Light Front Holography and AdS/QCD

University of Helsinki
April 29, 2008

AdS/QCD

Stan Brodsky, SLAC & IPPP

Goal:

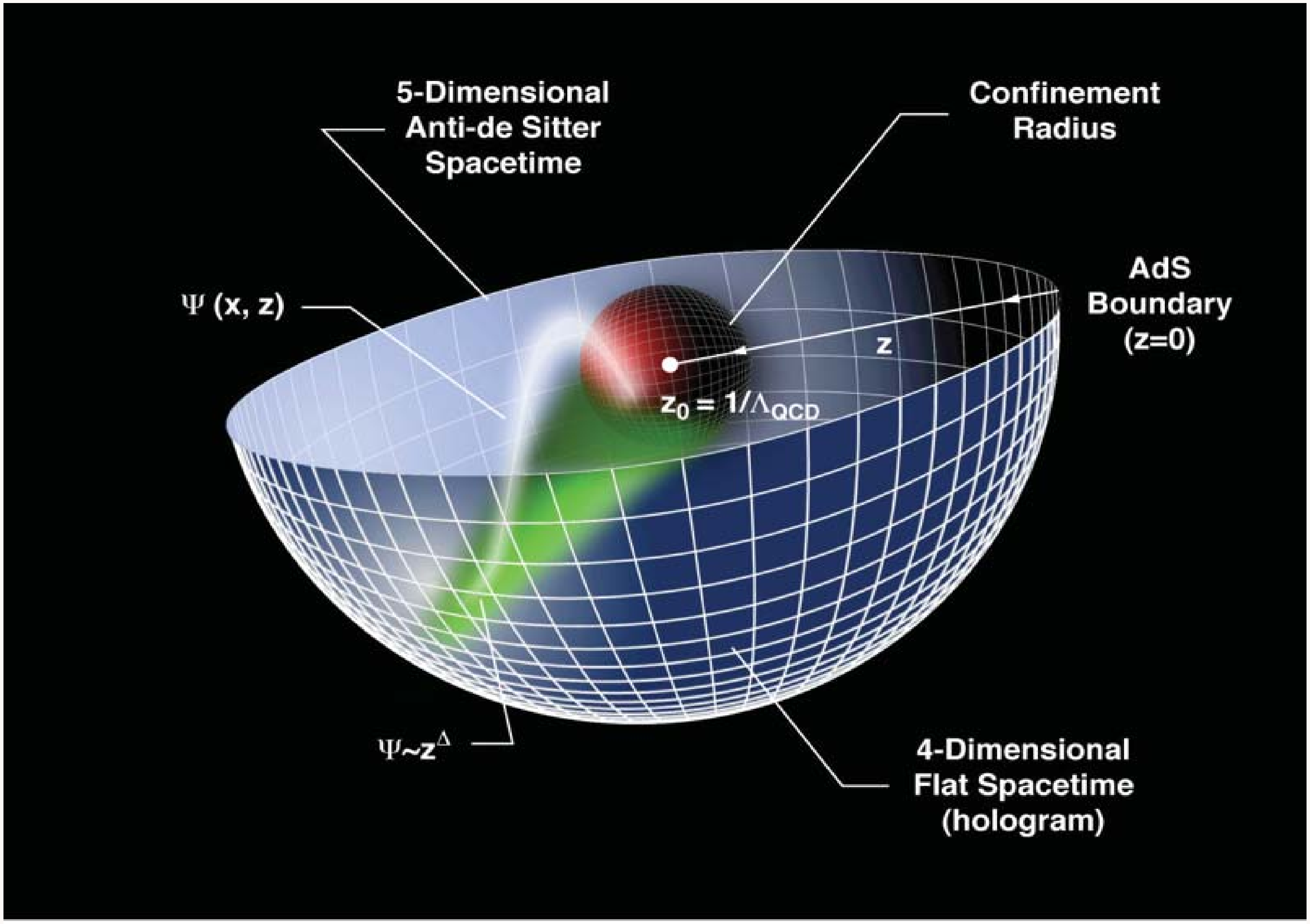
- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**

- **Analogous to the Schrodinger Theory for Atomic Physics**

- *AdS/QCD Light-Front Holography*

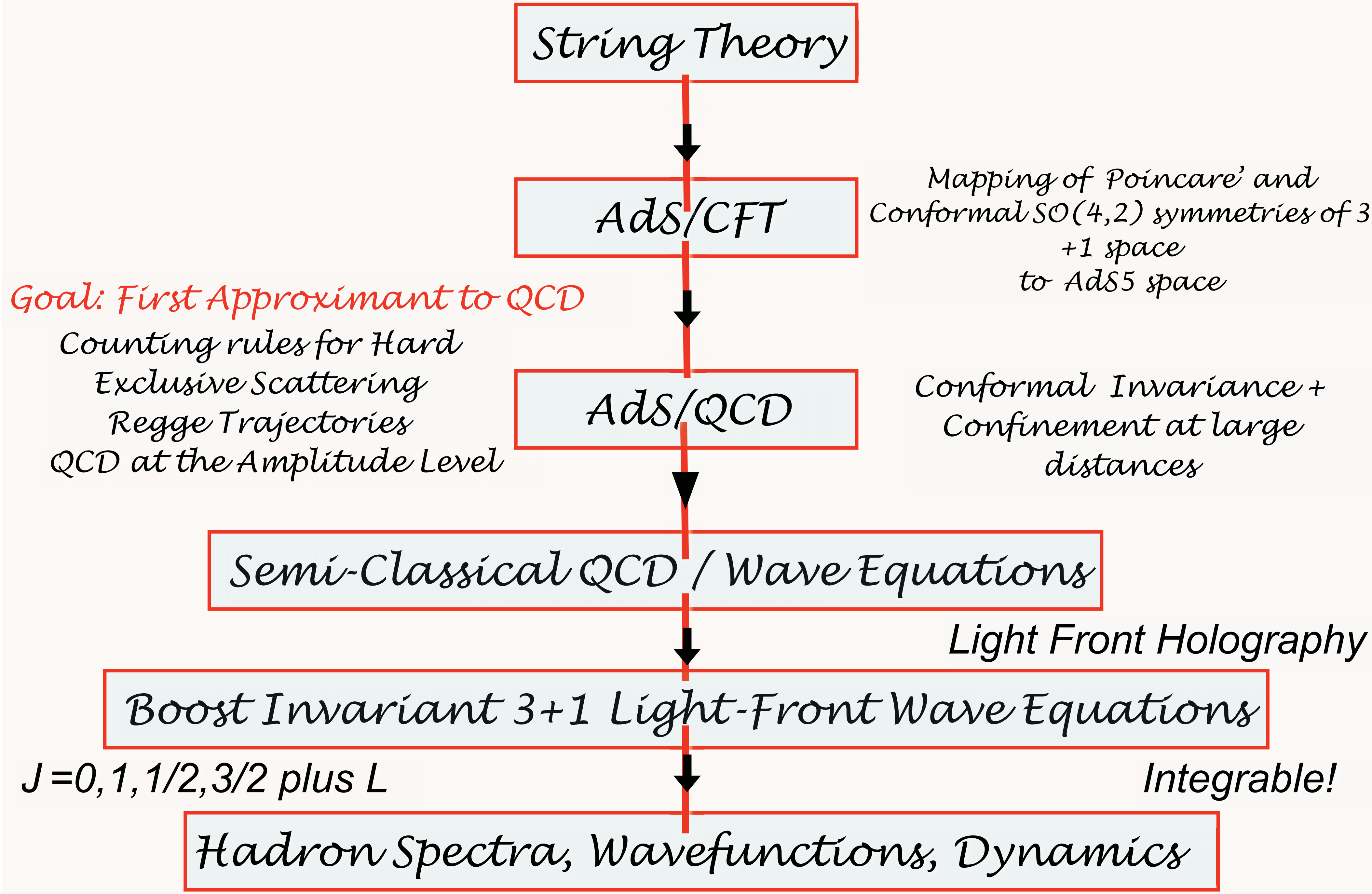
- *Hadronic Spectra and Wavefunctions*

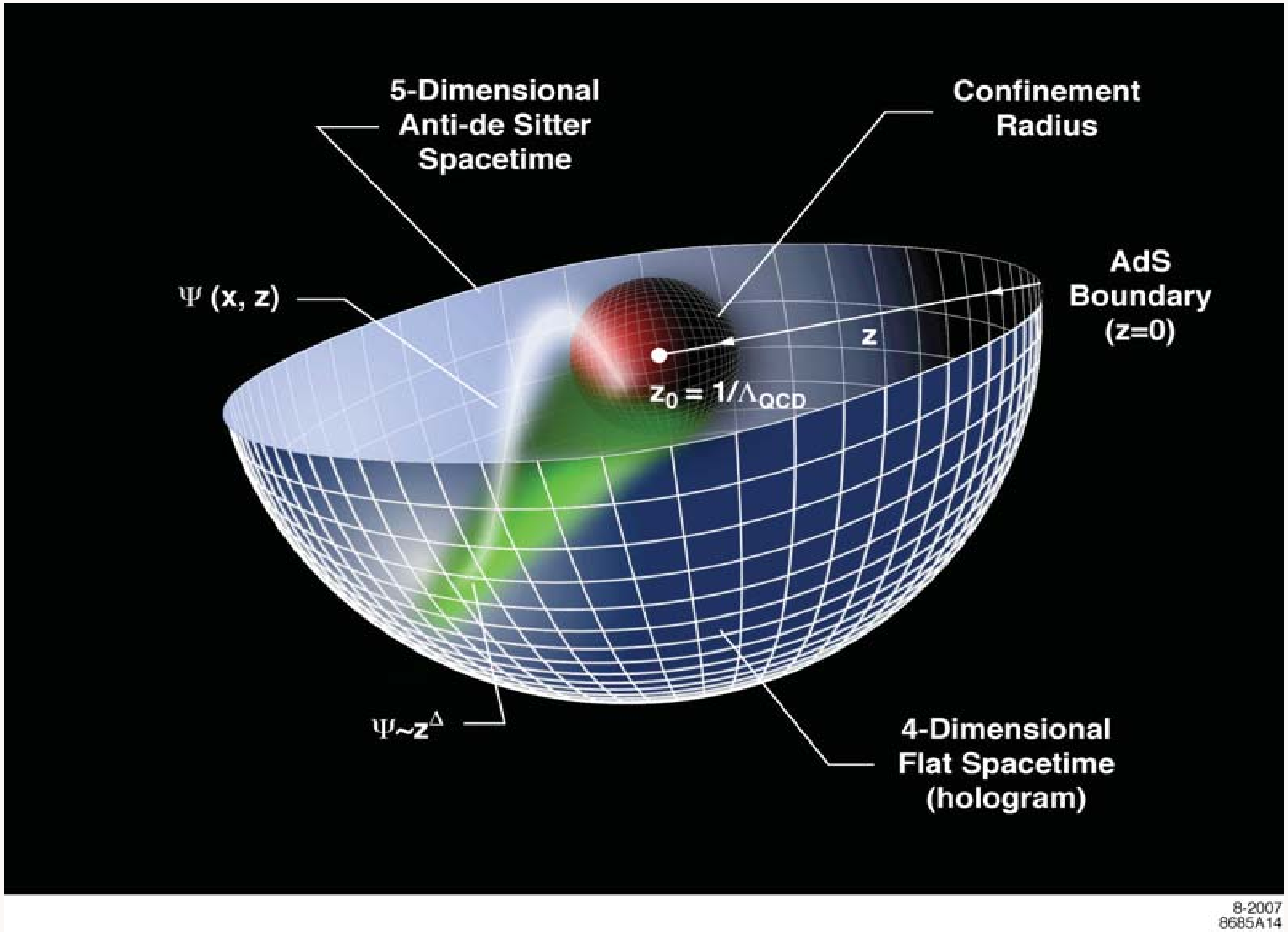
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond


String Theory
AdS/CFT
Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level
AdS/QCD
Conformal Invariance + Confinement at large distances
Semi-Classical QCD / Wave Equations
Light Front Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics



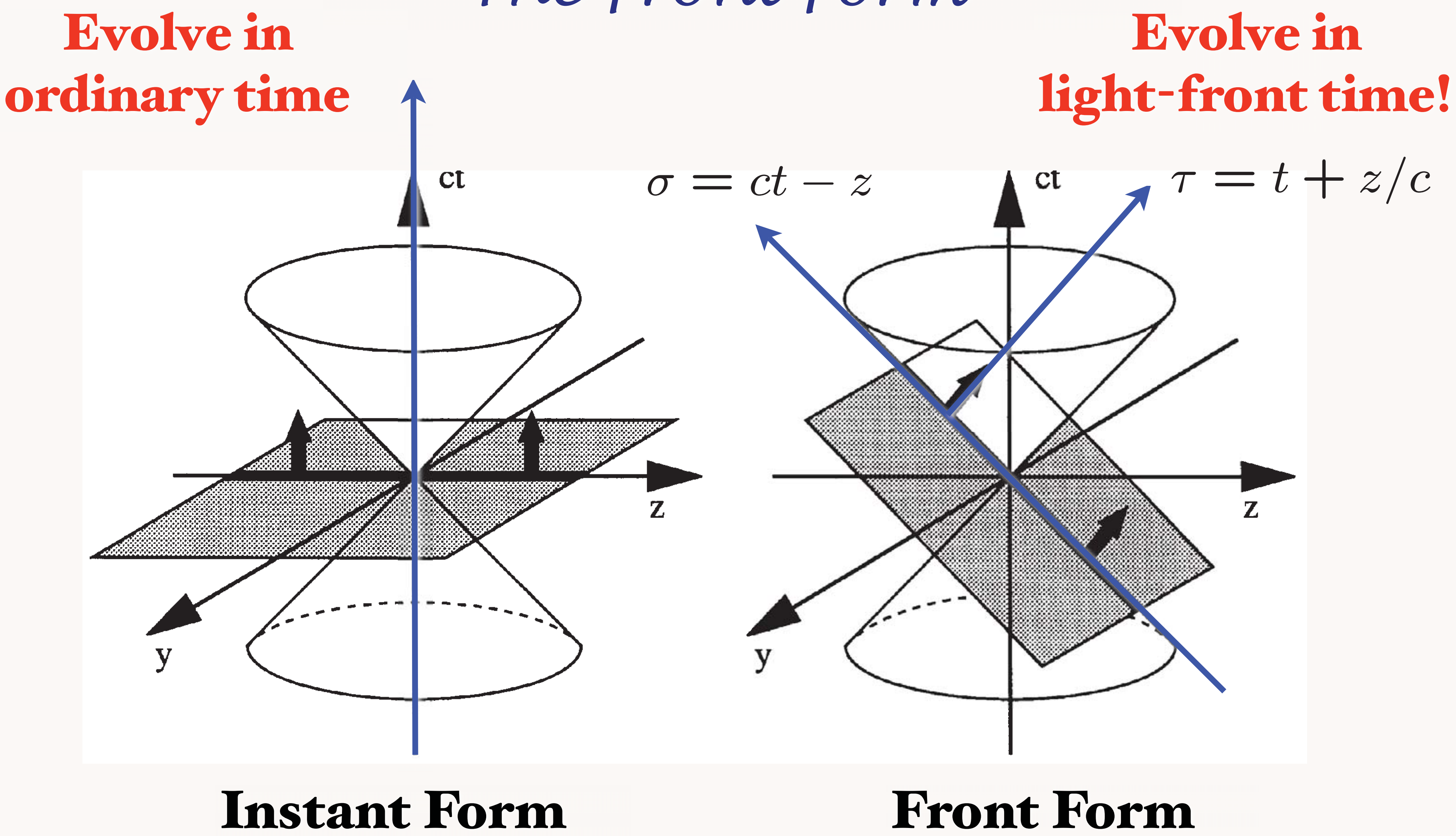
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

