff}$



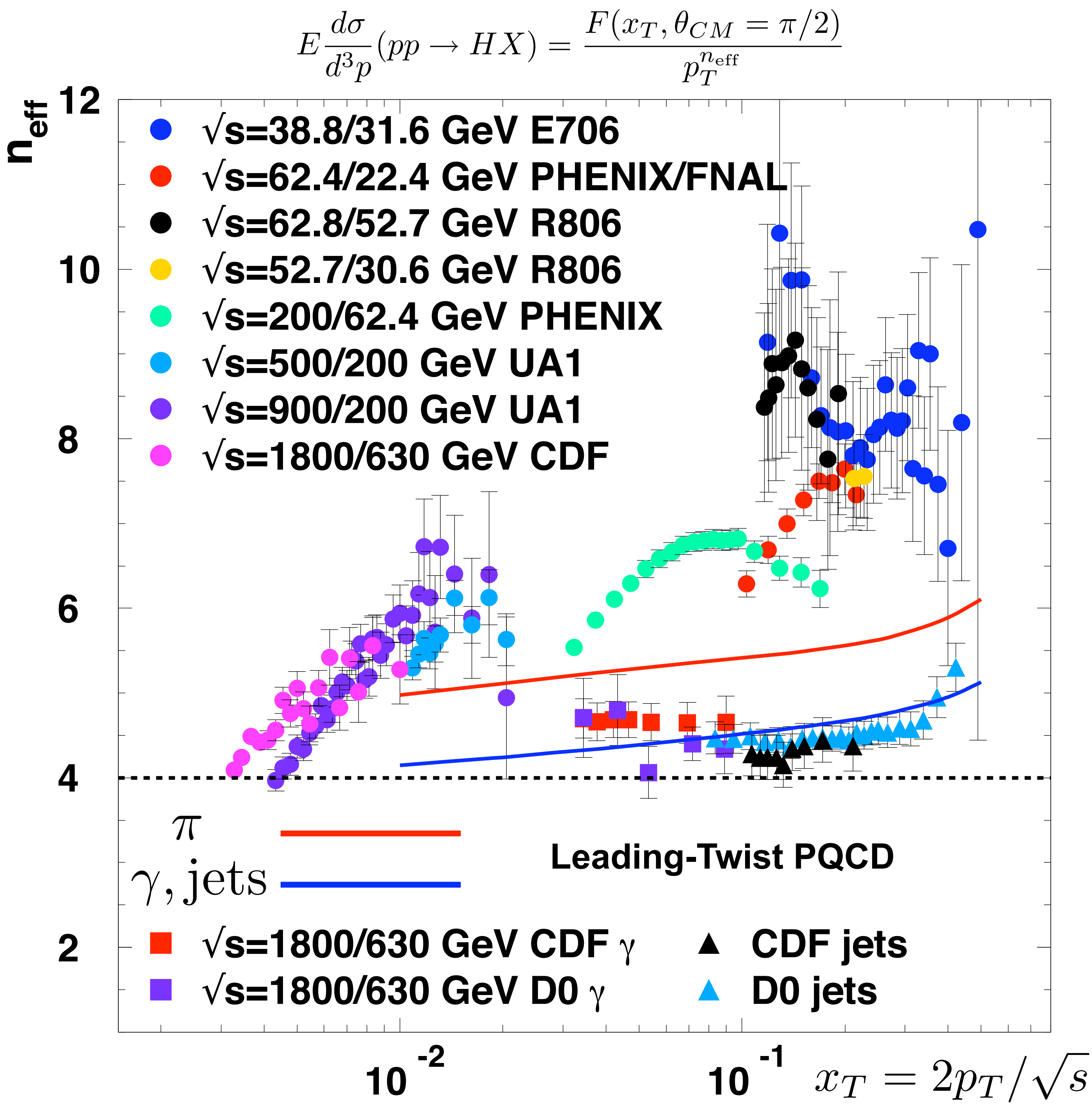
Clear evidence for higher-twist contributions

