Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

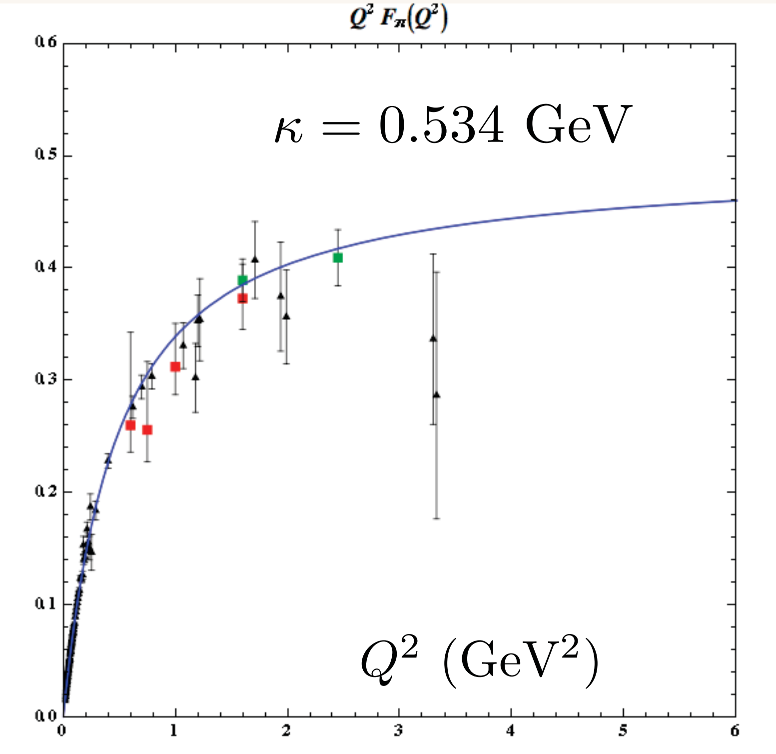
$$\begin{aligned}
F(Q^2) &= \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \\
F(Q^2) &= \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)\cdots\left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

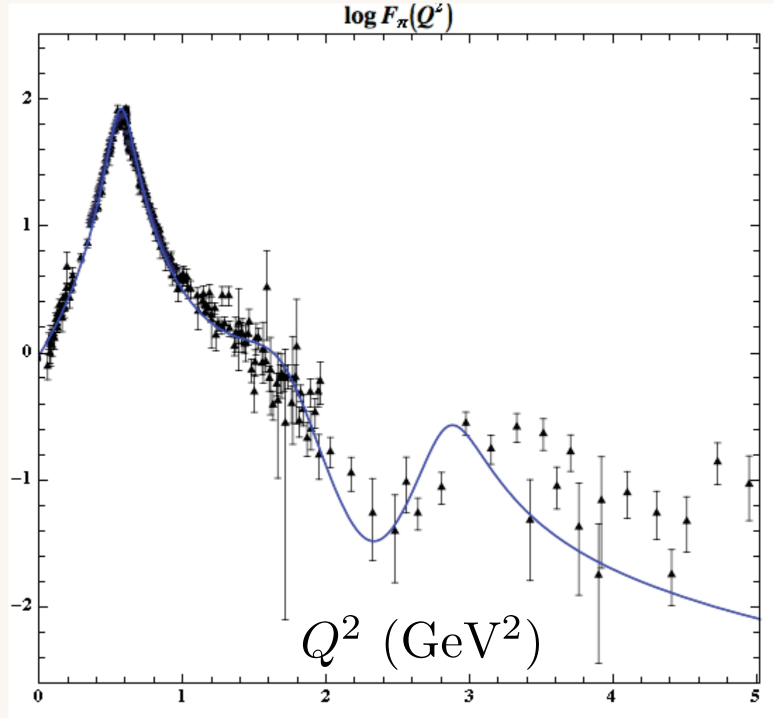
- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

Spacelike and timelike pion form factor

Preliminary

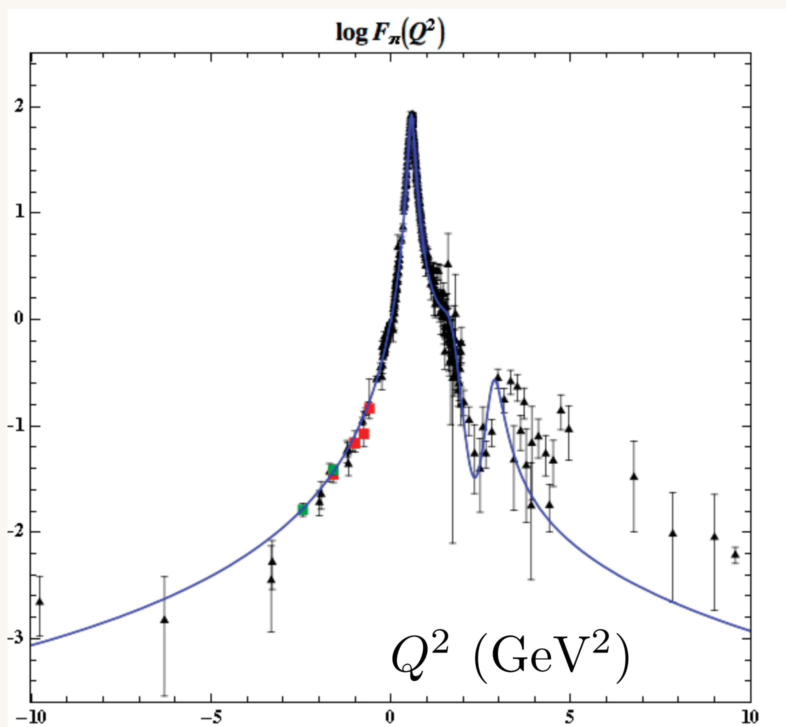




$$|\pi >= \psi_{q\bar{q}}|q\bar{q} > + \, \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q} >$$

$$\Gamma_\rho = 120 \text{ MeV}, \; \Gamma'_\rho = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 15\%$$



Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) \left|\Phi_{\pi^+}(z)\right|^2,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\!\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\!\left(zQ \sqrt{\frac{1-x}{x}}\right) \left|\Phi_{\pi^+}(z)\right|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4}}$$

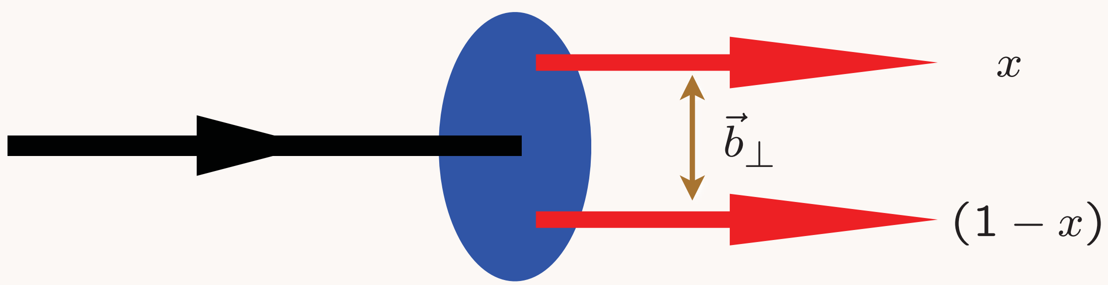
with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) AdS$_5$

$$\psi(x,\vec{b}_\perp) \Longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \Longleftrightarrow z$$

$\psi(x,\vec{b}_\perp)$



$$\psi(x,\vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

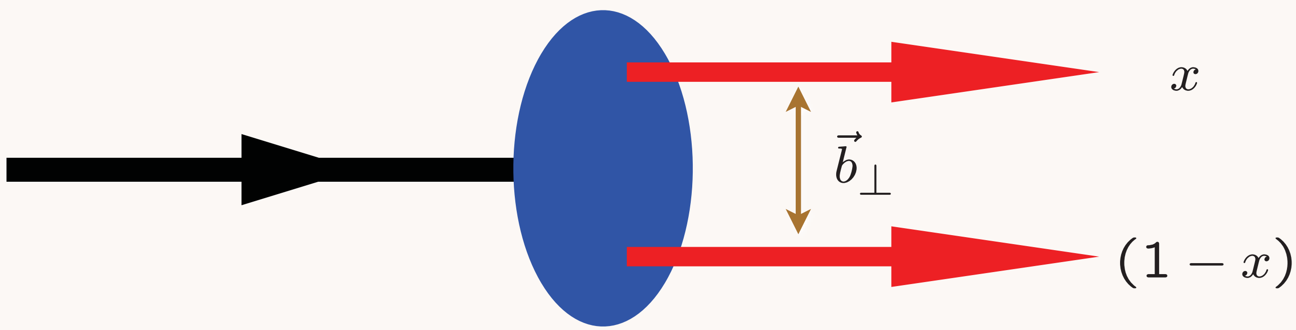
Identical to LF Holography obtained from electromagnetic current

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp\, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

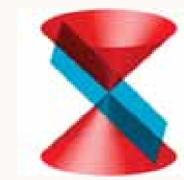
- Find ($L = |M|$)

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Big) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\bar{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

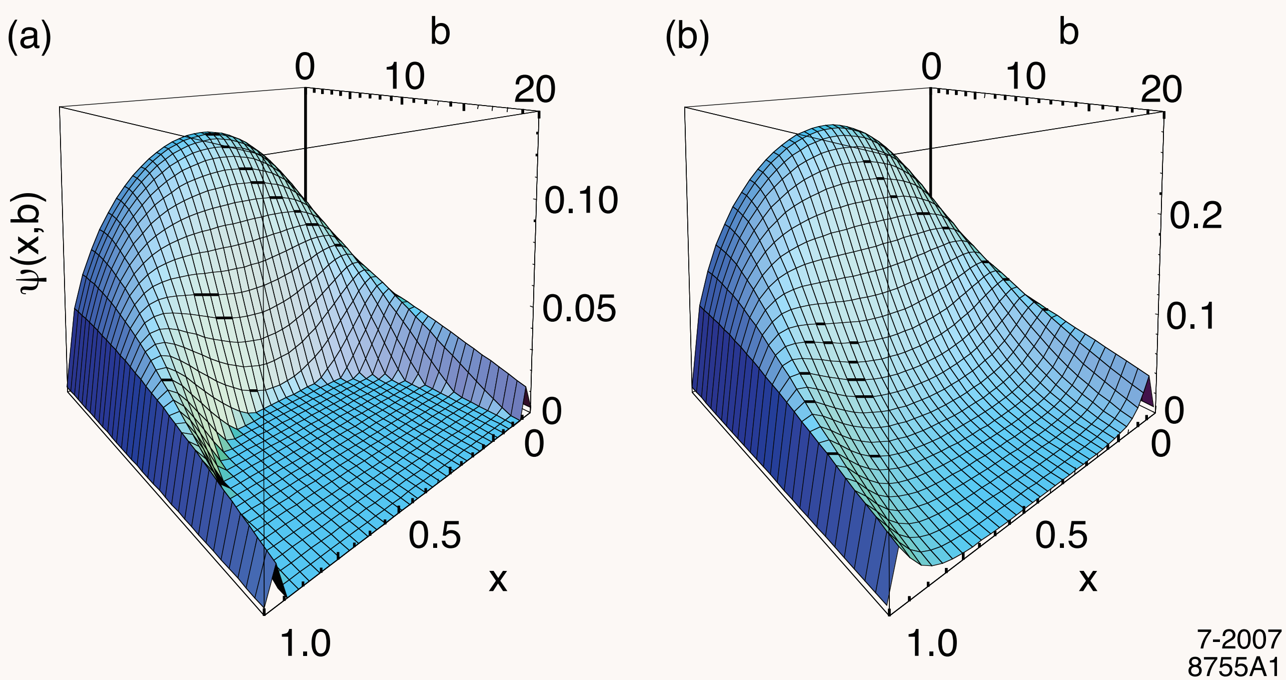


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \to \lambda x^\mu, \; z \to \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+ dx^- - dx_\perp{}^2$

and

$$ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2) \text{ for } x^+ = 0.$$

