

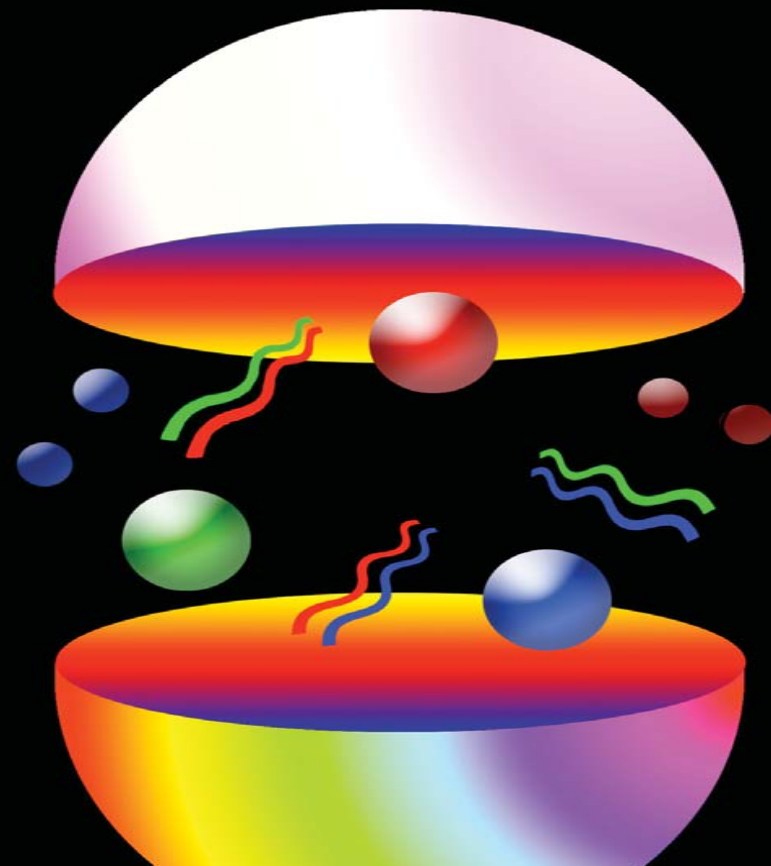
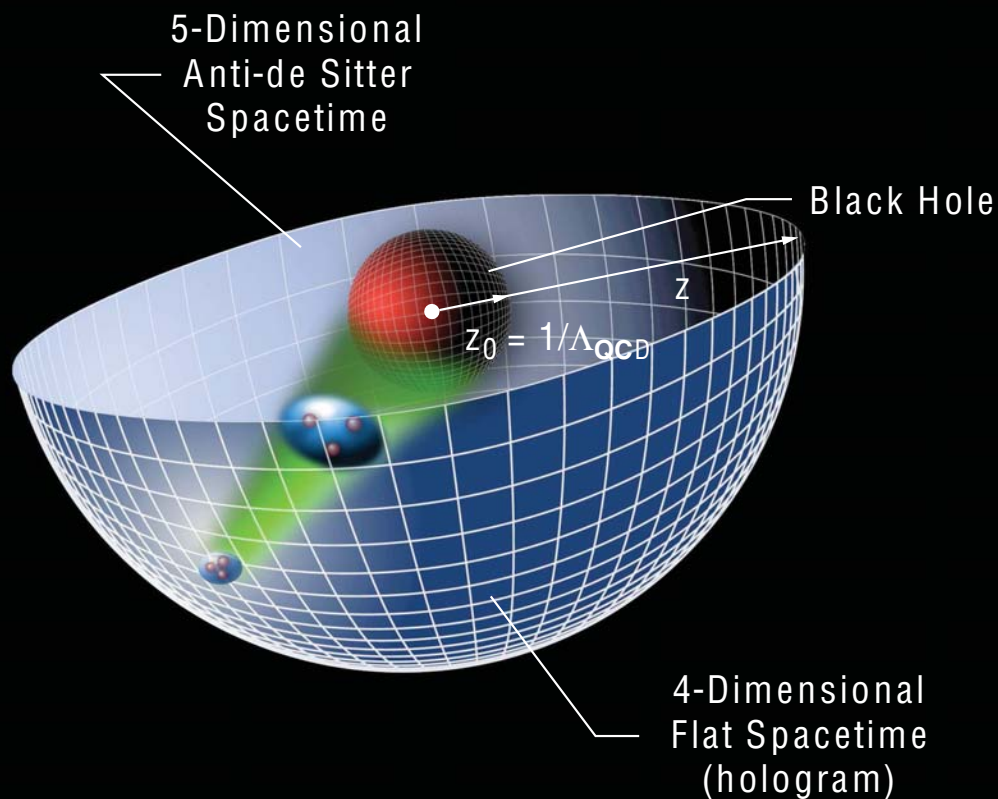
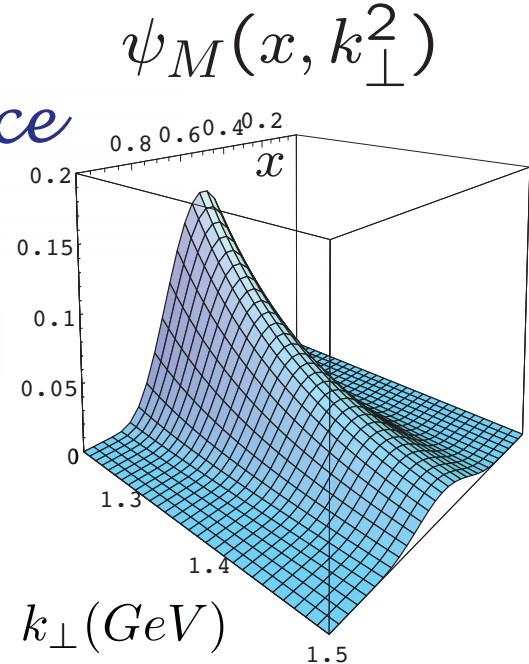


*The AdS/CFT Correspondence
and Light-Front QCD*

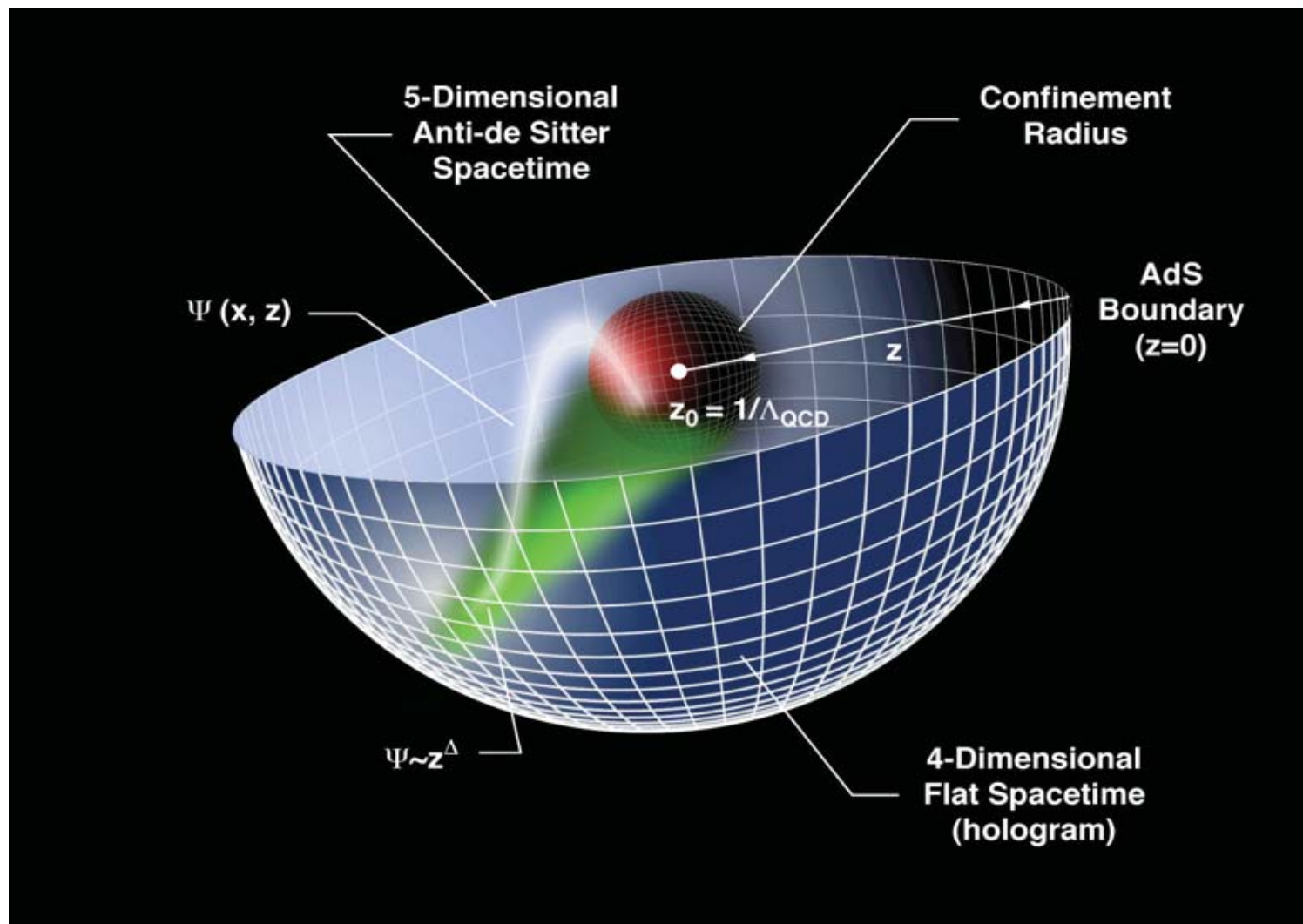
Stan Brodsky, SLAC

Columbia University

February 18, 2008



Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

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February 18, 2008**

QCD on the LF

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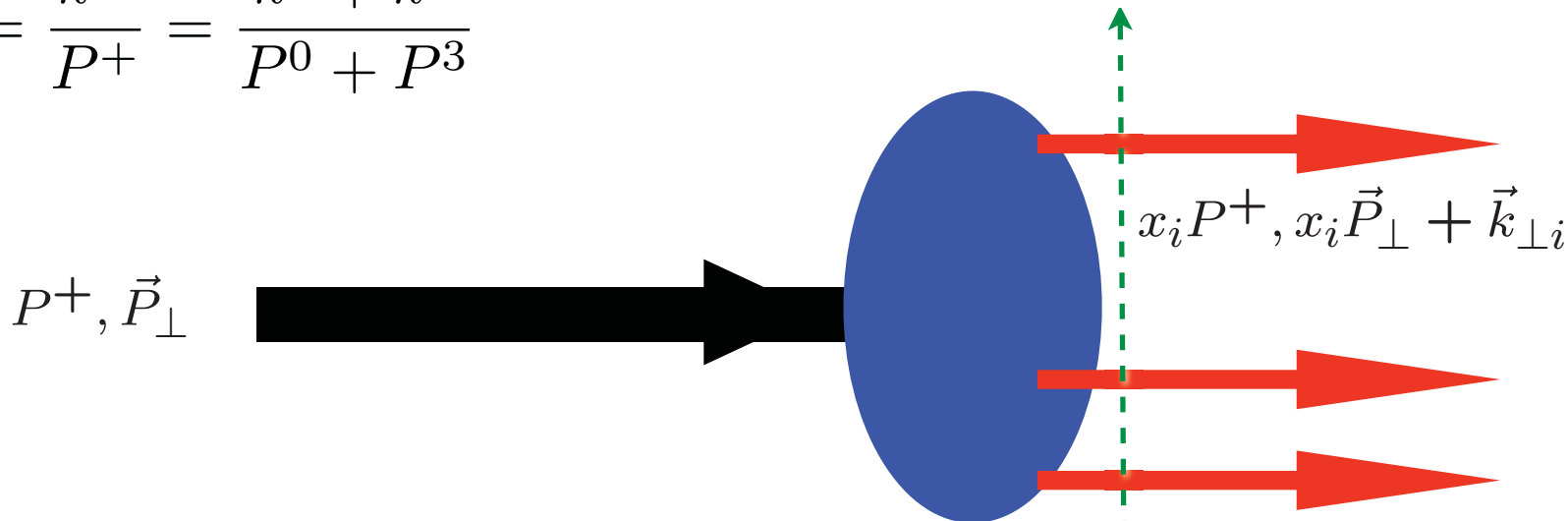
Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Holographic Model*
- *Light-Front Hadron Wavefunctions*

