Light-Front Holographic QCD and the Bound State Structure of Hadrons

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1 Introduction

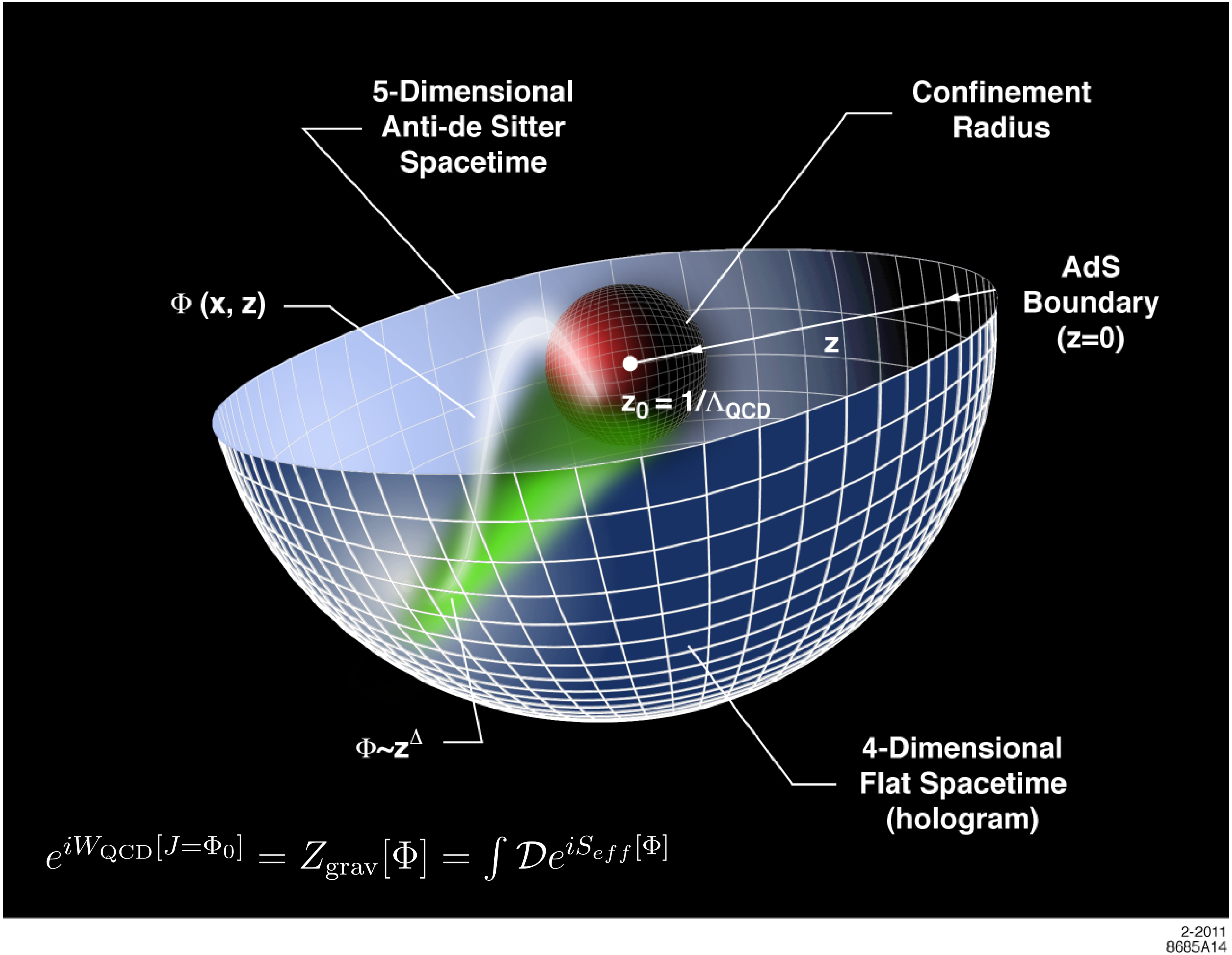
Gauge Gravity Correspondence and Light-Front QCD

- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in physical spacetime has led to a semiclassical approximation for strongly-coupled QCD, which provides physical insights into its non-perturbative dynamics

- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...