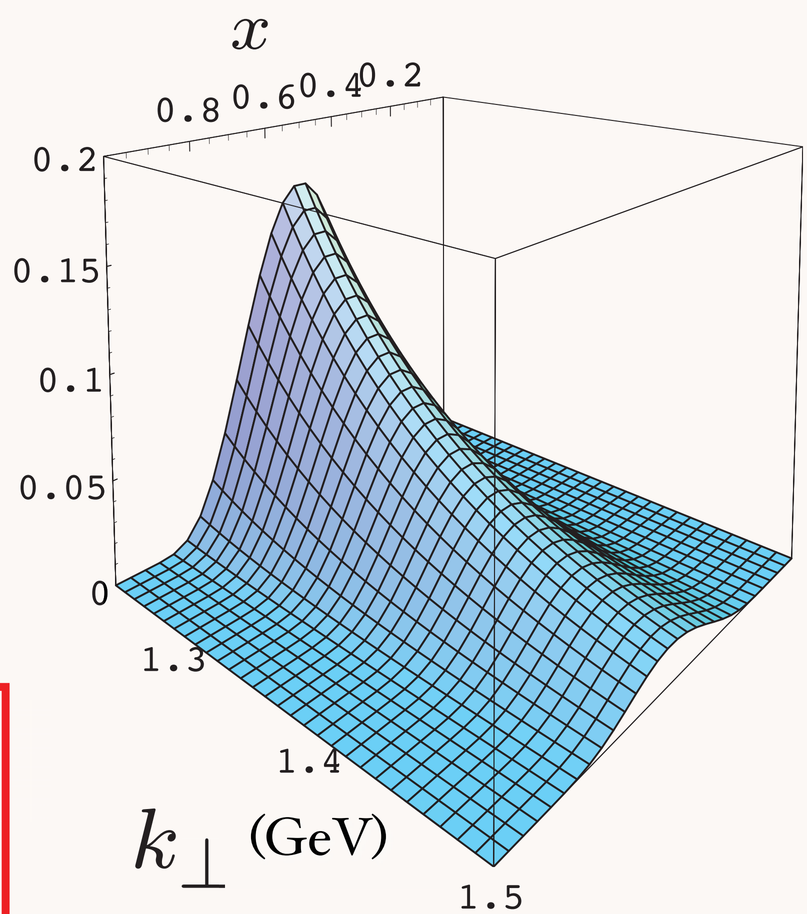
Light-Front/AdS5 Duality

- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

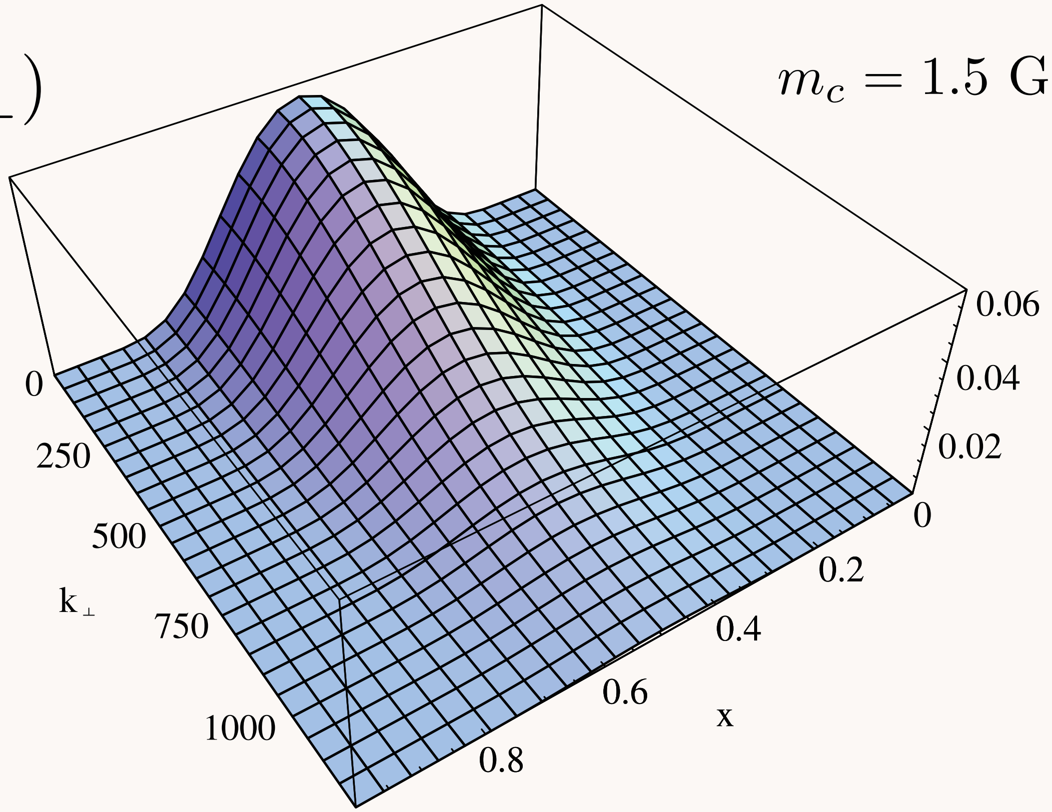
PHYSICAL REVIEW D **80,** 055014 (2009)

Meson wave function from holographic models

Alfredo Vega,[1] Ivan Schmidt,[1] Tanja Branz,[2] Thomas Gutsche,[2] and Valery E. Lyubovitskij[2,*]
[1]*Departamento de Física y Centro de Estudios Subatómicos, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*
[2]*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D–72076 Tübingen, Germany*

$\psi_{c\bar{c}}(x, \vec{k}_\perp)$

$m_c = 1.5 \text{ GeV}, \kappa = 0.894 \text{ GeV}$



$$\psi^{(1)}_{q_1\bar{q}_2}(x, \mathbf{k}_\perp) = \frac{4\pi A_1}{\kappa_1\sqrt{x(1-x)}} \exp\left(-\frac{\mathbf{k}_\perp^2}{2\kappa_1^2 x(1-x)} - \frac{\mu_{12}^2}{2\kappa_1^2}\right).$$

$$\mu_{12}^2 = \frac{m_1^2}{x} + \frac{m_2^2}{1-x}.$$

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle.$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}} R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

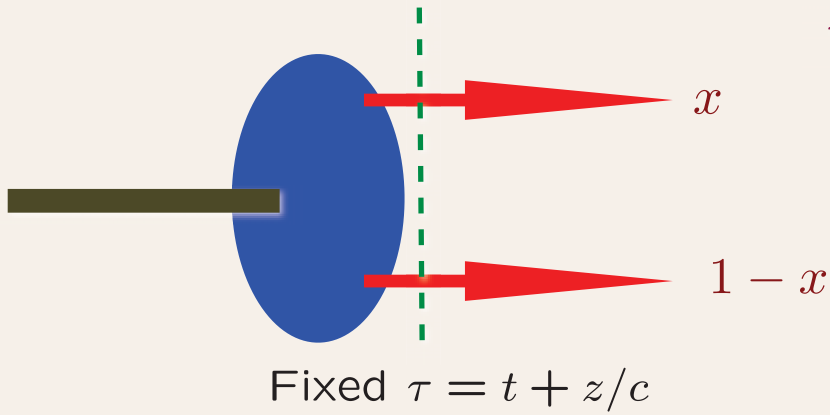
$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{QCD} = 91.7\,\mathrm{MeV}, \;\; f_\pi^{SW} = \frac{\sqrt{3}}{8} \kappa = 81.2\,\mathrm{MeV},$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \ \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

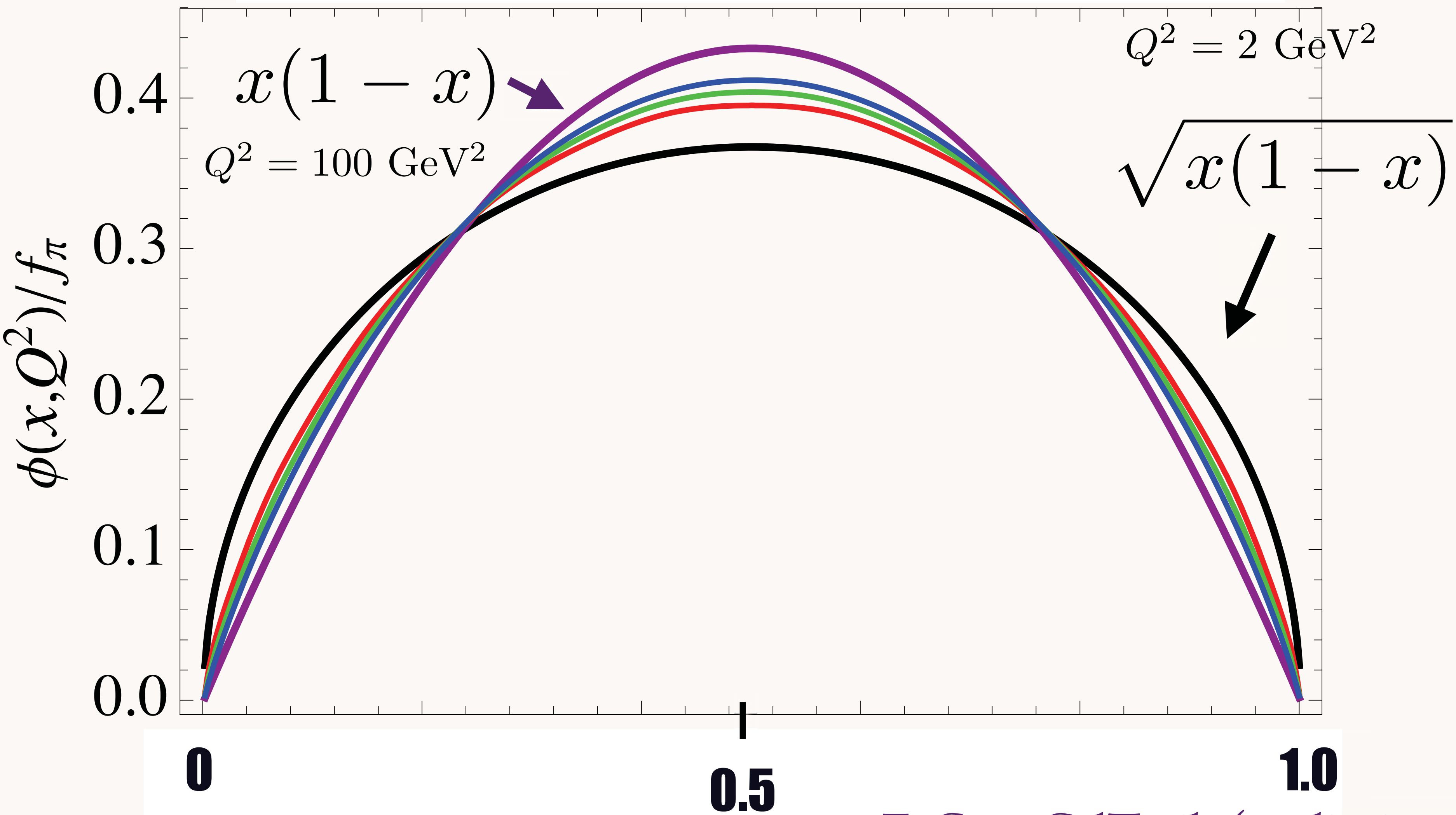
$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ — Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ — Braun et al.

