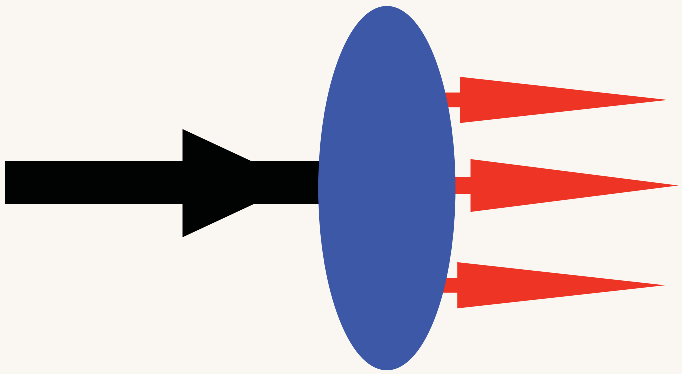
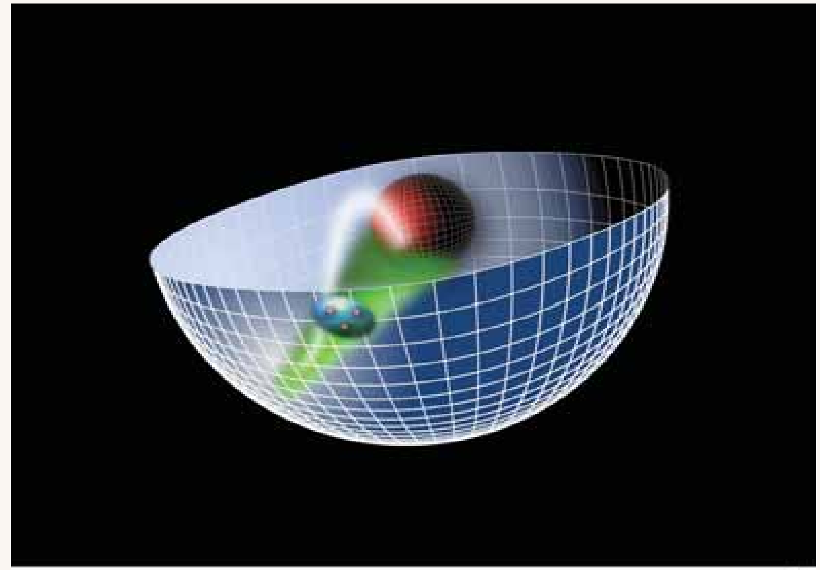
AdS/QCD, Light-Front Holography, and the Chiral Condensate



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Origin Of Mass
May 3-7, 2010

Stan Brodsky SLAC NATIONAL ACCELERATOR LABORATORY & ***CP3 -Origins***

CP3 - Origins
Particle Physics & Origin of Mass

Odense, Denmark

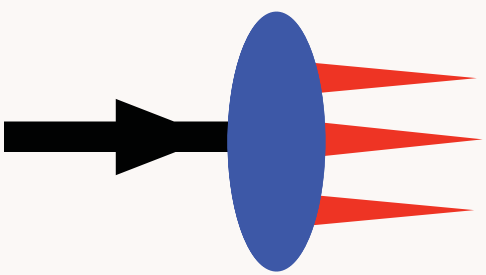
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable

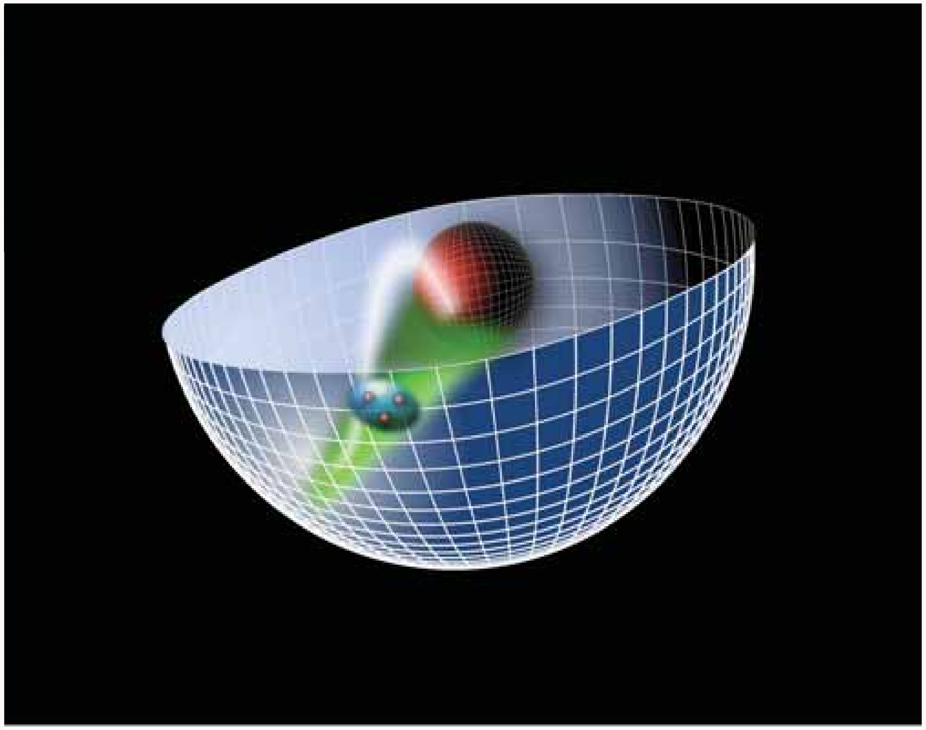
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond and Alexandre Deur

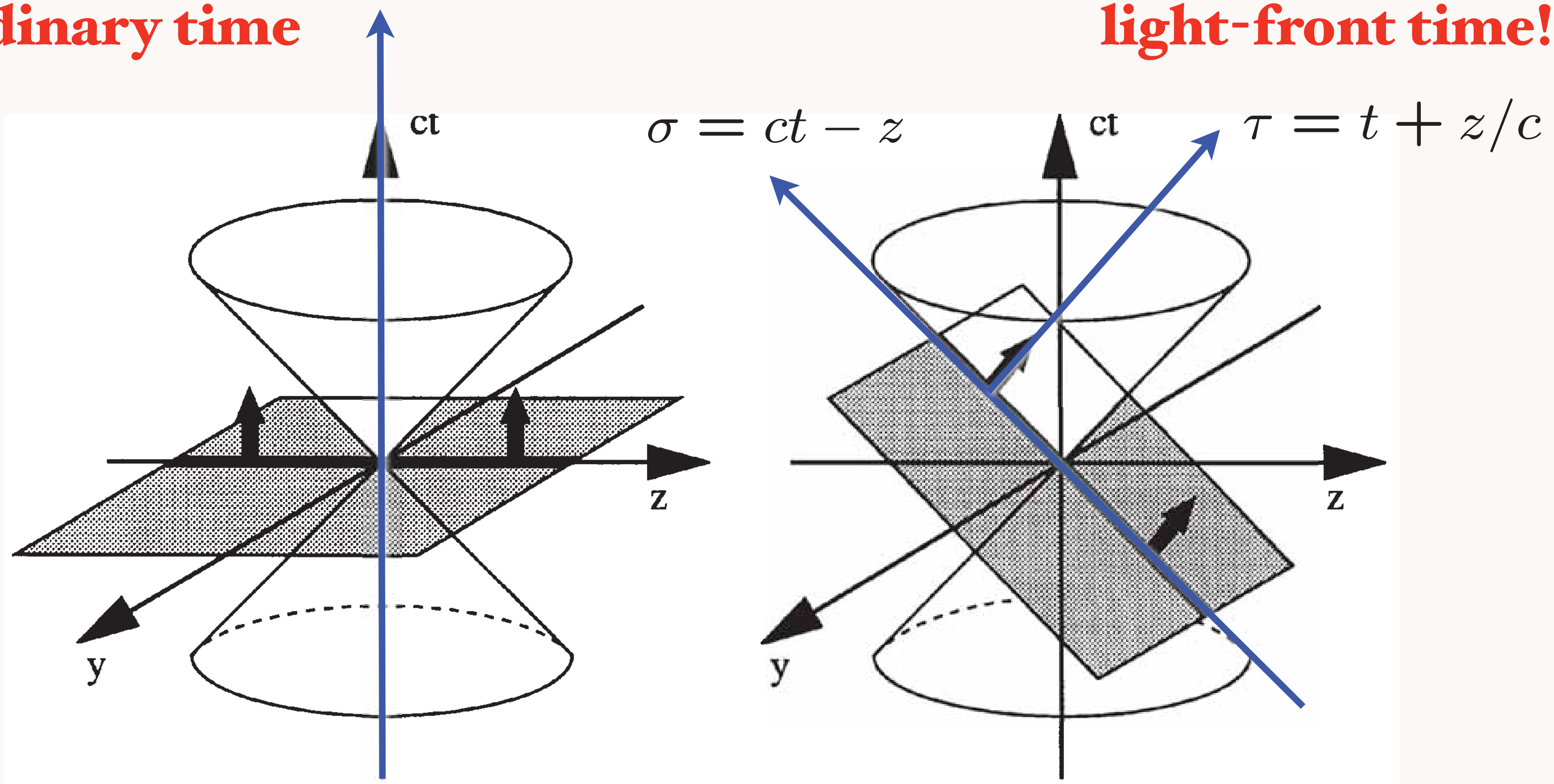
Central problem for strongly-coupled gauge theories

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same Light Front time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Causal, Trivial Vacuum

DIS, Form Factors, DVCS, etc. measure proton WF at fixed

- QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ, $\mathbf{A}_\perp$

$$\begin{aligned}
P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}\,\gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi + \text{interactions} \\
P^+ &= \int dx^- d^2\mathbf{x}_\perp \,\overline{\psi}\,\gamma^+ i\partial^+\psi \\
\mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \,\overline{\psi}\,\gamma^+ i\nabla_\perp\psi
\end{aligned}$$

- LF Hamiltonian P^- generates LF time translations

$$\left[\psi(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi(x)$$

and the generators P^+ and $\mathbf{P}_\perp$ are kinematical

Light-Front Bound State Hamiltonian Equation

$$\tau = t + z/c$$

- Construct light-front invariant Hamiltonian for the composite system: $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$H_{LF} \, |\psi_H\rangle = \mathcal{M}_H^2 \, |\psi_H\rangle$$

- State $|\psi_H(P^+, \mathbf{P}_\perp, J_z)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian:

$$|\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle, \qquad |n\rangle = \begin{cases} |uud\rangle \\ |uudg\rangle \\ |uud\bar{q}q\rangle \quad \cdots \end{cases}$$

where $k_i^2 = m_i^2, \;\; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each component i

- Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

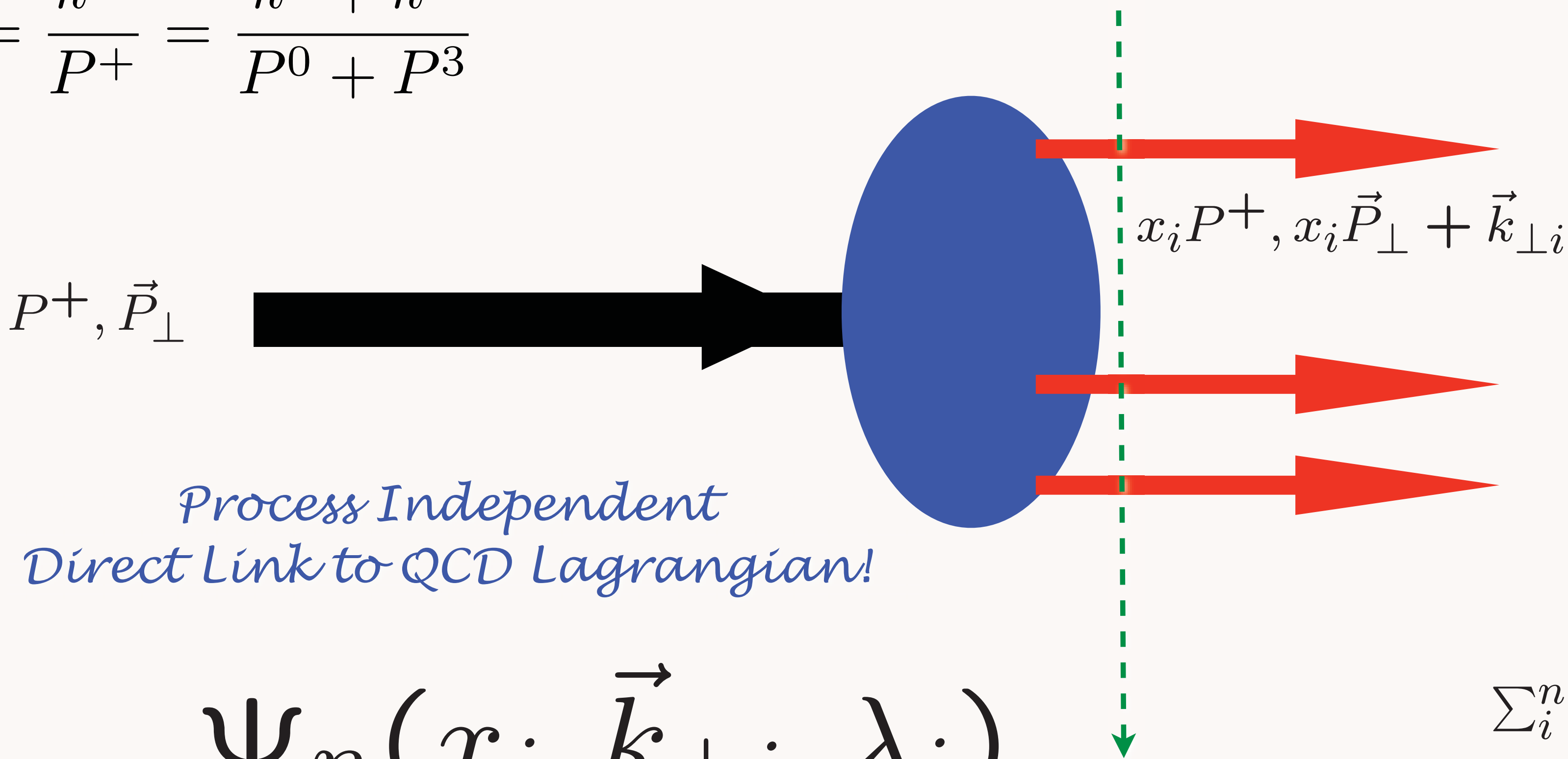
$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