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

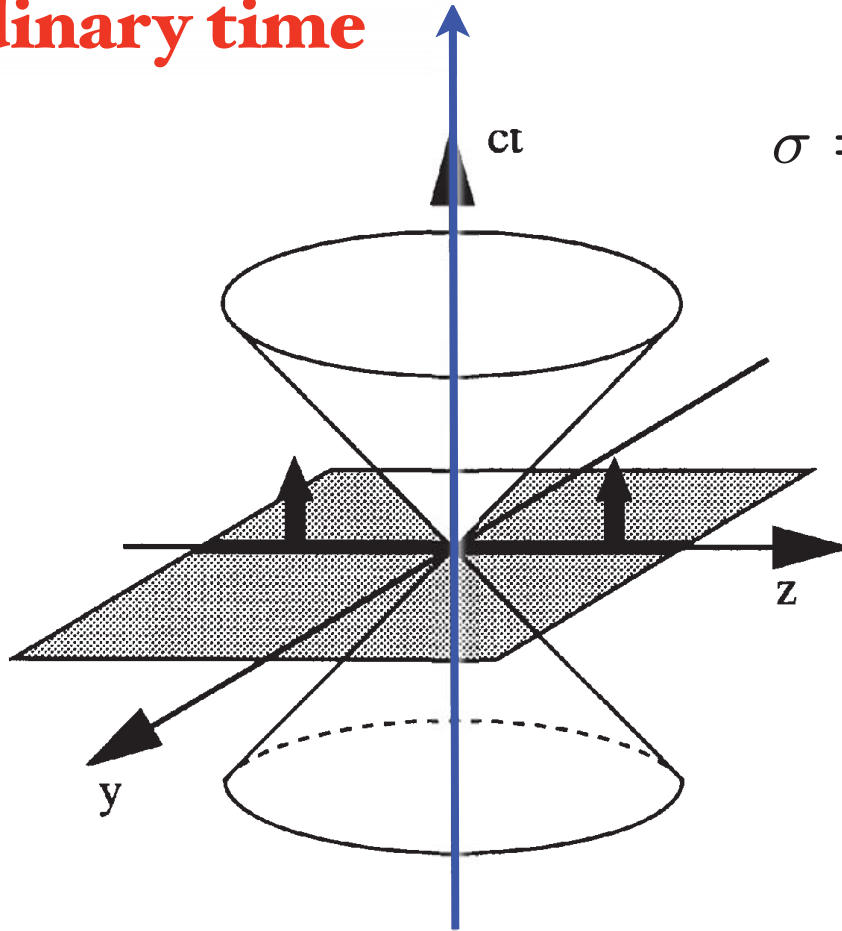
$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

Invariant under boosts! Independent of p^μ

Dirac's Amazing Idea: The Front Form

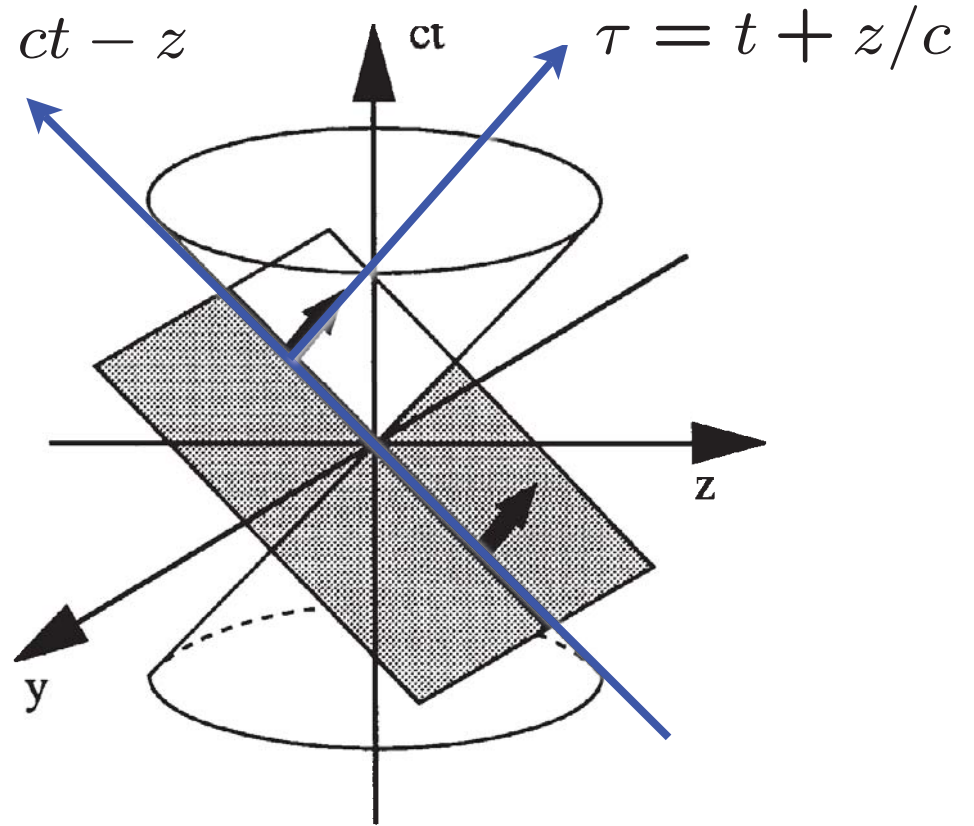
**Evolve in
ordinary time**



Instant Form

**Evolve in
light-front time!**

$$\sigma = ct - z$$



$$\tau = t + z/c$$

Front Form

*'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing*

...A moment standing still for ever.

James Montgomery 1833

Sed fugit, interea, fugit irreparabile tempus.

VIRG. Georg. iii. 284.

The poetical works of James Montgomery

TIME.

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TIME:

A RHAPSODY.

Sed fugit, interea, fugit irreparabile tempus.
VING. *Georg.* III. 284.

'Tis a mistake: time flies not,
He only hovers on the wing:
Once born, the moment dies not,
'Tis an immortal thing;
While all is change beneath the sky,
Fix'd like the sun as learned sages prove,
Though from our moving world he seems to move,
'Tis time stands still, and we that fly.

There is no past; from nature's birth,
Days, months, years, ages, till the end
Of these revolving heavens and earth,
All to one centre tend;
And, having reach'd it late or soon,
Converge,—as in a lens, the rays,
Caught from the fountain-light of noon,
Blend in a point that blinds the gaze:
—What has been is, what is shall last;
The present is the *focus* of the past;
The future, perishing as it arrives,
Becomes the present, and itself survives.

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MISCELLANIES.

Time is not *progress*, but *amount*;
One vast accumulating store,
Laid up, not lost;—we do not count
Years *gone* but *added* to the score
Of wealth untold, to clime nor class confined,
Riches to generations lent,
For ever spending, never spent,
The' august inheritance of all mankind.
Of this, from Adam to his latest heir,
All in due turn their portion share,
Which, as they husband or abuse,
Their souls they win or lose.

Though history, on her faded scrolls,
Fragments of facts, and wrecks of names enrols,
Time's indefatigable fingers write
Men's meanest actions on their souls,
In lines which not himself can blot:
These the last day shall bring to light,
Though through long centuries forgot,
When hearts and sepulchres are bared to sight.

Then, having fill'd his measure up,
Amidst his own assembled progeny,
(All that have been, that are, or yet may be,)
Before the great white throne,
To Him who sits thereon,
Time shall present the' amalgamating cup,
In which, as in a crucible,
He hid the moments as they fell,

TIME.

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More precious than Golconda's gems,
Or stars in angels' diadems,
Though to our eyes they seem'd to pass
Like sands through his symbolic glass:
But now, the process done,
Of millions multiplied by millions, none
Shall there be wanting,—while by change
Ineffable and strange,
All shall appear at once, all shall appear as one.

Ah! then shall each of Adam's race,
In that concenter'd instant, trace,
Upon the tablet of his mind,
His whole existence in a thought combined,
Thenceforth to part no more, but be
Impictured on his memory;
—As in the image-chamber of the eye,
Seen at a glance, in clear perspective, lie
Myriads of forms of ocean, earth, and sky.

Then shall be shown, that but in name
Time and eternity were both the same;
A point which life nor death could sever,
A moment standing still for ever.

1833.

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

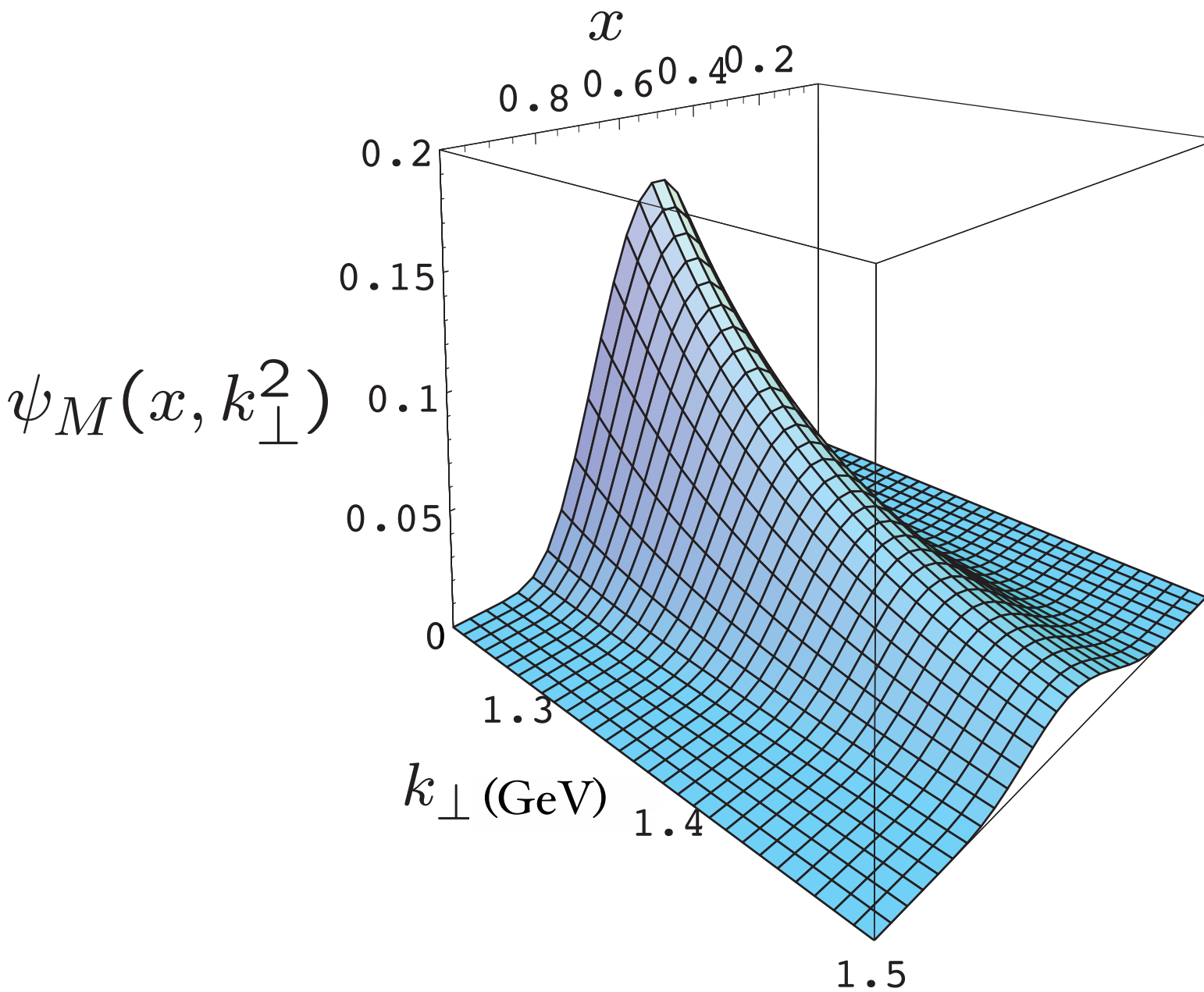
$$\Psi(x, k_{\perp}) \quad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^{μ}

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

*Remarkable new insights from AdS/CFT,
the duality between conformal field
theory and Anti-de Sitter Space*

