Light-Front Holography and AdS/QCD

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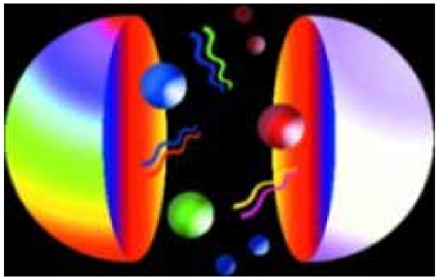

Slac Theory Seminar

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Outline

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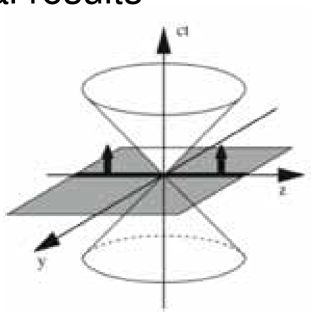
1 Introduction

- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom
- Recent developments inspired by the AdS/CFT correspondence (Maldacena 1998) between string states in AdS space and conformal field theories in physical space-time have led to analytical insights into the confining dynamics of QCD
- Description of strongly coupled gauge theory using a dual gravity description!
- Strings describe spin-J extended objects (no quarks). QCD degrees of freedom are pointlike particles and hadrons have orbital angular momentum: how can they be related? How can we map string states into partons?
- Light-front quantization is the ideal framework to describe hadronic structure in terms of quark and gluon degrees of freedom
- Simple vacuum structure allows unambiguous definition of the partonic content of a hadron: partons in a hadronic state are described by light-front wave functions which encode the hadronic properties

2 Light Front Dynamics

- Different possibilities to parametrize space-time Dirac (1949)
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different ”times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



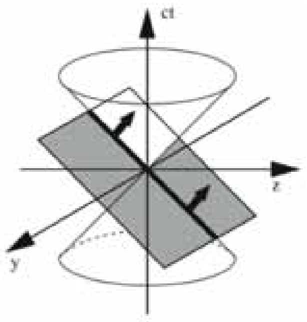
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ_+, $\mathbf{A}_\perp$

$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+}\psi_+ + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+\psi_+ \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp\psi_+ \end{aligned}$$

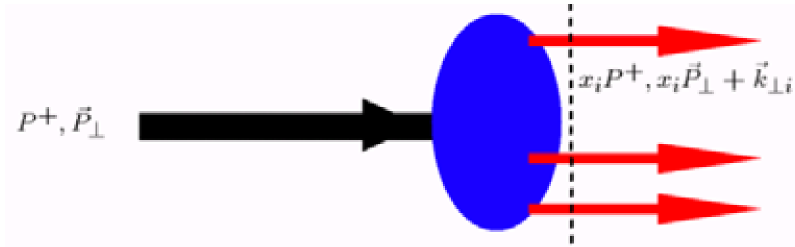
where the integral is over the initial surface $x^+ = 0$

- LF energy P^- generates LF time translations

$$\left[\psi_+(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi_+(x)$$

and the generators P^+ and $\mathbf{P}_\perp$ are kinematical

Light-Front Fock Representation



- Light-front Lorentz invariant Hamiltonian for the composite system

$$H_{LF} = P^2 = P^- P^+ - \mathbf{P}_\perp^2$$

- H_{LF} has eigenstates $|\psi_H(P)\rangle = |\psi_H(P^+, \mathbf{P}_\perp, S_z)\rangle$ and eigenmass $\mathcal{M}_H^2$, the mass spectrum of the color-singlet states of QCD:

$$H_{LF} \, |\psi_H\rangle = \mathcal{M}_H^2 \, |\psi_H\rangle$$

- State $\,|\psi_H\rangle$ is an expansion in multi-particle Fock states $\,|n\rangle$ of the free light-front Hamiltonian

$$|\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle$$

- Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

- Complete basis of Fock-states $|n\rangle$ constructed by applying free-field creation operators to the vacuum state $|0\rangle$, $P^+|0\rangle = 0$, $\mathbf{P}_\perp|0\rangle = 0$, with no particle content
- Dirac field ψ_+, expanded in terms of ladder operators on the initial surface $x^+ = x^0 + x^3$

$$\psi_+(x)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x} \right]$$

with u and v light-cone spinors

- Use commutation relations

$$\left\{ b(q), b^\dagger(q') \right\} = \left\{ d(q), d^\dagger(q') \right\} = (2\pi)^3 \, \delta(q^+ - {q'}^+) \delta^{(2)}\big(\mathbf{q}_\perp - \mathbf{q}'_\perp\big)$$

- Find

$$P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{m^2 + \mathbf{q}_\perp^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

- One parton state: $|q\rangle = \sqrt{2q^+} \, b^\dagger(q)|0\rangle$

- Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle\psi_H(P')|H_{LF}|\psi_H(P)\rangle = \mathcal{M}_H^2\langle\psi_H(P')|\psi_H(P)\rangle$$

- Find

$$\mathcal{M}_H^2 = \sum_n \int \big[dx_i\big]\big[d^2\mathbf{k}_{\perp i}\big] \sum_\ell \left(\frac{m_\ell^2 + \mathbf{k}_{\perp \ell}^2}{x_q}\right) \left|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})\right|^2 + \text{interactions}$$

- Phase space normalization of LFWFs

$$\sum_n \int \big[dx_i\big]\,\big[d^2\mathbf{k}_{\perp i}\big]\,\left|\psi_{n/h}(x_i, \mathbf{k}_{\perp i})\right|^2 = 1$$

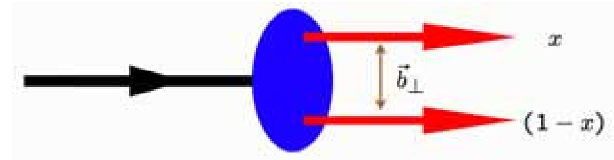
- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

$$\mathcal{M}_H^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_{n/H}^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{m_\ell^2 - \nabla_{\mathbf{b}_{\perp \ell}}^2}{x_q}\right) \psi_{n/H}(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j}\,|\psi_n(x_j, \mathbf{b}_{\perp j})|^2 = 1$$

3 Semiclassical Approximation to QCD



- Consider a two-parton hadronic bound state in the limit $m_q \to 0$

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{\mathbf{k}_\perp^2}{x(1-x)} \left|\psi(x,\mathbf{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\mathbf{b}_\perp \, \psi^*(x,\mathbf{b}_\perp) \left(-\nabla^2_{\mathbf{b}_{\perp\ell}}\right) \psi(x,\mathbf{b}_\perp) + \text{interactions}$$

- Functional dependence on invariant mass for a given Fock state

$$\mathcal{M}_n^2 = \left(\sum_{a=1}^{n} k_a^\mu\right)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2}{x_a} \quad \to \quad \frac{\mathbf{k}_\perp^2}{x(1-x)}$$

  the measure of the off-mass shell energy $\mathcal{M}^2 - \mathcal{M}_n^2$

- Boost invariant variable in transverse space : $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Semiclassical approximation: LF dynamics depends only on the boost invariant variable ζ and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$: $\psi(x,\mathbf{k}_\perp) \to \phi(\zeta)$

- Normalization for the LF mode $\phi(\zeta) = \langle\zeta|\phi\rangle$: $\langle\phi|\phi\rangle = \int d\zeta \, |\langle\zeta|\phi\rangle|^2 = 1$

