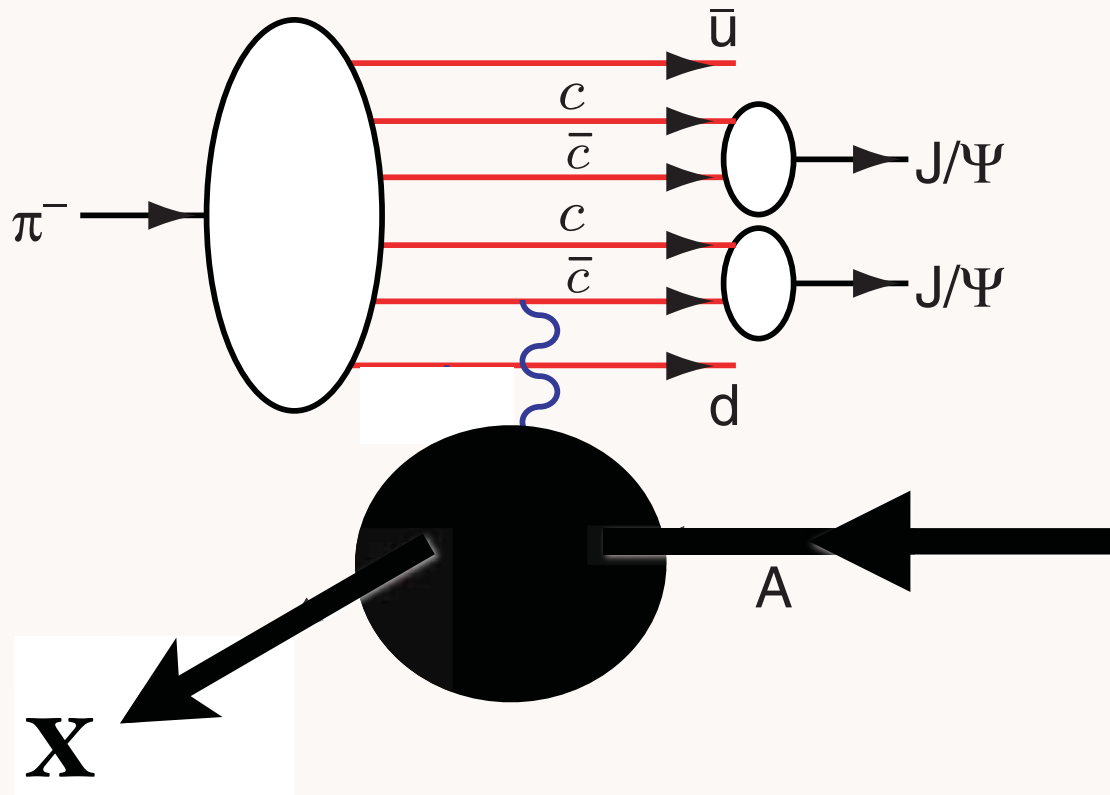


# Production of Two Charmonia at High $x_F$



All events have  $x_{\psi\psi}^F > 0.4$  !

**Excludes 'color drag' model**

$$\pi A \rightarrow J/\psi J/\psi X$$

Intrinsic charm contribution to double quarkonium hadroproduction <sup>\*</sup>

R. Vogt <sup>a</sup>, S.J. Brodsky <sup>b</sup>

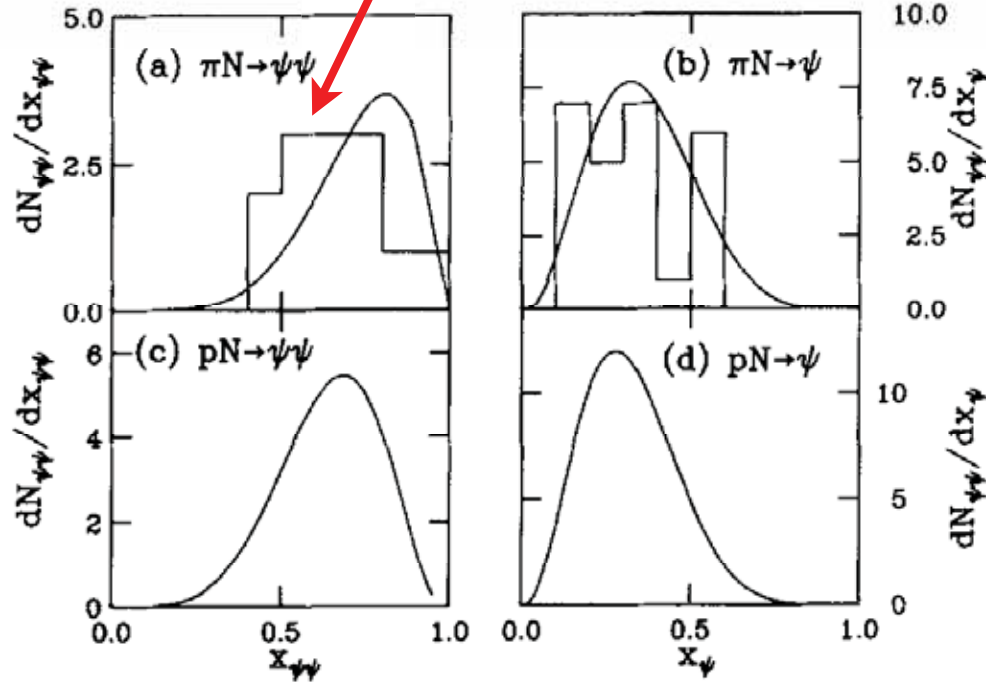
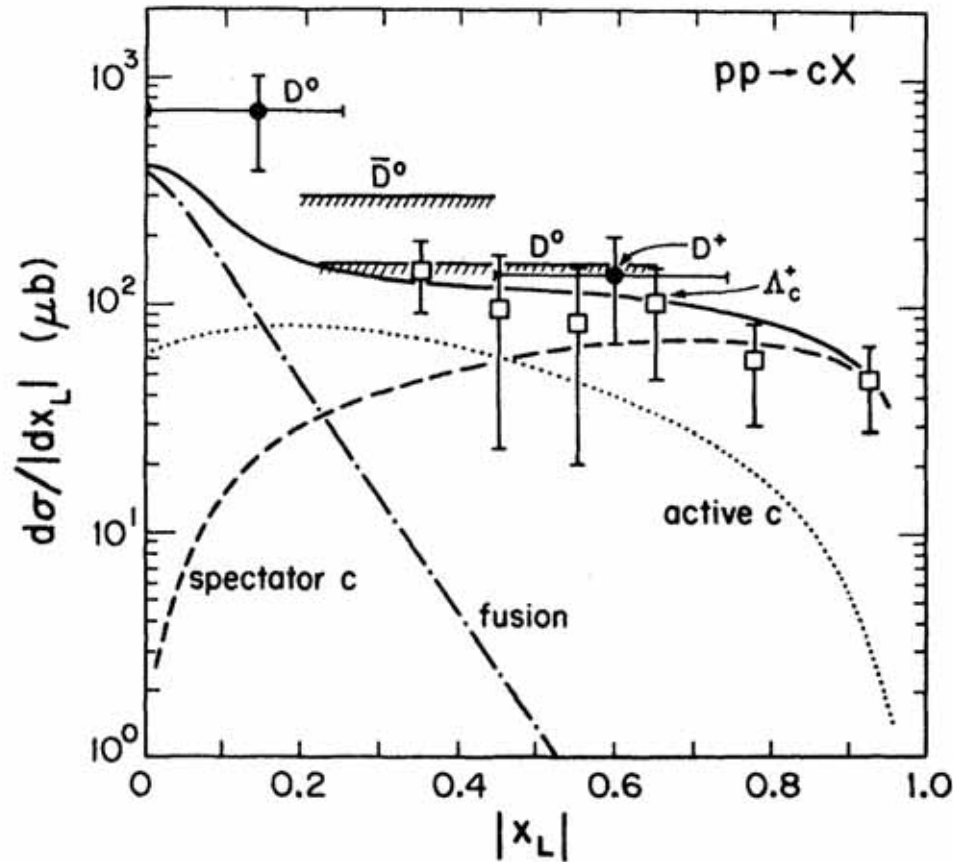


Fig. 3. The  $\psi\psi$  pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of  $J/\psi$ 's from the pairs are shown in (b) and (d). Our calculations are compared with the  $\pi^- N$  data at 150 and 280 GeV/c [1]. The  $x_{\psi\psi}$  distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single  $J/\psi$ 's is twice the number of pairs.

The probability distribution for a general  $n$ -parton intrinsic  $c\bar{c}$  Fock state as a function of  $x$  and  $k_T$  written as

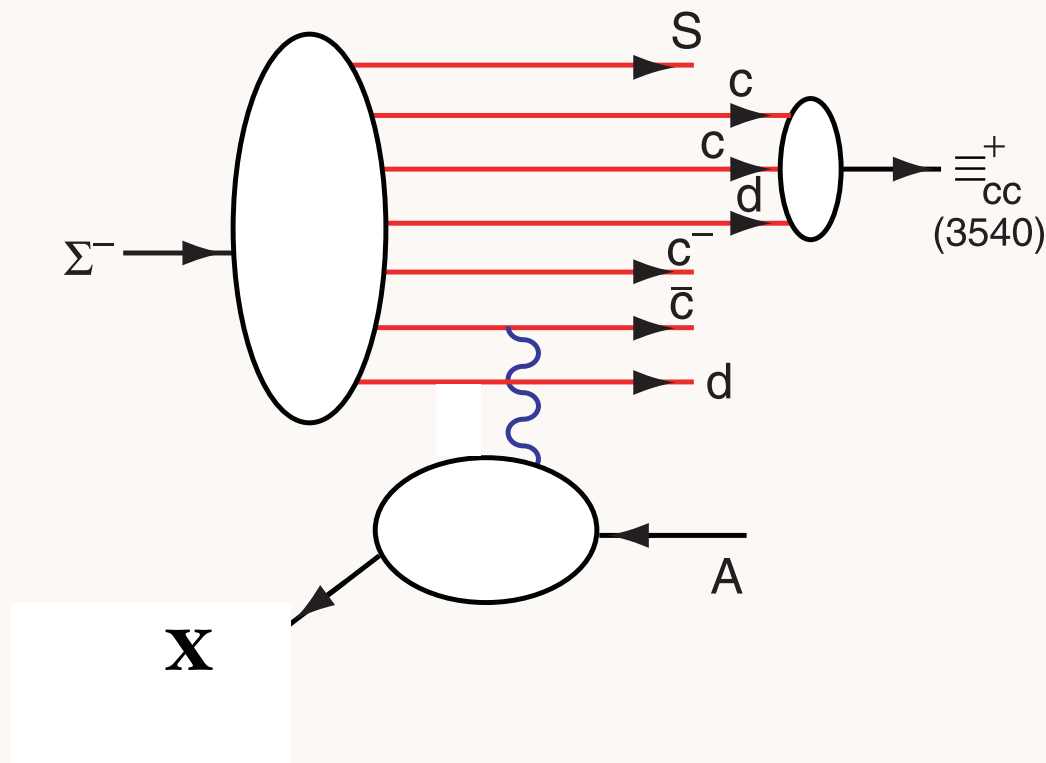
$$\frac{dP_{ic}}{\prod_{i=1}^n dx_i d^2k_{T,i}} = N_n \alpha_s^4 (M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n k_{T,i}) \delta(1 - \sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$

**NA3 Data**



*Model similar to  
Intrinsic Charm*

V. D. Barger, F. Halzen and W. Y. Keung,  
 “The Central And Diffractive Components Of Charm Pro-  
 duction,”  
 Phys. Rev. D 25, 112 (1982).

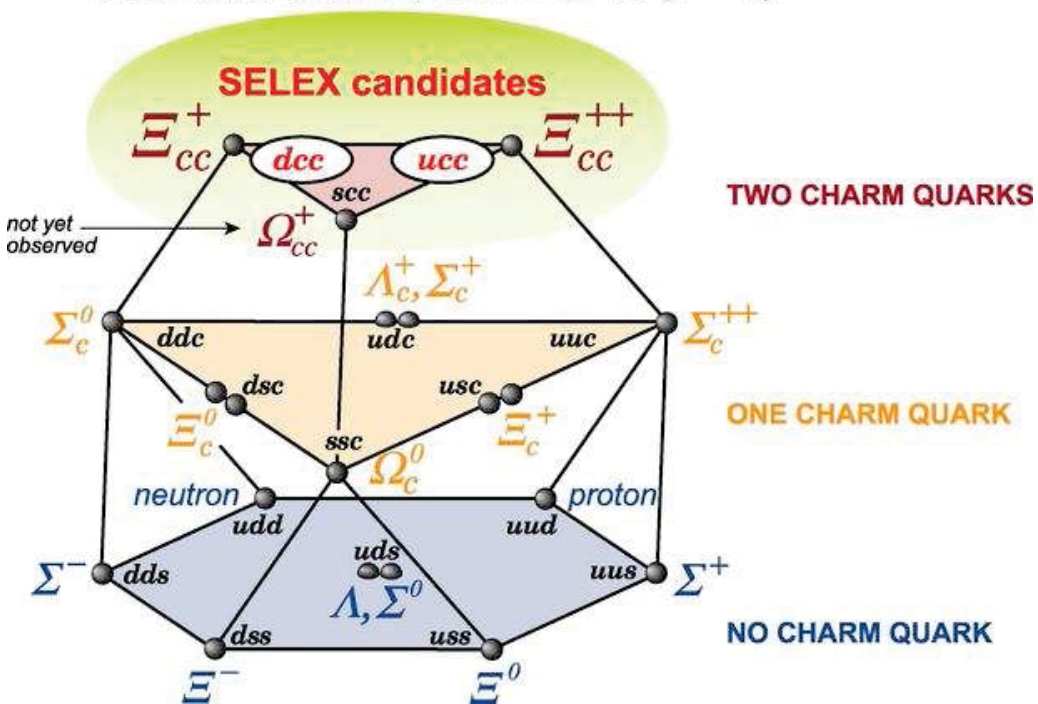


## *Production of a Double-Charm Baryon*

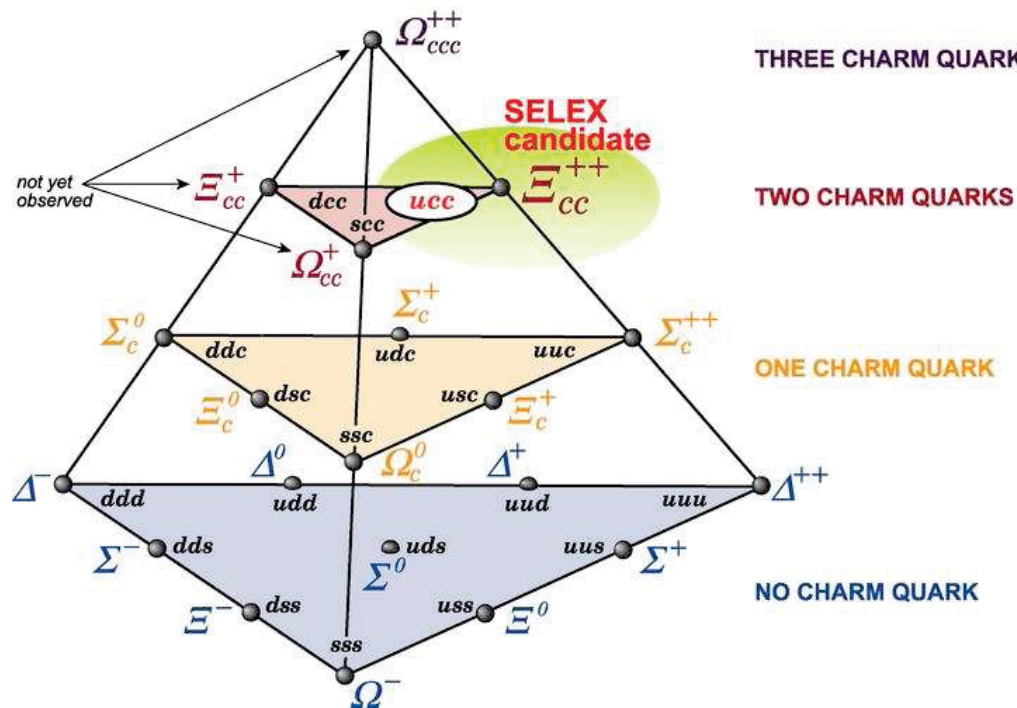
**SELEX high  $x_F$**        $\langle x_F \rangle = 0.33$

# Doubly Charmed Baryons

BARYONS WITH LOWEST SPIN ( $J = 1/2$ )

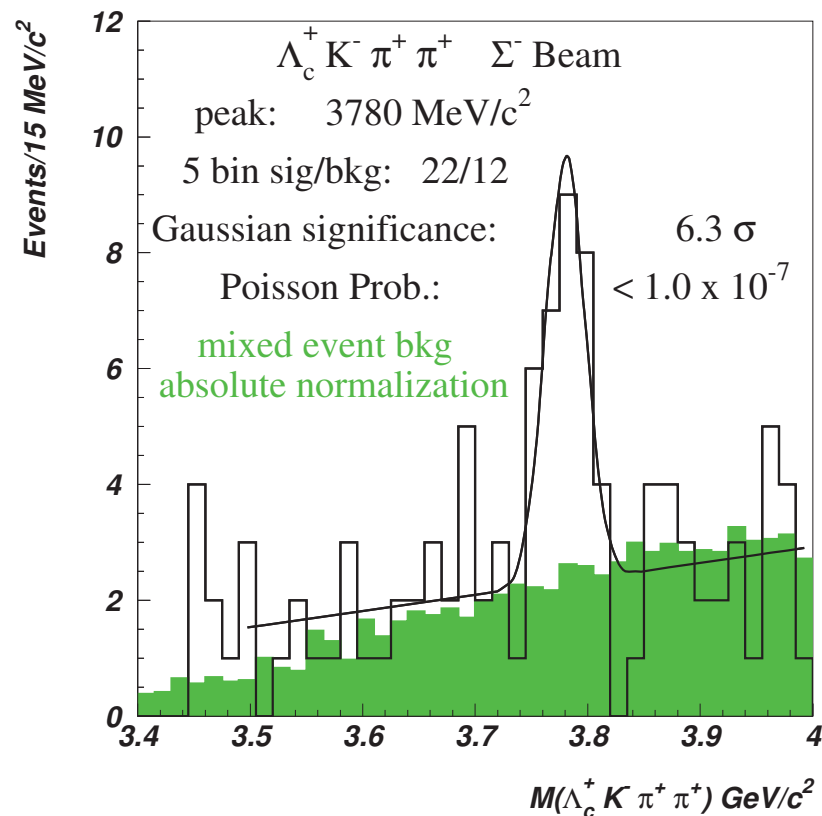


BARYONS WITH HIGHEST SPIN ( $J = 3/2$ )

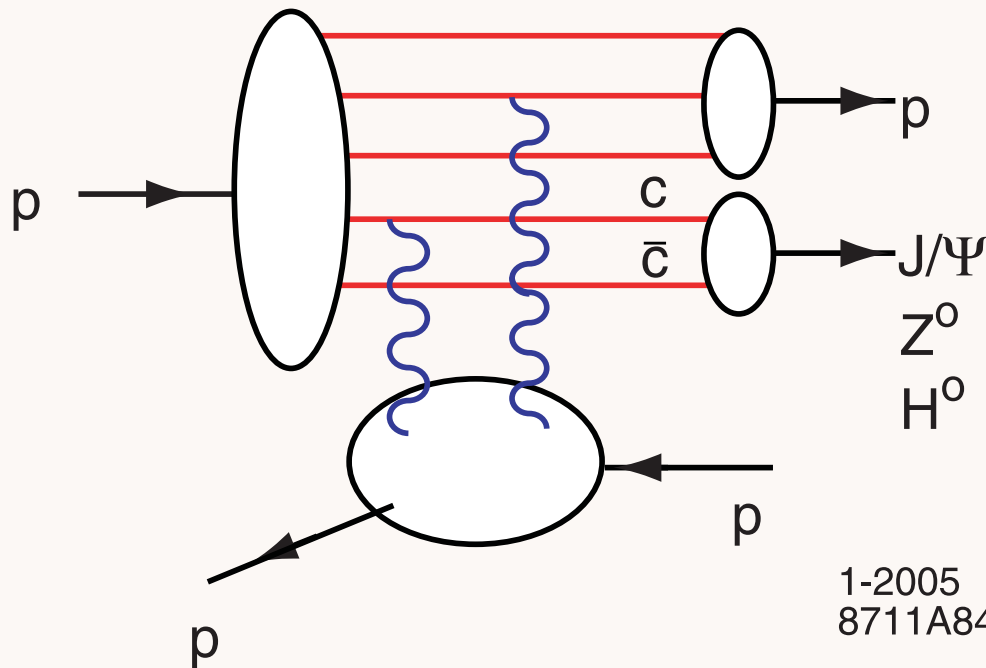


$$\Xi_{cc}(3780)^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$$

- Re-Analyzed Data
- Restrict to  $\Sigma^-$ -Beam
- Peak wider than Resolution
- Half decay to  $\Xi_{cc}^+(3520)$
- Still working on Details



# Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$p p \rightarrow J/\psi p p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

Exclusive Diffractive  
High- $X_F$  Higgs Production

Kopeliovitch, Schmidt, Soffer, sjb

1-2005  
8711A84

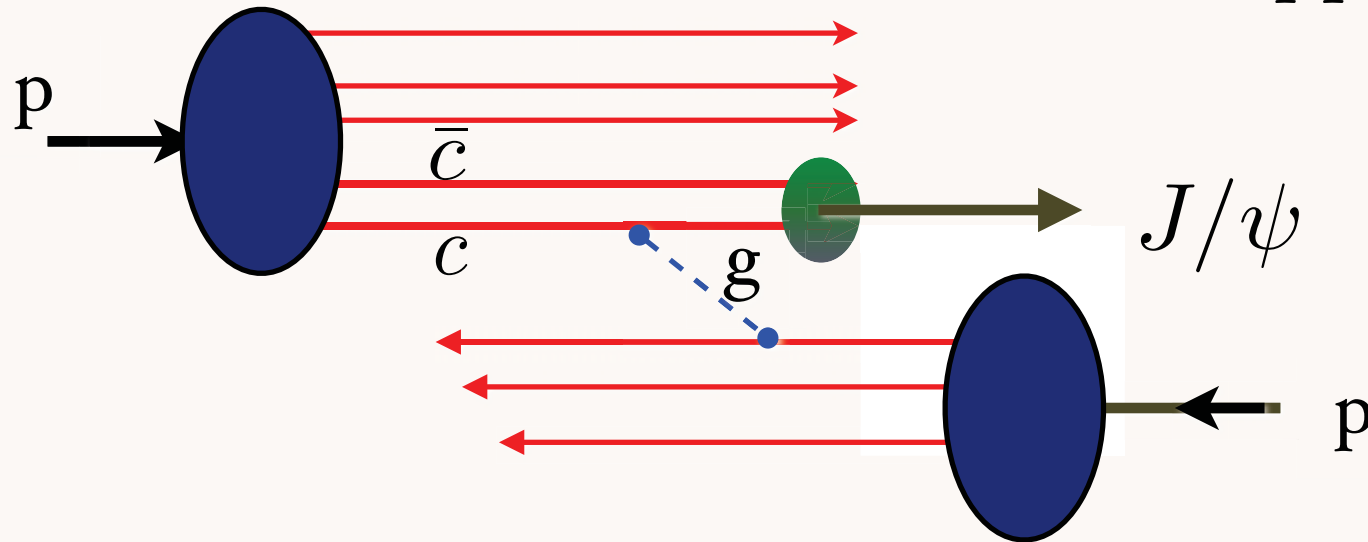
Intrinsic  $c\bar{c}$  pair formed in color octet  $8_C$  in proton wavefunction Large Color Dipole

Collision produces color-singlet  $J/\psi$  through color exchange

RHIC Experiment

# Intrinsic Charm Mechanism for Inclusive High- $x_F$ Quarkonium Production

$$pp \rightarrow J/\psi X$$



Goldhaber, Kopeliovich, Soffer, Schmidt, sjb

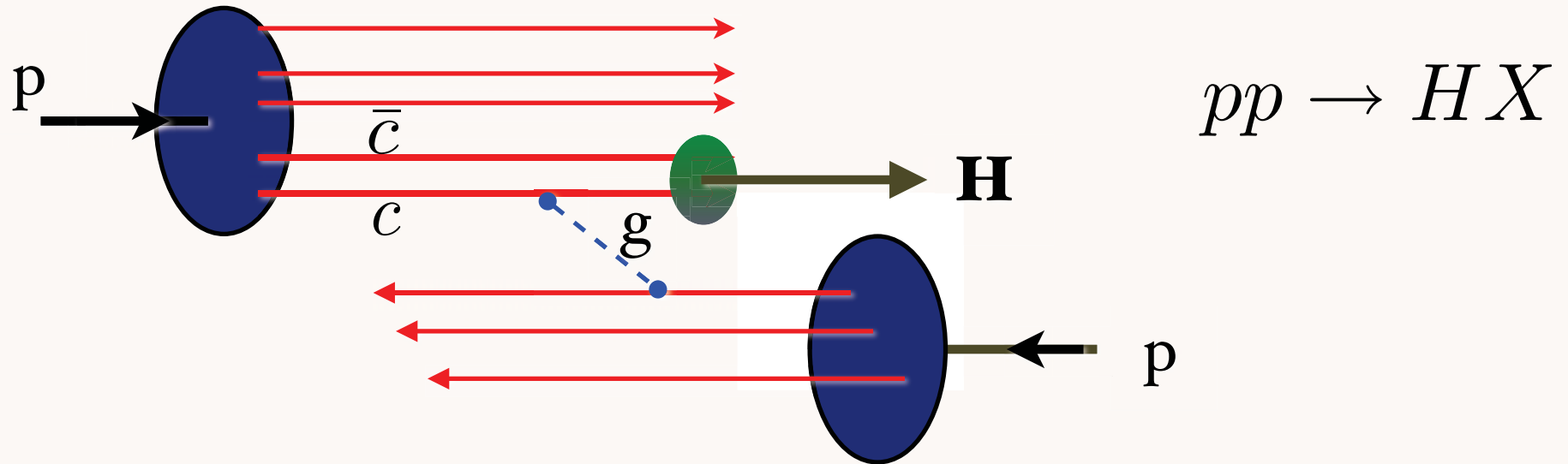
**Quarkonia can have 80% of Proton Momentum!**

