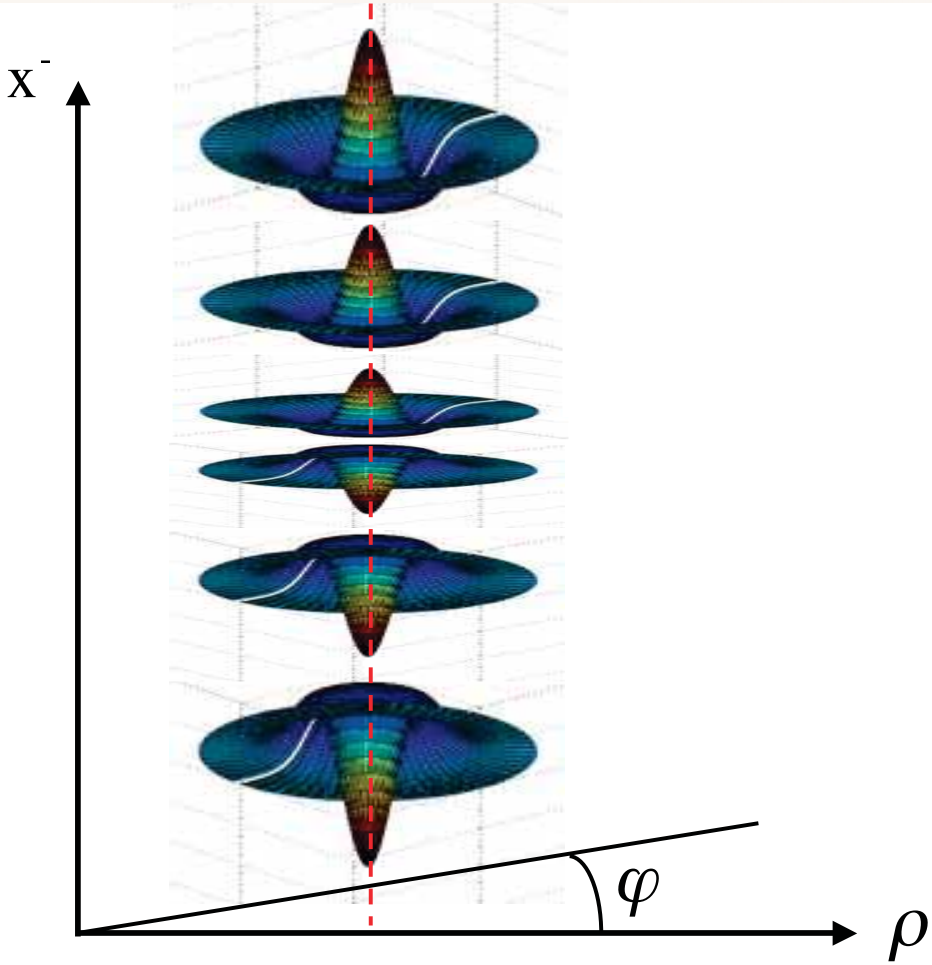
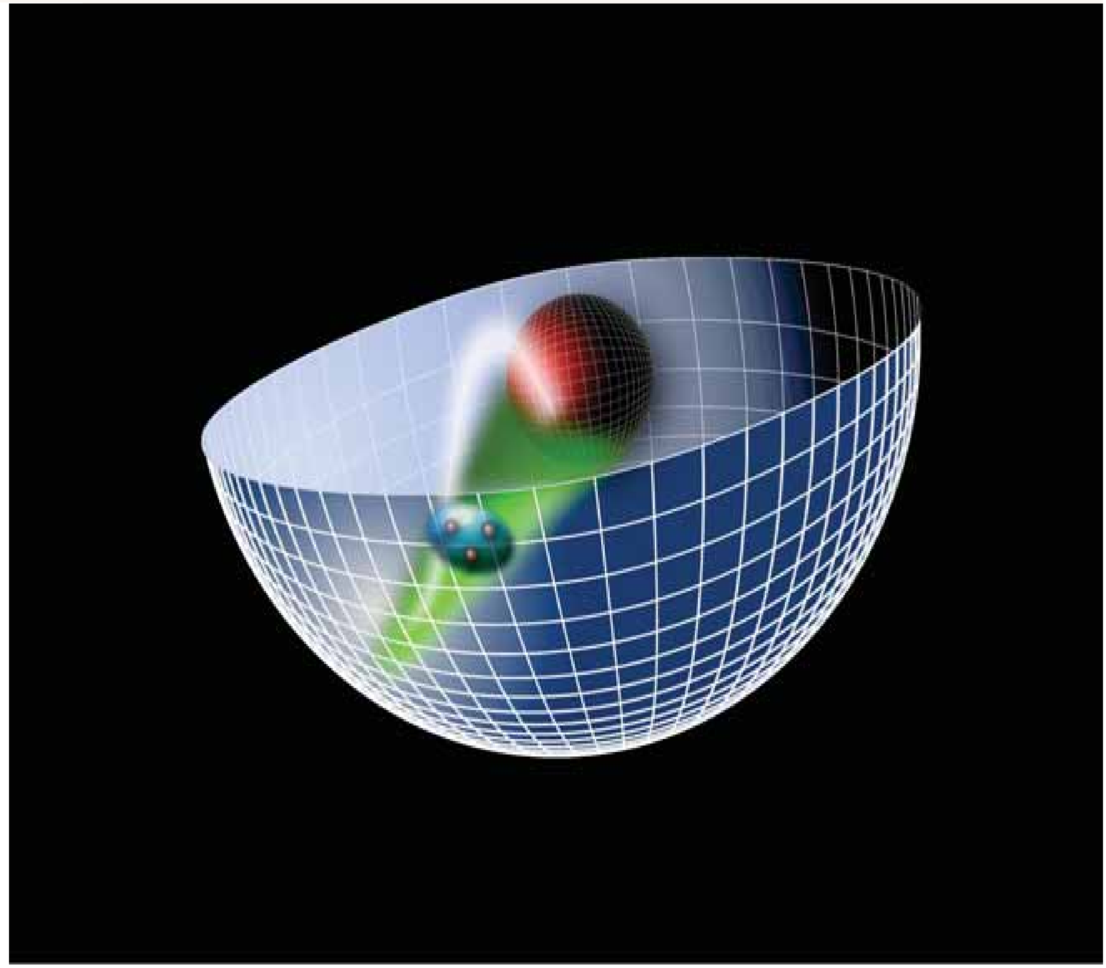
Exascale Computing and Light Front QCD





Stan Brodsky, SLAC

Exascale Workshop December 2009

Scientific Challenges for Understanding the Quantum Universe and the Role of Computing at Extreme Scale

Impact of Exascale Computing on LF QCD

E-flop

10^{18} floating-point calculations per second

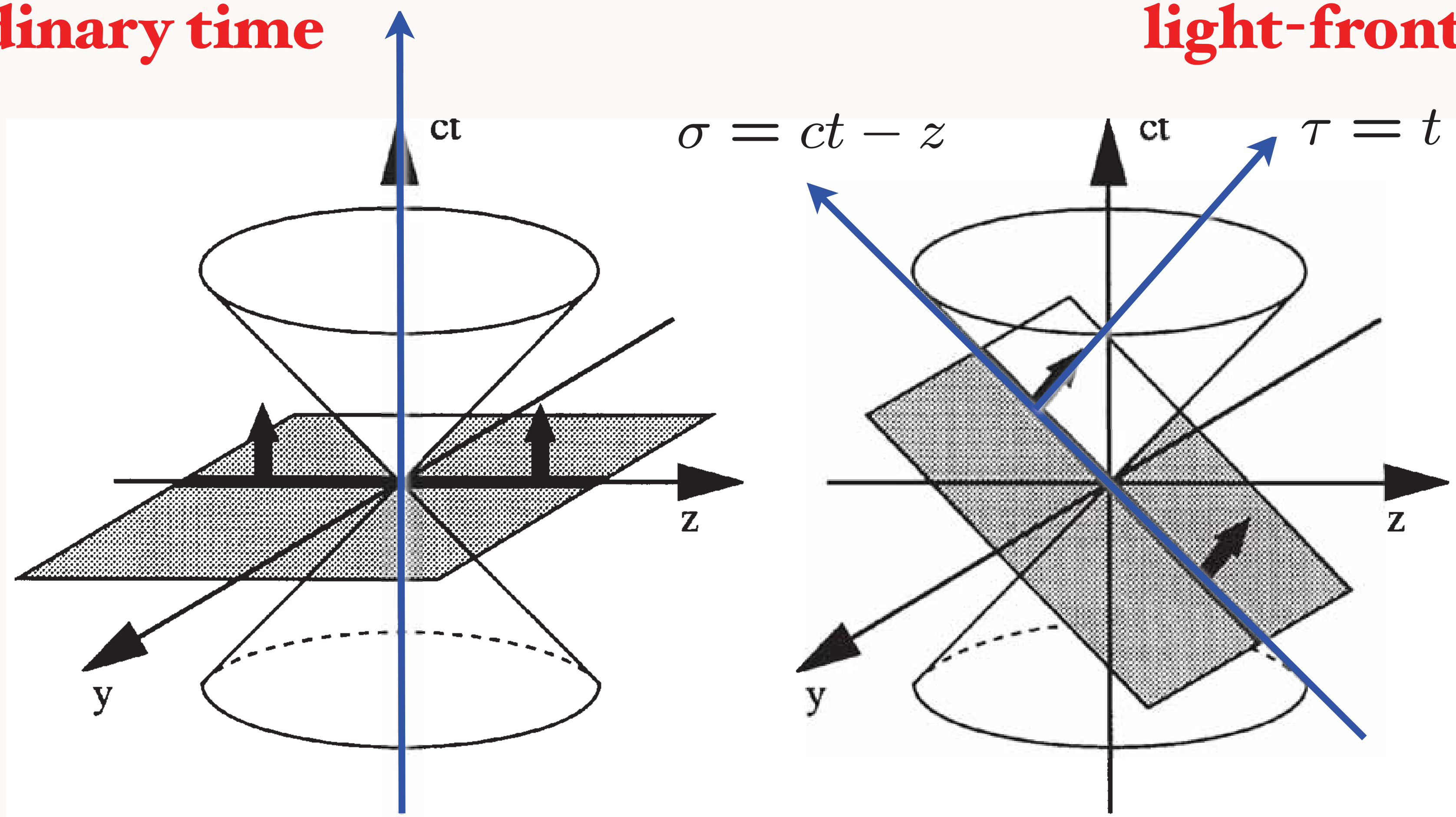
- Goal: Nonperturbative Solutions to QCD
- DLCQ and AdS/QCD-generated solutions of QCD(3+1)
- Hadronization at the Amplitude Level
- Computer Simulation of Complex QCD and other quantum field theory processes

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



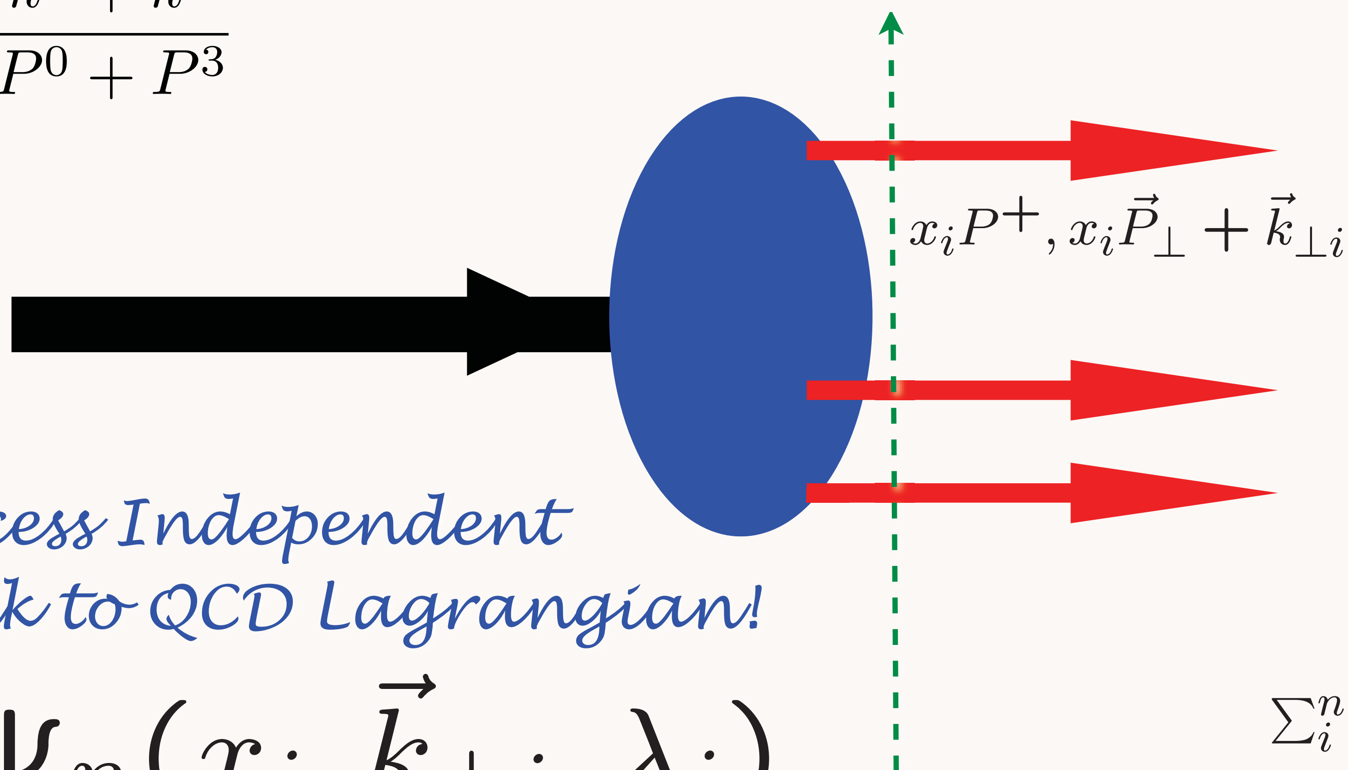
Instant Form **Front Form**

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



Process Independent

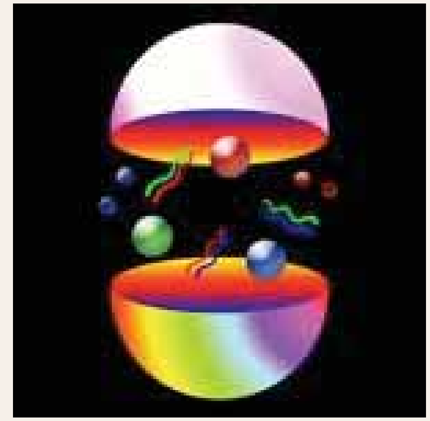
Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

