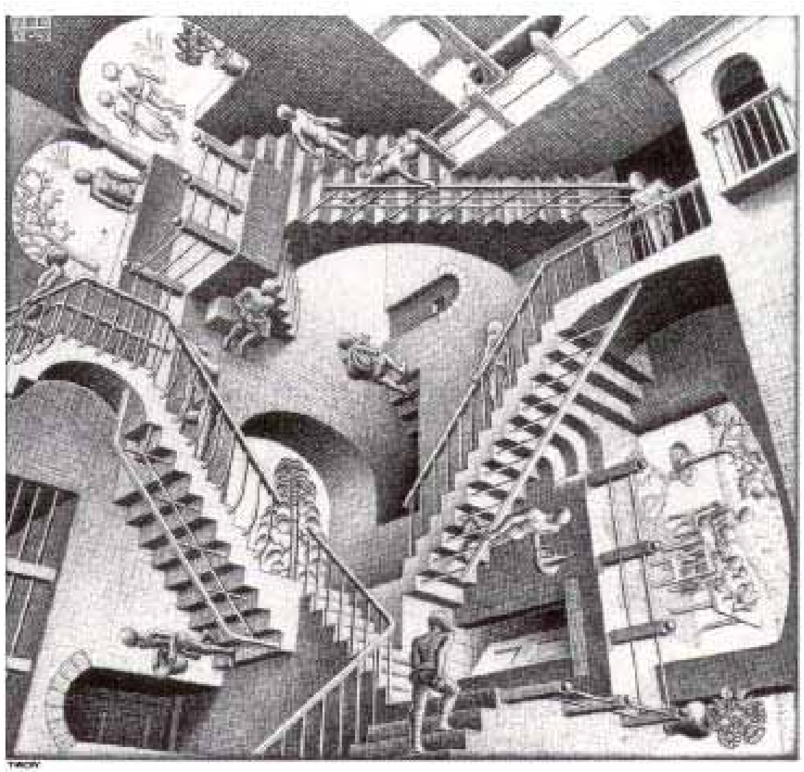
Introduction to Large & Universal Extra Dimensions



M.C.Escher, Relativity (1953)

T. Rizzo, KIPAC-GLAST

2/27/08

There are many reasons to study extra dimensions…..
In addition to the potential motivation from string theory, mostly they provide ways to address well-known issues by offering enormous model building opportunities…

- addressing the hierarchy problem[1, 2]
- producing electroweak symmetry breaking without a Higgs boson[3]
- the generation of the ordinary fermion and neutrino mass hierarchy, the CKM matrix and new sources of CP violation[4]
- TeV scale grand unification or unification without SUSY while suppressing proton decay[5]
- new Dark Matter candidates and a new cosmological perspective[6, 7]
- black hole production at future colliders as a window on quantum gravity[8]

⋮

But better still these ideas are both fun & experimentally testable!!!

The `modern era' of extra dimensions is now 10 years old!

Can we learn anything about Extra dims from 'classical' considerations? YES!

e.g.,

Consider a particle of mass M in 5D 'cartesian' co-ords + assume Lorentz Invariance holds in 5D

$\longrightarrow$ Then $\boxed{p^2 = M^2}$ but $p^2 = ?$

$$\boxed{p^2 = p_0^2 - \vec{p}^{\,2} \pm p_5^2}$$

{ $O(4,1)$ or $O(3,2)$? }

- p_0^2: Energy
- $\vec{p}^{\,2}$: 3-momentum
- $\pm$: sign?
- p_5^2: momentum in 5th dim

+	time-like
−	space-like
	Extra dim

Is there any preference? Yes ... rewrite..

$$p^2 = M^2 \rightarrow \quad p_0^2 - \vec{p}^{\,2} = p_\mu p^\mu = M^2 \mp p_5^2 \Rightarrow$$

- if $p_5^2 > M^2$ (why not?) AND I choose a time-like E.D. then $\boxed{p_\mu p^\mu < 0}$

$\rightarrow$ a **tachyon** w/ possible causality problems!

To avoid tachyons we generally choose all extra dims space-like $\therefore$ **only 1 time dimension**

- Thinking about <u>one</u> extra dimension →

∵ A simple extension of 4D ? $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \, [dx_5^2]$

A <u>flat</u>, <u>space-like</u> <u>extra dim</u>

(no complicated metric tensors etc)

5D Klein Gordon Equation (real scalar) : $\partial_A \partial^A \Phi(x_\mu, y) = 0$, with $\partial_A \partial^A = \partial_\mu \partial^\mu - \partial_y^2$, $A = (\mu, y)$

∴ do sep. of variables $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$

⟹ Kaluza-Klein (KK) decomposition

→→ $\sum_n (\chi_n \partial_\mu \partial^\mu \phi_n - \phi_n \partial_y^2 \chi_n) = 0$

now <u>if</u> $\partial_y^2 \chi_n = -m_n^2 \chi_n$ then

$\sum_n \chi_n \underbrace{(\partial_\mu \partial^\mu + m_n^2)\phi_n}_{0} = 0$ } a set of indep. equations

∞ set of **massive** scalar states : **A KK tower**

- m_n takes on <u>discrete</u> values ⟺ extra dimension is <u>COMPACT</u> - of <u>finite</u> size !!

➡ They are <u>small</u> + that's why we don't 'see' them

Action Approach

$$S = \int d^4x \int_{y_1}^{y_2} dy \; \underbrace{\tfrac{1}{2}\partial_A \Phi \, \partial^A \Phi}_{\tfrac{1}{2}\partial_\mu \Phi \, \partial^\mu \Phi \; - \; \tfrac{1}{2}\partial_y \Phi \, \partial_y \Phi}$$

let $\Phi \equiv \sum_n \chi_n(y)\, \phi_n(x_\mu)$ Then

$$S = \int d^4x \int_{y_1}^{y_2} dy \left\{ \tfrac{1}{2} \sum_{nm} \overset{①}{\chi_n \chi_m} \, \partial_\mu \phi_n \partial^\mu \phi_m - \tfrac{1}{2} \sum_{nm} \phi_n \phi_m \overset{②}{\partial_y \chi_n \partial_y \chi_m} \right\}$$

To 'Diagonalize'

① $\int_{y_1}^{y_2} dy \; \chi_n \chi_m = \delta_{nm}$

② integrate-by-parts : $\int_{y_1}^{y_2} dy \; \partial_y \chi_n \partial_y \chi_m$

$$= \underbrace{\chi_m \partial_y \chi_n \Big|_{y_1}^{y_2}}_{\overset{!!}{0}} - \int_{y_1}^{y_2} dy \; \chi_m \underbrace{\partial_y^2 \chi_n}_{}$$

Boundary Conditions !

now $\underline{\text{if}}$ $= -m_n^2 \chi_n$

Then $S = \int d^4x \sum_n \left(\tfrac{1}{2} \partial_\mu \phi_n \partial^\mu \phi_n - \tfrac{1}{2} m_n^2 \phi_n^2 \right)$

– n massive scalars

$+ \boxed{(\partial_y^2 + m_n^2)\chi_n = 0}$, $\chi_n = A_n e^{i m_n y} + B_n e^{-i m_n y}$

$m_n = ?$

non-compact $\sim e^{i p_x x}$

p_x momentum along an infinite dimension

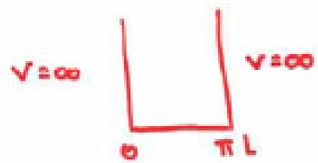
p is **continuous**

Compact cases $\leftrightarrow$ quantization of momentum $\leftrightarrow$ mass

$(\partial_y^2 + m_n^2)\chi_n = 0$ $\qquad \chi_n = A_n e^{i m_n y} + B_n e^{-i m_n y}$

general solution

From QM : (1) "particle in a box": BC's ??