- Functional relation: $\frac{|\phi|^2}{\zeta} = \frac{2\pi}{x(1-x)} |\psi(x, \mathbf{b}_\perp)|^2$

- Invariant mass $\mathcal{M}^2$ in terms of LF mode $\phi(\zeta, \varphi) \sim f(\varphi)\phi(\zeta)$

$$\begin{aligned}\mathcal{M}^2 &= \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)\, U(\zeta)\, \phi(\zeta) \\ &= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)\end{aligned}$$

where the interaction terms are summed up in the effective potential $U(\zeta)$ and the orbital angular momentum has the $SO(2)$ Casimir representation $SO(N) \sim S^{N-1}:\ L(L+N-2)$

$$\langle\varphi|L|f\rangle = \frac{1}{i}\frac{\partial}{\partial\varphi}\langle\varphi|f\rangle = L f(\varphi), \quad \phi(\zeta, \varphi) \sim e^{\pm iL\varphi}\phi(\zeta)$$

- LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)}$$

Conformal Algebraic Structure , Integrability and Stability Conditions

- Consider the potential (hard wall)

$$U(\zeta) = 0 \quad \text{if} \quad \zeta \leq \frac{1}{\Lambda_{\rm QCD}}, \qquad U(\zeta) = \infty \quad \text{if} \quad \zeta > \frac{1}{\Lambda_{\rm QCD}}$$

- If $L^2 > 0$ the LF Hamiltonian, H_{LF}, is written as a bilinear form (Bargmann 1949)

$$H_{LF}^{L}(\zeta) = \Pi_L^\dagger(\zeta)\Pi_L(\zeta)$$

in terms of the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\right)$$

and its adjoint

$$\Pi_L^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta}\right)$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}$$

- Conformal algebraic structure !

- If $L^2 \geq 0$ the Hamiltonian is positive definite

$$\langle\phi\left|H_{LF}^{L}\right|\phi\rangle = \int d\zeta\, |\Pi_L \phi(z)|^2 \geq 0$$

and thus $\mathcal{M}^2 \geq 0$

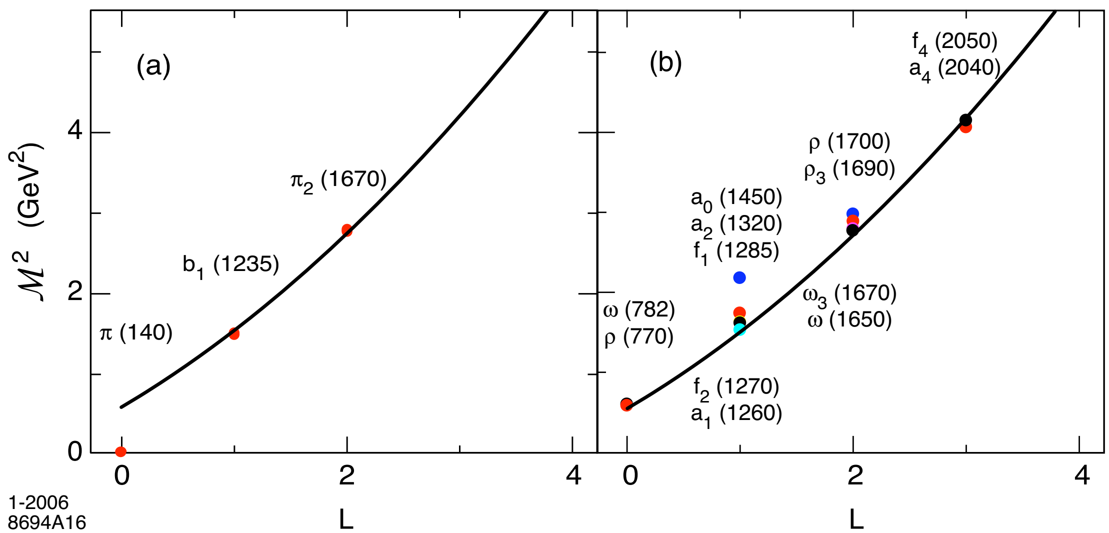
- If $L^2 < 0$ the Hamiltonian cannot be written as a bilinear form and the Hamiltonian is not bounded from below (“Fall-to-the-center” problem in Q.M.)

- Critical value of the potential corresponds to $L = 0$, the lowest possible stable state

- Orbital excitations constructed by the L-th application of the raising operator $a_L^\dagger = -i\Pi_L$ on the ground state, $a^\dagger|L\rangle \sim |L+1\rangle$:

$$\begin{aligned} \phi_L(\zeta) &= \langle\zeta|L\rangle = C_L\sqrt{\zeta}\,(-\zeta)^L \left(\frac{1}{\zeta}\frac{d}{d\zeta}\right)^L J_0(\zeta\mathcal{M}) \\ &= C_L\sqrt{\zeta} J_L\left(\zeta\mathcal{M}\right) \end{aligned}$$

- Mode spectrum from boundary conditions $\phi\Big(\zeta = \frac{1}{\Lambda_{\rm QCD}}\Big) = 0$, thus $\mathcal{M}^2 = \beta_{Lk}\Lambda_{\rm QCD}$



Light meson orbital spectrum in a hard wall holographic model for $\Lambda_{QCD} = 0.32\ \mathrm{GeV}$

Non-Conformal Extension of Algebraic Integrability

- Consider the extension of the conformal operator algebra by constructing the generator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right)$$

and its adjoint

$$\Pi_L^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{L+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)$$

with commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2} - 2\kappa^2$$

- The LF Hamiltonian

$$H_{LF} = \Pi_L^\dagger \Pi_L + C$$

is positive definite $\langle\phi|H_{LF}|\phi\rangle \geq 0$ for $L^2 \geq 0$, and $C \geq -4\kappa^2$

- Identify the zero mode ($C = -4\kappa^2$) with the pion

- Orbital and radial excited states are constructed from the ladder operators from the $L = 0$ state.
- Light-front Hamiltonian equation

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle,$$

leads to effective LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

with effective potential

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L-1)$$

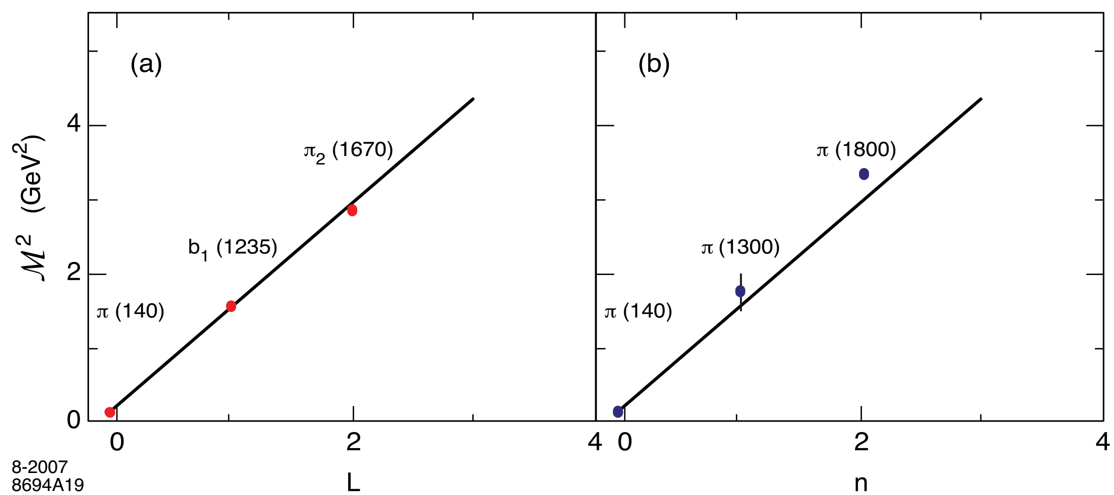
eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+L)$$

and eigenfunctions

$$\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L\left(\kappa^2\zeta^2\right).$$

- Transverse oscillator in the LF plane with $SO(2)$ rotation subgroup has Casimir L^2 representing rotations in the transverse LF plane.