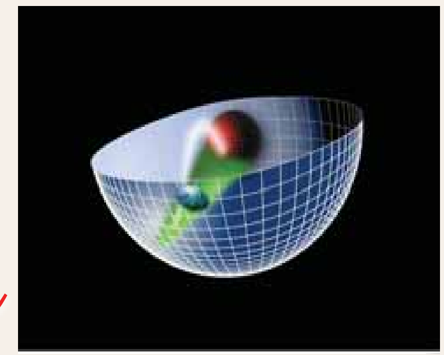
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ



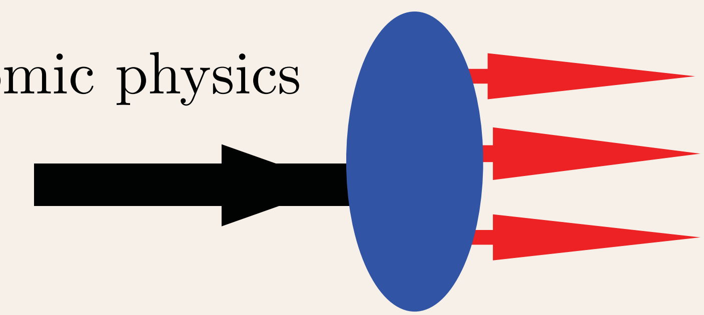
- Quarks and Gluons:
  Fundamental constituents of hadrons and nuclei
- *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: *AdS/QCD*



- *Light-Front Holography:*
  First Approximation to QCD
- *Hadronization at the Amplitude Level*
- *Light Front Wavefunctions:*

Analogous to Schrödinger wavefunctions of atomic physics

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$P^{\pm} = P^0 \pm P^z$$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

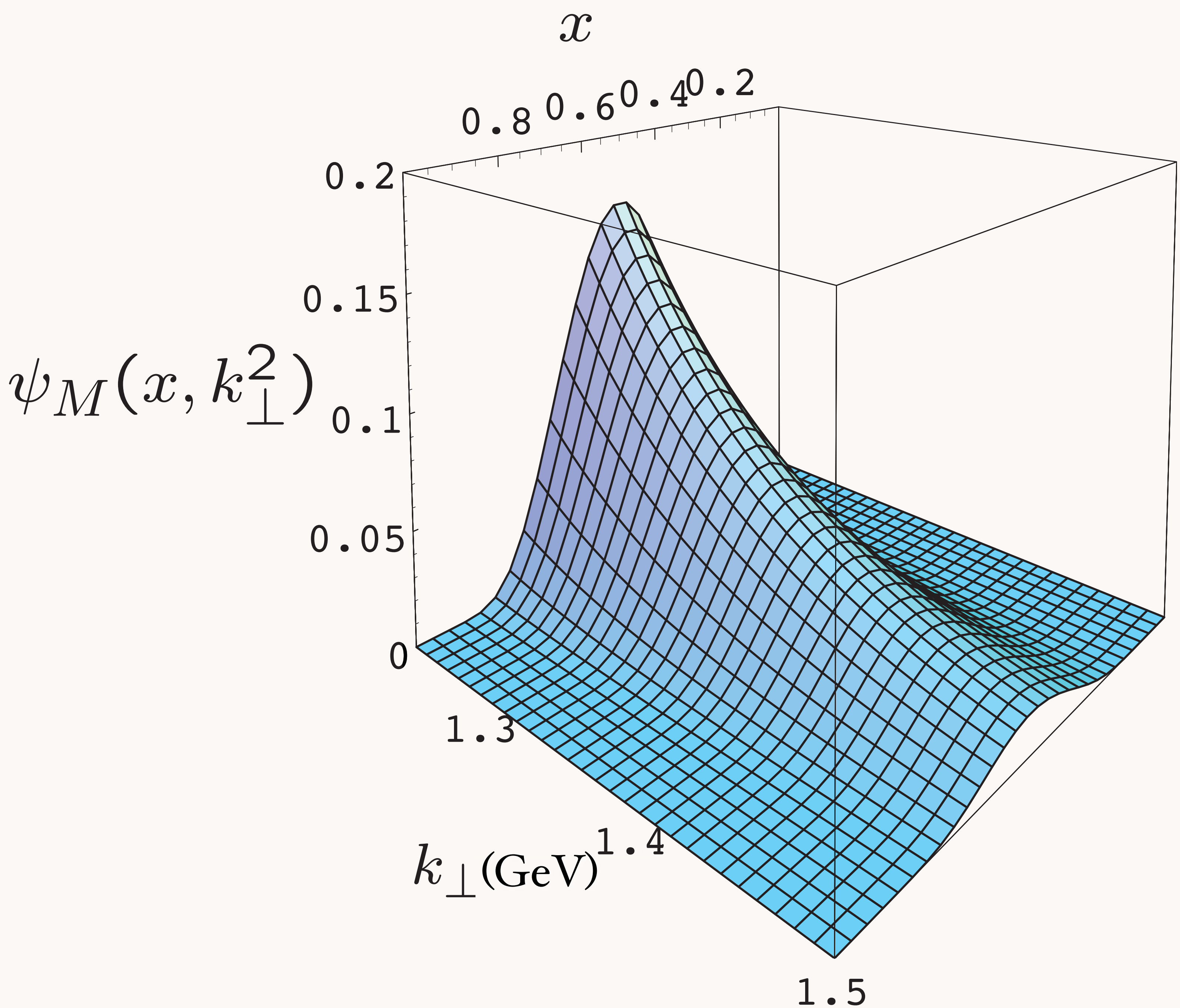
Invariant under boosts

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

$$H_{LF} = P^+P^- - P_\perp^2$$

Remarkable new insights from AdS/CFT,
the duality between conformal field theory
and Anti-de Sitter Space

