Light-Front Hadron Dynamics and AdS/QCD Correspondence

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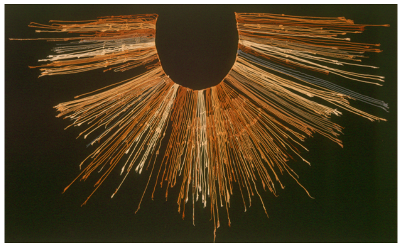

From Strings to Things: String Theory Methods in QCD and Hadron Physics

Institute for Nuclear Theory

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1 Introduction

- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom
- Recent developments using the AdS/CFT correspondence between string states in AdS space and conformal field theories in physical space-time have renewed the hope of finding an analytical approximation to describe the confining dynamics of QCD, at least in its strongly coupling regime
- Original correspondence between $\mathcal{N} = 4$ SYM at large N_C and the low energy supergravity approximation to Type IIB string on $\text{AdS}_5 \times S^5$ Maldacena, hep-th/9711200
- Description of strongly coupled gauge theory using a dual gravity description!
- QCD is fundamentally different from SYM theories where all matter is in the adjoint rep of $SU(N_C)$, and is non-conformal. Is there a dual string theory to QCD?

Strongly Coupled QCD and Effective Gravity Description

- Semi-classical correspondence as a first approximation to quasi-conformal QCD (strongly coupled at all scales)
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS space into the modes propagating inside AdS
- Strings describe extended objects (no quarks). QCD degrees of freedom are pointlike particles: how can they be related? How can we map string states into partons?
- Eigenvalues of normalizable modes inside AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for distribution of quarks at a given scale
- Non-normalizable modes are related to external currents: they probe the cavity interior. Also couple to boundary QCD interpolating operators

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- $x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z

- A distance $L_{\rm AdS}$ shrinks by a warp factor as observed in Minkowski space $(dz = 0)$:

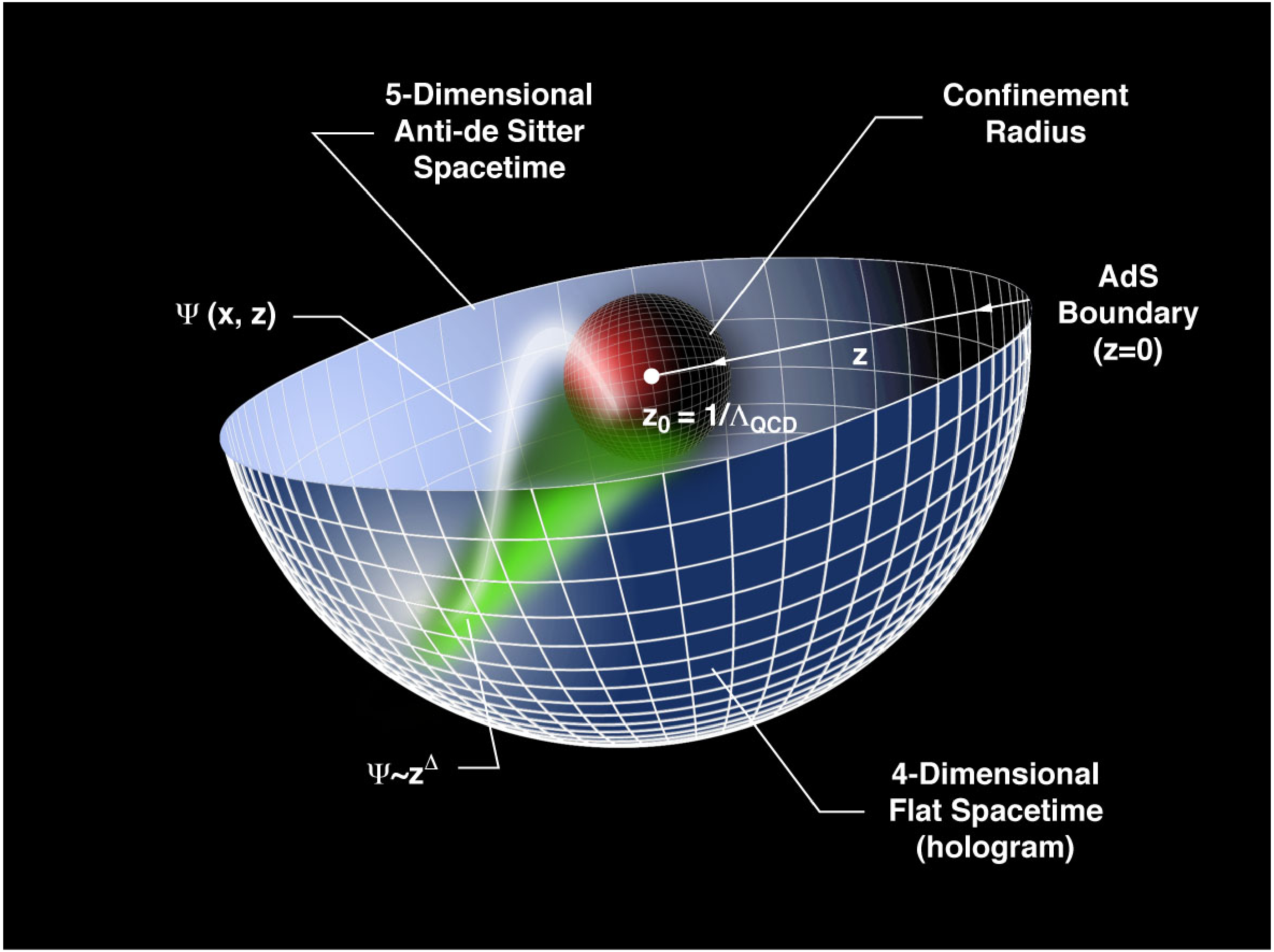
$$L_{Minkowski} \sim \frac{z}{R} \, L_{\rm AdS}$$

- Different values of z correspond to different scales at which the hadron is examined: AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit

- There is a maximum separation of quarks and a maximum value of z at the IR boundary

- Truncated AdS/CFT model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001)

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ usual Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006)



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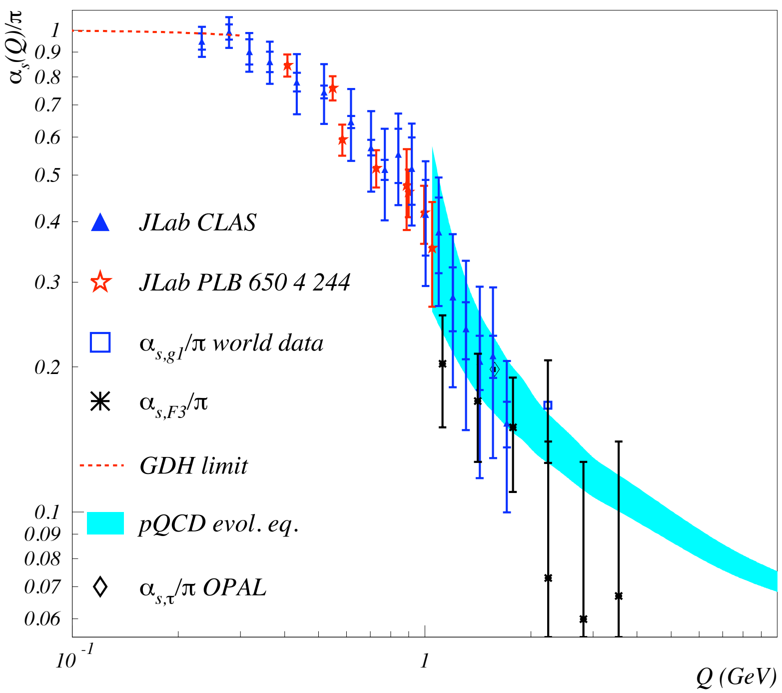
Conformal QCD Window in Exclusive Processes

- In the semi-classical approximation to QCD with massless quarks and no quantum loops the β function is zero and the approximate theory is scale and conformal invariant
- Is α_s frezed in the IR ? D-S Equation Alkofer, Fischer, LLanes-Estrada, Papavassiliou . . .
- New JLAB extraction of effective $\alpha_s(Q^2)$ from CLAS spin structure data shows lack of Q^2 dependence in the low Q^2 region Deur *et al.*, arXiv:0803.4119: $\frac{\alpha_{s,g_1}}{\pi} = \left(1 - \frac{6\,\Gamma_1^{p-n}}{g_A}\right)$, $\Gamma_1^{p-n}(Q^2)$ Bjorken sum



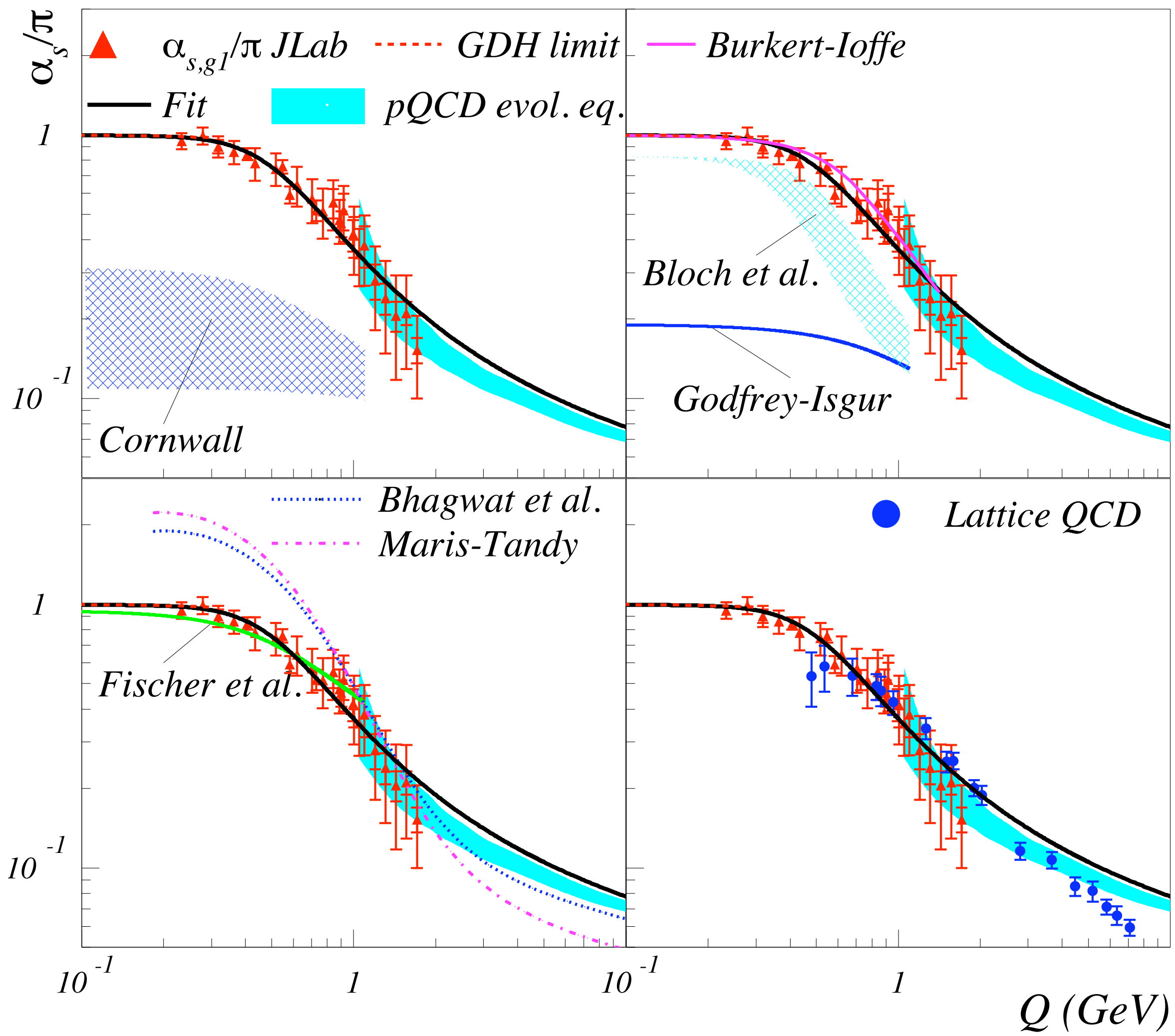


Fig: Infrared conformal window (from Deur *et al.*, arXiv:0803.4119)