DIS, Form Factors, DVCS, etc. measure proton LFWF

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



Process Independent
Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

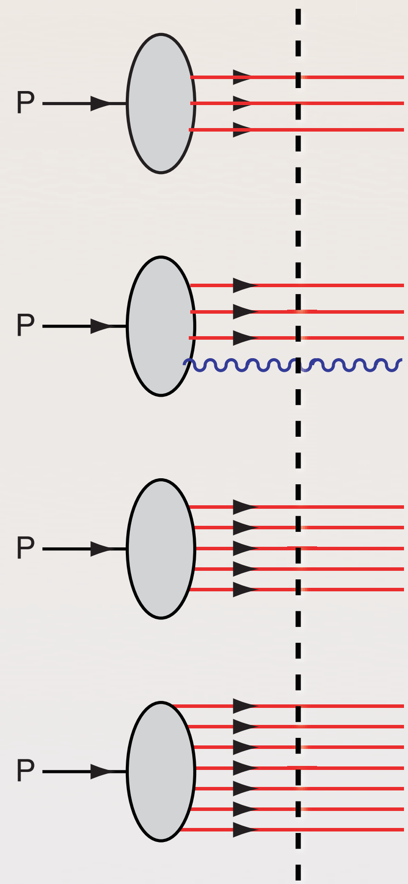
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

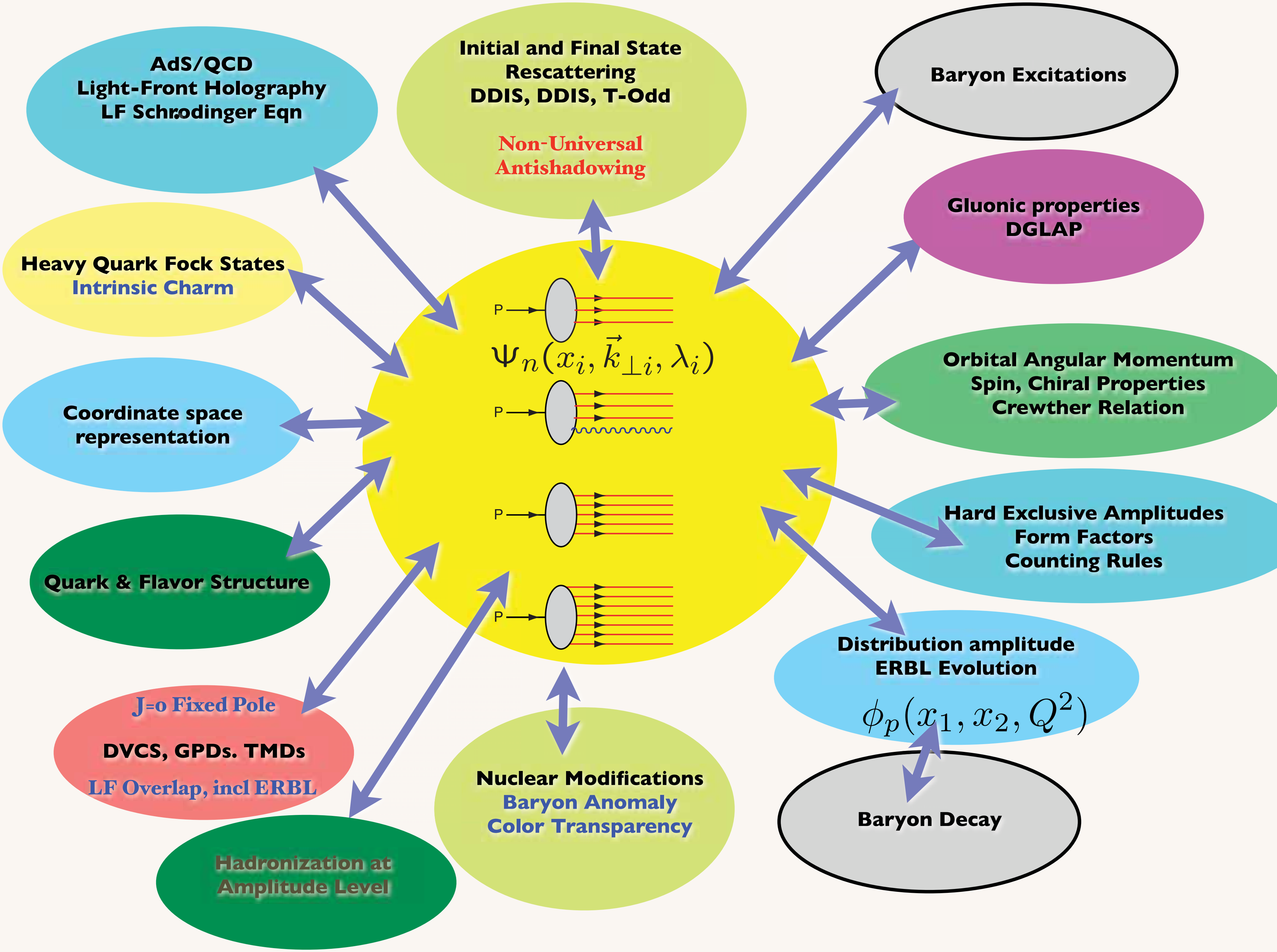
Intrinsic heavy quarks

$c(x), b(x)$ at high x

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

QCD and the LF Hadron Wavefunctions



Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved at every vertex
and
LF Fock state by Fock State!

LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

Light-Front QCD

Heisenberg Matrix Formulation

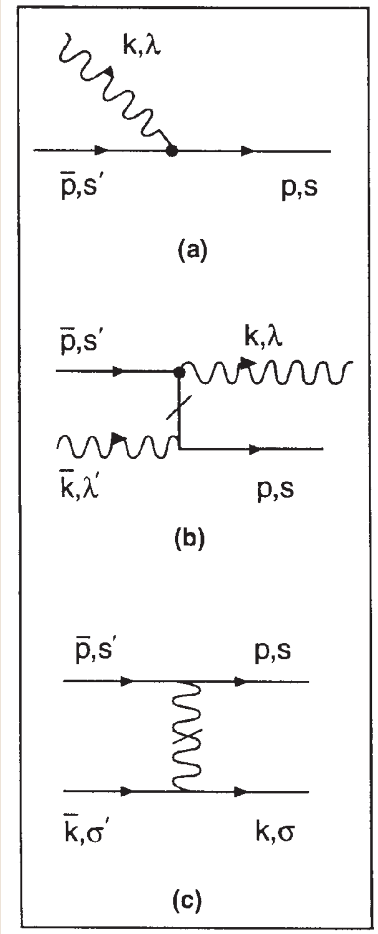
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

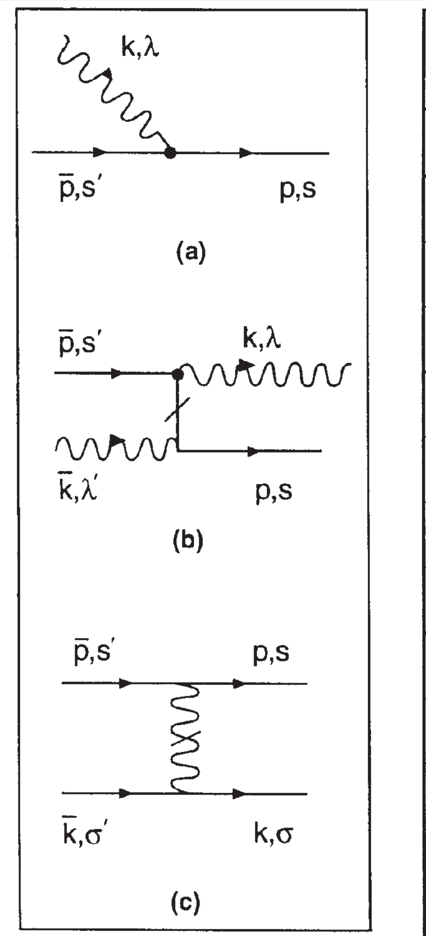
Light-Front QCD

Heisenberg Matrix Formulation

$$\boxed{H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>}$$

H.C. Pauli & sjb

Discretized Light-Cone Quantization



n	Sector	1 qq̄	2 gg	3 qq̄ g	4 qq̄ qq̄	5 gg g	6 qq̄ gg	7 qq̄ qq̄ g	8 qq̄ qq̄ qq̄	9 gg gg	10 qq̄ gg g	11 qq̄ qq̄ gg	12 qq̄ qq̄ qq̄ g	13 qq̄ qq̄ qq̄ qq̄
1	qq̄					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	qq̄ g								•	•		•	•	•
4	qq̄ qq̄		•			•				•	•		•	•
5	gg g	•			•			•	•			•	•	•
6	qq̄ gg								•				•	•
7	qq̄ qq̄ g	•	•			•				•				•
8	qq̄ qq̄ qq̄	•	•	•		•	•			•	•			
9	gg gg	•		•	•			•	•			•	•	•
10	qq̄ gg g	•	•		•				•				•	•
11	qq̄ qq̄ gg	•	•	•		•				•				•
12	qq̄ qq̄ qq̄ g	•	•	•	•	•	•			•	•			
13	qq̄ qq̄ qq̄ qq̄	•	•	•	•	•	•	•		•	•	•		

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

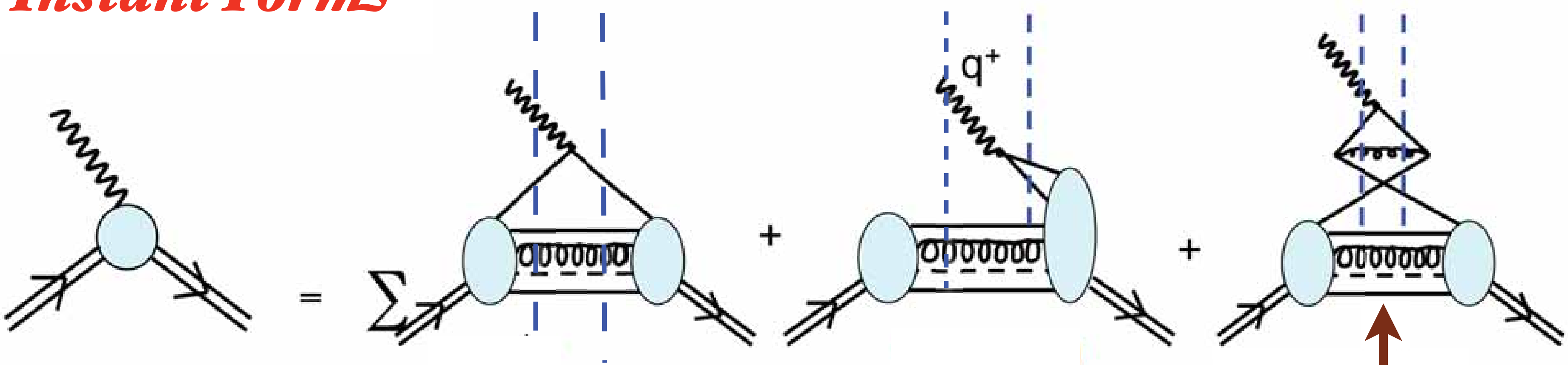
DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

Light-Front QCD Features and Phenomenology

- Trivial Vacuum
- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

Calculation of Form Factors in Equal-Time Theory

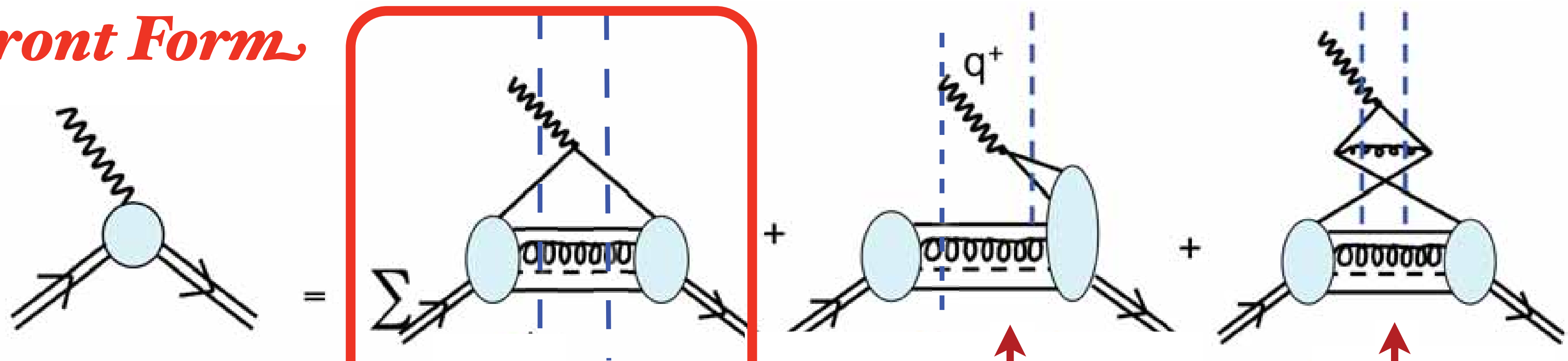
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

Absent for $q^+ = 0$

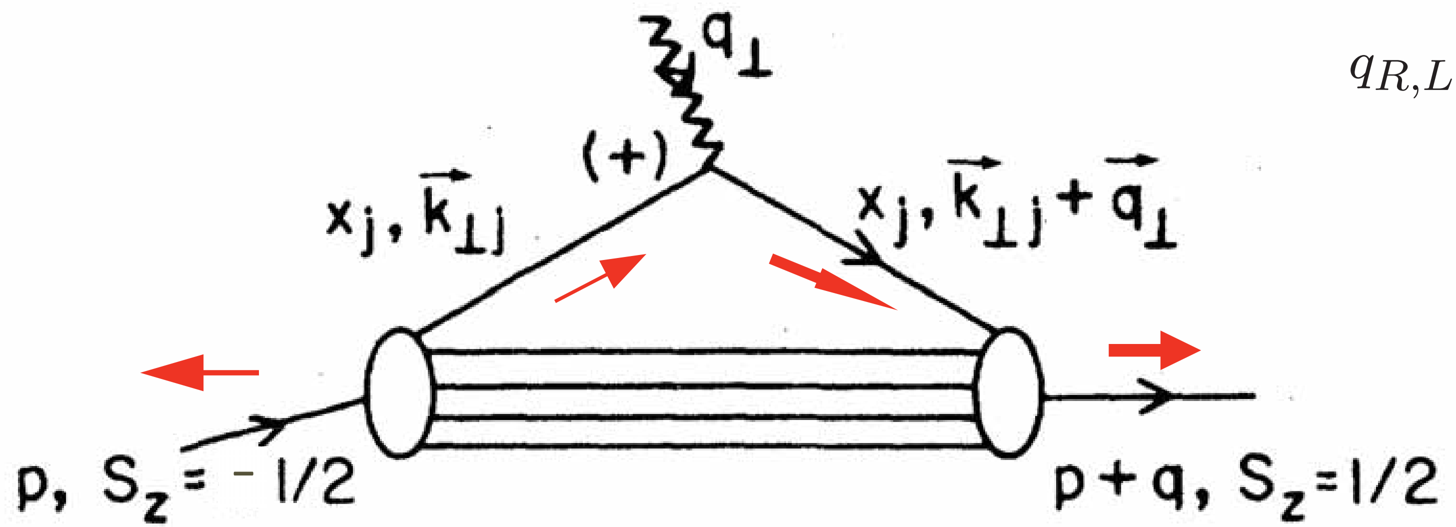
zero !!