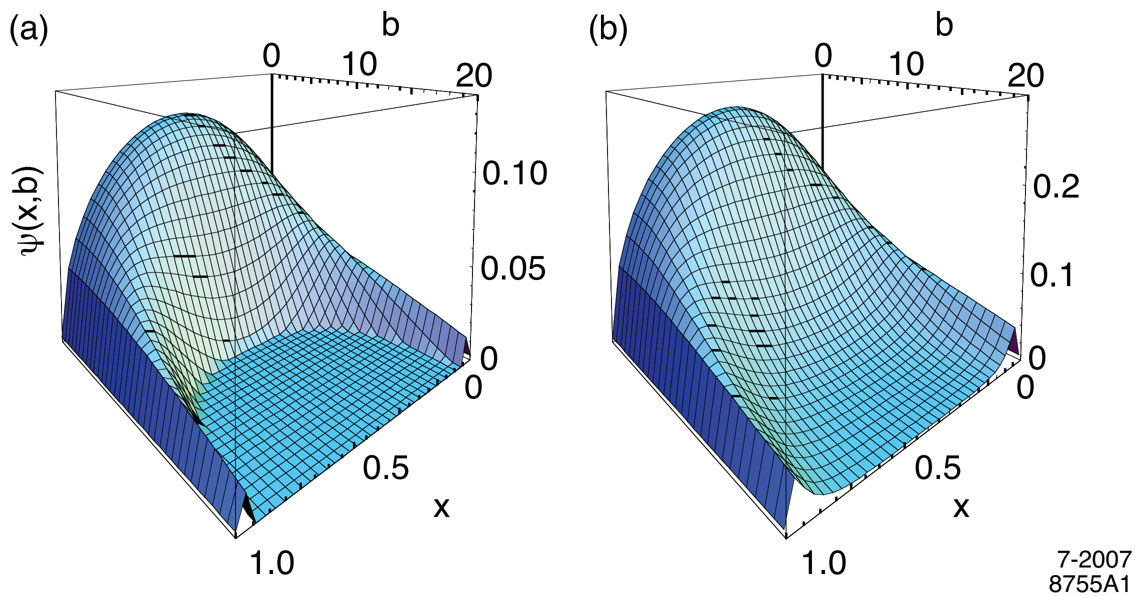
Light meson orbital (a) and radial (b) spectrum in a transverse oscillator holographic model for $\kappa = 0.6$ GeV.

Pion LFWF

- Two parton LFWF bound state:

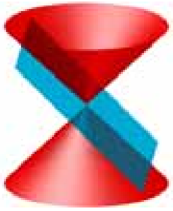
$$\psi^{HW}_{\overline{q}q/\pi}(x, \mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi} J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\, |\mathbf{b}_\perp| \beta_{L,k} \Lambda_{\rm QCD}\right) \theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right)$$

$$\psi^{SW}_{\overline{q}q/\pi}(x, \mathbf{b}_\perp) = \kappa^{L+1} \sqrt{\frac{2n!}{(n+L)!}}\, [x(1-x)]^{\frac{1}{2}+L} |\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right)$$



Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV

Recap



- LF Hamiltonian equation in QCD

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$$

is a LF wave equation for ϕ

$$\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement}\Big)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Invariant LF variable ζ allows separation of dynamics of quark and gluon binding from kinematics of constituent spin and internal orbital angular momentum
- LF impact variable ζ measures the separation of quark and gluon constituents within the hadron

4 Gauge Gravity Correspondence

- Substitute $\Phi(\zeta) \sim \zeta^{3/2}\phi(\zeta), \quad \zeta \to z$ in the conformal LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

- Find:

$$\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0,$$

with $(\mu R)^2 = -4 + L^2$, the wave equation describing the propagation of a string mode in AdS$_5$!

- Isomorphism of $SO(4,2)$ group of conformal QCD with generators $P^\mu, M^{\mu\nu}, D, K^\mu$ with the group of isometries of AdS$_5$ space

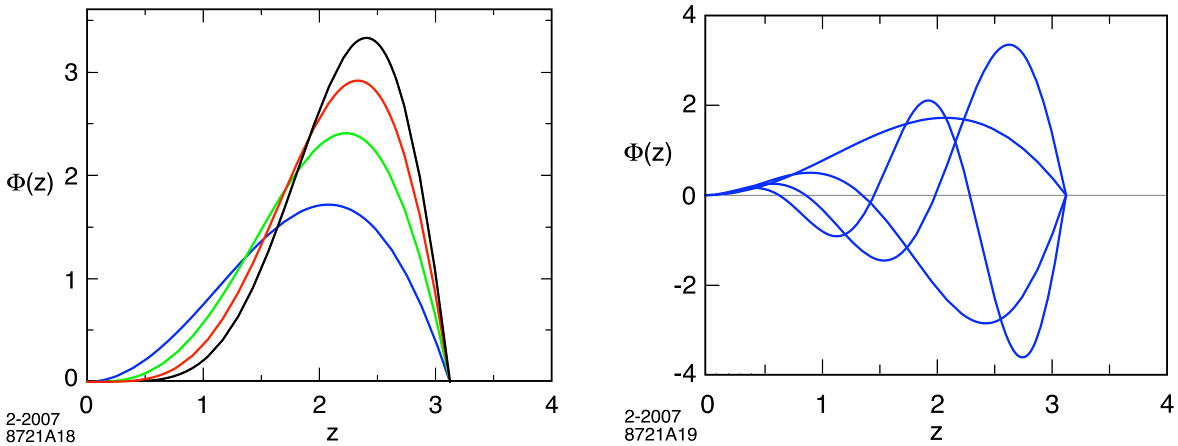
$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

- Conformal dimension Δ of AdS mode Φ given in terms of 5-dim mass by $(\mu R)^2 = \Delta(\Delta - 4)$. Thus $\Delta = 2 + L$ in agreement with the twist scaling dimension of a two parton object in QCD

- Truncated AdS/CFT model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001)

$\Phi(z) \sim z^{\Delta}$
$z \to 0$



Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

- Excitation spectrum hard-wall model: $\mathcal{M}_n(L) \sim L + 2n$

- Smooth cutoff: transverse oscilator model equivalent to the introduction of a background dilaton field $\varphi(z) = \kappa^2 z^2$ (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006)