- Light-front holography provides a remarkable connection between the equations of motion in AdS and the bound-state LF Hamiltonian equation in QCD [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space (Dim isometry group of AdS$_{d+1}$ is $(d+1)(d+2)/2)$


5-Dimensional
Anti-de Sitter
Spacetime
Confinement
Radius
AdS
Boundary
(z=0)
Φ (x, z)
z
$z_0 = 1/\Lambda_{QCD}$
$\Phi \sim z^\Delta$
4-Dimensional
Flat Spacetime
(hologram)
$e^{iW_{\rm QCD}[J=\Phi_0]} = Z_{\rm grav}[\Phi] = \int \mathcal{D} e^{iS_{eff}[\Phi]}$
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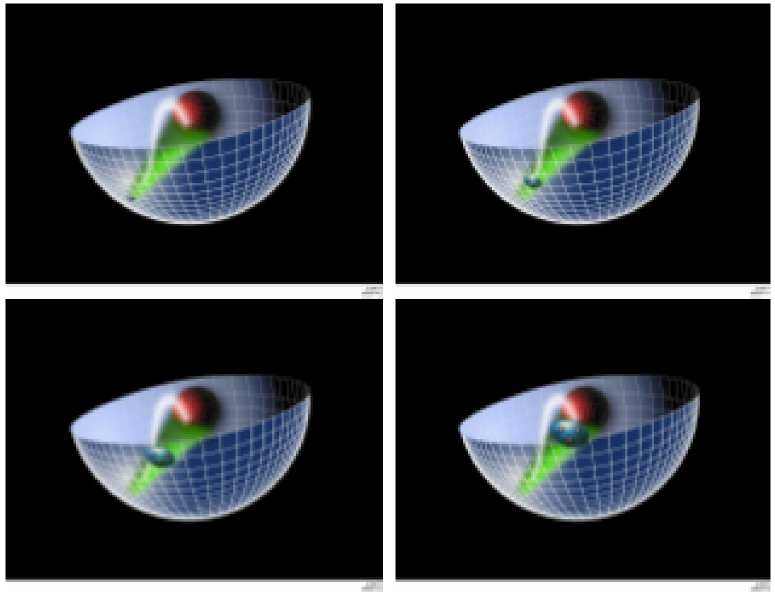
- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

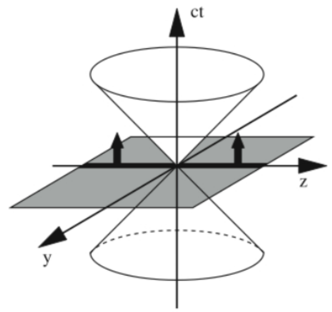
$$L_{\rm Minkowski} \sim \frac{z}{R}\, L_{\rm AdS}$$



- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space
- Short distances $x_\mu x^\mu \to 0$ map to UV conformal AdS$_5$ boundary $z \to 0$
- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



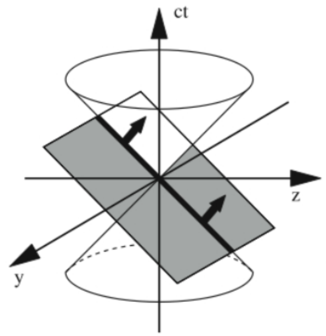
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

