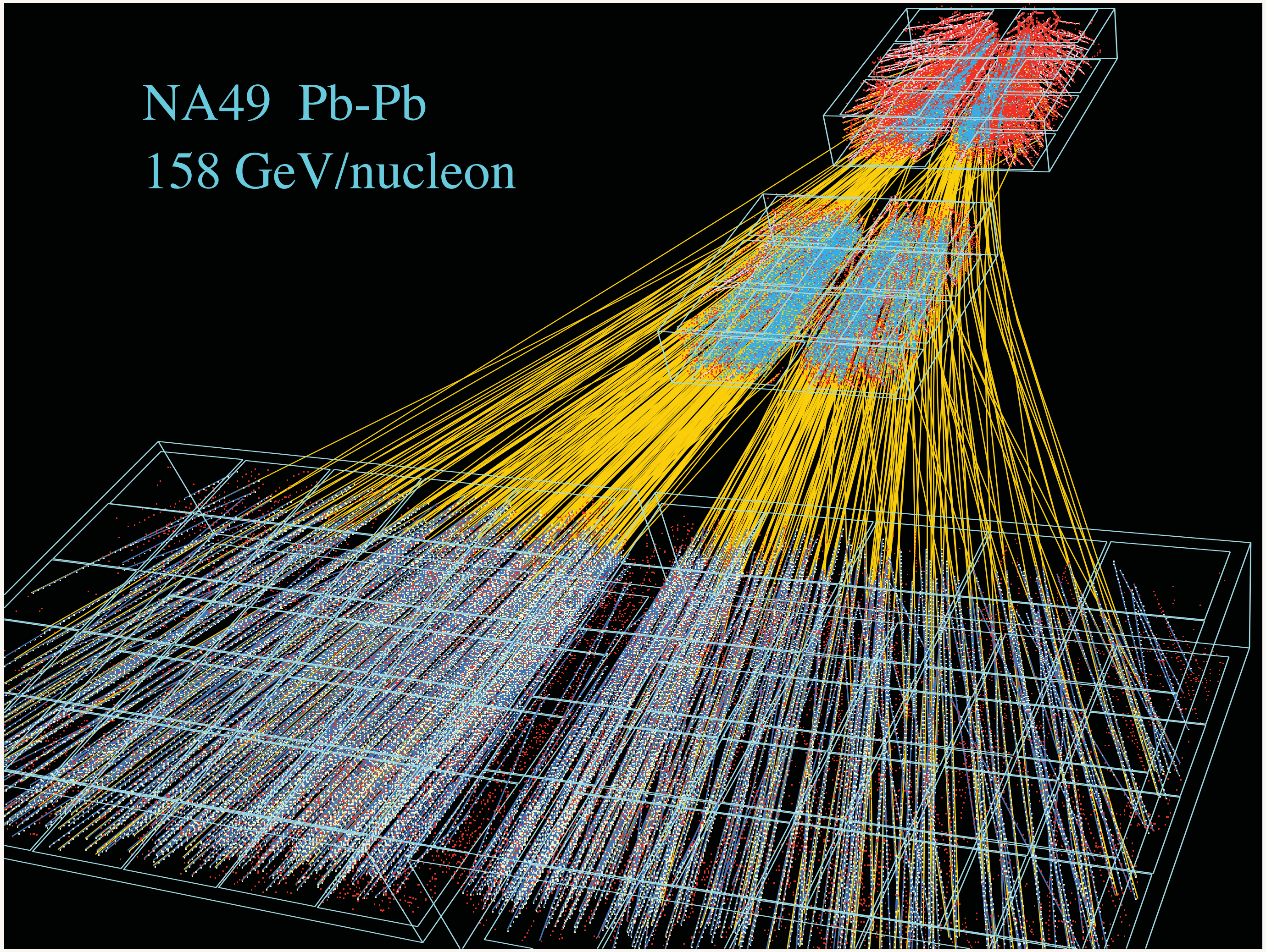
Light-Front Holography and Hadronization at the Amplitude Level



with
Robert Shrock
and
Guy de Teramond

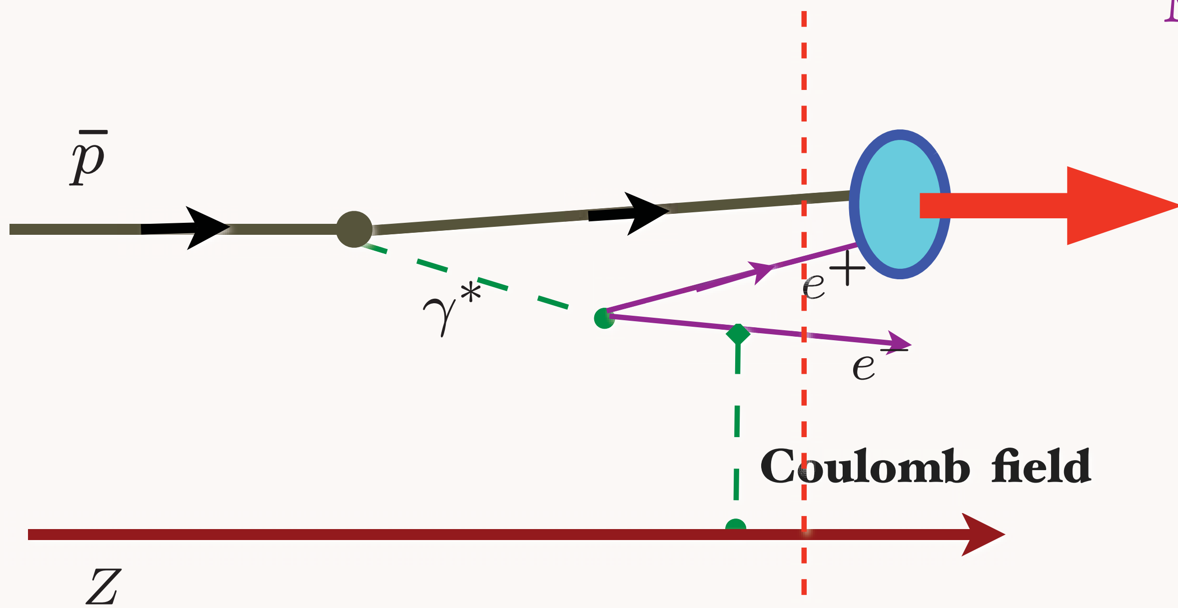
Stan Brodsky , SLAC/IPPP

Rutherford Workshop: New Ideas on Hadronization May 30, 2008

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell ***co-moving positron and antiproton***

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

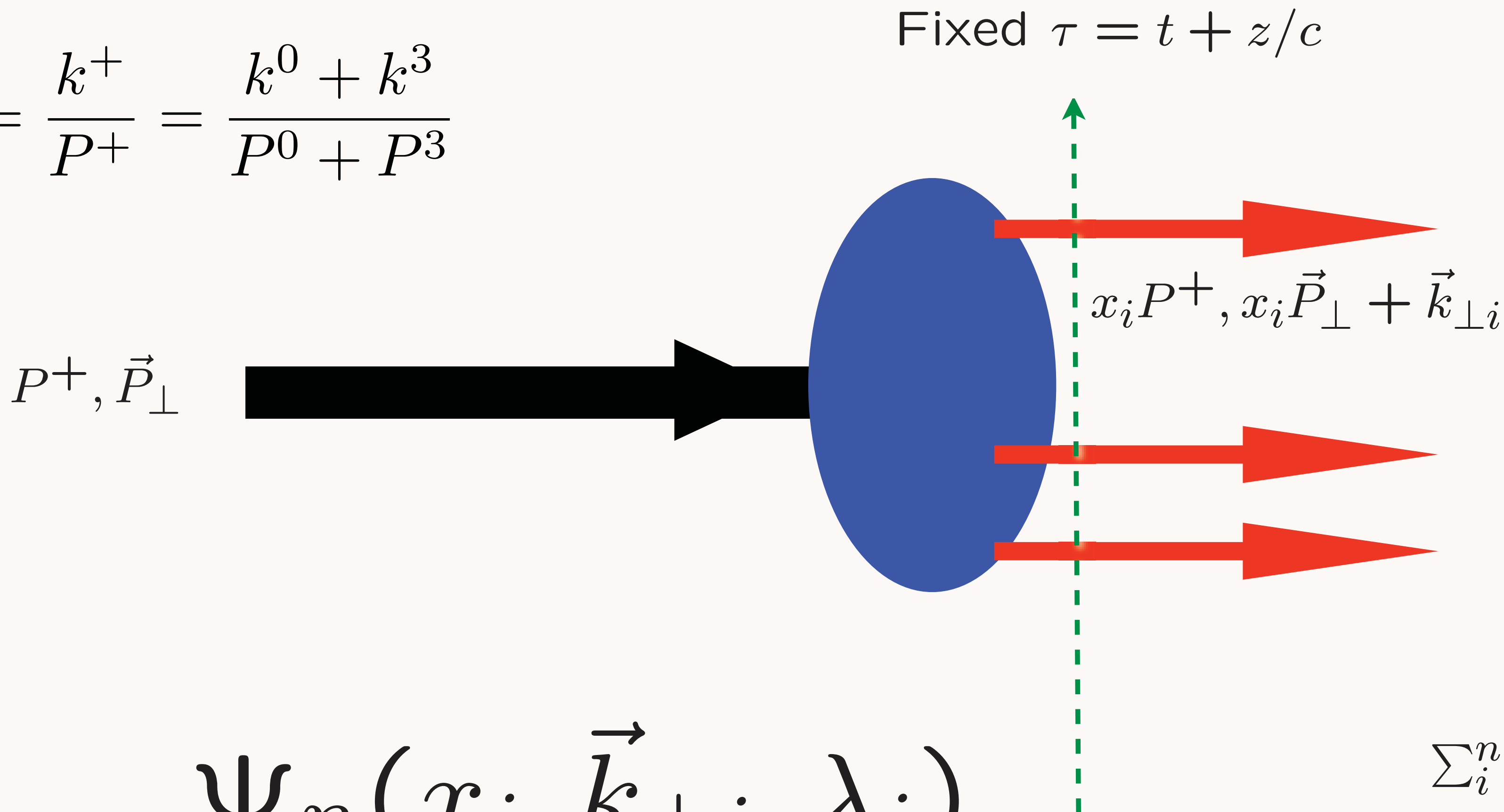
$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Procedures for QCD should be valid for QED

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

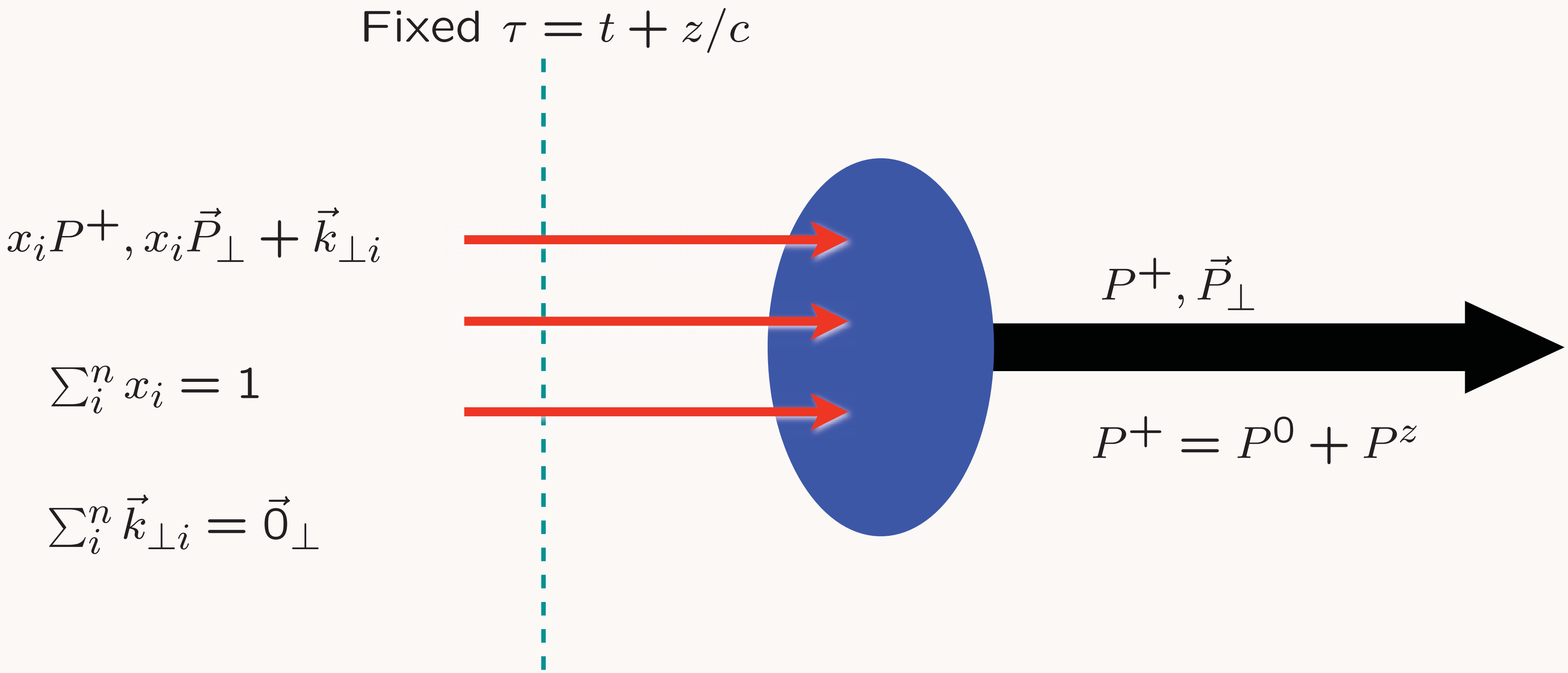
Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Light-Front Wavefunctions