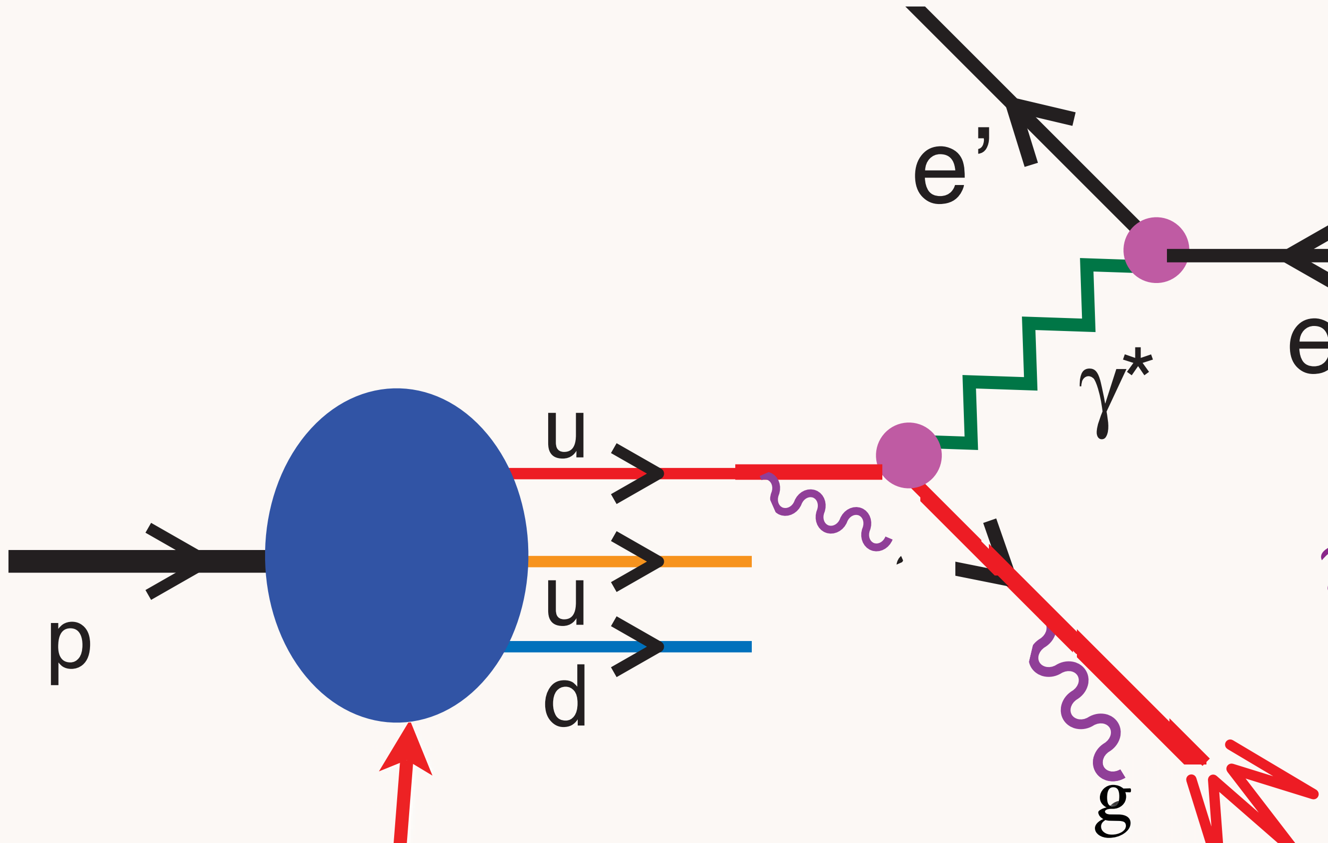
Prediction from AdS/QCD: Meson LFWF



"Soft Wall" model

de Teramond, sjb

Deep Inelastic Electron-Proton Scattering



jet

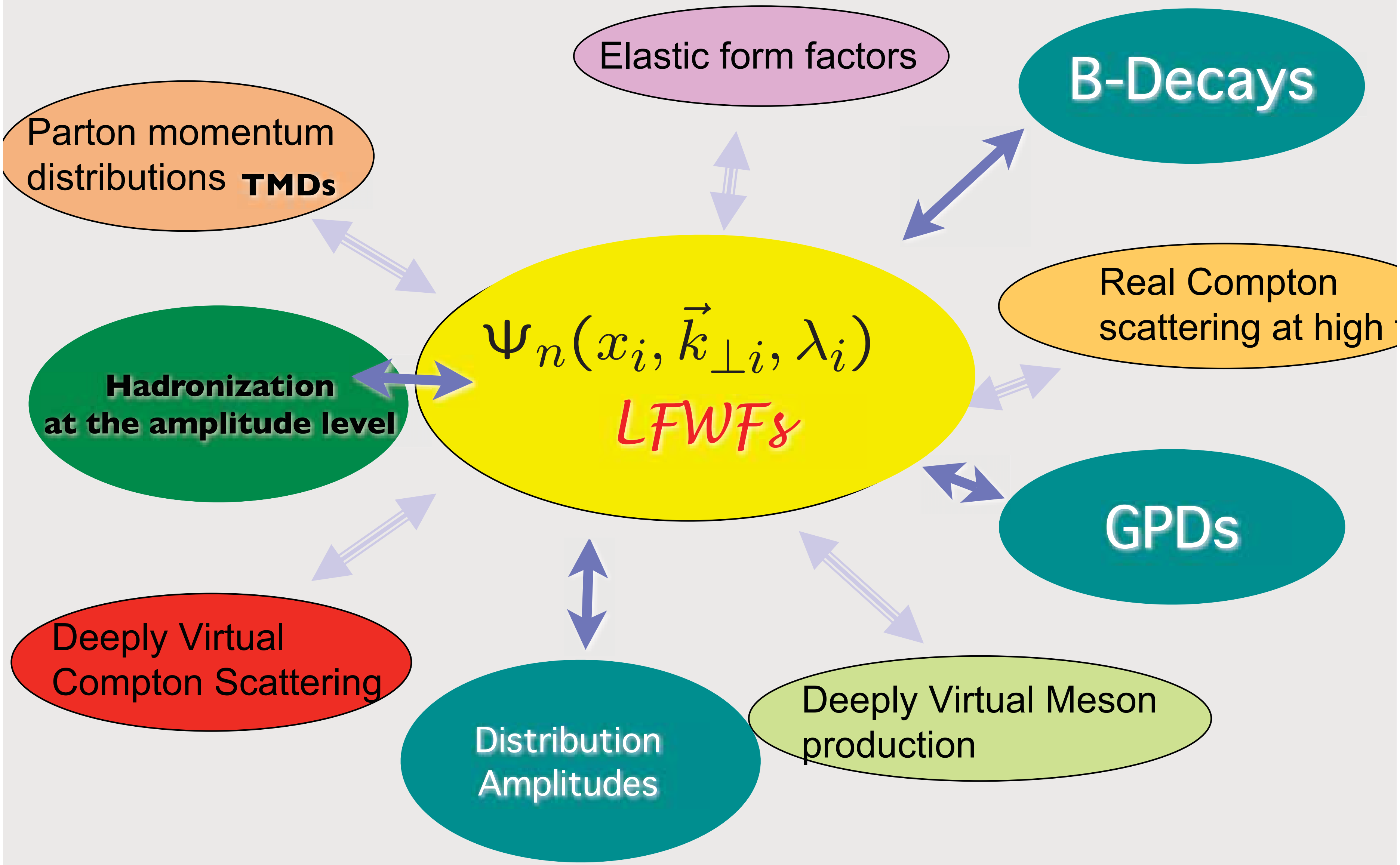
Gluonic Bremmstrahlung

DGLAP Evolution

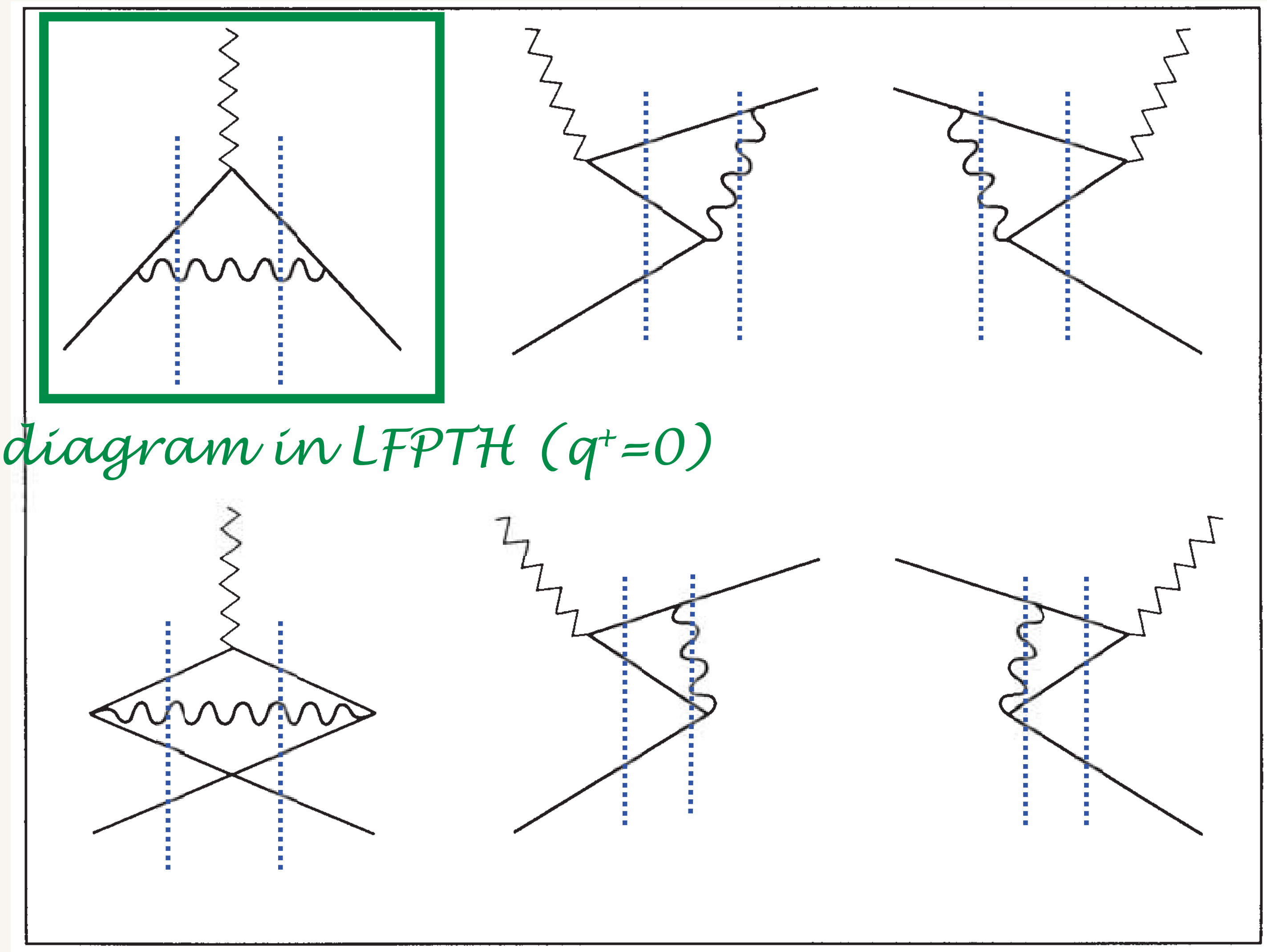
Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular momentum ....

Light-Front Quantization: Rigorous realization of IMF

A Unified Description of Hadron Structure



Calculation of lepton g-2 in TOPTH (Instant form)



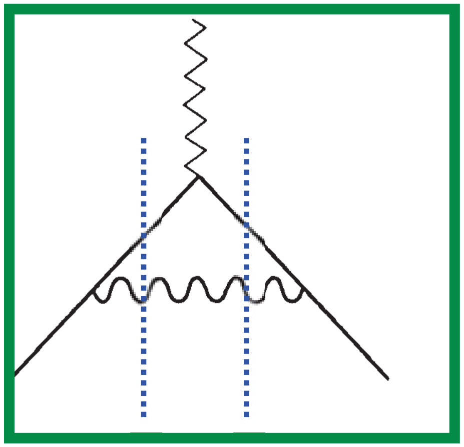
Only diagram in LFPTH (q^+=0)

$n!$ diagrams at order e^n

energy denominators:
frame-dependent and non-analytic

$$\sqrt{(\vec{p}+\vec{q}-\vec{k})^2+m^2}$$

Calculation of lepton g-2 in LFPTH (Front form)



Only diagram in LFPTH

$$\mathcal{M}^2_{intermediate} = \sum_i^n \frac{k^2_{\perp i} + m^2_i}{x_i}$$

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

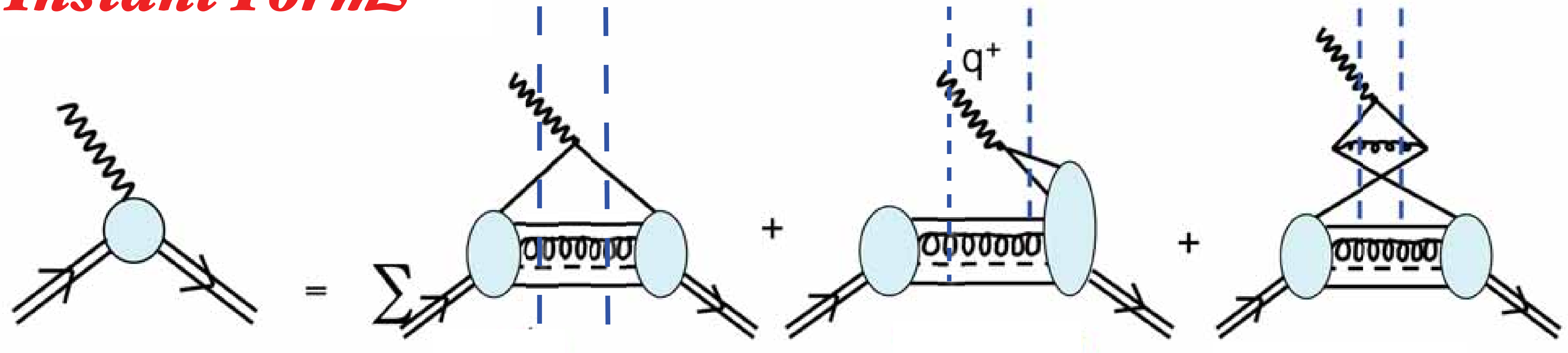
Order n diagrams at order e^n

energy denominators:

frame-dependent and analytic

Calculation of Form Factors in Equal-Time Theory

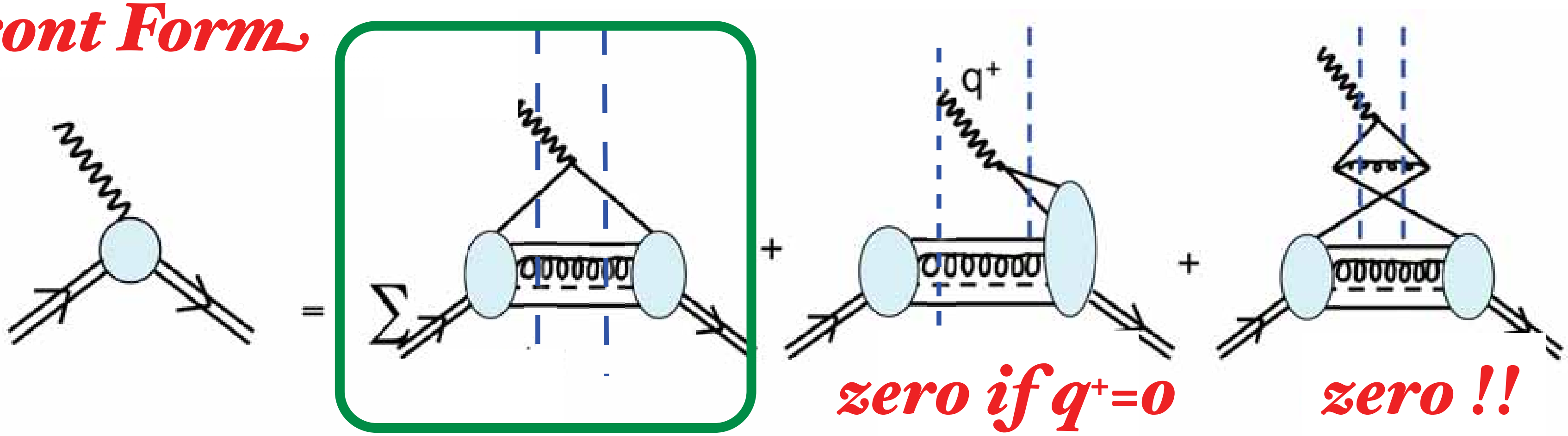
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



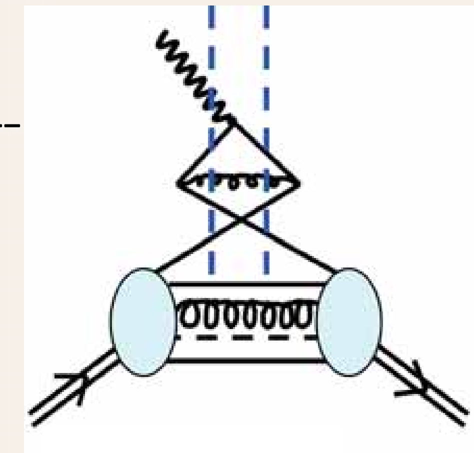
zero if $q^+=0$

zero !!

Only diagram in LFPTH

Calculation of Hadron Form Factors
Instant Form

- Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum



- Pair creation from vacuum occurs at any time before probe acts -- acausal

- Knowledge of hadron wavefunction insufficient to compute current matrix elements

- Requires dynamical boost of hadron wavefunction -- unknown except at weak binding

- Complex vacuum even for QED

- None of these complications occur for quantization at fixed LF time (front form)

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

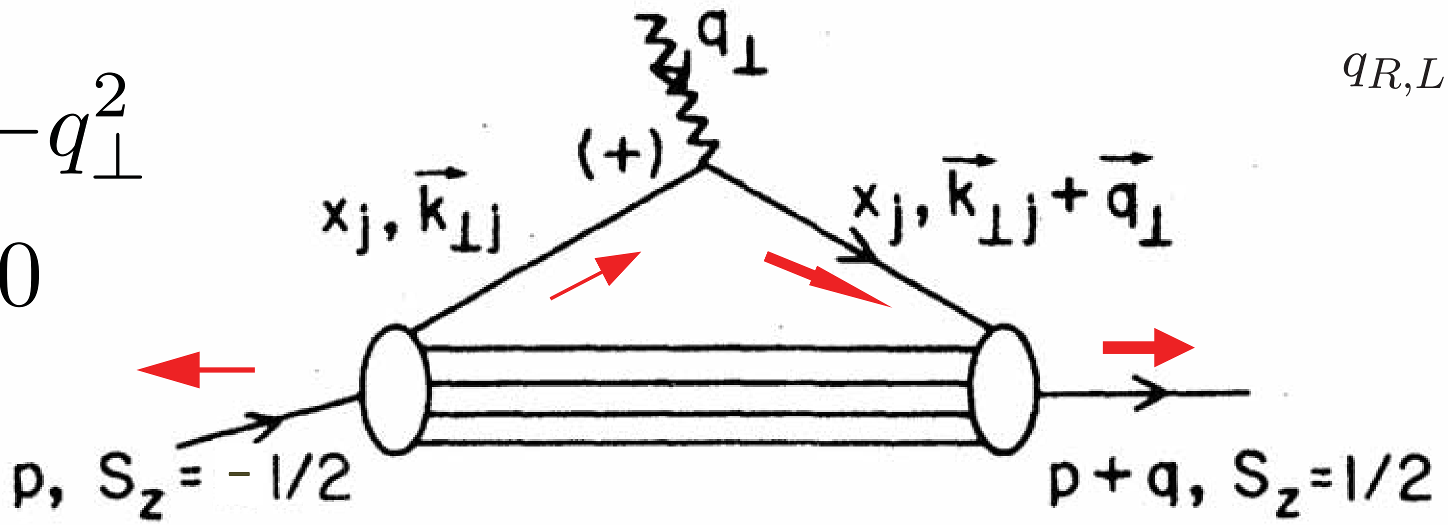
Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q^2 = -q_\perp^2$$

