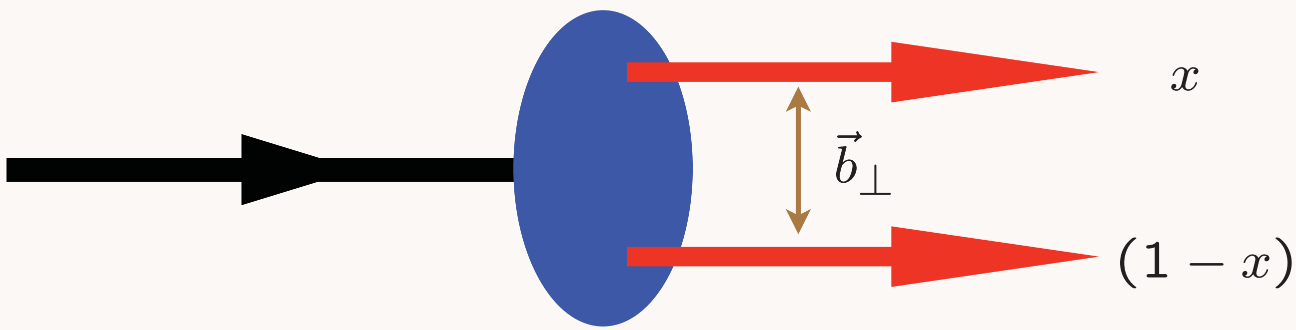
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



$$U(z)=\kappa^4 z^2+2\kappa^2(L+S-1)$$

soft wall confining potential

G. de Teramond, sjb

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

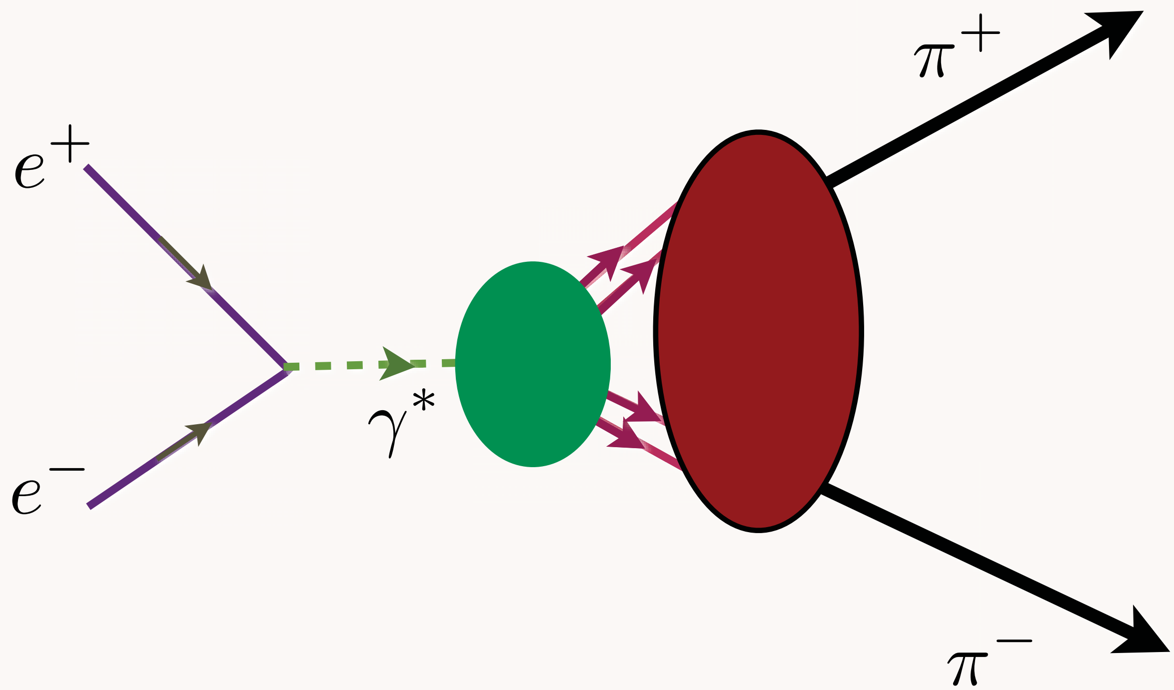
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current bring in higher Fock states and more vector meson poles



Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

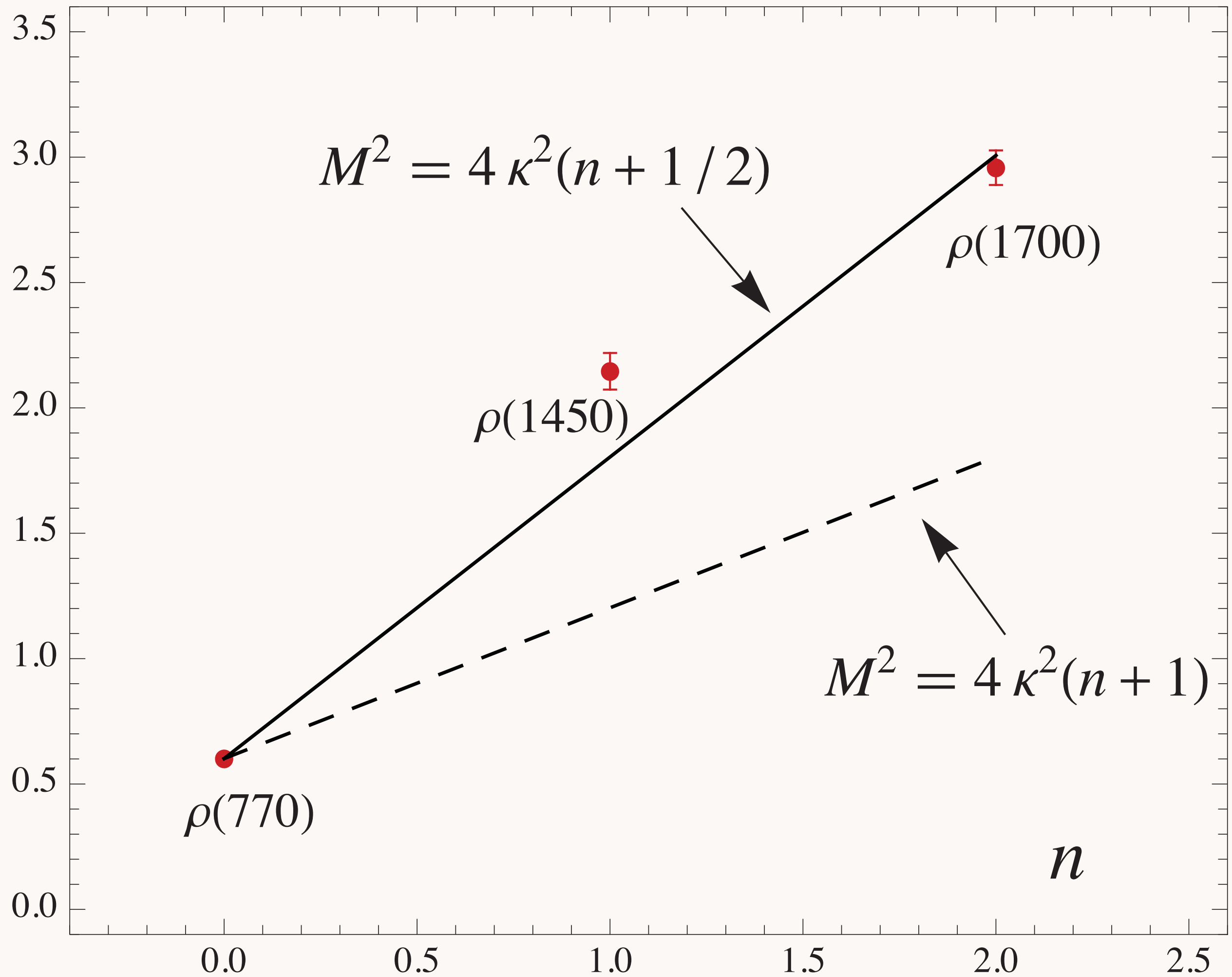
$$\cdots$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}, \quad N,$$

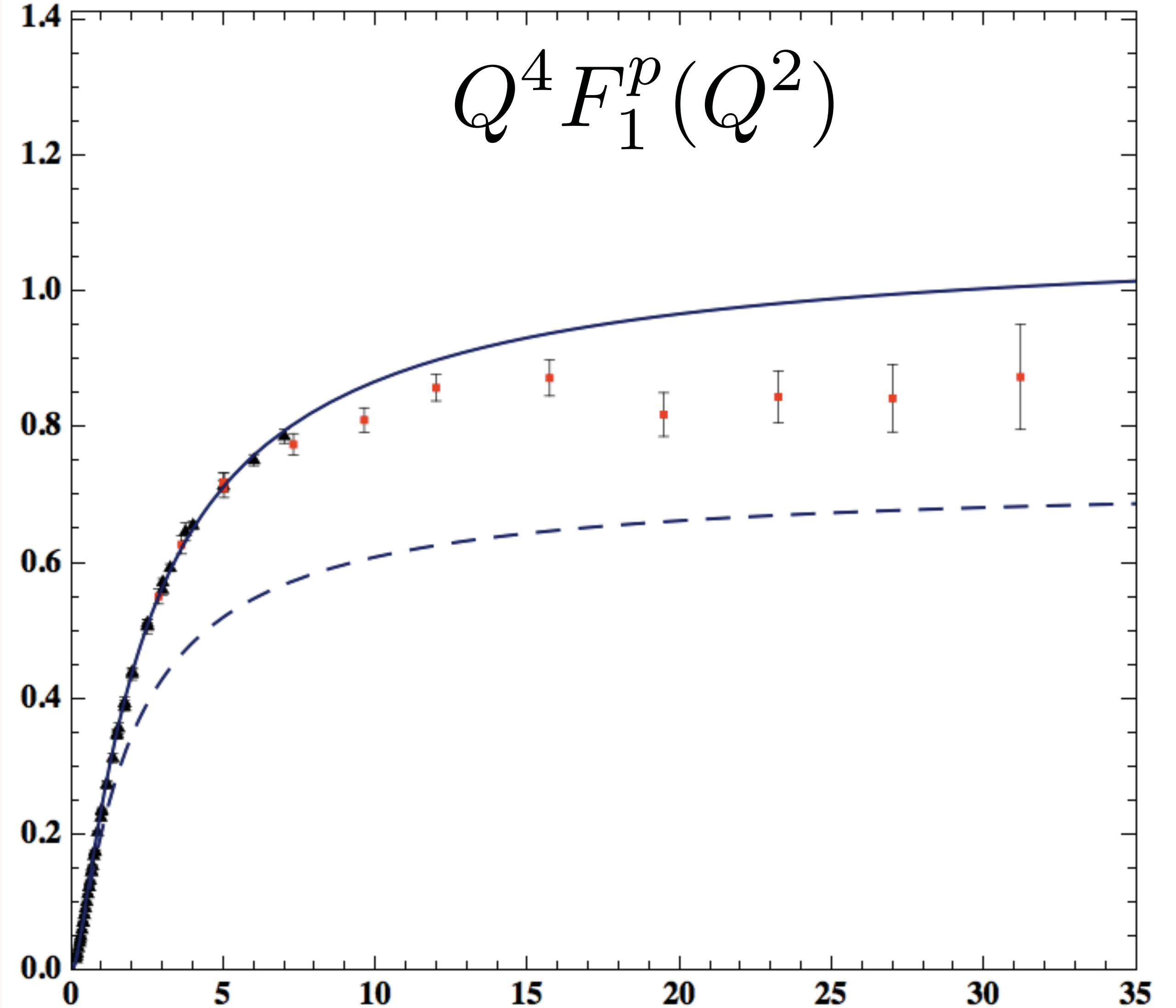
Positive Dilaton Background $\exp\left(+\kappa^2 z^2\right)$ $\qquad \mathcal{M}_n^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \qquad Q^2 \to \infty$$

