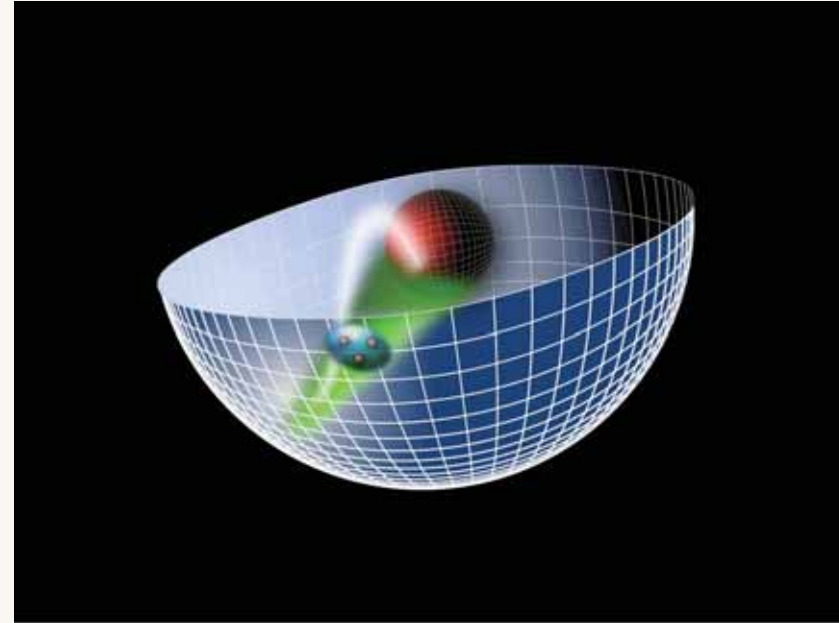
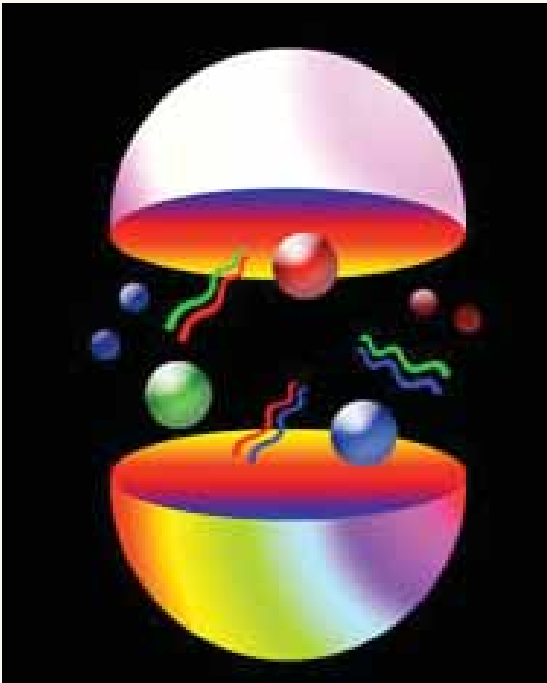


AdS/CFT and QCD



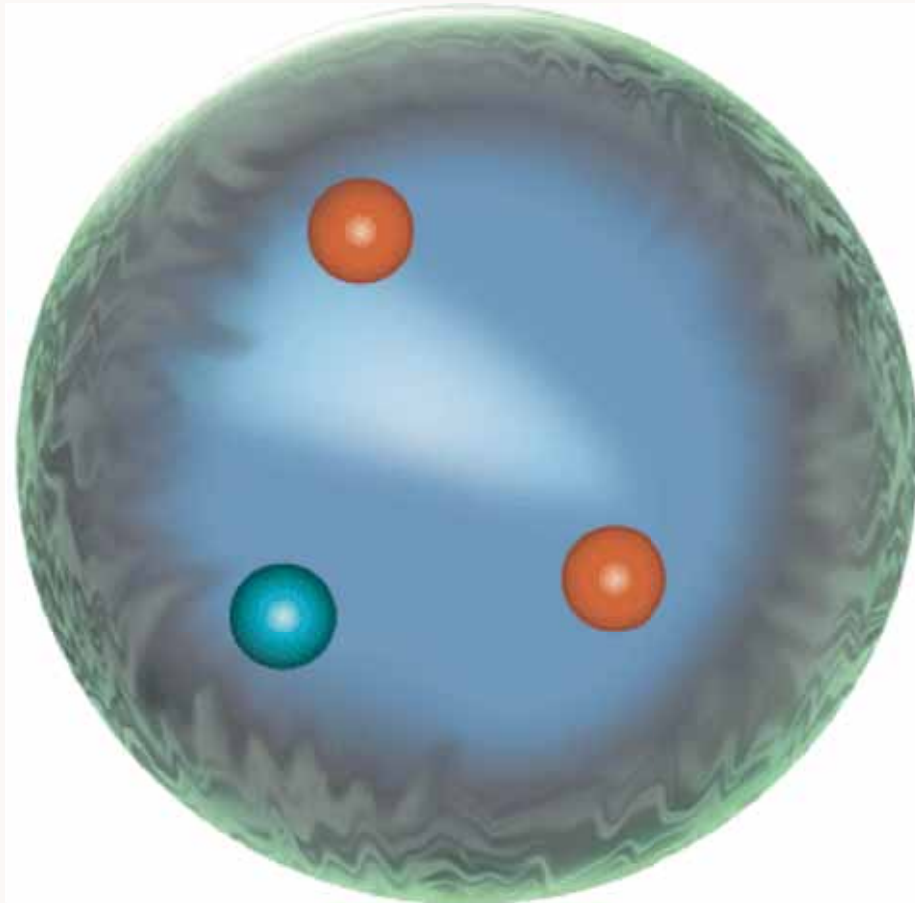
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Quarks in the Proton

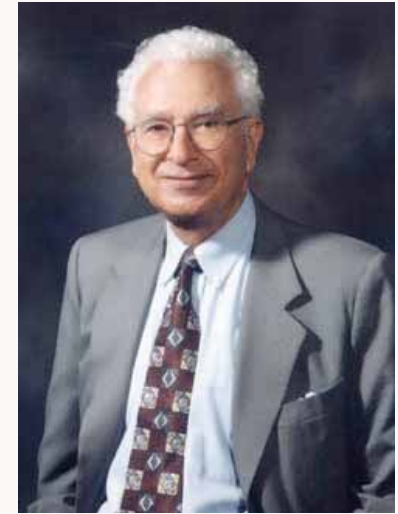


Bjorken: Scaling



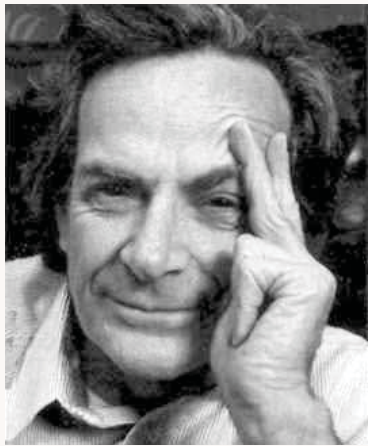
$$p = (u u d)$$

← 1 fm →
 $10^{-15}m = 10^{-13}cm$



Gell Mann: Eightfold Way

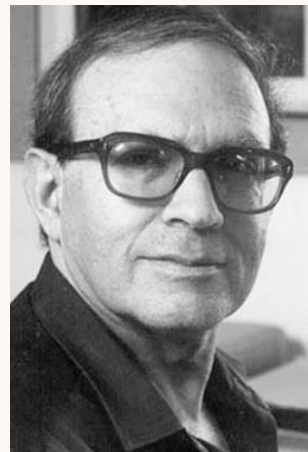
"Three quarks for Muster Mark!"



Feynman: Parton model



Zweig: "Aces, Deuces, Treys"



Ne'eman

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Current Algebra: Quarks and What Else?

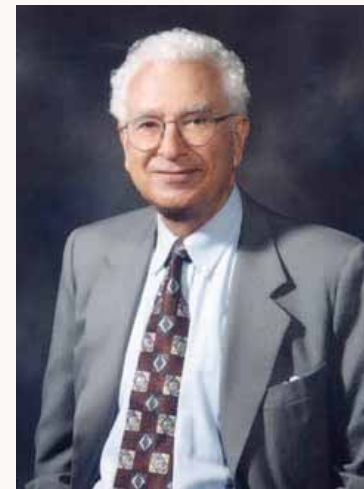


Harald Fritzsch*†

and

Murray Gell-Mann**†

CERN, Geneva, Switzerland



Proceedings of the XVI International Conference on High Energy Physics, Chicago, 1972. Volume 2, p. 135 (J. D. Jackson, A. Roberts, eds.)

Fritzsch and Gell-Mann introduce 'Color'



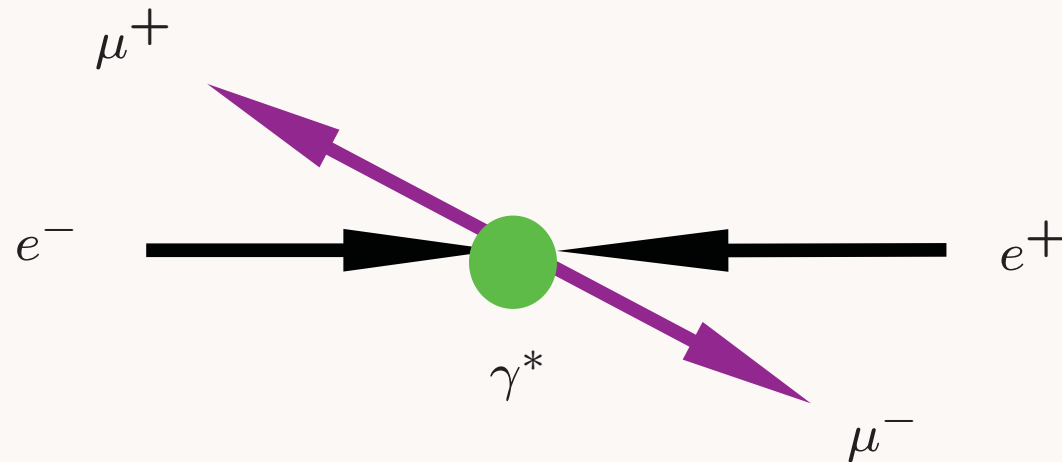
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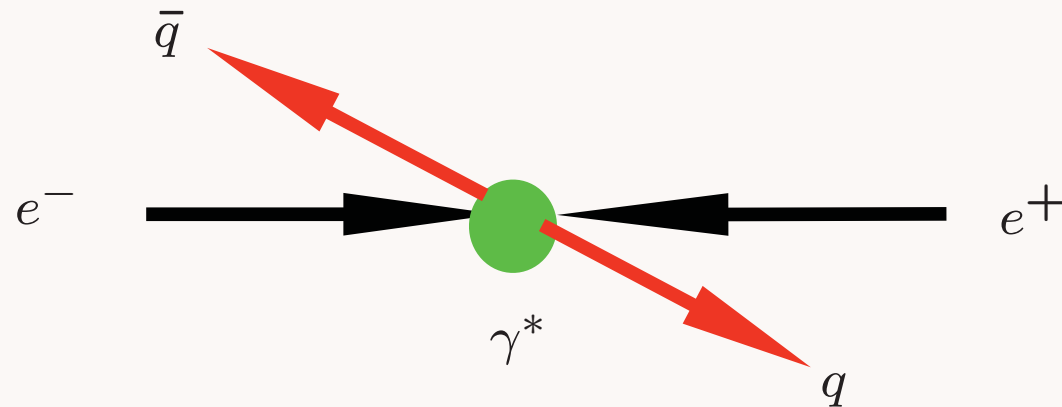
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Electron-Positron Annihilation



$$e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-$$

Electron-Positron Annihilation

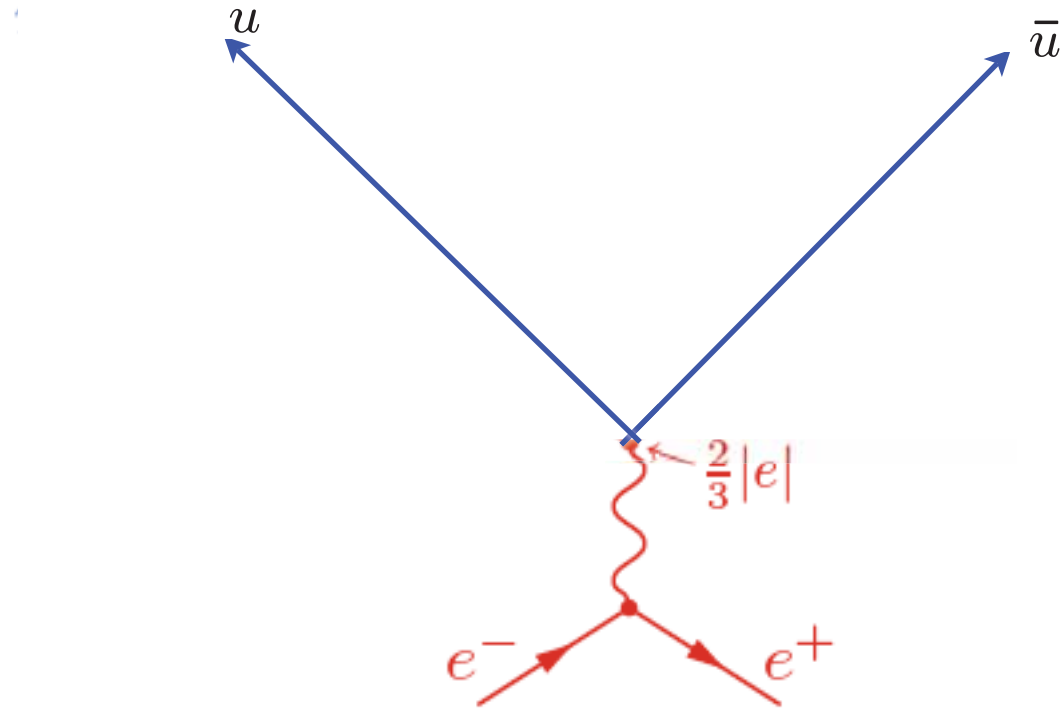


$$e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$$

Rate proportional to quark charge squared
and number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

How to Count Quarks



For $10 \text{ GeV} < E_{\text{cm}} < 40 \text{ GeV}$,

$$\frac{e^+e^- \rightarrow \text{hadrons}}{e^+e^- \rightarrow \mu^+\mu^-} = 3 \times \left[\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right]$$

↑ ↑ ↑ ↑ ↑
colors d u s c b

SPEAR Electron-Positron Collider



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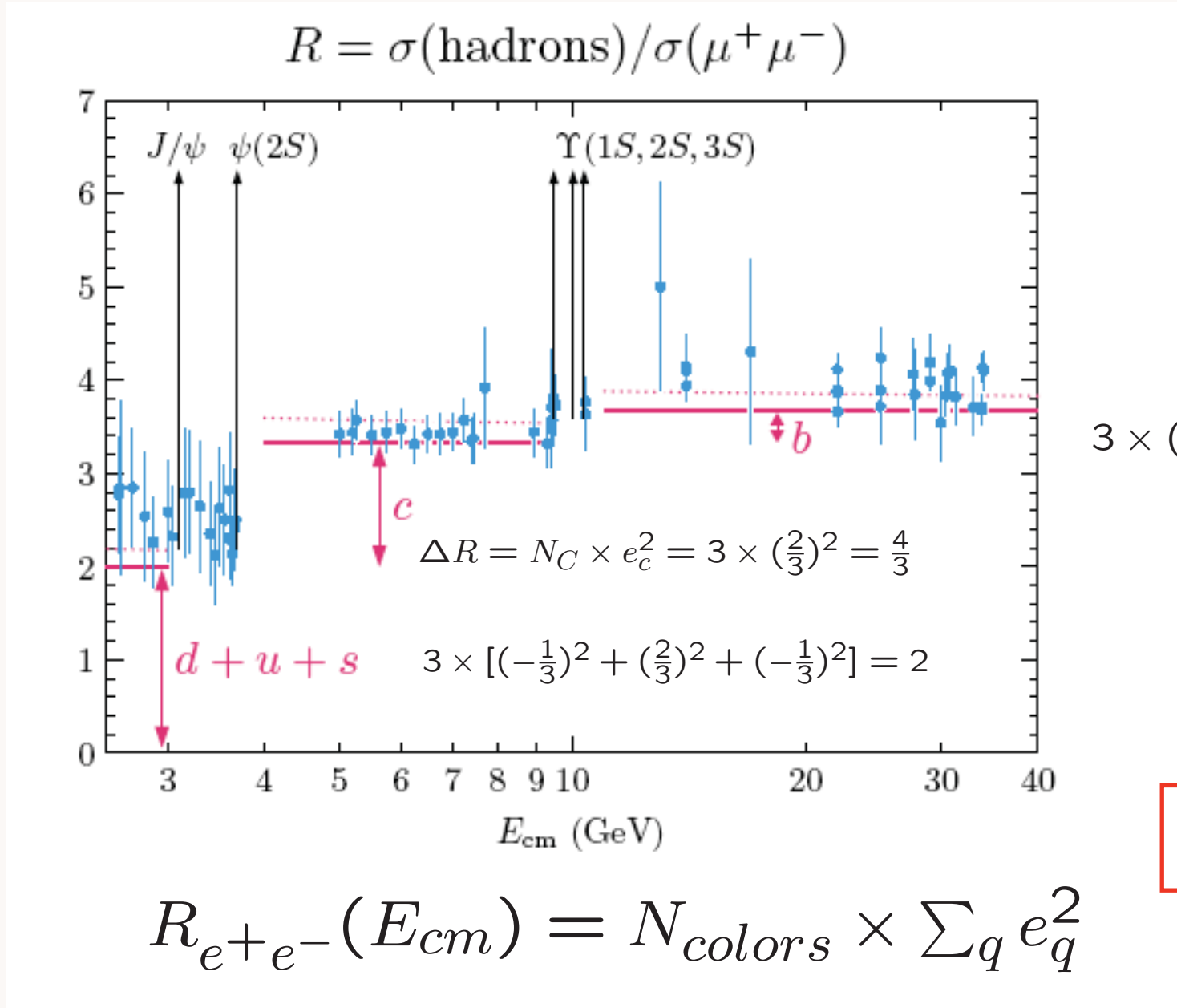
7

