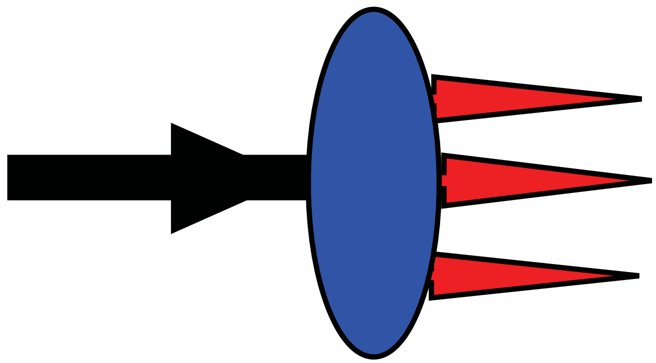
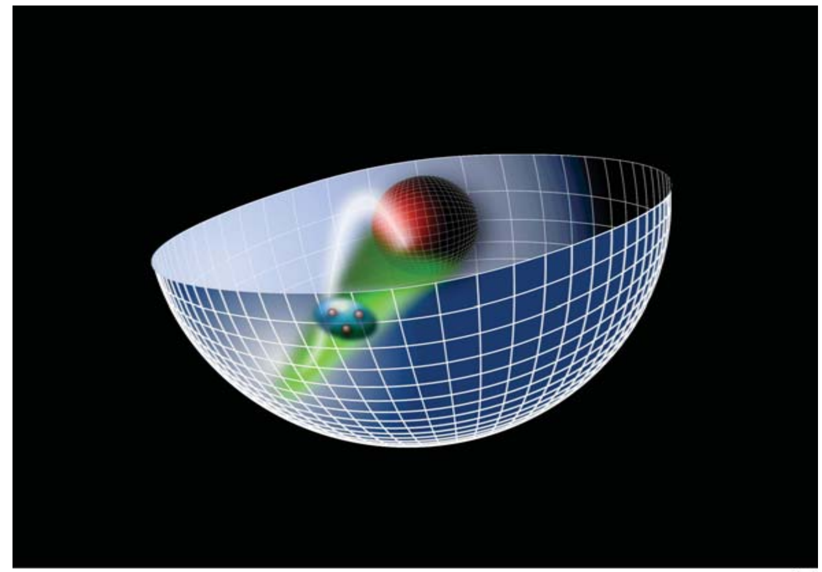
The AdS/QCD Correspondence and Exclusive Processes



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



The 4th Workshop on Exclusive Reactions at High Momentum Transfer

Thomas Jefferson National Accelerator Facility *May 18-21, 2010*

Stan Brodsky SLAC NATIONAL ACCELERATOR LABORATORY & ***CP3-Origins***

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

de Teramond, Deur, Shrock, Roberts, Tandy

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

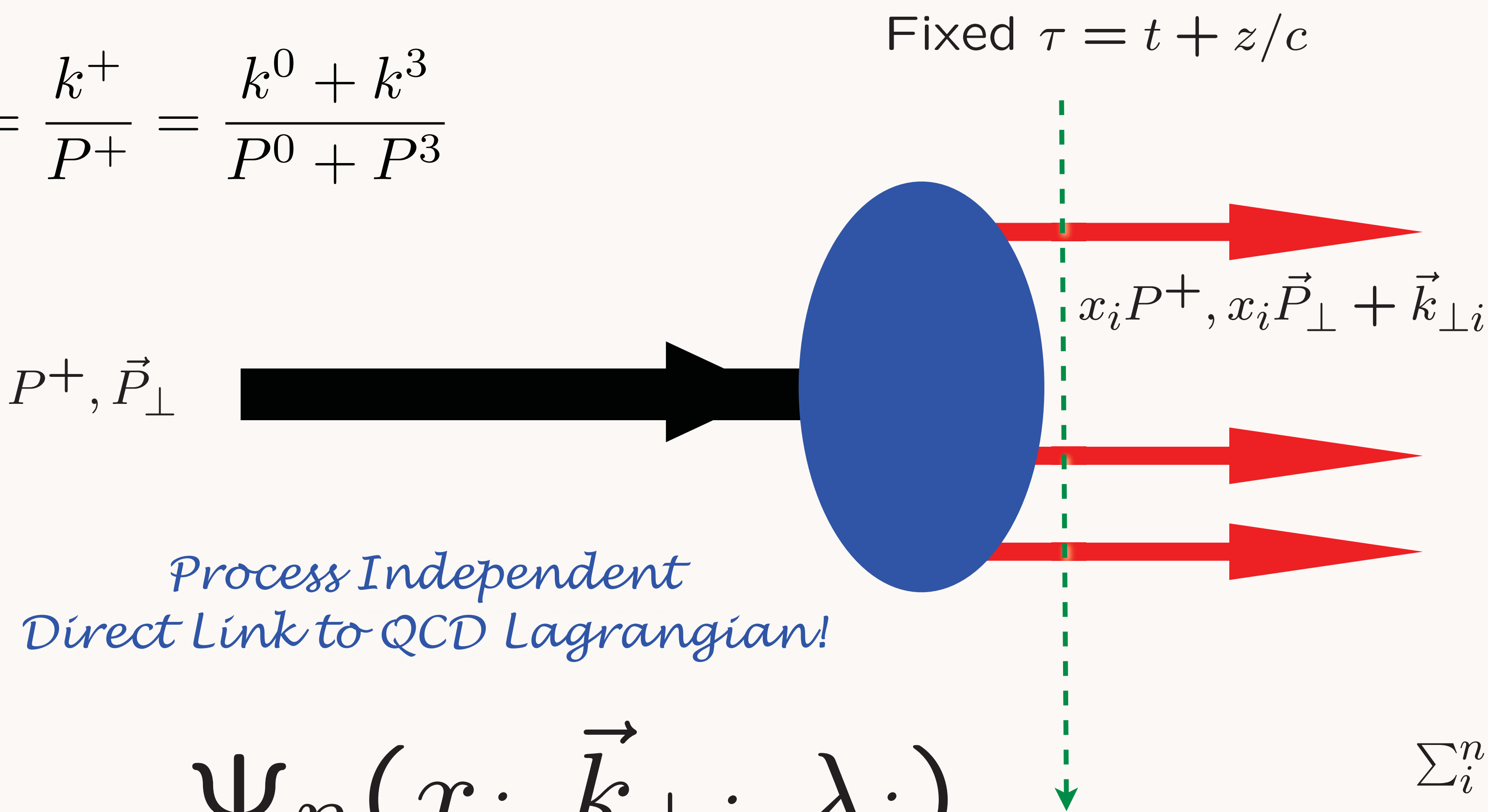
Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Process Independent
Direct Link to QCD Lagrangian!

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

Jaffe definition
LC gauge

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

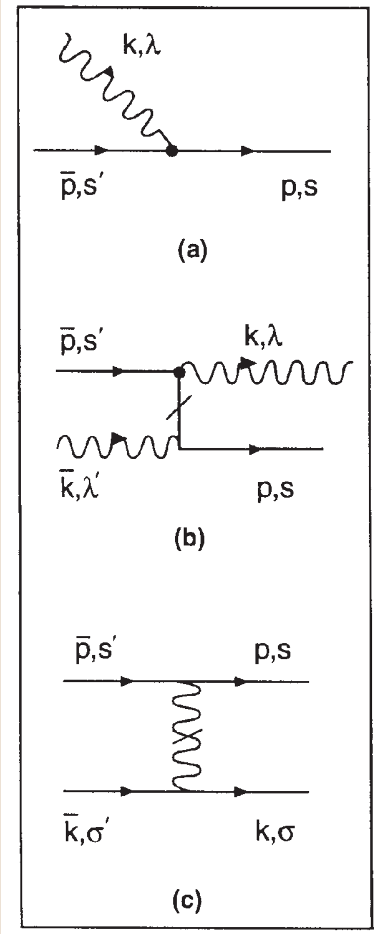
n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Light-Front formalism links dynamics to spectroscopy

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

Physical gauge: $A^+ = 0$



Heisenberg Matrix Formulation

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

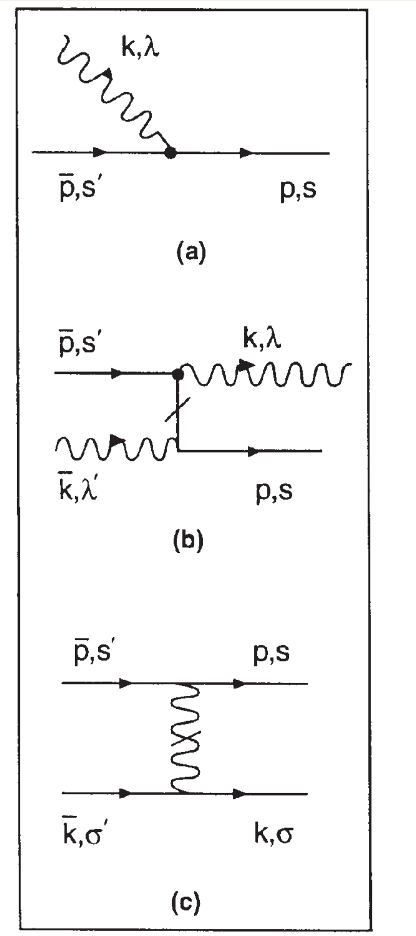
Light-Front QCD

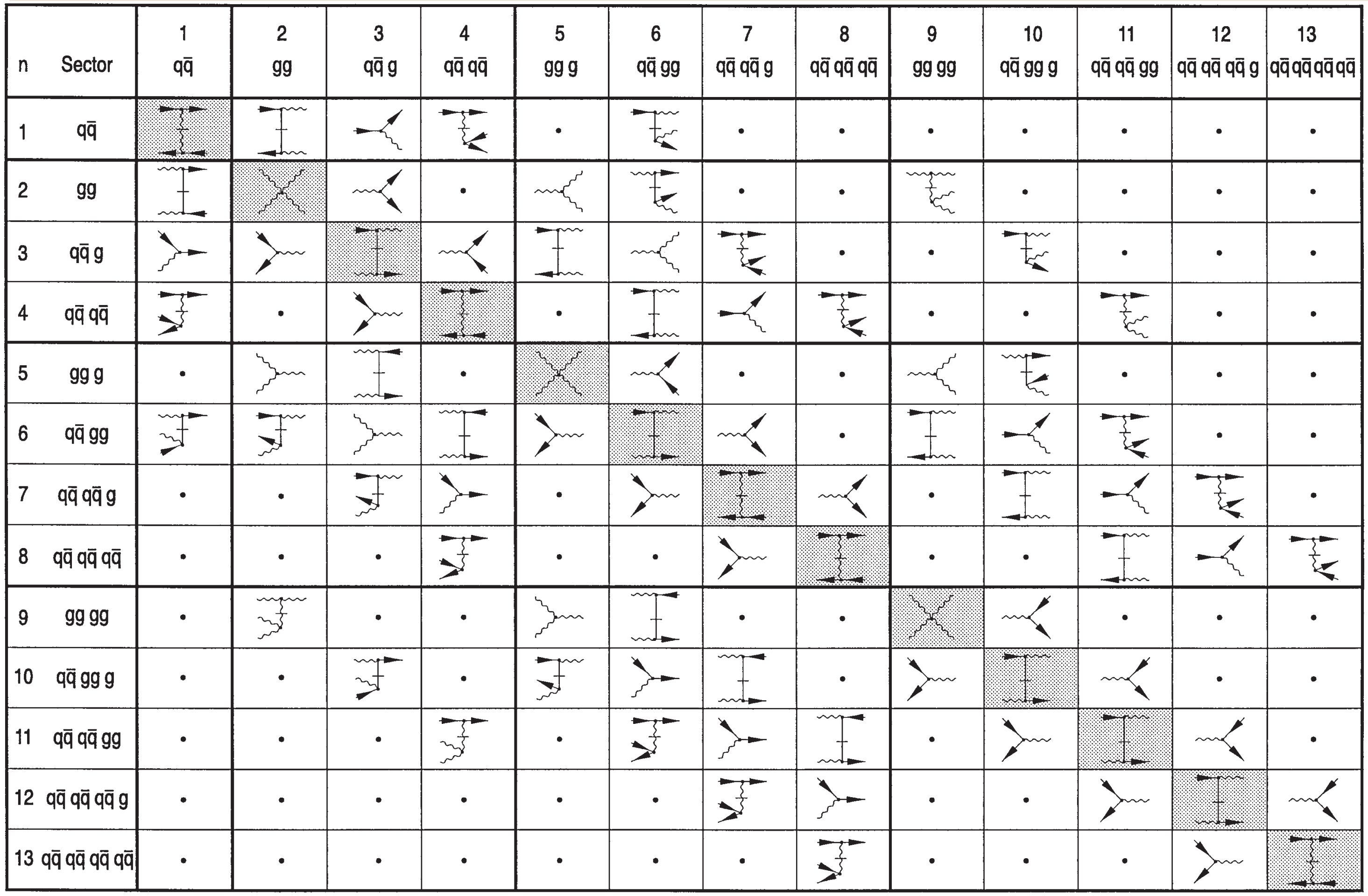
Heisenberg Matrix Formulation

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

H.C. Pauli & sjb

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

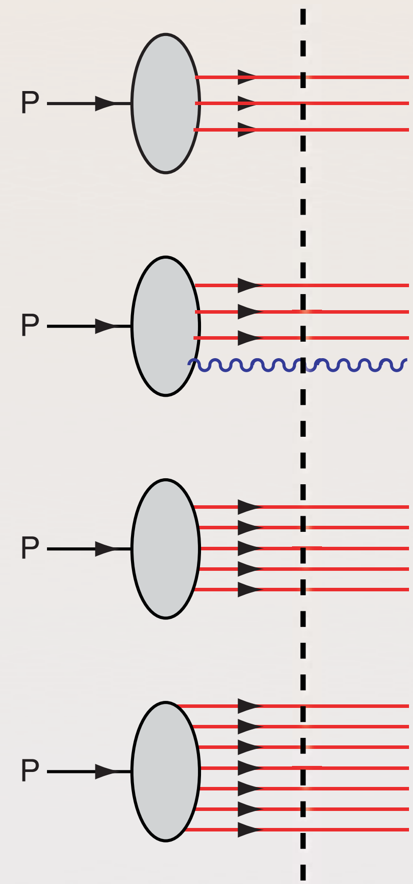
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