Prediction from AdS/CFT: Meson LFWF



**“Soft Wall”
model**

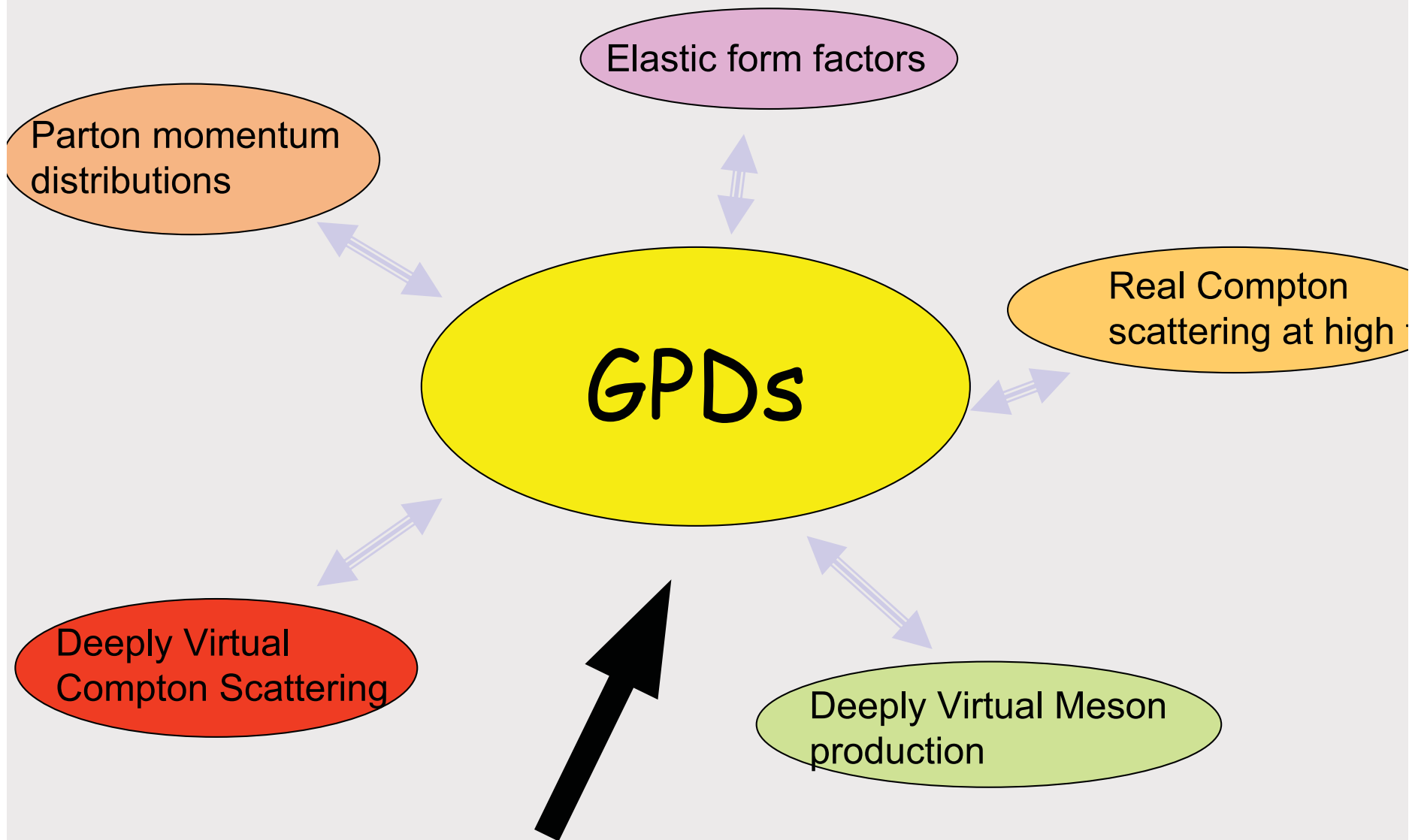
de Teramond, sjb

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QCD on the LF

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A Unified Description of Hadron Structure



Light Front Wavefunctions

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Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role of ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

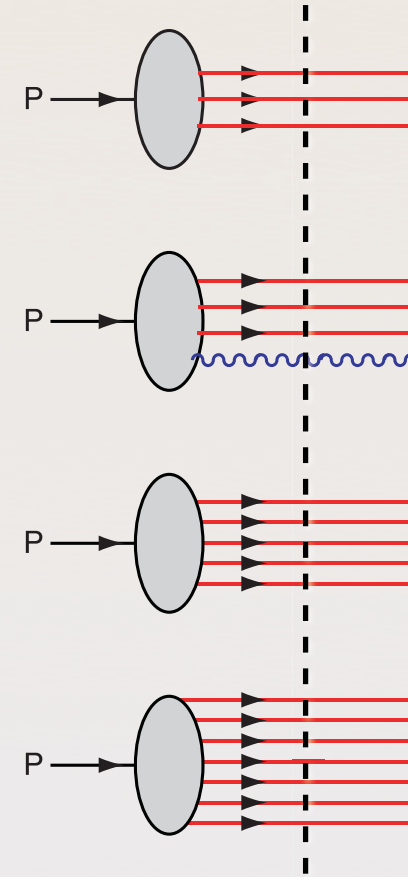
$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks

$$\bar{u}(x) \neq \bar{d}(x)$$

Mueller: BFKL DYNAMICS

$$\bar{s}(x) \neq s(x)$$



Fixed LF time

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Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

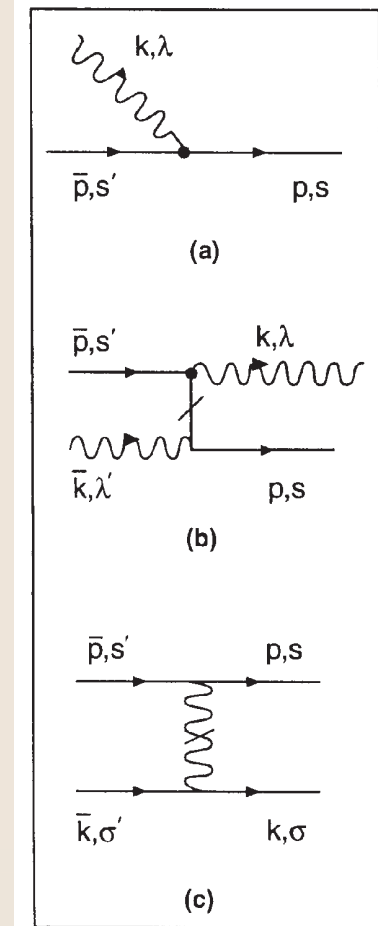
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions



DLCQ: Periodic BC in x^- . Discrete k^+ ; frame-independent truncation

Light-Front QCD

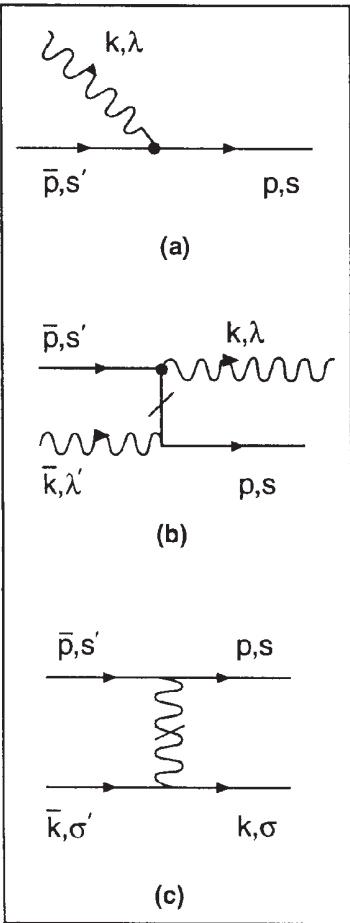
Heisenberg Matrix Formulation

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization

n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 ggg	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gggg	10 q \bar{q} ggg	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	ggg
6	q \bar{q} gg							.	.					.
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gggg
10	q \bar{q} ggg
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g			
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}		



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h} |\psi_h\rangle$ of the LF Hamiltonian $H_{LF} = P^2 = P^+ P^- - \mathbf{P}_\perp^2$, $H_{LF}|P\rangle = \mathcal{M}^2|P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac '49; Pauli and Pinsky, sjb Phys. Rept. 1988).

- Fock components

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}) = \langle n; x_i, \mathbf{k}_{\perp i}, |\psi_h(P^+, \mathbf{P}_\perp)\rangle,$$

frame independent and encode hadron properties in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+ / P^+$ and $\mathbf{k}_{\perp i}$ are the relative coordinates of parton i in Fock-state n

$$\sum_{i=1}^n x_i = 1 \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

- Define transverse position coordinates $x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^n \mathbf{b}_{\perp i} = 0, \quad \sum_{i=1}^n x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp.$$

Discretized Light-Cone Quantization

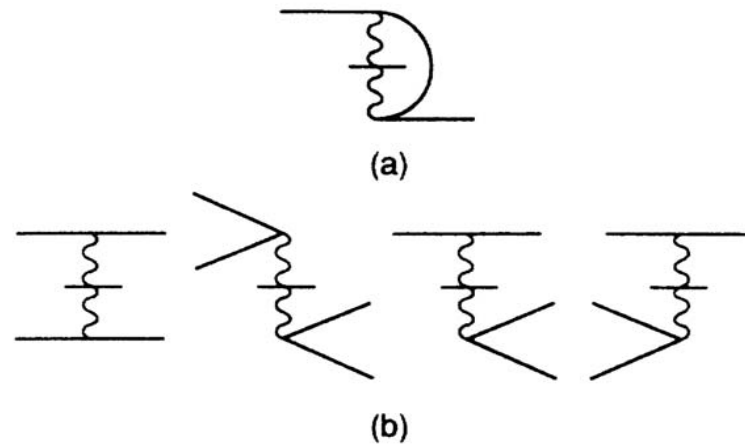
- Periodic boundary condition in x^-
- discrete positive plus momenta
- frame-independent formulation
- no fermion doubling
- covariant limit on Fock space

$$P^+ = \frac{2\pi}{L} K$$

$$k_i^+ = \frac{2\pi}{L} n_i$$

$$\sum_i k_i^+ = P^+$$

$$\sum_i n_i = K$$



QCD(I+I) Interaction vertices.

$$\frac{L}{2\pi} \frac{g^2}{\pi} \frac{1}{2} \left[\delta_{c_4}^{c_2} \delta_{c_1}^{c_3} - \frac{1}{N} \delta_{c_4}^{c_3} \delta_{c_1}^{c_2} \right]$$

$$\times \sum_{n_i = 1/2, 3/2, \dots} \frac{\delta_{n_1 + n_2, n_3 + n_4}}{(n_1 + n_3)^2} b_{n_4}^{\dagger c_4} b_{n_3, c_3} d_{n_2, c_2}^{\dagger} d_{n_1}^{c_1} .$$

PRD 41, 3814 (1990)

Light-cone-quantized QCD in 1+1 dimensions

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Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

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Max Plank Institute for Nuclear Physics, D-6900 Heidelberg 1, Germany

(Received 5 February 1990)

The QCD light-cone Hamiltonian in one space and one time dimension is diagonalized in a discrete momentum-space basis. The hadronic spectrum and wave functions for various coupling constants, numbers of color, and baryon number are computed.

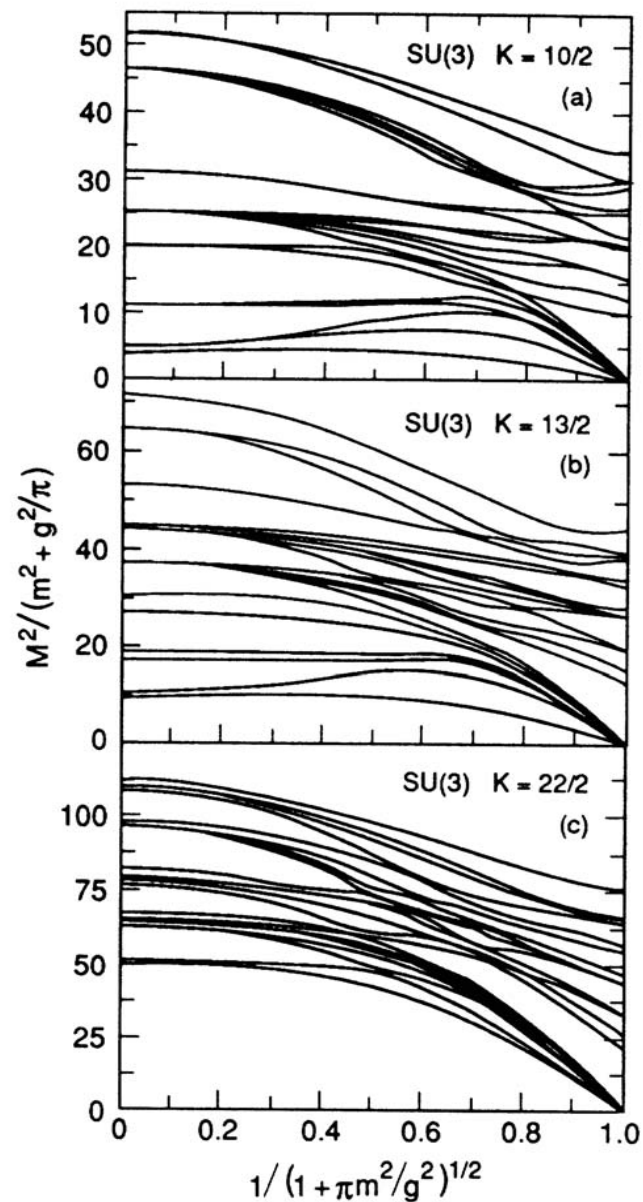
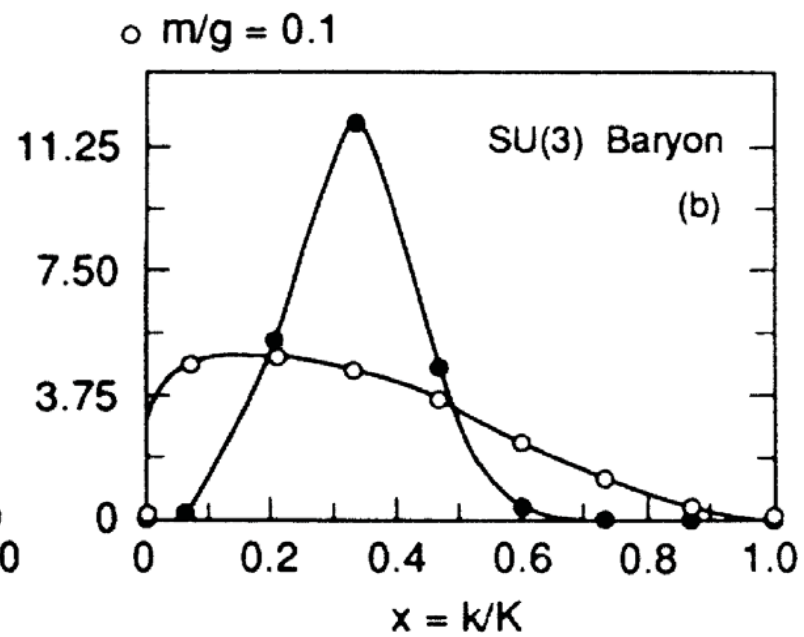
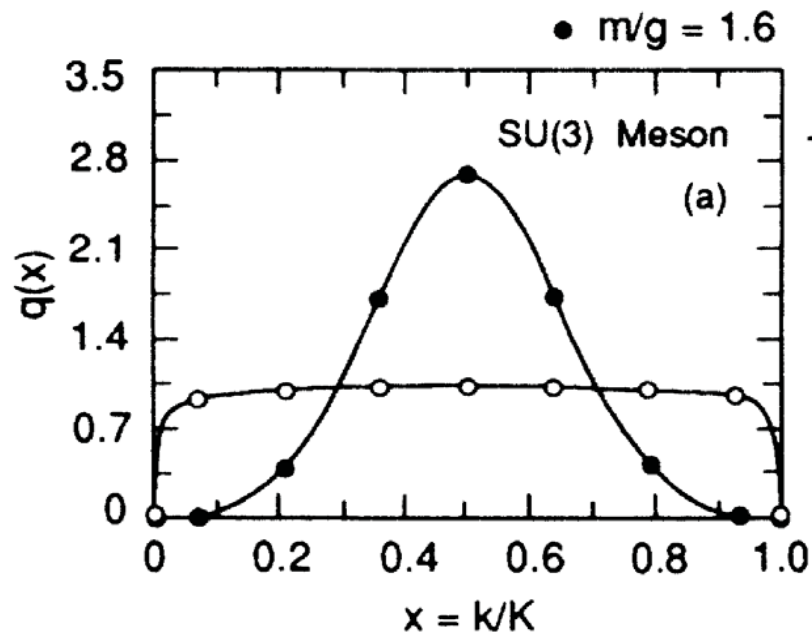