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$$J/\psi = (c\bar{c})_{1S}$$

How to Count Quarks

$$\Upsilon = (b\bar{b})_{1S}$$

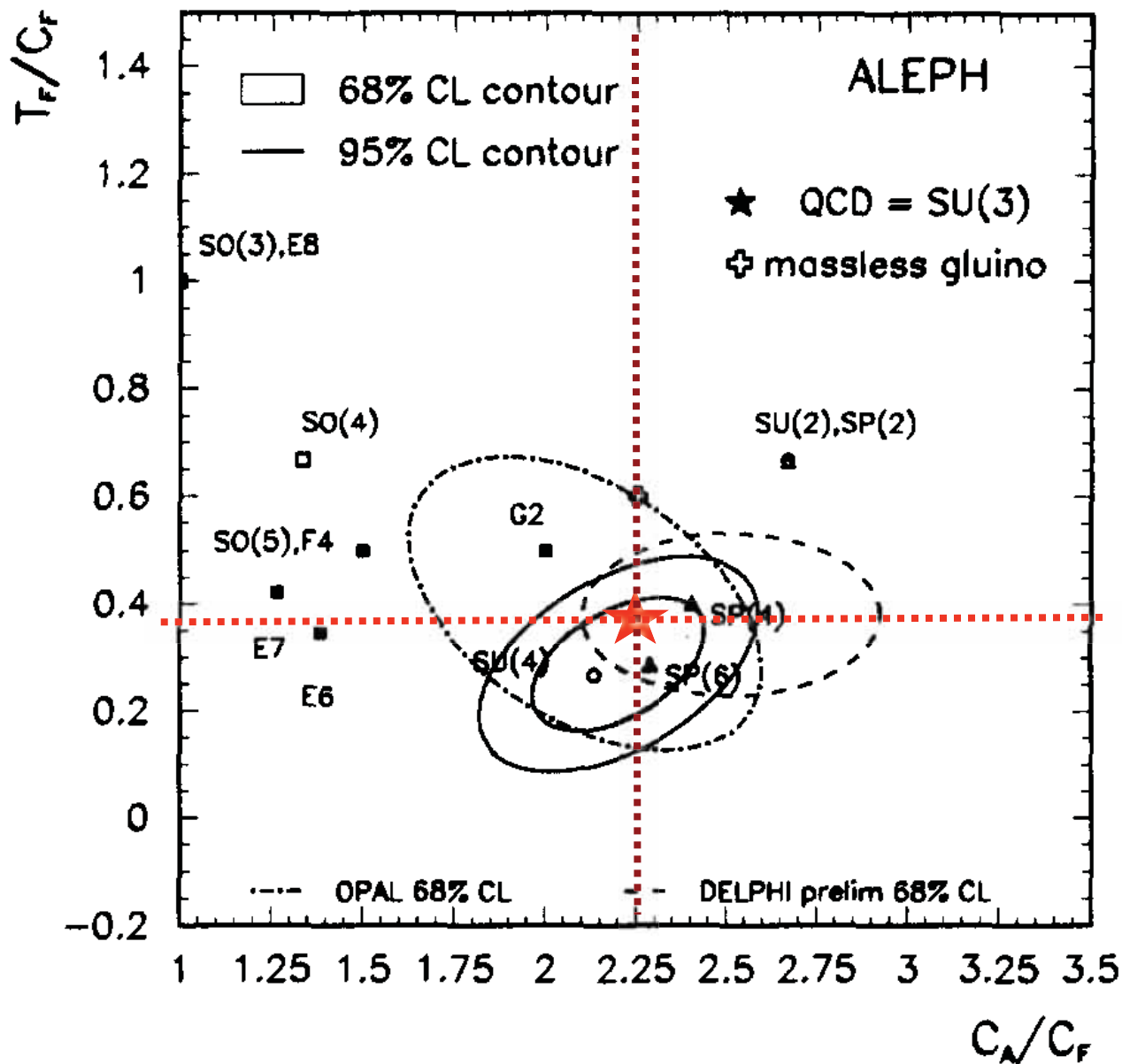


$$3 \times \left(-\frac{1}{3}\right)^2 = \frac{1}{3}$$

$$\Delta R = N_C \times e_c^2 = 3 \times \left(\frac{2}{3}\right)^2 = \frac{4}{3}$$

$$3 \times \left[\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = 2$$

$$N_C = 3$$



Ratio of Hadron Multiplicities from Quark and Gluon Jets determine color group!

$$\frac{n_g^H}{n_q^H} = \frac{C_A}{C_F} = \frac{N_C}{(N_C^2 - 1)/2N_C} = \frac{9}{4}$$

$$\frac{T_F}{C_F} = \frac{1/2}{(N_C^2 - 1)/2N_C} = \frac{3}{8}$$

$$SU(N_C)$$

$$N_C = 3$$

★ QCD

A Measurement of the QCD color factors and a limit on the light gluino.

By ALEPH Collaboration ([R. Barate et al.](#)). Published in *Z.Phys.C76:1-14,1997*.

Hadron Multiplicity in Color Gauge Theory Models.

[Stanley J. Brodsky \(SLAC\)](#) , [J.F. Gunion \(UC, Davis\)](#) . SLAC-PUB-1749, UCD-76-5, May 1976. 13pp.
Published in *Phys.Rev.Lett.37:402-405,1976*.



Volume 47B, number 4

PHYSICS LETTERS

26 November 1973

ADVANTAGES OF THE COLOR OCTET GLUON PICTURE[☆]

H. FRITZSCH^{*}, M. GELL-MANN and H. LEUTWYLER^{**}

California Institute of Technology, Pasadena, Calif. 91109, USA

Received 1 October 1973

It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

Fritzsch, Gell-Mann, and Leutwyler introduce ‘Quantum Chromodynamics’ (QCD) as the Gauge Theory of the Strong Interactions

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QCD Lagrangian

The diagram shows the QCD Lagrangian L_{QCD} enclosed in a red box. Labels with arrows point to various parts of the equation:

- gluon dynamics** points to the first term: $-\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu})$
- quark kinetic energy + quark-gluon dynamics** points to the second term: $\sum_{f=1}^{nf} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f$
- mass term** points to the third term: $\sum_{f=1}^{nf} m_f \bar{\Psi}_f \Psi_f$
- QCD color charge** points to the g^2 in the denominator of the first term.
- field strength tensor** points to $G^{\mu\nu}$ in the first term.
- covariant derivative** points to D_μ in the second term.
- quark field** points to Ψ_f in the second term.

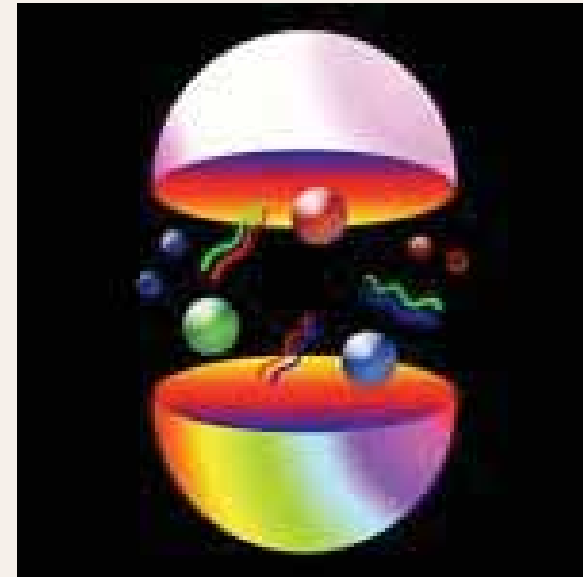
$$L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{nf} m_f \bar{\Psi}_f \Psi_f$$

*Yang-Mills Gauge Principle:
Invariance under Color
Rotation and Phase Change
at Every Point of Space and
Time*

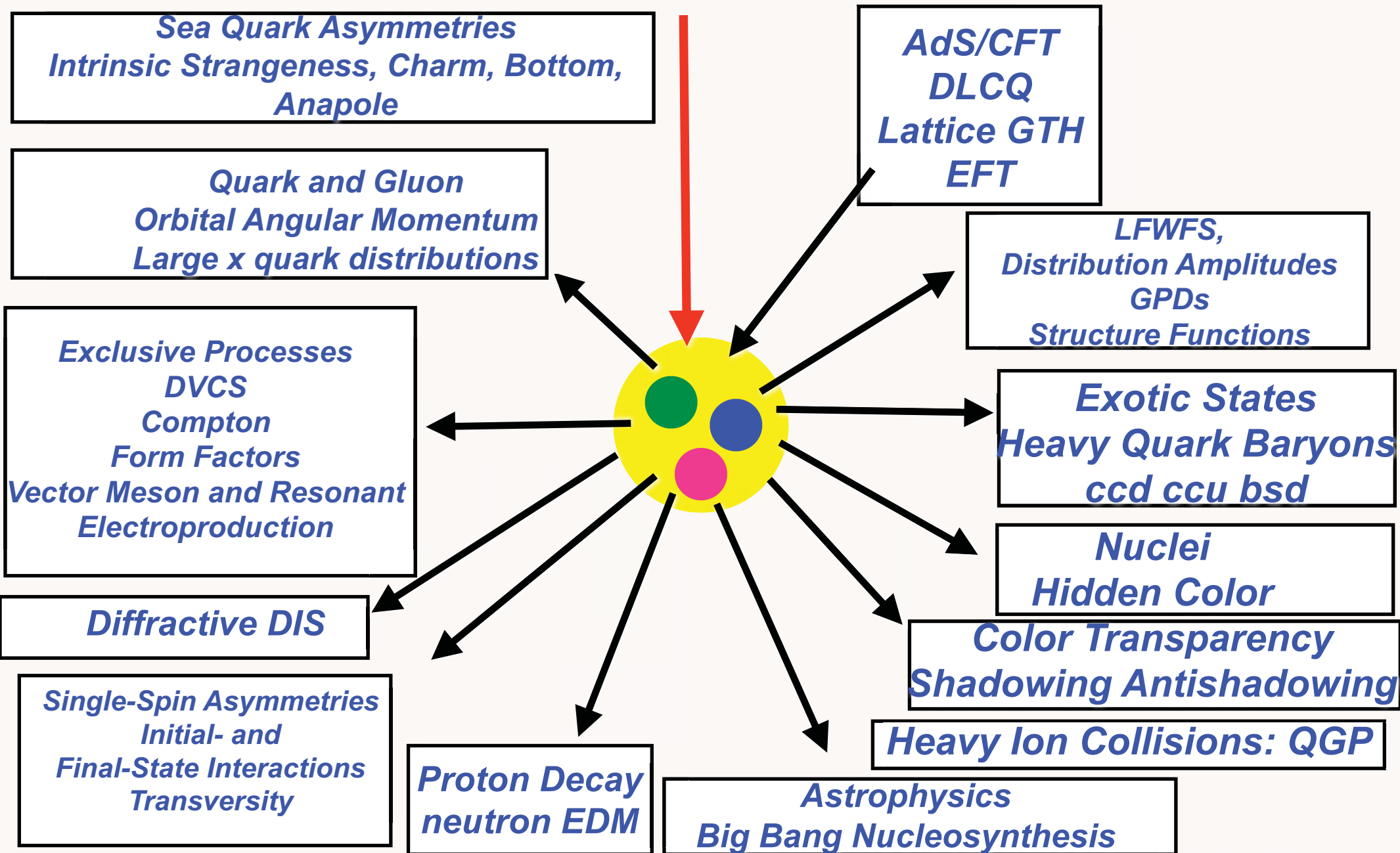
Dimensionless Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

The World of Quarks and Gluons:

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of *Quantum Chromodynamics (QCD)*
- New Insights from higher space-time dimensions: Light-Front Holography: AdS/CFT



QCD Lagrangian



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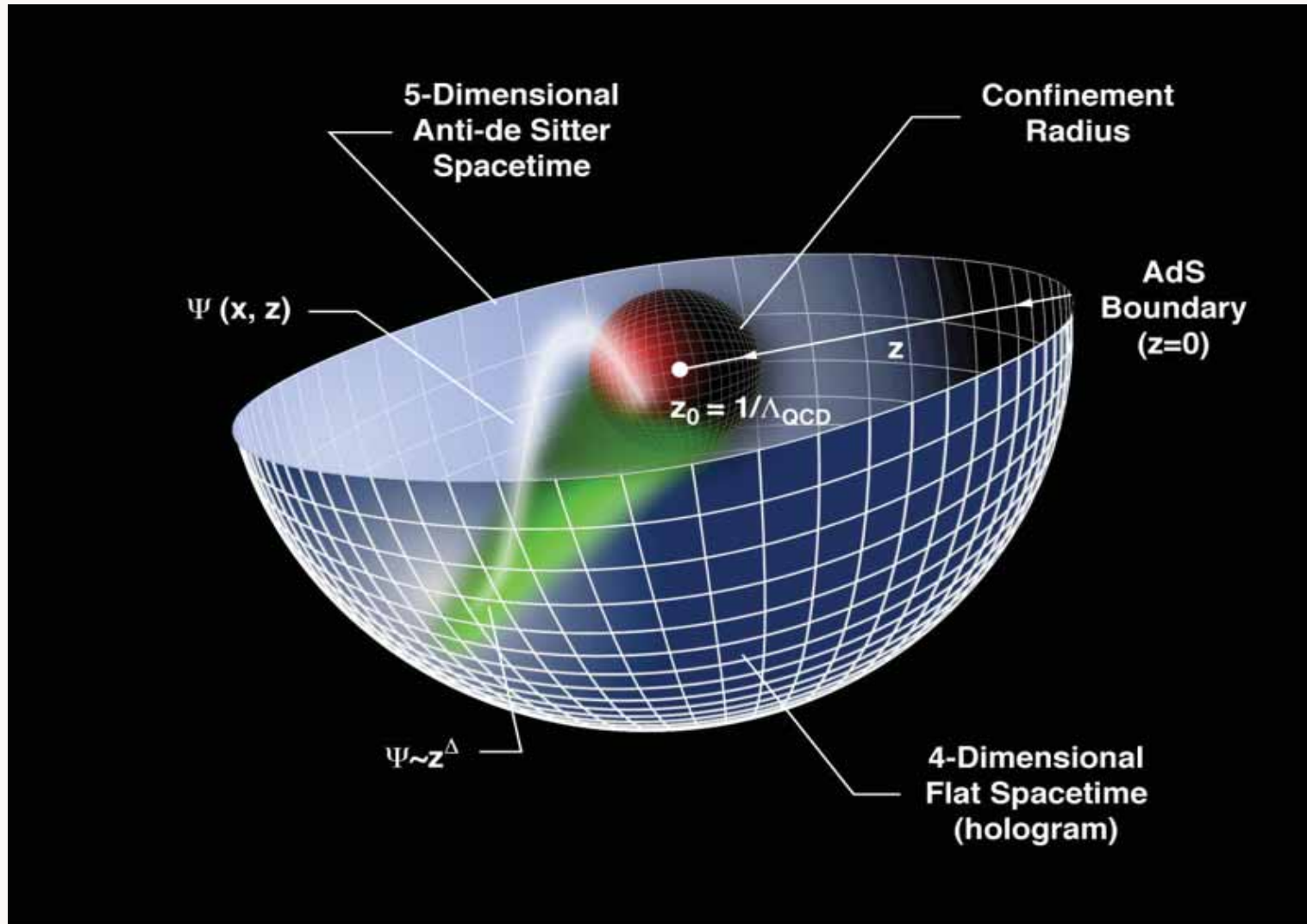
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- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.
- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, QGP, CGL, ...