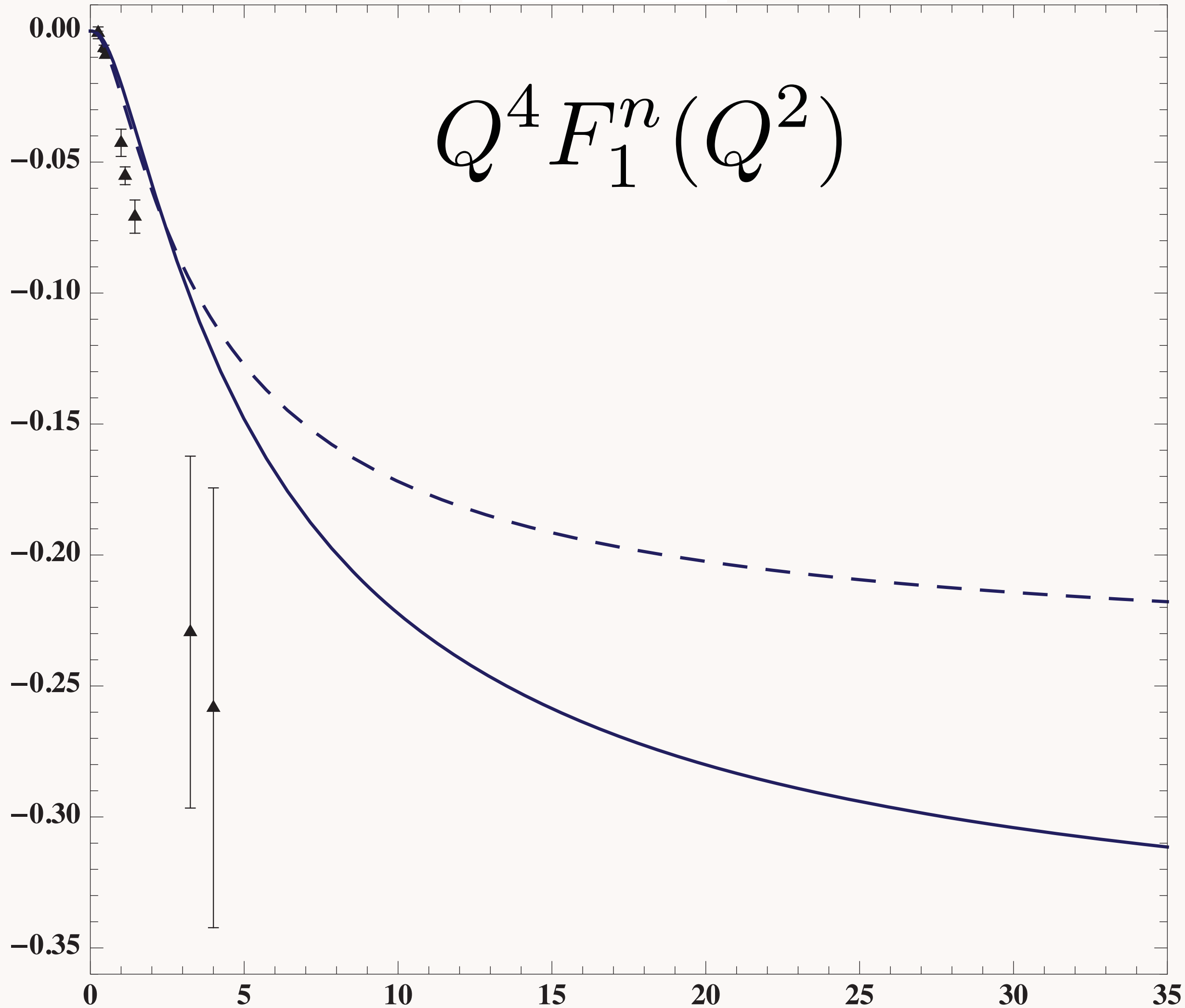
Constituent Counting



Vector meson radial trajectories in a negative (dashed line, $\kappa = 0.3877$ GeV) and positive dilaton backgrounds (continuous line, $\kappa = 0.5484$ GeV).



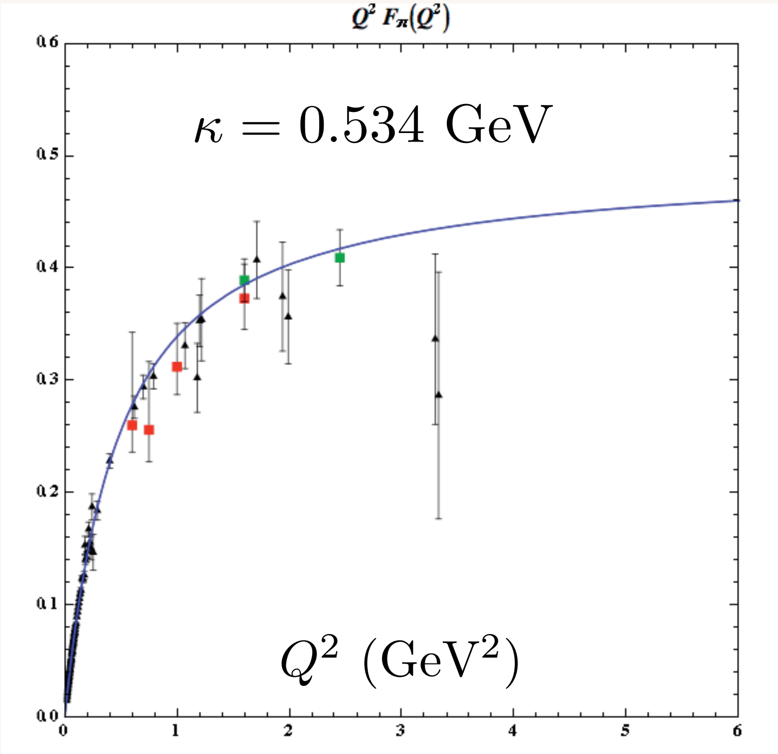
$Q^4 F_p^1(Q^2)$ in a negative (dashed line, $\kappa = 0.3877$ GeV) and positive dilaton backgrounds (continuous line, $\kappa = 0.5484$ GeV). The data compilation is from Diehl.

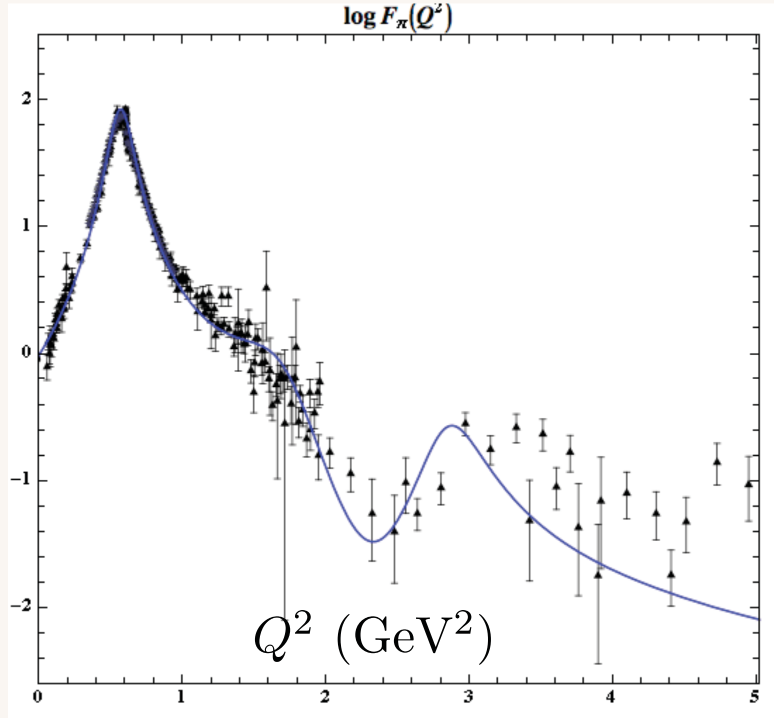


$Q^4 F_n^1(Q^2)$ in a negative (dashed line, $\kappa = 0.3877$ GeV) and positive dilaton backgrounds (continuous line, $\kappa = 0.5484$ GeV). The data compilation is from Diehl.