We will consider both holographic models

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form



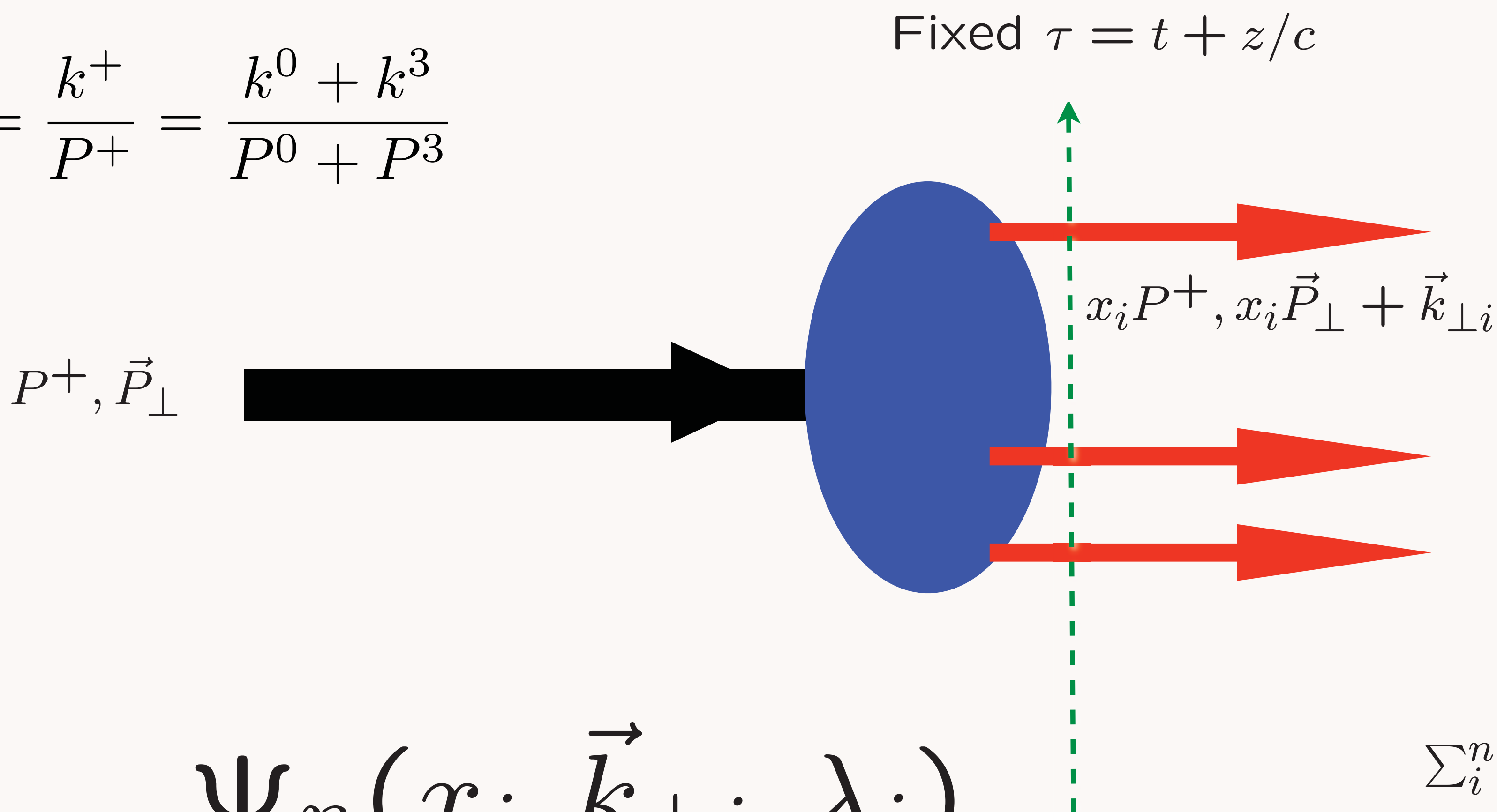
Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

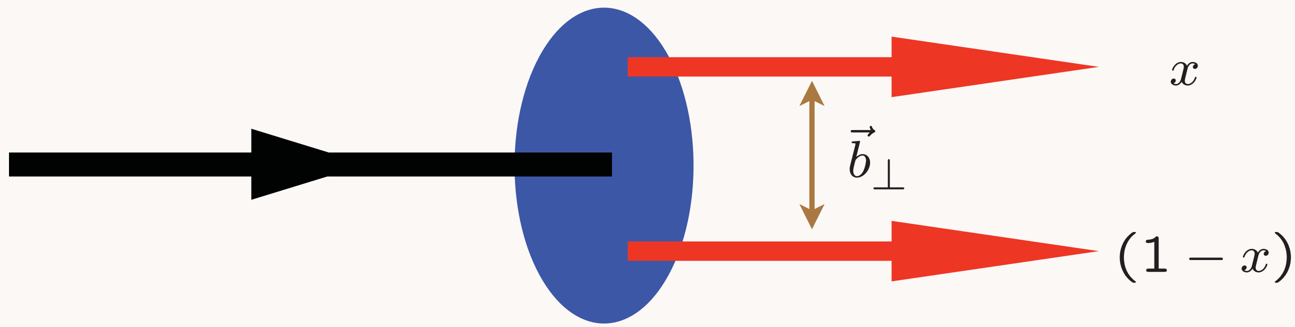
Nonzero Anomalous Moment -->Nonzero orbital angular momentum

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements: **em and gravitational!**

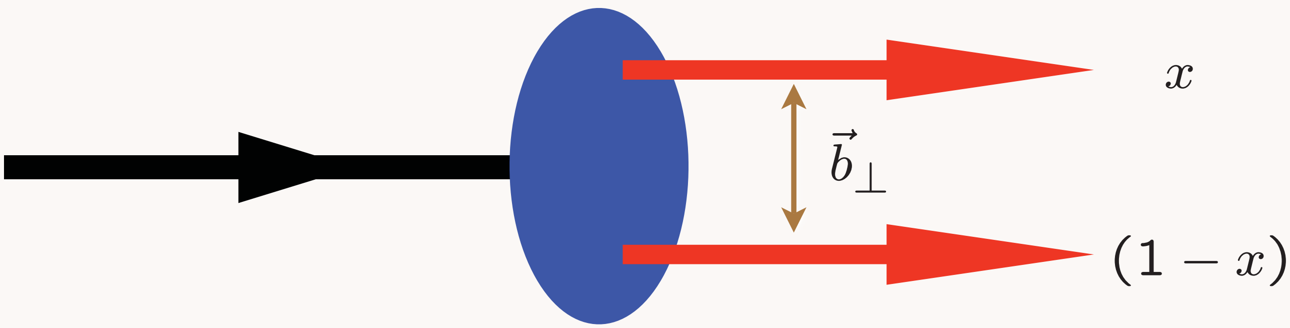
Light-Front Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation — Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



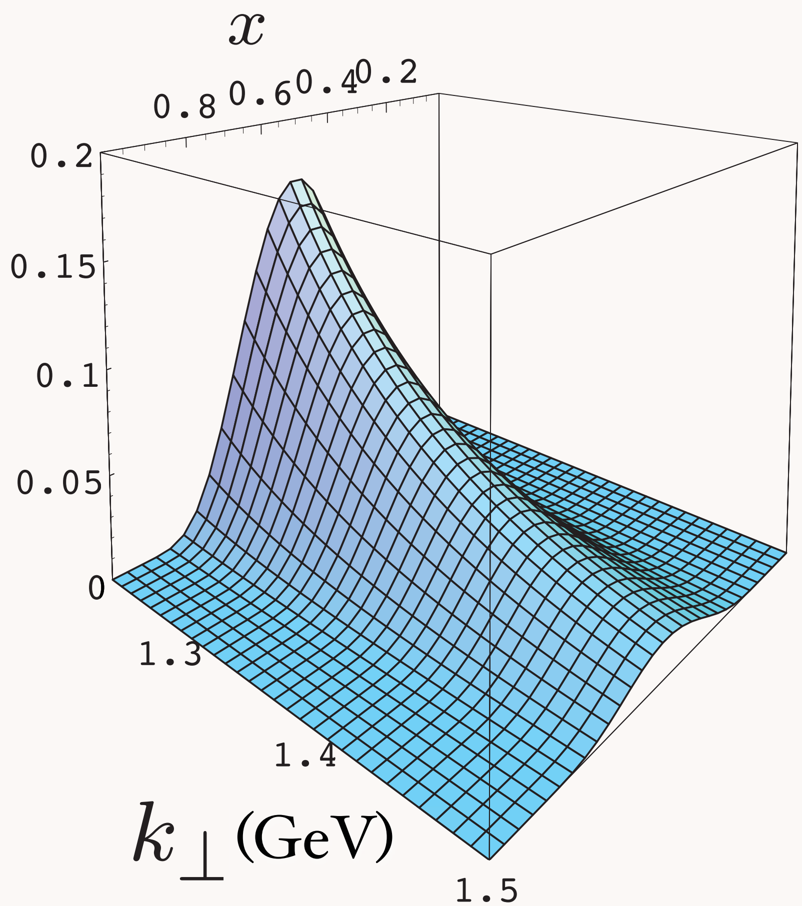
Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$ confining potential:

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb



$\psi_M(x, k_\perp^2)$

"Soft Wall" model

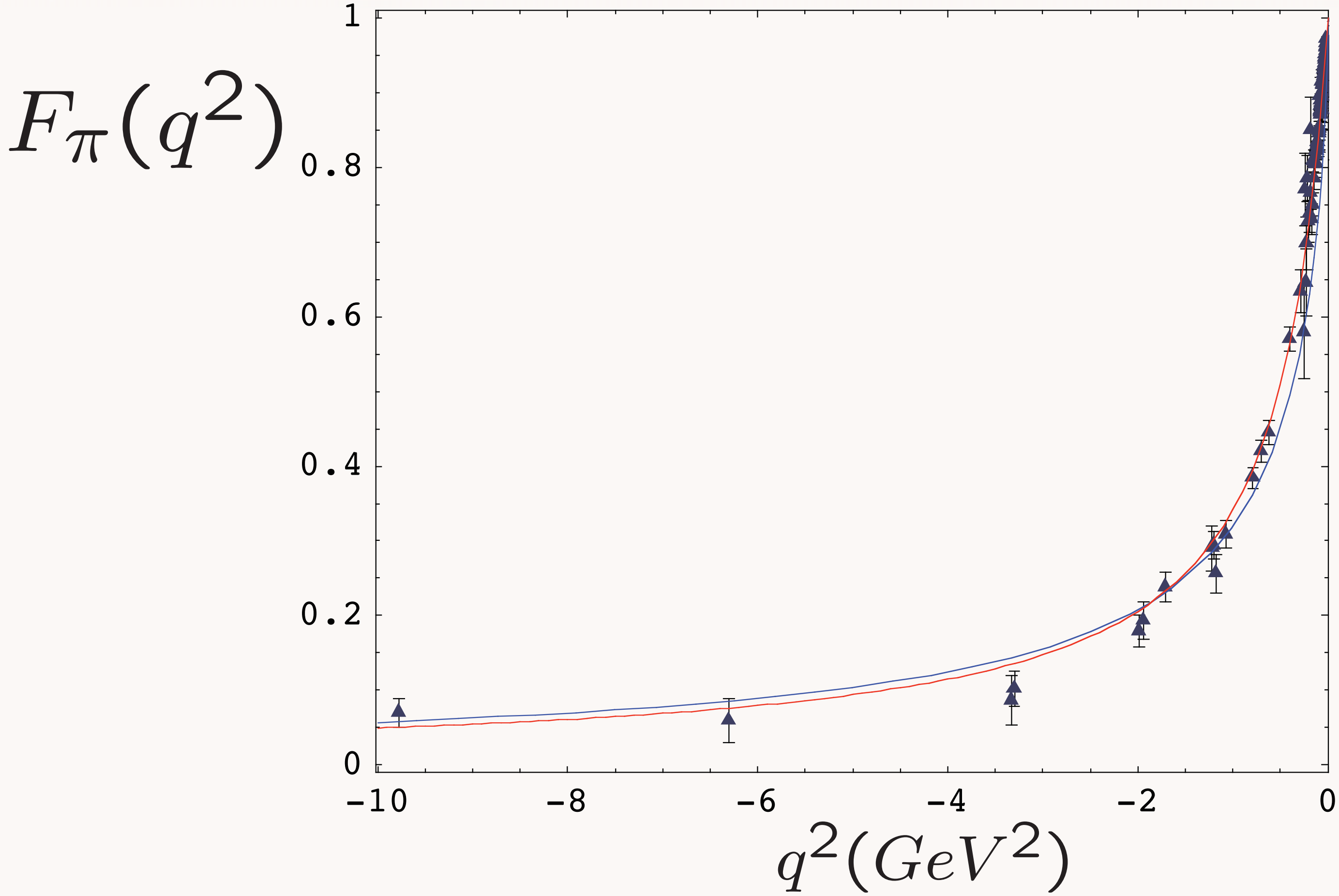
$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Light-Front QCD

Heisenberg Matrix Formulation

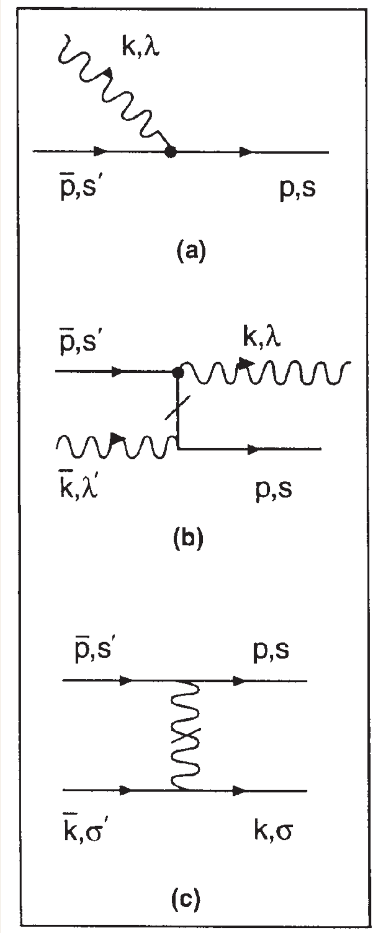
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h >= \Sigma_n \psi_{n/h} |n >$: the LF Hamiltonian $H_{LF} = P^2 = P^+ P^- - \mathbf{P}_\perp^2$, $H_{LF}|P\rangle = \mathcal{M}^2 |P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac '49; Pauli and Pinsky, sjb Phys. Rept. 1988).