*Truth is stranger than fiction,
but it is because Fiction is
obliged to stick to possibilities.*

—Mark Twain

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

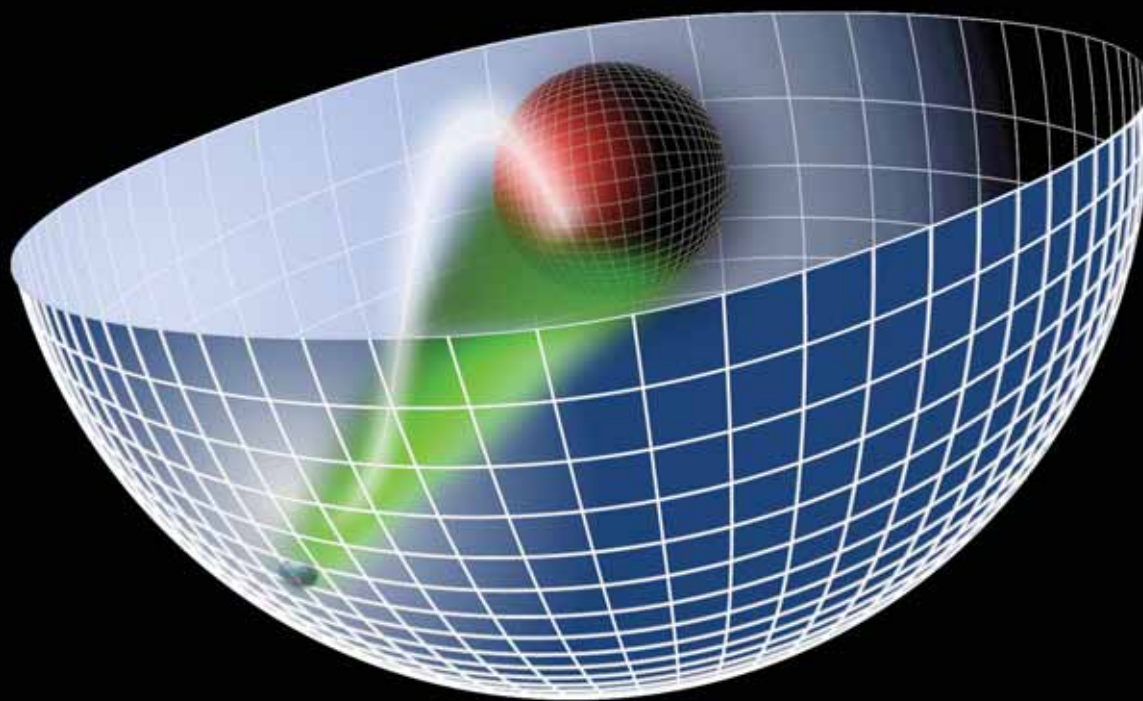
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Goal:

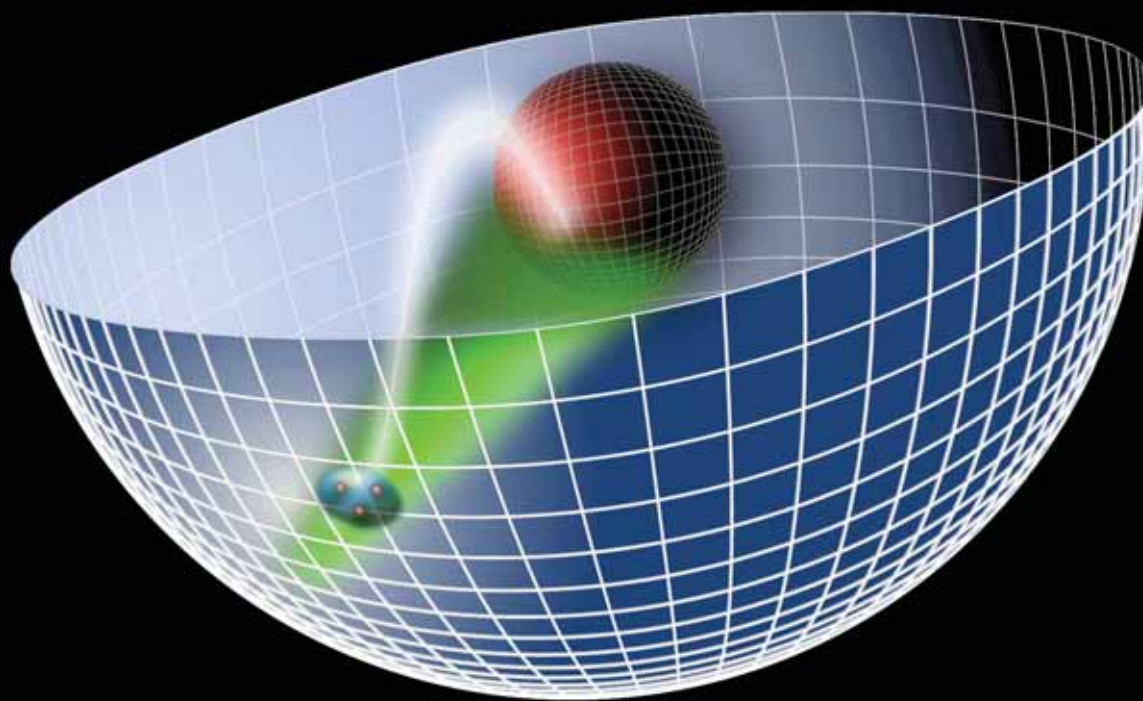
- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*
- *Hadronization at the Amplitude Level*



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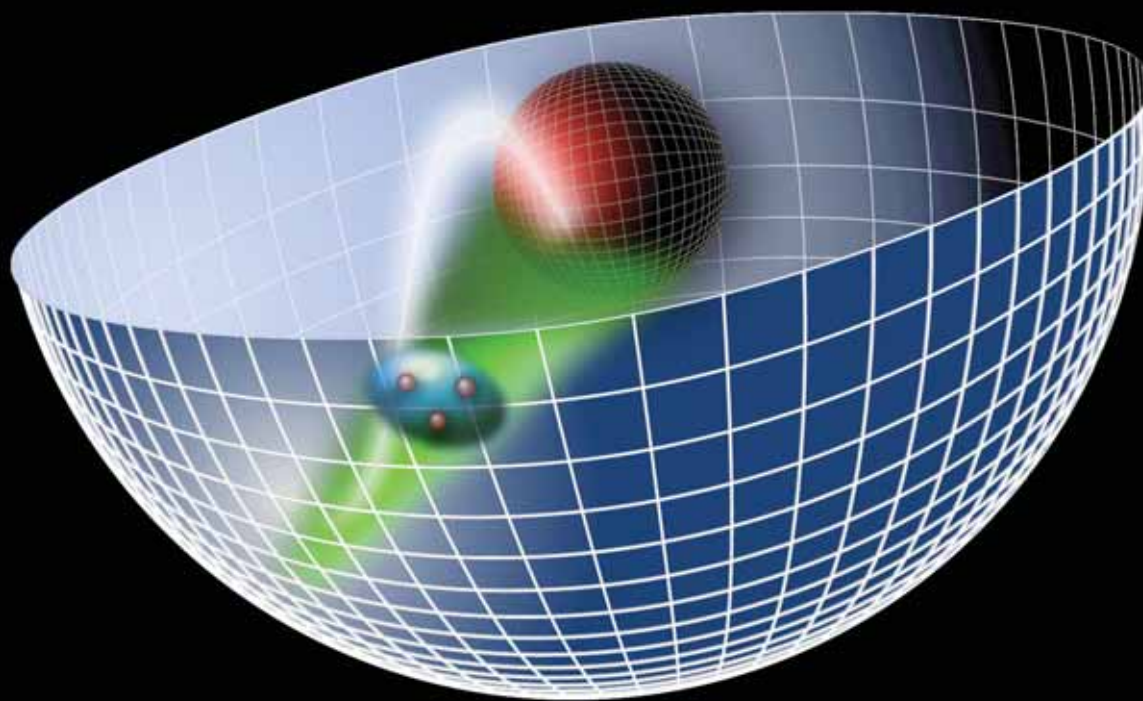
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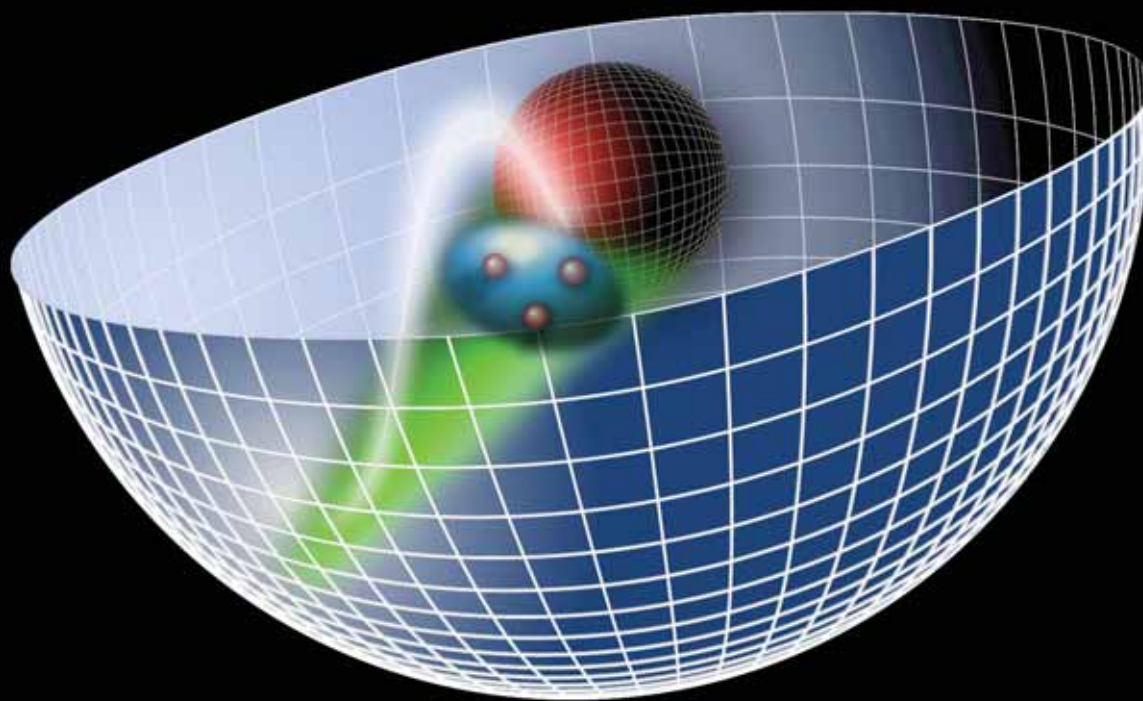
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Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of $SO(4,2)$


$SO(4,2)$ has a mathematical representation on AdS_5

Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure



$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S^5$ to conformal $N=4$ SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

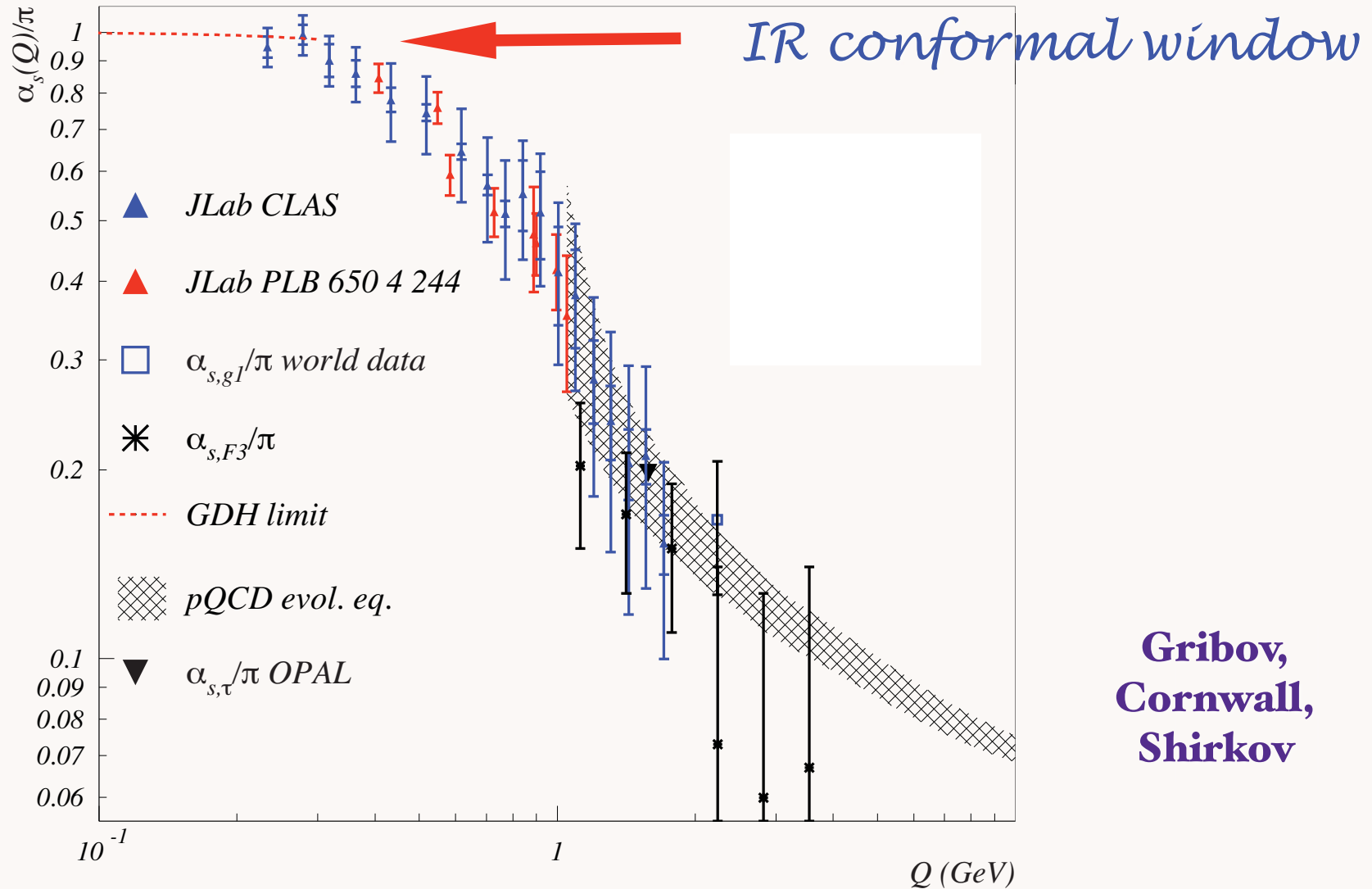
- **Conformal window in the IR:**

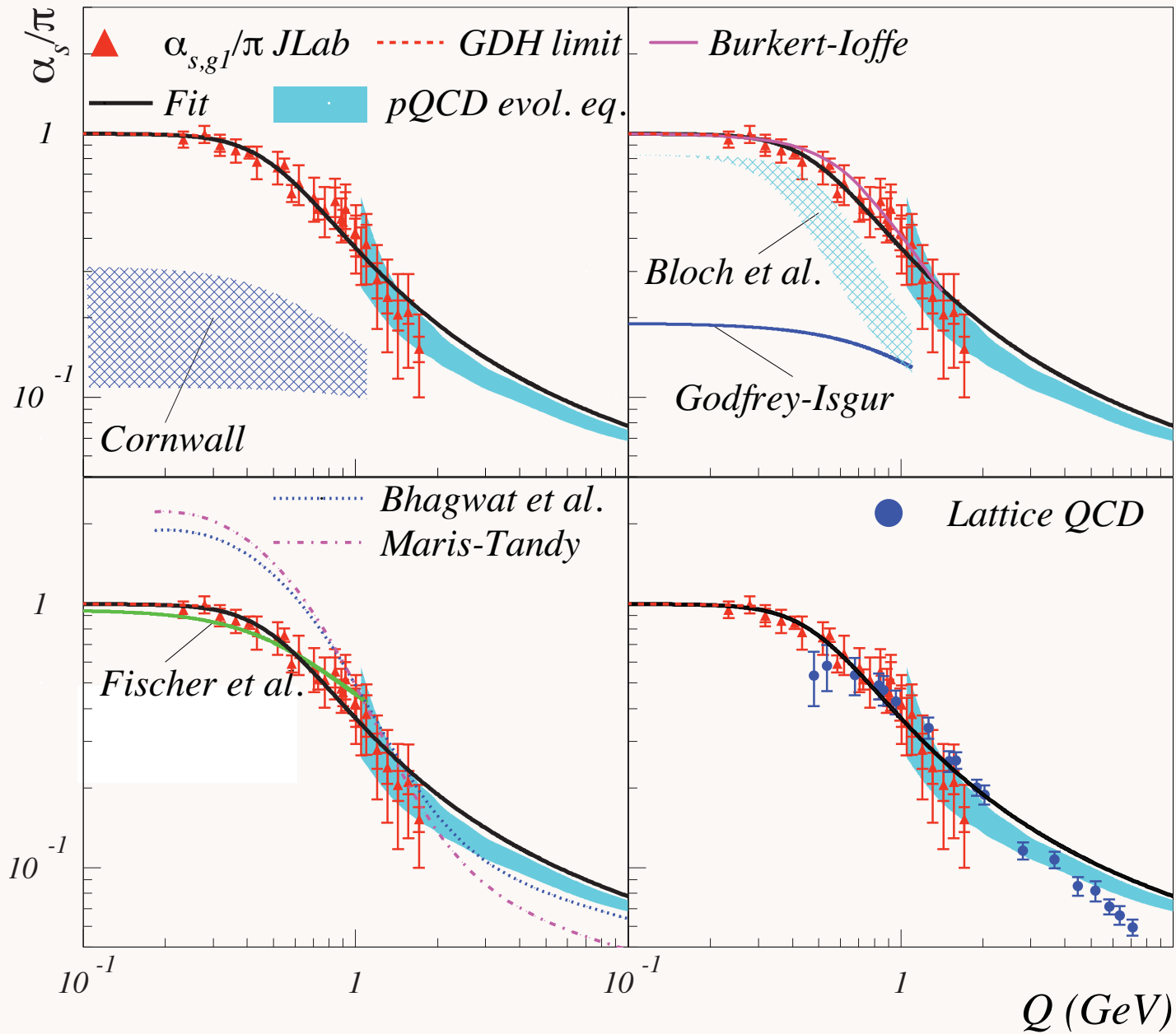
$$\alpha_s(Q^2) \simeq \text{const at small } Q^2$$

- **Use mathematical mapping of the conformal group $SO(4,2)$ to AdS_5 space**

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi} \right]$$





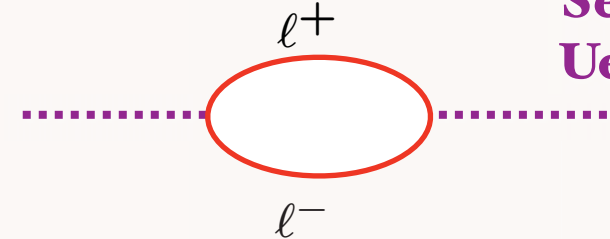
IR Conformal Window for QCD?

- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

Shrock,
de Teramond,
sjb

- **Confined gluons and quarks have maximum wavelength**
- **Decoupling of QCD vacuum polarization at small Q^2**

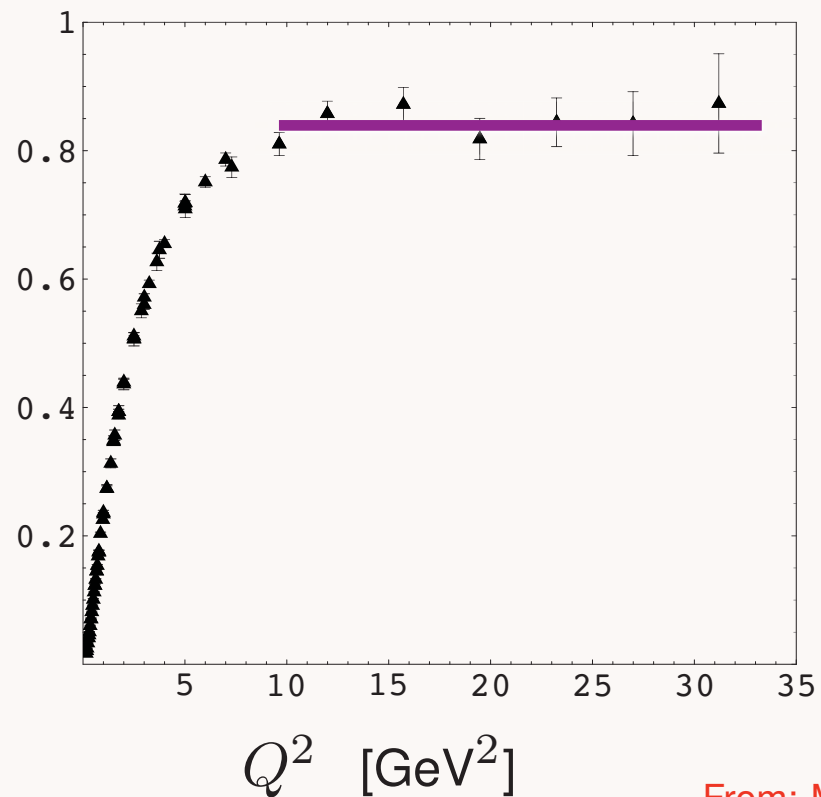
$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$



Serber-
Uehling

- **Justifies application of AdS/CFT in strong-coupling conformal window**

$Q^4 F_1^p(Q^2)$ [GeV⁴]



$$F_1(Q^2) \sim [1/Q^2]^{n-1}, \quad n = 3$$

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

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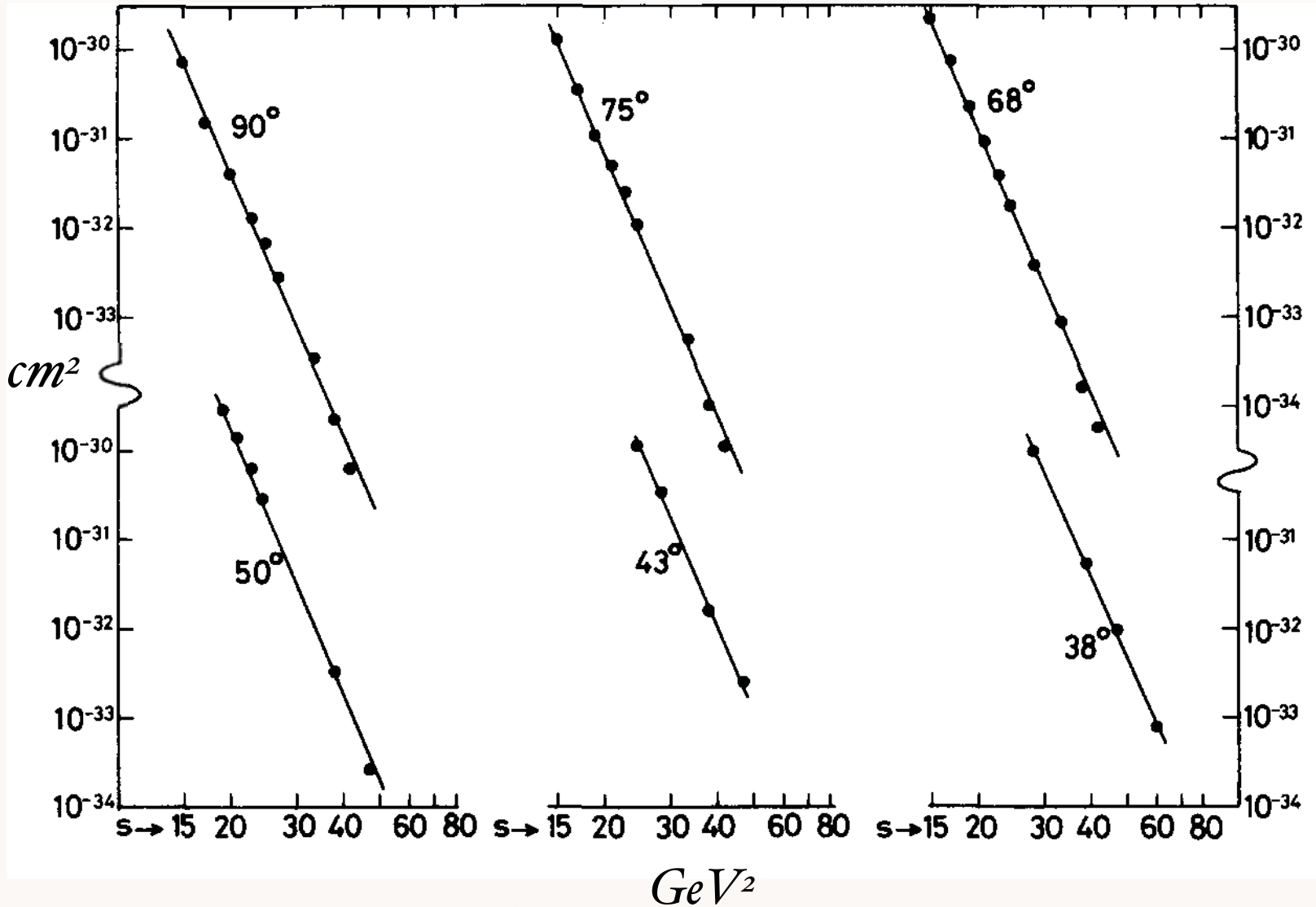
Quark Counting Rules for Exclusive Processes

- Power-law fall-off of the scattering rate reflects degree of compositeness
- The more composite -- the faster the fall-off
- Power-law counts the number of quarks and gluon constituents
- Form factors: probability amplitude to stay intact
- $F_H(Q) \propto \frac{1}{(Q^2)^{n-1}}$ **n = # elementary constituents**

Quark-Counting : $\frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}}$

$n = 4 \times 3 - 2 = 10$

P.V. LANDSHOFF and J.C. POLKINGHORNE



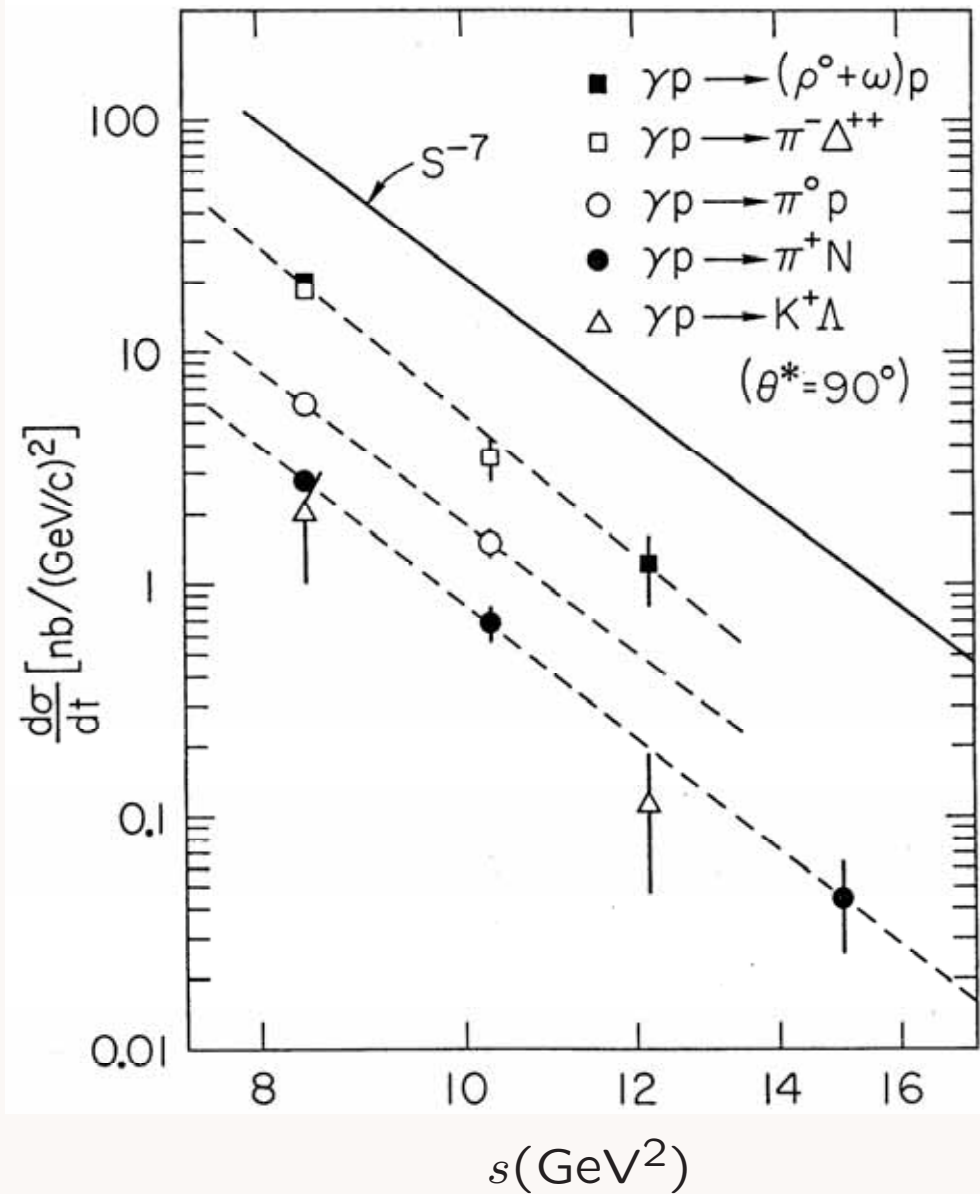
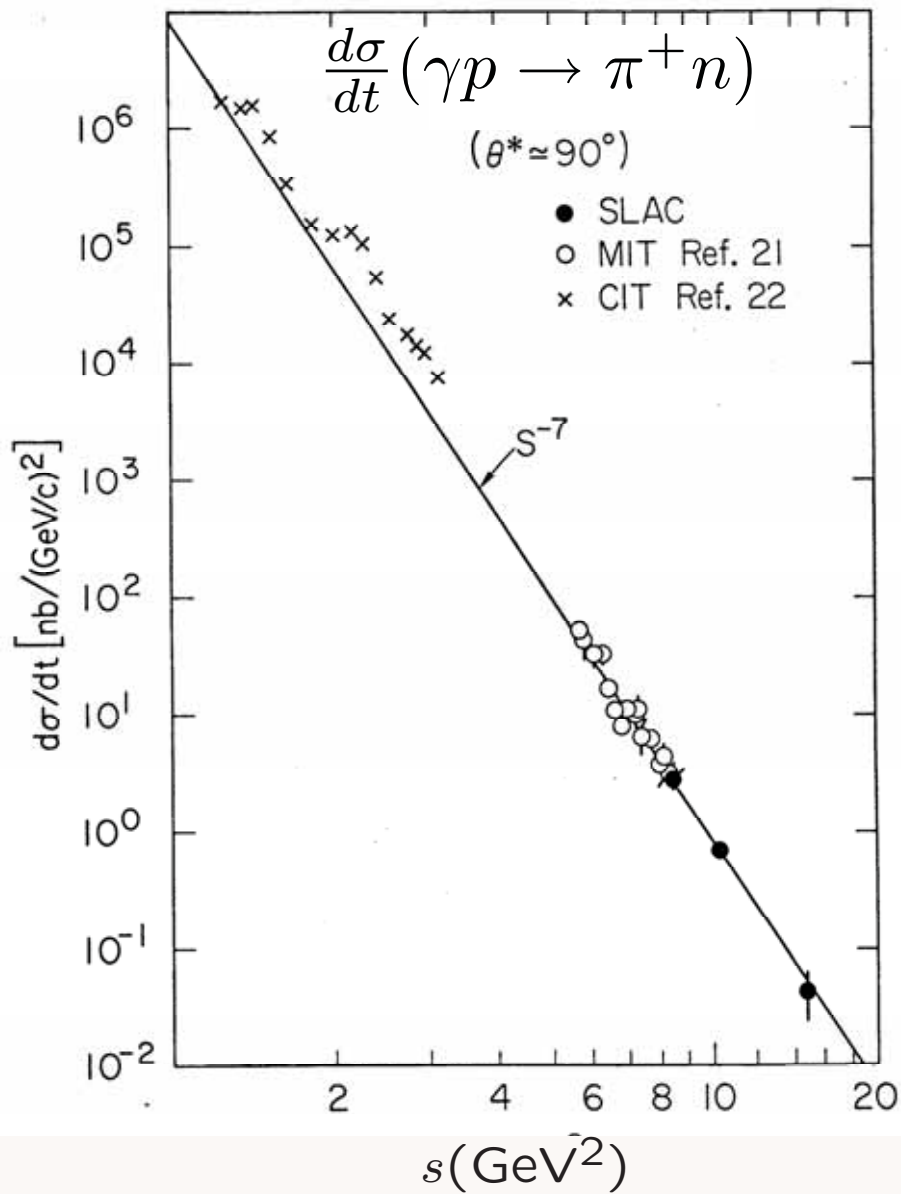
Best Fit
 $n = 9.7 \pm 0.5$
 Reflects underlying conformal scale-free interactions

Angular distribution -- quark interchange

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Conformal Invariance:

$$\frac{d\sigma}{dt} (\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7}$$

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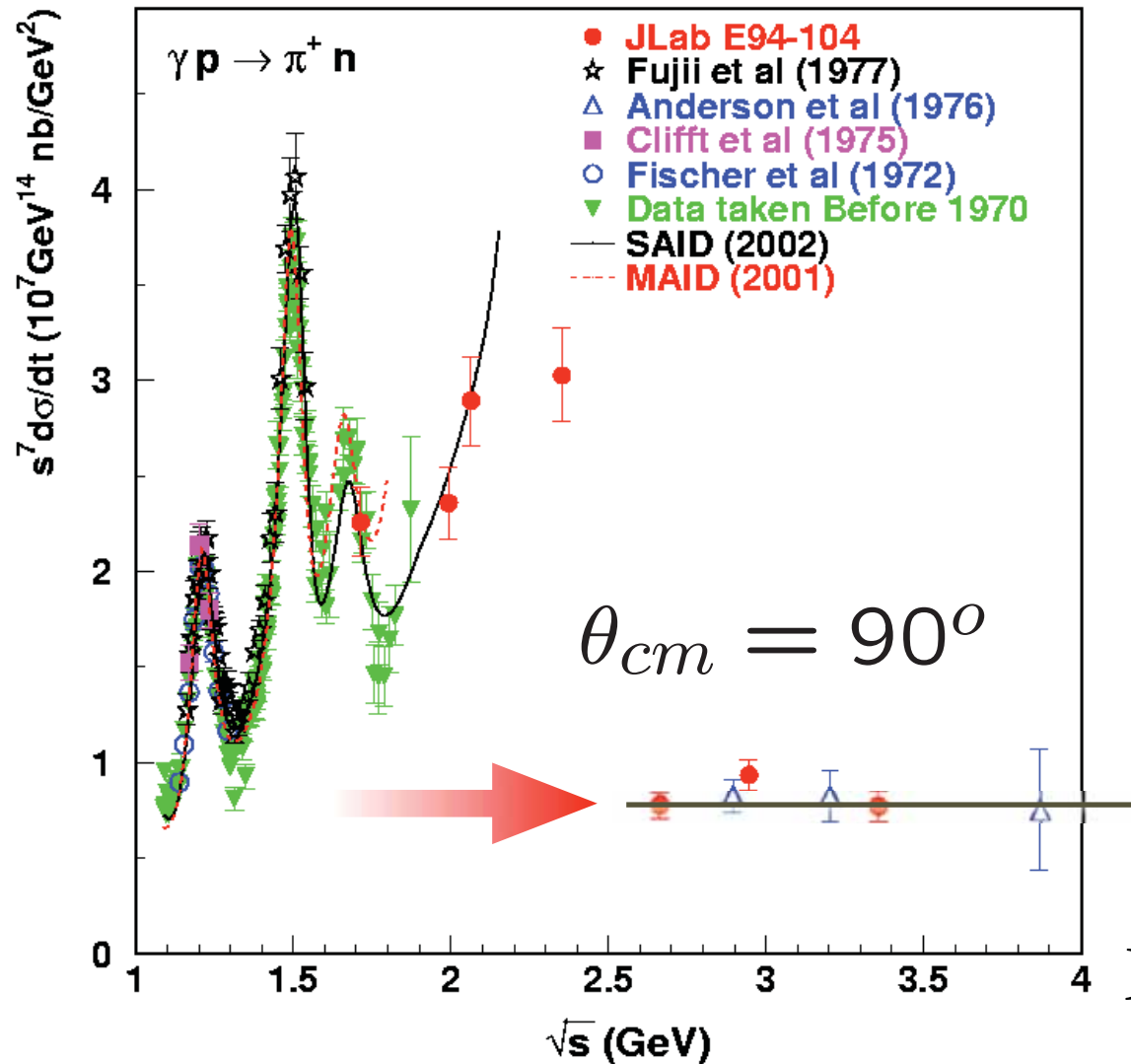
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Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Tavkelidze



$$s^7 d\sigma/dt(\gamma p \rightarrow \pi^+ n) \sim \text{const}$$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

$$s^{n_{tot}-2} \frac{d\sigma}{dt}(A+B \rightarrow C+D) = F_{A+B \rightarrow C+D}(\theta_{CM})$$

$$s^7 \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1 + 3 + 2 + 3 = 9$$

No sign of running coupling

Conformal invariance

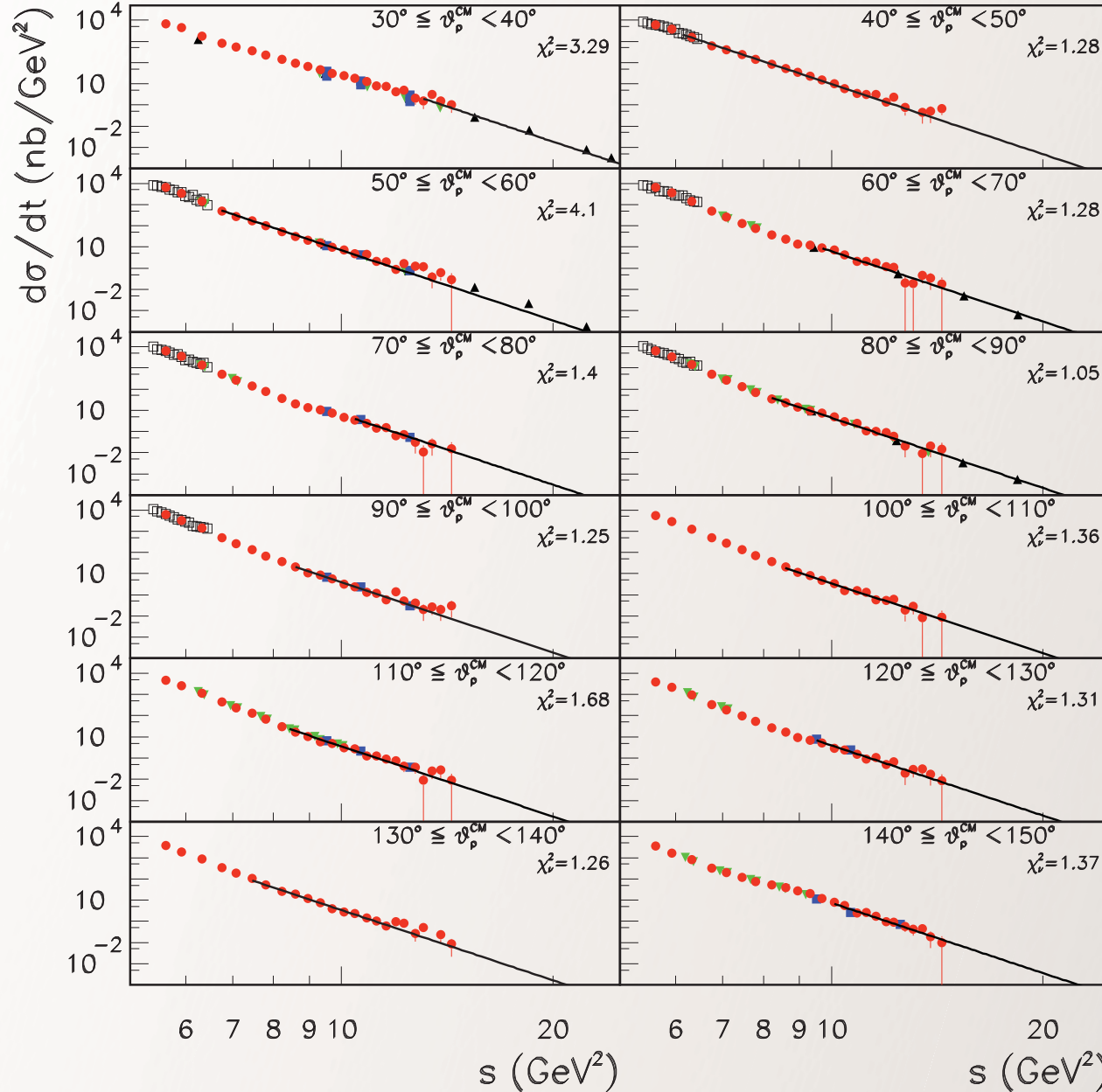
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Deuteron Photodisintegration

J-Lab



PQCD and AdS/CFT:

$$s^{n_{tot}-2} \frac{d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B \rightarrow C+D}(\theta_{CM})$$

$$s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM})$$

$$n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11$$

Reflects conformal invariance

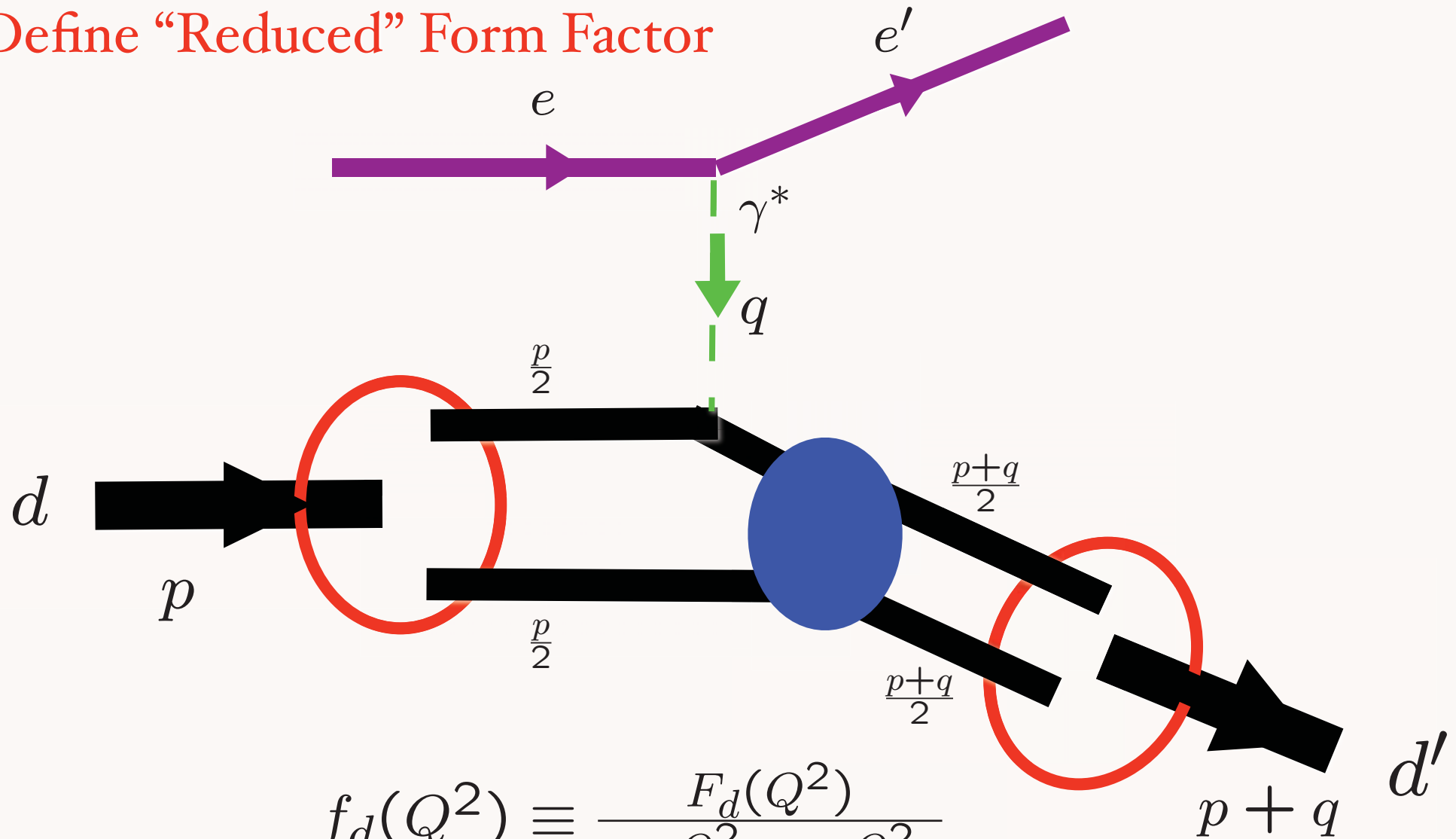
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Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n^d - \gamma_m^d} \left[1 + \mathcal{O} \left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-(2/5) C_F/B}$$

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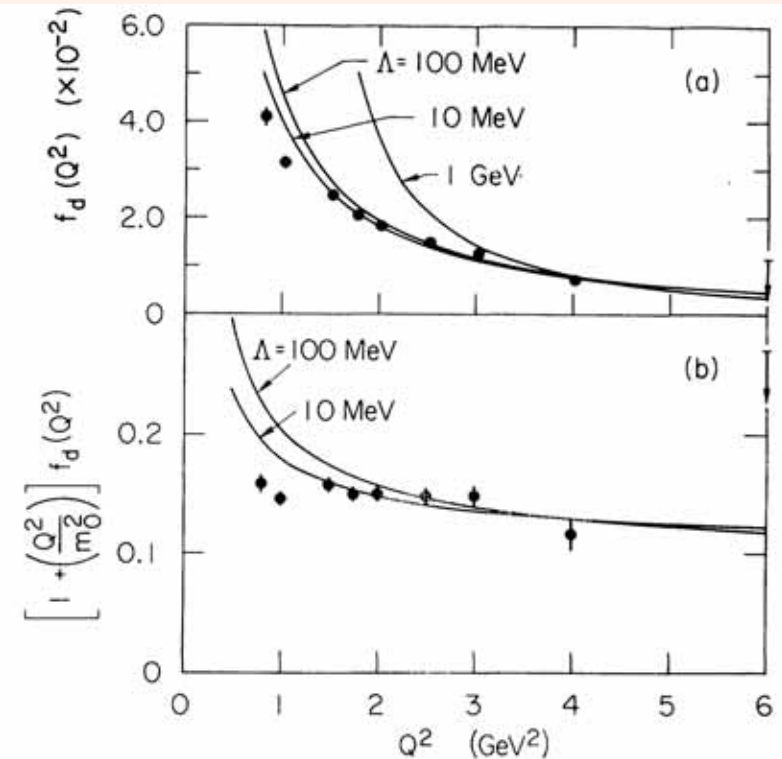
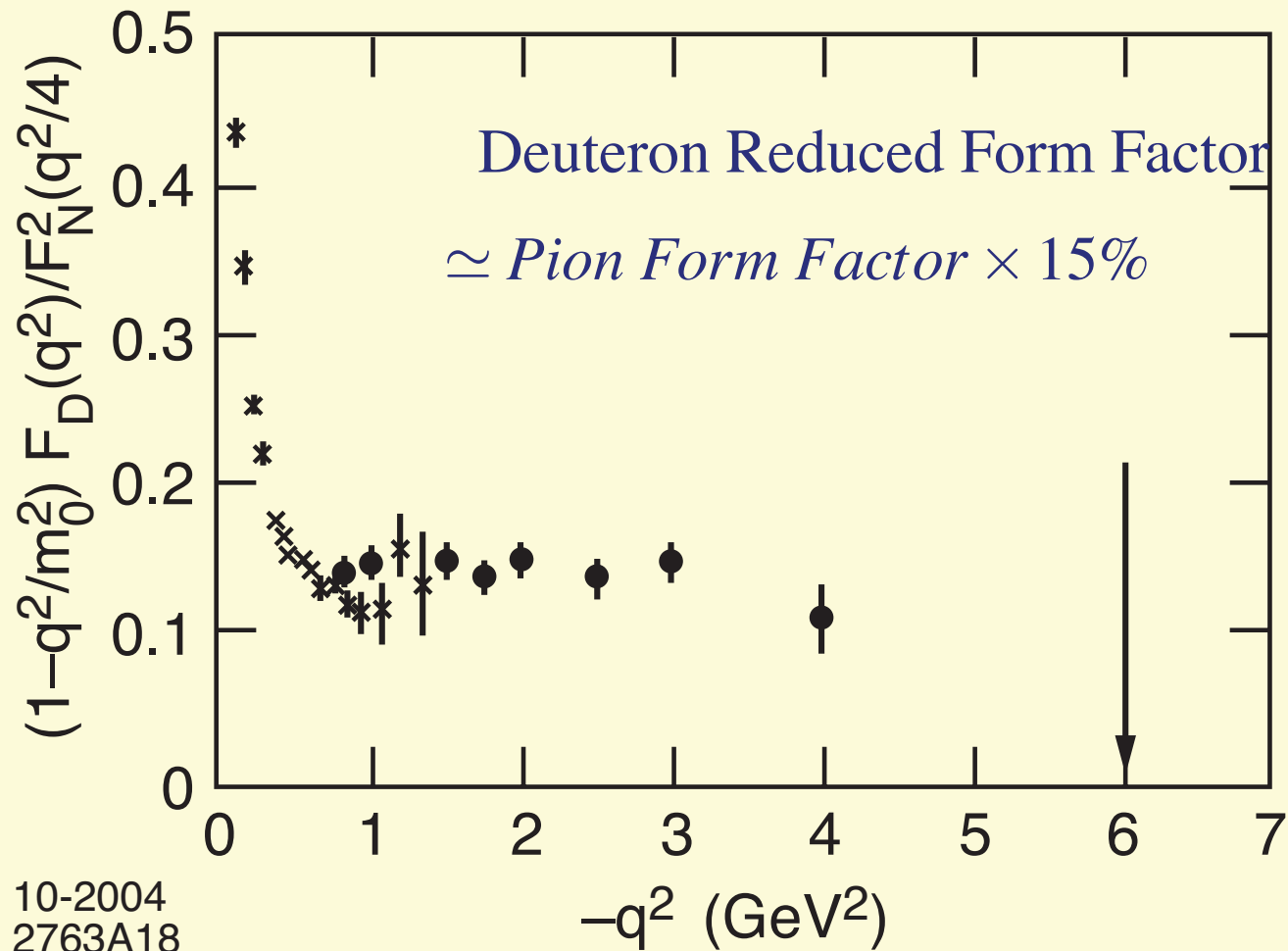


FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2) [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/B}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71 \text{ GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4 \text{ GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/B}$ with the above data. The value $m_0^2 = 0.28 \text{ GeV}^2$ is used (Ref. 8).



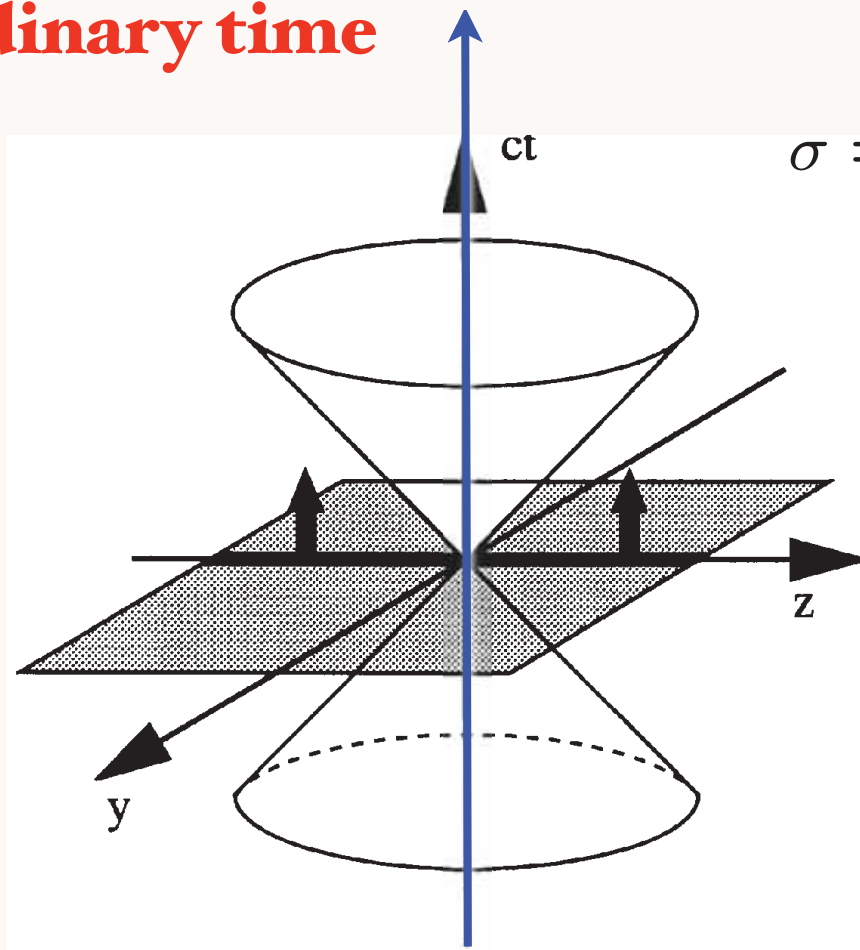
- 15% Hidden Color in the Deuteron

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is $|n\ p\rangle$
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- **Predict** $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$ at high Q^2

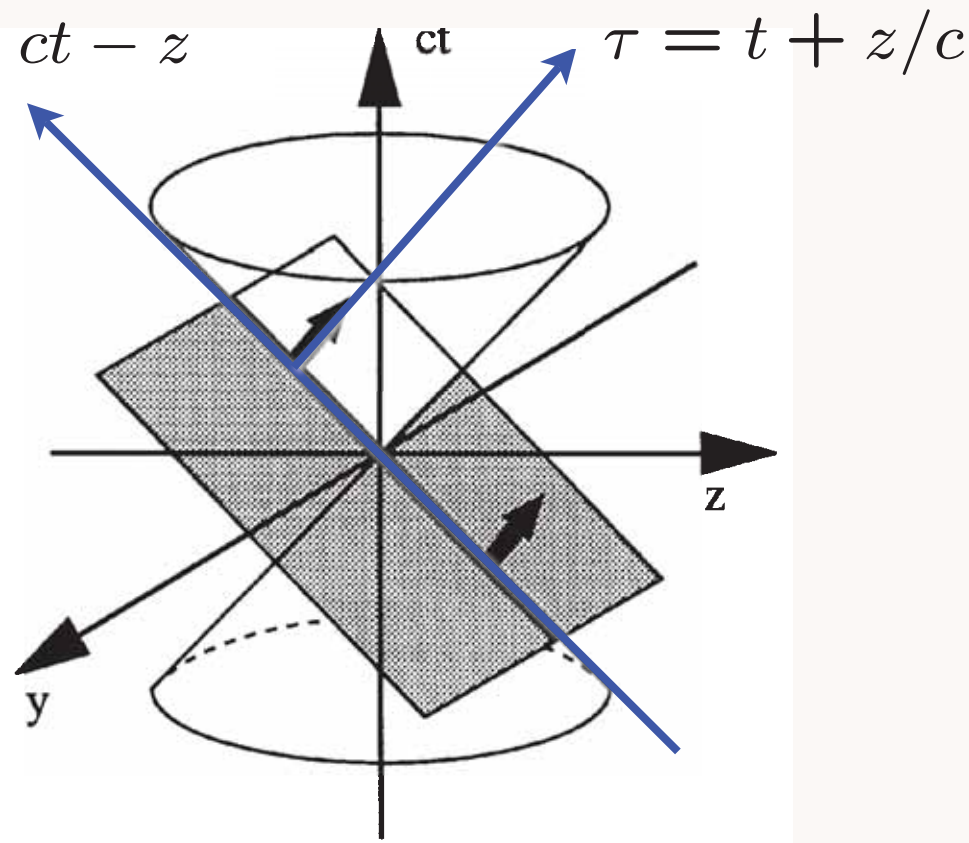
Dirac's Amazing Idea: The Front Form

**Evolve in
ordinary time**

**Evolve in
light-front time!**



$$\sigma = ct - z$$



$$\tau = t + z/c$$

Instant Form

Front Form

*Each element of
flash photograph
illuminated
at same LF time*

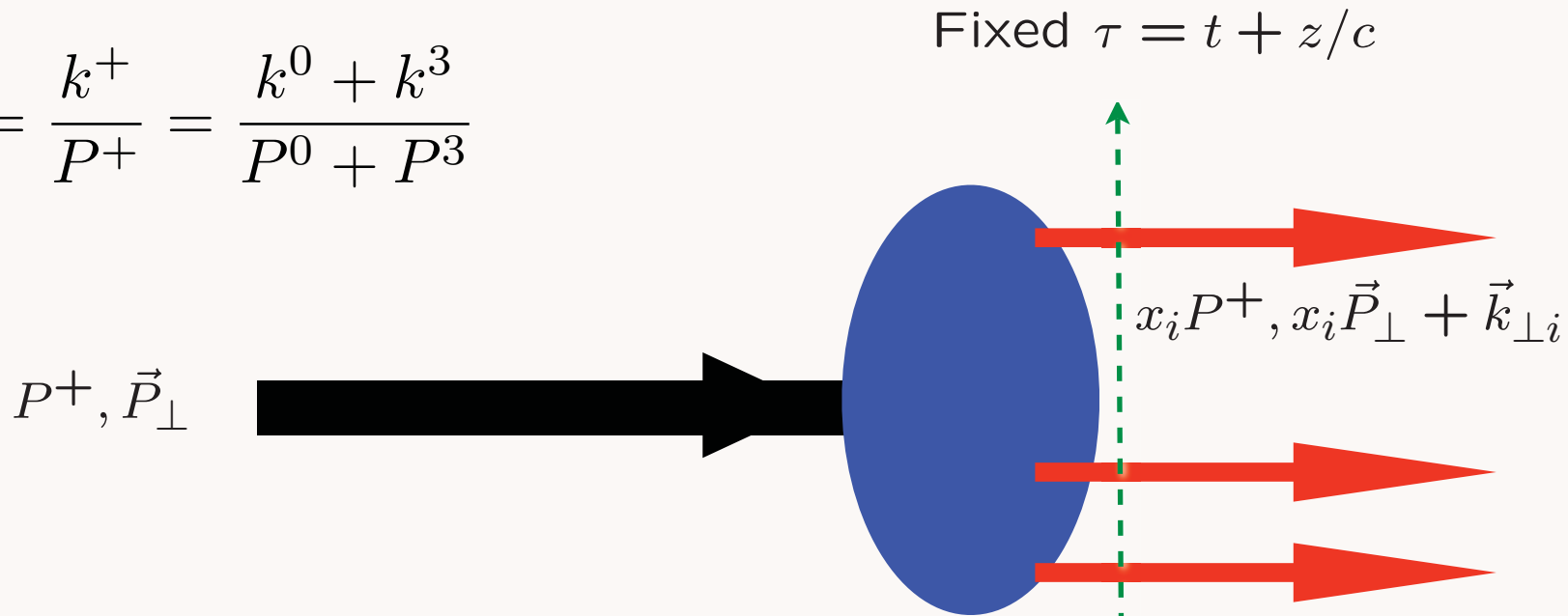
$$\tau = t + z/c$$



HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of p^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

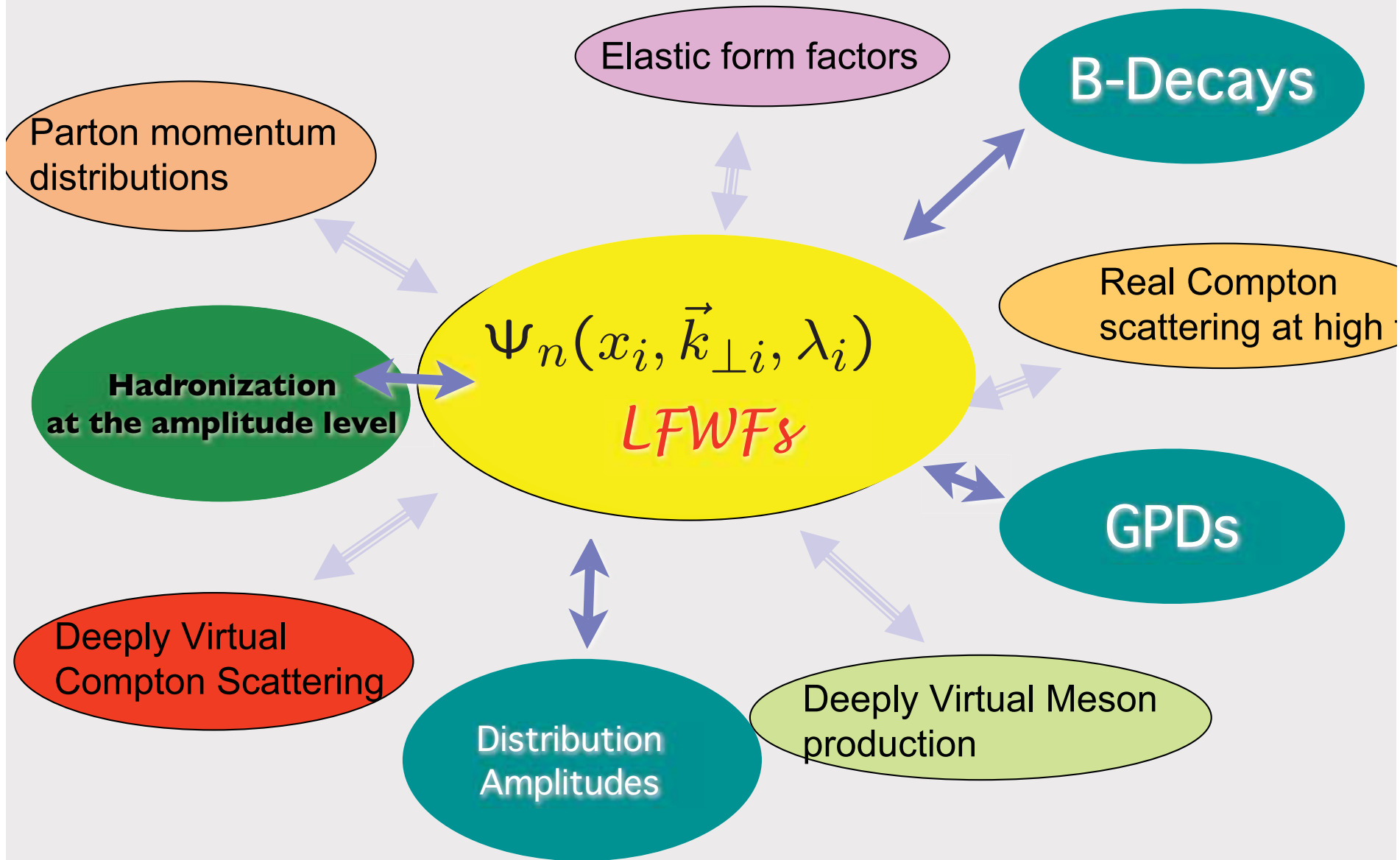
Conserved
LF Fock state by Fock State

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

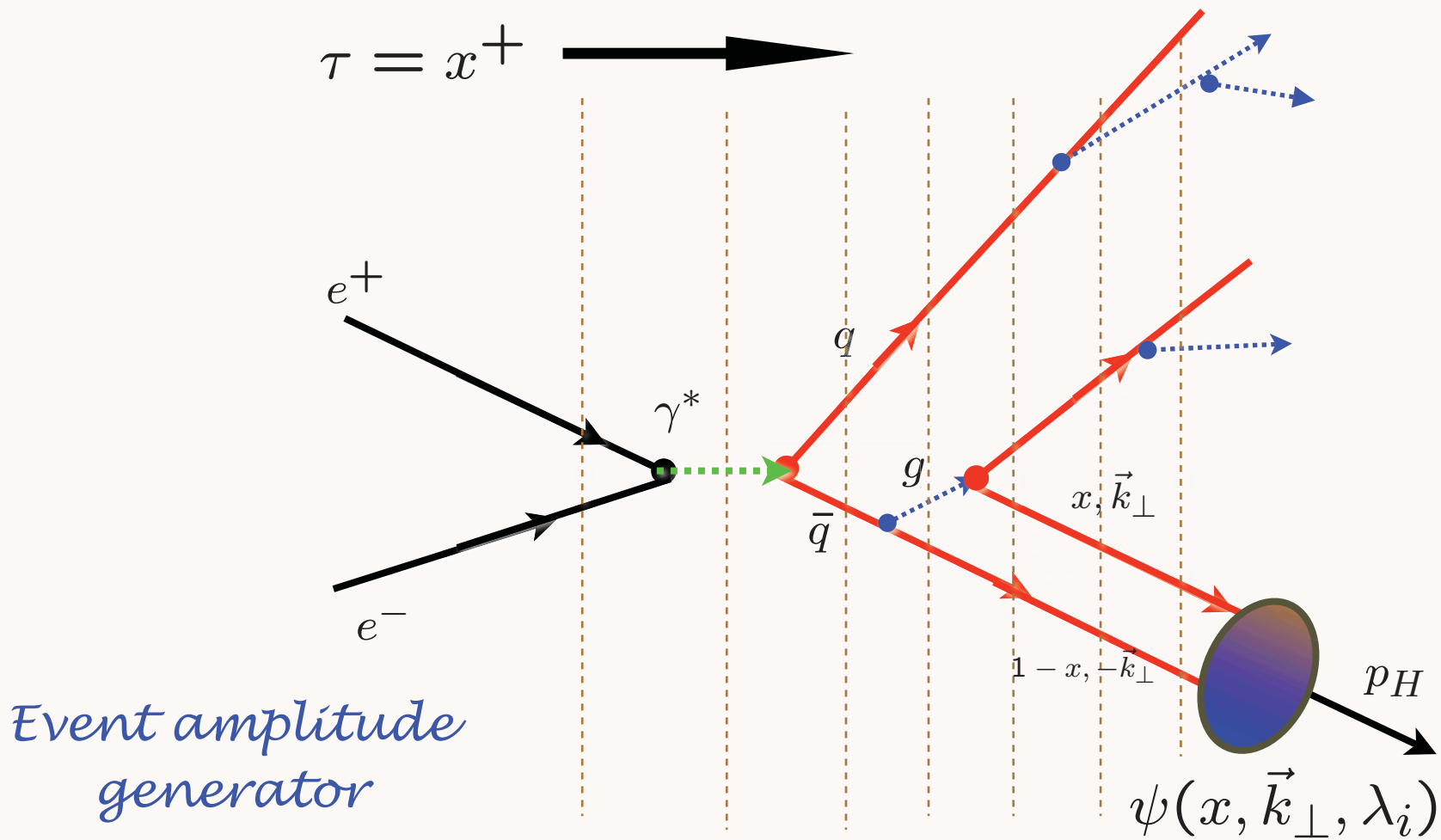
n-1 orbital angular momenta

Nonzero Anomalous Moment --> Nonzero orbital angular momentum

A Unified Description of Hadron Structure



Hadronization at the Amplitude Level

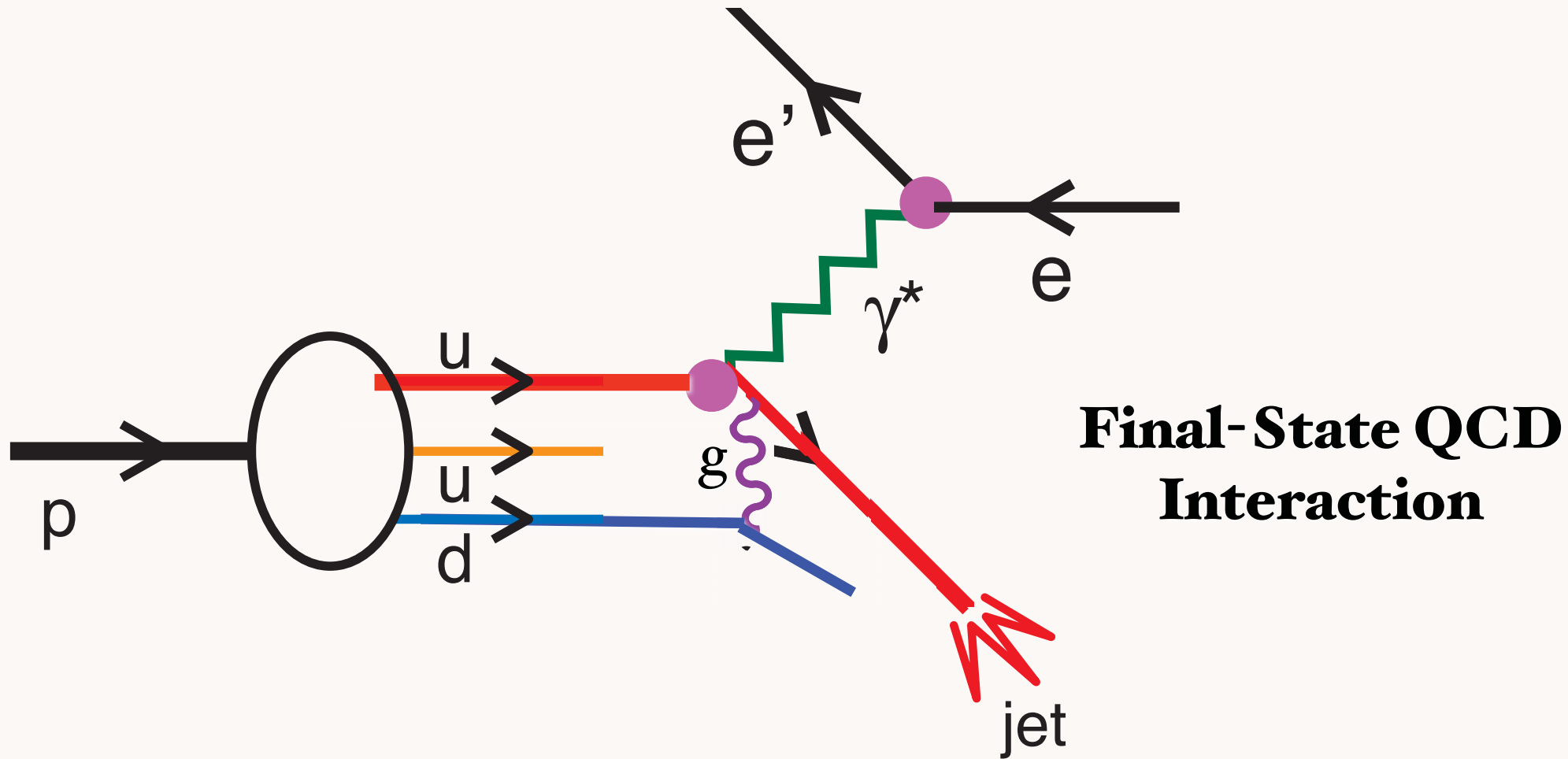


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes, distribution amplitudes, direct subprocesses, hadronization.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs: Diffractive DIS, Sivers effect

Deep Inelastic Electron-Proton Scattering



*Conventional wisdom:
Final-state interactions of struck quark can be neglected*

Single-spin asymmetries

Leading Twist Sivers Effect

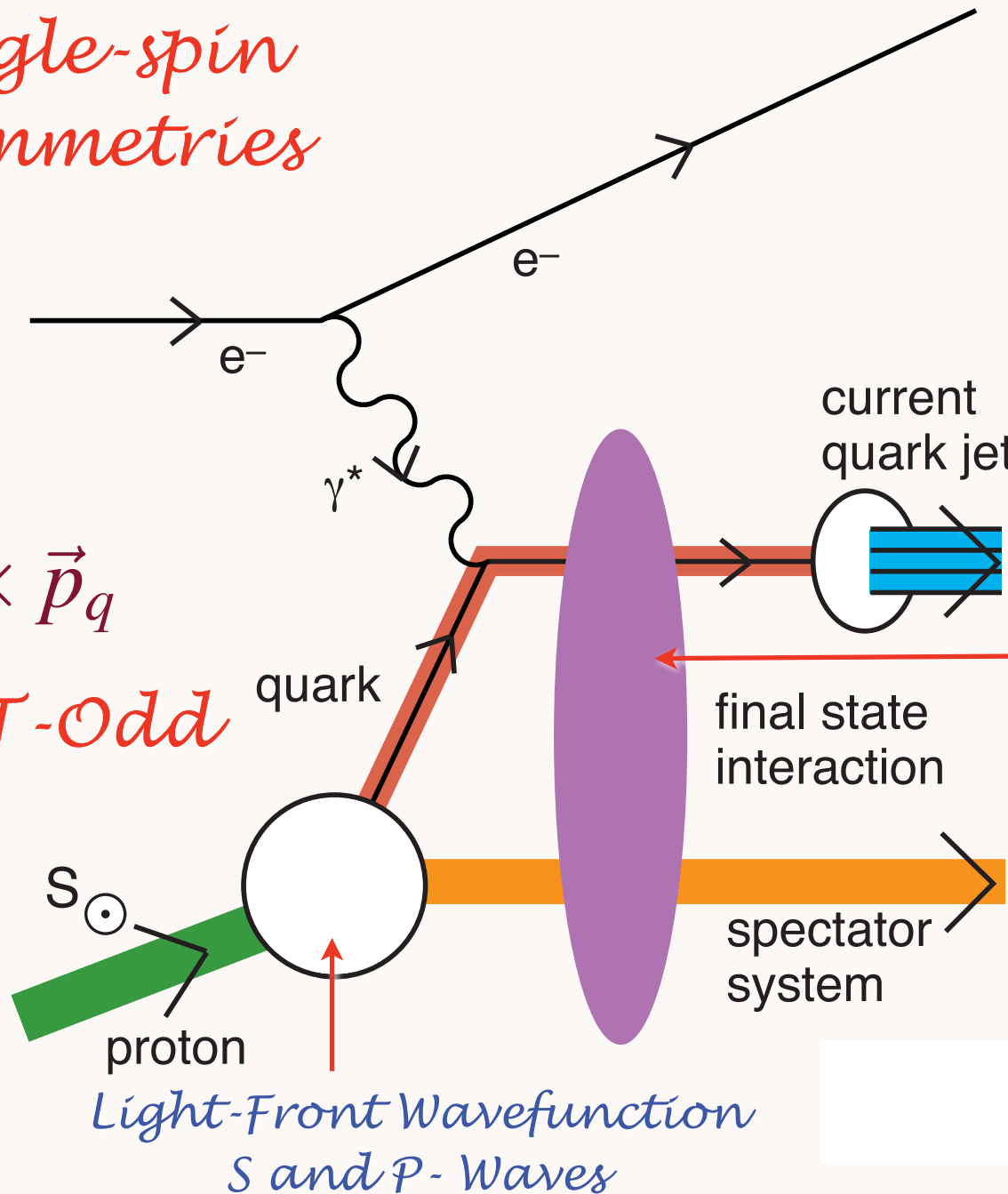
Hwang,
Schmidt, sjb

Collins, Burkardt
Ji, Yuan

*QCD S- and P-
Coulomb Phases
--Wilson Line*

$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

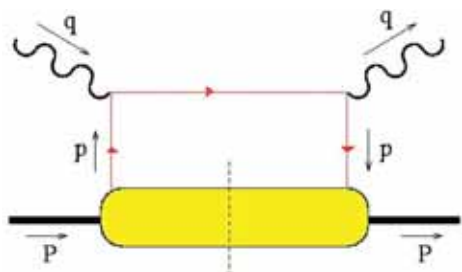
Pseudo-T-Odd



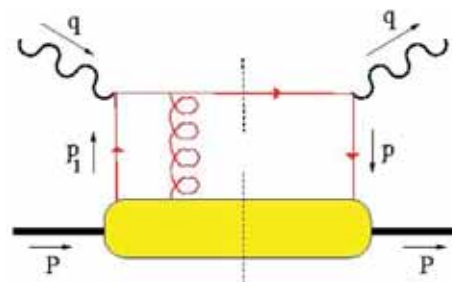
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AdS/QCD
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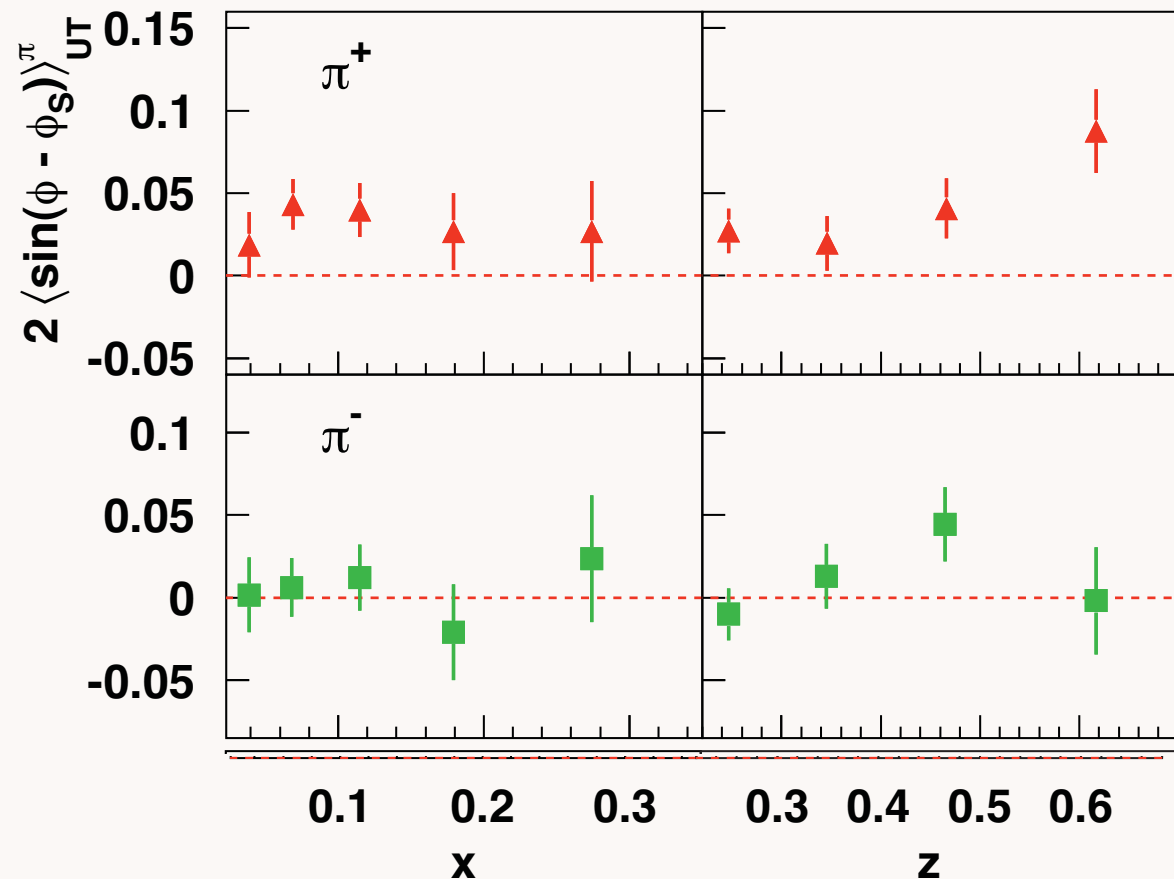
can interfere with



and produce a T-odd effect!
(also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

Sivers asymmetry from HERMES



- First evidence for non-zero Sivers function!
- \Rightarrow presence of non-zero **quark orbital angular momentum!**
- **Positive** for π^+ ...
Consistent with zero for π^- ...

Gamberg: Hermes data compatible with BHS model

Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment

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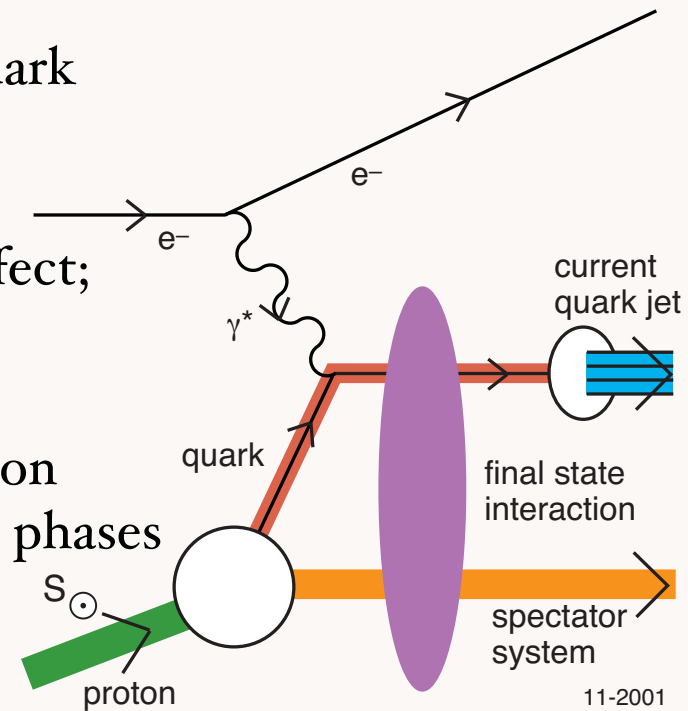
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Final-State Interactions Produce Pseudo-T-Odd (Sivers Effect)

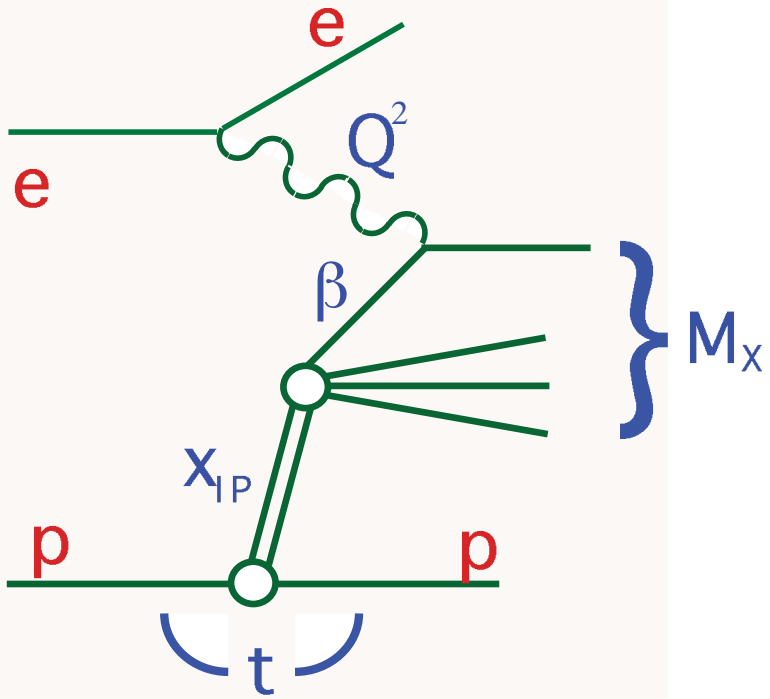
- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

$$\mathbf{i} \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

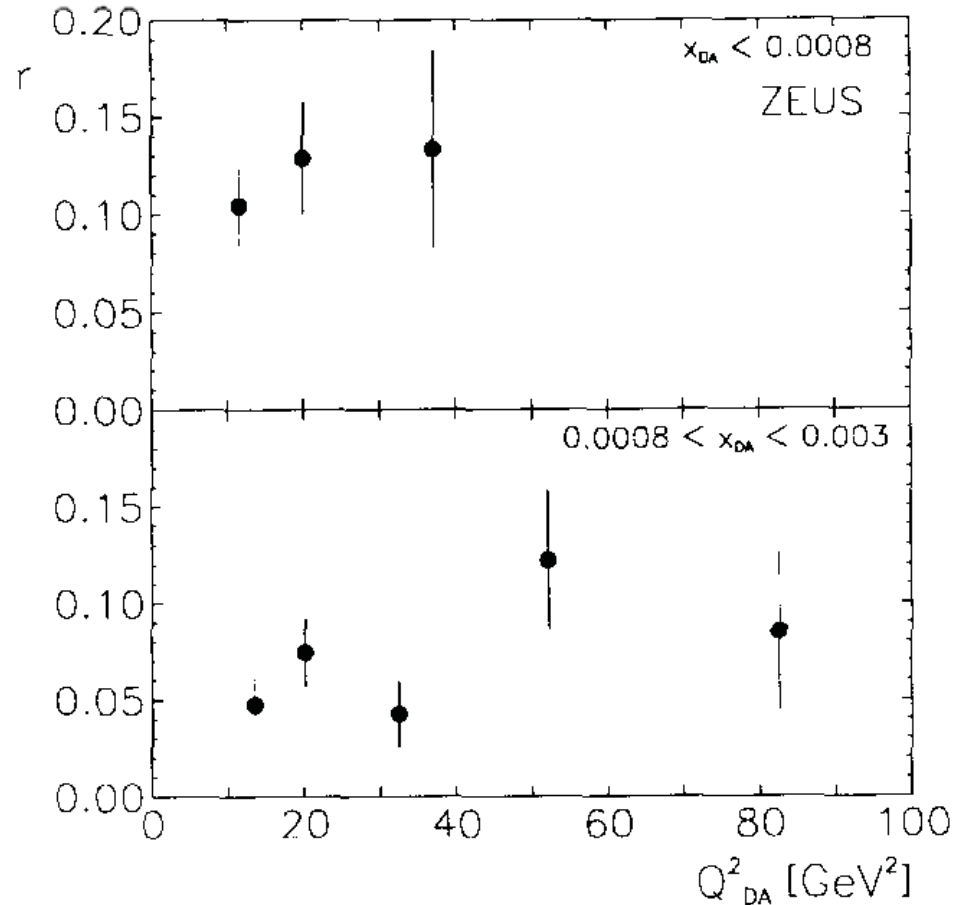


11-2001
8624A06

Remarkable observation at HERA



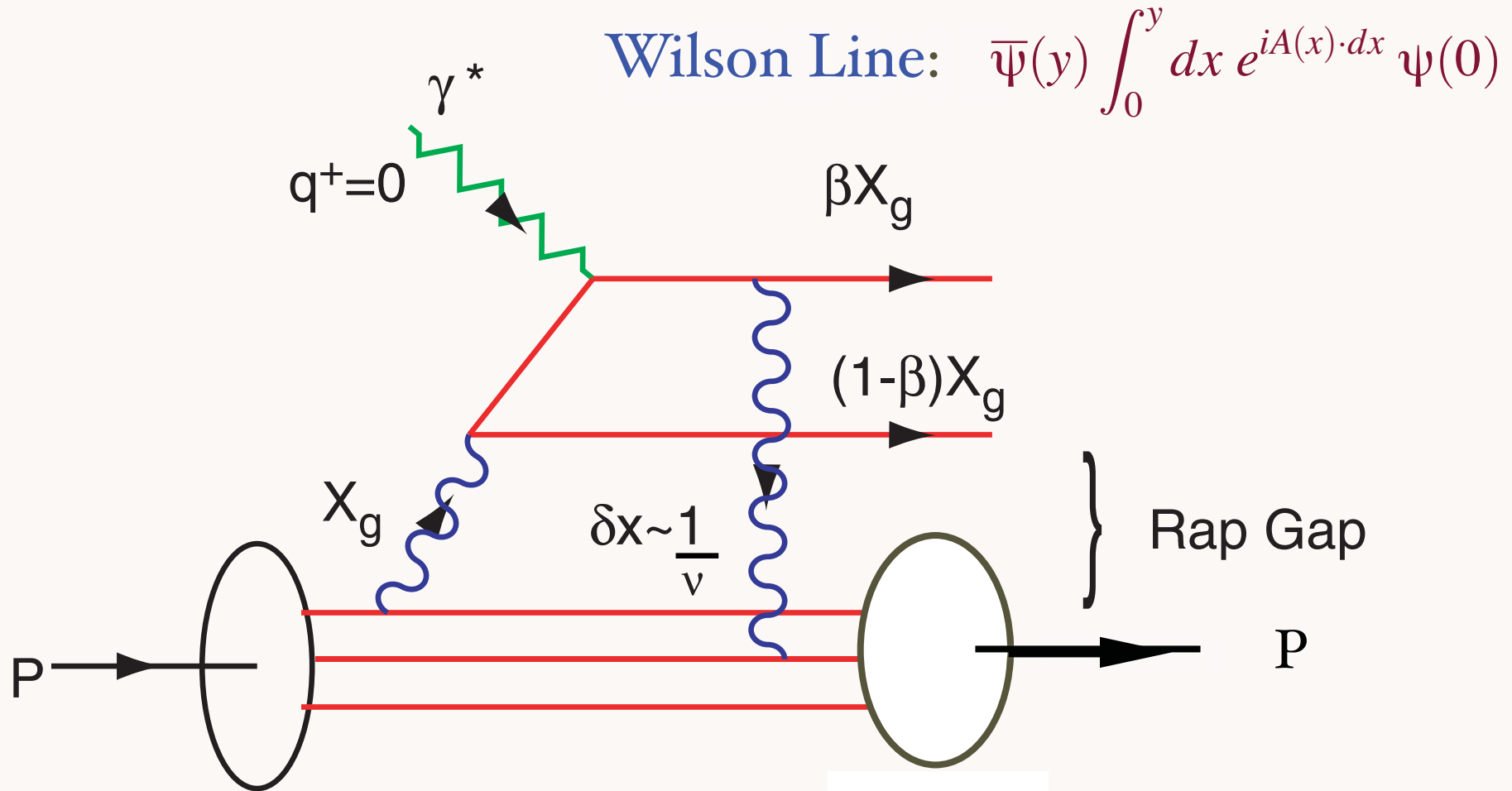
*10% to 15%
of DIS events
are
diffractive!*



Fraction r of events with a large rapidity gap, $\eta_{\max} < 1.5$, as a function of Q_{DA}^2 for two ranges of x_{DA} . No acceptance corrections have been applied.

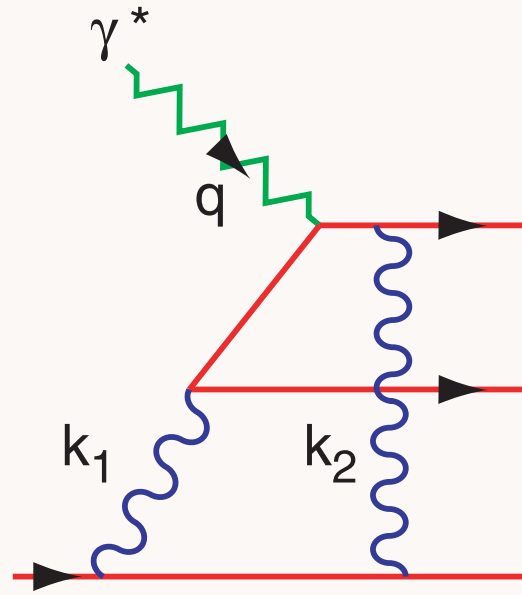
M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993).

QCD Mechanism for Rapidity Gaps

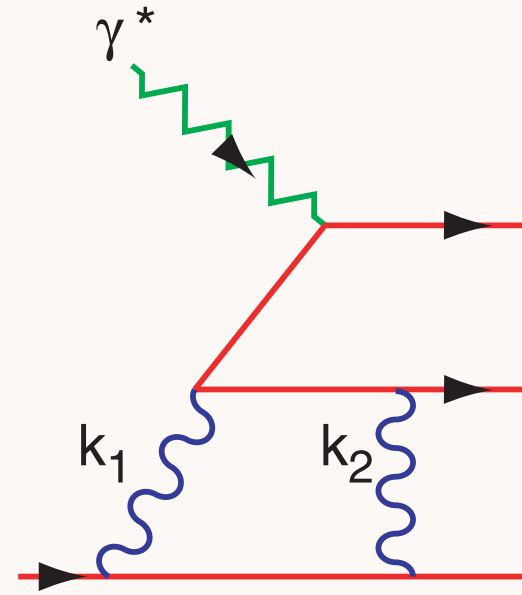


Reproduces lab-frame color dipole approach

Final State Interactions in QCD

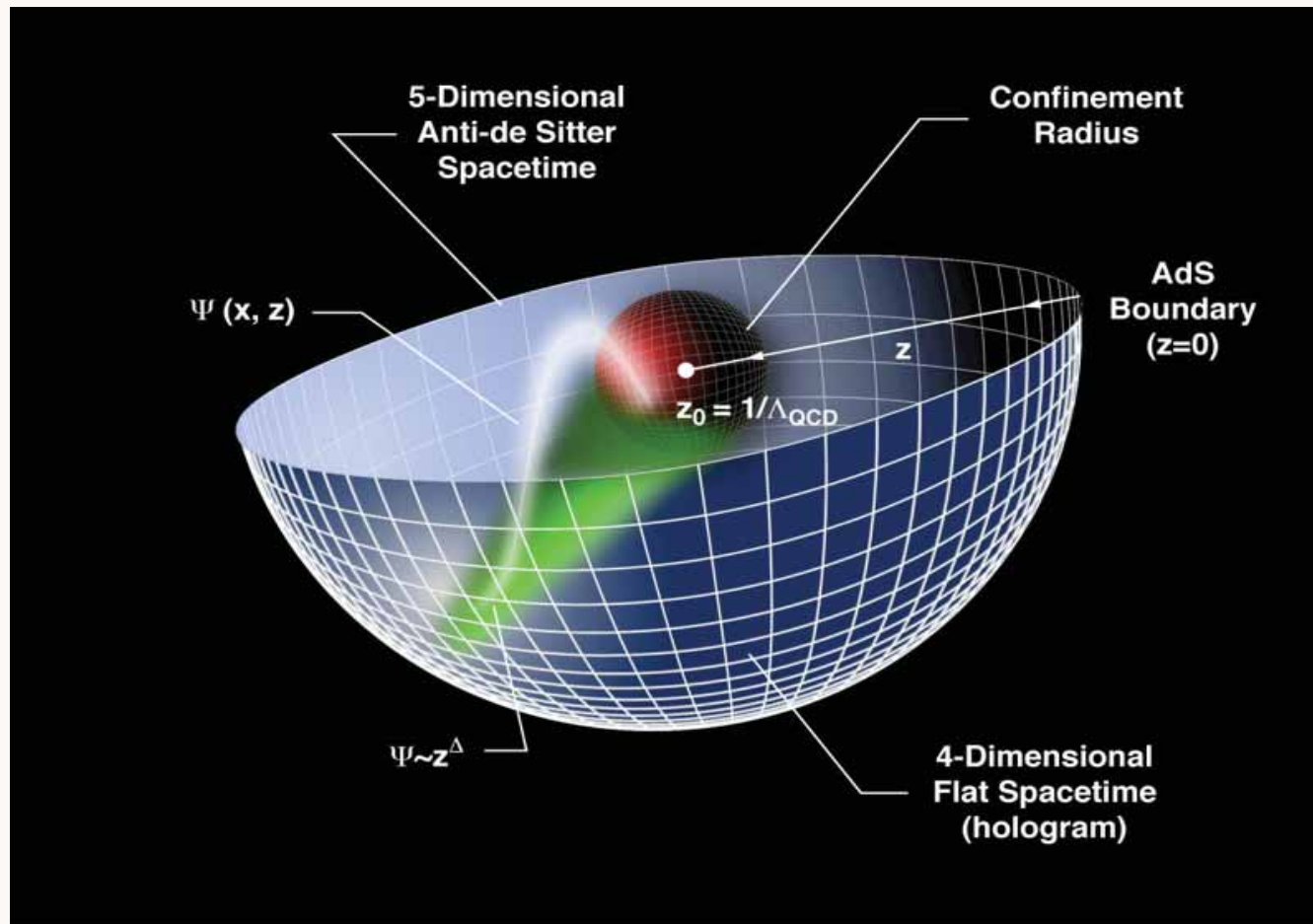


Feynman Gauge



Light-Cone Gauge

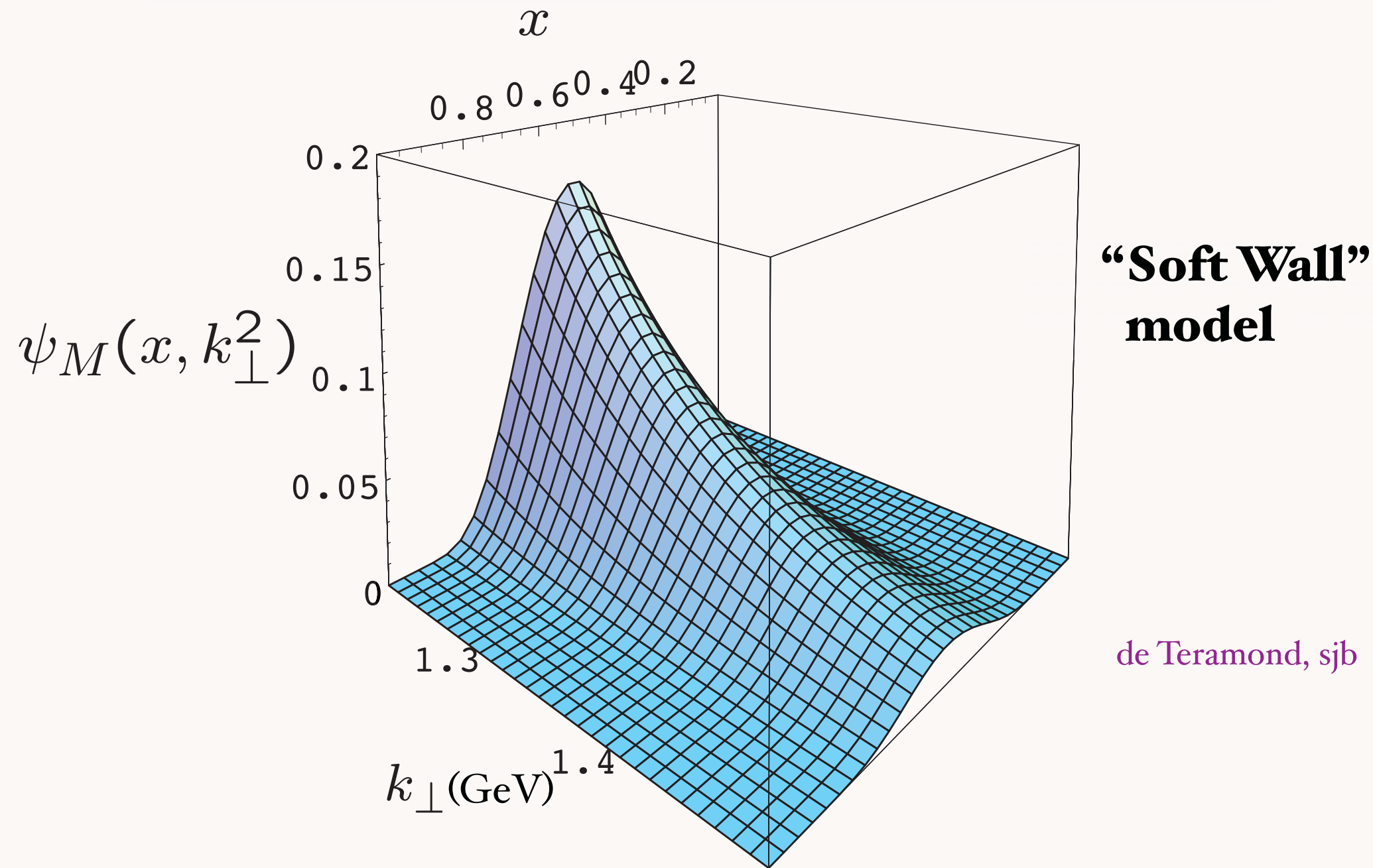
Result is Gauge Independent



- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

We will consider both holographic models

Prediction from AdS/CFT: Meson LFWF



**“Soft Wall”
model**

de Teramond, sjb

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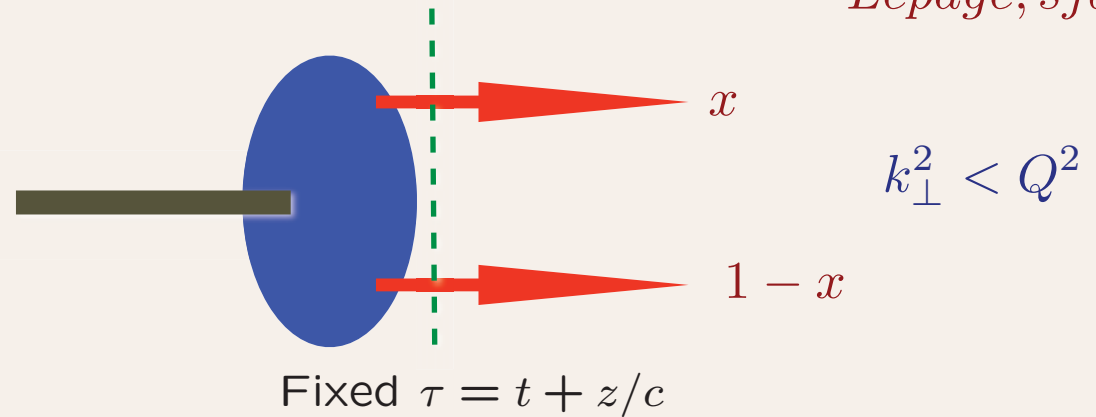
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Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb

Frishman, Lepage, Sachrajda, sjb

Peskin Braun

Efremov, Radyushkin Chernyak etal

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_{\perp})$$

Prediction from AdS/CFT: Meson LFWF

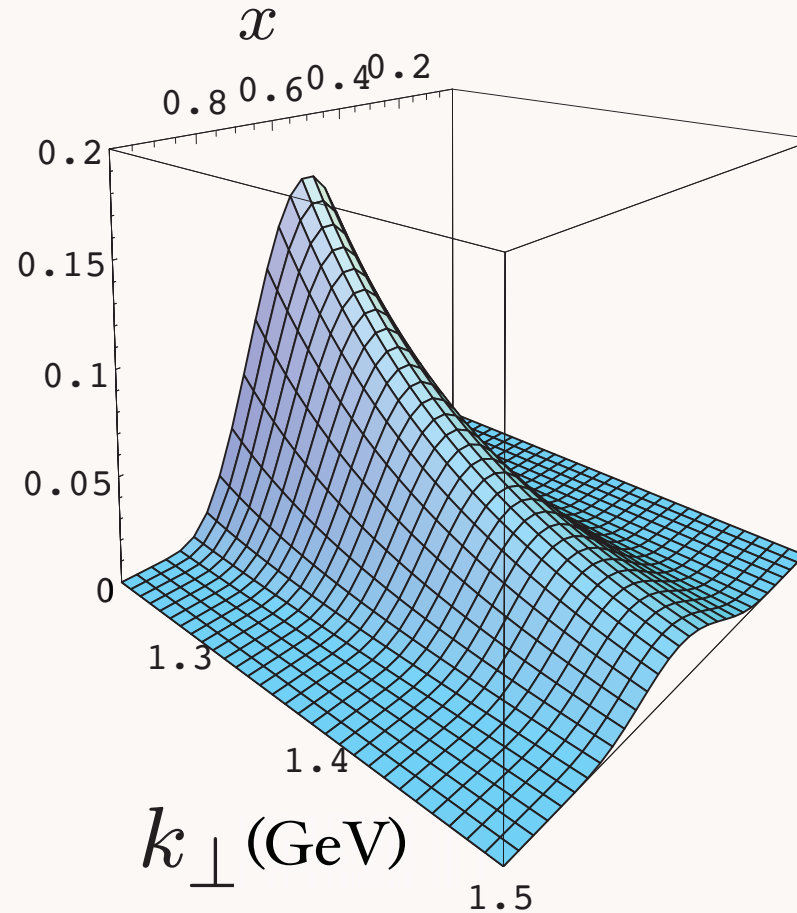
de Teramond, sjb

**“Soft Wall”
model**

$$\kappa = 0.375 \text{ GeV}$$

massless quarks

$$\psi_M(x, k_{\perp}^2)$$



$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

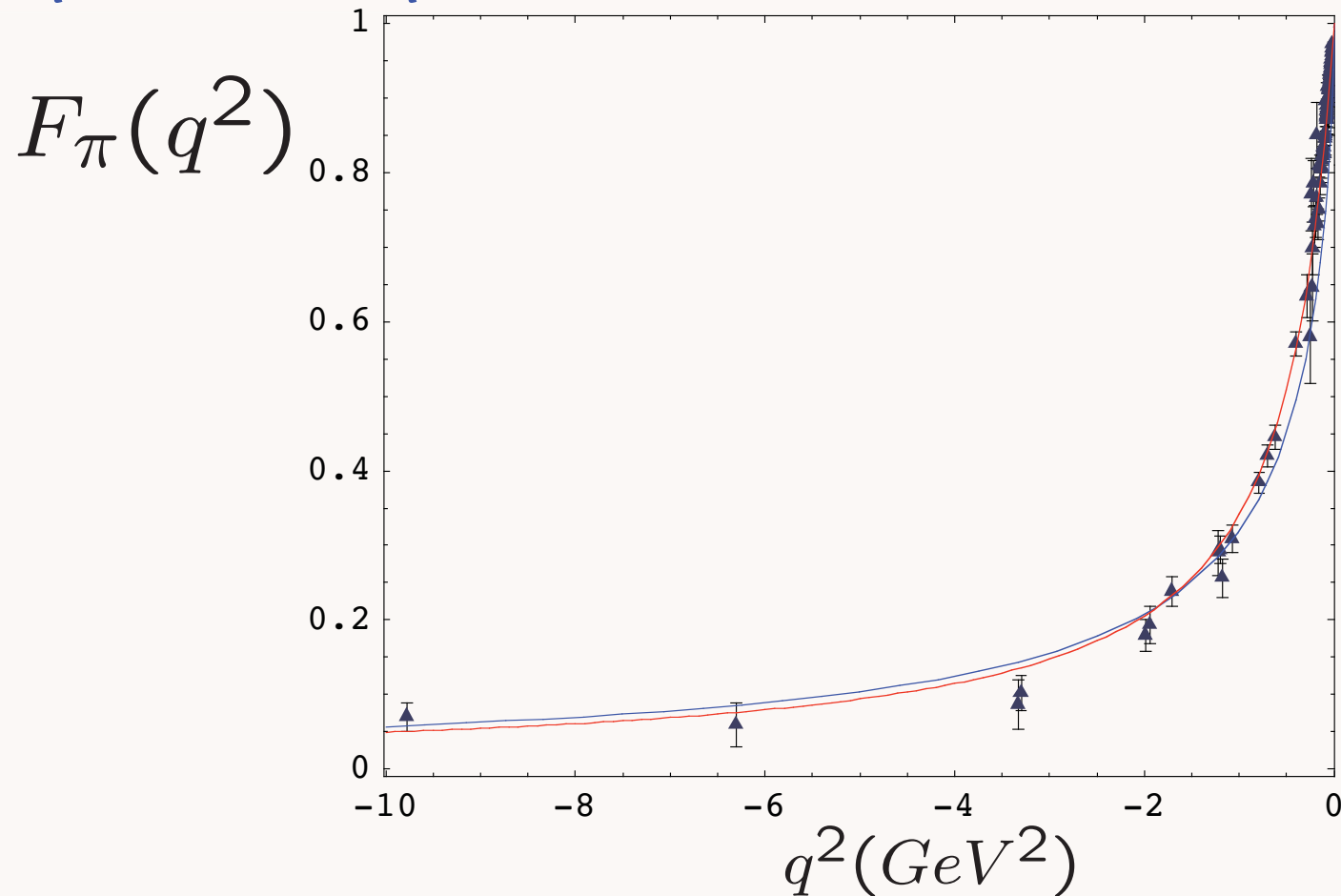
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

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Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

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de Teramond, sjb
See also: Radyushkin
Stan Brodsky
SLAC & IPPP