Spacelike and timelike pion form factor

Preliminary

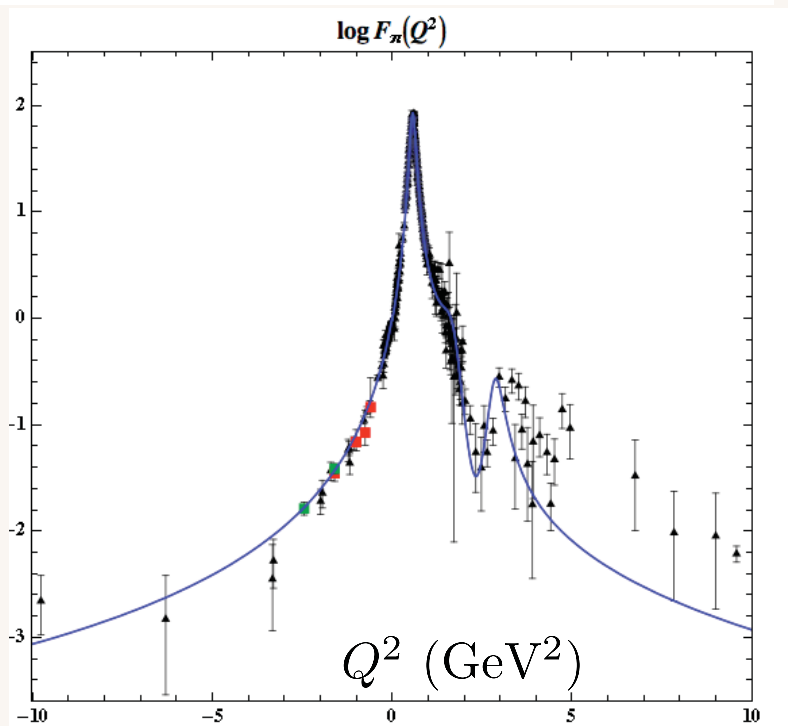




$$|\pi> = \psi_{q\bar{q}}|q\bar{q}> + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}>$$

$$\Gamma_\rho = 120 \text{ MeV}, \; \Gamma'_\rho = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 15\%$$



Carl E. Carlson Zainul Abidin

AdS/CFT now extensive field---apologies for all omitted references
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: "fancy"
Start from string theory, develop QCD analogs on lower dimensional branes — Sakai & Sugimoto

"Bottom-up"
Anticipate what 5D Lagrangian must be (guess), directly involving desired rho, pi, a1, ... fields and connect to matching QCD structures — Erlich et al.; Da Rold & Pomarol

EM form factors in "bottom-up" approach — Brodsky & de Teramond; Radyushkin & Grigoryan

Gravitational form factors in bottom-up approach — Zainul Abidin & me

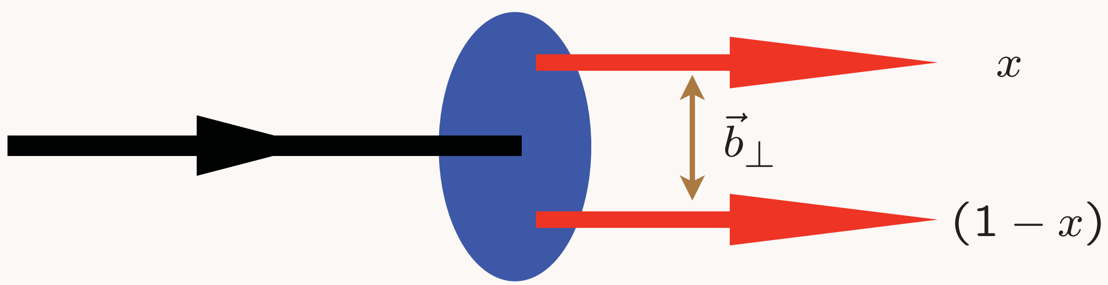
Soft-wall — Karch, Katz, Son, and Stephanov; Batell, Gherghetta, and Sword

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{2}} \quad \longleftrightarrow \quad z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

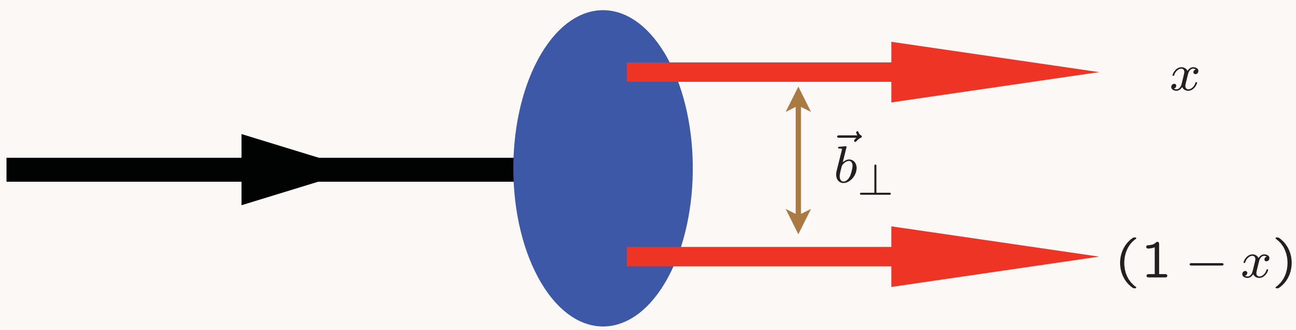
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2=x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions}.$$

Change variables

$$(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi>= E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x) b^2_\perp$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

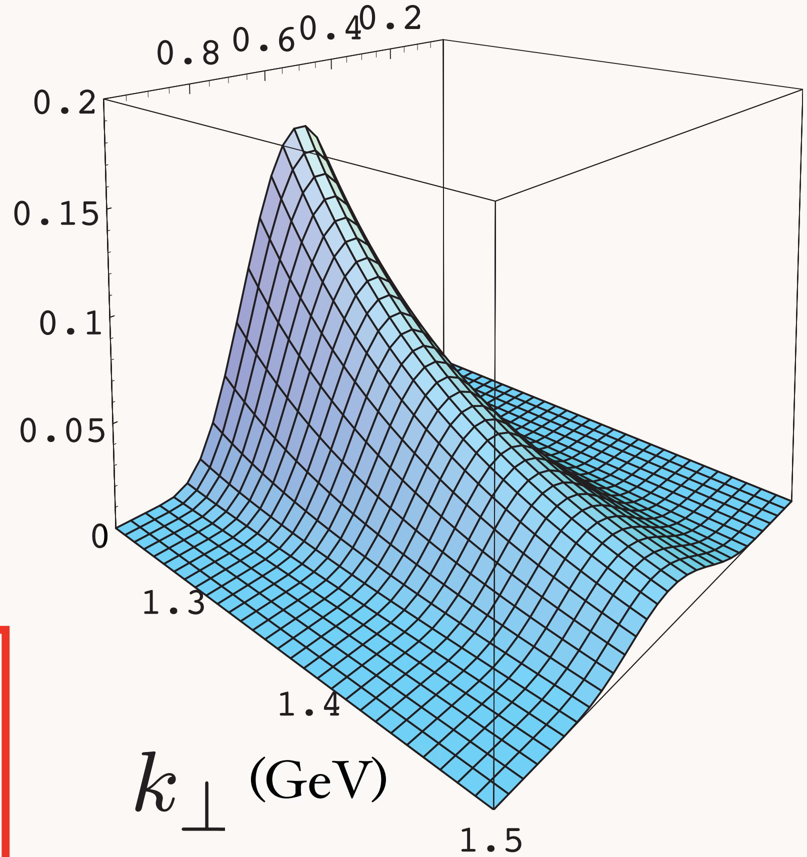
Confining AdS/QCD potential

Semiclassical first approximation to QCD

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

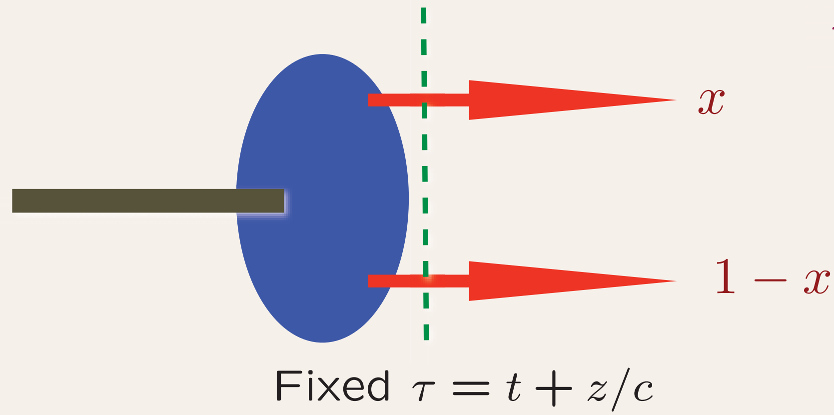
Connection of Confinement to TMDs

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

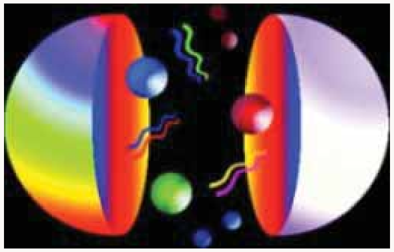
Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ — Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ — Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



From Nick Evans

- Action for massive fermionic modes on AdS_5:

$$S[\overline{\Psi}, \Psi] = \int d^4x\, dz\, \sqrt{g}\, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right] \Psi(x^\ell) = 0$$

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2}\left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_-\right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm\left(z = 1/\Lambda_{QCD}\right) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k}\, \Lambda_{QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k}\, \Lambda_{QCD}$$

  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi^\dagger_L(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi^\dagger_L(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD} \right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k} \Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k} \Lambda_{\rm QCD},$$

with a scale independent mass ratio.

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

Hoyer's Linear Potential

- Solution $(\mu R = \nu + 1/2,\, d = 4)$
  $\nu = L + 1$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2)$$