$\Phi(z) \sim z^\Delta$
$z \to 0$

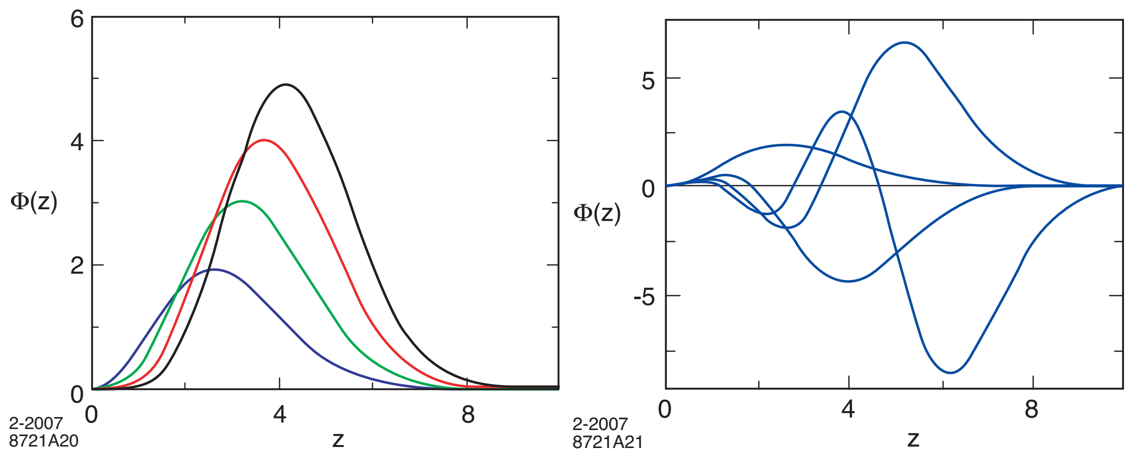


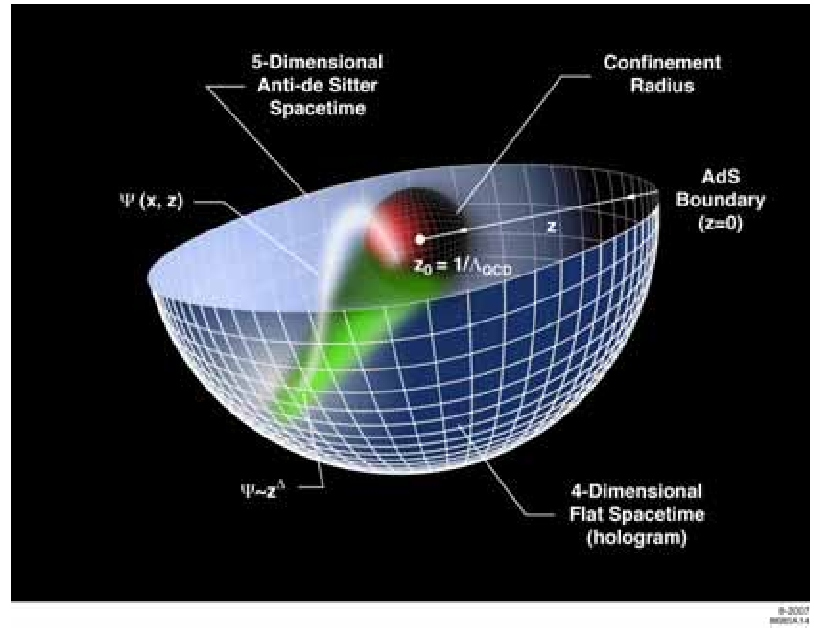
Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

- Excitation spectrum soft-wall: $\mathcal{M}_n^2(L) \sim L + n$, usual Regge dependence

Use the isometries of AdS space to map the local interpolating operators at the UV boundary of AdS space into the modes propagating inside AdS:

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$

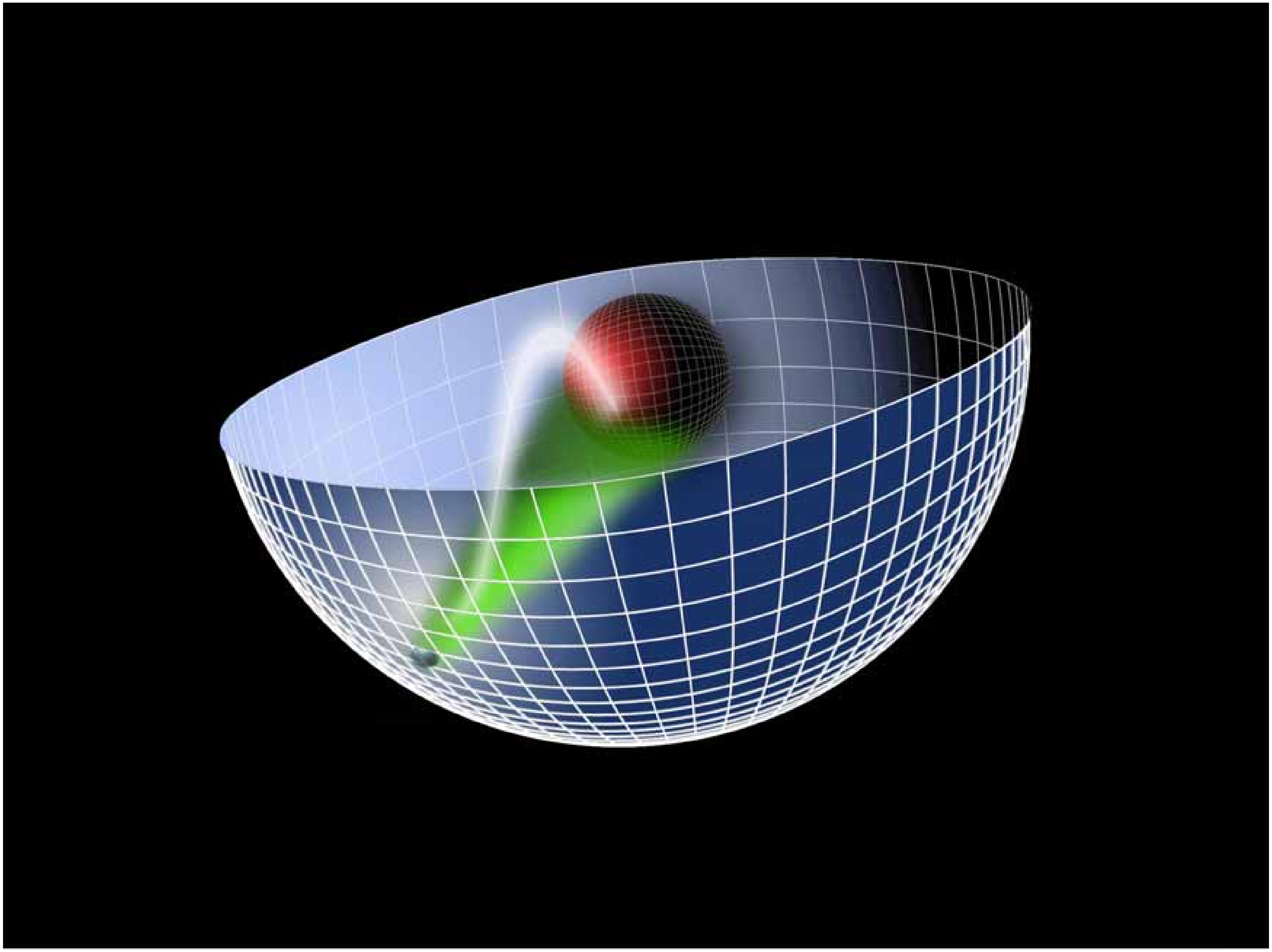
$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$



