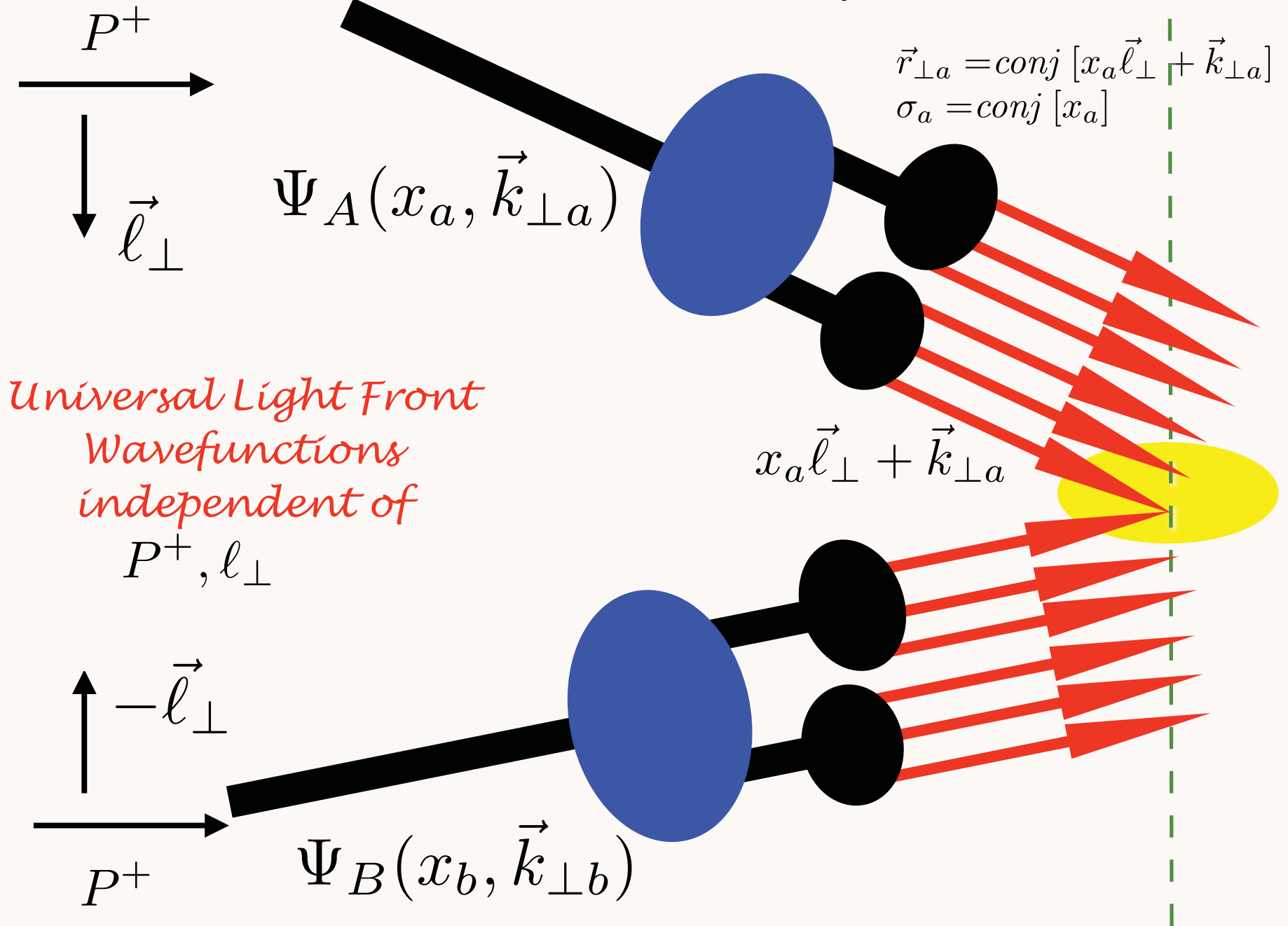


Interaction of  $a$  and  $b$  when  $\vec{r}_{\perp a} \simeq \vec{r}_{\perp b}$  and  $\sigma_a \simeq \sigma_b$

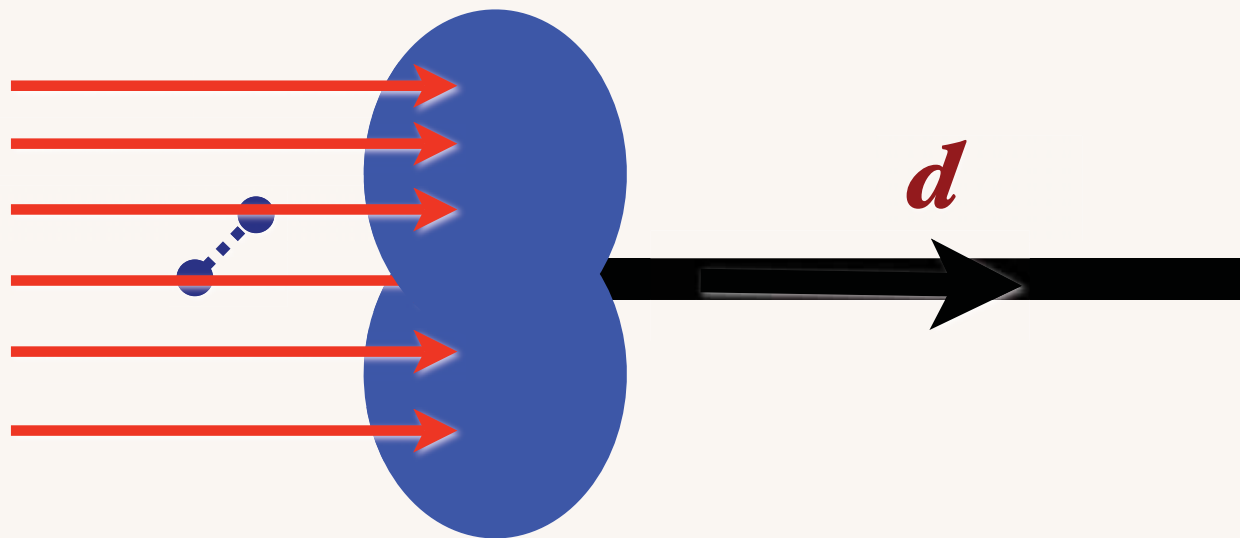


*Universal Light Front  
Wavefunctions  
independent of  
 $P^+, \ell_{\perp}$*

# Features of LF T-Matrix Formalism

## “Event Amplitude Generator”

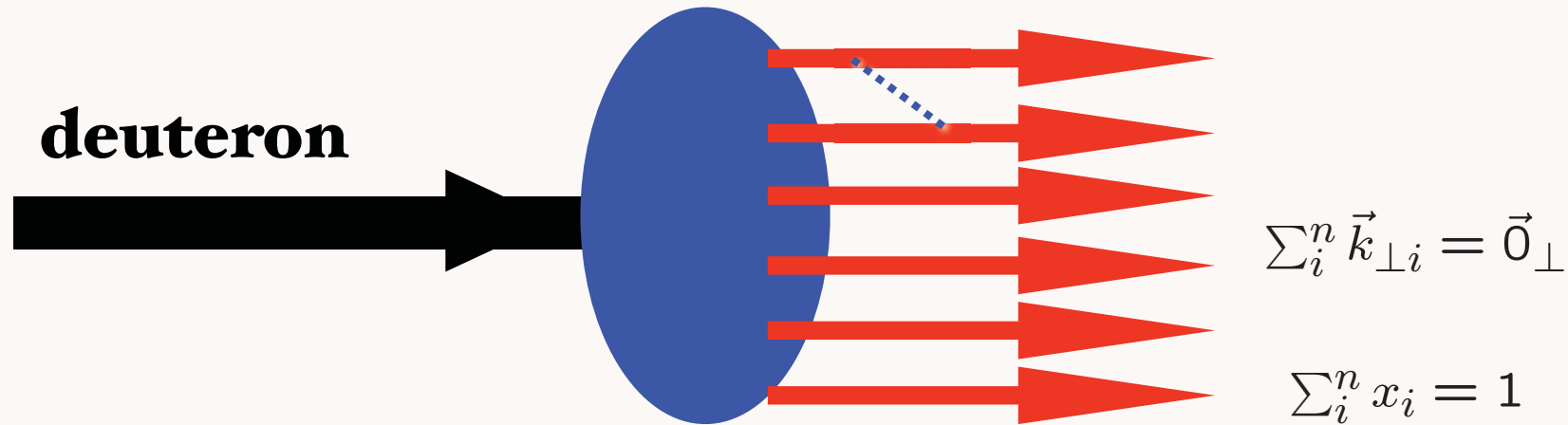
- Hidden Color: Six-quark color-singlet Fock states of deuteron from hard gluon exchange:
- Deuteron LFWF not always product of nucleon clusters



# Hidden Color of Deuteron

## Evolution of 5 color-singlet Fock states

$$\Psi_n^{\mathbf{d}}(x_i, \vec{k}_{\perp i}, \lambda_i)$$



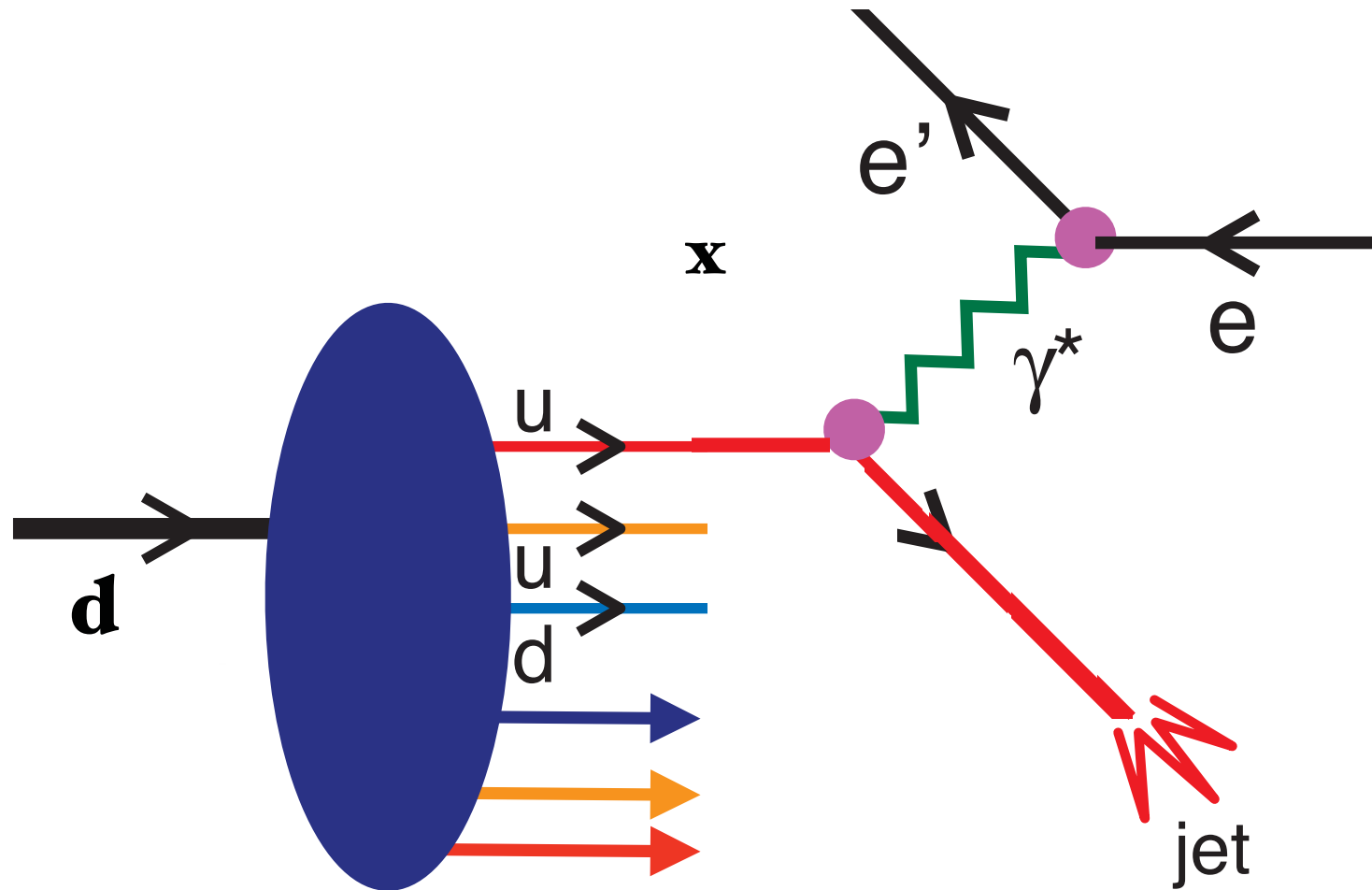
$$\Phi_n(x_i, Q) = \int^{k_{\perp i}^2 < Q^2} \prod' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j})$$

**Ji, Lepage, sjb**

5 X 5 Matrix Evolution Equation for deuteron  
distribution amplitude

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is  $|n\ p\rangle$
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- **Predict**  $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$  at high  $Q^2$

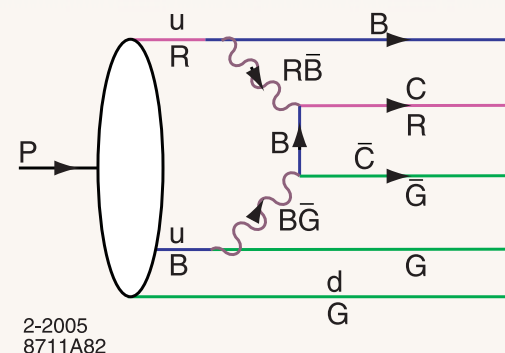
# Deep Inelastic Electron-Deuteron Scattering