$$\Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

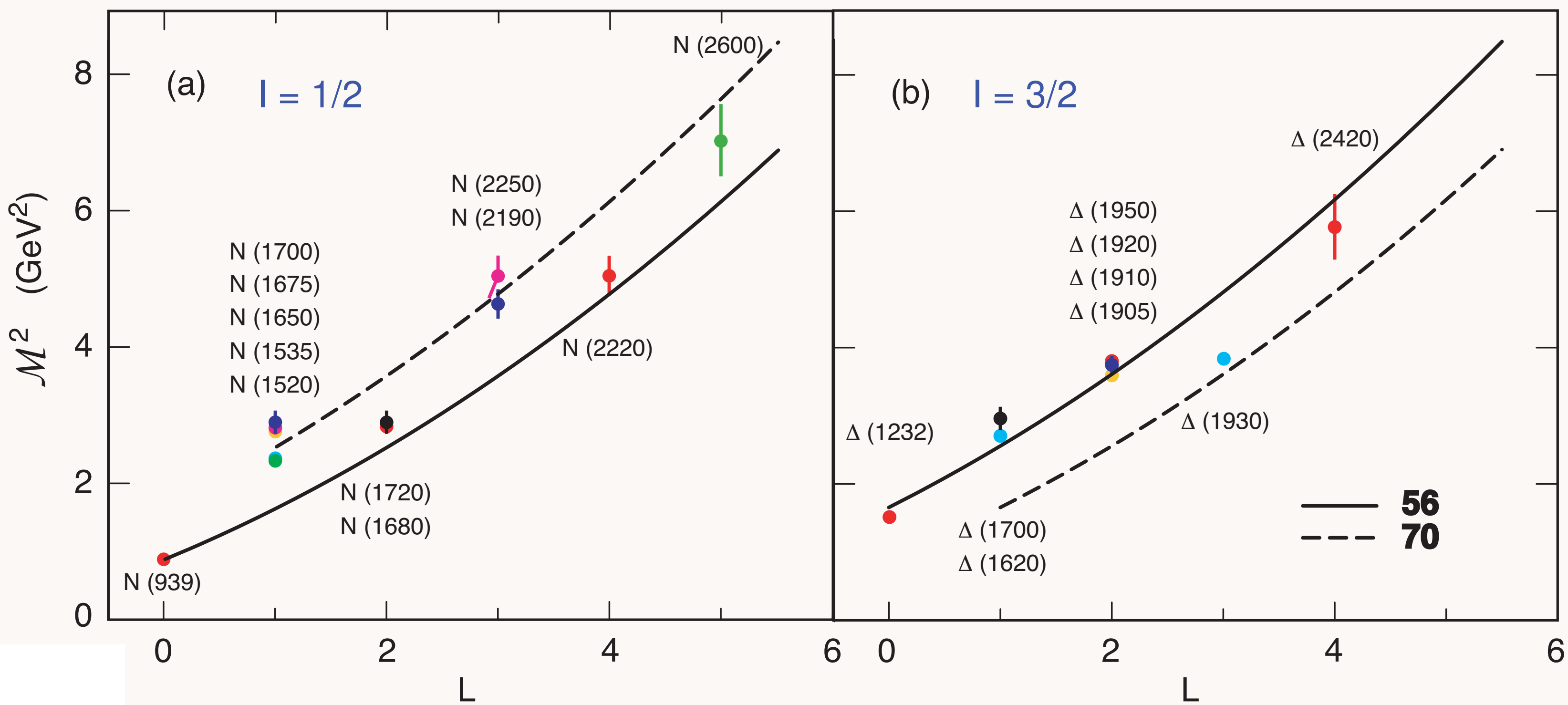


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

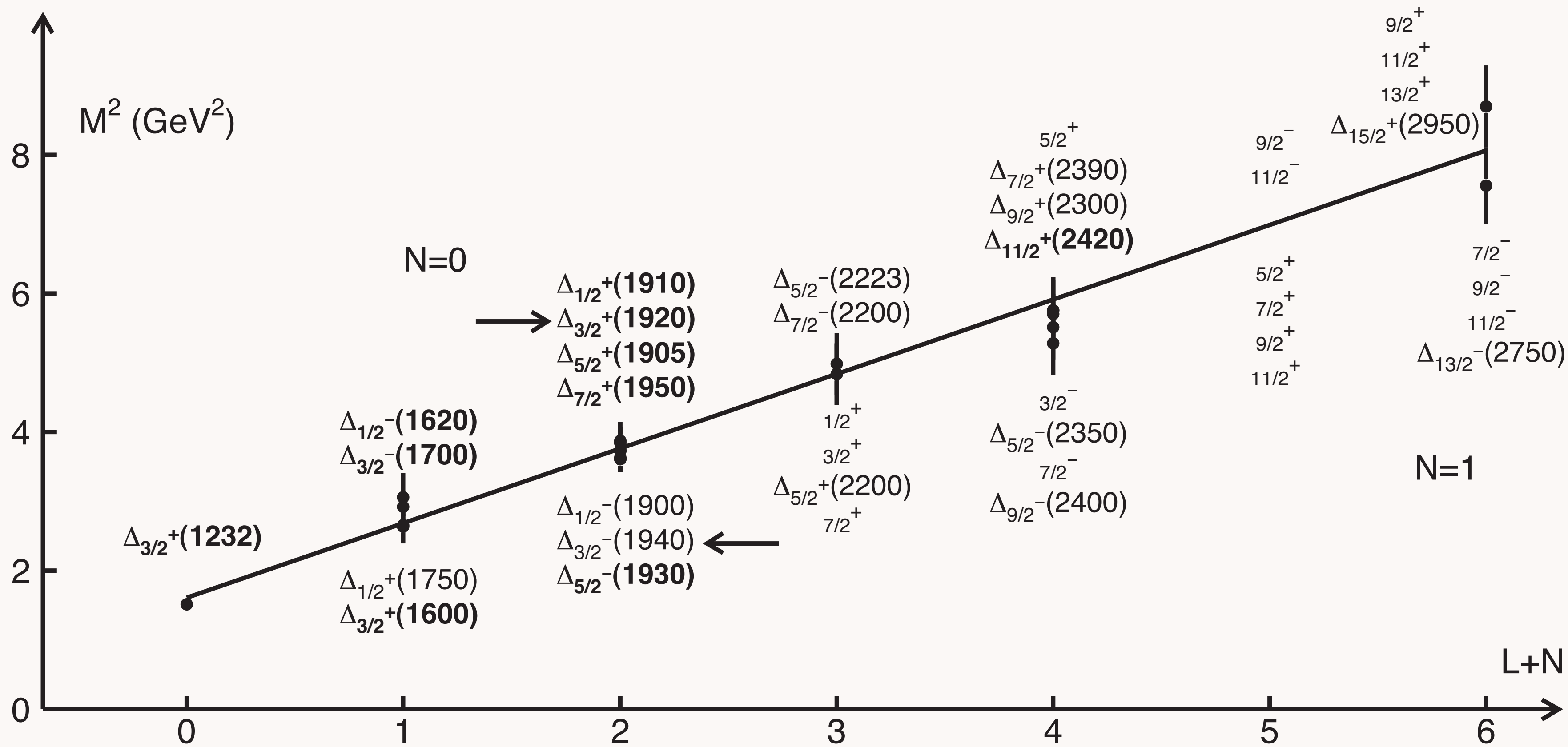
$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

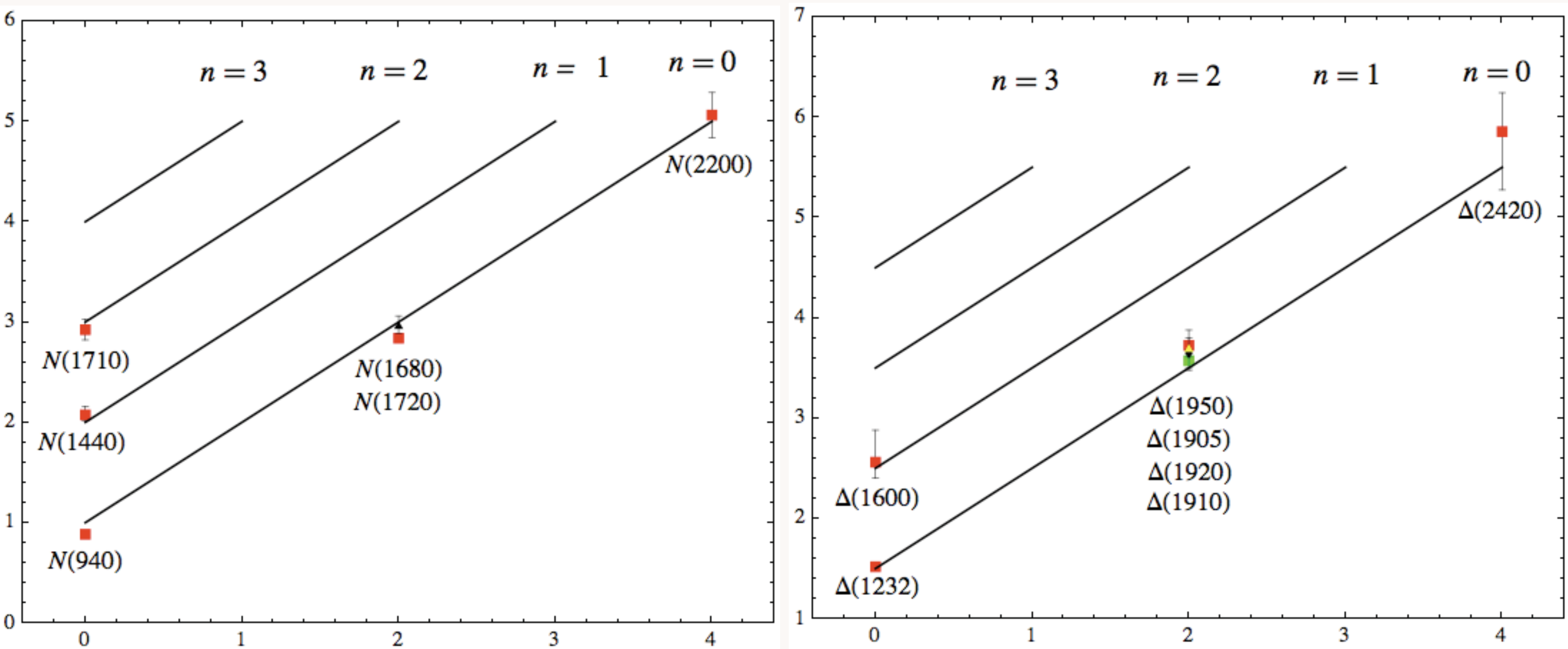
• **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$

$\mathcal{M}^2$



L

Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

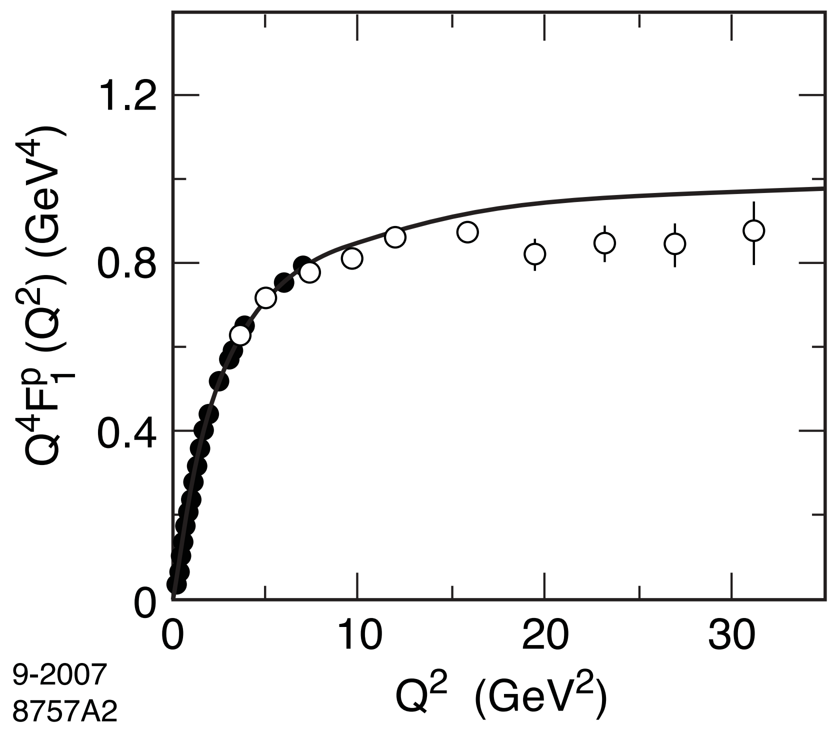
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