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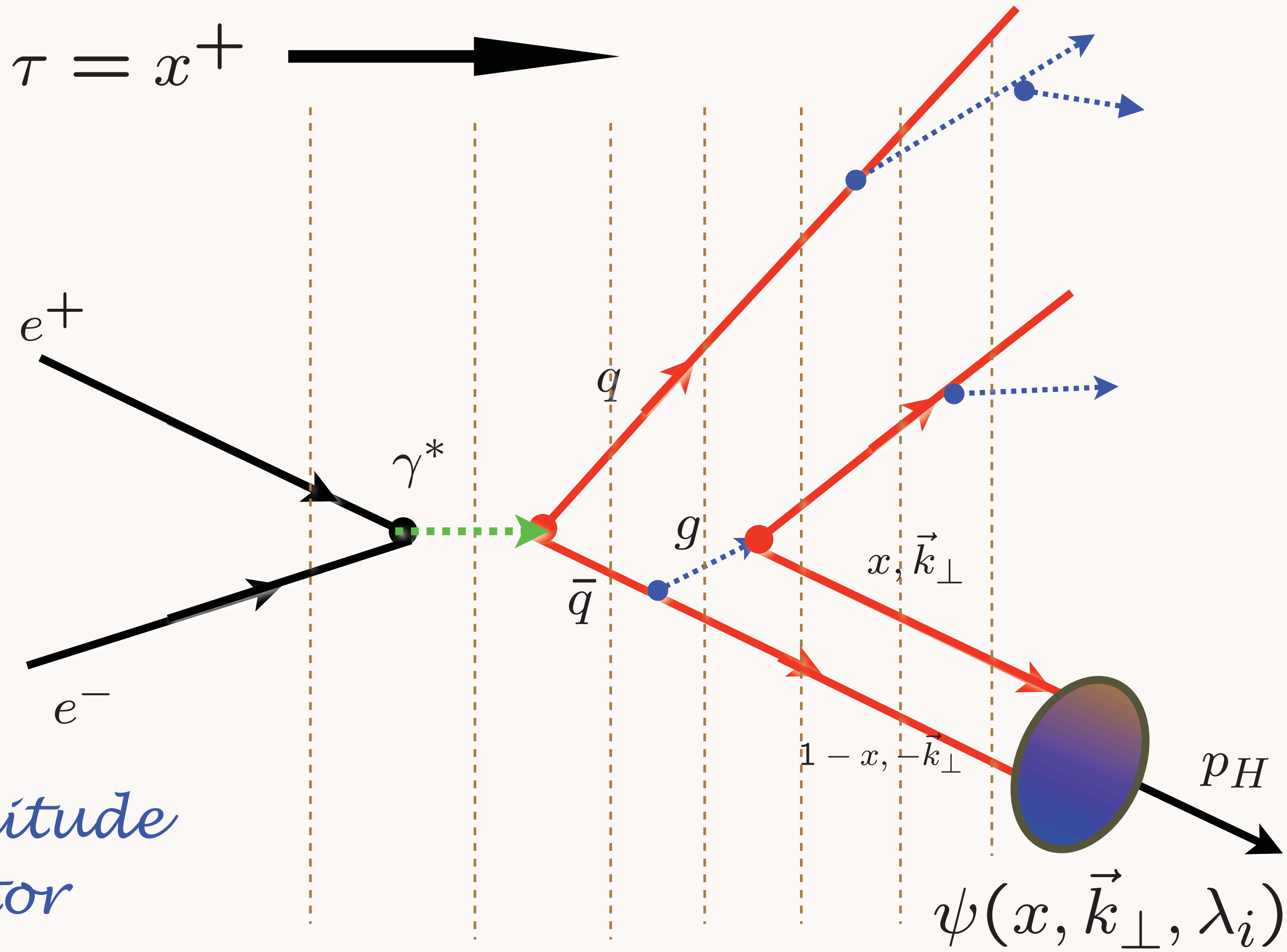
Invariant under boosts! Independent of P^μ

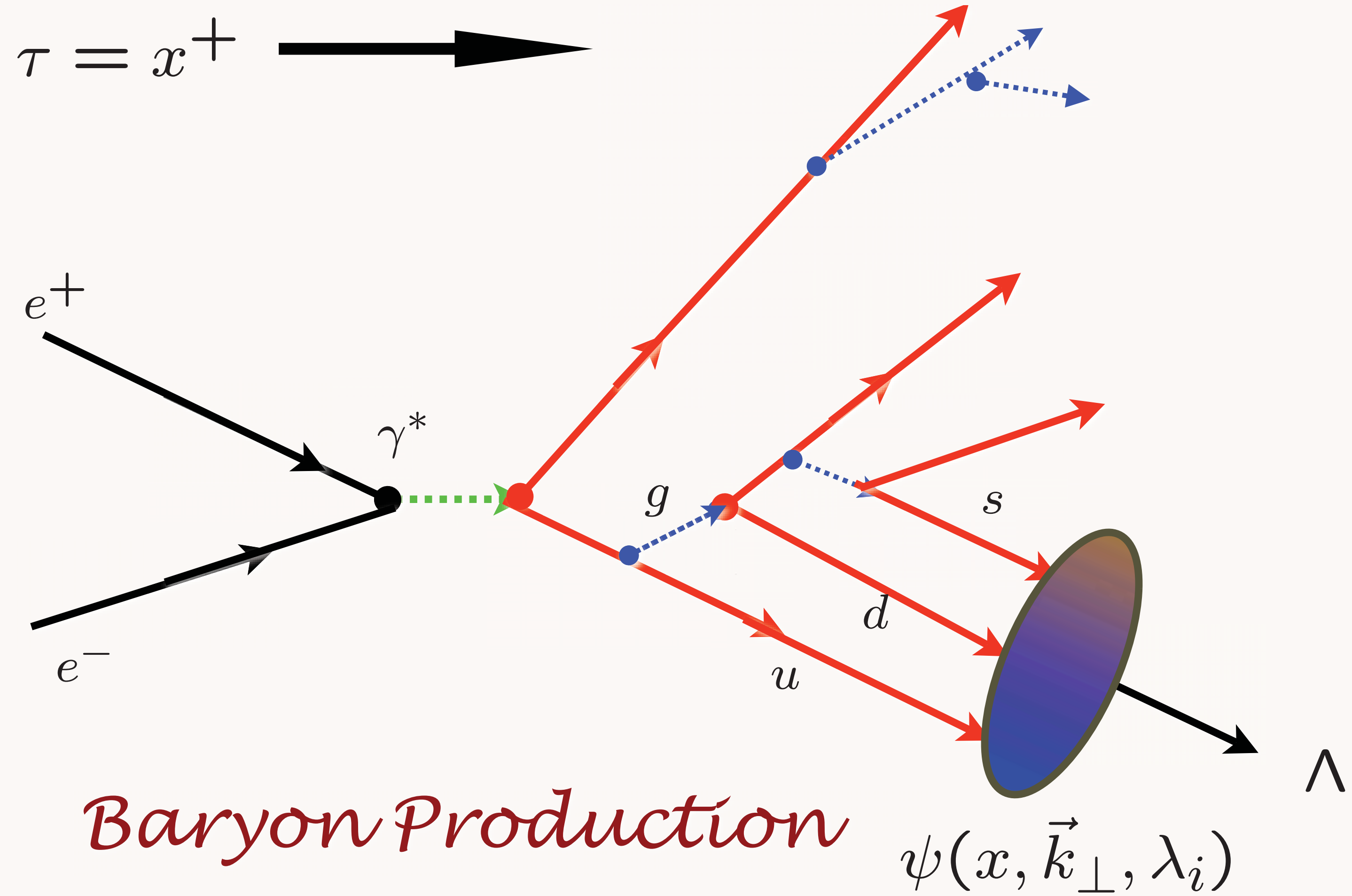
Hadronization at the Amplitude Level



Event amplitude generator

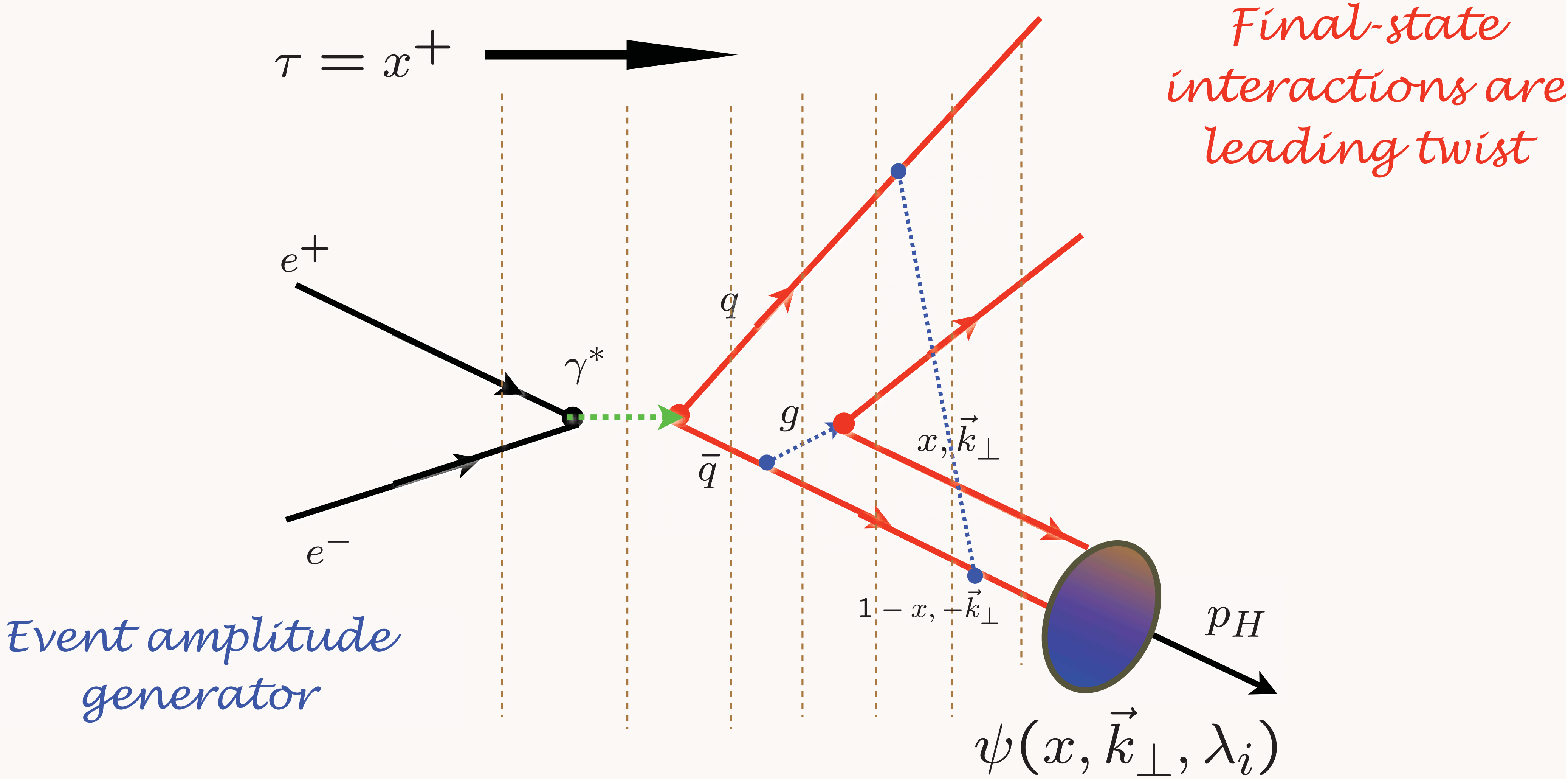
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



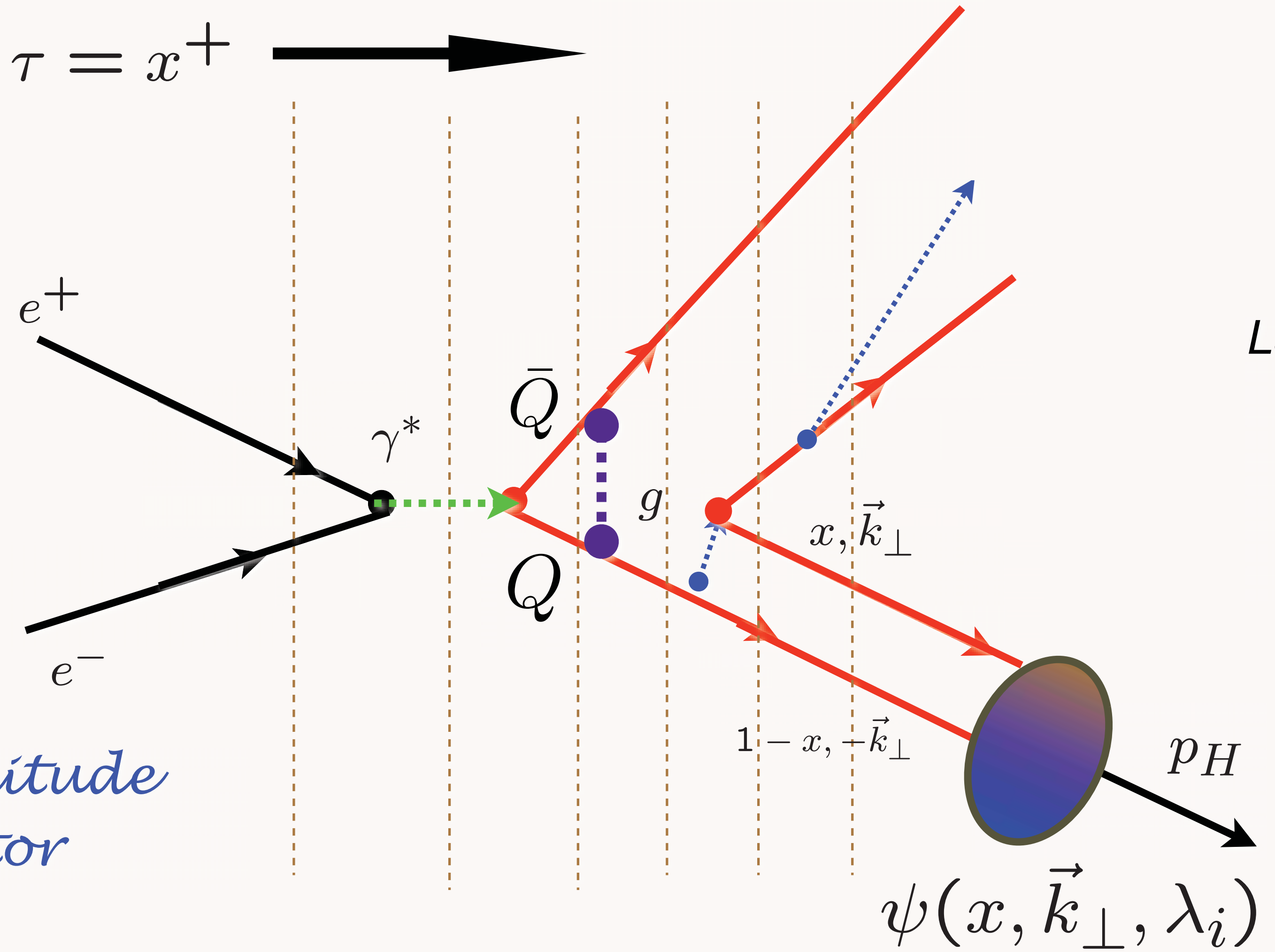
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



