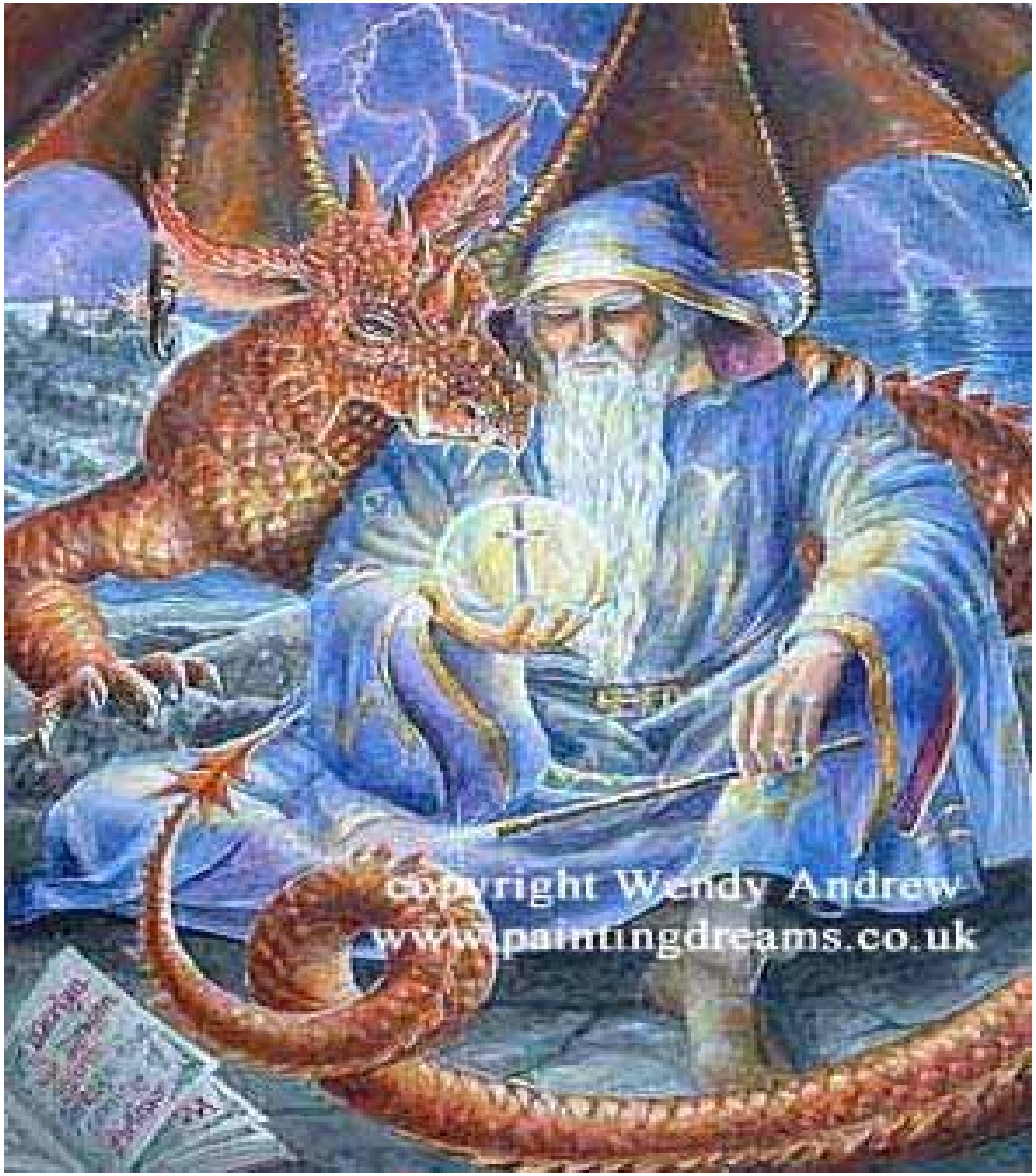
Matrix Elements for $\beta\beta_{0\nu}$



T. Rizzo

10/06/09

$\rightarrow$ The size of the nuclear matrix element is the largest theoretical uncertainty in the extraction of effective Majorana neutrino masses from the (??) observation of neutrinoless double beta decay assuming light neutrinos. However, it appears that the size of the nuclear community working on this problem is rather small.

$$\tau_{1/2}^{-1} = G(Q,Z,A) < m_\nu^{eff} >^2 |M_N|^2$$

Known kinematic function

Nuclear matrix elements

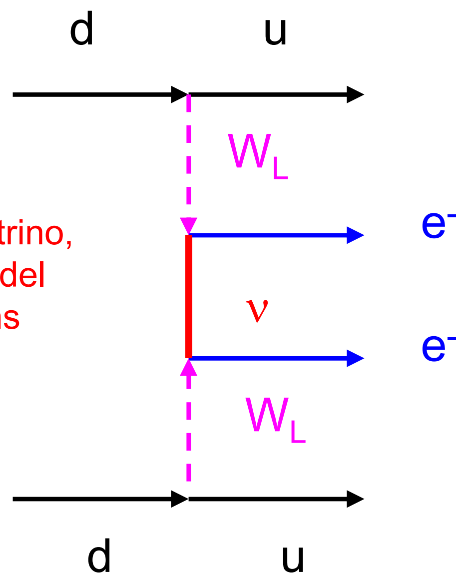
$$\rightarrow\rightarrow \quad < m_\nu^{eff} >^2 = |\Sigma\, m_j U_{ej}^2|^2$$

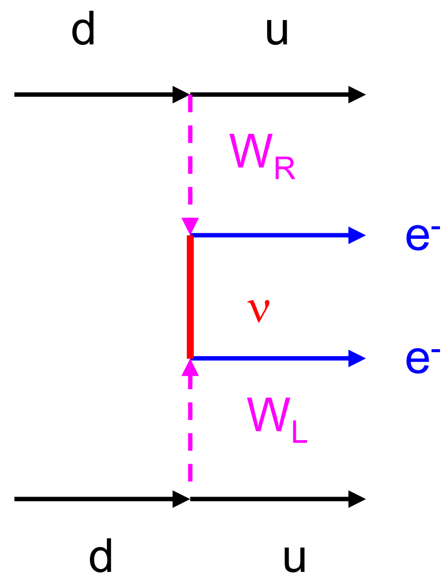
We Must Remember

that although the observation of $\beta\beta_{0\nu}$ is certainly a clear signal for $\Delta L=2$ interactions (this is a theorem by Schechter & Valle, '82), it is not always true that the rate for the $\beta\beta_{0\nu}$ process is directly related to the Majorana neutrino mass values in any simple way as many other virtual processes can also contribute to these reactions:

All these diagrams can in principle contribute to the decay amplitude

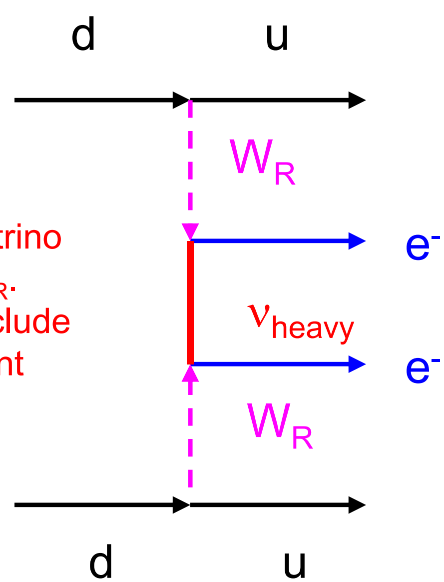
Light Majorana neutrino, only Standard Model weak interactions

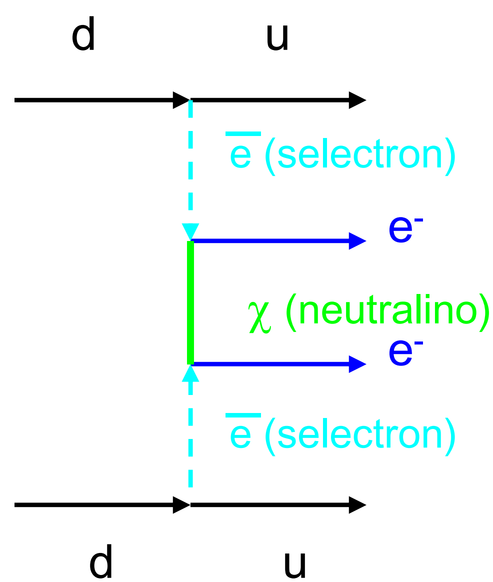




Light or heavy Majorana neutrino. Model extended to include right-handed W_R. Mixing extended between the left and right-handed neutrinos.