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$

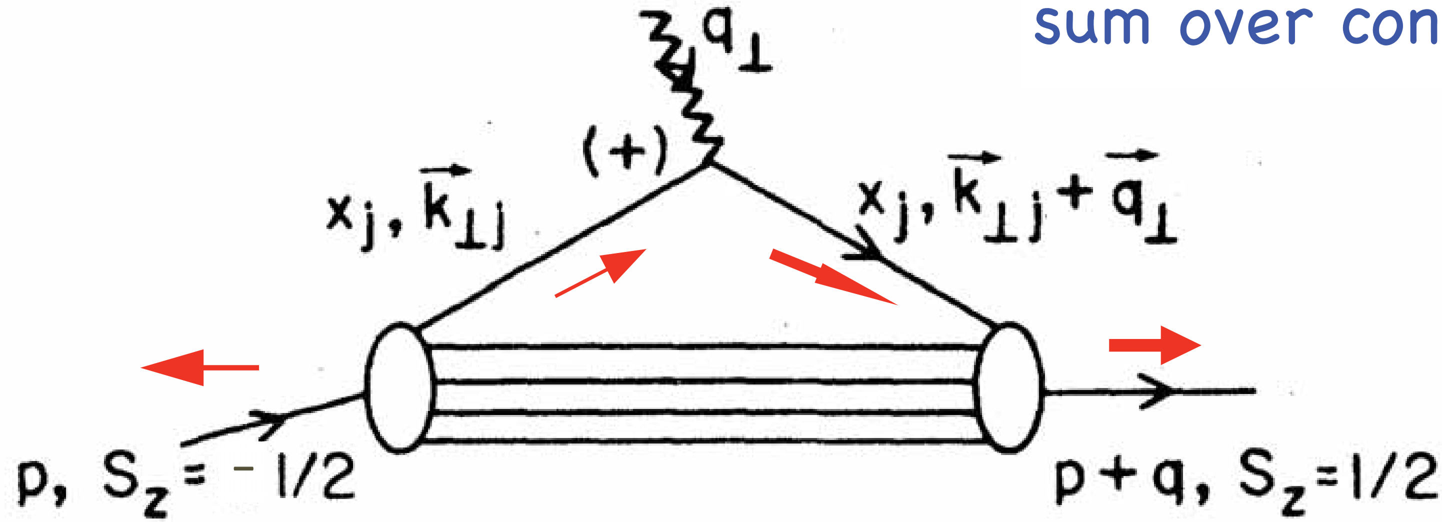
Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

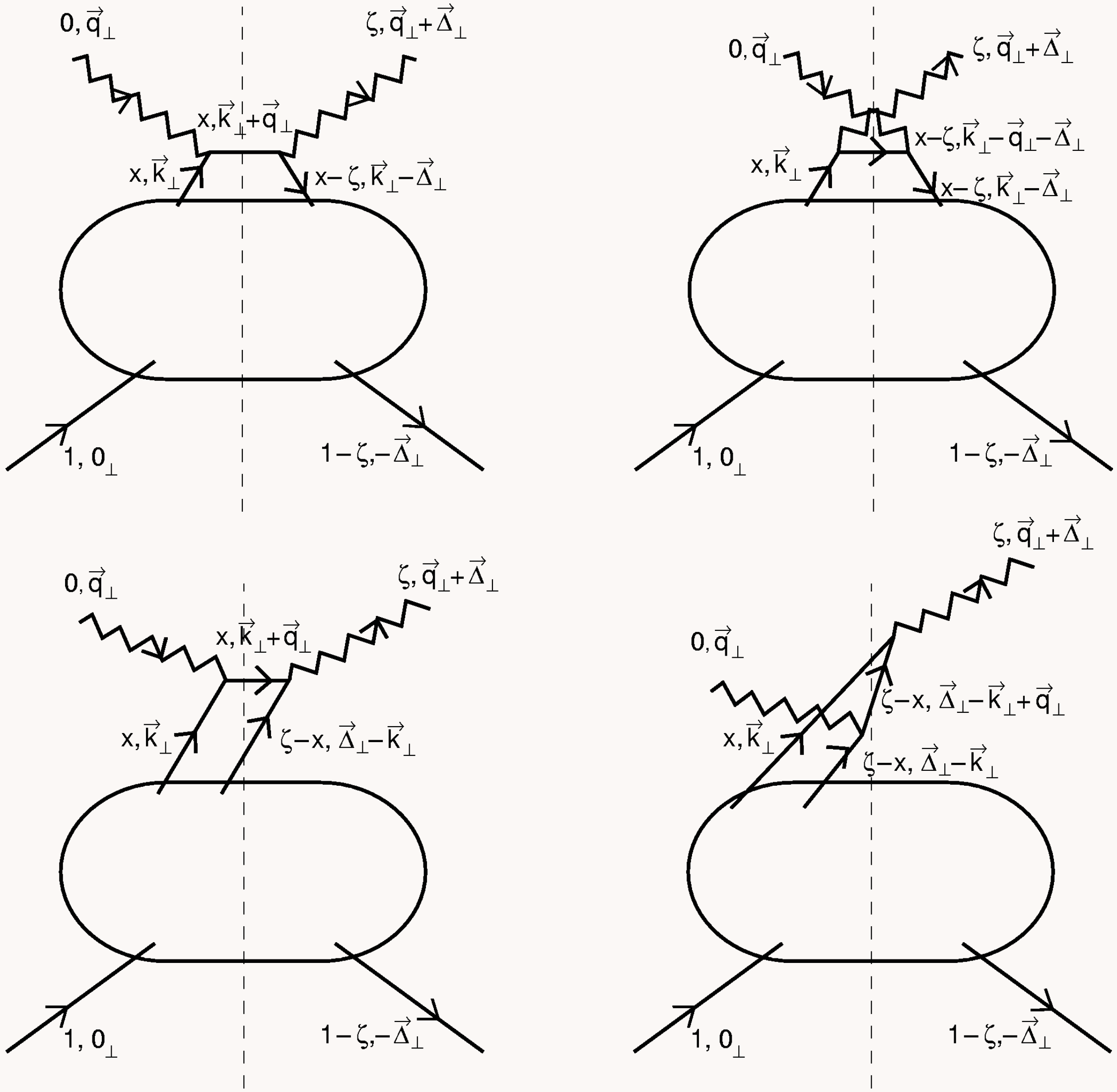
sum over constituents



Hwang, Schmidt, sjb; Holstein et al

$\boxed{B(0) = 0}$

Each Fock State



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M}E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n}\sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\,16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$

$$\times\,\delta(x-x_1)\,\psi^{\uparrow *}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\,\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2,\ldots,n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t) \right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t) \right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

Quark angular momentum (Ji's sum rule)

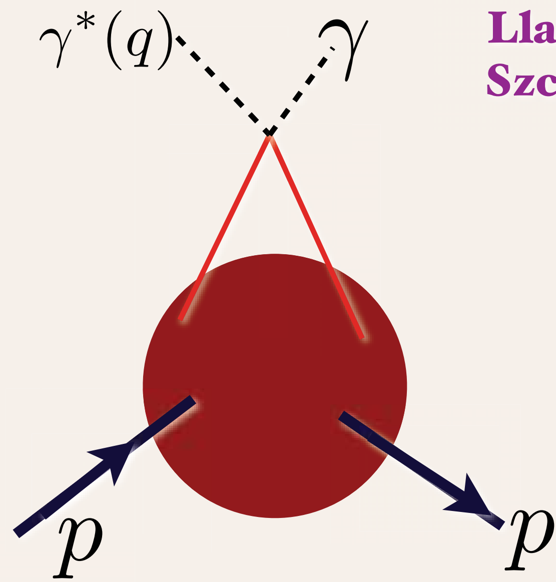
$$J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0) \right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

J=0 Fixed pole in real and virtual Compton scattering

Damashek, Gilma
Close, Gunion, sjb
Llanes-Estrada,
Szczepaniak, sjb

Effective two-photon contact term



Seagull for scalar quarks

Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of Q² at fixed t

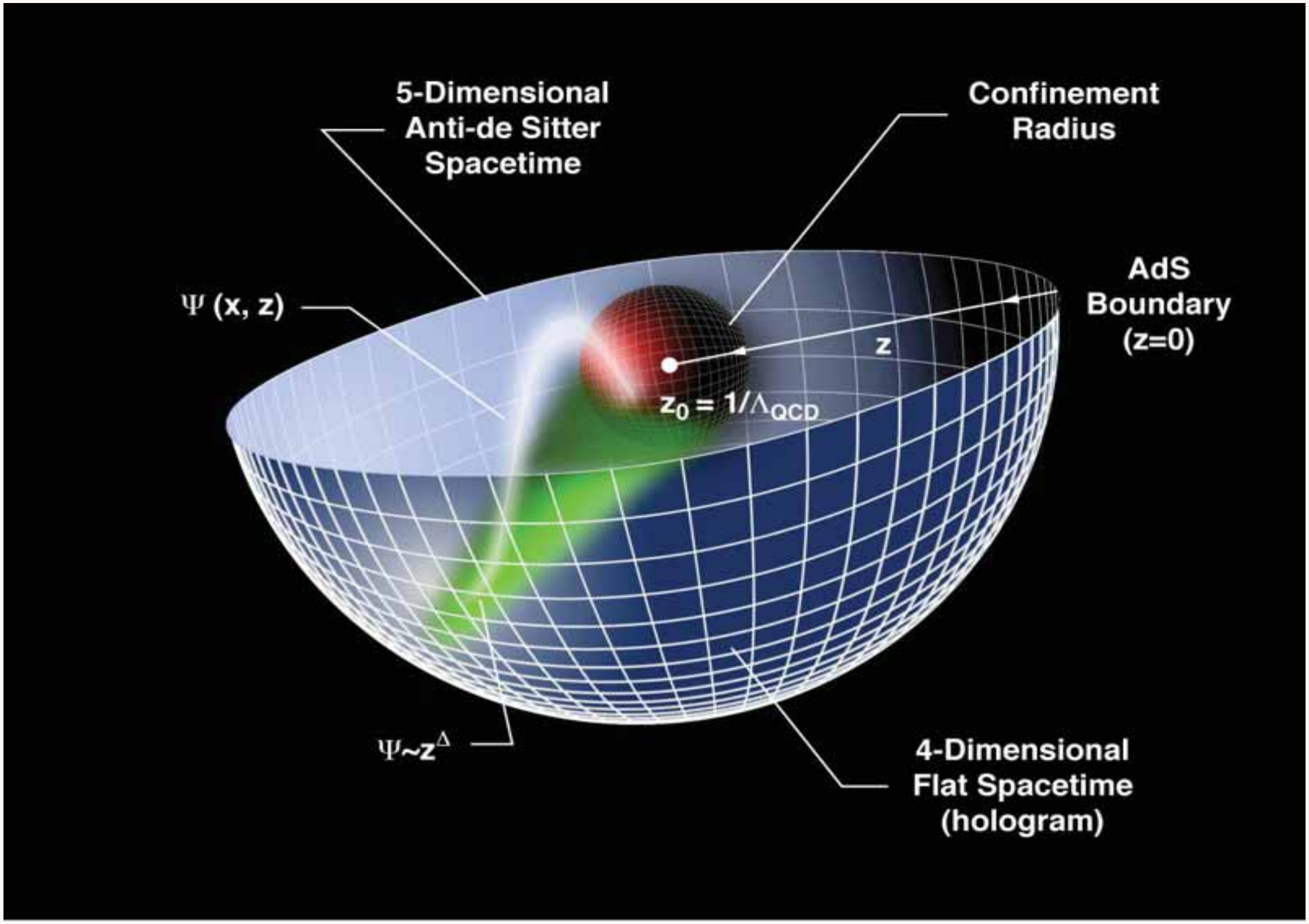
<1/x> Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

Q^2-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt}(\gamma^* p \to \gamma p) = F^2(t)$$

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

- **Analogous to the Schrödinger Theory for Atomic Physics**

- *AdS/QCD Light-Front Holography*

- *Hadronic Spectra and Light-Front Wavefunctions*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