*Color-octet IC interacts at front surface of nucleus*

**IC can explain large excess of quarkonia at large  $x_F$ , A-dependence**



# Intrinsic Charm Mechanism for Inclusive High- $X_F$ Higgs Production



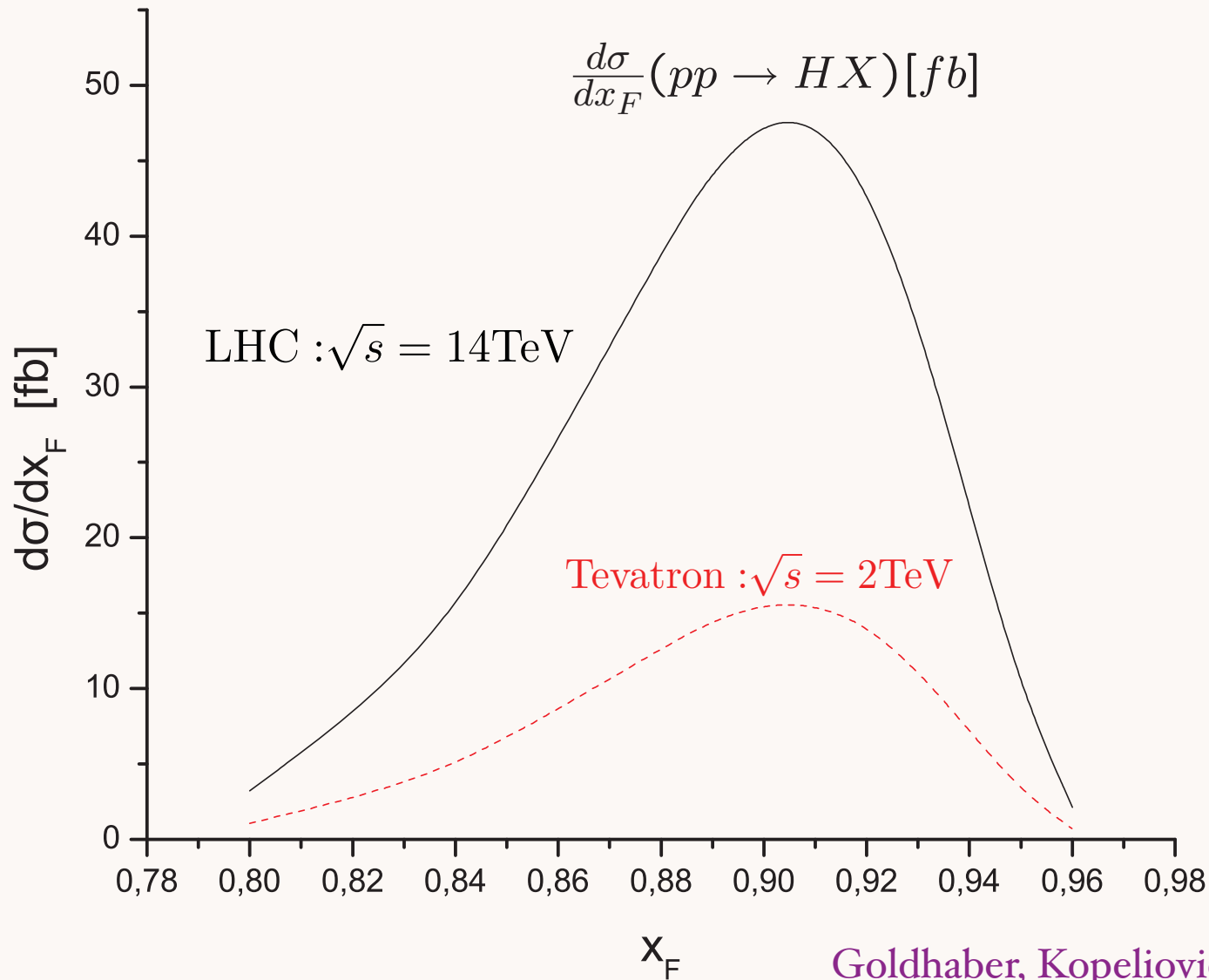
Goldhaber, Kopeliovich, Schmidt, sjb

**Also: intrinsic bottom, top**

**Higgs can have 80% of Proton Momentum!**

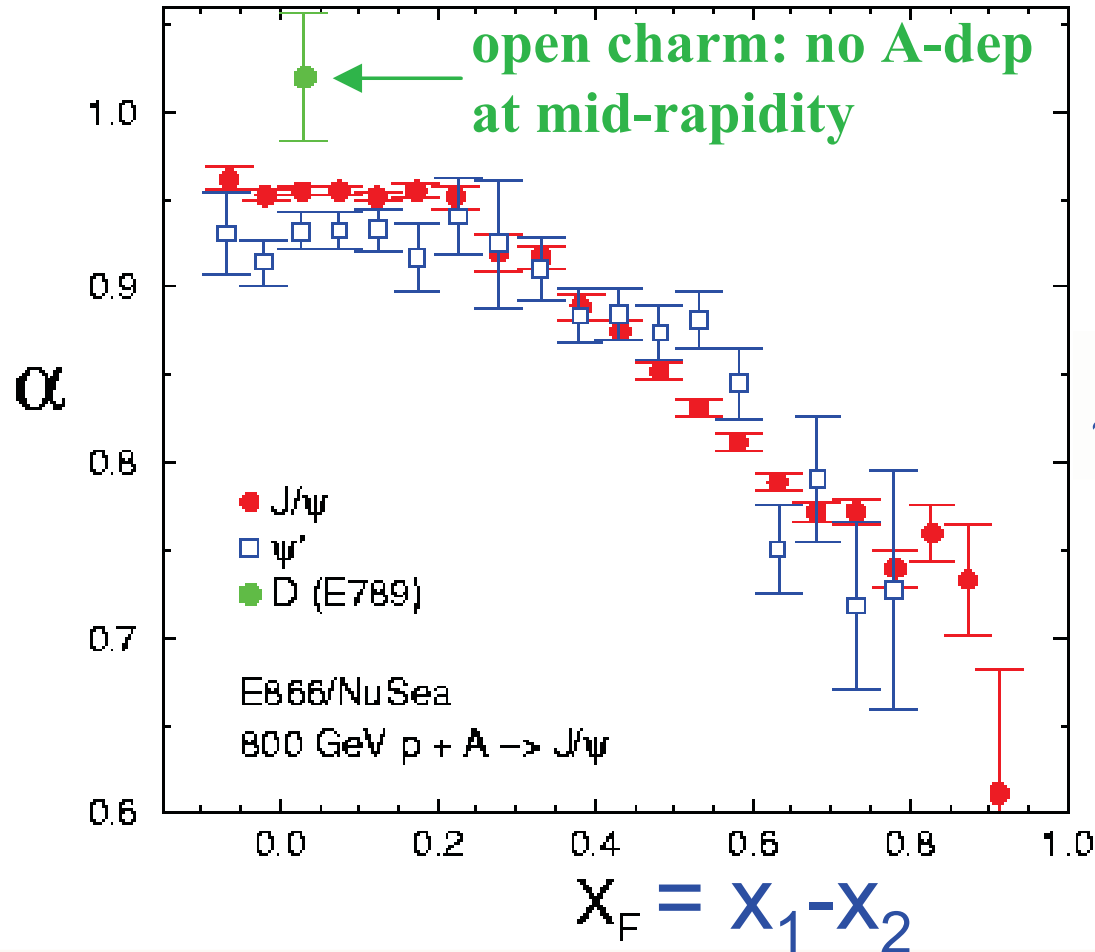
*New search strategy for Higgs*

# Intrinsic Bottom Contribution to Inclusive Higgs Production



Goldhaber, Kopeliovich, Schmidt, sjb

800 GeV p-A (FNAL)  $\sigma_A = \sigma_p * A^\alpha$   
*PRL 84, 3256 (2000); PRL 72, 2542 (1994)*



$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$$

*Remarkably Strong Nuclear Dependence for Fast Charmonium*

*Violation of PQCD Factorization*

Violation of factorization in charm hadroproduction.

[P. Hoyer](#), [M. Vanttinen](#) ([Helsinki U.](#)), [U. Sukhatme](#) ([Illinois U., Chicago](#)) . HU-TFT-90-14, May 1990. 7pp.

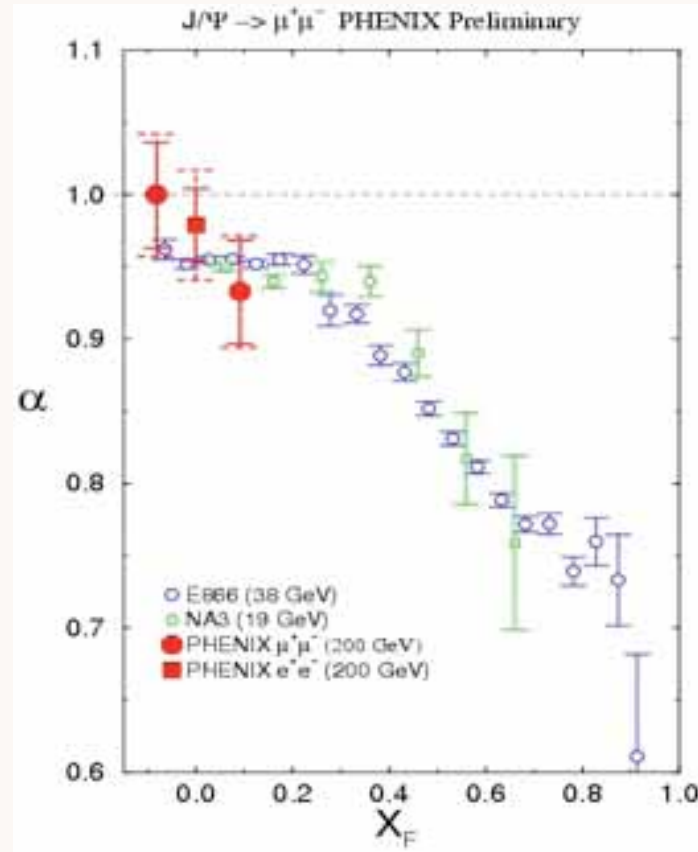
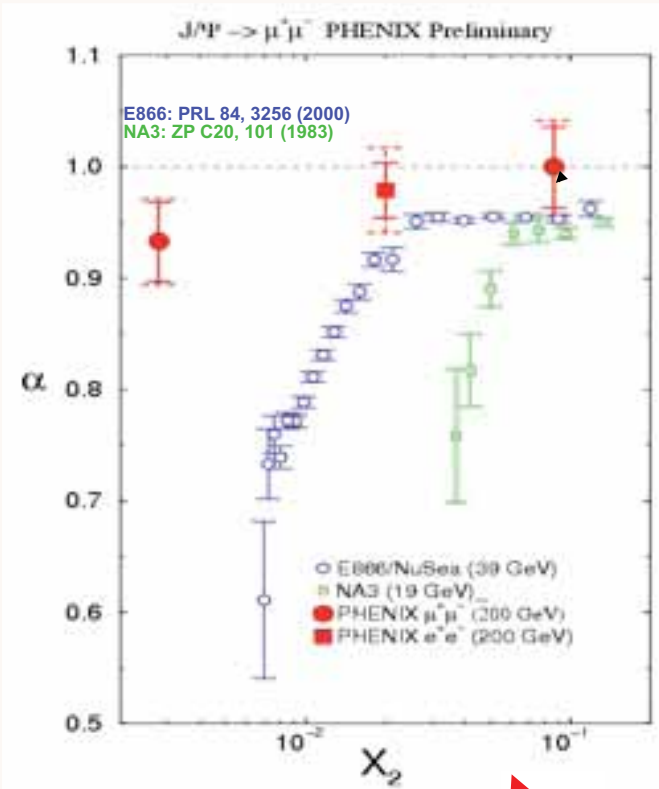
Published in Phys.Lett.B246:217-220,1990

**IC Explains large excess of quarkonia at large  $x_F$ , A-dependence**

# J/ψ nuclear dependence vrs rapidity, $x_{Au}$ , $x_F$

M.Leitch

## PHENIX compared to lower energy measurements



Huge  
"absorption"  
effect



Klein, Vogt, PRL 91:142301, 2003  
 Kopeliovich, NP A696:669, 2001

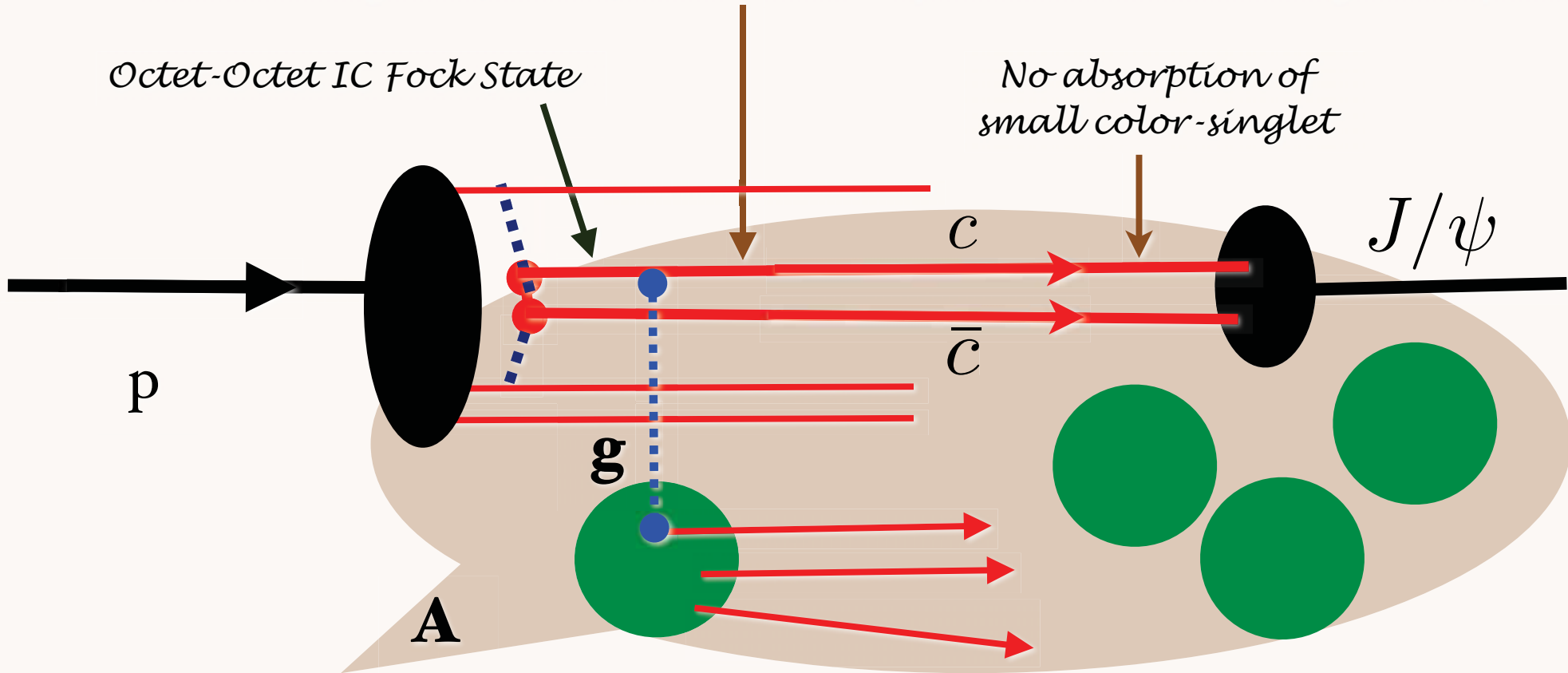
*Violates PQCD  
factorization!*

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)$$

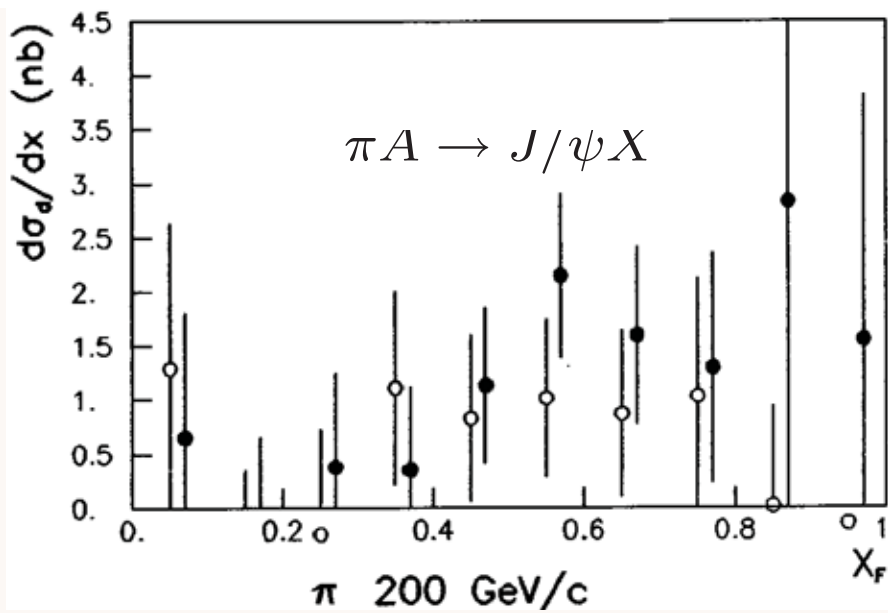
Hoyer, Sukhatme, Vanttinen

*Color-Opaque IC Fock state  
interacts on nuclear front surface*

*Scattering on front-face nucleon produces color-singlet  $c\bar{c}$  pair*

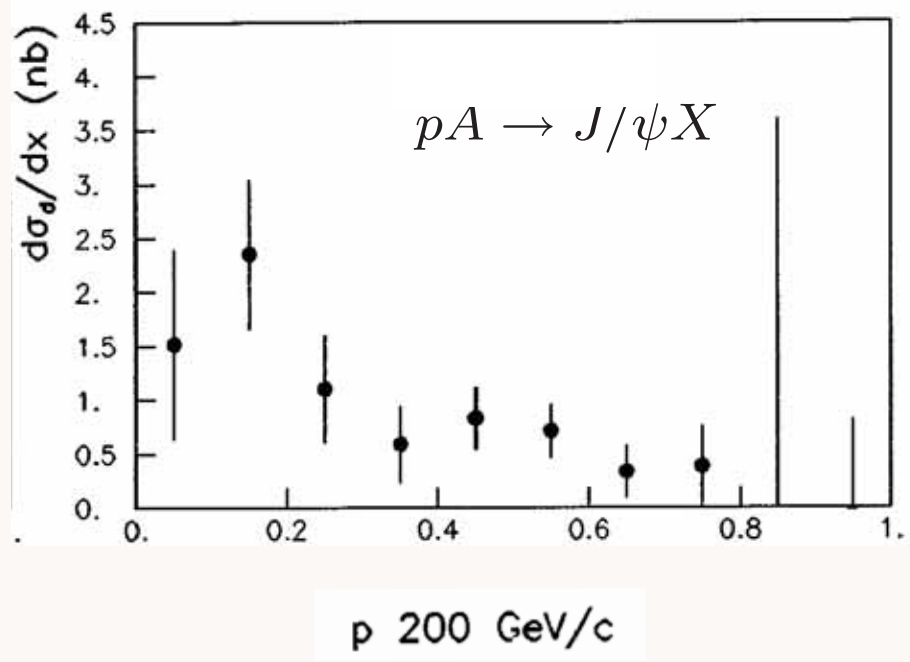


$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$



$A^{2/3}$  component

J. Badier et al, NA3



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

**Excess beyond conventional PQCD subprocesses**

- IC Explains Anomalous  $\alpha(x_F)$  not  $\alpha(x_2)$  dependence of  $pA \rightarrow J/\psi X$   
(Mueller, Gunion, Tang, SJB)
- Color Octet IC Explains  $A^{2/3}$  behavior at high  $x_F$  (NA3, Fermilab) *Color Opacity*  
(Kopeliovitch, Schmidt, Soffer, SJB)
- IC Explains  $J/\psi \rightarrow \rho\pi$  puzzle  
(Karliner, SJB)
- IC leads to new effects in  $B$  decay  
(Gardner, SJB)

## Higgs production at $x_F = 0.8$

## *Why is Intrinsic Charm Important for Flavor Physics?*

- **New perspective on fundamental nonperturbative hadron structure**
- **Charm structure function at high  $x$**
- **Dominates high  $x_F$  charm and charmonium production**
- **Hadroproduction of new heavy quark states such as  $ccu$ ,  $ccd$  at high  $x_F$**
- **Intrinsic charm -- long distance contribution to penguin mechanisms for weak decay**
- **Novel Nuclear Effects from color structure of IC, Heavy Ion Collisions**
- **New mechanisms for high  $x_F$  Higgs hadroproduction**
- **Dynamics of  $b$  production: LHCb**
- **Fixed target program at LHC: produce  $bbb$  states**



# Light-Front Wavefunctions

Dirac's Front Form: Fixed  $\tau = t + z/c$

$$\Psi(x, k_{\perp}) \quad x_i = \frac{k_i^+}{P^+}$$

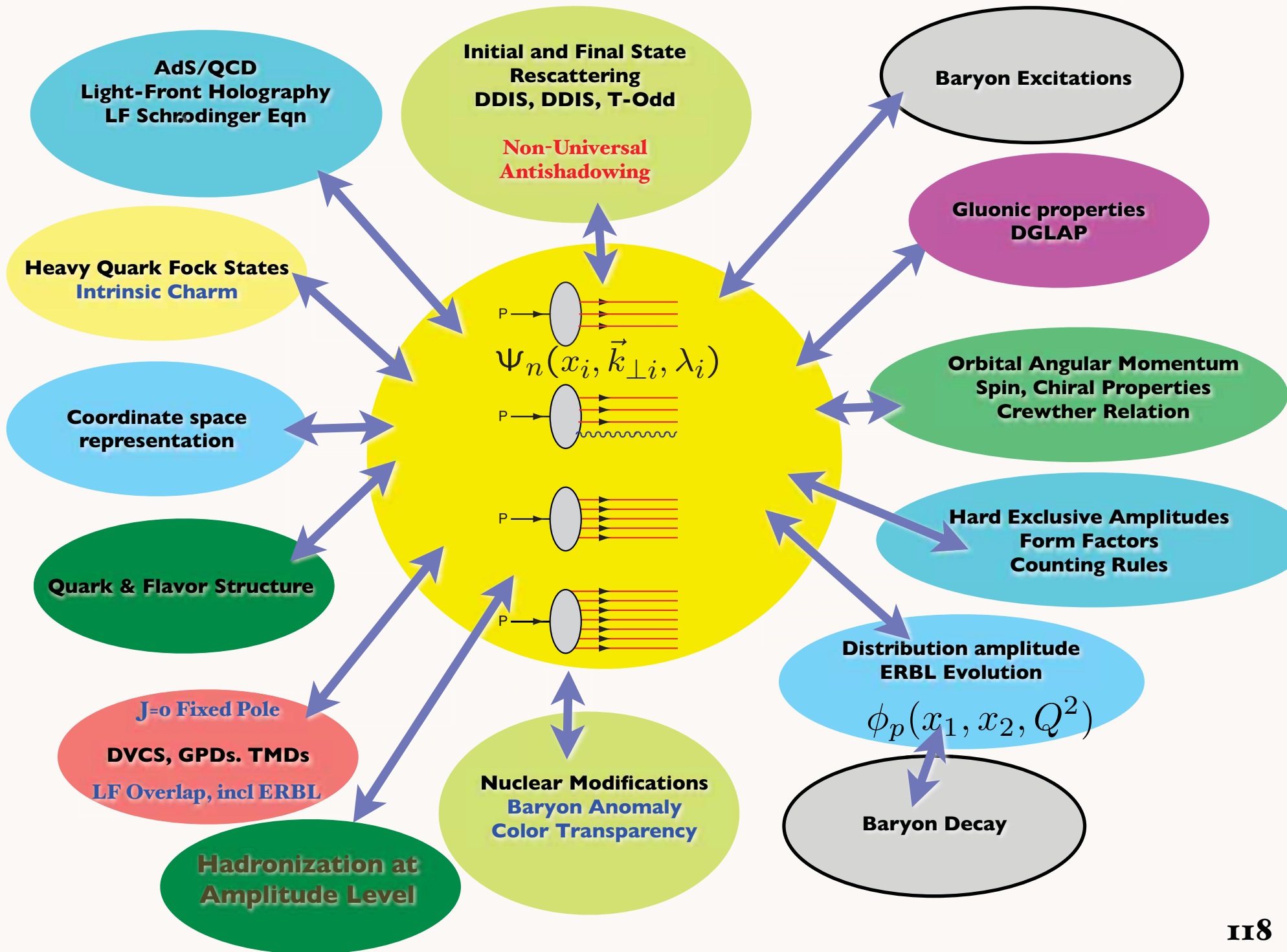
*Invariant under boosts. Independent of  $P^{\mu}$*