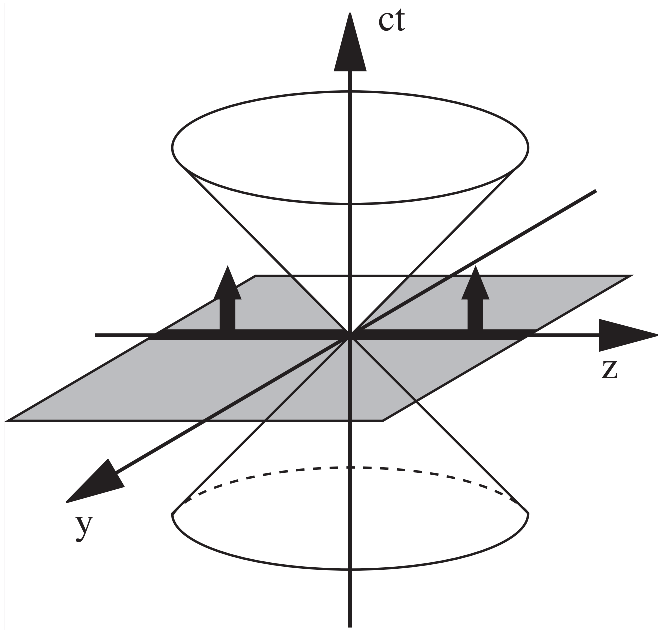
$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

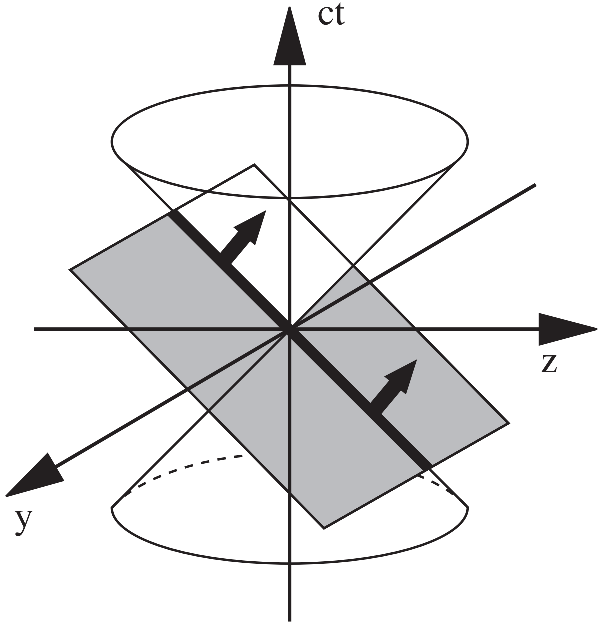
On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$



The instant form

$$\tilde{x}^0 = ct$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = z$$

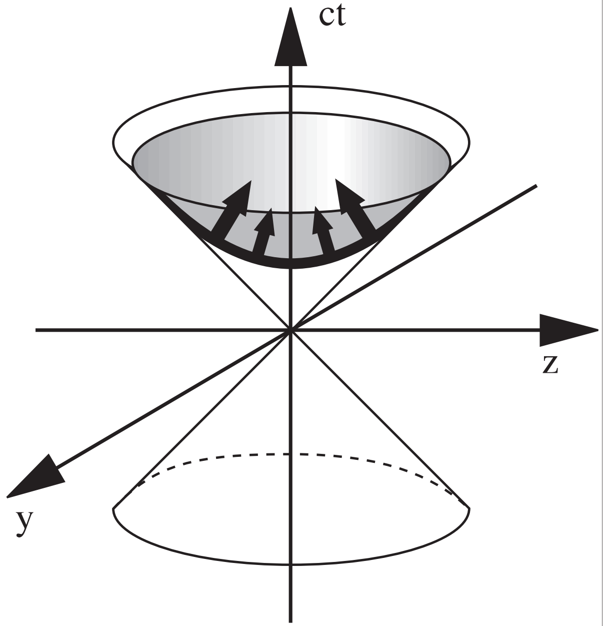
$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



The front form

$$\tilde{x}^0 = ct + z$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = ct - z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$