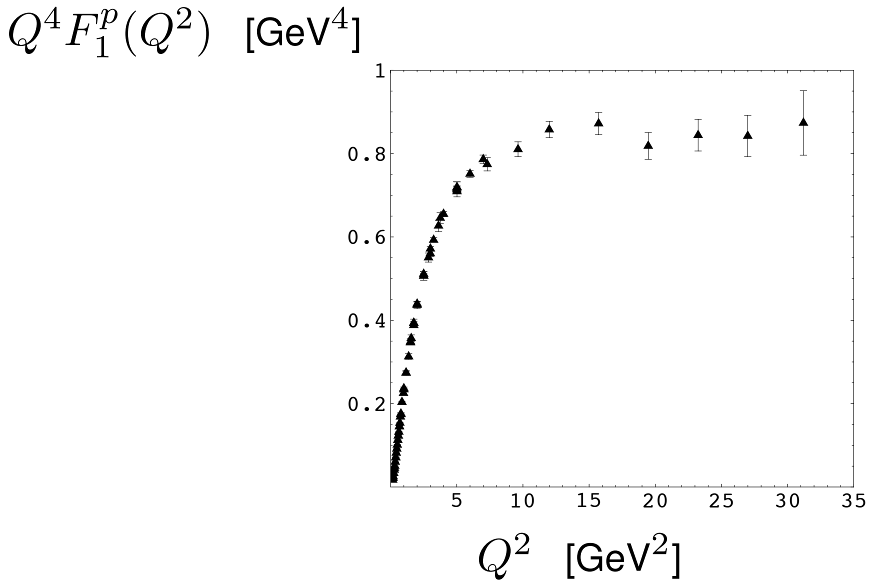
- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies
Brodsky and Farrar (1973); Matveev *et al.* (1973)

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001)

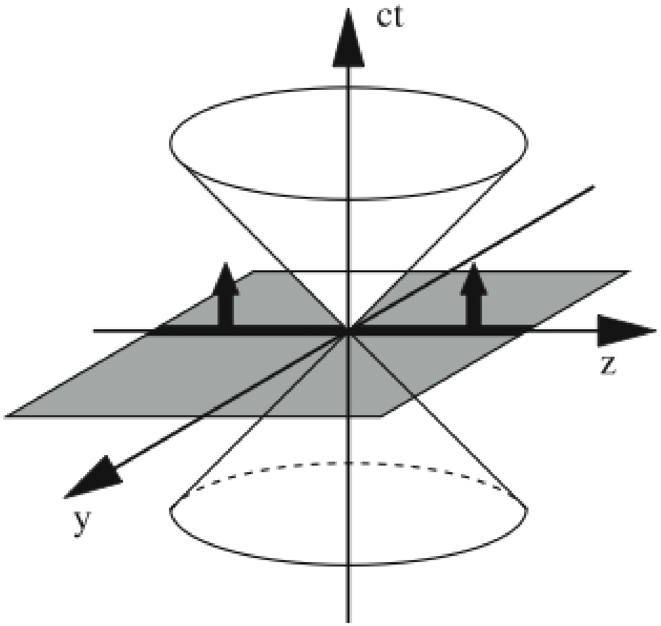
- Example: Dirac proton form factor: $F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \; n = 3$



From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

2 Light Front Dynamics

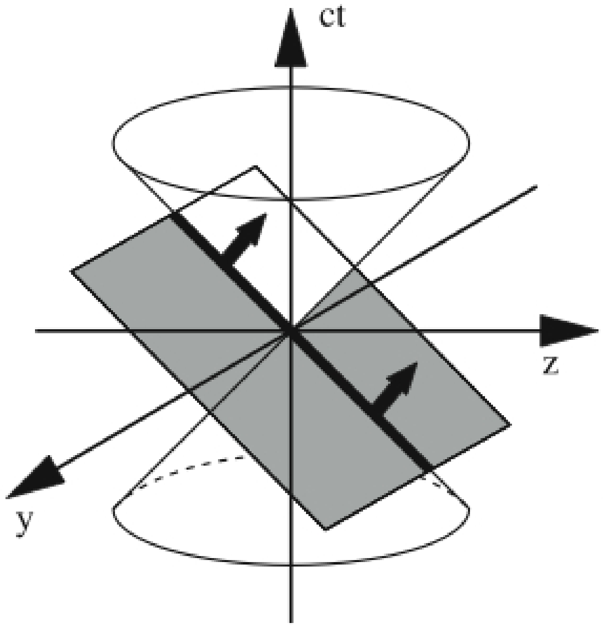
- Different possibilities to parametrize space-time in terms of general coordinates $\overline{x}(x)$ (excluding all related by a Lorentz transformation)
- According to Dirac there are no more than three different parametrization of space-time, the *instant form*, the *front form* and the *point form*, Dirac (1949)
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one
- *Front form*: hypersurface is tangent to the light cone
- *Point form*: hypersurface is an hyperboloid