Fermilab, ISR data

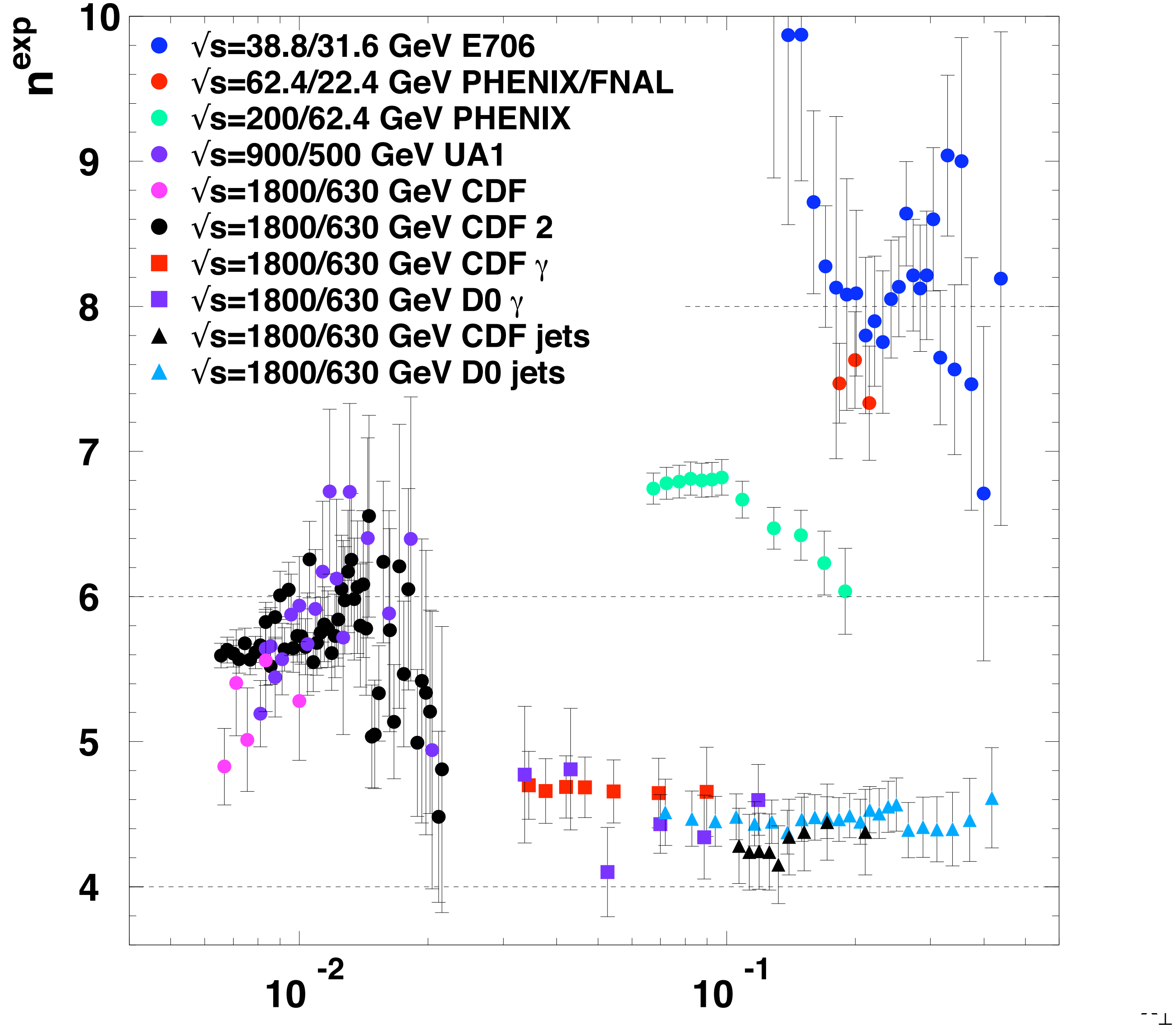
$n_{\rm eff}$



Continuous Rise of n_{eff}

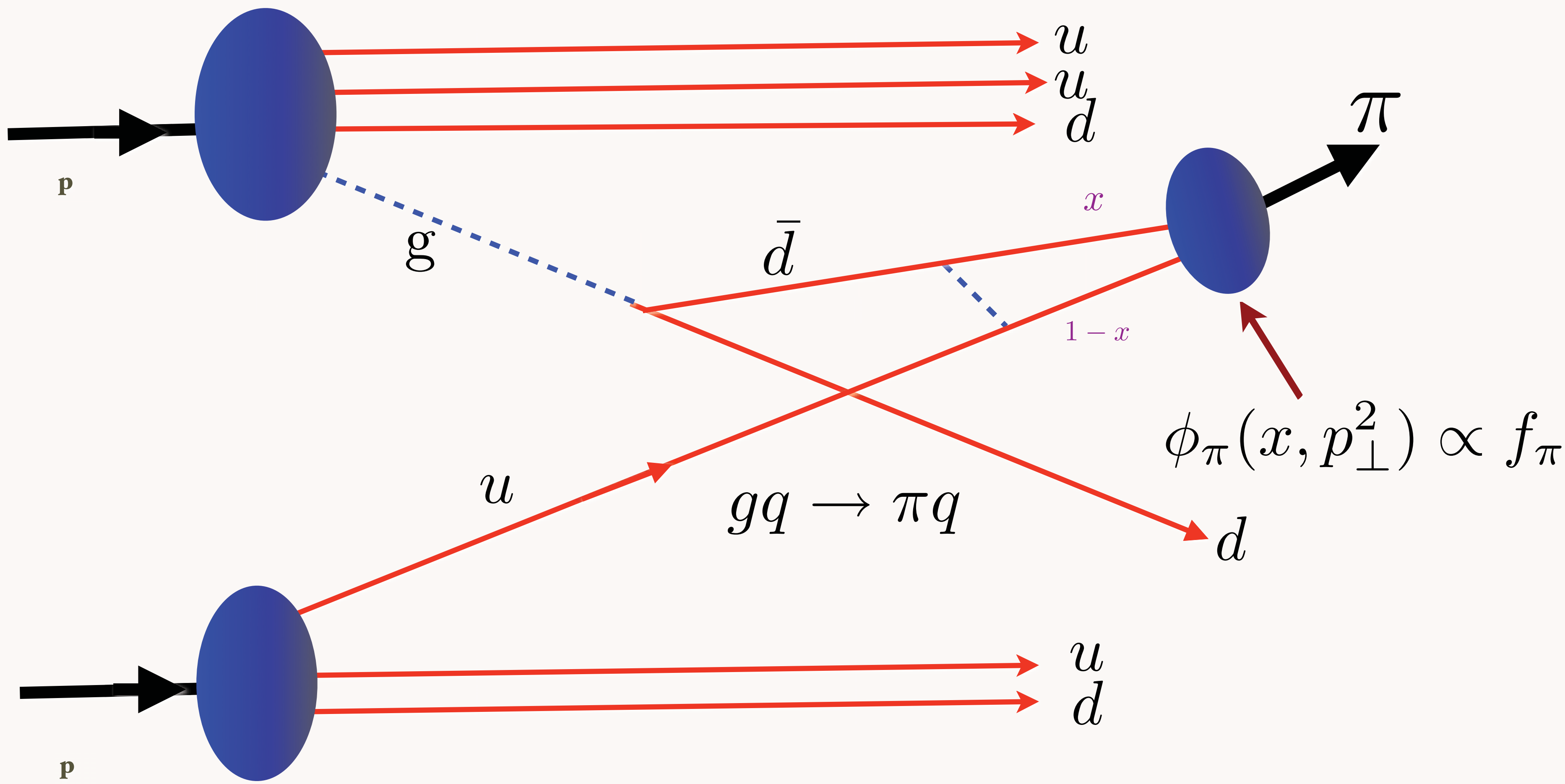


$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\rm eff}}}$
$n_{\rm eff}$
12
10
8
6
4
2
√s=38.8/31.6 GeV E706
√s=62.4/22.4 GeV PHENIX/FNAL
√s=62.8/52.7 GeV R806
√s=52.7/30.6 GeV R806
√s=200/62.4 GeV PHENIX
√s=500/200 GeV UA1
√s=900/200 GeV UA1
√s=1800/630 GeV CDF
π
γ, jets
Leading-Twist PQCD
√s=1800/630 GeV CDF γ
√s=1800/630 GeV D0 γ
CDF jets
D0 jets