$$q^+ = 0$$



$$q_{R,L} = q^x \pm i q^y$$

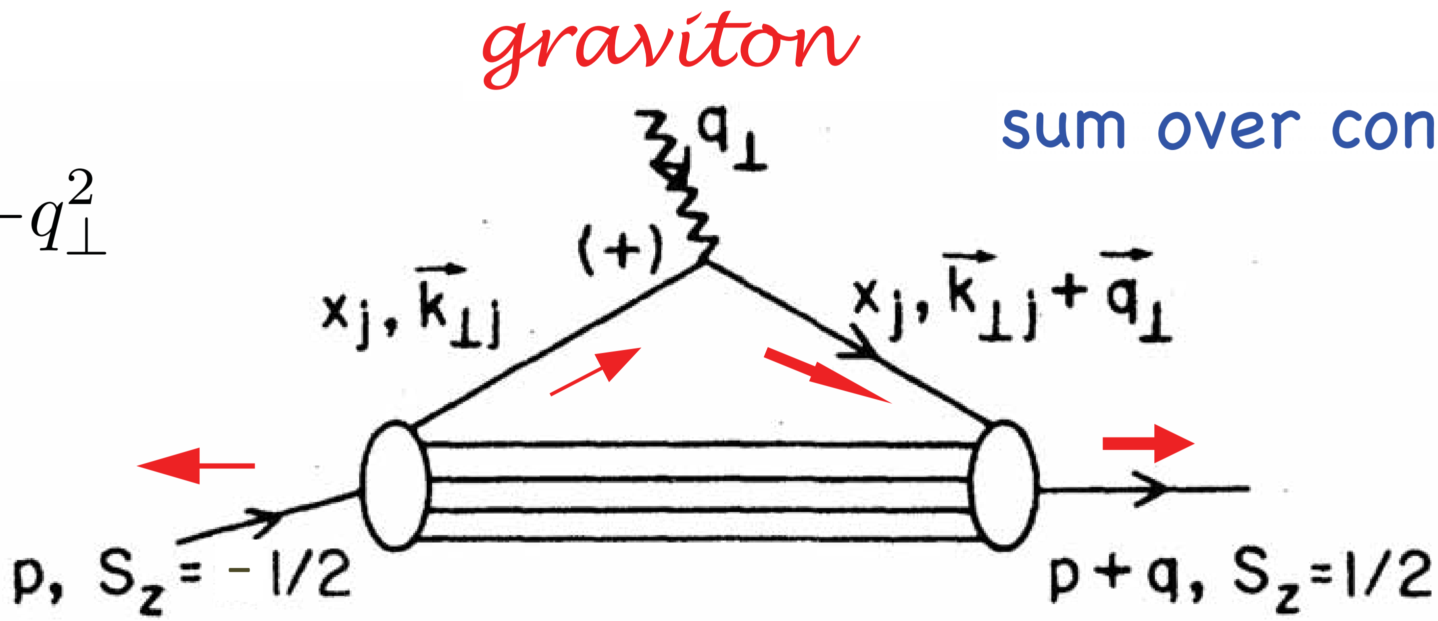
Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Checked to $\mathcal{O}\alpha^3$ *in QED* Roskies, Suaya, sjb

Anomalous gravitomagnetic moment B(0)

Okun, Kobzarev, Teryaev: *B(0) Must vanish because of Equivalence Theorem*

$$q^2 = -q_\perp^2$$



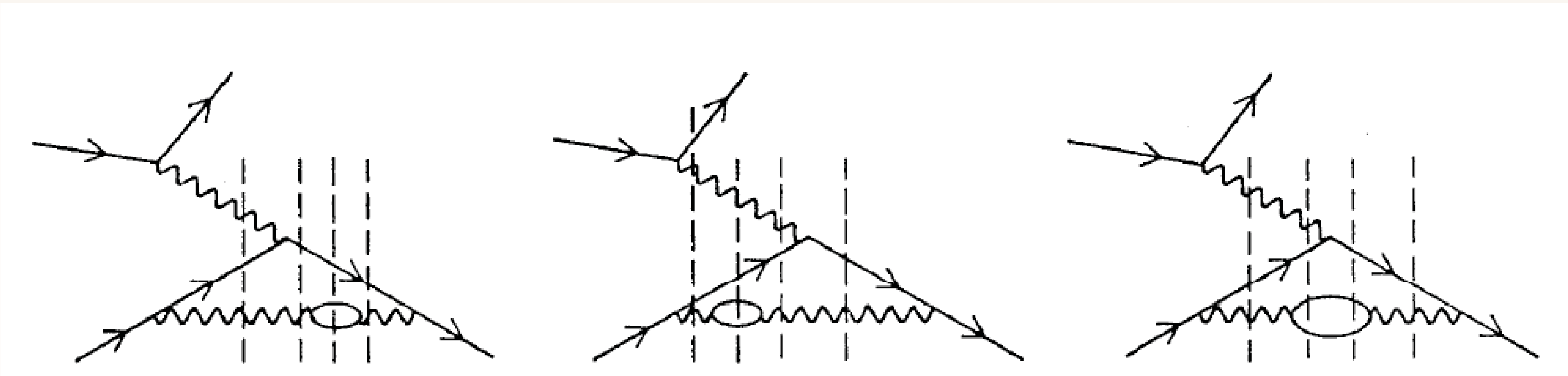
sum over constituents

Hwang, Ma, Schmidt, sjb; Holstein et al

B(0) = 0

Each Fock State

Quantum Electrodynamics and Renormalization Theory in the Infinite Momentum Frame.
Stanley J. Brodsky (SLAC) , Ralph Roskies (Yale U.) , Roberto Suaya (SLAC) . SLAC-PUB-1278, Jul 1973. 71pp.
Published in **Phys.Rev.D8:4574,1973**.



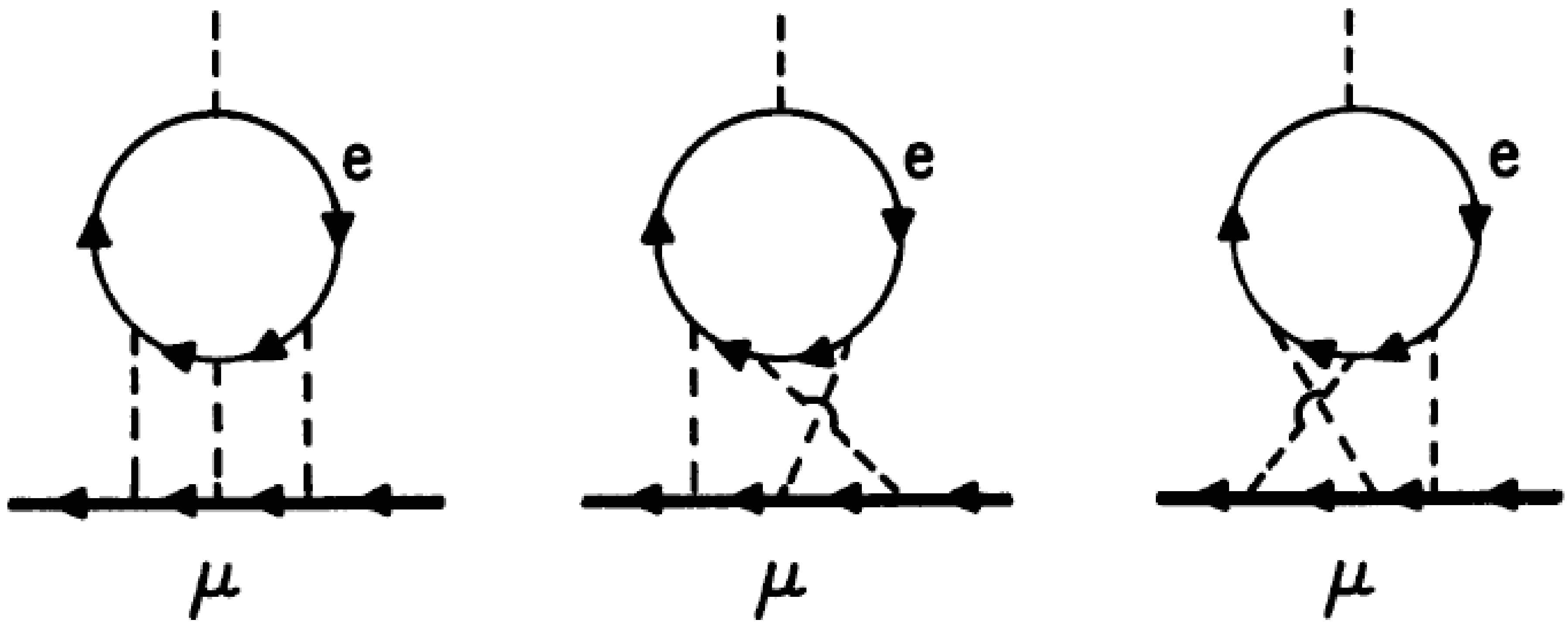
Subtract diagrams with “alternate denominators”: removes UV divergences.
Subtraction terms associated with renormalization constants

Calculated g-2 of leptons to two loops in LFPTH, part of three loops

Photon - Photon Scattering Contribution To The Sixth Order Magnetic Moments Of The Muon And Electron.

Janis Aldins, Toichiro Kinoshita (Cornell U., LNS) , Stanley J. Brodsky, A.J. Dufner (SLAC) . SLAC-PUB-0701, Jan 1970. 58pp.

Published in **Phys.Rev.D1:2378,1970**.



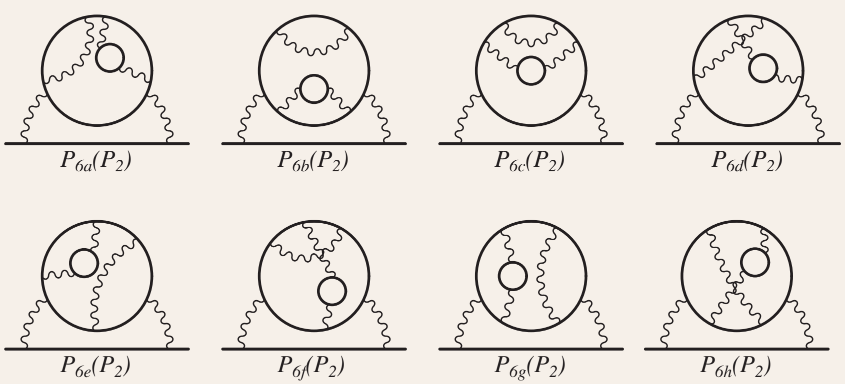
$$\Delta a_{\text{photon-photon}} = [(6.4 \pm 0.1)\, \ell n\, (m_\mu/m_e) + \text{const.}]\, (\alpha/\pi)^3.$$

Exascale: Application to QED Perturbation Theory

- Goal: Use QED LF Hamiltonian perturbation theory to compute lepton g-2 to 6 loops or higher
- Alternate denominator renormalization method

Roskies, Suaya, sjb

- Gauge invariant: Compare Feynman and LC gauges
- Cluster Factorization: Subgraph format only needs to be computed once: Retain “History”



Tenth-Order Lepton Anomalous Magnetic Moment — Second-Order Vertex Containing Two Vacuum Polarization Subdiagrams, One Within the Other

Tatsumi Aoyama,[1] Masashi Hayakawa,[2] Toichiro Kinoshita,[3] and Makiko Nio[4,*]

Light-Front QCD

Heisenberg Matrix Formulation

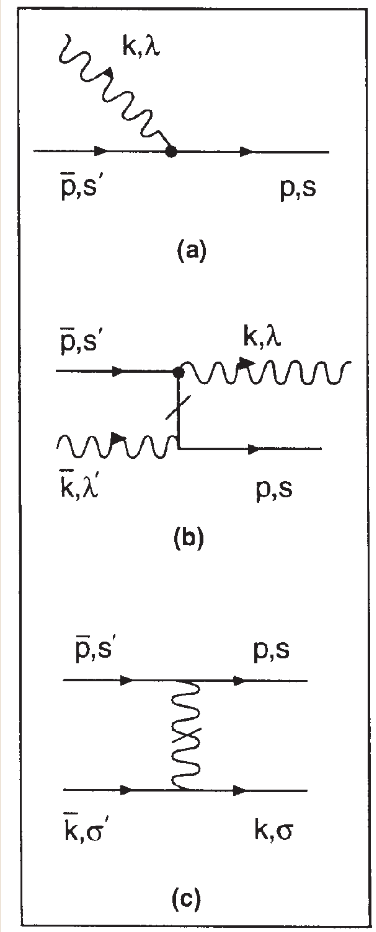
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

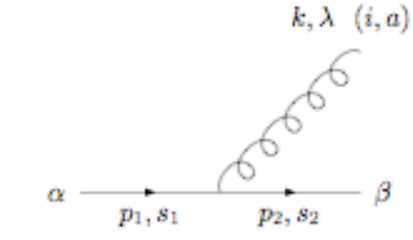
$$H^{QCD}_{LF} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$

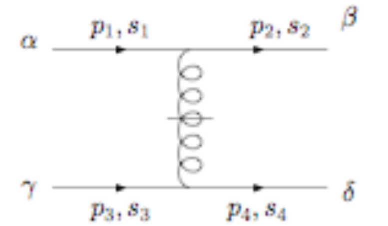


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

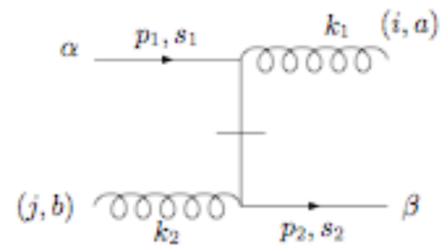
DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