$$\frac{\pi \alpha_s}{\beta}$$

Large Threshold Corrections Sommerfeld-Schwinger Factor

Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front QCD

Heisenberg Matrix Formulation

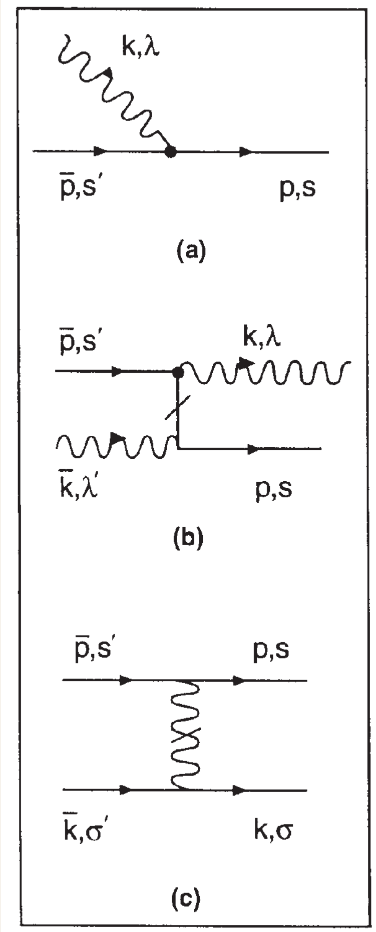
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

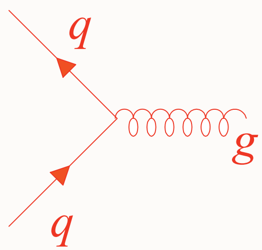


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

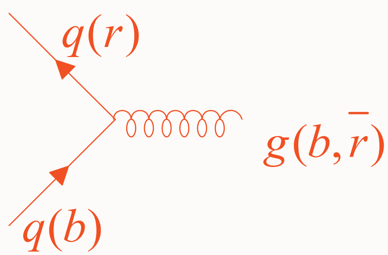
Fundamental Couplings

Only quarks and gluons involve basic vertices: Quark-gluon vertex

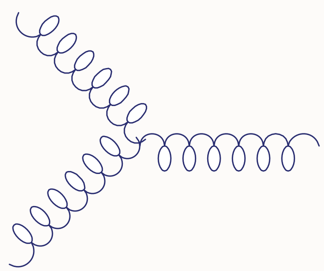


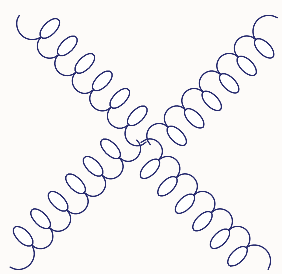
Similar to QED

More exactly



Gluon vertices

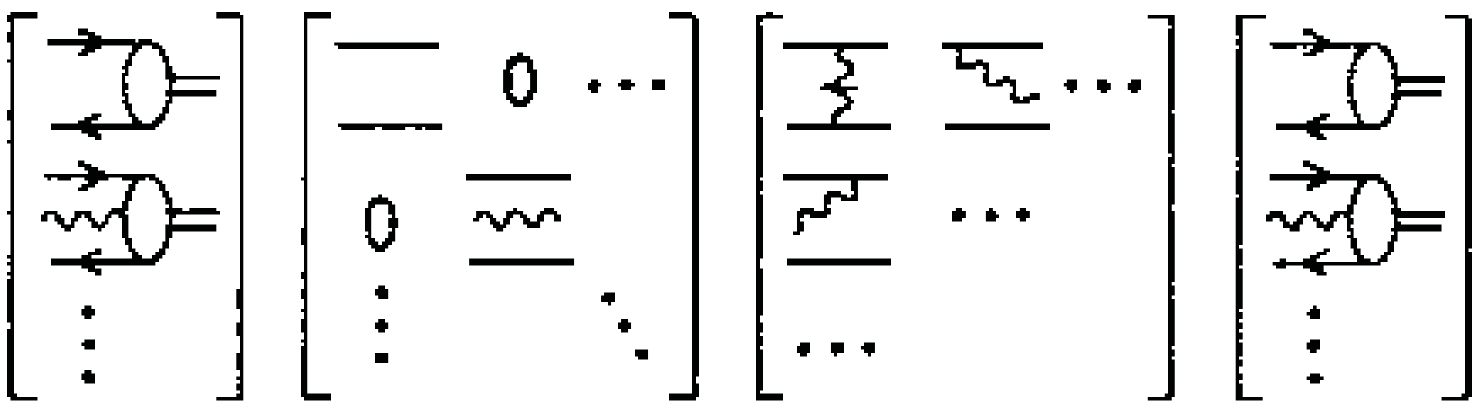




colored particles couple to gluons

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

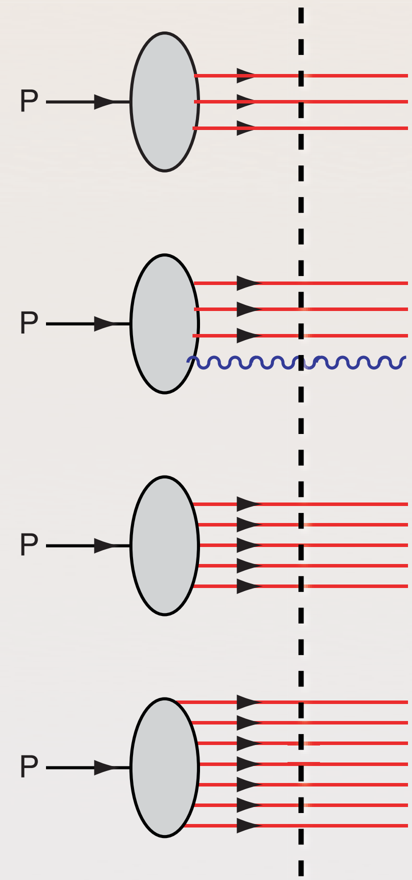
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

$$\bar{u}(x) \neq \bar{d}(x)$$

Mueller: BFKL DYNAMICS

$$\bar{s}(x) \neq s(x)$$

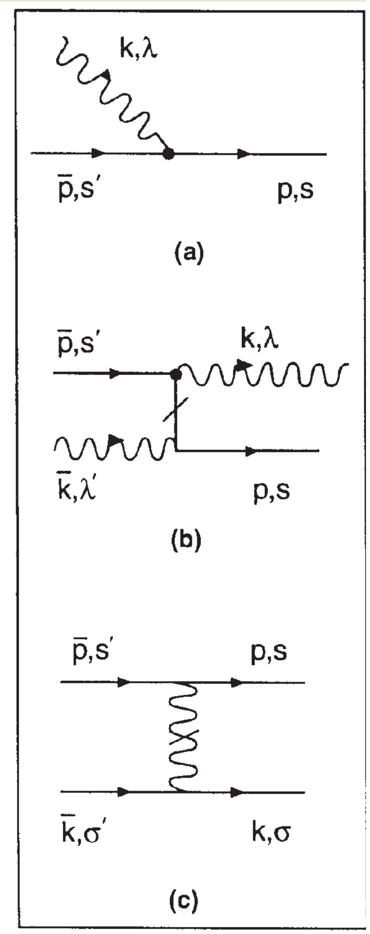
Light-Front QCD

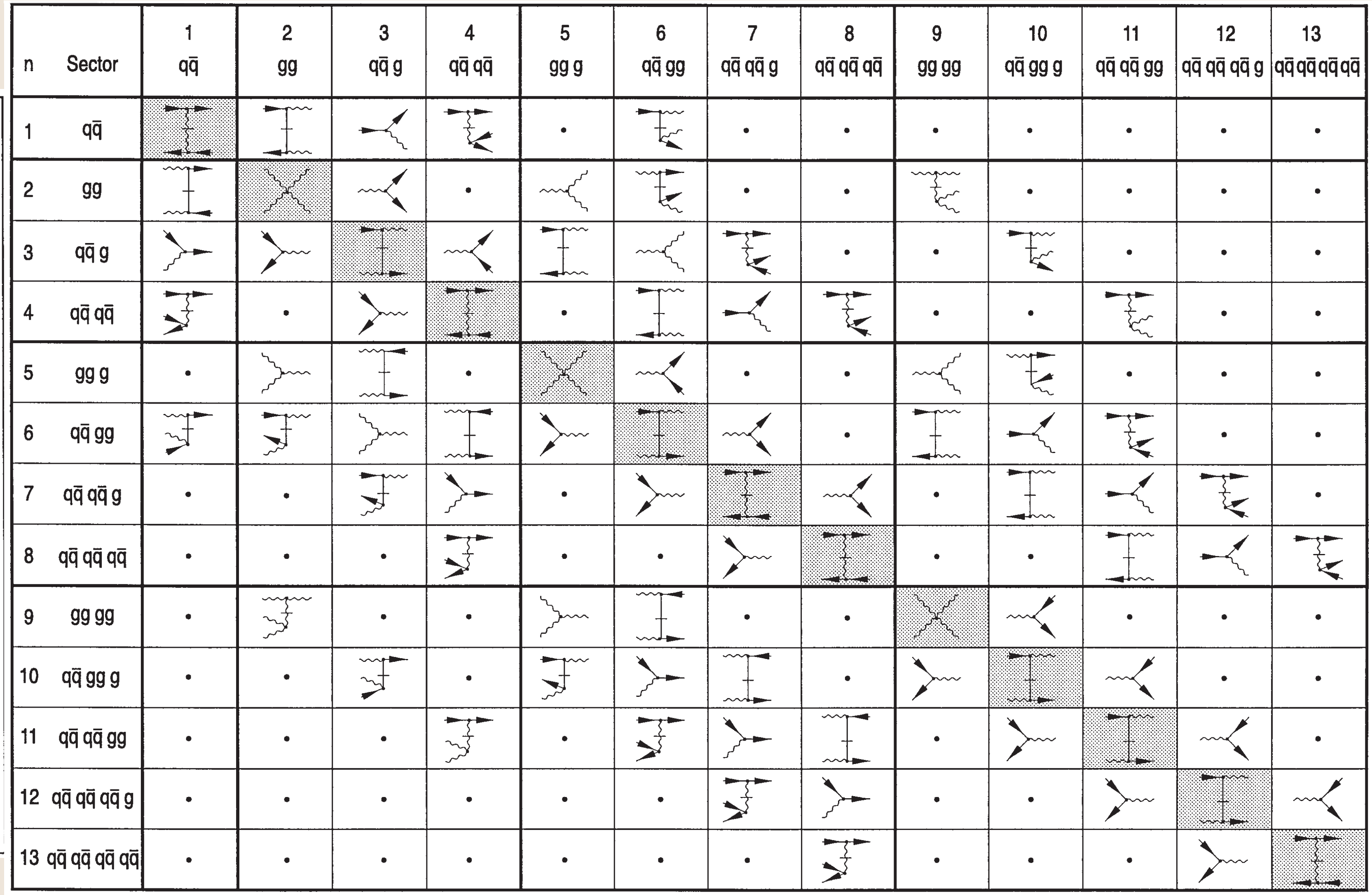
Heisenberg Matrix Formulation

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

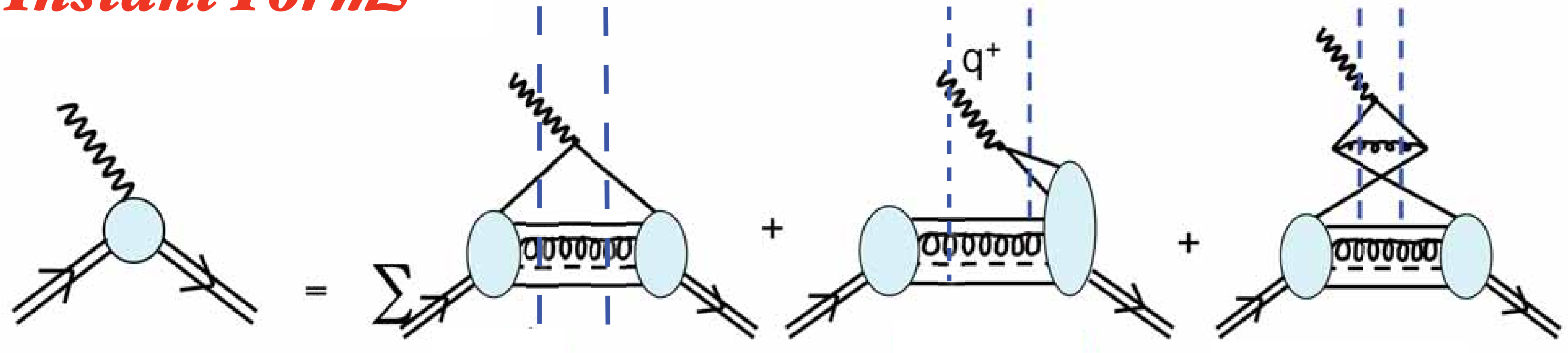
Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