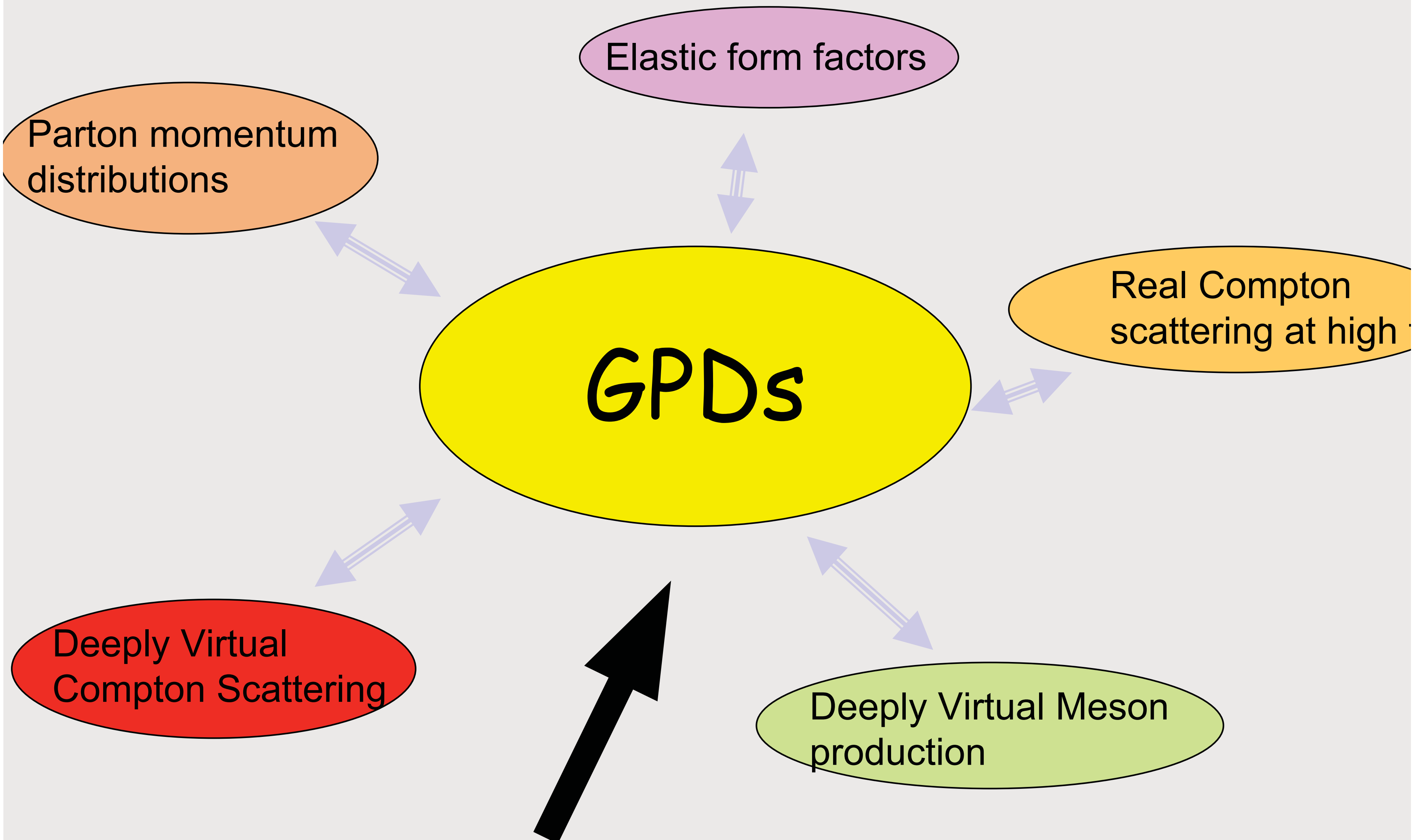
Fixed LF time

Intrinsic heavy quarks

$$\bar{u}(x) \neq \bar{d}(x)$$

$$\bar{s}(x) \neq s(x)$$

A Unified Description of Hadron Structure



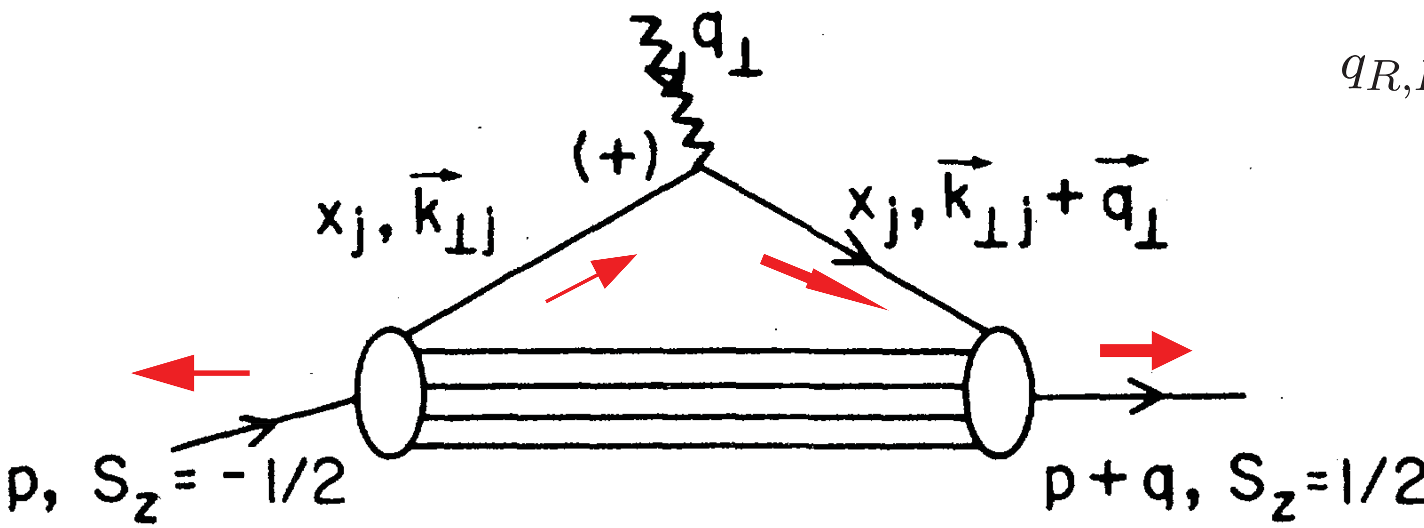
Light Front Wavefunctions

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

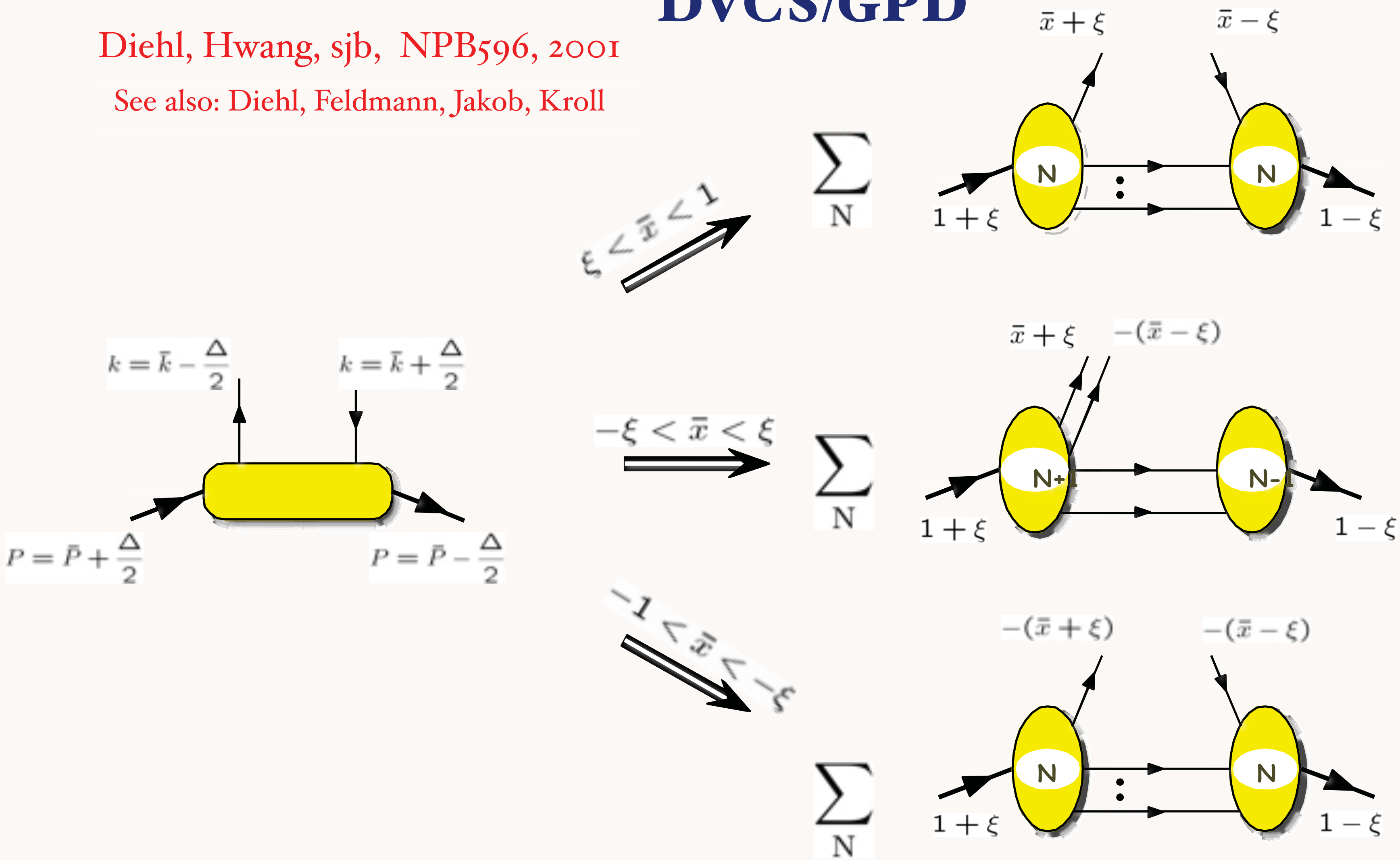
Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



DGLAP region

ERBL region

DGLAP region

Example of LFWF representation of GPDs (n+1 => n-1)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1\to n-1)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_{\perp j}\right)$$

$$\times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_\perp)$$

$$\times \delta(x - x_1) \psi^{\uparrow *}_{(n-1)}(x'_i, \vec{k}'_{\perp i}, \lambda_i) \psi^{\downarrow}_{(n+1)}(x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},$$

where $i = 2, \ldots, n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

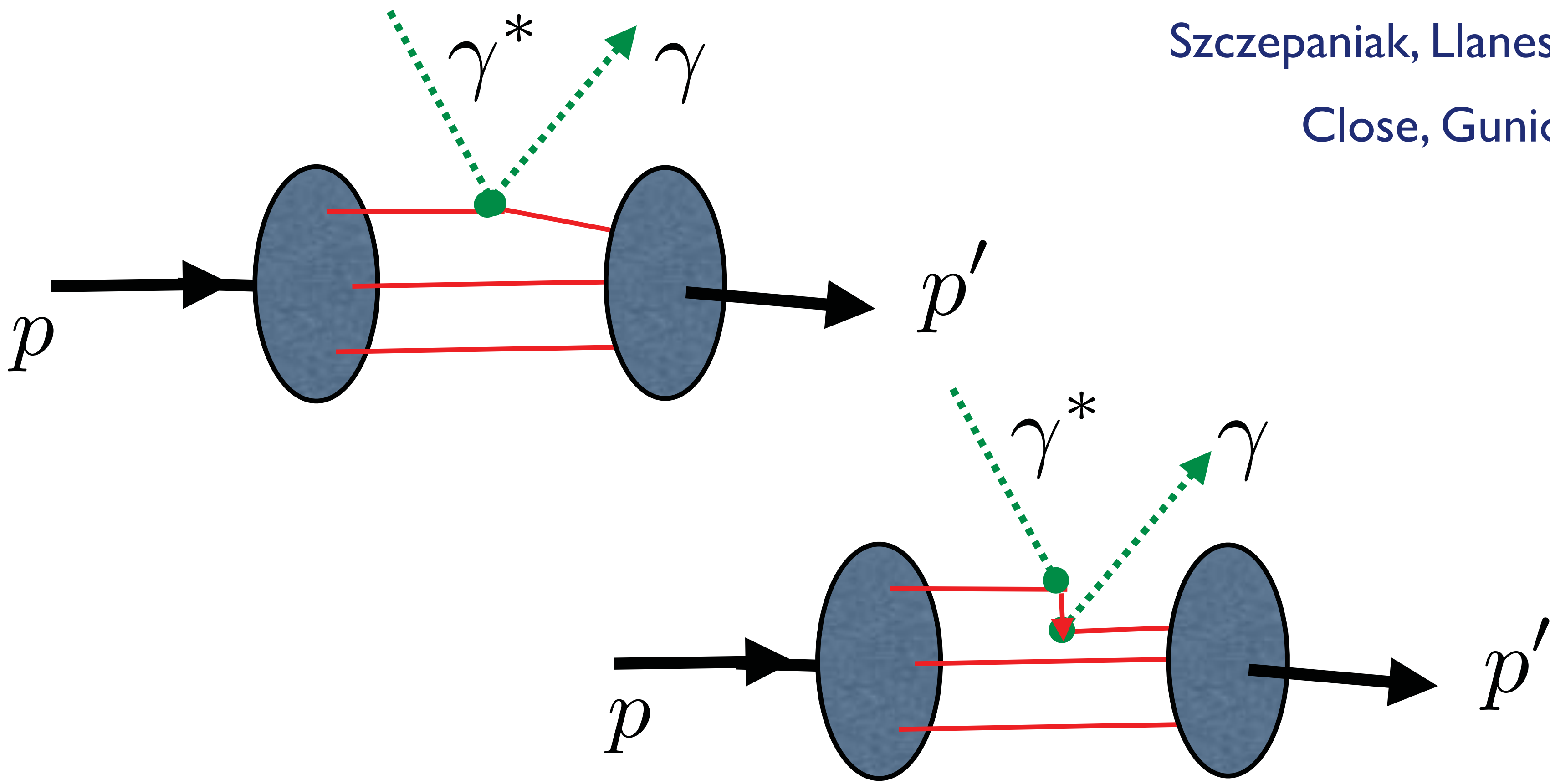
$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta} \vec{\Delta}_\perp.$$

J=0 Fixed Pole Contribution to DVCS

- J=o fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

Szczepaniak, Llanes-Estrada, sjb

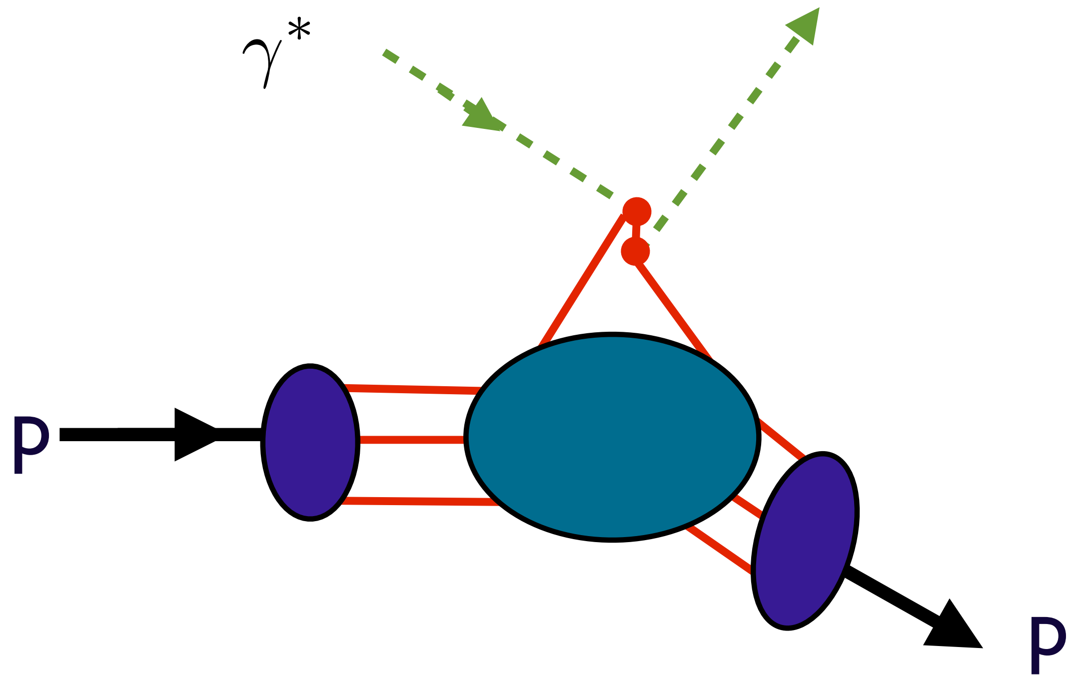
Close, Gunion, sjb



Real amplitude, independent of Q^2 at fixed t

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$



Seagull interaction (instantaneous quark exchange or Z-graph)

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

Hard Reggeon Domain

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s^{\alpha}_R(t)\beta_R(t)$$

$$\alpha_R(t) \to 0$$

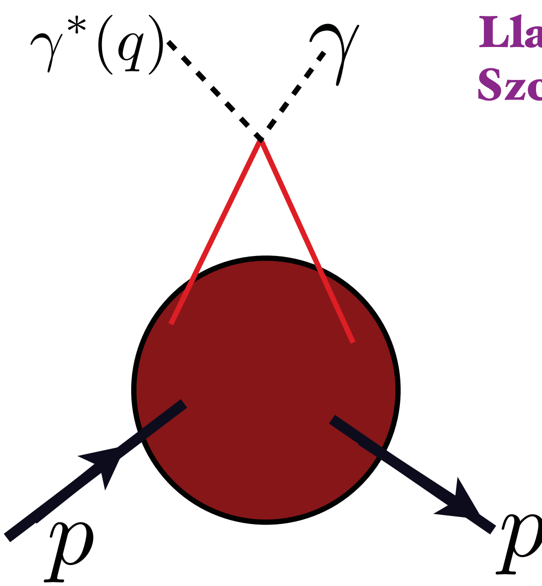
Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

J=0 Fixed pole in real and virtual Compton scattering

**Damashek, Gilman
Close, Gunion, sjb
Llanes-Estrada,
Szczepaniak, sjb**



Effective two-photon contact term

Seagull for scalar quarks

Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of Q^2 at fixed t

<1/x> Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

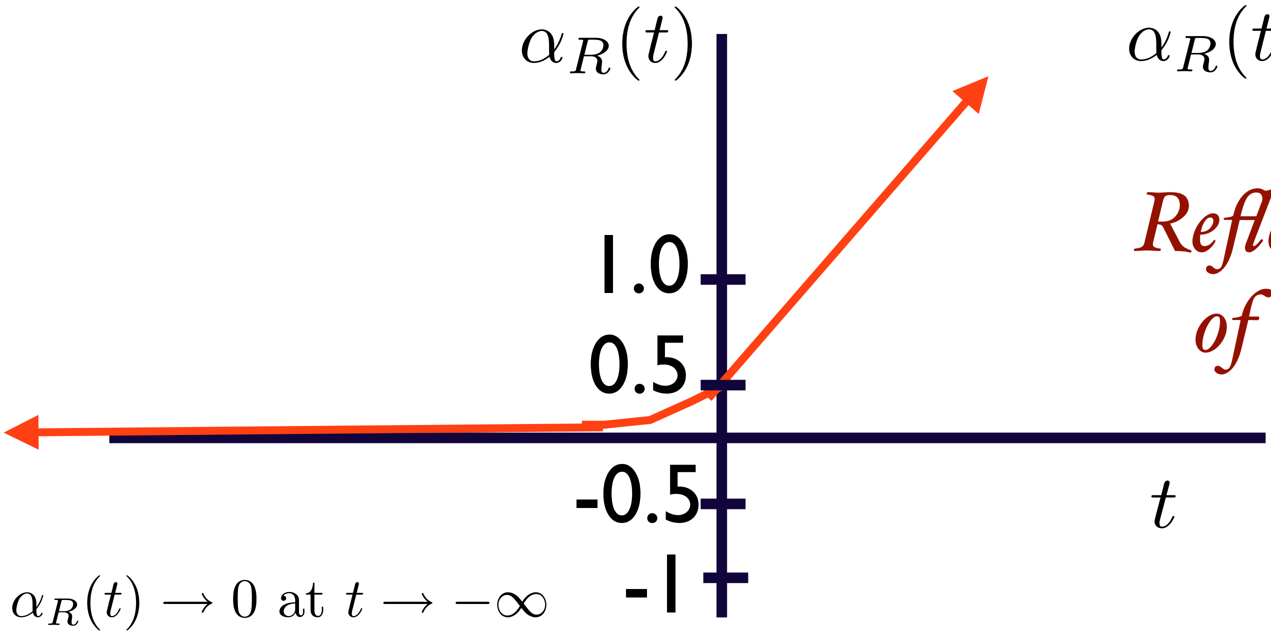
No ambiguity in D-term

Q^2-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt}(\gamma^* p \to \gamma p) = F^2(t)$$

Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t) \beta_R(t) \qquad s >> -t, Q^2$$



$$\alpha_R(t) \to 0 \text{ at } t \to -\infty$$

J=0 fixed pole

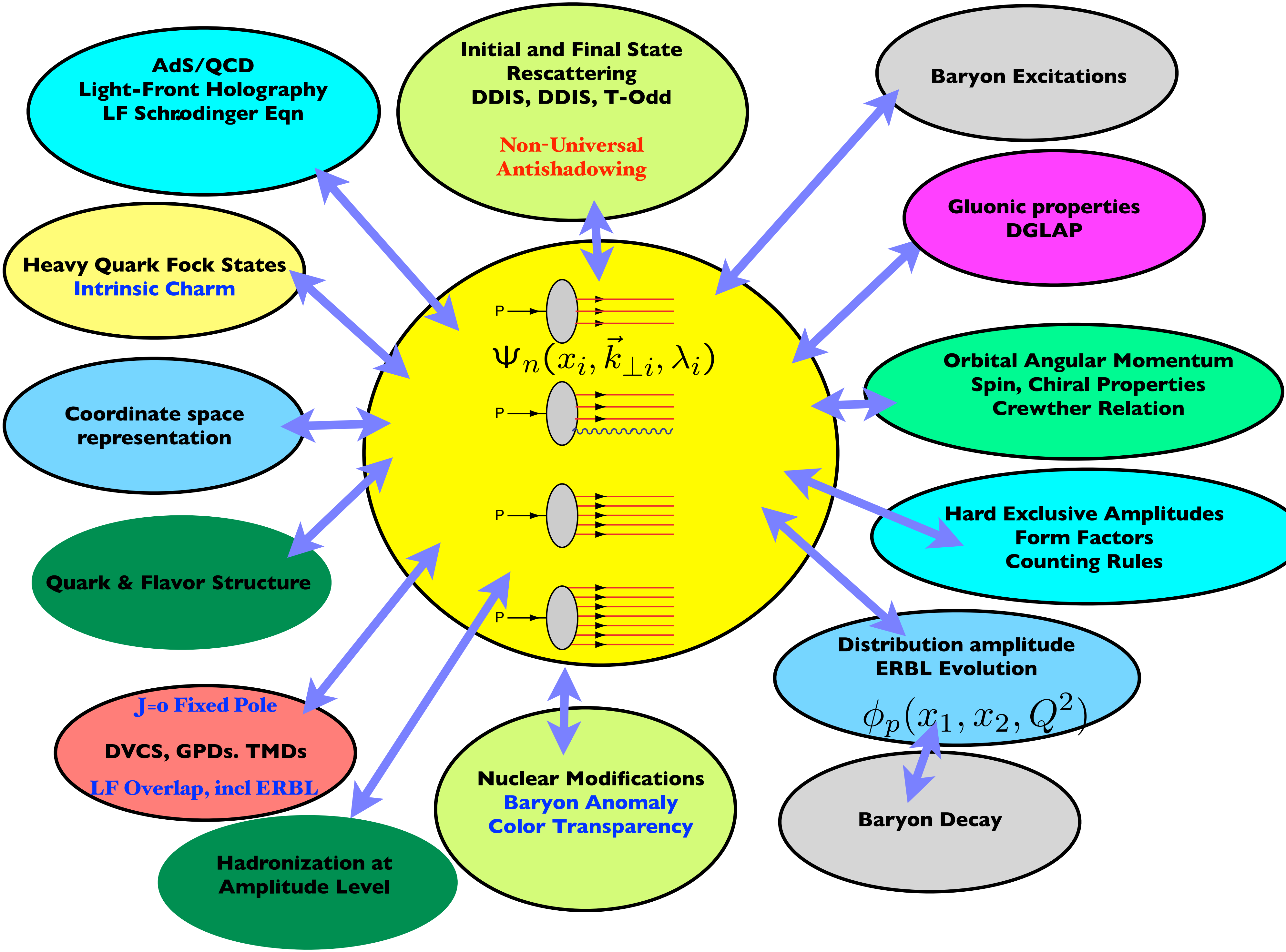
Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