The instant form

$$\tilde{x}^0 = ct$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = z$$

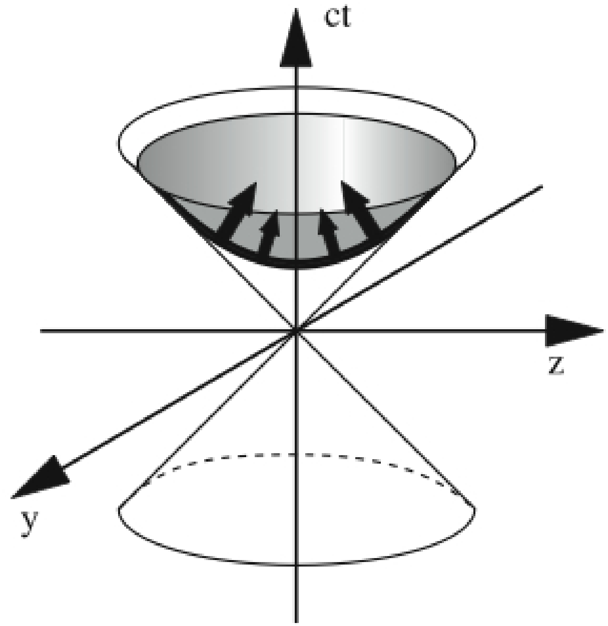
$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



The front form

$$\tilde{x}^0 = ct + z$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = ct - z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$



The point form

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau \cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau \sinh\omega \sin\theta \cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau \sinh\omega \sin\theta \sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau \sinh\omega \cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2 \sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2 \sinh^2\omega \sin^2\theta \end{pmatrix}$$

Light-Front Fock Representation

- Light-front expansion constructed by quantizing QCD at fixed light-cone time $\tau = t + z/c$ and forming the invariant light-front Hamiltonian (Brodsky, Pauli and Pinski, Phys. Rept. **301** 299 (1998)) :

$$H_{LF} = P^+ P^- - \vec{P}_\perp^2,$$

where $P^\pm = P^0 \pm P^z$

- Momentum generators P^+ and $\vec{P}_\perp$ are kinematical (independent of the interactions) and $P^- = i\frac{d}{d\tau}$ generates light-front time translations

- Eigenvalues of H_{LF} give the mass spectrum of the color-singlet hadronic states:

$$H_{LC} \mid \psi_H\rangle = \mathcal{M}_H^2 \mid \psi_H\rangle$$

- State $\mid \psi_h\rangle$ is an expansion in multi-particle Fock eigenstates $\mid n\rangle$ of the free light-front Hamiltonian:

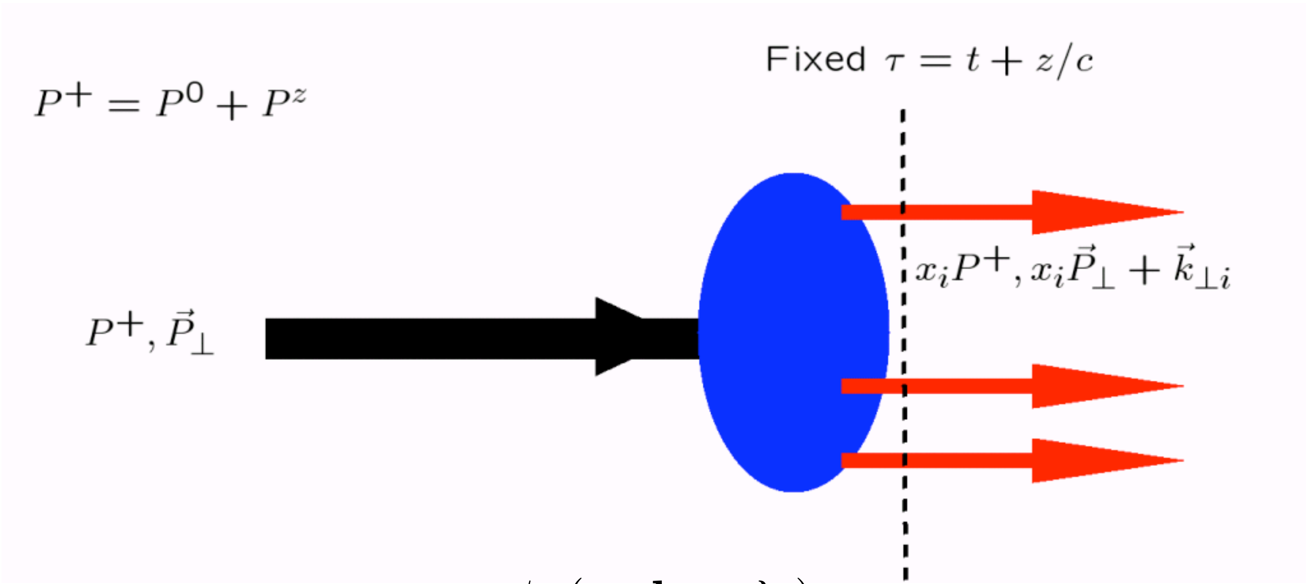
$$|\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle$$

- Proton:

$$|P\rangle = \psi_{uud/P}|uud\rangle + \psi_{uudg/P}|uudg\rangle + \psi_{uud\bar{q}q/P}|uud\bar{q}q\rangle \ldots$$

- Fock components $\psi_{n/h}(x_i, \mathbf{k}_{\perp i})$ are independent of the total momentum P^+ and $\mathbf{P}_\perp$ of the hadron and depend only on the relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin component λ_i

$$\sum_{i=1}^{n} x_i = 1, \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0.$$



$\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i)$

- Complete basis of Fock-states $|n\rangle$ constructed by applying free-field creation operators to the vacuum state $|0\rangle$: $P^+|0\rangle = 0, \; \mathbf{P}_\perp|0\rangle = 0$