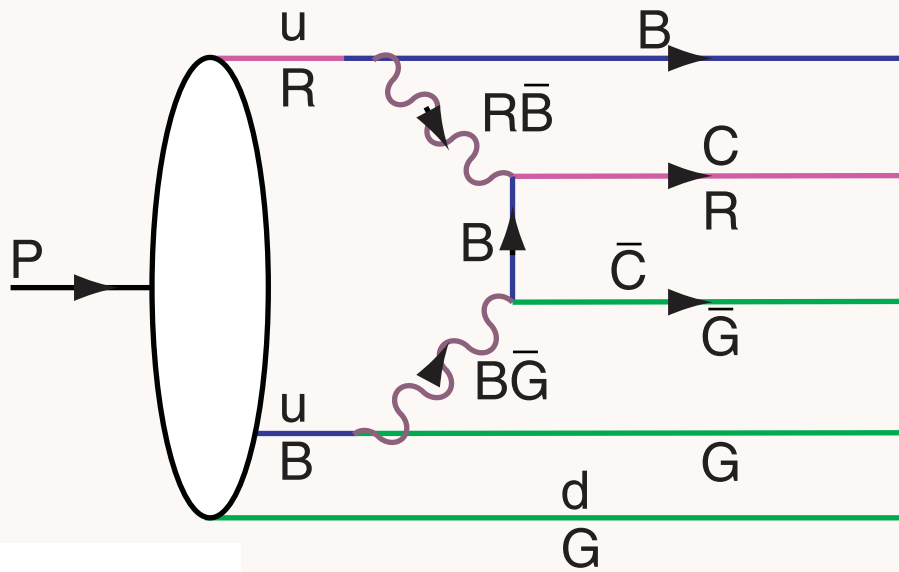


*Hidden color: excited target spectator system,  
No nucleon spectator*

# Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability  $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$      $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$      $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests





$|uudc\bar{c}\rangle$  Fluctuation in Proton

QCD: Probability  $\sim \frac{\Lambda_{QCD}^2}{M_Q^2}$

$|e^+e^-l^+l^-\rangle$  Fluctuation in Positronium

QED: Probability  $\sim \frac{(m_e\alpha)^4}{M_l^4}$

OPE derivation - M.Polyakov et al.

$$\langle p | \frac{G_{\mu\nu}^3}{m_Q^2} | p \rangle \text{ vs. } \langle p | \frac{F_{\mu\nu}^4}{m_l^4} | p \rangle \sim c\bar{c} \text{ in Color Octet}$$

Distribution peaks at equal rapidity (velocity)  
Therefore heavy particles carry the largest momentum fractions

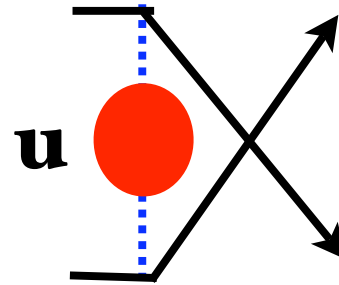
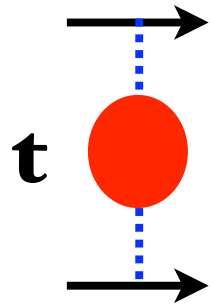
$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

*High x charm!*

*Charm at Threshold*

# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \rightarrow ee}(++; ++)=\frac{8\pi s}{t}\alpha(t)+\frac{8\pi s}{u}\alpha(u)$$



$$\alpha(t)=\frac{\alpha(0)}{1-\Pi(t)}$$

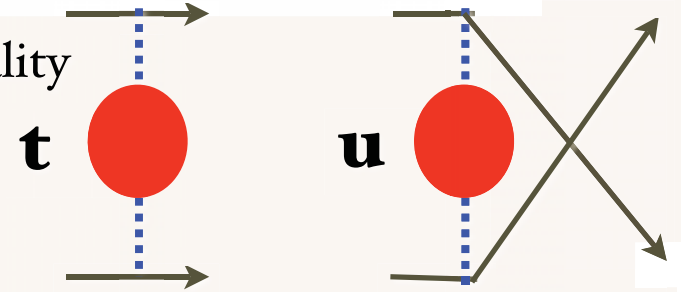
**Gell Mann-Low Effective Charge**



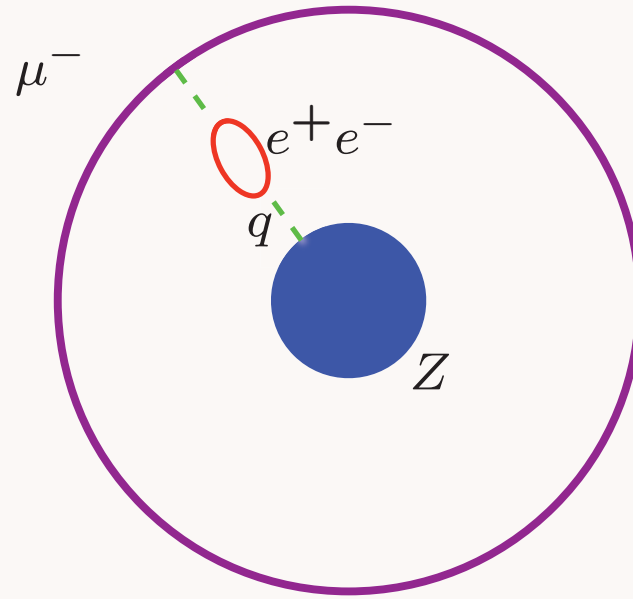
# Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

- Two separate physical scales:  $t, u =$  photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!



# Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

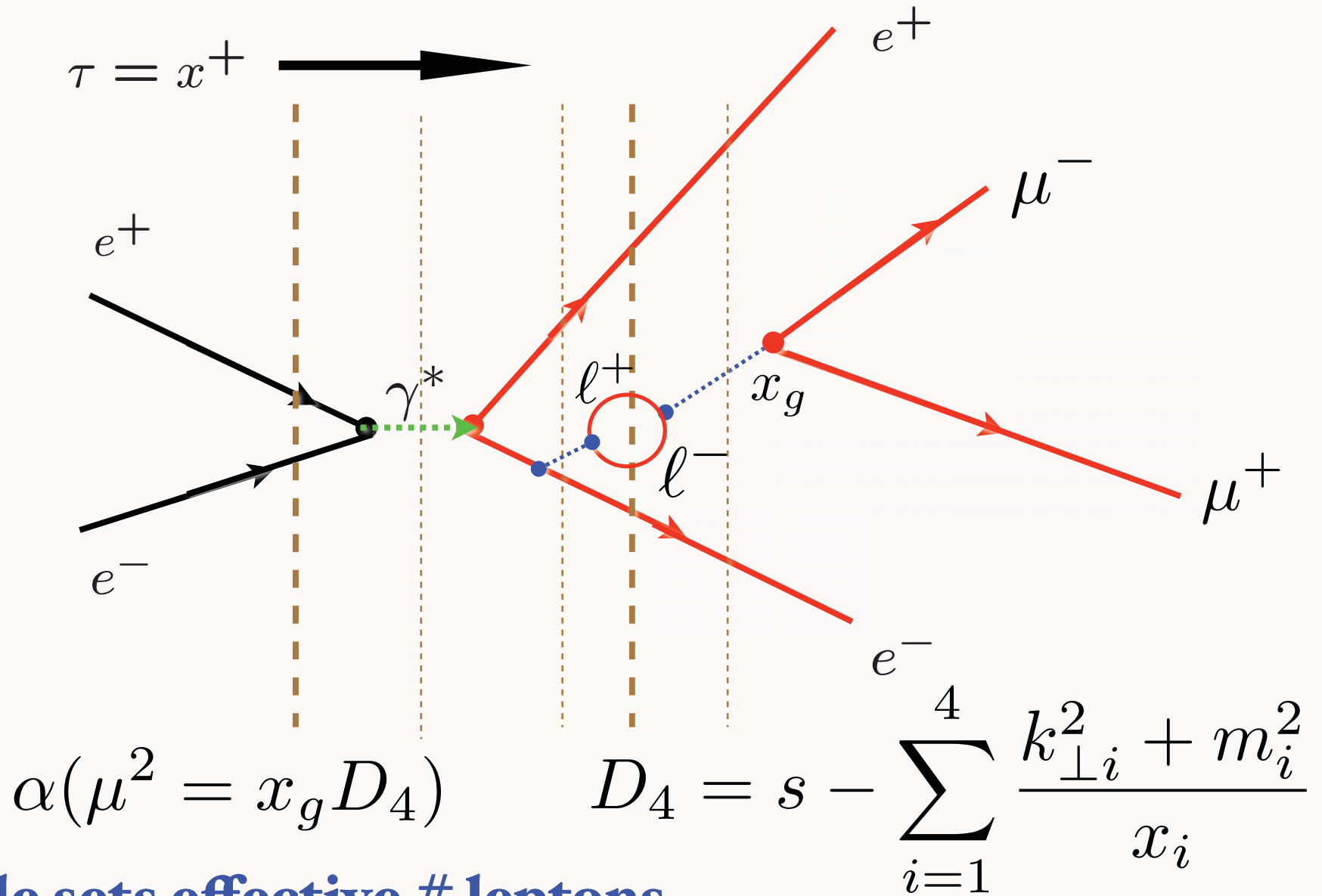
$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

**Scale is unique: Tested to ppm**

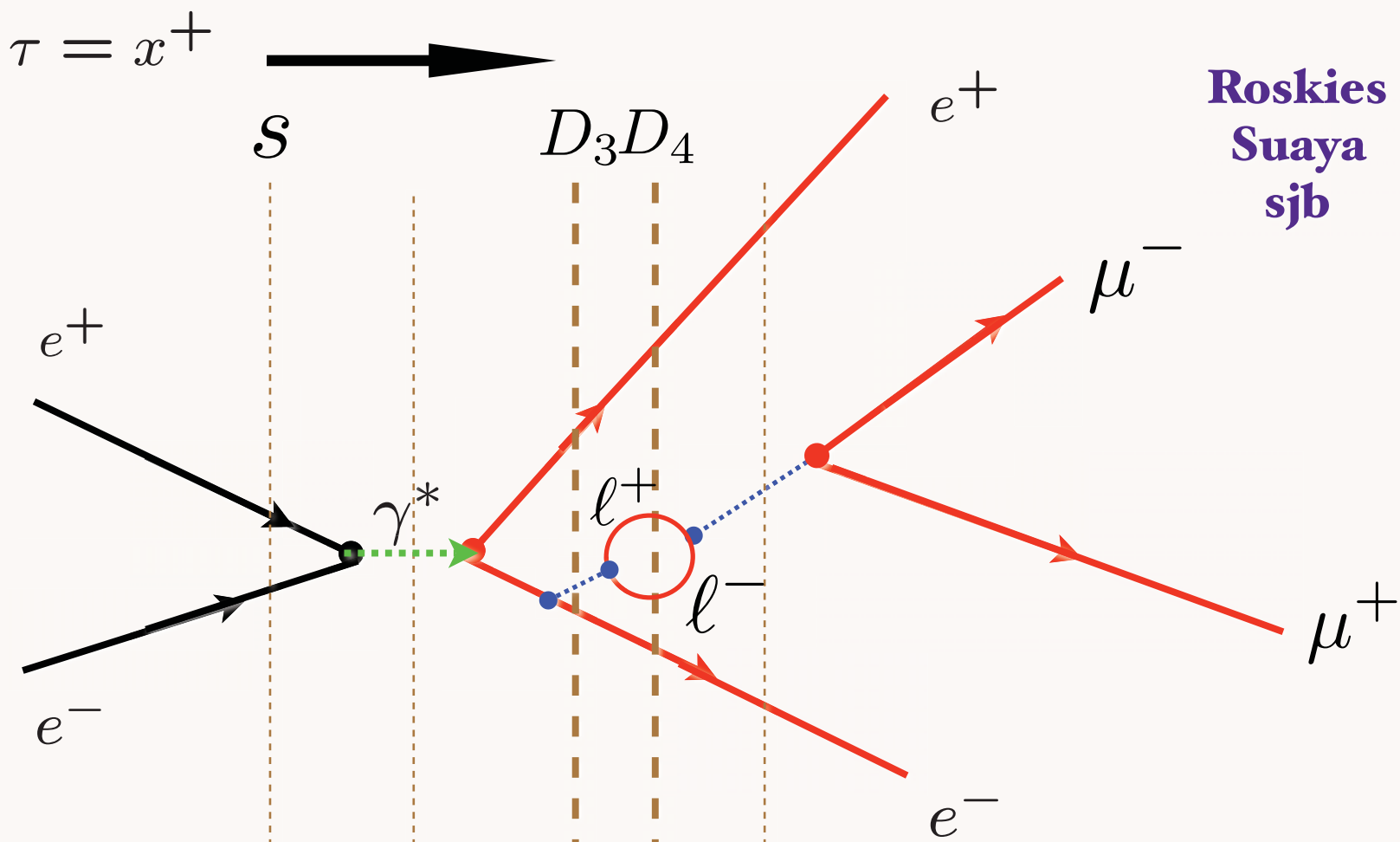
Gyulassy: Higher Order VP verified to 0.1% precision in  $\mu$  Pb

# QED Renormalization Scale Setting in LFPth



**Scale sets effective # leptons**

# Alternate Denominator: UV Subtraction Method



Roskies  
Suaya  
sjb

$$T_{ren} : \frac{1}{s - D_4} \rightarrow \frac{1}{s - D_4} - \frac{1}{D_3 - D_4}$$

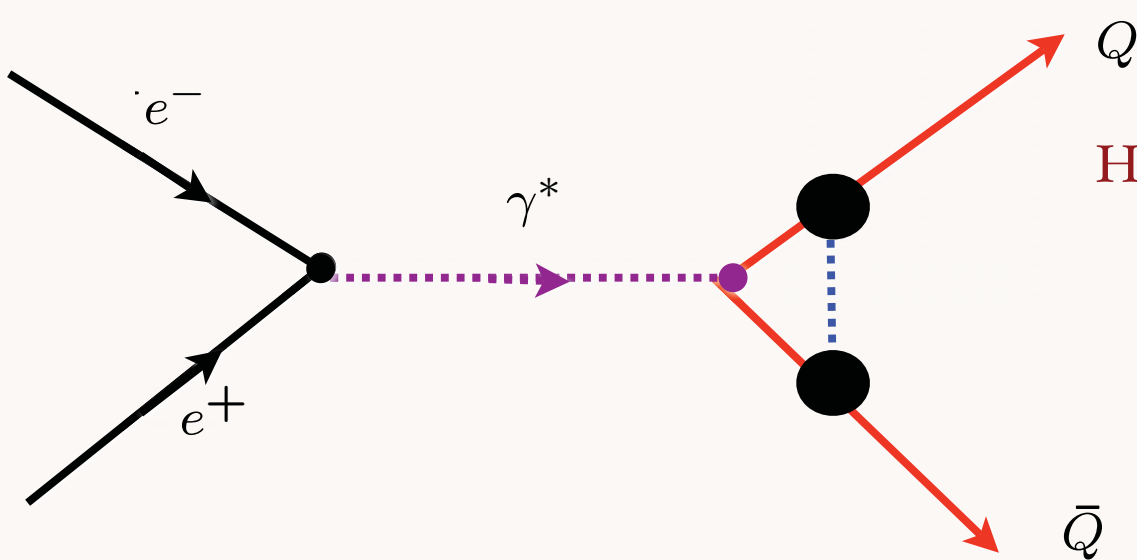
$$D_4 = s - \sum_{i=1}^4 \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