ERBL Evolution of Pion Distribution Amplitude

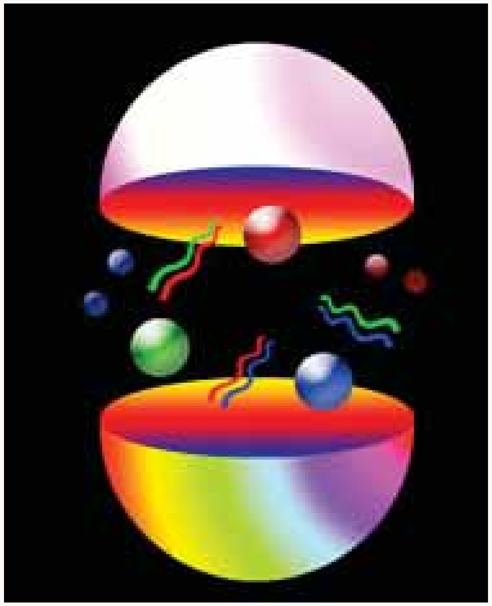


F. Cao, GdT, sjb (preliminary)

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

Baryons in AdS/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g}\, \overline{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x, z).$$

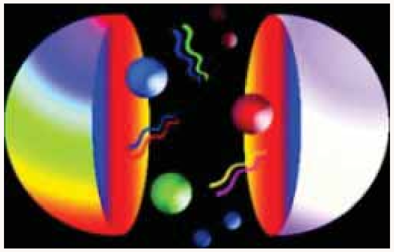
- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x, z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right] \Psi(x^\ell) = 0.$$

- Baryons Spectrum in ”bottom-up” holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



From Nick Evans

- Action for massive fermionic modes on AdS_5:

$$S[\overline{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{QCD}\right) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{QCD}$$

  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha \Pi(\zeta) \psi(\zeta) = \mathcal{M} \psi(\zeta),$$

where $H_{LF} = \alpha \Pi$ and the operator

$$\Pi_L(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L+1}{\zeta^2} \gamma_5.$$

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

- Solution $(\mu R = \nu + 1/2,\, d = 4)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2)$$
$$\Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ

by shifting dimensions

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i .$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta} \left[J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD} \right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k} \Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k} \Lambda_{\rm QCD},$$

with a scale independent mass ratio.

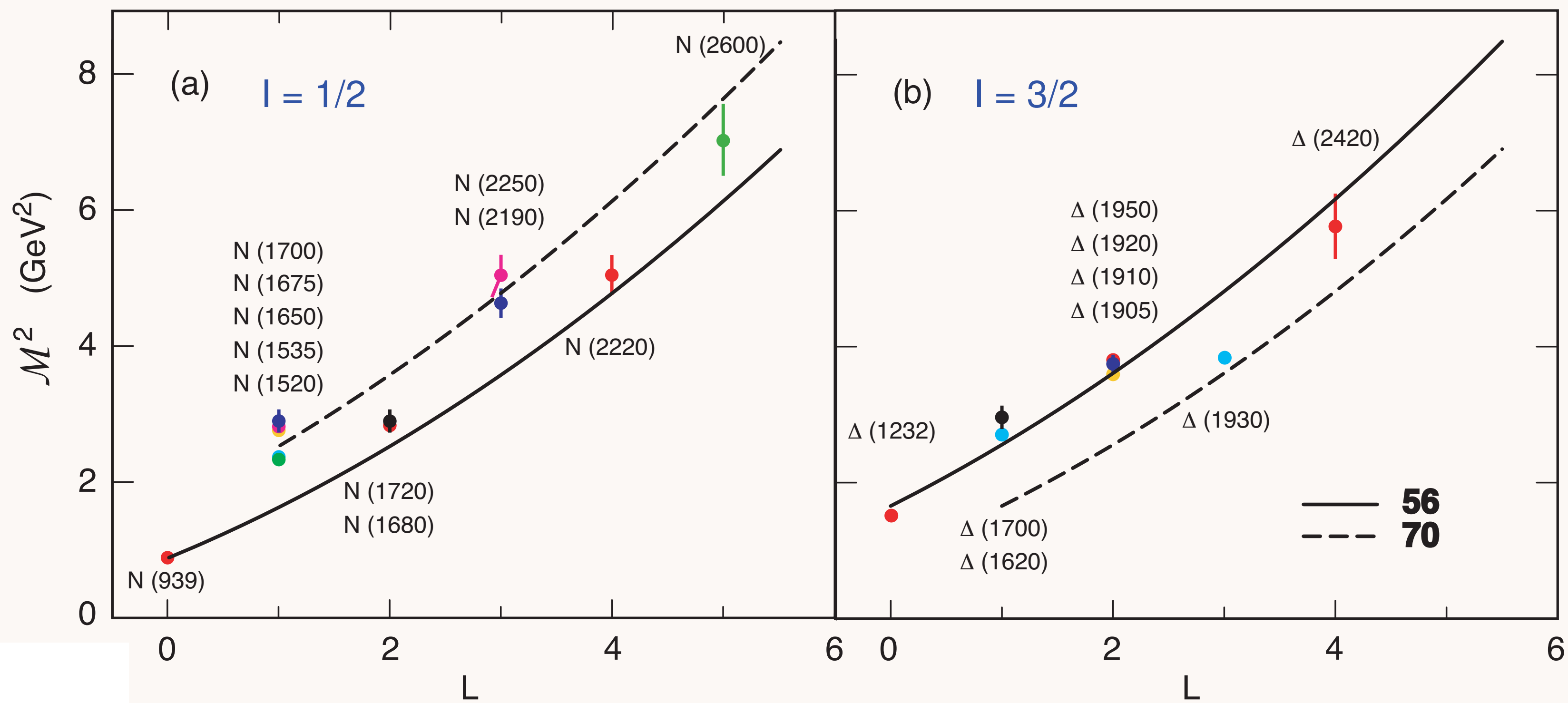


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

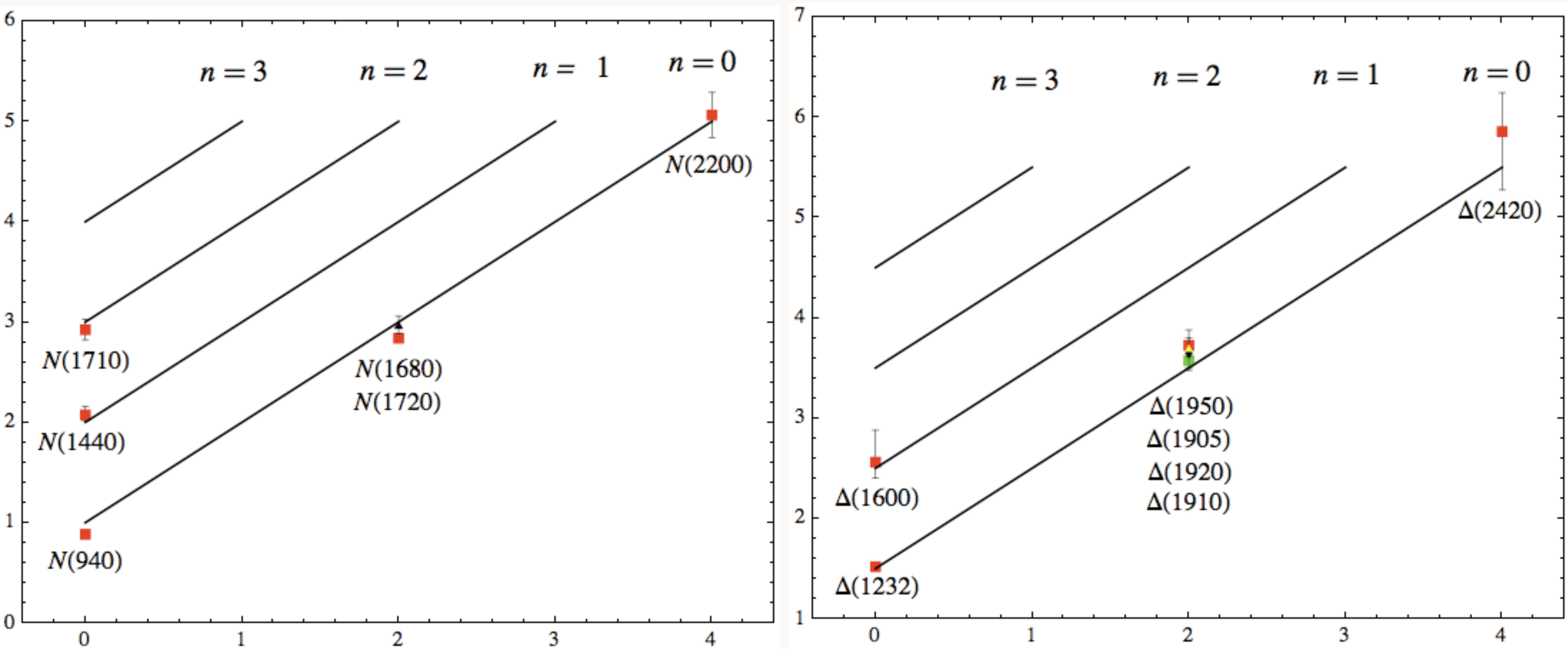
• **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