- A distance $L_{\rm AdS}$ shrinks by a warp factor as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R}\, L_{\rm AdS}$$

- Different values of z correspond to different scales at which the hadron is examined
- AdS boundary at $z \to 0$ correspond to the $Q \to \infty$ UV zero separation limit
- There is a maximum separation of quarks and a maximum value of z at the IR boundary



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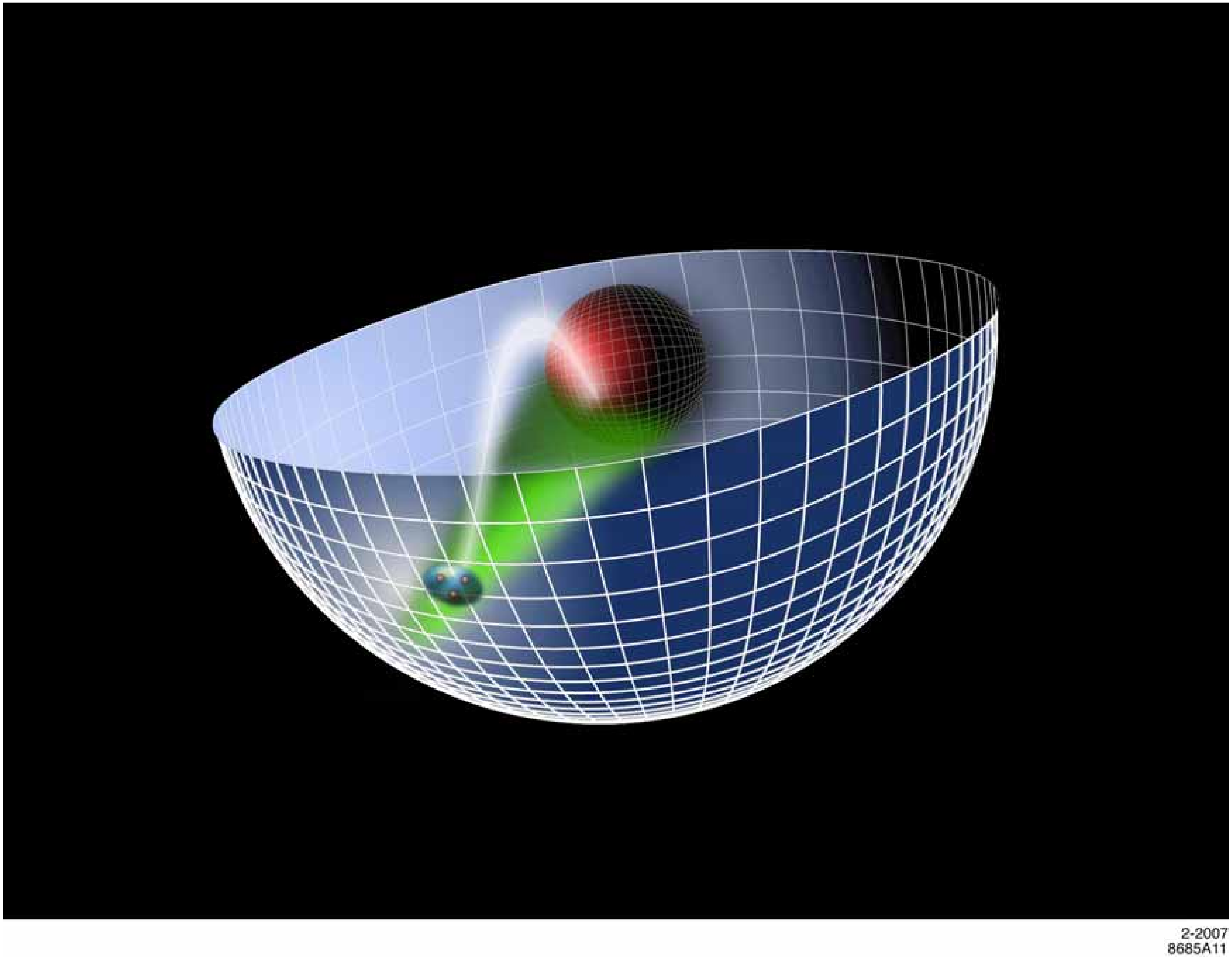