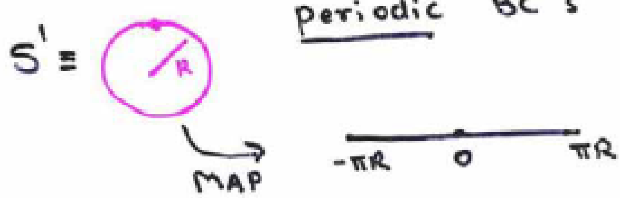


$\chi_n(0, \pi L) = 0 \rightarrow C_n \sin ny/L$

and $\boxed{m_n = n/L}$; $n = 1, 2, \ldots$

(2) motion in a circle (or orbital angular momentum)

periodic BC's



$\chi_n(y + 2\pi R) = \chi(y)$

$\Downarrow$

$\chi_n = A_n \cos ny/R + B_n \sin ny/R$

$n = 0, 1, 2 \ldots$

$\boxed{m_n = n/R}$

Observe : KK masses $\sim \frac{1}{\text{size}}$ of extra dim

$\rightarrow$ a slight variation ...

S^1/Z_2 ORBIFOLD

↑ a manifold w/ discrete symmetr(ies)

$Z_2: \quad y \leftrightarrow -y$

here Z_2 is just parity on the interval...

$$\chi_n = \begin{cases} A_n \cos ny/R \\ B_n \sin ny/R \end{cases} \quad \text{OR} \quad (m_n = n/R)$$

.... even vs odd under Z_2

- Higher Dim Field Decomposition :

we saw a 5D scalar → a tower of 4D scalars ....

What about a 5-vector A^M ??? or symmetric tensor h^{MN}

- We've 'done' this before .... !!

Lorentz (4D)		Rotations (3D)
scalar	⟷	scalar
4-vector A^μ	→	$\vec{A}, \phi$
tensor $F^{\mu\nu}$	→	$\vec{E}, \vec{B}$

do it again for 5D...

→ 5D scalar → 4D scalar(s) (this we know)

5D vector : $A^M \to (A^\mu, A^5)_n$ — tower of vectors + scalars in 4D

Symmetric tensor : $h^{MN} \to (h^{\mu\nu}, h^{\mu 5}, h^{55})_n$ — tensors + vectors + scalars generalize to 4+n dimensions

$$A^{L} = (A^{\mu}, A^{i})_{n_i} \quad (i = 1 \ldots n)$$

A^{μ} ← 4-vector; A^{i} ← 4-scalars

$$h^{LP} = (h^{\mu\nu}, h^{\mu i}, h^{ij})_{n_i}$$

$h^{\mu i}$ ← n 4-vectors; h^{ij} ← $\frac{1}{2}n(n+1)$ 4-scalars

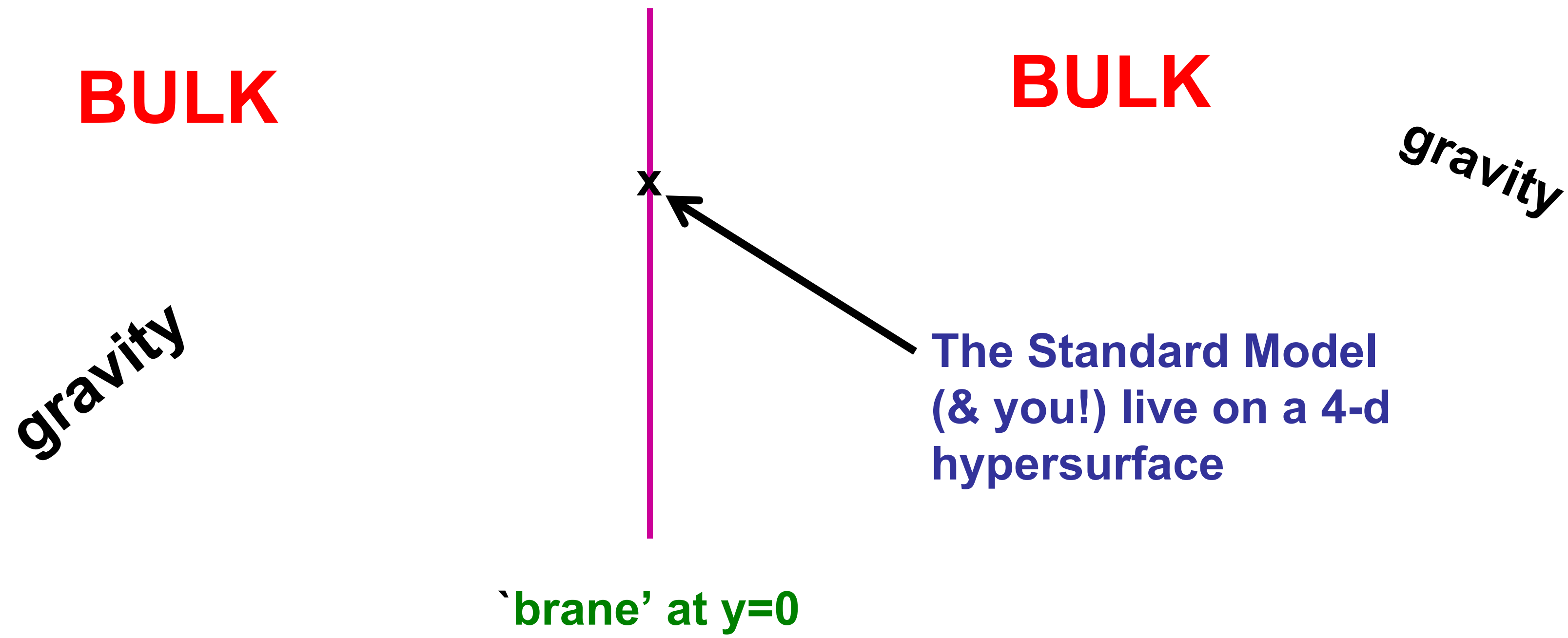
Messages: there are many ways to pick BC's and ways to compactify - the number only grows as we add more dimensions ...

Common Choices: S^1, S^2 ... S^n

- S^1 (circle)
- S^2 usual sphere
- S^n n-sphere in n+1 dim space

$\underbrace{S^1 \times S^1}_{T^2} \times \ldots S^1 \rightarrow$ torus or 'doughnut'

ADD



The Arkani-Hamed, Dimopoulos & Dvali (ADD) scenario postulates that only gravity can propagate in extra dimensions while the SM lives on a hypersurface at y=0 in a D=4+n-dimensional space. These extra n-dimensions are compactified & have a volume $V_{n.}$

The purpose of this model is to address the hierarchy problem: i.e., why are the weak and Planck scales so different...

Gauss' Law for gravity in n-dimensions tells us several things:

$$\bar{M}_{pl}^2 = V_n \, M_*^{n+2} \quad * $$ (to be derived below)

Here M_* is the true, D-dimensional fundamental scale of gravity. If $M_* \sim 1$ TeV the hierarchy problem `goes away'.