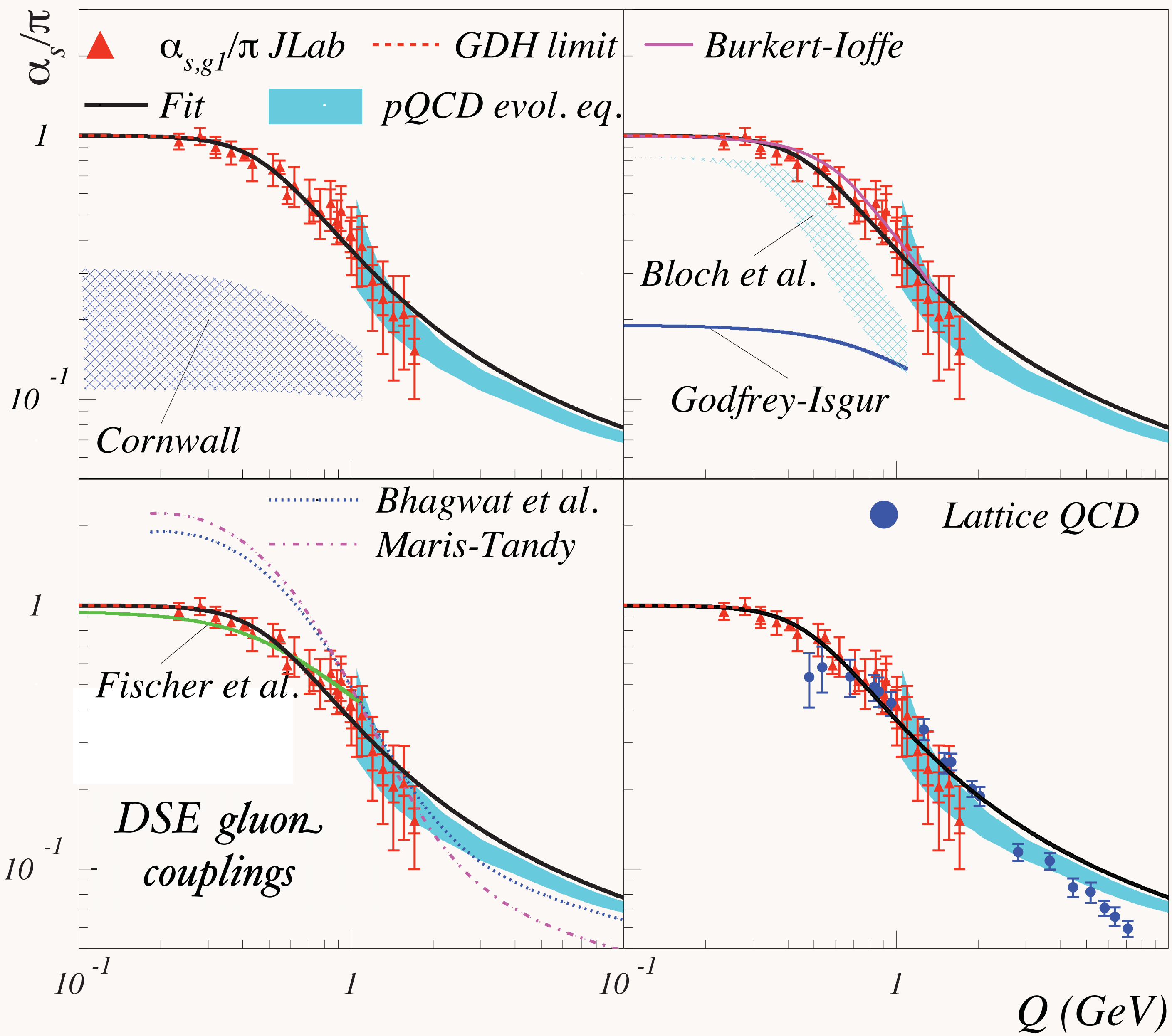
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

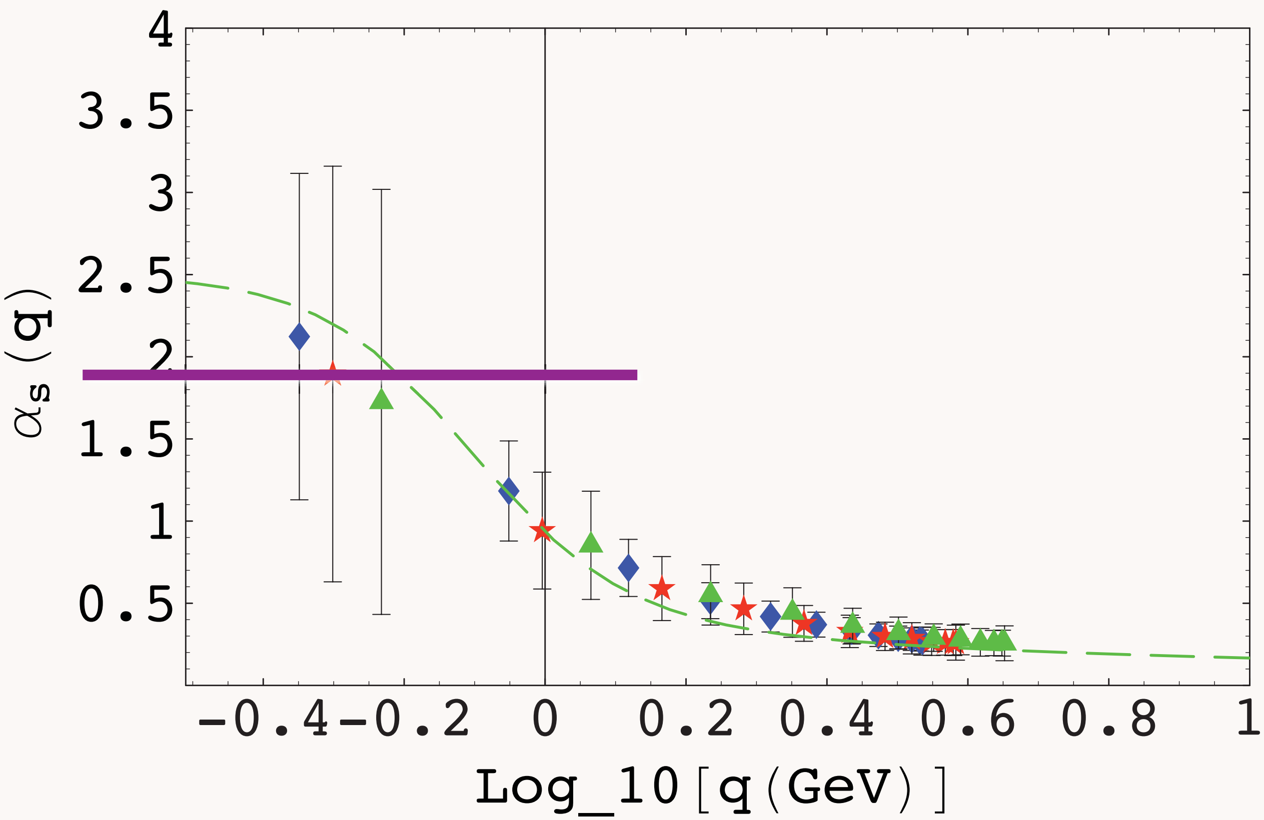
Deur, Korsch, et al.



Conformal Behavior of QCD in Infrared

- Does α_s develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .

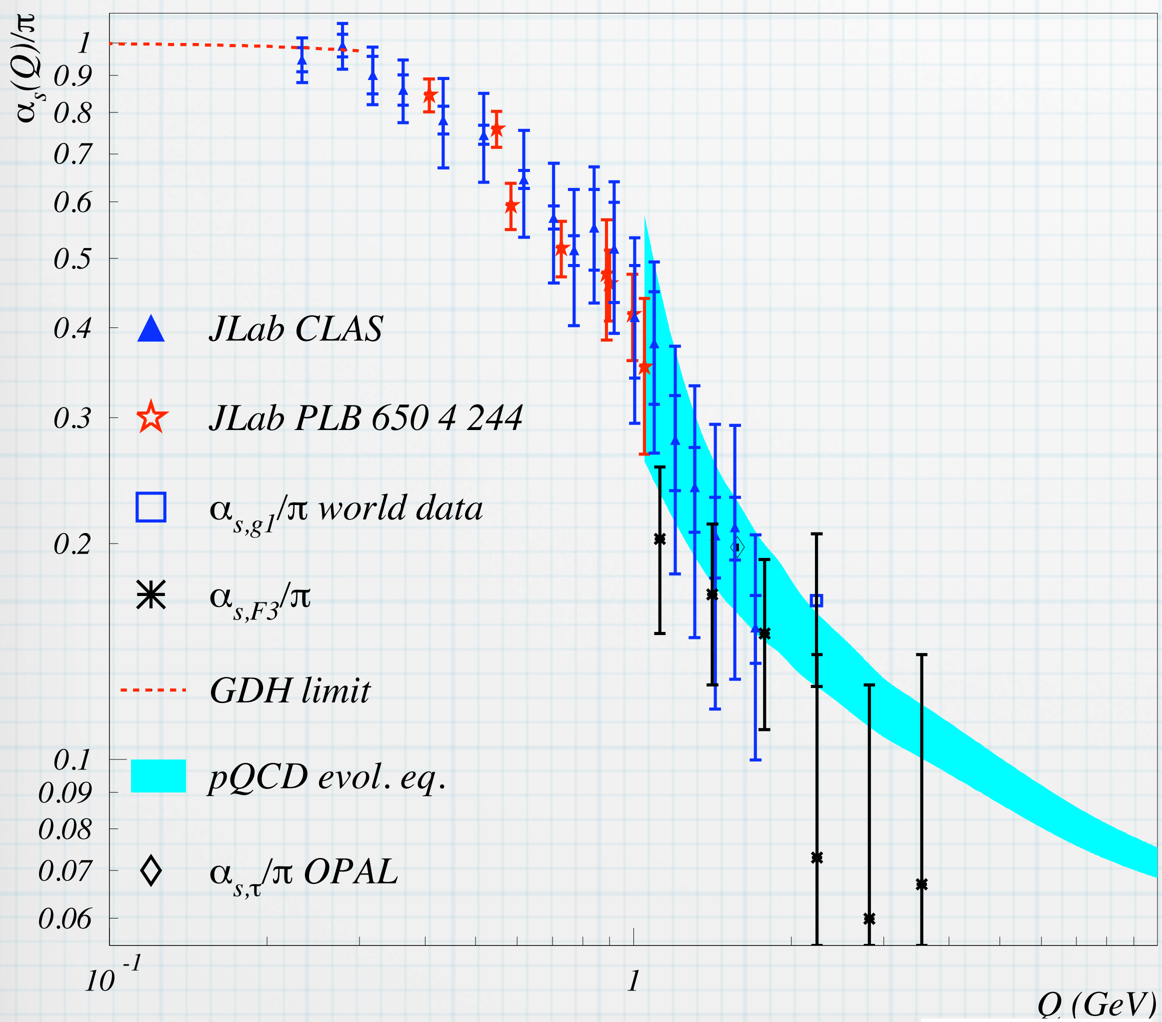
- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).



Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx\, \left(g_1^p(x,Q^2) - g_1^n(x,Q^2)\right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi}\right)$$



g1 = spin dependent structure function (from inelastic ep scattering

Data from EG1 exp., at JLab CLAS (2008)

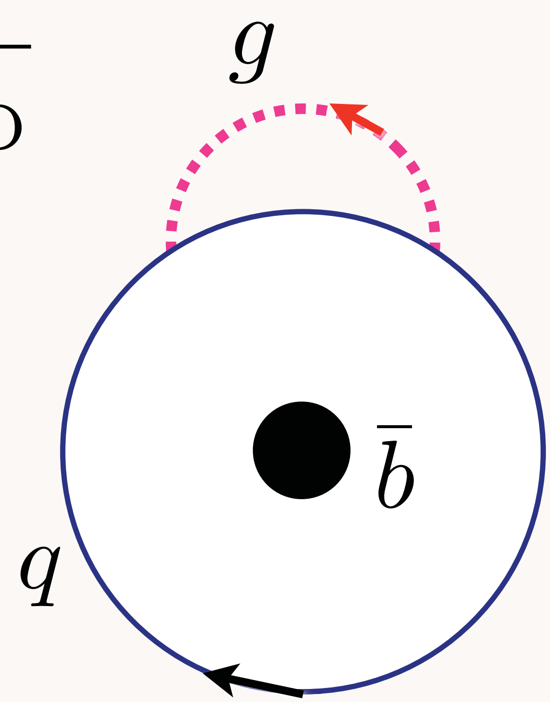
α_s runs only modestly at small Q^2

Fig. from 0803.4119, Duer et al.

Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{QCD}} \qquad \lambda < \Lambda_{QCD}$$



B-Meson

gluon and quark propagators cutoff in IR
because of color confinement

R. Shrock, sjb

Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain** $(x-y)^2 < \Lambda^{-2}_{QCD}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

Shrock, sjb

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

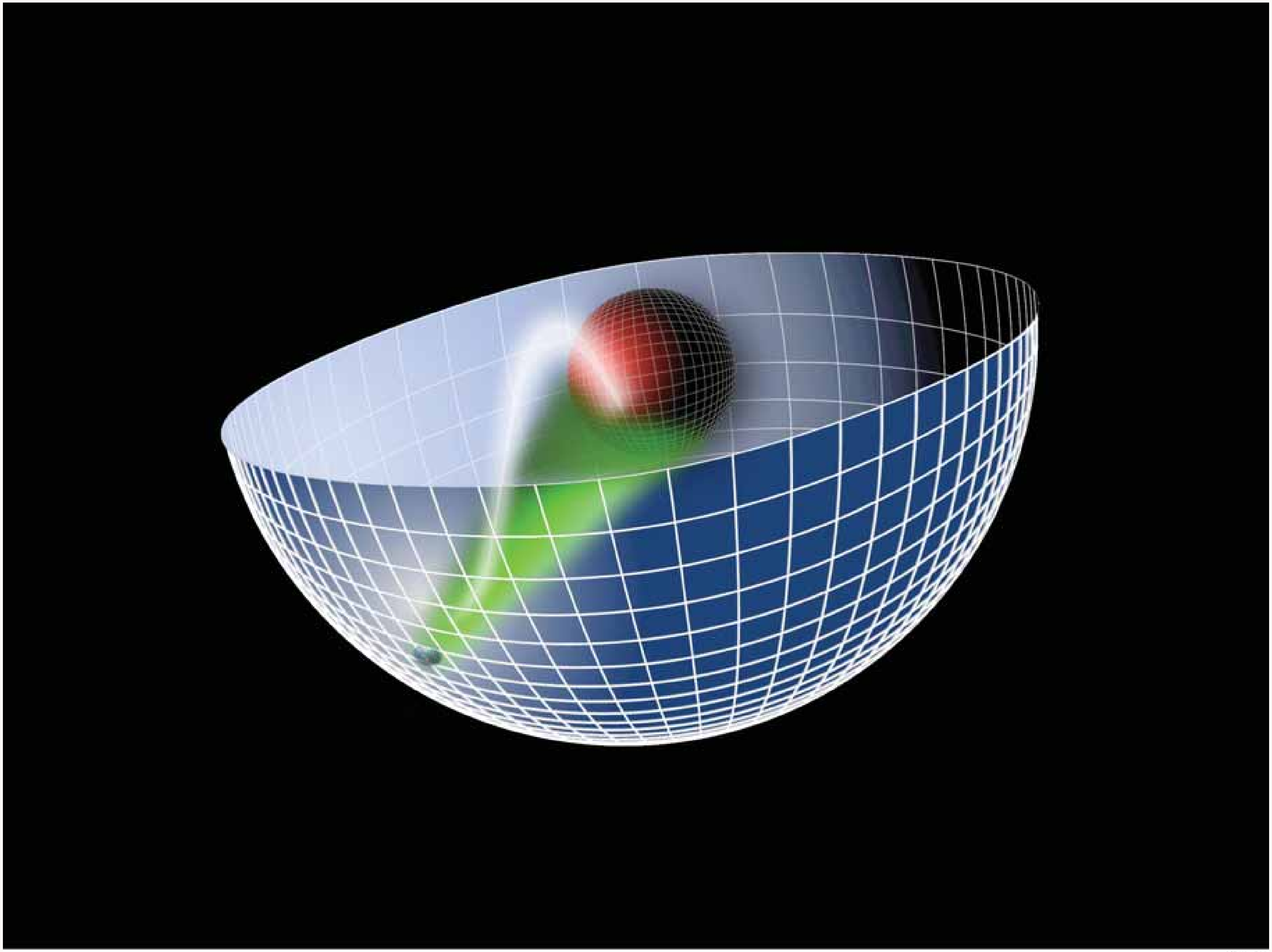
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