$\lim N_C \rightarrow 0$  at fixed  $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

QCD  $\rightarrow$  Abelian Gauge Theory

*Analytic Feature of SU(Nc) Gauge Theory*

*Scale-Setting procedure for QCD  
must be applicable to QED*



Hoang, Kuhn, Teubner, sjb

$$\begin{aligned}
 F_1 + F_2 &= 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \\
 &\approx \left( 1 - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \right) \left( 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} \right)
 \end{aligned}$$

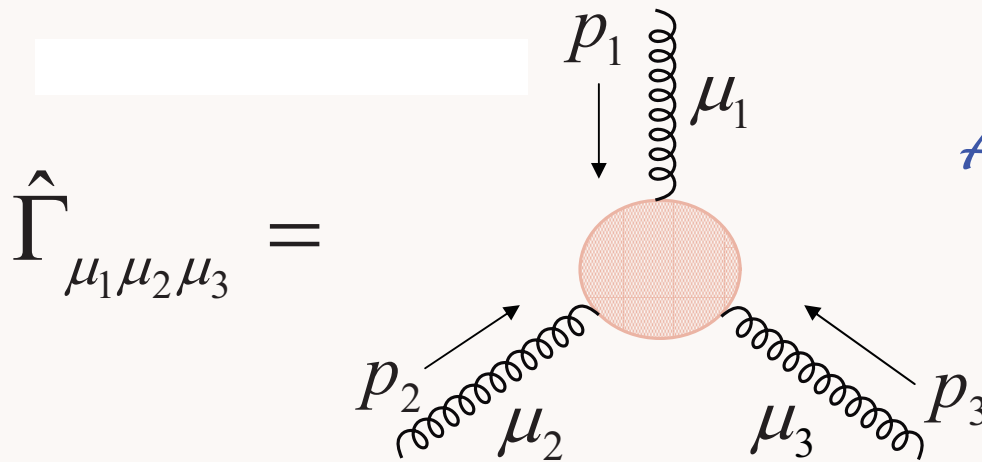
## *Example of Multiple BLM Scales*

Angular distributions of massive quarks and leptons close to threshold.

# General Structure of the Three-Gluon Vertex

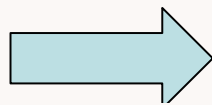
"THE FORM-FACTORS OF THE GAUGE-INVARIANT THREE-GLUON VERTEX"

M. Binger, sjb

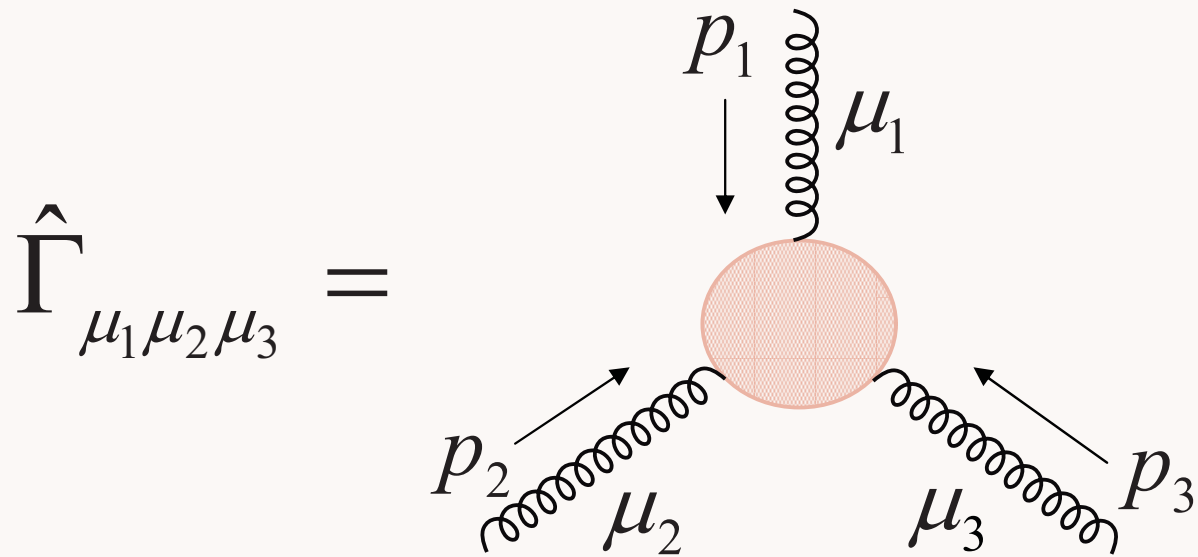


*Analytic calculation:  
general masses, spin*

3 index tensor  $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$  built out of  $g_{\mu\nu}$  and  $p_1, p_2, p_3$   
with  $p_1 + p_2 + p_3 = 0$



14 basis tensors and form factors



H. J. Lu

$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$



## Properties of the Effective Scale

$$Q_{\text{eff}}^2(a, b, c) = Q_{\text{eff}}^2(-a, -b, -c)$$

$$Q_{\text{eff}}^2(\lambda a, \lambda b, \lambda c) = |\lambda| Q_{\text{eff}}^2(a, b, c)$$

$$Q_{\text{eff}}^2(a, a, a) = |a|$$

$$Q_{\text{eff}}^2(a, -a, -a) \approx 5.54 |a|$$

$$Q_{\text{eff}}^2(a, a, c) \approx 3.08 |c| \quad \text{for } |a| \gg |c|$$

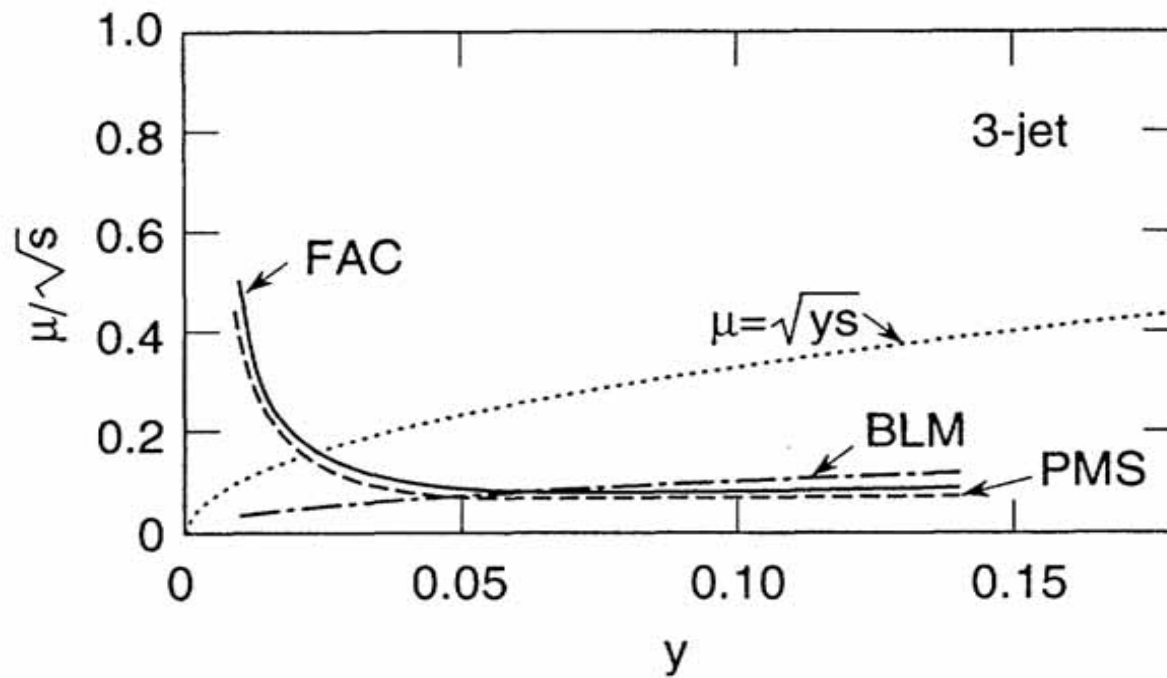
$$Q_{\text{eff}}^2(a, -a, c) \approx 22.8 |c| \quad \text{for } |a| \gg |c|$$

$$Q_{\text{eff}}^2(a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for } |a| \gg |b|, |c|$$

*Surprising dependence on Invariants*

# BLM Method

- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit ( $N_c = 0$ )
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...



Kramer & Lampe

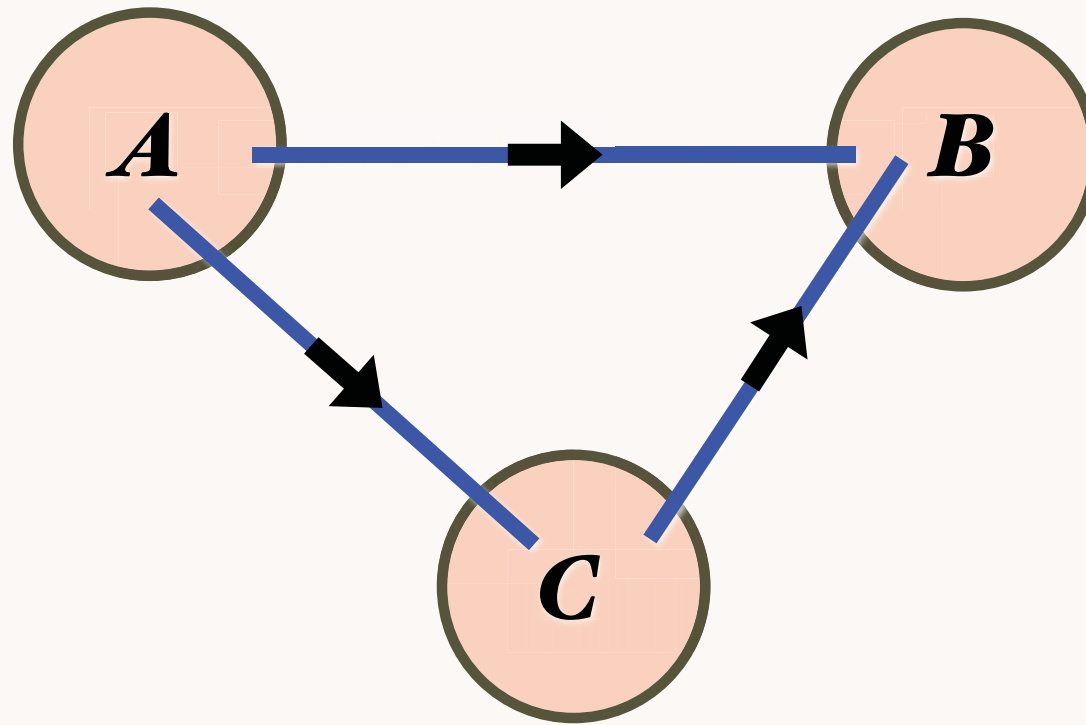
## Three-Jet Rate

The scale  $\mu/\sqrt{s}$  according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and  $\sqrt{y}$  (dotted) procedures for the three-jet rate in  $e^+e^-$  annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low  $y$ . In particular, the latter two methods predict increasing values of  $\mu$  as the jet invariant mass  $\mathcal{M} < \sqrt{(ys)}$  decreases.

## Other Jet Observables:

Rathsman

# *Transitivity Property of Renormalization Group*



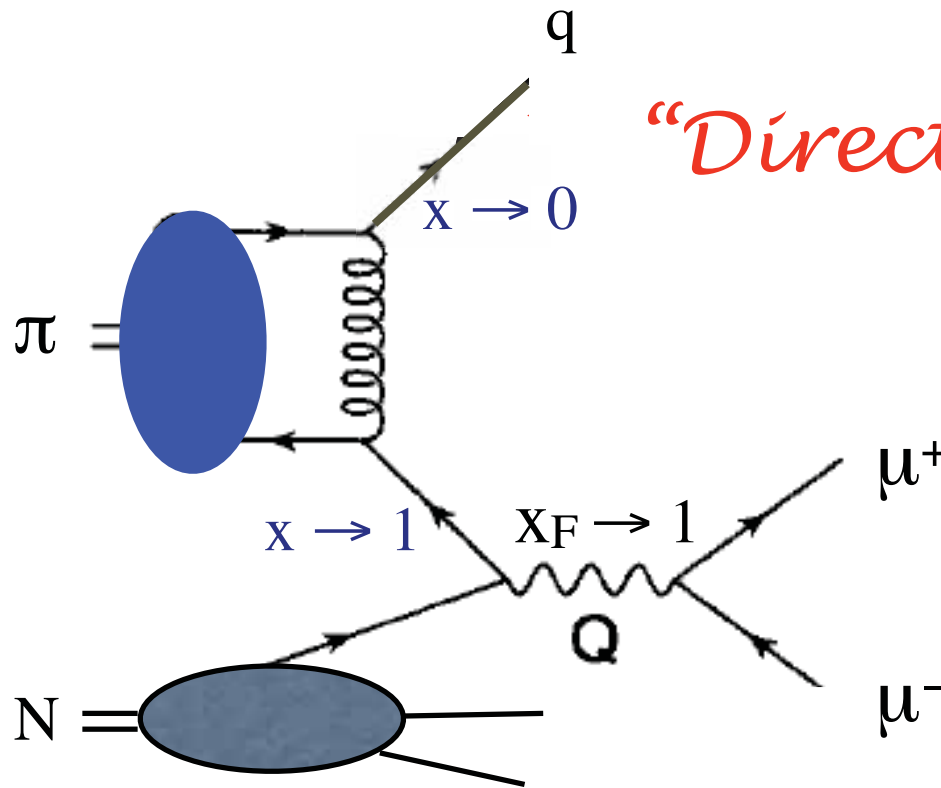
$A \rightarrow C$     $C \rightarrow B$    *identical to*    $A \rightarrow B$