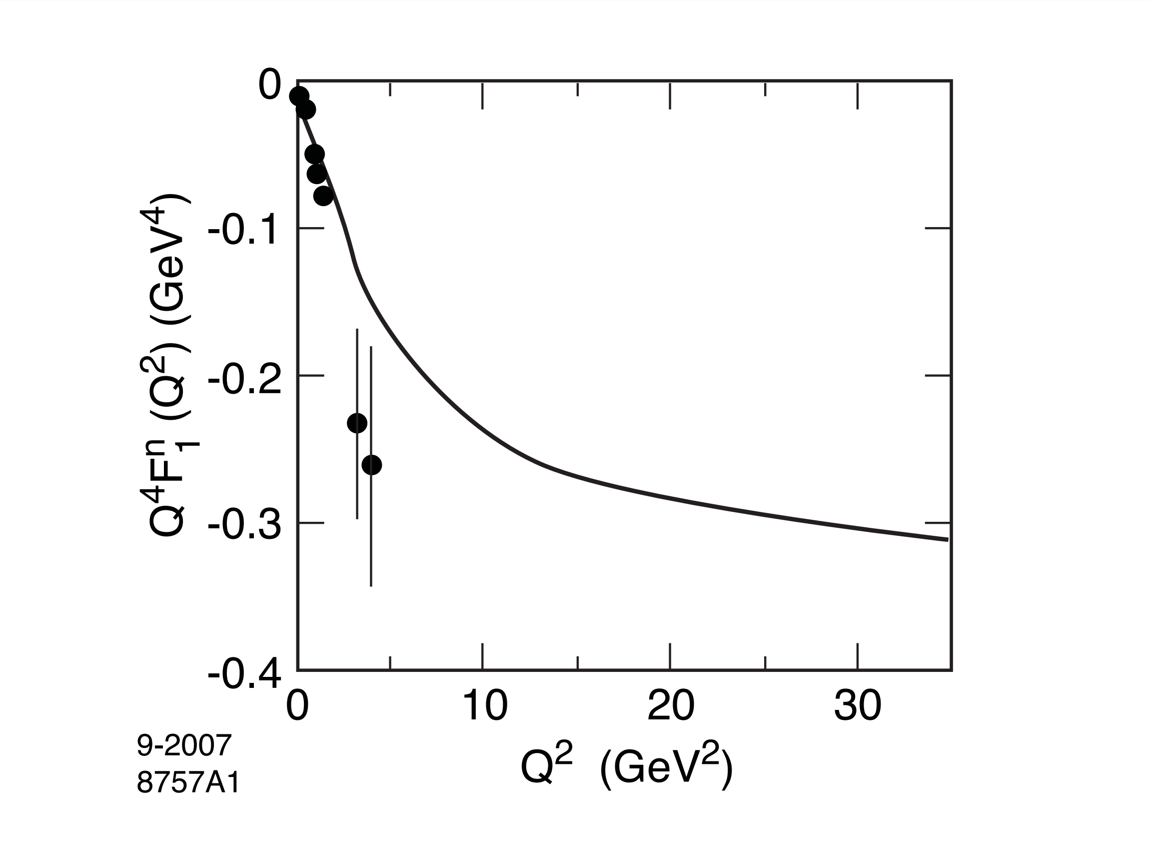
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

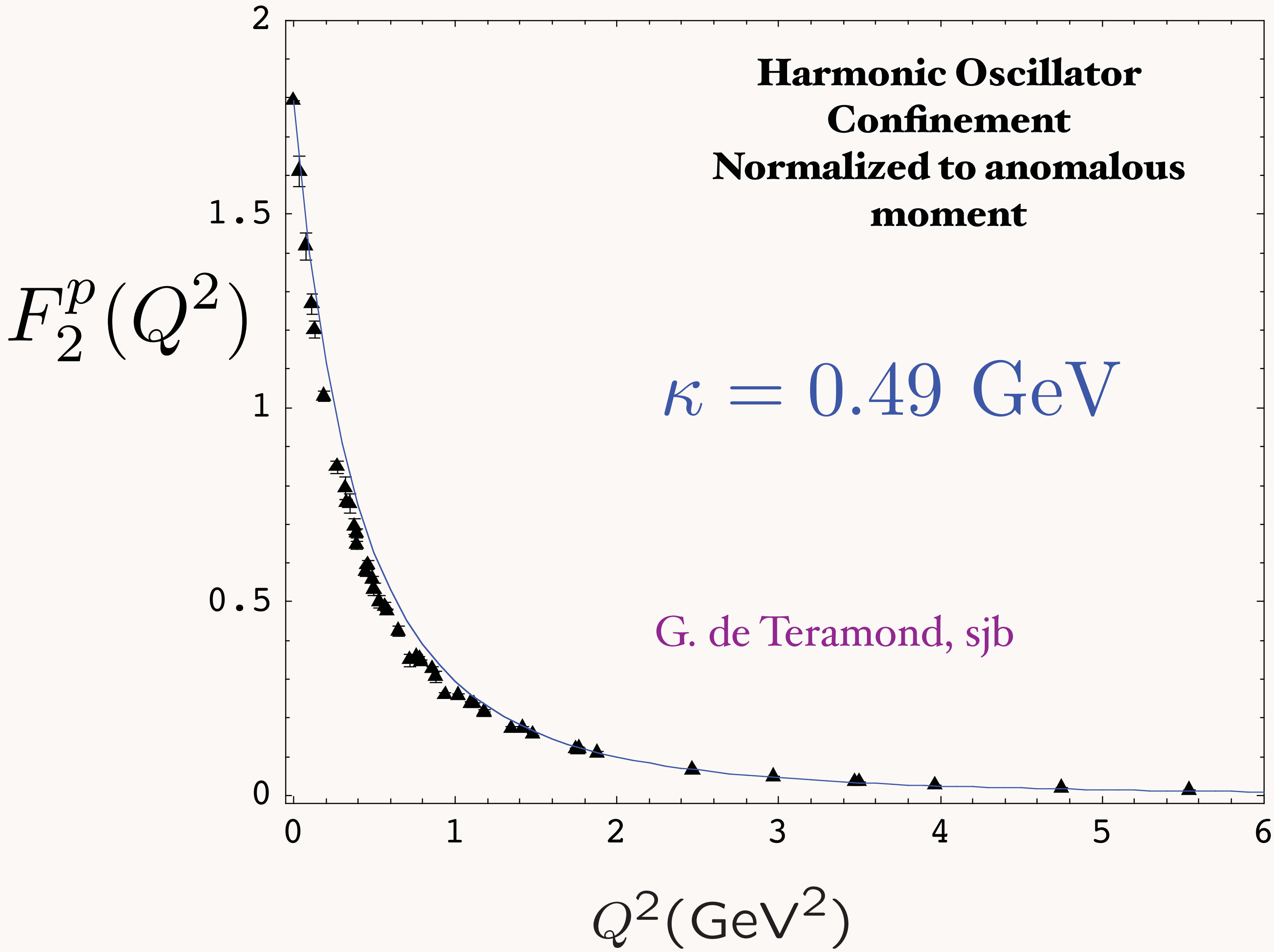


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

↓

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

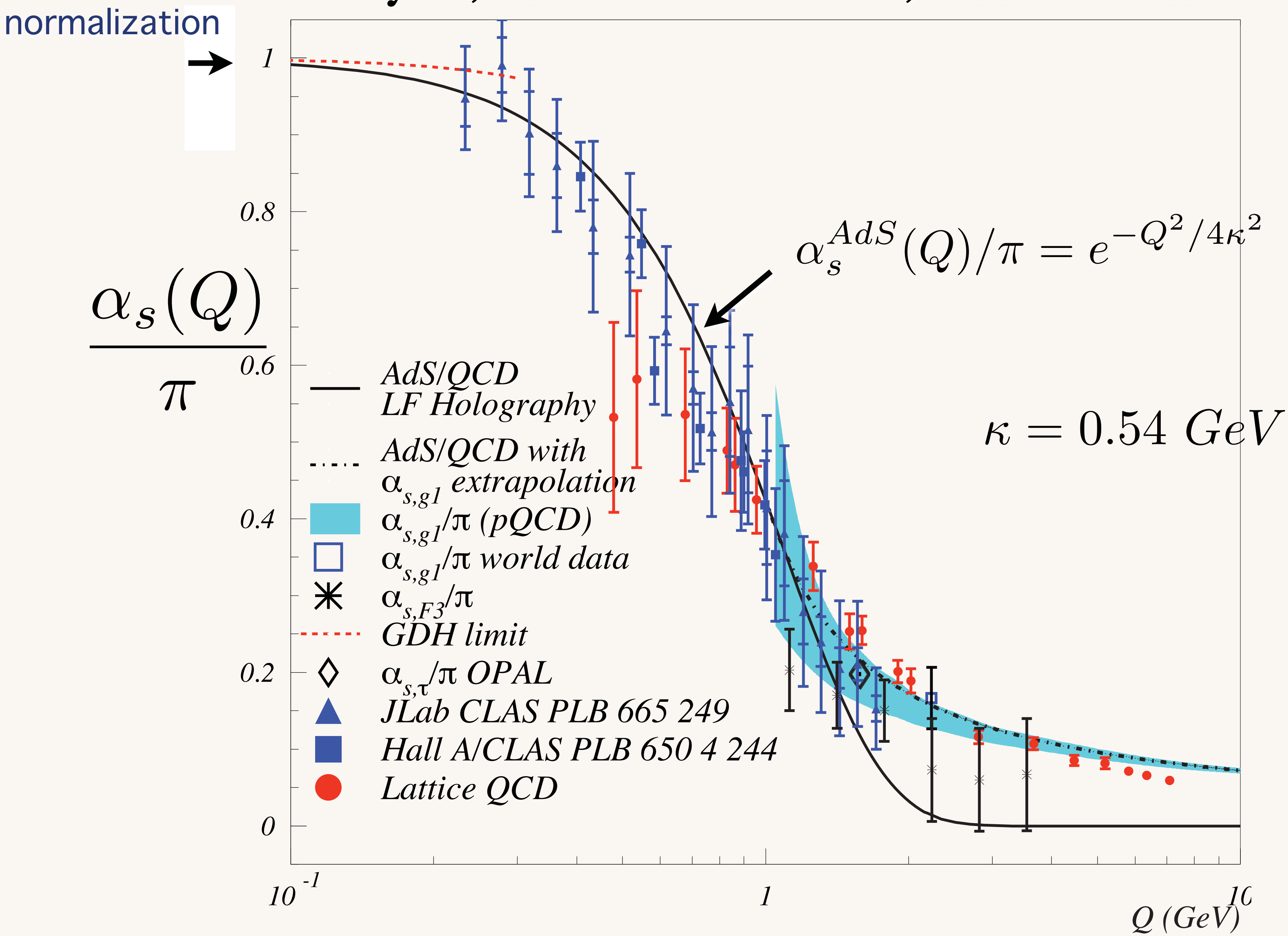
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