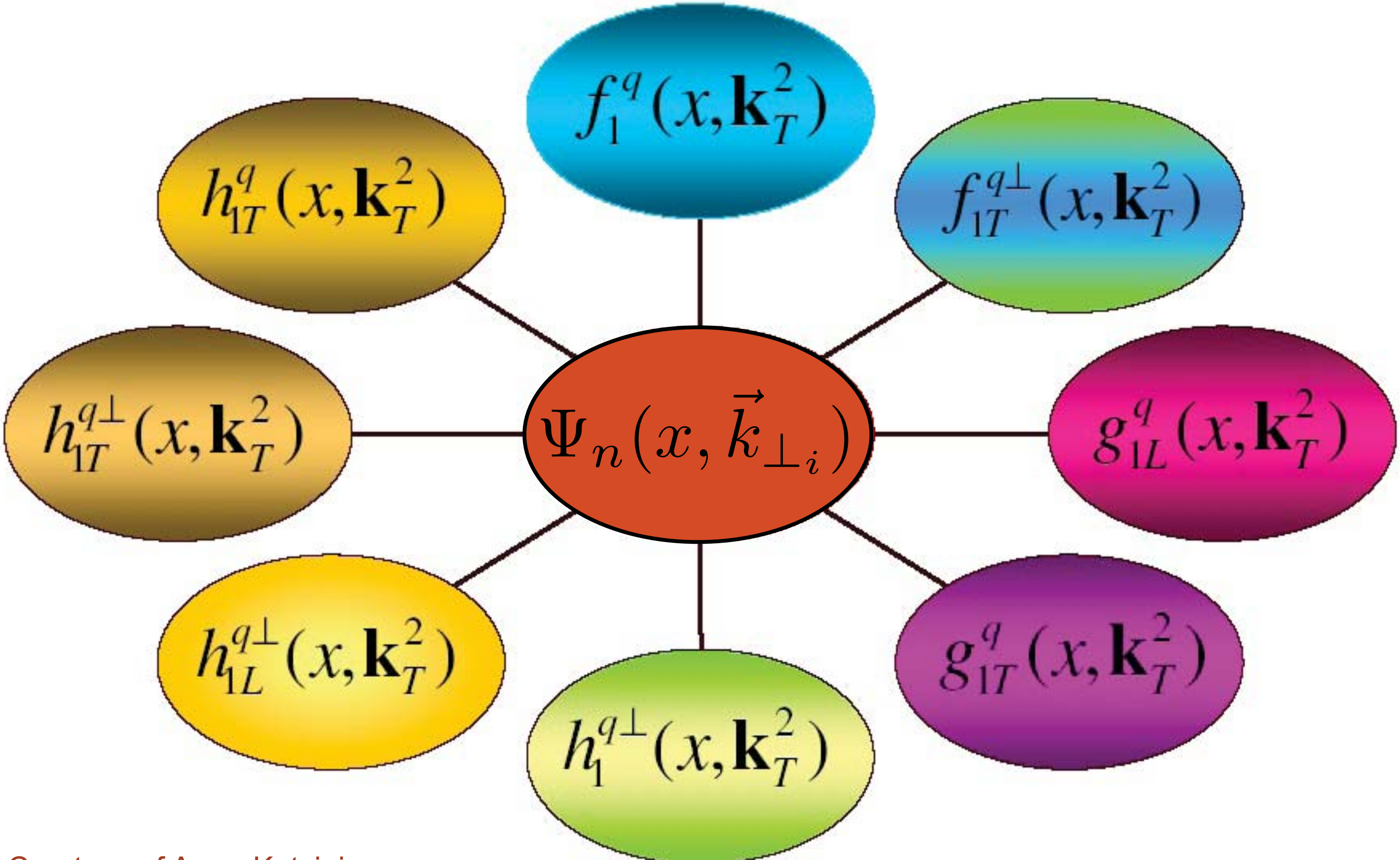
$$\frac{d\sigma}{dt}(\gamma^* p \to \gamma p) \to \frac{1}{s^2} \beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

Fundamental test of QCD

QCD and the LF Hadron Wavefunctions



8 leading-twist spin-$k_\perp$ dependent distribution functions



Courtesy of Aram Kotzinian

Single-spin asymmetries

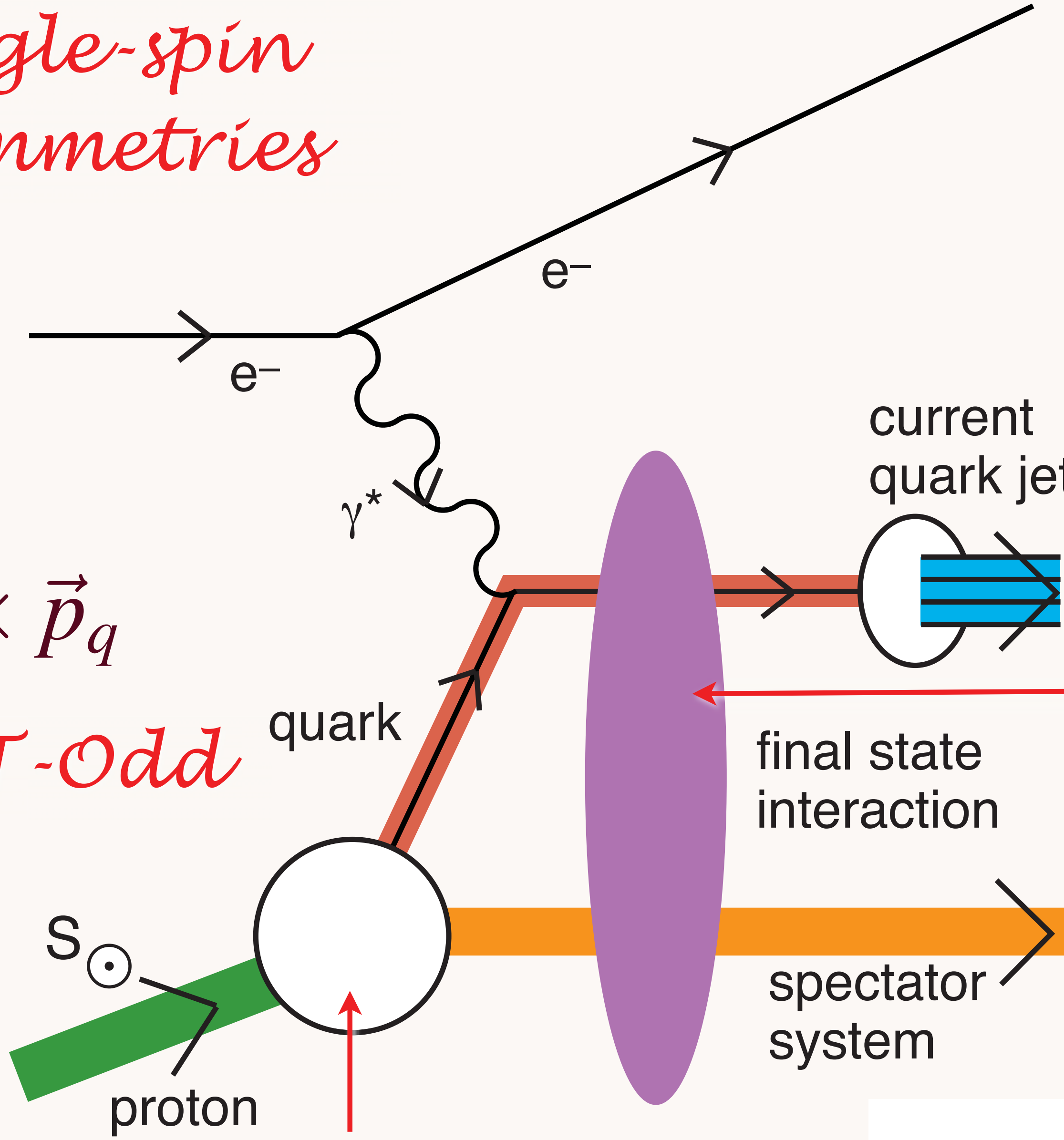
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd



QCD S- and P-Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

Leading-Twist Rescattering Violates pQCD Factorization!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

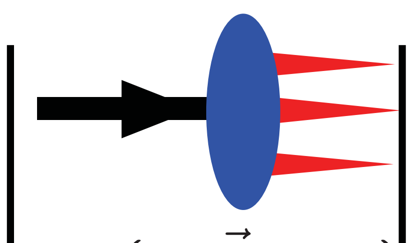
Hwang, Schmidt, sjb,

Mulders, Boer

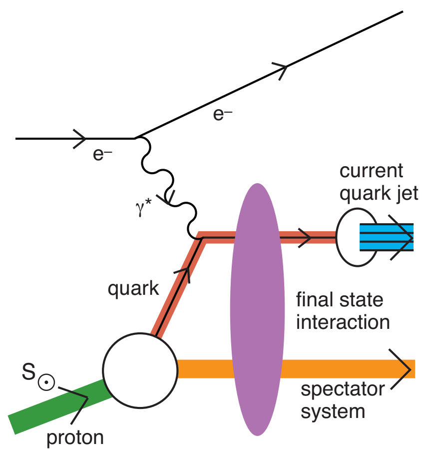
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\left|\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$



Applications of Nonperturbative Running Coupling from AdS/QCD

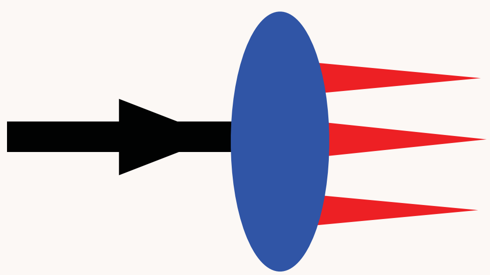
- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

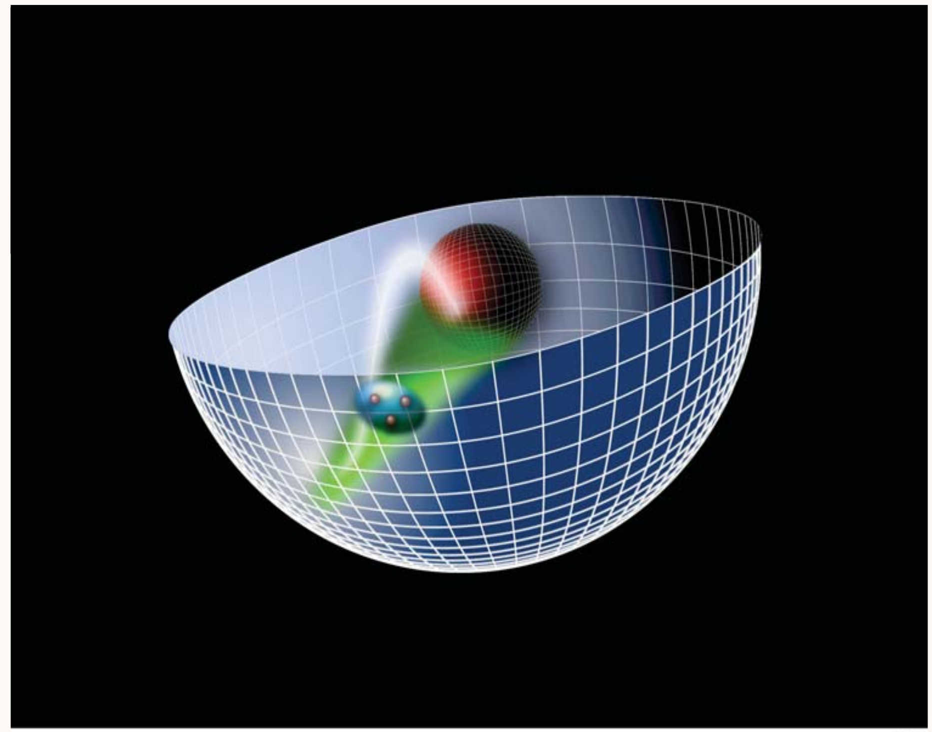
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR



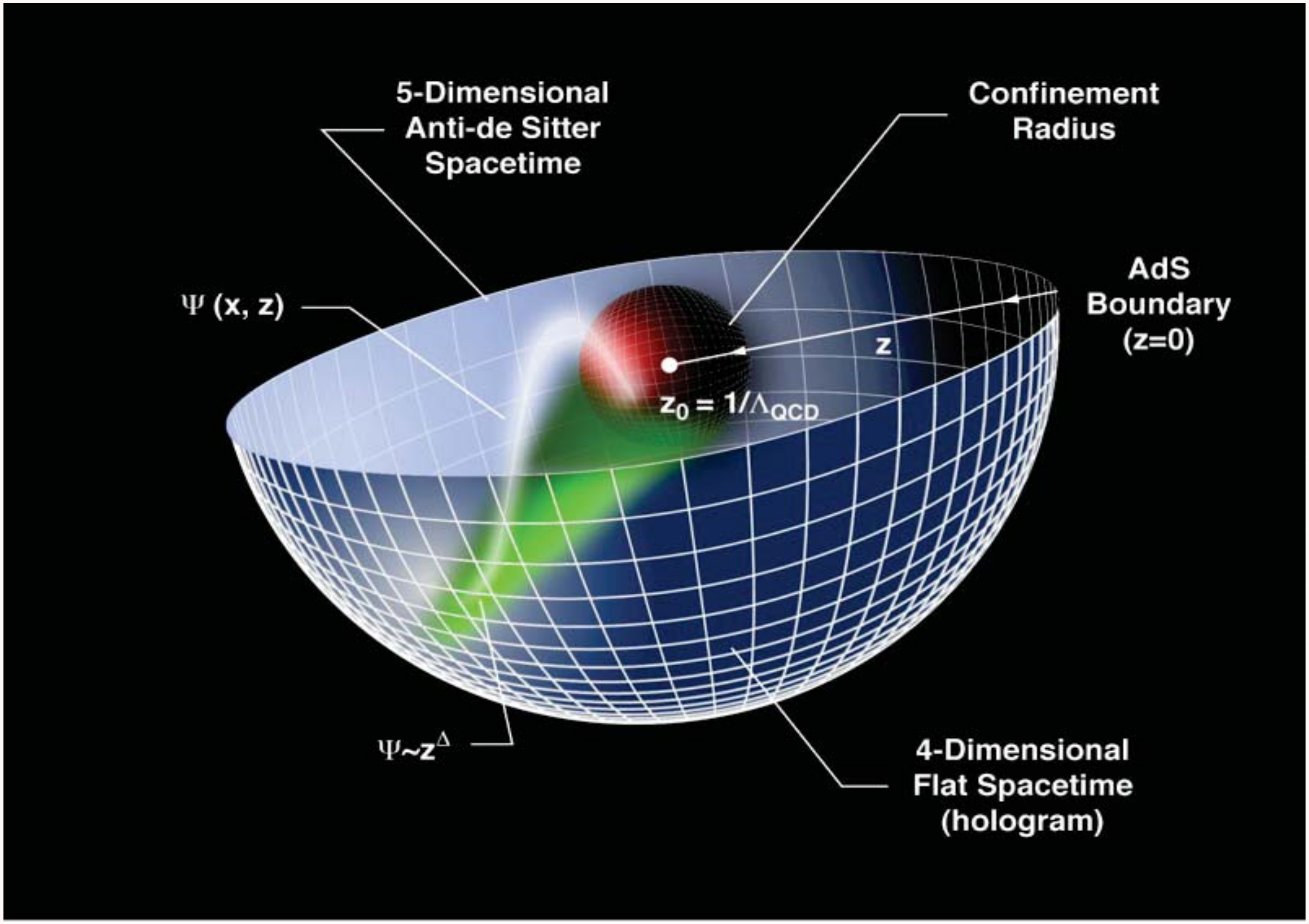
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond and Alexandre Deur

Central problem for strongly-coupled gauge theories

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

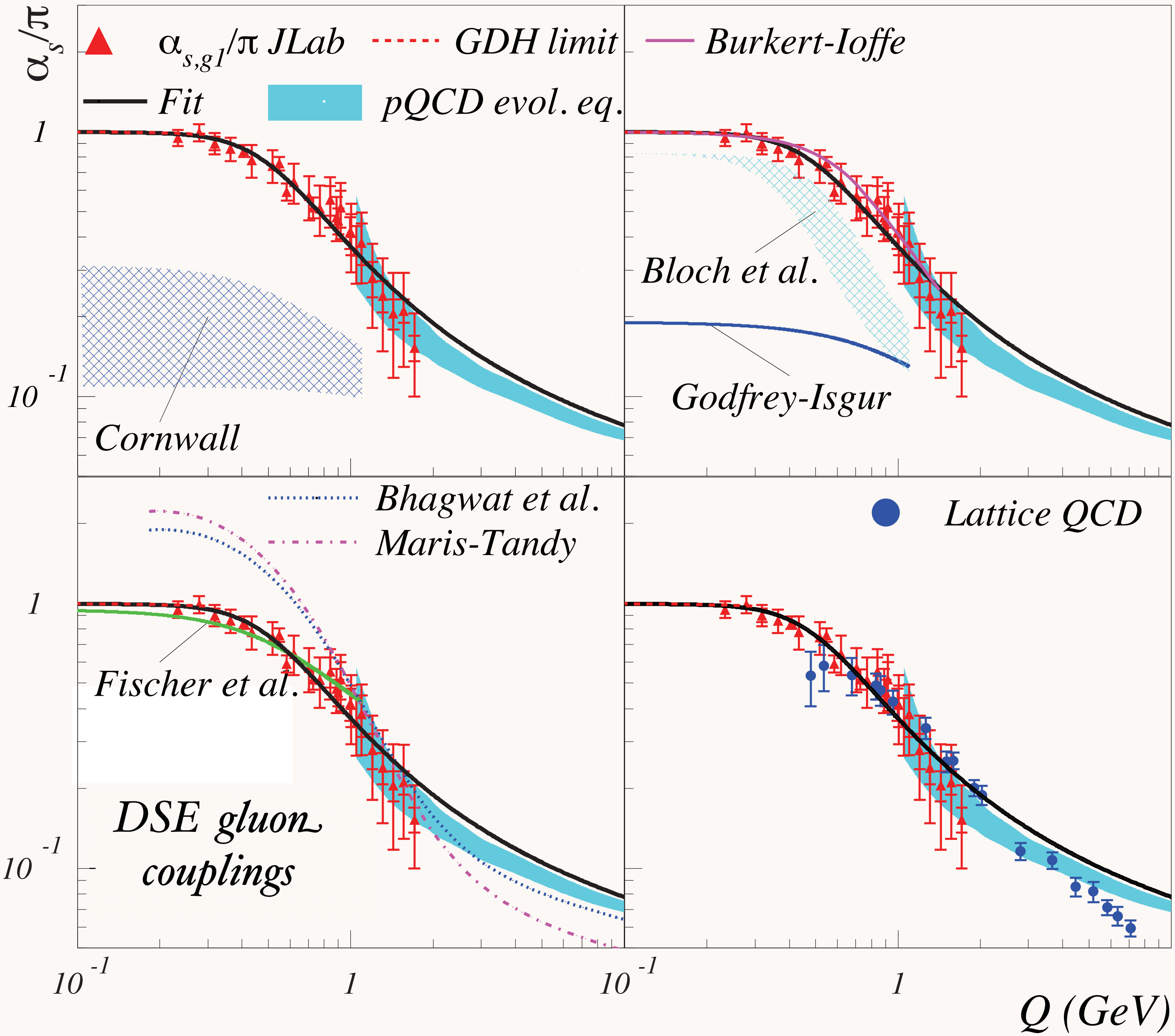
Map AdS_5 X S_5 to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