- Fock components

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}) = \left\langle n;\, x_i, \mathbf{k}_{\perp i}, \middle| \psi_h(P^+, \mathbf{P}_\perp) \right\rangle,$$

frame independent and encode hadron properties in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+/P^+$ and $\mathbf{k}_{\perp i}$ are the relative coordinates of parton i in Fock-state n

$$\sum_{i=1}^{n} x_i = 1 \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0.$$

- Define transverse position coordinates $x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp.$$

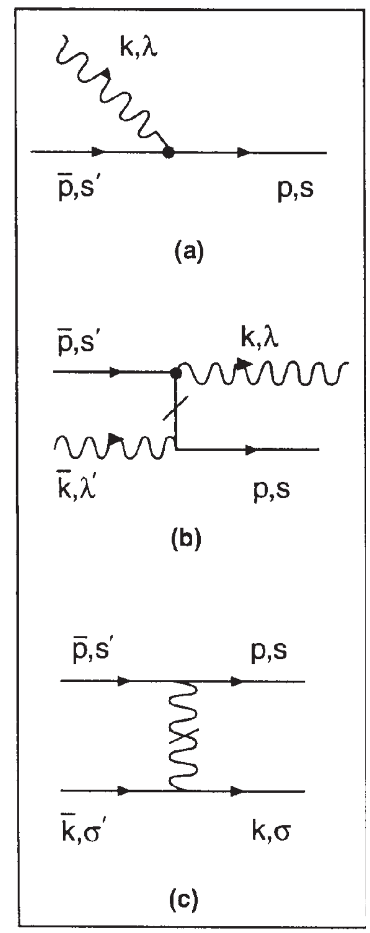
Light-Front QCD

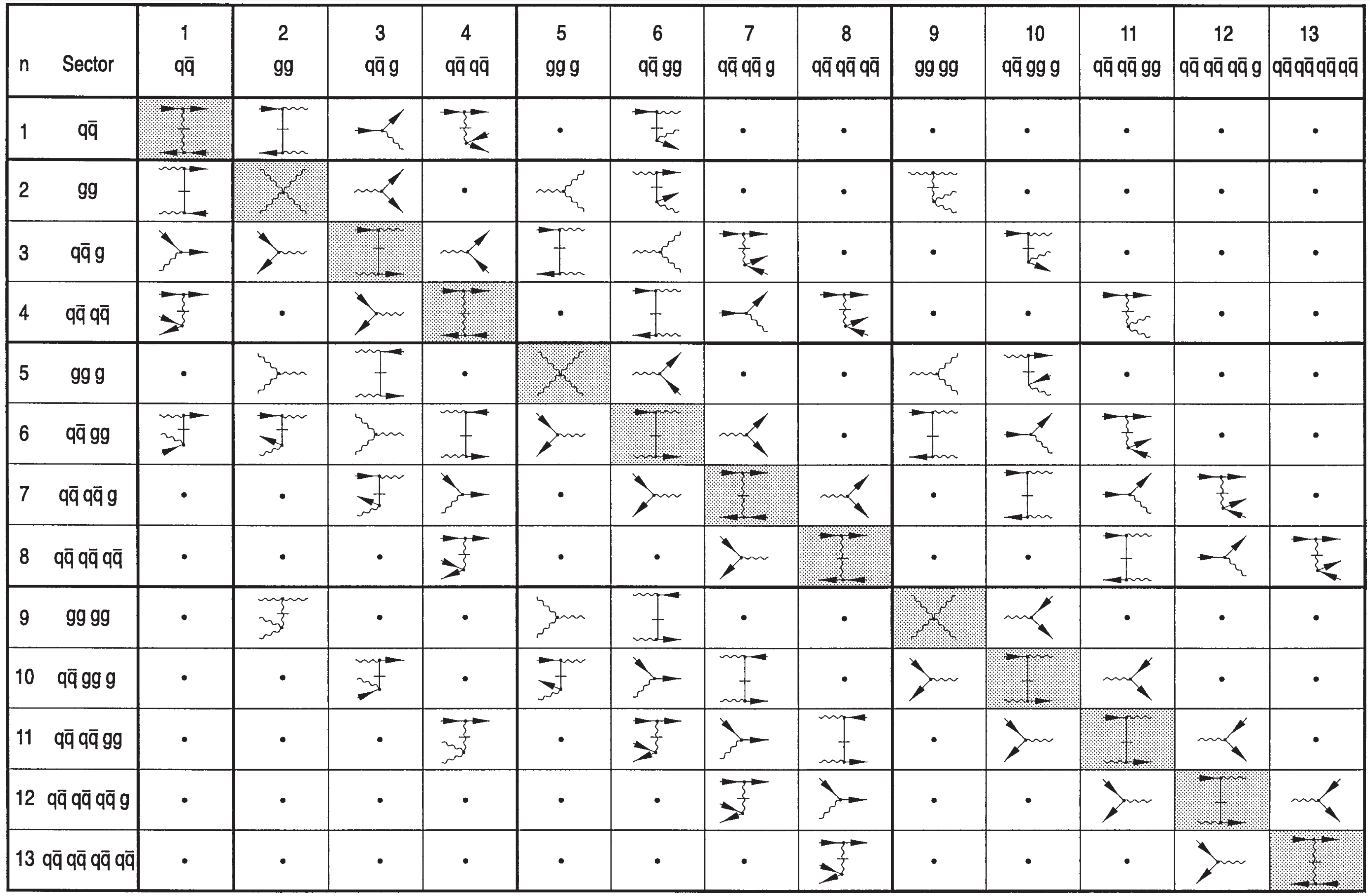
Heisenberg Matrix Formulation

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization





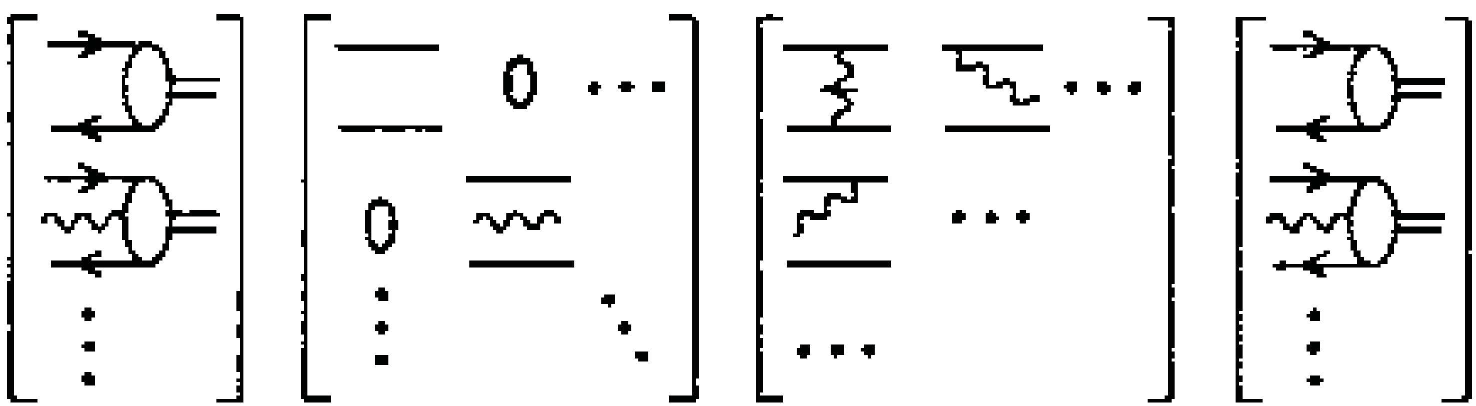
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

LIGHT-FRONT SCHRODINGER EQUATION

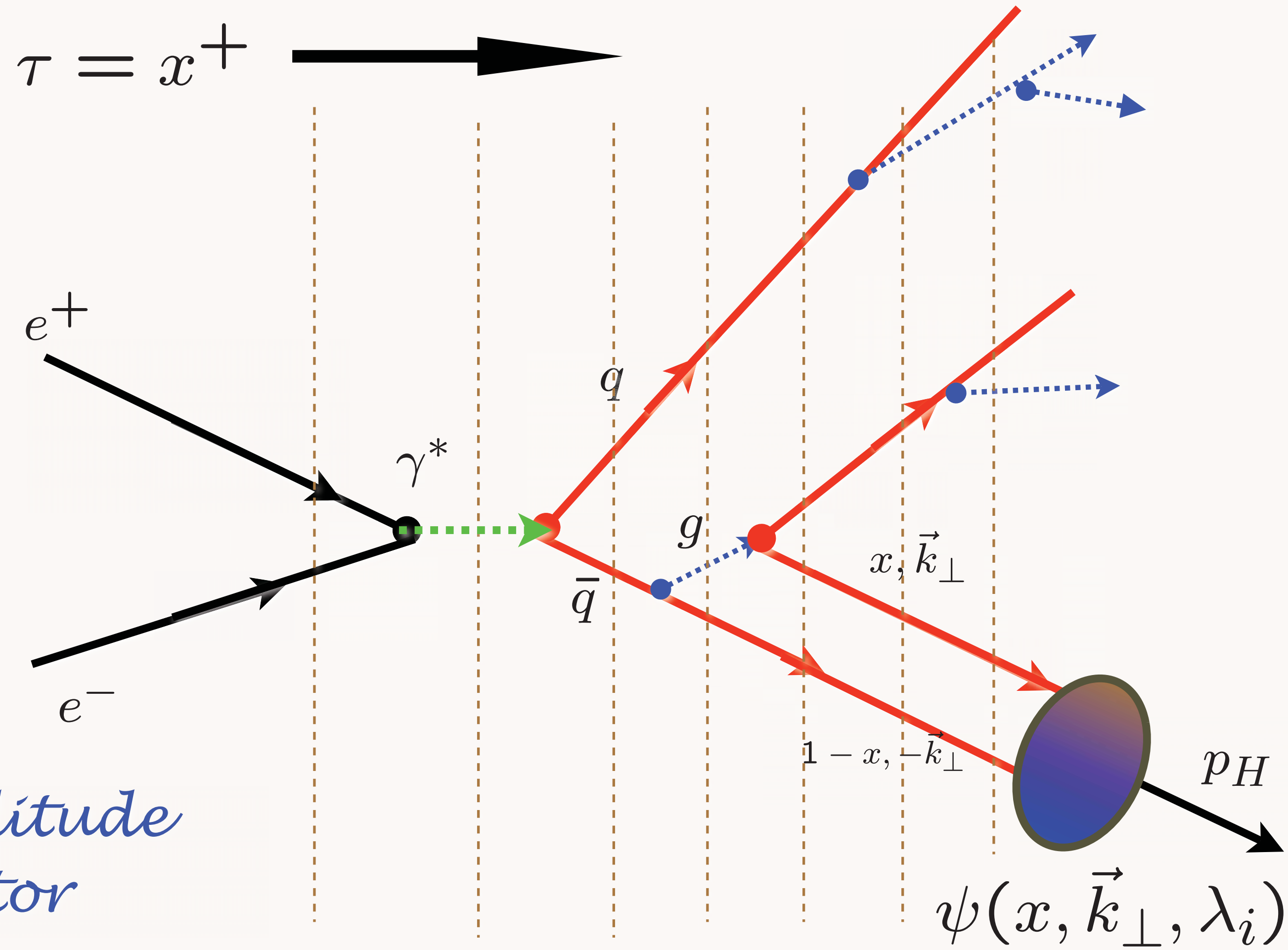
$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

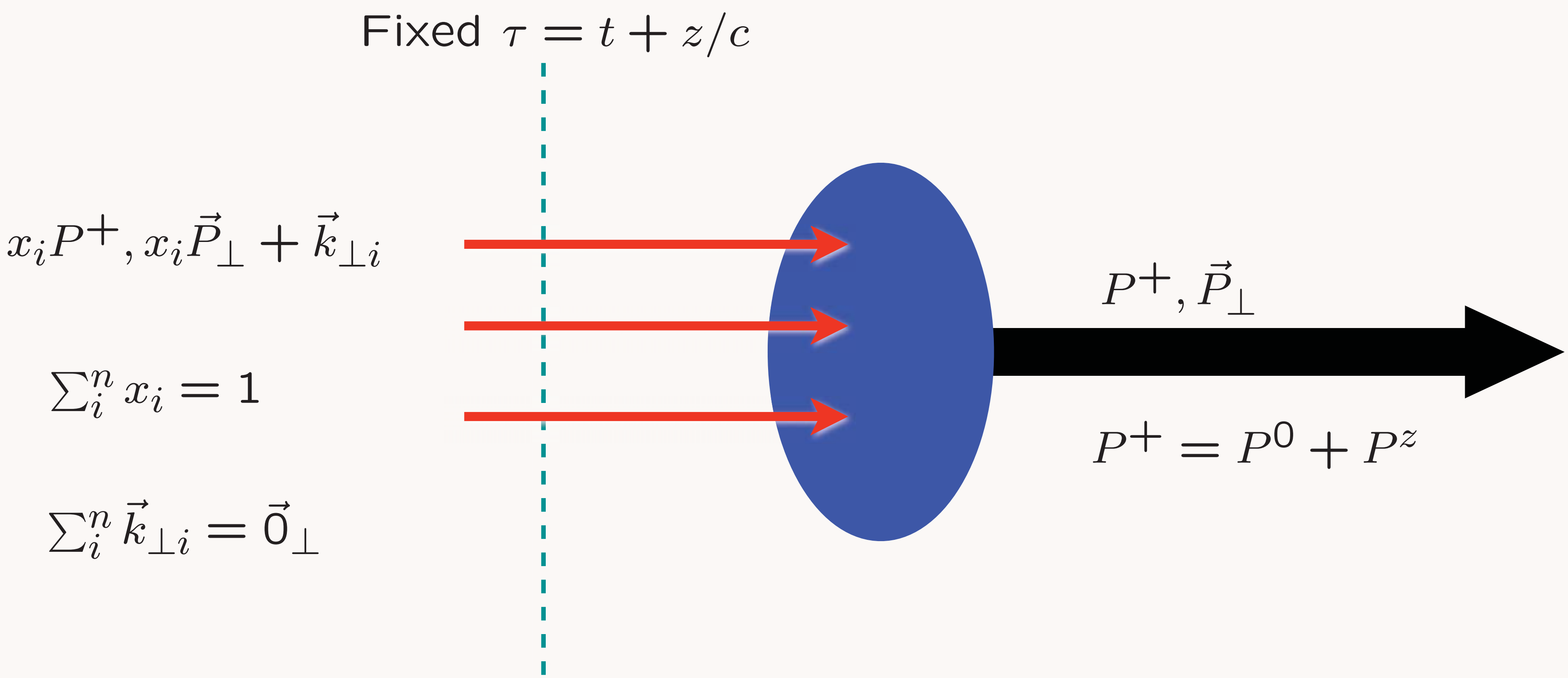
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