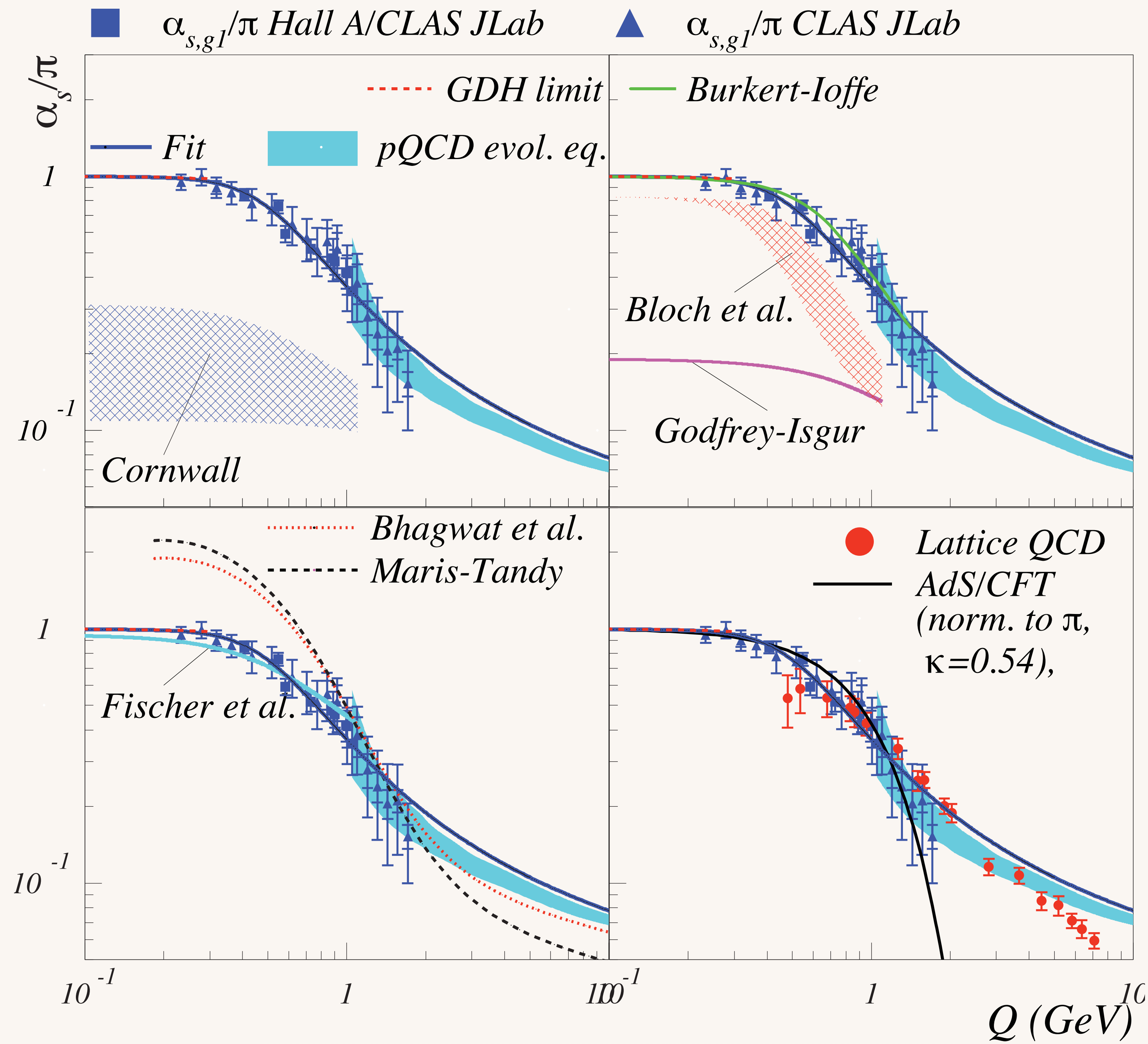
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

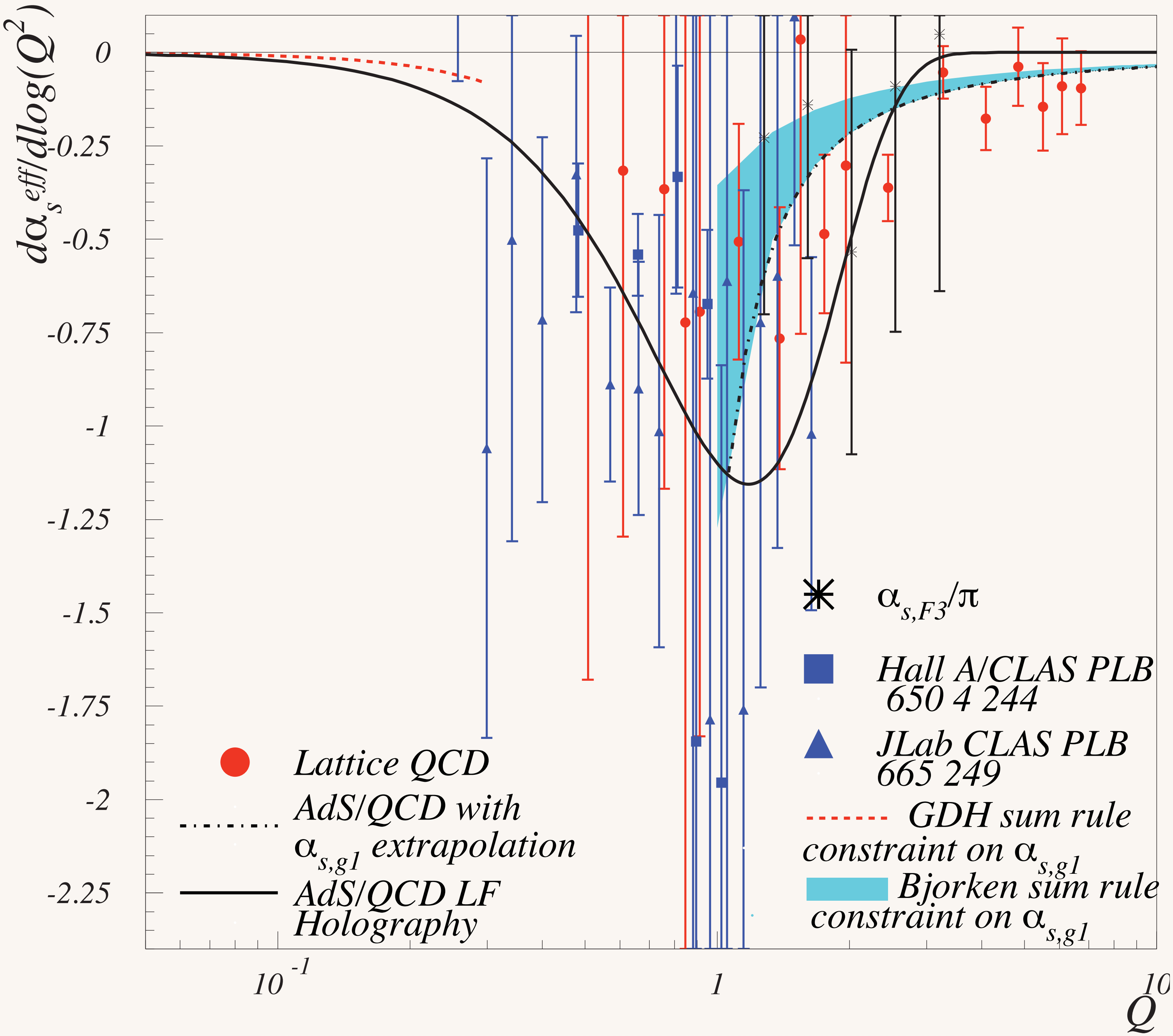


Deur, de Teramond, sjb, (preliminary)



Deur, de Teramond, sjb, (preliminary)

$$\beta^{AdS}(Q^2) = \frac{d}{d\log Q^2}\alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb, (preliminary)

Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

Single-spin asymmetries

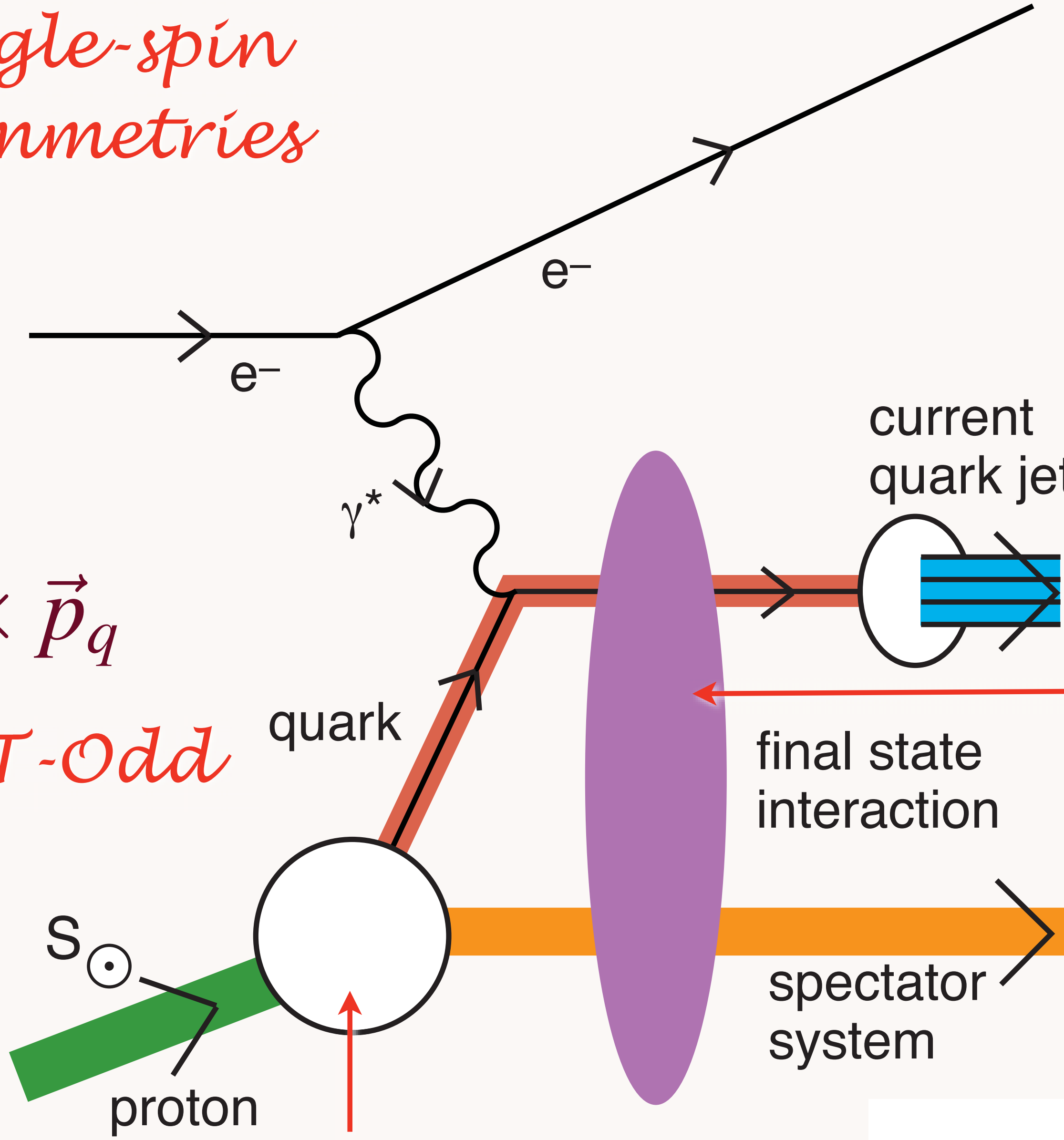
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



QCD S- and P-Coulomb Phases --Wilson Line

Leading-Twist Rescattering Violates pQCD Factorization!