Heavy Majorana neutrino interacting with W_R. Model extended to include right-handed current interactions.



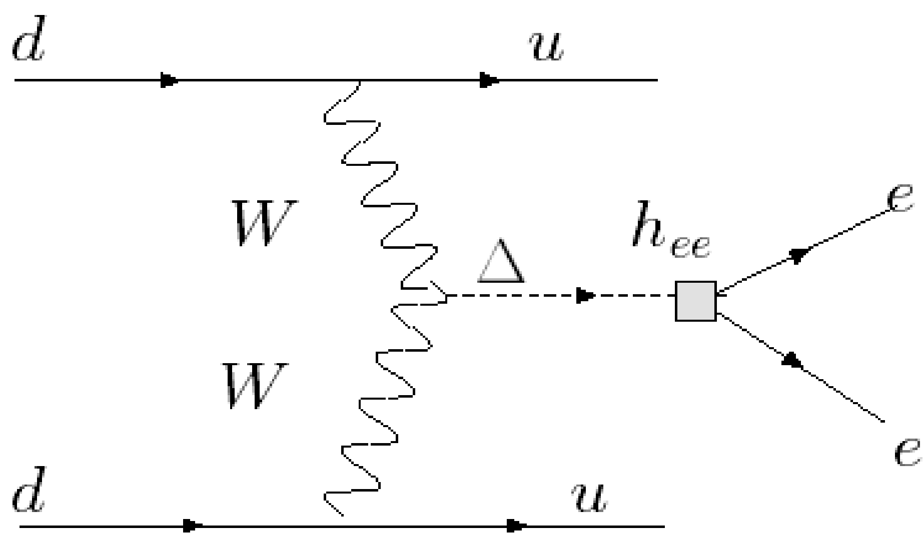


Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

Vogel

Another Possibility: Left-Right Symmetric Model (LRSM)

$$\mathcal{L}_{\delta^{\pm\pm}_{L,R}} = \frac{g}{2}\left[\delta^{++}_{L,R}\,\overline{l^c}\,(h_{L,R}\,P_{L,R})\,l + \delta^{--}_{L,R}\,\bar{l}\left(h^\dagger_{L,R}\,P_{R,L}\right)l^c\right]$$



The model includes a doubly charged Higgs that couples to leptons as shown

This is an example of 0νββ decay mediated by this coupling. The amplitude scales like

$$\frac{g_2^3 \quad h_{ee}}{M^3_{W_R} M^2_\Delta}$$

Another example is the exchange of heavy right-handed ν_R and two W_R that scales like

$$\frac{g_2^4}{M^4_{W_R} M_{\nu_R}}$$

In both cases the amplitude scales like $1/\Lambda^5$ with $\Lambda \sim M_{W(R)} \sim M_\Delta \sim M_{\nu\,(R)}$

Vogel

In the amplitudes for these other processes the direct relationship of the observed rate to the ν Majorana masses is lost. The nuclear matrix elements in these 'unusual' cases are even less well studied & well known than in the more familiar case of light neutrino exchange.

Measurements of $\beta\beta_{0\nu}$ in several nuclei (as well as of the electron angular correlations*) may be necessary to pin down the 'correct 'mechanism & help with the nuclear uncertainties.

I will, however, assume that the direct 'light Majorana neutrino' assumption holds in the analysis that follows.

* see next slide

An Aside :

The angular correlation, i.e., the distribution of the angle between the two outgoing leptons, is given by

$d\Gamma \sim 1 - K \cos \phi$ with $K \sim \langle \beta_1 \beta_2 \rangle$ the product of their speeds in the 'light neutrino' scenario

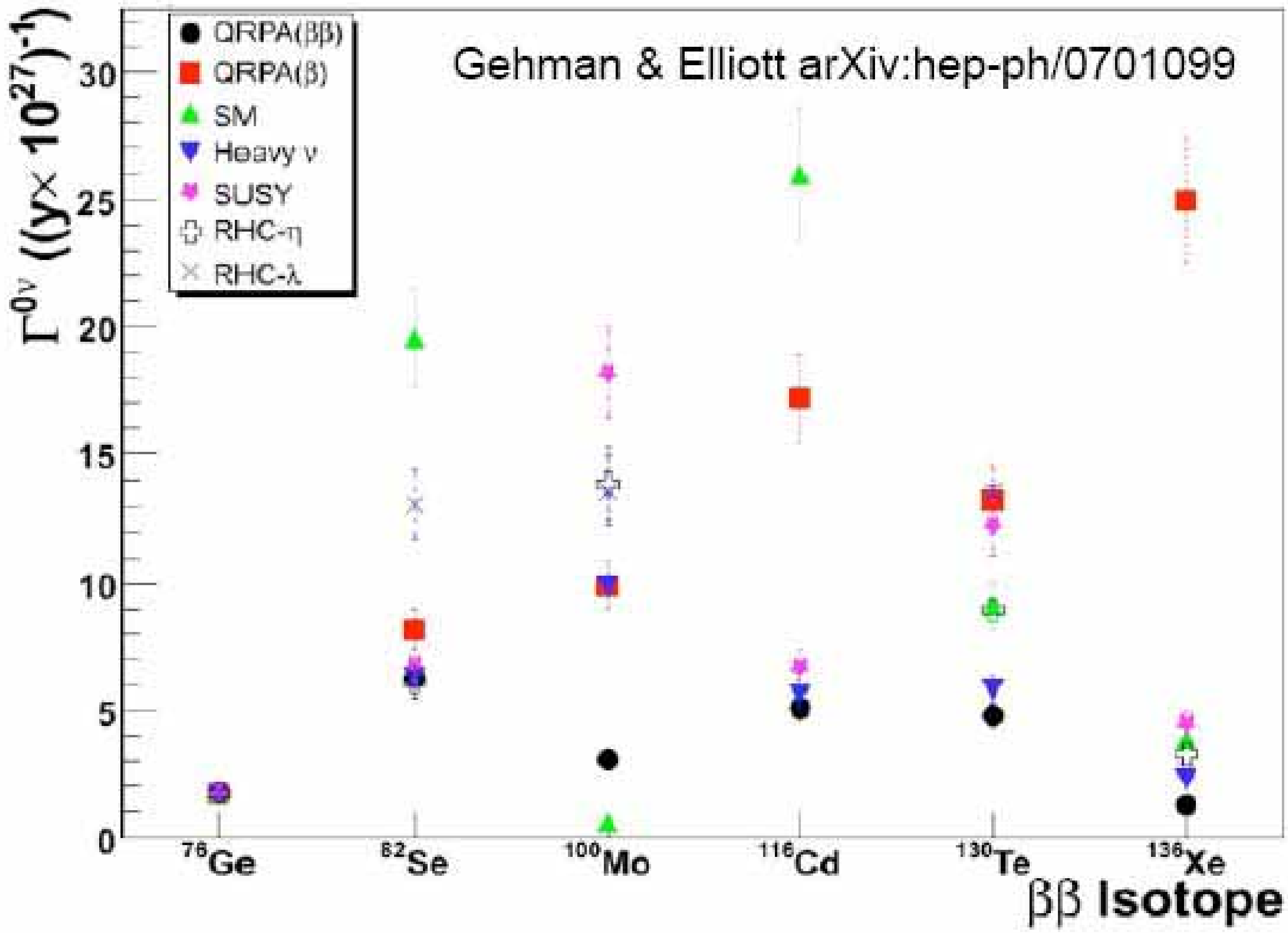
One finds that K~ 0.84 ±0.03 for the various nuclei depending upon A, Z & the Q values…a fairly narrow range.

For more exotic physics models, K can lie ANYWHERE in the range between +1 and -1 so measuring the K value above is reasonably restrictive on model building.