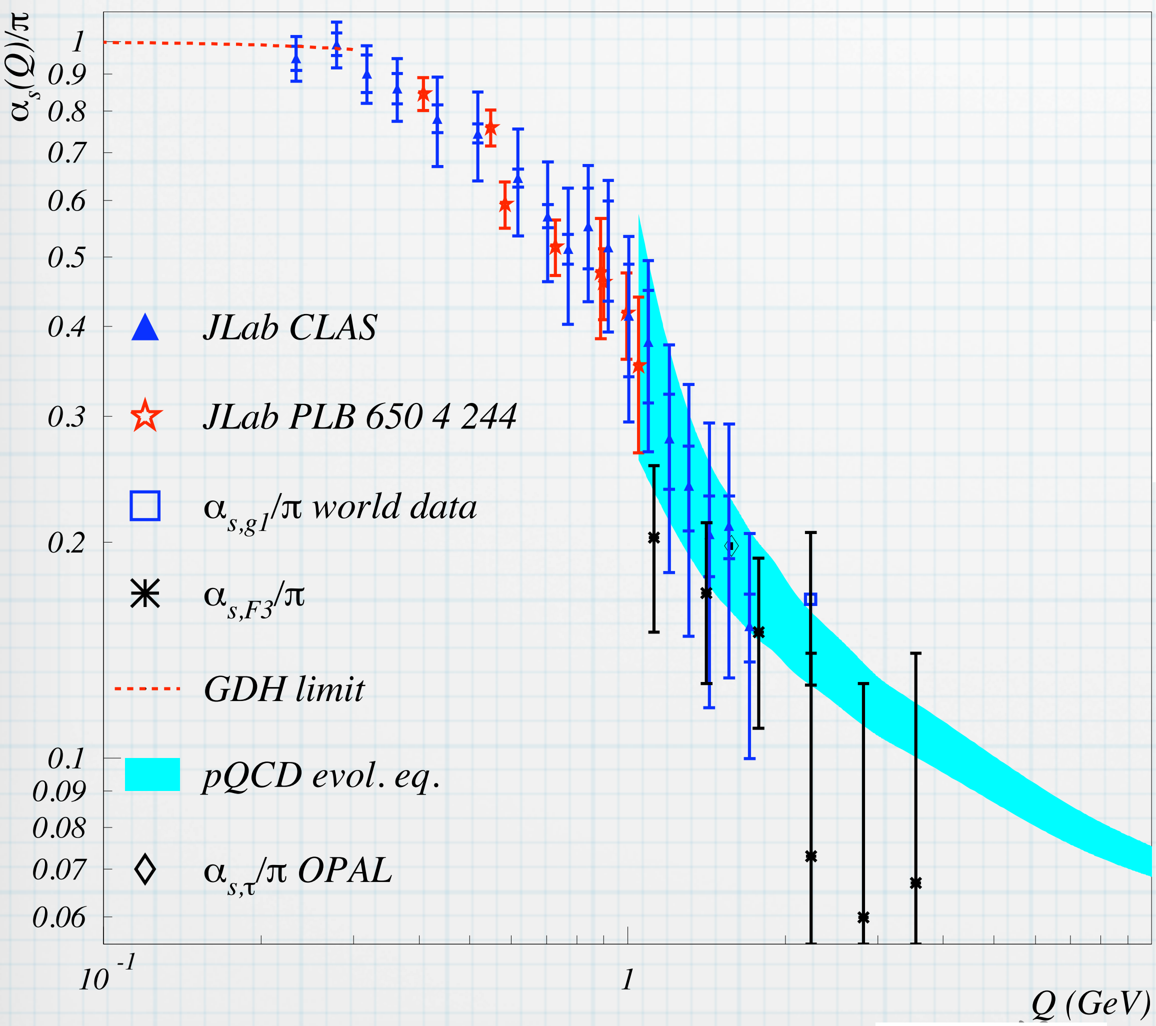
Deur, Korsch, et al.



Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left(g_1^p(x,Q^2) - g_1^n(x,Q^2)\right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi}\right)$$



g1 = spin dependent structure functio

Recent JLab data from EG1(2008), CLAS, and Hall A

α_s runs only modestly at small Q^2

Fig. from 0803.4119, Duer et al.

Maximal Wavelength of Confined Fields

$$(x-y)^2 < \Lambda^{-2}_{QCD}$$

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

Shrock, sjb

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

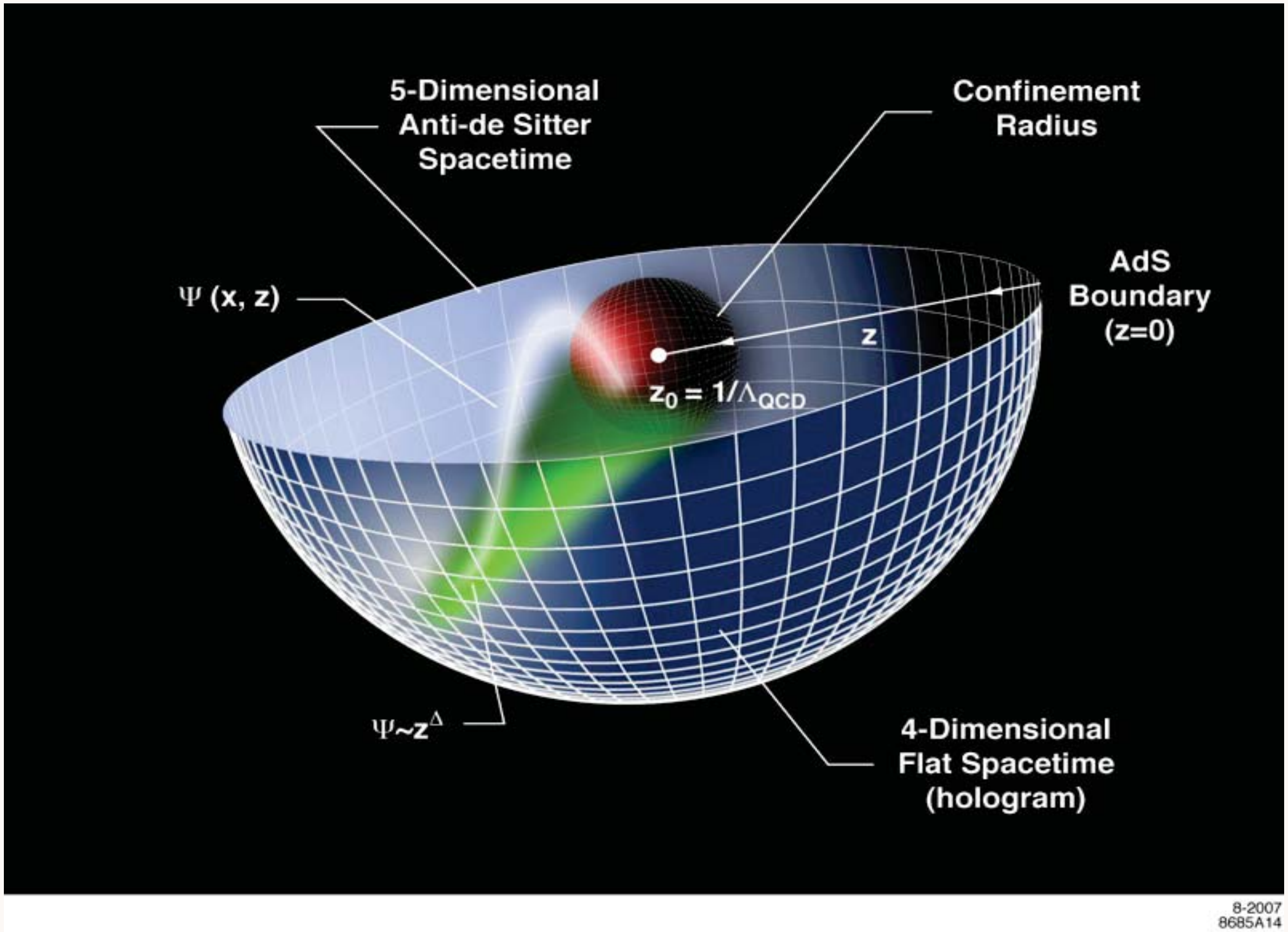
$x^\mu \rightarrow \lambda x^\mu, \; z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal Invariance + Confinement at large distances

↓

Semi-Classical QCD / Wave Equations

↓

Light Front Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS_{d+1}:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

L: light-front orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_c}) = 0.$$

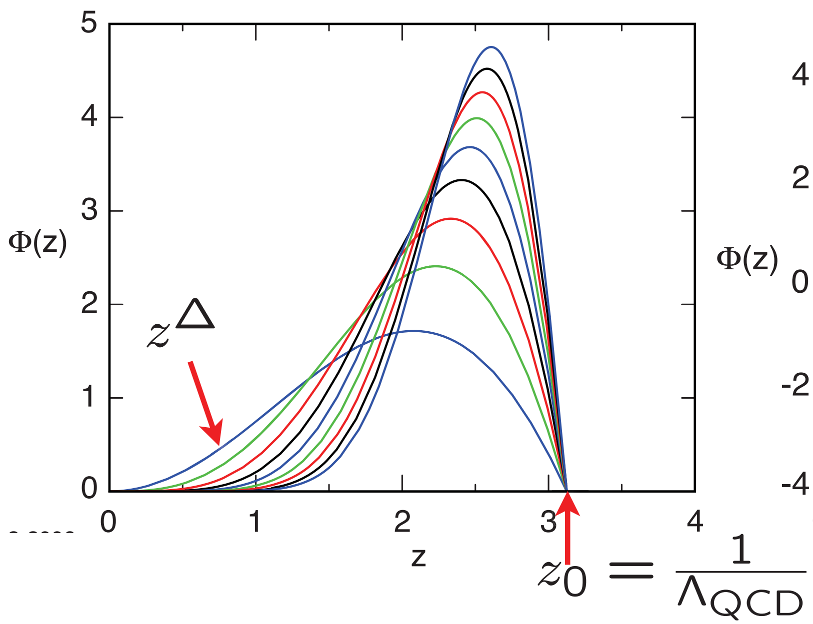
Match fall-off at small z to conformal twist-dimension at short distances

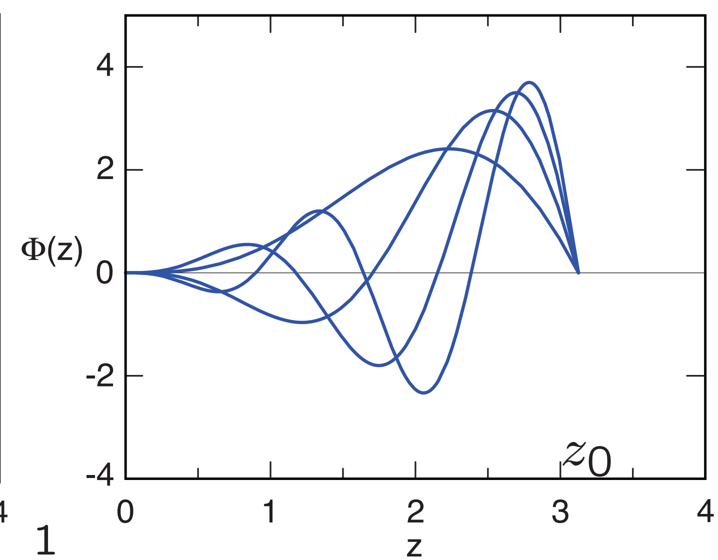
twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$





$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

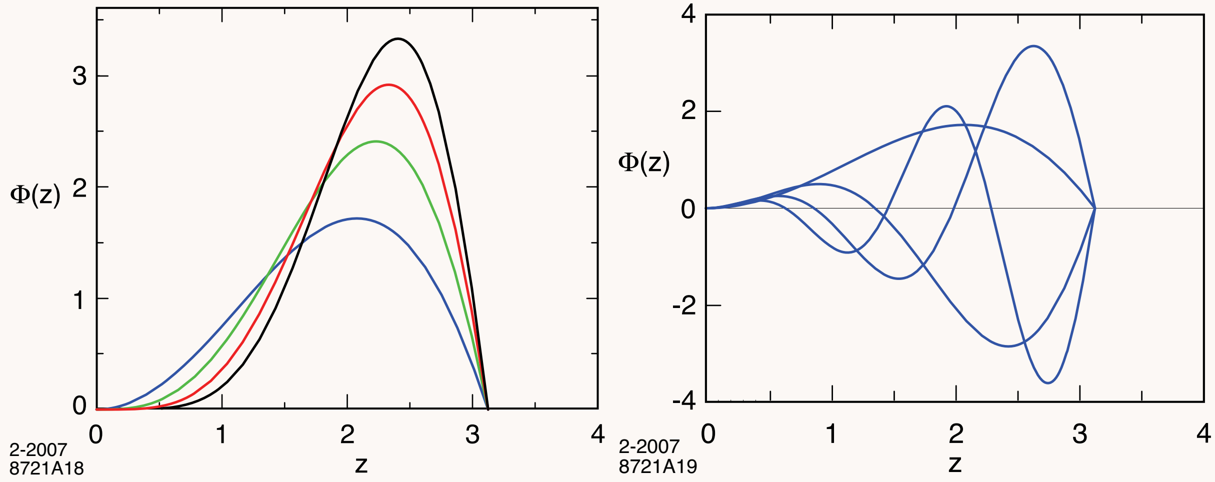


Fig: Orbital and radial AdS modes in the hard wall model for Λ_{QCD} = 0.32 GeV .

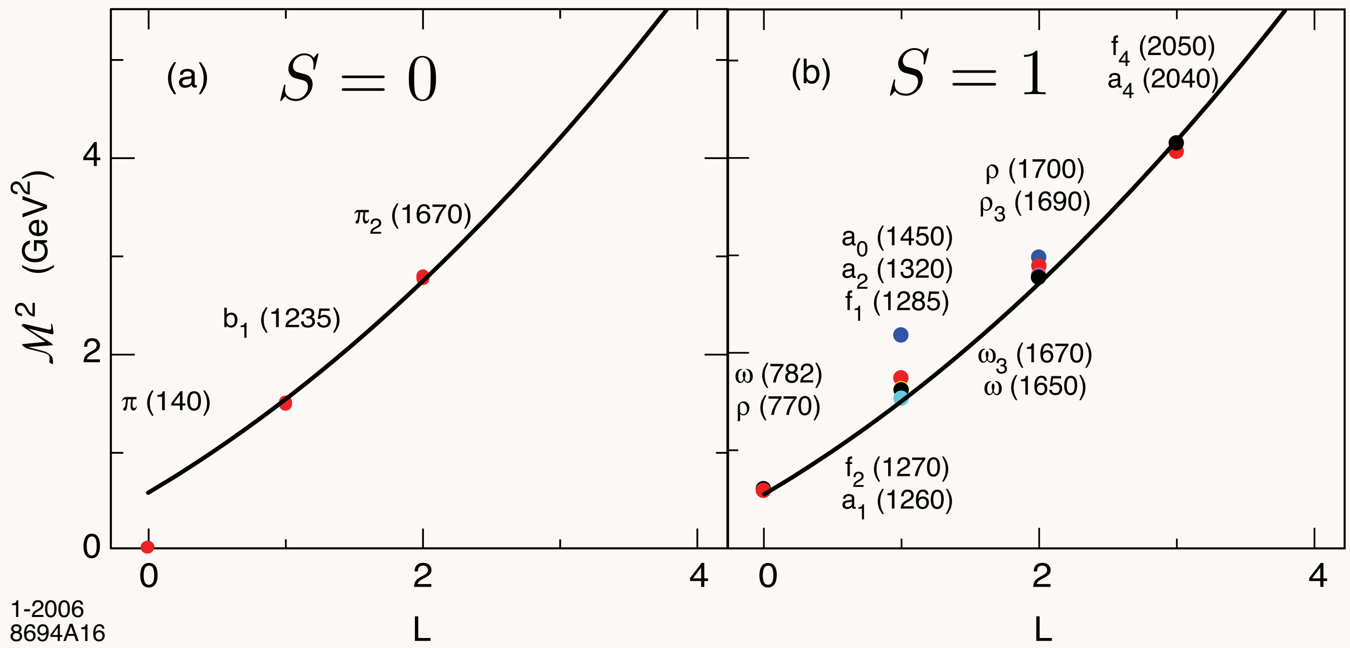


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Soft-Wall Model

$$S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \qquad \varphi(z) = \pm \kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(3 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action Dilaton-Modified AdS_5

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

Quark separation increases with L

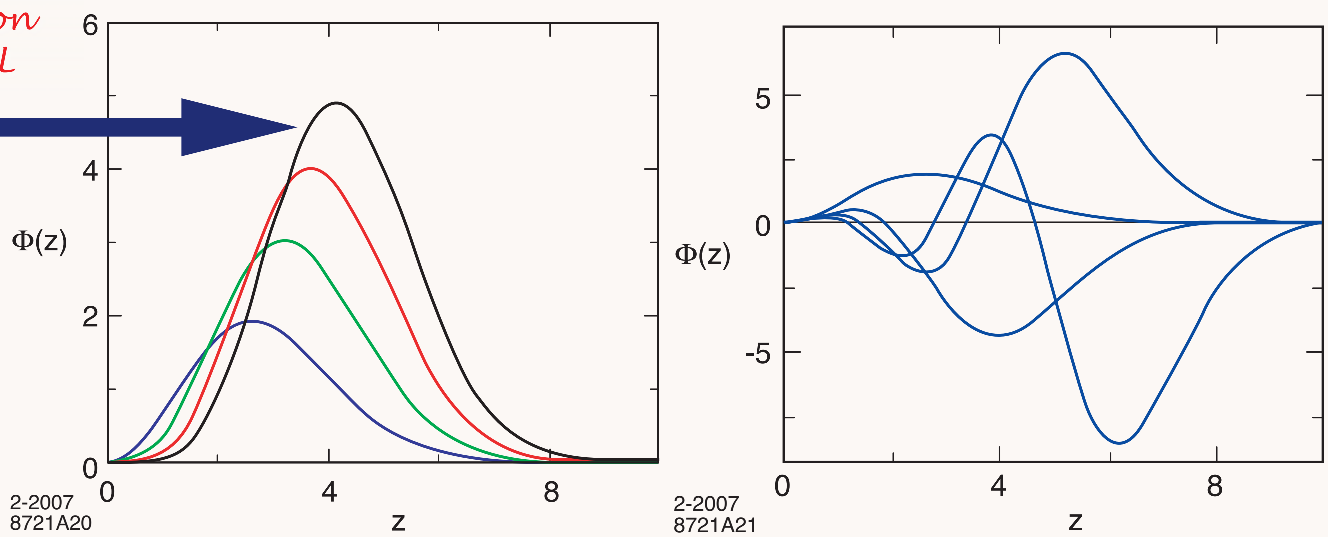
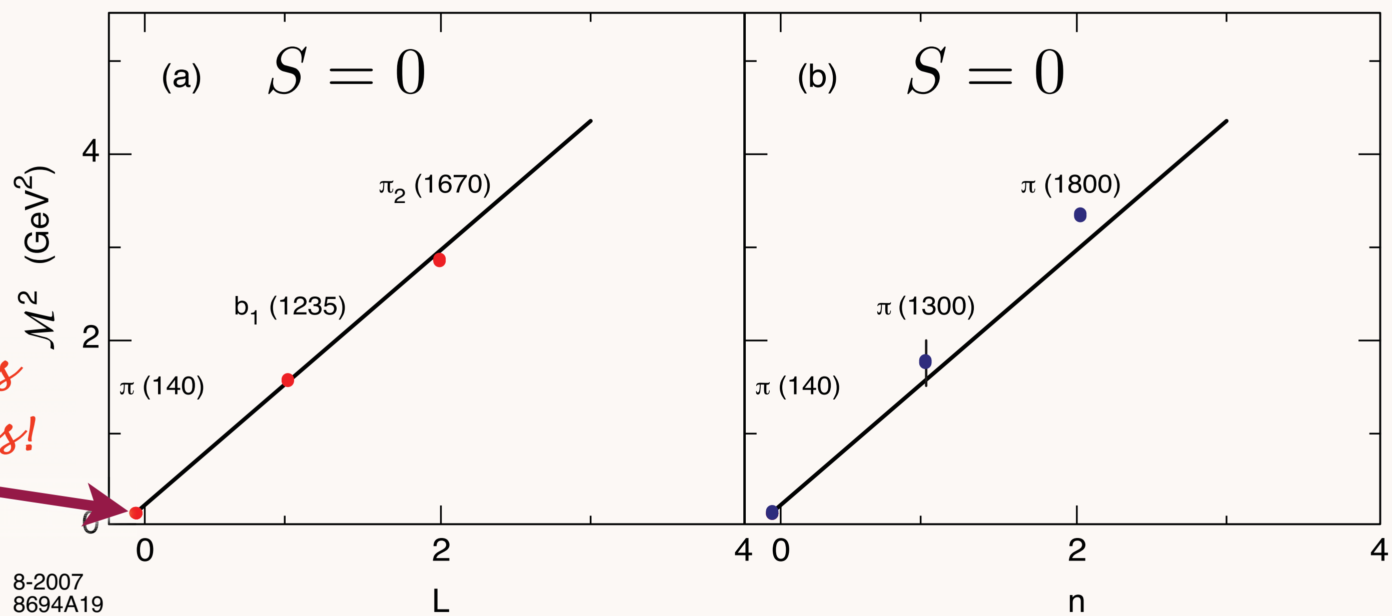


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Pion has zero mass!