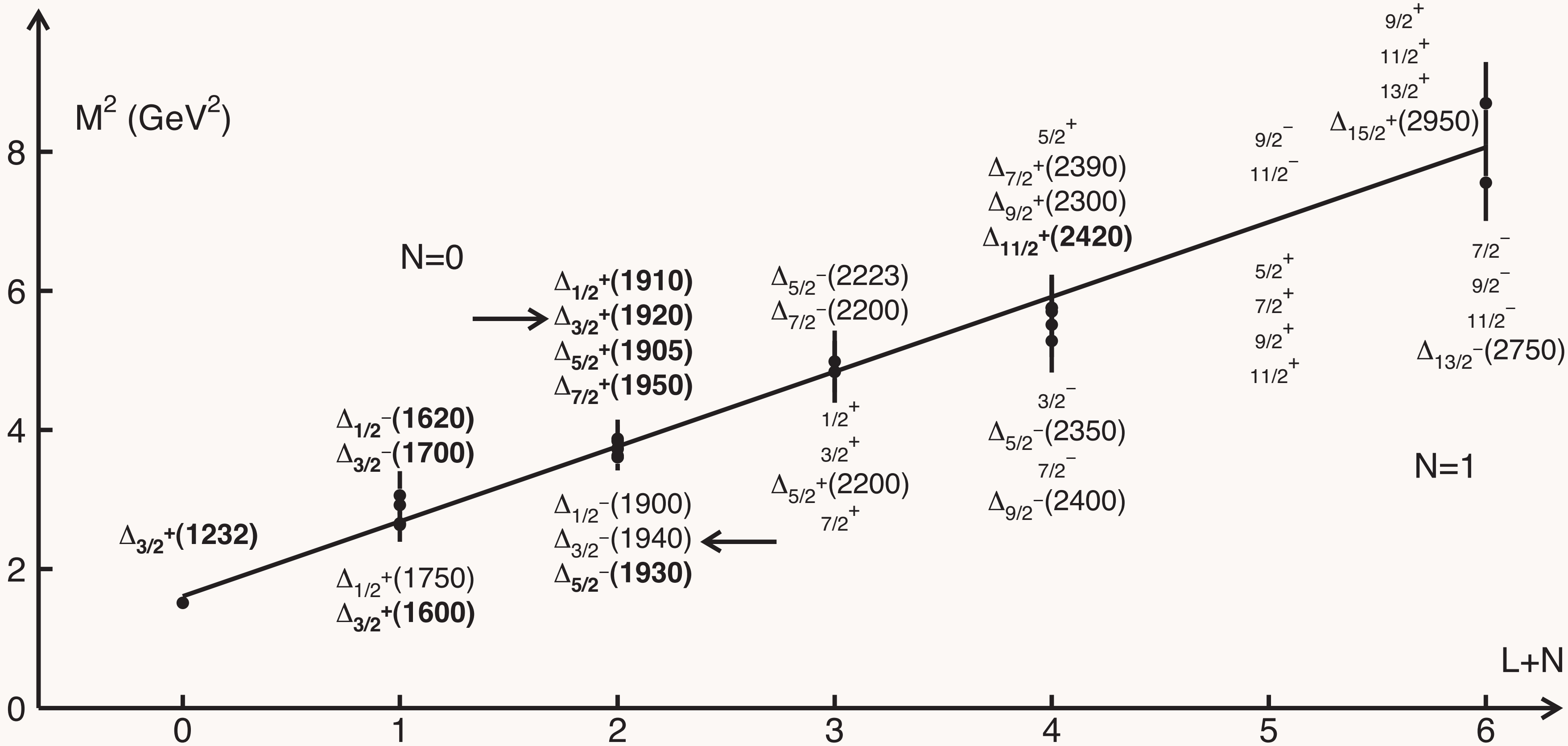
$2\kappa^2$ for $\Delta S = 1$

$\mathcal{M}^2$



L

Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

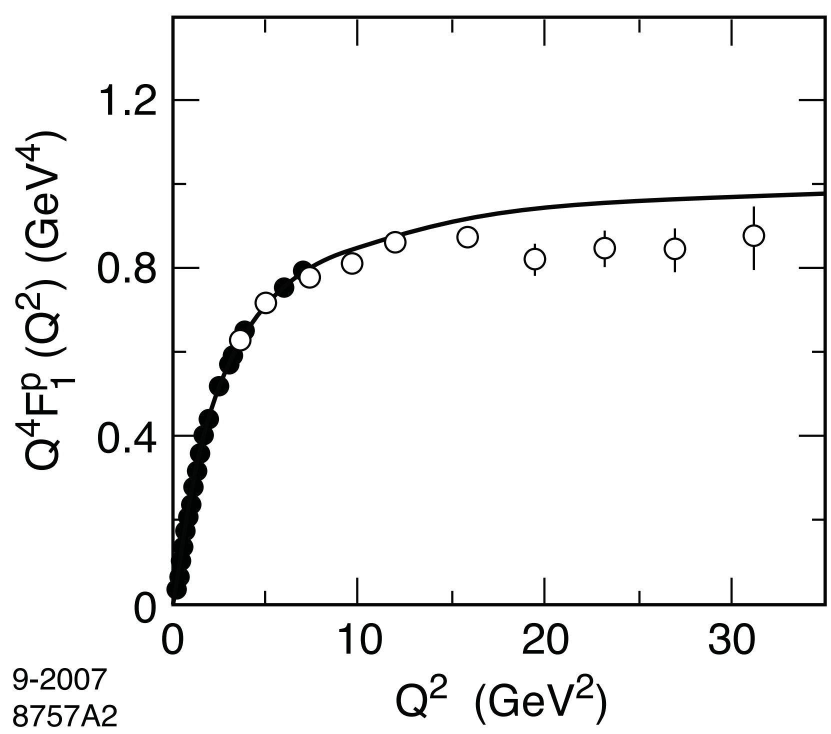
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

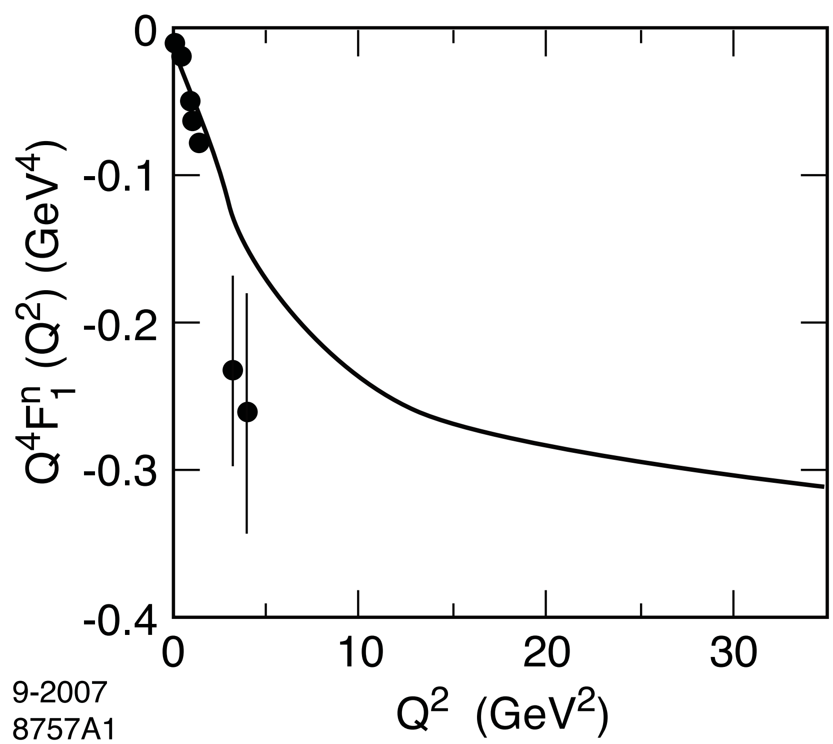
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

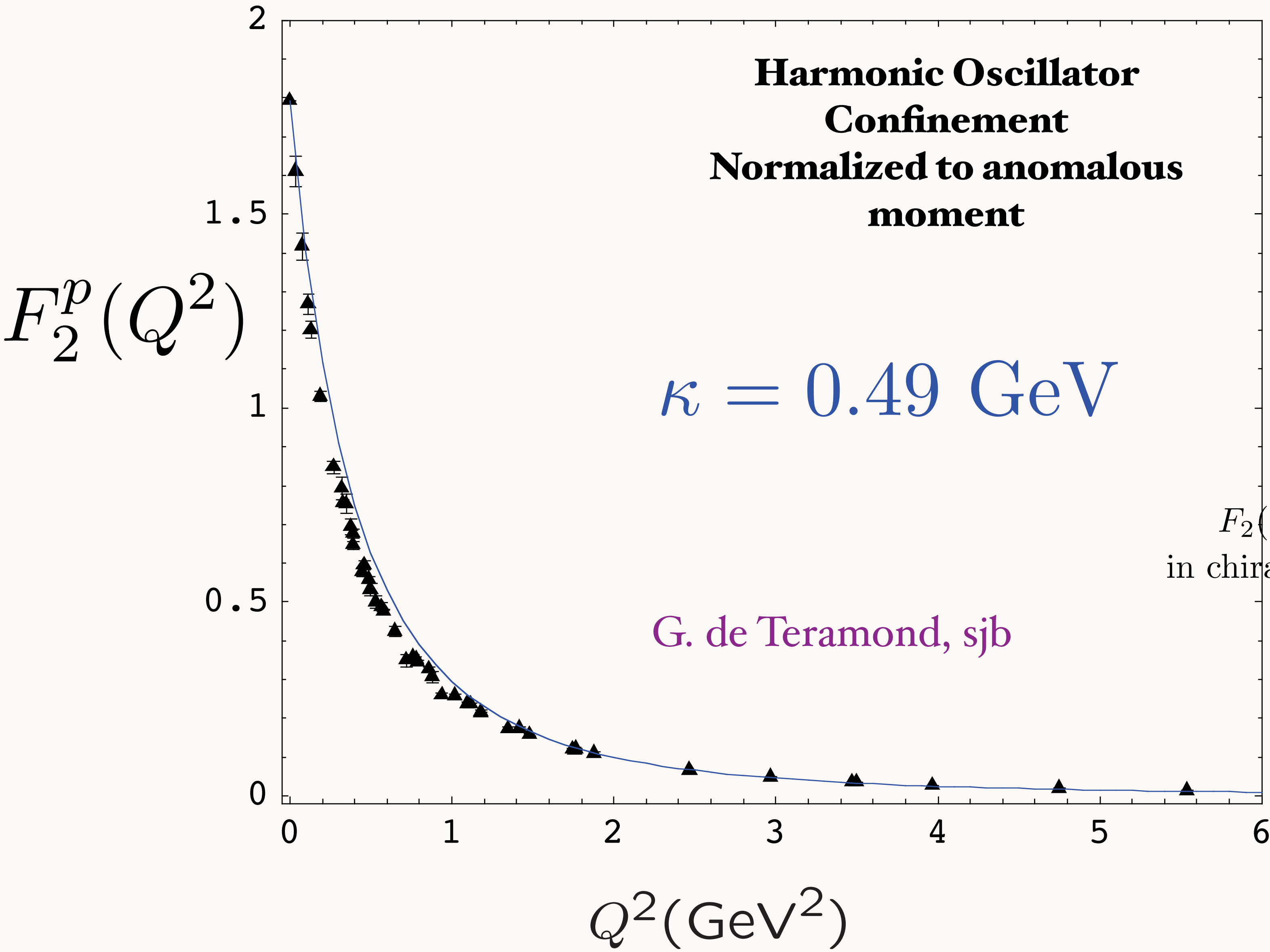


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



AdS/QCD No chiral divergence!