10^{-2}
10^{-1}
$x_T = 2p_T/\sqrt{s}$



- Significant increase of the hadron $n^{\rm exp}$ with $x_\perp$
  - $n^{\rm exp} \simeq 8$ at large $x_\perp$
- Huge contrast with photons and jets!

Higher-Twist Contribution to Hadron Production

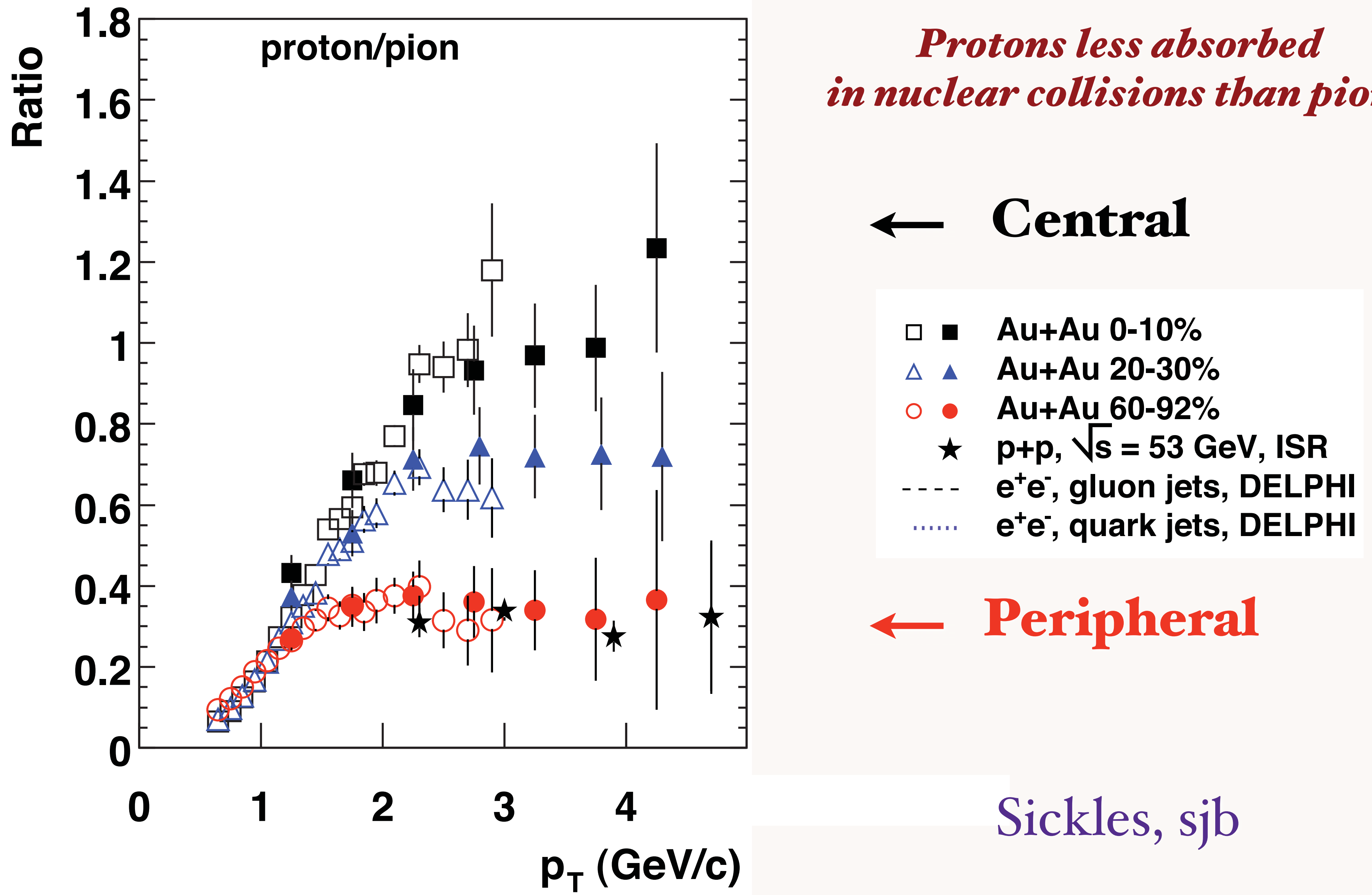


$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

No Fragmentation Function

S. S. Adler *et al.* PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Baryon Anomaly: Particle ratio changes with centrality!