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

$$m_q = 0$$

Higher-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

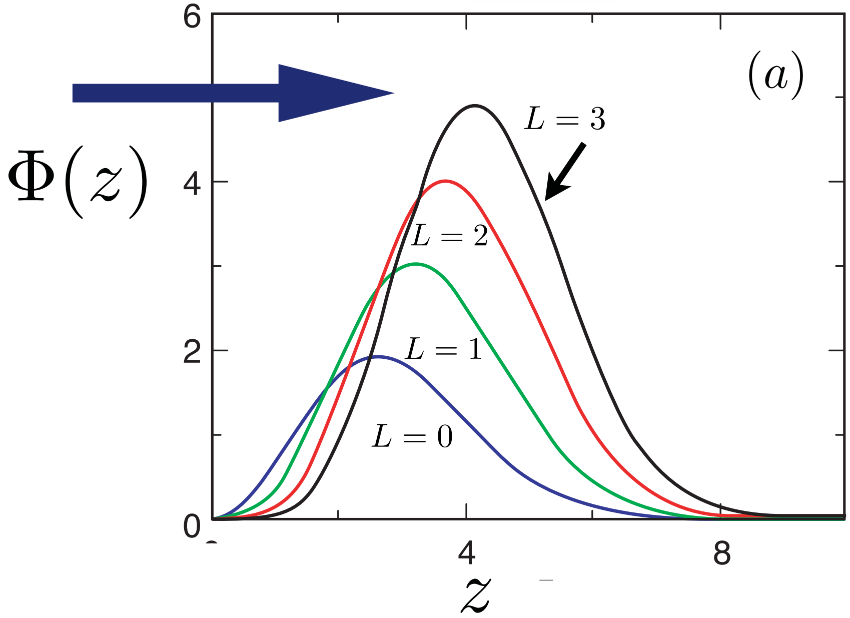
$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

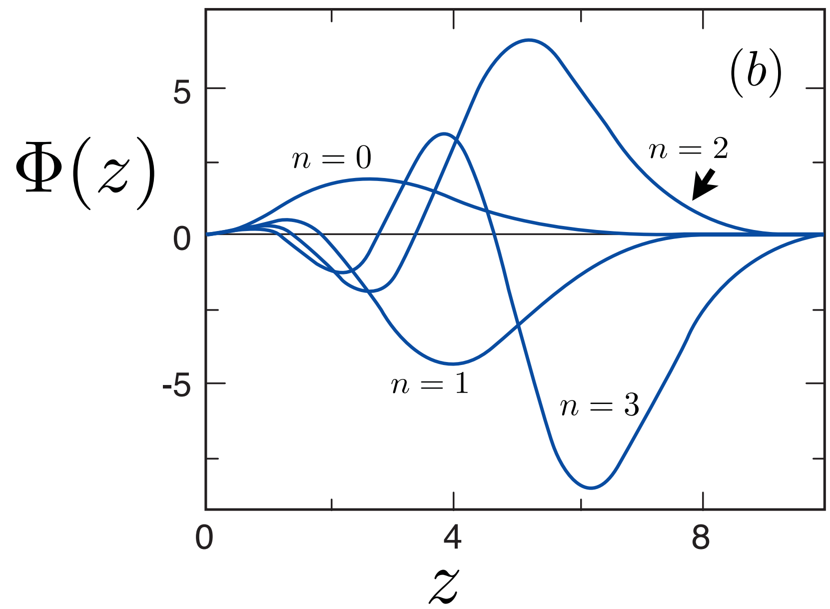
we find the LF wave equation

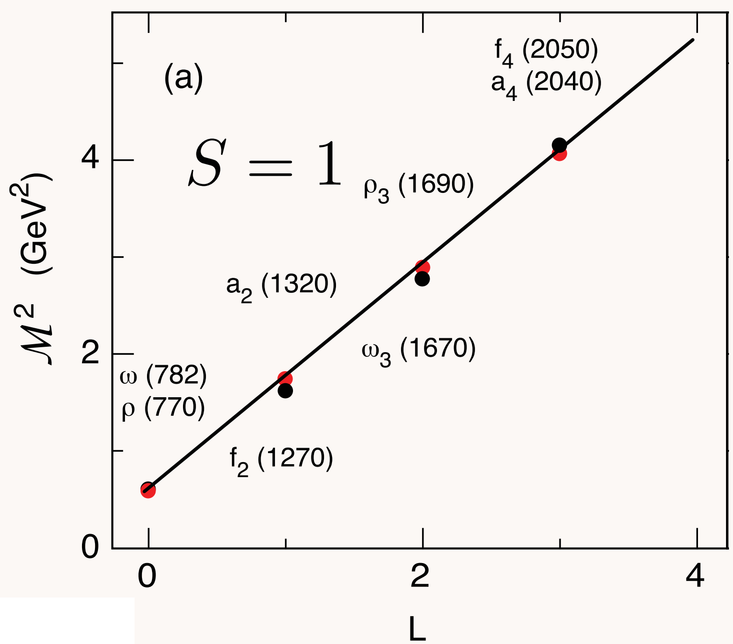
$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

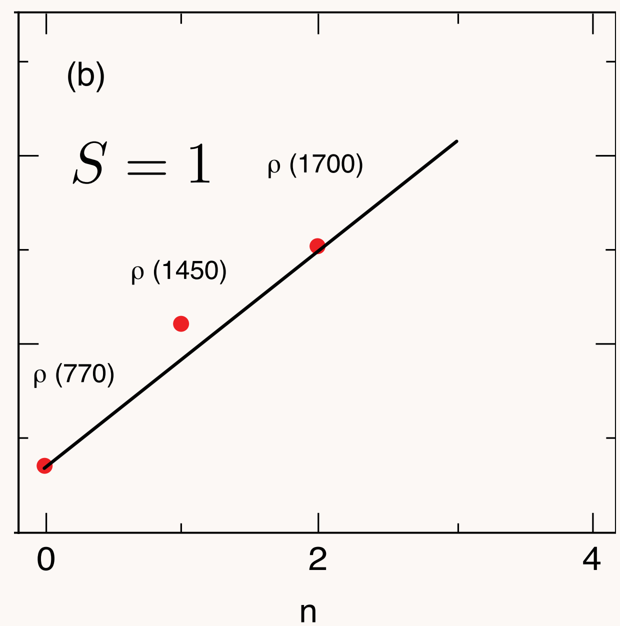
with $(\mu R)^2 = -(2-J)^2 + L^2$

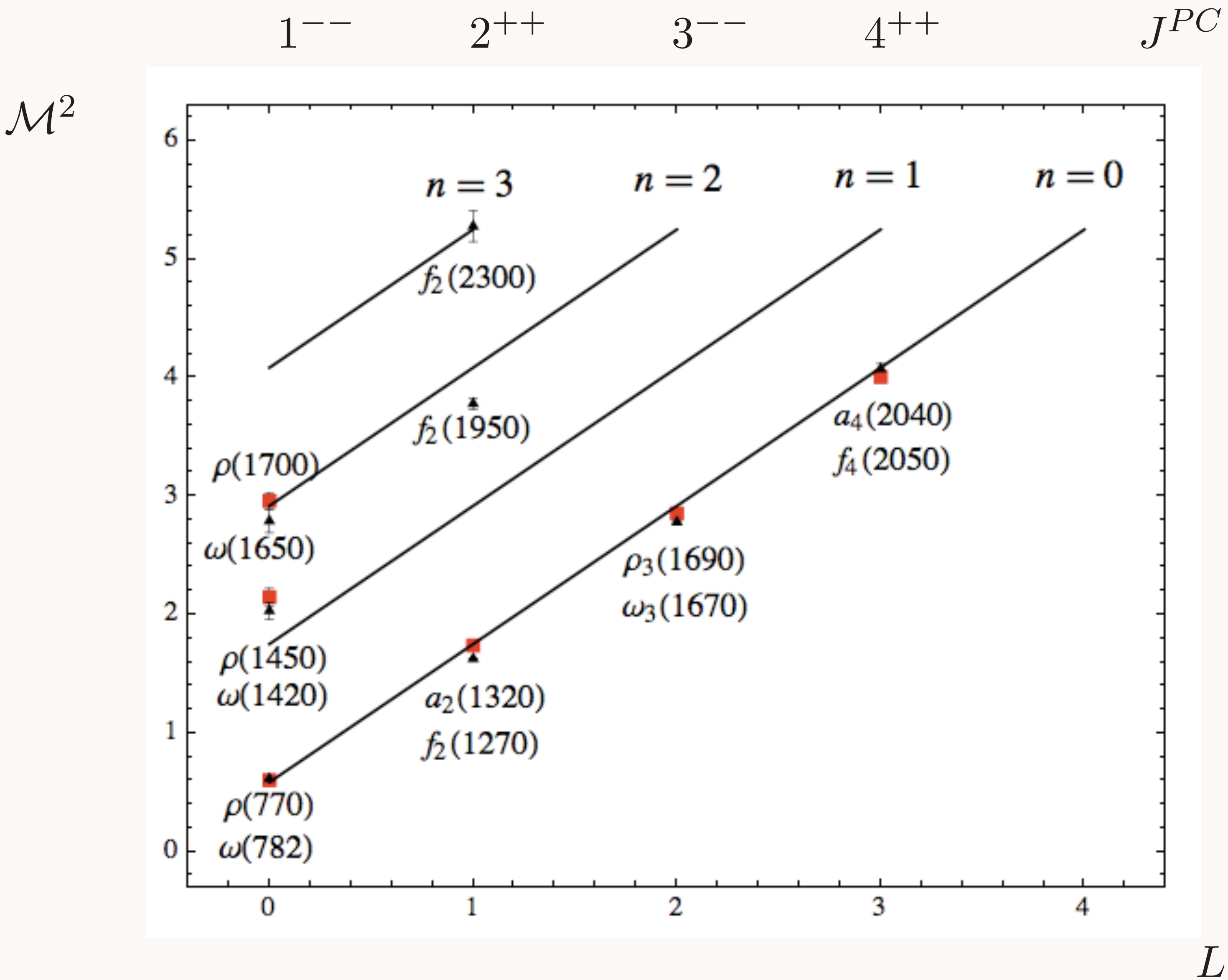
Quark separation increases with L











Parent and daughter Regge trajectories for the $I=1$ ρ-meson family (red)

and the $I=0$ ω-meson family (black) for $\kappa=0.54$ GeV

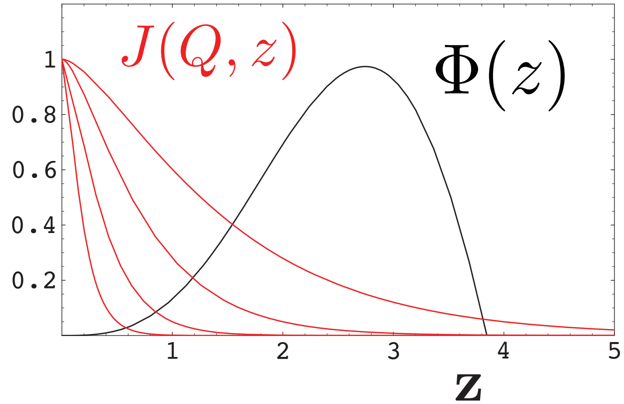
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

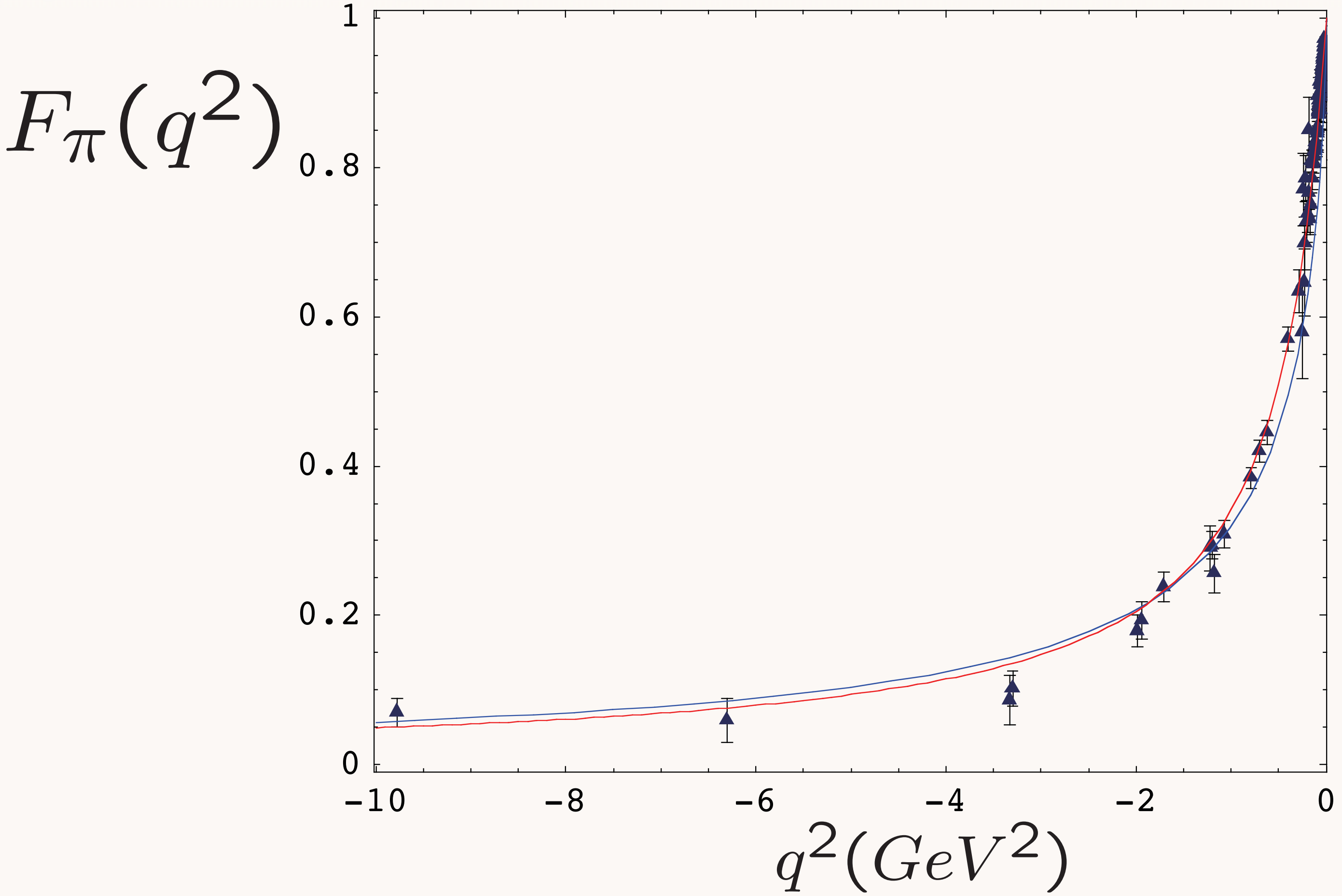
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp) \, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp \, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp \, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0\left(bqx\right) \, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} \, H(Q^2,z) \, |\Phi_\pi(z)|^2 \, ,$$

Abidin & Carlson

where $H(Q^2,z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2,z)$

$$H(Q^2,z) = 2 \int_0^1 x \, dx \, J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} \, J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

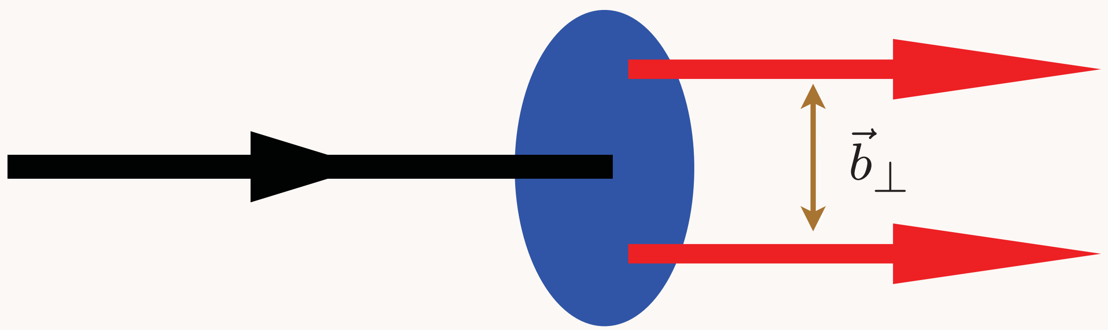
$$\left|\tilde\psi_{q\bar q/\pi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

Identical to LF Holography obtained from electromagnetic current

LF(3+1) AdS$_5$

$\psi(x, \vec{b}_\perp)$ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ z



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

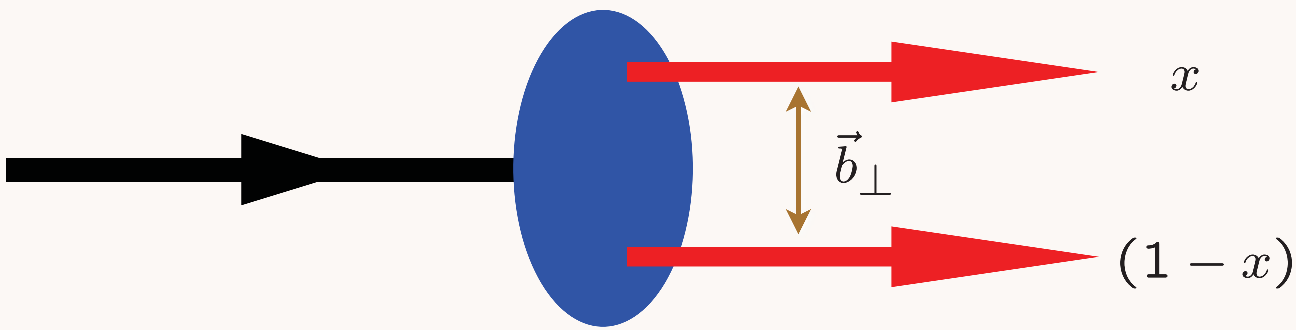
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

soft wall
confining potential:

G. de Teramond, sjb

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

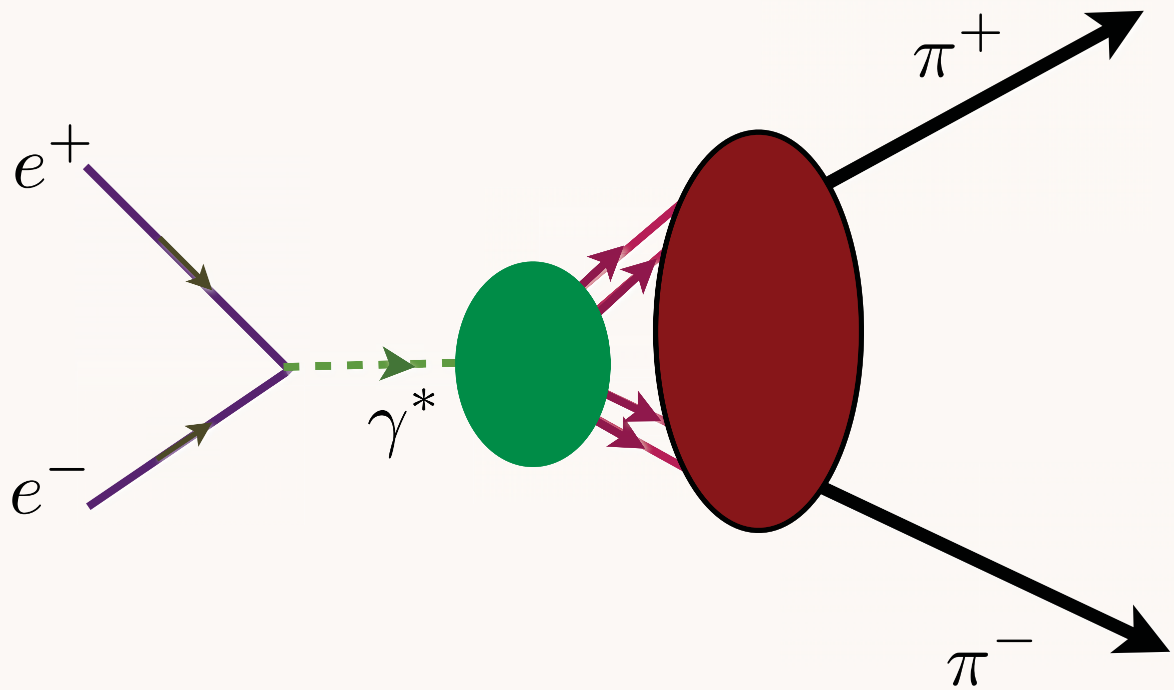
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current bring in higher Fock states and more vector meson poles



Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \qquad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \qquad N = 3,$$

$$\cdots$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \qquad N,$$

Positive Dilaton Background $\exp\left(+\kappa^2 z^2\right)$ $\qquad \mathcal{M}_n^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \qquad Q^2 \to \infty$$

Constituent Counting

Carl E. Carlson　　Zainul Abidin

AdS/CFT now extensive field---apologies for all omitted references
Original 1997 Maldacena paper has 6016 citations

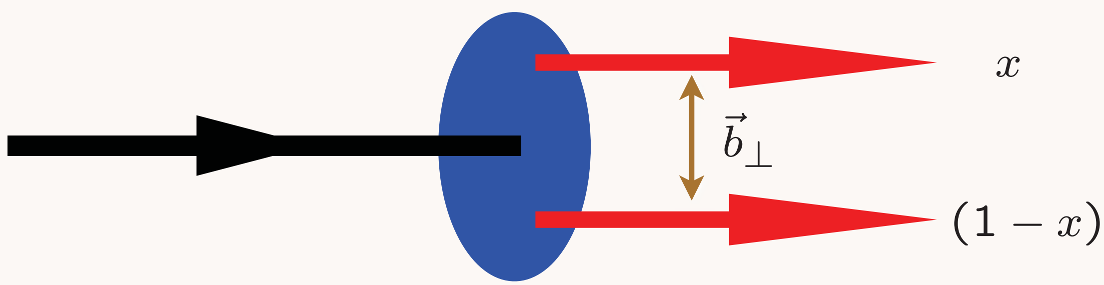
Calculations of form factors: "fancy" Start from string theory, develop QCD analogs on lower dimensional branes	Sakai & Sugimoto
"Bottom-up" Anticipate what 5D Lagrangian must be (guess), directly involving desired rho, pi, a1, ... fields and connect to matching QCD structures	Erlich et al. Da Rold & Pomarol
EM form factors in "bottom-up" approach	Brodsky & de Teramond Radyushkin & Grigoryan
Gravitational form factors in bottom-up approach	Zainul Abidin & me
Soft-wall	Karch, Katz, Son, and Stephanov Batell, Gherghetta, and Sword

LF(3+1) AdS5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \longleftrightarrow z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

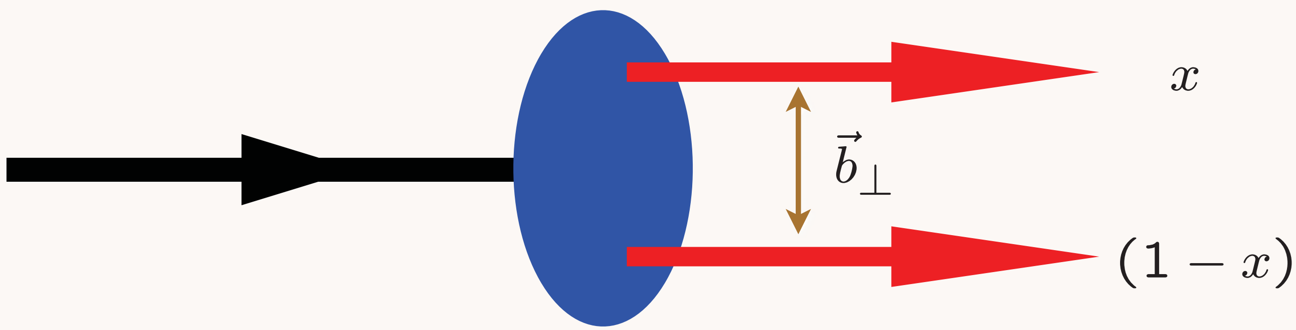
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables

$$(\vec{\zeta},\varphi), \; \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp : \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

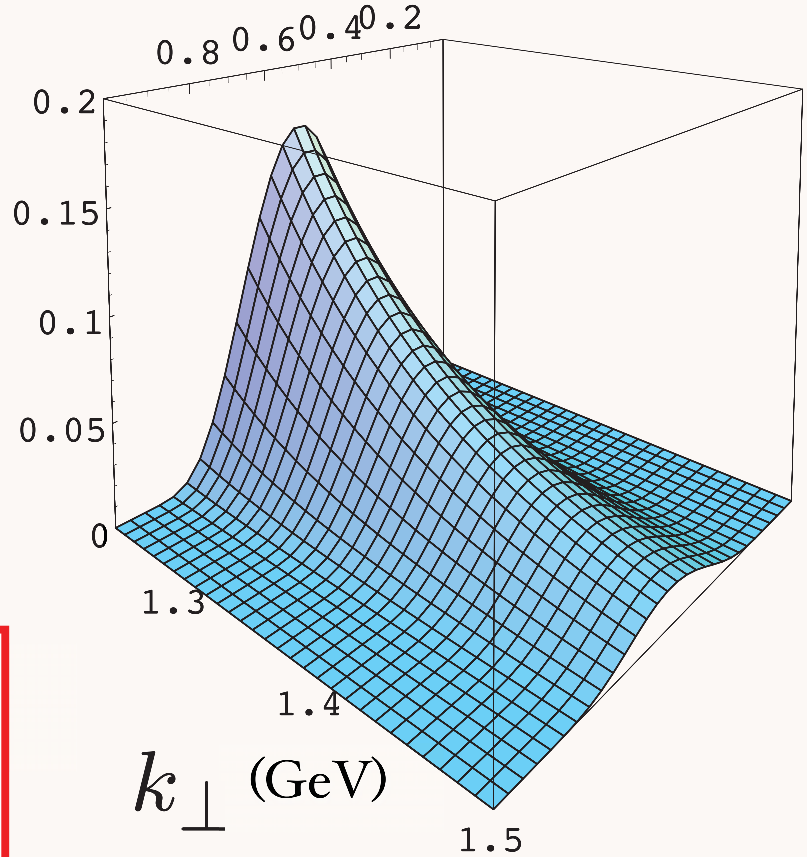
Confining AdS/QCD potential

Semiclassical first approximation to QCD

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



$k_\perp$ (GeV)

"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling $k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

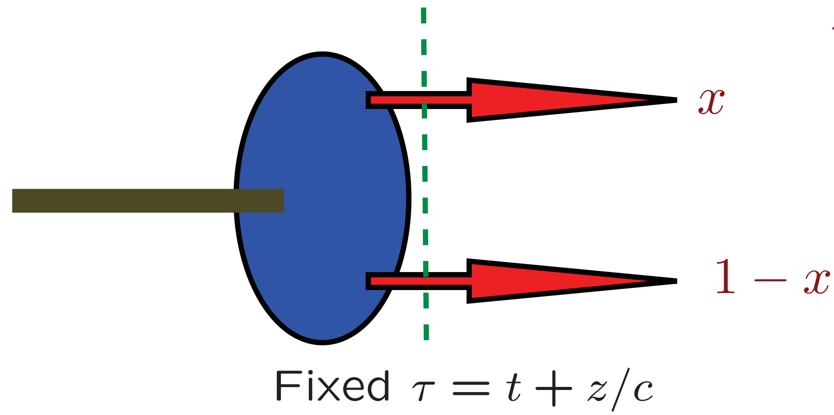
Connection of Confinement to TMDs

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

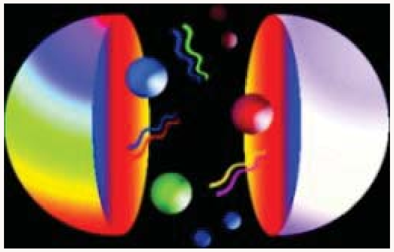
Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



From Nick Evans

- Action for massive fermionic modes on AdS_5:

$$S[\overline{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{\rm QCD}\right) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{\rm QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{\rm QCD}$$

  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

Hoyer's Linear Potential

- Solution $(\mu R = \nu + 1/2,\, d = 4)$
  $\nu = L + 1$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2)$$
$$\Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

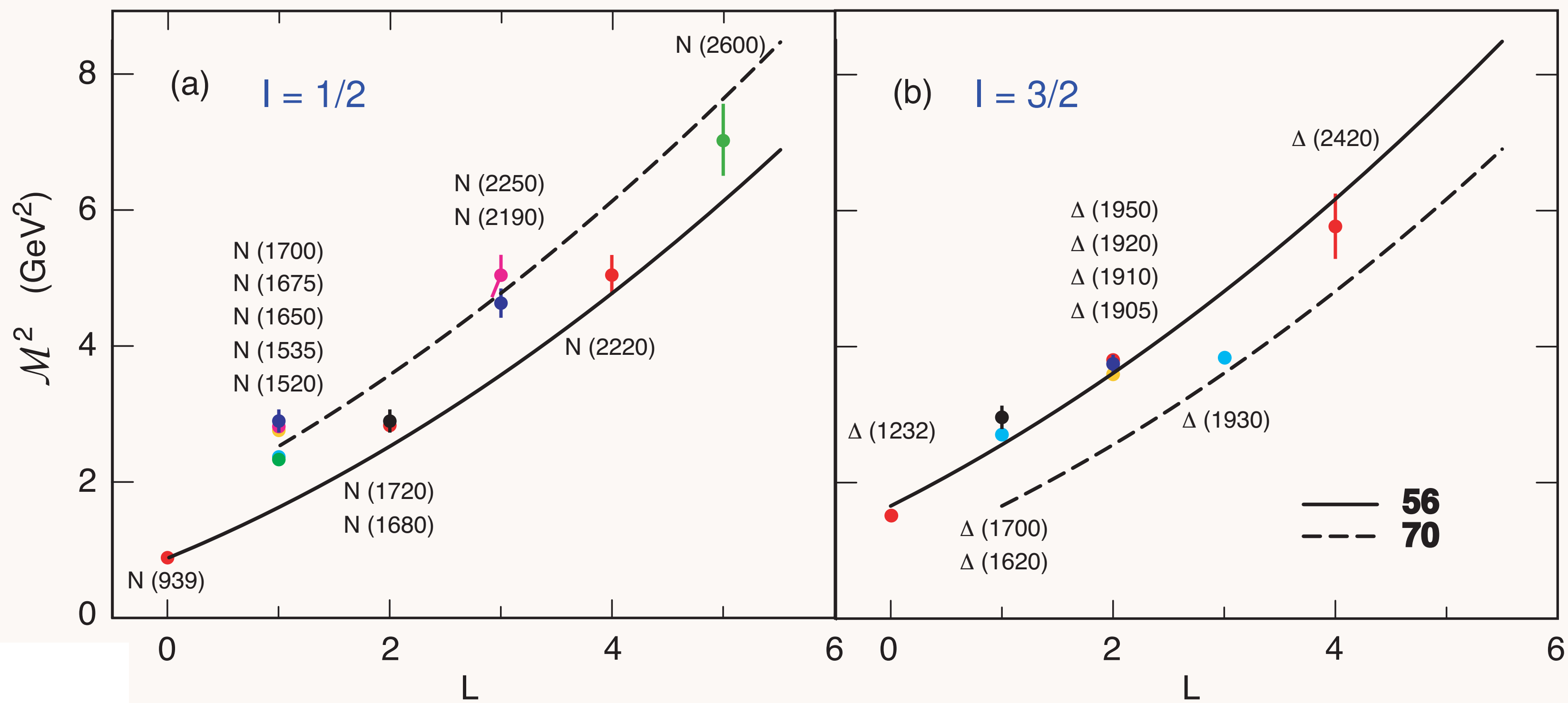


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

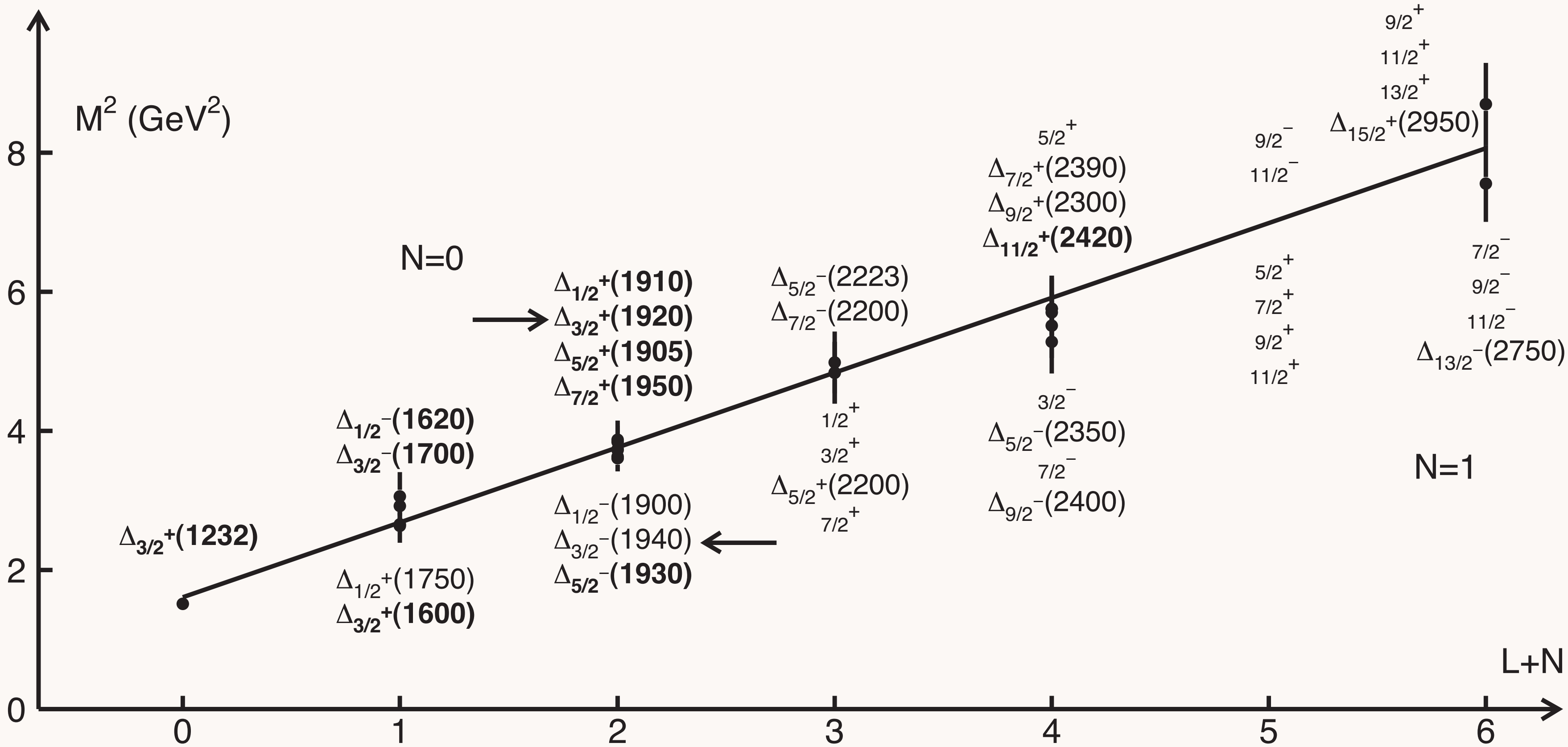
$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

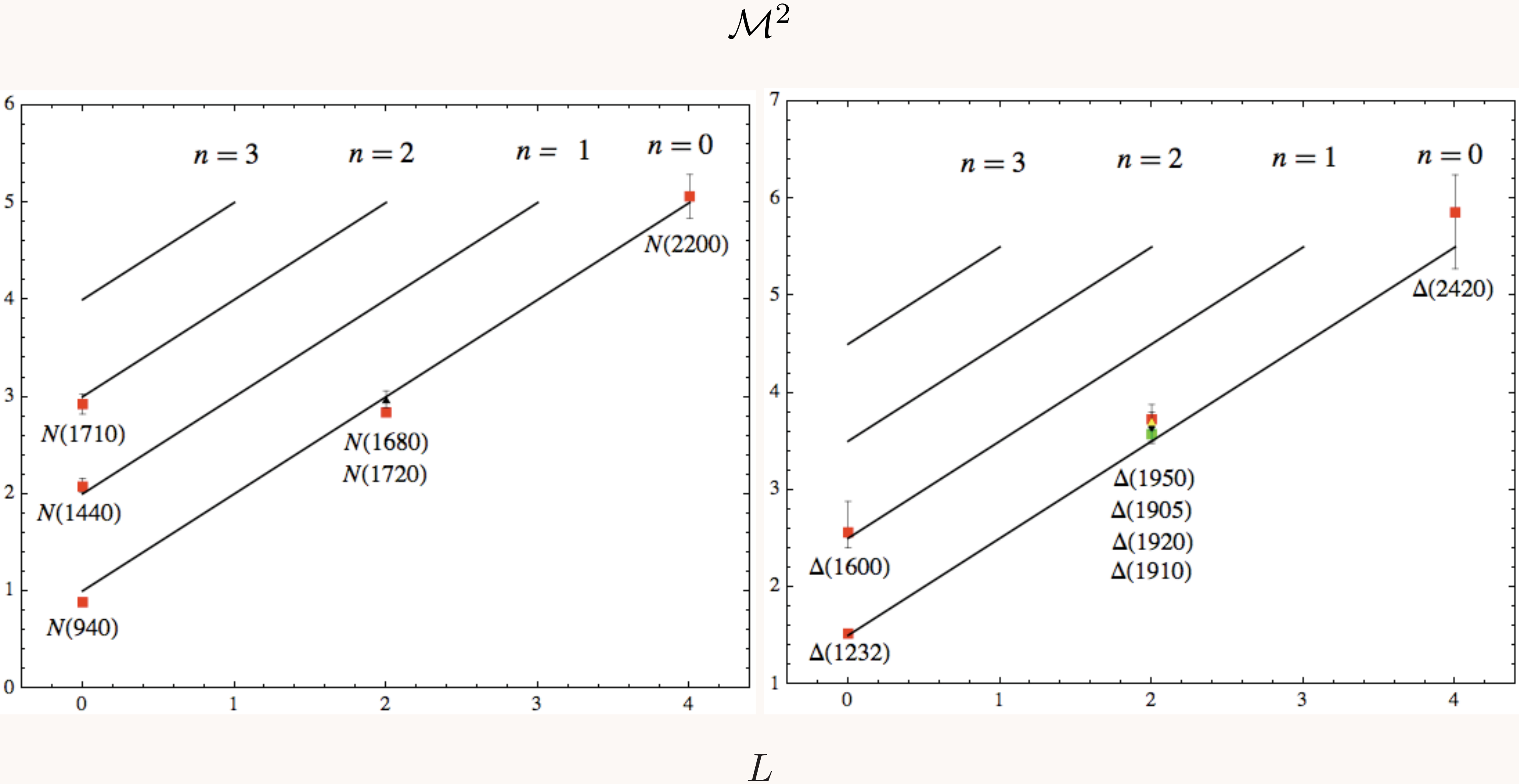


E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

- **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

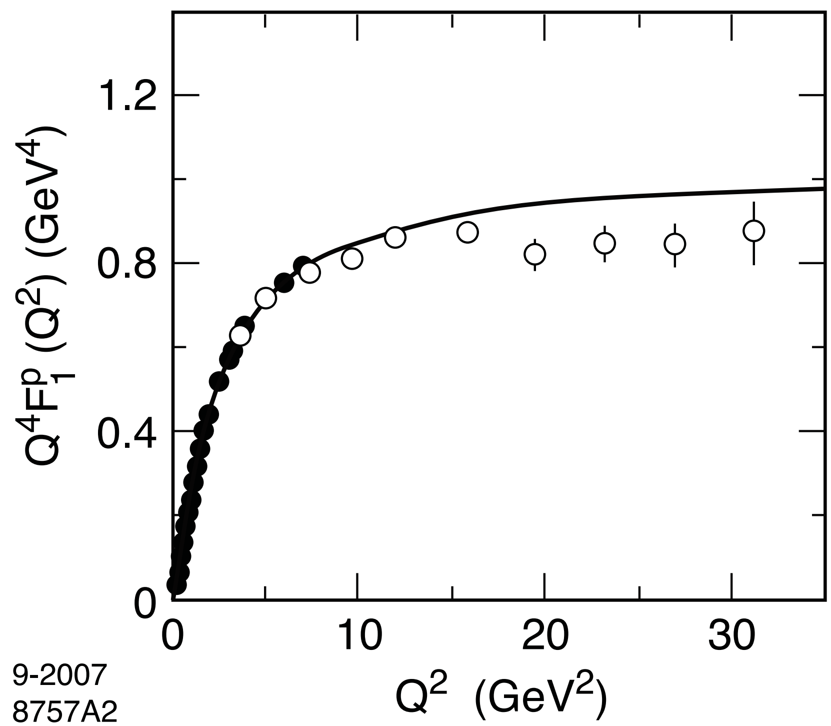
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