HELEN BRADLEY - PHOTOGRAPHY

Calculation of Form Factors in Equal-Time Theory

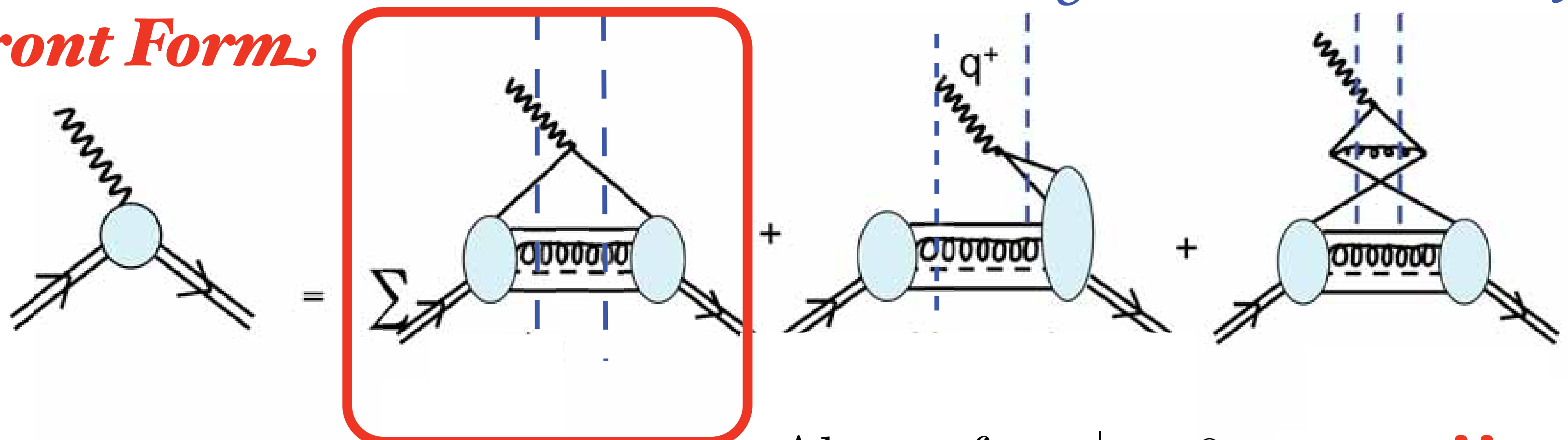
Instant Form



Need vacuum-induced currents

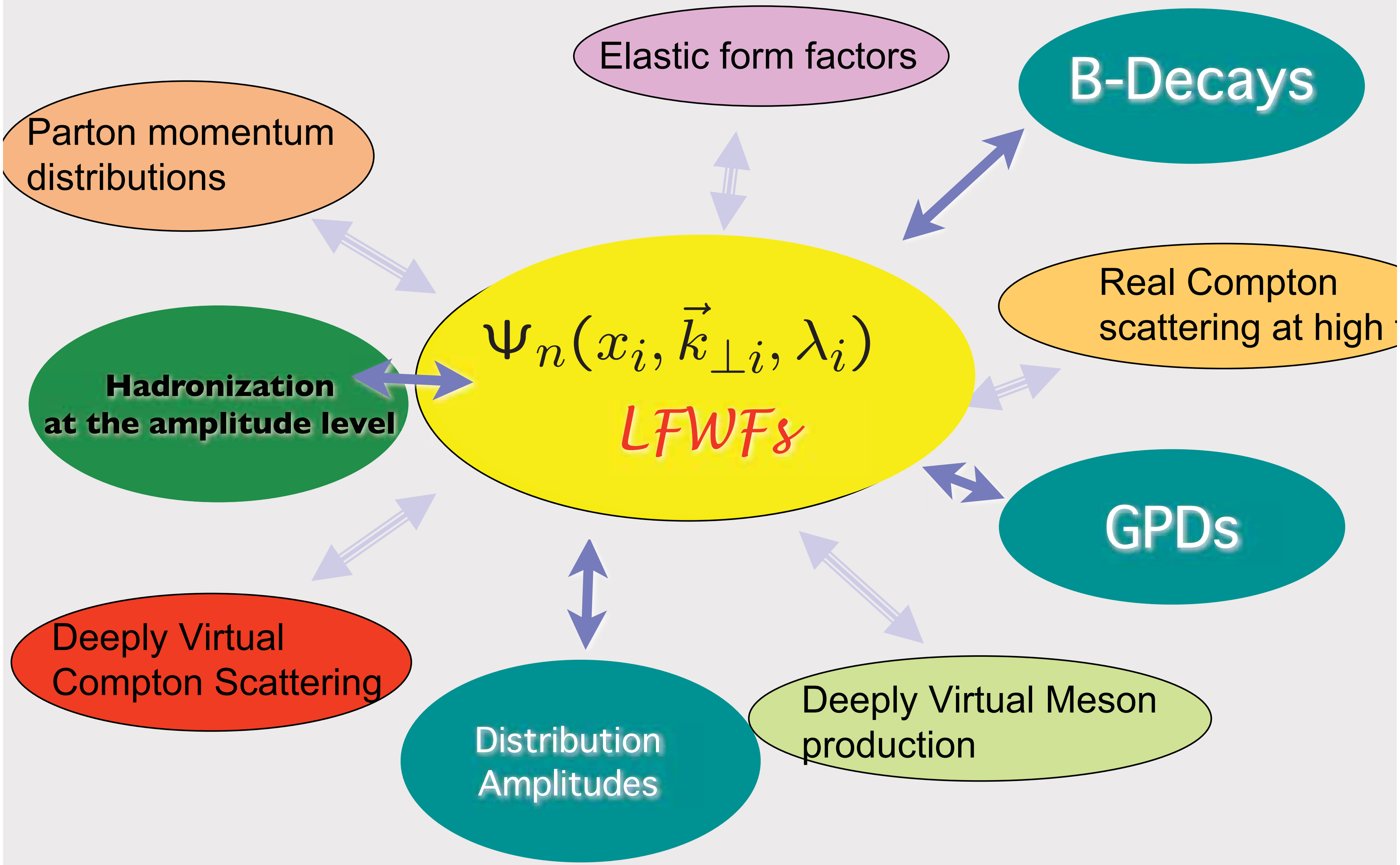
Calculation of Form Factors in Light-Front Theory

Front Form



Absent for $q^+ = 0$ ***zero !!***

A Unified Description of Hadron Structure

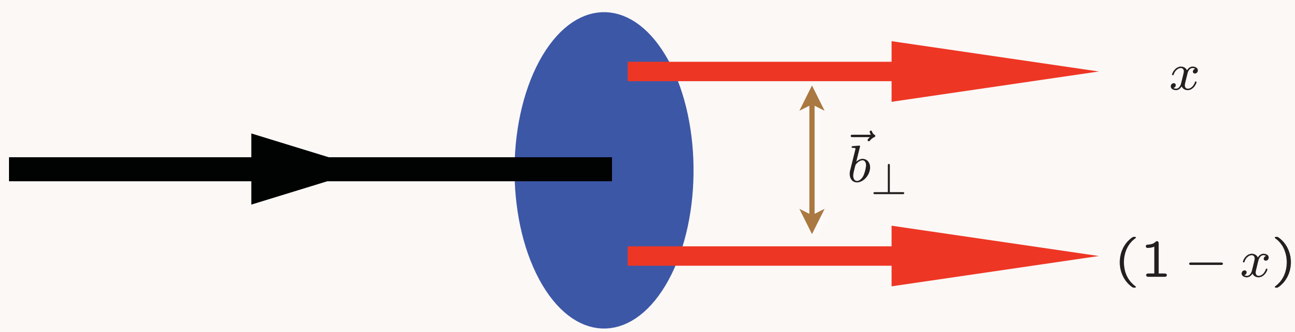


LF(3+1) — AdS$_5$

$\psi(x, \vec{b}_\perp)$ ⟷ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ ⟷ z

$\psi(x, \vec{b}_\perp)$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements: **em and gravitational!**

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

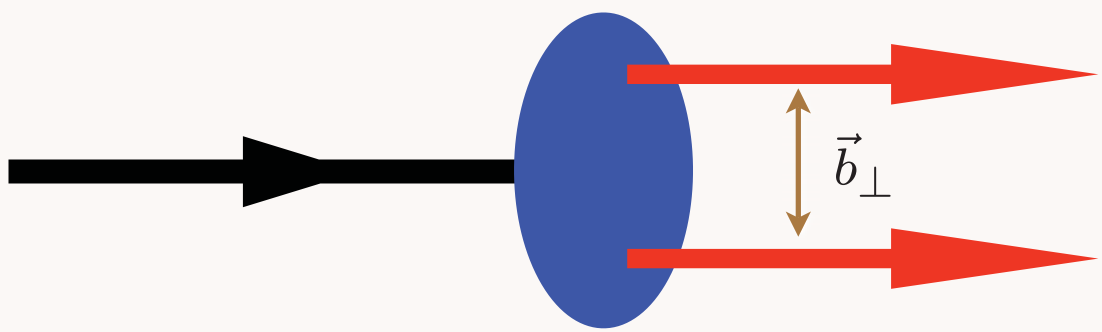
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



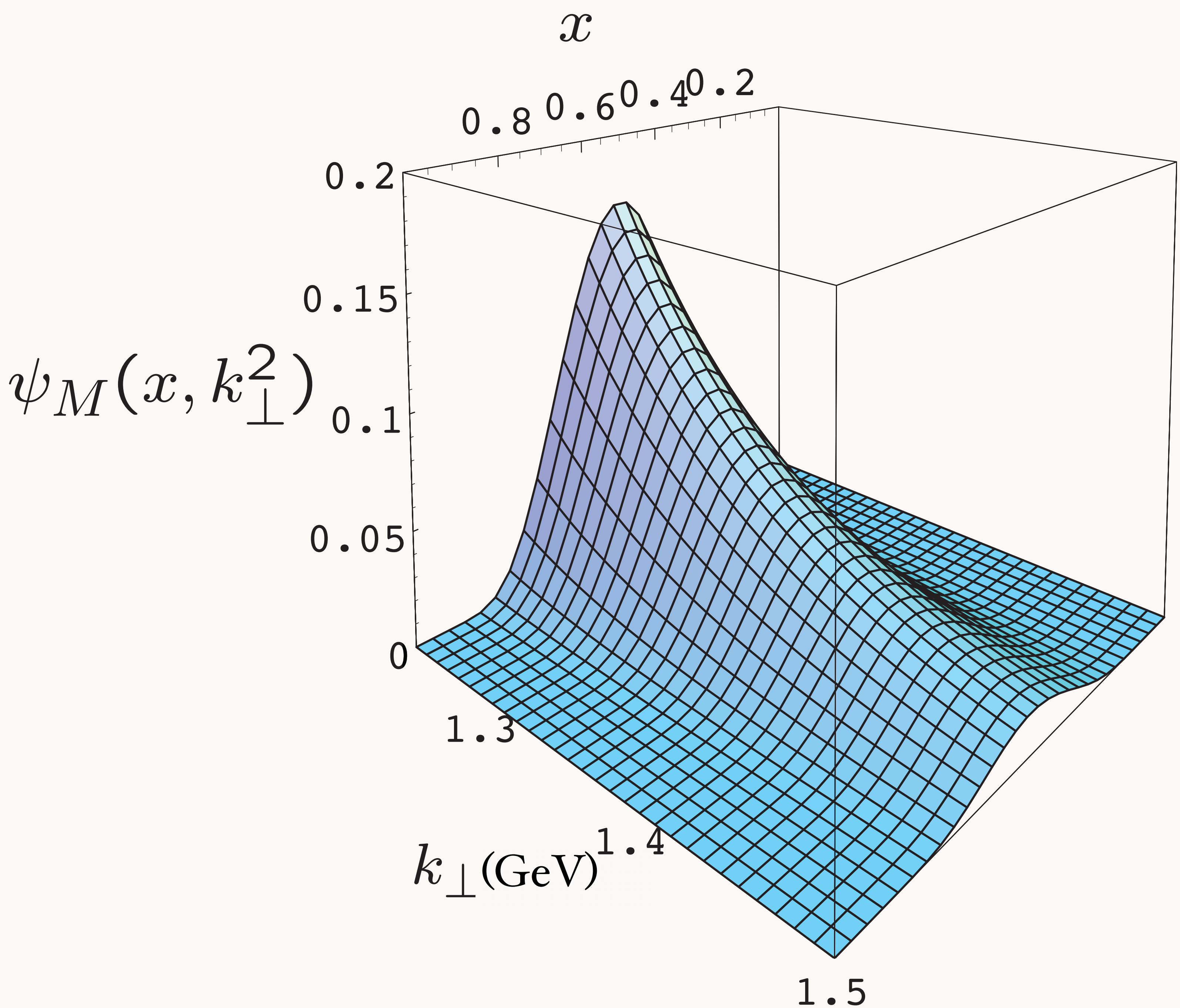
Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2} .$$

$$+\kappa^4\zeta^2$$

confining potential:

Prediction from AdS/CFT: Meson LFWF

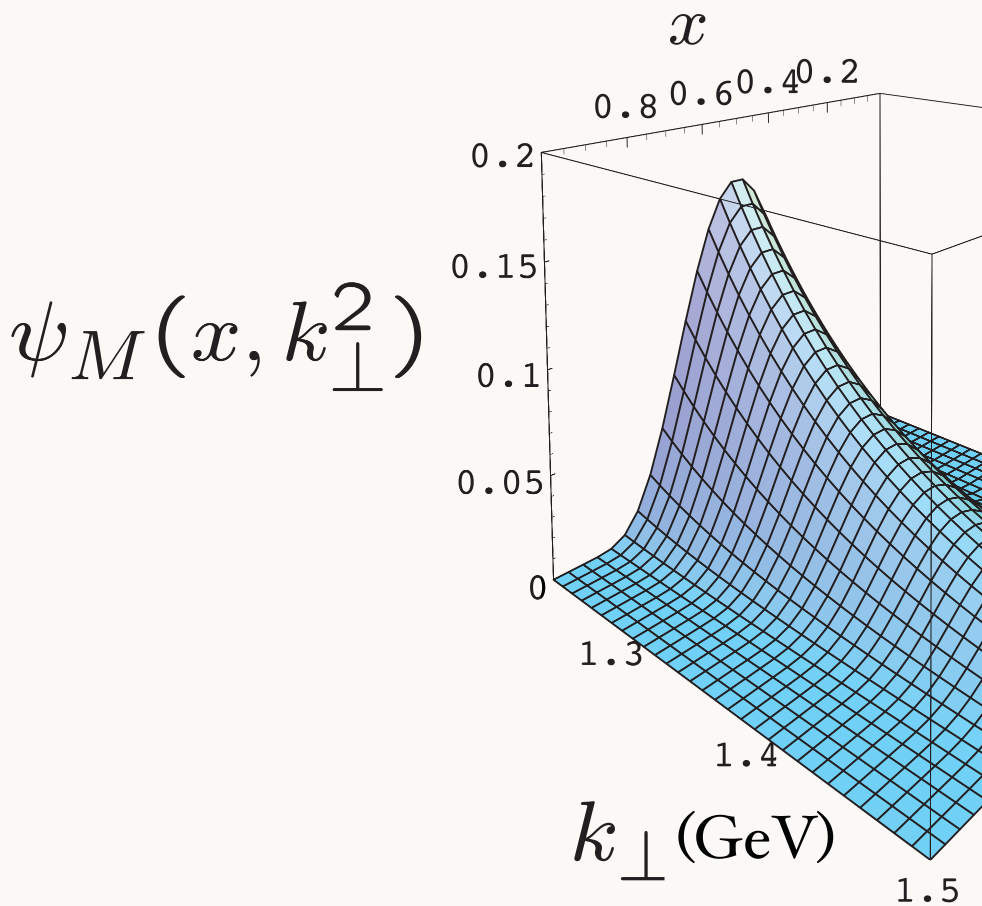


"Soft Wall" model

de Teramond, sjb

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb



"Soft Wall" model

$\kappa = 0.375$ GeV

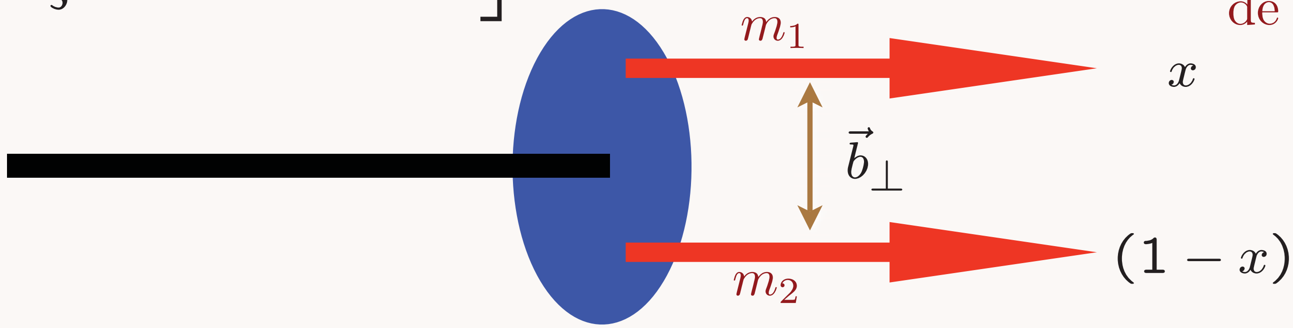
massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x,\mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

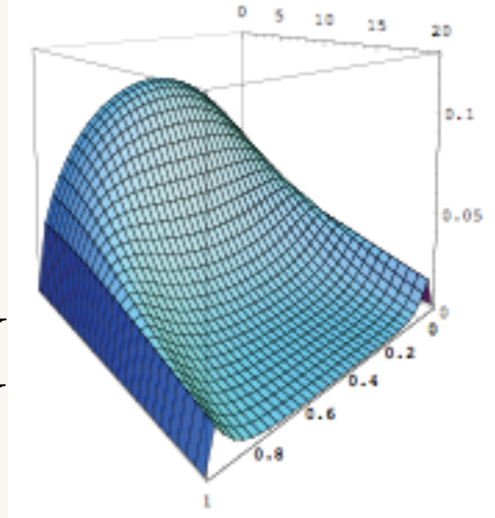
$$\psi(x,\mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

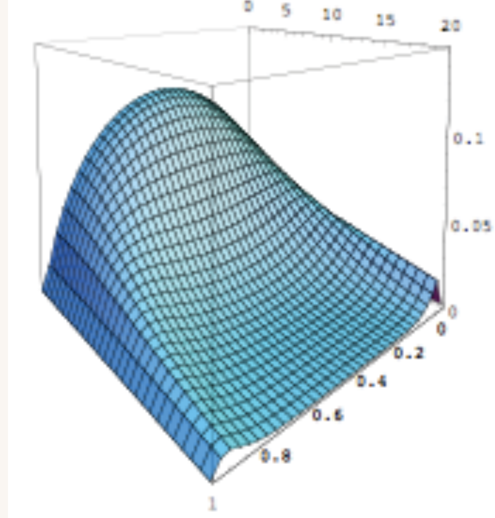
$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