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

*Direct connection to QCD Lagrangian*

*Remarkable new insights from AdS/CFT,  
the duality between conformal field theory  
and Anti-de Sitter Space*

# QCD and the LF Hadron Wavefunctions

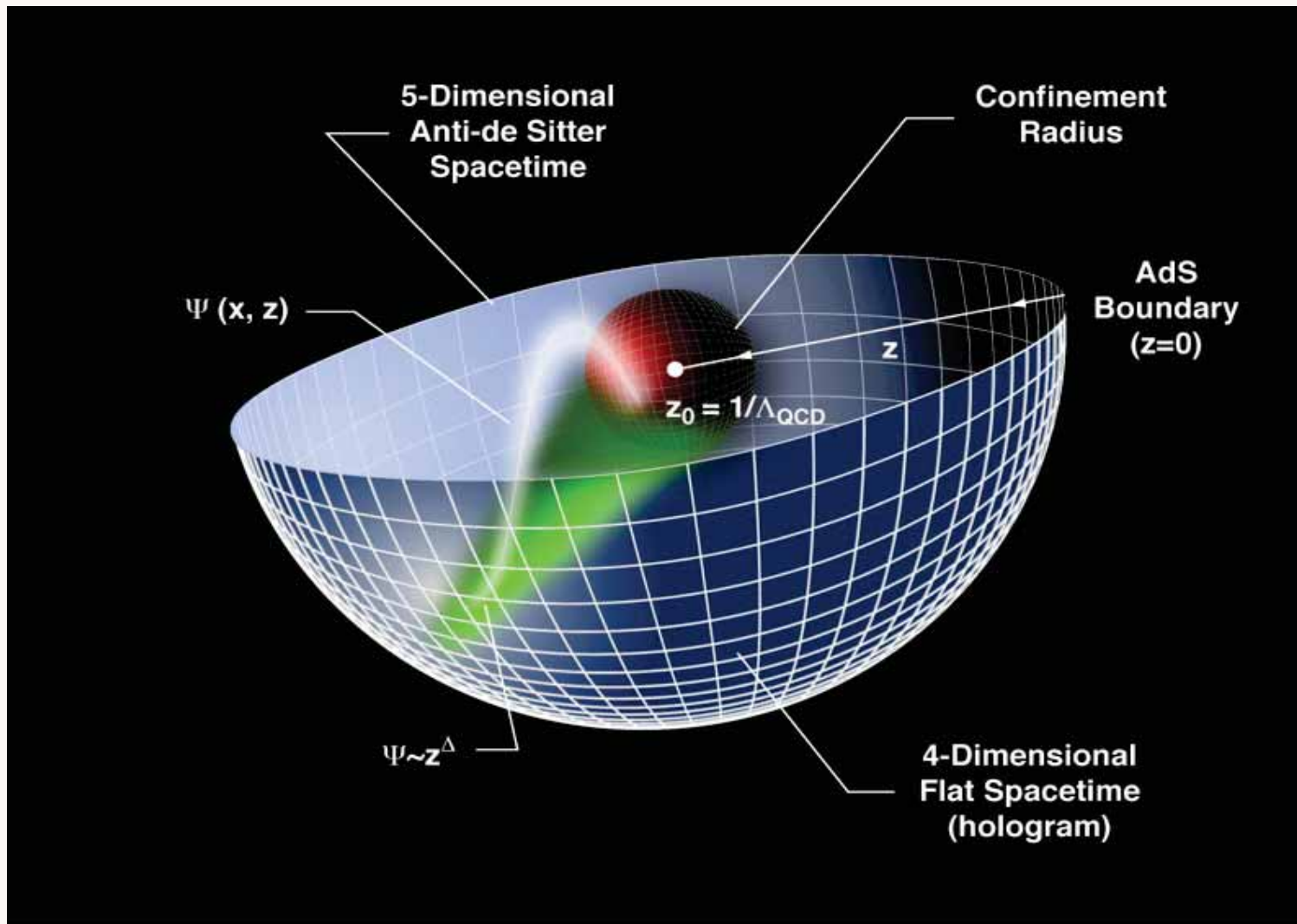


# *Goal: an analytic first approximation to QCD*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

**de Teramond, Deur, Shrock, Roberts, Tandy**

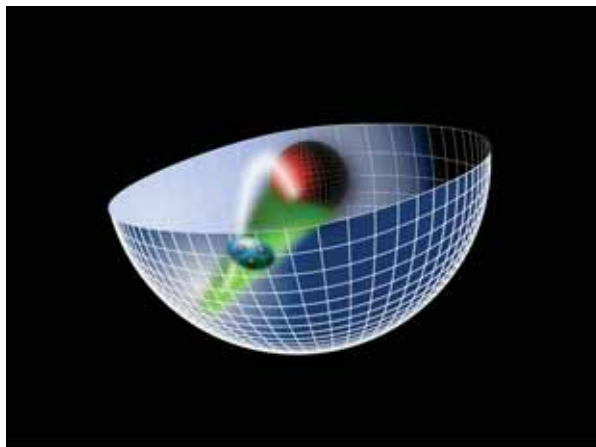
# Applications of AdS/CFT to QCD



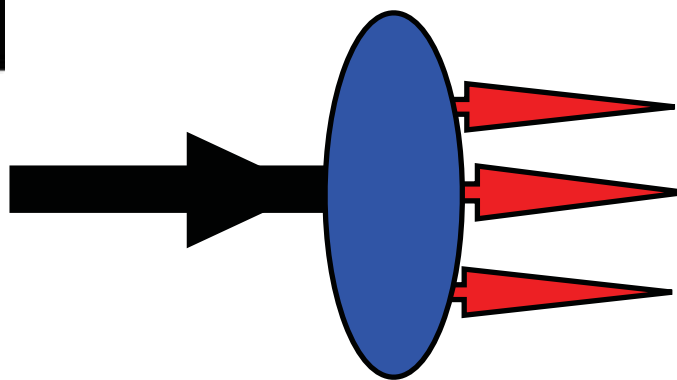
*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

**in collaboration with Guy de Teramond**

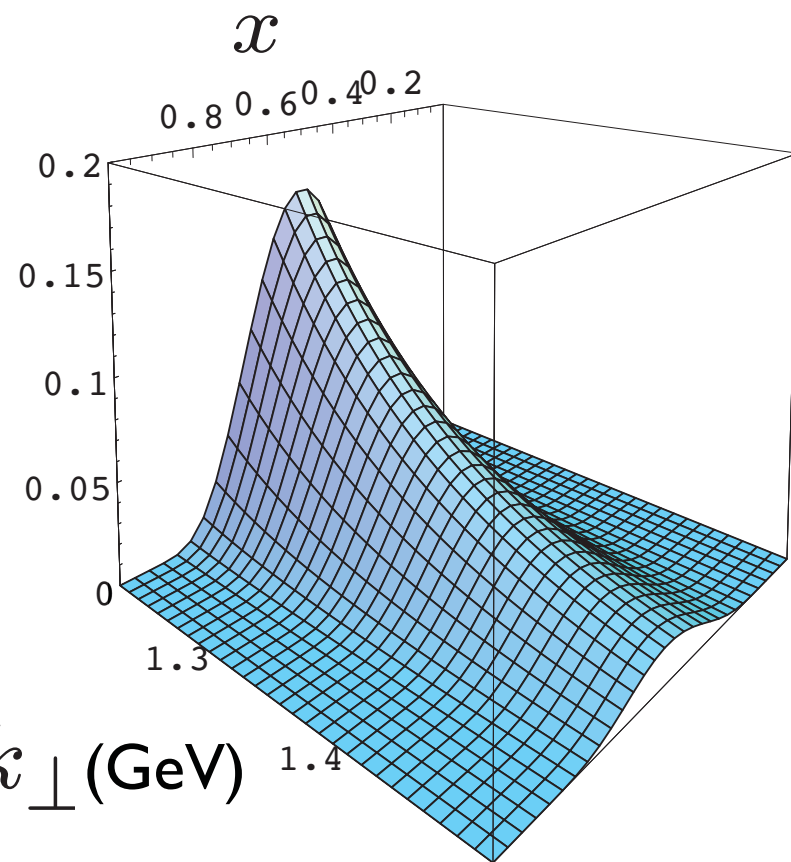
$$\phi(z)$$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

Schrödinger Wavefunctions  
of Hadron Physics

String Theory



AdS/CFT

Mapping of Poincare' and Conformal  $SO(4,2)$  symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering  
Regge Trajectories

Conformal behavior at short distances  
+ Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$  plus  $L$

Integrable!

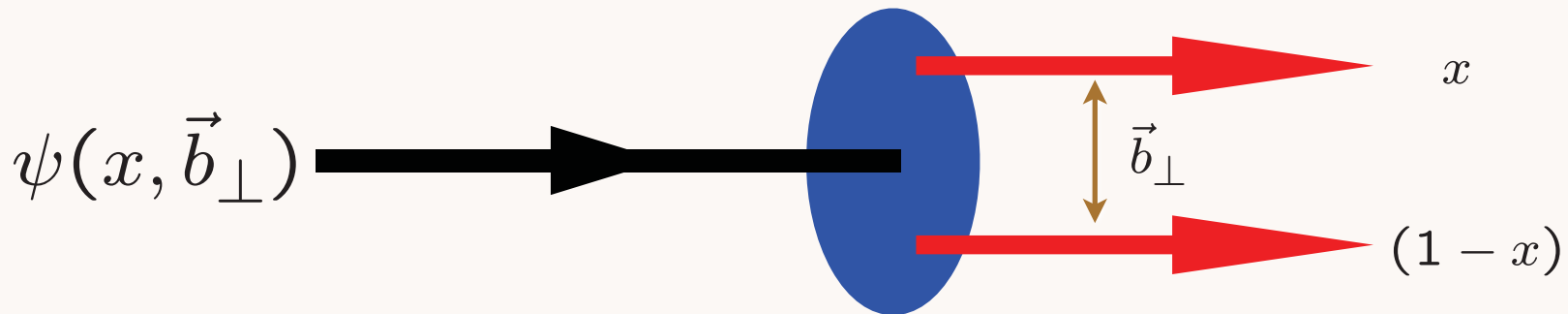
Hadron Spectra, Wavefunctions, Dynamics

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

*Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

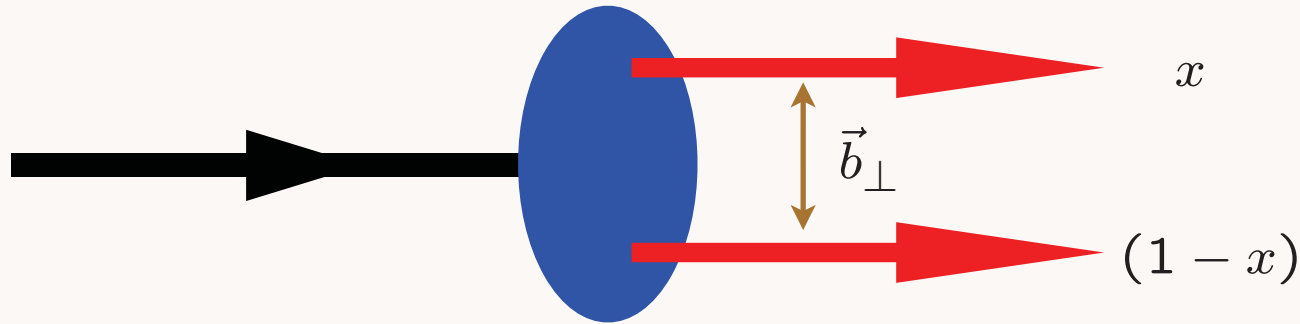
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



G. de Teramond, sjb

$$U(\zeta) = \kappa^4 \zeta^2$$

*soft wall  
confining potential:*



$$H_{QED}$$

*QED atoms: positronium  
and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

*Coupled Fock states*

$$\left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$\left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, l) \right] \psi(r) = E \psi(r)$$

*Spherical Basis*  $r, \theta, \phi$

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

**Bohr Spectrum**

*Semiclassical first approximation to QED*

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

*Coupled Fock states*

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

*Effective two-particle equation*

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

*Azimuthal Basis*  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

*Semiclassical first approximation to QCD*

*Confining AdS/QCD potential*

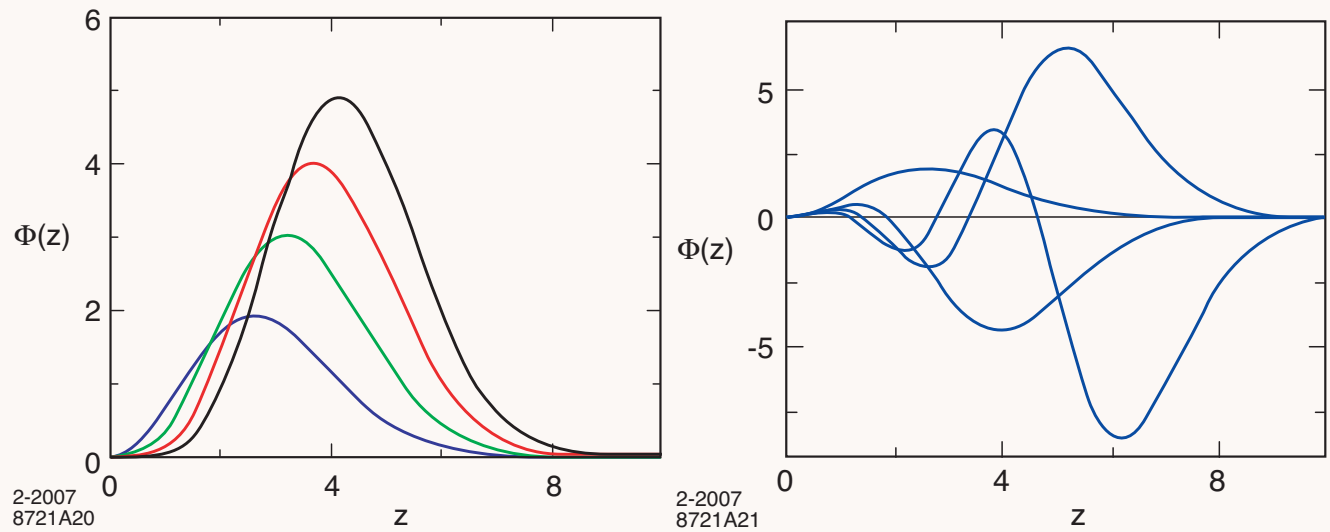
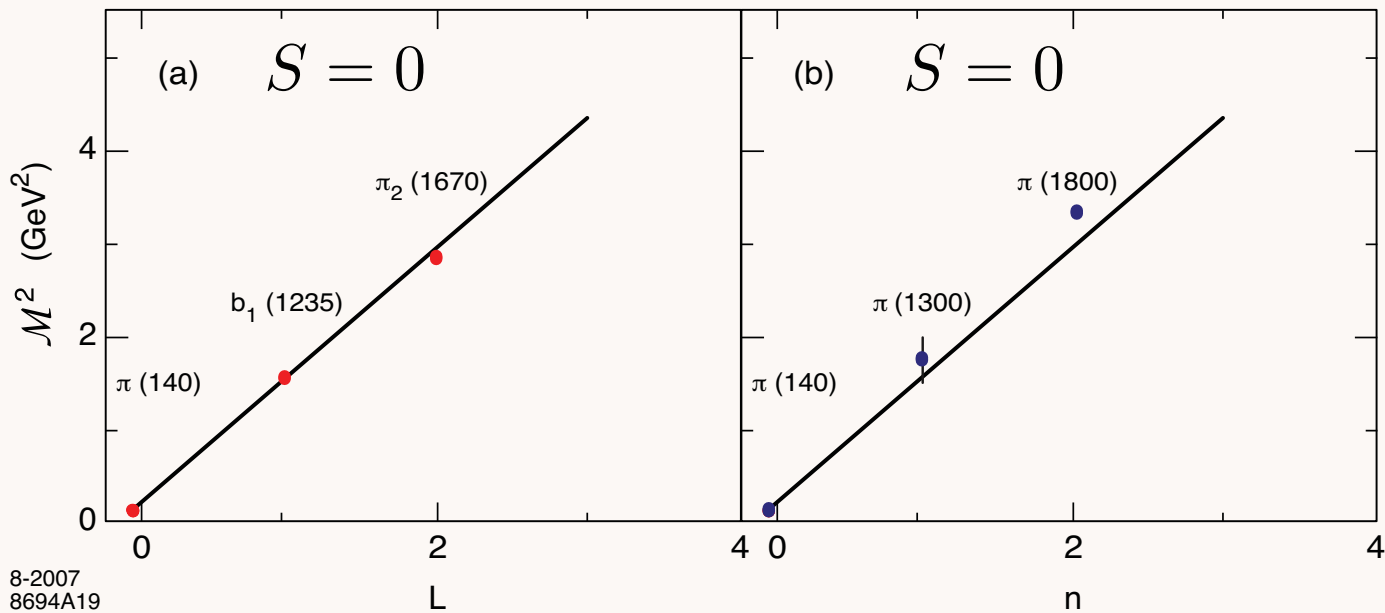


Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

*Soft Wall Model*



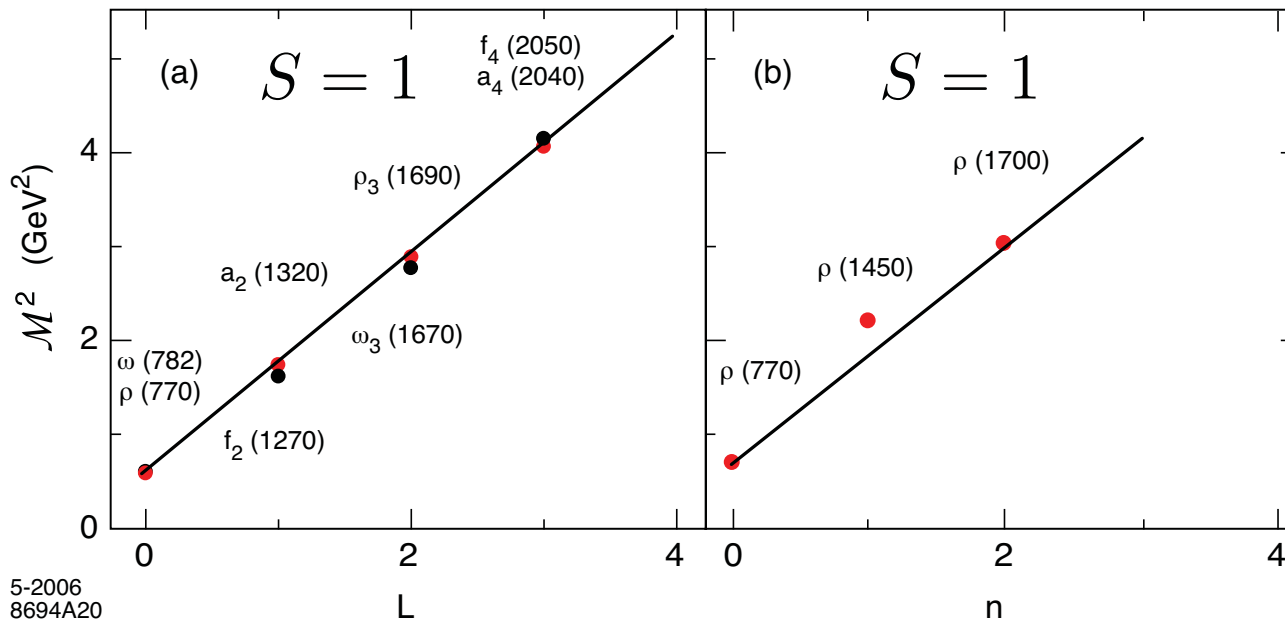
Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

- Effective LF Schrödinger wave equation

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L + S - 1) \right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)$$

with eigenvalues  $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$ . *Same slope in n and L*

- Compare with Nambu string result (rotating flux tube):  $M_n^2(L) = 2\pi\sigma(n + L + 1/2)$ .

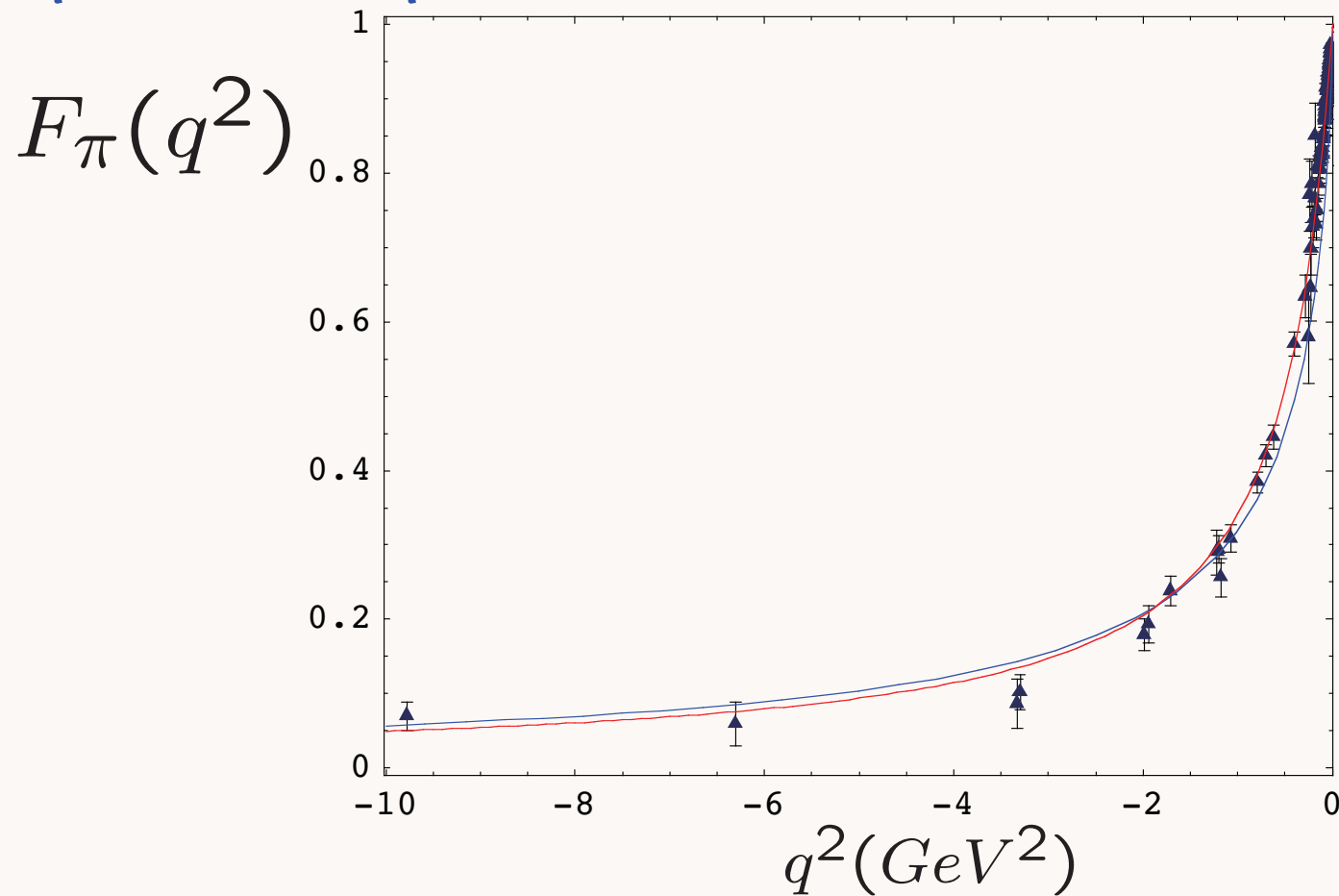


Kirchbach:  
Conformal  
symmetry

Vector mesons orbital (a) and radial (b) spectrum for  $\kappa = 0.54$  GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Fazio, Jugeau and Nicotri( 2007).