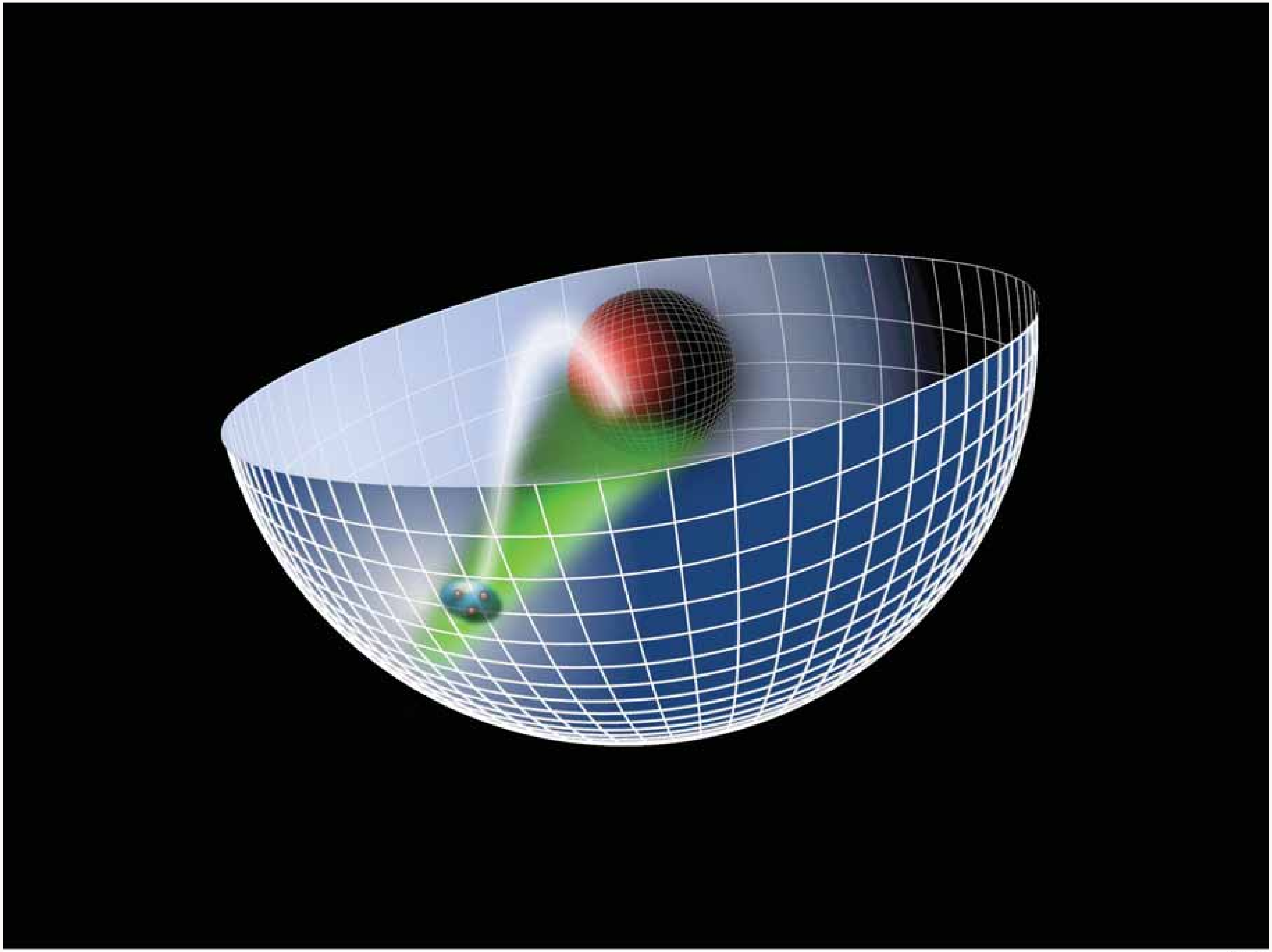
- Different values of z correspond to different scales at which the hadron is examined.

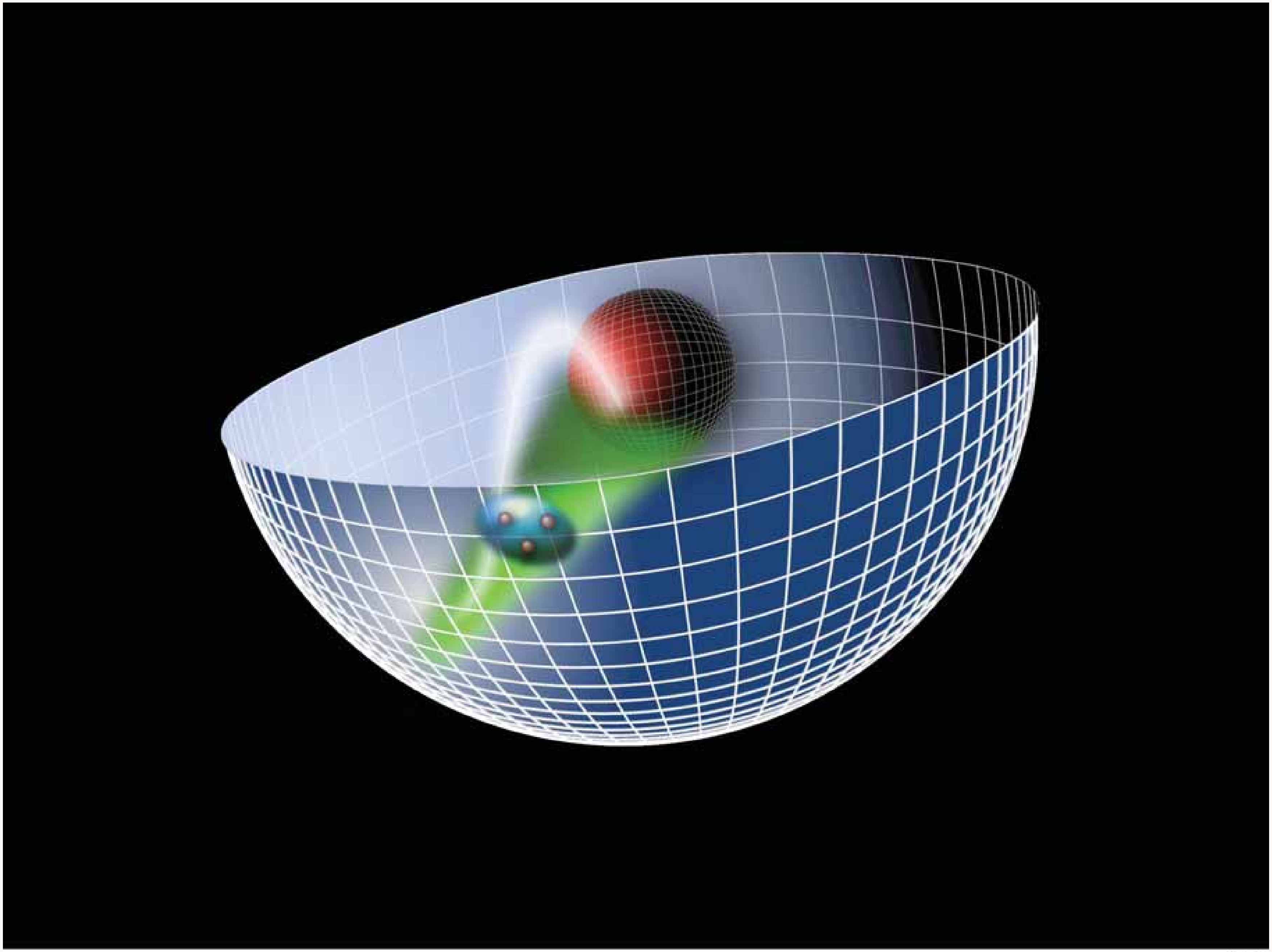
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

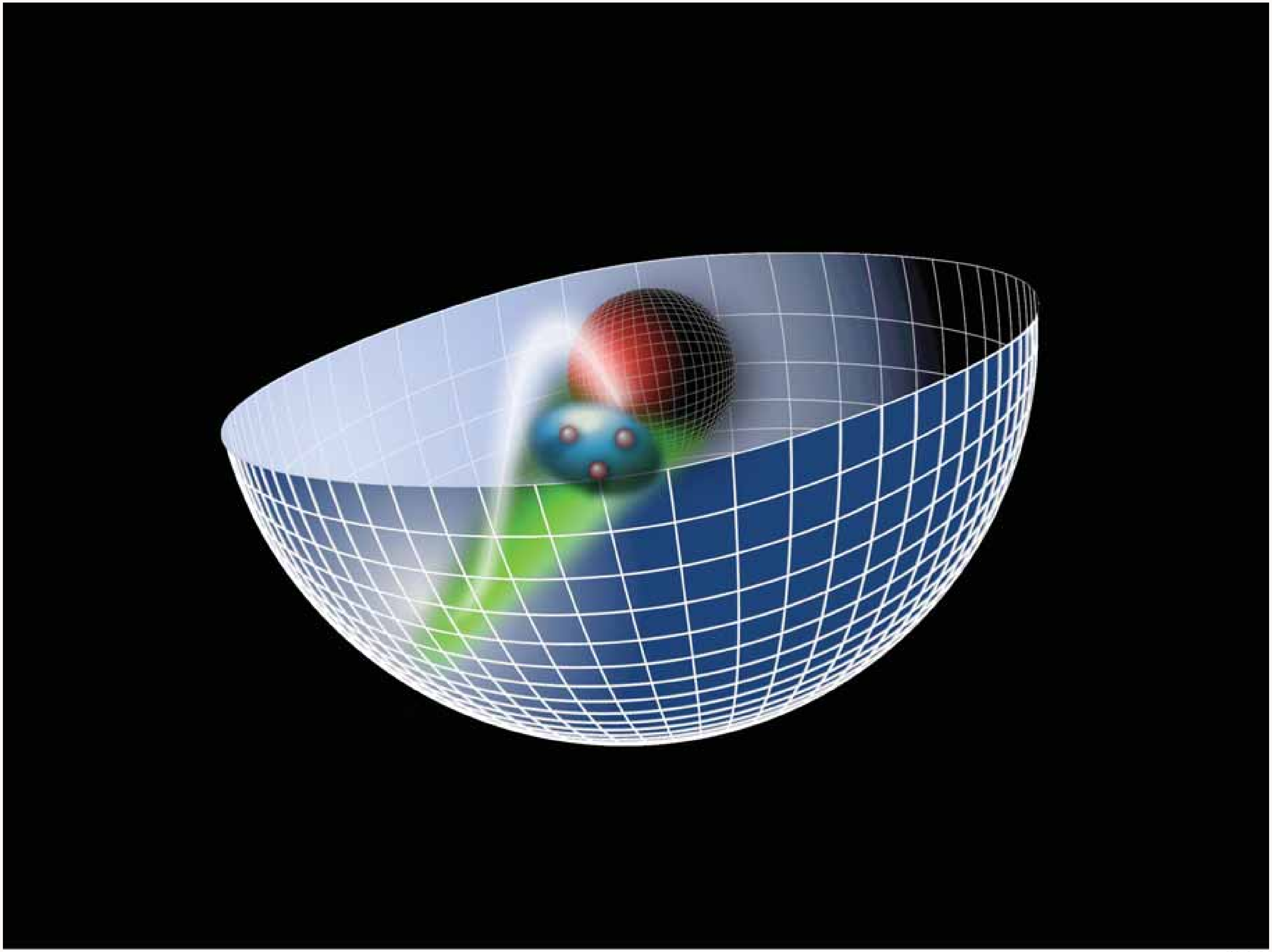
$x^2 = x_\mu x^\mu$: invariant separation between quarks

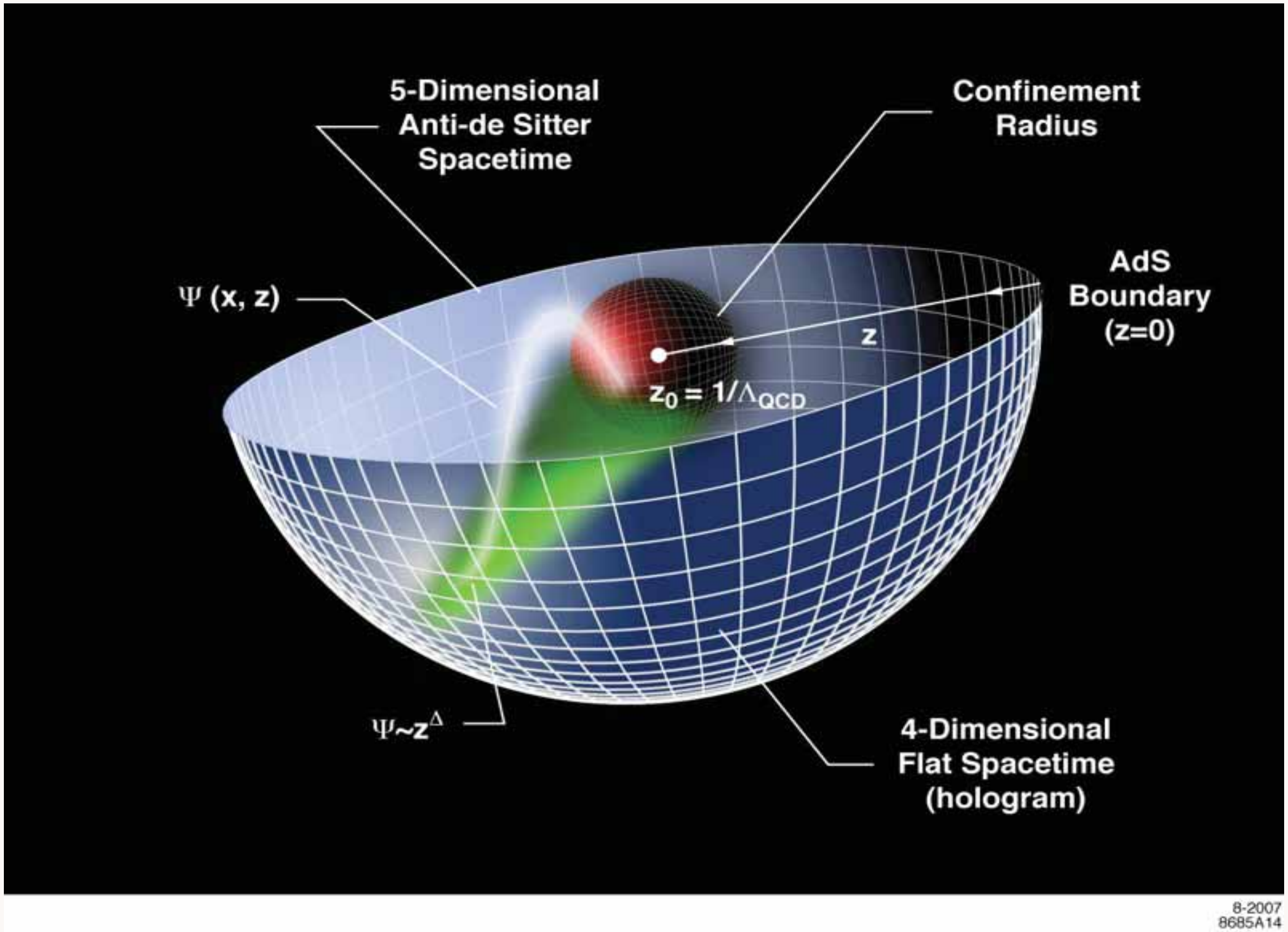
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.











- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2 \qquad z \to \lambda z$
- Match solutions at small z to conformal twist dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS_{d+1}:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\boxed{\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.}$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

L: light-front orbital angular momentum

Derived from variation of Action in AdS_5

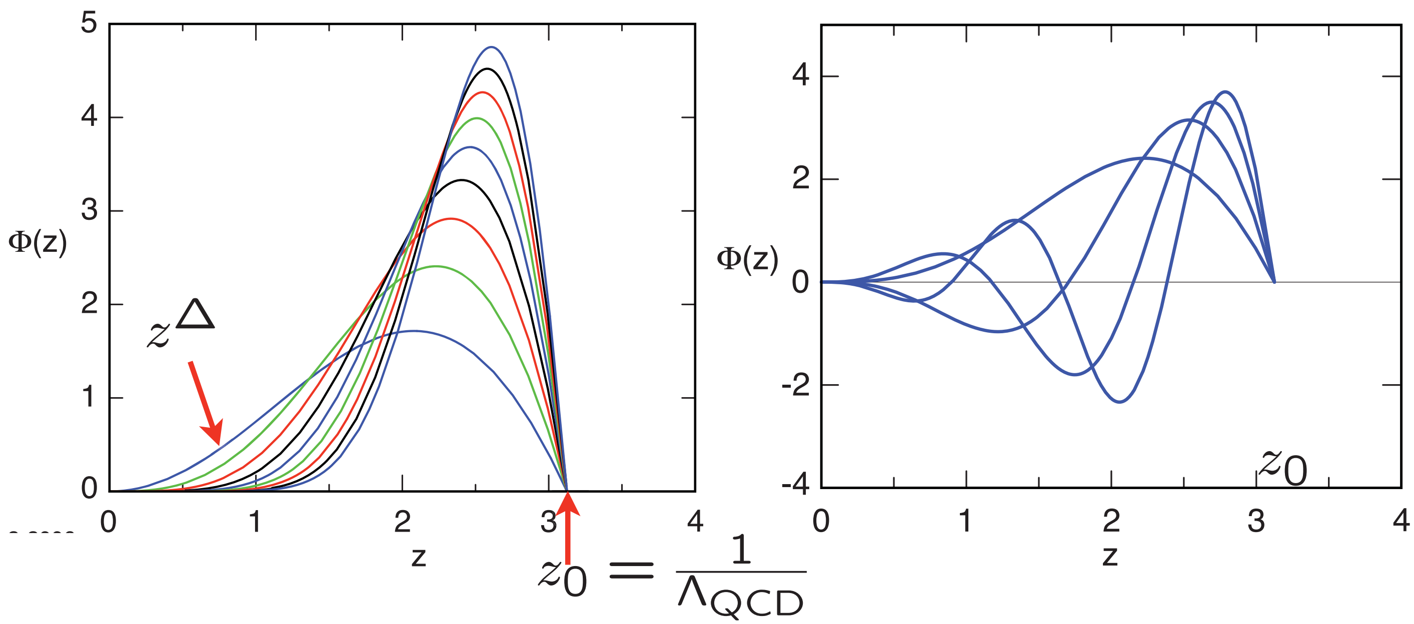
Hard wall model: truncated space

$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_\mathrm{c}}) = 0.$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

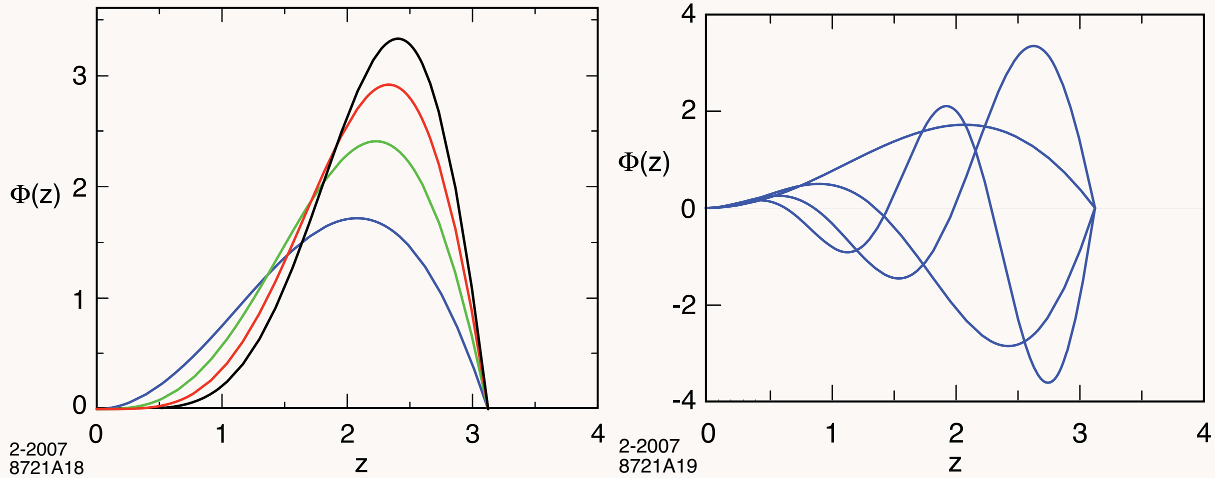


Fig: Orbital and radial AdS modes in the hard wall model for Λ_{QCD} = 0.32 GeV .

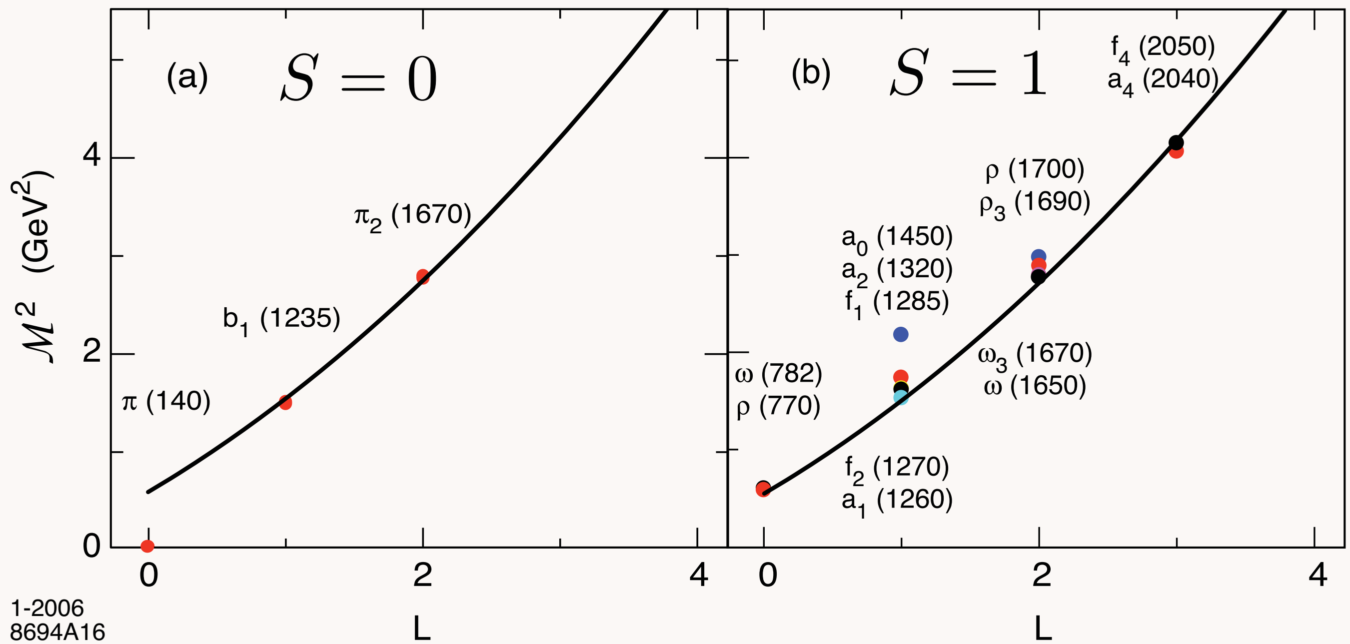


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Soft-Wall Model

$$S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \qquad \varphi(z) = \pm\kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2\partial_z^2 - \left(3 \mp 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

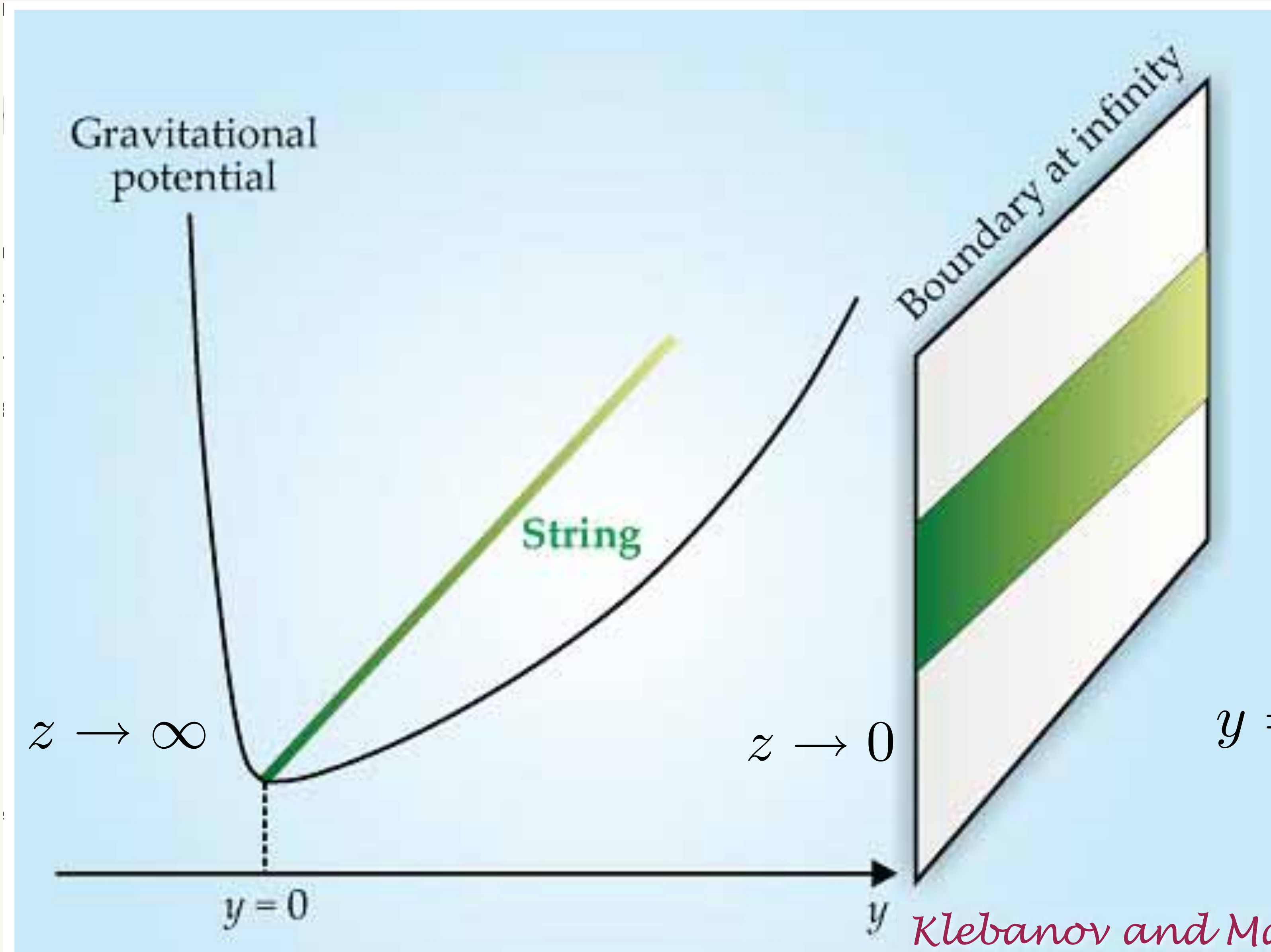
- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$