Elementary vertices in LF gauge

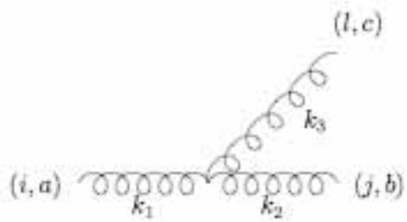


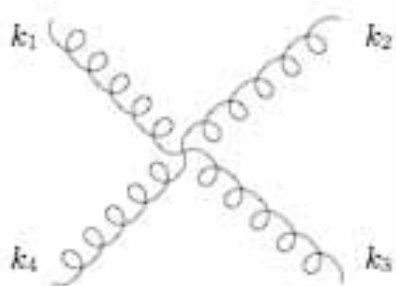


H_{qqgg1}

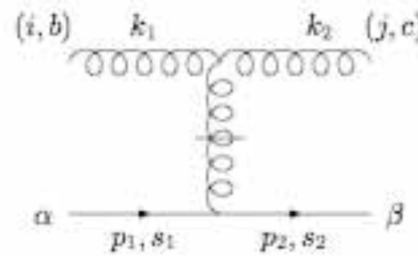


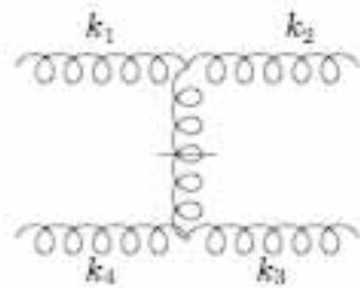
QED & QCD





H_{qqgg2}





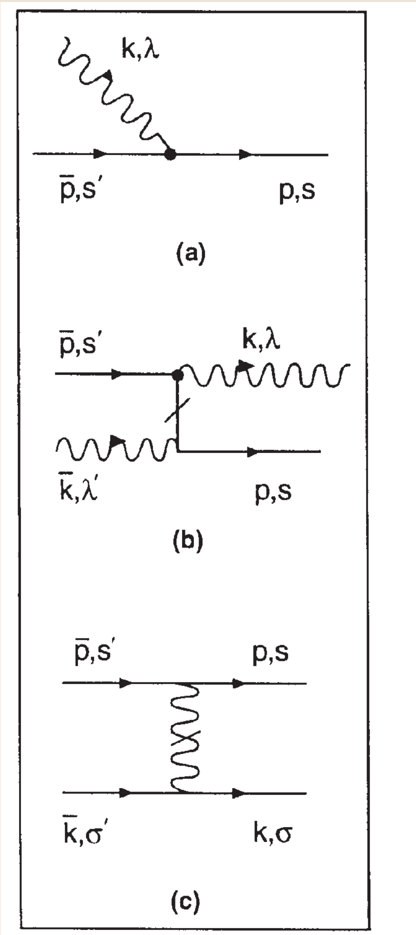
QCD

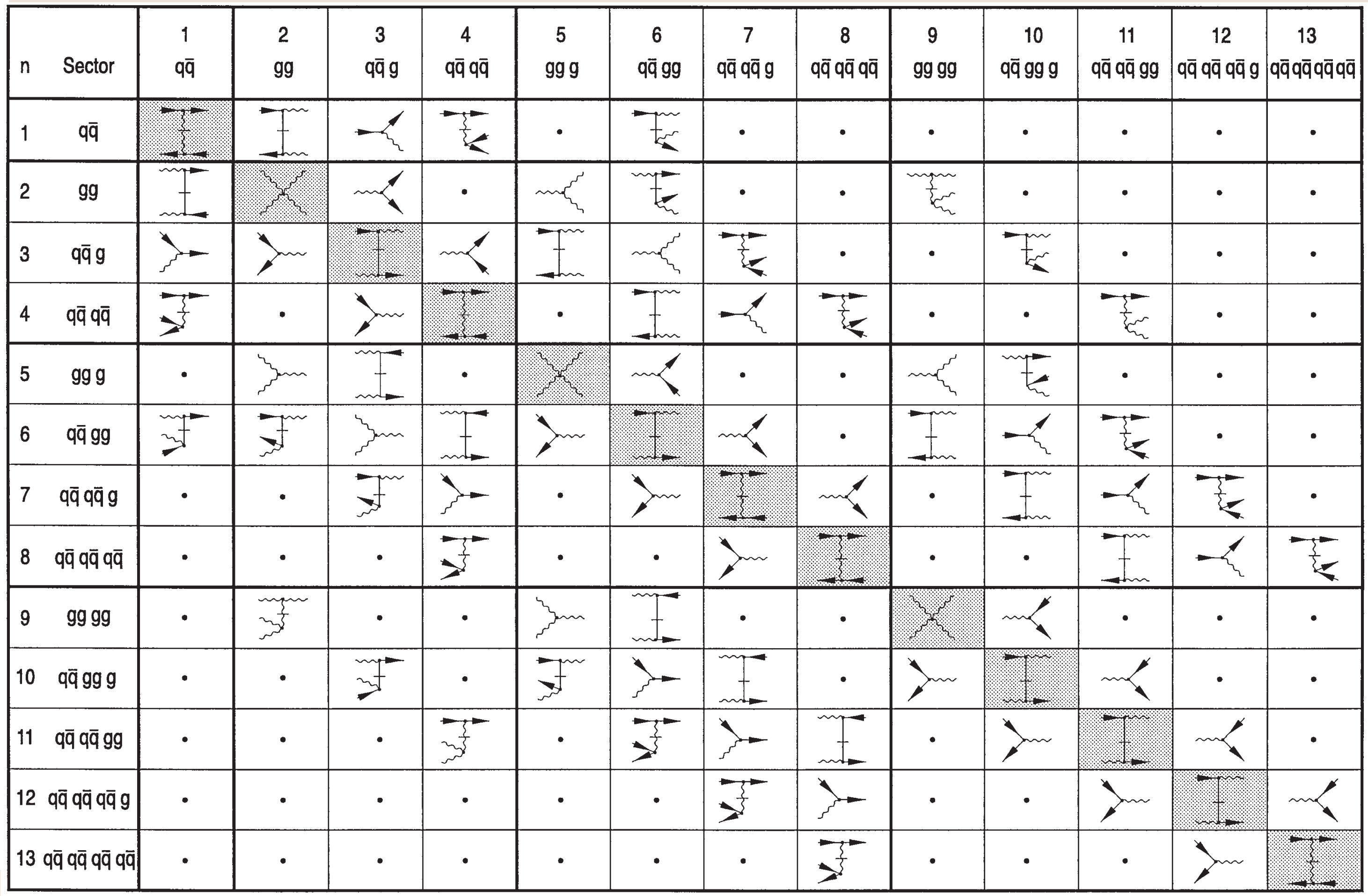
Light-Front QCD

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

H.C. Pauli & sjb

Heisenberg Matrix Formulation





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Angular Momentum on the Light-Front

A^+=0 gauge:

No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

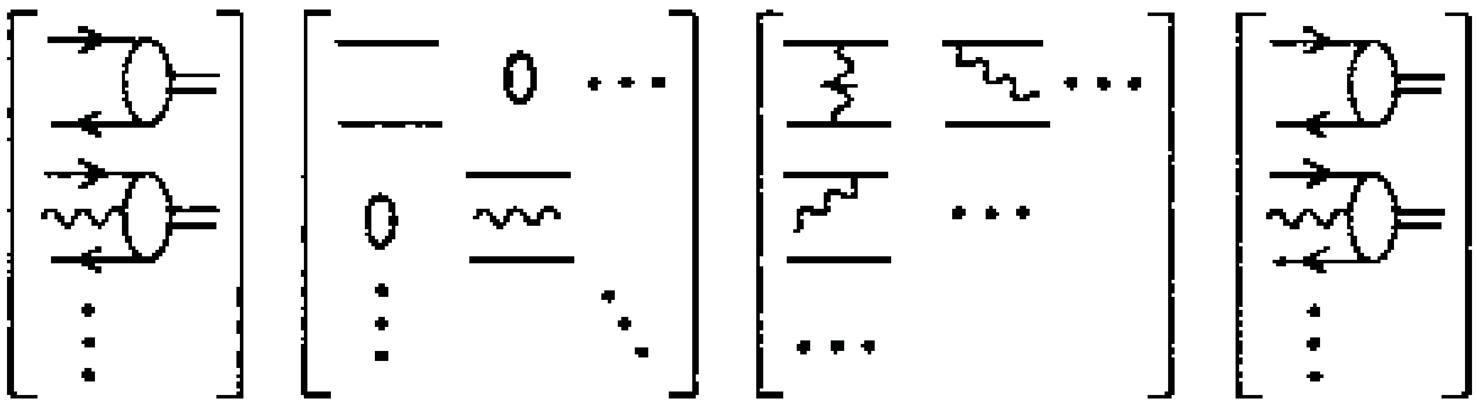
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment requires
Nonzero orbital angular momentum

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

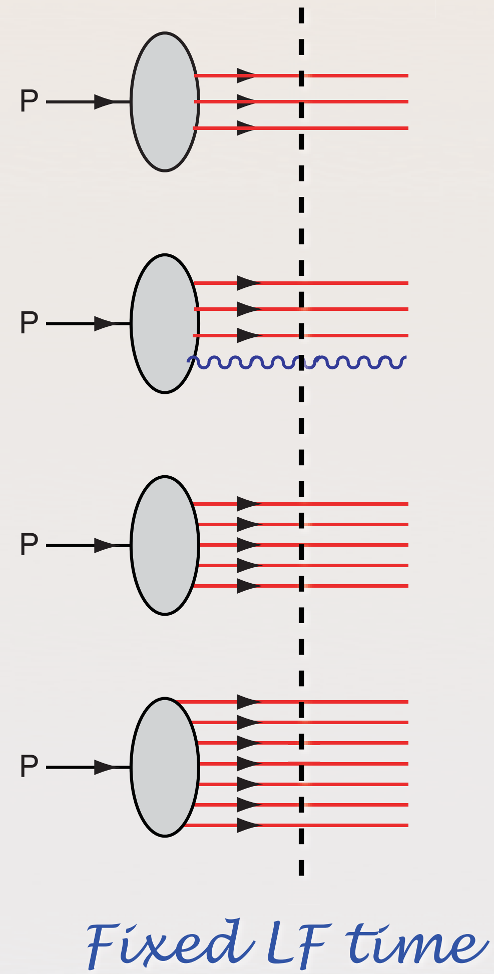
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

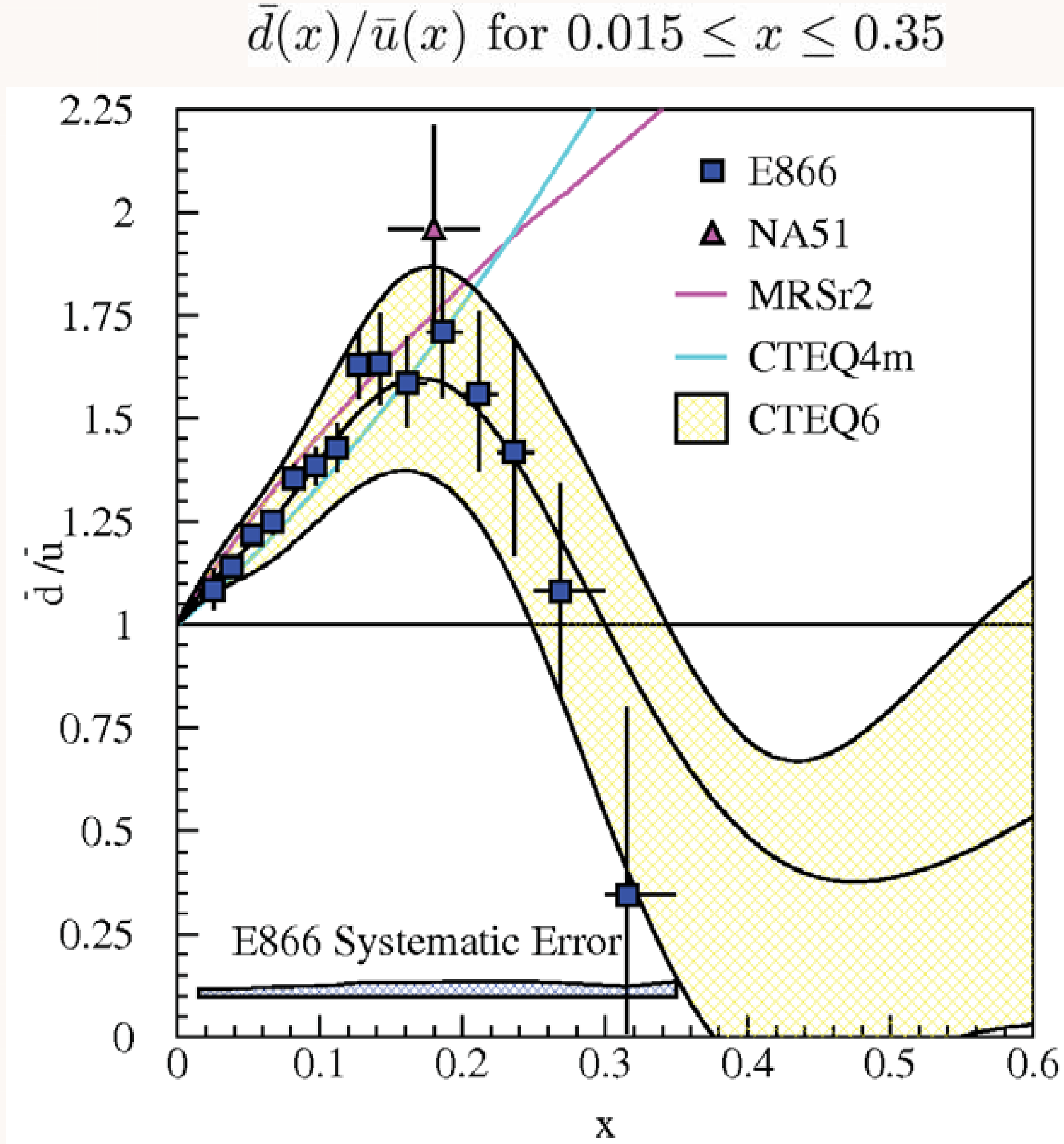
$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks,

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Higher Fock State of Proton is Complex