# Spacelike pion form factor from AdS/CFT



Data Compilation  
Baldini, Kloe and Volmer

— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

de Teramond, sjb  
See also: Radyushkin

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator  $\Pi$

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint  $\Pi^\dagger$ , with commutation relations

$$\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),$$

$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2).$$

- Eigenvalues

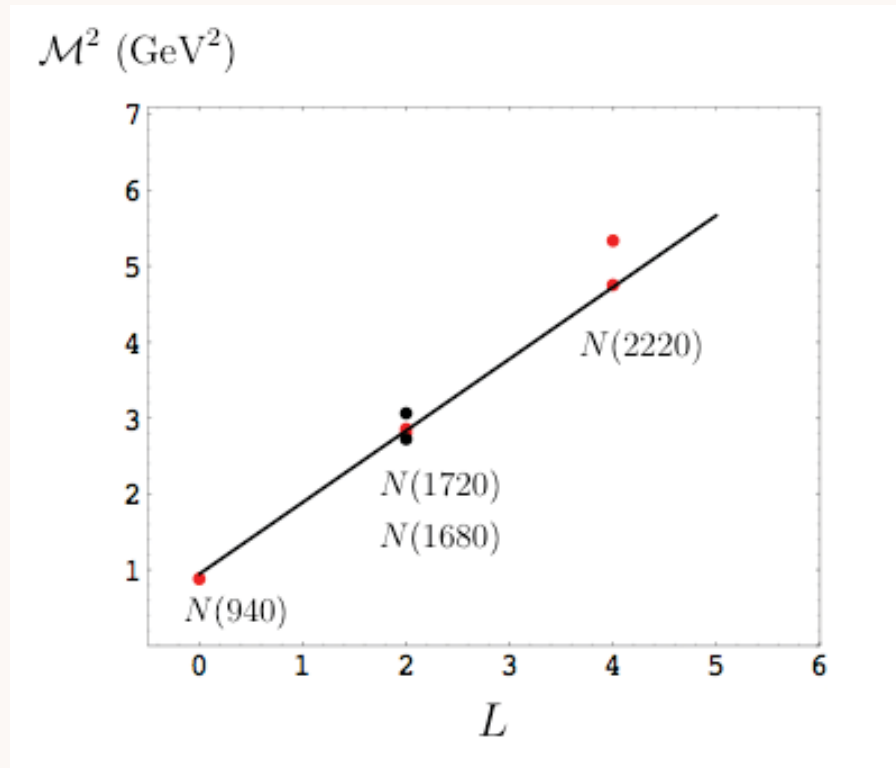
$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Kirchbach:  
Conformal  
symmetry

- Baryon: twist-dimension  $3 + L$  ( $\nu = L + 1$ )

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1).$$

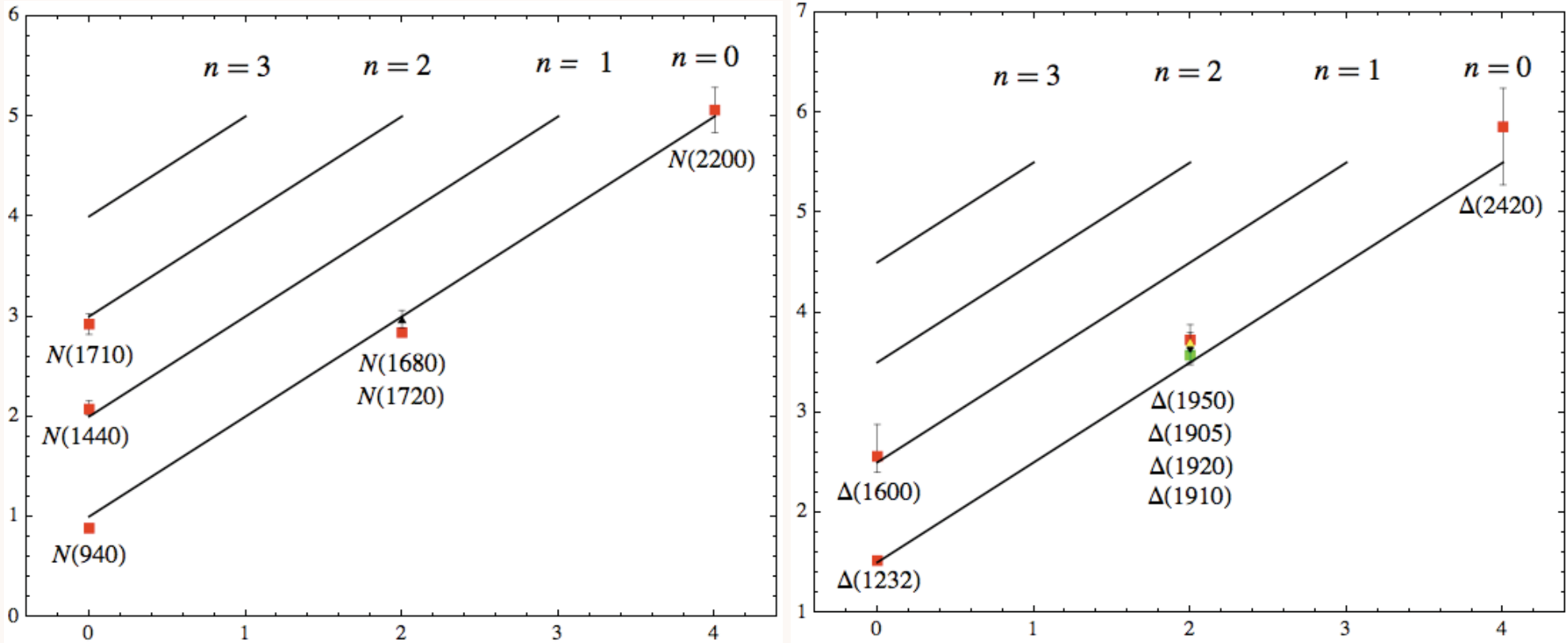


Proton Regge Trajectory  $\kappa = 0.49\text{GeV}$

- $\Delta$  spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

$4\kappa^2$  for  $\Delta n = 1$   
 $4\kappa^2$  for  $\Delta L = 1$   
 $2\kappa^2$  for  $\Delta S = 1$

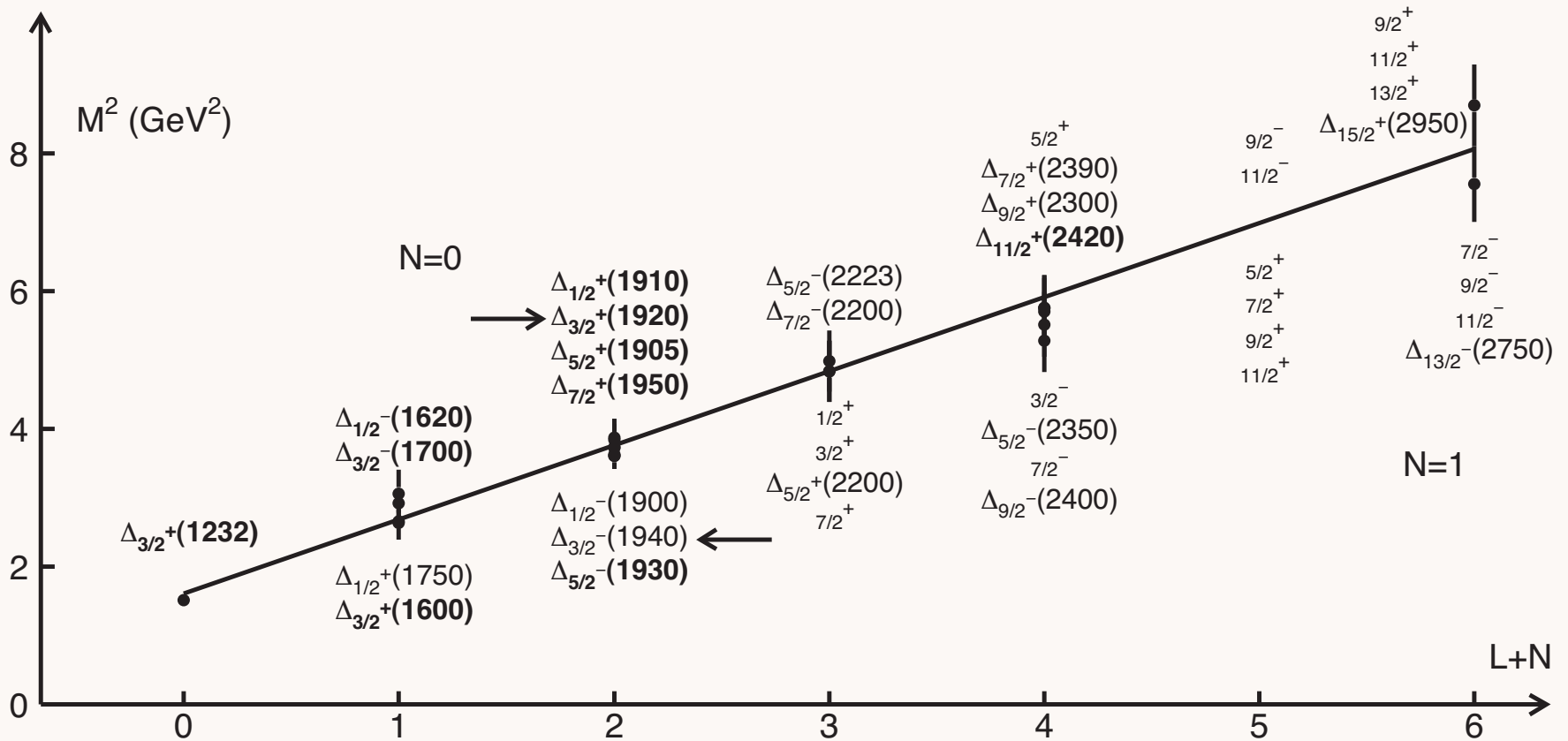
$$\mathcal{M}^2$$



$$L$$

Parent and daughter 56 Regge trajectories for the  $N$  and  $\Delta$  baryon families for  $\kappa = 0.5$  GeV





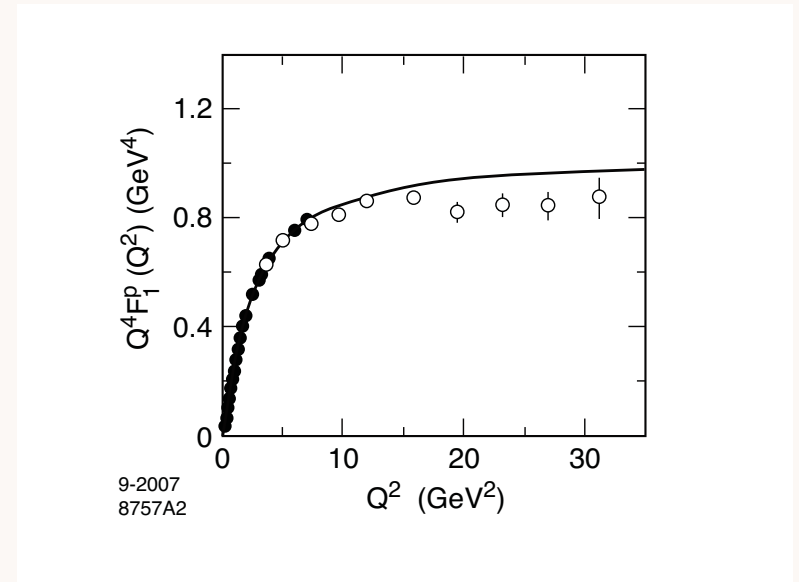
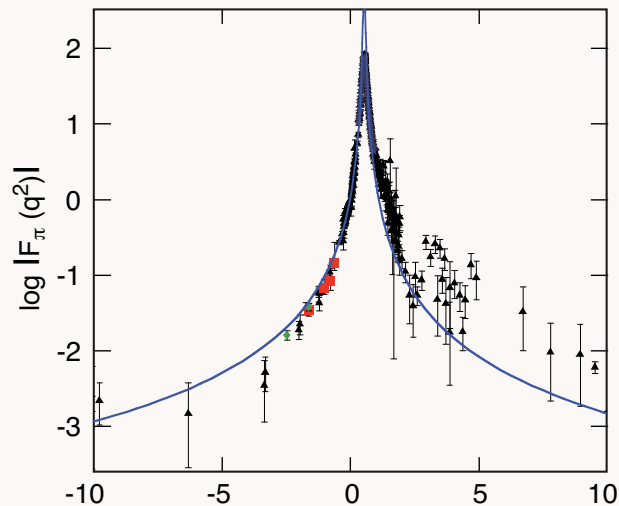
E. Klempt *et al.*:  $\Delta^*$  resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.

H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

## Other Applications of Light-Front Holography

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form factors of composite hadrons
- $n$ -parton holographic mapping
- Heavy flavor mesons



hep-th/0501022  
hep-ph/0602252  
arXiv:0707.3859  
arXiv:0802.0514  
arXiv:0804.0452

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory.

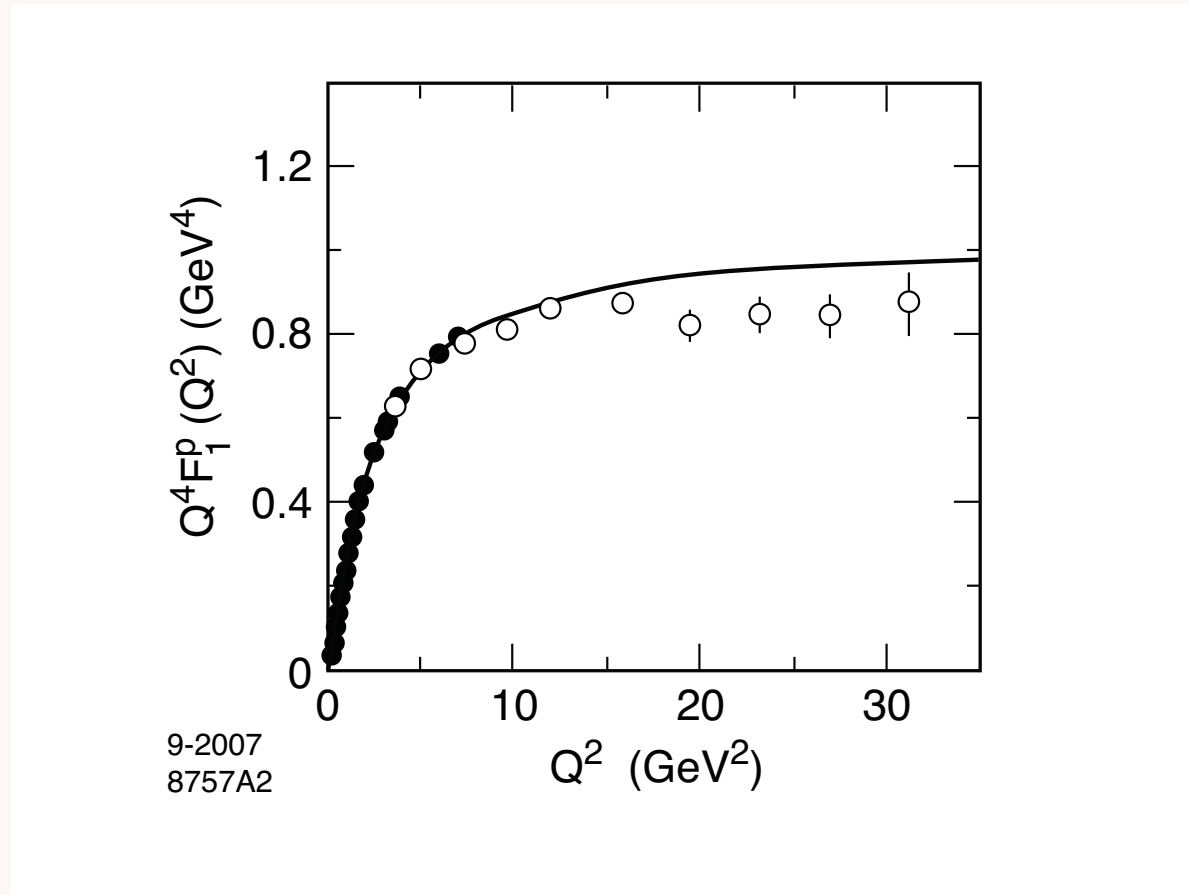
- Choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\psi_+(\zeta)$  and  $\psi_-(\zeta)$  correspond to nucleons with  $J^z = +1/2$  and  $-1/2$ .
- For  $SU(6)$  spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

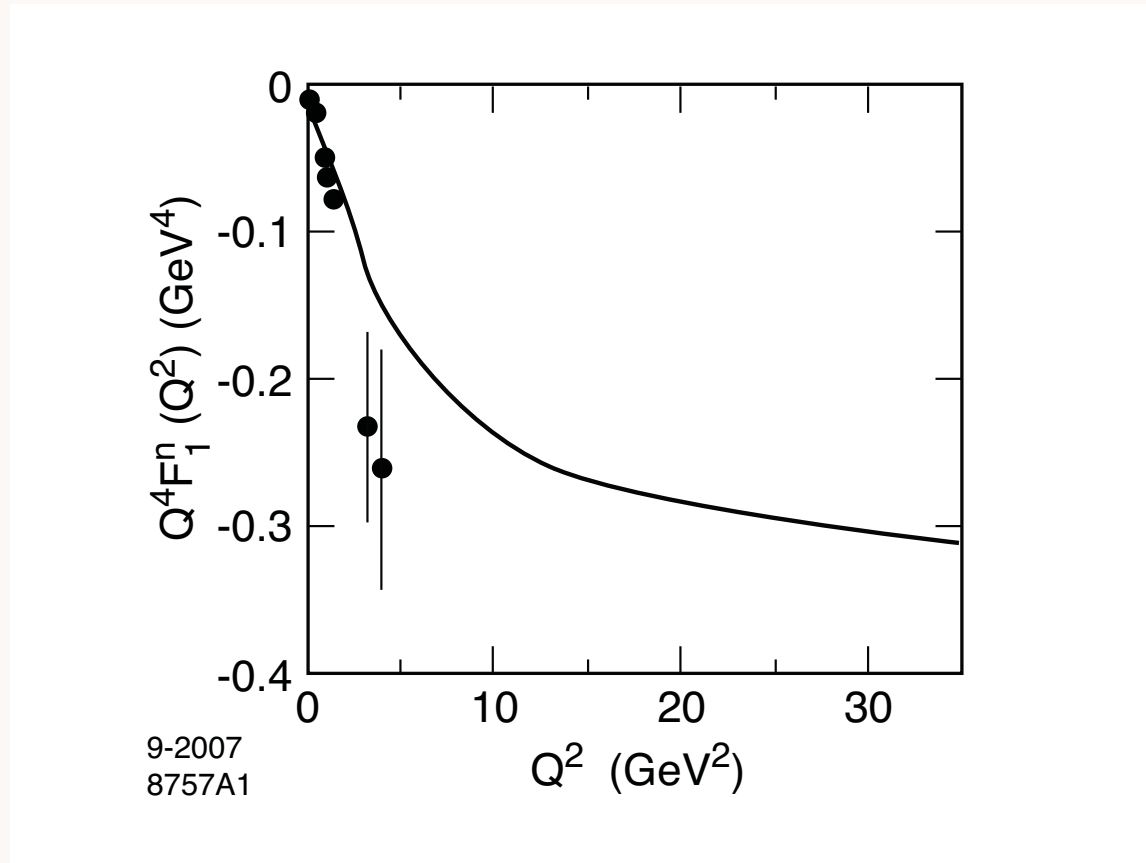
where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ .

- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  Proton  $\tau = 3$



SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$  Neutron  $\tau = 3$

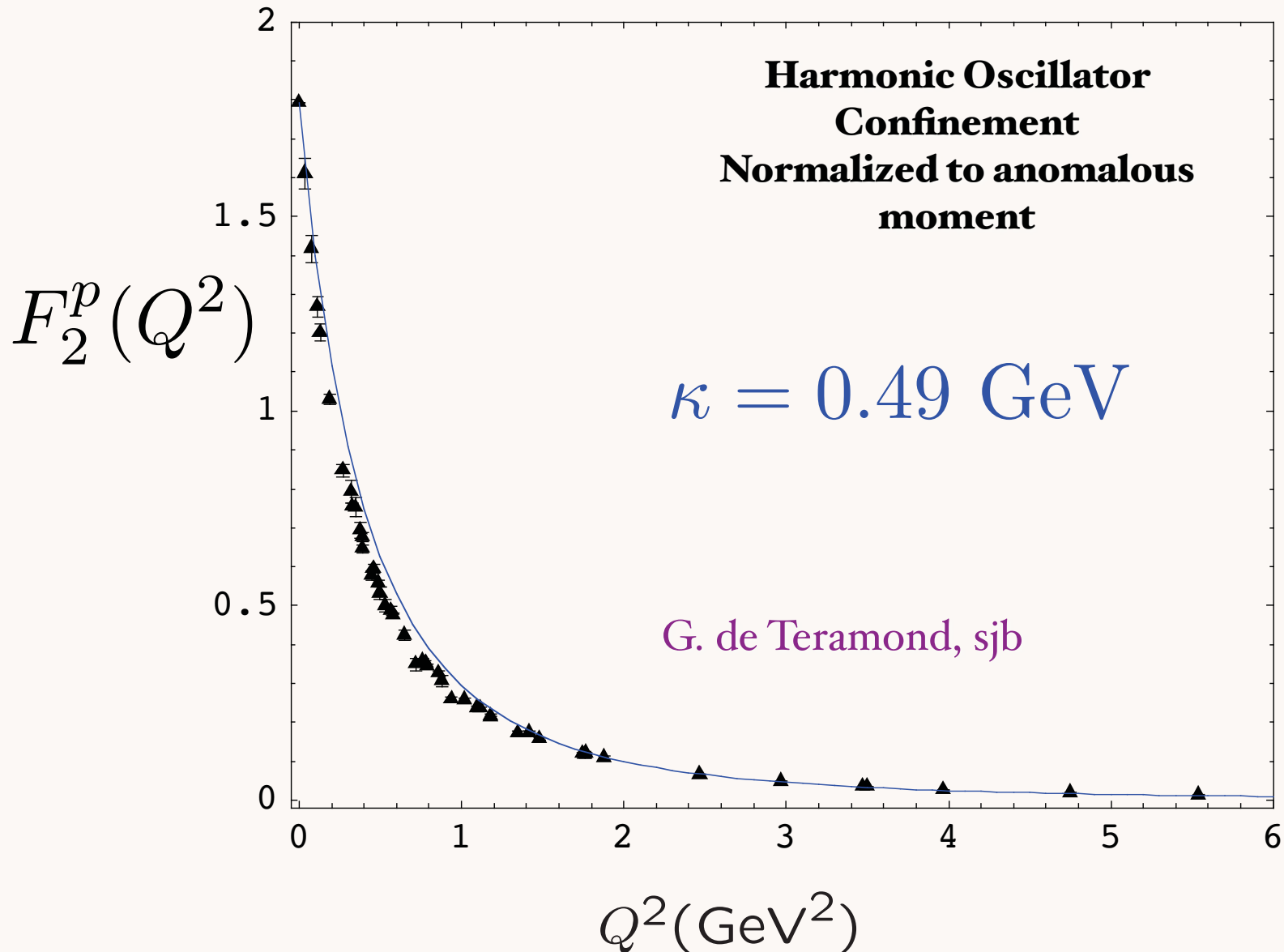


SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

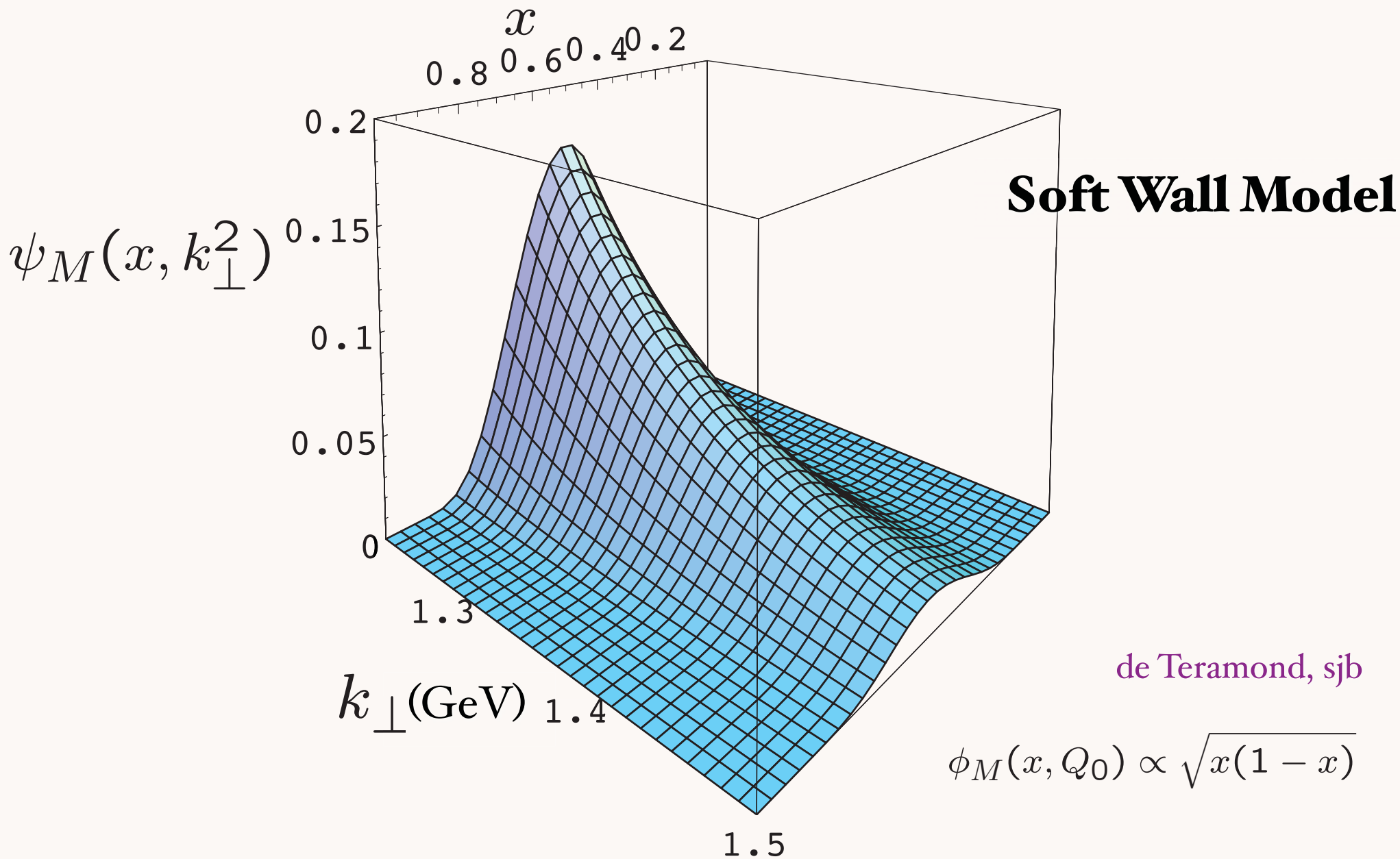
# Spacelike Pauli Form Factor

Preliminary

From overlap of  $L = 1$  and  $L = 0$  LFWFs



# Prediction from AdS/CFT: Meson LFWF



de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD prediction for  $F_{\pi}(Q^2)$  by 16/9

# Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

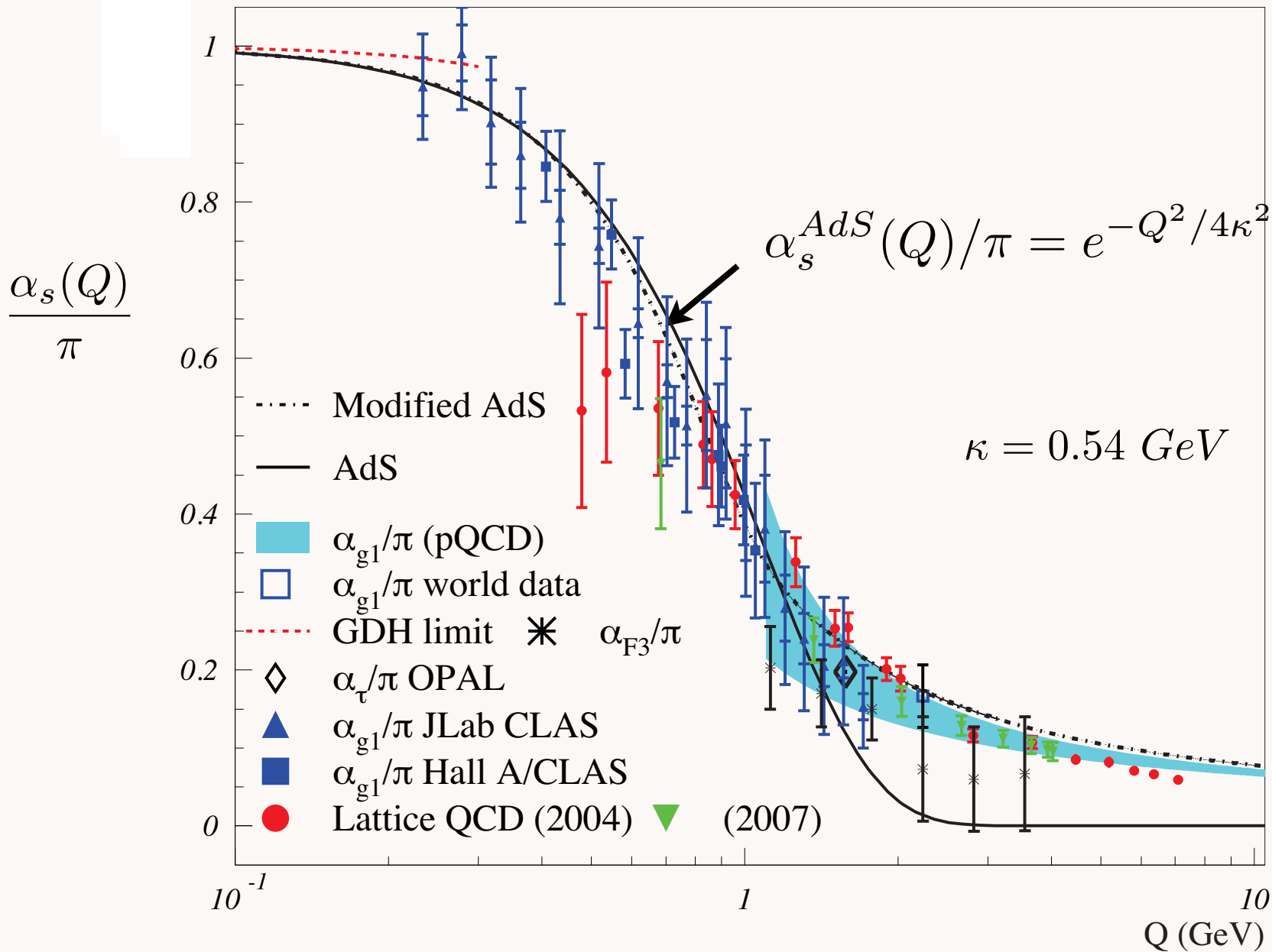
Braun et al.



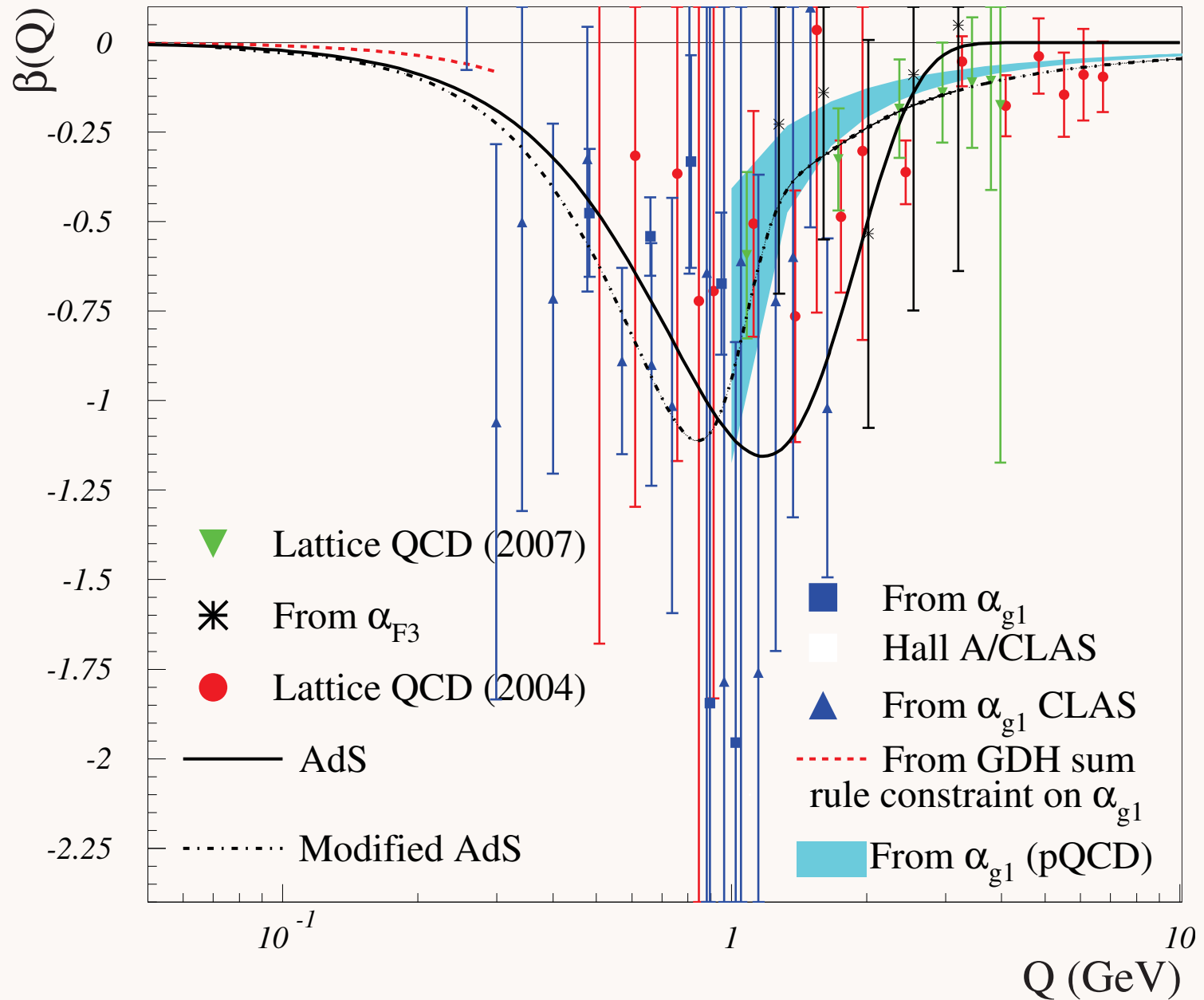
# *Features of AdS/QCD LF Holography*

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS<sub>5</sub>**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite  $N_c = 3$ : Baryons built on 3 quarks -- Large  $N_c$  limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

Analytic, defined at all scales, IR Fixed Point

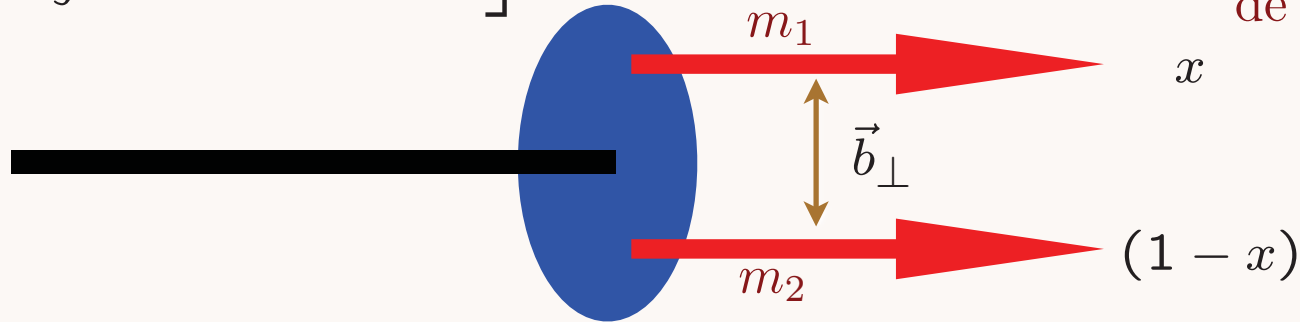


$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

*Holographic Variable*

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

*LF Kinetic Energy in momentum space*

*Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared*

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

## Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa \sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2} \left( \frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right)}$$

*LF WF in impact space: soft-wall model with massive quarks*

$$\psi(x, \mathbf{b}_\perp) = \frac{c \kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2} \kappa^2 x(1-x) \mathbf{b}_\perp^2 - \frac{1}{2\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]}$$

$$z \rightarrow \zeta \rightarrow \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]$$

$J/\psi$  $\psi_{J/\psi}(x, b)$ 

*LFWF peaks at*

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

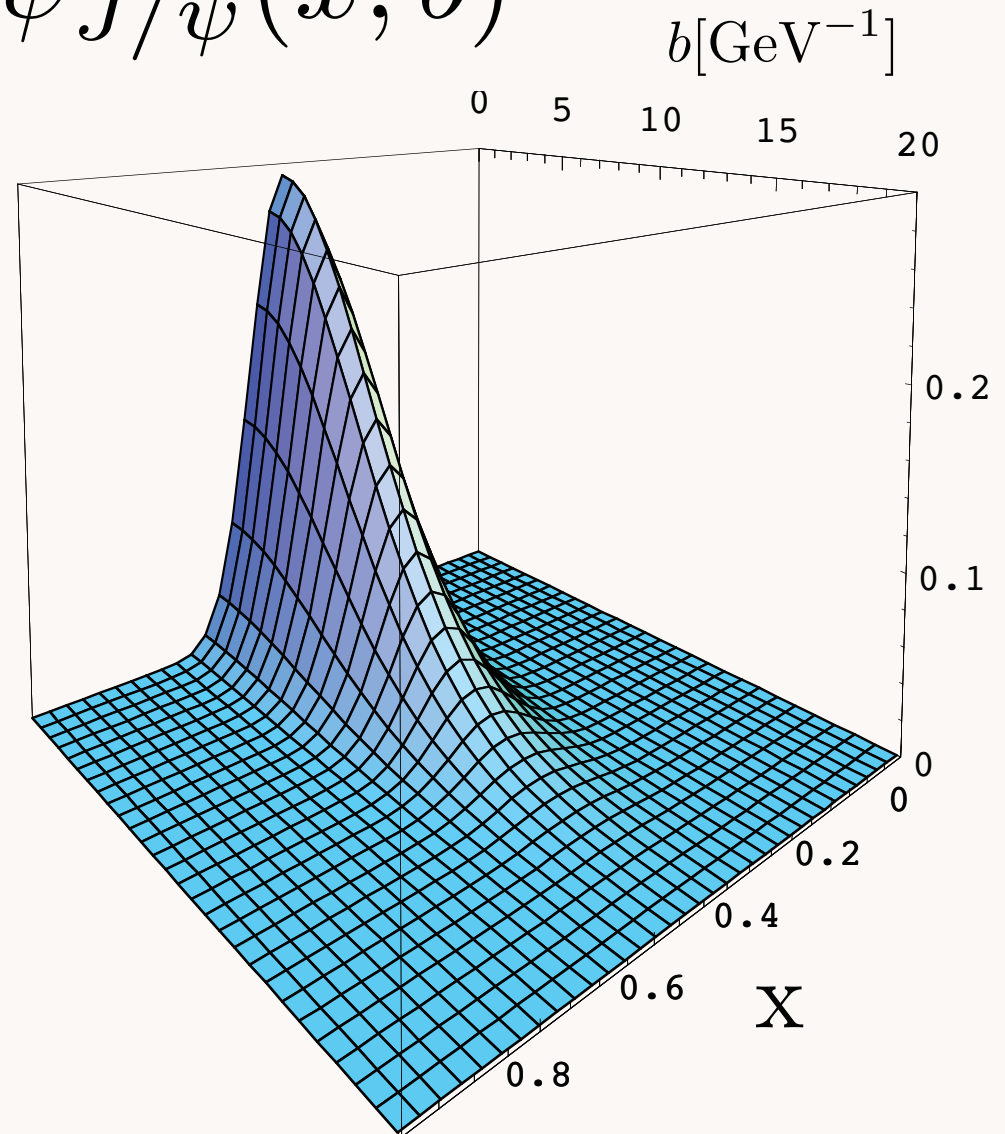
where

$$m_{\perp i} = \sqrt{m^2 + k_{\perp}^2}$$

*minimum of LF  
energy  
denominator*

$$\kappa = 0.375 \text{ GeV}$$

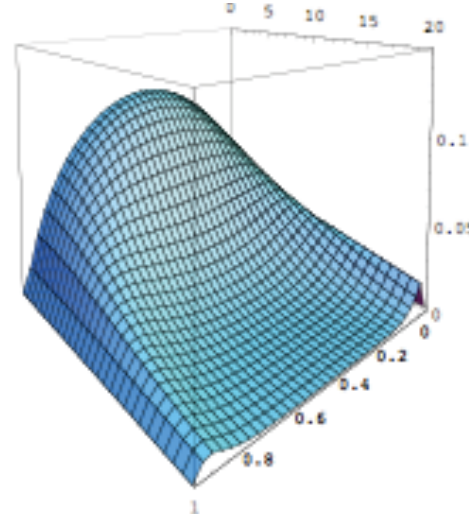
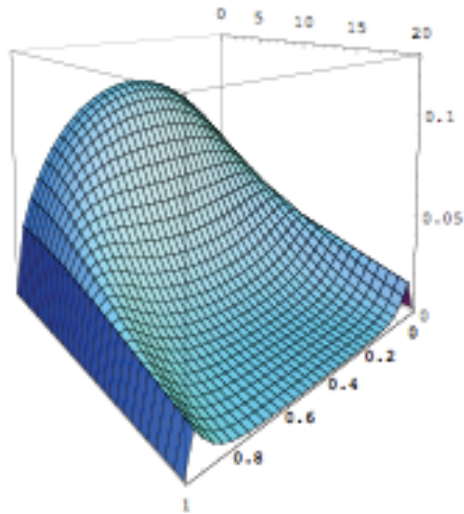
$$m_a = m_b = 1.25 \text{ GeV}$$



$$|\pi^+\rangle = |u\bar{d}\rangle$$

$$m_u = 2 \text{ MeV}$$

$$m_d = 5 \text{ MeV}$$

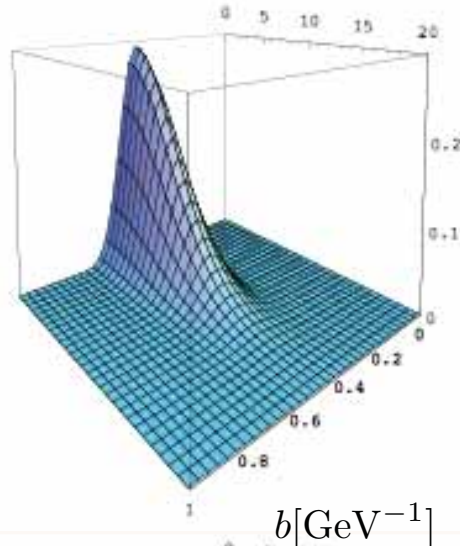
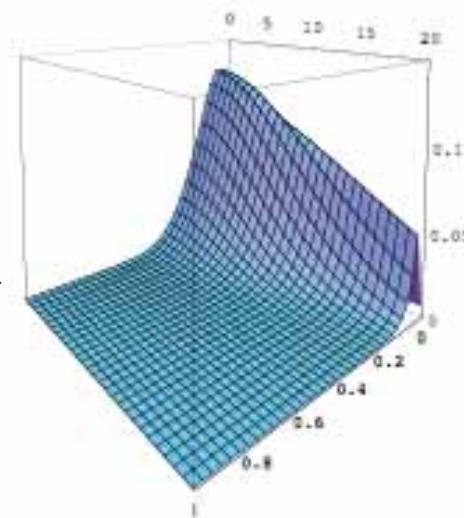


$$|K^+\rangle = |u\bar{s}\rangle$$

$$m_s = 95 \text{ MeV}$$

$$|D^+\rangle = |c\bar{d}\rangle$$

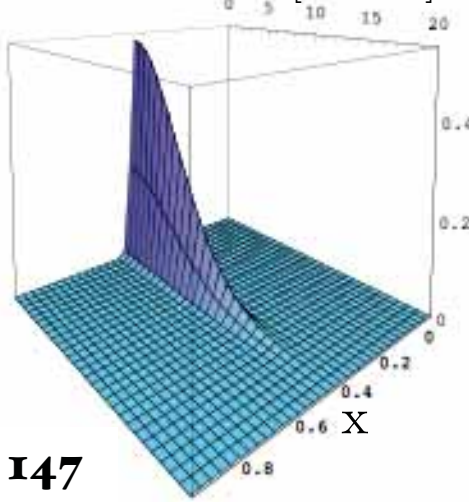
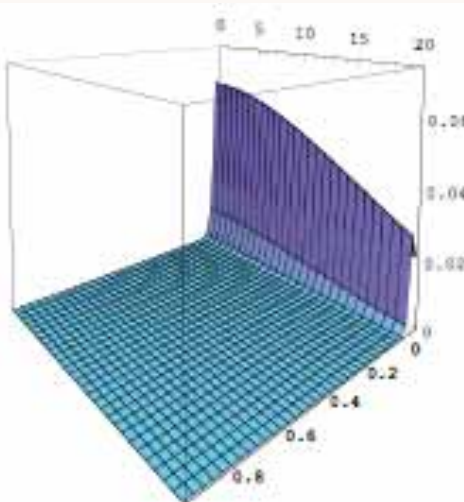
$$m_c = 1.25 \text{ GeV}$$



$$|\eta_c\rangle = |c\bar{c}\rangle$$

$$|B^+\rangle = |u\bar{b}\rangle$$

$$m_b = 4.2 \text{ GeV}$$



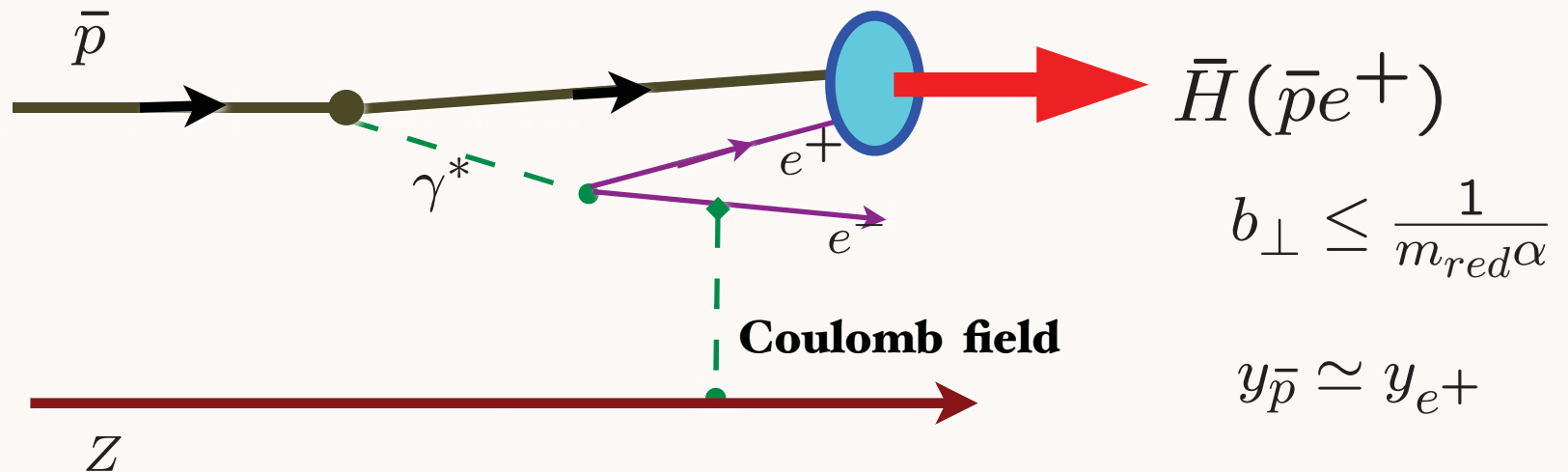
$$|\eta_b\rangle = |b\bar{b}\rangle$$

$$\kappa = 375 \text{ MeV}$$

# Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



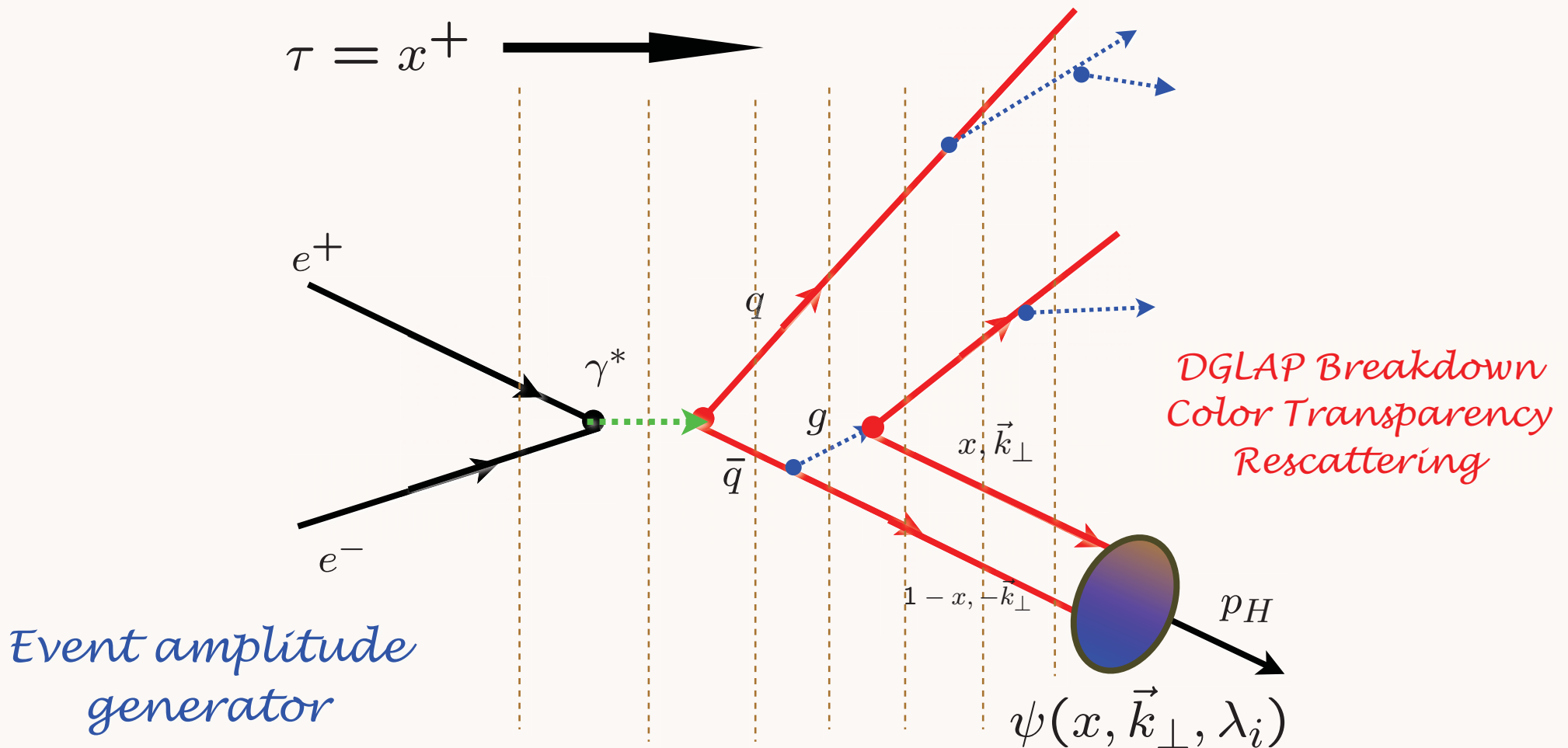
*Coalescence of off-shell co-moving positron and antiproton.*