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$
in chiral perturbation theory

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

↓

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Light-Front QCD

Heisenberg Matrix Formulation

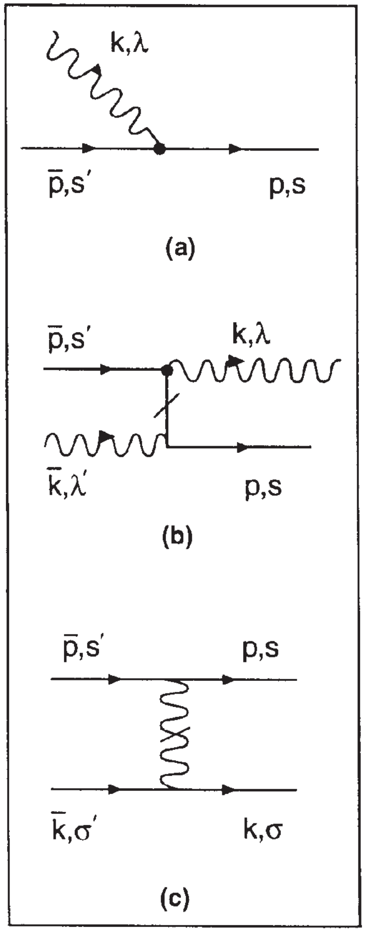
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

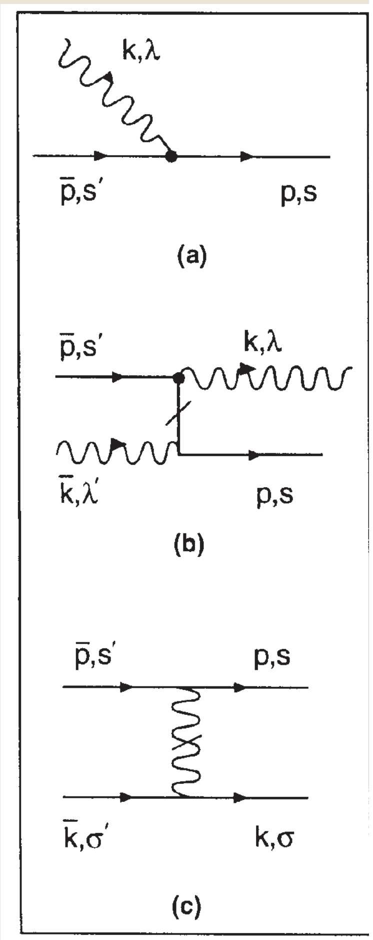
$$H_{LF}^{QCD} |\Psi_h >= \mathcal{M}_h^2 |\Psi_h >$$

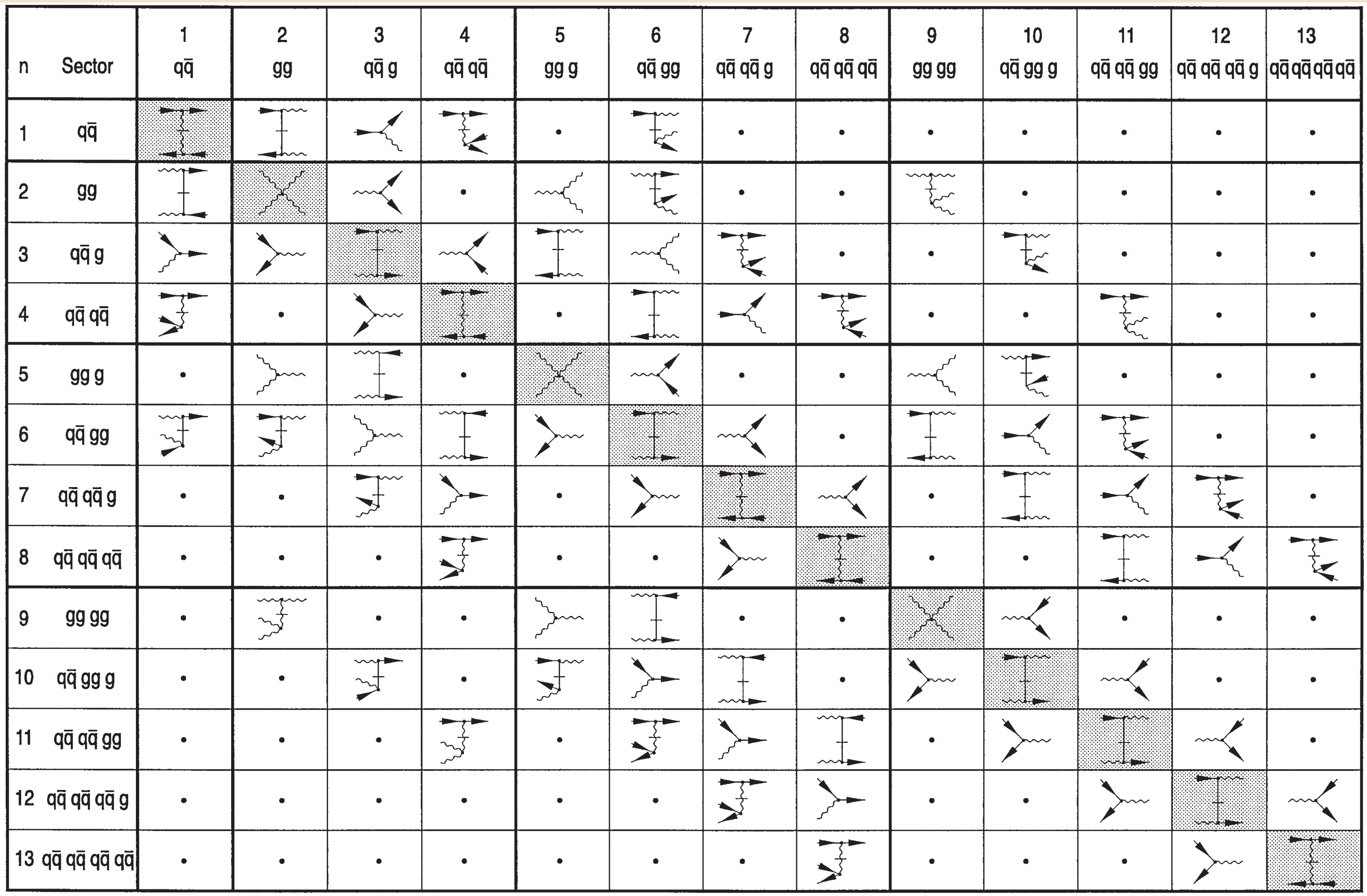


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1

Pauli, Hornbostel, Hiller, McCartor, sjb

- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
  G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

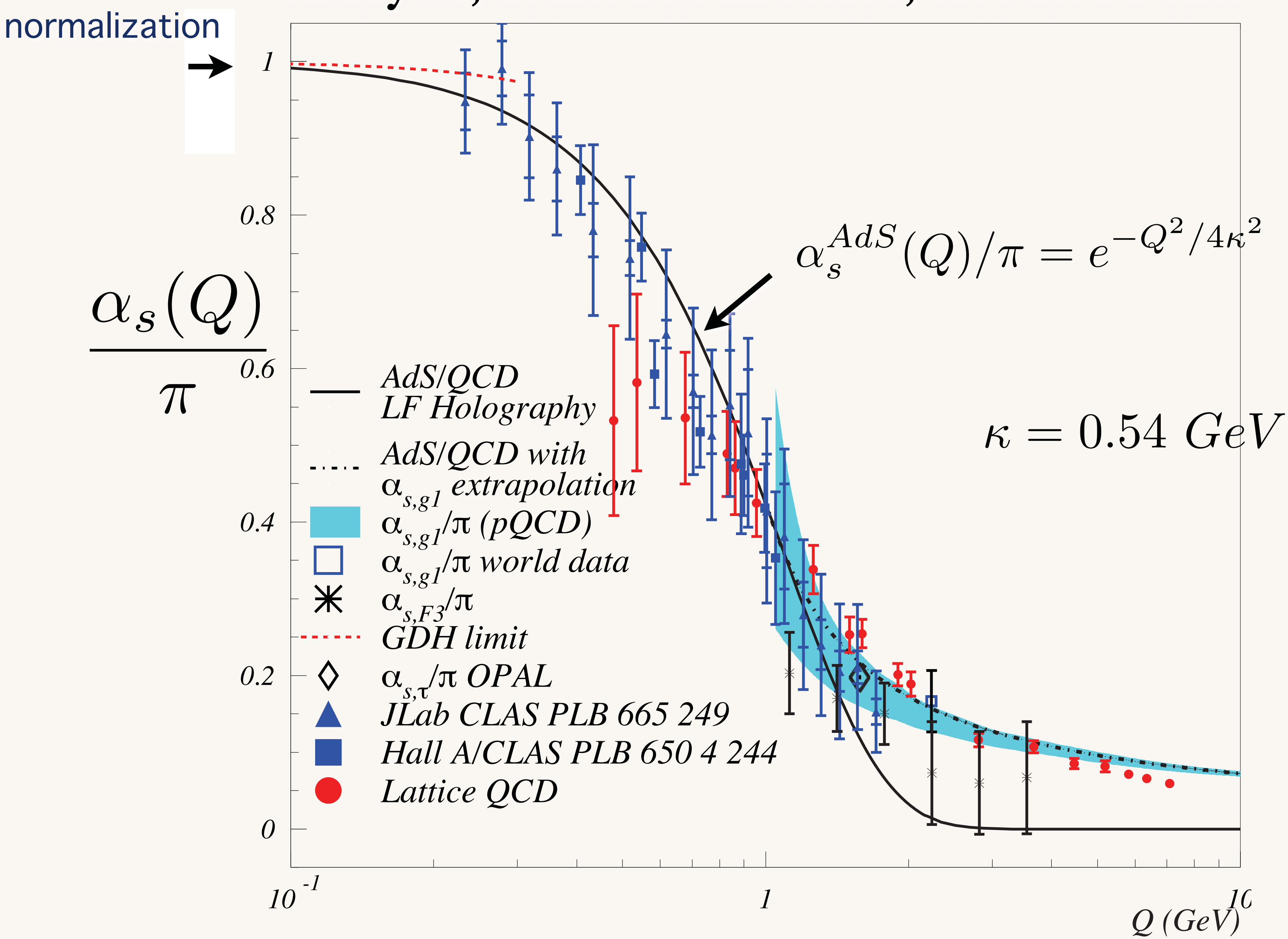
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