- Dirac field ψ_+, $\psi_\pm = \Lambda_\pm \psi$, $\Lambda_\pm = \gamma^0\gamma^\pm$, and the transverse field $\mathbf{A}_\perp$ in the $A^+ = 0$ gauge, expanded in terms of quark and gluon ladder operators on the transverse plane with coordinates $x^- = x^0 - x^3$ and $\mathbf{x}_\perp$ at fixed light-front time $x^+ = x^0 + x^3$

$$\psi_+(x)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x} \right],$$

with u and v light-cone spinors

- Commutation relations

$$\left\{ b(q), b^\dagger(q') \right\} = \left\{ d(q), d^\dagger(q') \right\} = (2\pi)^3 \, \delta(q^+ - {q'}^+) \delta^{(2)}\big(\mathbf{q}_\perp - \mathbf{q}'_\perp\big)$$

- One parton state: $|q\rangle = \sqrt{2q^+}\, b^\dagger(q)|0\rangle$

Electromagnetic Form Factor of Composite Hadrons

- EM FF defined by matrix elements of the current operator $J^+(x) = \sum_q e_q \overline{\psi}(x)\gamma^+\psi(x)$

$$\left\langle P' \left| J^+(0) \right| P \right\rangle = 2 \left(P + P'\right)^+ F(Q^2), \quad Q = P' - P$$

- Particle number representation

$$J^+ = \sum_q e_q \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \int \frac{dq'^+ d^2\mathbf{q}'_\perp}{(2\pi)^3} \left\{b^\dagger(q) b(q') + d^\dagger(q) d(q')\right\}$$

- Drell-Yan-West (DYW) expression for meson form factor

$$F(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \sum_j e_j \psi^*_{n/P'}(x_i, \mathbf{k}'_{\perp i}) \psi_{n/P}(x_i, \mathbf{k}_{\perp i}),$$

where $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + (1 - x_i)\, \mathbf{q}_\perp$ for a struck quark and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\, \mathbf{q}_\perp$ for each spectator

- Phase space normalization of LFWFs

$$\sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \left|\psi_{n/h}(x_i, \mathbf{k}_{\perp i})\right|^2 = 1$$

- Transverse position coordinates $x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0, \qquad \sum_{i=1}^{n} x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp$$

- LFWF $\psi_n(x_j, \mathbf{k}_{\perp j})$ expanded in terms of $n-1$ independent coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$

$$\psi_n(x_j, \mathbf{k}_{\perp j}) = (4\pi)^{\frac{n-1}{2}} \prod_{j=1}^{n-1} \int d^2\mathbf{b}_{\perp j} \exp\Big(i \sum_{j=1}^{n-1} \mathbf{b}_{\perp j} \cdot \mathbf{k}_{\perp j}\Big) \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \left|\tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2 = 1$$

- The form factor has the exact representation (DYW)

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i \mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big) |\tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})|^2$$

Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\left\langle P' \left|\Theta^{++}(0)\right| P\right\rangle = 2\left(P^+\right)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $S_{\rm QCD} = \int d^4x \sqrt{g}\,\mathcal{L}_{\rm QCD}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta S_{\rm QCD}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \tfrac{1}{2}\overline{\psi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\,\psi - g^{\mu\nu}\overline{\psi}\left(i\not{D} - m\right)\psi - G^{a\,\mu\lambda} G^{a\,\nu}{}_{\lambda} + \tfrac{1}{4} g^{\mu\nu} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0,\ g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2}\sum_f \overline{\psi}^f(x)\gamma^+ \overleftrightarrow{\partial}^+ \psi^f(x)$$

- Particle number representation

$$\Theta^{++} = \frac{1}{2} \sum_f \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \int \frac{dq'^+ d^2\mathbf{q}'_\perp}{(2\pi)^3} \left(q^+ + q'^+\right) \left\{ b^{f\dagger}(q) b^f(q') + d^{f\dagger}(q) d^f(q') \right\}$$

- Gravitational form-factor in momentum space

$$A(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \sum_f x_f \, \psi^*_{n/P'}(x_i, \mathbf{k}'_{\perp i}) \psi_{n/P}(x_i, \mathbf{k}_{\perp i}),$$

where $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + (1 - x_i)\, \mathbf{q}_\perp$ for a struck quark and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\, \mathbf{q}_\perp$ for each spectator

- Gravitational form-factor in impact space

$$A(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_f x_f \exp\Big(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big) \left|\tilde\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

3 Semi-Classical Correspondence

Correspondence between a gravity theory in AdS$_{d+1}$ and the strong coupling limit of a conformal field theory at the $z=0$ boundary Gubser, Klebanov and Polyakov (1998); Witten (1998)

- d-dim QCD generating functional in presence of external source $J(x) = \Phi_0(x)$

$$Z_{\rm QCD}[\Phi_0(x)] = e^{iW_{\rm QCD}[\Phi_0]} = \int \mathcal{D}[\psi, \overline{\psi}, A] \exp\left\{ iS_{\rm QCD} + i \int d^d x \Phi_0 \mathcal{O} \right\},$$

  with $\mathcal{O}$ a hadronic local interpolating operator ($\mathcal{O} = G^a_{\mu\nu} G^{a\mu\nu}, \overline{q}\gamma_5 q, \cdots$)

- $d+1$-dim gravity partition function for scalar field in AdS$_{d+1}$: $\Phi(x,z)$

$$Z_{grav}[\Phi(x,z)] = e^{iS_{eff}[\Phi]} = \int \mathcal{D}[\Phi] e^{iS[\Phi]}$$

- Boundary condition for full theory (True for QCD ?):

$$Z_{grav}\left[\Phi(x, z=0) = \Phi_0(x)\right] = Z_{\rm QCD}\left[\Phi_0\right]$$

- Semi-classical effective approximation

$$W_{\rm QCD}\left[\phi_0\right] = S_{eff}\left[\Phi(x,z)|_{z=0} = \Phi_0(x)\right]_{\rm on-shell}$$