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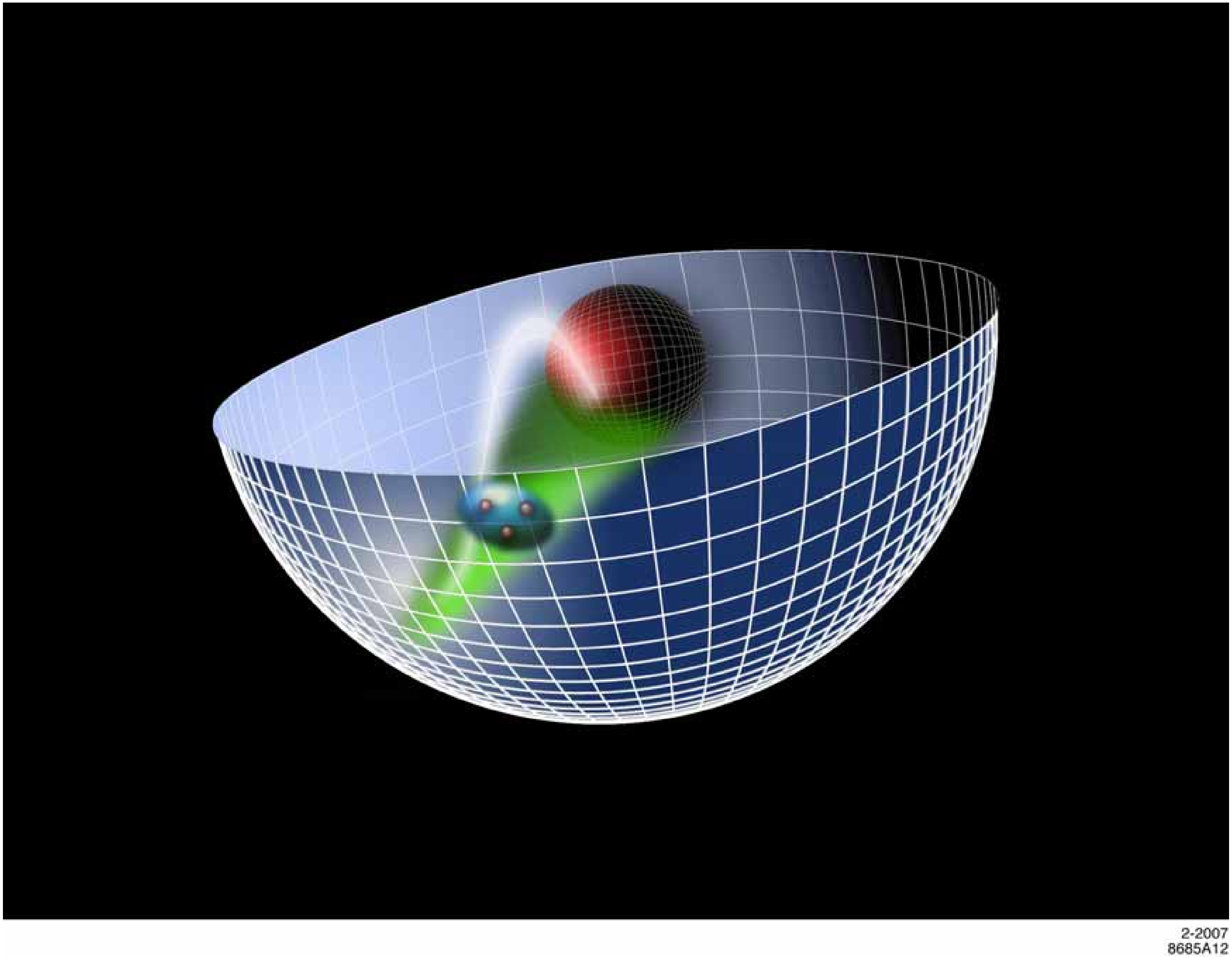
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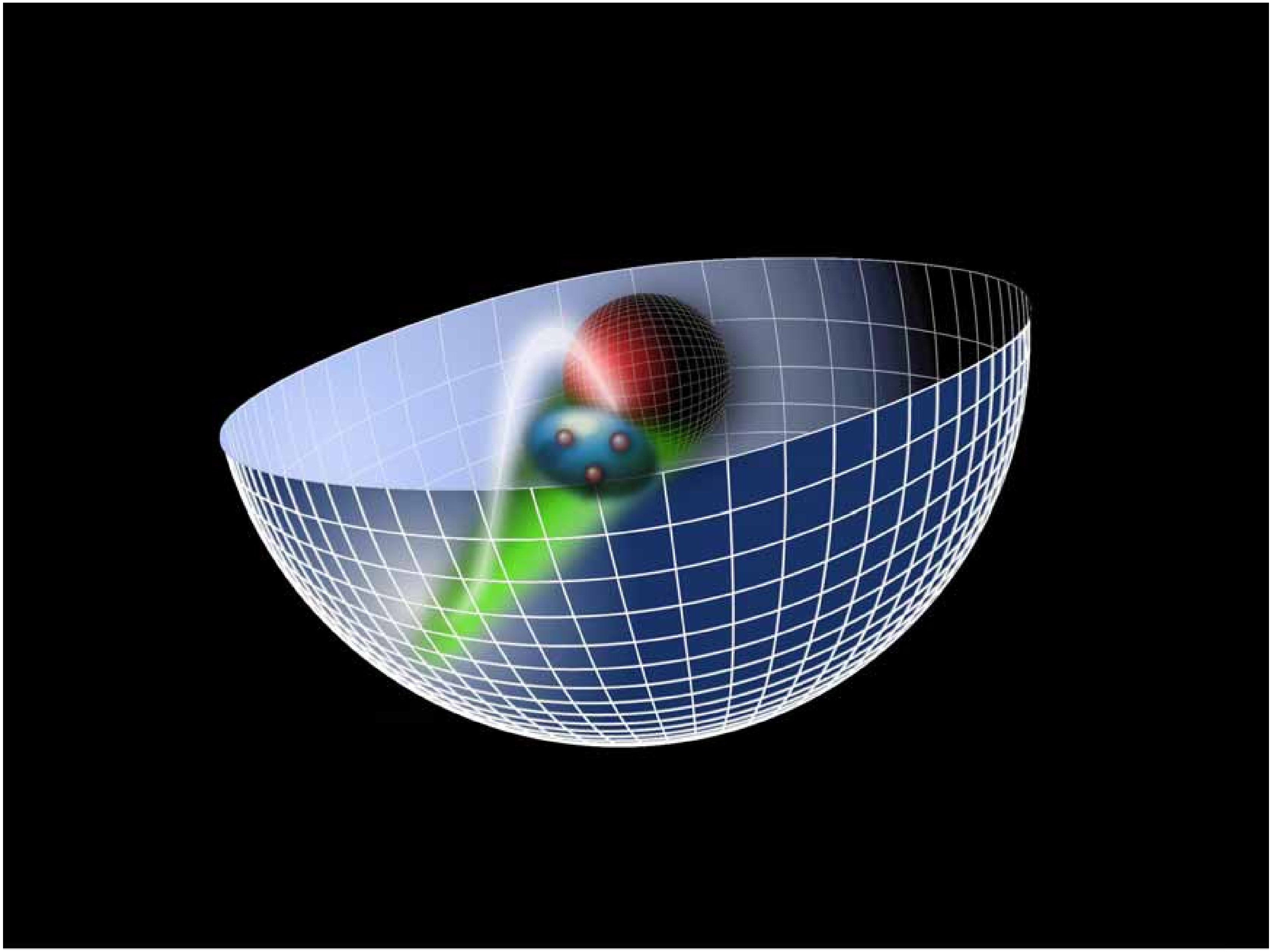

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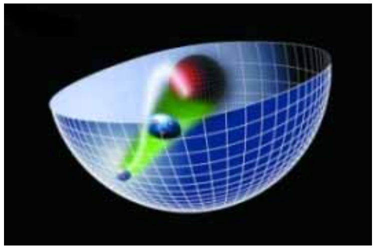
Gravity Action

$$\mathcal{R}_{ik\ell m} = -\frac{1}{R^2}\left(g_{i\ell}g_{km} - g_{im}g_{k\ell}\right)$$



- AdS$_{d+1}$ metric $x^\ell = (x^\mu, z)$:

$$ds^2 = g_{\ell m} dx^\ell dx^m = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- Action for gravity coupled to scalar field in AdS$_{d+1}$

$$S = \int d^{d+1}x\sqrt{g}\,\Big(\underbrace{\frac{1}{\kappa^2}\left(\mathcal{R} - 2\Lambda\right)}_{S_G} + \underbrace{\frac{1}{2}\left(g^{\ell m}\partial_\ell\Phi\partial_m\Phi - \mu^2\Phi^2\right)}_{S_M}\Big)$$

with $\Lambda = -\frac{d(d-1)}{2R^2}$ and $\sqrt{g} = (\frac{R}{z})^{d+1}$

- Equations of motion

$$\mathcal{R}_{\ell m} - \frac{1}{2}g_{\ell m}\mathcal{R} - \Lambda g_{\ell m} = 0$$

$$z^3\partial_z\left(\frac{1}{z^3}\partial_z\Phi\right) - \partial_\rho\partial^\rho\Phi - \left(\frac{\mu R}{z}\right)^2\Phi = 0$$

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$
- Factoring out dependence of string mode $\Phi_P(x,z)$ along x^μ-coordinates

$$\left[z^2\partial_z^2 - (d-1)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0$$

- Solution:

$$\Phi(z) = C z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}\,(z\mathcal{M})$$

- Conformal dimension

$$\Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2}\,\right)$$

- Normalization

$$R^{d-1}\int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^{d-1}}\,\Phi^2(z) = 1$$

5 Higher-Spin Modes

Hard Wall Model

- Spin J-field on AdS represented by rank-J totally symmetric tensor field $\Phi(x,z)_{\ell_1\cdots\ell_J}$
- Action in AdS$_{d+1}$

$$S_M = \frac{1}{2}\int d^{d+1}x\sqrt{g}\,\left(\partial_\ell \Phi_{\ell_1\cdots\ell_J}\partial^\ell \Phi^{\ell_1\cdots\ell_J} - \mu^2 \Phi_{\ell_1\cdots\ell_J}\Phi^{\ell_1\cdots\ell_J} + \dots\right)$$