$$|\pi^+ >= |u\bar{d}>$$

$$m_u = 2\ MeV$$
$$m_d = 5\ MeV$$



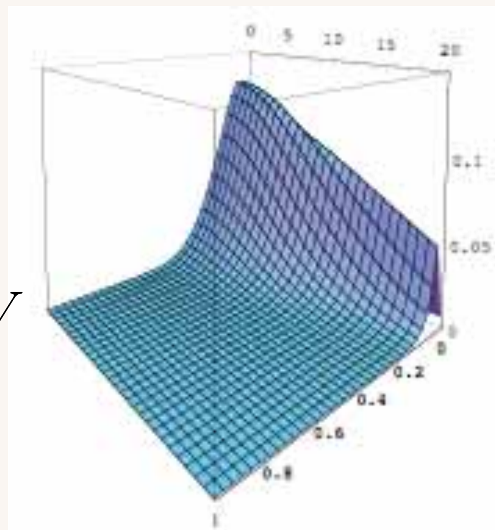


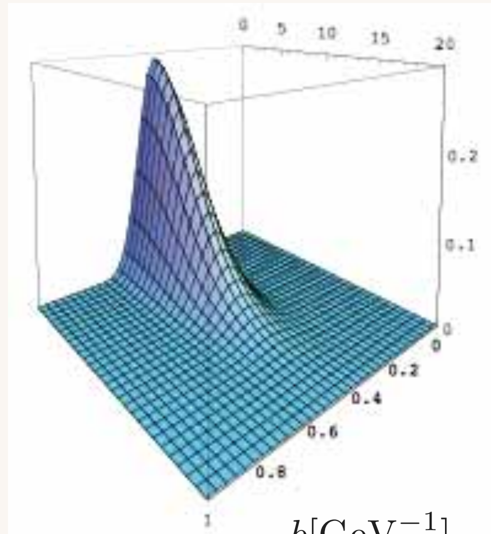
$$|K^+ >= |u\bar{s}>$$

$$m_s = 95\ MeV$$

$$|D^+ >= |c\bar{d}>$$

$$m_c = 1.25\ GeV$$

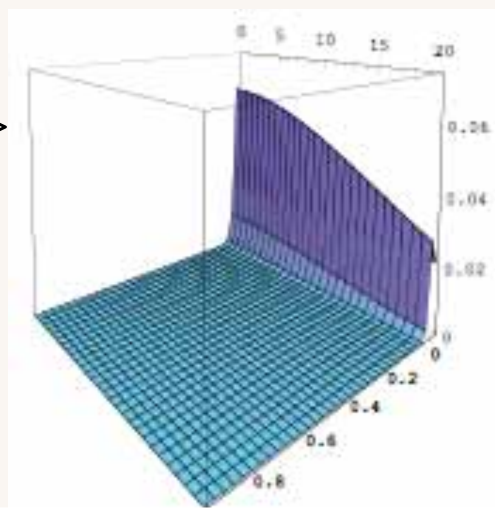


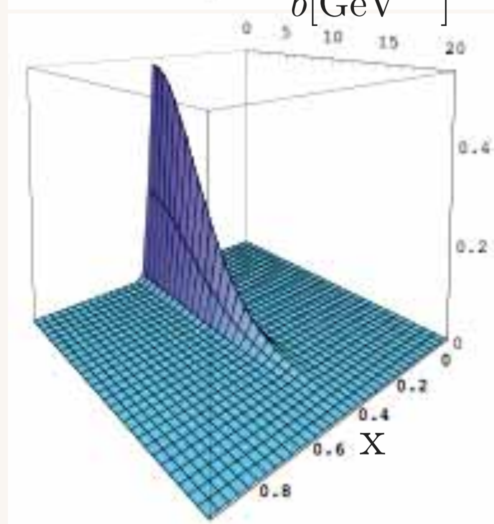


$$|\eta_c >= |c\bar{c}>$$

$$|B^+ >= |u\bar{b}>$$

$$m_b = 4.2\ GeV$$

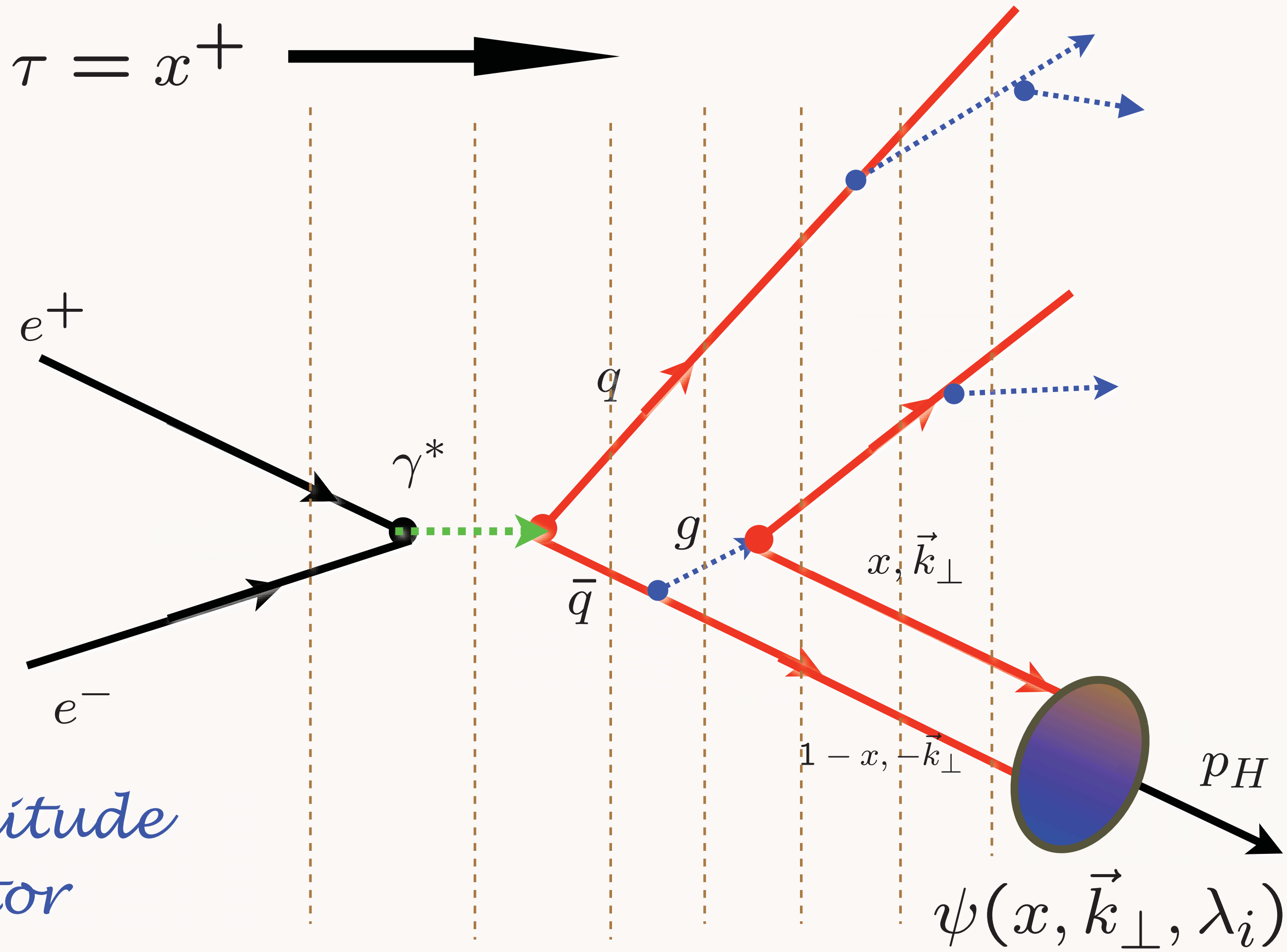




$$|\eta_b >= |b\bar{b}>$$

$$\kappa = 375\ MeV$$

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

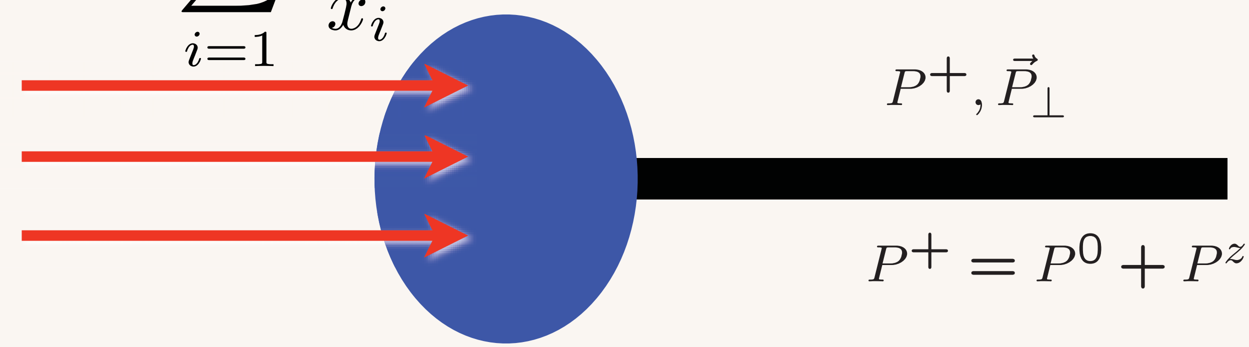
For each color-singlet cluster

If $\mathcal{M}_n^2 \leq \Lambda_{QCD}^2$ coalesce to hadron

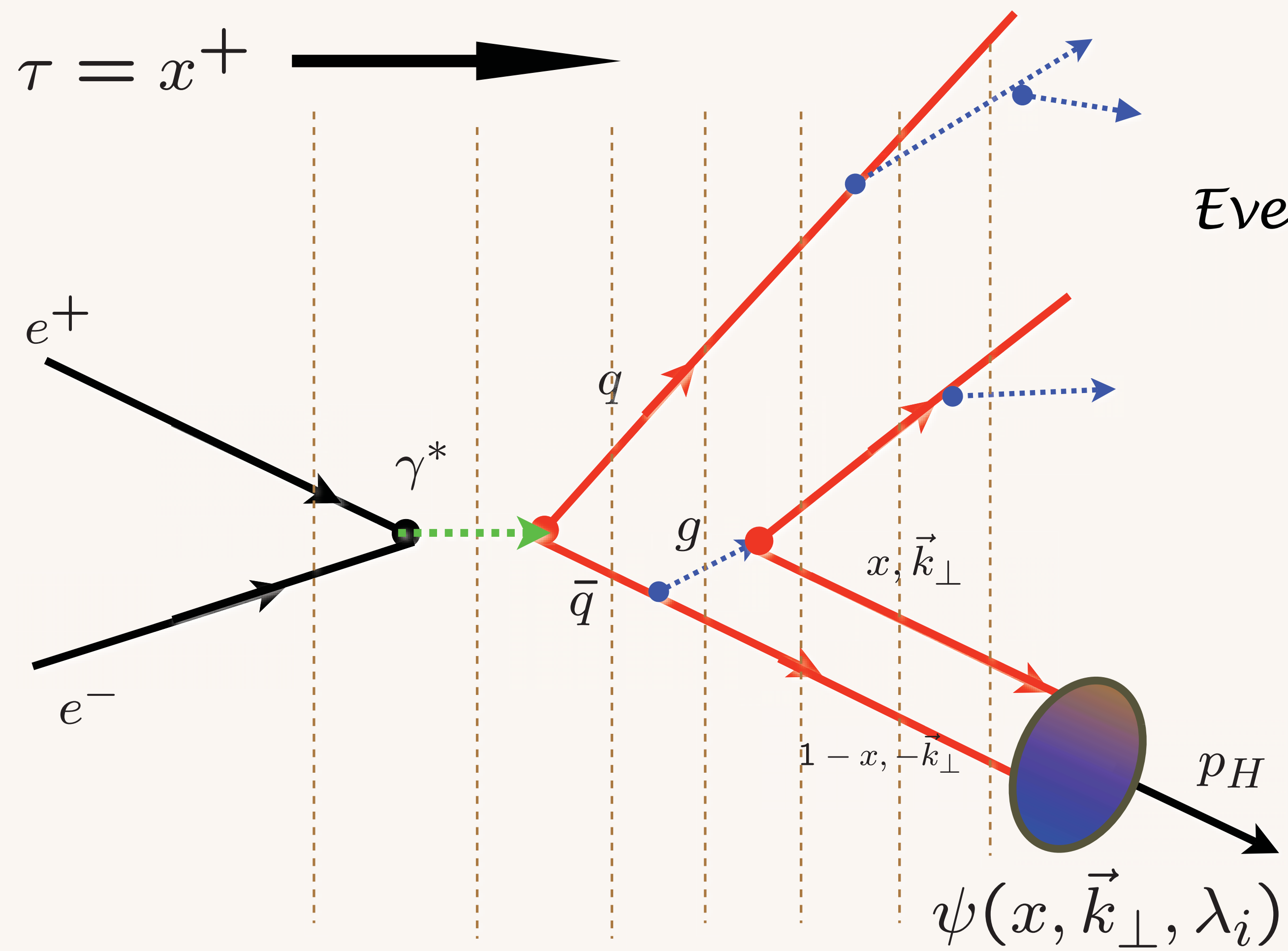
If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ continue to evolve

avoids gluon avalanche in jet evolution, heavy hadron decays

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2}{x_i}$$



Hadronization at the Amplitude Level



Event amplitude generator

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

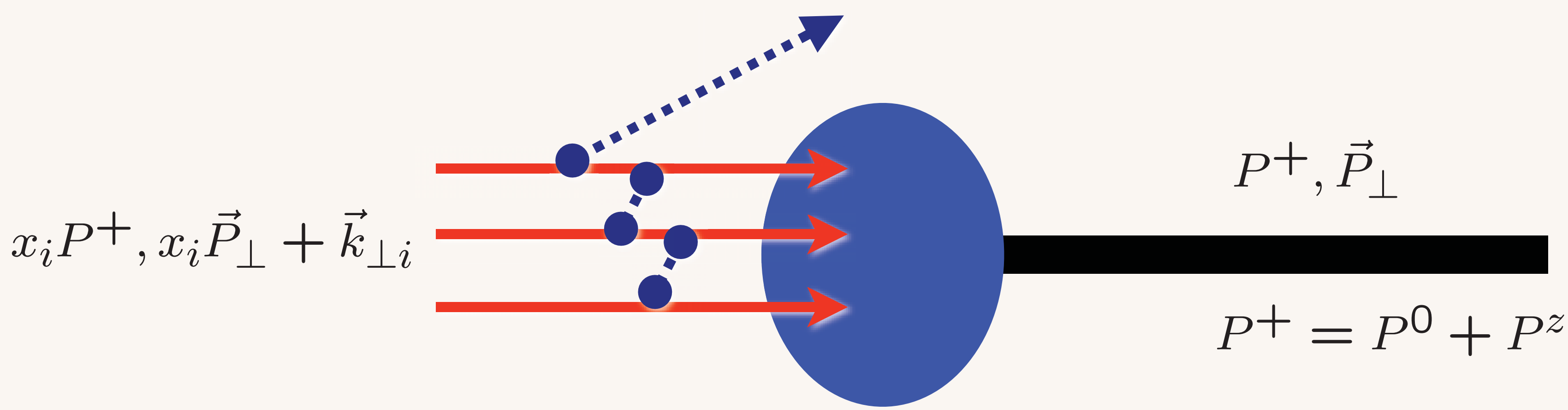
AdS/QCD Hard Wall Confinement:

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange

- Factorization Scale for structure functions and fragmentation functions set: $\mu_{fact} = \Lambda_{QCD}$

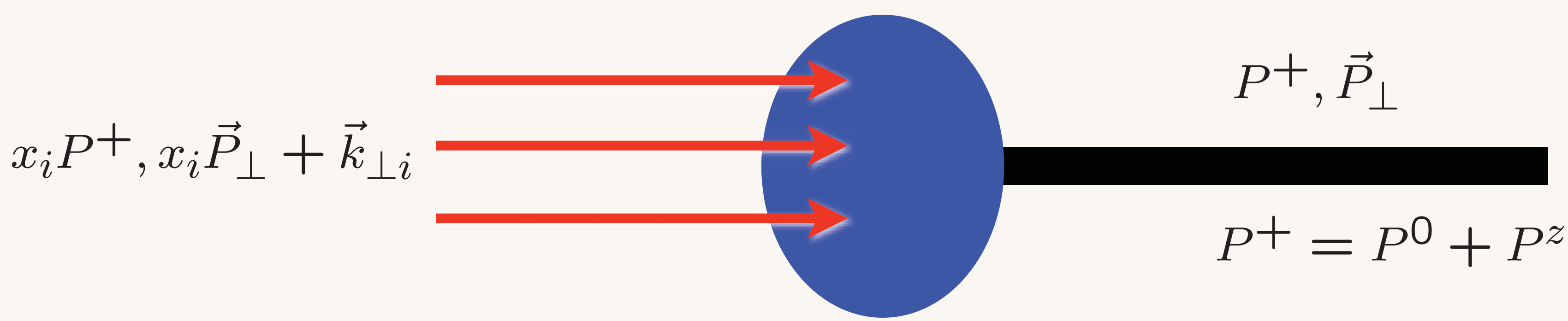


Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of P^+ and P^z
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using "alternating denominator" method
- Multiple renormalization scales (BLM)

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



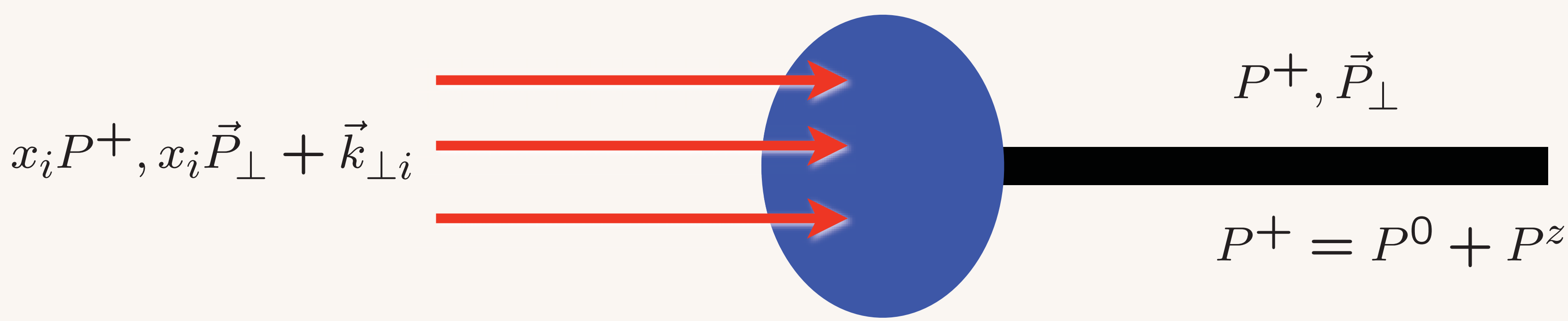
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Includes Effects of Initial and Final State Interactions from gluon exchange
- Sivers, Collins, Boer-Mulders Effects
- Diffractive Channels
- Heavy quark threshold corrections
- Intrinsic Heavy Quark Effects
- s(x) versus anti-s(x) asymmetry

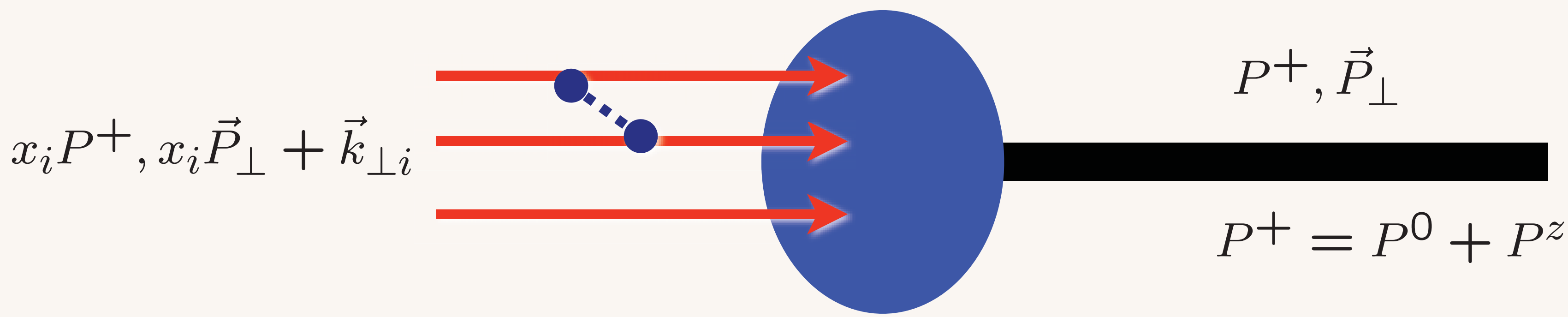
Features of LF T-Matrix Formalism

"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- Generates PQCD Hard Tail of LFWF at high x and high transverse momentum
- Dimensional Counting rules and Color Transparency for Hard Exclusive Channels
- Counting rules for structure functions and fragmentation functions at large x and z:

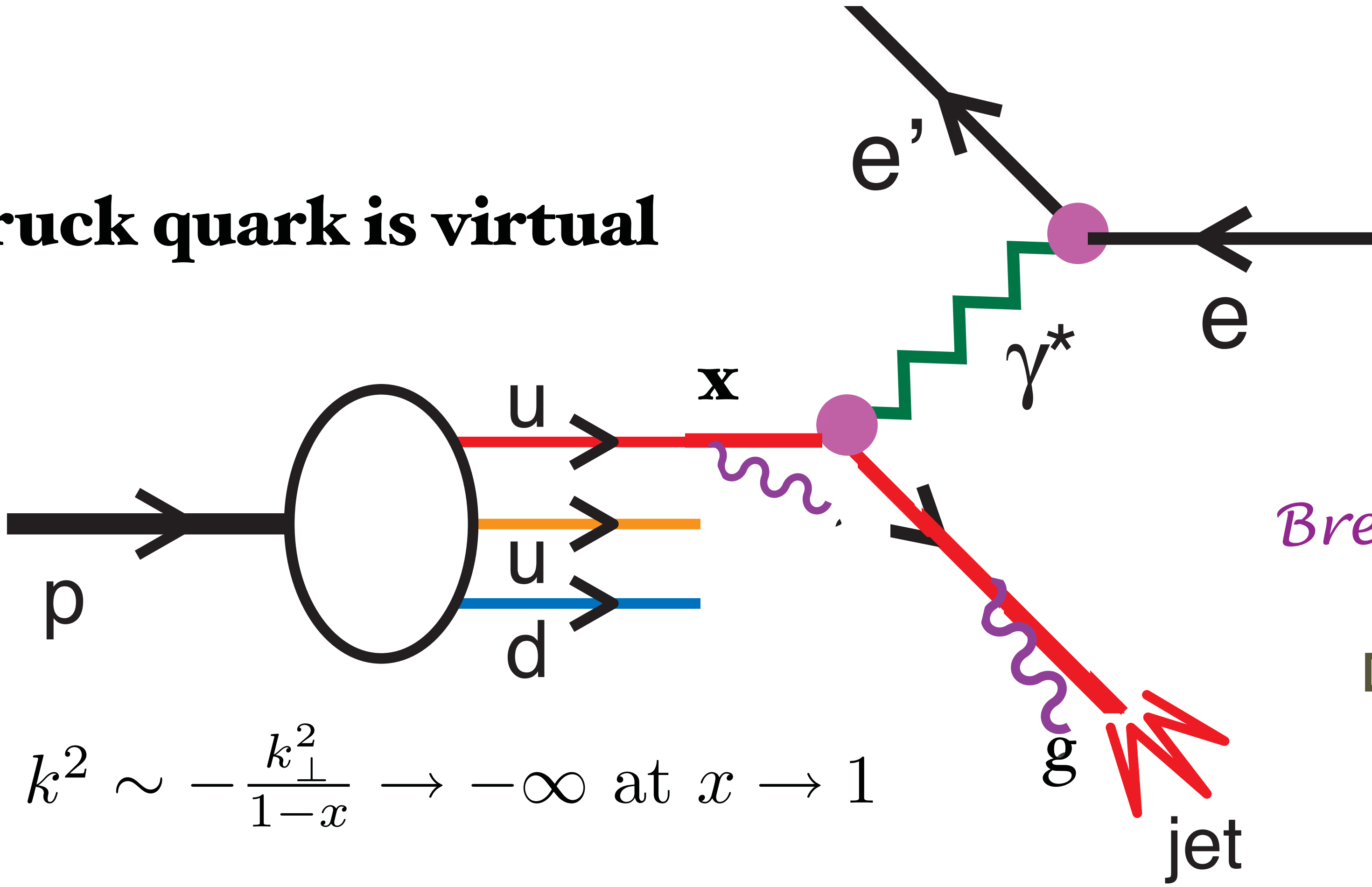
$$(1-x)^{2n_{spect}-1}, (1-z)^{2n_{spect}-1}$$