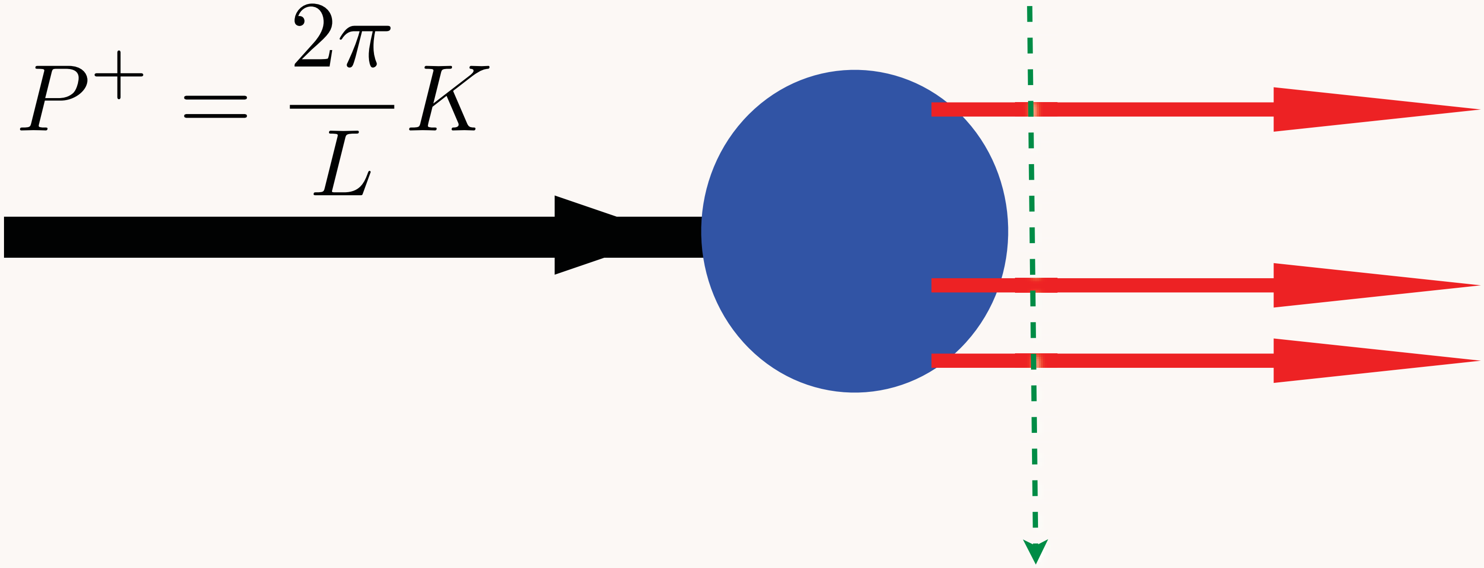
- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

Intrinsic glue, sea, heavy quarks

$$P^+ = \frac{2\pi}{L} K$$



$$\sum_{i=1}^{N} k_i^+ = P^+$$

Periodic Boundary Conditions in x^-

$$k_i^+ = \frac{2\pi}{L} n_i \qquad \sum_{i=1}^{N} n_i = K$$

$$\sum_{i=1}^{N} [x_i = \frac{k_i^+}{P^+} = \frac{n_i}{K}] = 1$$

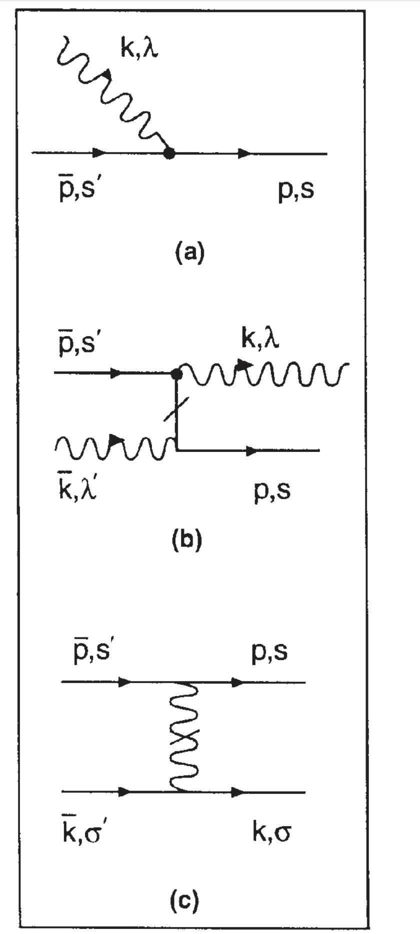
Light-Front QCD

Heisenberg Matrix Formulation

$$H^{QCD}_{LF} |\Psi_h>= \mathcal{M}^2_h |\Psi_h>$$

H.C. Pauli & sjb

Discretized Light-Cone Quantization



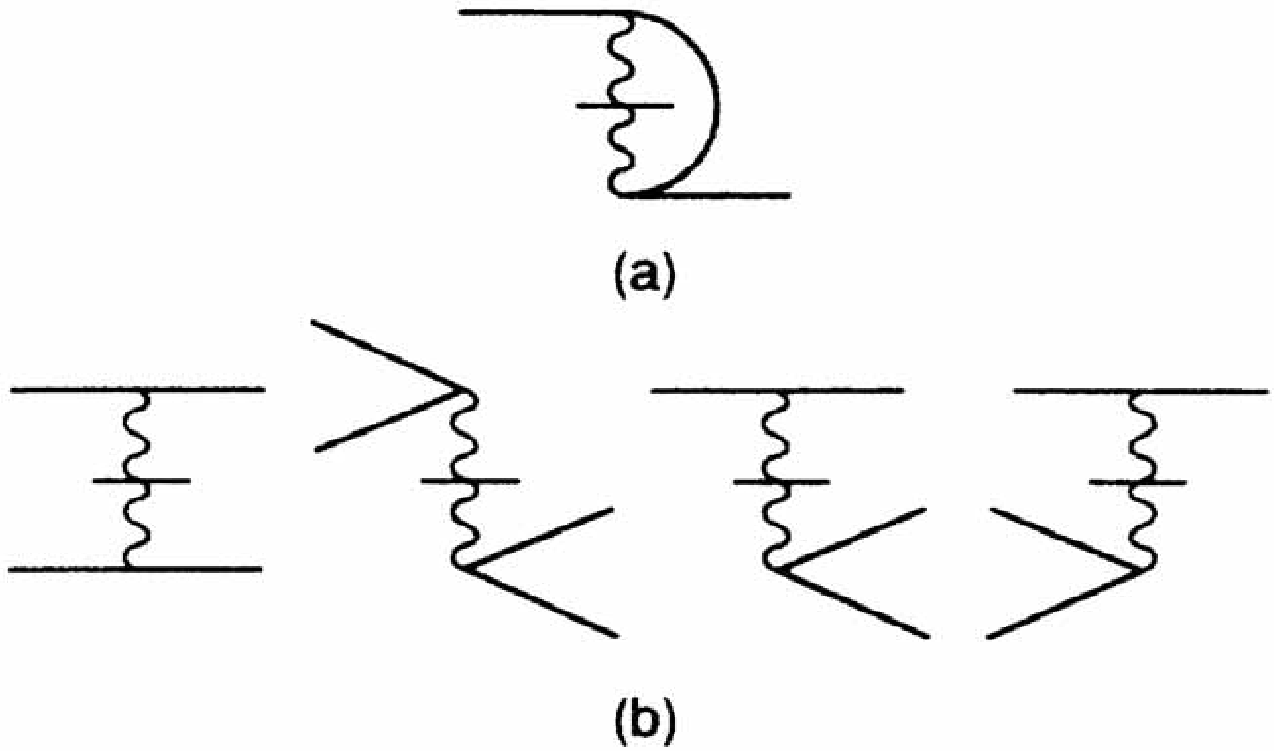
n	Sector	1 qq̄	2 gg	3 qq̄ g	4 qq̄ qq̄	5 gg g	6 qq̄ gg	7 qq̄ qq̄ g	8 qq̄ qq̄ qq̄	9 gg gg	10 qq̄ gg g	11 qq̄ qq̄ gg	12 qq̄ qq̄ qq̄ g	13 qq̄ qq̄ qq̄ qq̄
1	qq̄					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	qq̄ g								•	•		•	•	•
4	qq̄ qq̄		•			•				•	•		•	•
5	gg g	•			•			•	•			•	•	•
6	qq̄ gg								•				•	•
7	qq̄ qq̄ g	•	•			•				•				•
8	qq̄ qq̄ qq̄	•	•	•		•	•			•	•			
9	gg gg	•		•	•			•	•			•	•	•
10	qq̄ gg g	•	•		•				•				•	•
11	qq̄ qq̄ gg	•	•	•		•				•				•
12	qq̄ qq̄ qq̄ g	•	•	•	•	•	•			•	•			
13	qq̄ qq̄ qq̄ qq̄	•	•	•	•	•	•	•		•	•	•		

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

Interactions in QCD(1+1)

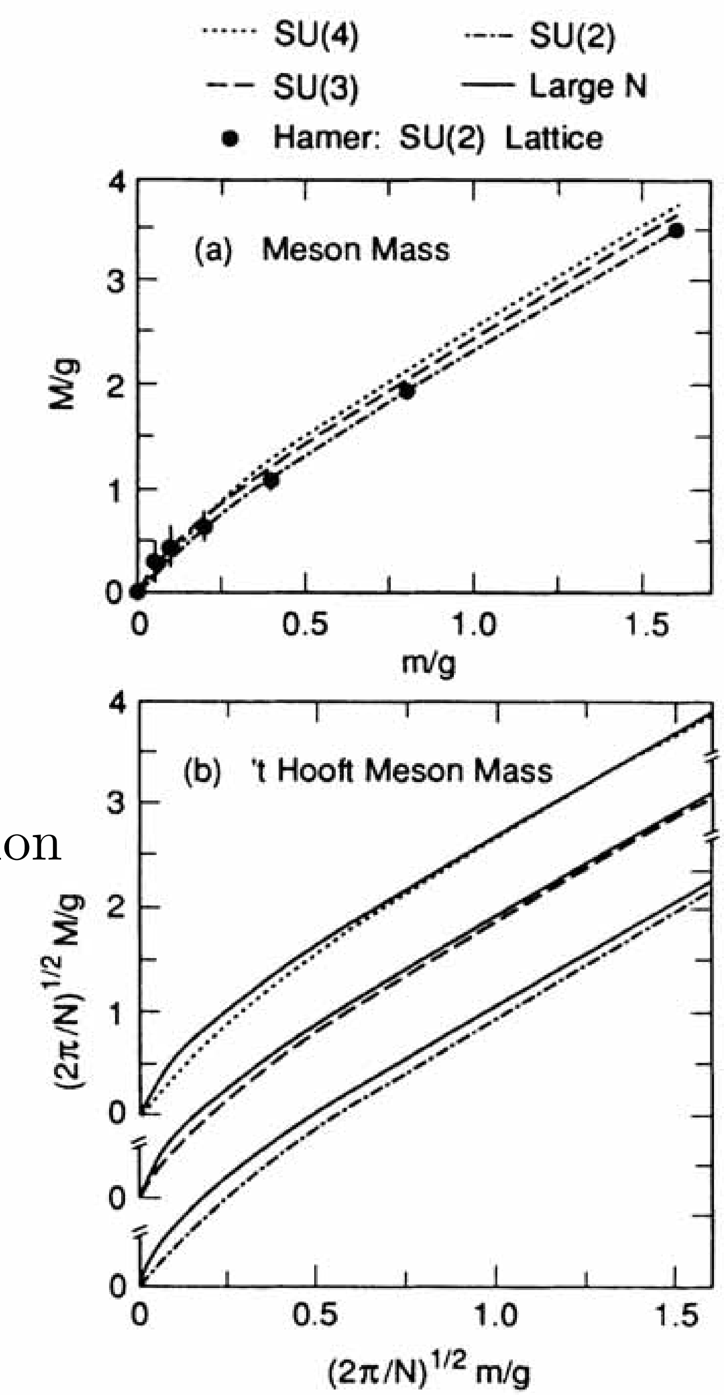


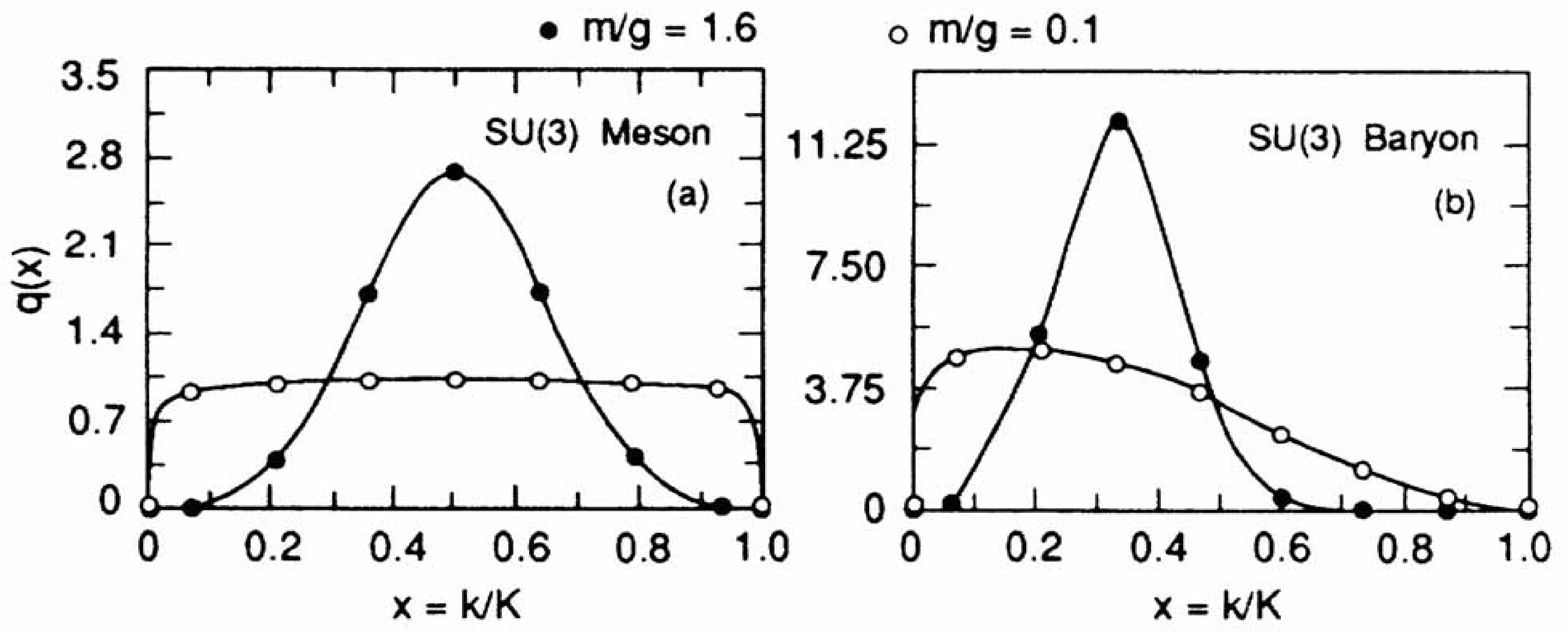
$$\frac{L}{2\pi}\frac{g^2}{\pi}\frac{1}{2}\left[\delta^{c_2}_{c_4}\delta^{c_3}_{c_1}-\frac{1}{N}\delta^{c_3}_{c_4}\delta^{c_2}_{c_1}\right]$$

$$\times\sum_{n_i=1/2,3/2,\ldots}\frac{\delta_{n_1+n_2,n_3+n_4}}{(n_1+n_3)^2}b^{\dagger c_4}_{n_4}b_{n_3,c_3}d^{\dagger}_{n_2,c_2}d^{c_1}_{n_1}$$