*Relation of observables independent of intermediate scheme  $C$*

$$\pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F$$

In the limit where  $(1-x_F)Q^2$  is fixed as  $Q^2 \rightarrow \infty$

Entire pion wf  
contributes to  
hard process



*“Direct” Subprocess*

Virtual photon is  
longitudinally  
polarized

Berger and Brodsky, PRL 42 (1979) 940

$$\pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

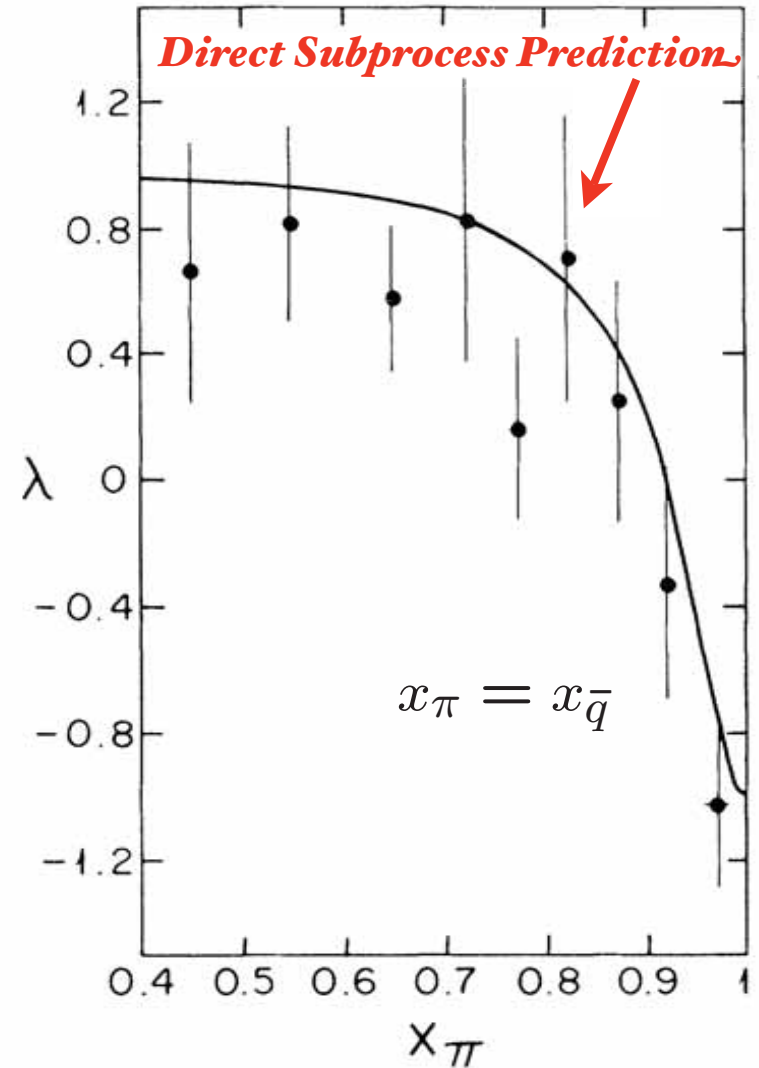
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[ (1 - x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2$$

*Dramatic change in  
angular distribution at  
large  $x_F$*

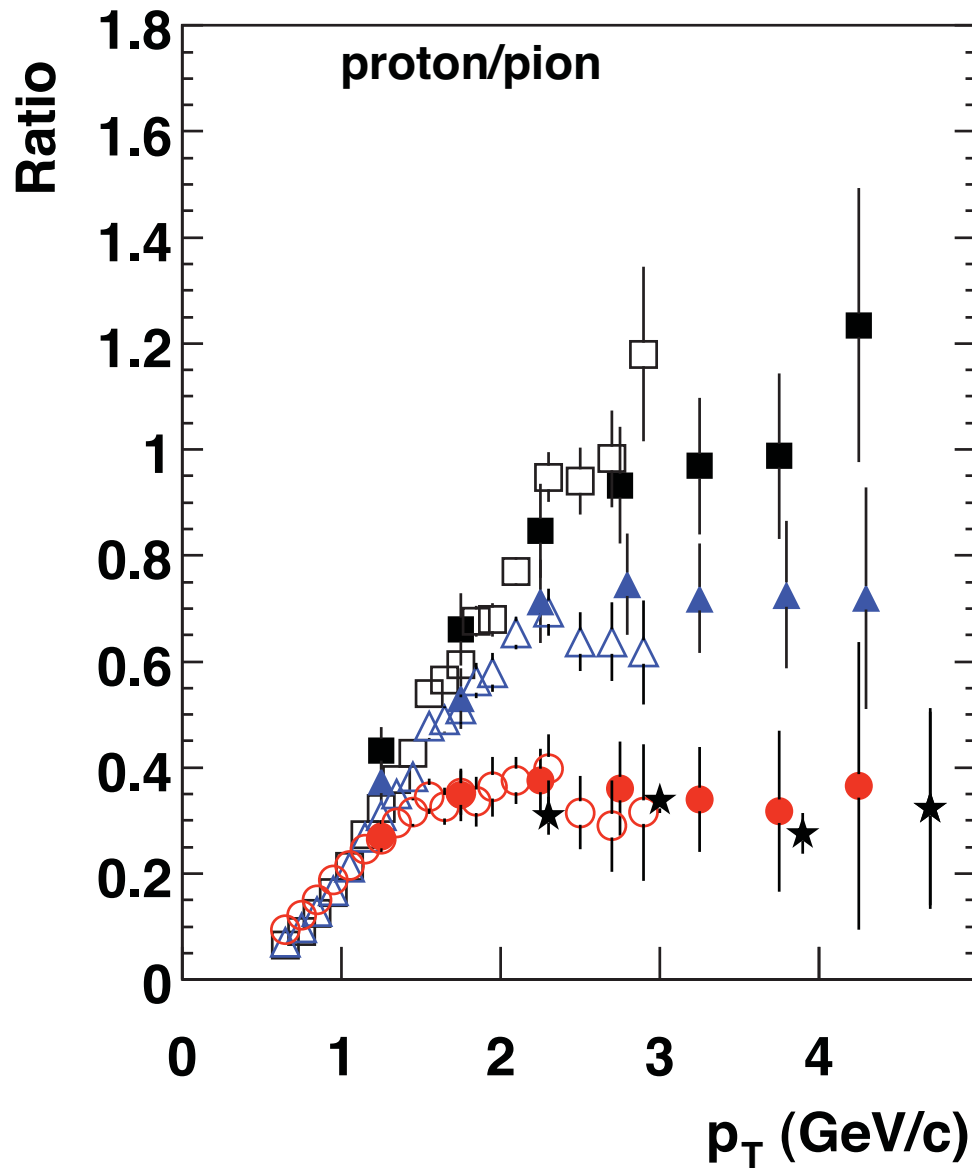
**Example of a higher-twist  
direct subprocess**



Chicago-Princeton  
Collaboration

Phys.Rev.Lett.55:2649,1985

*Particle ratio changes with centrality!*



*Protons less absorbed  
in nuclear collisions than pions*

← **Central**

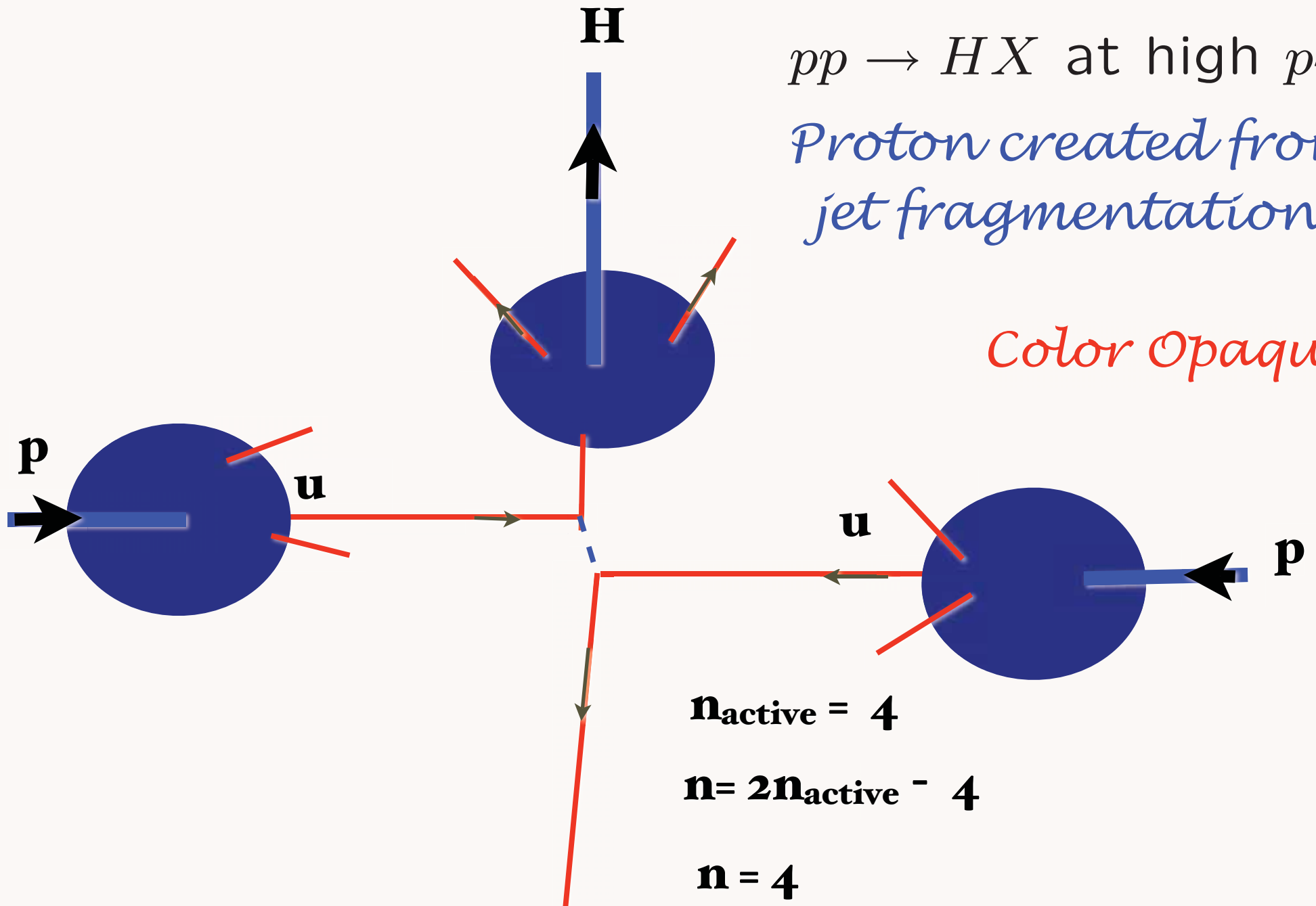
- ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ● Au+Au 60-92%
- ★ p+p,  $\sqrt{s} = 53$  GeV, ISR
- e<sup>+</sup>e<sup>-</sup>, gluon jets, DELPHI
- ..... e<sup>+</sup>e<sup>-</sup>, quark jets, DELPHI

← **Peripheral**

**A. Sickles and SJB**

$pp \rightarrow HX$  at high  $p_T$   
*Proton created from  
 jet fragmentation*

*Color Opaque*



$$n_{\text{active}} = 4$$

$$n = 2n_{\text{active}} - 4$$

$$n = 4$$



*Crucial Test of Leading -Twist QCD:  
Scaling at fixed  $x_T$*

$$x_T = \frac{2p_T}{\sqrt{s}}$$

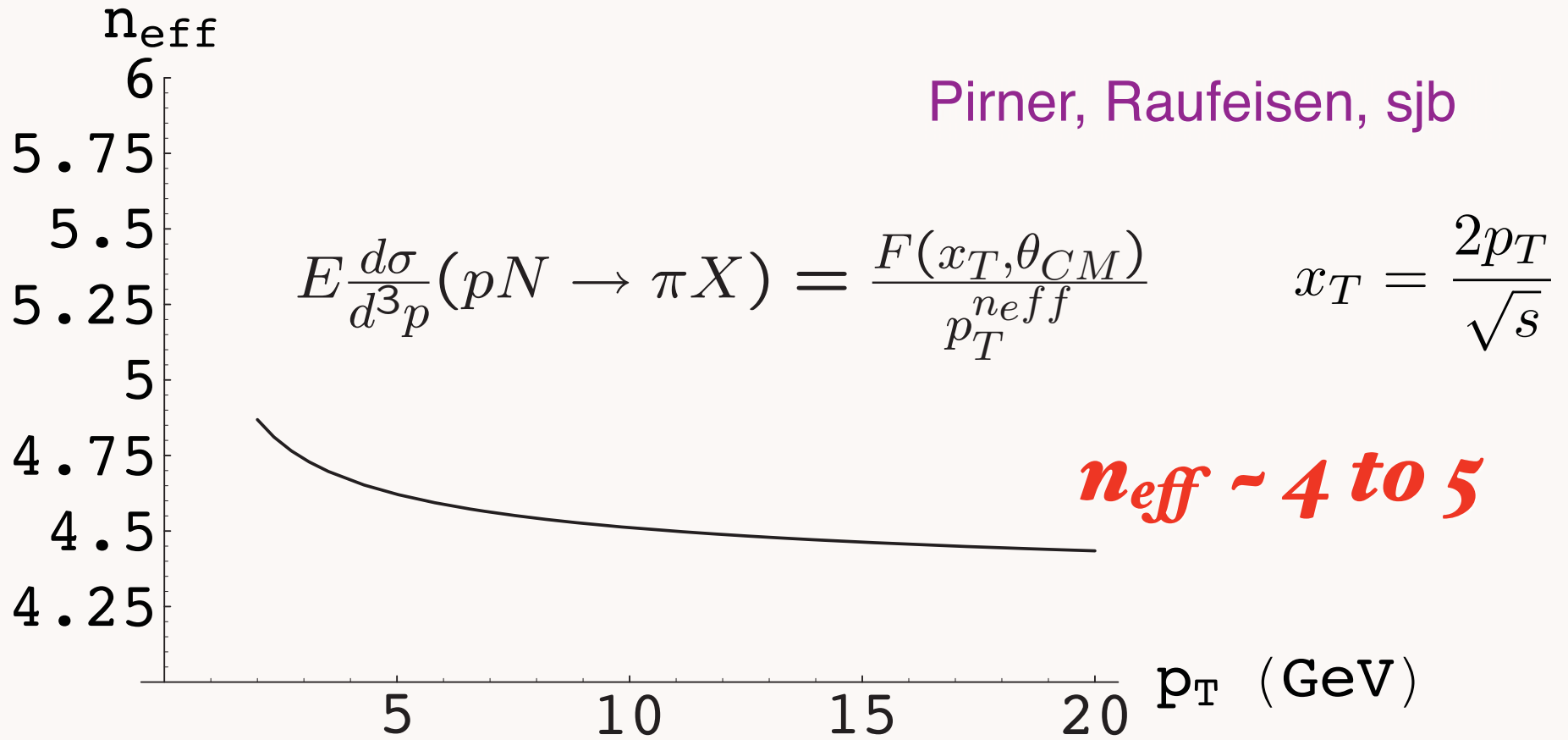
$$E \frac{d\sigma}{d^3p} (pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