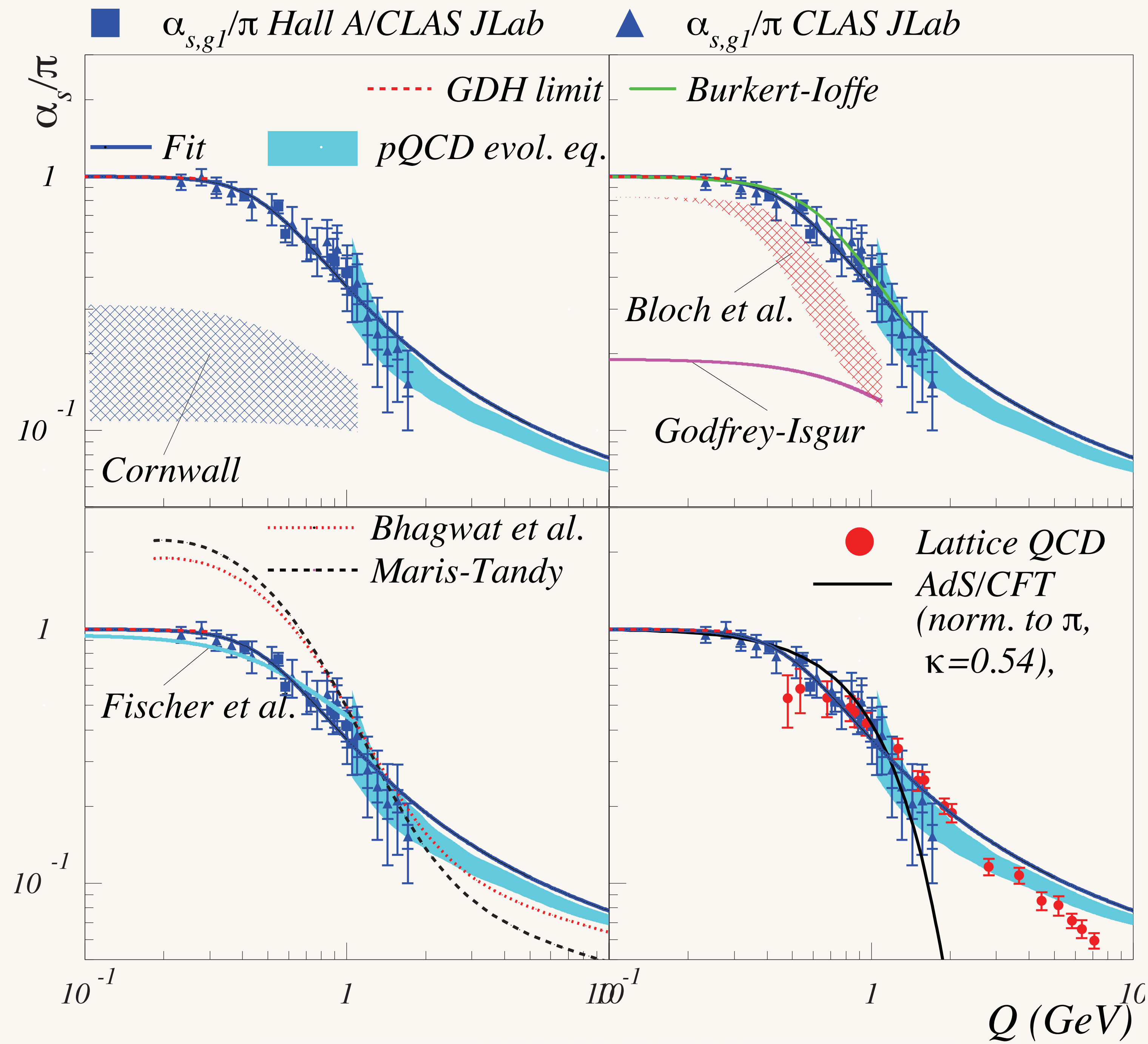
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

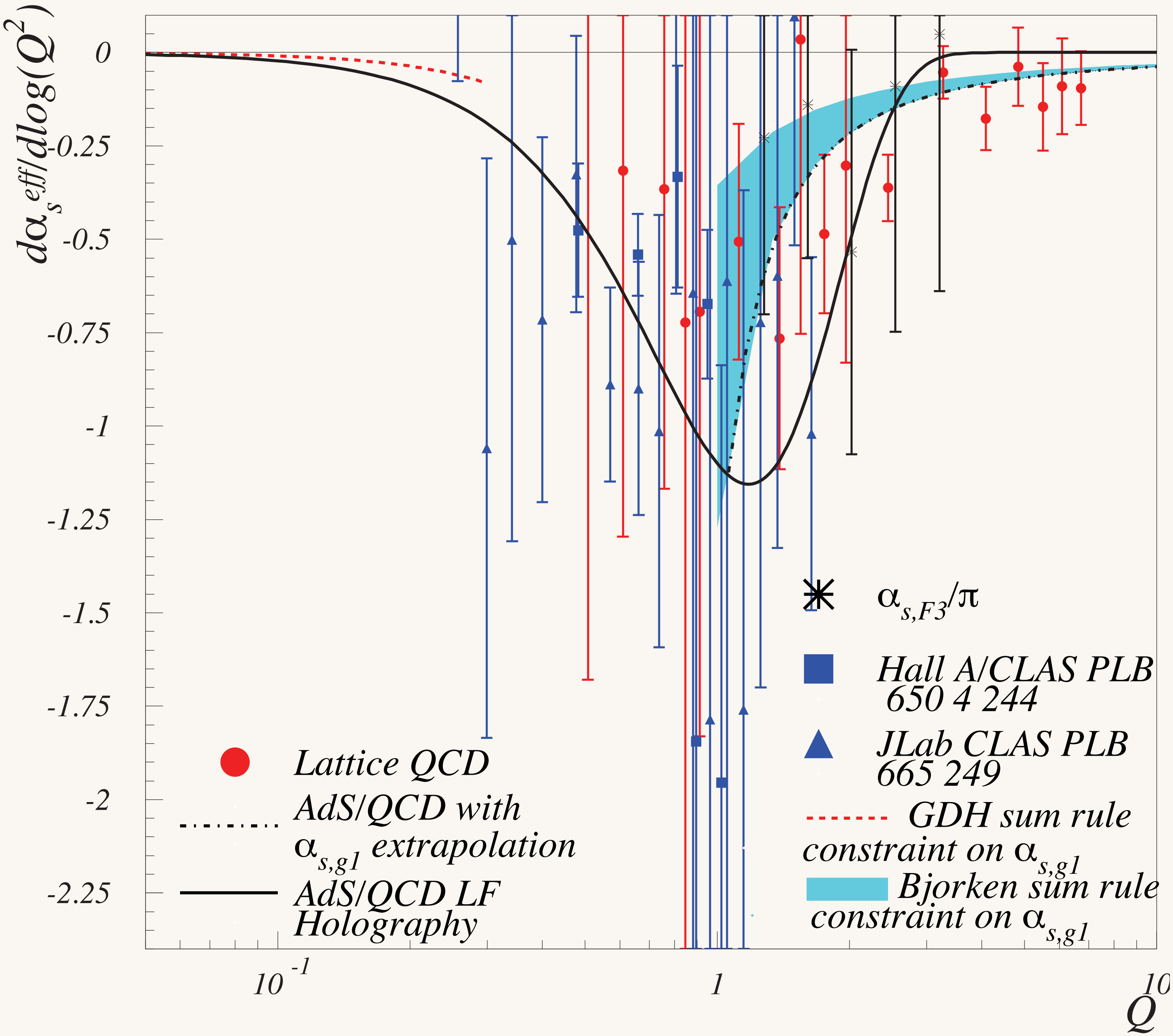


Deur, de Teramond, sjb, (preliminary)



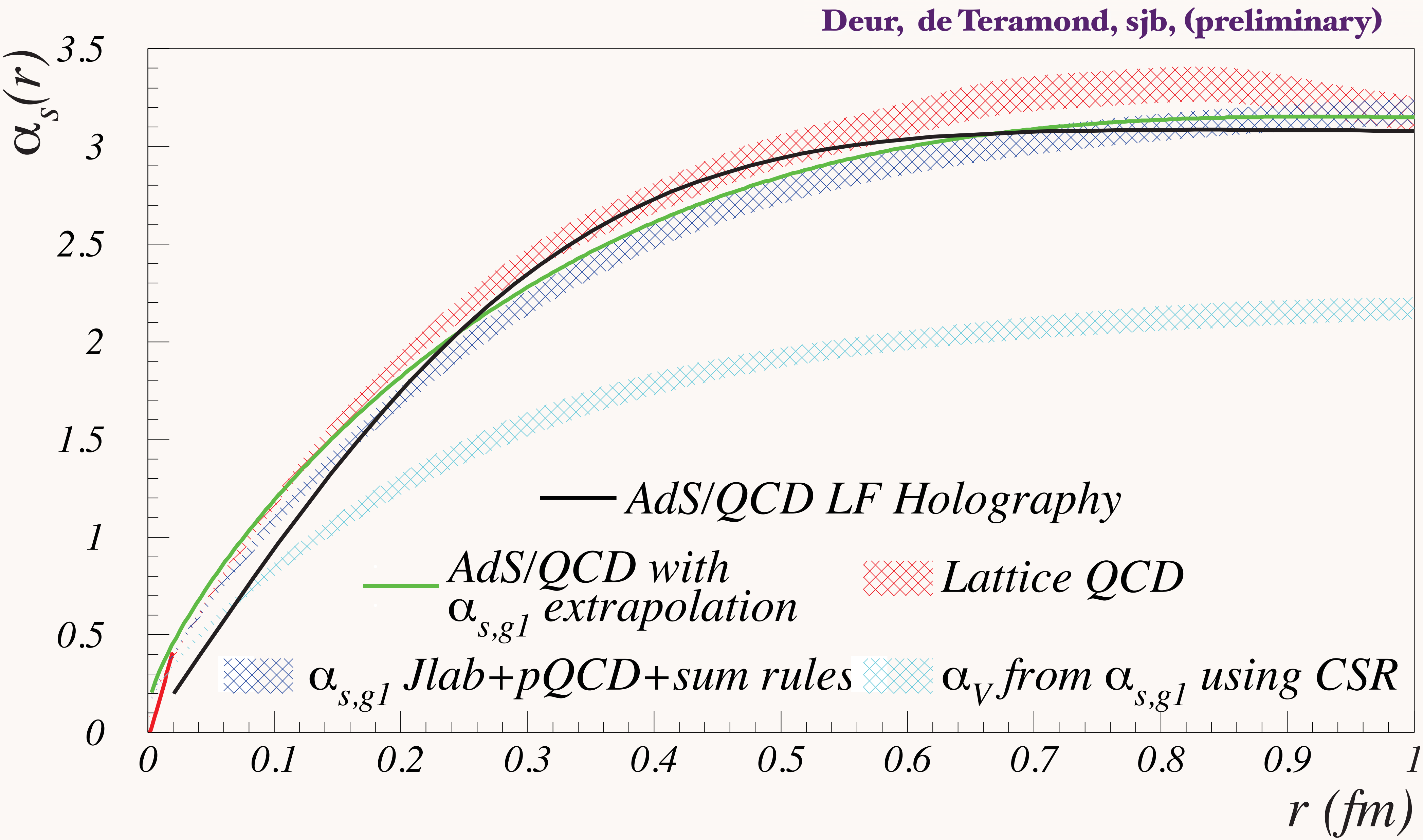
Deur, de Teramond, sjb, (preliminary)

$$\beta^{AdS}(Q^2) = \frac{d}{d\log Q^2}\alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2}e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb, (preliminary)