FIG. 2. Spectra for $N=3$, baryon number $B=0, 1$, and 2 as a function of g/m ; K fixed.



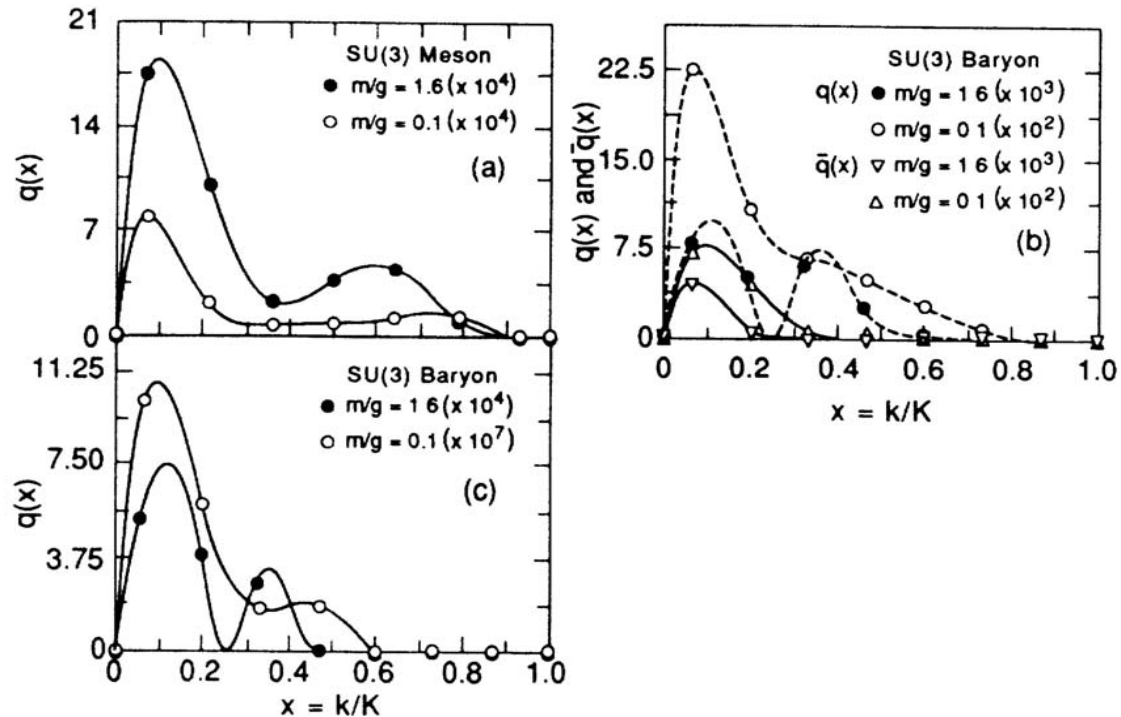


FIG. 4. Higher-Fock contributions to $N=3$ structure functions. (a) Lightest meson. (b) Lightest baryon, including anti-quarks. (c) Baryon: contribution from two extra quark pairs. The curves are intended to guide the eye.

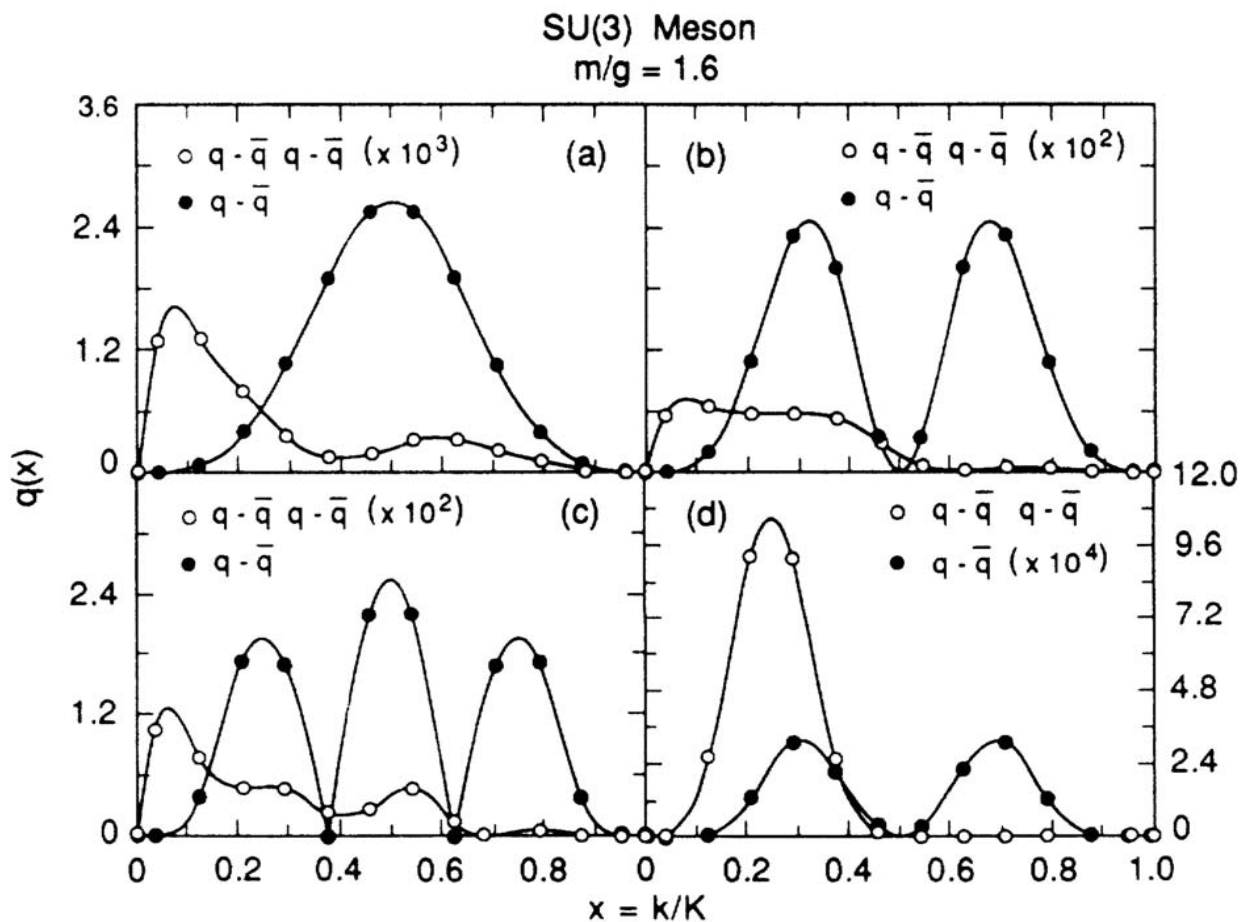
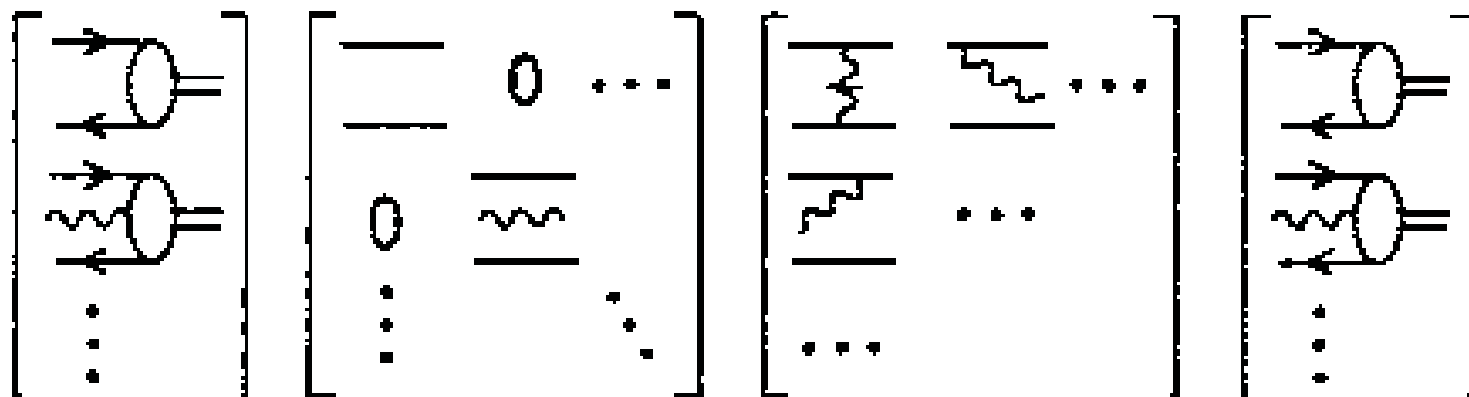


FIG. 5. (a)–(c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, $2K = 24$. (d) Eleventh state.

LIGHT-FRONT SCHRÖDINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$$A^+ = 0$$

G.P. Lepage, sjb

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Angular Momentum on the Light-Front

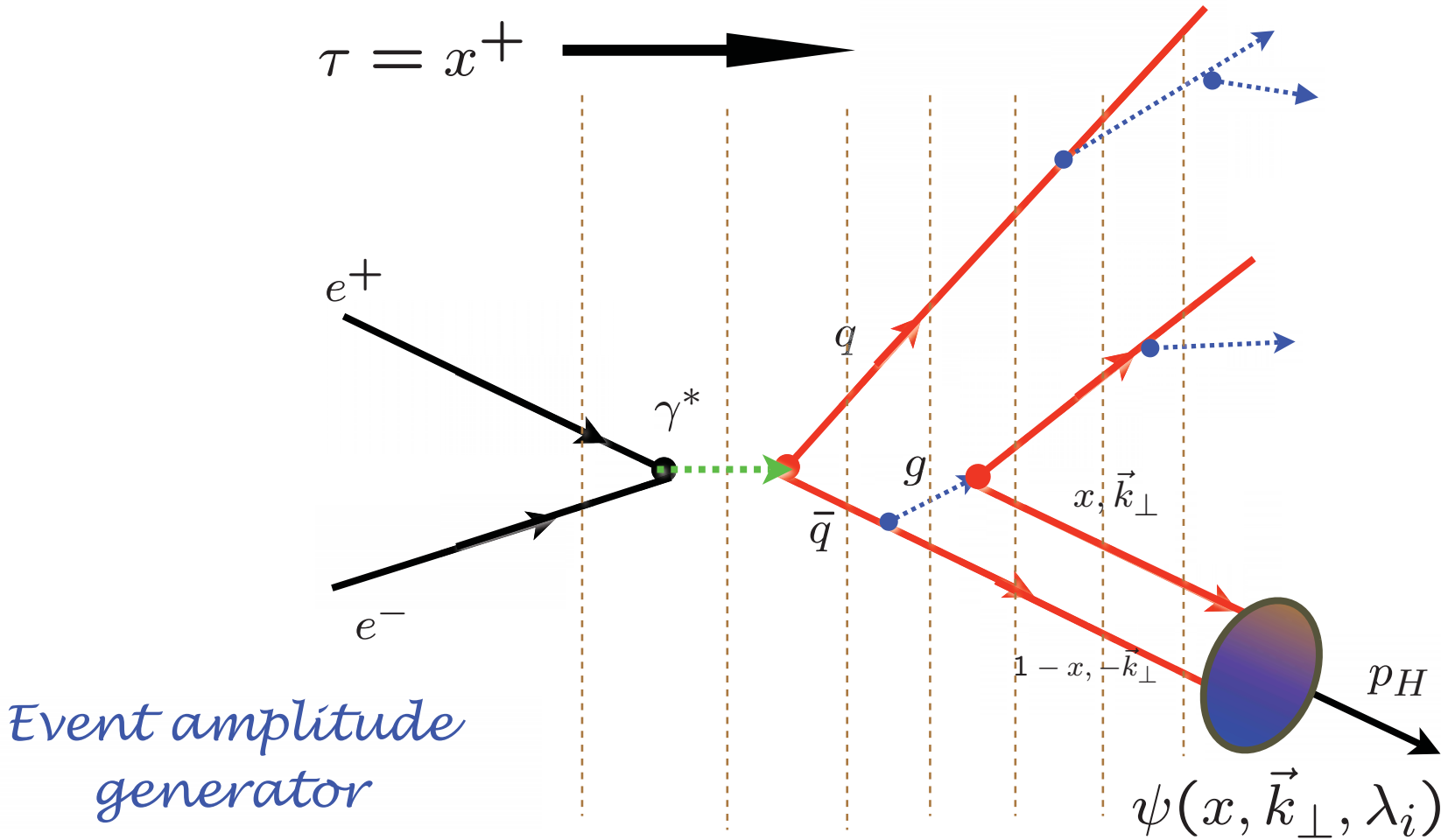
$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right) \quad n-1 \text{ orbital angular momenta}$$