***Parton model:  $n_{eff} = 4$***

***As fundamental as Bjorken scaling in DIS***

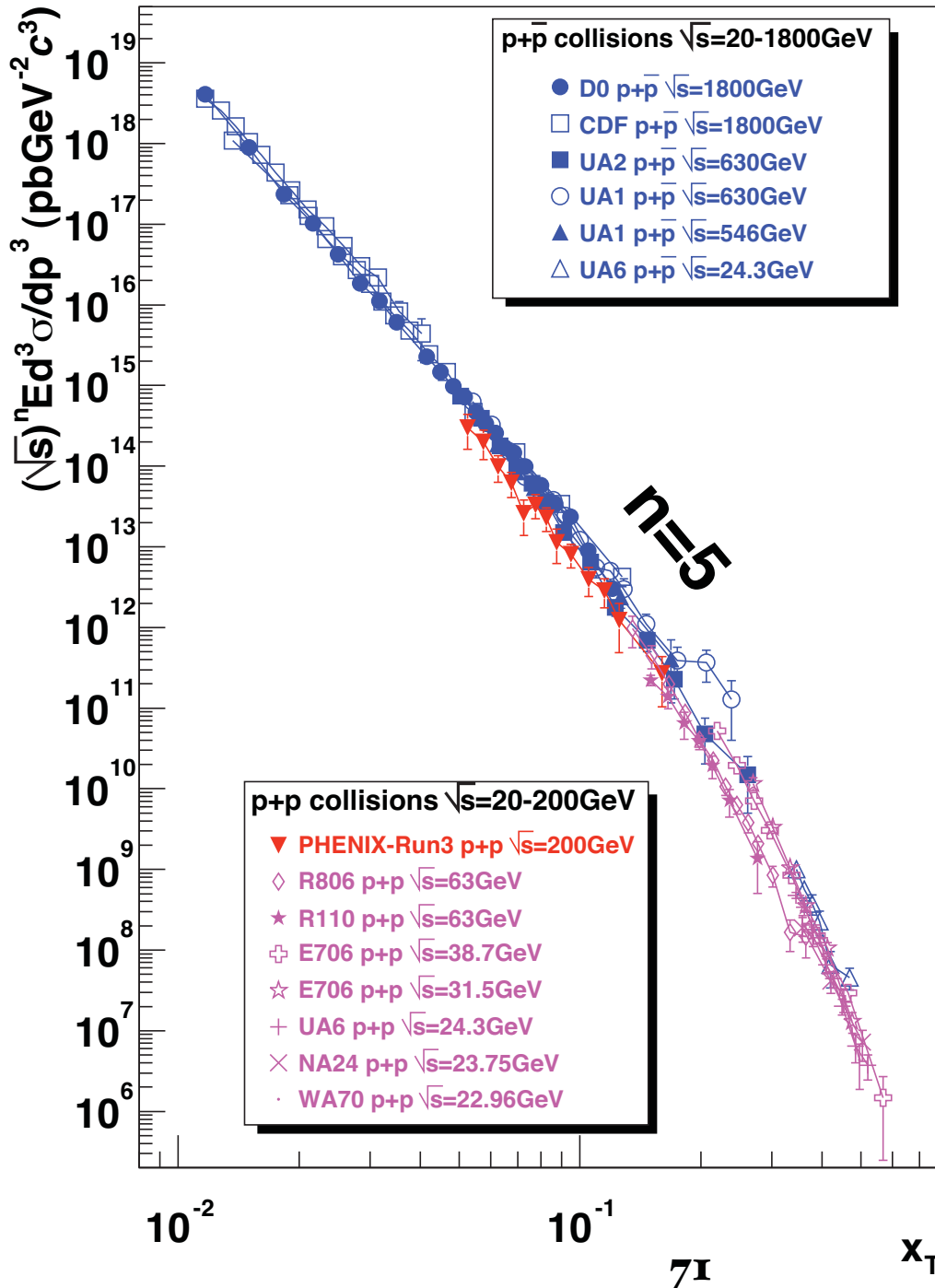
**Conformal scaling:  $n_{eff} = 2 n_{active} - 4$**

*QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling*



*Key test of PQCD: power-law fall-off at fixed  $x_T$*

$$\sqrt{s}^n E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) \text{ at fixed } x_T$$

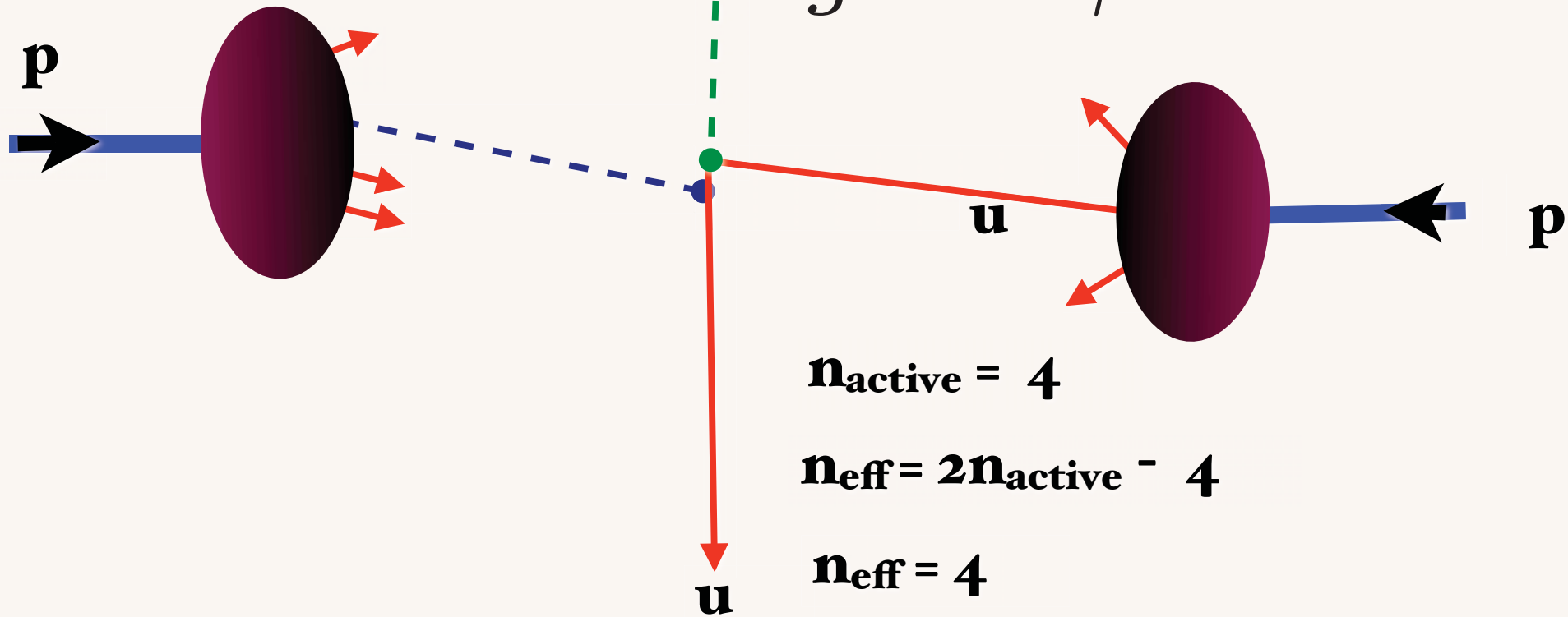


**Scaling of direct  
photon  
production  
consistent with  
PQCD**

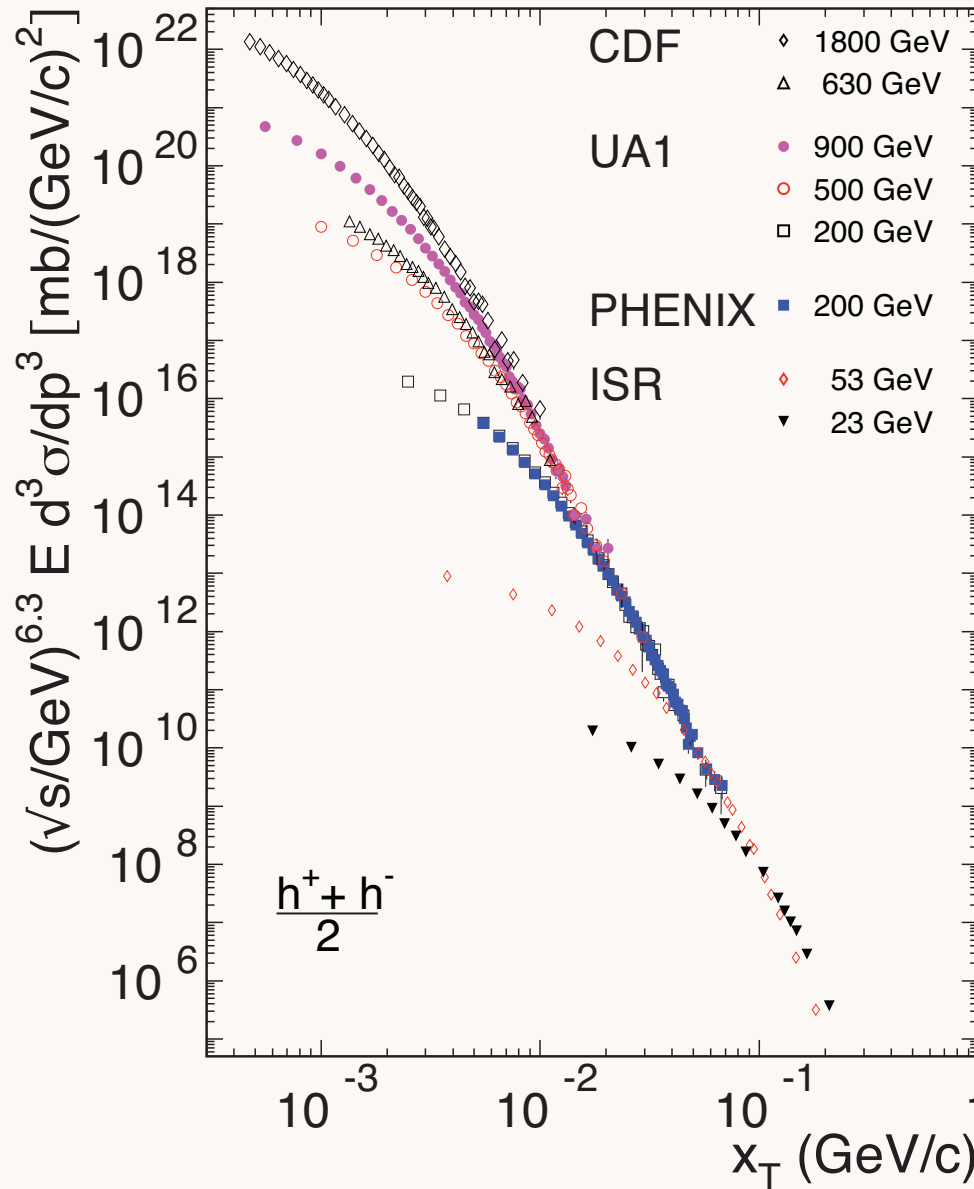
$pp \rightarrow \gamma X$

$$E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$gu \rightarrow \gamma u$



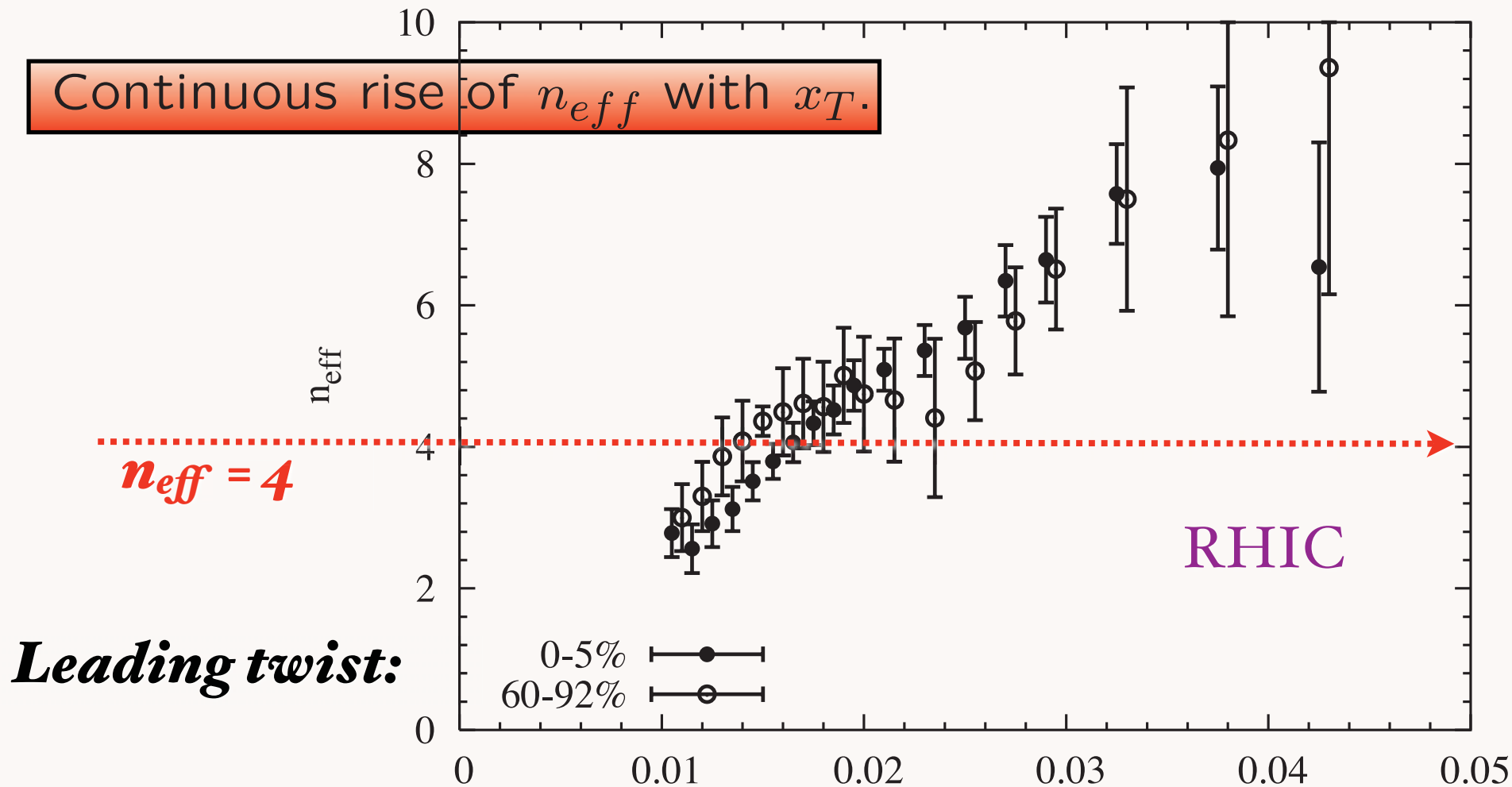
$$\sqrt{s}^{6.3} \times E \frac{d\sigma}{d^3p} (pp \rightarrow H^\pm X) \text{ at fixed } x_T$$