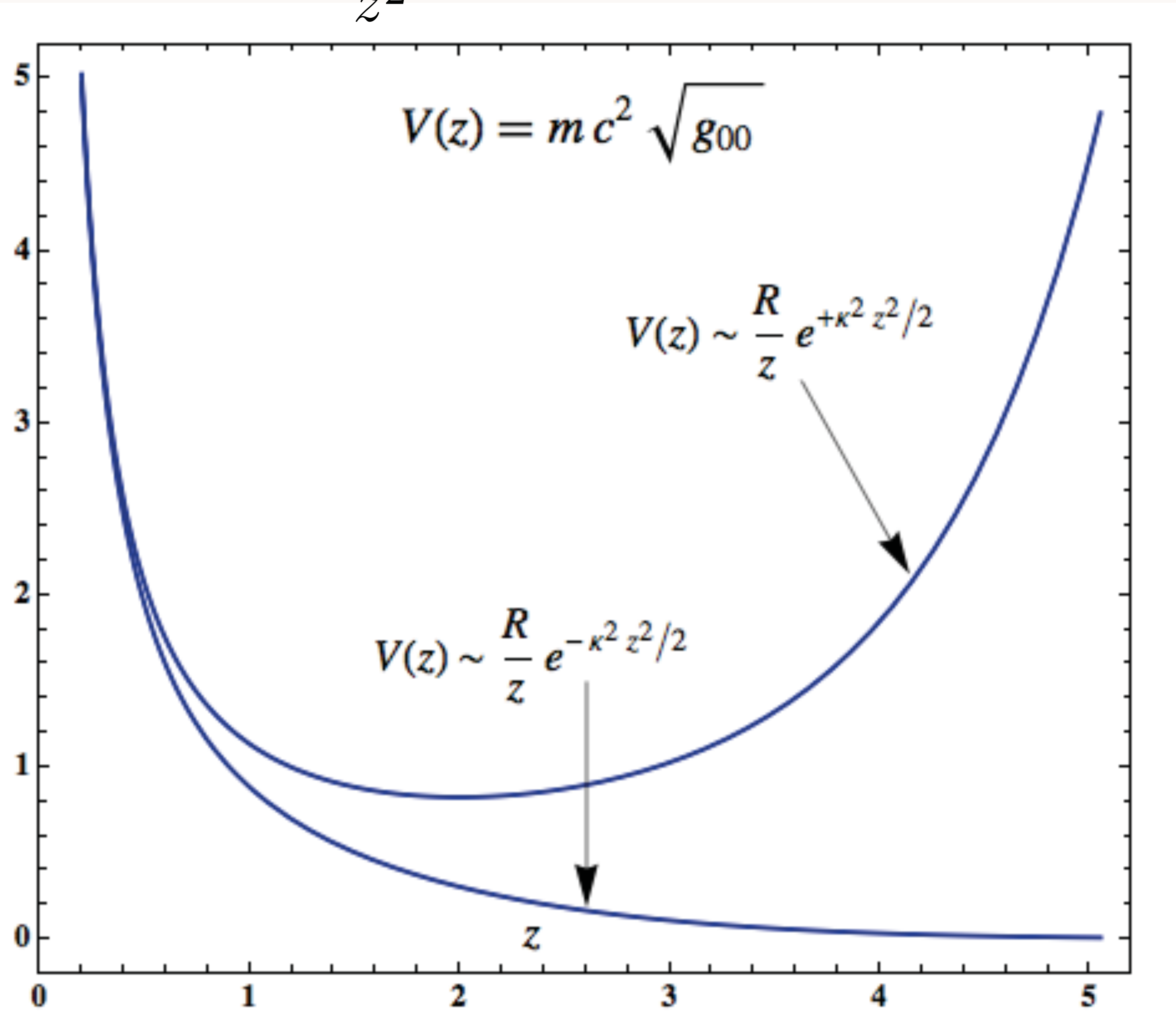


$y = R/z$

Klebanov and Maldacena

$$ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2$$

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2}(dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$



Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action Dilaton-Modified AdS_5

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

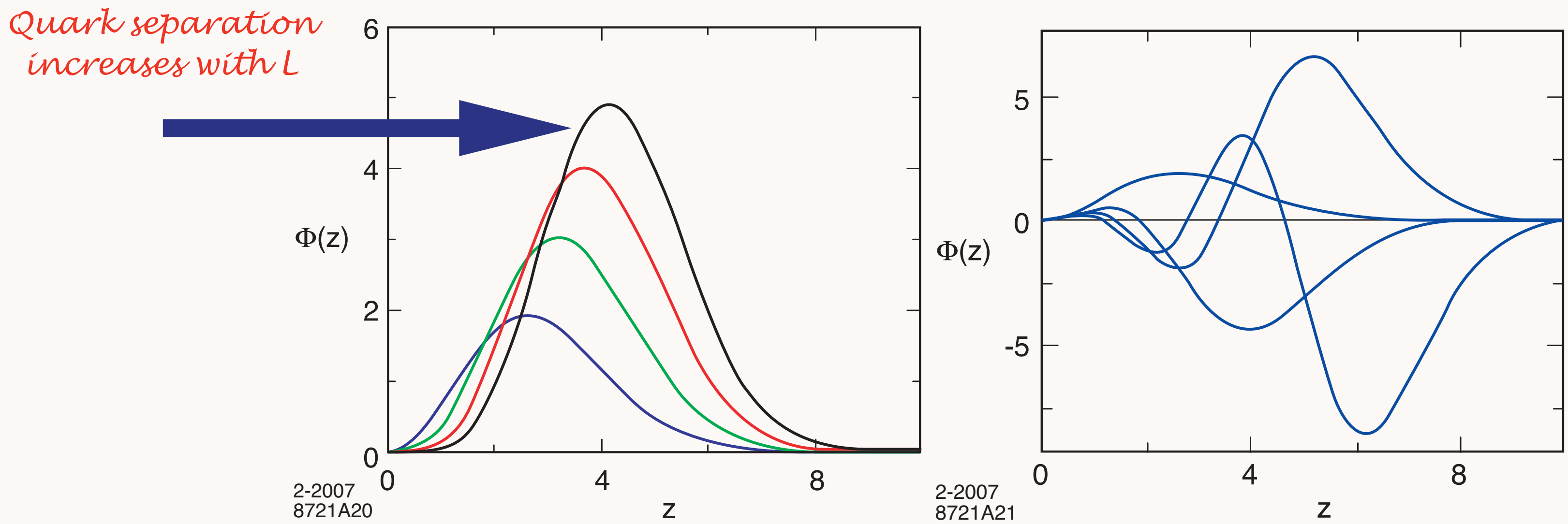
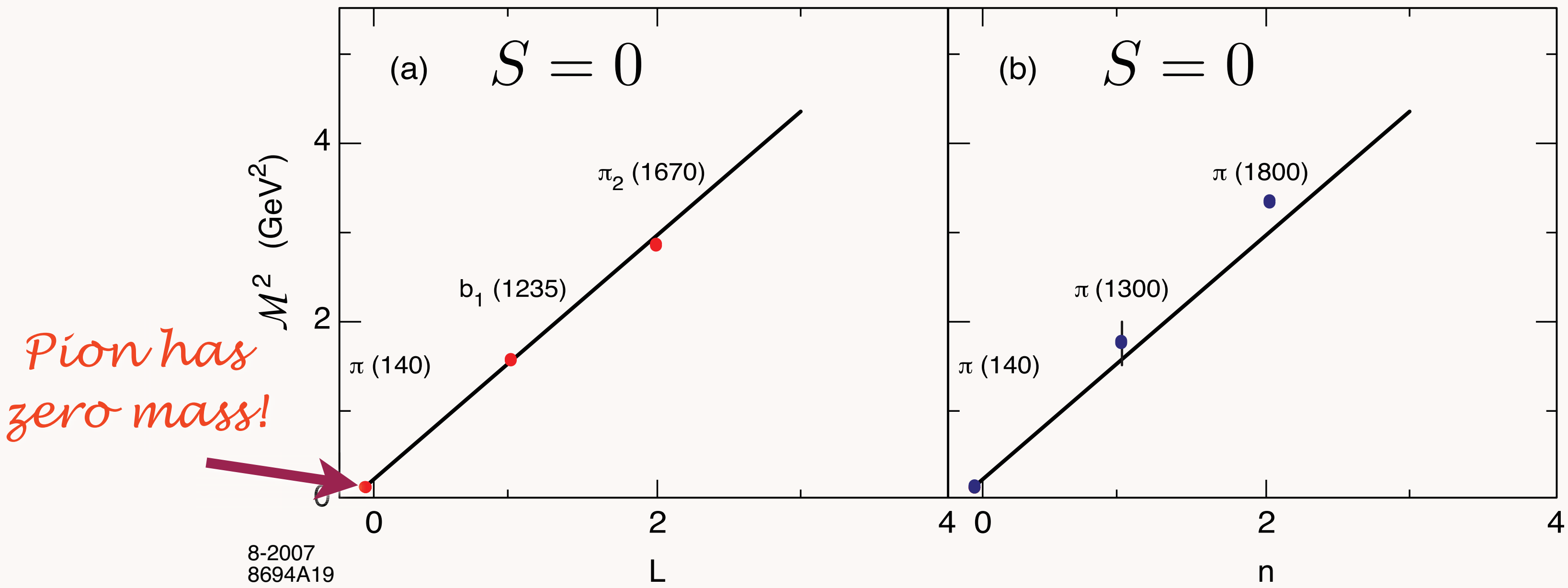


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

$$m_q = 0$$

Higher-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

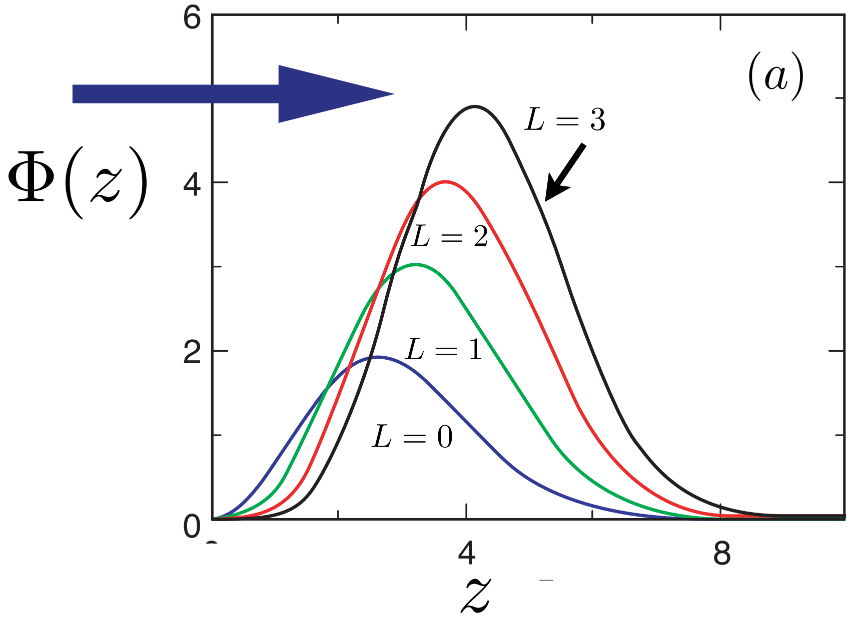
$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

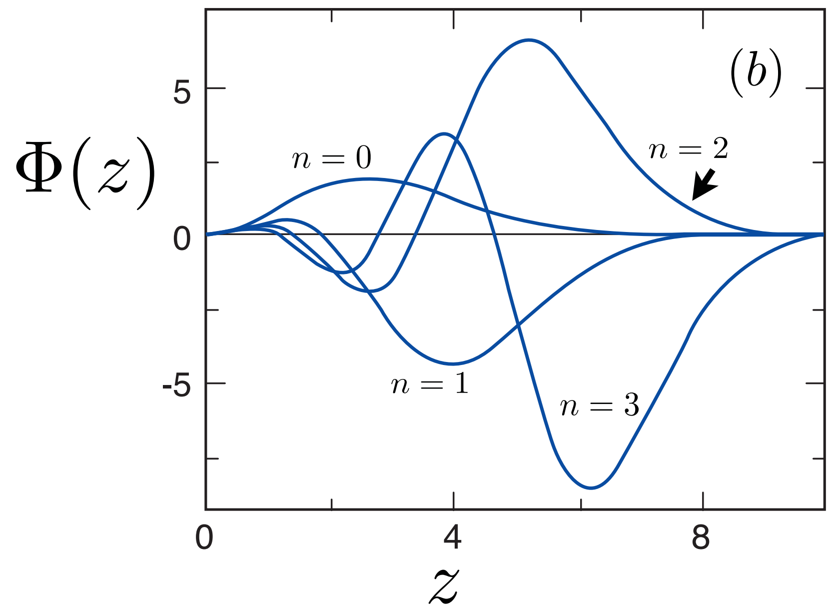
we find the LF wave equation

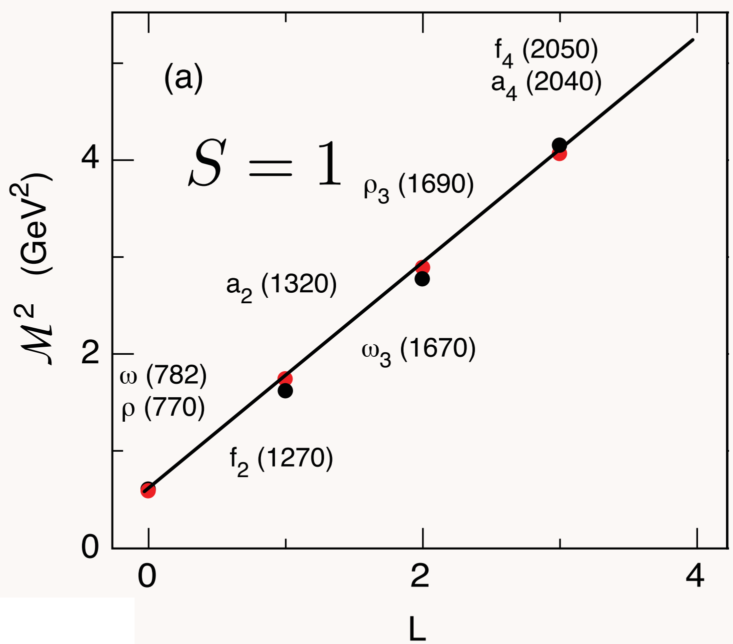
$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