See Ali, Borisov & Zhuridov

0νββ-decay as a Probe of LNV Interactions

If 0νββ is observed, then measurements on 3-4 multiple isotopes might be able to distinguish potential physics mechanisms

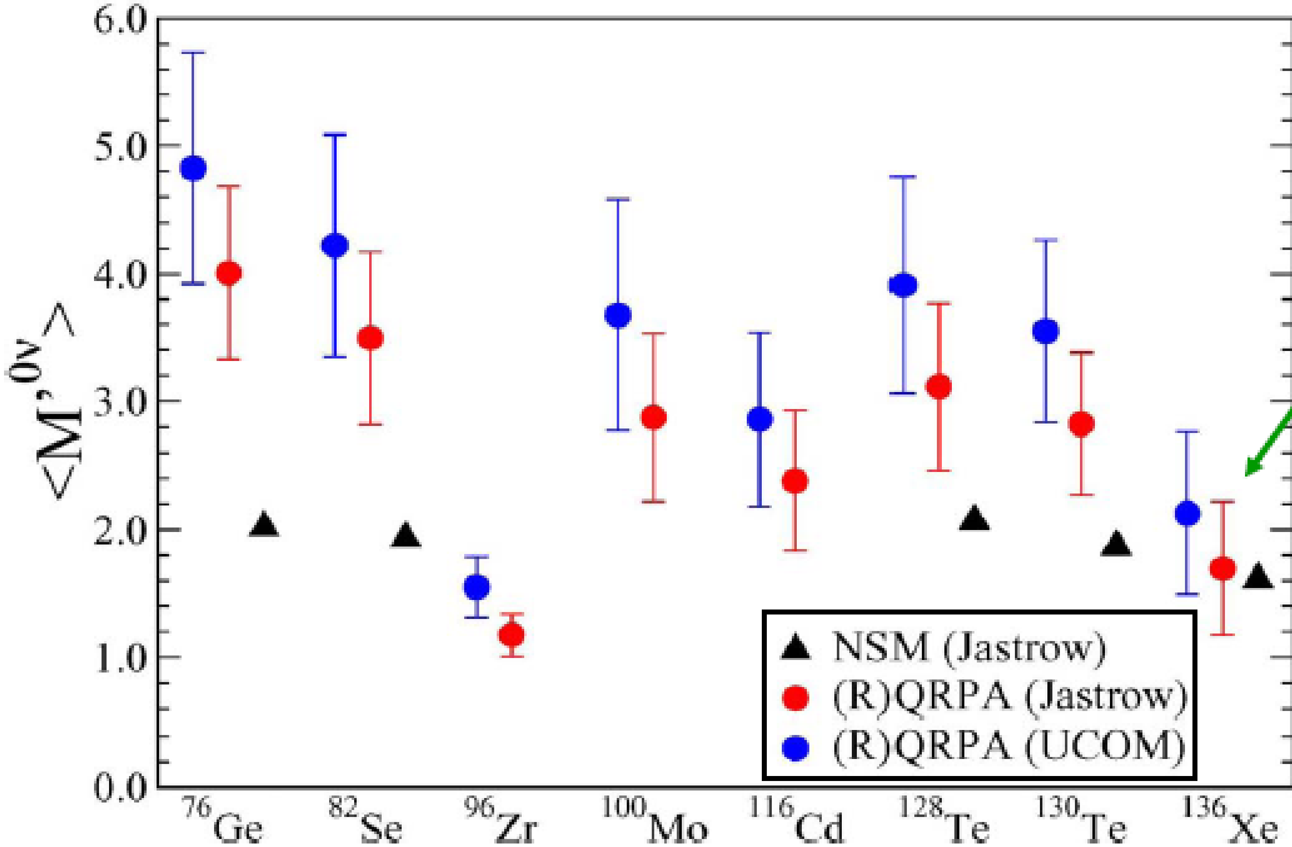


Comparison assumes a single dominant mechanism.

Requires results from 3-4 isotopes & calculation of NME to ~20%

Also see
Deppisc & Päs
arXi:hep-ph/0612165

Wilkerson

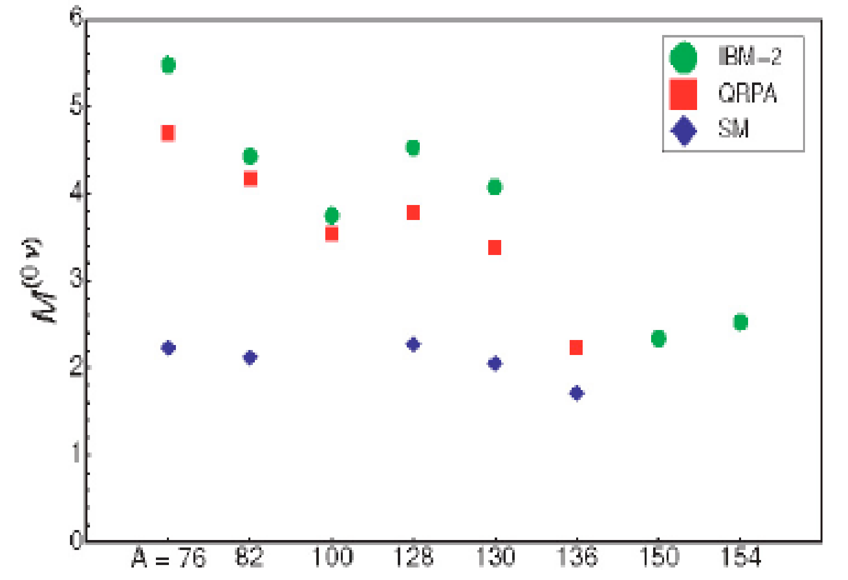


Stolen from Giorgio's SSI lecture

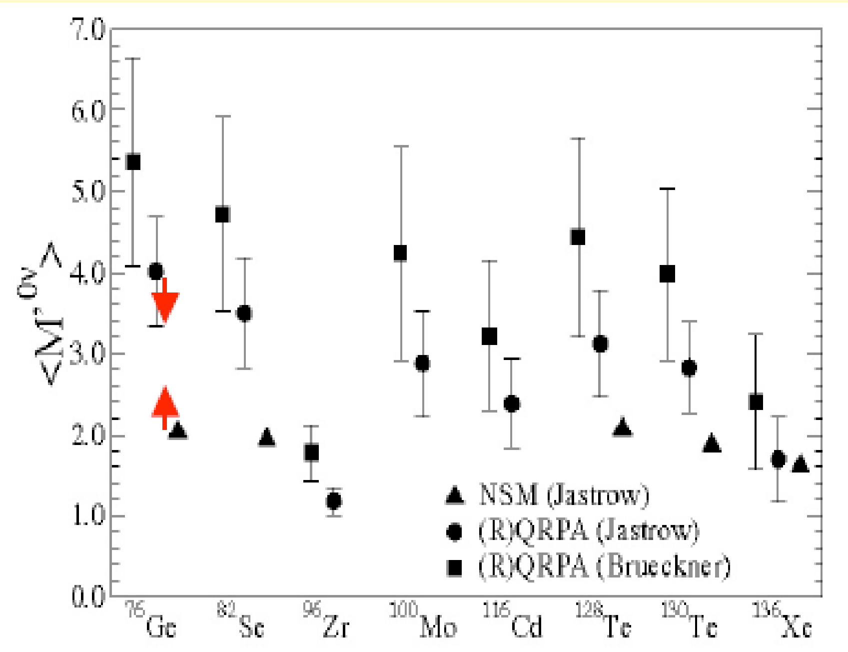
Lower bound on $T_{1/2}$ used for ^{136}Xe

F. Simkovic et al. Phys. Rev. C 77 (2008) 045503

What are these matrix elements? What techniques are being used to obtain them? What are the origins of these sizeable uncertainties? I will start from essentially complete ignorance (which accurately reflects my knowledge!) & then outline what is going on here from the point of view of a HE theorist .. It's not possible to do any of the calculations in real time.



E.g., Barea and Iachello, PRC 79, 044301 (2009)



E.g., Simkovic et al, arXiv:0902.0331

Update from Lisi @MEDEX09

The spread among NME from different nuclear models (QRPA, SM, IBM) is an indicator of **unknown** theoretical systematics. However, within each given model, the two (currently) largest source of correlated uncertainties are actually **known**:

(1) s.r.c. effects (Jastrow? UCOM? Others?)
(2) g_A variations (Quenched? Unquenched?)