*Wavefunction maximal at small impact separation and equal rapidity*

*“Hadronization” at the Amplitude Level*

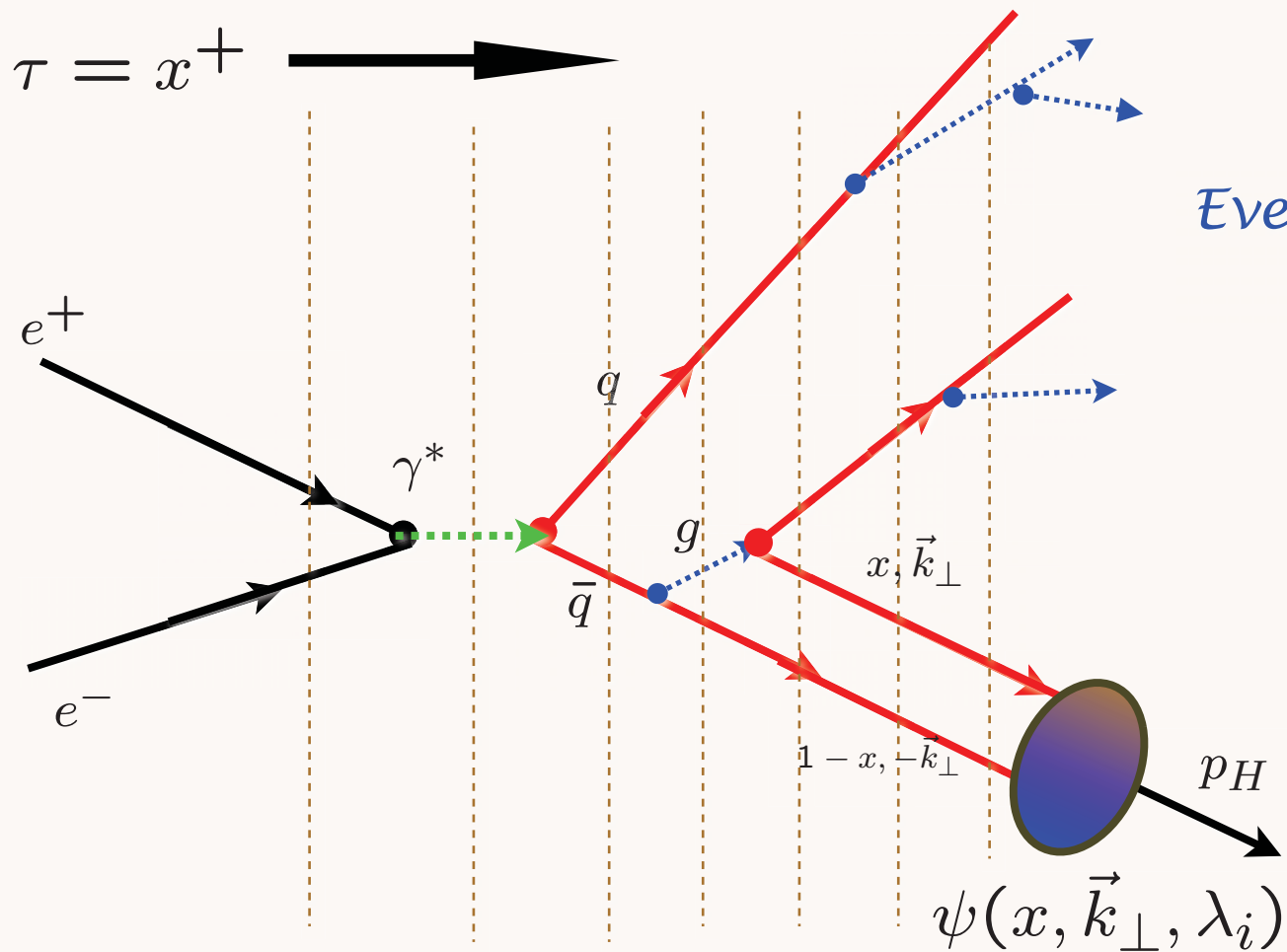


# Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Hadronization at the Amplitude Level

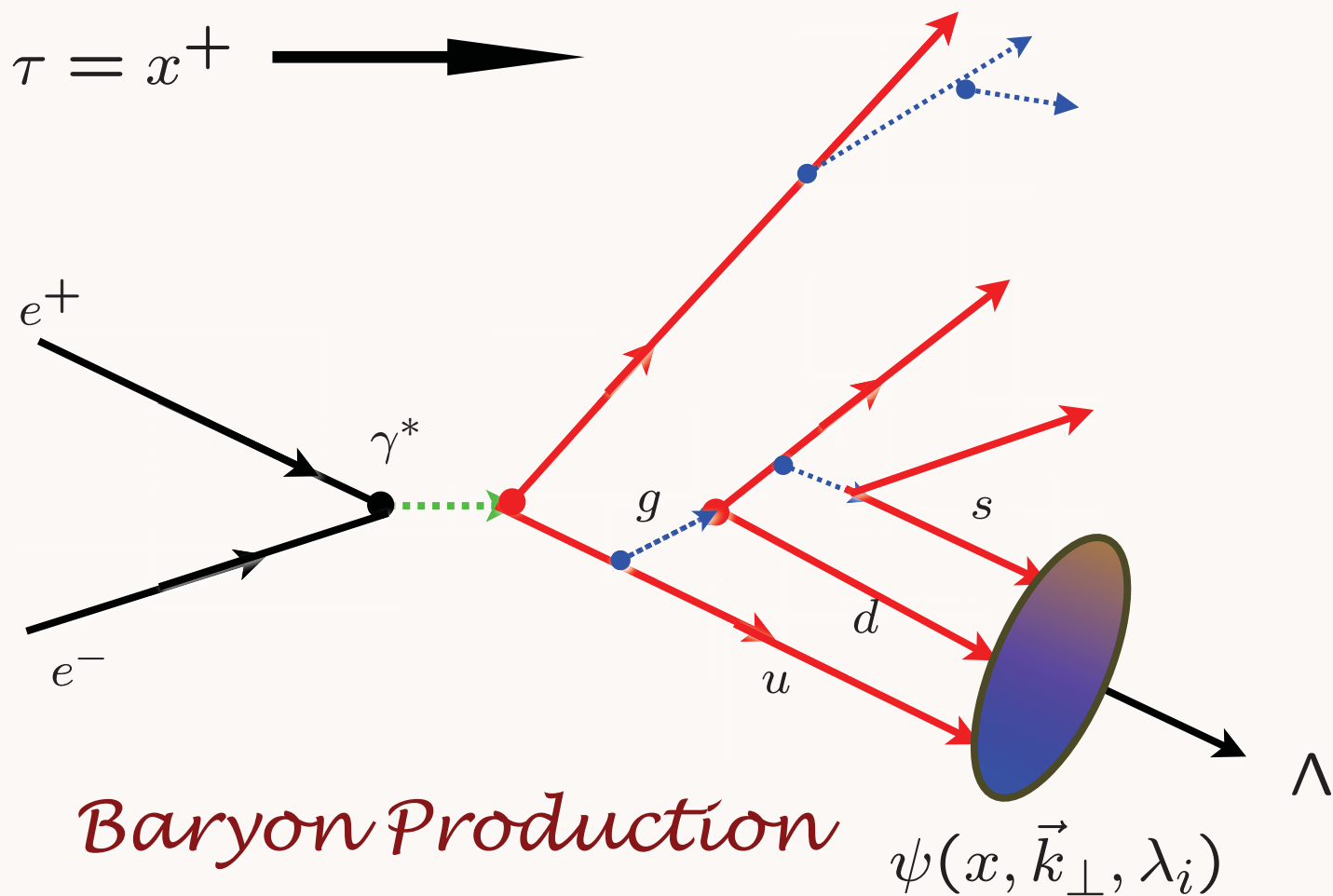


*AdS/QCD*  
*Hard Wall*  
*Confinement:*

Capture if  $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda_{QCD}^2}$   
 i.e.,  

$$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda_{QCD}^2$$

# Hadronization at the Amplitude Level

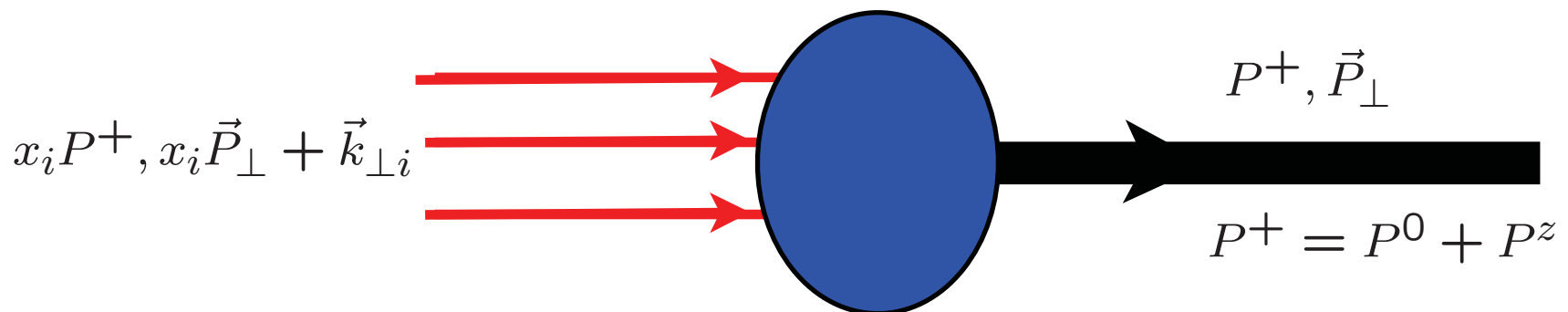


Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

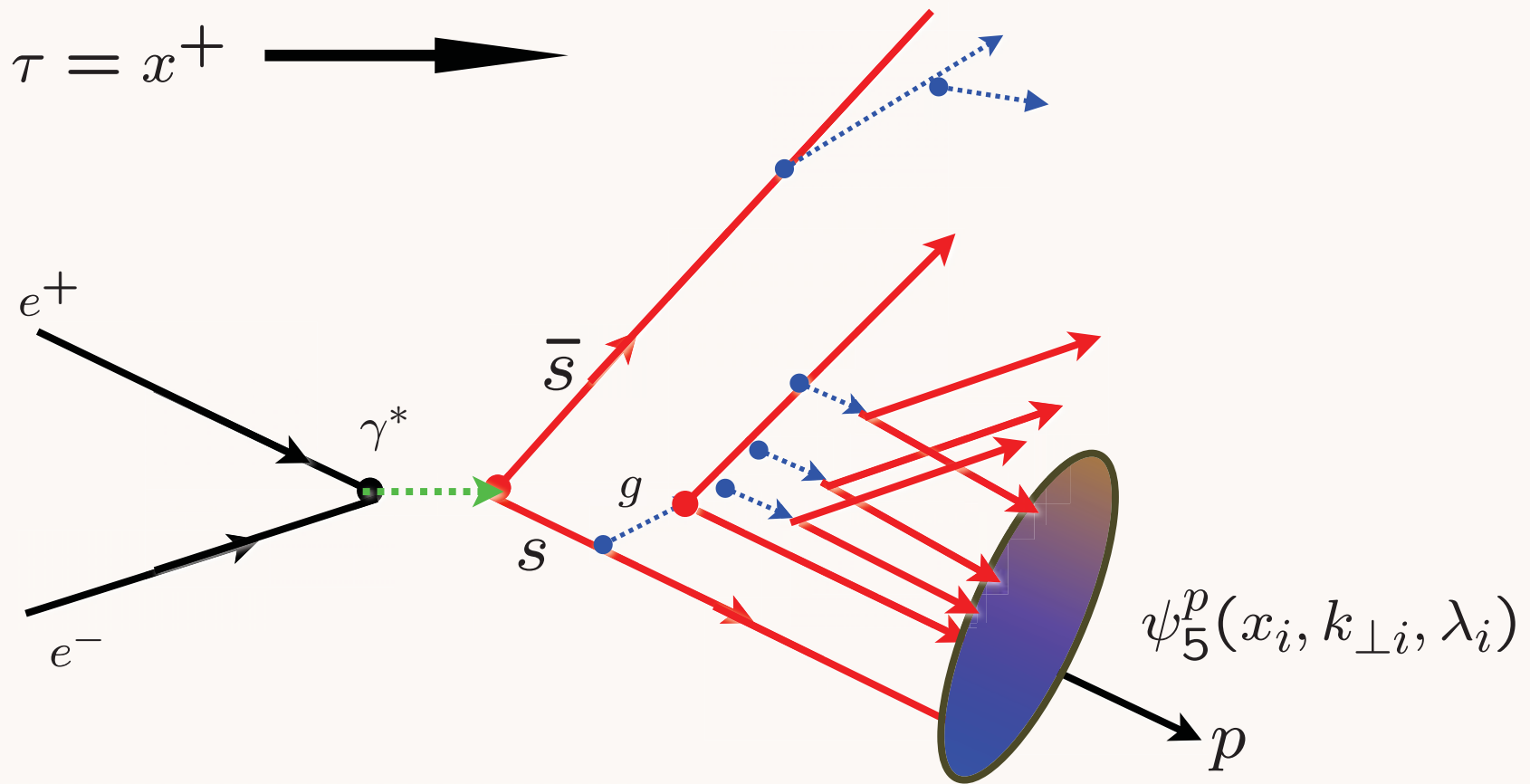
# Features of LF T-Matrix Formalism

## “Event Amplitude Generator”

- **Same principle as antihydrogen production: off-shell coalescence**
- **coalescence to hadron favored at equal rapidity, small transverse momenta**
- **leading heavy hadron production: D and B mesons produced at large z**
- **hadron helicity conservation if hadron LFWF has  $L^z = 0$**
- **Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin**



# Hadronization at the Amplitude Level

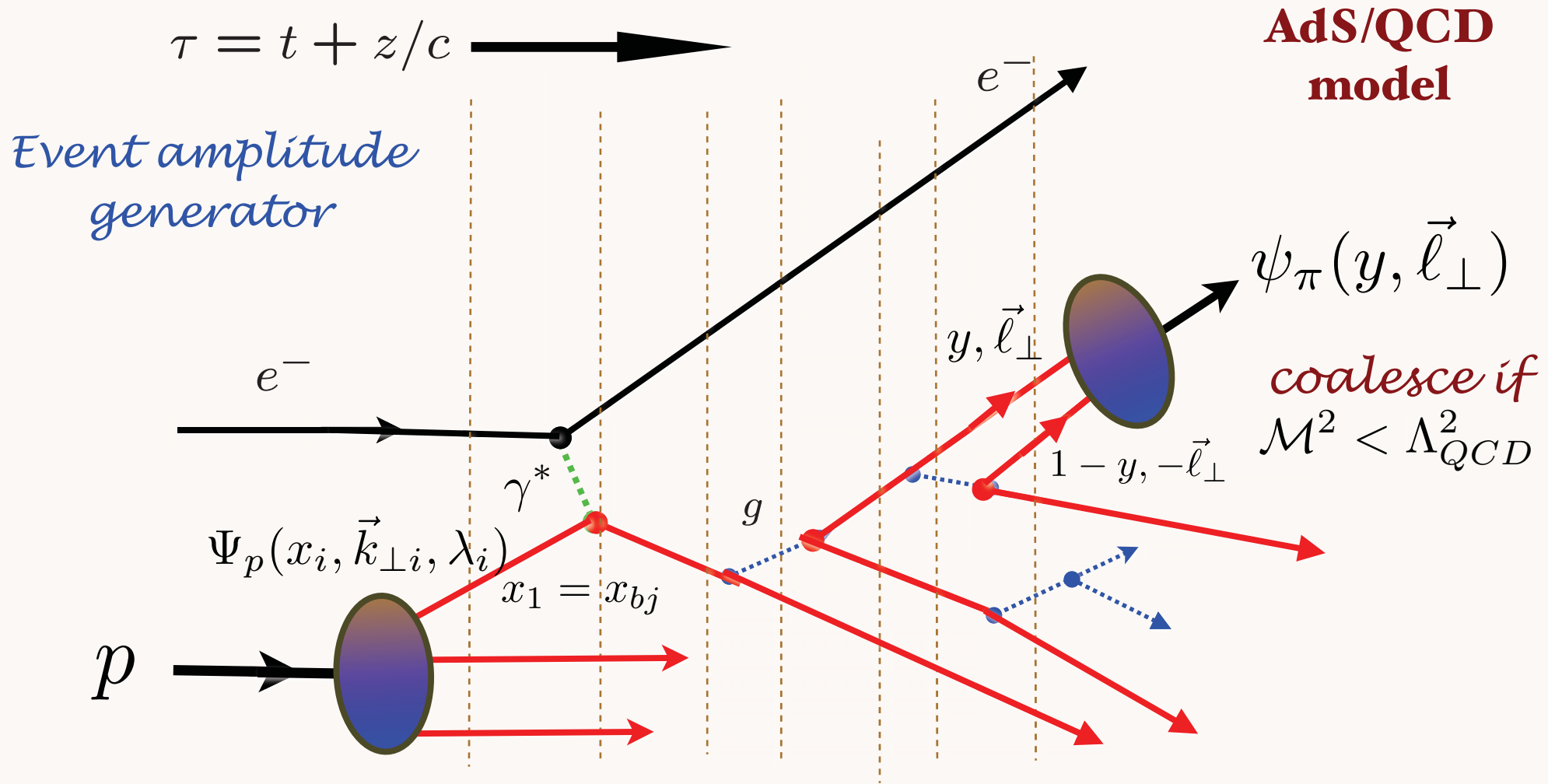


**Higher Fock State Coalescence**  $|uuds\bar{s}\rangle$

**Asymmetric Hadronization!**  $D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z)$

B-Q Ma, sjb

# Jet Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via Light-Front Wavefunctions**

# Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion ( $m_q = 0$ )
- Regge Trajectories: universal slope in  $n$  and  $L$
- Valid for all integer  $J$  &  $S$ .
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large  $N_c$  limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize  $H_{LF}$  on AdS basis

# Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different  $L^z$**
- **Proton: equal probability**  $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$   
 $J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z = 0 \rangle$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at  $z=0$ .**



# *An analytic first approximation to QCD*

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable  $\zeta$  conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter  $\kappa$**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

*“One of the gravest puzzles of  
theoretical physics”*

DARK ENERGY AND  
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA  
Kavil Institute for Theoretical Physics, University of California,  
Santa Barbara, CA 93106, USA  
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

*QCD Problem Solved if Quark and Gluon condensates reside  
within hadrons, not vacuum!*

**R. Shrock, sjb**

arXiv:0905.1151 [hep- th], Proc. Nat'l. Acad. Sci., (in press);

“Condensates in Quantum Chromodynamics and the Cosmological Constant”

## Chiral magnetism (or magnetohydrochironics)

Aharon Casher and Leonard Susskind

*Tel Aviv University Ramat Aviv, Tel-Aviv, Israel*

(Received 20 March 1973)

### I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.<sup>1</sup> Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.<sup>2</sup> A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.<sup>3</sup>

*Light-Front  
Formalism*

PHYSICAL REVIEW C **82**, 022201(R) (2010)

## New perspectives on the quark condensate

Stanley J. Brodsky,<sup>1,2</sup> Craig D. Roberts,<sup>3,4</sup> Robert Shrock,<sup>5</sup> and Peter C. Tandy<sup>6</sup>

<sup>1</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

<sup>2</sup>*Centre for Particle Physics Phenomenology: CP<sup>3</sup>-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

<sup>3</sup>*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>4</sup>*Department of Physics, Peking University, Beijing 100871, China*

<sup>5</sup>*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

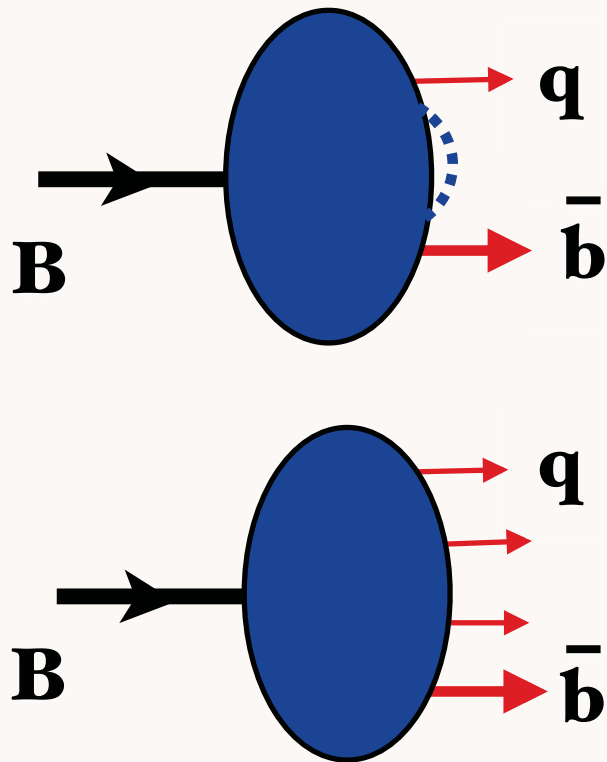
<sup>6</sup>*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

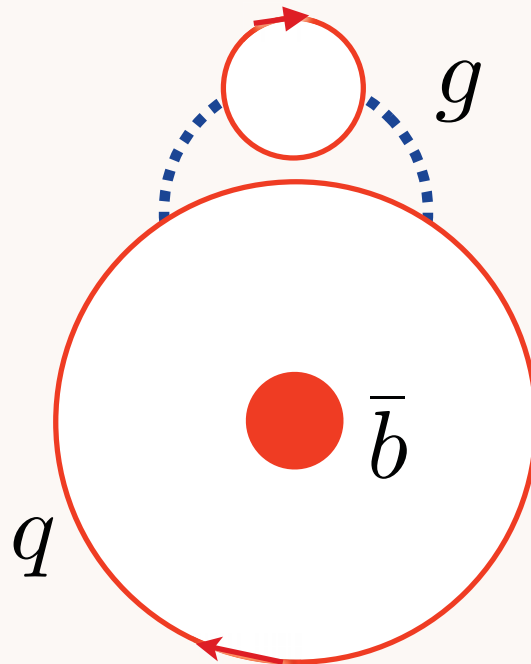
We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

*Simple physical argument  
for "in-hadron" condensate*

Roberts, Shrock, Tandy, sjb



**Gribov pairs**



*B-Meson*

*Use Dyson-Schwinger Equation for bound-state quark propagator:  
find confined condensate*

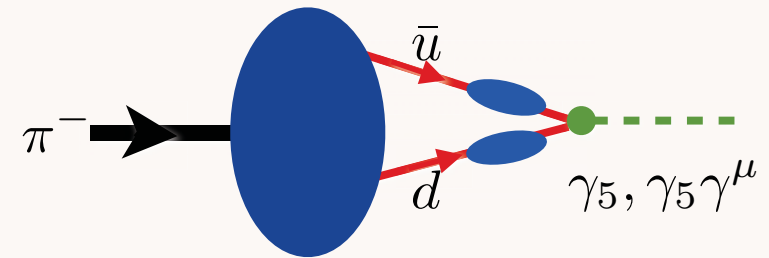
$$\langle B | \bar{q}q | B \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle$$

# Bethe-Salpeter Analysis

Maris,  
Roberts, Tandy

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

$f_H$  Meson Decay Constant  
 $T_H$  flavor projection operator,  
 $Z_2(\Lambda)$ ,  $Z_4(\Lambda)$  renormalization constants  
 $\mathcal{S}(p)$  dressed quark propagator  
 $\Gamma_H(q; P) = F.T. \langle H | \psi(x_a) \bar{\psi}(x_b) | 0 \rangle$   
 Bethe-Salpeter bound-state vertex amplitude.