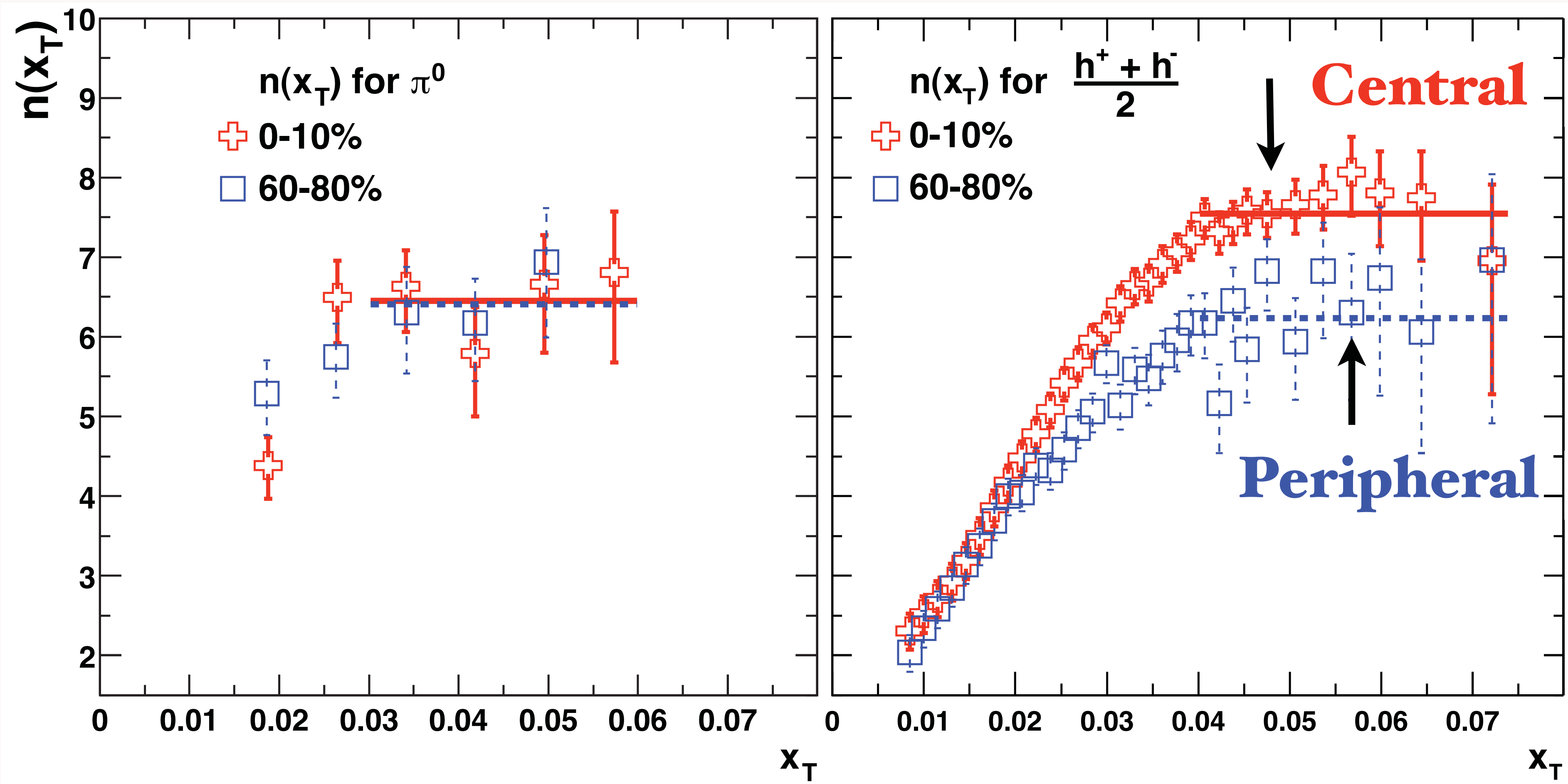
Protons less absorbed in nuclear collisions than pions