If the compactified space has a typical size ~ R, then

$\phi(r>>R) \sim 1/r$ while $\phi(r<<R) \sim (R/r)^n \, 1/r$

To say more we need to know the nature of the compactified space

* Care required!! note *reduced* Planck scale here

A Derivation of the ADD Relation

$$S_{n+4} = \int d^4x \; dy \frac{1}{M_*^{1+n/2}} h_{\mu\nu}(x,y) T^{\mu\nu}(x,y),$$

but

$$h_{\mu\nu}(x,y) = \sum_l h_{\mu\nu}^{(l)}(x)\chi^{(l)}(y), \quad T^{\mu\nu}(x,y) = T^{\mu\nu}(x)\delta(y),$$

with

$$\chi^{(l)}(y) = \frac{1}{\sqrt{V_n}} e^{ily/R}.$$

Trivially integrate over y to obtain S_4:

$$S_4 = \int d^4x \frac{1}{\sqrt{V_n}} \frac{1}{M_*^{1+n/2}} \sum_l h_{\mu\nu}^{(l)}(x) T^{\mu\nu}(x),$$

so to recover GR for the massless graviton, $l = 0$, it must be that

$$\overline{M}_{Pl}^2 = V_n M_*^{n+2}.$$

Another derivation follows from the Einstein-Hilbert action…

In D=4+n dimensions we have

$$S_{n+4} = \frac{M_*^{n+2}}{2} \int d^4x \; dy \sqrt{-g} R_{n+4} \, ,$$

Substitute the graviton KK decomposition & demand we recover GR for the zero-mode after integration over the extra dimensions & we again get the identical relationship

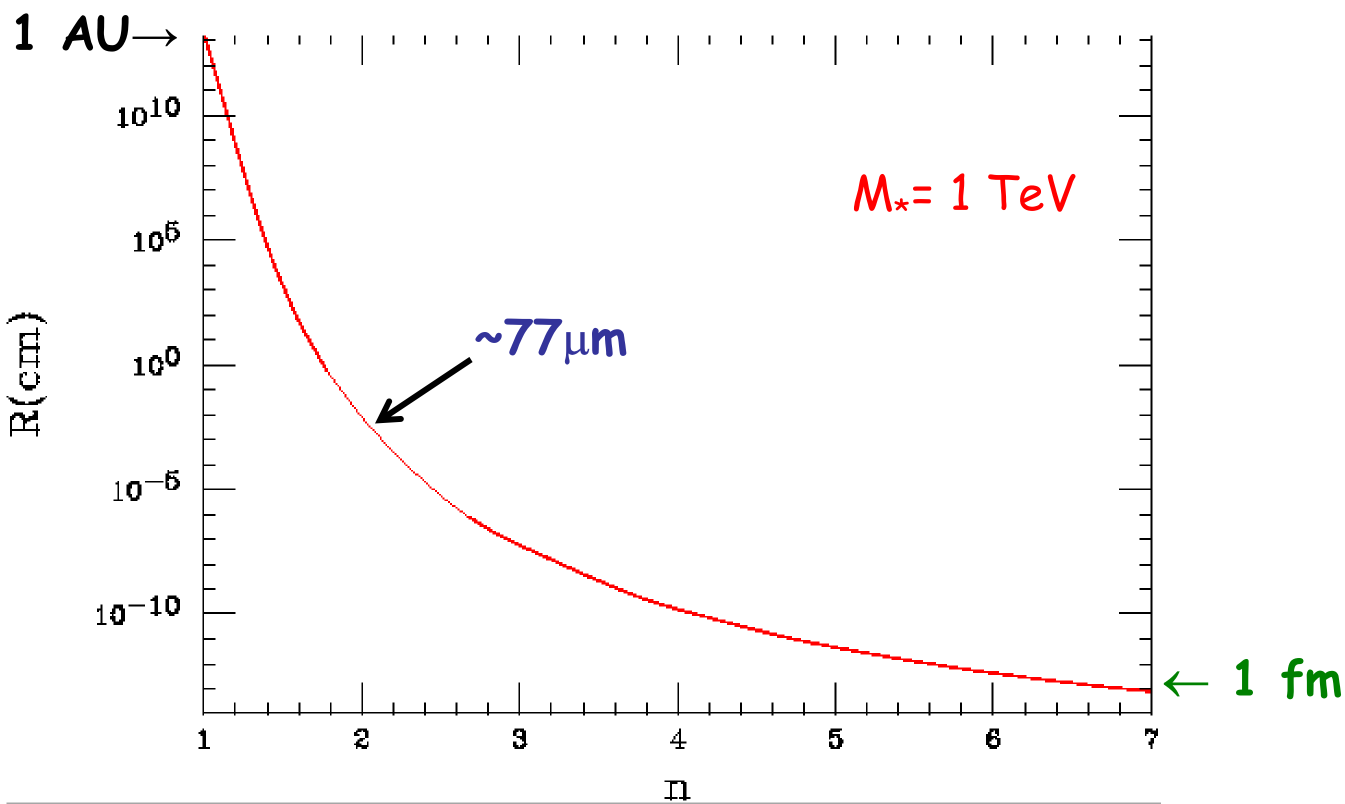
This also tells us that M_* , which appears in the action and graviton coupling to matter, is the correct D=4+n dimensional mass scale

ADD use a toroidal compactification: $T^n = S^1 \times S^1 \times \ldots$

If all these circles have the same size then $V_n = (2\pi R)^n$

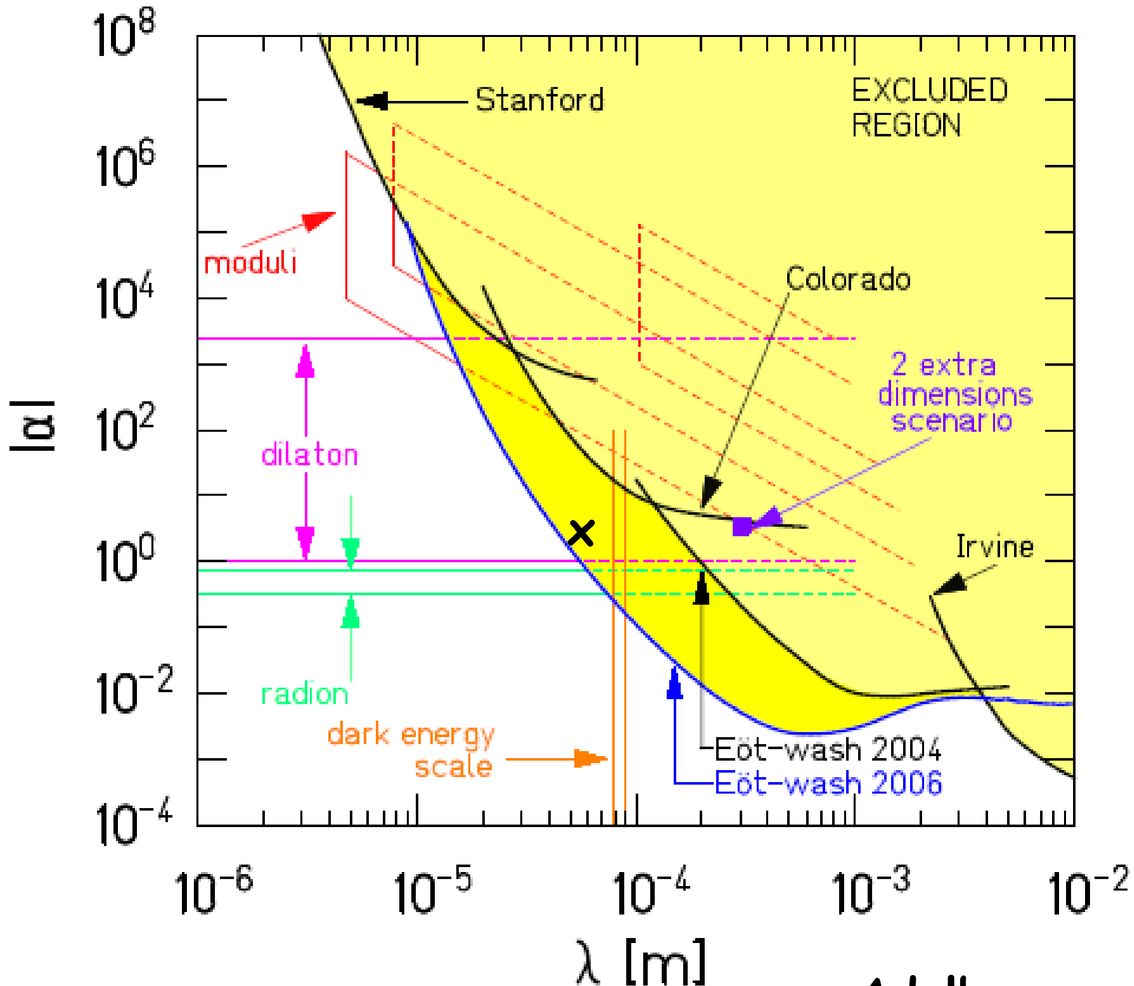
Thus if $M_* \sim 1$ TeV, R is calculable up to a factor of

$$(1\ \mathrm{TeV}/M_*)^{(n+2)/n}$$



There are well-known constraints on this radius by looking for deviations from Newton's Law….note the prediction below for n=2. Putting in the reduced Planck scale & keeping all the π's, and use of M_* is important!