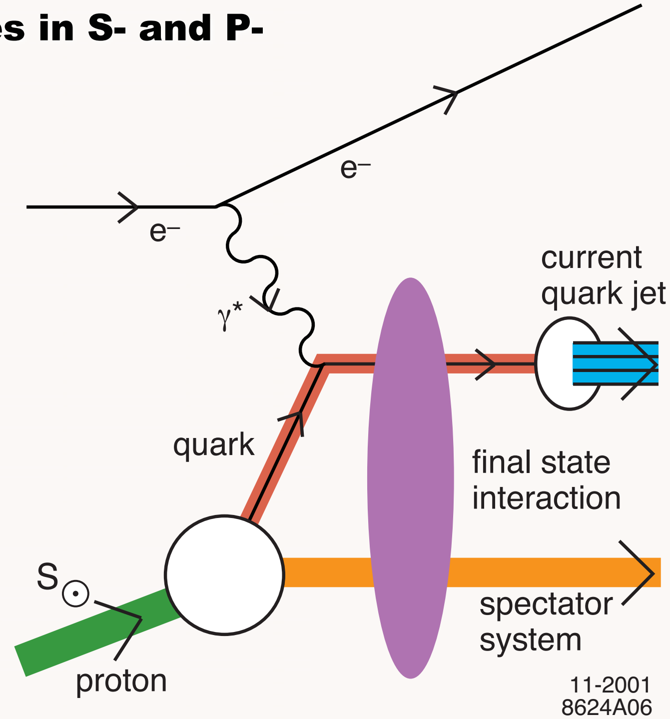
Light-Front Wavefunction S and P- Waves

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**



Pasquini, Xiao, Yuan, sjb

Mulders, Boer　　**Qiu, Sterman**

Static	Dynamic
• Square of Target LFWFs	Modified by Rescattering: ISI & FSI
• No Wilson Line	Contains Wilson Line, Phases
• Probability Distributions	No Probabilistic Interpretation
• Process-Independent	Process-Dependent - From Collision
• T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
• No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
• Sum Rules: Momentum and J^z	Sum Rules Not Proven
• DGLAP Evolution; mod. at large x	DGLAP Evolution
• No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

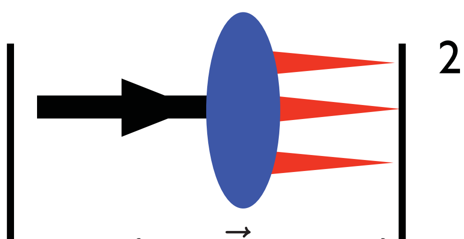
Hwang, Schmidt, sjb,

Mulders, Boer

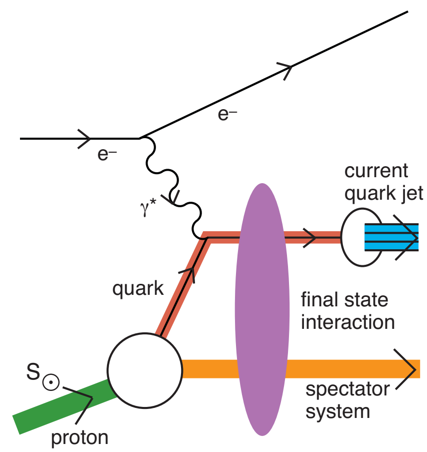
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



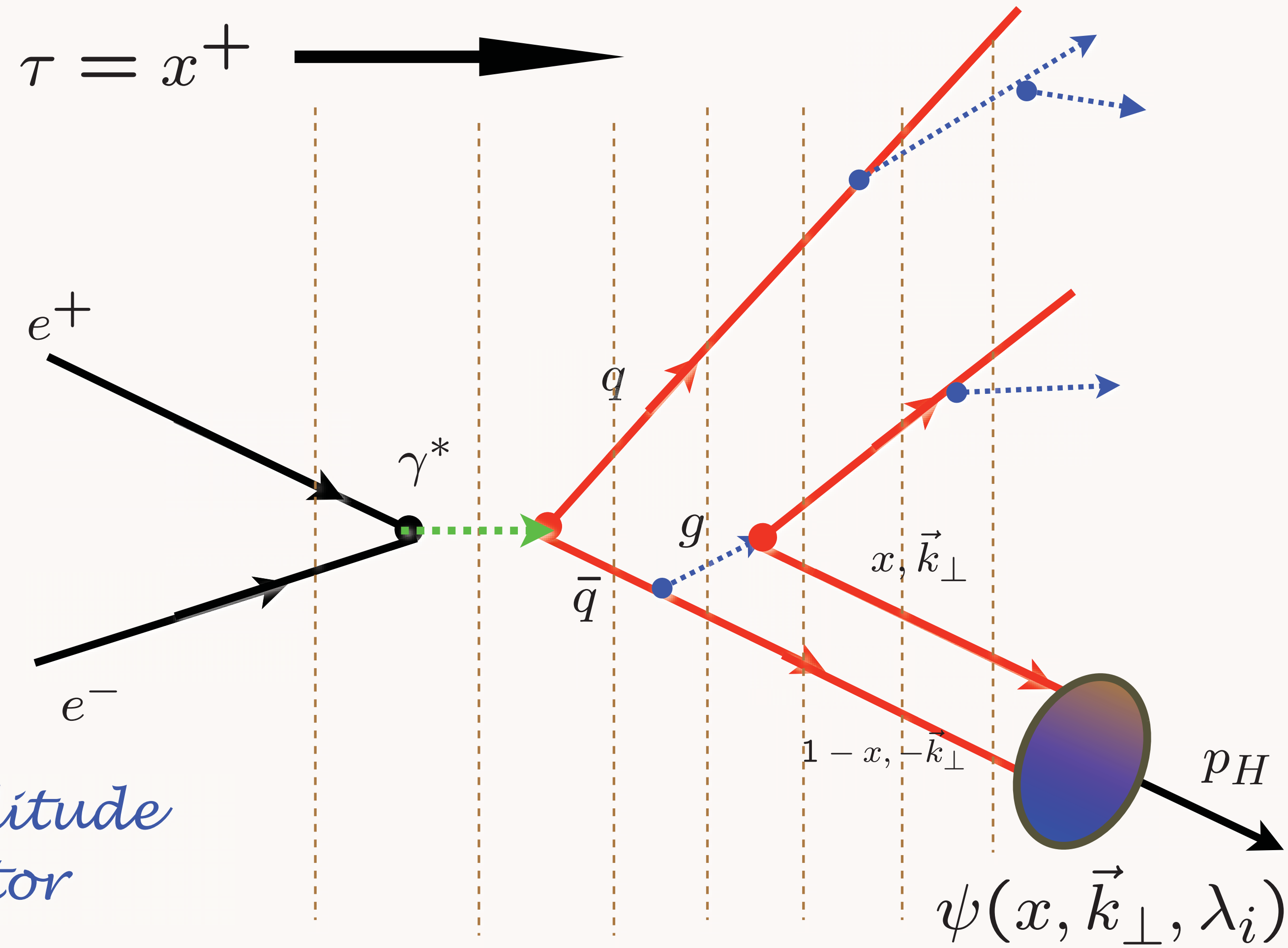
Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion (m_q =0)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S. Spectrum is independent of S**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1 — **Pauli, Hornbostel, Hiller, McCartor, sjb**
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

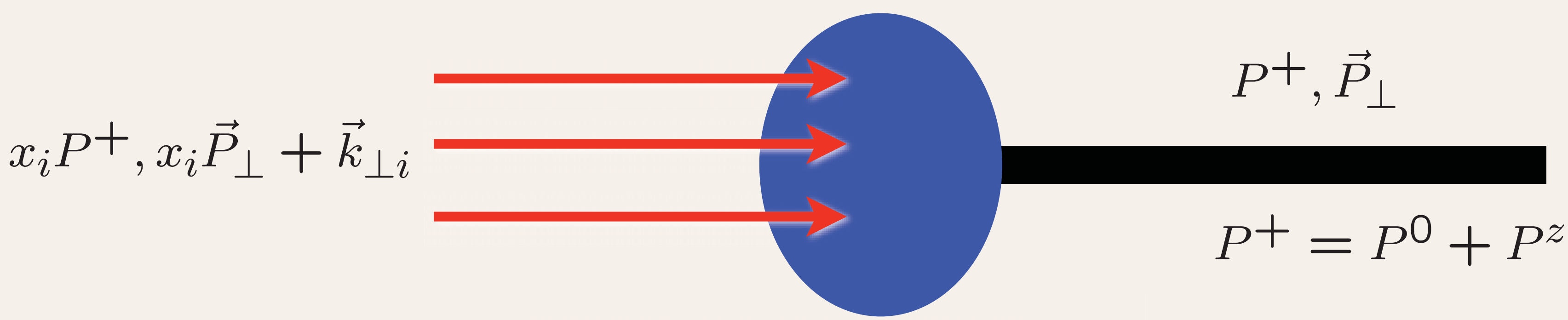
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda^2_{QCD}$$

- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement



Chiral Symmetry Breaking in AdS/QCD

Erlich et al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz\ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**

- **AdS/QCD: confined condensate**

- **Suggests "In-Hadron" Condensates**

de Teramond, Shrock, sjb

-

In presence of quark masses the Holographic LF wave equation is ($\zeta = z$)

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{1}$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \tag{2}$$

The parameter a is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x}\langle\bar{\psi}\psi\rangle z^3,} \tag{3}$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2\sum_i m_i \langle\bar{\psi}\psi\rangle\langle z^2\rangle + \langle\bar{\psi}\psi\rangle^2\langle z^4\rangle, \tag{4}$$

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$.