← **Central**

← **Peripheral**

Sickles, sjb

$$\sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV}$$

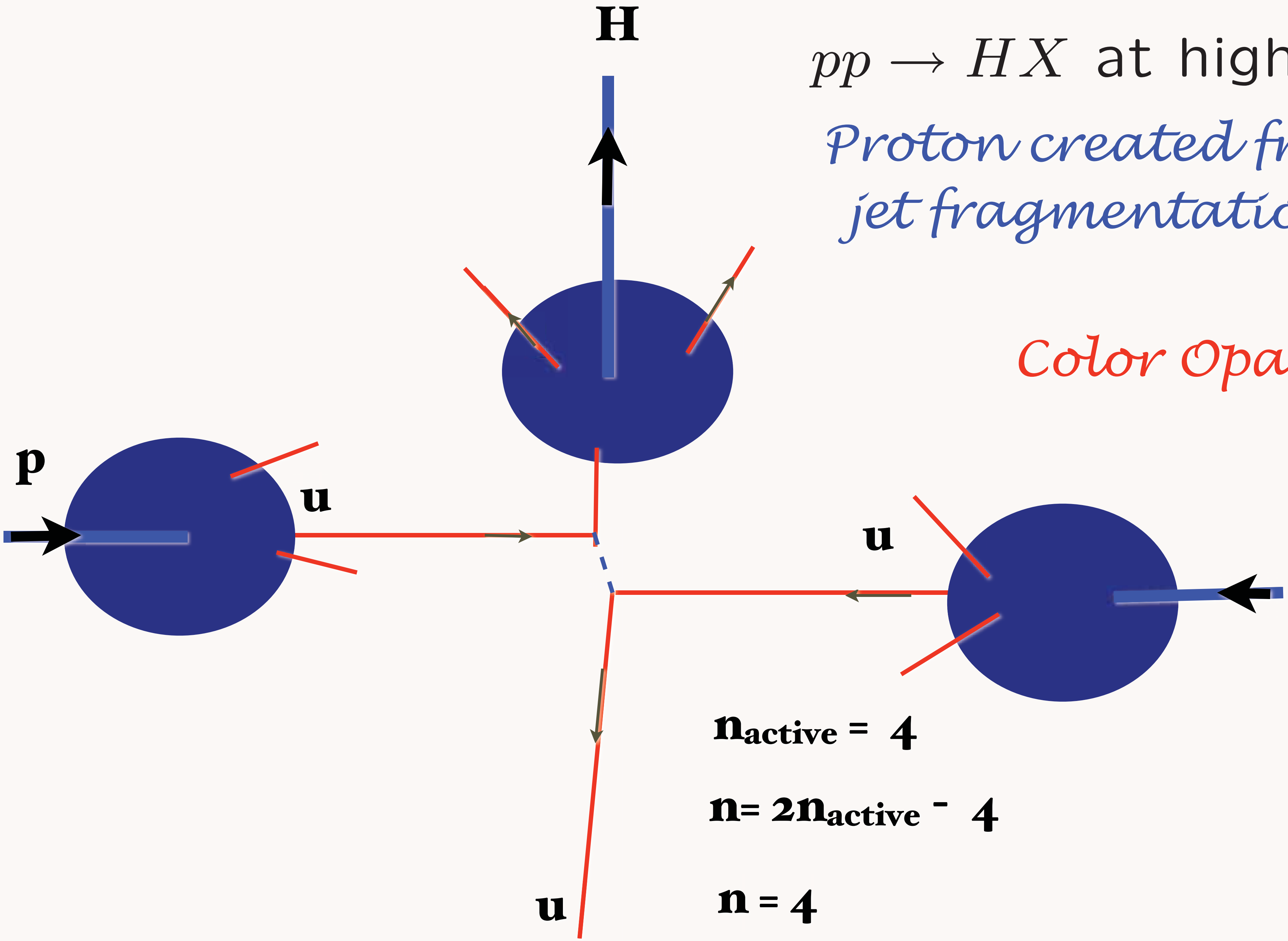


Proton power changes with centrality !

$pp \to HX$ at high p_T

Proton created from jet fragmentation

Color Opaque



$n_{active} = 4$

$n = 2n_{active} - 4$

$n = 4$

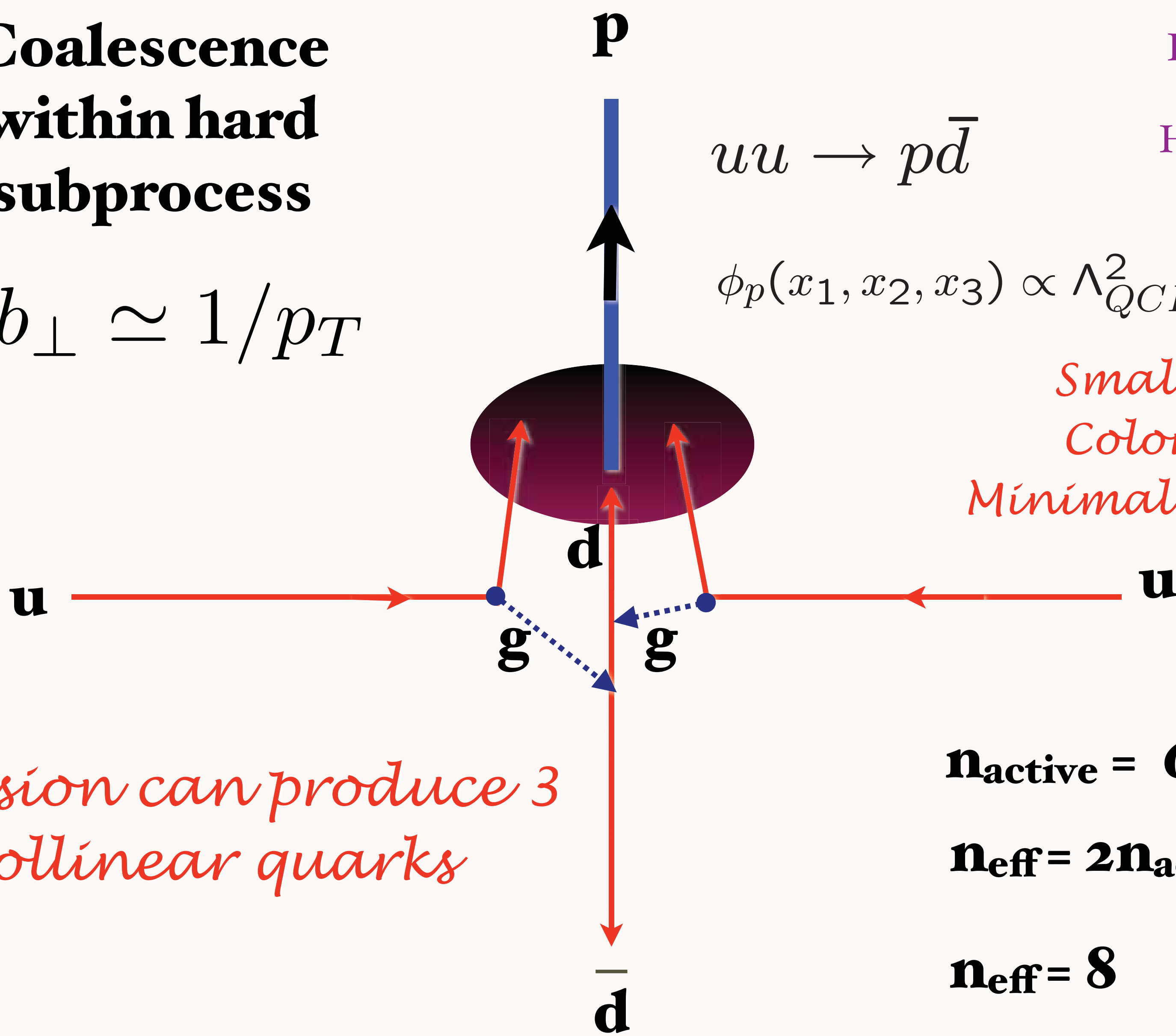
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