- Each hadronic state of total spin J is dual to a normalizable string mode

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{-iP\cdot x}\,\Phi(z)_{\mu_1\cdots\mu_J}$$

  with four-momentum P_μ, spin polarization indices along the 3+1 physical coordinates and hadronic invariant mass $P_\mu P^\mu = \mathcal{M}^2$

- For string modes with all indices along Poincaré coordinates, $\Phi_{z\mu_2\cdots\mu_J} = \Phi_{\mu_1 z\cdots\mu_J} = \cdots = 0$ and appropriate subsidiary conditions system of coupled differential equations from S_M reduce to a homogeneous wave equation for $\Phi(z)_{\mu_1\cdots\mu_J}$

- Define the spin-J field $\Phi_{\mu_1 \cdots \mu_J}$ from the scalar mode Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Normalization   Hong, Yoon and Strassler (2006)

$$R^{d-2J-1} \int_0^{z_{max}} \frac{dz}{z^{d-2J-1}} \, \Phi_J^2(z) = 1.$$

- Substituting in the AdS wave equation for Φ

$$\left[z^2 \partial_z^2 - (d-1-2J) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

upon fifth-dimensional mass rescaling $(\mu R)^2 \to (\mu R)^2 - J(d-J)$

- Conformal dimension of J-mode

$$\Delta = \frac{1}{2}\left(d + \sqrt{(d-2J)^2 + 4\mu^2 R^2}\right)$$

and thus $(\mu R)^2 = (\Delta - J)(\Delta - d + J)$

- Upon substitution $z \to \zeta$ and

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} \Phi_J(\zeta)$$

we recover the QCD LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$ for $d = 4$

- Total orbital decoupling in the HW model

- For $L^2 \geq 0$ the LF Hamiltonian is positive definite $\langle \phi_J | H_{LF} | \phi_J \rangle \geq 0$ and we find the stability bound $(\mu R)^2 \geq -(2-J)^2$

- The scaling dimensions are $\Delta = 2 + L$ independent of J in agreement with the twist scaling dimension of a two parton bound state in QCD

Note: p-forms

- In tensor notation EOM for a p-form in AdS$_{d+1}$are $p+1$ coupled differential equations   I'Yi (1998)

$$\left[z^2\partial_z^2 - (d+1-2p)z\,\partial_z - z^2\mathcal{M}^2 - (\mu R)^2 + d + 1 - 2p\right]\Phi_{z\alpha_2\cdots\alpha_p} = 0$$

$$\cdots$$

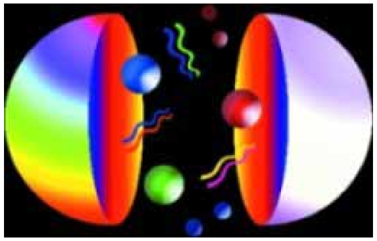
$$\left[z^2\partial_z^2 - (d-1-2p)z\,\partial_z - z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_{\alpha_1\alpha_2\cdots\alpha_p} = 2z\left(\partial_{\alpha_1}\Phi_{z\alpha_2\cdots\alpha_p} + \partial_{\alpha_2}\Phi_{\alpha_1 z\cdots\alpha_p} + \cdots\right)$$

- For modes with all indices along the Poincaré coordinates $\Phi_{z\alpha_2\cdots\alpha_p} = \Phi_{\alpha_1 z\cdots\alpha_p} = \cdots = 0$

$$\left[z^2\partial_z^2 - (d{-}1{-}2p)z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_{\alpha_1\cdots\alpha_p} = 0$$

with $(\mu R)^2 = (\Delta - p)(\Delta - d + p)$

6 Fermionic Modes



From Nick Evans

- Baryons Spectrum in "bottom-up" holographic QCD

  (GdT and SJB: hep-th/0409074, hep-th/0501022)

- Conformal metric $x^\ell = (x^\mu, z)$:

$$ds^2 = g_{\ell m} dx^\ell dx^m = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \, \sqrt{g} \, \overline{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x, z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

Holographic Light-Front Representation

- Upon the substitution $\Psi(z) \sim z^2\psi(z), \;\; z \to \zeta$ we find

$$H_{LF}|\psi\rangle = \mathcal{M}|\psi\rangle$$

with $H_{LF} = \alpha\,\Pi$ and $\mu R = \nu + \frac{1}{2}$

- The operator

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5\right)$$

and its adjoint $\Pi_\nu^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \frac{2\nu + 1}{\zeta^2}\gamma_5$$

- In the Weyl representation ($i\alpha = \gamma_5\beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

- Baryon: twist-dimension $3+L \quad (\nu = L+1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i$$

- Solution to Dirac eigenvalue equation

$$\left(\alpha \Pi(\zeta) - \mathcal{M}\right) \psi(\zeta) = 0,$$

is

$$\psi(\zeta) = C \sqrt{\zeta} \left[J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right]$$

Baryonic modes propagating in AdS space have two components: orbital L and $L+1$

- Hadronic mass spectrum determined from IR boundary conditions

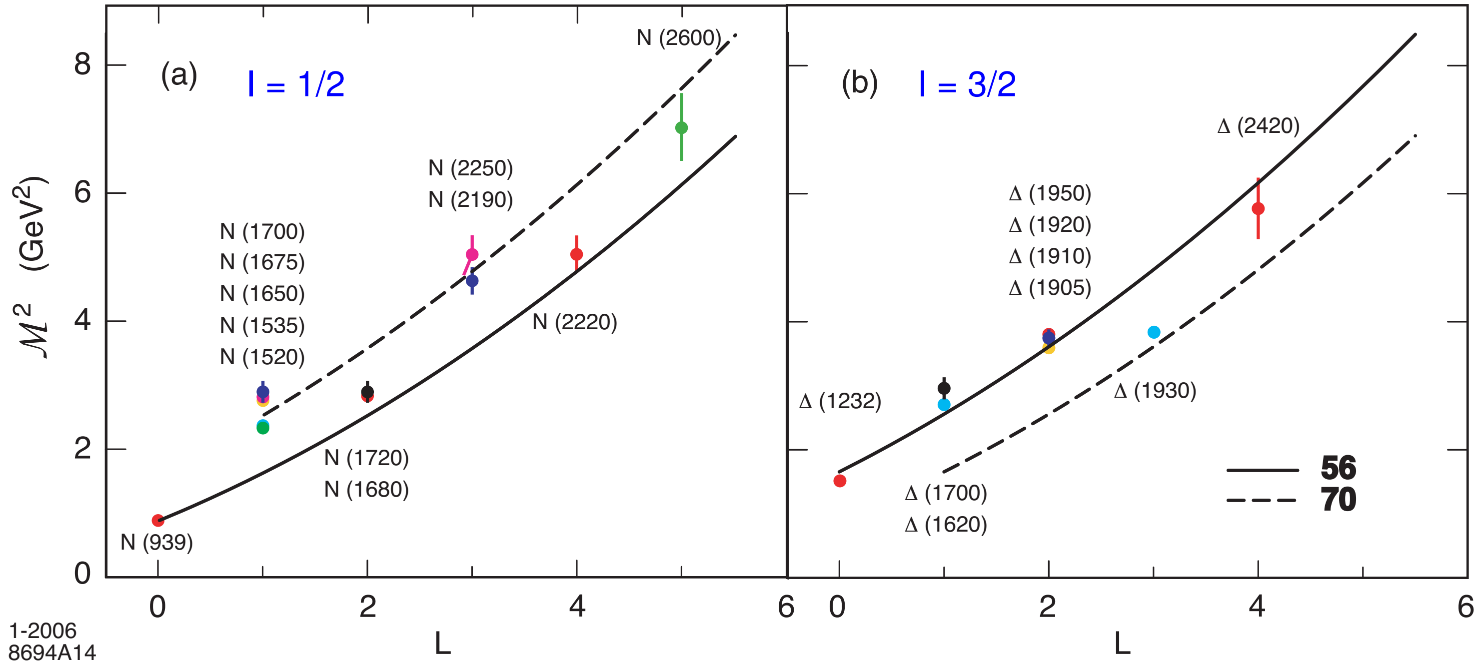
$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{L,k} = \beta_{L+1,k} \Lambda_{\rm QCD}, \quad \mathcal{M}^-_{L,k} = \beta_{L+2,k} \Lambda_{\rm QCD}$$