Tannenbaum

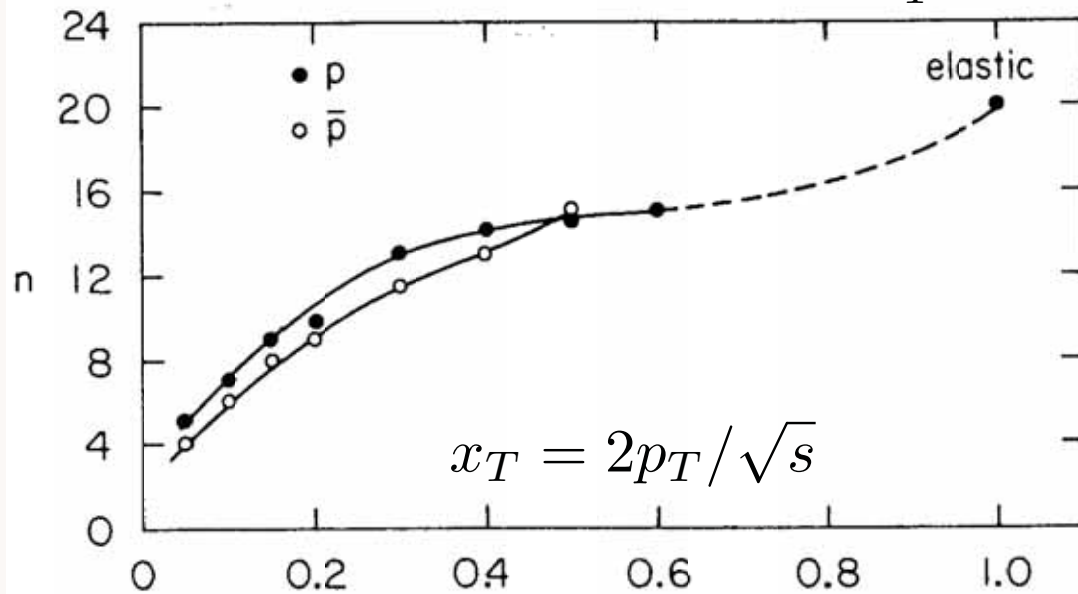
**Scaling  
inconsistent with  
PQCD**

Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available  $p_T$  range. Shown are data for central (0 – 5%) and for peripheral (60 – 90%) collisions.



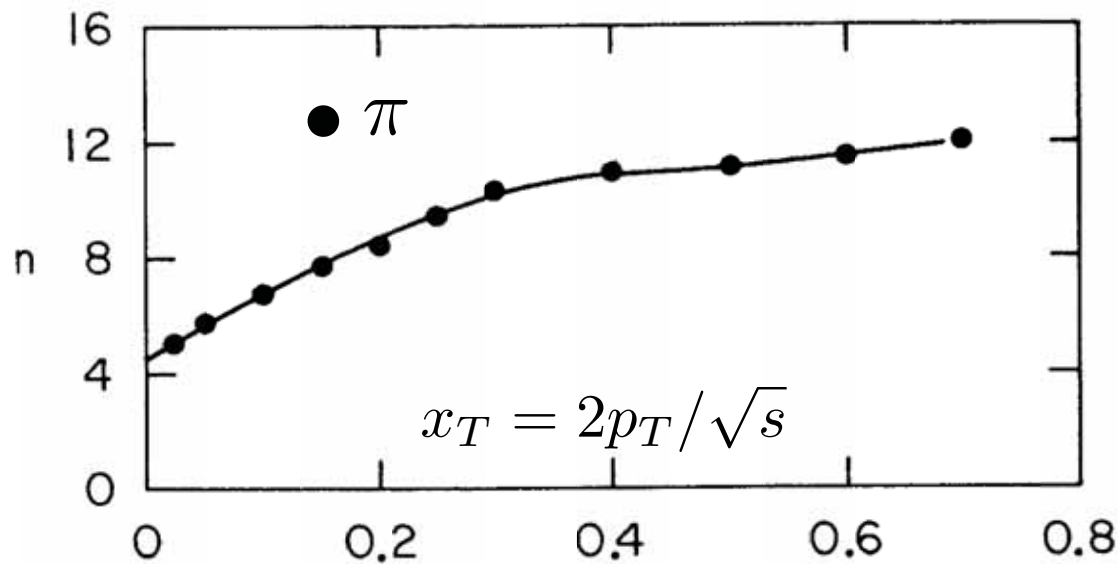
$$E \frac{d\sigma}{d^3p}(pN \rightarrow pX) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}} x_T$$

$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$

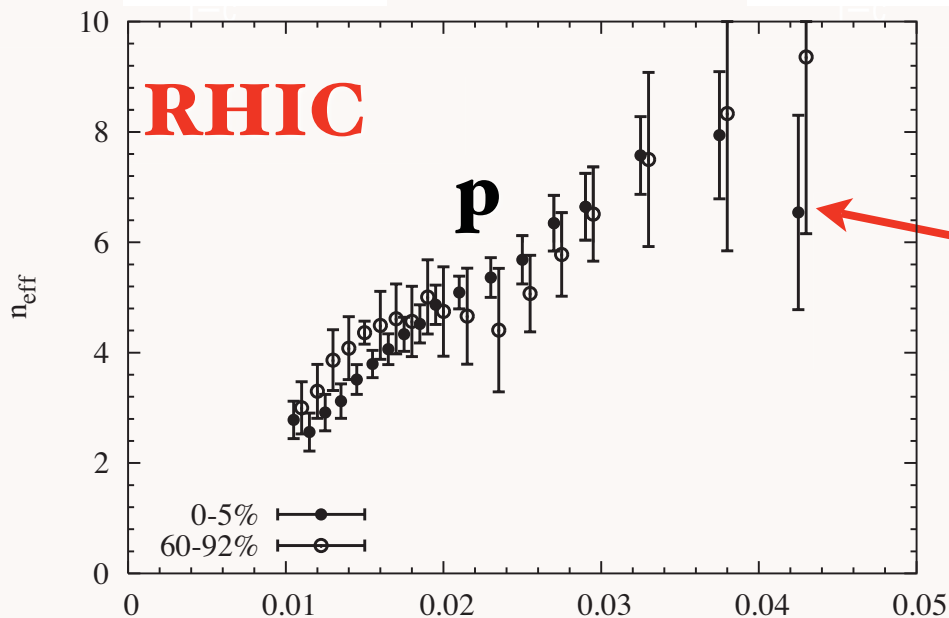
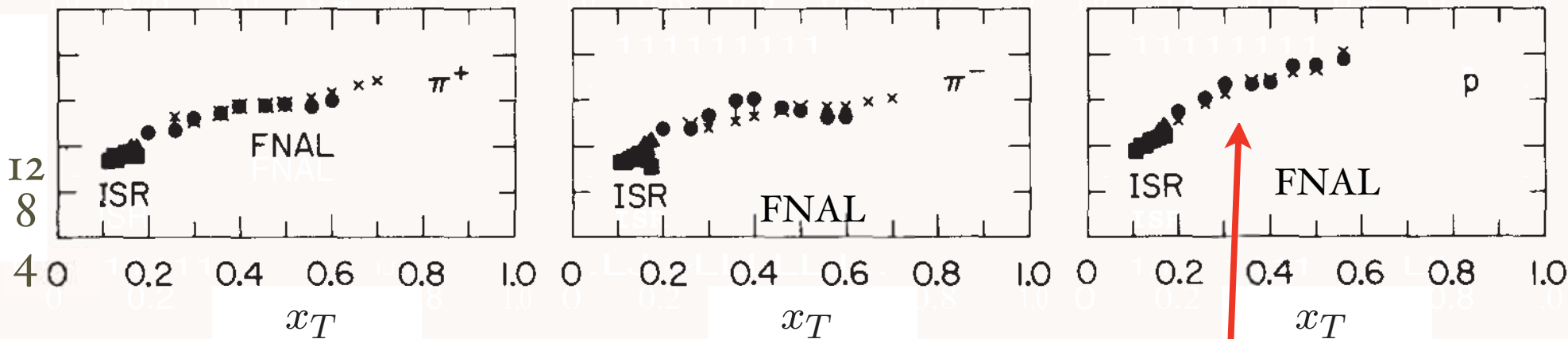


*Clear evidence  
for higher-twist  
contributions*

**J. W. Cronin, SSI 1974**



$$E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{CM})}{n_{eff} p_T}$$



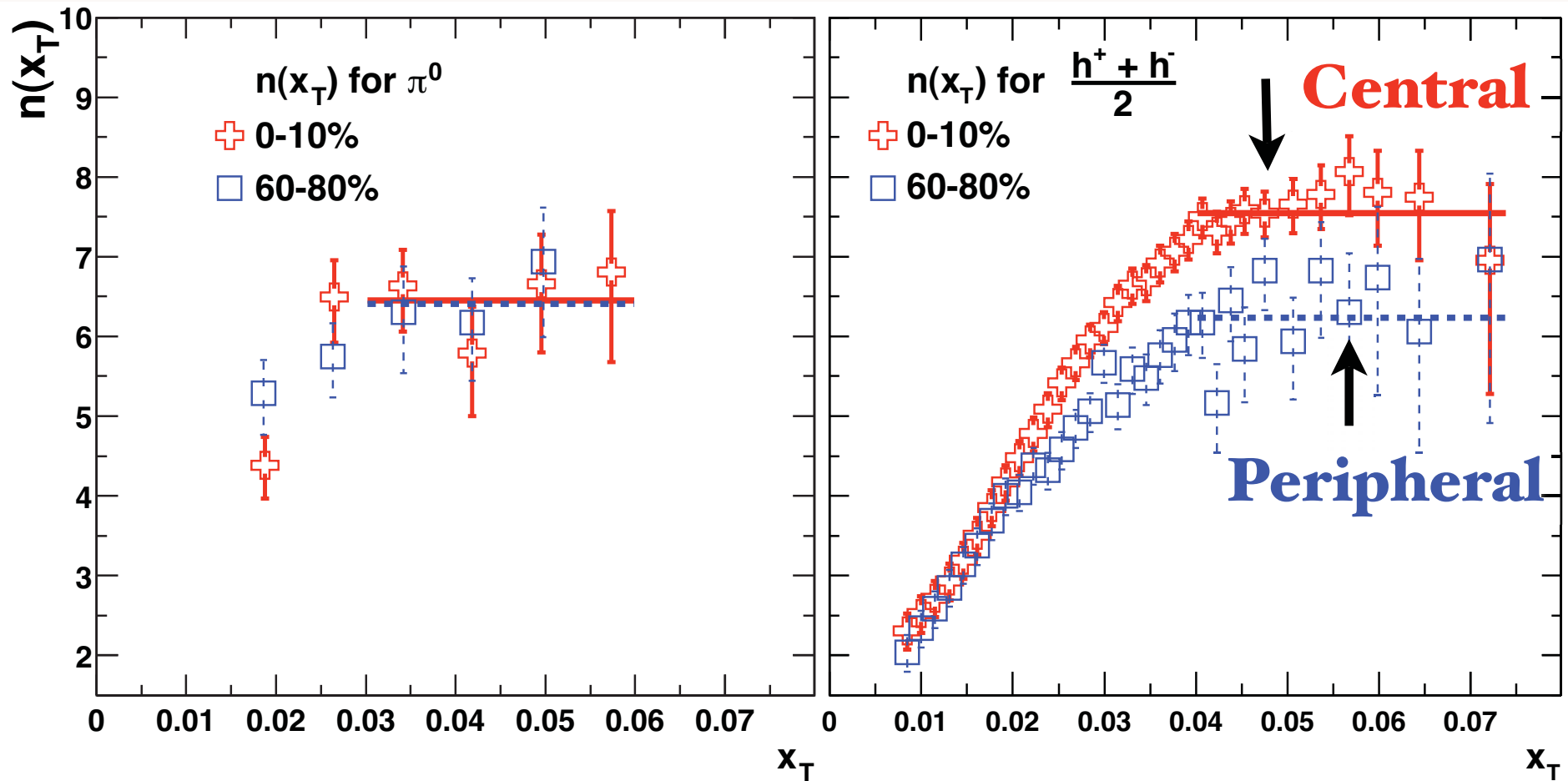
$$E \frac{d\sigma}{d^3p} (pp \rightarrow pX) = \frac{F(x_T, \theta_{CM})}{p_T^{12}}$$

$$E \frac{d\sigma}{d^3p} (pp \rightarrow pX) = \frac{F(x_T, \theta_{CM})}{p_T^8}$$