Hadronization at the Amplitude Level

Deep Inelastic Electron-Proton Scattering

Struck quark is virtual



$$k^2 \sim -\frac{k_\perp^2}{1-x} \to -\infty \text{ at } x \to 1$$

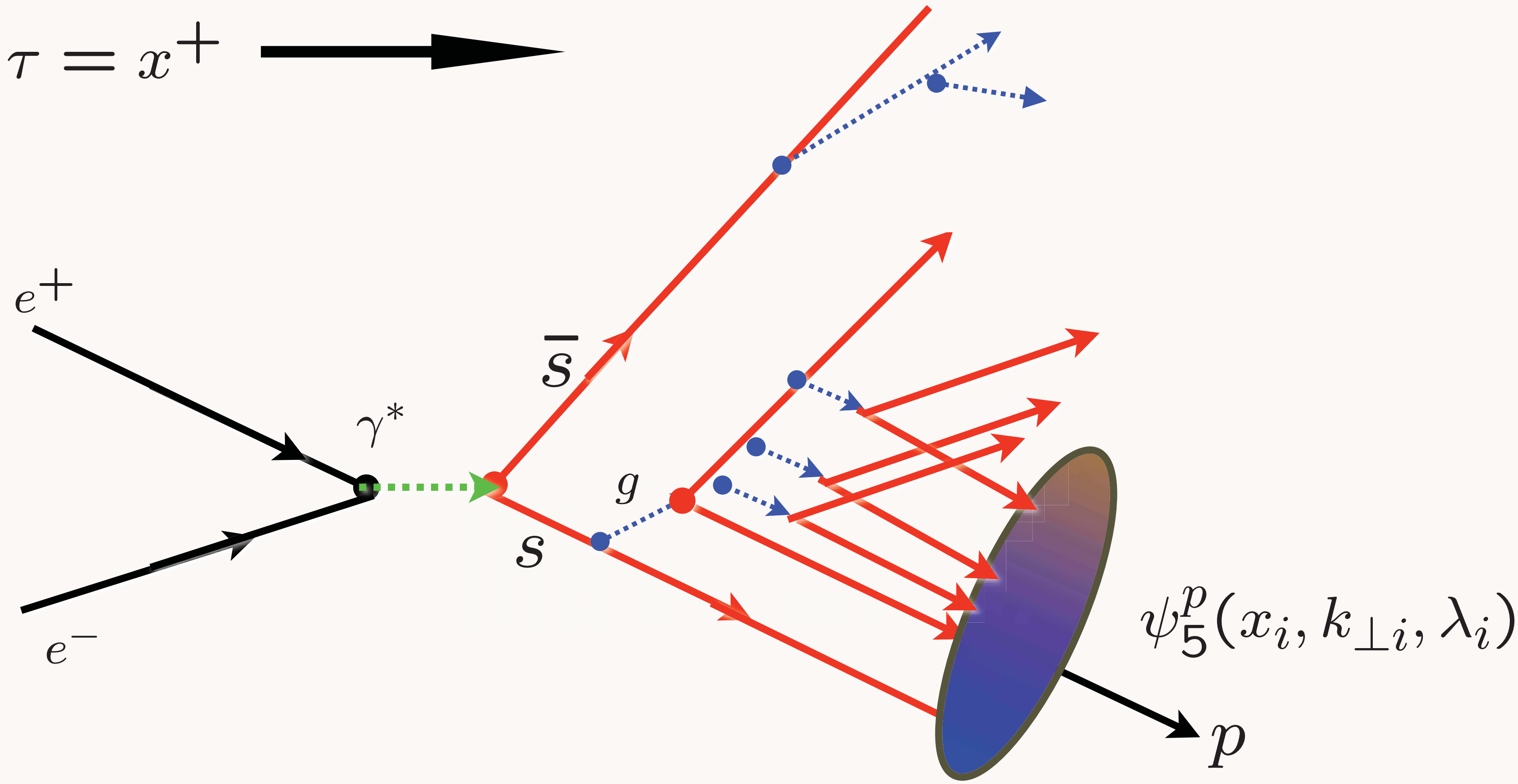
Gluonic Bremmstrahlung

DGLAP Evolution

Off-shell Effect: Breakdown of DGLAP at x~1 !

Off-shell Effect: Breakdown of DGLAP at z~1 !

Hadronization at the Amplitude Level



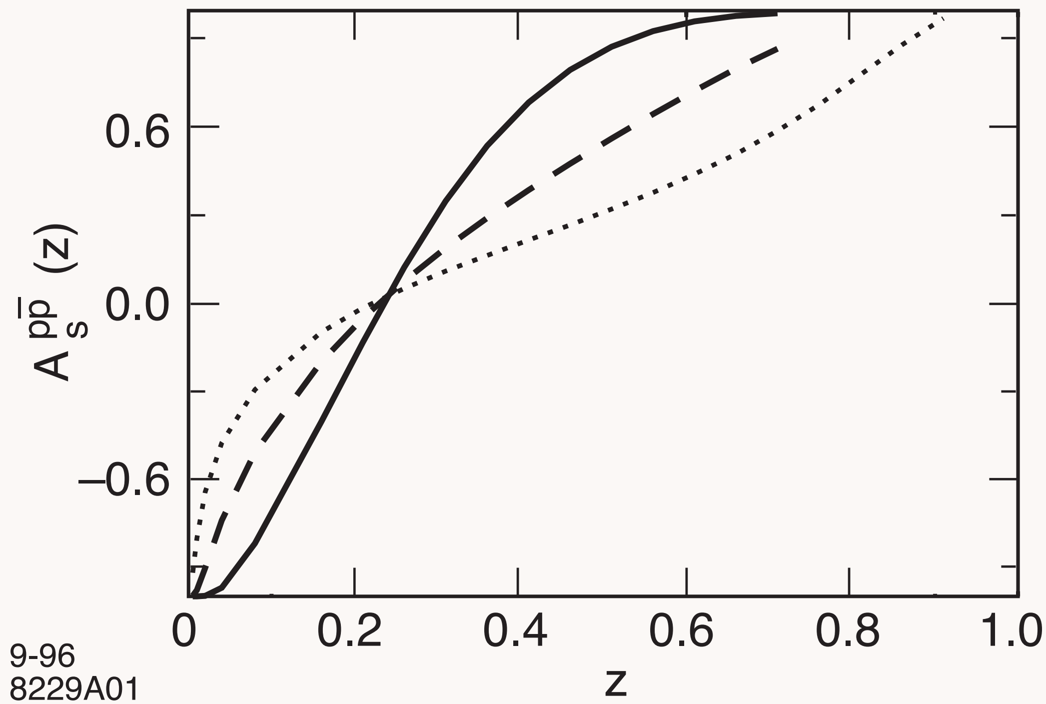
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s \to p}(z) \neq D_{s \to \bar{p}}(z)$

B-Q Ma, sjb

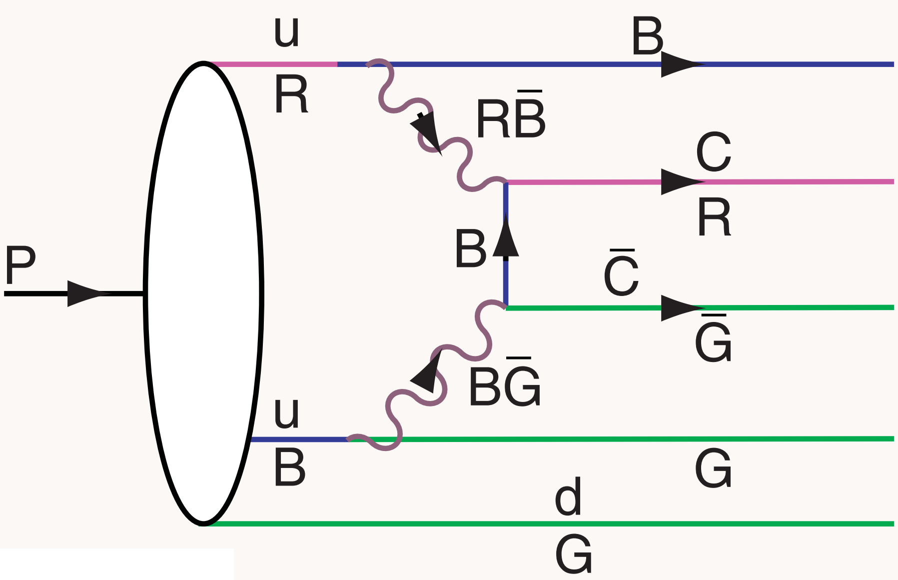
$$D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$$

B-Q Ma, sjb



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to \bar{p}}(z)}{D_{s\to p}(z) + D_{s\to \bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$



$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

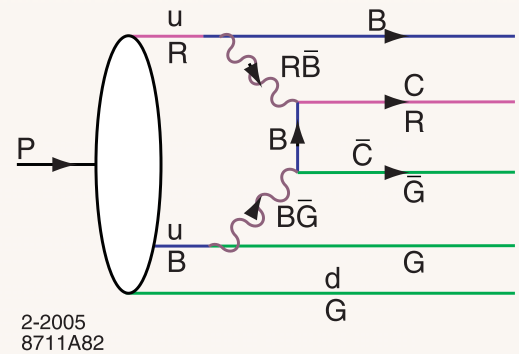
$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm!

Hoyer, Peterson, Sakai, sjb

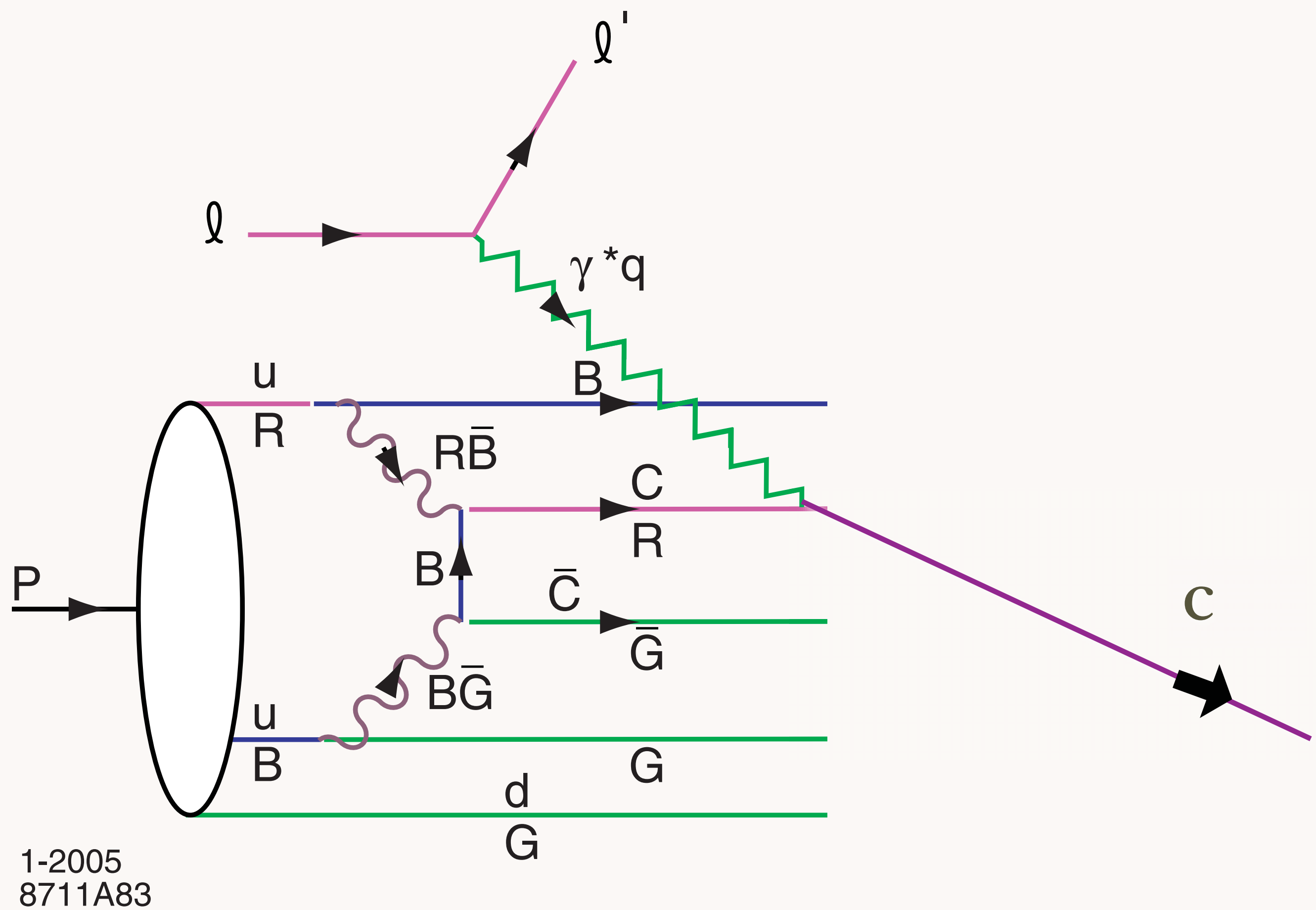
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