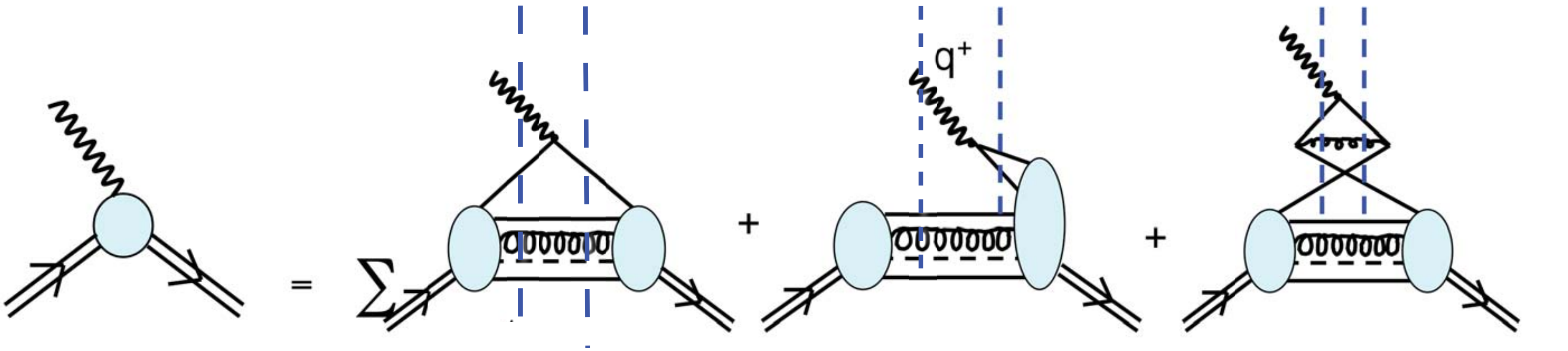
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

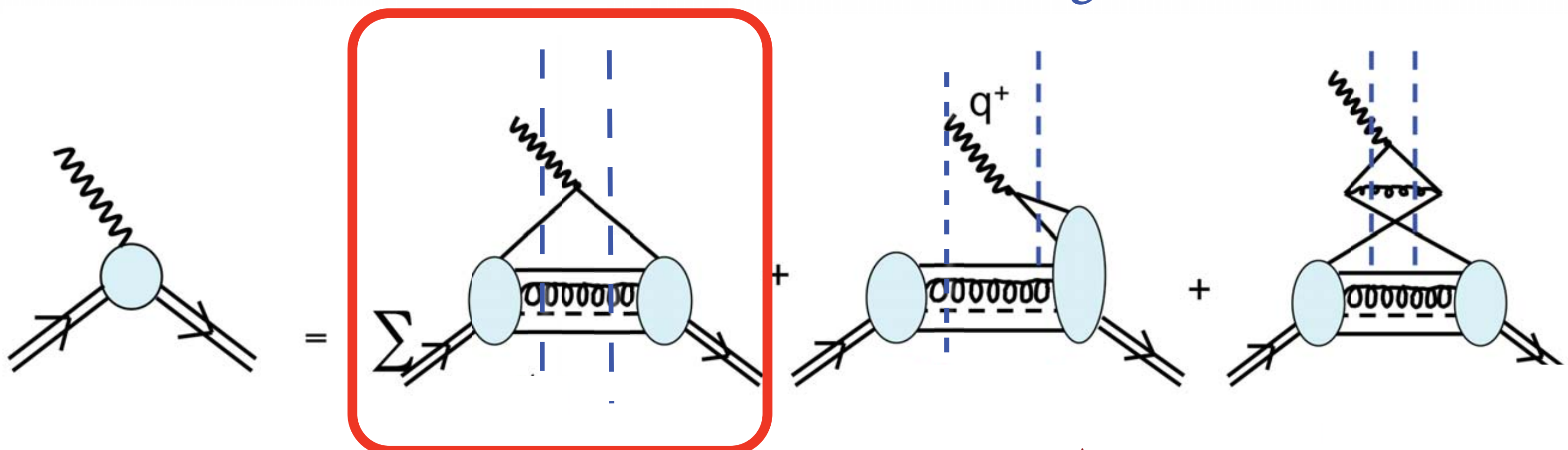
Invariant under boosts! Independent of P^μ

Calculation of Form Factors in Equal-Time Theory



Need vacuum fluctuations

Calculation of Form Factors in Light-Front Theory



zero for $q^+ = 0$ ***zero !!***

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

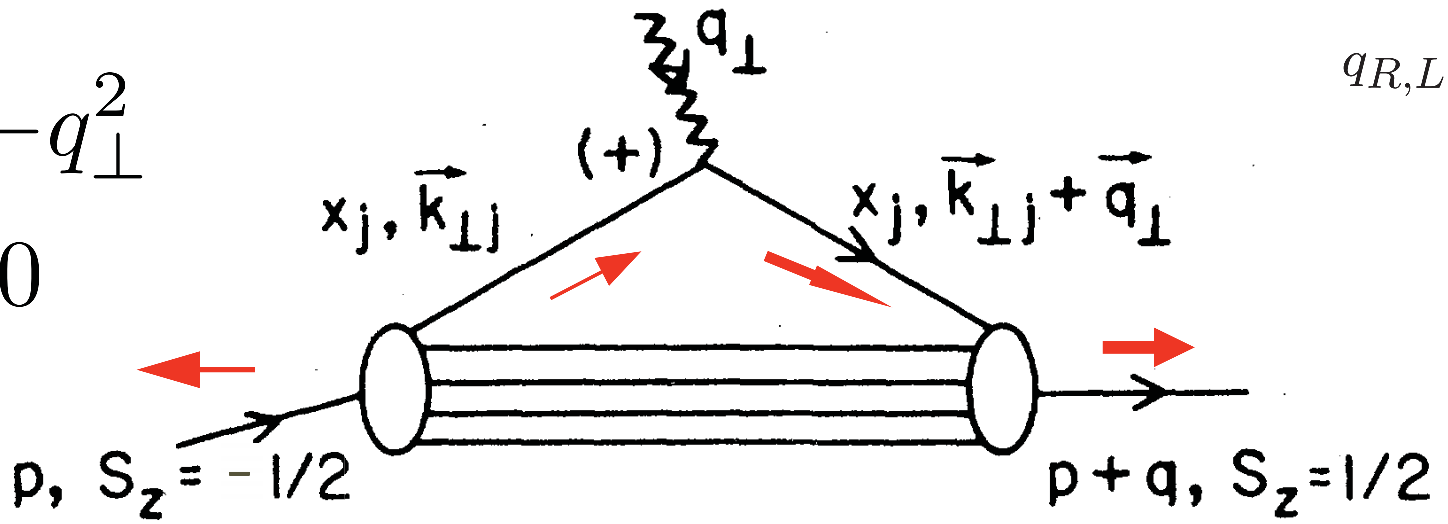
$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q^2 = -q_\perp^2$$

$$q^+ = 0$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

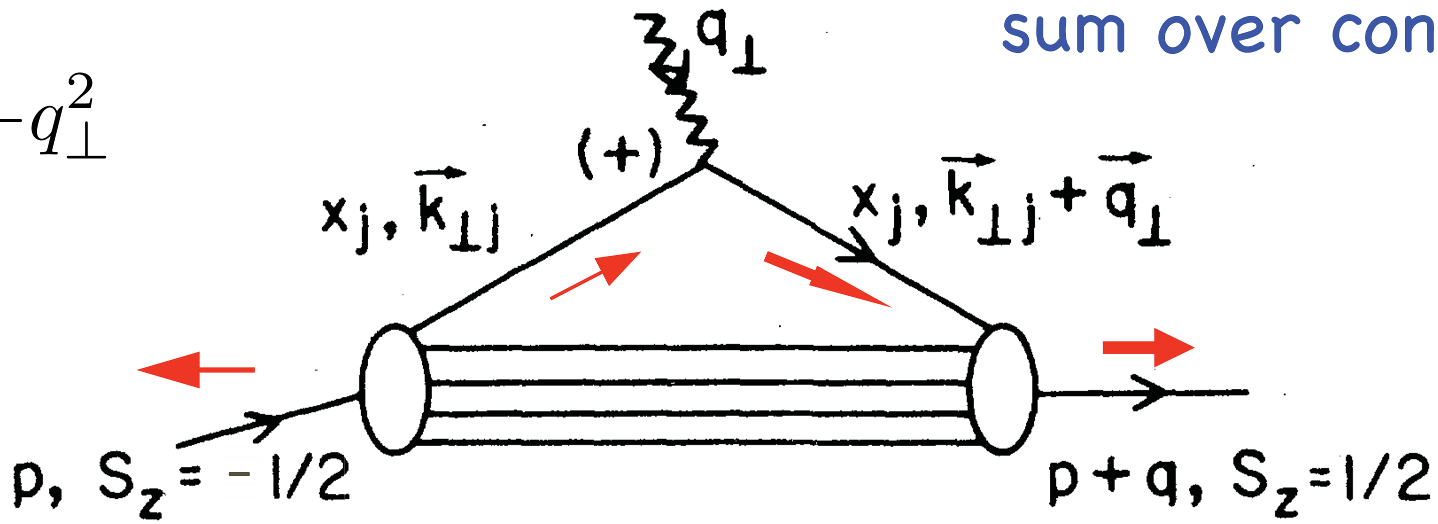
Anomalous gravitomagnetic moment B(0)