M_*=2 TeV is fine!!

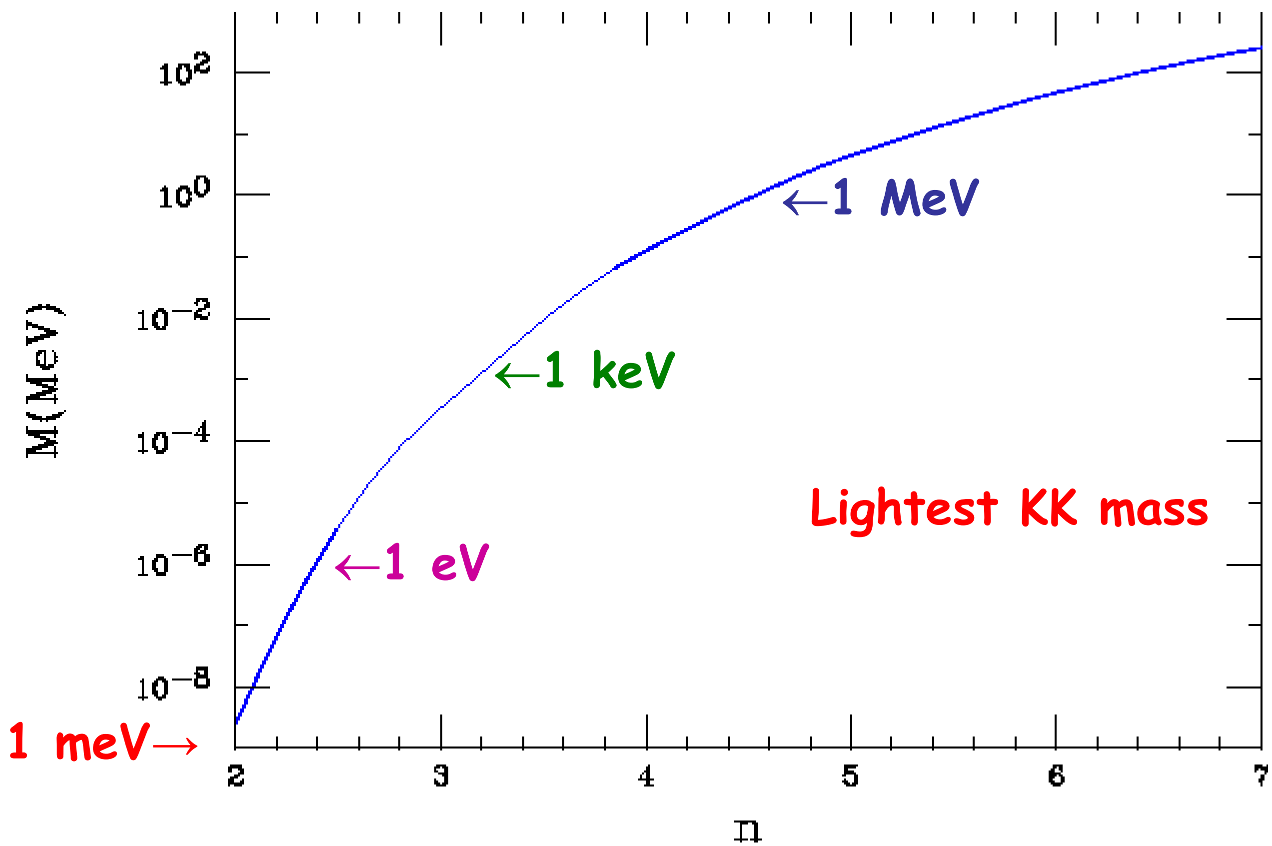


Adelberger etal

In this case, the KK graviton masses are given by

$$M_k^2 = k.k/R^2, \qquad k=(k_1, \ldots, k_n)$$

Thus the *lightest* (massive) KK states have a mass =1/R



Comments:

- Not all the dimensions need to be the same size
- The compactification manifold itself need not be so trivial
- Watch out for alternative notation & normalization in the literature -> consistency issues!! This has been a nightmare for theorists & experimenters
- The reason why the SM is brane localized is now clear: SM particles, e.g., the photon, do not have (observable!!) KK excitations w/ masses below about ~1TeV

How do these massive gravitons decay?

There are 2 interesting decay components:

- Heavy gravitons may decay to, e.g., pairs of lighter gravitons, as long as n-dimensional momentum is conserved and the phase space is available, via the triple graviton coupling in GR…this is non-trivial.

(Momentum is conserved along each of these directions since any motion corresponds to a sum of that along a series of orthogonal circles and QM enforces the individual angular momentum conservation for each case)

This can lead to complex decay patterns..

- Furthermore, and perhaps more importantly, all gravitons can also decay through a universal coupling to the SM fields on the y=0 brane given by

$$\mathcal{L} = -\frac{1}{\overline{M}_{Pl}} \sum_n G_n^{\mu\nu} T_{\mu\nu} \,,$$

So we can go ahead and calculate graviton lifetimes as a function of their mass…

A canonical graviton decay width is controlled by the quantity

$\Gamma_0 = m^3/(80\pi\overline{M}_{pl}^2)$,   m is the KK mass

which corresponds to a time scale

τ_0 = 31 *Gyr* (100 MeV/m)^3

The actual decay rate depends upon the available phase space & the number of open modes. The decay rate into photons (1), each neutrino species (0.5), electron & muons (0.5 each), u and d quarks (1.5 each) are all proportional to Γ_0 (apart from phase space)

Note that the decay to hadrons via *gg* has a rate $8\Gamma_0$.

These hadronic modes only open once the 2 pion mass threshold is reached.

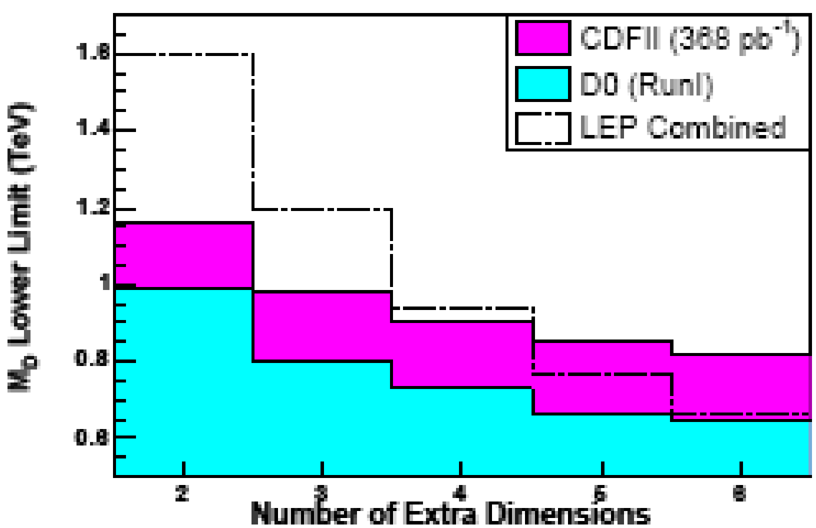
Constraints I

Clearly, for n>2 there are no table-top limits so we look for constraints elsewhere. E.g., at colliders, towers of KK gravitons can be emitted in SM processes:

$q\bar{q}\to gG$, $qg\to qG$, $gg\to gG$ can occur at the Tevatron/LEP

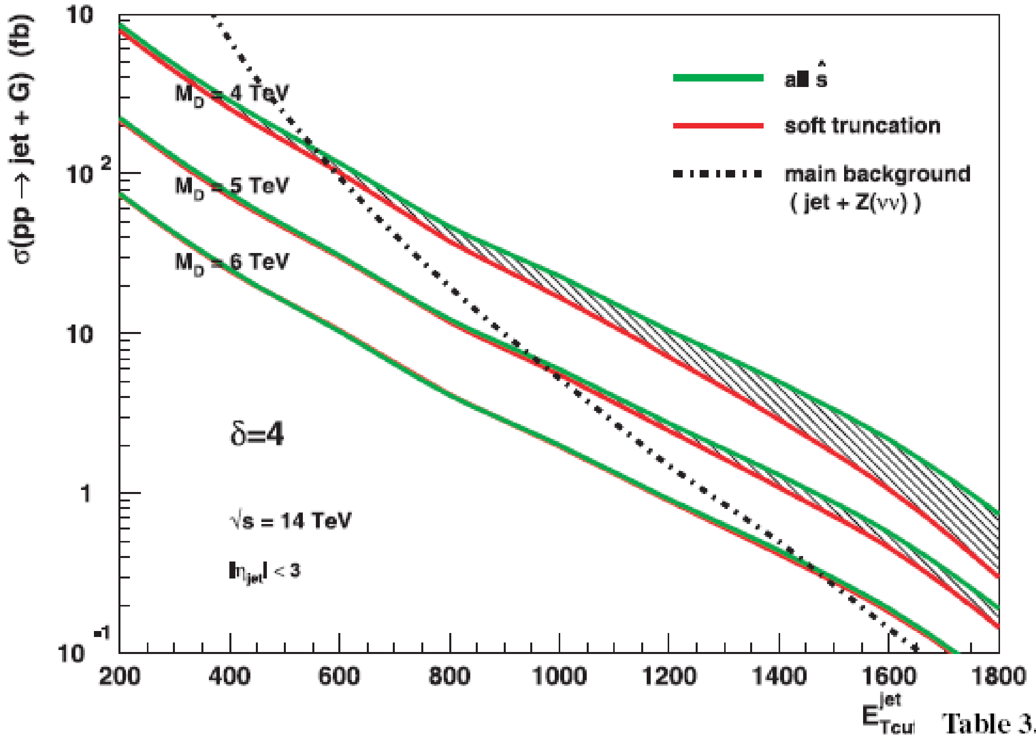
$e^+e^-\to\gamma G$ at LEPII/ILC

The gravitons appear as missing energy since they interact so weakly in the detector



There are serious SM backgrounds that need to be accounted for in these searches..

Note $M_* = M_D\,(2\pi)^{-n/n+2}$

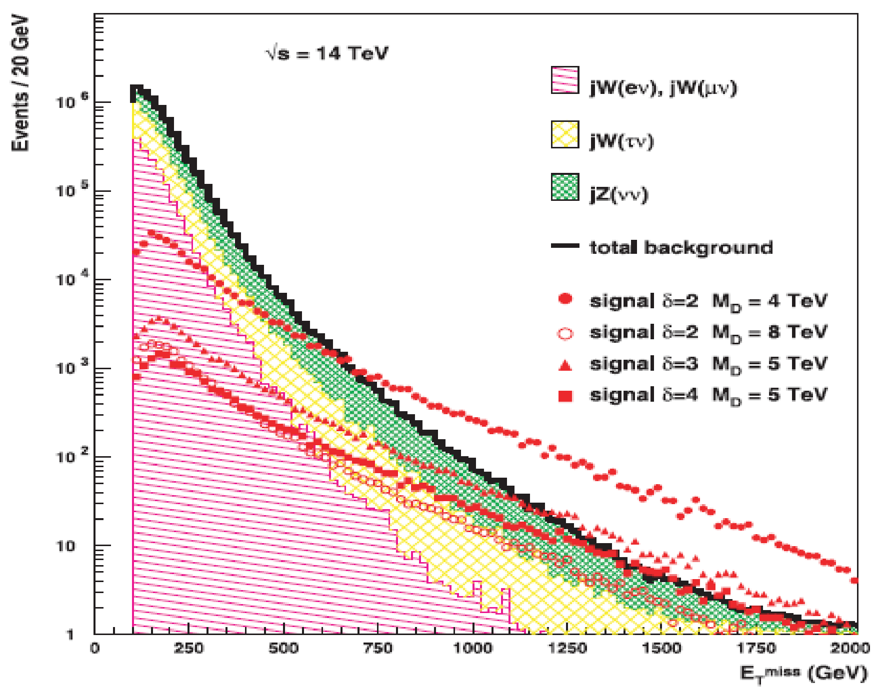


Detailed LHC/ATLAS study by Vacavant & Hinchliffe…

note M_D ↓

Table 3. 5-sigma discovery limits on M_D (TeV) for various scenarios.

δ	$\sqrt{s} = 14$ TeV 100 fb^{-1}	$\sqrt{s} = 14$ TeV 1000 fb^{-1}	$\sqrt{s} = 28$ TeV 100 fb^{-1}	$\sqrt{s} = 28$ TeV 1000 fb^{-1}
2	9	12	15	19
3	7	8	12	14
4	6	7	10	12



Graviton exchange between SM fields also leads to valuable constraints

Constraints II

As you know there are also constraints from both astrophysics & cosmology on ADD extra dimensions:

- Overproduction of KK's by gravi-bremsstrahlung in NN scattering can lead to early matter domination which reduces the allowed age of the universe[#]. The bounds might be avoided by some new cosmological evolution between ~ 1MeV, respecting nucleosynthesis and the QCD phase transition temperature, T_{QCD}. Furthermore, if the KK production scale were as high as ~1GeV, the dominant modes would decay quickly enough to soften these bounds by a substantial amount[@]. This softening would also apply to the case of KK production in SN.

As a non-expert, I think the jury is still out here but I'm happy to be convinced one way or the other…

[#]Fairbairn [@]Macesanu & Trodden

- Cooling of SN by KK production & gamma backgrounds produced by graviton KKs trapped in the NS remnant halo can lead to very strong constraints[$].

These bounds are summarized in the next set of tables…again notational confusion has crept into the literature

[$]Hannestad & Raffelt

Constraints II (cont)

`Age of the Universe'

Table 1: Minimum value of M_f to prevent $t_{today} < 12.8Gyr$ for various T_{QCD}

Number of extra dimensions	T_{QCD}=170MeV	T_{QCD}=135MeV	T_{QCD}=100MeV
2	1,000TeV	390TeV	86TeV
3	59TeV	26TeV	7.4TeV
4	9.0TeV	4.4TeV	1.5TeV