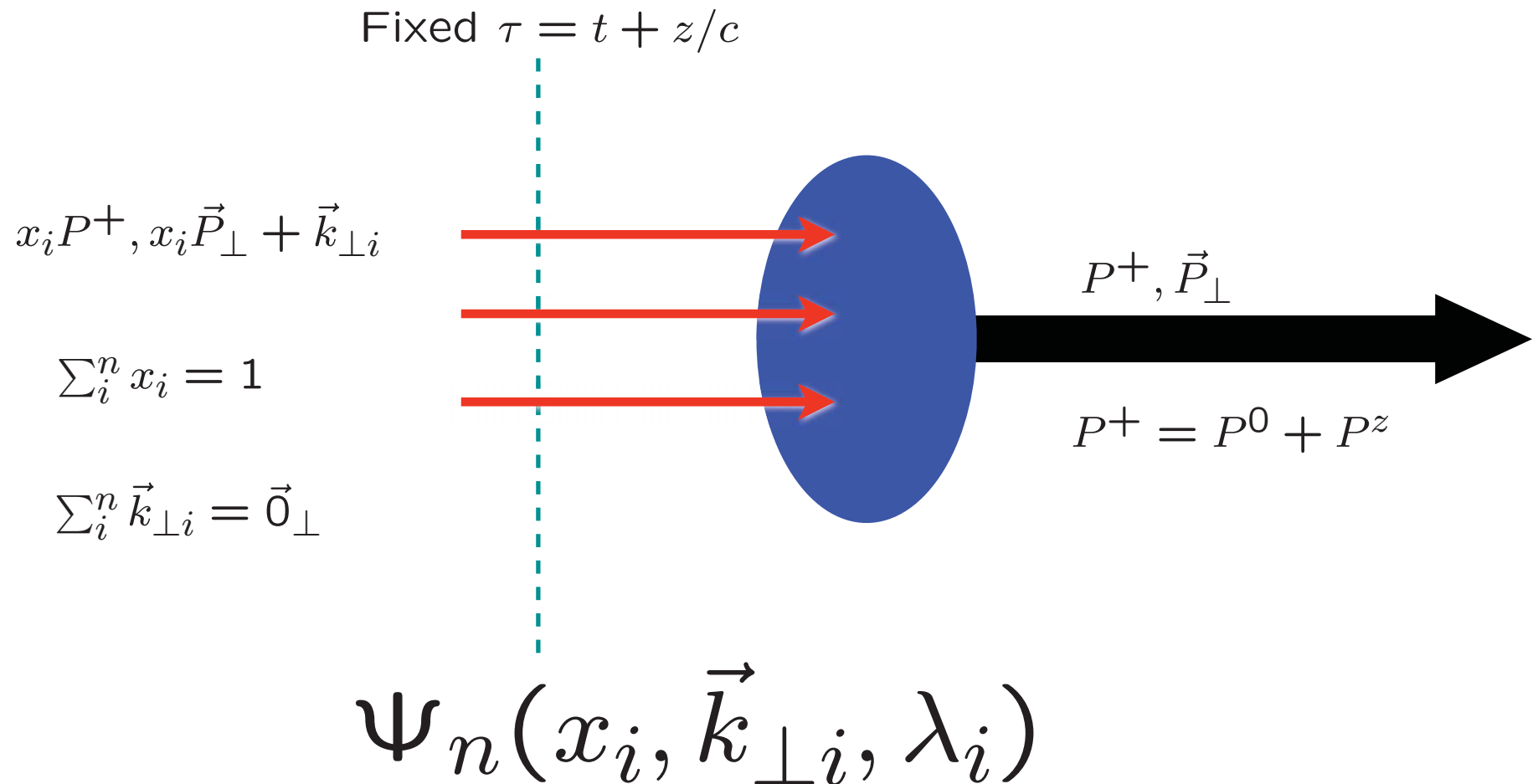
Nonzero Anomalous Moment --> Nonzero orbital angular momentum

Hadronization at the Amplitude Level



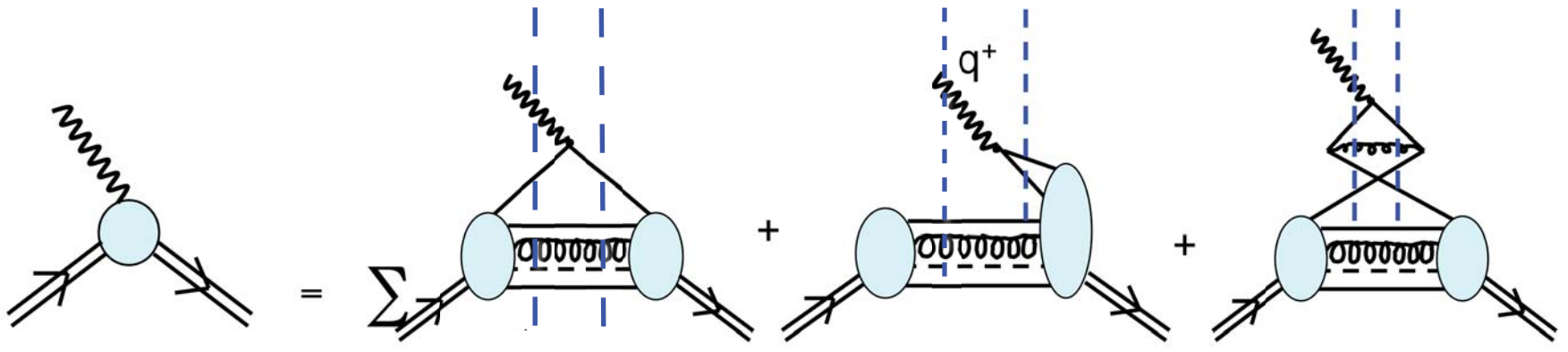
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions



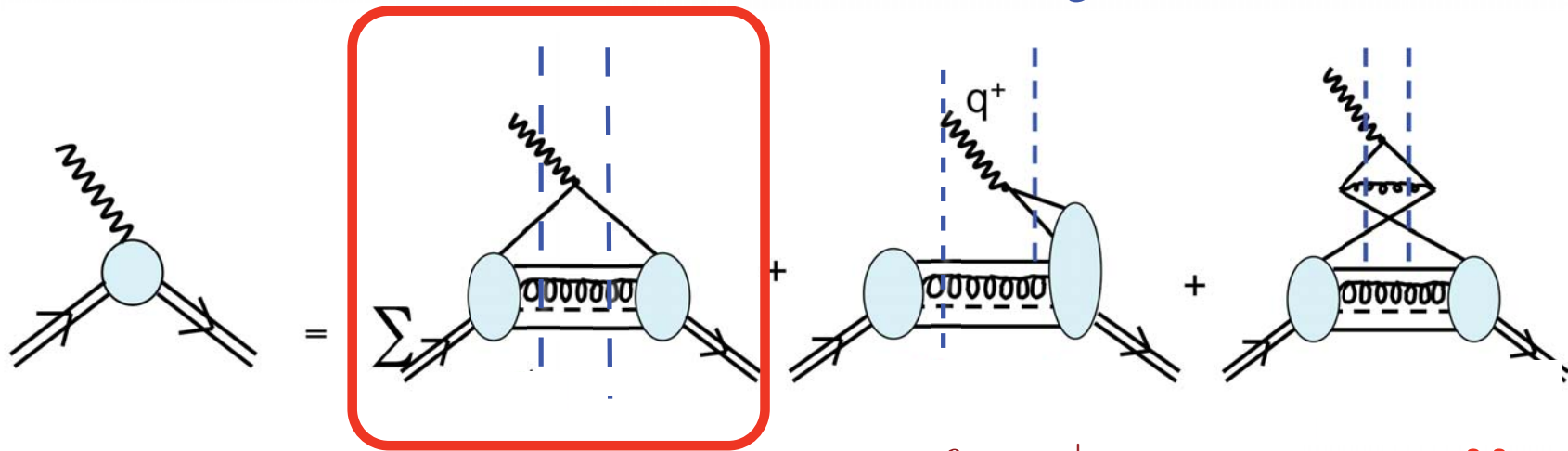
Invariant under boosts! Independent of P^μ

Calculation of Form Factors in Equal-Time Theory



Need vacuum fluctuations

Calculation of Form Factors in Light-Front Theory



zero for $q^+ = 0$ **zero !!**

Problems with the Instant Form

- Need n simultaneous measurements to specify initial condition for system of n partons
- Boosts are dynamical and complex; not product of Wigner boosts! McGee, Primack, Osborne, sjb)
- Wavefunction insufficient to determine current matrix elements -- need to couple to vacuum
- $N!$ time-ordered frame-dependent diagrams to calculate covariant Feynman amplitude of order g^N .

$$D = E_{init} - \sum_n \sqrt{\vec{k}^2 + m^2 + i\epsilon}$$

Lorentz boost of composite systems

$$S_a(\Lambda) = \exp \left(\frac{1}{2} \vec{\alpha}_a \cdot \hat{V} \tanh^{-1} V \right)$$

$$= \sqrt{\frac{E+m}{2m}} \left(1 + \frac{\vec{\alpha}_a \cdot \vec{P}}{E+m} \right).$$

Crucial even for
infinitesimal boosts



$$\left(\begin{array}{c} 1 + \frac{\vec{\sigma}_a \cdot \vec{P}}{m+E} \quad \frac{\vec{\sigma}_a \cdot \vec{p}}{2m_a+k_a} \\ \vec{\sigma}_a \cdot \left(\frac{\vec{P}}{m+E} + \frac{\vec{p}}{2m_a+k_a} \right) \end{array} \right) \otimes \left(\begin{array}{c} 1 - \frac{\vec{\sigma}_b \cdot \vec{P}}{m+E} \quad \frac{\vec{\sigma}_b \cdot \vec{p}}{2m_b+k_b} \\ \vec{\sigma}_b \cdot \left(\frac{\vec{P}}{m+E} - \frac{\vec{p}}{2m_b+k_b} \right) \end{array} \right).$$

The Electromagnetic Interactions Of Loosely Bound Composite Systems.
[Stanley J. Brodsky](#), [Joel R. Primack](#) (SLAC) . SLAC-PUB-0440, Jun 1968. 9pp.
Published in *Phys.Rev.*174:2071-2073,1968.

The Electromagnetic Interactions of Composite Systems.
[Stanley J. Brodsky](#), [Joel R. Primack](#) (SLAC) . SLAC-PUB-0512, Oct 1968. 88pp.
Published in *Annals Phys.*52:315-365,1969.