Okun, Kobzarev, Teryaev: B(0) Must vanish because of Equivalence Theorem

graviton

sum over constituents

$$q^2 = -q_\perp^2$$

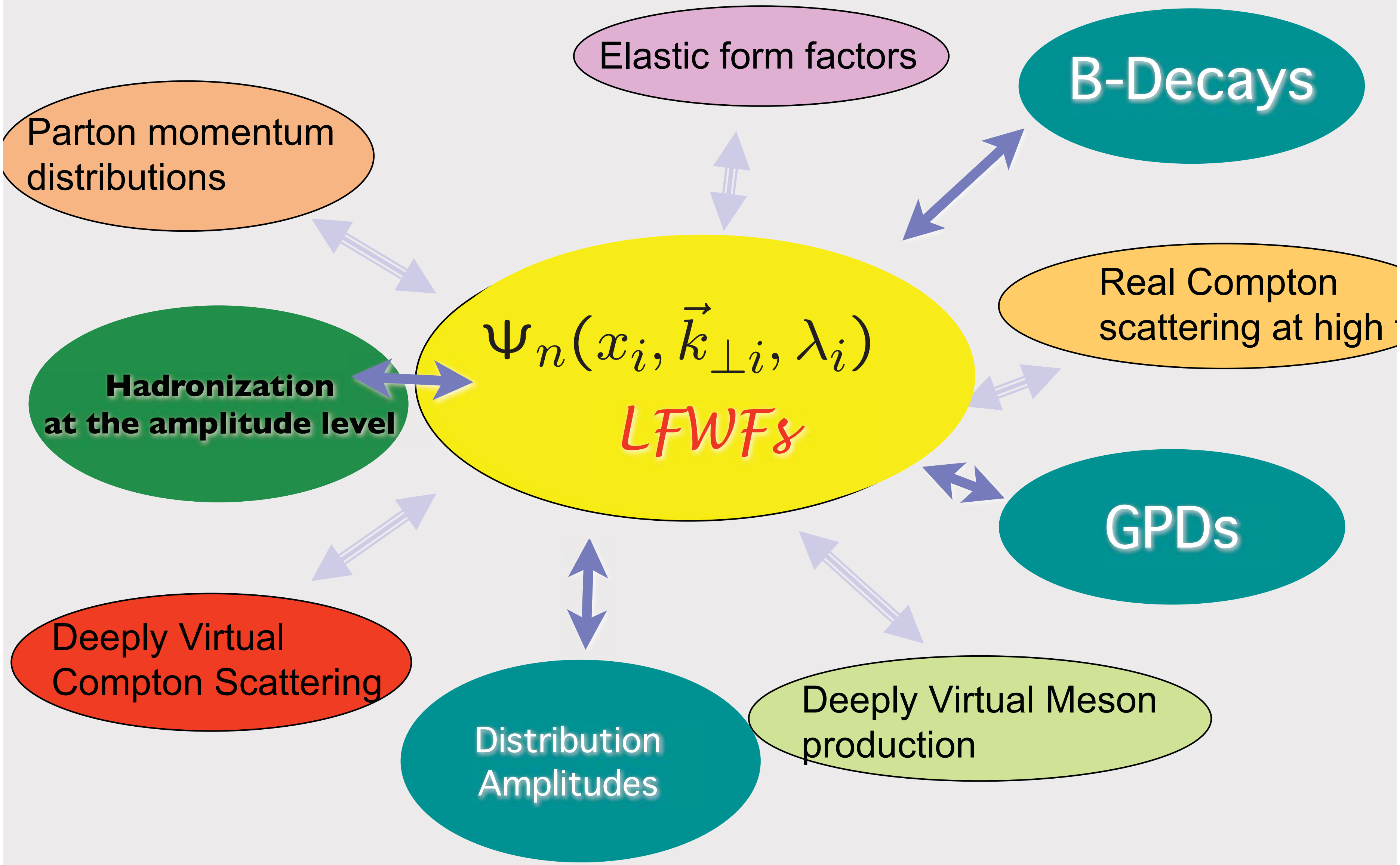


Hwang, Schmidt, sjb;
Holstein et al

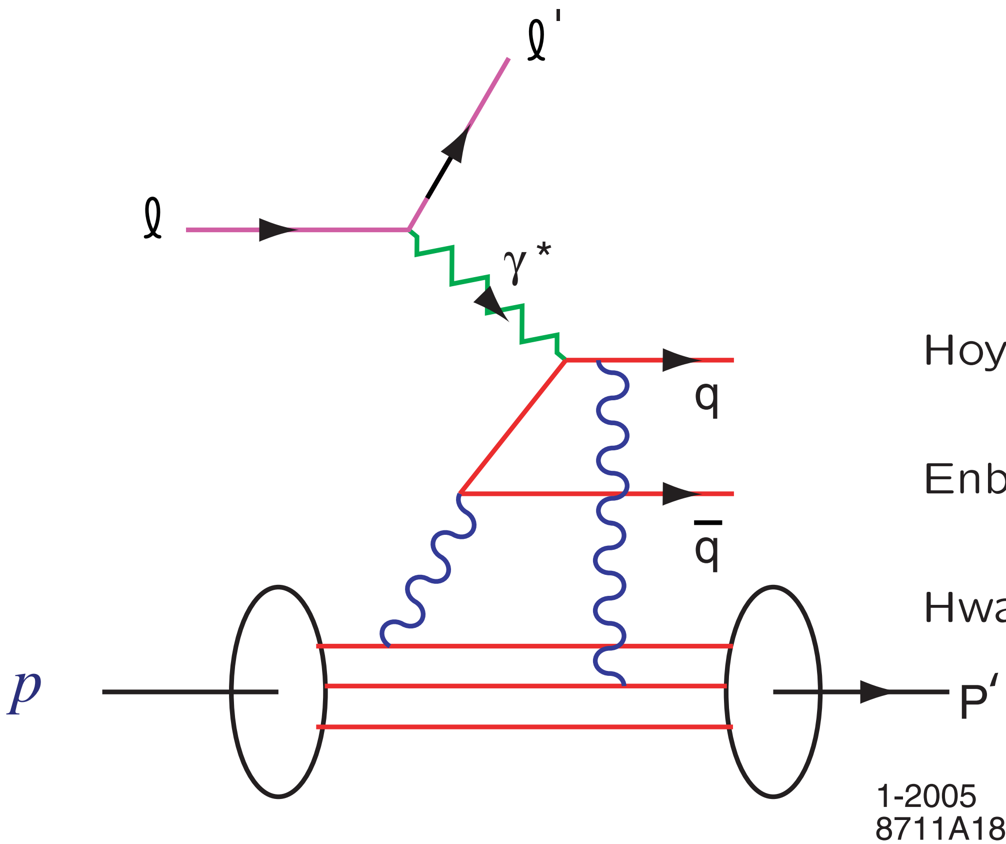
B(0) = 0

Each Fock State

A Unified Description of Hadron Structure



Final-State Interaction Produces Diffractive DIS



Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM

Enberg, Hoyer, Ingelman, SJB

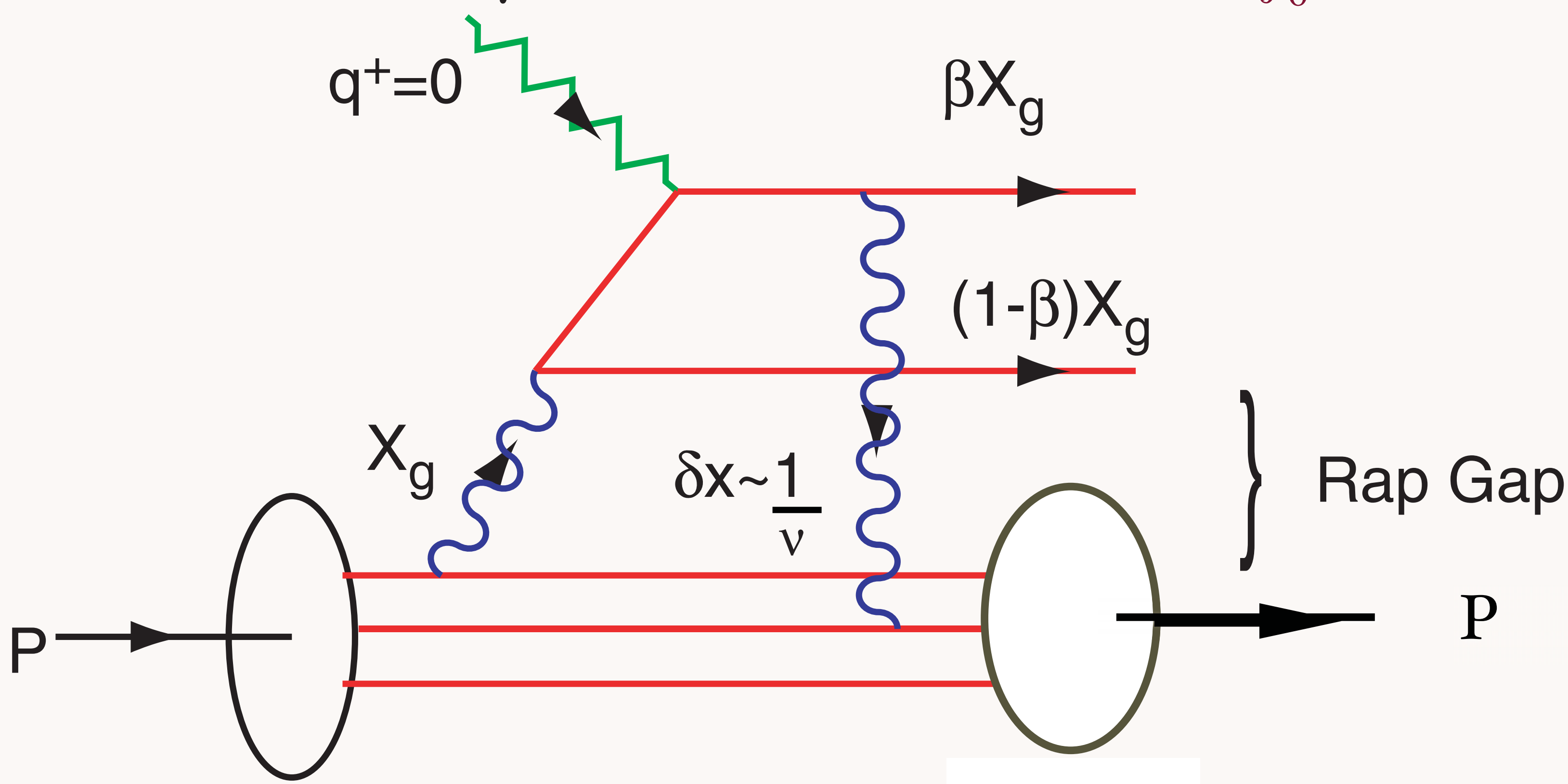
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

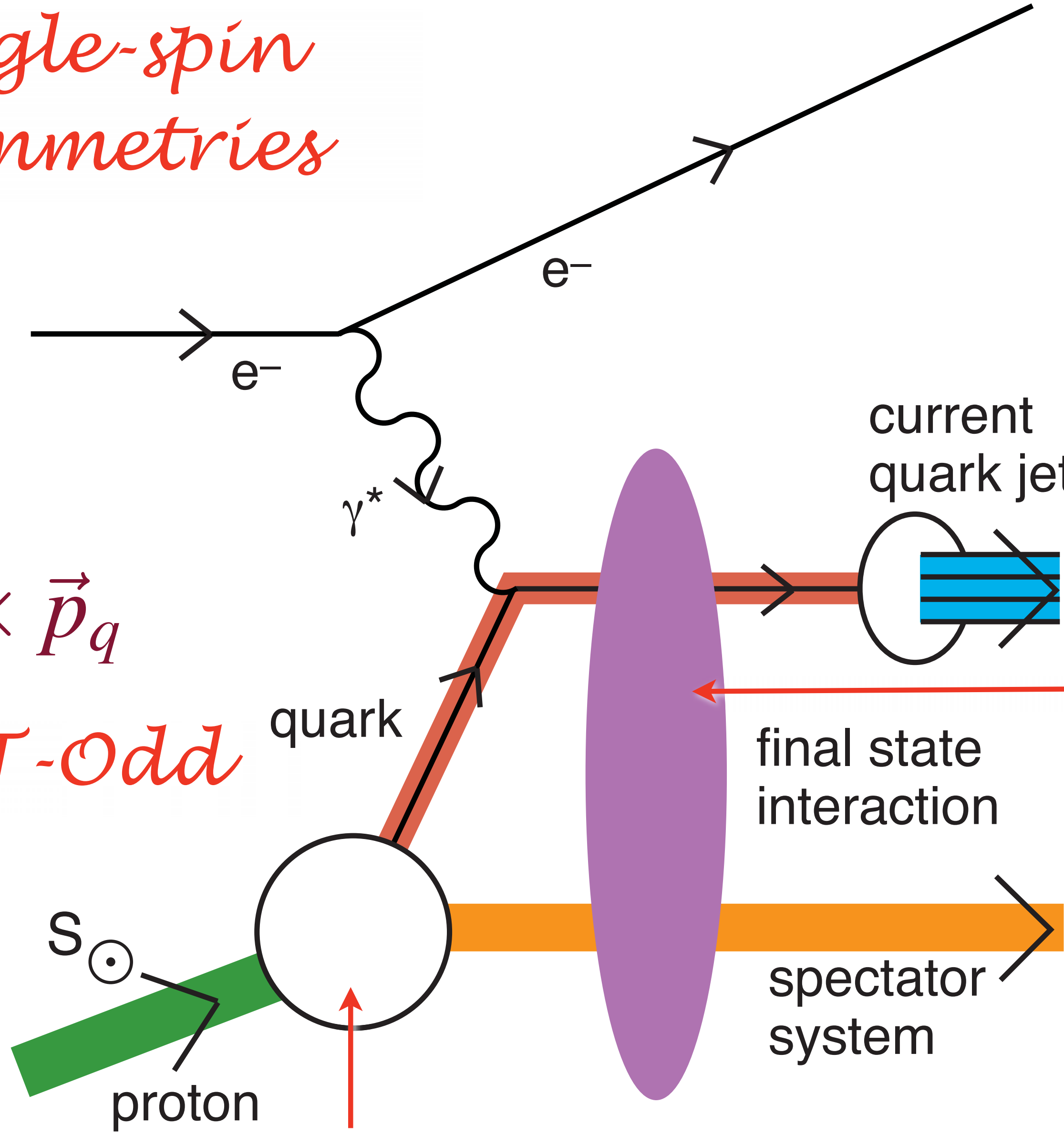
Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



Reproduces lab-frame color dipole approach

Single-spin asymmetries

Leading-Twist Sivers Effect



$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

Double Initial-State Interactions generate anomalous $\cos 2\phi$

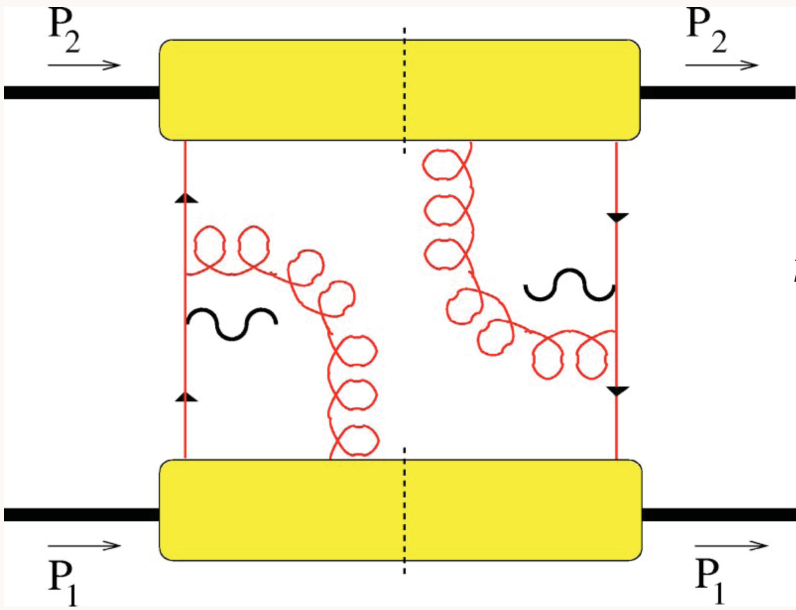
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$

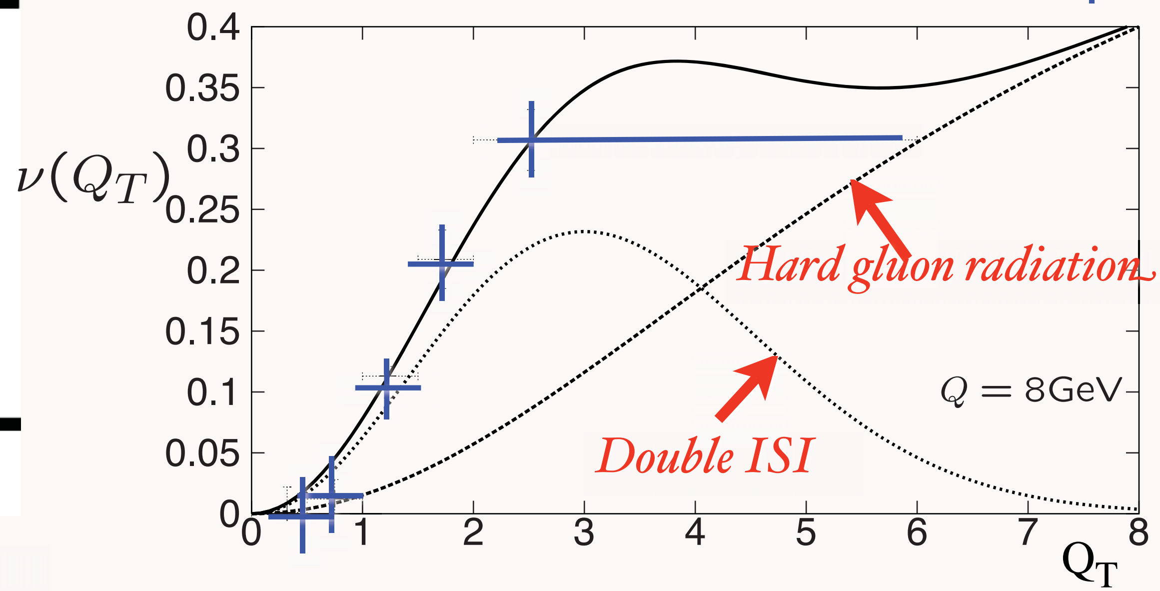
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^{\perp}(\pi)\, h_1^{\perp}(N)$$



Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



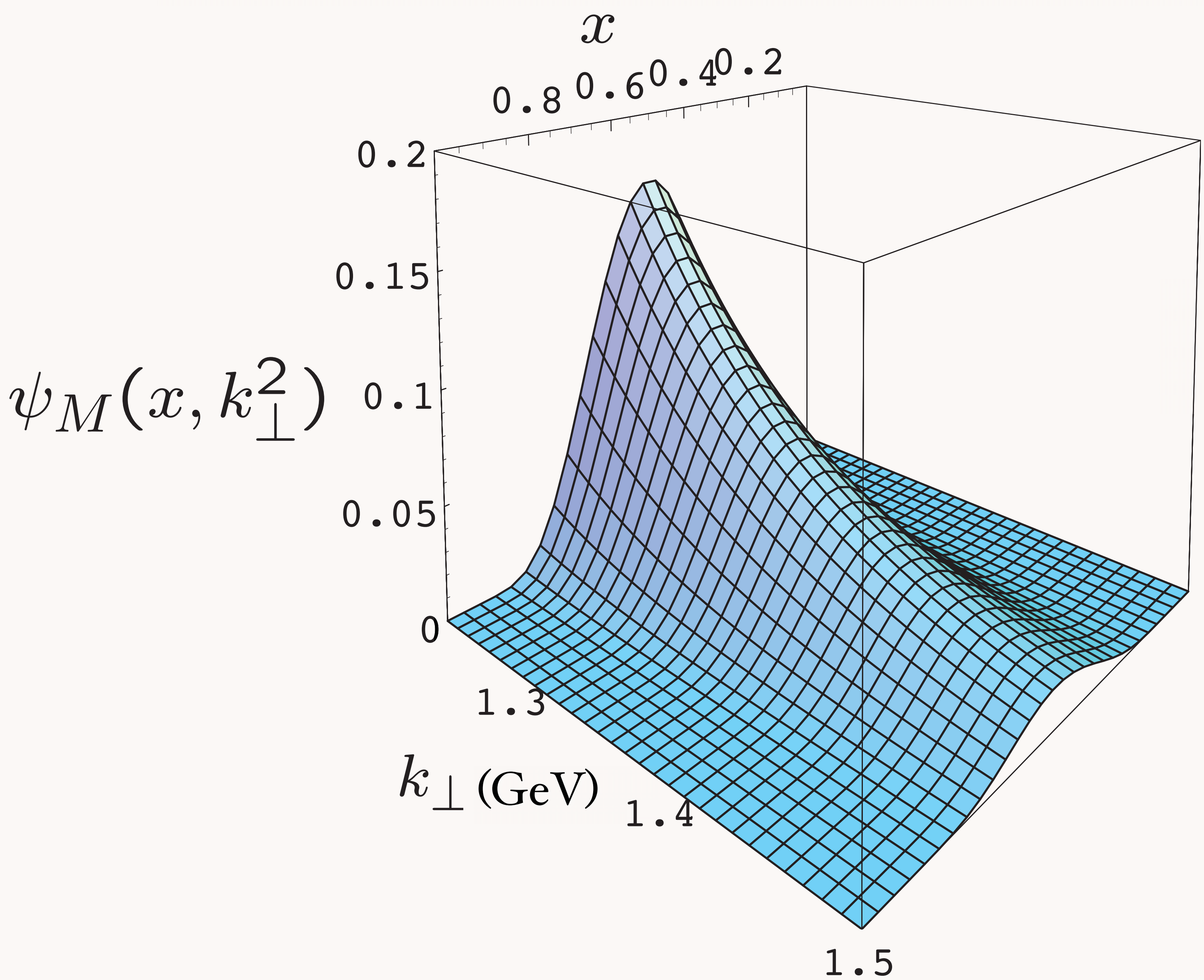
Model: Boer,

Physics of Rescattering

- Diffractive DIS
- Non-Unitary Correction to DIS: Structure functions are not probability distributions
- Nuclear Shadowing, Antishadowing- Not in Target WF
- Single Spin Asymmetries -- opposite sign in DY and DIS
- DY $\cos 2\phi$ distribution at leading twist from double ISI-- not given by PQCD factorization -- breakdown of factorization!
- Wilson Line Effects not 1 even in LCG
- Must correct hard subprocesses for initial and final-state soft gluon attachments
- Corrections to Handbag Approximation in DVCS!

Hoyer, Marchal, Peigne, Sannino, sjb

Prediction from AdS/CFT: Meson LFWF



"Soft Wall" model

de Teramond, sjb

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

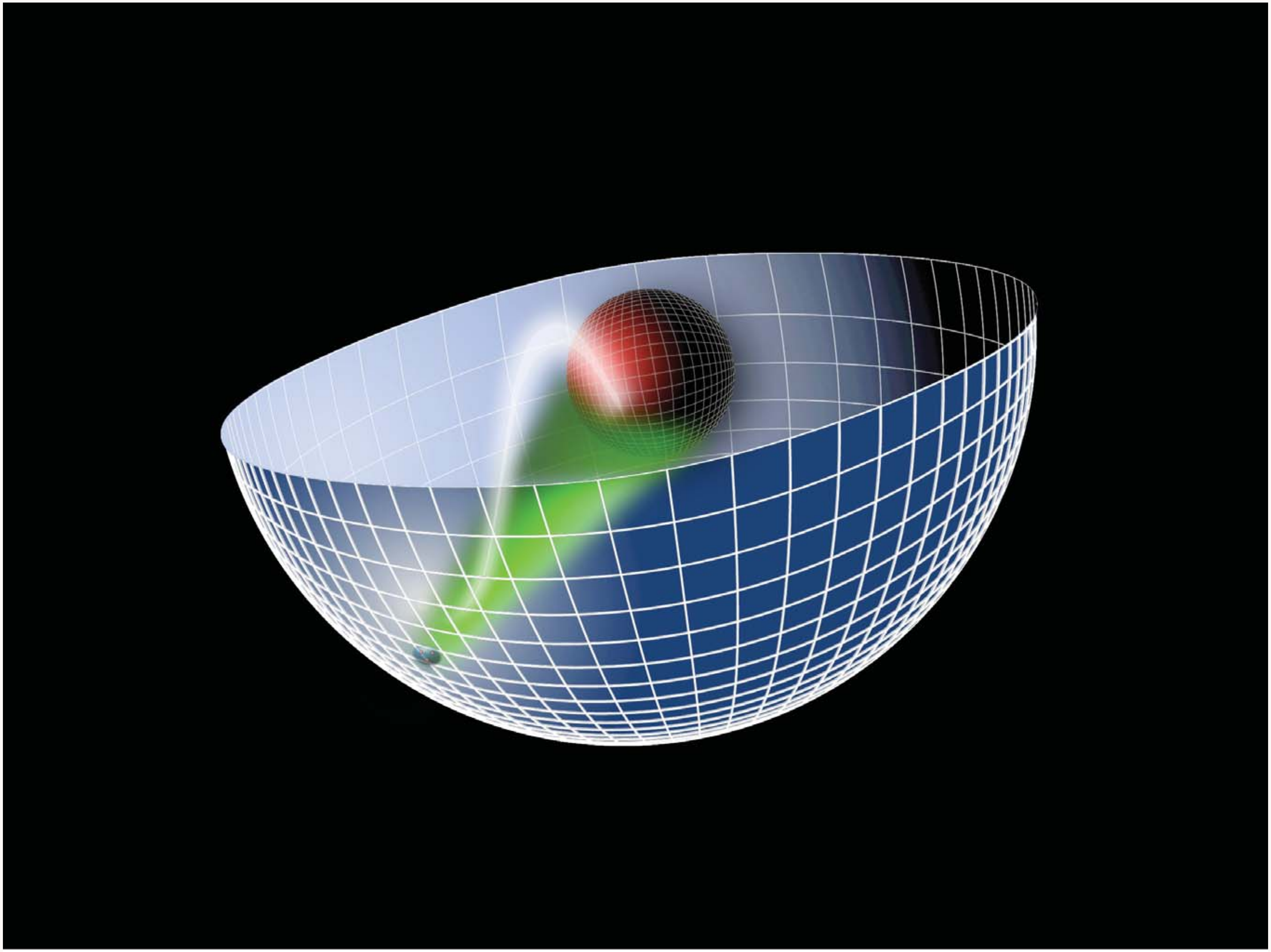
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

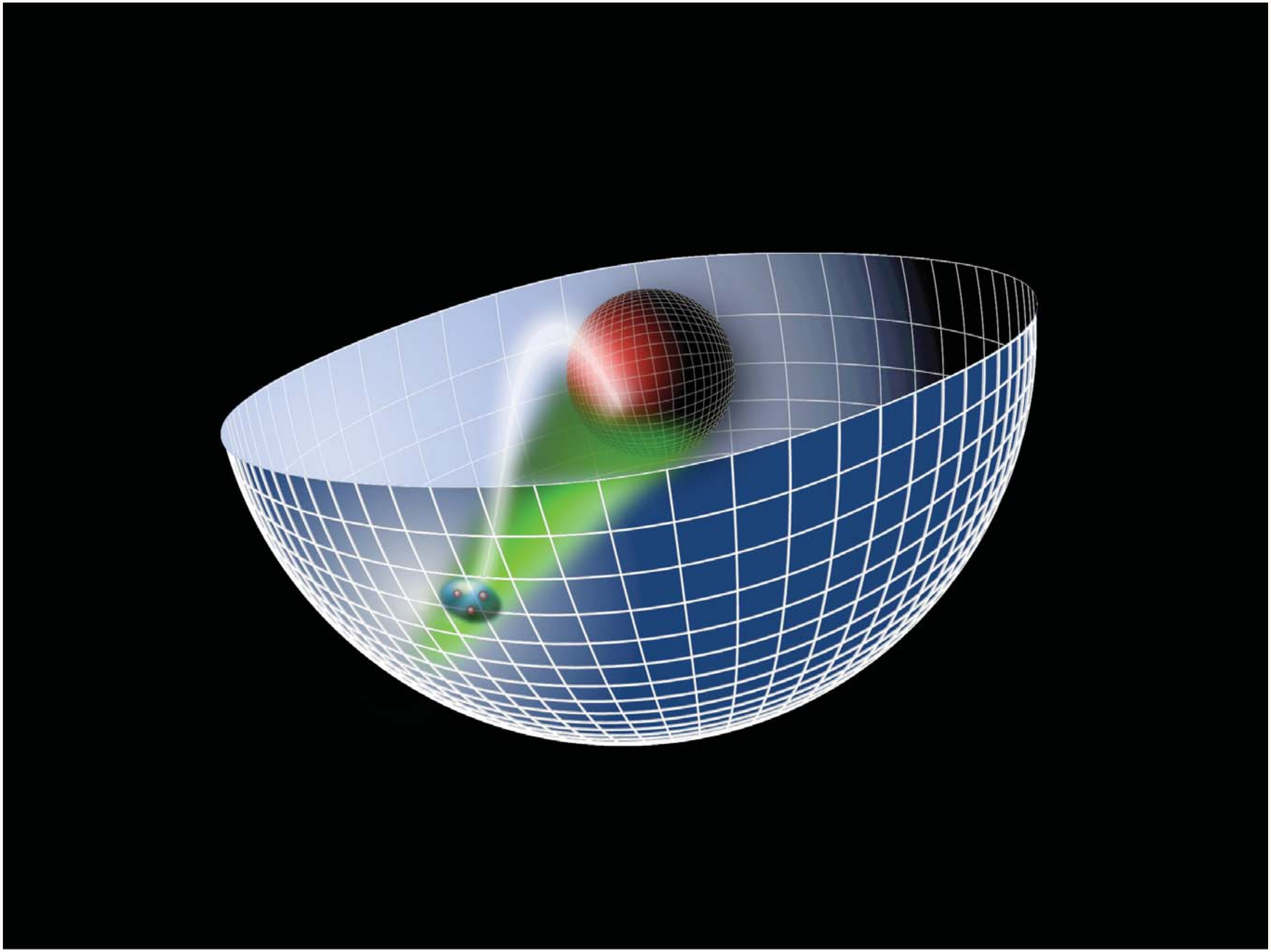
- Different values of z correspond to different scales at which the hadron is examined.

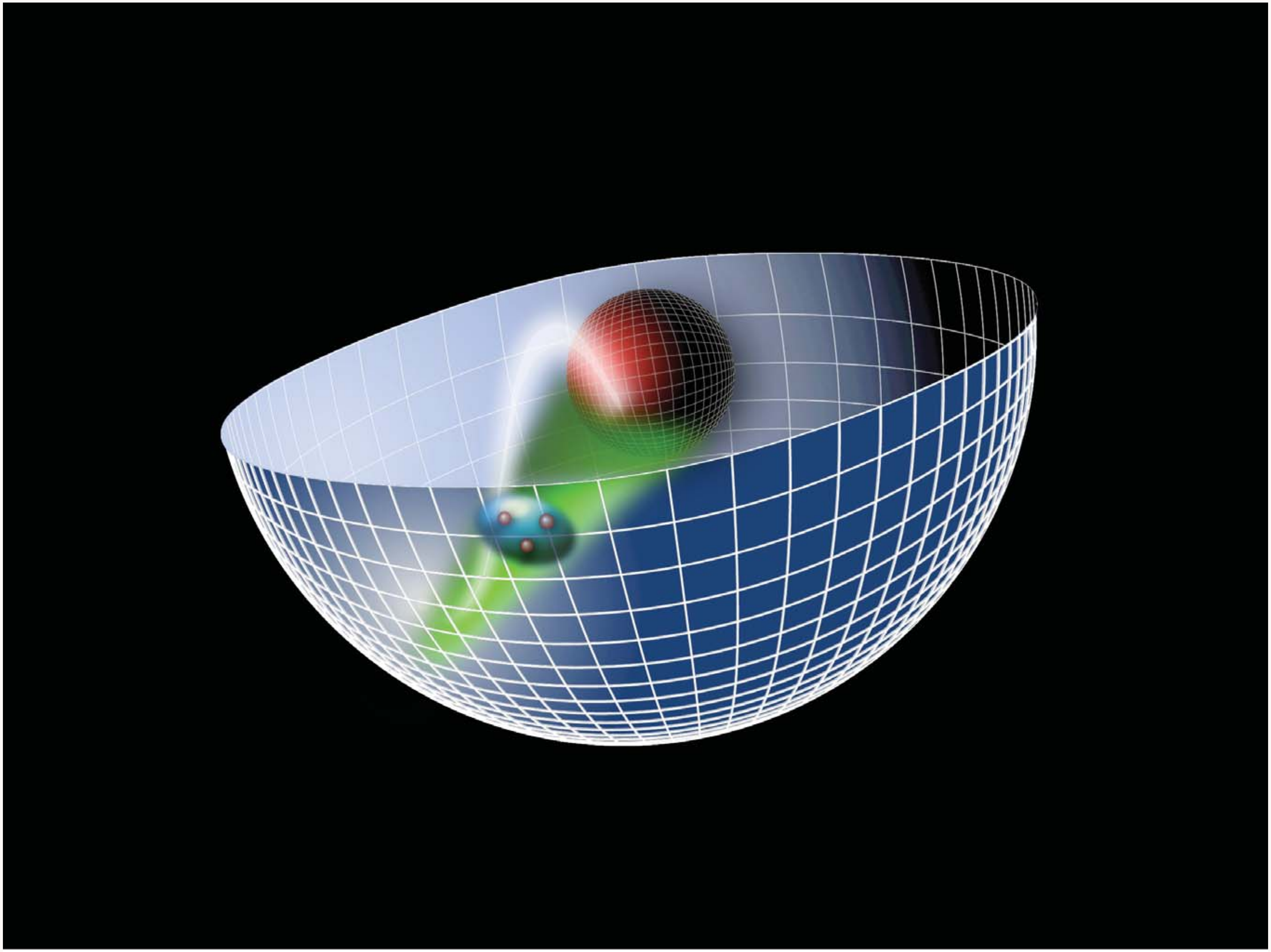
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