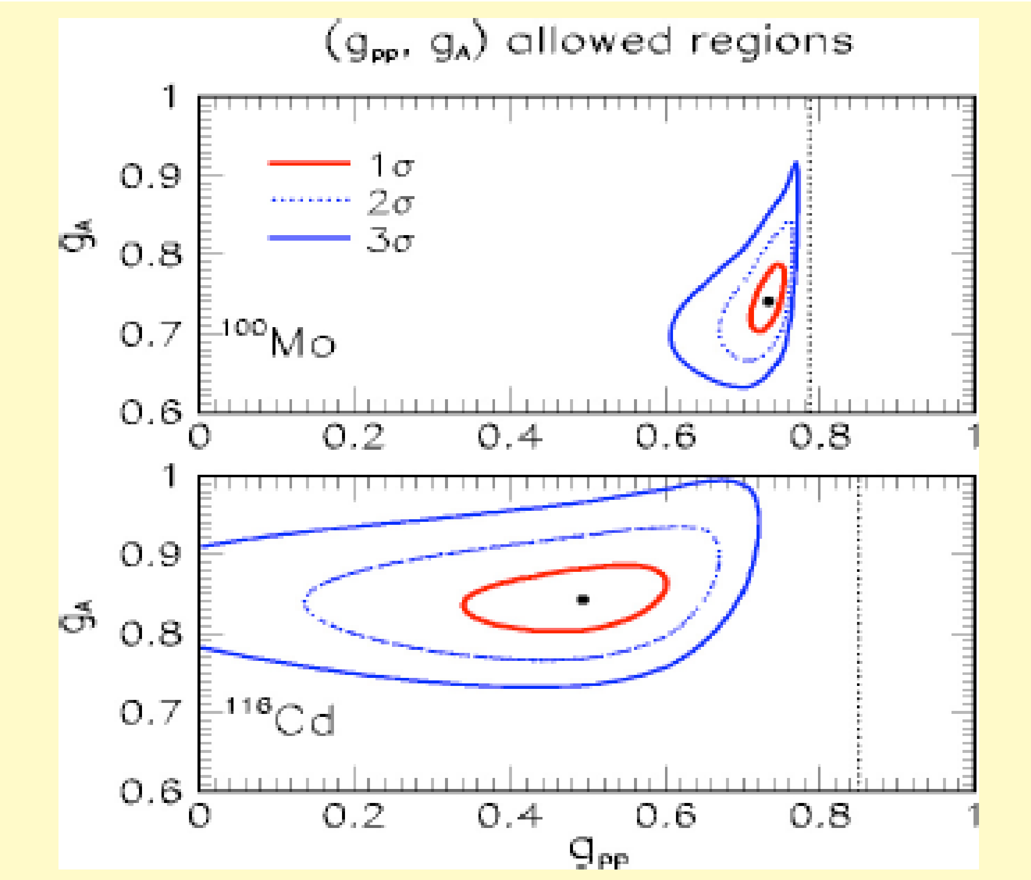
Quenching issues are particularly embarassing… Let me be provocative:

After decades of research on weak interactions in nuclei, it should be time to abandon crude recipes like "take either g_A~1 or ~1.25, and evaluate the difference." More properly, one should get a set of g_A expt/theo estimates (with errors), which might well be somewhat different in different nuclei (since g_A is an effective coupling in nuclear medium).

CONCLUSIONS:

In general, progress in this area will require keeping track of all known sources of uncertainties in each nuclear physics model, computing their correlated effects in both standard and nonstandard particle physics scenarios, & constrain them by using as many weak interact. and nuclear structure data as possible

Lisi advocates that, within each NME model, one should do a 'simultaneous fit' to over-constrain the free parameters using all of the existing data & extract their values…for some reason this has not been an accepted approach within the nuclear community. Doing this within each model framework has improved, & will continue to improve, the agreement among the NME predictions for $\beta\beta_{0\nu}$.



A toy analysis for QRPA picks out parameter values not commonly considered by most authors !!!

CLEAR PROGRESS !!!

No Comment…

The Matrix Element of What??

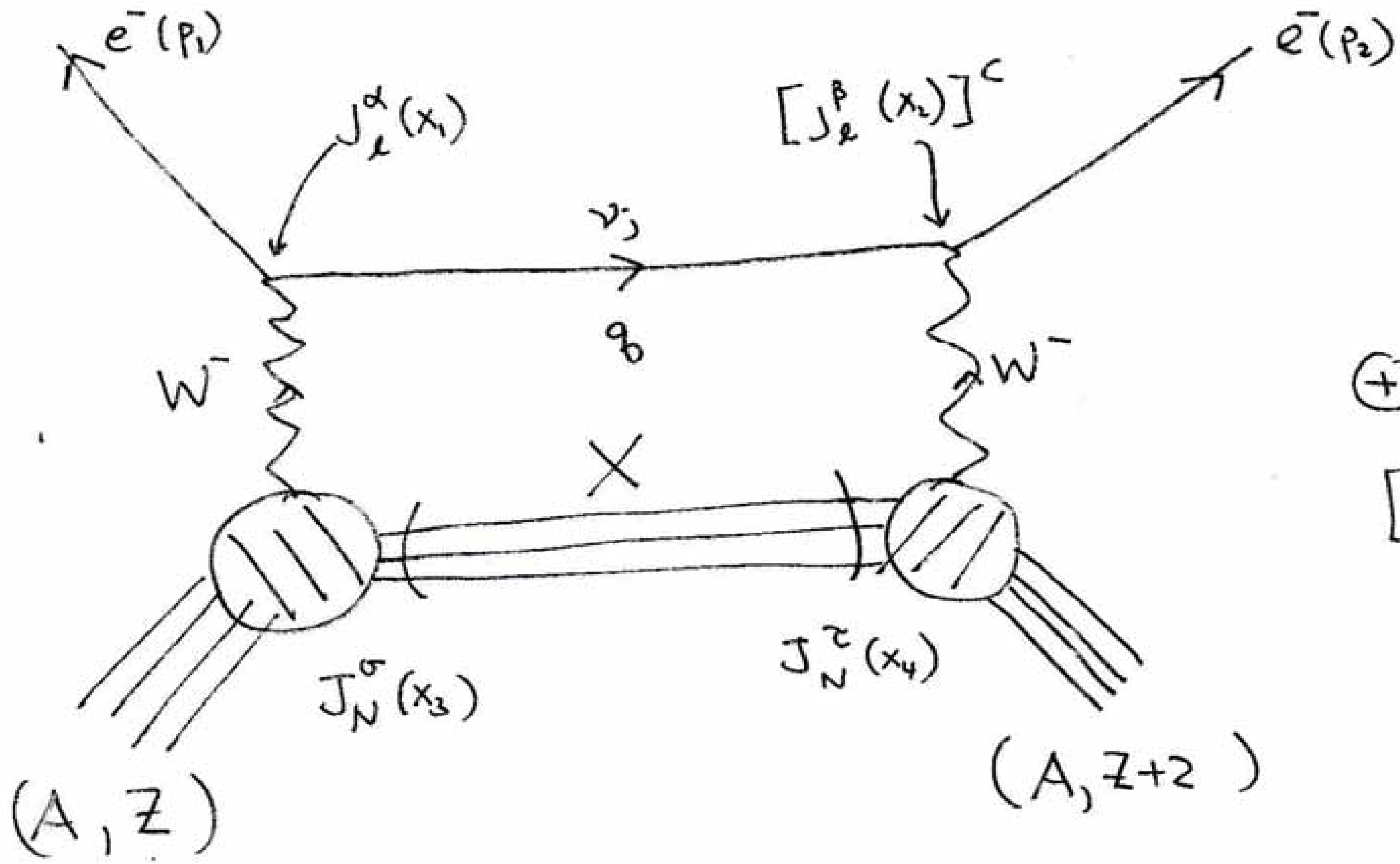
The calculations of, and those leading to, the nuclear matrix elements are the result of a long series of approximations. ..

Some of these are clearly valid while others are 'reasonable' & allow for the further explicit calculation. Many approximations are associated with the ratios of the various energy scales in the problem.

Others either seem questionable or, at least to me, unclear without further study.

In all theses calculations the nucleus is considered to be a system of nucleons existing in some mean 'background' field. Pairs of nucleons may also experience a residual interaction.

Any good theorist starts with a Feynman diagram



$\oplus$ $(1 \leftrightarrow 2)$

[a factor of 2]

(in the usual approximation to be made below)

- "Collapse" the W-propagator as $M_W^2 \gg (E_{nuclear})^2$

$$\to \quad \sim g^{\alpha\sigma} g^{\beta\tau} \delta(x_1-x_3)\,\delta(x_2-x_4)\, \underbrace{\frac{1}{4}\left(g/\sqrt{2}\right)^4 \frac{1}{M_W^4}}_{G_F^2}$$

(+ integrate over $d^4x_3\, d^4x_4$)

This is an excellent approximation !