$$b_\perp \simeq 1/p_T$$

$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$



Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles, sjb

Small color-singlet
Color Transparent
Minimal same-side energy

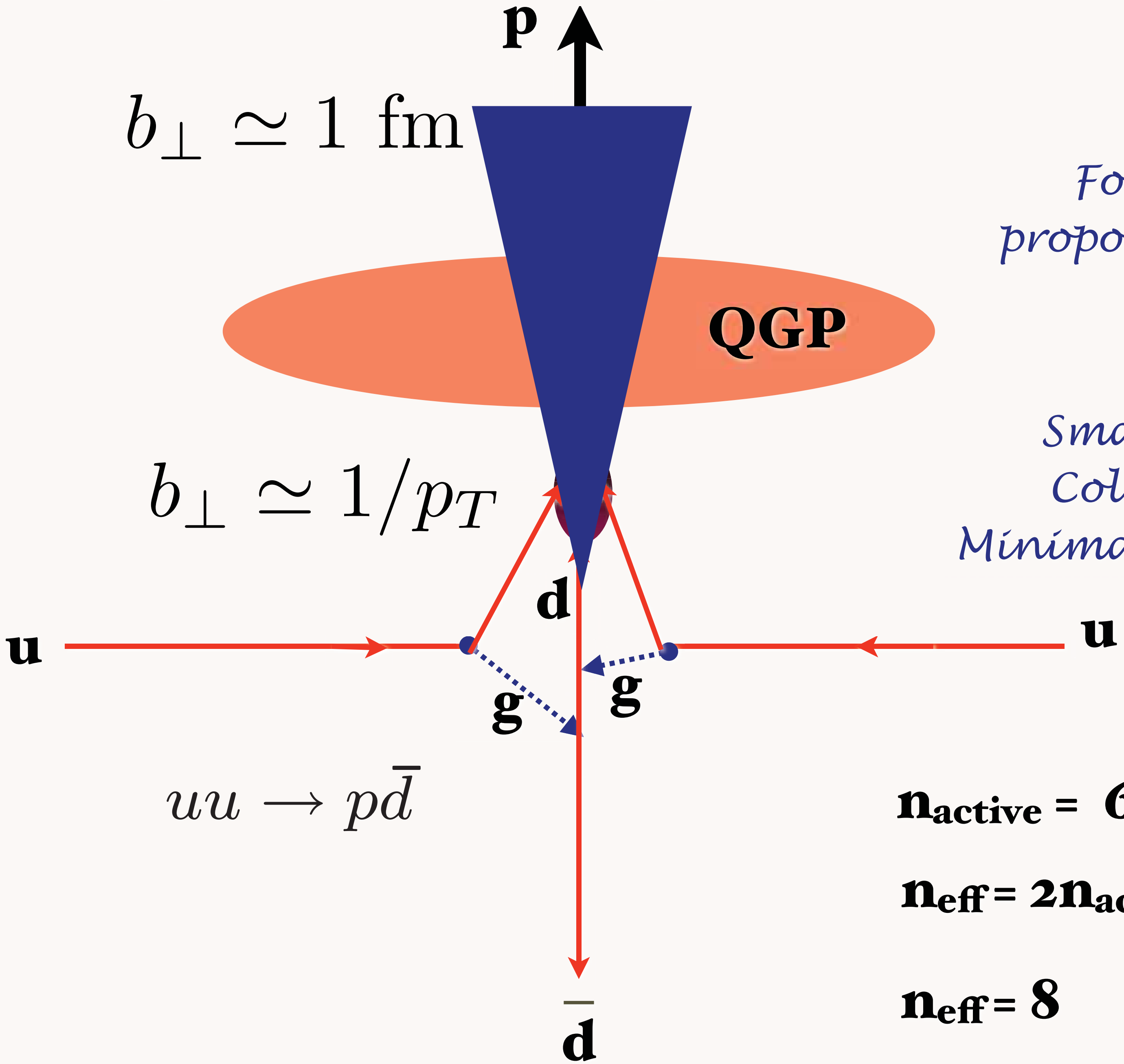
Collision can produce 3 collinear quarks