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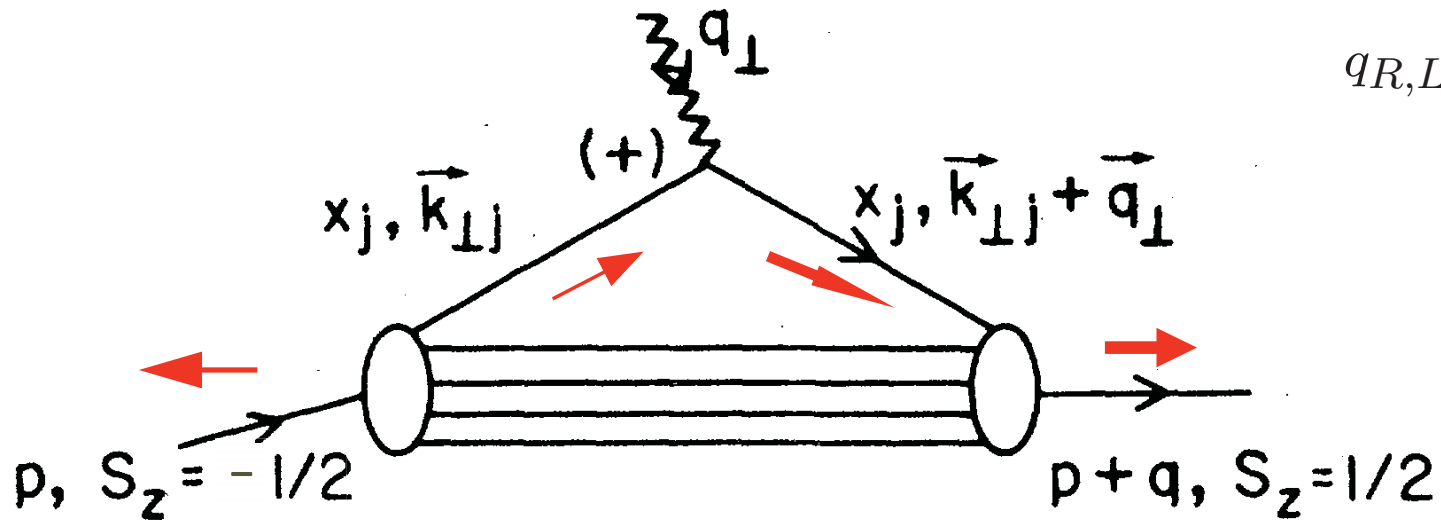
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$



Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

Gardner, Hwang, sjb,

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle = \bar{U}(P', \lambda') \left[F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu\alpha} q_\alpha + F_3(q^2) \frac{-1}{2M} \sigma^{\mu\alpha} \gamma_5 q_\alpha \right] U(P, \lambda)$$

$$\kappa = \frac{e}{2M} [F_2(0)] , \quad d = \frac{e}{M} [F_3(0)]$$

Find: $F_3(q^2) = F_2(q^2) \times \tan \phi$

CP-violating phase



Fock state by Fock state

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$, $q^+ = 0$ frame, $q^2 < 0$
 imply ($q^{R/L} \equiv q^1 \pm iq^2$):

Gardner, Hwang, sjb,

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{i}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

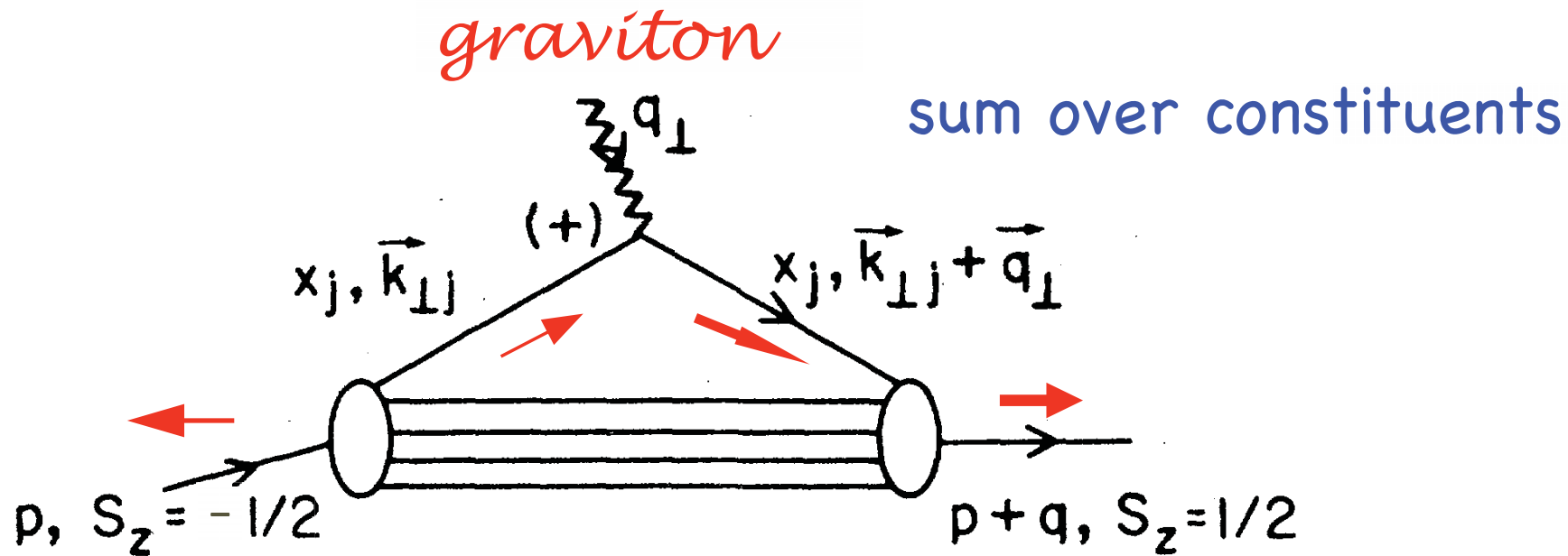
$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$ for the struck constituent j and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp$ for each spectator ($i \neq j$). $q^+ = 0 \implies$ only $n' = n$.

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

$$F_3(q^2) = F_2(q^2) \times \tan \phi$$

Anomalous gravitomagnetic moment $B(0)$

Okun, Kobzarev, Teryaev: $B(0)$ Must vanish because of Equivalence Theorem



Hwang, Schmidt, sjb;
Holstein et al

$$B(0) = 0$$

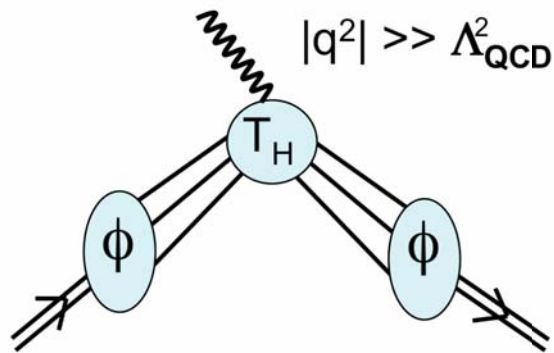
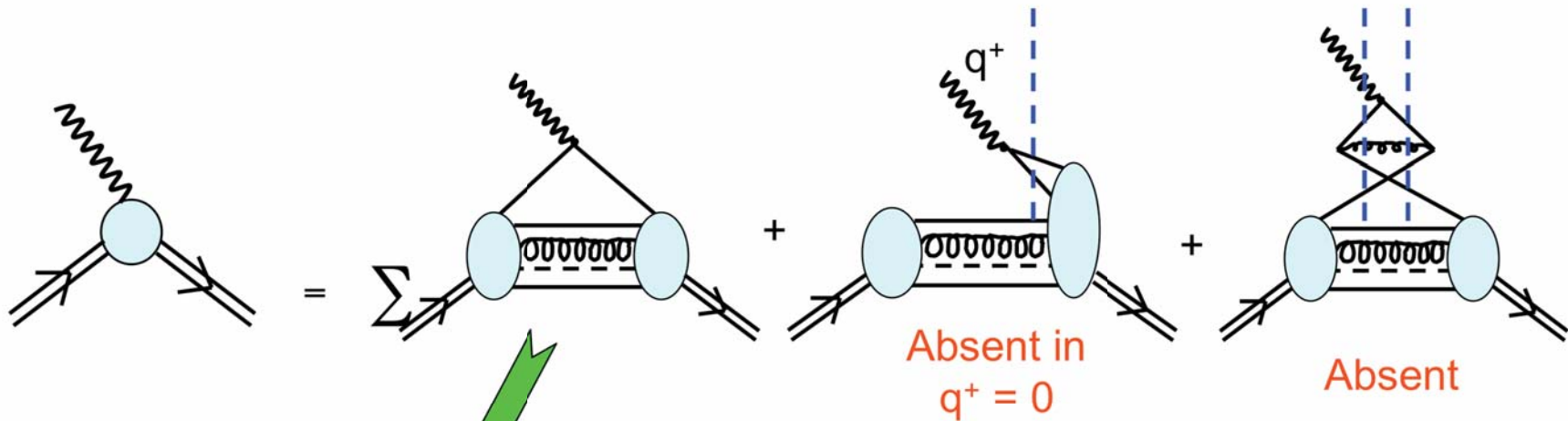
Each Fock State

Advantages of LF QCD

- Lorentz frame invariant; no boosts
- No fermion doubling
- Minkowski space
- Complete set of eigensolutions, bound state and continuum spectroscopy
- LFWFs, observables, simple spin properties
- Physical gauge: ghost free
- Zero modes instead of VEVs
- QED(3+1), QCD(1+1)
- Perturbation theory tractable, renormalization (alternate denominators)
- Relativistic statistical physics

QCD at the Amplitude Level

LFD in Exclusive Processes



$$T_H = \sum \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \dots \end{array} \right]$$

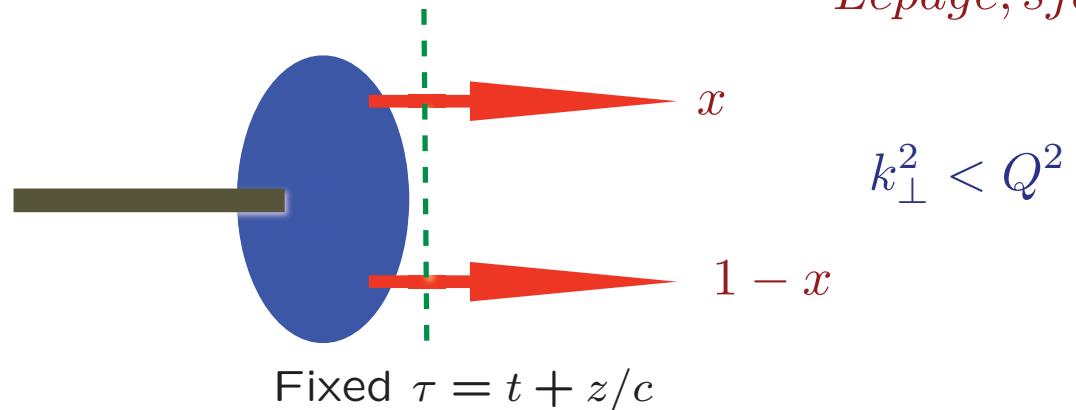
$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i)$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb

Frishman, Lepage, Sachrajda, sjb

Peskin Braun

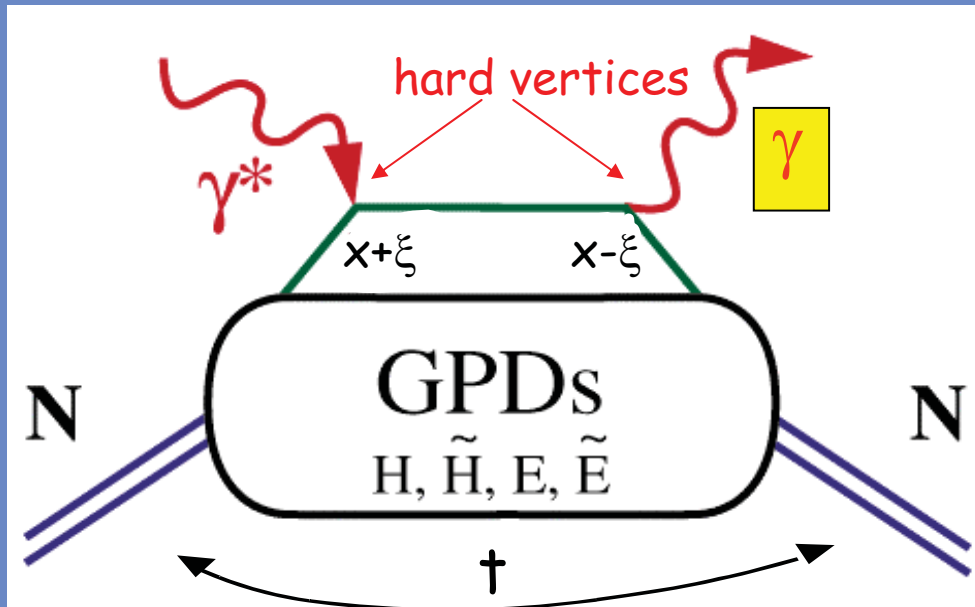
Efremov, Radyushkin Chernyak etal

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

GPDs & Deeply Virtual Exclusive Processes - New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)



x - quark momentum fraction

ξ - longitudinal momentum transfer

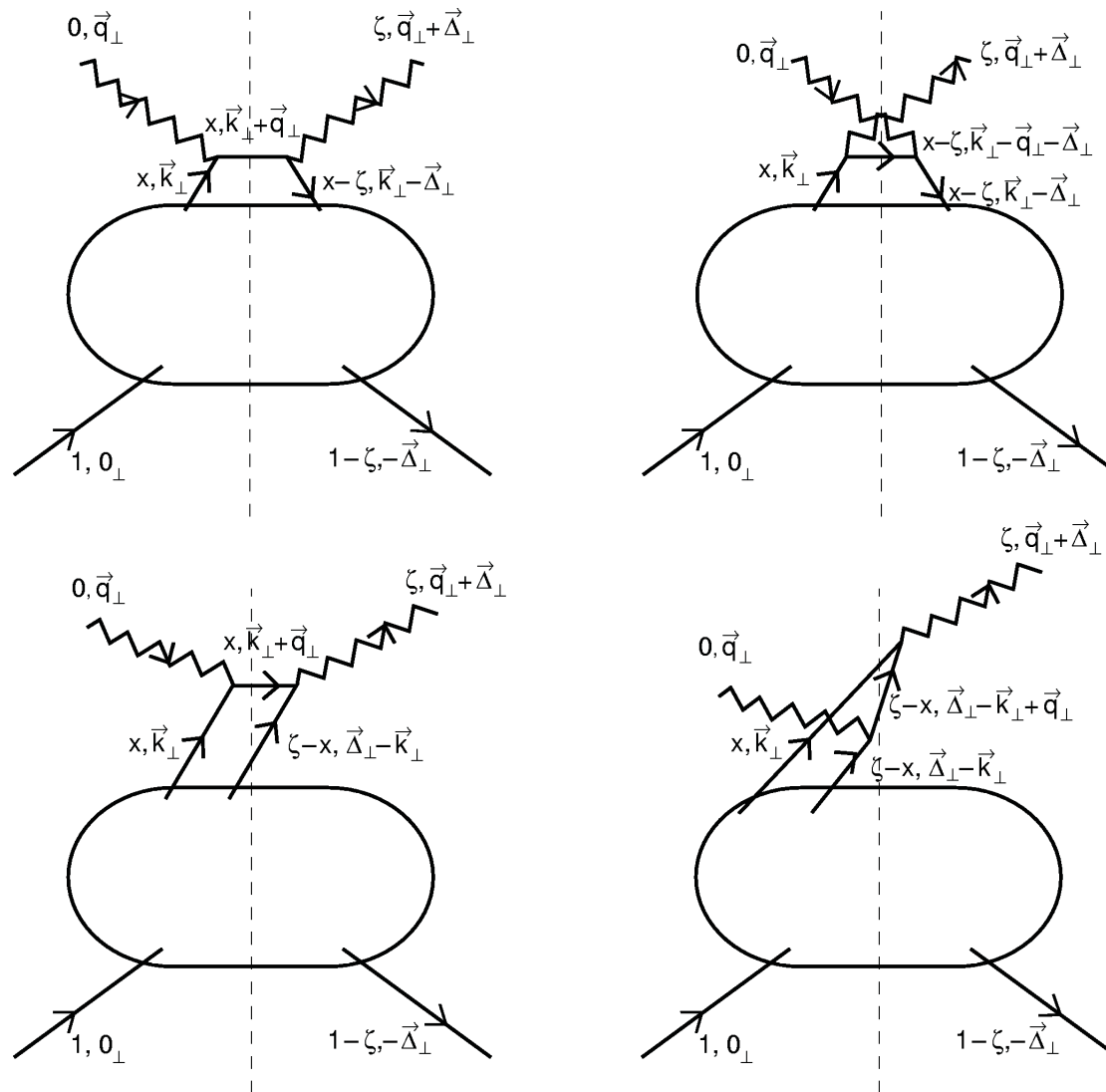
$\sqrt{-t}$ - Fourier conjugate to transverse impact parameter

$H(x, \xi, t), E(x, \xi, t), \dots$ "Generalized Parton Distributions"

Quark angular momentum (Ji sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^1 x dx [H^q(x, \xi, 0) + E^q(x, \xi, 0)]$$

X. Ji, Phys.Rev.Lett.78,610(1997)



Light-cone wavefunction representation of deeply virtual Compton scattering [☆]

Stanley J. Brodsky ^a, Markus Diehl ^{a,1}, Dae Sung Hwang ^b

Example of LFWF representation of GPDs ($n \Rightarrow n$)

Diehl, Hwang, sjb

$$\begin{aligned} & \frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i \Delta^2}{2M} E_{(n \rightarrow n)}(x, \zeta, t) \\ &= (\sqrt{1-\zeta})^{2-n} \sum_{n, \lambda_i} \int \prod_{i=1}^n \frac{dx_i d^2 \vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \vec{k}_{\perp j}\right) \\ & \quad \times \delta(x - x_1) \psi_{(n)}^{\uparrow*}(x'_1, \vec{k}'_{\perp 1}, \lambda_1) \psi_{(n)}^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i), \end{aligned}$$

where the arguments of the final-state wavefunction are given by

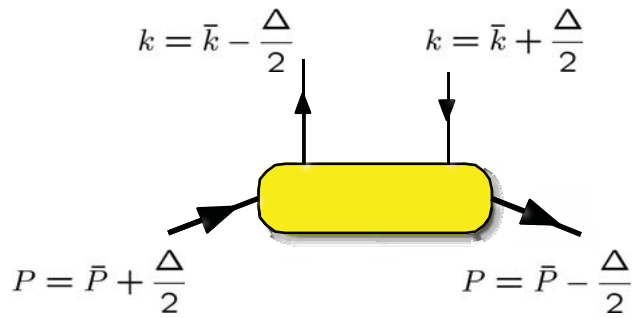
$$\begin{aligned} x'_1 &= \frac{x_1 - \zeta}{1 - \zeta}, & \vec{k}'_{\perp 1} &= \vec{k}_{\perp 1} - \frac{1 - x_1}{1 - \zeta} \vec{\Delta}_{\perp} & \text{for the struck quark,} \\ x'_i &= \frac{x_i}{1 - \zeta}, & \vec{k}'_{\perp i} &= \vec{k}_{\perp i} + \frac{x_i}{1 - \zeta} \vec{\Delta}_{\perp} & \text{for the spectators } i = 2, \dots, n. \end{aligned}$$

Light-Front Wave Function Overlap Representation

DVCS/GPD

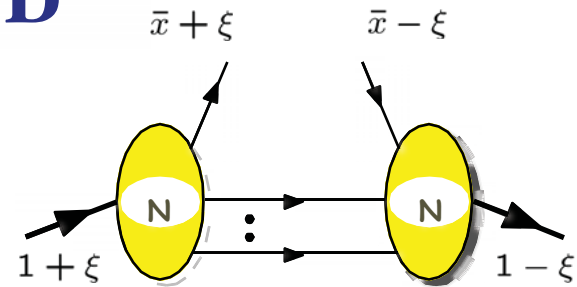
Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



$\xi < \bar{x} < 1$

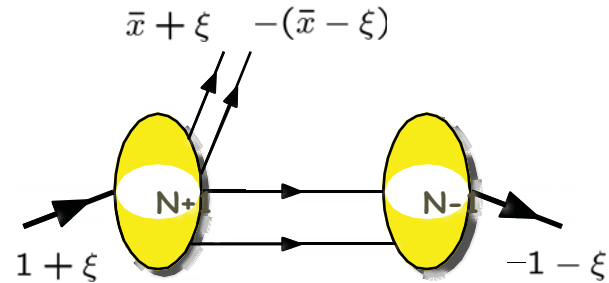
$$\sum_N$$



DGLAP region

$-\xi < \bar{x} < \xi$

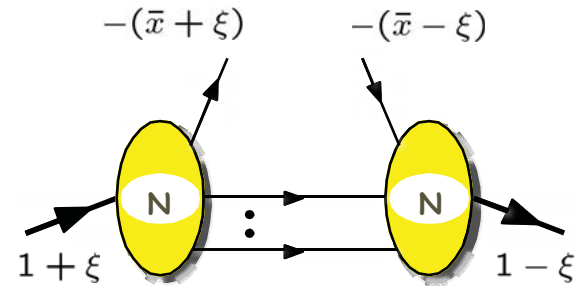
$$\sum_N$$



ERBL region

$-1 < \bar{x} < -\xi$

$$\sum_N$$



DGLAP region

$N=3$ VALENCE QUARK \Rightarrow Light-cone Constituent quark model

$N=5$ VALENCE QUARK + QUARK SEA \Rightarrow Meson-Cloud model

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QCD on the LF

Stan Brodsky, SLAC

Link to DIS and Elastic Form Factors

DIS at $\xi=t=0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta \bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^1 dx \sum_q [H^q(x, \xi, t)] = F_1(t) \text{ Dirac f.f.}$$

$$\int_{-1}^1 dx \sum_q [E^q(x, \xi, t)] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^1 dx \tilde{H}^q(x, \xi, t) = G_{A,q}(t), \quad \int_{-1}^1 dx \tilde{E}^q(x, \xi, t) = G_{P,q}(t)$$

$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x, \xi, t)$

Verified using
LFWFs
Diehl, Hwang, sjb

Quark angular momentum (Ji's sum rule)

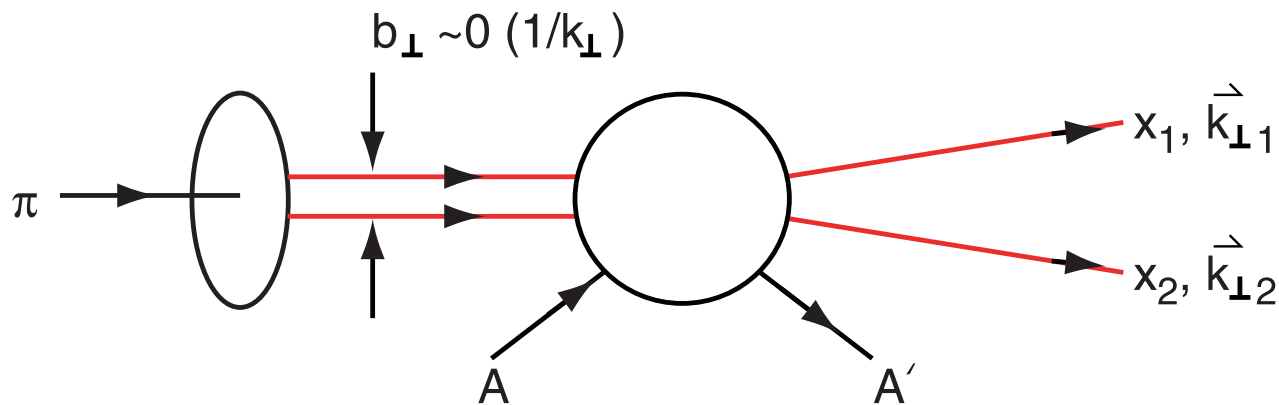
$$J^q = \frac{1}{2} - J^G = \frac{1}{2} \int_{-1}^1 x dx [H^q(x, \xi, 0) + E^q(x, \xi, 0)]$$

X. Ji, Phys.Rev.Lett.78,610(1997)

Diffractive Dissociation of Pion into Quark Jets

Mueller, sjb
Frankfurt Miller Strikman

E791 Ashery et al.



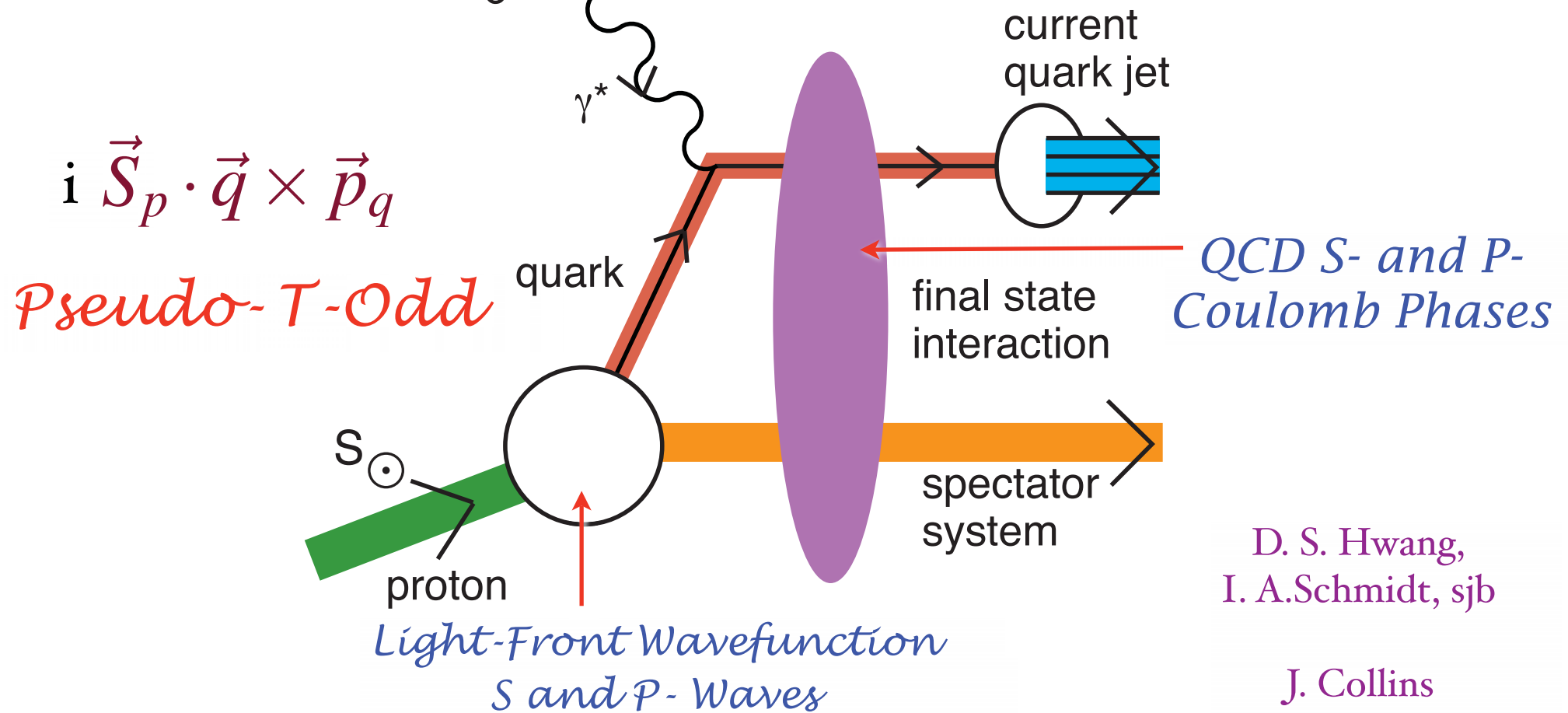
$$M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp})$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!