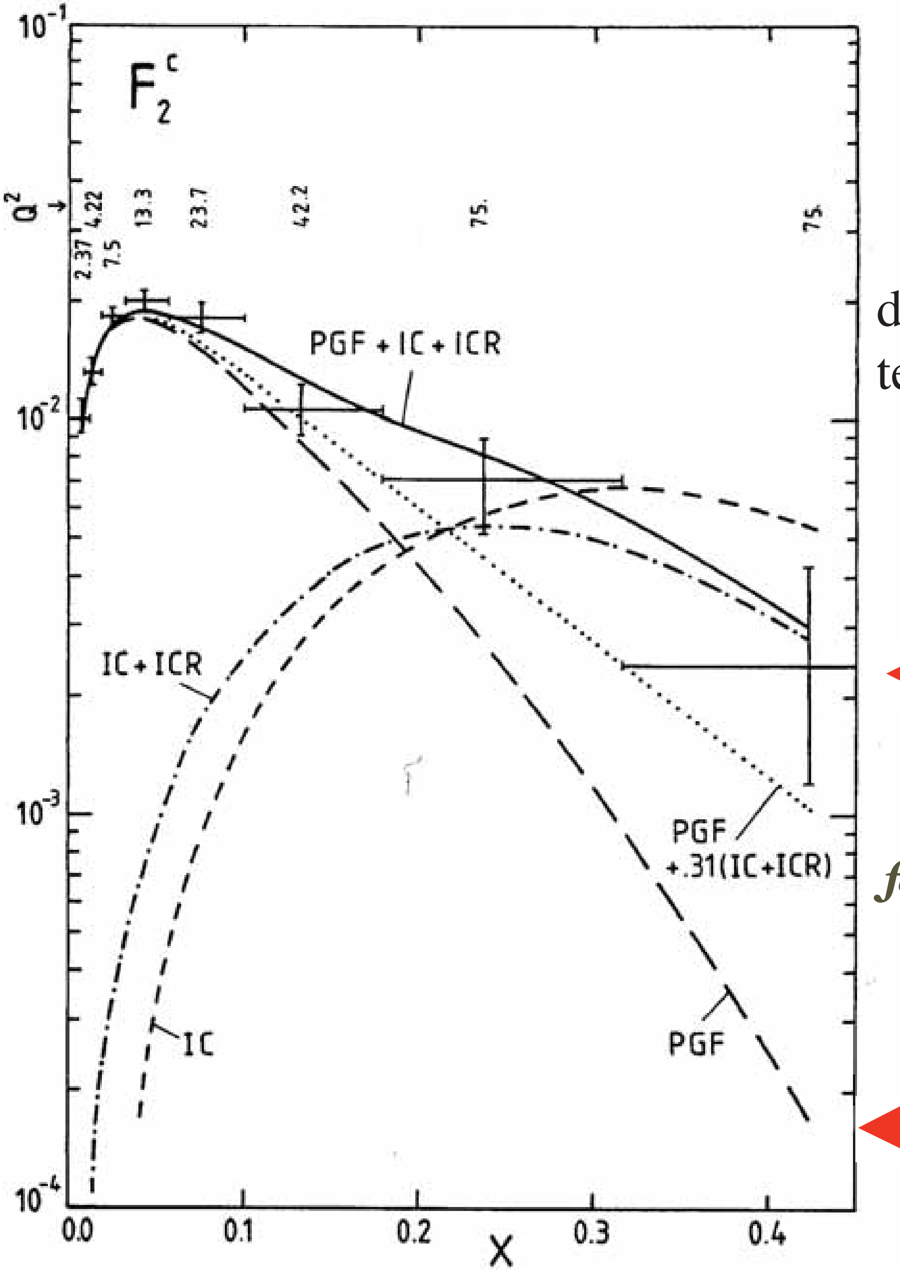
- Rigorous prediction of QCD, OPE
- Color - Octet + Color - Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

Measure c(x) in Deep Inelastic Lepton-Proton Scattering



Hoyer, Peterson, SJB

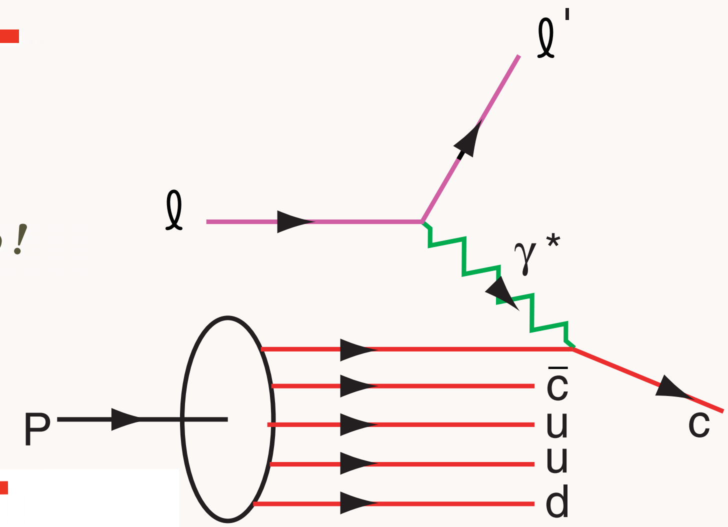
Measurement of Charm Structure Function



J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \rightarrow J/\psi X$

- High x_F $pp \rightarrow J/\psi J/\psi X$

- High x_F $pp \rightarrow \Lambda_c X$

- High x_F $pp \rightarrow \Lambda_b X$

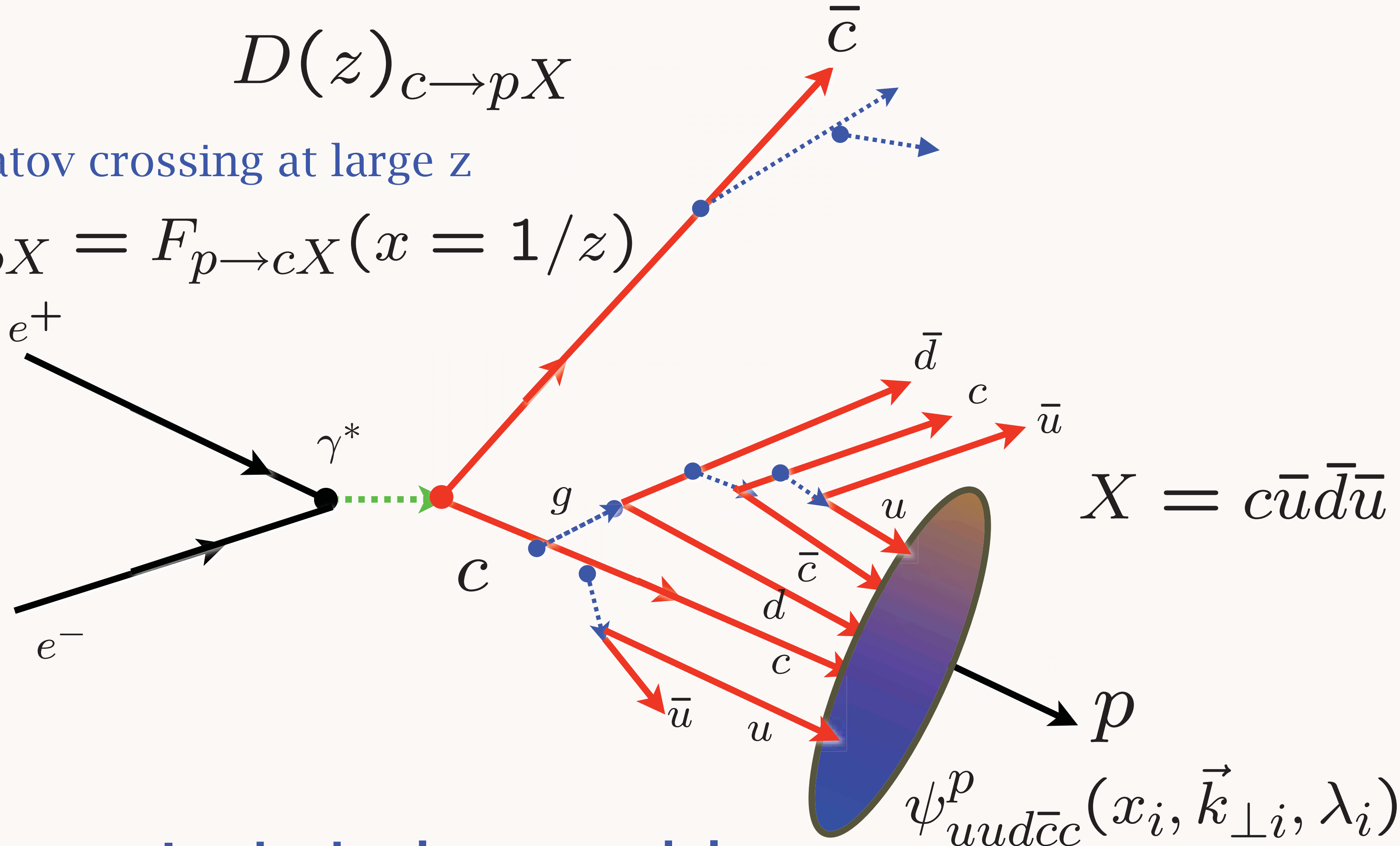
- High x_F $pp \rightarrow \Xi(ccd) X$ (SELEX)

Timelike Test of Charm Distribution in Proton

$$D(z)_{c\to pX}$$

Gribov-Lipatov crossing at large z

$$zD(z)_{c\to pX} = F_{p\to cX}(x = 1/z)$$

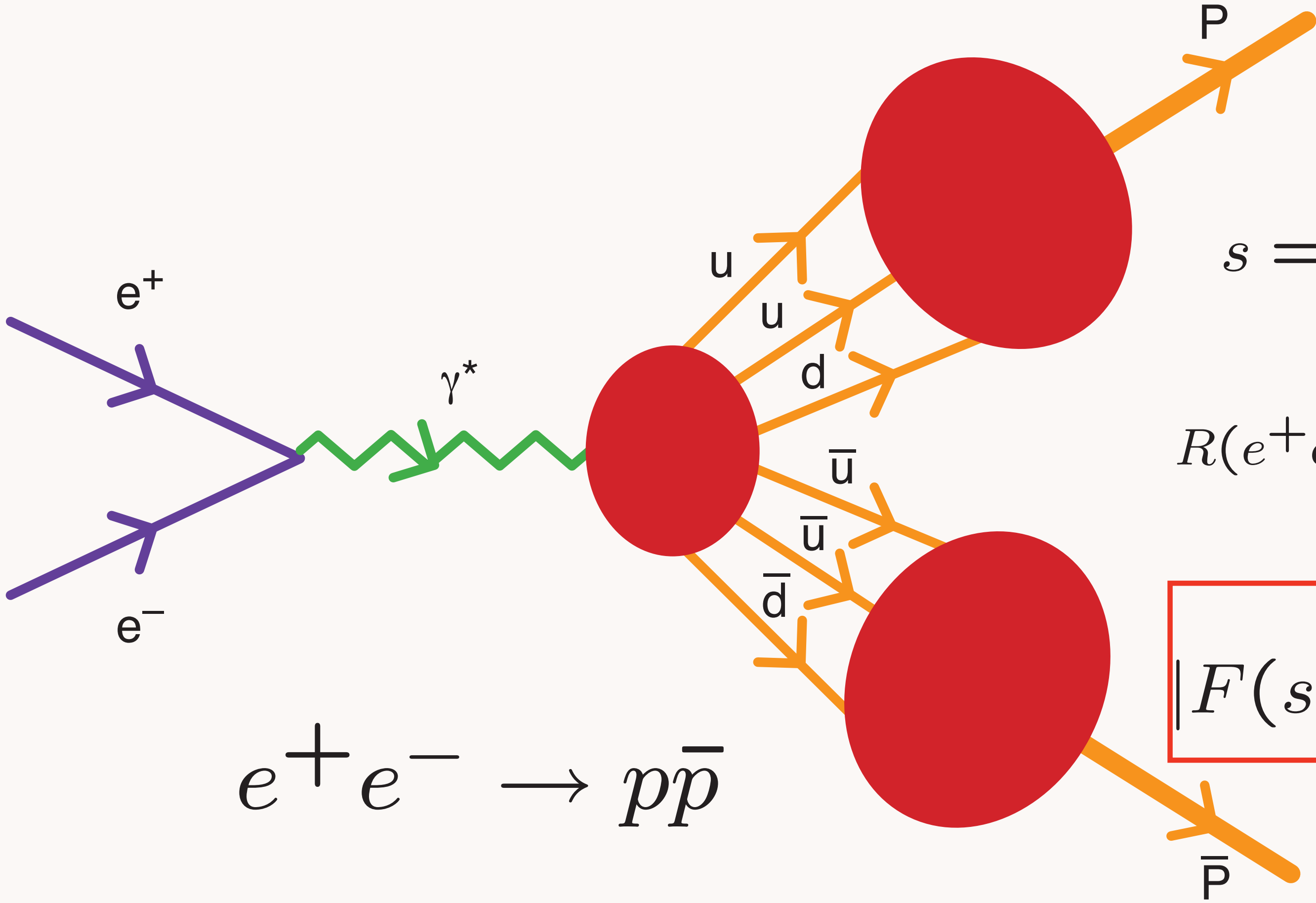


Intrinsic charm model:
predict proton at same rapidity as charm quark: high z

$$z_i \propto m_{\perp i} = \sqrt{m_i^2 + k_\perp^2}$$

Exclusive Processes

What if we ask for a specific final state?



$$s = (E_{e^+} + E_{e^-})^2$$

$$R(e^+e^- \rightarrow H\bar{H}) \propto |F(s)|^2$$

$$|F(s)| \propto [\tfrac{1}{s}]^{n_q - 1}$$

$$e^+e^- \longrightarrow p\bar{p}$$

Probability decreases with number of constituents!