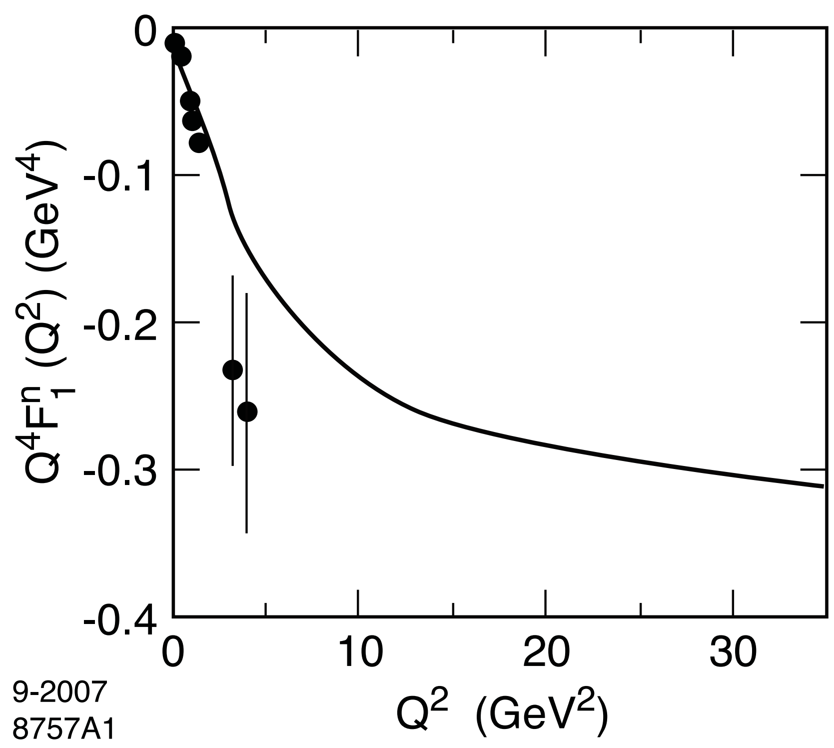
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

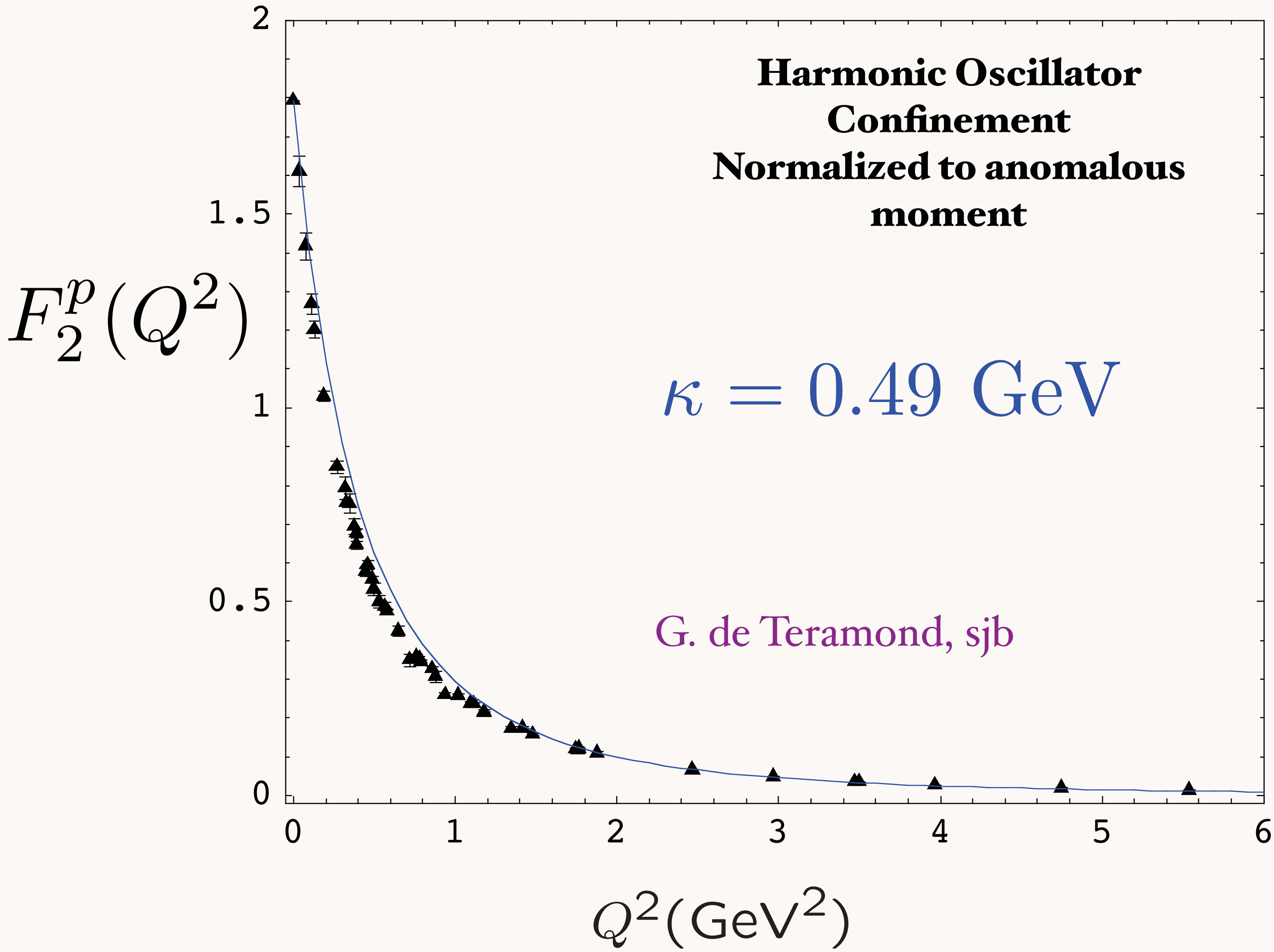


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

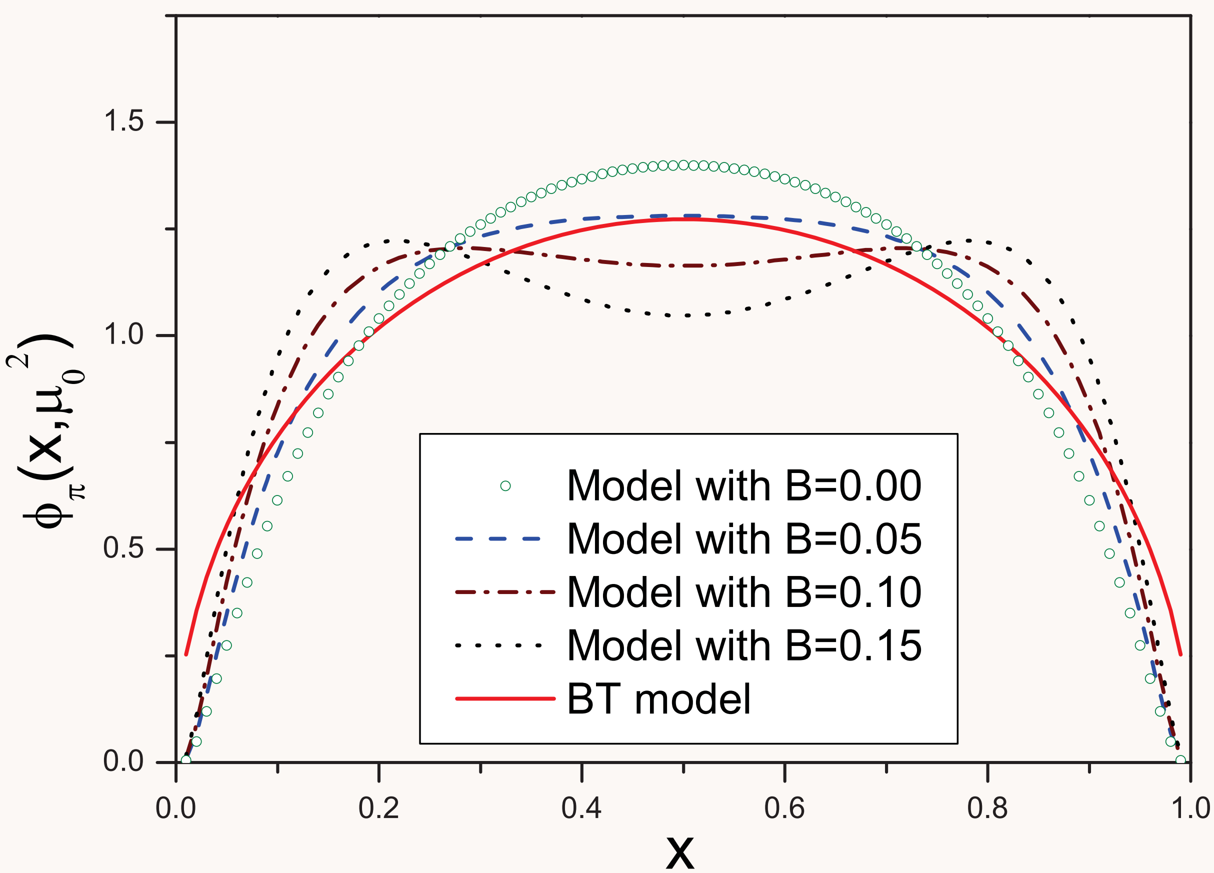
Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



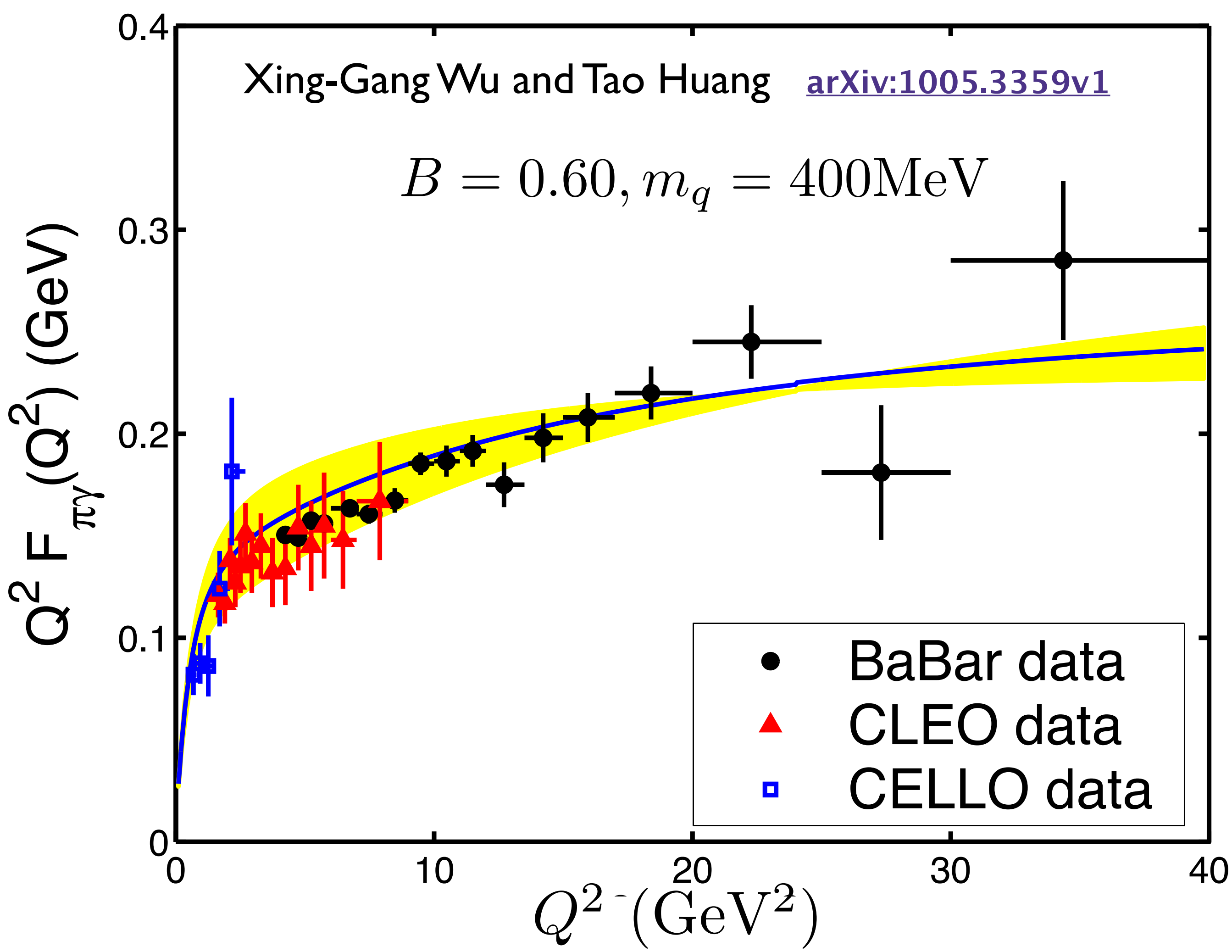
Xing-Gang Wu and Tao Huang arXiv:1005.3359v1



$$\phi_M(x,\mu_0^2) = \mathcal{C}\sqrt{x(1-x)}\exp\left[-\frac{1}{2\kappa^2}\left(\frac{m_u^2}{x}+\frac{m_d^2}{1-x}\right)\right]\left[1-\exp\left(-\frac{\mu_0^2}{2\kappa^2 x(1-x)}\right)\right]$$

Comparison of CZ and AdS/QCD (BT) distribution amplitudes

$$\Psi^R_{q\bar{q}}(x,\mathbf{k}_\perp) = A\left(1 + B\times C_2^{3/2}(2x-1)\right)\exp\left[-\frac{\mathbf{k}_\perp^2 + m_q^2}{8\beta^2 x(1-x)}\right],$$



$$F^{(V)}_{\pi\gamma}(Q^2) = \frac{1}{4\sqrt{3}\pi^2}\int_0^1\int_0^{x^2Q^2}\frac{dx}{xQ^2}\left[1 - \frac{C_F\alpha_s(Q^2)}{4\pi}\left(\ln\frac{\mu_f^2}{xQ^2+k_\perp^2} + 2\ln x + 3 - \frac{\pi^2}{3}\right)\right]$$
$$\Psi_{q\bar{q}}(x,k_\perp^2)dk_\perp^2,$$

Lepage and Sjb; Li, Mishima, Nadi

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

↓

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

Holography

↓

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

↓

Hadron Spectra, Wavefunctions, Dynamics

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

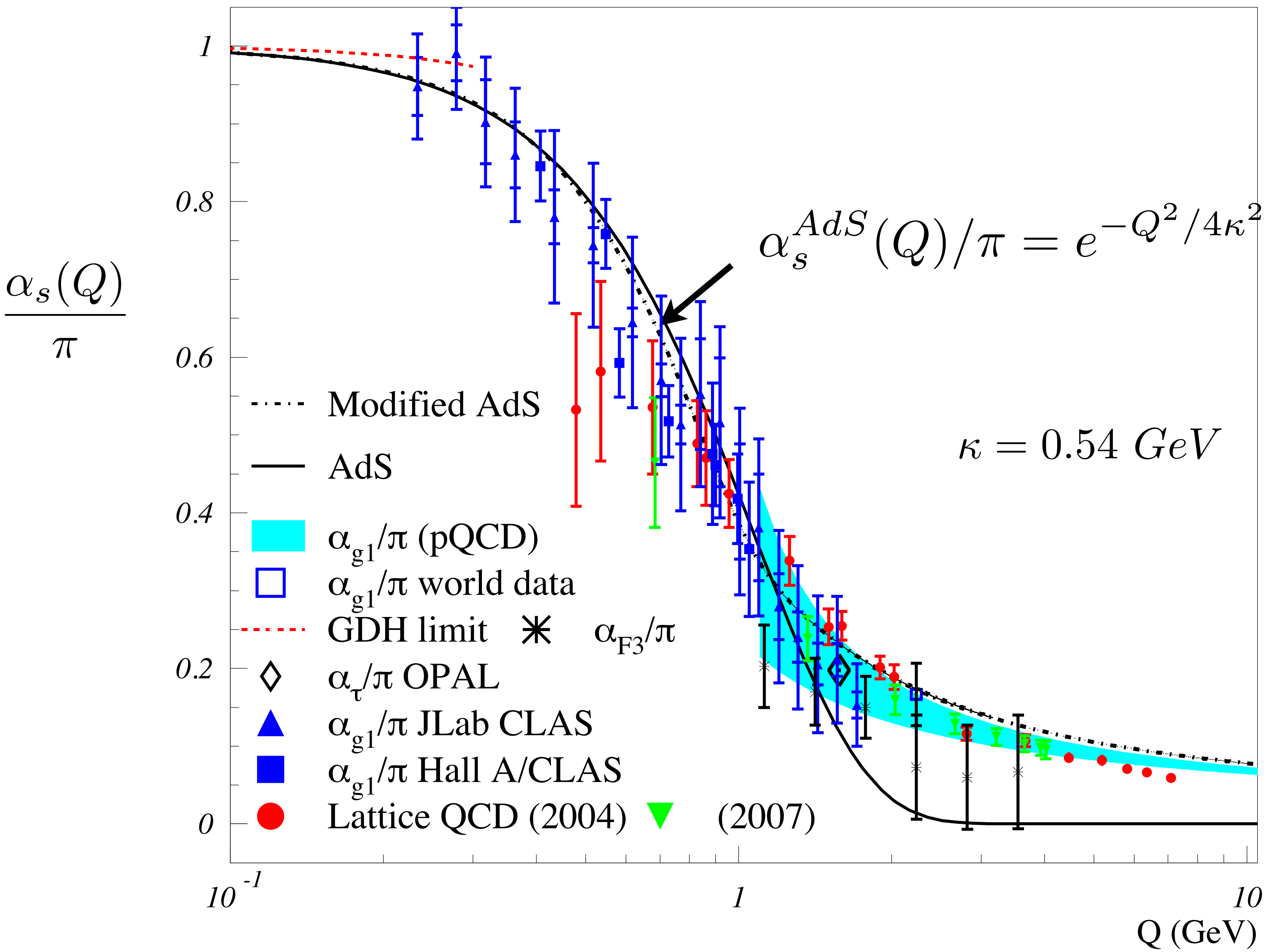
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