with a scale independent mass ratio

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$



Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0$$

in terms of the matrix-valued operator Π_ν

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right)$$

- Commutation relations for fermionic generators

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2\right)\gamma_5$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2)\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

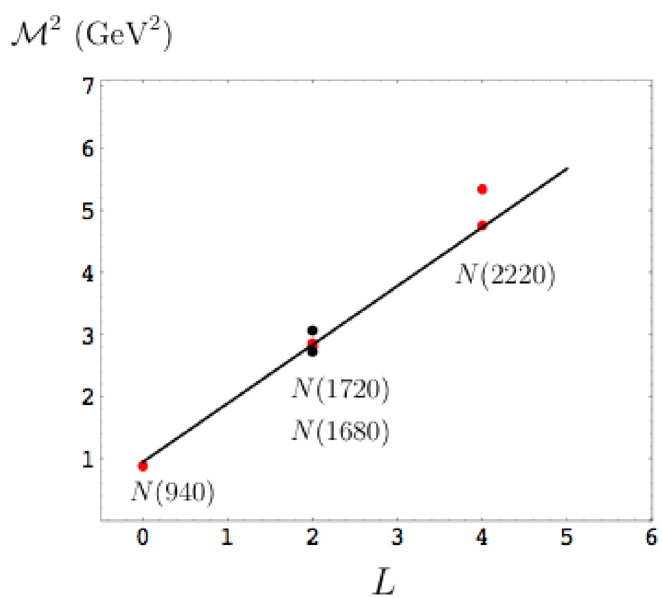
- Equivalent to Dirac equation in AdS space $x^\ell = (x^\mu, z)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + U(z)\right]\Psi(x^\ell) = 0.$$

in presence of a linear confining potential $U(z) = \kappa^2 z$!

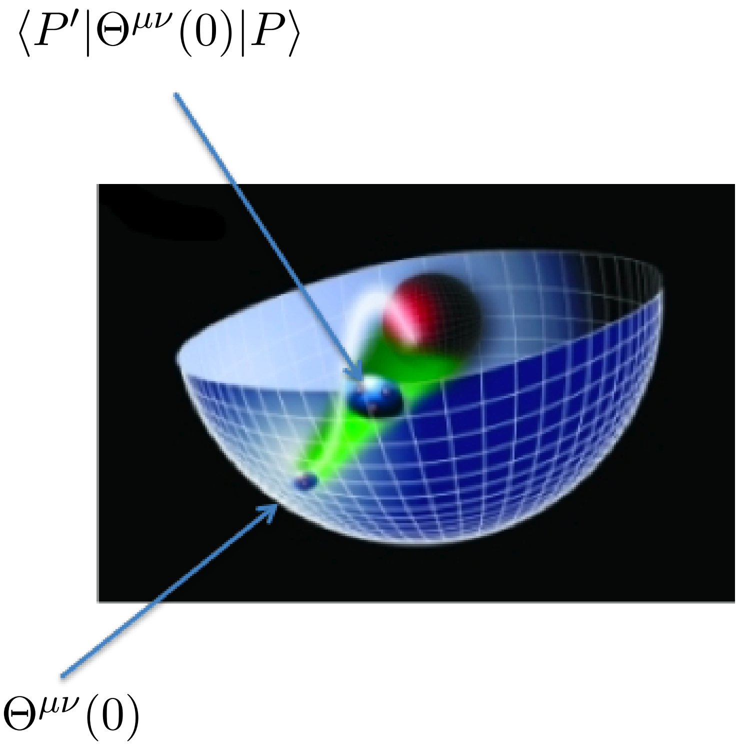
- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1).$$



Proton Regge Trajectory $\kappa = 0.49$ GeV

7 Conclusions



- Holographic duality requires a higher dimensional warped space. Space with negative curvature and a 4-dim boundary: AdS_5

- Local operators like hadronic interpolating operators $\mathcal{O}$, the energy-momentum tensor $\Theta^{\mu\nu}$, the EM current J^μ and the QCD Lagrangian $\mathcal{L}_{\mathrm{QCD}}$ are defined in terms of quark and gluon fields at the AdS_5 boundary

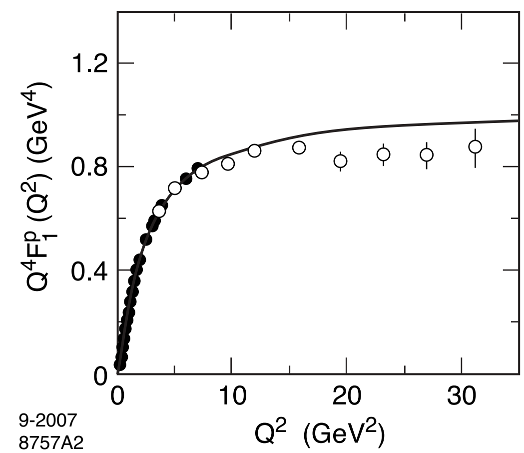
- Hadronic transition matrix elements like $\langle P'|\Theta^{\mu\nu}|P\rangle$ probes the hadronic wave function $\Phi(z)$ at $z \sim 1/Q \quad (Q = P' - P)$

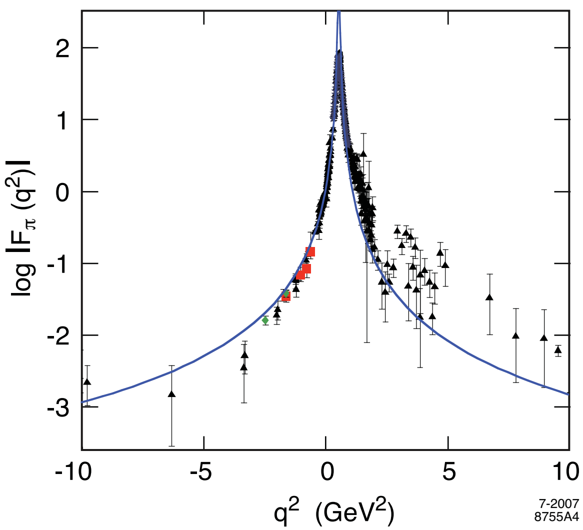
- Eigenvalues of normalizable modes inside AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for distribution of quarks at a given scale.

- Non-normalizable modes are related to external currents: they probe the cavity interior. Also couple to boundary QCD interpolating operators.

Other Applications of Light-Front Holography

- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form-factors of composite hadrons
- n-parton LFWF with massive quarks





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