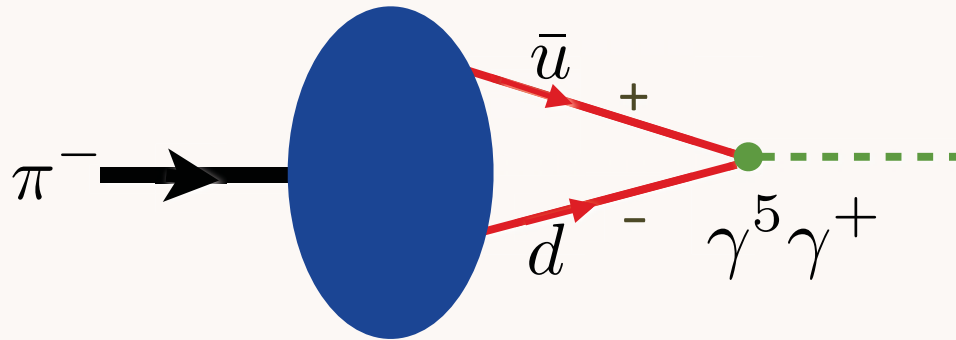
$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q} \rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

*In-Hadron Condensate!*

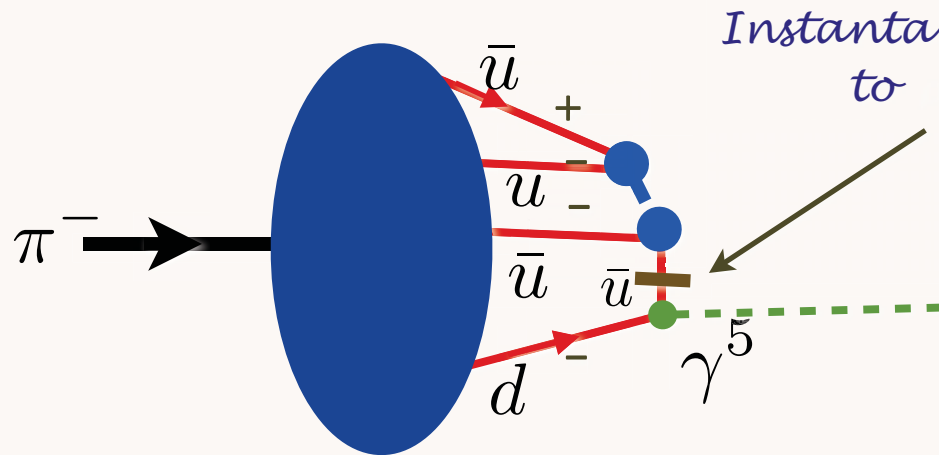
$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q$$

$$m_\pi^2 \propto (m_q + m_{\bar{q}}) / f_\pi \quad \text{GMOR}$$

# Higher Light-Front Fock State of Pion Simulates DCSB

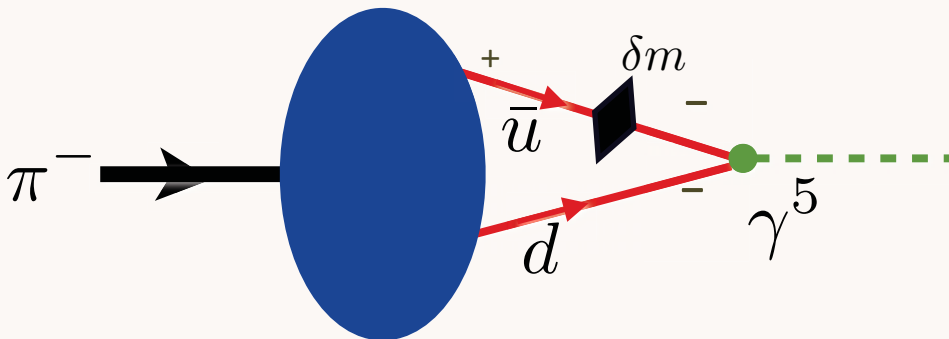


$$f_\pi P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle$$



*Instantaneous quark propagator contribution to  $\pi$  derived from higher Fock state*

$$i\rho_\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$



*Higher Fock state acts like mass insertion*

# Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

$-0.005 \pm 0.003$  from  $\tau$  decay.

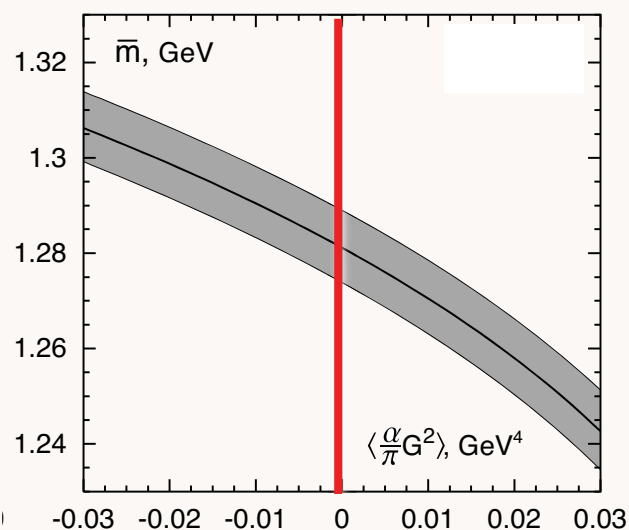
Davier et al.

$+0.006 \pm 0.012$  from  $\tau$  decay.

Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$  from charmonium sum rules

Ioffe, Zyablyuk



*Consistent with zero  
vacuum condensate*



*Quark and Gluon condensates reside  
within hadrons, not vacuum*

Casher and Susskind

Maris, Roberts, Tandy

Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant --  
Eliminates 45 orders of magnitude conflict**

**R. Shrock, sjb**

**ArXiv:0905.1151**

# “One of the gravest puzzles of theoretical physics”

## DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA  
Kavil Institute for Theoretical Physics, University of California,  
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“Condensates in Quantum Chromodynamics and the Cosmological Constant”

*Quark and Gluon condensates reside within  
hadrons, not LF vacuum*

**Maris, Roberts,  
Tandy**

**Casher  
Susskind**

- **Bound-State Dyson-Schwinger Equations**
- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**
- **Finite size phase transition - infinite # Fock constituents**
- **AdS/QCD Description -- CSB is in-hadron Effect**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant**

**Shrock, sjb**

*“Confined QCD Condensates”*

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, BFKL scale)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

**Roberts, Shrock, Tandy, and sjb**

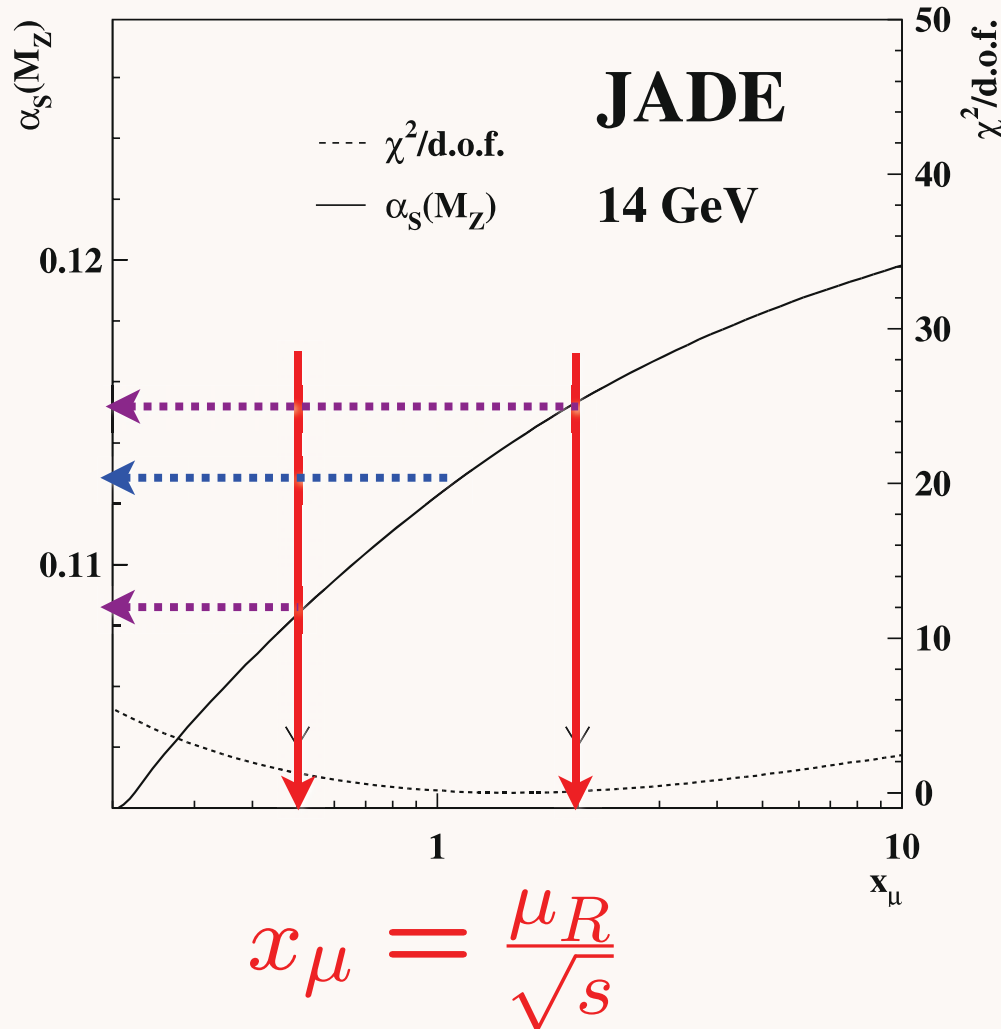
# Pervasive Myth in PQCD

- *Renormalization Scale is Arbitrary*

# Measurement of the strong coupling $\alpha_S$ from the four-jet rate in $e^+e^-$ annihilation using JADE data

J. Schieck<sup>1,a</sup>, S. Bethke<sup>1</sup>, O. Biebel<sup>2</sup>, S. Kluth<sup>1</sup>, P.A.M. Fernández<sup>3</sup>, C. Pahl<sup>1</sup>,  
The JADE Collaboration<sup>b</sup>

Eur. Phys. J. C 48, 3–13 (2006)



The theoretical uncertainty, associated with missing higher order terms in the theoretical prediction, is assessed by varying the renormalization scale factor  $x_\mu$ . The predictions of a complete QCD calculation would be independent of  $x_\mu$ , but a finite-order calculation such as that used here retains some dependence on  $x_\mu$ . The renormalization scale factor  $x_\mu$  is set to 0.5 and two. The larger deviation from the default value of  $\alpha_S$  is taken as systematic uncertainty.

$\alpha_S(M_{Z0})$  and the  $\chi^2/\text{d.o.f.}$  of the fit to the four-jet rate as a function of the renormalization scale  $x_\mu$  for  $\sqrt{s} = 14$  GeV to 43.8 GeV. The arrows indicate the variation of the renormalization scale factor used for the determination of the systematic uncertainties

*PMS & FAC inapplicable*

## *Conventional wisdom concerning scale setting*

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess
- with an arbitrary range
- Factorization scale should be taken equal to renormalization scale

$$\mu_R = Q$$

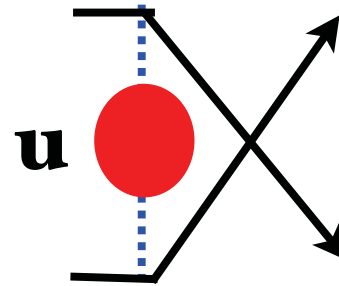
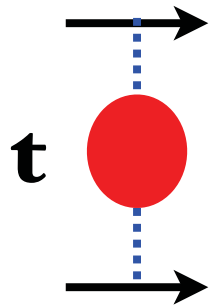
$$Q/2 < \mu_R < 2Q$$

$$\mu_F = \mu_R$$

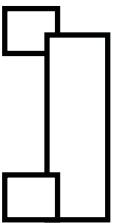
*These assumptions are untrue in QED  
and thus they cannot be true for QCD!*

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$



$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$



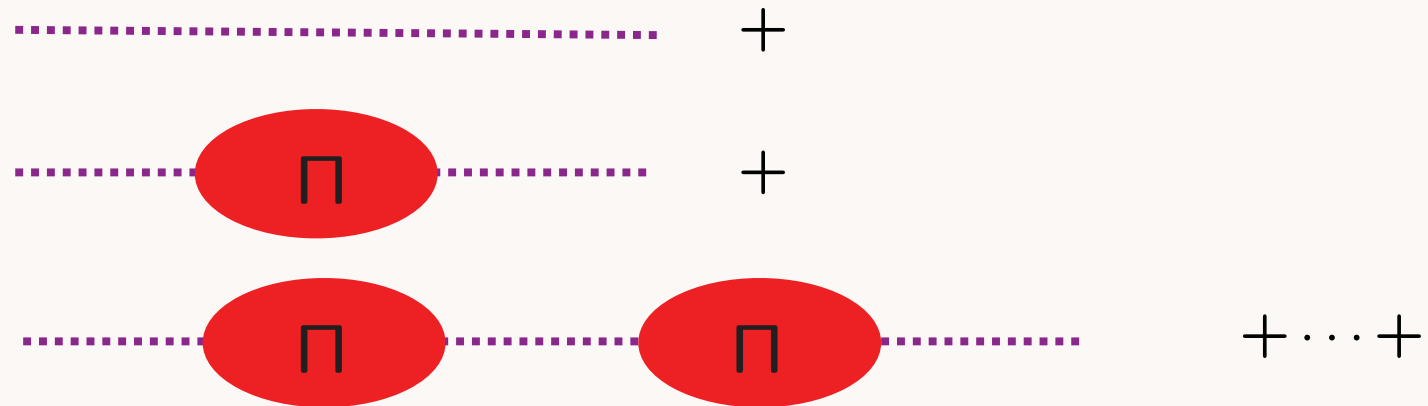
## Gell Mann-Low Effective Charge



# QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

*All-orders lepton loop corrections to dressed photon propagator*



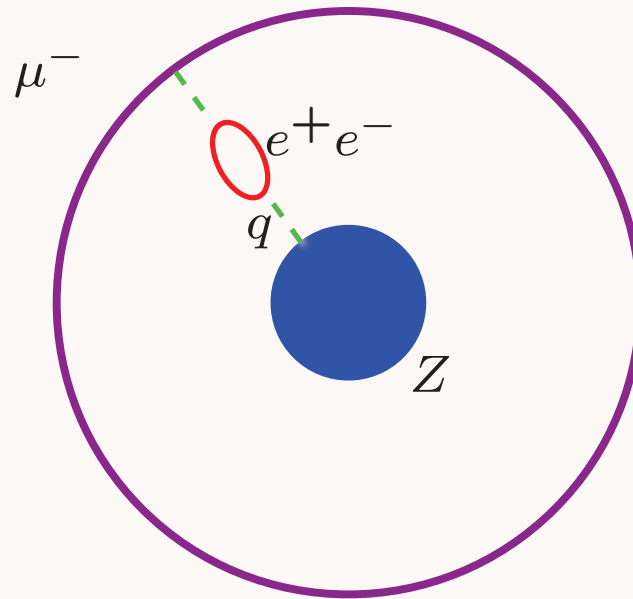
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)}$$

$$\Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

*Initial scale  $t_0$  is arbitrary -- Variation gives RGE Equations*

*Physical renormalization scale  $t$  not arbitrary*

## Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

**Scale is unique: Tested to ppm**

Gyulassy: Higher Order VP verified to 0.1% precision in  $\mu$  Pb

Must recover QED result using  $\alpha_S^{\overline{MS}}(\mu^2)$

$$\alpha(q^2) = \alpha(q_0^2) \frac{(1 - \Pi(q_0^2))}{(1 - \Pi(q^2))} \quad \text{where } \Pi(q^2 = 0) = 0$$

$$\Pi(q^2) = \text{.....} \bigcirc \text{.....}$$

Identical QED result if

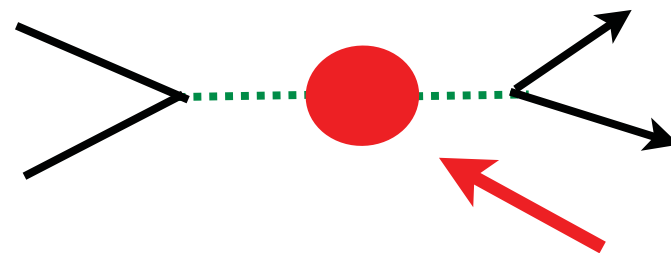
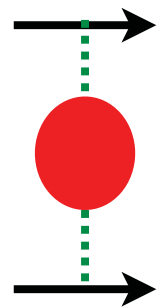
$$\ln\left(-\frac{\mu^2}{m^2}\right) = 6 \int_0^1 d\alpha [\alpha(1 - \alpha)] \ln\left(1 - \frac{q_0^2 \alpha(1 - \alpha)}{m^2}\right)$$

**Dae Sung Hwang, sjb**  $\mu^2 = q_0^2 e^{-5/3}$  at large  $q_0^2$

**$q_0^2$ : Normalization point**

# Electron-Positron Scattering in QED

$$M_{e^+e^- \rightarrow e^+e^-}(s, t) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi t}{s} \alpha(s)$$



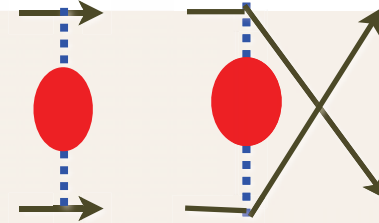
*Running Coupling is  
Complex for Timelike  
Argument*

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

**Gell Mann-Low Running Charge  
sums all vacuum polarization insertions**

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$



- **No renormalization scale ambiguity!**
- **Two separate physical scales:  $t, u =$  photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**
- **If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result! Scheme independent.**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**
- **Two separate physical scales.**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**
- **If one chooses a different scale, one must sum an infinite number of graphs -- but then recover same result!**

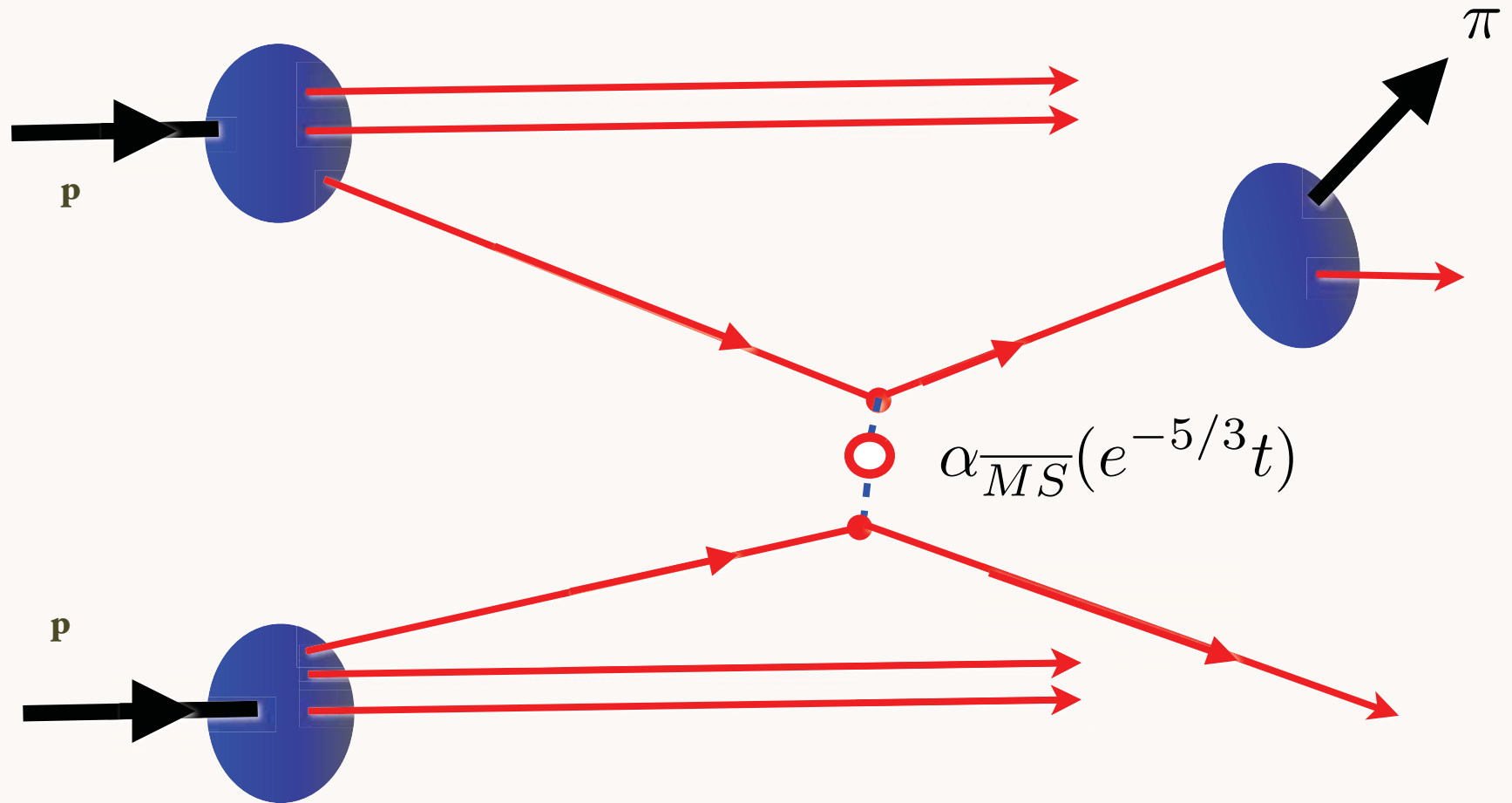
$\lim N_C \rightarrow 0$  at fixed  $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

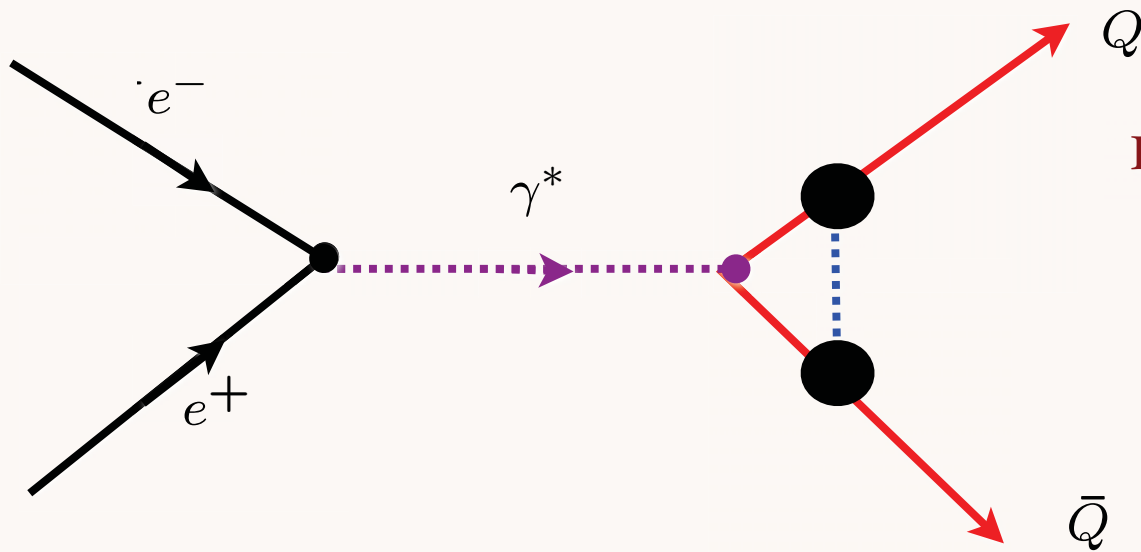
QCD  $\rightarrow$  Abelian Gauge Theory

*Analytic Feature of  $SU(N_c)$  Gauge Theory*

*Scale-Setting procedure for QCD  
must be applicable to QED*

# Renormalization Scale-Setting Not Ambiguous





Hoang, Kuhn, Teubner, sjb

$$\begin{aligned}
 F_1 + F_2 &= 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \\
 &\cong \left( 1 - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \right) \left( 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} \right)
 \end{aligned}$$

## *Example of Multiple BLM Scales*

Angular distributions of massive quarks and leptons close to threshold.



# On the elimination of scale ambiguities in perturbative quantum chromodynamics

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*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305\**

G. Peter Lepage

*Institute for Advanced Study, Princeton, New Jersey 08540*

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*Fermilab, Batavia, Illinois 60510*

(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the  $\Upsilon$ . Our analysis calls into question recent determinations of the QCD coupling constant based upon  $\Upsilon$  decay.