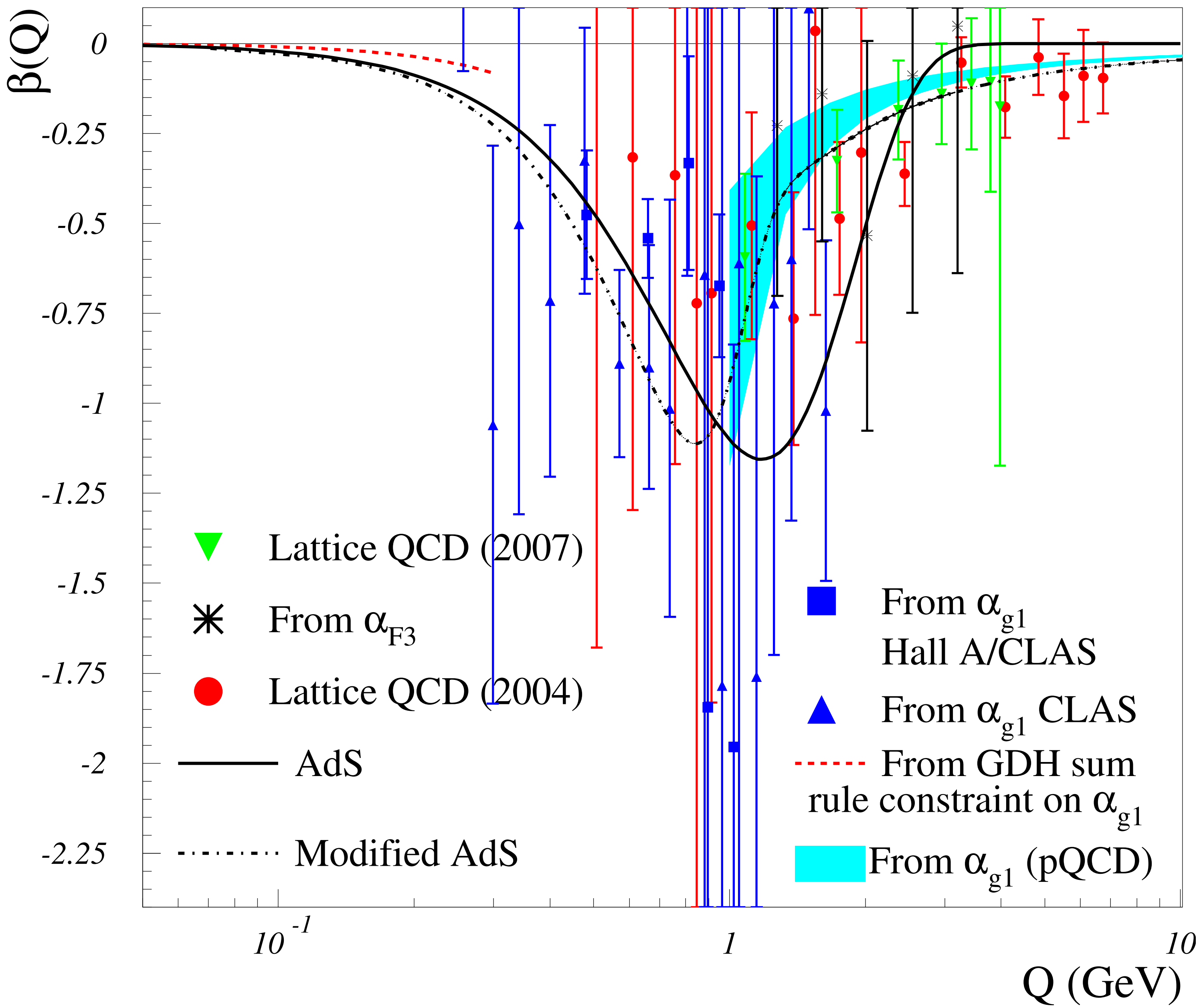
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Deur, de Teramond, sjb

$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$

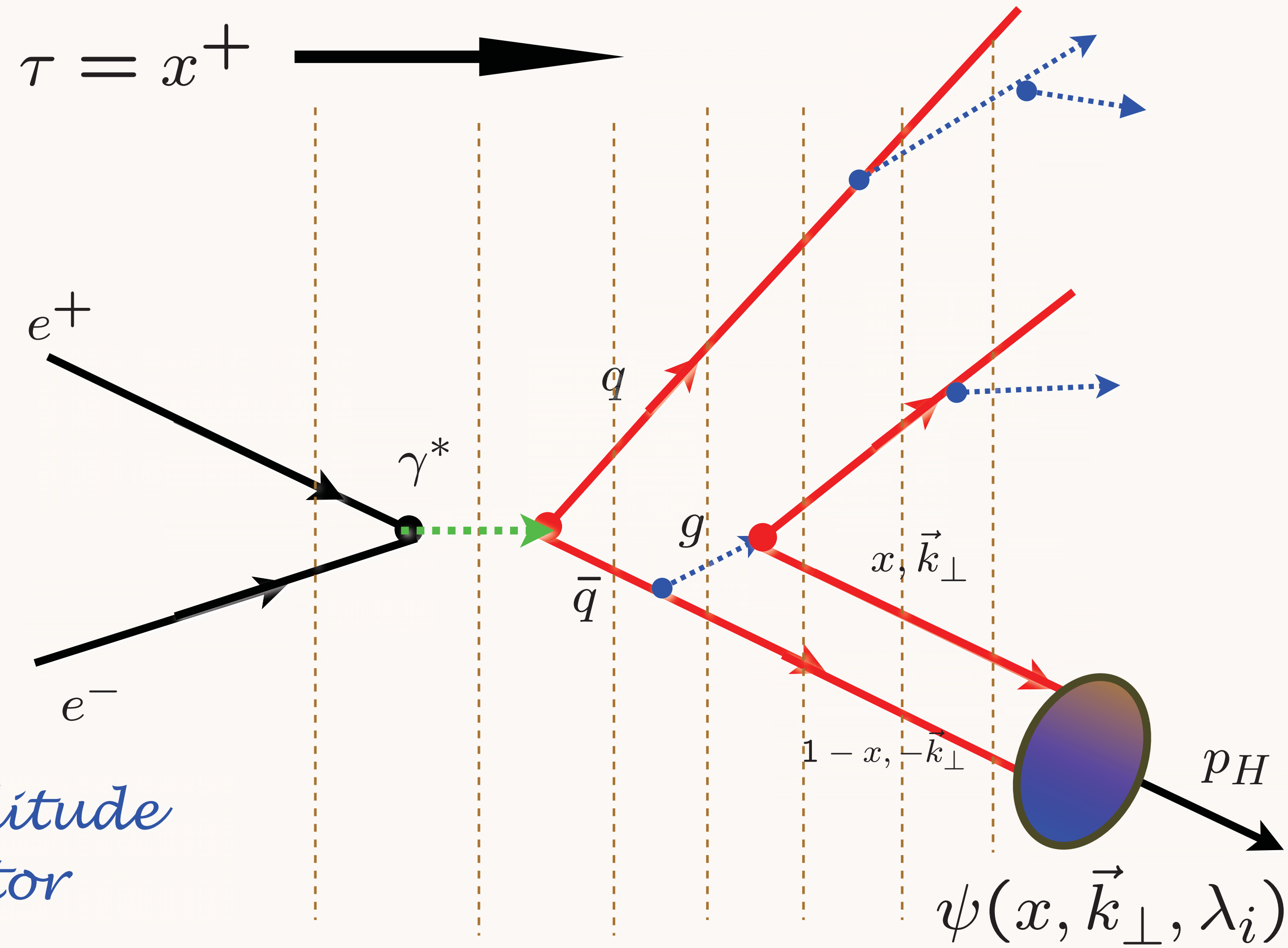


Deur, de Teramond, sjb

Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion (m_q =0)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S. Spectrum is independent of S**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Chiral Symmetry Breaking in AdS/QCD

Erlich et al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2 m_q < \bar\psi \psi > \times \int dz \; \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD: confined condensate**
- **Suggests "In-Hadron" Condensates**

de Teramond, Shrock, sjb

In presence of quark masses the Holographic LF wave equation is $(\zeta = z)$

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \tag{1}$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \tag{2}$$

The parameter a is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x}\langle\bar{\psi}\psi\rangle z^3,} \tag{3}$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2\sum_i m_i \langle\bar{\psi}\psi\rangle\langle z^2\rangle + \langle\bar{\psi}\psi\rangle^2\langle z^4\rangle, \tag{4}$$

where we have used the sum over fractional longitudinal momentum $\sum_i x_i = 1$.

Mass shift from dynamics inside hadronic boundary

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

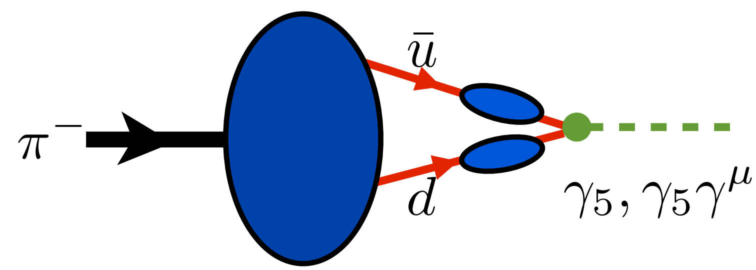
Light-Front Formalism

Bethe-Salpeter Analysis

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \; \frac{1}{2}\left[T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P + q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P - q))\right]$$

Maris, Roberts, Tandy

f_H Meson Decay Constant
T_H flavor projection operator,
$Z_2(\Lambda)$, $Z_4(\Lambda)$ renormalization constants
$S(p)$ dressed quark propagator
$\Gamma_H(q;P) = F.T.\langle H|\psi(x_a)\bar{\psi}(x_b)|0\rangle$
Bethe-Salpeter bound-state vertex amplitude.



$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q}\rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \; \frac{1}{2}\left[T_H \gamma_5 \mathcal{S}(\frac{1}{2}P + q))\Gamma_H(q;P)\mathcal{S}(\frac{1}{2}P - q))\right]$$

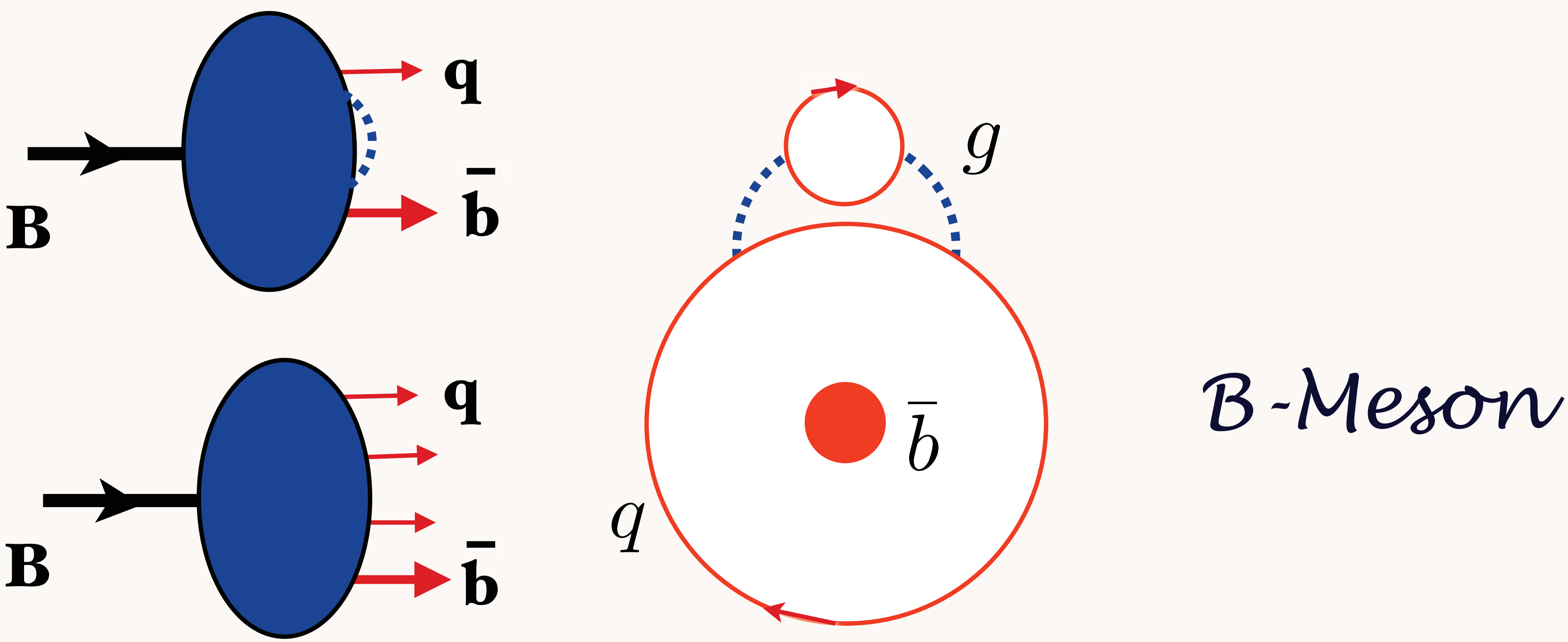
In-Hadron Condensate!

$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \qquad \mathcal{M}_H = \sum_{q\in H} m_q$$

$$m_\pi^2 \propto (m_q + m_{\bar{q}})/f_\pi \qquad \textit{GMOR}$$

Simple physical argument for "in-hadron" condensate

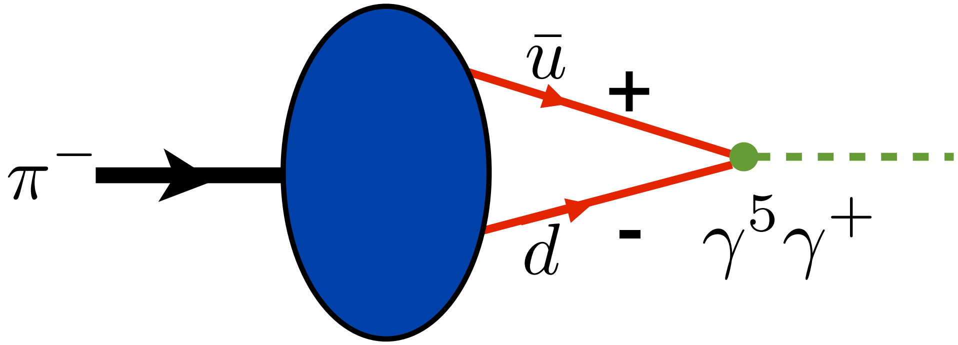
Roberts, Shrock, Tandy, sjb



Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

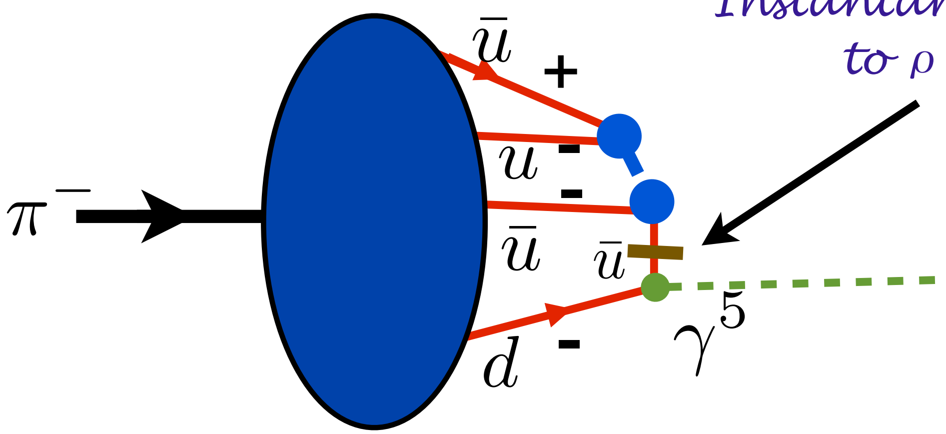
$$< B|\bar{q}q|B > \text{ not } < 0|\bar{q}q|0 >$$

Higher Light-Front Fock State of Pion Simulates DCSB

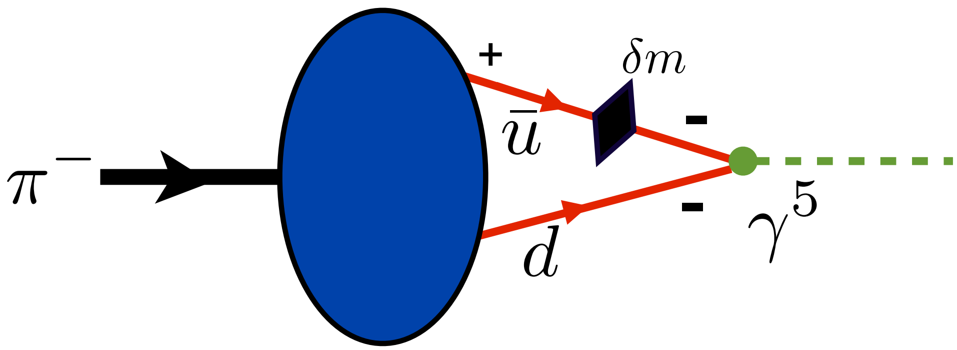


$$f_\pi P^+ =< 0|\bar{q}\gamma^5\gamma^+ q|\pi >$$

Instantaneous quark propagator contribution to ρ_π derived from higher Fock state



$$i\rho_\pi =< 0|\bar{q}\gamma^5 q|\pi >$$



Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb

Essence of the vacuum quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1]SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309
[2]Centre for Particle Physics Phenomenology: CP^3-Origins, University of Southern Denmark, Odense 5230 M, Denmark
[3]Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
[4]Department of Physics, Peking University, Beijing 100871, China
[5]C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794
[6]Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind   Maris, Roberts, Tandy   Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

R. Shrock, sjb

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb

arXiv:0905.1151 [hep- th], Proc. Nat’l. Acad. Sci., (in press);
``Condensates in Quantum Chromodynamics and the Cosmological Constant.”

Quark and Gluon condensates reside within hadrons, not LF vacuum

Maris, Roberts, Tandy

Casher Susskind

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant

Shrock, sjb

"Confined QCD Condensates"

Determinations of the vacuum Gluon Condensate

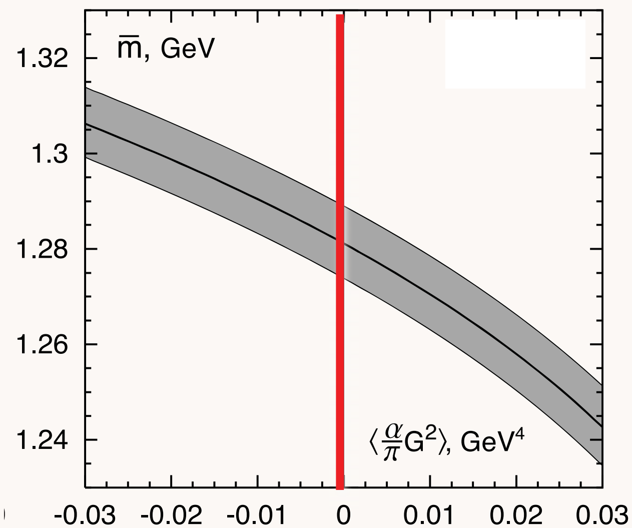
$$< 0|\frac{\alpha_s}{\pi} G^2|0 > [\text{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



Consistent with zero vacuum condensate

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, ...)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

Roberts, Shrock, Tandy, and sjb

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L+S-1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

An analytic first approximation to QCD
AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter κ**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**