$\mathbf{n_{active} = 6}$

$\mathbf{n_{eff} = 2n_{active} - 4}$

$\mathbf{n_{eff} = 8}$

Baryon made directly within hard subprocess



Formation Time proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

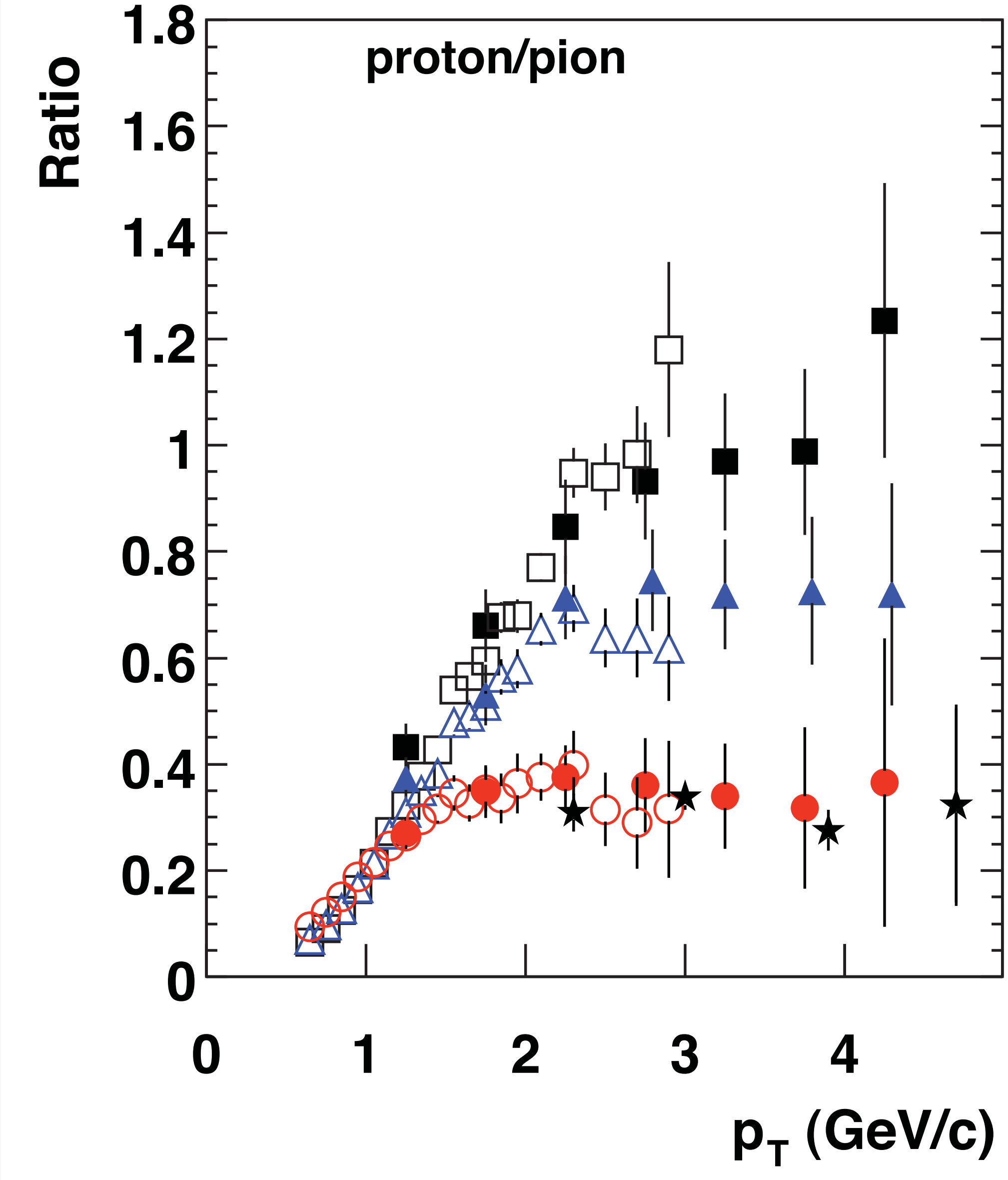
$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!



Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process

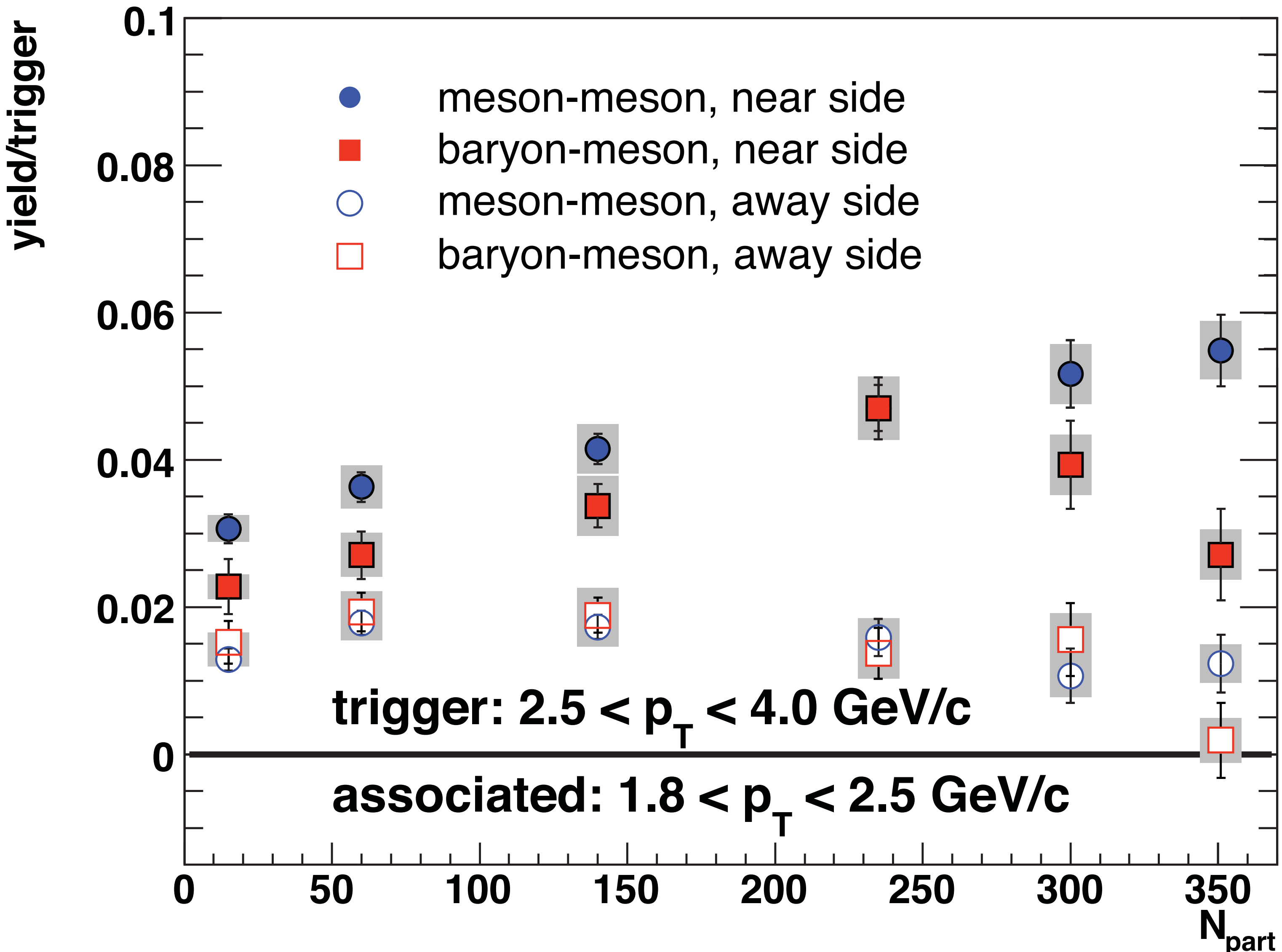
← **Central**

← **Peripheral**

Evidence for Direct, Higher-Twist Subprocesses

- Anomalous power behavior at fixed x_T
- Protons more likely to come from direct subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Exclusive-inclusive connection at $x_T = 1$

Anne Sickles



proton trigger:
same-side
particles
decreases with
centrality

Proton production more dominated by color-transparent direct high-n_{eff} subprocesses

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S. Spectrum is independent of S
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓

Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Chiral Symmetry Breaking in AdS/QCD

We consider the action of the X field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} \left(g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2 \right), \tag{1}$$

Ehrlich, Katz, Son, Stephanov
Babington, Erdmenger, Evans, Kirsch, Guralnik, Thelfall

with equations of motion

$$z^3 \partial_z \left(\frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left(\frac{\mu_X R}{z} \right)^2 X = 0. \tag{2}$$

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$\left[z^2 \partial_z^2 - 3z\, \partial_z + 3 \right] X(z) = 0. \tag{3}$$

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3, \tag{4}$$

where A and B are determined by the boundary conditions.

de Teramond, Shrock, sjb

$$A \propto m_q \qquad\qquad B \propto < \bar{\psi} \psi >$$

Expectation value taken inside hadron

Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz\ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD: confined condensate**
- **"In-Hadron" Condensates**

de Teramond, Shrock, sjb

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

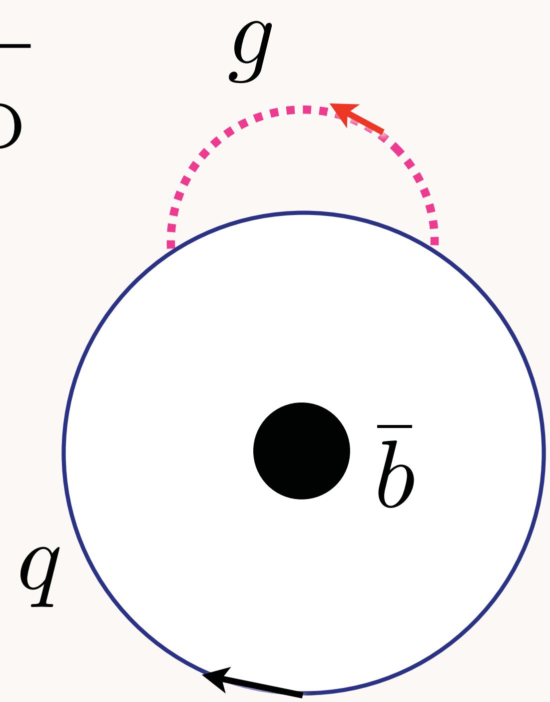
On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

Light-Front (Front Form) Formalism

- Casher & Susskind model shows that spontaneous chiral symmetry breaking can occur in the finite domain of a hadronic LFWF
- Infinite number of partons required, but this is a feature of QCD LFWFs --
- Regge behavior of DIS due to $x^{-\alpha_R}$ behavior of structure functions (LFWFs squared)
- A.H. Mueller: BLKL Pomeron derived from the multi-gluon Fock States of the quarkonium LFWF
- F. Antonuccio, S. Dalley, sjb: Construct soft-gluon LFWF via ladder operators
- LF Vacuum Trivial up to zero modes

Maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{\rm QCD}} \qquad \lambda < \Lambda_{\rm QCD}$$



B-Meson

Shrock, sjb

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

$$< \bar b|\bar q q|\bar b > \text{ not } < 0|\bar q q|0 >$$

Quark and Gluon condensates reside within hadrons, not LF vacuum

- **Bound-State Dyson-Schwinger Equations**

Maris, Roberts, Tandy

- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**

Casher Susskind

- **Finite size phase transition - infinite # Fock constituents**
- **AdS/QCD Description -- CSB is in-hadron Effect**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant -- reduction by 45 orders of magnitude!**

Shrock, sjb

"Confined QCD Condensates"

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind   Roberts et al.   Shrock and sjb

- Bound-State Dyson-Schwinger Equations   Roberts et al.
- AdS/QCD
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict   Shrock and sj

Quark and Gluon condensates reside within hadrons, not vacuum

- **Light Front Vacuum trivial up to zero modes**
- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs** — Casher and Susskind
- **Bound-State Bethe-Salpeter Equations** — Roberts et al.
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict** — Shrock and sjb
- **"In-Hadron" Condensates**

- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Provides Conformal Template
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model

Shrock and sjb