Mass shift from dynamics inside hadronic boundary

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

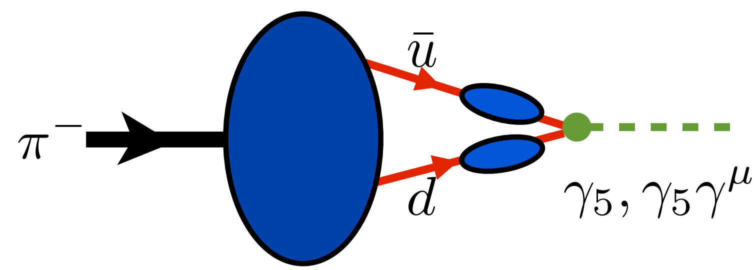
Light-Front Formalism

Bethe-Salpeter Analysis

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4q}{(2\pi)^4}\; \frac{1}{2}\left[T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P+q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P-q))\right]$$

Maris, Roberts, Tandy

f_H Meson Decay Constant
T_H flavor projection operator,
$Z_2(\Lambda)$, $Z_4(\Lambda)$ renormalization constants
$S(p)$ dressed quark propagator
$\Gamma_H(q;P) = F.T.\langle H|\psi(x_a)\bar\psi(x_b)|0\rangle$
Bethe-Salpeter bound-state vertex amplitude.



$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q}\rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4q}{(2\pi)^4}\; \frac{1}{2}\left[T_H \gamma_5 \mathcal{S}(\frac{1}{2}P+q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P-q))\right]$$

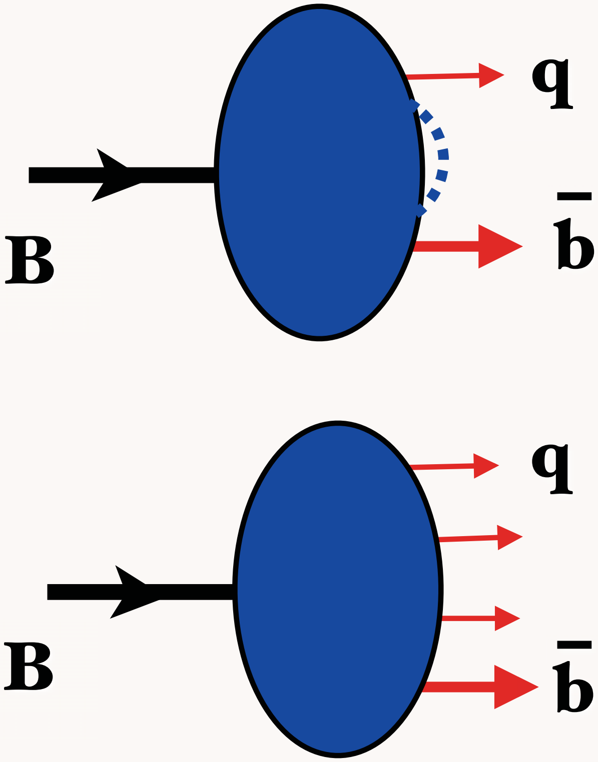
In-Hadron Condensate!

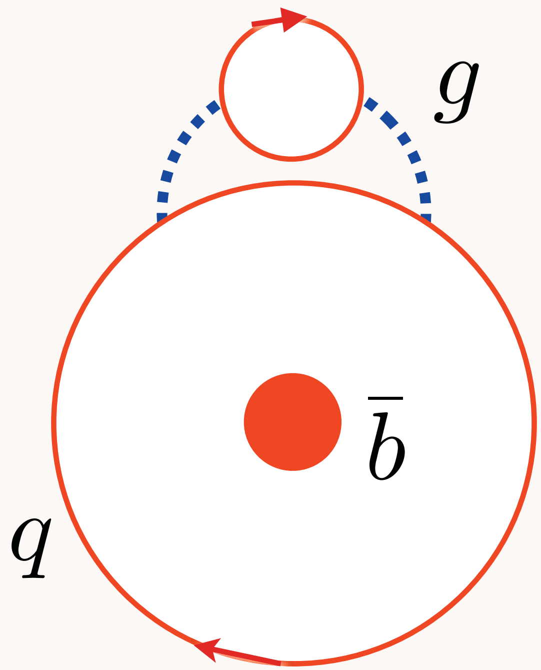
$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \qquad \mathcal{M}_H = \textstyle\sum_{q\in H} m_q$$

$$m_\pi^2 \propto (m_q + m_{\bar{q}})/f_\pi$$ *GMOR*

Simple physical argument for "in-hadron" condensate

Roberts, Shrock, Tandy, sjb



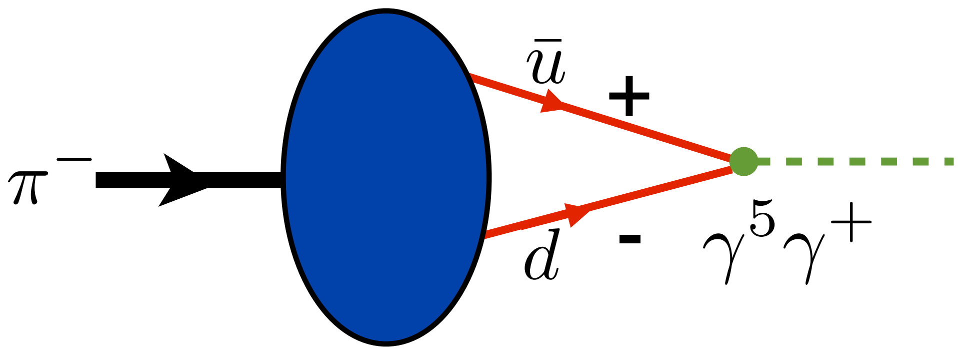


B-Meson

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

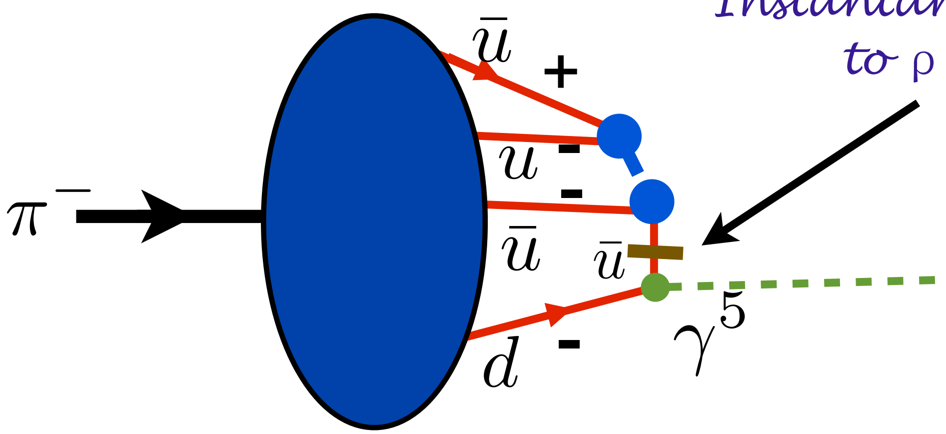
$$< B|\bar{q}q|B > \text{ not } < 0|\bar{q}q|0 >$$

Higher Light-Front Fock State of Pion Simulates DCSB

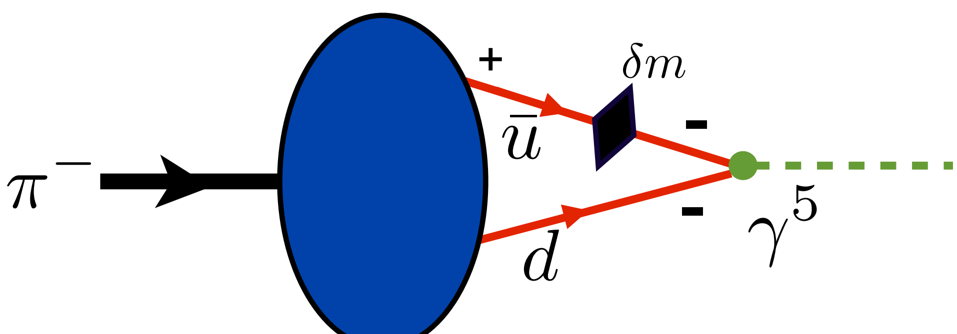


$$f_\pi P^+ =< 0|\bar{q}\gamma^5\gamma^+ q|\pi >$$

Instantaneous quark propagator contribution to ρ_π derived from higher Fock state



$$i\rho_\pi =< 0|\bar{q}\gamma^5 q|\pi >$$



Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb

Essence of the vacuum quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

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[2]Centre for Particle Physics Phenomenology: CP[3]-Origins,
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[3]Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
[4]Department of Physics, Peking University, Beijing 100871, China
[5]C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794
[6]Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind   Maris, Roberts, Tandy   Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

R. Shrock, sjb

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb

Quark and Gluon condensates reside within hadrons, not LF vacuum

Maris, Roberts, Tandy

Casher Susskind

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant

Shrock, sjb

"Confined QCD Condensates"

Determinations of the vacuum Gluon Condensate

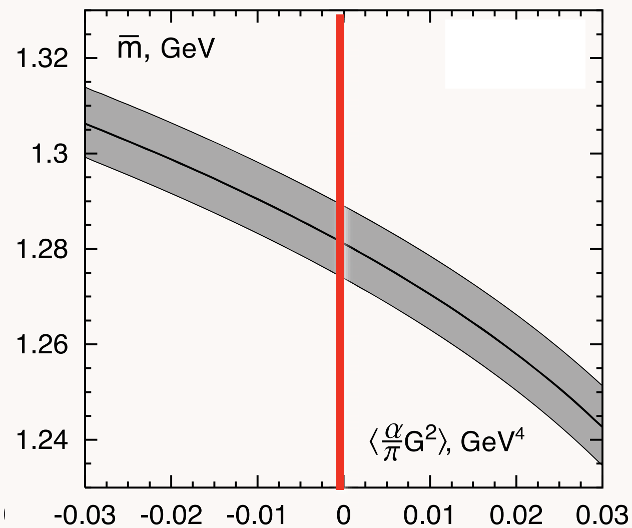
$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



Consistent with zero vacuum condensate

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, ...)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

Shrock and sjb

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

Conformal Template

- **BLM scale-setting: Retain conformal series; nonzero β-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.**
- **Commensurate Scale Relations between observables based on conformal template -- prime tests of QCD independent of theory conventions**
- **BLM scales for three-gluon coupling; multi-scale problems**
- **Analytic scheme for coupling unification**
- **Direct high p_T processes, baryon anomaly, intrinsic heavy quarks**
- **IR Fixed point -- conformal symmetry motivation for AdS/CFT**
- **Light-Front Schrödinger Equation: analytic first approximation to QCD**
- **Dilaton-modified AdS_5: Predict Hadron Spectra, Light-Front Wavefunctions, Form Factors, Hadronization at Amplitude Level**
- **Non-Perturbative running QCD coupling -- new range of QCD phenomena such as Sivers and Diffractive DIS fraction calculable; IR fixed point**

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable with DLCQ Methods