Single-spin asymmetries

Leading-Twist Sivers Effect



D. S. Hwang,
I. A. Schmidt, sjb

J. Collins

Light-Front Quantization of the Standard Model

$$\phi(x) = \frac{1}{\sqrt{2}} v + \varphi = \frac{1}{\sqrt{2}} ([v + h(x)] + i\eta(x))$$

No Higgs VEV!

Goldstone field

$k^+ = 0$ zero mode

A Unitary and renormalizable theory of the standard model in ghost free light cone gauge.

P. Srivastava and sjb

Phys.Rev.D66:045019,2002.

hep-ph/0202141

Decoupling of gravity to the Higgs zero mode

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QCD on the LF

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$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

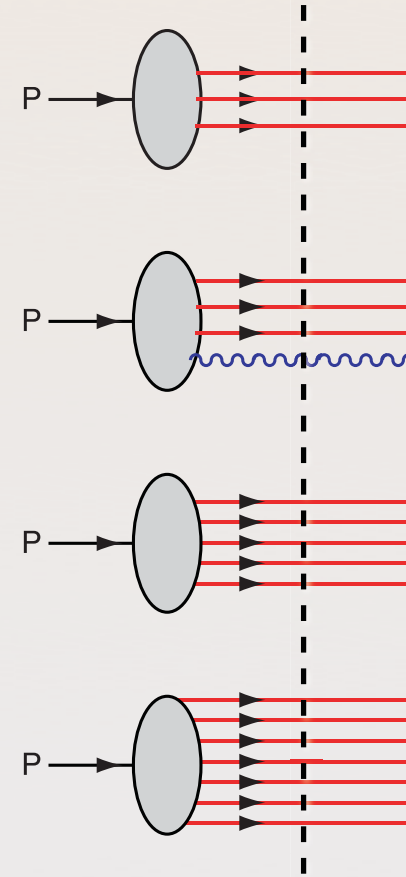
$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks

$$\bar{u}(x) \neq \bar{d}(x)$$

Mueller: BFKL DYNAMICS

$$\bar{s}(x) \neq s(x)$$



Fixed LF time

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QCD on the LF

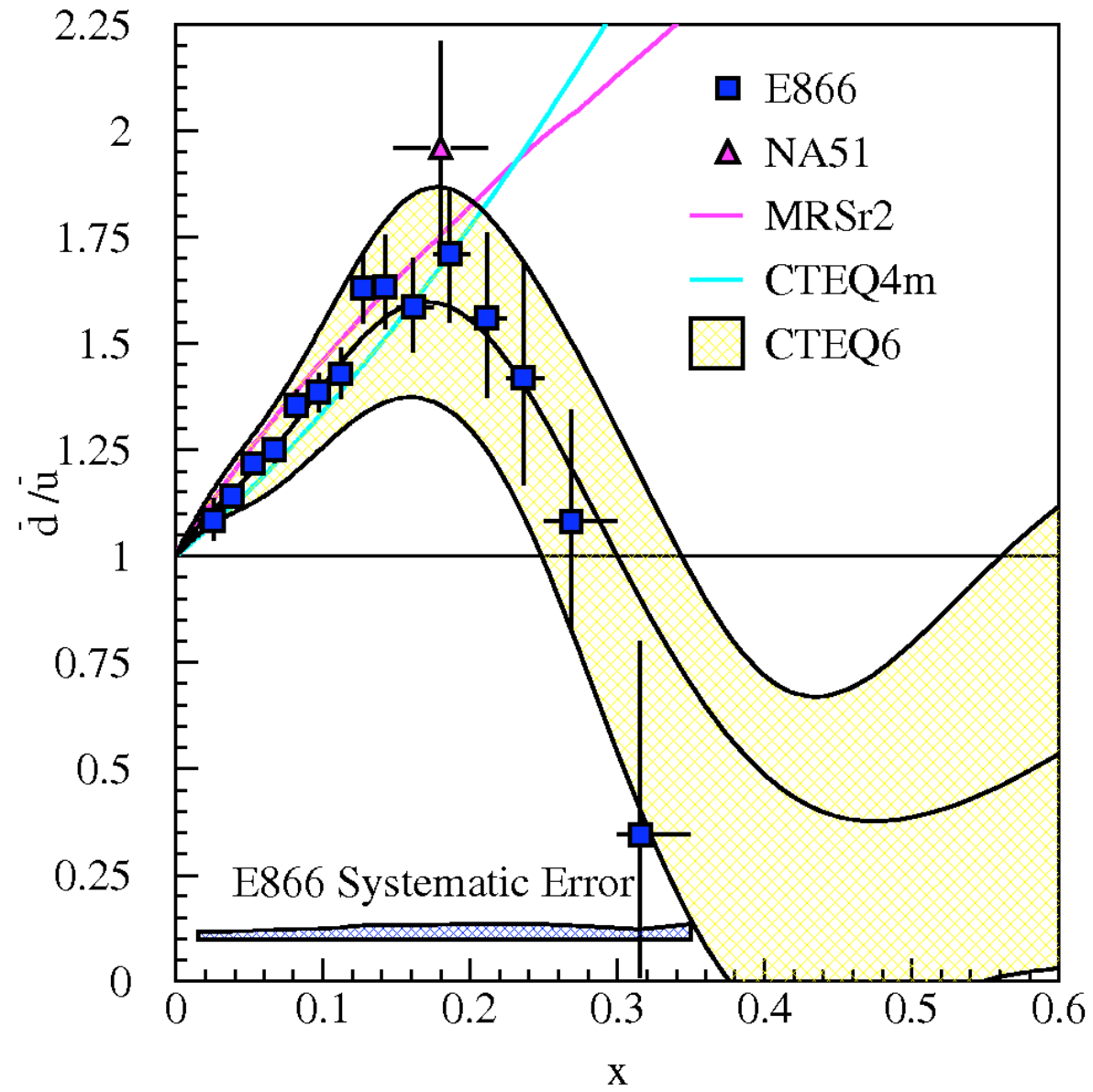
Light Antiquark Flavor Asymmetry

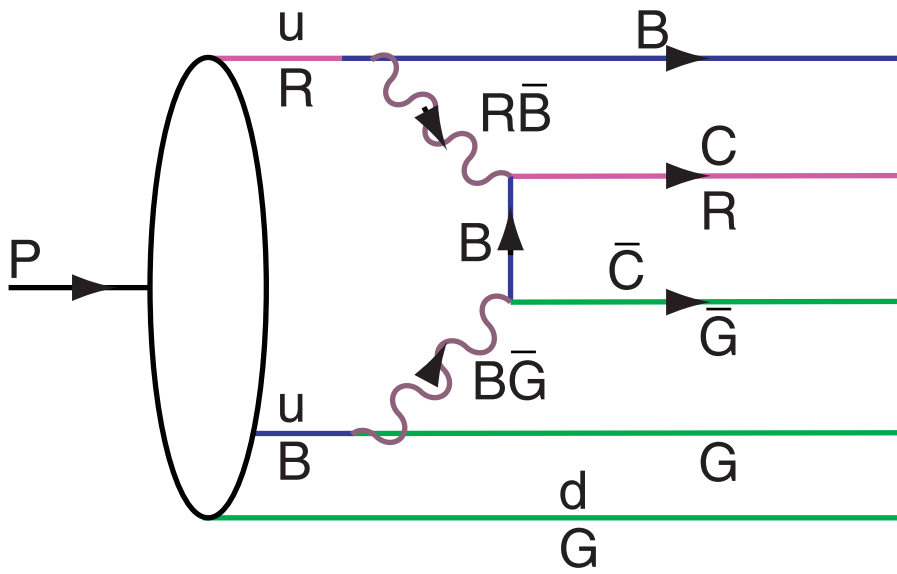
- Naïve Assumption from gluon splitting:

$$\bar{d}(x) = \bar{u}(x)$$

- E866/NuSea (Drell-Yan)

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$





$|uudc\bar{c}\rangle$ Fluctuation in Proton

QCD: Probability $\sim \frac{\Lambda_{QCD}^2}{M_Q^2}$

$|e^+e^-l^+l^-\rangle$ Fluctuation in Positronium

QED: Probability $\sim \frac{(m_e\alpha)^4}{M_l^4}$

OPE derivation - M.Polyakov et al.

$$\langle p | \frac{G_{\mu\nu}^3}{m_Q^2} | p \rangle \text{ vs. } \langle p | \frac{F_{\mu\nu}^4}{m_l^4} | p \rangle$$

$c\bar{c}$ in Color Octet

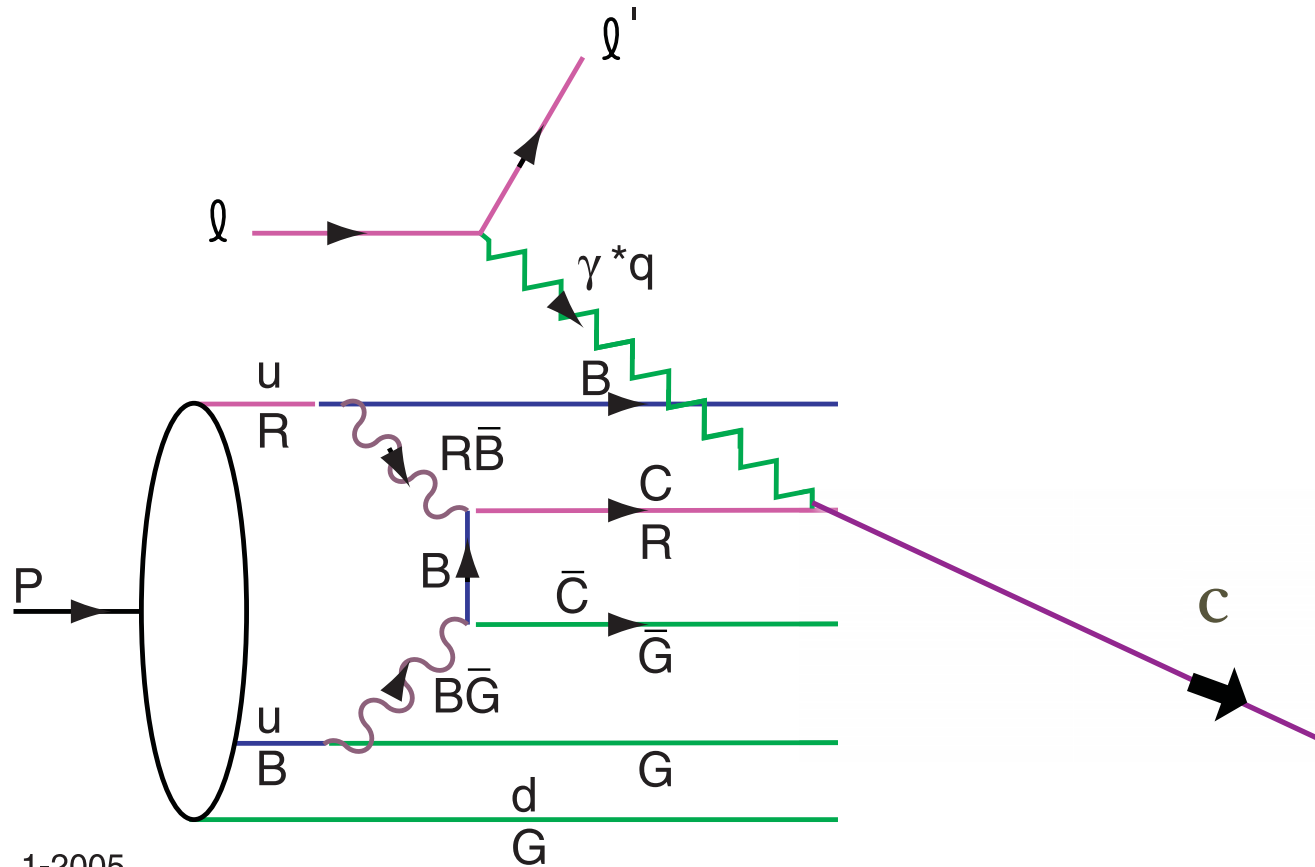
Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm!

Hoyer, Peterson, Sakai, sjb

Measure $c(x)$ in Deep Inelastic Lepton-Proton Scattering



1-2005
8711A83

Hoyer, Peterson, SJB

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