- Treat the external e^- as a plane wave + apply the Coulomb corrections later as is done in ordinary β-decay process

Standard approach

- $J_\ell^\alpha(x_1) \equiv \bar{e}(x_1)\gamma_\alpha \{ U^L_{ej} P_L + \underbrace{\xi U^R_{ej}}_{=0 \text{ in the SM}} P_R \} \nu_j(x_1)$

Majorana neutrino

$\Rightarrow$ insert into the 'leptonic' part of the diagram - it looks like

$$\sim \int \frac{d^4q}{(2\pi)^4} e^{-iq(x_1-x_2)} \sum_j \underbrace{\bar{e}(x_1)}_{\sim e^{-ip_1x_1}} \gamma_\alpha \{ \;\} \, i \frac{\not{q} + m_j}{q^2 - m_j^2} \{ \;\} \gamma_\beta \underbrace{e^c(x_2)}_{\sim e^{-ip_2x_2}}$$

virtual ν momenta must be integrated over

neutrino propagator

$$\sum_j \Big[\underline{\underline{m_j}} \{ (U^L_{ej})^2 P_L + \xi^2 (U^R_{ej})^2 P_R \} + \xi \not{q}\, U^L_{je} U^R_{je} \underbrace{(P_L + P_R)}_{=1} \Big]$$

Note the extra terms here..

Let's look a bit closer:

$$\sum_j m_j \left\{ \underbrace{(U^L_{ej})^2 P_L}_{\text{SM piece}} + \underbrace{\xi^2 (U^R_{ej})^2 P_R}_{\substack{\text{RH current piece} \\ \text{(ignore)}}} \right\} + \underbrace{\xi \,\%\, U^L_{ej} U^R_{ej}}_{\substack{\text{GIM} \\ \text{Suppressed} \\ \text{(ignore)}}} \rightarrow \sum_j \frac{|U_{je}|^2}{q^2 - m_j^2} \text{ tiny!}$$

Majorana ν' masses

for majorana ν's $U^R_{ej} = U^{L*}_{ej}$

· What about the $\gamma_\alpha \gamma_\beta$ structure?

$$\left[\gamma_\alpha \gamma_\beta = \underbrace{\tfrac{1}{2}(\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha)}_{g_{\alpha\beta}} + \underbrace{\cancel{\tfrac{1}{2}(\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)}}_{\text{'odd'}} \right] \otimes \underbrace{J^\alpha_N J^\beta_N}_{\text{even}} \rightarrow J^\alpha_N J_{\alpha N}$$

$$\cdot \int d^4 q \rightarrow \underbrace{\int \vec{q}^{\,2} d|\vec{q}|}_{\int q^2 dq} \cdot 2\pi \cdot \int_{-1}^{1} e^{i \vec{q} \cdot \vec{r}} d(\cos\theta) \underbrace{\int dq_0 \, e^{-i q_0 (t_1 - t_2)} \frac{1}{q_0^2 - (\vec{q}^{\,2} + m_j^2)}}_{\sim \frac{\pi}{q_0} e^{-i q_0 (t_1 - t_2)}}$$

$\vec{q} \cdot \vec{r} = |\vec{q}||\vec{r}| \cos\theta$

do integral after we know tensor structure of nuclear "operator"

w/ $q_0^2 = \vec{q}^{\,2} + m_j^2$

remember this guy

What about the hadronic side ?

$$\langle f | J_N^\mu(x_1)\, J_N^\nu(x_2) | i \rangle = \sum_n \langle f | J^\mu(\vec{x}_1) | n \rangle \langle n | J^\nu(\vec{x}_2) | i \rangle \underbrace{e^{-i(E_f-E_n)x_{10}}}_{\sim 1_1} \underbrace{e^{-i(E_n-E_i)x_{10}}}_{\sim 1_2}$$

- Take the time components of the electron's wave functions + the result of the q_0-integral above - put it all together + integrate over $\underline{dt_1}\ \underline{dt_2}$ adding the $(1 \leftrightarrow 2)$ electron interchanged graph

$$\rightarrow (2\pi)\, \delta(E_f + E_{e_1} + E_{e_2} - E_i) \sum_n \left[\frac{\langle f | J_N^\mu(\vec{x}_1) | n \rangle \langle n | J_\mu^N(\vec{x}_2) | i \rangle}{q_0 (E_n + q_0 + E_{e_2} - E_i)} + (1 \leftrightarrow 2) \right]$$

- now $q_0 = (|\vec{q}|^2 + m_j^2)^{1/2} \simeq |\vec{q}| = q \sim 1/\text{Ave nucleon spacing} \simeq 100\ \text{MeV} \gg E_n$

So we approximate E_n as some average value $\bar{E}$ and replace "$\sum_n |n\rangle\langle n|$" by $\boxed{1.}$ This is called the closure approximation + is good to $\sim 15\%$. to go further we need to know $\boxed{J_N^\mu}$..

10-15% ??

$$J^\mu_N \sim \bar{\Psi}\, \tau^+ \left\{ g_V(q^2)\gamma^\mu - g_A(q^2)\gamma^\mu\gamma^5 - i g_M(q^2)\frac{\sigma^{\mu\nu} q_\nu}{2M_p} + g_P(q^2)\, q^\mu \gamma_5 \right\} \Psi$$

(go to Ch.10 of B_j + D)

isospin operator

vector FF

axial-vector FF

magnetic FF

pseudoscalar FF

nucleon wavefunction

is the most general nucleon current

Non-Relativistic Approximation.

$E_{kin} << M$

$$J^\mu_N \sim \sum_n \tau^+_n \left(g^{\mu 0} J_0(\vec{q}^{\,2}) + g^{\mu k} J_k(\vec{q}^{\,2}) \right) \qquad (\vec{q}^{\,2} = q^2)$$

$$J_0(\vec{q}^{\,2}) = g_V(q^2)$$

$$\vec{J}_n(q^2) = g_A(q^2)\,\vec{\sigma}_n + i g_M(q^2)\frac{\vec{\sigma}_n \times \vec{q}}{2M_p} - g_P(q^2)\frac{\vec{q}\,(\vec{\sigma}_n \cdot \vec{q})}{2M_p}$$

Goldberger-Trieman Relation

$$g_P(q^2) = \frac{2M_p}{q^2 + m_\pi^2}\, g_A(q^2)\left\{1 + O(1\%)\right\}$$

weak magnetism:

$$g_M(q^2) = (\mu_p - \mu_n)\, g_V(q^2)$$

nucleon magnetic moments

Form Factor Scales : $g_V(q^2) = g_V \left\{1 + q^2/\Lambda_V^2\right\}^{-2}$, $g_A(q^2) = g_A \left\{1 + q^2/\Lambda_A^2\right\}^{-2}$

$g_V \simeq 1$ $\qquad$ $\Lambda_V^2 \simeq 0.71\,\text{GeV}^2$ $\qquad$ $g_A \simeq 1.25$?? $\qquad$ $\Lambda_A^2 \simeq 1.09\,\text{GeV}^2$

(may not be true in a large nucleus) ~20-40% quenching?

$$J_N^\mu \otimes J_\mu^N \sim \boxed{\Omega} = \tau_+\tau_+\left\{\left(-h_F + h_{GT}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\right) - h_T S_{12}\right\} \quad \left(S_{12} = 3\,\vec{\sigma}_1\cdot\hat{q}\;\vec{\sigma}_2\cdot\hat{q} - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$

Fermi $\qquad$ Gamow-Teller $\qquad$ Tensor $\qquad$ $\sim 3\cos^2\theta - 1$

$$h_F \equiv g_V(q^2)$$

$$h_{GT} \equiv g_A^2(q^2)\left[1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] + \frac{2}{3}\frac{g_M^2(q^2)\,q^2}{4M_p^2}$$

$$h_T \equiv g_A^2(q^2)\left[\frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right] + \frac{1}{3}\frac{g_M^2(q^2)\,q^2}{4M_p^2}$$

now $\displaystyle\int_{-1}^{1} e^{iqrz}\,dz\,\left[1,\ 3z^2-1\right] \sim \left[j_0(qr),\ j_2(qr)\right]$ Spherical Bessel Functions

& put it all together to get…

Assuming only $0^+ \to 0^+$ transitions so that the outgoing e^-'s are in the S-wave state + so that $e^{i(\vec{p}_1\cdot\vec{r}_1 + \vec{p}_2\cdot\vec{r}_2)} \simeq 1$, we get

(Including the p-waves & Coulomb corrections is straightforward)

$$M_{0\nu} = \frac{2R}{\pi g_A^2} \langle f | \int_0^\infty q^2 dq \sum_{a,b} \frac{j_0(q r_{ab})\left[-h_F + h_{GT}\, \vec{\sigma}_a \cdot \vec{\sigma}_b\right] + j_2(q r_{ab}) h_T \hat{S}_{ab}}{q + \bar{E} - (E_i + E_f)/2} \tau_a^+ \tau_b^+ | i \rangle$$

where $\underline{R}$ is the 'nuclear radius' [a source of 'errors' as various values used!]