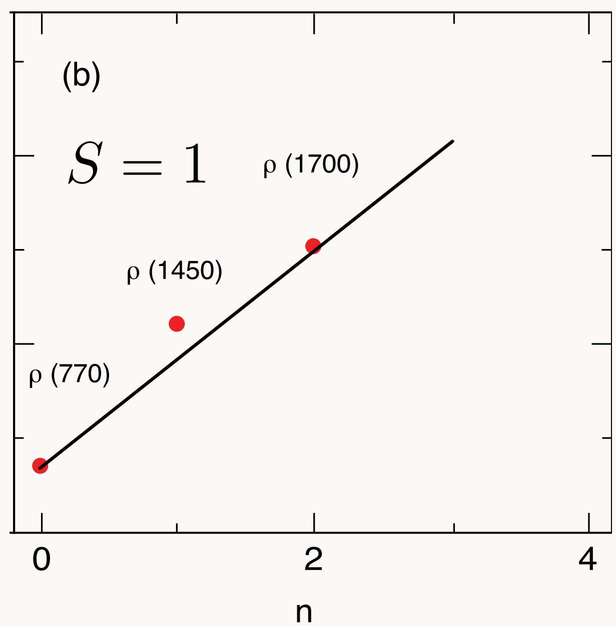
with $(\mu R)^2 = -(2 - J)^2 + L^2$

Quark separation increases with L



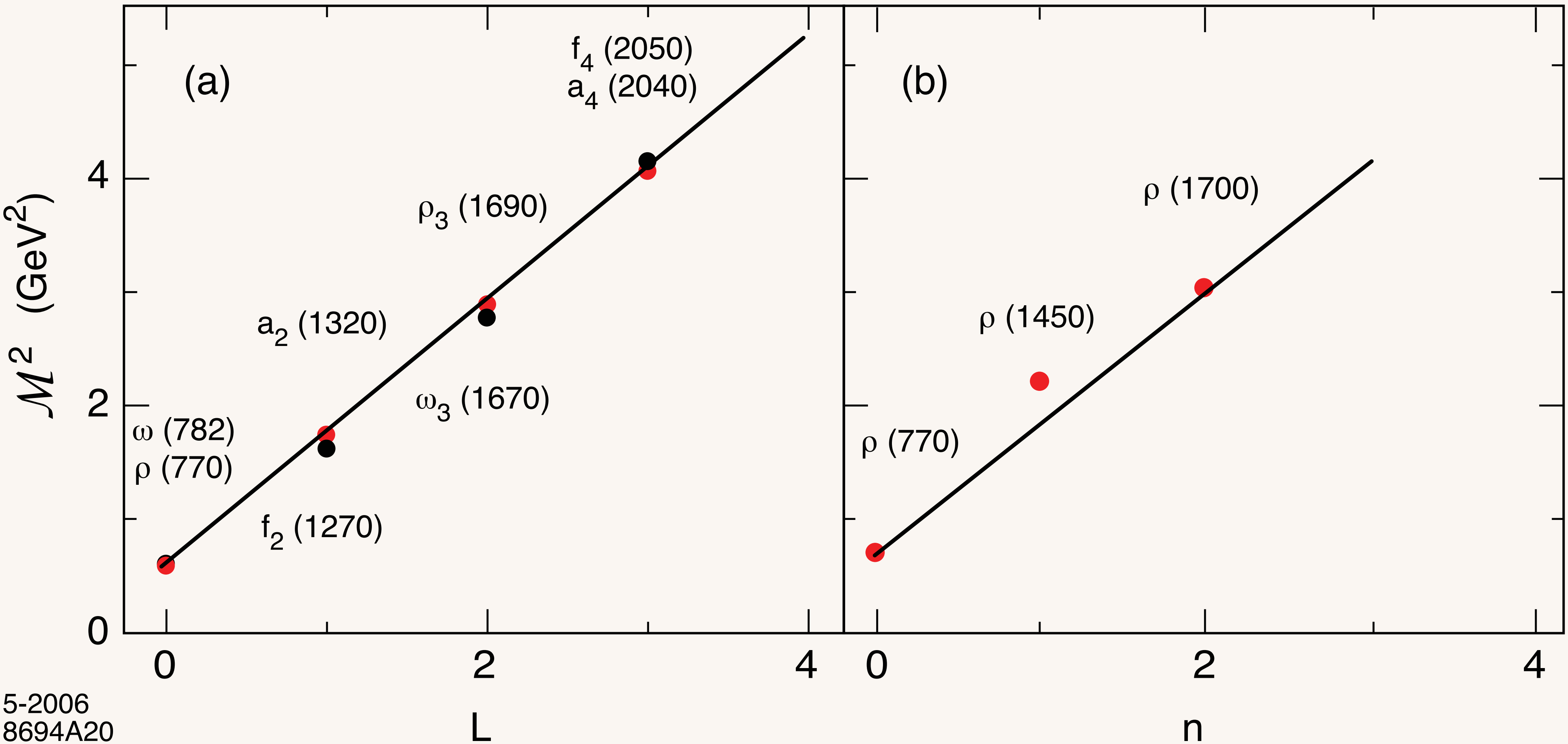


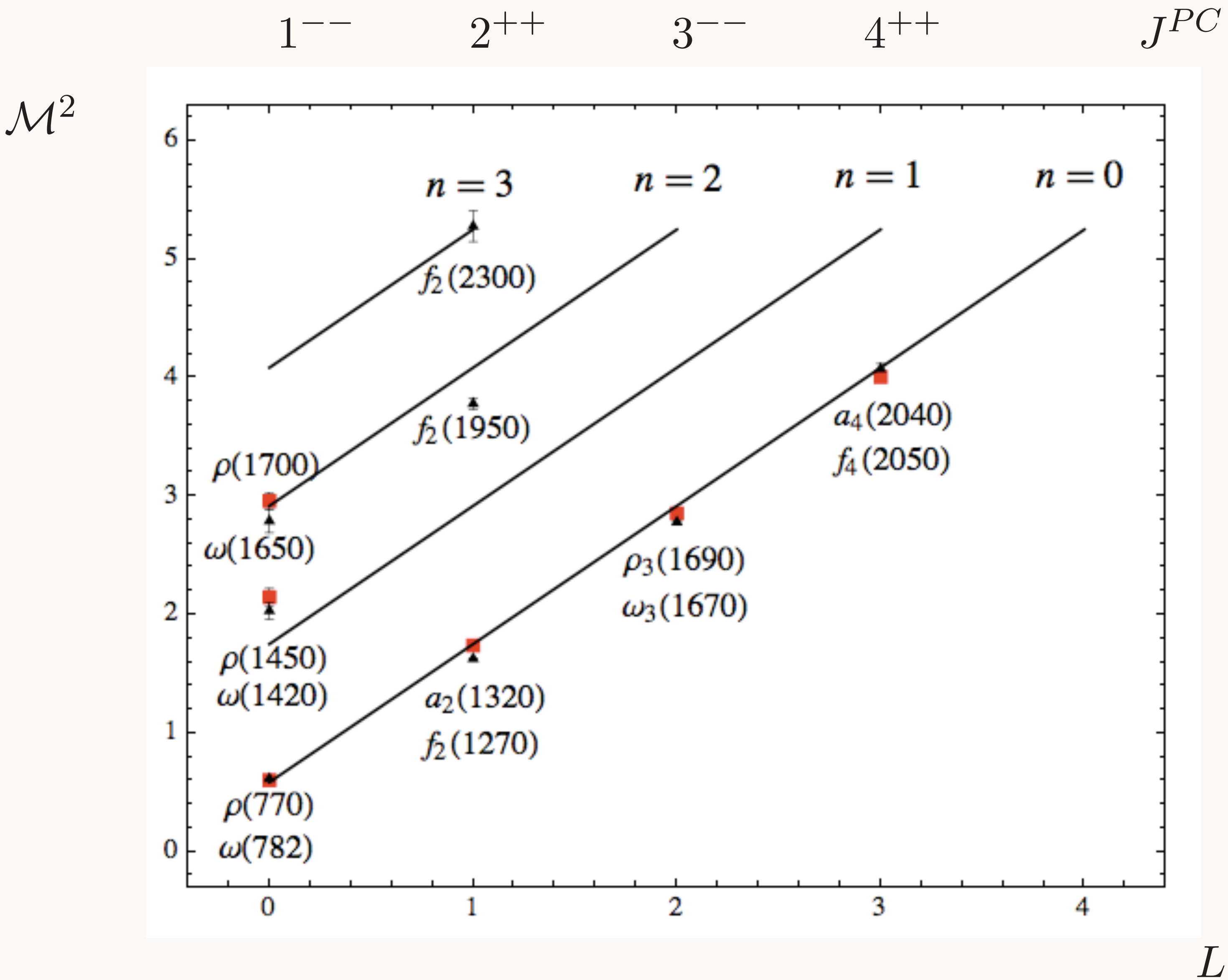




$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$





Parent and daughter Regge trajectories for the $I=1$ ρ-meson family (red)

and the $I=0$ ω-meson family (black) for $\kappa=0.54$ GeV

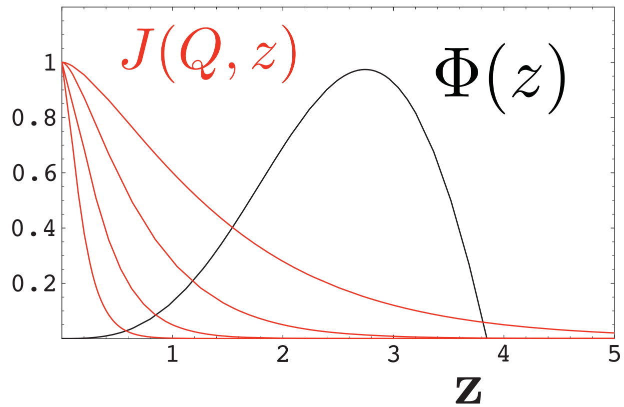
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



Polchinski, Strassler
de Teramond, sjb

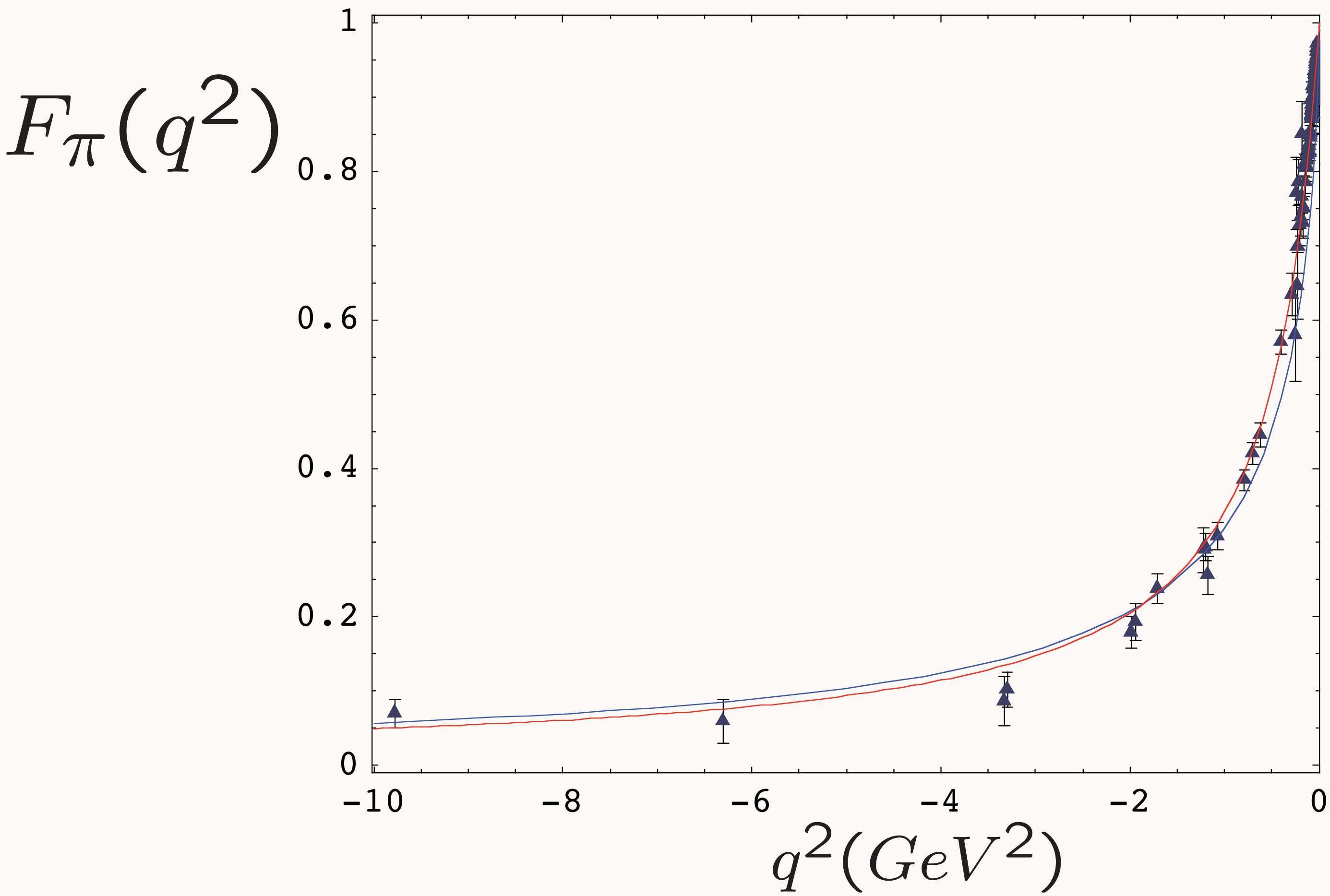
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp) \, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp \, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp \, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3}\, J(Q^2, z)\, |\Phi_{\pi^+}(z)|^2\,,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx\, J_0\!\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3}\, J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

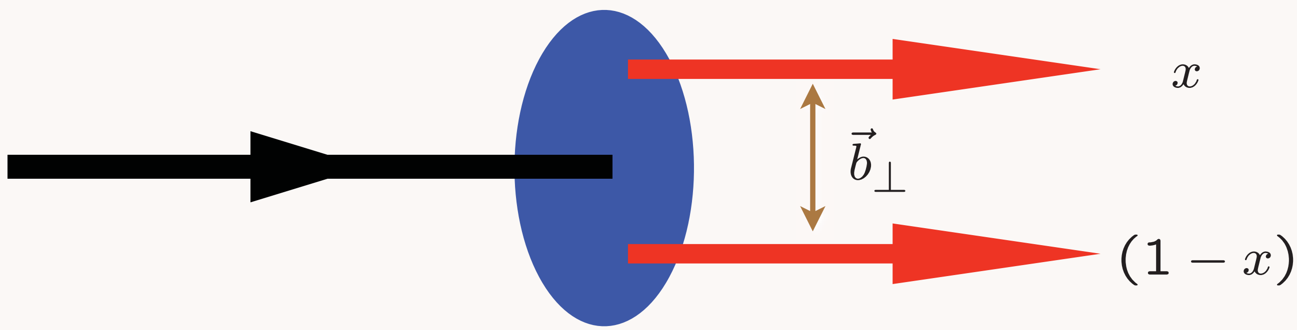
$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

with $\zeta = z,\ 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) $\quad\quad$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements