$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

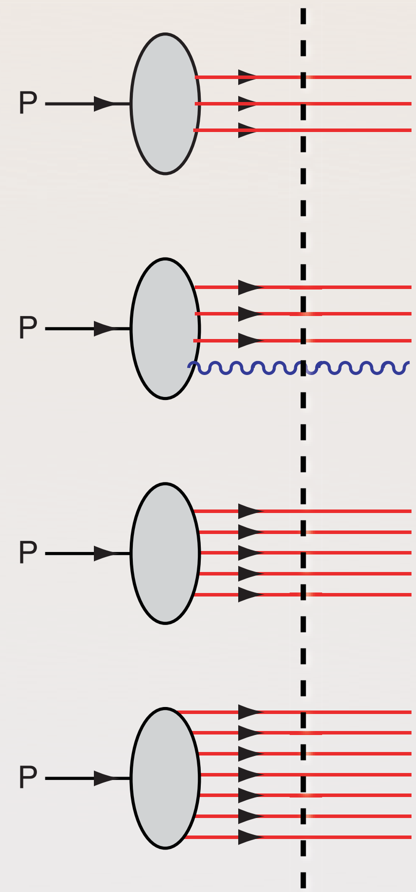
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

$\bar{u}(x) \neq \bar{d}(x)$

Mueller: BFKL DYNAMICS

$\bar{s}(x) \neq s(x)$

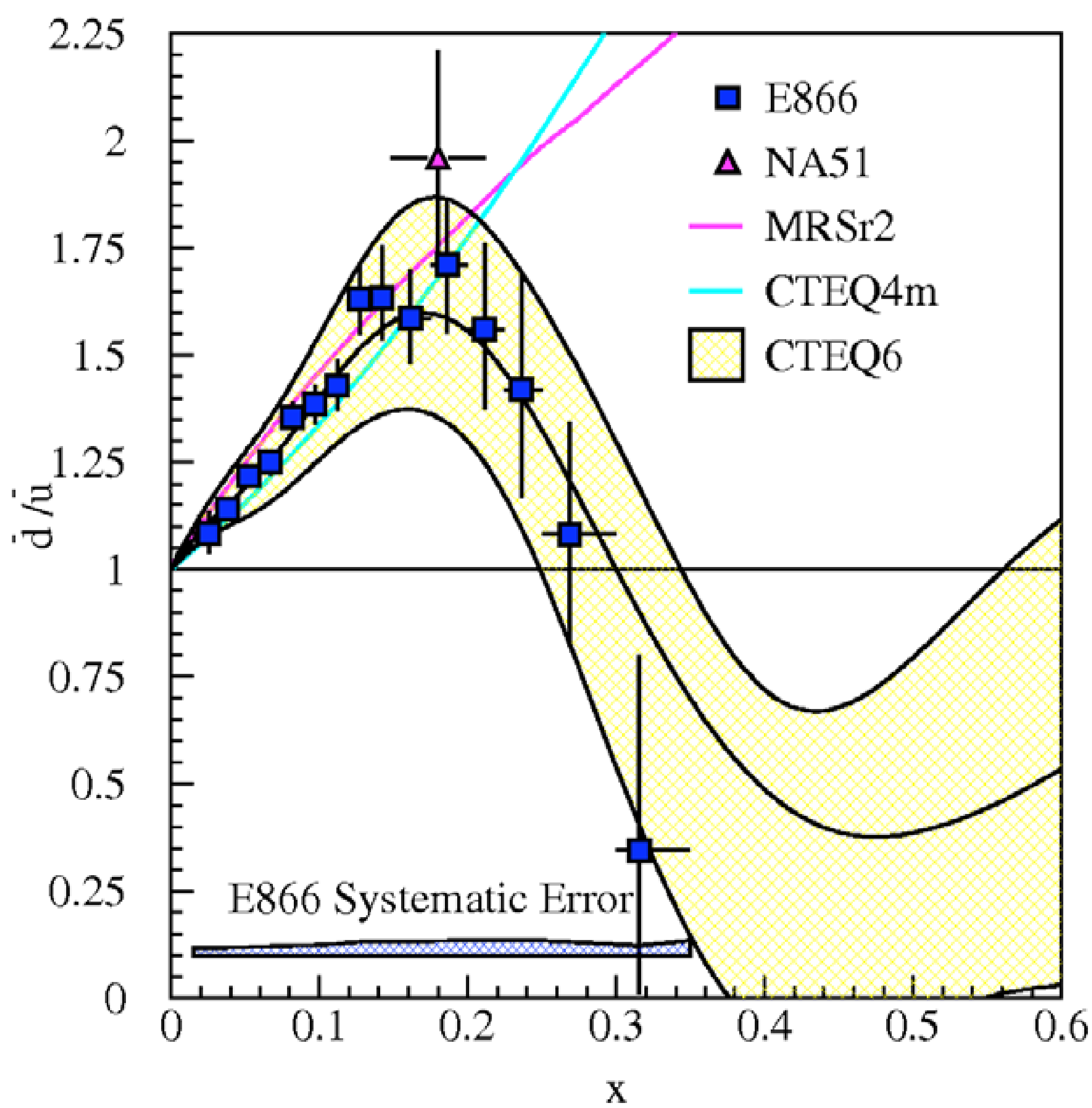
Light Antiquark Flavor Asymmetry

- Naïve Assumption from gluon splitting:

$$\bar{d}(x) = \bar{u}(x)$$

- E866/NuSea (Drell-Yan)

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \leq x \leq 0.35$



Light-Front QCD

Heisenberg Matrix Formulation

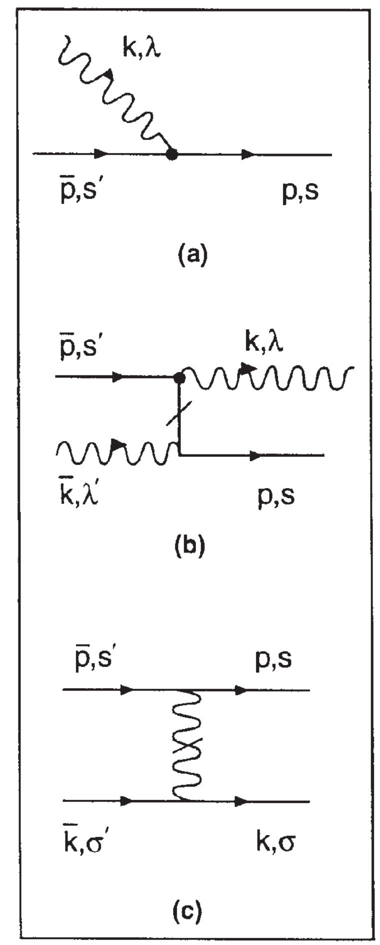
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

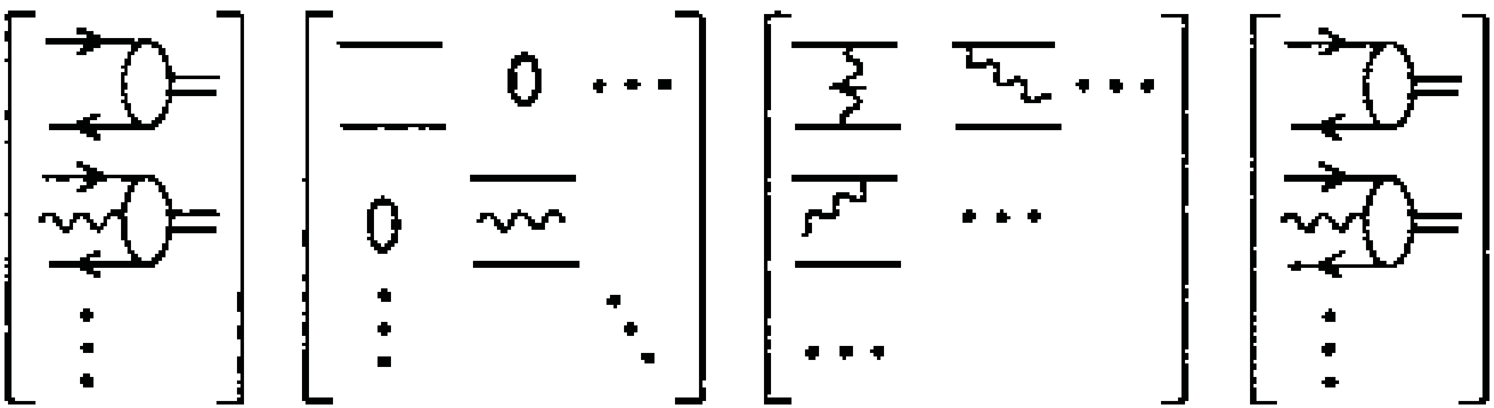


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar q/\pi} \\ \psi_{q\bar q g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar q|\,V\,|q\bar q\rangle & \langle q\bar q|\,V\,|q\bar q g\rangle & \cdots \\ \langle q\bar q g|\,V\,|q\bar q\rangle & \langle q\bar q g|\,V\,|q\bar q g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar q/\pi} \\ \psi_{q\bar q g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

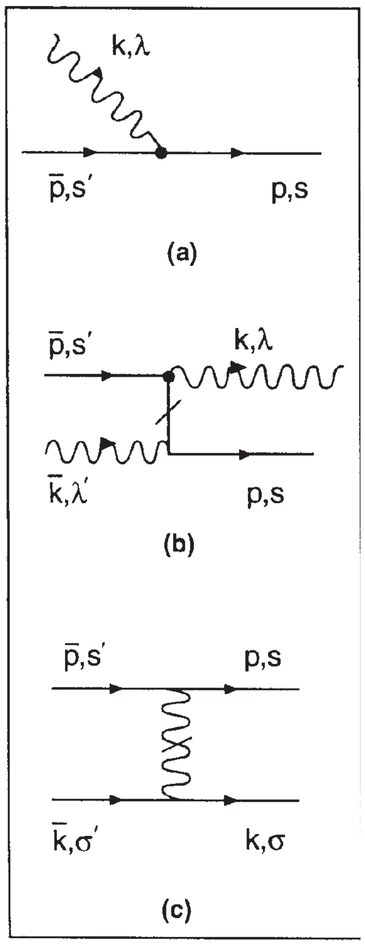
Light-Front QCD

Heisenberg Matrix Formulation

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization



n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}$ g	4 $q\bar{q}$ $q\bar{q}$	5 gg g	6 $q\bar{q}$ gg	7 $q\bar{q}$ $q\bar{q}$ g	8 $q\bar{q}$ $q\bar{q}$ $q\bar{q}$	9 gg gg	10 $q\bar{q}$ gg g	11 $q\bar{q}$ $q\bar{q}$ gg	12 $q\bar{q}$ $q\bar{q}$ $q\bar{q}$ g	13 $q\bar{q}$ $q\bar{q}$ $q\bar{q}$ $q\bar{q}$
1	$q\bar{q}$					·		·	·	·	·	·	·	·
2	gg				·			·	·		·	·	·	·
3	$q\bar{q}$ g								·	·		·	·	·
4	$q\bar{q}$ $q\bar{q}$		·			·				·	·		·	·
5	gg g	·			·			·	·			·	·	·
6	$q\bar{q}$ gg								·				·	·
7	$q\bar{q}$ $q\bar{q}$ g	·	·			·				·				·
8	$q\bar{q}$ $q\bar{q}$ $q\bar{q}$	·	·	·		·	·			·	·			
9	gg gg	·		·	·			·	·			·	·	·
10	$q\bar{q}$ gg g	·	·		·				·				·	·
11	$q\bar{q}$ $q\bar{q}$ gg	·	·	·		·				·				·
12	$q\bar{q}$ $q\bar{q}$ $q\bar{q}$ g	·	·	·	·	·	·			·	·			
13	$q\bar{q}$ $q\bar{q}$ $q\bar{q}$ $q\bar{q}$	·	·	·	·	·	·	·		·	·	·		

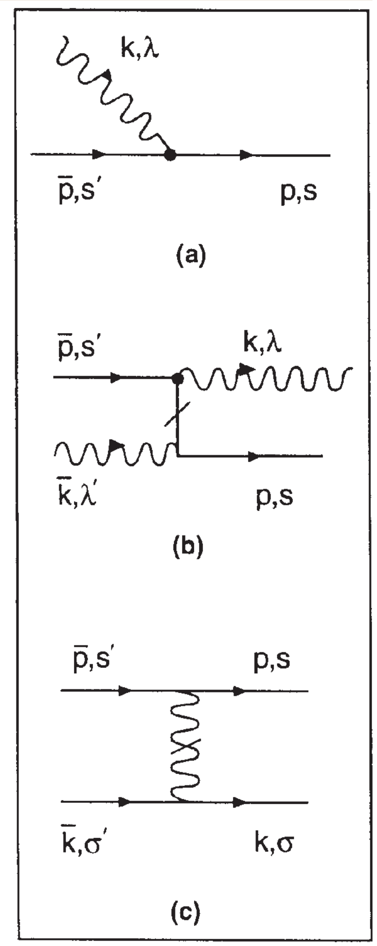
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

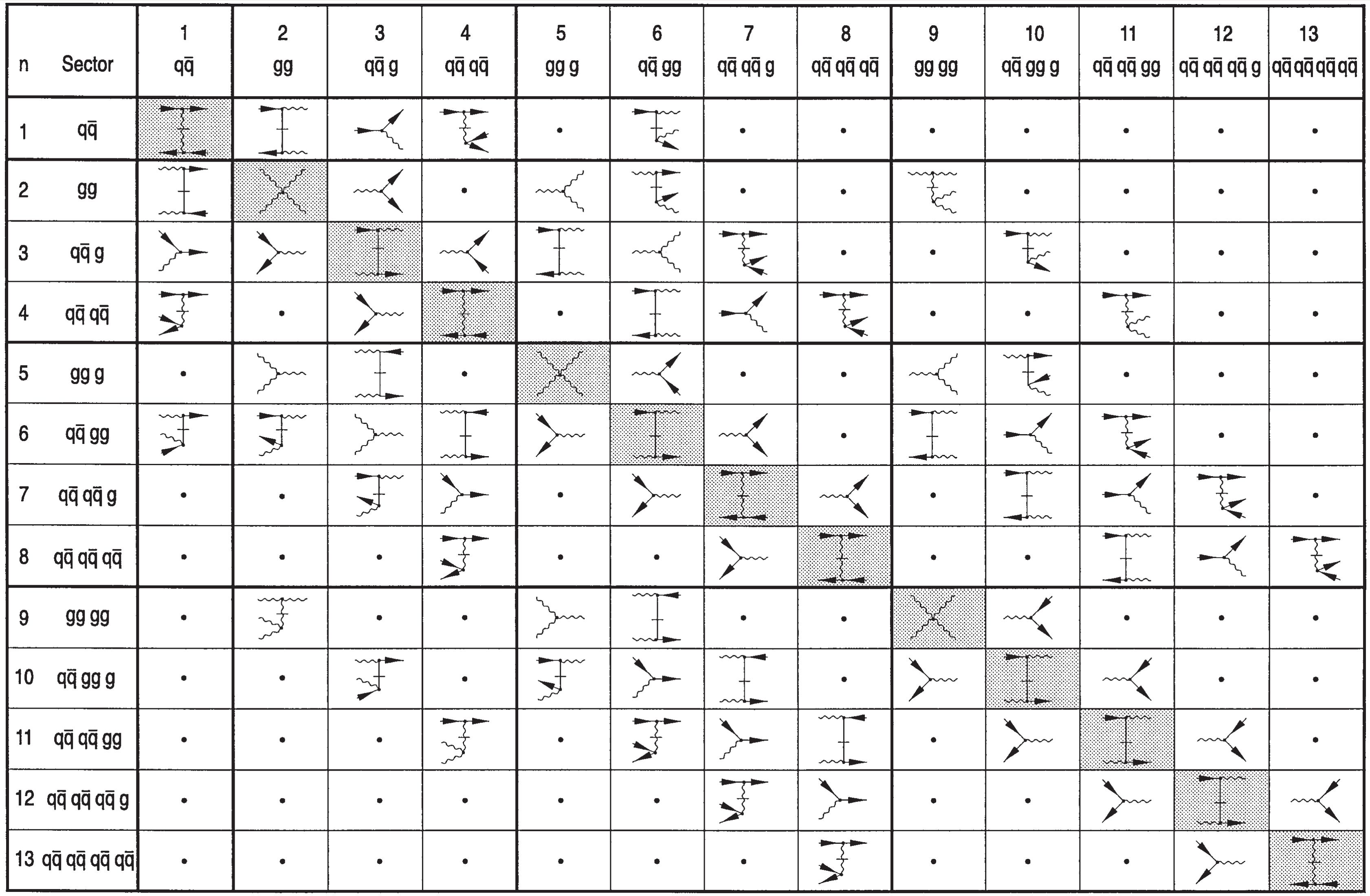
H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC} |\Psi_h\rangle = \mathcal{M}^2_h \; |\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant: generates all Fock states
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5

- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$

- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

$$V(z) = -\frac{1-4L^2}{4z^2}$$

Interpret L as orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

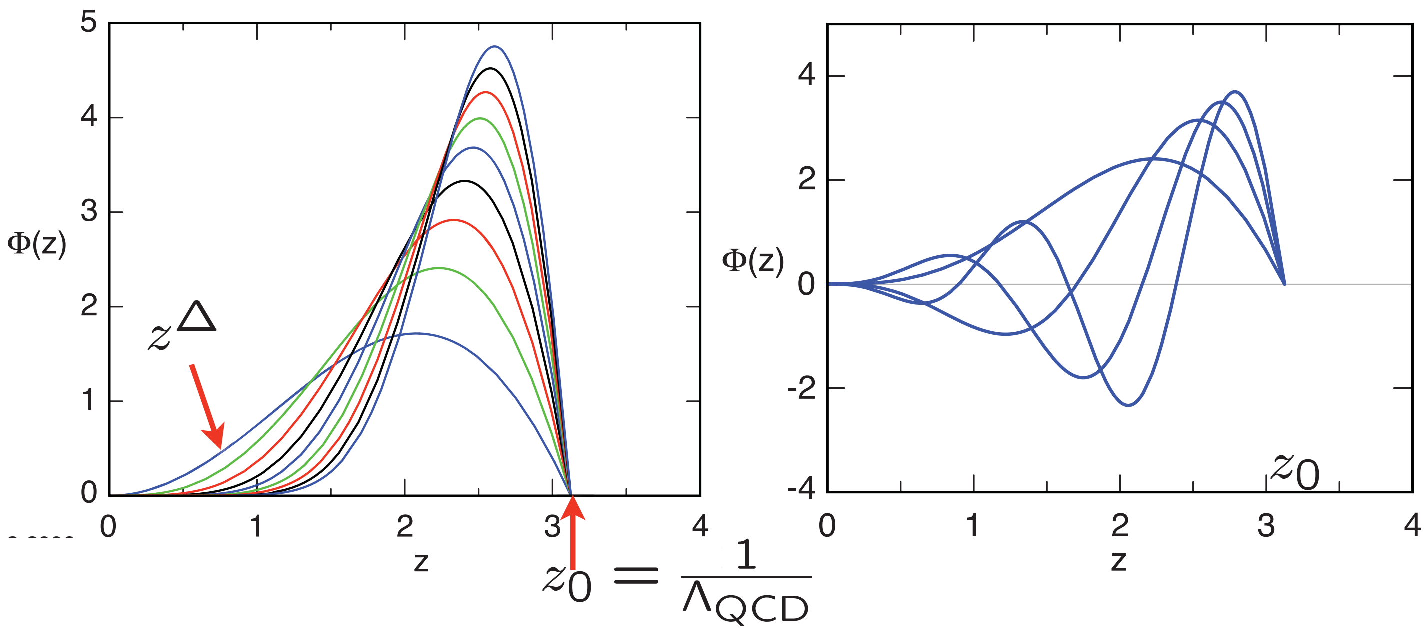
Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

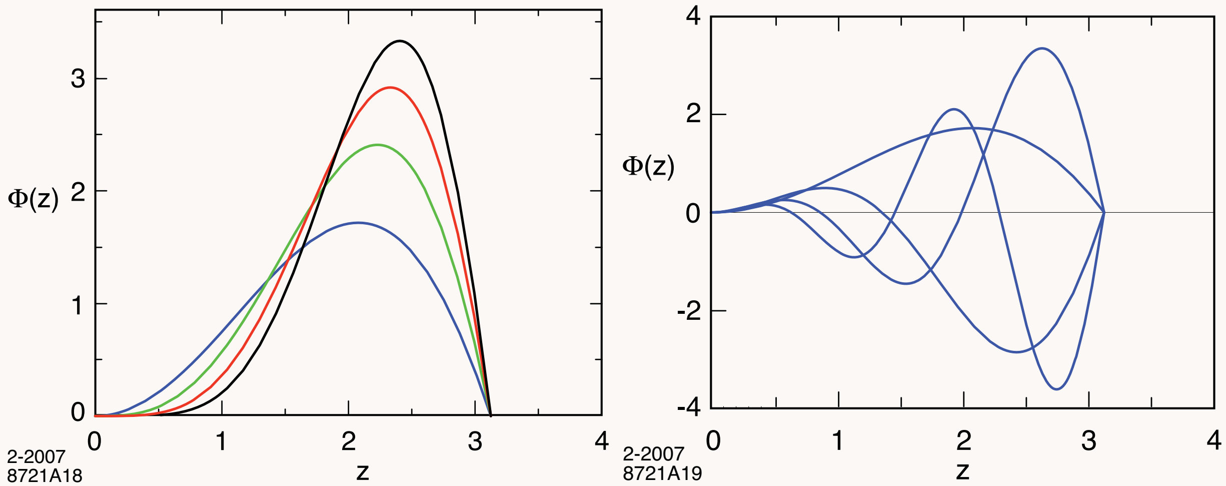


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

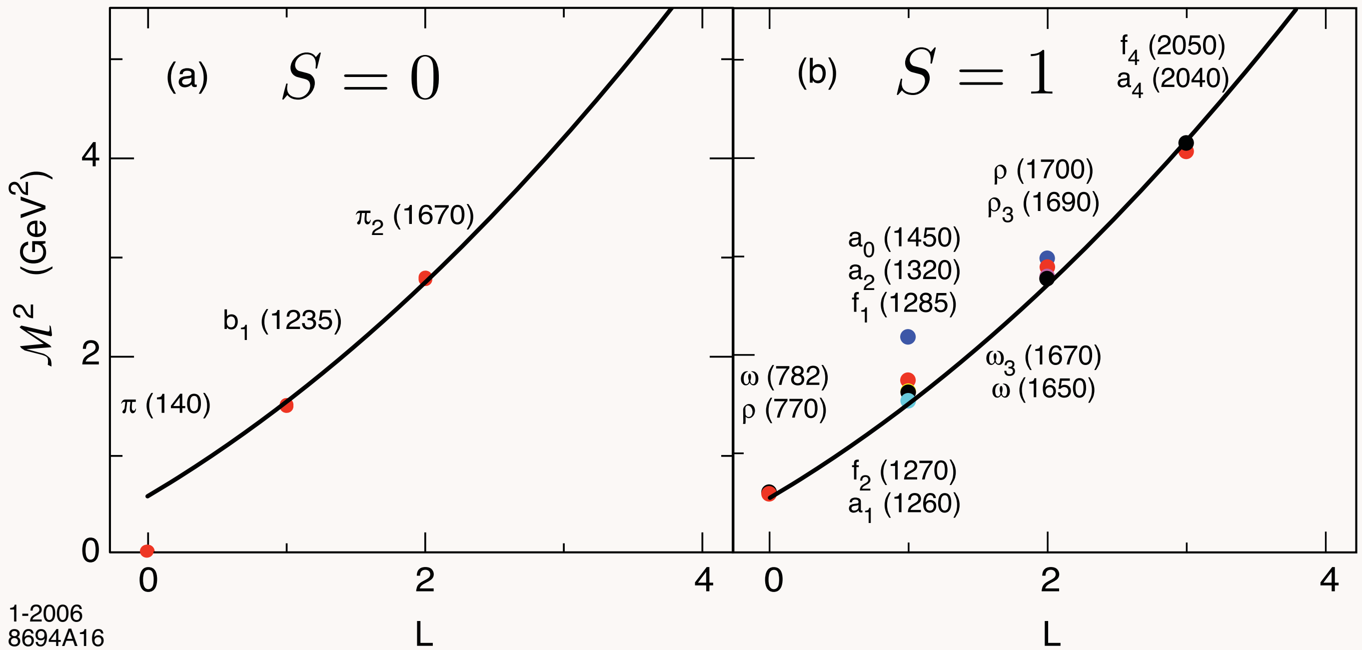


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32 \,\mathrm{GeV}$

Let $\Phi(z) = z^{3/2}\phi(z)$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Hard wall model: truncated space

$$V(z) = -\frac{1-4L^2}{4z^2} \qquad \phi(z = z_0 = 1/\Lambda_0) = 0$$

Soft wall model: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Derived from variation of Action in AdS_5

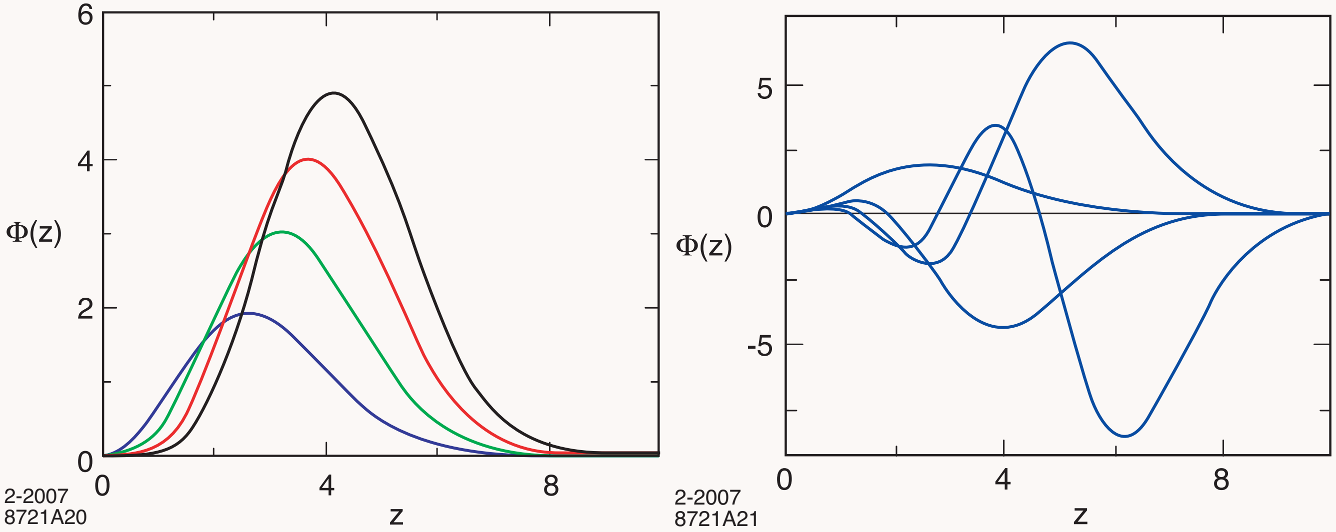
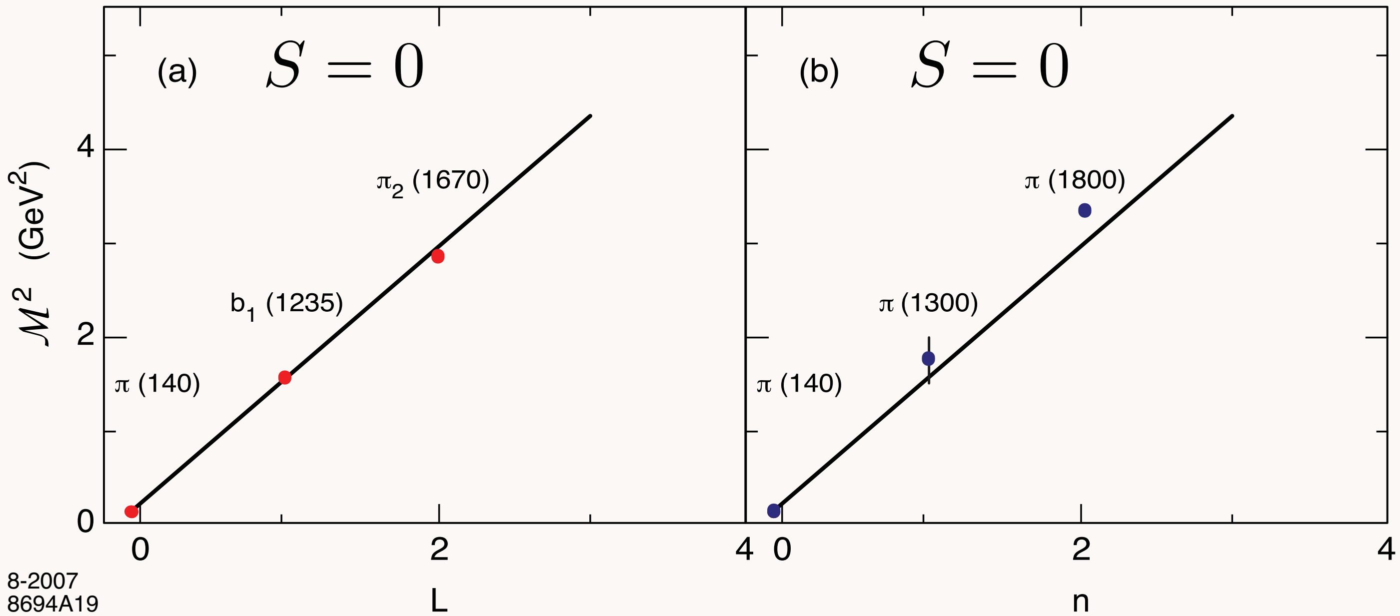


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Soft-wall model

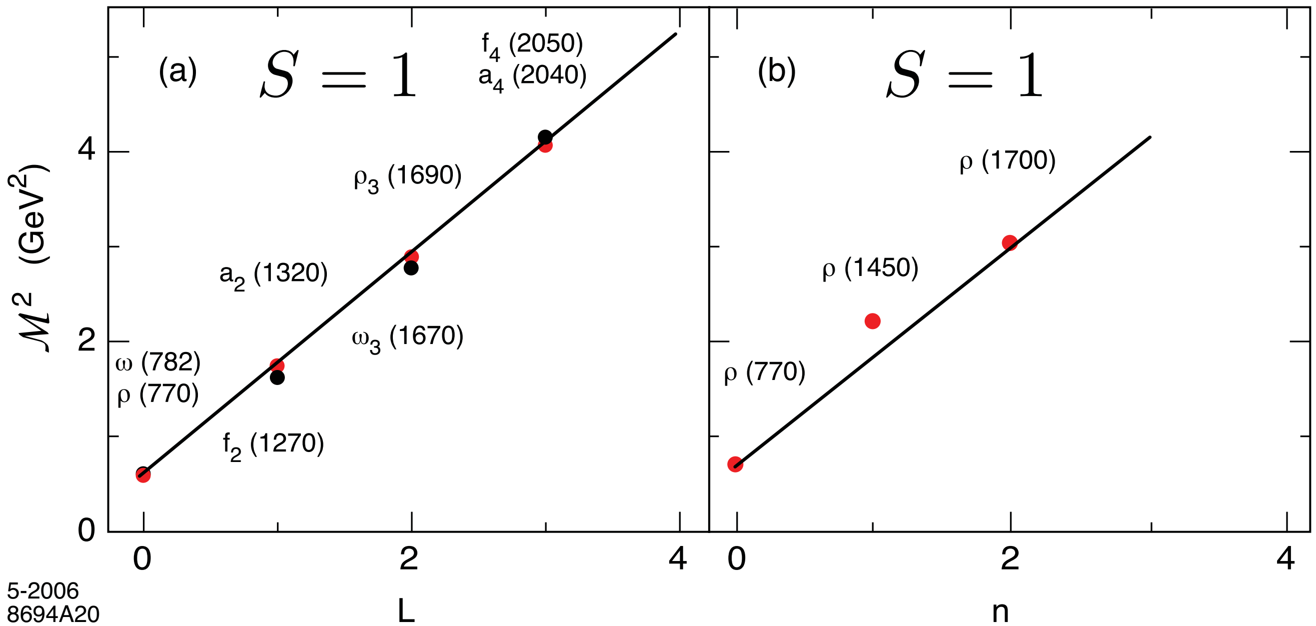
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$.

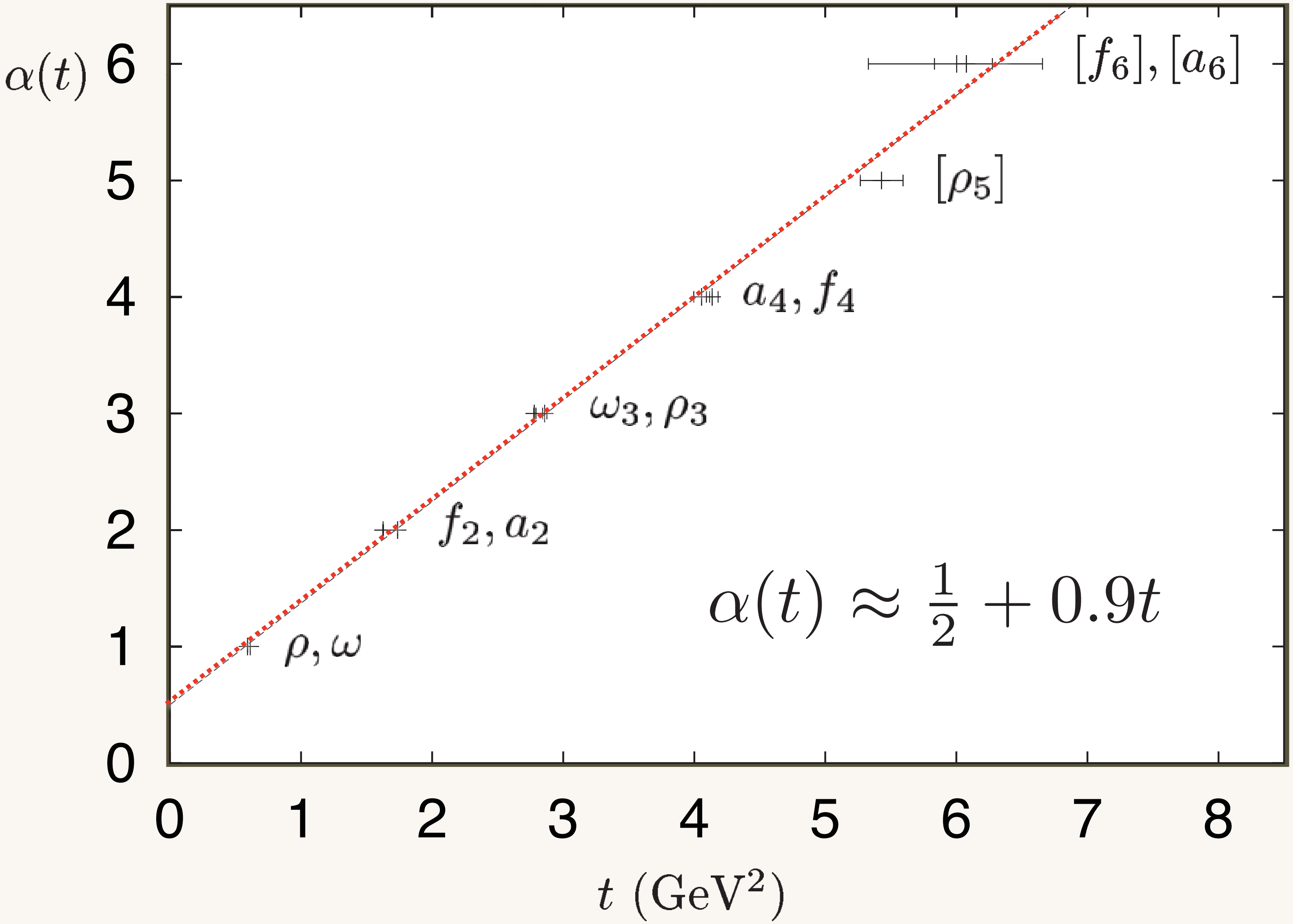
Same slope in n and L

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n + L + 1/2)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

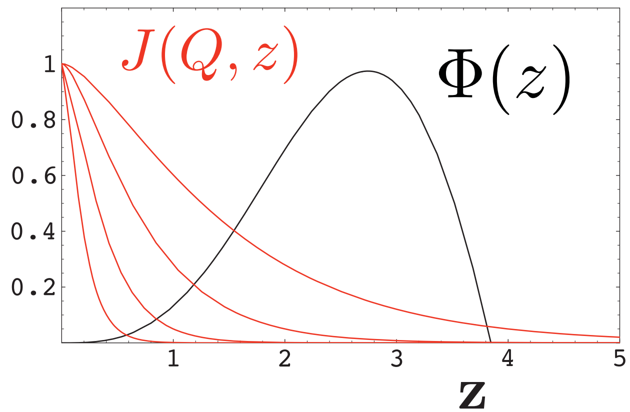
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb
Andreev

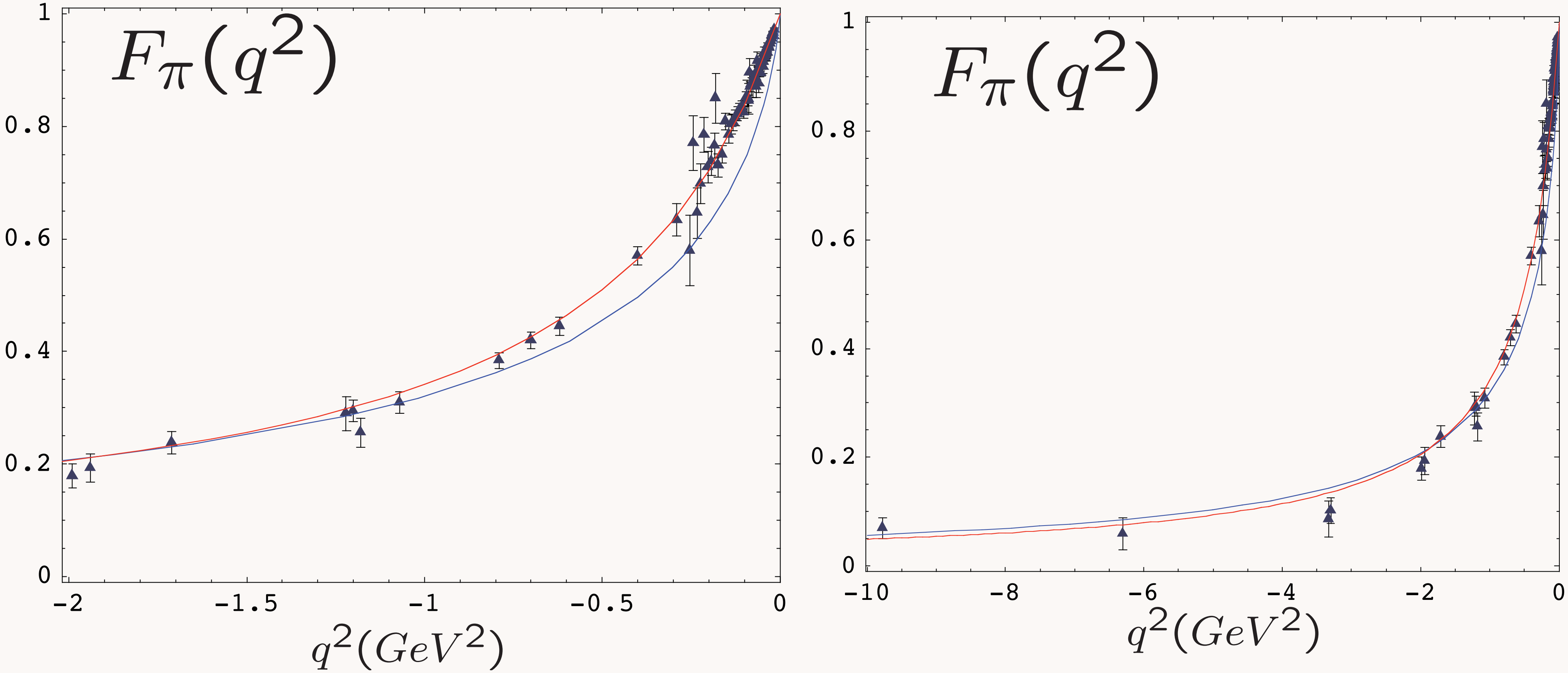
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ
scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

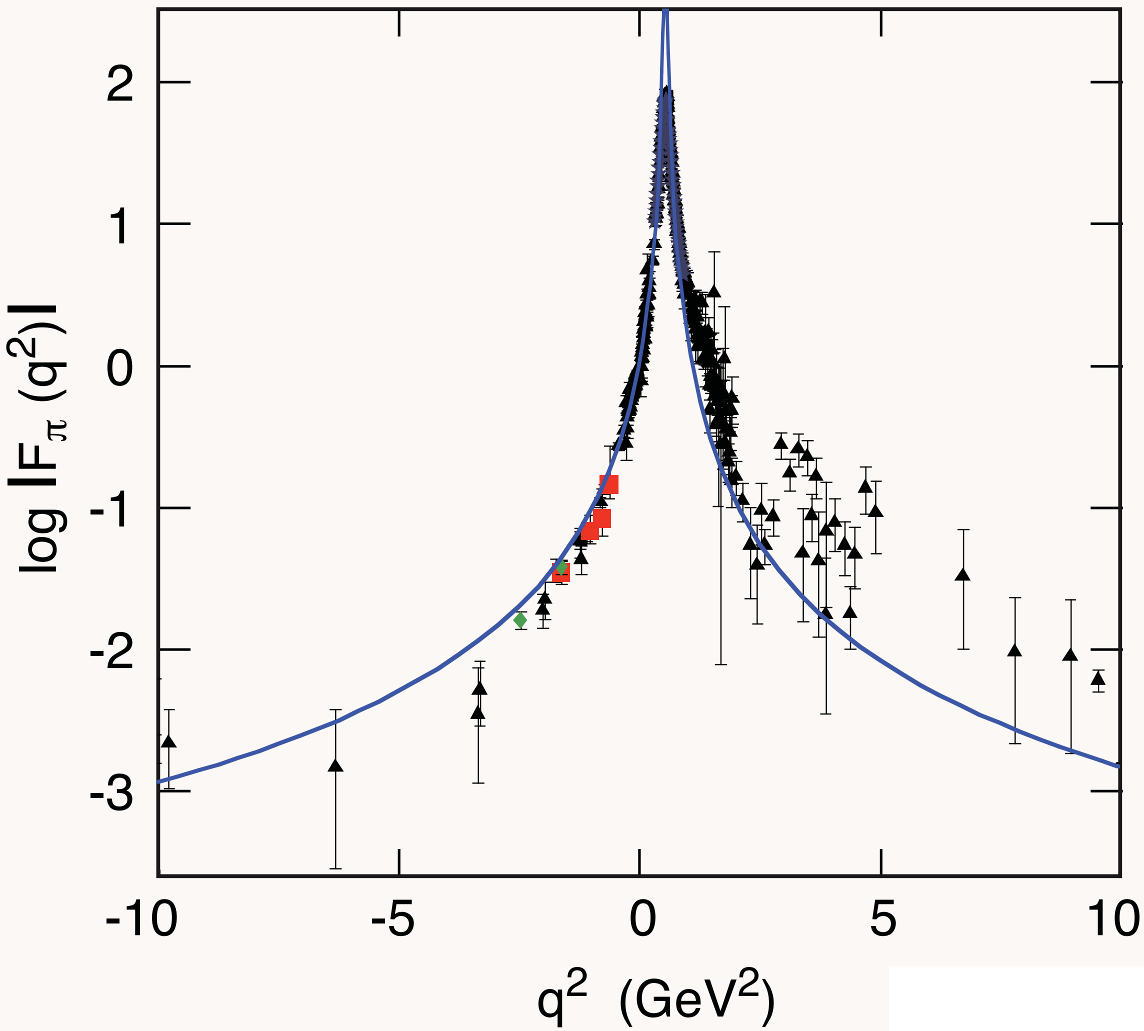
SW: Harmonic Oscillator Confinement

HW: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb

- Analytical continuation to time-like region $q^2 \to -q^2$ $\quad M_\rho = 2\kappa = 750 \text{ MeV}$
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \, |\Phi_{\pi^+}(z)|^2 ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\!\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\!\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

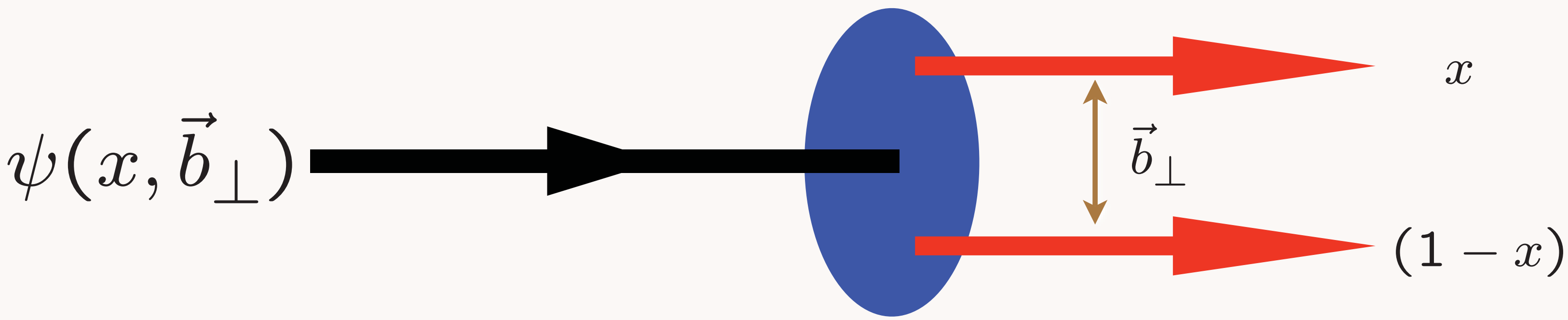
$$\boxed{\left|\tilde\psi_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) ⟷ AdS_5

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

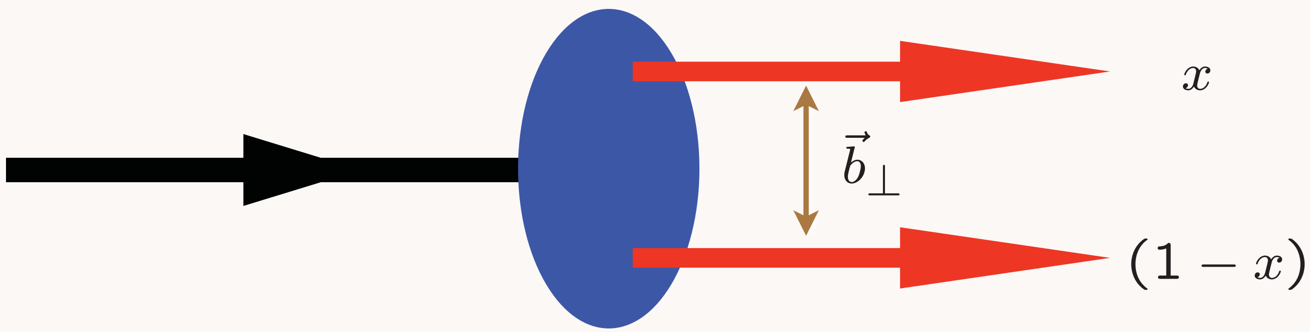
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation   Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} \, H(Q^2, z) \left| \Phi_\pi(z) \right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\!\left(zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} \, J_0\!\left(zQ \sqrt{\frac{1-x}{x}} \right) \left| \Phi_\pi(z) \right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{\left| \Phi_\pi(\zeta) \right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \to \lambda x^\mu, \; z \to \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+ dx^- - dx_\perp{}^2$

and

$$ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2) \text{ for } x^+ = 0.$$

Light-Front AdS_5 Duality

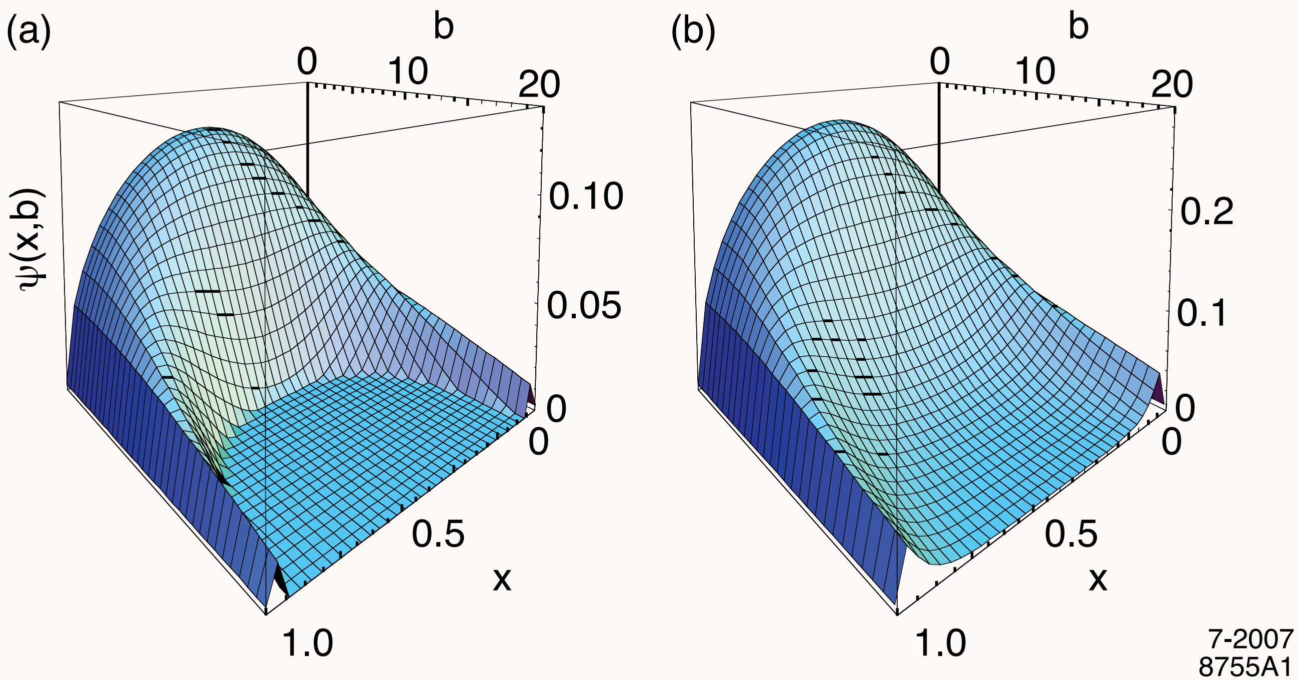
- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L + N - 2)$].

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

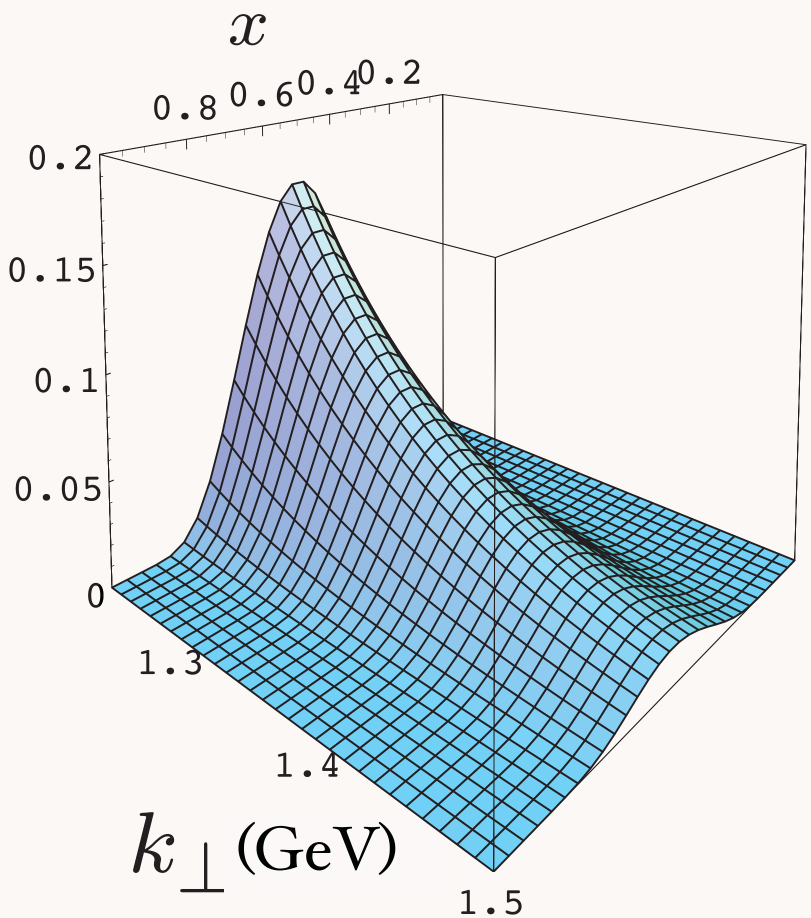


Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

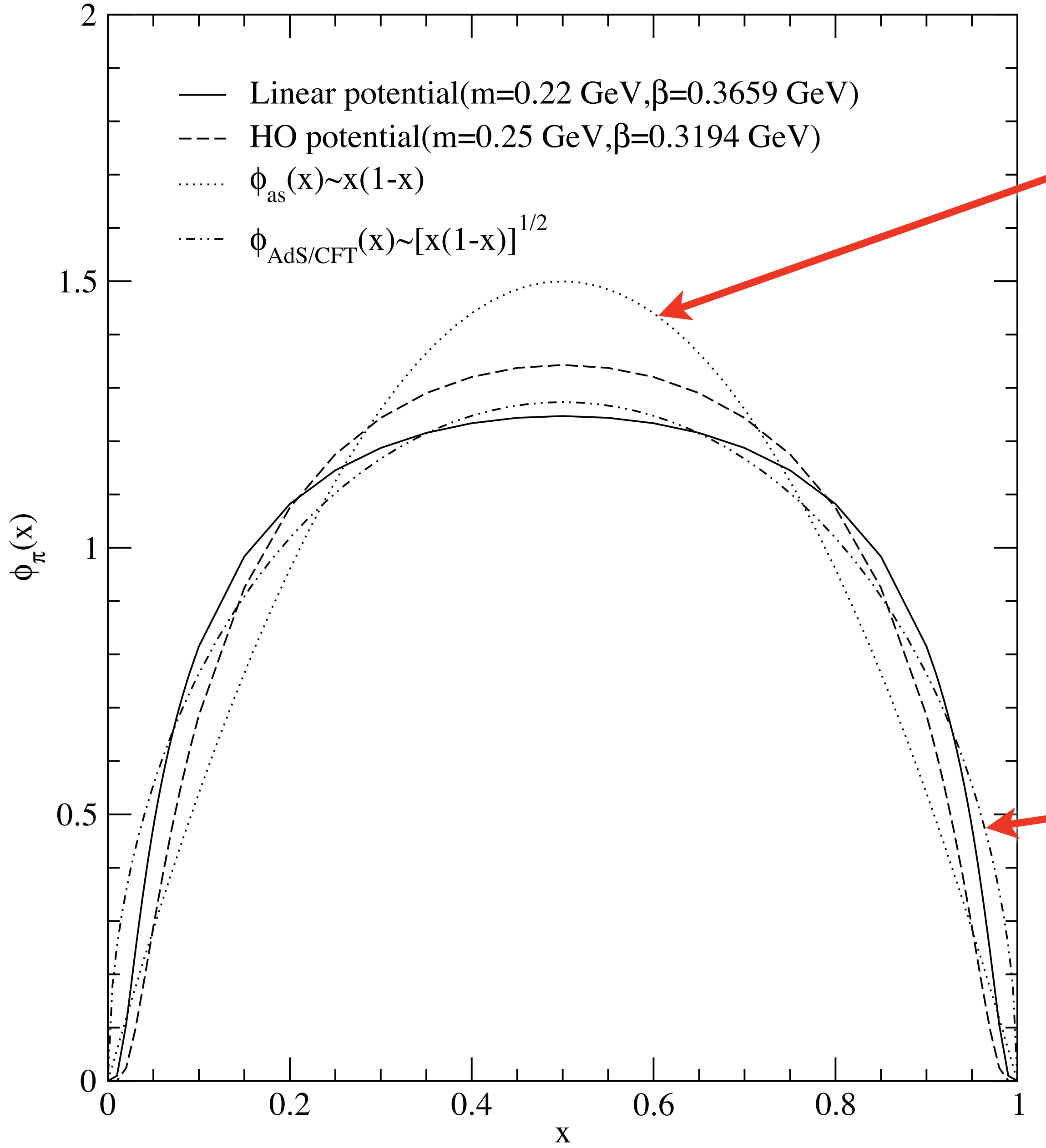
$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.



$$\phi_{asympt} \sim x(1-x)$$

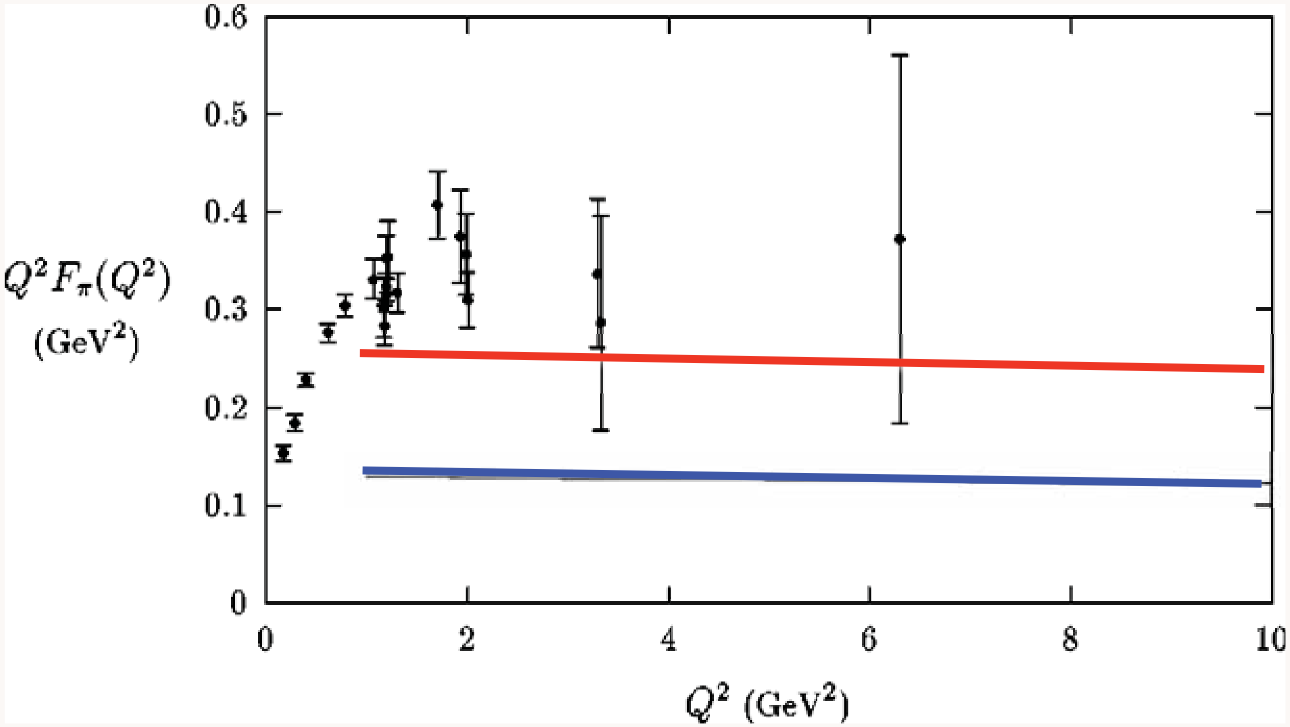
AdS/CFT:

$$\phi(x, Q_0) \propto \sqrt{x(1-x)}$$

Lepage, sjb

C. Ji, A. Pang, D. Robertson, sjb
Choi, Ji

$$F_\pi(Q^2) = \int_0^1 dx\,\phi_\pi(x) \int_0^1 dy\,\phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



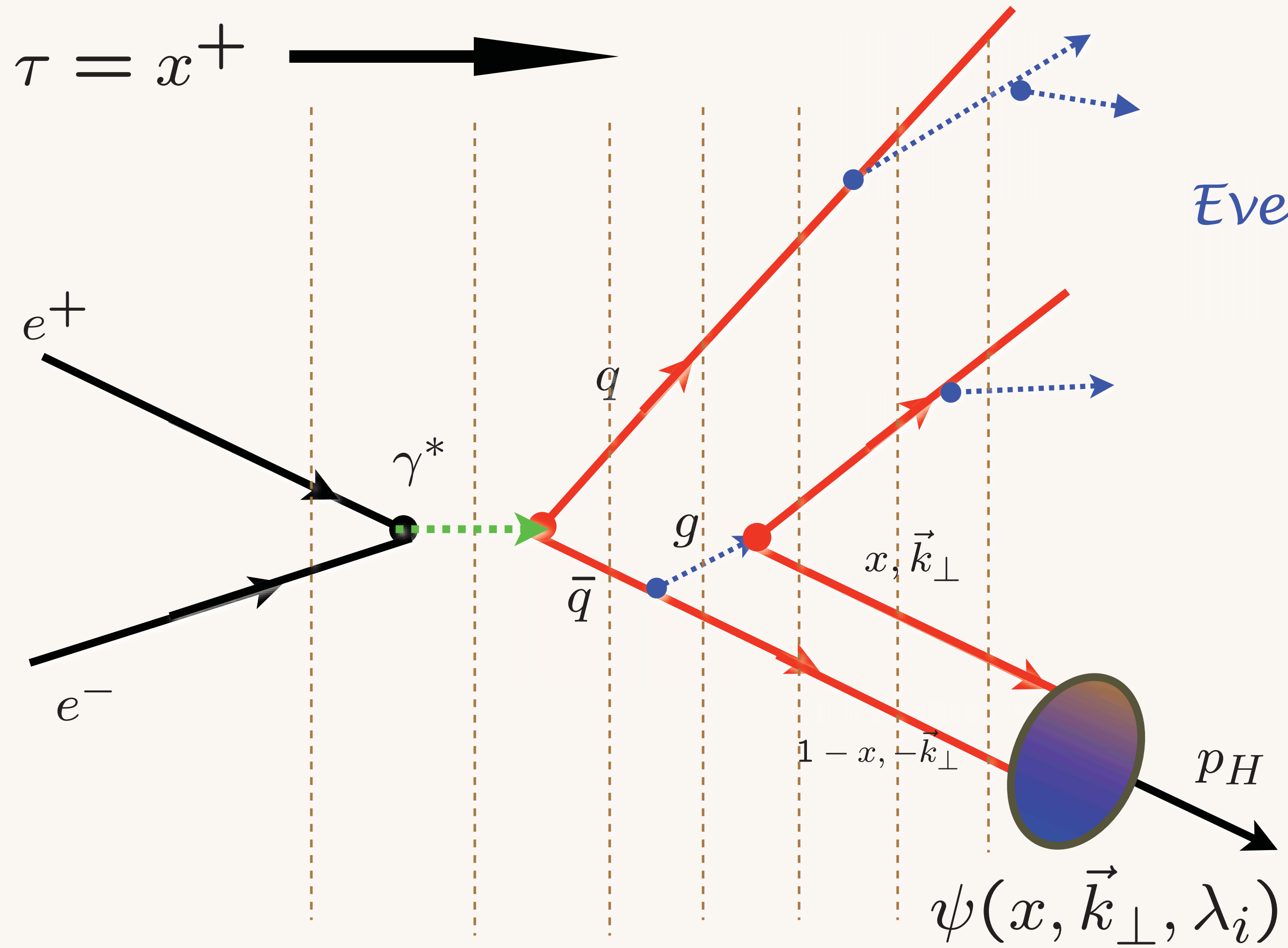
$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

$\phi_{asymptotic} \propto x(1-x)$

Normalized to f_π

AdS/CFT: Increases PQCD leading twist prediction for $F_\pi(Q^2)$ by factor 16/9

Hadronization at the Amplitude Level



Event amplitude generator

AdS/QCD Hard Wall

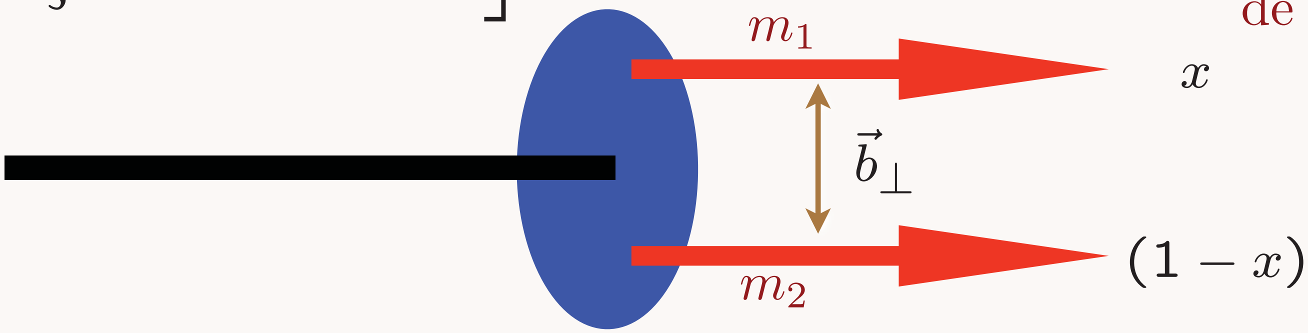
Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

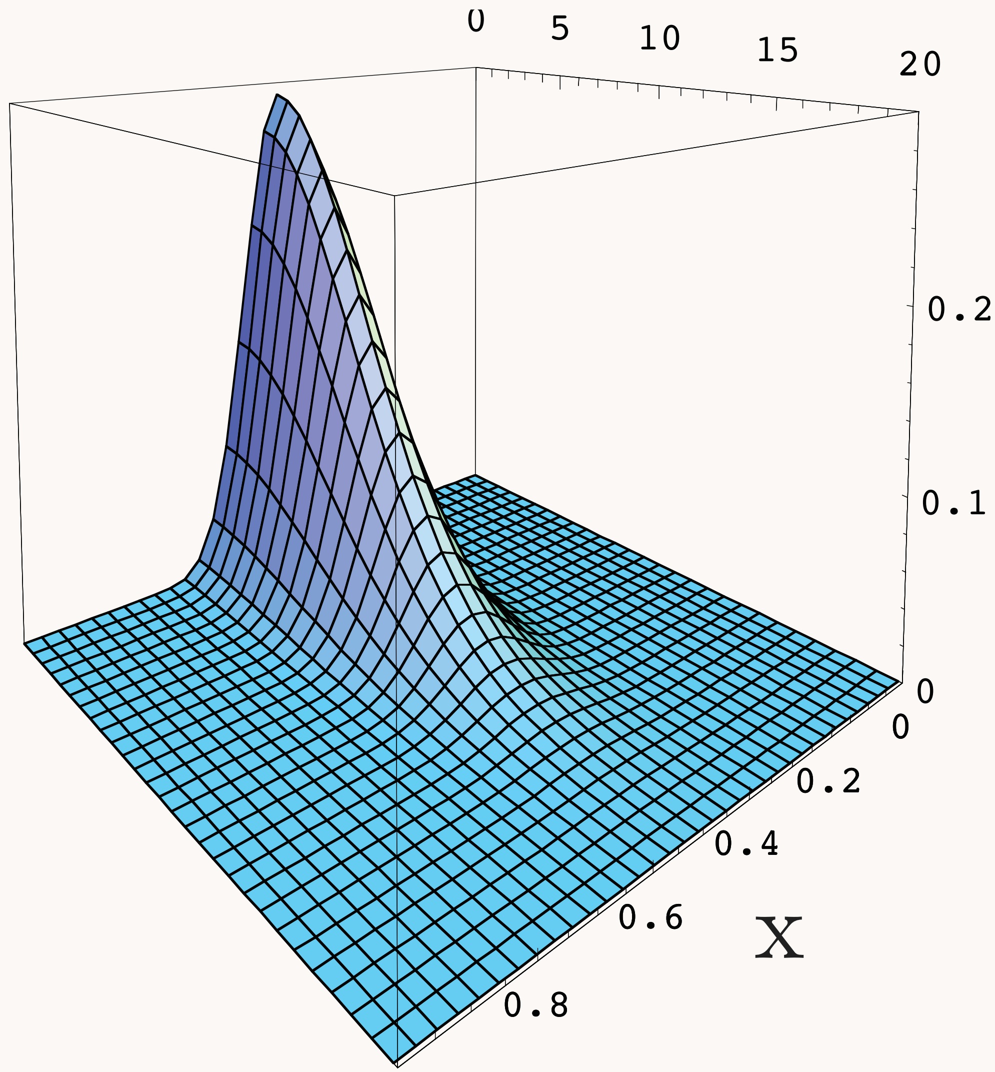
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

$\psi_{J/\psi}(x,b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

$$|\pi^+ >= |u\bar{d}>$$

$$m_u = 2\ MeV$$
$$m_d = 5\ MeV$$

$$|K^+ >= |u\bar{s}>$$

$$m_s = 95\ MeV$$

$$|D^+ >= |c\bar{d}>$$

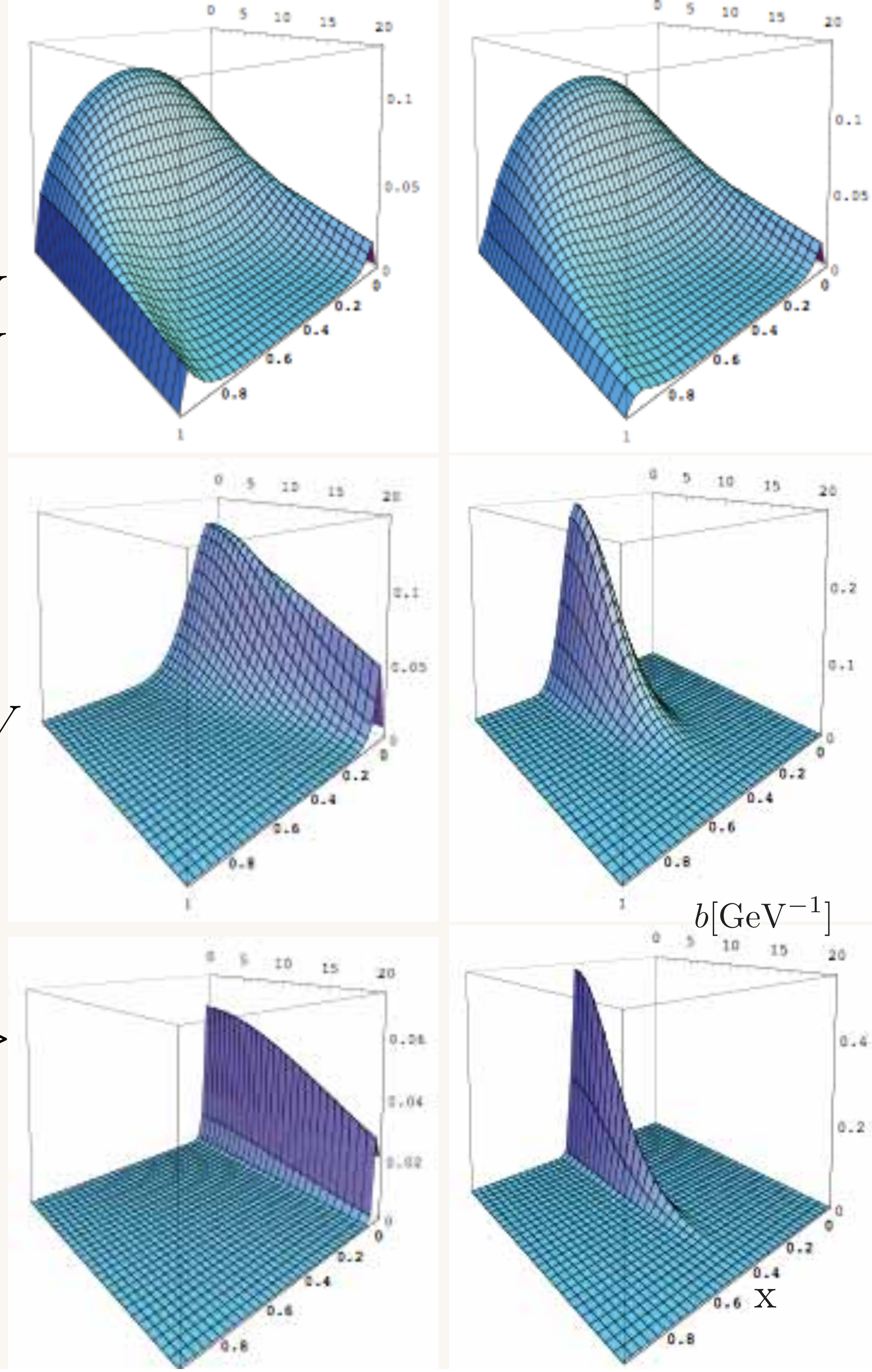
$$m_c = 1.25\ GeV$$

$$|\eta_c >= |c\bar{c}>$$

$$|B^+ >= |u\bar{b}>$$

$$m_b = 4.2\ GeV$$

$$|\eta_b >= |b\bar{b}>$$

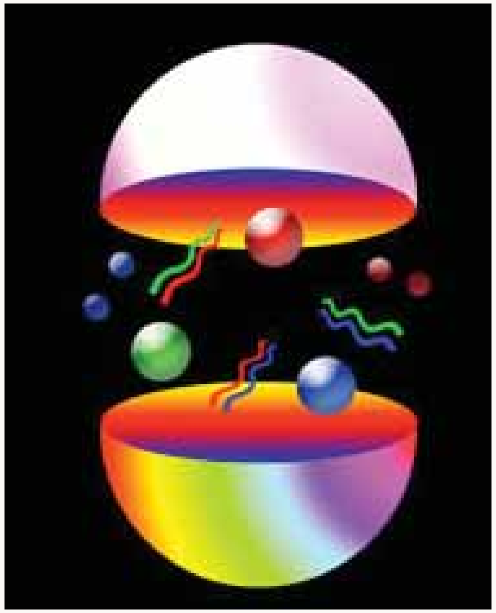


$$\kappa = 375\ MeV$$

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g}\, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right] \Psi(x^\ell) = 0.$$

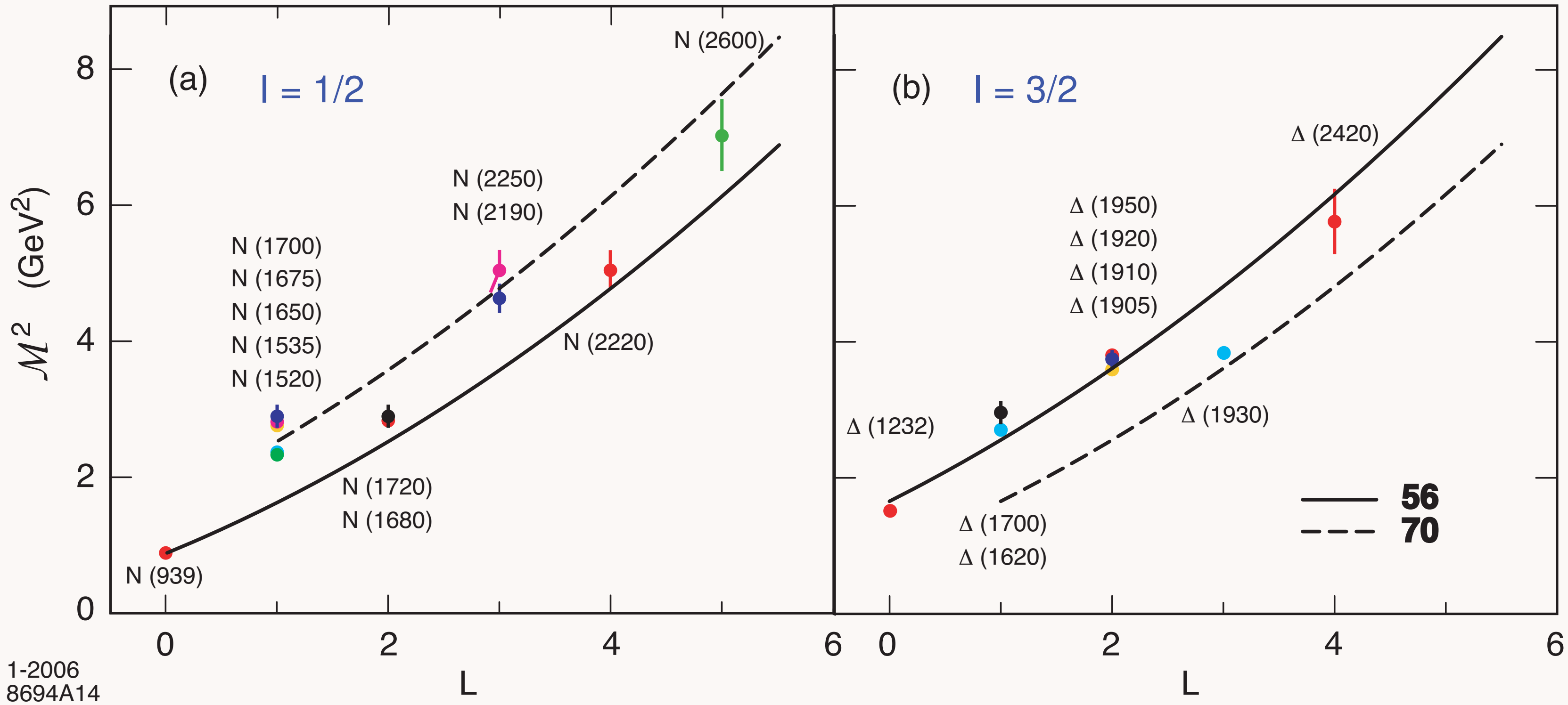


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

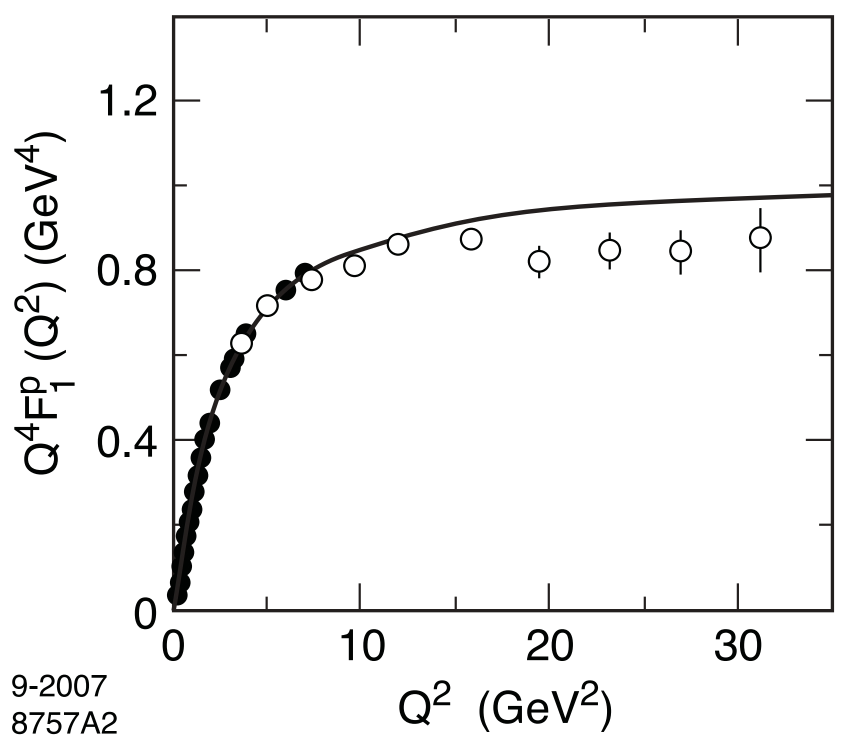
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Proton } \tau = 3}$

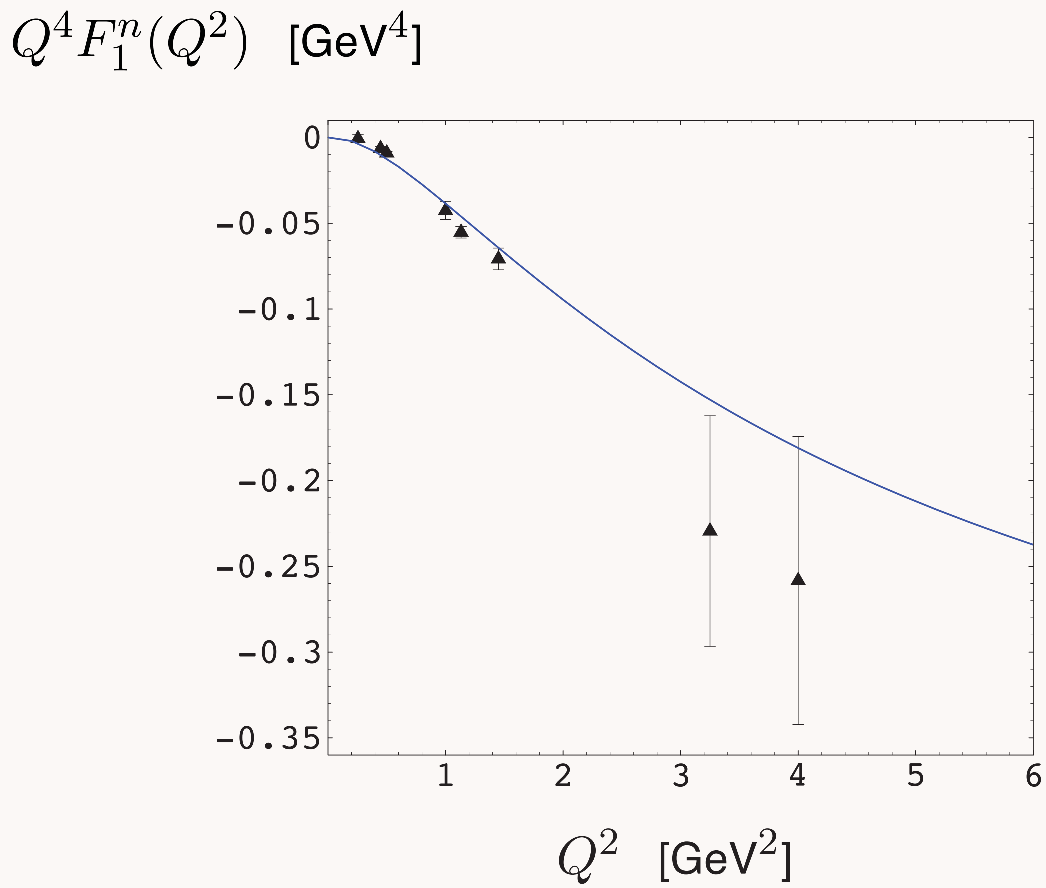


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

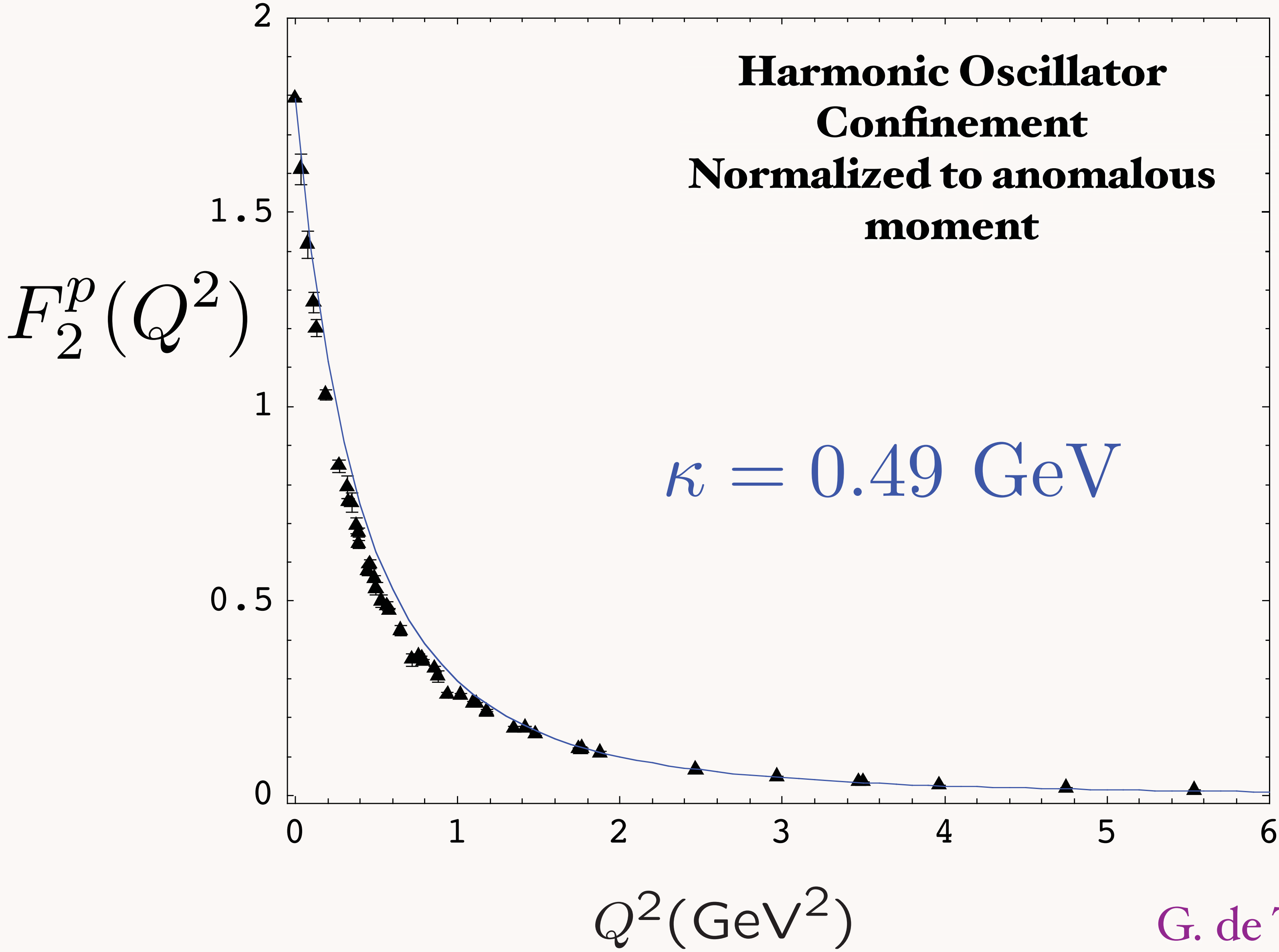


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

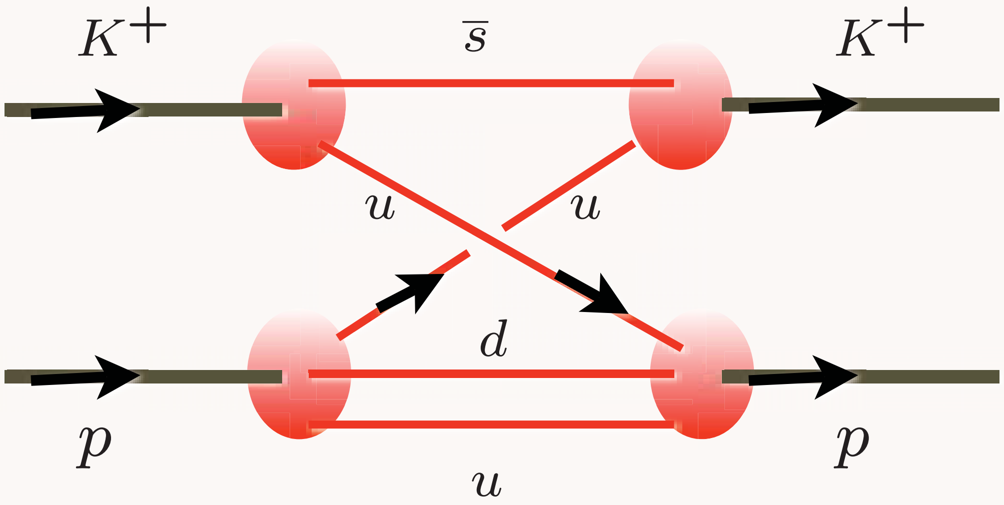
Holographic Connection between LF and AdS/CFT

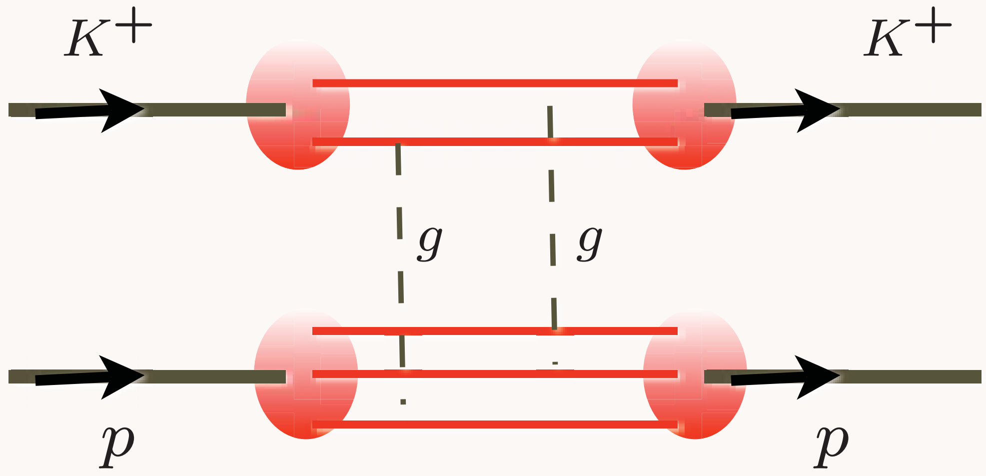
- Predictions for hadronic spectra, light-front wavefunctions, interactions
- Deduce meson and baryon wavefunctions, distribution amplitude, structure function from holographic constraint
- Identification of Orbital Angular Momentum Casimir for SO(2): LF Rotations
- Extension to massive quarks

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb





Quark Interchange
(Spin exchange in atom-atom scattering)

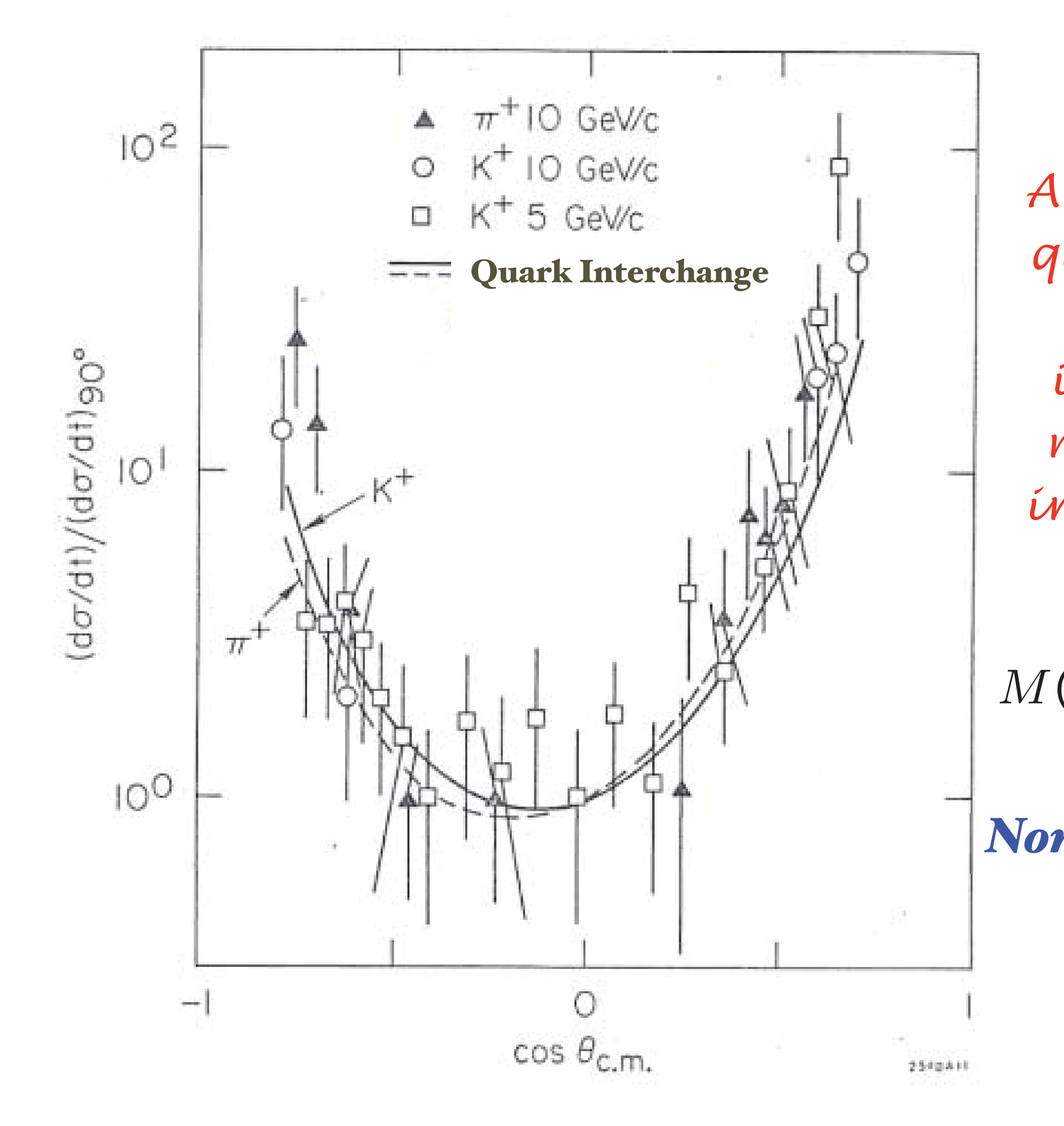
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
University of Minnesota, Minneapolis, Minnesota 55455

D. S. Barton, G. Bunce, A. S. Carroll, and Y. I. Makdisi
Brookhaven National Laboratory, Upton, New York 11973

and

S. Gushue[e] and J. J. Russell
Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

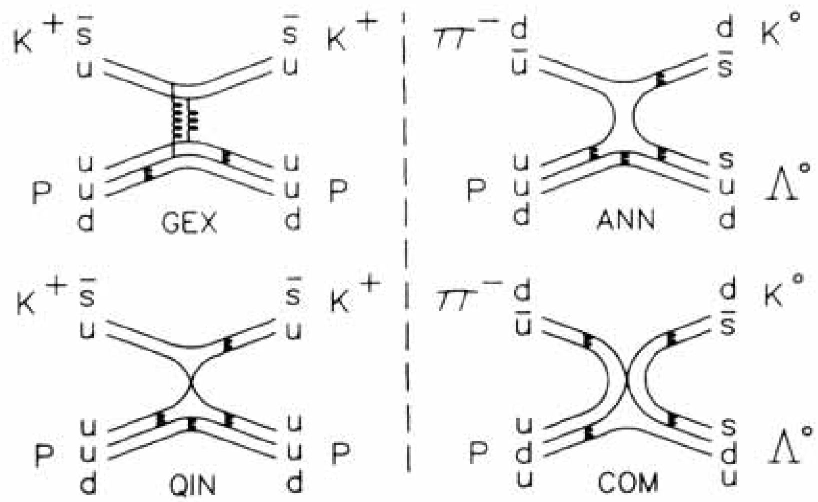
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \to pp^{\pm}$.



New Perspectives on QCD Phenomena from AdS/CFT

- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory

- New Way to Implement Conformal Symmetry

- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances

- Remarkable predictions for hadronic spectra, wavefunctions, interactions

- AdS/CFT provides novel insights into the quark structure of hadrons

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

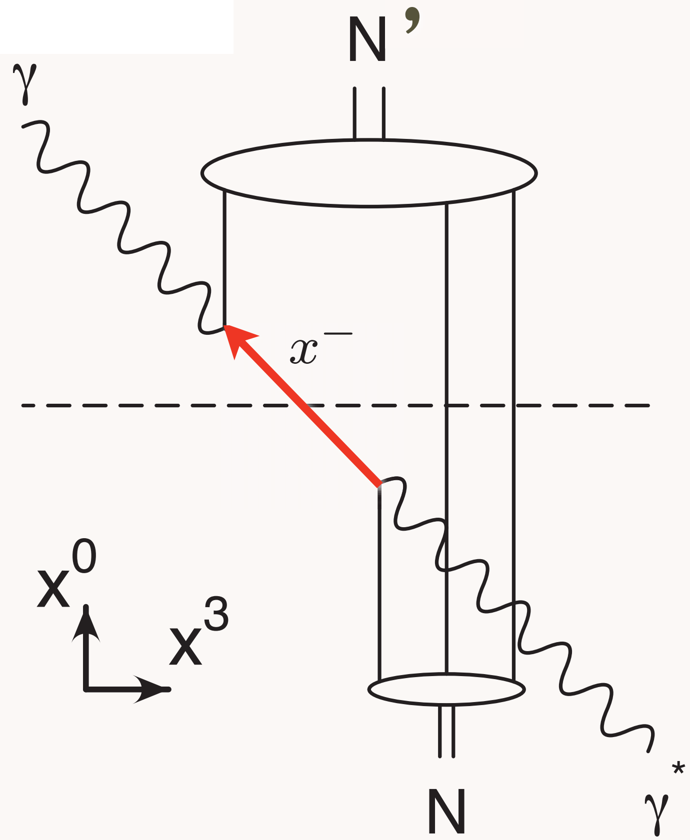
$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role of ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

Space-time picture of DVCS



$\sigma = \frac{1}{2}x^- P^+$

$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness, $\xi = \frac{Q^2}{2p.q}$
the longitudinal momentum transfer

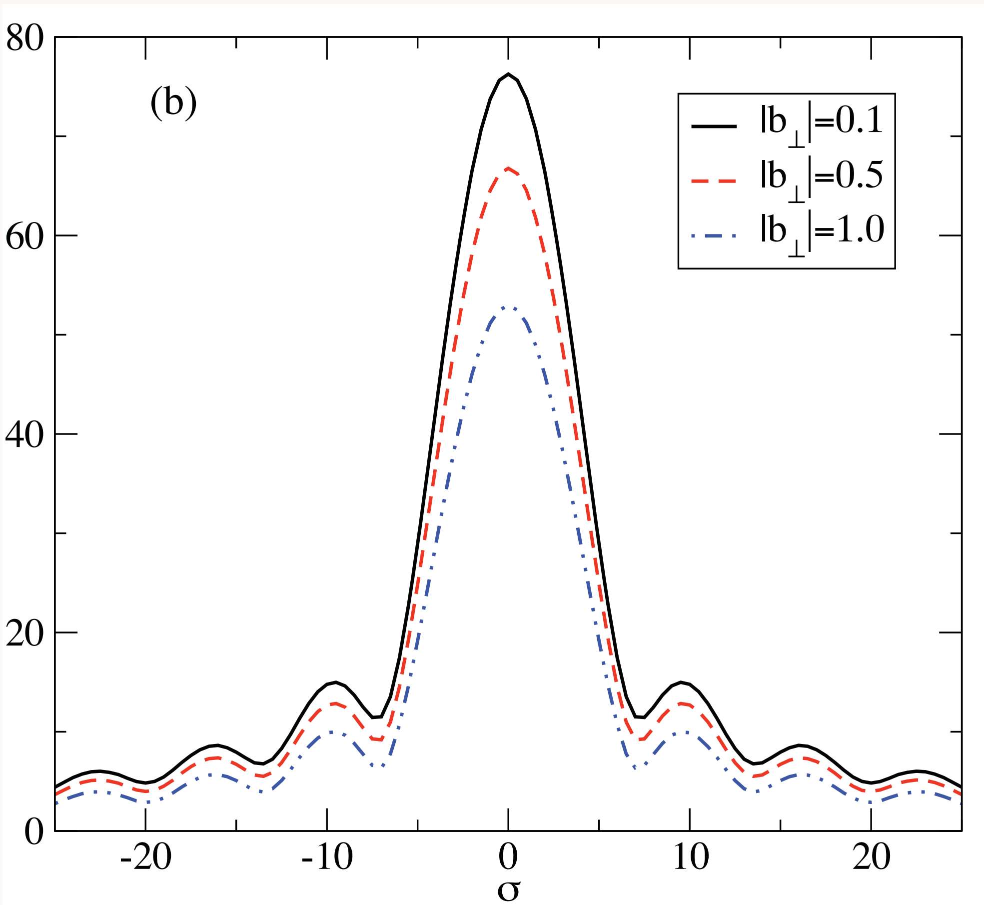
S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma,\vec{b}_\perp) = \frac{1}{2\pi}\int d\xi e^{i\frac{1}{2}\xi\sigma}\tilde{A}(\xi,\vec{b}_\perp)$$

$$\sigma = \frac{1}{2}x^- P^+ \quad \xi = \frac{Q^2}{2p.q}$$

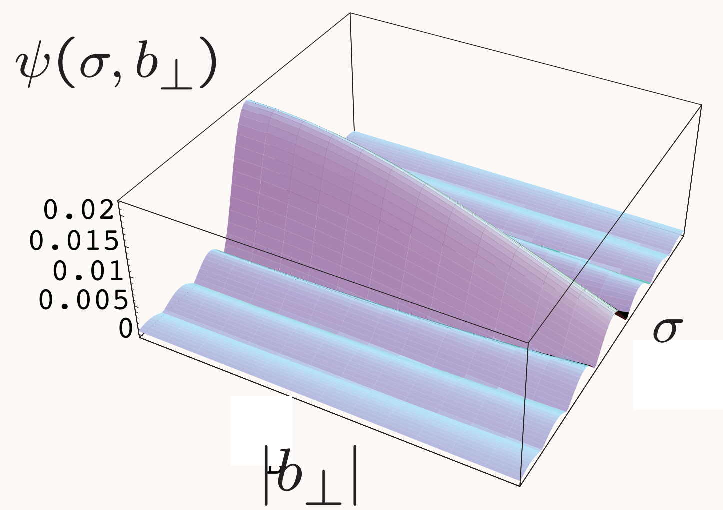


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$



Quark and Gluon condensates reside within hadrons, not vacuum

Shrock, sjb

- Bound-State Dyson-Schwinger Equations
- LF vacuum trivial up to $k^+ = 0$ zero modes
- Analogous to finite size superconductor
- Implications for cosmological constant -- reduction by 55 orders of magnitude!

Confined QCD Condensates

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3 +1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances

+ Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

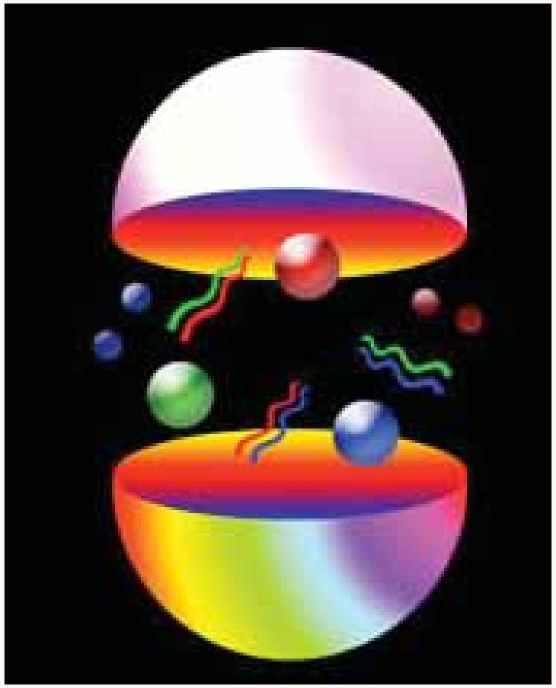
Boost Invariant 3+1 Light-Front Wave Equations

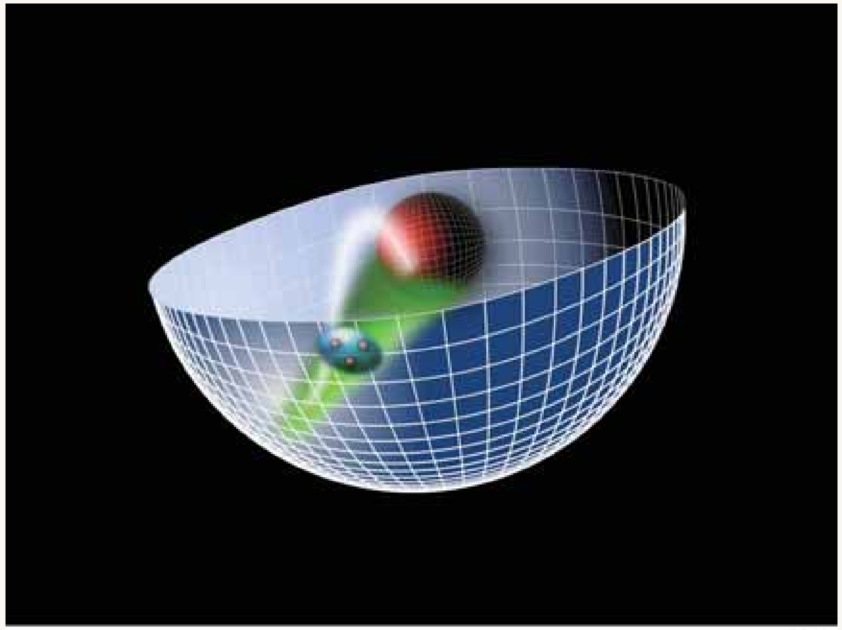
J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Congratulations Harald!





Stan Brodsky SLAC/IPPP

Fritzsch Symposium LMU June 6, 2008

Light-Front Holography and AdS/QCD Correspondence.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-13220, Apr 2008. 14pp.
e-Print: **arXiv:0804.3562** [hep-ph]

Light-Front Dynamics and AdS/QCD Correspondence: Gravitational Form Factors of Composite Hadrons.
Stanley J. Brodsky (SLAC) , Guy F. de Teramond (Ecole Polytechnique, CPHT & Costa Rica U.) . SLAC-PUB-13192, Apr 2008. 12pp. e-Print: **arXiv:0804.0452** [hep-ph]

AdS/CFT and Light-Front QCD.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-13107, Feb 2008. 38pp.
Invited talk at International School of Subnuclear Physics: 45th Course: Searching for the "Totally Unexpected" in the LHC Era, Erice, Sicily, Italy, 29 Aug - 7 Sep 2007.
e-Print: **arXiv:0802.0514** [hep-ph]

AdS/CFT and Exclusive Processes in QCD.
Stanley J. Brodsky, Guy F. de Teramond . SLAC-PUB-12804, Sep 2007. 29pp. Temporary entry
e-Print: **arXiv:0709.2072** [hep-ph]

Light-Front Dynamics and AdS/QCD Correspondence: The Pion Form Factor in the Space- and Time-Like Regions.
Stanley J. Brodsky (SLAC) , Guy F. de Teramond (Costa Rica U. & SLAC) . SLAC-PUB-12554, SLAC-PUB-12544, Jul 2007. 20pp.
Published in **Phys.Rev.D77:056007,2008**.
e-Print: **arXiv:0707.3859** [hep-ph]