*Trend consistent with RHIC at small  $x_T$*



$$\sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV}$$



*Proton power changes with centrality !*

# Baryon can be made directly within hard subprocess

## Coalescence within hard subprocess

$$b_{\perp} \simeq 1/p_T$$

Bjorken

Blankenbecler, Gunion, sjb

Berger, sjb

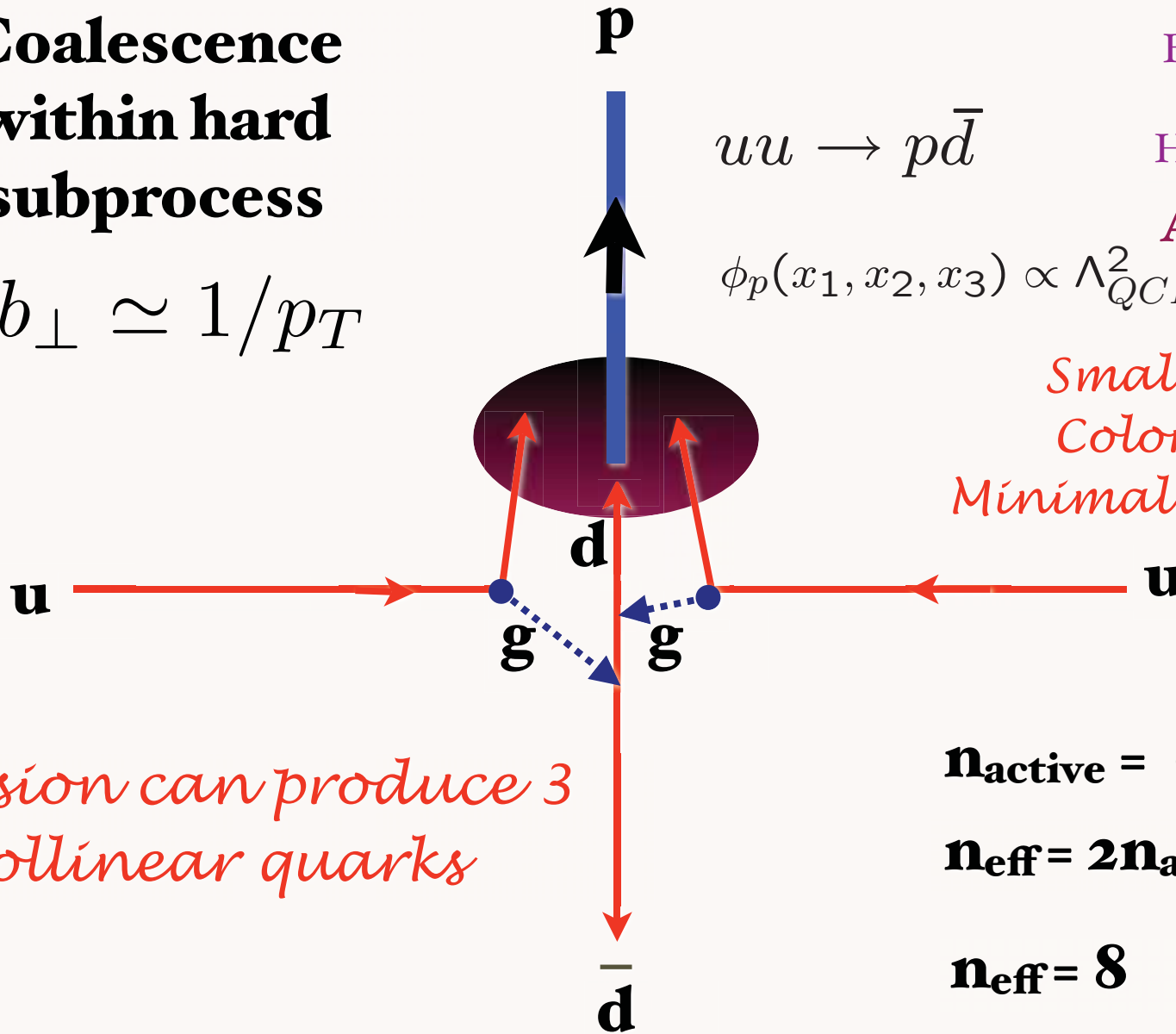
Hoyer, et al: Semi-Exclusive

**A. Sickles and SJB**

$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



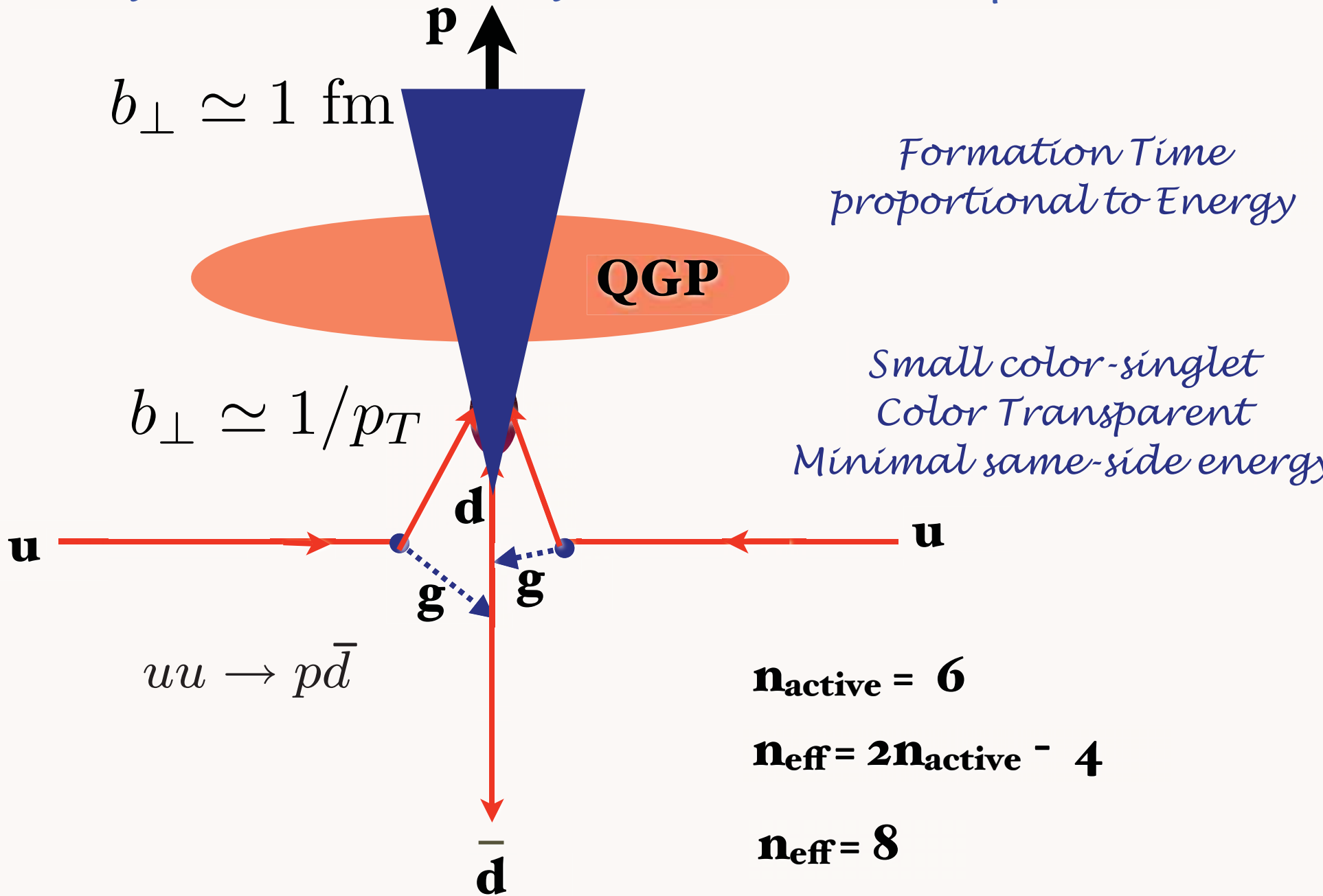
*Collision can produce 3 collinear quarks*

$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

*Baryon made directly within hard subprocess*



Dimensional counting rules provide a simple rule-of-thumb guide for the power-law fall-off of the inclusive cross section in both  $p_T$  and  $(1 - x_T)$  due to a given subprocess:

$$E \frac{d\sigma}{d^3p}(AB \rightarrow CX) \propto \frac{(1 - x_T)^{2n_{spectator} - 1}}{p_T^{2n_{active} - 4}}$$

where  $n_{active}$  is the “twist”, i.e., the number of elementary fields participating in the hard subprocess, and  $n_{spectator}$  is the total number of constituents in  $A, B$  and  $C$  not participating in the hard-scattering subprocess. For example, consider  $pp \rightarrow pX$ . The leading-twist contribution from  $qq \rightarrow qq$  has  $n_{active} = 4$  and  $n_{spectator} = 6$ . The higher-twist subprocess  $qq \rightarrow p\bar{q}$  has  $n_{active} = 6$  and  $n_{spectator} = 4$ . This simplified model provides two distinct contributions to the inclusive cross section

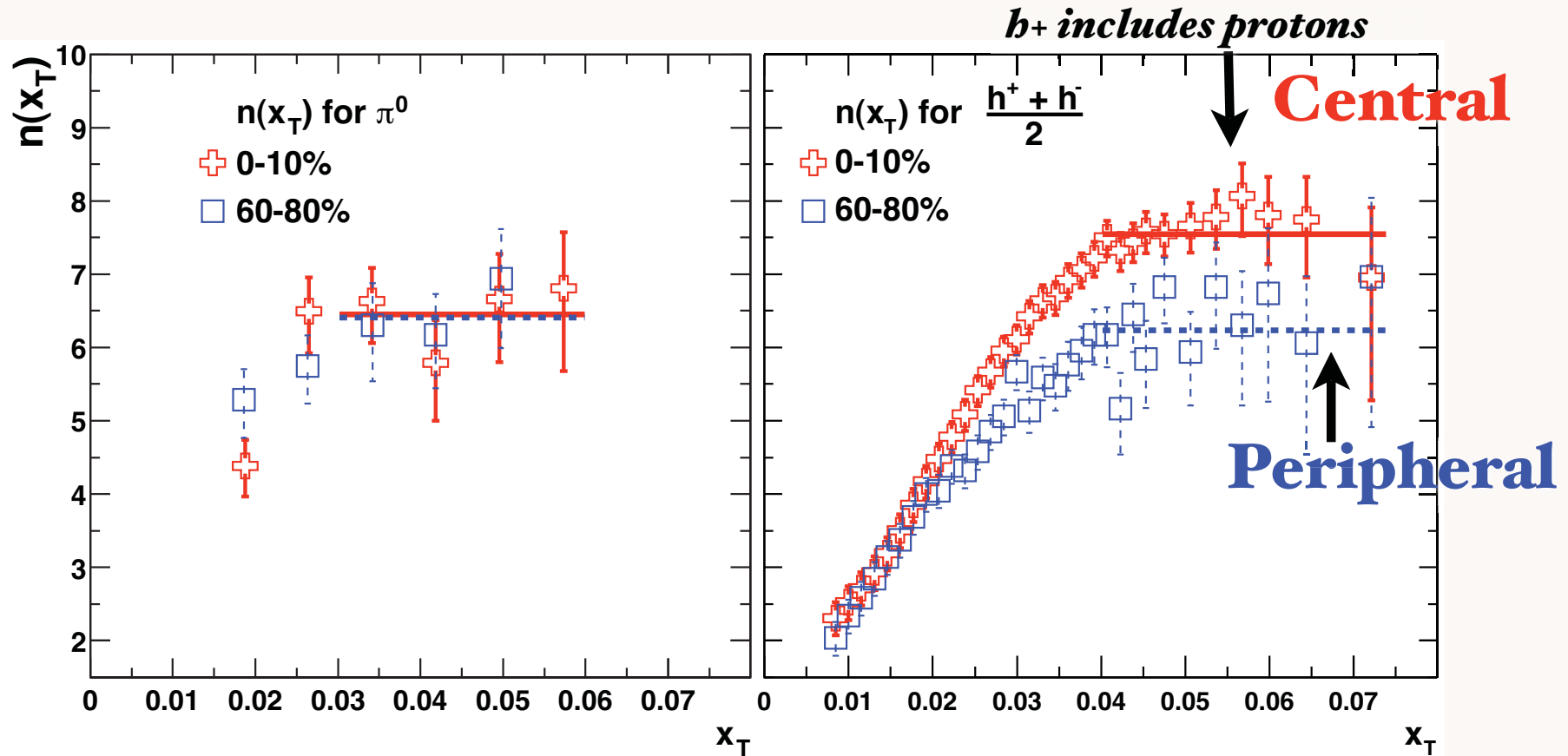
$$\frac{d\sigma}{d^3p/E}(pp \rightarrow pX) = A \frac{(1 - x_T)^{11}}{p_T^4} + B \frac{(1 - x_T)^7}{p_T^8}$$

and  $n = n(x_T)$  increases from 4 to 8 at large  $x_T$ .

*Small color-singlet  
Color Transparent  
Minimal same-side energy*

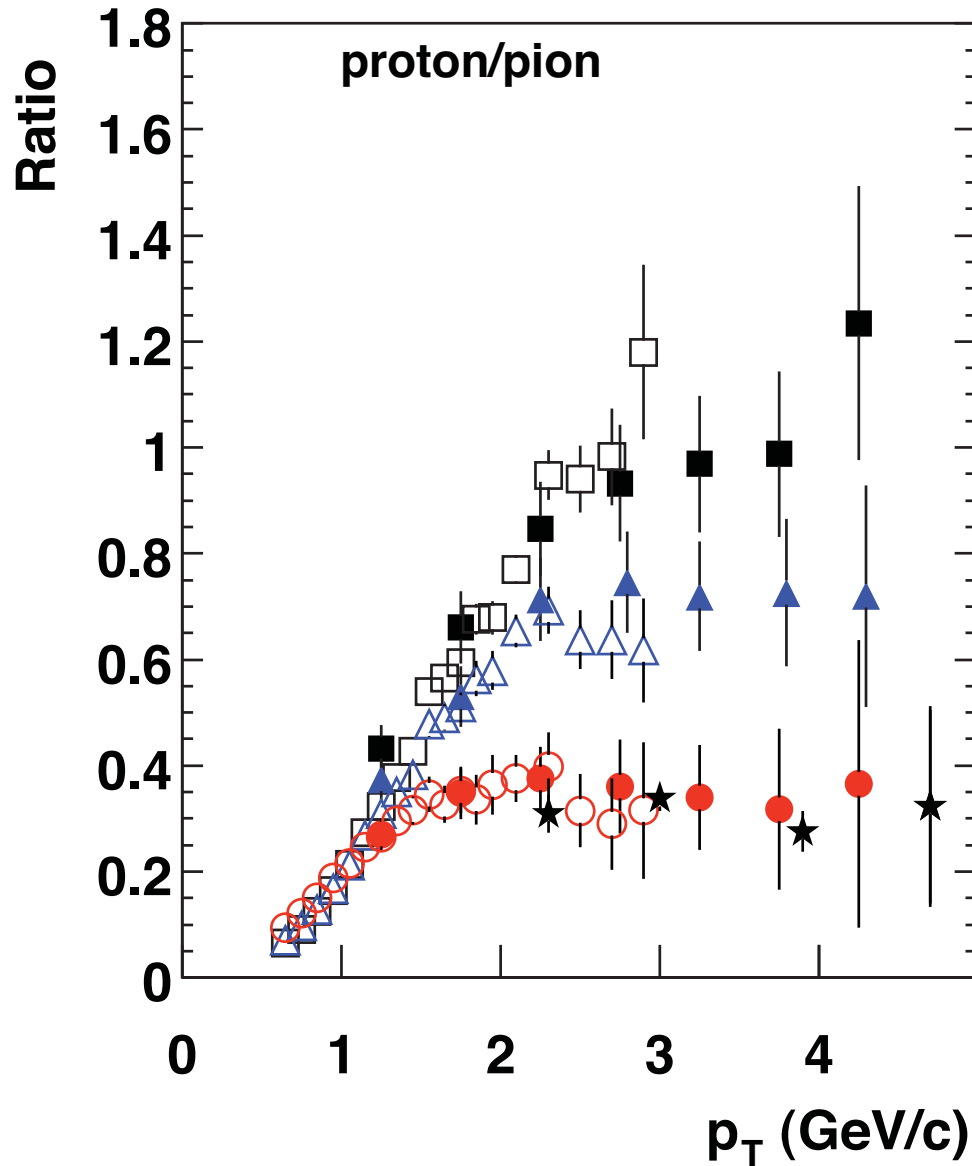
Power-law exponent  $n(x_T)$  for  $\pi^0$  and  $h$  spectra in central and peripheral Au+Au collisions at  $\sqrt{s_{NN}} = 130$  and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev. C* **69**, 034910 (2004) [nucl-ex/0308006].