BLM: Choose  $\mu_R$  in  $\alpha_s$  to absorb all  $\beta$  terms

# BLM Scale Setting

$$\beta_0 = 11 - \frac{2}{3}n_f$$

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \left( -\frac{3}{2}\beta_0 A_{\text{VP}} + \frac{33}{2}A_{\text{VP}} + B \right) + \dots \right]$$

*$n_f$  dependent coefficient identifies quark loop VP contribution*

by

$$\rho = C_0 \alpha_{\overline{\text{MS}}}(Q^*) \left[ 1 + \frac{\alpha_{\overline{\text{MS}}}(Q^*)}{\pi} C_1^* + \dots \right],$$

where

Conformal coefficient - independent of  $\beta$

$$Q^* = Q \exp(3A_{\text{VP}}),$$

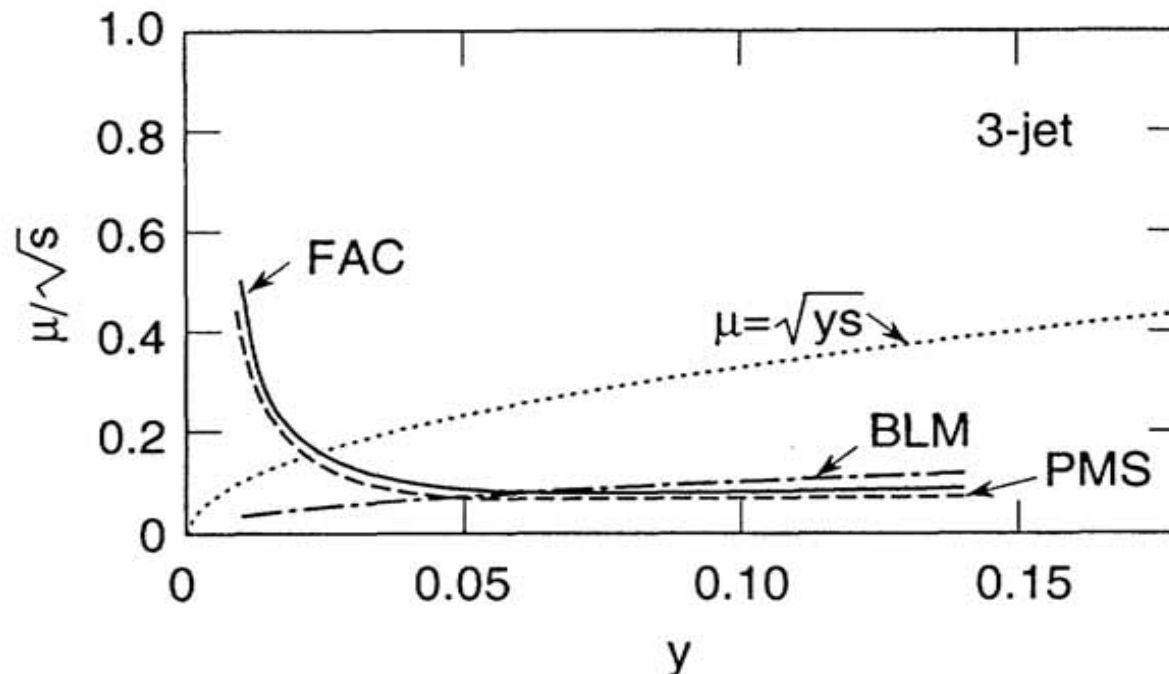
$$C_1^* = \frac{33}{2}A_{\text{VP}} + B.$$

The term  $33A_{\text{VP}}/2$  in  $C_1^*$  serves to remove that part of the constant  $B$  which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by  $\beta_0 = 11 - \frac{2}{3}n_f$ .

*Use skeleton expansion:  
Gardi, Grunberg, Rathsmann, sjb*

# Features of BLM Scale Setting

- **All terms associated with nonzero beta function summed into running coupling**
- **BLM Scale  $Q^*$  sets the number of active flavors**
- **Only  $n_f$  dependence required to determine renormalization scale at NLO**
- **Result is scheme independent:  $Q^*$  has exactly the correct dependence to compensate for change of scheme**
- **Result independent of starting scale**
- **Correct Abelian limit**
- **Resulting series identical to conformal series!**
- **Renormalon  $n!$  growth of PQCD coefficients from beta function eliminated!**
- **In general, BLM scale depends on all invariants**



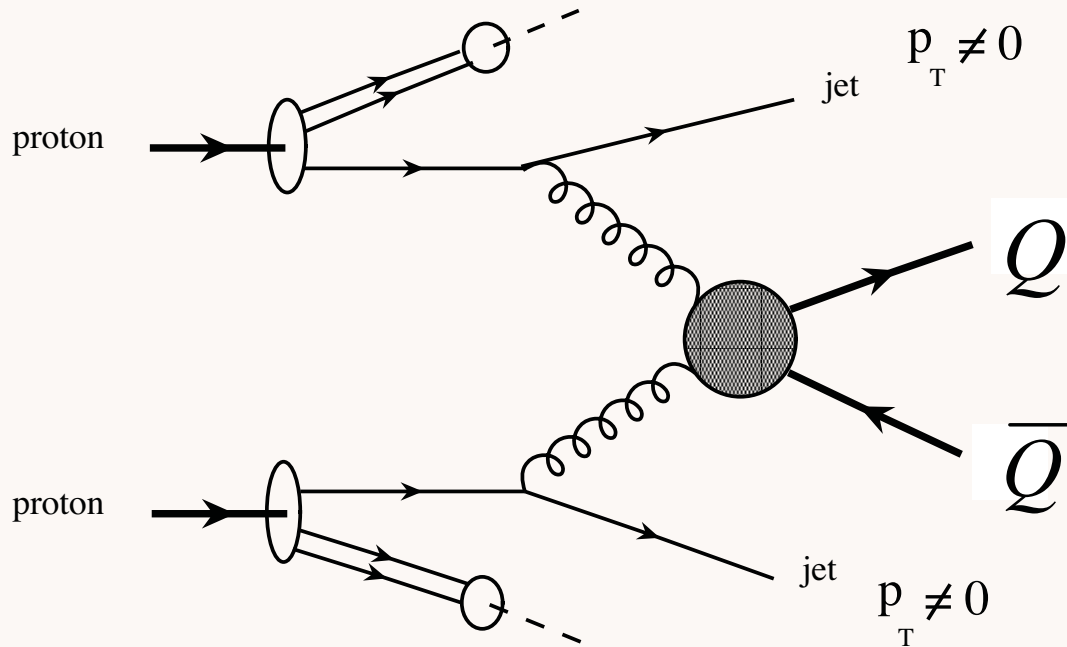
**Kramer &  
Lampe**

*Three-Jet rate in electron-positron annihilation*

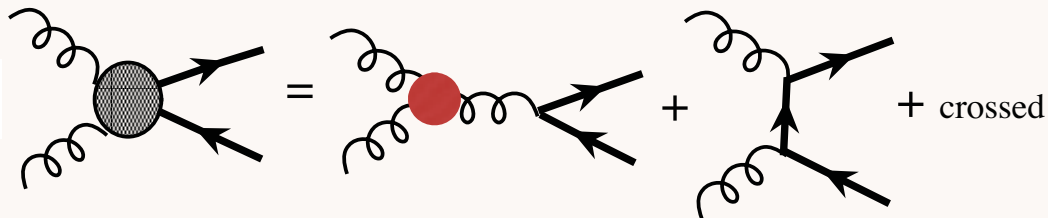
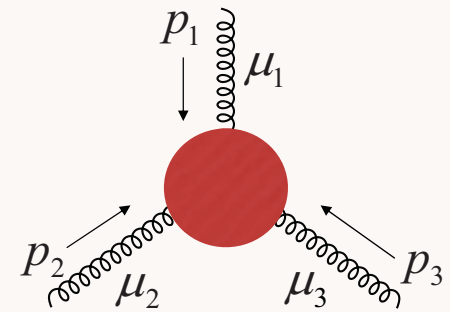
The scale  $\mu/\sqrt{s}$  according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and  $\sqrt{y}$  (dotted) procedures for the three-jet rate in  $e^+e^-$  annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low  $y$ . In particular, the latter two methods predict increasing values of  $\mu$  as the jet invariant mass  $\mathcal{M} < \sqrt{(ys)}$  decreases.

*Other Jet Observables:* **Rathsman**

# Heavy Quark Hadroproduction



**3-gluon  
coupling  
depends on 3  
physical scales**



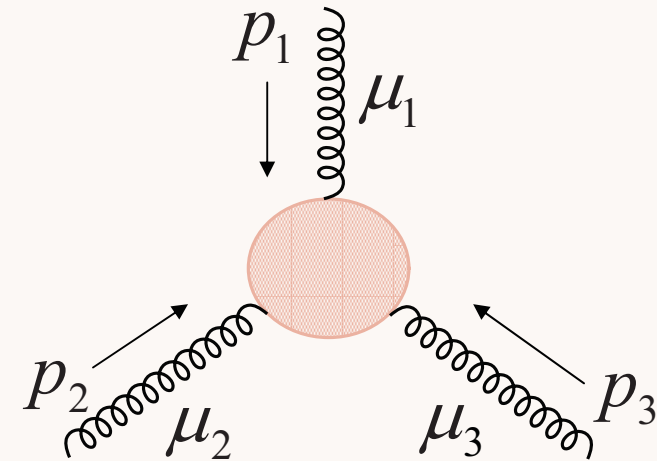
# The Renormalization Scale Problem

$$\rho(Q^2) = C_0 + C_1\alpha_s(\mu_R) + C_2\alpha_s^2(\mu_R) + \dots$$

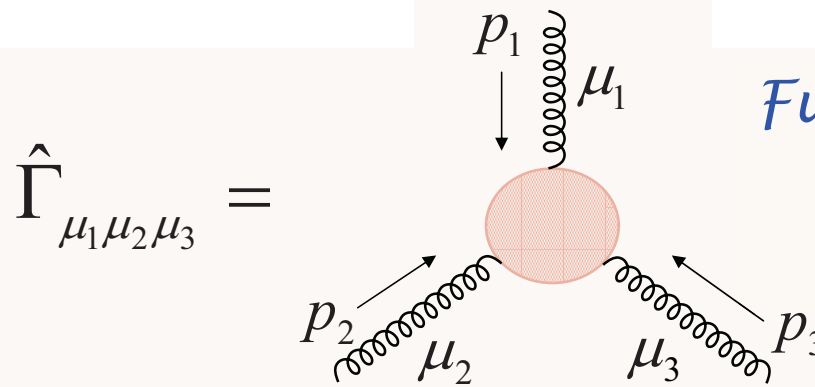
$$\mu_R^2 = CQ^2$$

*Is there a way to set the renormalization scale  $\mu_R$  ?*

*What happens if there are multiple physical scales ?*



# General Structure of the Three-Gluon Vertex



*Full analytic calculation,  
general masses, spin  
Pinch Scheme*

3 index tensor  $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$  built out of  $g_{\mu\nu}$  and  $p_1, p_2, p_3$   
with  $p_1 + p_2 + p_3 = 0$

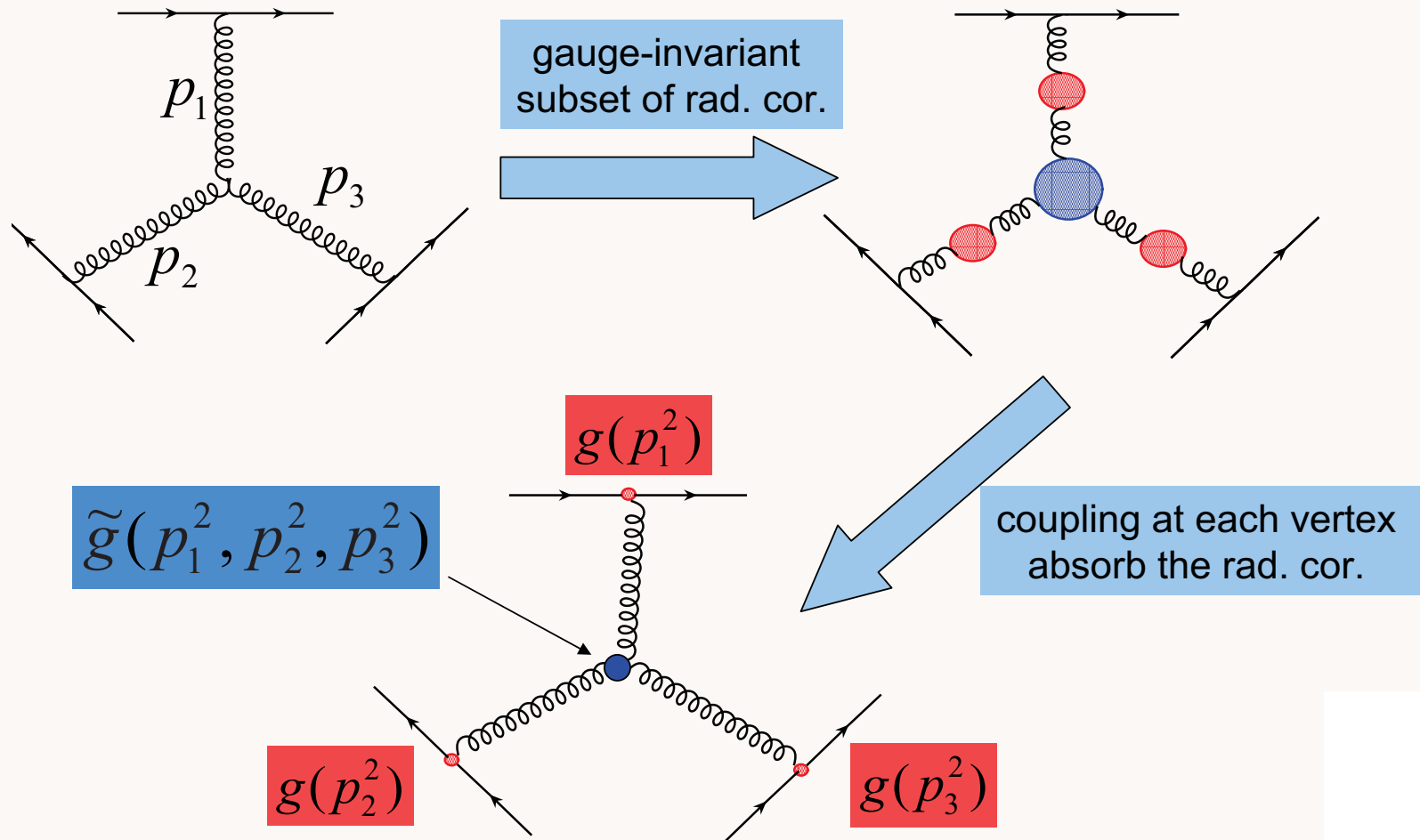
➡ 14 basis tensors and form factors

PHYSICAL REVIEW D 74, 054016 (2006)

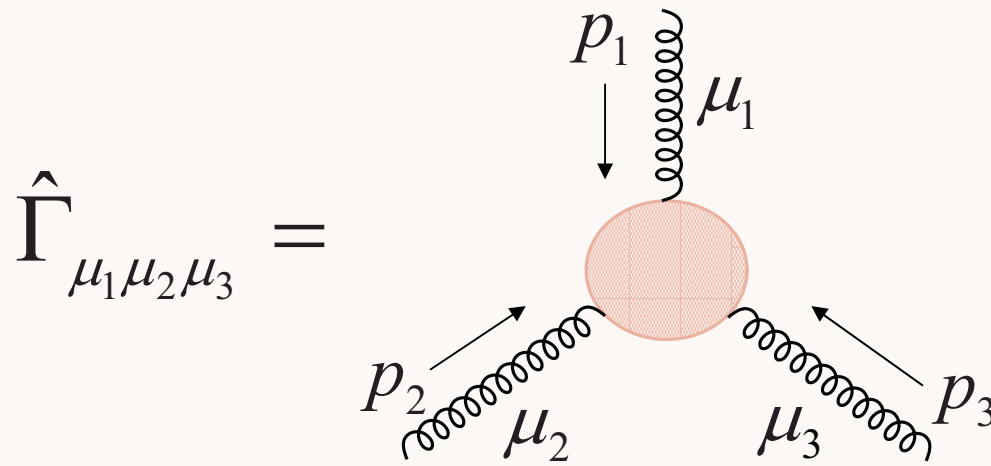
**Form factors of the gauge-invariant three-gluon vertex**

Michael Binger\* and Stanley J. Brodsky†

# Multi-scale Renormalization of the Three-Gluon Vertex







H. J. Lu

$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

# Properties of the Effective Scale

$$Q_{eff}^2(a, b, c) = Q_{eff}^2(-a, -b, -c)$$

$$Q_{eff}^2(\lambda a, \lambda b, \lambda c) = |\lambda| Q_{eff}^2(a, b, c)$$

$$Q_{eff}^2(a, a, a) = |a|$$

$$Q_{eff}^2(a, -a, -a) \approx 5.54 |a|$$

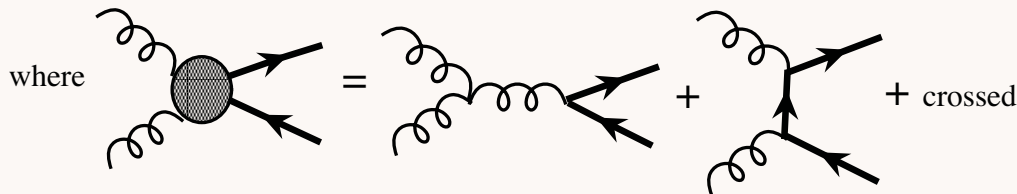
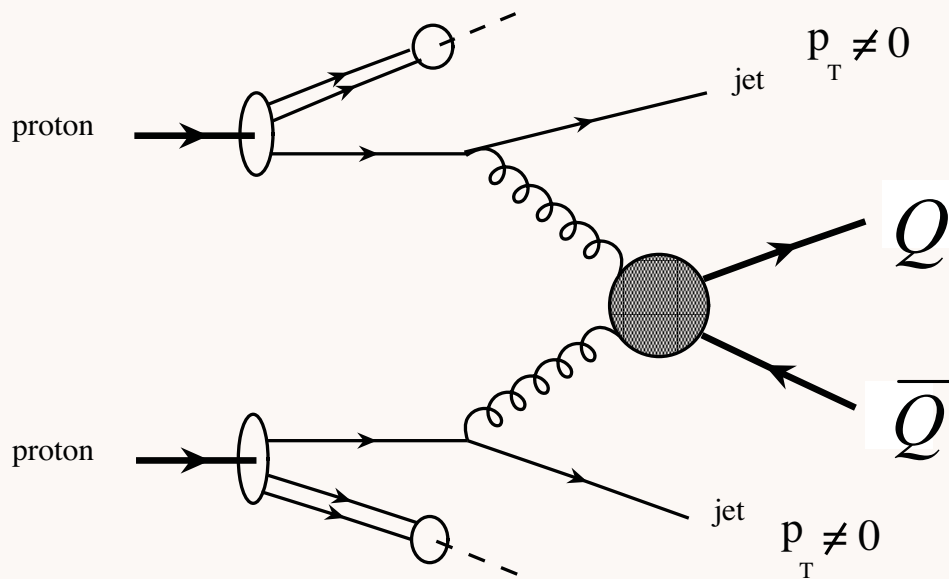
$$Q_{eff}^2(a, a, c) \approx 3.08 |c| \quad \text{for } |a| \gg |c|$$

$$Q_{eff}^2(a, -a, c) \approx 22.8 |c| \quad \text{for } |a| \gg |c|$$

$$Q_{eff}^2(a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for } |a| \gg |b|, |c|$$

*Surprising dependence on Invariants*

# Heavy Quark Hadro-production



- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale  
➔ much larger cross section than  $\overline{MS}$  with scale  $\mu_R = M_{Q\bar{Q}}$  or  $M_Q$
- Future : repeat analysis using the full mass-dependent results and include all form factors

**Expect that this approach accounts for most of the one-loop corrections**

# *Relate Observables to Each Other*

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

# Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\begin{aligned}
\frac{\alpha_R(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left( \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^2 \left[ \left( \frac{41}{8} - \frac{11}{3} \zeta_3 \right) C_A - \frac{1}{8} C_F + \left( -\frac{11}{12} + \frac{2}{3} \zeta_3 \right) f \right] \\
& + \left( \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^3 \left\{ \left( \frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2 \right) C_A^2 + \left( -\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5 \right) C_A C_F - \frac{23}{32} C_F^2 \right. \\
& + \left[ \left( -\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2 \right) C_A + \left( -\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5 \right) C_F \right] f \\
& \left. + \left( \frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2 \right) f^2 + \left( \frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc}}{C_F d(R)} \frac{(\sum_f Q_f)^2}{\sum_f Q_f^2} \right\}.
\end{aligned}$$

$$\begin{aligned}
\frac{\alpha_{g_1}(Q)}{\pi} = & \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} + \left( \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^2 \left[ \frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right] \\
& + \left( \frac{\alpha_{\overline{\text{MS}}}(Q)}{\pi} \right)^3 \left\{ \left( \frac{5437}{648} - \frac{55}{18} \zeta_5 \right) C_A^2 + \left( -\frac{1241}{432} + \frac{11}{9} \zeta_3 \right) C_A C_F + \frac{1}{32} C_F^2 \right. \\
& \left. + \left[ \left( -\frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5 \right) C_A + \left( \frac{133}{864} + \frac{5}{18} \zeta_3 \right) C_F \right] f + \frac{115}{648} f^2 \right\}.
\end{aligned}$$

**Eliminate MSbar,  
Find Amazing Simplification**

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

*Geometric Series in Conformal QCD*

*Generalized Crewther Relation*

Lu, Kataev, Gabadadze, Sjb

# *Generalized Crewther Relation*

$$\left[1 + \frac{\alpha_R(s^*)}{\pi}\right] \left[1 - \frac{\alpha_{g_1}(q^2)}{\pi}\right] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

*Conformal relation true to all orders in  
perturbation theory*

*No radiative corrections to axial anomaly*

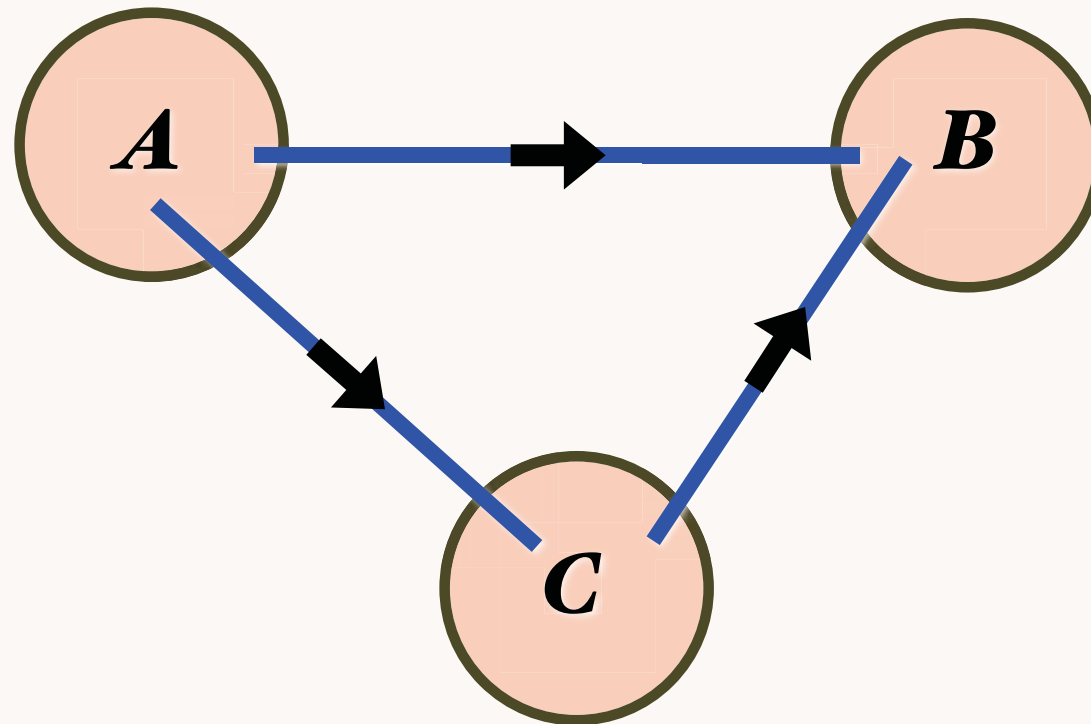
*Nonconformal terms set relative scales (BLM)*

*Analytic matching at quark thresholds*

*No renormalization scale ambiguity!*



# *Transitivity Property of Renormalization Group*



$$A \rightarrow C \quad C \rightarrow B \quad \textit{identical to} \quad A \rightarrow B$$

*Relation of observables independent of intermediate scheme C*

## *Conventional wisdom concerning scale setting*

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess
- with an arbitrary range
- Factorization scale should be taken equal to renormalization scale

$$\mu_R = Q$$

$$Q/2 < \mu_R < 2Q$$

$$\mu_F = \mu_R$$

*These assumptions are untrue in QED  
and thus they cannot be true for QCD!*

*Worse: result is scheme dependent!*

# QCD Myths

- **Anti-Shadowing is Universal**
- **ISI and FSI are higher twist effects and universal**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **heavy quarks only from gluon splitting**
- **renormalization scale cannot be fixed**
- **QCD condensates are vacuum effects**
- **Infrared Slavery**
- **Nuclei are composites of nucleons only**
- **Real part of DVCS arbitrary**