- Near the boundary of AdS_{d+1} space $z \to 0$:

$$\Phi(x,z) \to z^{\Delta}\Phi_+(x) + z^{d-\Delta}\Phi_-(x)$$

- $\Phi_-(x)$ is the boundary limit of non-normalizable mode (source): $\Phi_- = \Phi_0$
- $\Phi_+(x)$ is the boundary limit of the normalizable mode (physical states)
- Using the equations of motion AdS action reduces to a UV surface term

$$S_{eff} = \frac{R^{d-1}}{4} \lim_{z \to 0} \int d^d x \, \frac{1}{z^{d-1}} \, \Phi \partial_z \Phi$$

- S_{eff} is identified with the boundary functional W_{CFT}

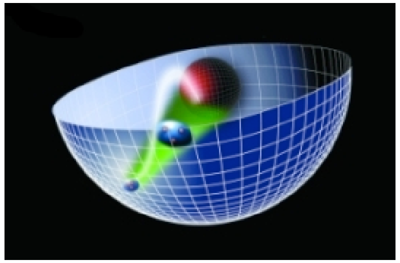
$$\langle \mathcal{O} \rangle_{\Phi_0} = \frac{\delta W_{CFT}}{\delta \Phi_0} = \frac{\delta S_{\rm eff}}{\delta \Phi_0} \sim \Phi_+(x)$$

Balasubramanian *et. al.* (1998), Klebanov and Witten (1999)

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP \cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$
- For small-z $\Phi(z) \sim z^{\Delta}$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P | \mathcal{O} | 0 \rangle \neq 0$

Gravity Action

$$\mathcal{R}_{ik\ell m} = -\frac{1}{R^2}\left(g_{i\ell}g_{km} - g_{im}g_{k\ell}\right)$$



- AdS$_{d+1}$ metric $x^\ell = (x^\mu, z)$:

$$ds^2 = g_{\ell m} dx^\ell dx^m = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- Action for gravity coupled to scalar field in AdS$_{d+1}$ $\left(\Lambda = -\frac{d(d-1)}{2R^2}\right)$:

$$S = \int d^{d+1}x \sqrt{g}\,\Big(\underbrace{\frac{1}{\kappa^2}\left(\mathcal{R} - 2\Lambda\right)}_{S_G} + \underbrace{g^{\ell m}\partial_\ell \Phi^* \partial_m \Phi - \mu^2 \Phi^* \Phi}_{S_M}\Big)$$

- Equations of motion

$$\mathcal{R}_{\ell m} - \frac{1}{2} g_{\ell m}\mathcal{R} - \Lambda g_{\ell m} = 0$$

$$z^3 \partial_z \left(\frac{1}{z^3}\partial_z \Phi\right) - \partial_\rho \partial^\rho \Phi - (\mu R)^2 \Phi = 0$$

Electromagnetic Transition Matrix Elements in AdS

- Hadronic matrix element for EM coupling with AdS mode Φ, $J^\ell = \frac{1}{\sqrt{g}}\frac{\delta S_I}{\delta A^\ell}$:

$$\langle P'|M|P\rangle = \mathcal{Q}\int d^4x\, dz\, \sqrt{g}\, A^\ell(x,z)\Phi^*_{P'}(x,z)\overleftrightarrow{\partial}_\ell \Phi_P(x,z)$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z\partial_z\left(\frac{1}{z}\partial_z\right) - Q^2\right] J(Q,z) = 0,$$

subject to boundary conditions $J(Q=0,z) = J(Q,z=0) = 1$

- Solution

$$J(Q,z) = zQK_1(zQ)$$

- Substitute hadronic modes $\Phi(x,z)$ in the transition matrix element $\langle P'|M|P\rangle$

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \quad \Phi(z) \to z^\Delta, \;\; z \to 0$$

- Find the transition amplitude

$$\langle P'|M^\mu|P\rangle = 2(P+P')^\mu F(Q^2)$$

- EM form-factor $F(Q^2)$ is the overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with non-normalizable mode $J(Q,z)$ dual to external source [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int_0^{\Lambda^{-1}_{\rm QCD}} \frac{dz}{z^3}\, \Phi(z) J(Q,z) \Phi(z)$$

- Since $K_n(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x}$, the external source is suppressed inside AdS for large Q. Important contribution to the integral is from $z \sim 1/Q$, where $\Phi \sim z^\Delta$
- For large Q^2

$$F(Q^2) \to \left(\frac{1}{Q^2}\right)^{\Delta-1},$$

and the power-law ultraviolet point-like scaling is recovered [Polchinski and Susskind (2001)]

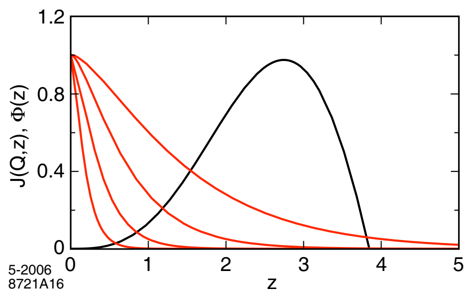


Fig: Suppression of external modes for large Q inside AdS. Red curves: $J(Q,z)$, black: $\Phi(z)$

Gravitational Transition Matrix Elements in AdS

- Consider a small deformation of the metric about AdS background $g_{\ell m}$: $\overline{g}_{\ell m} = g_{\ell m} + h_{\ell m}$ and expand S_M, $\Theta^{\ell m}(x^\ell) = -\frac{2}{\sqrt{g}} \frac{\delta S_M}{\delta g_{\ell m}(x^\ell)}$:

$$S_M[h_{\ell m}] = S_M[0] \underbrace{- \frac{1}{2} \int d^{d+1}x \sqrt{g}\, h_{\ell m} \Theta^{\ell m}}_{S_I} + \mathcal{O}(h^2),$$

where

$$\Theta^{\ell m} = \partial^\ell \Phi^* \partial^m \Phi + \partial^m \Phi^* \partial^\ell \Phi - g^{\ell m} \left(\partial_n \Phi^* \partial^n \Phi - \mu^2 \Phi^* \Phi \right)$$