Some authors drop the tensor term [~30% effect!] to simplify further:

$$M_{0\nu} \simeq M_{GT} - \frac{g_V^2}{g_A^2} M_F$$

$$M_F \equiv \langle f | \sum_{ab} H \tau_a^+ \tau_b^+ | i \rangle , \quad M_{GT} \equiv \langle f | \sum_{ab} H \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle$$

with $H \simeq \frac{2R}{\pi r_{ab}} \int_0^\infty dq \frac{\sin(q r_{ab})}{q + \bar{E} - (E_i + E_f)/2}$ is the 'neutrino' potential

Comments

- As you can see, now that we have arrived at the form of the relevant operators, a reasonably large number of approx-imations have already been made. These are over & above any FURTHER approximations that are made in obtaining the values of the matrix elements themselves but most of these are not necessary to make.

- So far this has all been rather straightforward (??) although a bit messy. HOWEVER, this is where High Energy Physics ends & Nuclear Physics takes over --- which causes an immediate drastic increase in the murkiness of the nuclear matrix element calculations…

Comments (cont.)

- One apparently obvious & universal uncertainty in these matrix element calculations is the detailed knowledge required of both the initial & final state nuclear wave functions (as well as those for any of the possible intermediate states!). Remember that these are obtained from some assumed forms for the 'collective' nuclear potential and the N-N interaction which are not known a priori.

Matrix Element Evaluation

The matrix elements themselves are evaluated using several *FAMILIES* of techniques which have different strengths and weaknesses & are based on different physics assumptions:

QRPA = Quasiparticle Random Phase Approximation, which provides a 'straightforward' (!) calculational technique but allows only limited testing opportunities..most commonly used approach

NSM = Nuclear Shell Model, which is easily tested by nuclear spectroscopy, etc., but is difficult to apply to heavy nuclei

IBM = Interacting Boson Model, a modified shell-like model

Within EACH family are further competing approximations & techniques apparently based on the bias of the different authors

Aside II:

Constraints on $\beta\beta_{0\nu}$ nuclear matrix elements come from many sources - but one potentially important one is the corresponding $\beta\beta_{2\nu}$ process which is lepton number conserving & for which data exists. For the analogous $0^+ \rightarrow 0^+$ transition the rate for an even-even nucleus is given by

$$\tau_{1/2}^{-1} = F(Q,Z,A) \; |M_{GT}|^2$$

where F is a known function and the nuclear matrix element is essentially pure GT due to selection rules:

$$M_{GT}^{2\nu} = \sum_m \frac{\langle f||\sigma\tau_+||m\rangle\langle m||\sigma\tau_+||i\rangle}{E_m - (M_i + M_f)/2}$$

Here |m> label the 1^+ of the intermediate odd-odd nucleus. Note Fermi matrix elements are NOT constrained.

NSM

- At it's most basic level the NSM is the easiest to understand as it is very similar to atomic physics. SUPPOSE we knew the complete set of wavefunctions and energy levels of the original & final nuclei, as well as those for any of the possible inter-mediate states, in the $\beta\beta_{0\nu}$ process.

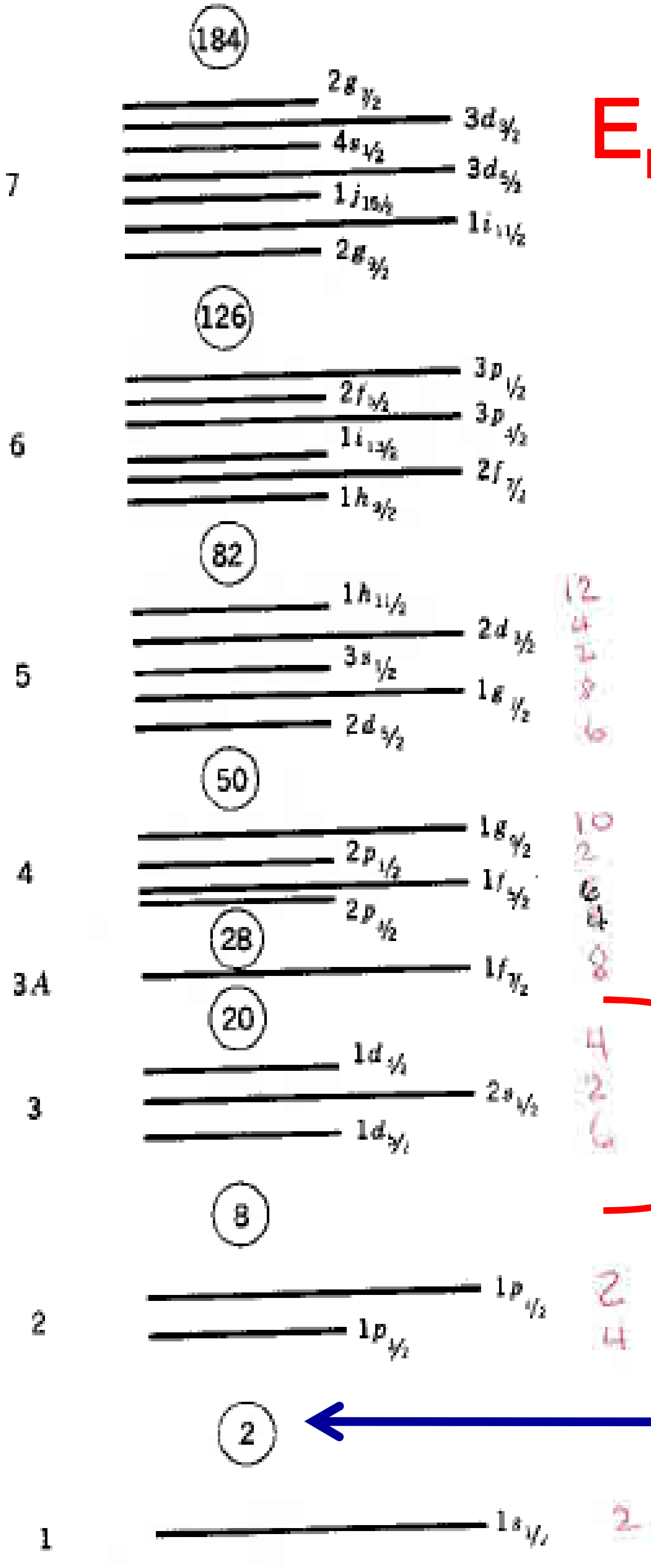
- We then could diagonalize the effective Hamiltonian & obtain the required matrix elements by numerical integration & then summing over all of (the very large number of) configurations.

- The goal of NSM (as I see it) is to 'explain' nuclear energy levels & to allow for the calculation of various transition rates in a manner following the analogy w/ atomic physics.

- To accomplish this they need to (numerically!) obtain the nuclear wavefunctions based on an assumed form of the common central potential (there are several possible choices, e.g., the Woods-Saxon potential or modified oscillator potential) & correcting for, e.g., the 2-body, spin-orbit, and velocity dependent interactions using experimental data from these & related nuclei as input employed in a self-consistent manner. This is clearly an art form as well as science!