Running Coupling for Static Potential from AdS/QCD



Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

Leading Twist Sivers Effect

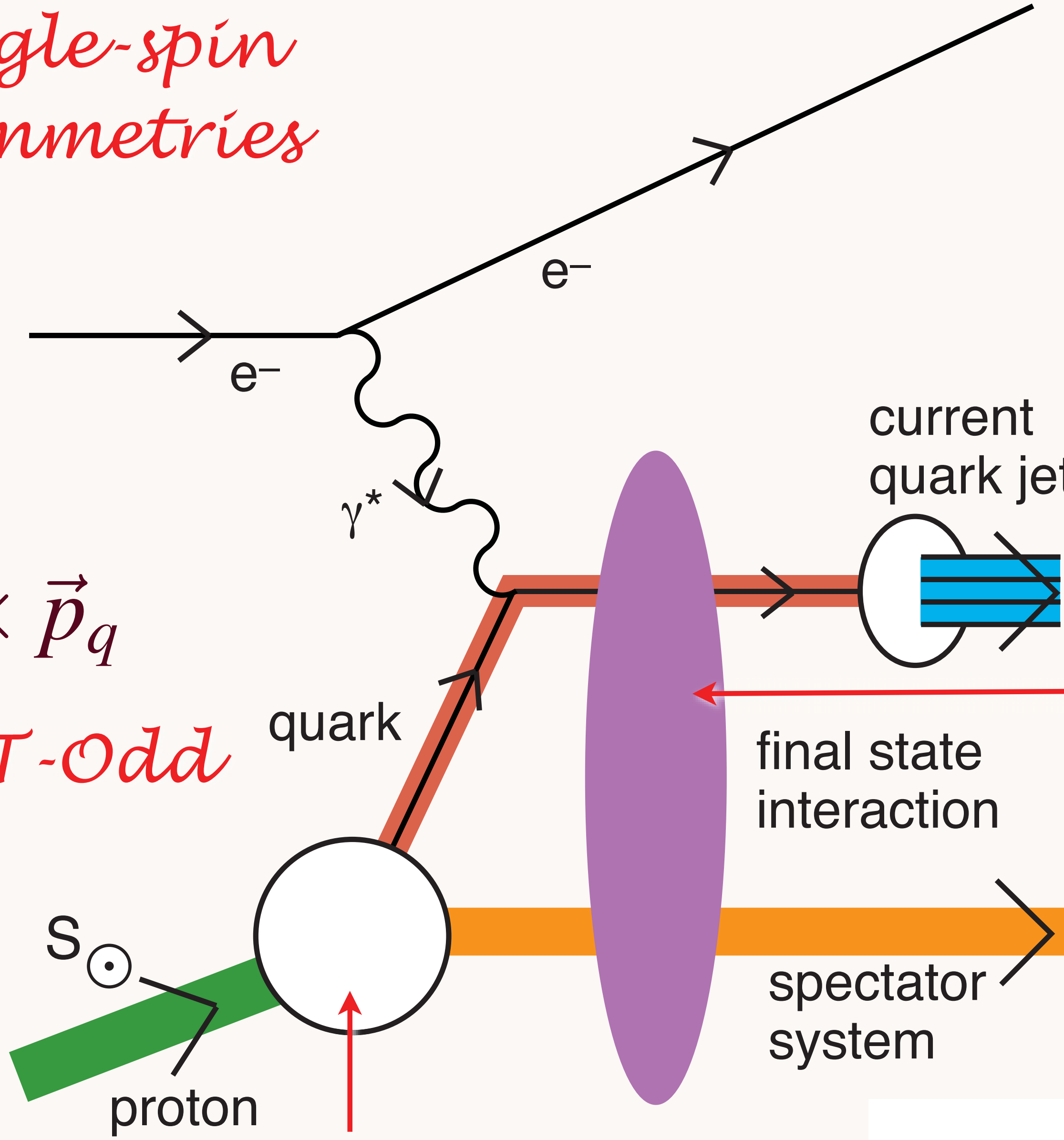
Single-spin asymmetries

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i \; \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



QCD S- and P-Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

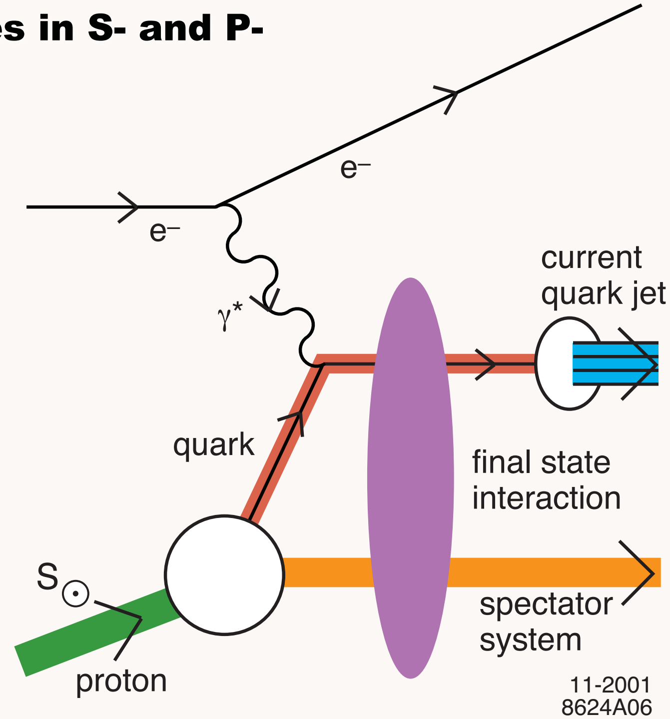
Leading-Twist Rescattering Violates pQCD Factorization!

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**



Pasquini, Xiao, Yuan, sjb
Mulders, Boer   Qiu, Sterman

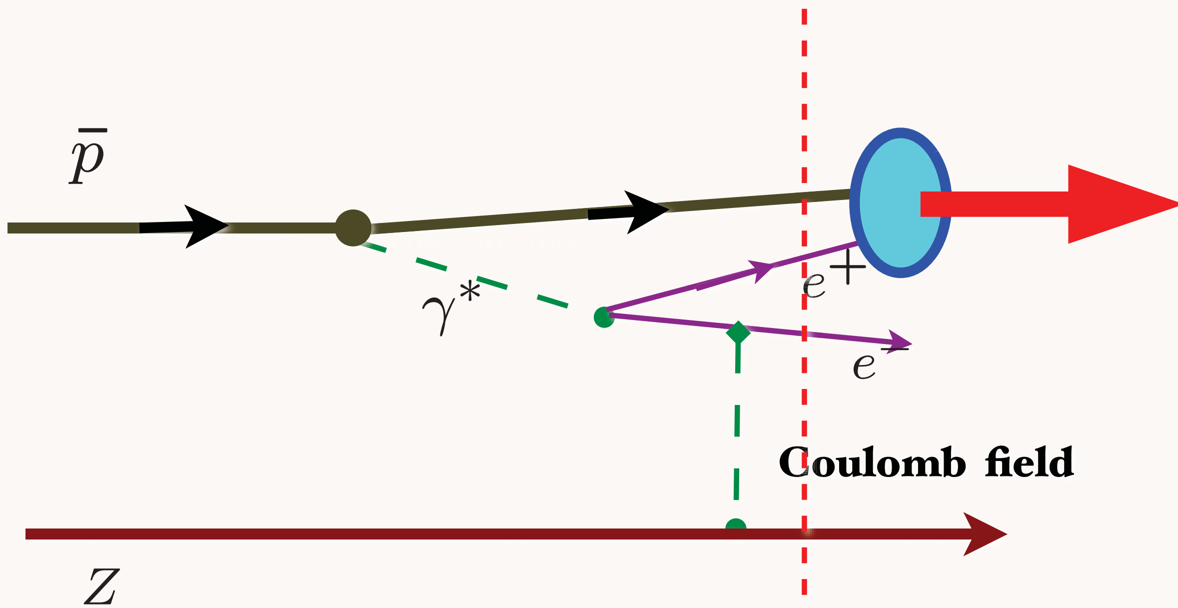
Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion (m_q =0)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S. Spectrum is independent of S**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$\bar{H}(\bar{p}e^+)$$