- Hadronic matrix element

$$\langle P' | T | P \rangle = \int d^4x\, dz \sqrt{g}\, h_{\ell m}(x, z) \partial^{(\ell} \Phi^*_{P'}(x, z) \partial^{m)} \Phi_P(x, z)$$

- Find propagation of gravitational probe inside AdS. Expand S_G ($\bar{g}_{\ell m} = g_{\ell m} + h_{\ell m}$) :

$$S_G[h_{\ell m}] = S_G[0] + \underbrace{\frac{1}{4\kappa^2}\int d^{d+1}x\,\sqrt{g}\left(\partial_n h^{\ell m}\partial^n h_{\ell m} - \frac{1}{2}\partial_\ell h\,\partial^\ell h\right)}_{S_h} + \mathcal{O}(h^2),$$

in the harmonic gauge $\partial_\ell h^\ell_m = \frac{1}{2}\partial_m h$

- Graviton with metric components along Minkowski coordinates $h_{zz} = h_{z\mu} = 0$. Equation of motion

$$z^3\partial_z\left(\frac{1}{z^3}\partial_z h_\mu^\nu\right) - \partial_\rho\partial^\rho h_\mu^\nu = 0,$$

in the transverse and traceless ($h = h^\mu_\mu = 0$) gauge

- Write

$$h_\mu^\nu(x,z) = \epsilon_\mu^\nu\, e^{-iq\cdot x} H(q^2,z)$$

with boundary conditions $H(q^2\!=\!0, z) = H(q^2, z\!=\!0) = 1$

- Solution

$$H(Q^2,z) = \frac{1}{2}Q^2 z^2 K_2(zQ)$$

- Substitute hadronic modes $\Phi(x,z)$ in the transition matrix element $\langle P'|T|P\rangle$

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z)$$

- Find the transition amplitude

$$\left\langle P' \left|T^{\mu\nu}\right| P\right\rangle = \left(P'^{\mu}P^{\nu} + P^{\mu}P'^{\nu}\right) A(Q^2)$$

- Gravitational form-factor $A(Q^2)$

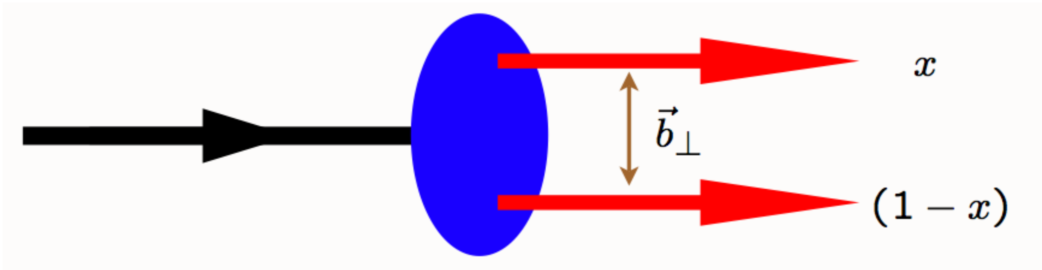
$$A(Q^2) = R^3 \int \frac{dz}{z^3}\, \Phi(z) H(Q^2,z) \Phi(z), \quad A(0) = 1$$

- At large Q^2

$$A(Q^2) \to \left(\frac{1}{Q^2}\right)^{\Delta - 1},$$

we recover ultraviolet point-like behavior responsible for power law scaling

4 Light-Front Mapping of String Amplitudes



- Consider LF holographic mapping of a two-parton bound state with LFWF $\psi_{\overline{q}q/\pi}(x, \mathbf{b}_\perp)$
- n-parton holographic mapping $\psi_{n/H}(x_i, \mathbf{b}_{\perp i})$ described in terms of effective single particle distribution (Soper): SJB and GdT, arXiv:0707.385 and arXiv:0804.045

Electromagnetic Form Factor

- Drell-Yan West electromagnetic form factor in impact space

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) \left|\tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- For a two-quark π^+ bound state $|u\overline{d}\rangle$ with charges $e_u = \frac{2}{3}$ and $e_{\overline{d}} = \frac{1}{3}$:

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\tilde{\psi}_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2,$$

where $F_\pi^+(q = 0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\tilde{\psi}_{u\bar{d}/\pi}(x, \zeta)\right|^2,$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \left|\Phi_{\pi^+}(z)\right|^2 ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) \left|\Phi_{\pi^+}(z)\right|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\bar{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4}}$$

with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

Gravitational Form Factor

- Gravitational form factor in impact space

$$A(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_f x_f \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) \left|\tilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- For a two-quark π^+ bound state $|u\overline{d}\rangle$ with longitudinal momentum fraction x and $1-x$

$$A_\pi(q^2) = 2\int_0^1 x dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\tilde{\psi}_{q\overline{q}/\pi}(x, \mathbf{b}_\perp)\right|^2,$$

where $A(q=0)=1$ $\left(\int_0^1 x dx \int d^2\mathbf{b}_\perp |\tilde{\psi}(x,\mathbf{b}_\perp)|^2 = \frac{1}{2}\right)$

- Integrating over angle we find

$$A_\pi(Q^2) = 4\pi \int_0^1 \frac{dx}{(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) |\tilde{\psi}_{q\overline{q}/\pi}(x,\zeta)|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} \, H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

which is identical to the result obtained from the EM form-factor

Example: Two-parton Pion LFWF

- Hard-Wall Model (P-S)

$$\tilde{\psi}^{HW}_{\bar{q}q/\pi}(x, \mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

- Soft-Wall Model (K-K-S-S)

$$\tilde{\psi}^{SW}_{\bar{q}q/\pi}(x, \mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right)$$