- The basic concepts are used in all NME calculations

Let’s review….



$$E_{njl} = E_0 + a(2n+l) - b[j(j+1) - l(l+1) - 3/4] + ..$$

Examples: O^{18} has 2 closed shells of p's & n's with 2 n's in the 3rd shell $= \nu(1d_{5/2})^2$

Xe^{136} has 6 closed n shells & has 4p's in excess of 5 closed shells $=\pi(2d_{5/2})^4$

If a shell is >1/2 full the excess is turned into 'holes' relative to the next full shell so that for O^{15} have $=\nu(1p_{1/2})^{-1}$

A 'shell' of roughly degenerate nucleons

'magic' number for closed shells

Quasi-particle = 1particle + 1 hole

$(1d_{5/2})^2$ is called a configuration of O^{18} but it is not the only possible one…

• 'Collisions' of 2 nucleons in the same shell can lead to some admixtures of shell model states. For example, in O^{18} the 2n's experience a pairing interaction so that the nucleus can be in a superposition of NSM basis states with the same total 2-particle total spin (here=0) and (here even) parity:

$$\Psi = a\,(d_{5/2})_0^2 + b\,(s_{1/2})_0^2 + c\,(d_{3/2})_0^2$$

with the relative weighting depending upon the strength of the interaction. The subscript on these terms tells us that these are all configurations with total J=0. The squares of the coefficients a-c , divided by the maximum number of possible nucleons in a given orbital are called occupation numbers , e.g.,

$V_{5/2}^2 = |a|^2/3$ for the $d_{5/2}$ configuration above

NSM II

Several issues now arise:

- (i) Given the uncertainties on the ‘input’ assumptions, the NSM wavefunctions, though leading to testable predictions, are not uniquely determined . E.g., variational techniques with different trial wavefunctions can yield similar results. (The full nuclear wavefunctions are then the symmetrized products of the single nucleon wavefunctions obtained via Slater determinants)

- (ii) They will certainly not be determined for all possible configurations of the initial, final & intermediate nuclei relevant for $\beta\beta_{0\nu}$. Data does not exist for all these states .

- (iii) The wavefunctions will not necessarily be probed by the nuclear spectroscopy data in the same range of ‘r_{12}’ ~ few (MeV) $^{-1}$ as do the ‘neutrino potentials’ ~(100 MeV) $^{-1}$.

NSM III

- (iv) There are obviously very many configurations to sum over & this is usually truncated to something ~ ($10^{10\text{-}11}$) due to CPU issues especially as the number of nucleons grows, i.e., for the heavier nuclei such as those employed in $\beta\beta_{0\nu}$ experiments.

Clearly, these calculations, though constrained by data, can at best be approximate – but, as these calculational techniques are improving (e.g., the use of Monte Carlo techniques) and more & more data is confronted by the NSM calculations, they can be viewed as reasonably trustworthy & the ones of choice in cases where they are applicable similar to what may see for large atoms.

Aside III

- A general correction that is employed in both NSM & QRPA calculations is accounting for the ‘overlap’ of the two initial nucleons’ wavefunctions due to the short ranged correlations from the neutrino potential, e.g., 2 nucleons occupying the ‘same place’.

- The Jastrow method rescales both the initial & final state wavefunctions at small separations by a brute-force ad-hoc common correction function which suppresses these overlaps. Unfortunately, this approach, though commonly used, does not conserve the norms of these states . This is ugly…

- The UCOM approach does something similar but through the use of a unitary transformation matrix which is ‘softer’ as well as norm-preserving & is thus a superior approach theoretically & is more favored by recent analyses & data fits.

QRPA

- The goal of QRPA is to reduce the complexities of the NSM calculations especially for heavier nuclei. To me, while it may reduce apparent calculational complexity, it's theoretical structure seems forbiddingly difficult to understand for an amateur.

- Similar to NSM the individual oscillator-like wavefunctions arise from the mean field core plus residual interactions obtained by summing over all of the N-N pair interactions & fit to the available data from the relevant nuclei. Many different N-N potentials are in use.

- QRPA comes in MANY, MANY different versions: RQRPA, Full-QRPA , SCQRPA, HQRPA , and on & on

QRPA II

- But QRPA goes further through the pairing of nuclei using BCS-like techniques & by then performing a double multipole expansion of the relevant nuclear matrix elements in terms of the J_I of the intermediate states which are restricted to be within ~10-20 MeV of either ground state & in terms of the relative J_R of the annihilating neutrons and final state protons. This makes the notation appear very ‘cumbersome’ to say the least!

- An additional interaction is included in QRPA between particles & holes which in principle differs in strength (g_{ph}) from the usual particle –particle interaction (see below). The value used comes data fitting.

QRPA III

- The nuclei are built up of a core plus bosonic quasiparticles which are point-like bound states (i.e., obeying Bose-Einstein statistics) of nucleon particles & holes..thus, the Exclusion Principle is violated(!) The Renormalized PQRA tries to take this into account & would seem to be superior but violates sum rules. While an arbitrarily large number of possible p,n single particle states may be included the subset of possible quasi-particle states included is computationally restricted.

- The number of states included in various calculations seem to vary significantly w/ up to ~30-40% influence on matrix elements..certainly MORE is better but harder. Some authors include only two shells in their model space while others extend this to 5.

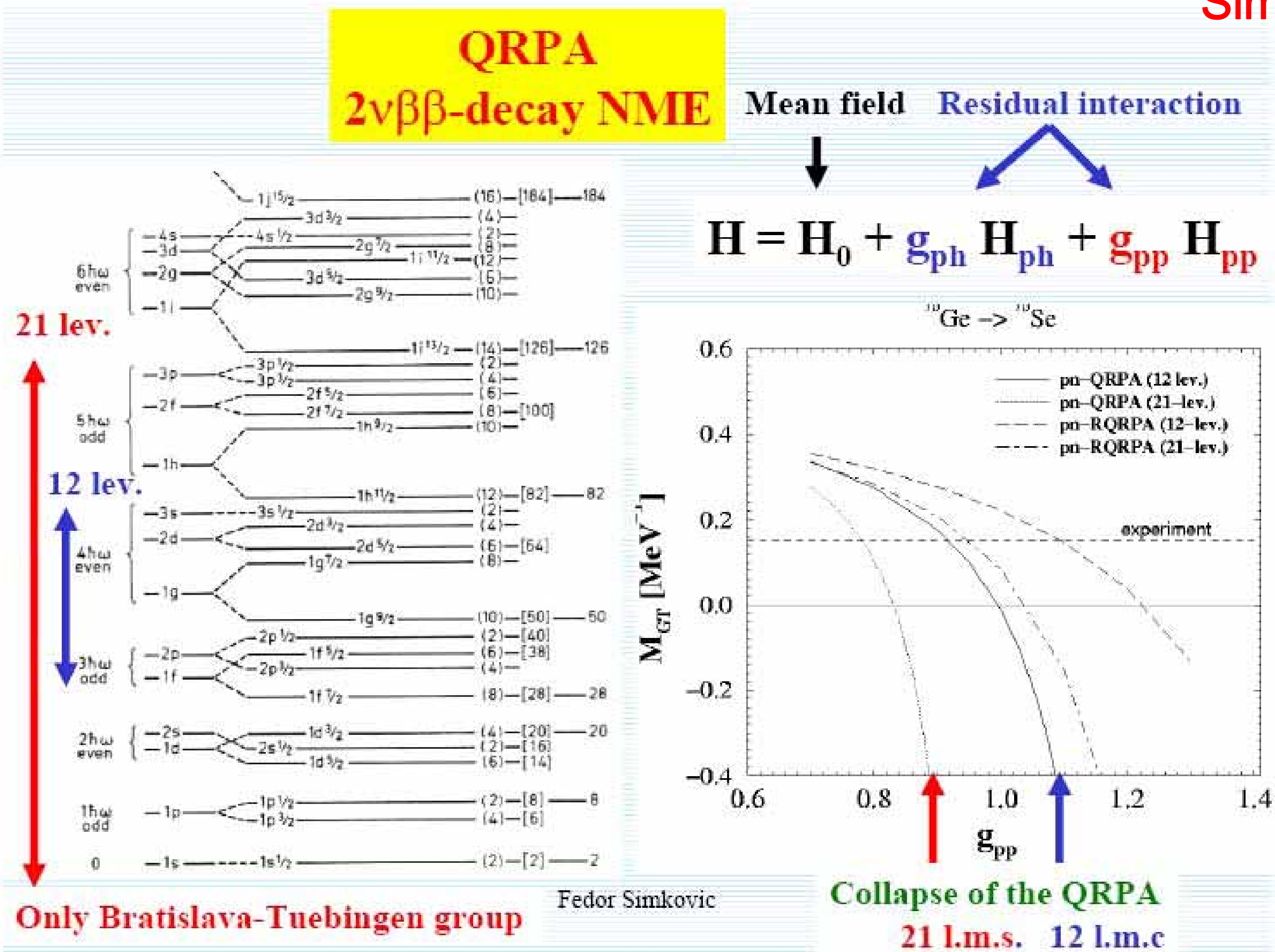
QRPA IV

- QRPA calculations are especially sensitive to deviations in the nuclear shape since multipole expansions are performed . Some authors have tried to accommodate deformations as a part of the QRPA formalism but these involve a far greater number of potentially contributing configurations.