$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

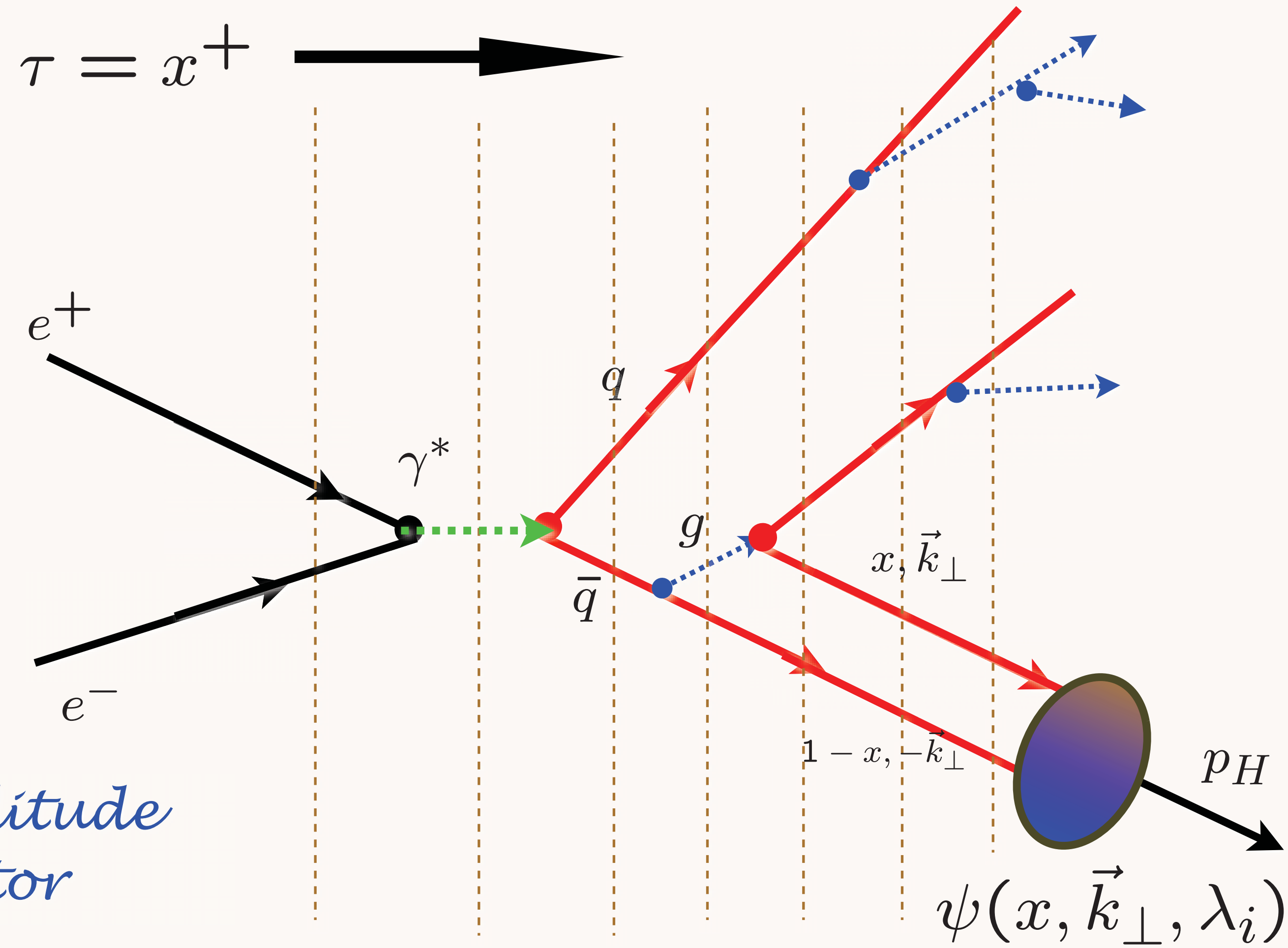
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell ***co-moving positron and antiproton***

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

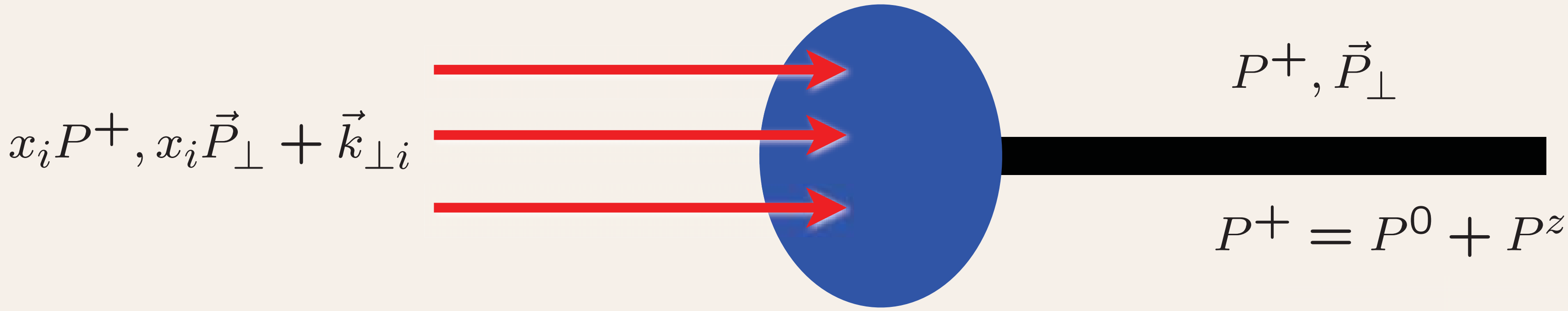
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

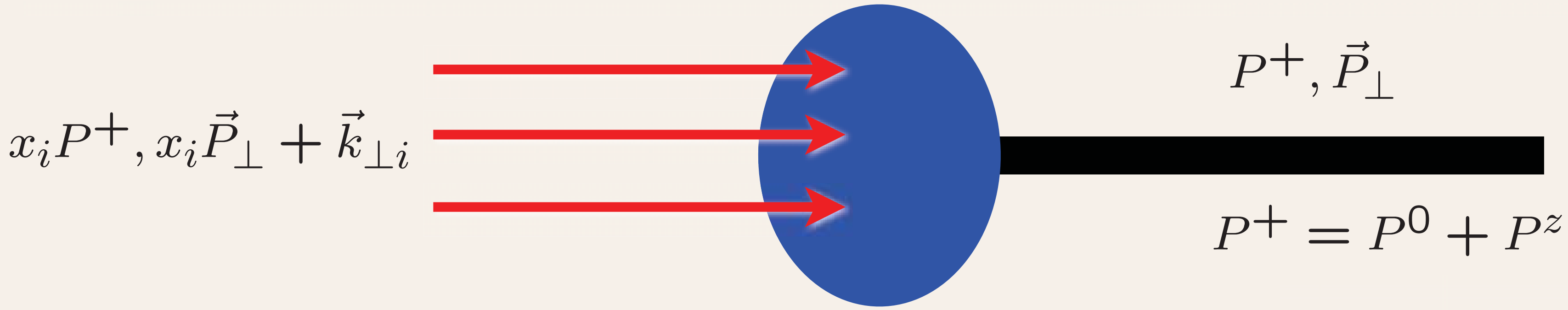
- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- **Same principle as antihydrogen production: off-shell coalescence**
- **coalescence to hadron favored at equal rapidity, small transverse momenta**
- **leading heavy hadron production: D and B mesons produced at large z**
- **hadron helicity conservation if hadron LFWF has $L^z = 0$**
- **Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin**



Chiral Symmetry Breaking in AdS/QCD

Erlich et al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2 m_q < \bar{\psi}\psi > \times \int dz \ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD: confined condensate**
- **Suggests "In-Hadron" Condensates**

de Teramond, Shrock, sjb

-

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

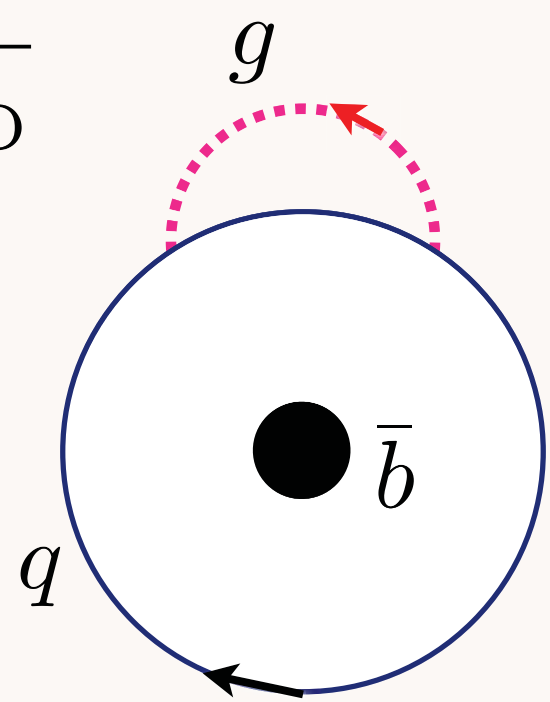
The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

Light-Front
(Front Form)
Formalism

Maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{QCD}} \qquad \lambda < \Lambda_{QCD}$$



B-Meson

Shrock, sjb

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

$$< \bar{b}|\bar{q}q|\bar{b} > \text{ not } < 0|\bar{q}q|0 >$$

Pion mass **and decay constant.**
Pieter Maris, Craig D. Roberts (Argonne, PHY) , Peter C. Tandy (Kent State U.) . ANL-PHY-8753-TH-97, KSUCNR-103-97, Jul 1997. 12pp.
Published in **Phys.Lett.B420:267-273,1998**.
e-Print: **nucl-th/9707003**

Pi- and K meson Bethe-Salpeter amplitudes.
Pieter Maris, Craig D. Roberts (Argonne, PHY) . ANL-PHY-8788-TH-97, Aug 1997. 34pp.
Published in **Phys.Rev.C56:3369-3383,1997**.
e-Print: **nucl-th/9708029**

Concerning the quark condensate.
K. Langfeld (Tubingen U.) , H. Markum (Vienna, Tech. U.) , R. Pullirsch (Regensburg U.) , C.D. Roberts (Argonne, PHY & Rostock U.) , S.M. Schmidt (Tubingen U. & HGF, Bonn) . ANL-PHY-10460-TH-2002, MPG-VT-UR-239-02, Jan 2003. 7pp.
Published in **Phys.Rev.C67:065206,2003**.
e-Print: **nucl-th/0301024**

"In-Meson Condensate"

$$-\langle \bar{q}q \rangle_\zeta^\pi = f_\pi \langle 0 | \bar{q}\gamma_5 q | \pi \rangle \, .$$

Valid even for $m_q \to 0$

f_π nonzero

In presence of quark masses the Holographic LF wave equation is ($\zeta = z$)

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{1}$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \tag{2}$$

The parameter a is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x}\langle\bar{\psi}\psi\rangle z^3,} \tag{3}$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2\sum_i m_i \langle\bar{\psi}\psi\rangle\langle z^2\rangle + \langle\bar{\psi}\psi\rangle^2\langle z^4\rangle, \tag{4}$$

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$.

Mass shift from dynamics inside hadronic boundary

Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations

Maris, Roberts, Tandy

- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs

Casher Susskind

- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant -- reduction by 45 orders of magnitude!

Shrock, sjb

"Confined QCD Condensates"

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, ...)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

Shrock and sjb

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

Conformal Template

- **BLM scale-setting: Retain conformal series; nonzero β-terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.**
- **Commensurate Scale Relations between observables based on conformal template -- prime tests of QCD independent of theory conventions**
- **BLM scales for three-gluon coupling; multi-scale problems**
- **Analytic scheme for coupling unification**
- **Direct high p_T processes, baryon anomaly, intrinsic heavy quarks**
- **IR Fixed point -- conformal symmetry motivation for AdS/CFT**
- **Light-Front Schrödinger Equation: analytic first approximation to QCD**
- **Dilaton-modified AdS_5: Predict Hadron Spectra, Light-Front Wavefunctions, Form Factors, Hadronization at Amplitude Level**
- **Non-Perturbative running QCD coupling -- new range of QCD phenomena such as Sivers and Diffractive DIS fraction calculable; IR fixed point**