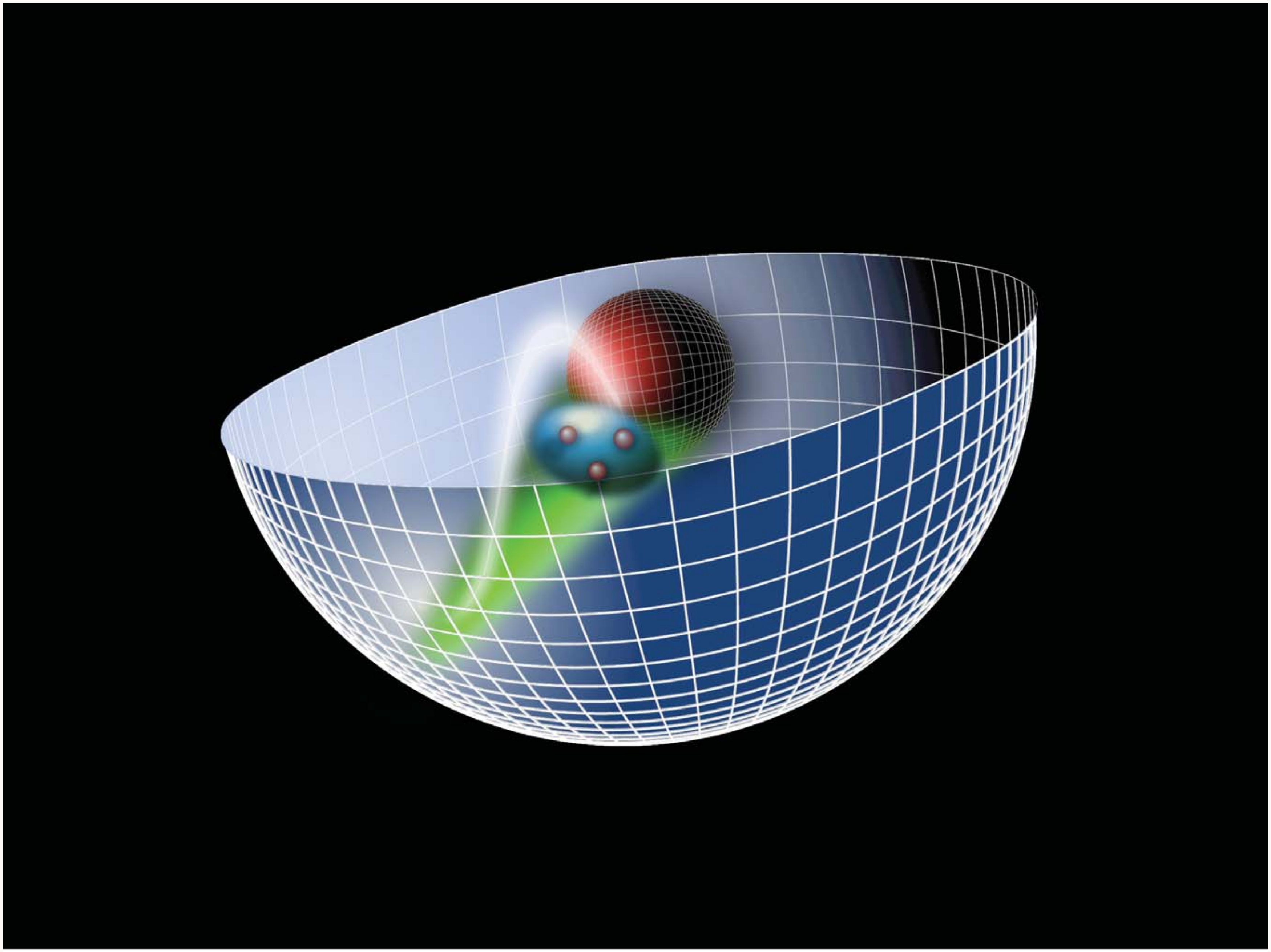
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.









AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map AdS_5 X S_5 to conformal N=4 SUSY

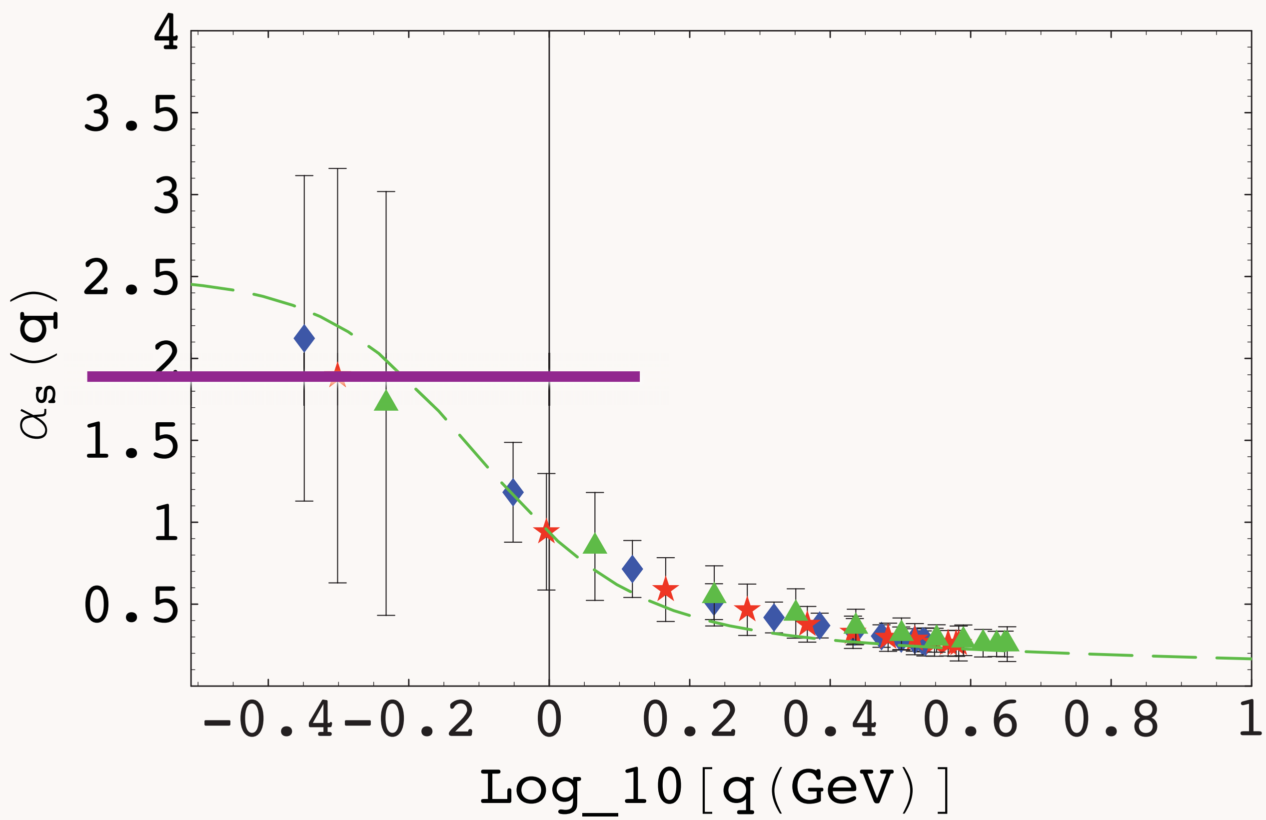
- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

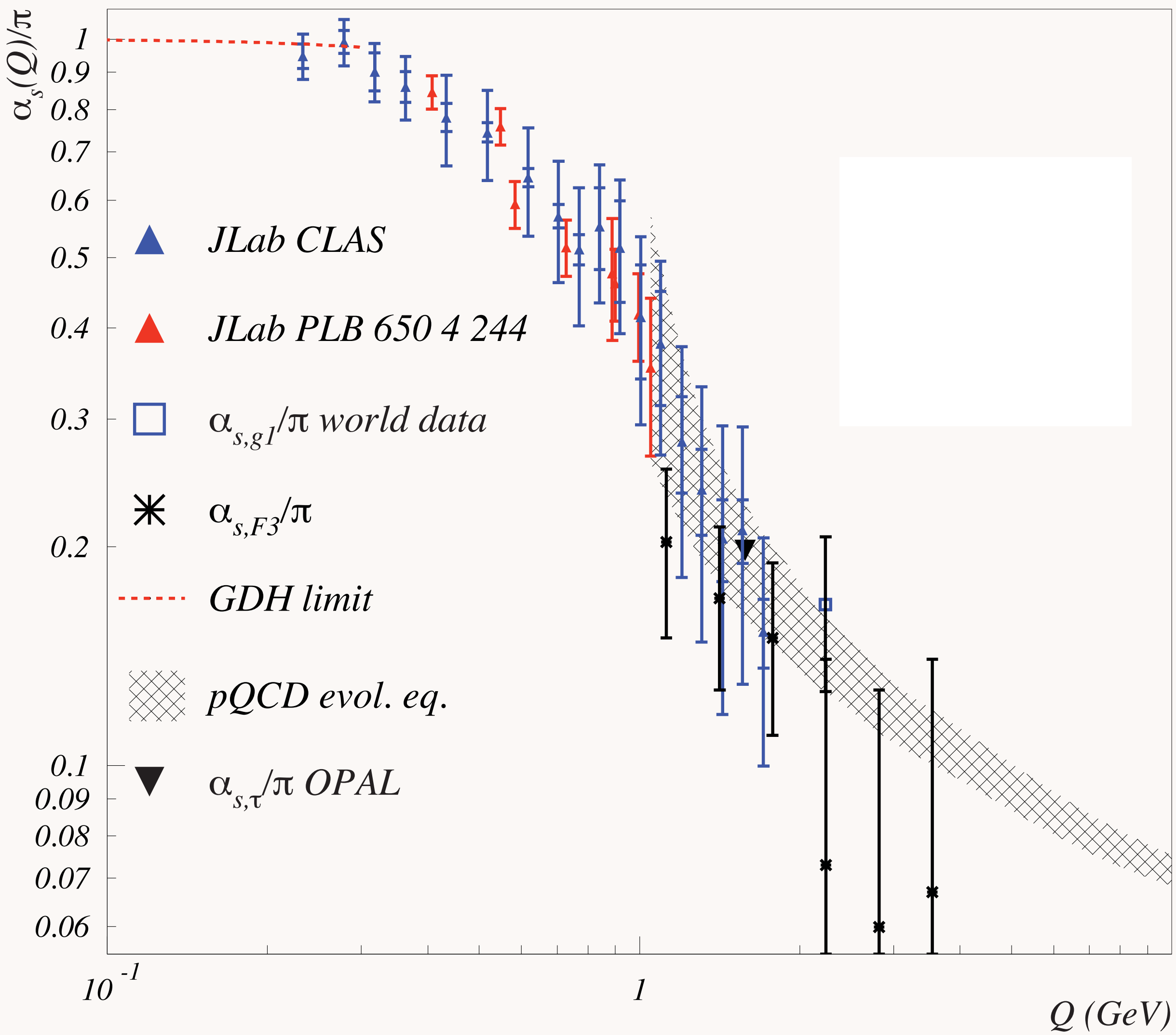
Conformal QCD Window in Exclusive Processes

- Does α_s develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .
- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).

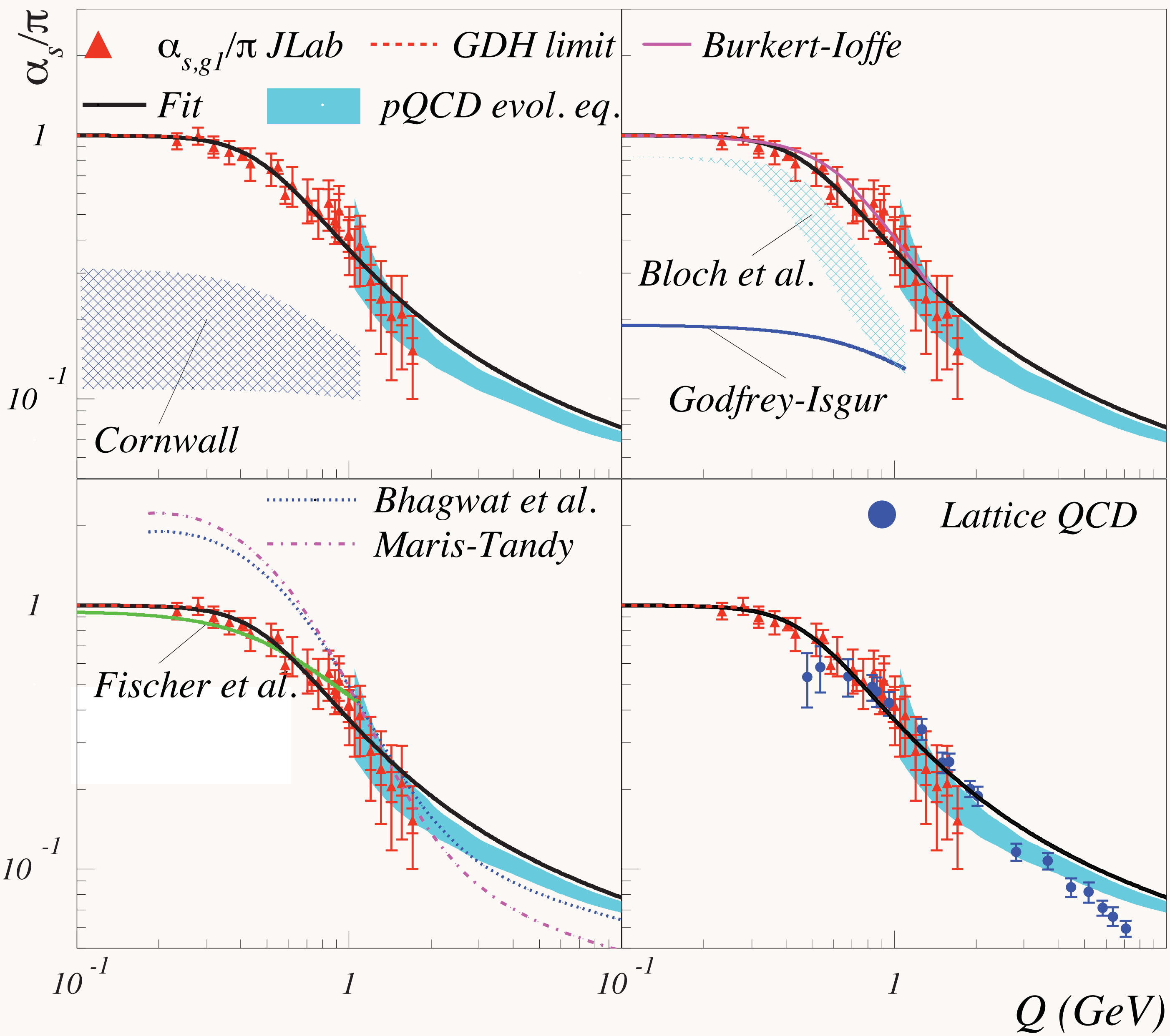


Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Deur, Korsch, et al.