Phase transition temperature

Fairbairn

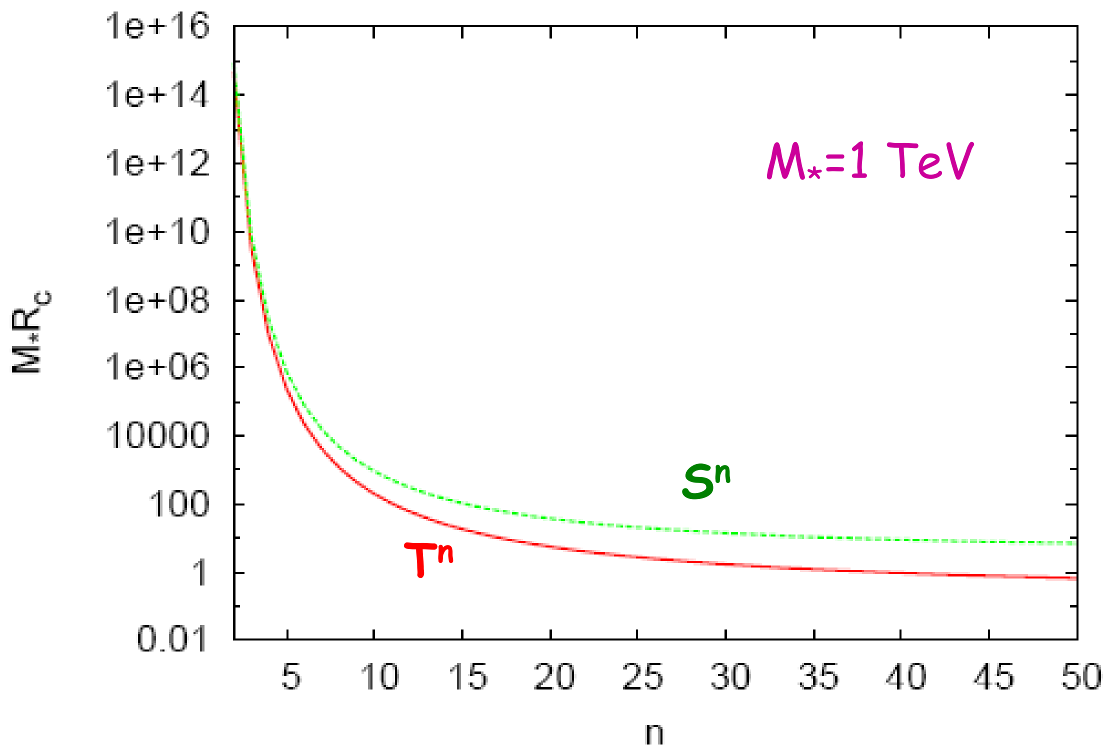
divide by this

Bounds on M*

n	1	2	3	4	5	6	7
$M/\overline{M}_{n+4}$	2.32	2.98	3.46	3.82	4.10	4.32	4.51
Neutrino Signal							
SN 1987A	7.4×10^2	8.9	0.66	1.18×10^{-1}	3.5×10^{-2}	1.44×10^{-2}	7.2×10^{-3}
EGRET γ-ray limits							
All cosmic SNe	3.4×10^3	28.	1.65	2.54×10^{-1}	6.8×10^{-2}	2.56×10^{-2}	1.21×10^{-2}
Cas A	7.7×10^2	14.5	1.24	2.34×10^{-1}	7.0×10^{-2}	2.80×10^{-2}	1.37×10^{-2}
PSR J0953+0755 / RX J185635−3754	**2.93×10^3**	**38.6**	**2.65**	**0.43**	**0.116**	**4.31×10^{-2}**	**1.98×10^{-2}**
Neutron-star excess heat							
PSR J0952+0755	1.61×10^5	7.01×10^2	25.5	2.77	0.57	0.17	6.84×10^{-2}

Hannestad & Raffelt

The problem with the ADD scenario (one of many) is that it does not really solve the hierarchy problem, i.e., to eliminate or explain the large ratio of the weak and Planck scales…it just hides this problem somewhere else



For any typical n, M_*R is an enormous number… other models are better for this.

NOTE

The hierarchy and flavor problems are best addressed within models with warped extra dimensions such as the Randall-Sundrum model. Unfortunately, a discussion of such scenarios is beyond the scope of this talk.

Universal Extra Dimensions (UED)

The goal here is NOT to address the hierarchy but for other model building purposes…there are several versions of this model the simplest being the case of one flat extra dimension compactified on an S^1/Z_2 orbifold which has a size 1/R ~ 1 TeV.

Orbifolding is a powerful tool and lets us make chiral (2-component) 4-d fermions from 5-d (4-component) ones.

SM fields will now have KK excitations with masses beginning at ~1/R

One extra dimension of radius R with $-\pi R \leq y \leq \pi R$ and a parity $y \to -y$ symmetry, i.e., *even* or *odd* states

All SM fields are `in the bulk', i.e., will have KK excitations

$M_n^2 = m_0^2 + (n/R)^2$, n=0,1,2,… where m_0 is the SM particle mass, are the excitation/tower masses. Even and odd parity states are degenerate.

→ The usual SM particles are the `zero modes' of the KK tower

SM Gauge and Higgs bosons are parity even, i.e., have zero modes (obviously)

KK Tower Structure

	EVEN	ODD
	etc…	
n = 2	---------	---------
n = 1	---------	---------
n = 0	---------	X

… only the even tower has a zero mode

$\psi_n \sim \cos ny/R \qquad \sim \sin ny/R$ ← KK wavefunctions, periodic BCs

· For fermions, even and odd towers BOTH exist and have opposite helicity. E.g., for SM doublets (singlets), even tower fields are LH (RH)

→ KK fermion excitations are similar to `vector-like' fermions

Model Parameters : only two → very predictive!

- 1/R, the KK mass scale How large is it ?

- Λ – the cutoff scale → UED is an effective theory and needs a cutoff. The practical application is that the tree level spectrum is highly degenerate so loop corrections to masses are important. They behave as a sum of terms that go like

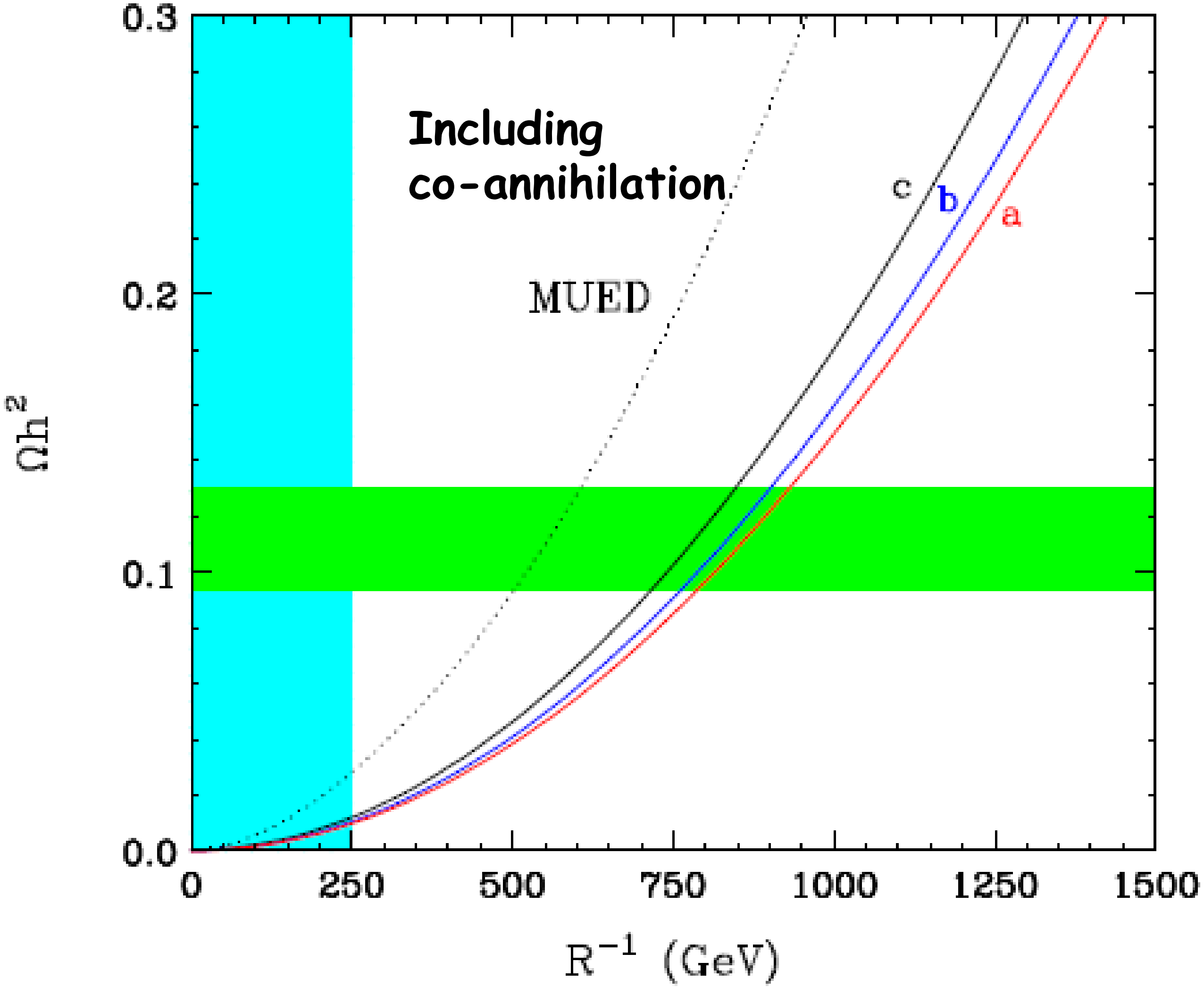
$$\Delta m^2 \sim N_i/R^2 \ (\alpha_i/4\pi) \log(\Lambda R) \qquad (\text{ with } \Lambda R \sim 20)$$