- The two other free parameters in PQRA include the overall N-N coupling strength: g_{pp} ~1. This can generally be fitted by making use of the corresponding 2ν decay mode. The last parameter is the axial-vector coupling g_A (~1??) with its associated quenching issue.

• If the N-N coupling becomes too strong then the calculations can 'collapse', i.e., fail in the interesting parameter region due to Exclusion Principle violations.

Simkovic



In the Closure Approximation…

$$M_K = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel \bar{f}(r_{12}) O_K \bar{f}(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0_f^+ || \widetilde{[c_{p'}^+ \tilde{c}_{n'}]_J} || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_i || [c_p^+ \tilde{c}_n]_J || 0_i^+ \rangle .$$

6j symbol

Relative nucleon pair angular momentum

Spin of intermediate state

Creation & annihilation operators for ~quasiparticle in state J

Jastrow src correction

Transition operator F,GT or T

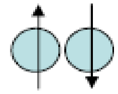
Intermediate states created from the initial& final states

IB(F)M*

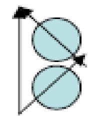
is an old idea ('74) which has only been recently ('09) applied to the problem of double-beta decay. A problem w/ the NSM is that there are SO many nucleons in very many possible states when A is large. The goal of IBM, like the QRPA scheme, is to simplify this structure so that the multitude of states entering into, e.g., a decay calculation can be significantly reduced while still getting the essential physics correct.

NSM tells us that low-lying states of E-E nuclei are made of nucleon pairs which are either total spin-0 or 2. IBM postulates only valence s- or d-bosons outside of an inert core instead of p's or n's . Extra unpaired nucleons remain as fermions in more general nuclei; IBM-2 treats p's & n's pairings separately.

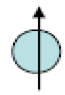
*Talks by F. Iachello & J. Barea @ MEDEX'09



J=0 s-boson



J=2 d-boson



Unpaired fermions

Iachello

All can be calculated at once using the compact expression:

$$V_{s_1,s_2}^{(\lambda)} = \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left[\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

current-current operators of different L

$$\lambda = 0, s_1 = s_2 = 0(F)$$
$$\lambda = 0, s_1 = s_2 = 1(GT)$$
$$\lambda = 2, s_1 = s_2 = 1(T)$$

Neutrino potentials

In second quantized form:

$$V_{s_1,s_2}^{(\lambda)} = -\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sum_{J} (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}} \sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$
$$\times G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1 j'_2; J)\left[\left(\pi_{j_1}^\dagger \times \pi_{j_2}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)}\right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

Iachello

The fermion operator V is mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j,j') d_{\pi,M}^\dagger$$

LO mapping from nucleon to pairs of bosons.

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2}\sum_{j_1}\sum_{j'_1} G_{s_1 s_2}^{(\lambda)}\left(j_1 j_1 j'_1 j'_1;0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

$$-\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2} \sqrt{1+\delta_{j_1 j_2}}\sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1 j'_2;2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

Neutron pairs annihilated
Proton pairs created

Modified matrix elements

The basis

$$\left(S^\dagger\right)^{\frac{n-v}{2}}\left(D^\dagger\right)^{\frac{v}{2}}|0\rangle$$

is constructed with operators:

$$S_\pi^\dagger = \sum_j \alpha_j \sqrt{\frac{\Omega_j}{2}}\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)}$$

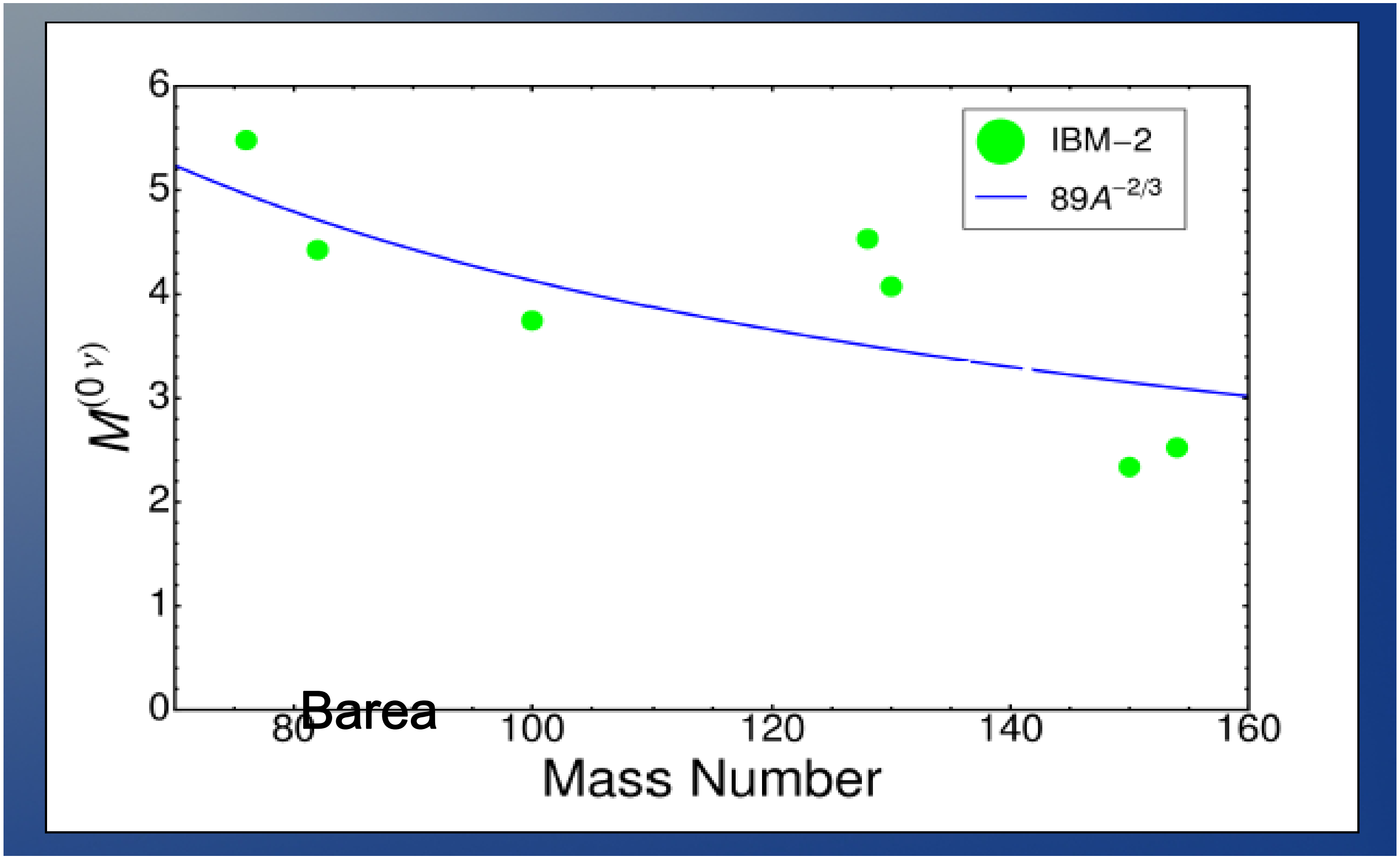
$$D_\pi^\dagger = \sum_{j \le j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}}\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)^{(2)}$$

with $\Omega_j = j + 1/2$.

now input the appropriate wavefunctions from fits to spectrum data

Guestimated error ~25%

- The predictions of IBM are generally slightly larger than, but in general agreement with, the QRPA predictions at the level of the expected ~25% uncertainty.



Conclusions:

My impression is that, certainly for the case of the XENON nucleus, by a comparison of the predictions, the matrix element uncertainties are apparently slowly shrinking through both more sophisticated calculations as well as systematic analyses using better common input data.

You EXOs are Smart and/or Lucky!

My bet is that by the time data is available the uncertainty on the NME for XENON will be at the ~20% level (or less) which seems to be more than adequate for neutrino mass extraction. How good these calculations will be for other nuclei, which are clearly necessary to test the underlying theory, is unclear. Observing this rare process will, of course, produce faster theoretical results.