QCD(1+1)

$QCD(1+1)$ lightest meson mass
(a) for $N_C = 2, 3, 4$
compared with lattice result for $N_C = 2$
(b) large N_C compared with 't Hooft solution

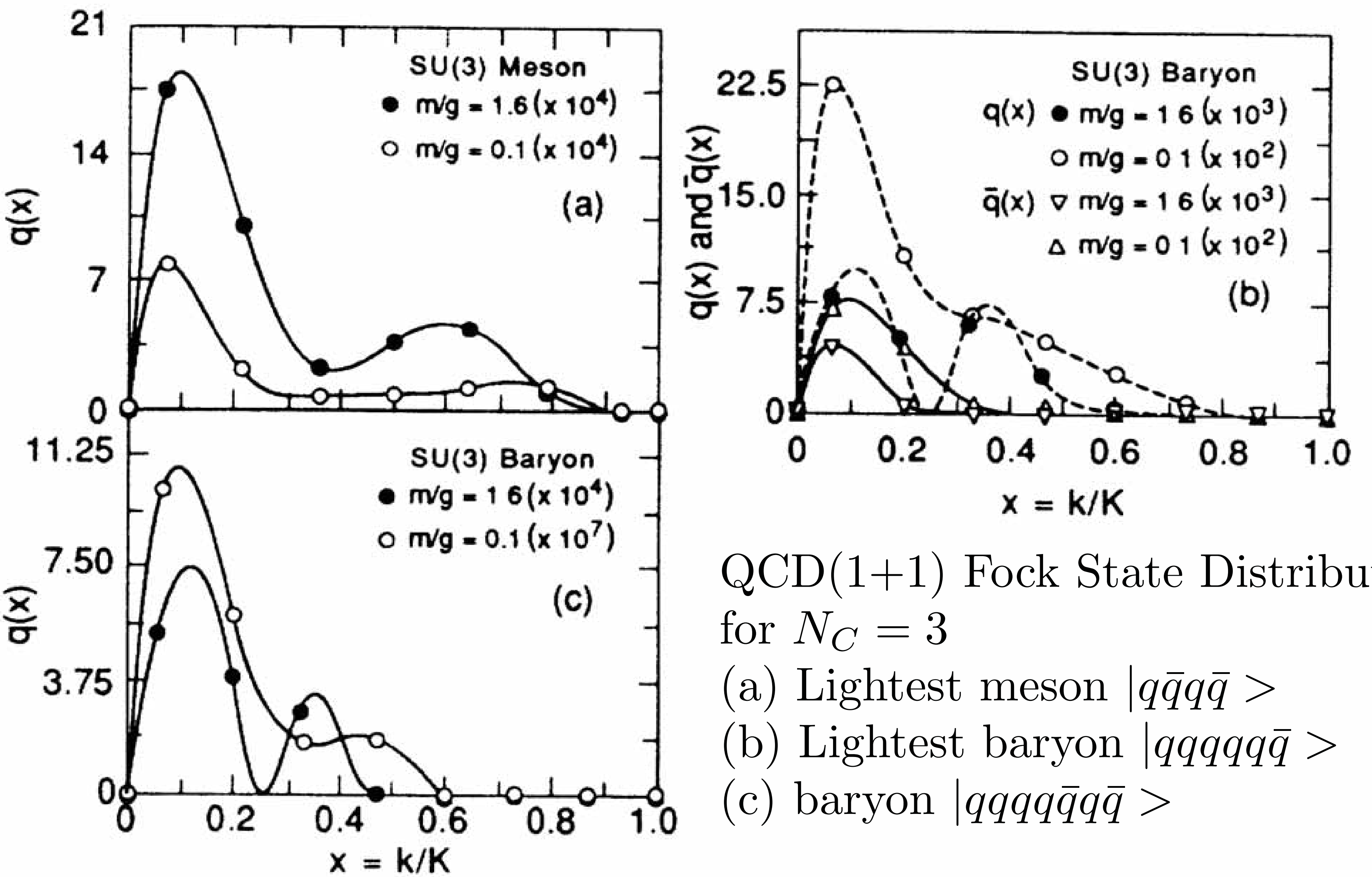




QCD(1+1) Valence Fock State Distributions for $N_C = 3$

(a) Lightest meson $|q\bar{q}>$

(b) Lightest baryon $|qqq>$



QCD(1+1) Fock State Distributions for $N_C = 3$

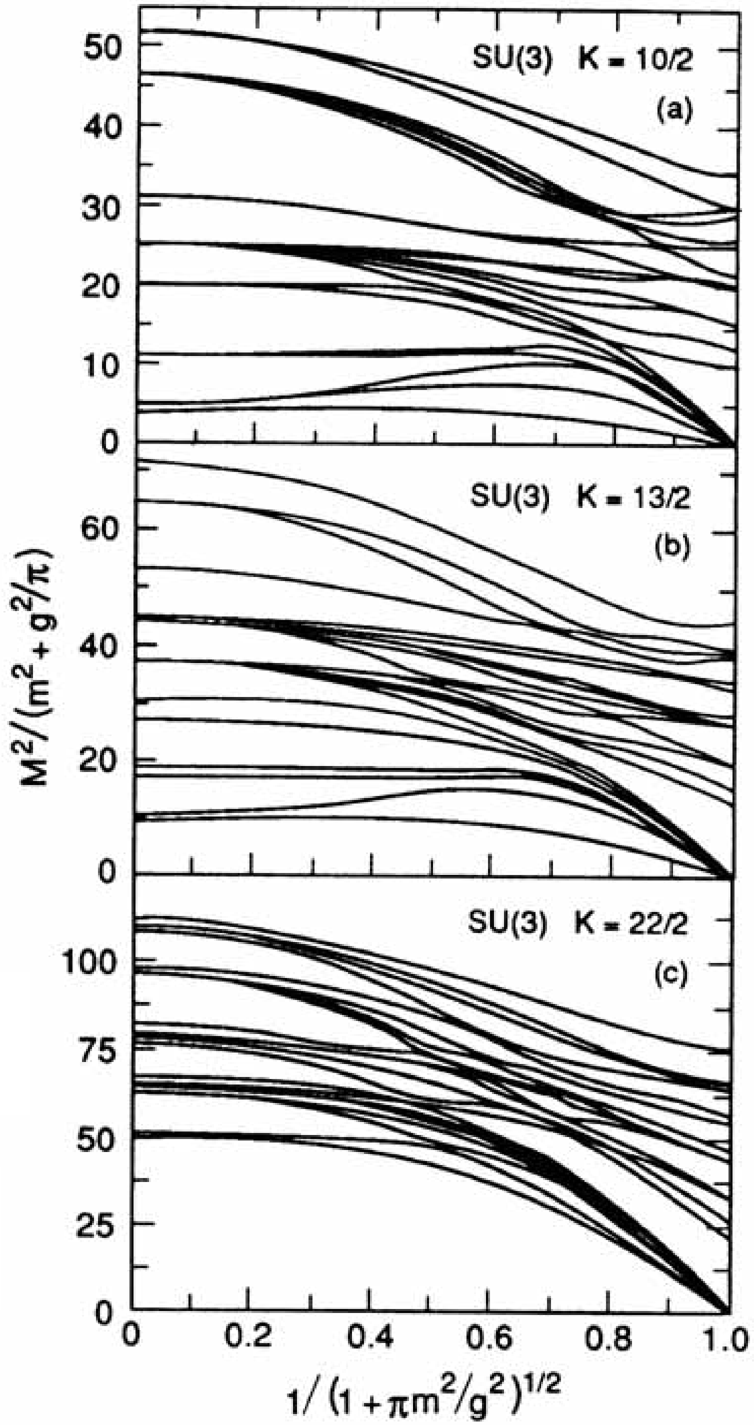
(a) Lightest meson $|q\bar{q}q\bar{q}>$

(b) Lightest baryon $|qqqqq\bar{q}>$

(c) baryon $|qqqq\bar{q}q\bar{q}>$

QCD(1+1)

Mass spectrum of QCD(1+1) for $N_C = 3$, baryon number $B = 0, 1, 2$ as a function of g/m,; K fixed.



Hornbostel, Pauli, sjb

Solve QCD(1+1) for any number of colors, flavors, quark masses

Many 1+1 quantum field theories solved using DLCQ

e.g., Klebanov, et al

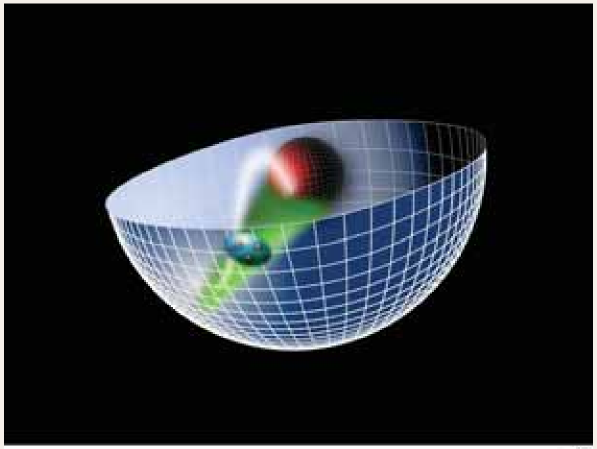
Discretized Light-Cone Quantization (DLCQ)

- Diagonalize LF Hamiltonian Hermetian Matrix on a Discrete Momentum Basis
- Rigorous Nonperturbative Solutions of QCD and other Quantum Field Theories
- Minkowski Space
- Lorentz-Frame Independent
- No Fermion Doubling
- Complete Solutions to QCD(1+1), many other theories
- SDLCQ: Solve Supersymmetric Theories: $H_{LF} = Q^2$ Hiller, Pinsky
- Non-perturbative renormalization: Pauli-Villars; SUSY, Glazek-Wilson, ...
- QCD(3+1): Computationally Challenging -- Exascale

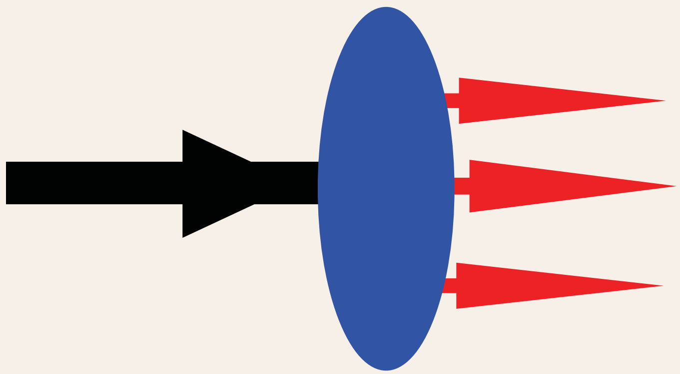
Future Exascale DLCQ Computations

- 10^{17} to 10^{18} floating point operations per second
- Memory 10 petabytes = 10^{16} bytes
- DLCQ: Sparse matrix diagonalization (10^9 x 10^9)
- DLCQ basis for hadrons with up to 4 constituents
- Integer basis may allow more efficient computation