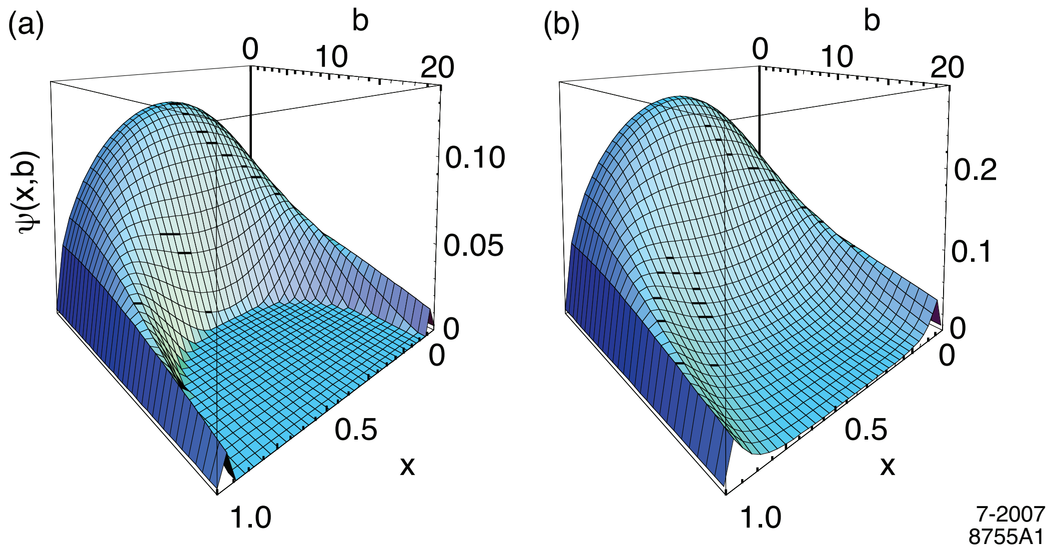
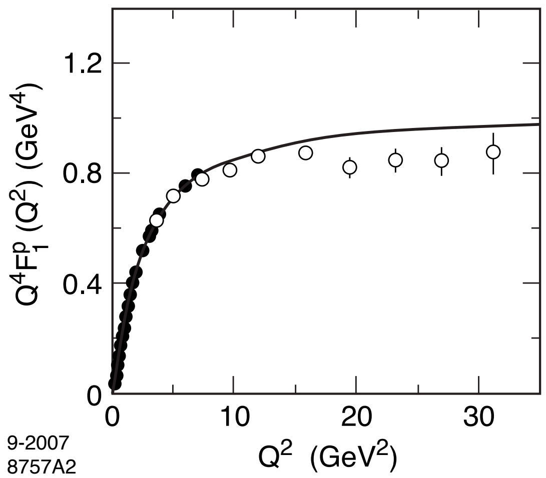


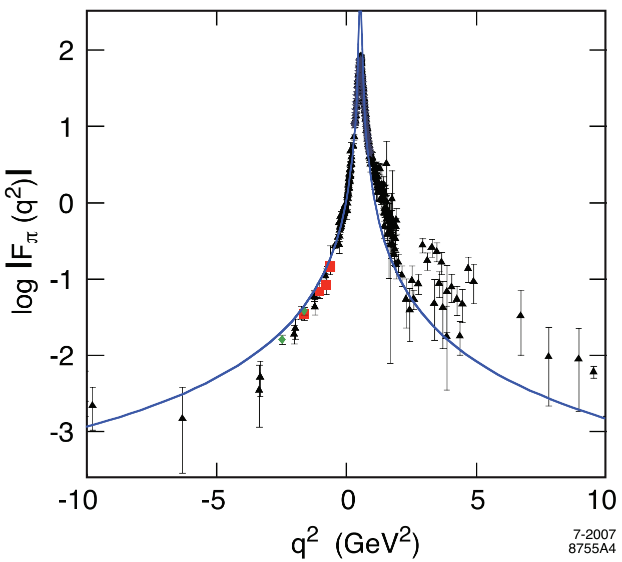
Fig: Ground state pion LFWF in impact space: (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV

Other Applications of Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- n-parton LFWF with massive quarks





hep-th/0501022

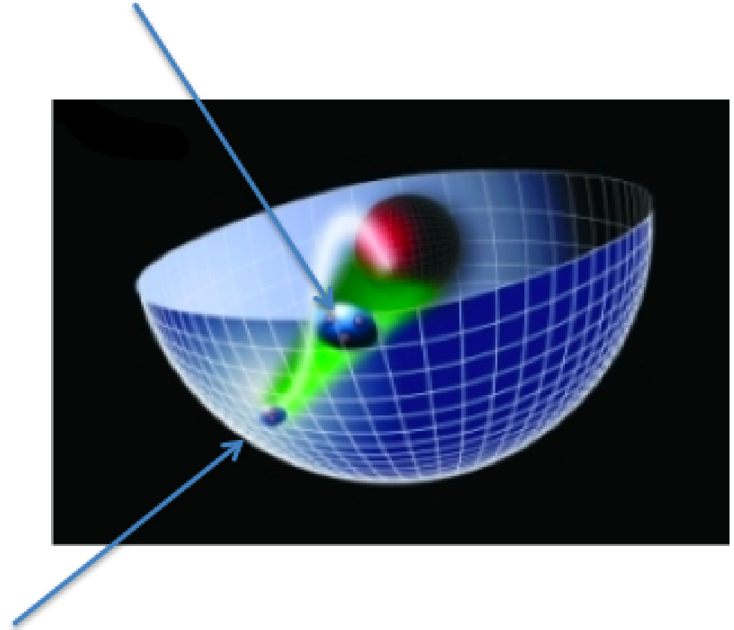
hep-ph/0602252

arXiv:0707.3859

arXiv:0802.0514

arXiv:0804.0452

5 Quark Hadron Duality in AdS/QCD



Local operators like $\Theta^{\mu\nu}$ are defined in terms of quark and gluon fields at AdS_5 boundary $z = 0$

$$\Updownarrow$$

Hadronic transition matrix element $\langle P'|\Theta^{\mu\nu}|P\rangle$ probes hadronic wave function $\Phi(z)$ at $z \sim 1/Q \quad (Q = P' - P)$