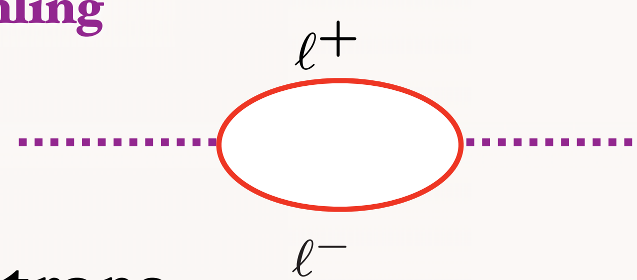
IR Fixed-Point for QCD?

- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory*
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**
- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Shrock, de Teramond, sjb

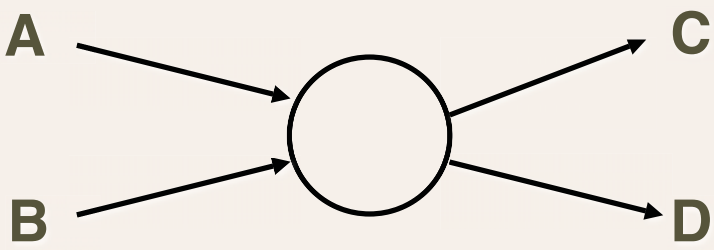
Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

Constituent Counting Rules



$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H-1}$$

$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

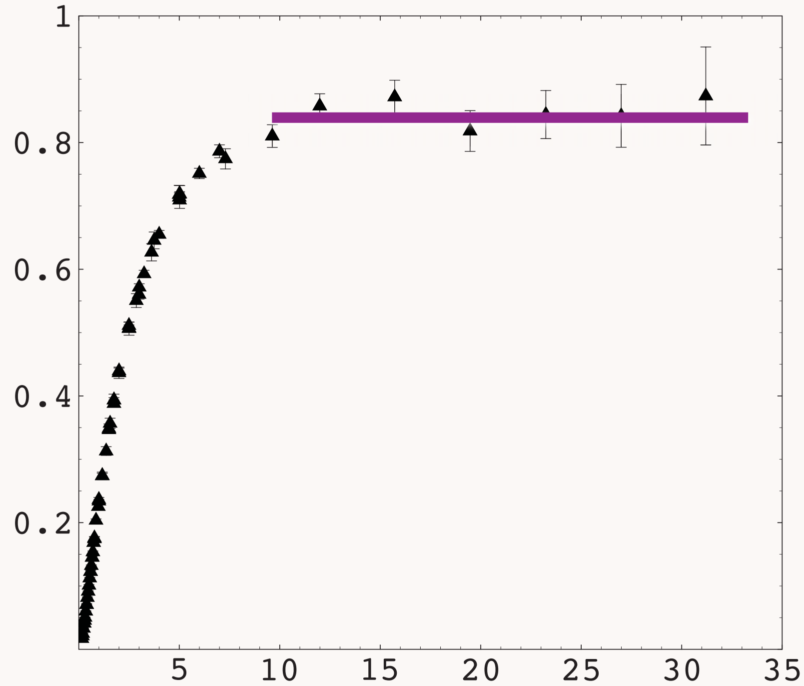
Farrar & sjb; Matveev, Muradyan, Tavkhelidze

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

$Q^4 F_1^p(Q^2)$ [GeV^4]



Q^2 [GeV^2]

$$F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \quad n = 3$$

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

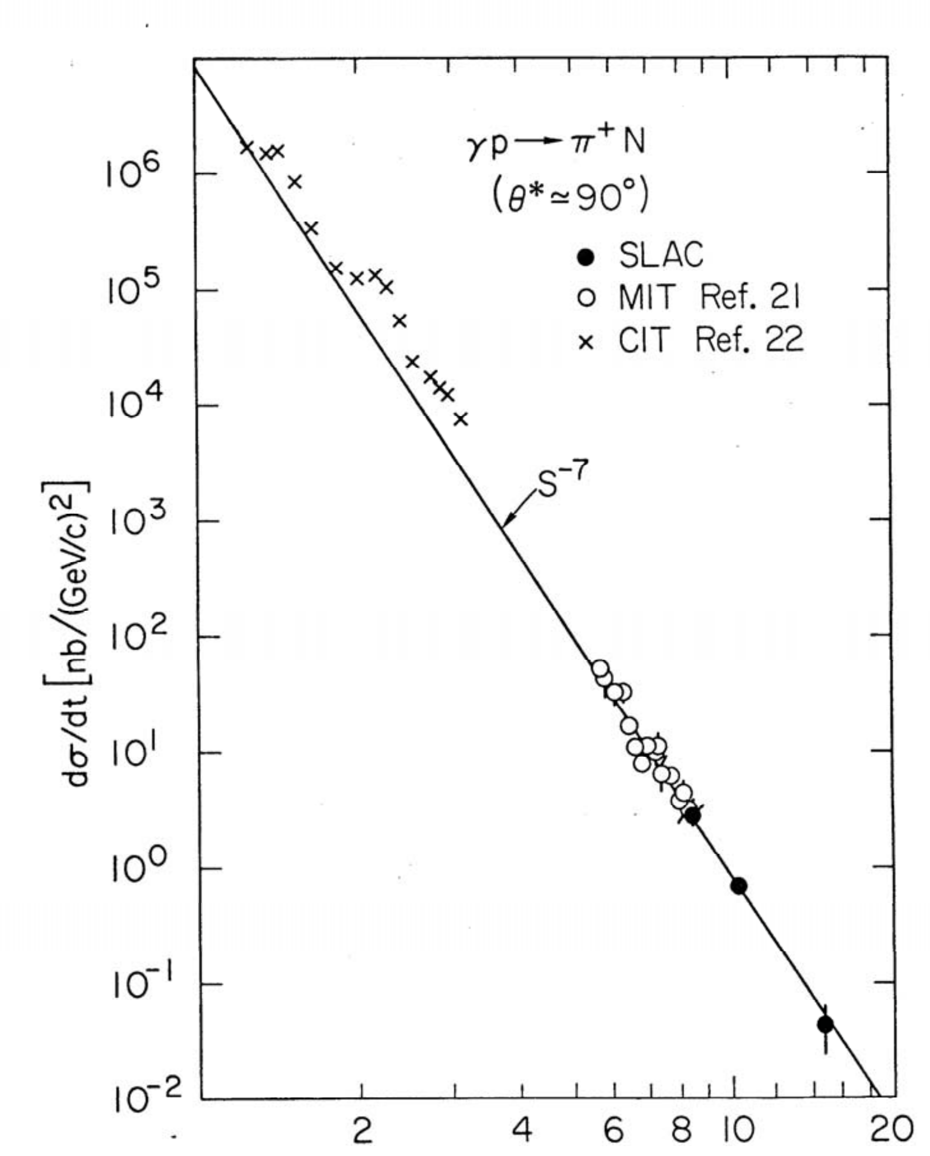
- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

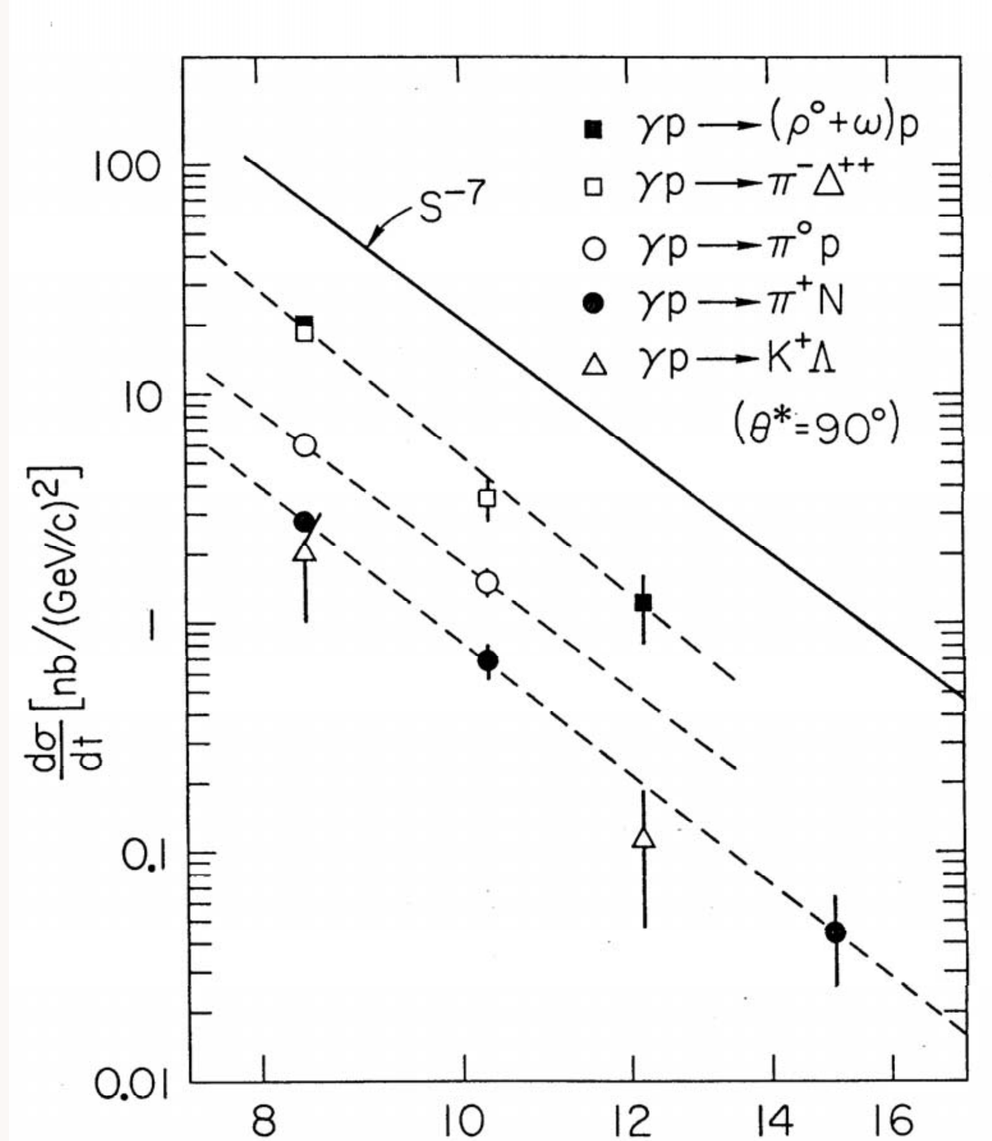
implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).



s(GeV²)



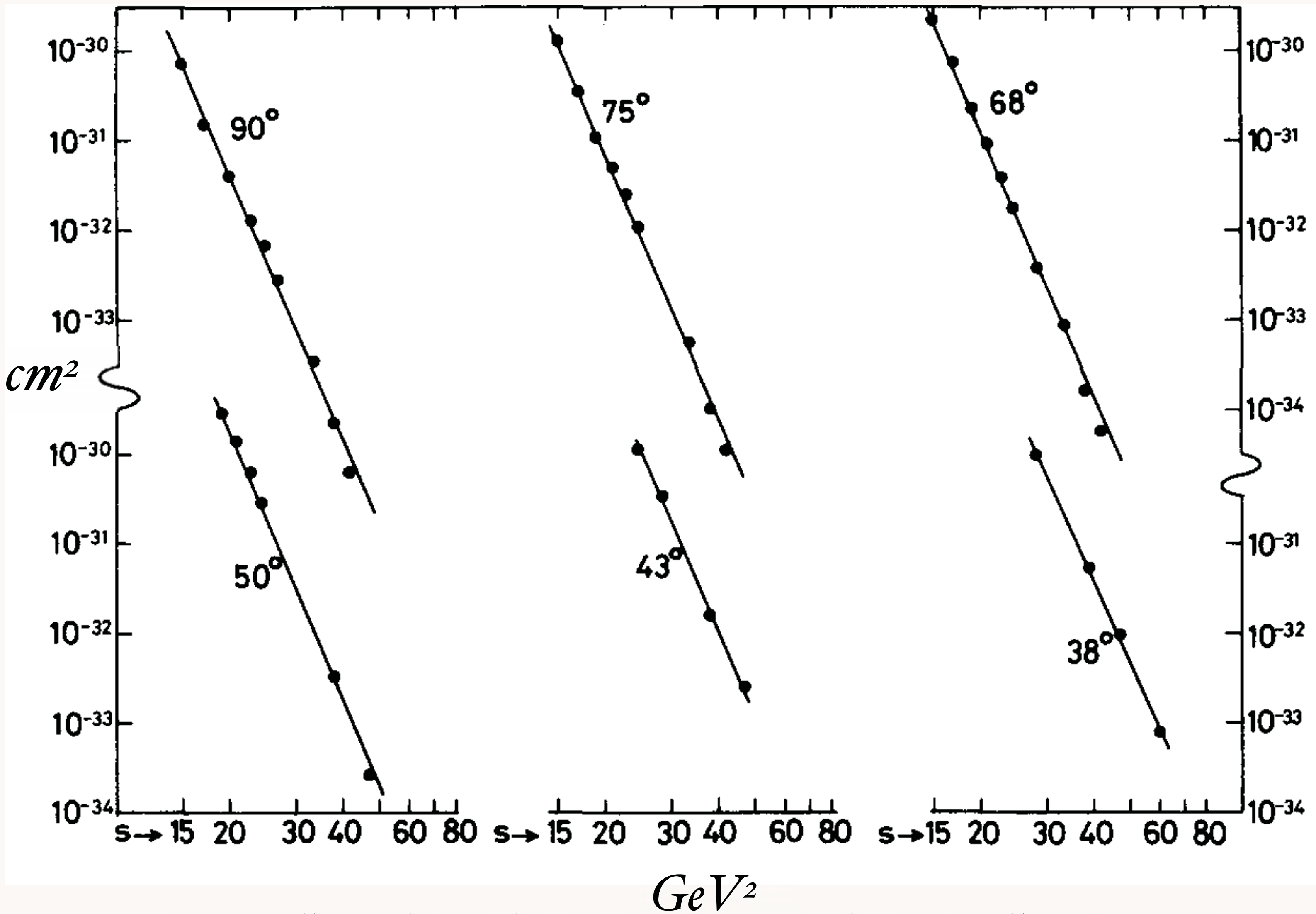
s(GeV²)

Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$

P.V. LANDSHOFF and J.C. POLKINGHORNE



Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

Angular distribution -- quark interchange