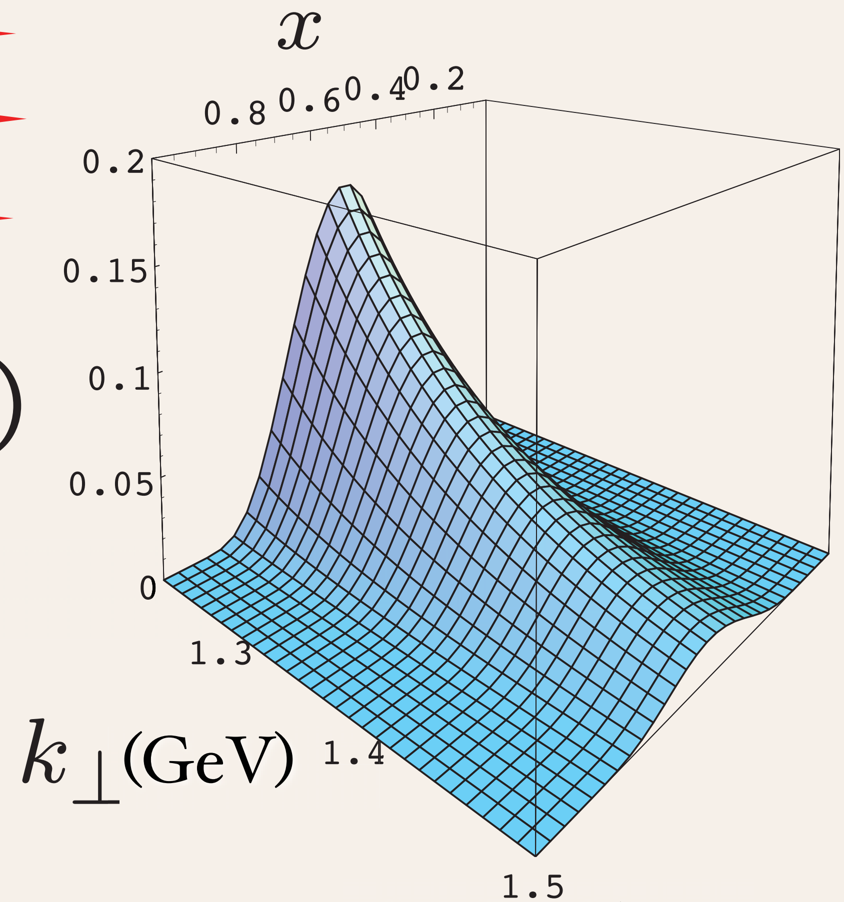
$\phi(z)$



- *Light-Front Holography*



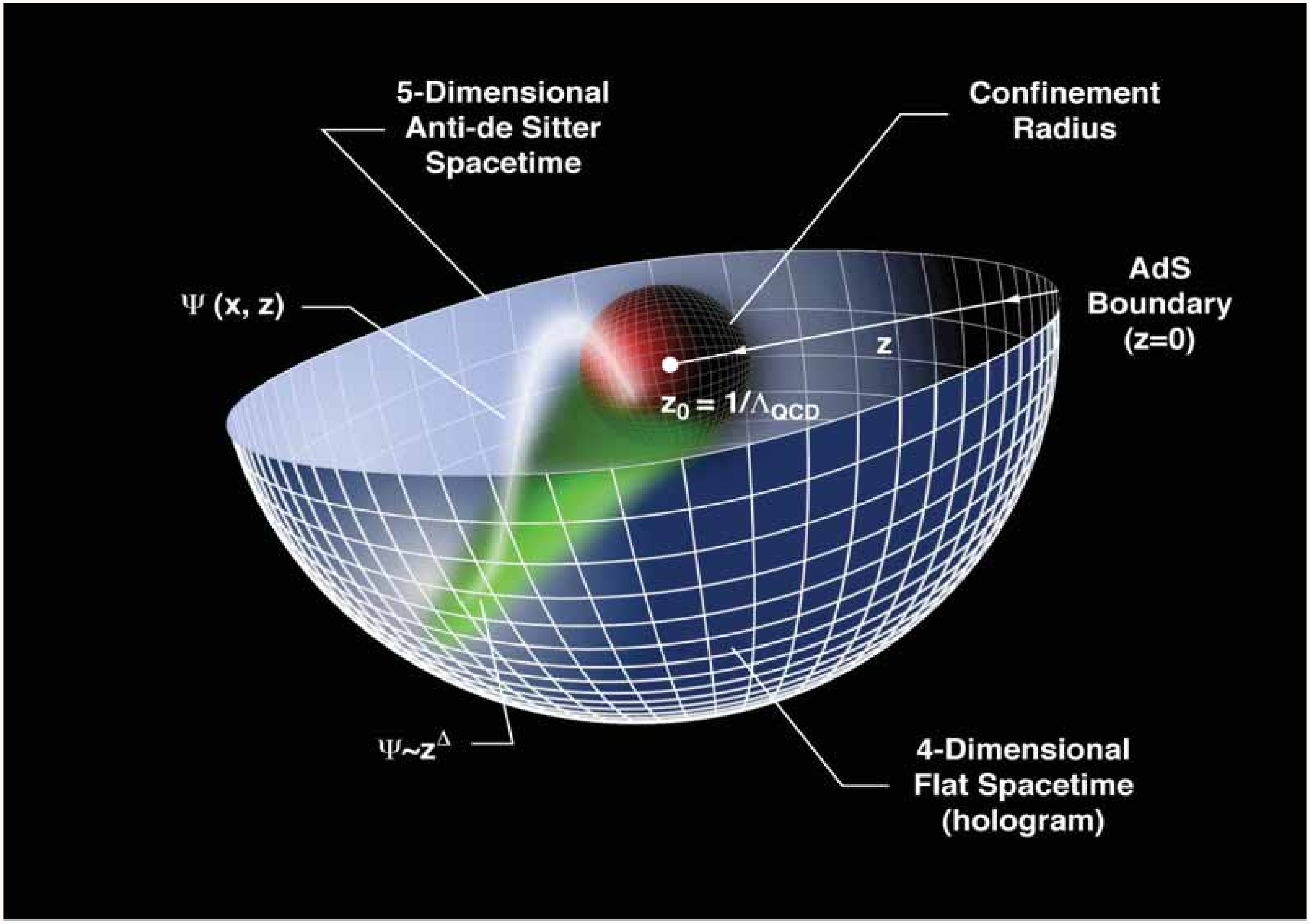
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

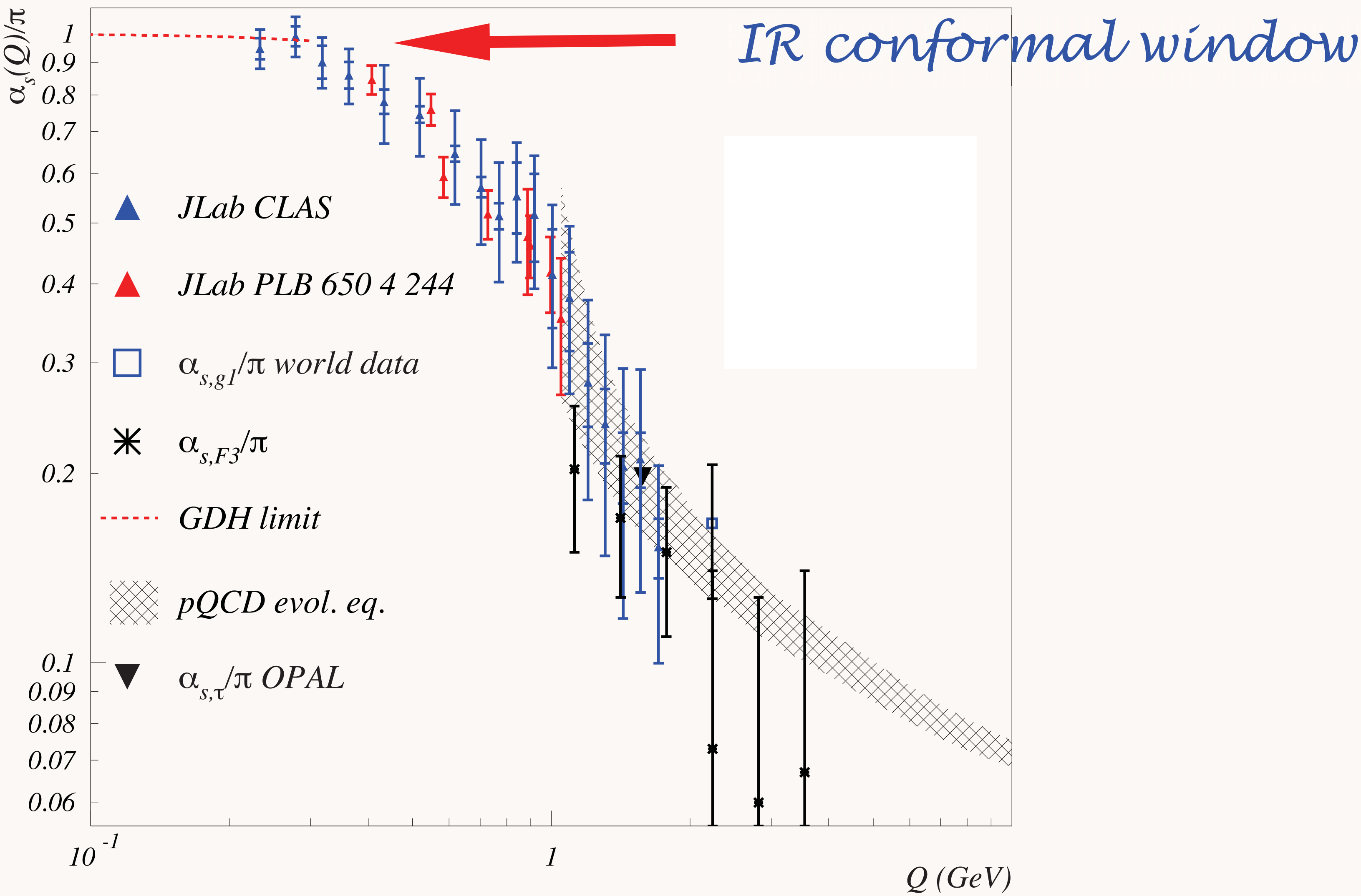
- **Analogous to the Schrodinger Theory for Atomic Physics**

- *AdS/QCD Light-Front Holography*

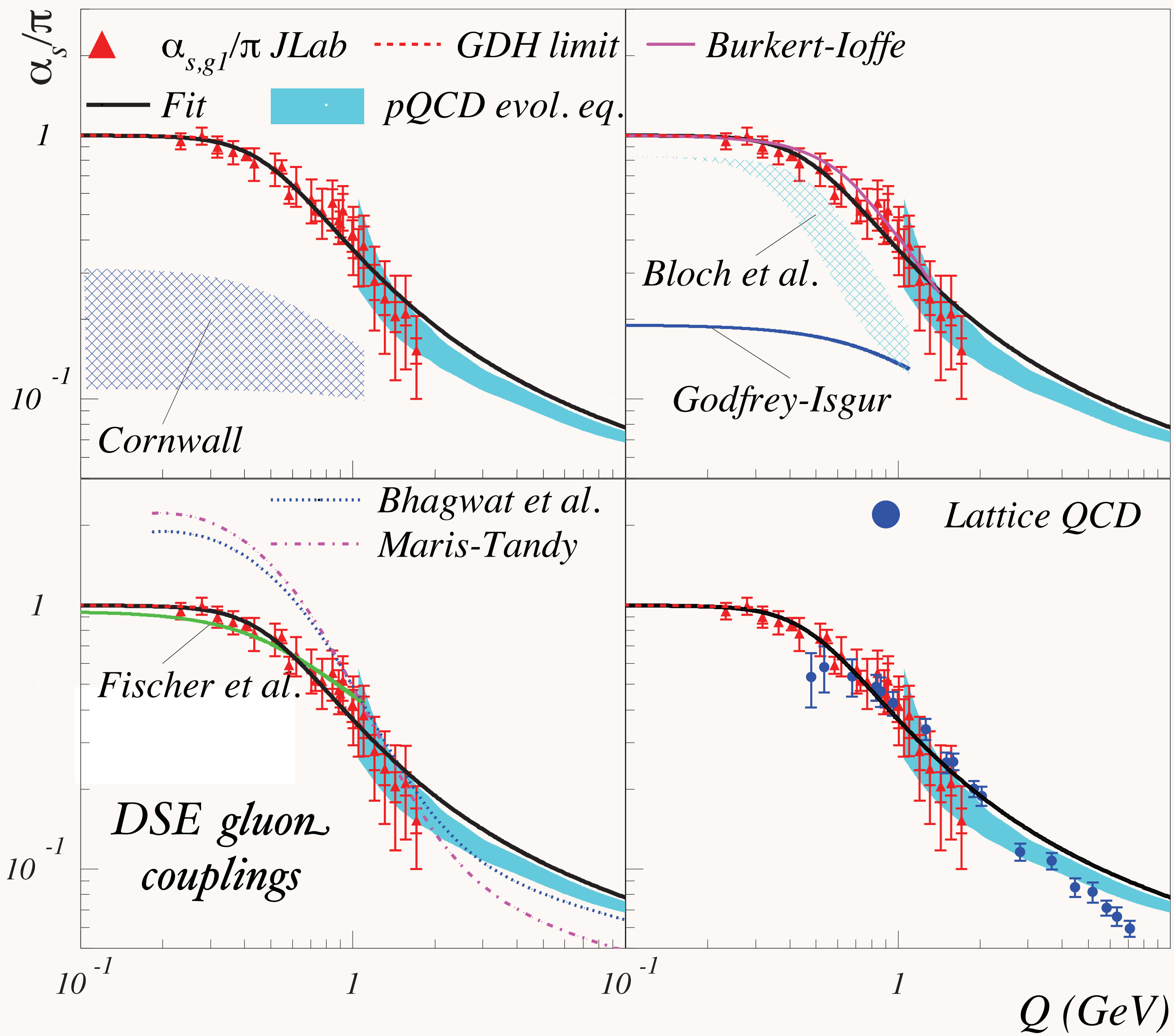
- *Hadronic Spectra and Light-Front Wavefunctions*

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Deur, Korsch, et al.



Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

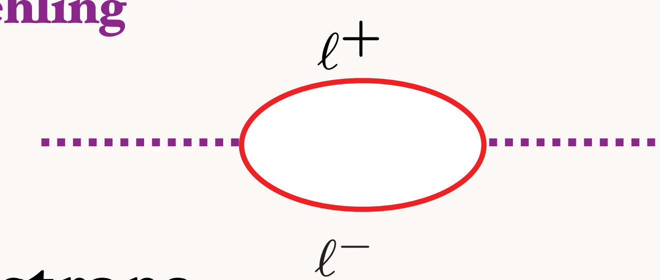
IR Conformal Window for QCD

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**
- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Shrock, de Teramond, sjb

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$ invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.