Note that there is only log sensitivity to Λ !

→ How large is 1/R and what does a realistic spectrum look like???

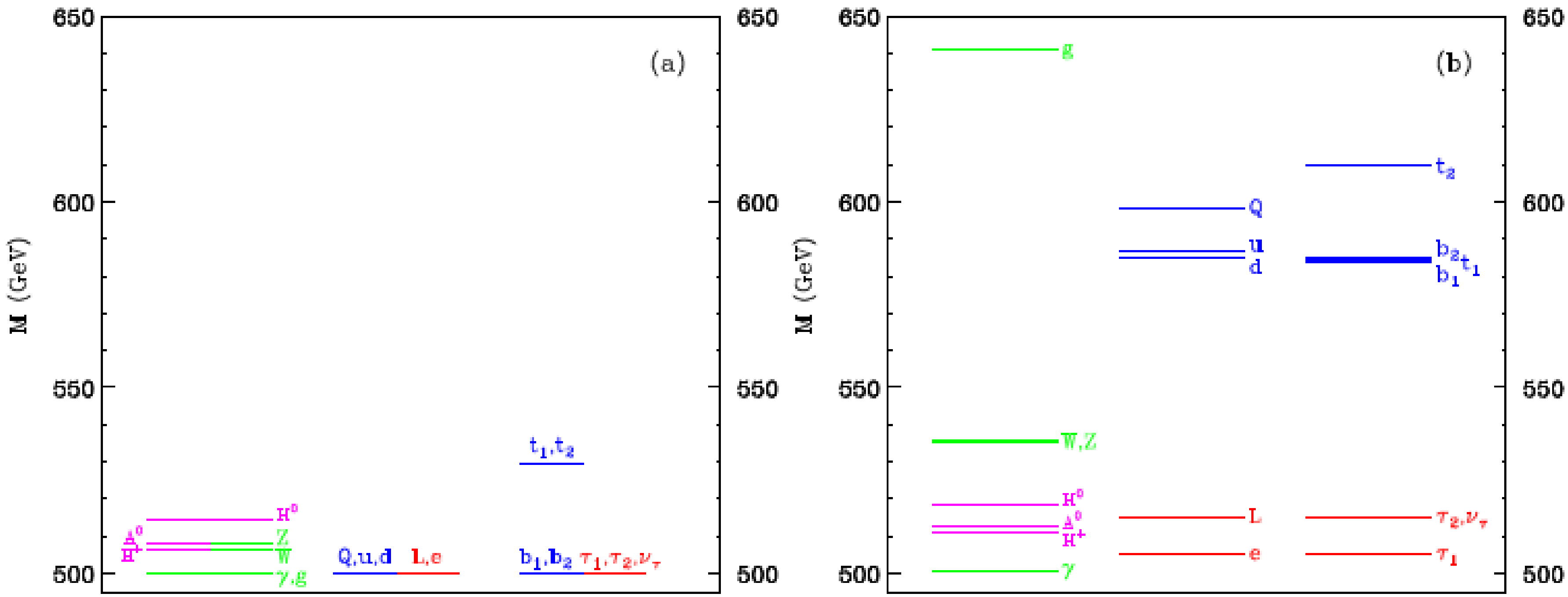
Precision EWK data → 1/R > 300 GeV

Dark Matter density → 450 < 1/R < 700 GeV (preferred)



Here is the effect of radiative corrections on the various particle masses for 1/R=500 GeV and ΛR=20

As can be seen these are substantial and important to the phenomenology



before after

Since all fermions have the same wavefunctions etc. and differ only in their zero mode masses there is an active GIM mechanism and all flavor interactions are controlled *only* by the CKM matrix…so these constraints are weak

→ All the new flavor physics comes in loops with KK tower fields in them

Comments:

- KK towers can talk to each other via Yukawa couplings and the usual Higgs field that generate SM fermion masses

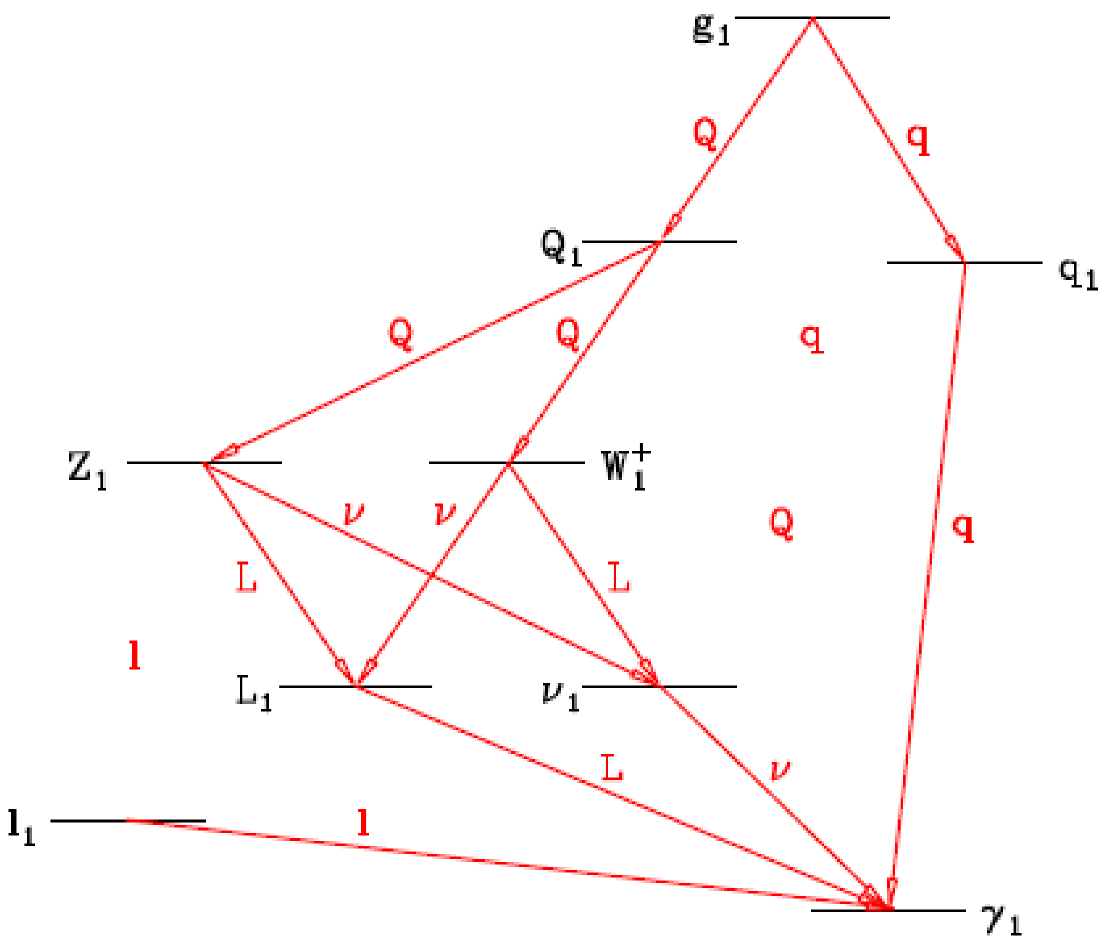
- The KK towers of the Higgs doublet whose zero modes are eaten by the SM W/Z remain in the spectrum