*Proton production dominated by color-transparent direct high  $n_{eff}$  subprocesses*

*Particle ratio changes with centrality!*

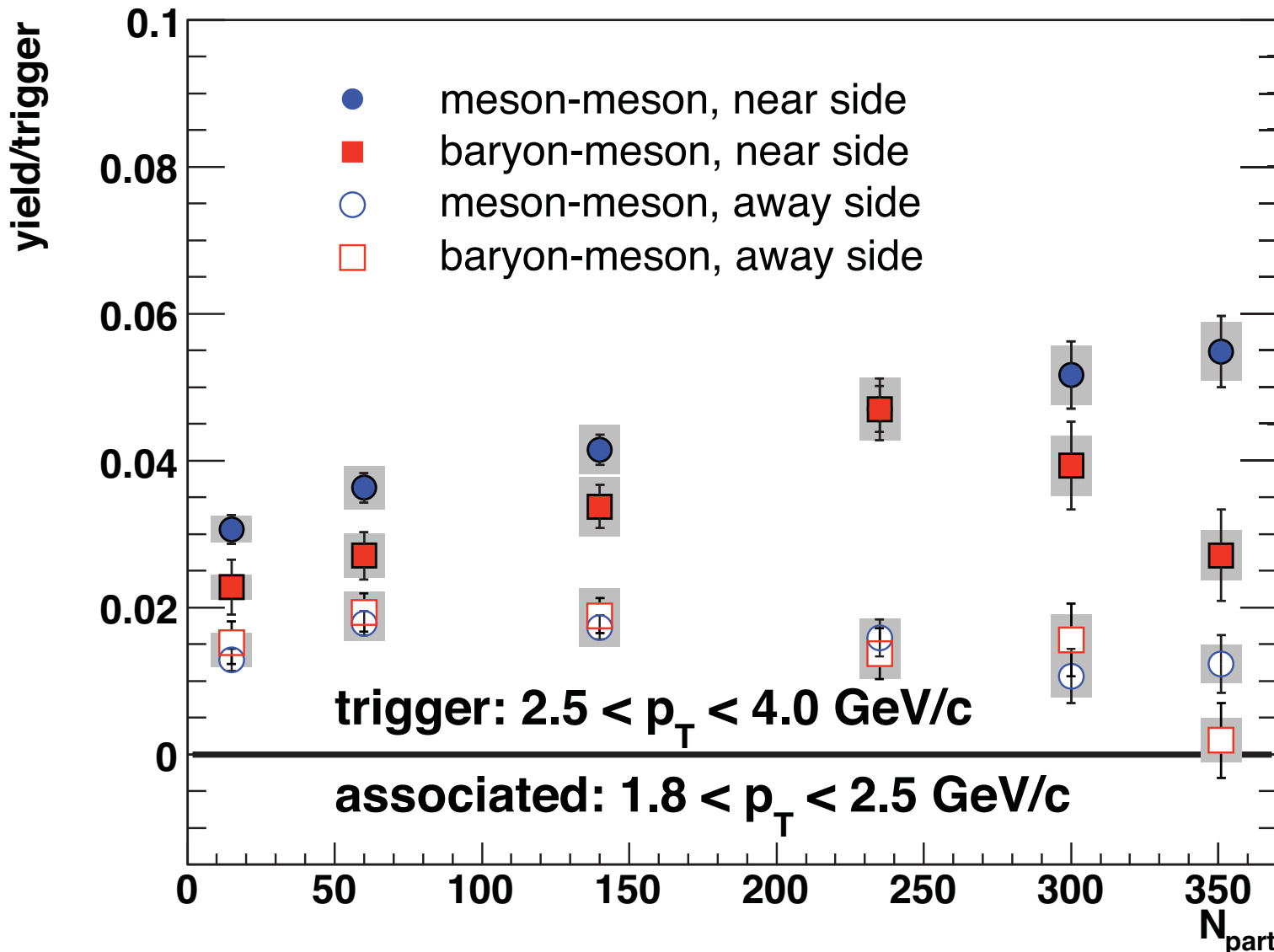


*Protons less absorbed  
in nuclear collisions than pions  
because of dominant  
color transparent higher twist process*

← **Central**

- ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ● Au+Au 60-92%
- ★ p+p,  $\sqrt{s} = 53$  GeV, ISR
- e<sup>+</sup>e<sup>-</sup>, gluon jets, DELPHI
- ..... e<sup>+</sup>e<sup>-</sup>, quark jets, DELPHI

← **Peripheral**

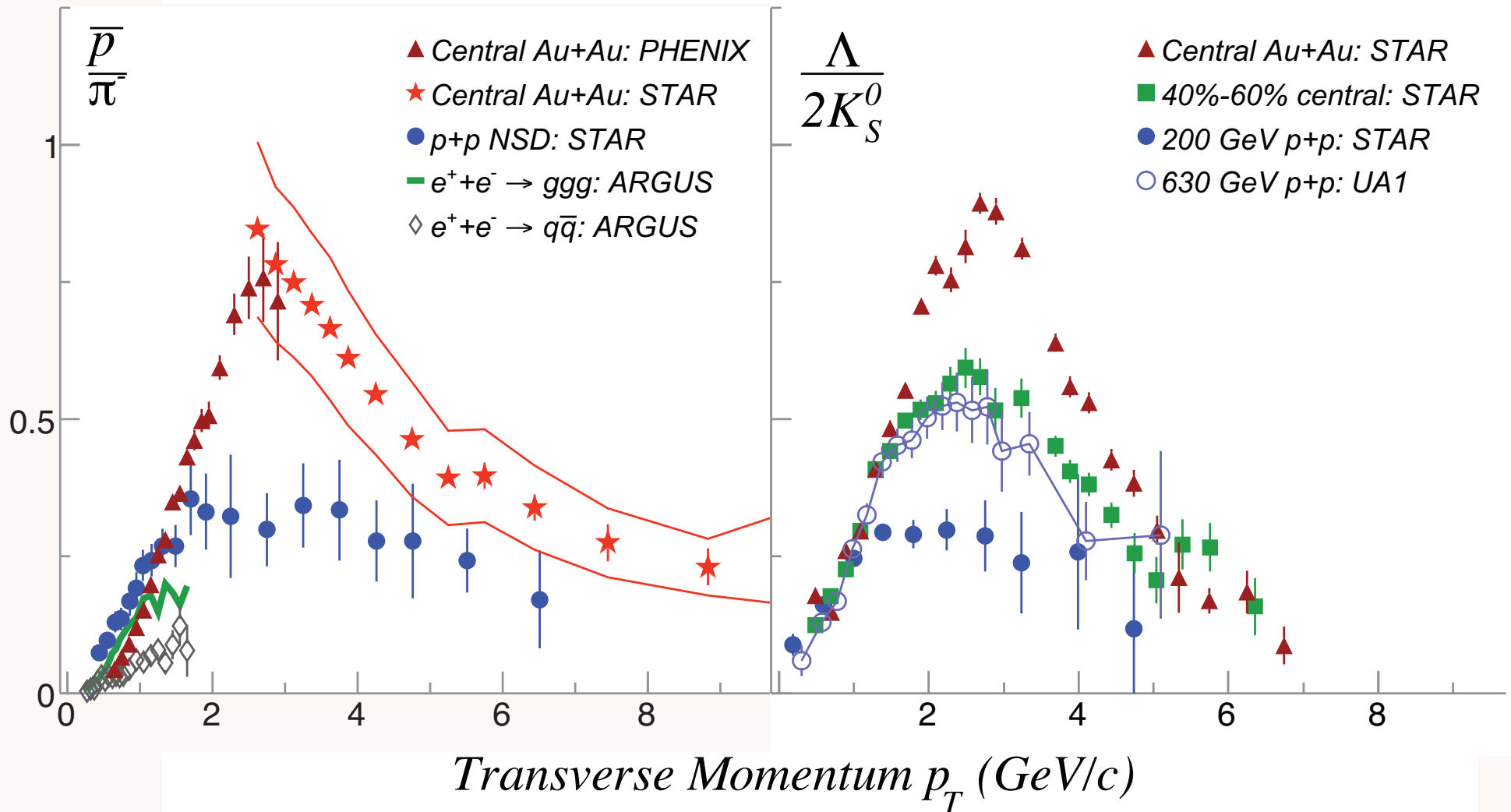


*proton trigger:  
# same-side particles  
decreases with centrality*



**Proton production more dominated by color-transparent direct high- $n_{eff}$  subprocesses**

Baryon to Meson Ratios



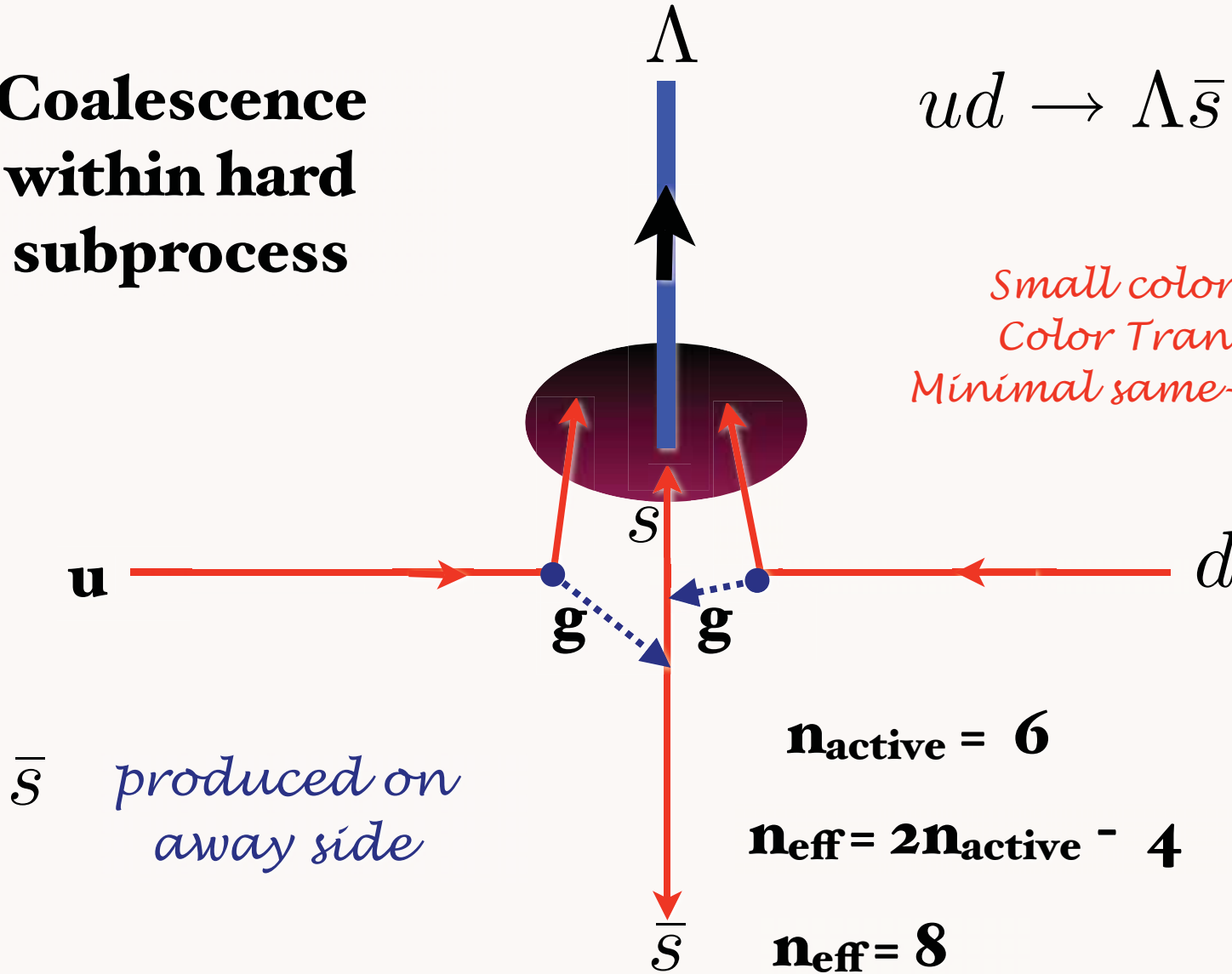


*Lambda can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

$$ud \rightarrow \Lambda \bar{s}$$

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



$\bar{s}$  *produced on  
away side*

# *Evidence for Direct, Higher-Twist Subprocesses*

- Anomalous power behavior at fixed  $x_T$
- Protons more likely to come from direct subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Exclusive-inclusive connection at  $x_T = 1$

- **Renormalization scale is not arbitrary; multiple scales, unambiguous at given order**
- **Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction**
- **Initial and final-state interactions are not always power suppressed in a hard QCD reaction**
- **LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent**
- **Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses**
- **Hadronization at the Amplitude Level**