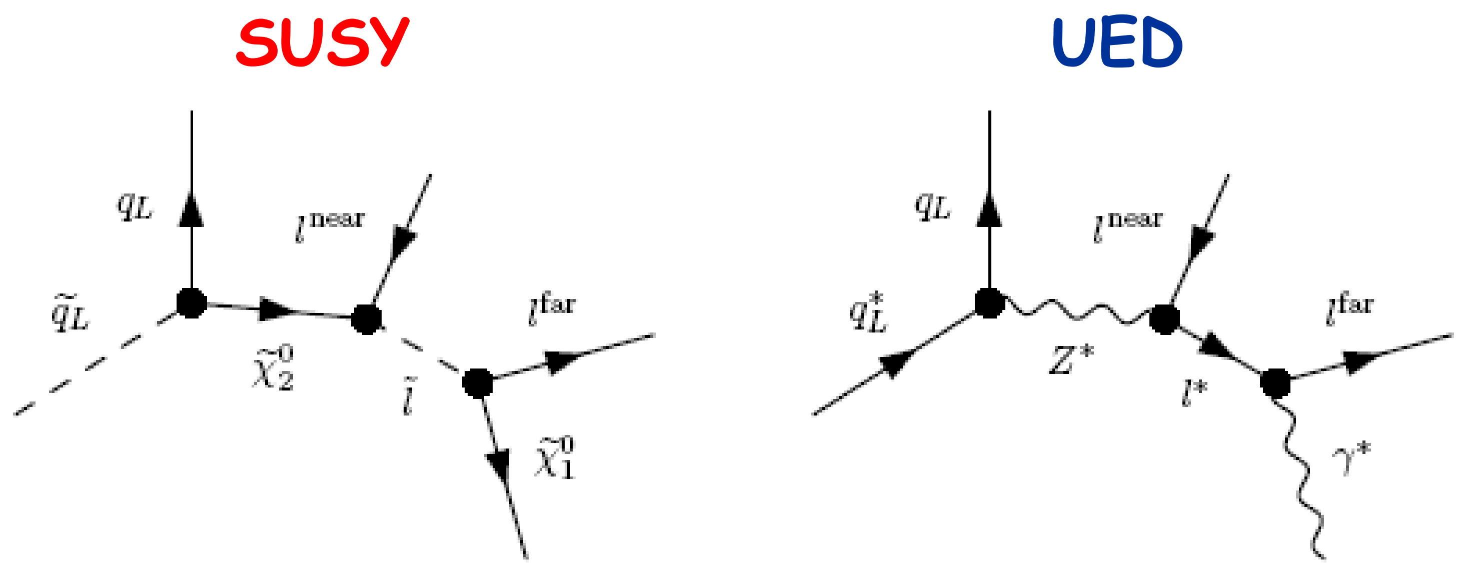
Due to the orbifolding mechanism the momentum is no longer conserved along the 5th dimension and the symmetry is reduced to a KK-parity, $P=(-1)^n$ which is an exact symmetry.

This means that the first-level KK states can only be pair produced at colliders and that the lightest KK state, which is ~ the KK excitation of the $U(1)_Y$ gauge boson, must be stable & is a good DM candidate – the LKP.

Sound familiar? This is a lot like SUSY & is now seen as a relatively common feature of many TeV scale models, e.g., Little Higgs.

The production and long decay chains of UED KK excitations at the LHC with masses in the above range looks *a lot* like SUSY...





These decay patterns look very similar except for the particle spin…note that because of the parity symmetry the lightest UED KK state is stable (LKP) like the LSP in SUSY...that's why it can be the dark matter.

These models are generally indistinguishable at LHC & might require ILC to do the job…info from the flavor sector may also be of some help here.

There are some differences for DM studies as can be imagined…

LKP & LSP DM annihilation, which occurs at non-relativistic velocities, is somewhat different due to the fact that one, the LSP, is a Majorana fermion while the other, the LKP, is a real spin-1 boson.

The dominant LSP pair annihilation to heavy fermion pairs via the Higgs is highly suppressed so co-annihilation channels are critical. In the LKP case there is no such suppression and light fermions are also allowed as final states. The favored LKP mass is thus somewhat higher which is good given the collider limits on 1/R.

Direct DM searches are also different in detail due to the spin of the two DM candidates.

Summary

- Extra dimensional models come in many different shapes and varieties which serve a number of distinct purposes. Many possibilities remain to be explored.

- The phenomenology of these models is quite sensitive to model details and assumptions

- The LHC will soon open up the possibility to directly produce TeV KK excitations of gravitons and/or SM particles. Will we know it?

- In the end, only experiment will tell us if any of these ideas are relevant to nature

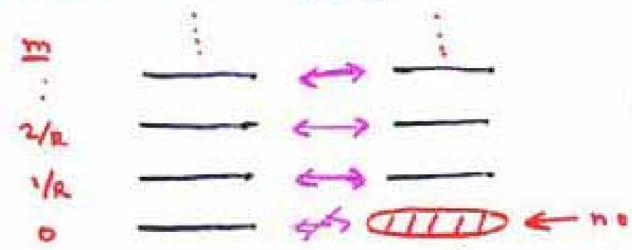
Backup Slides

KK - Goldstone mechanism :

∴ consider a gauge field in 5D → A^N....

a massless field in 5D has 3 polarizations
⇕
U(1) gauge invariance

⇒ put it on S^1/Z_2 :



4-vectors are Z_2 even...
scalars are odd

$A^\mu_n \sim \cos ny/R$ — ↑ vectors w/ 2 polarizations (massless!)

$A^5 \sim \sin ny/R$ — scalars

For $n \geq 1$ the vector eats the corresponding scalar to obtain a mass
∴ 3 polarisations in 4D

∴ in Physical (Unitary) gauge ......

→ massive tower ⊕ 1 massless mode
≡ zero mode

- The zero mode is massless since $A_5^{(0)}$ does not exist to be eaten ∴ U(1) is maintained

But : it was the **BC's** that killed

$A_5^{(0)}$ $\leftrightarrow$ link between BC's and breaking of gauge invariance

$\Rightarrow$ { non-ORBifold BC's can be used to break gauge symmetries !!

e.g.,

$y=0$ — $y=\pi R$

$\begin{cases} A^\mu = 0 & \partial A^\mu = 0 \\ \partial A_5 = 0 & A_5 = 0 \end{cases}$

$A^\mu \sim a_n \cos m_n y + b_n \sin m_n y$

$\rightarrow \begin{cases} a_n = 0 \\ m_n = (n + 1/2)/R \end{cases}$

$\Rightarrow m_0 = 1/2R \neq 0$!!

U(1) symmetry is broken! $A_5^{(0)}$ exists + is eaten by $A_\mu^{(0)}$

$\therefore$ The physics of extra dimensions is the physics of the KK excitations

$\rightarrow$ Some Models..