The point form

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau \cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau \sinh\omega \sin\theta \cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau \sinh\omega \sin\theta \sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau \sinh\omega \cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2 \sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2 \sinh^2\omega \sin^2\theta \end{pmatrix}$$

- QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ, $\mathbf{A}_\perp$

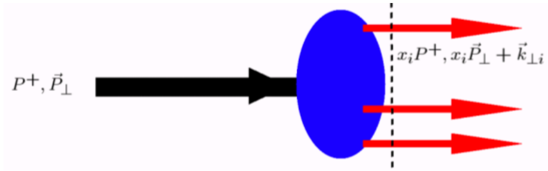
$$\begin{aligned}
P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}\,\gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi + \mathrm{interactions} \\
P^+ &= \int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\partial^+\psi \\
\mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\nabla_\perp\psi
\end{aligned}$$

- LF Hamiltonian P^- generates LF time translations

$$\left[\psi(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi(x)$$

and the generators P^+ and $\mathbf{P}_\perp$ are kinematical

Light-Front Fock Representation



- Dirac field ψ, expanded in terms of ladder operators on the initial surface

$$P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+}\right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \qquad |n\rangle = \{\, |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \ \cdots\}$$

with $k_i^2 = m_i^2, \ \ k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each constituent i in state n

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$
- Find

$$\mathcal{M}^2 = \sum_n \int [dx_i]\left[d^2\mathbf{k}_{\perp i}\right] \sum_\ell \left(\frac{\mathbf{k}_{\perp \ell}^2 + m_\ell^2}{x_q}\right) \left|\psi_n(x_i, \mathbf{k}_{\perp i})\right|^2 + \text{interactions}$$

- LFWF ψ_n represents a bound state which is off the LF energy shell $\mathcal{M}^2 - \mathcal{M}_n^2$

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^n k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a}$$

with $k_a^2 = m_a^2$ for each constituent

- Invariant mass M_n^2 key variable which controls the bound state: LFWF peaks at the minimum $\mathcal{M}_n^2$
- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \ldots, k_n) \to \phi_n\Big(\underbrace{(k_1 + k_2 + \cdots + k_n)^2}_{\mathcal{M}_n^2}\Big), \quad m_q \to 0$$

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

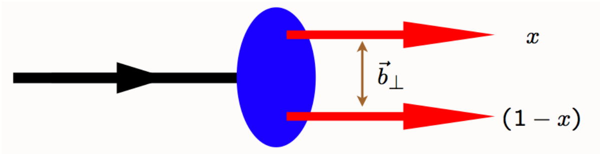
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = L^z)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time
- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in LF Hamiltonian EOM by applying the L-S formalism or evolution equations

3 Light-Front Holographic Mapping

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\quad \left(x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} \\ - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

$$[D_N, D_K] \Phi_{M_1 \cdots M_J} = - R^L_{M_1 N K} \Phi_{L \cdots M_J} - \cdots - R^L_{M_J N K} \Phi_{M_1 \cdots L}$$

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{-iP\cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu {=} M^2$

- Construct effective action in terms of spin-J modes Φ_J with only physical degrees of freedom

  H. G. Dosch, S. J. Brodsky, J. Erlich, and GdT (in progress)

- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \cdots A_J} = e^{M_1}_{A_1} e^{M_2}_{A_2} \cdots e^{M_J}_{A_J} \, \Phi_{M_1 M_2 \cdots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \cdots A_J}$$

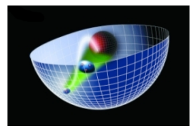
- Find effective action

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{N N'} \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \partial_N \hat{\Phi}_{\mu_1 \cdots \mu_J} \partial_{N'} \hat{\Phi}_{\mu_1' \cdots \mu_J'} \\ - \mu^2 \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \hat{\Phi}_{\mu_1 \cdots \mu_J} \hat{\Phi}_{\mu_1' \cdots \mu_J'} \Big)$$

  upon μ-rescaling

- Variation of the action gives AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$

$$\boxed{\left[- \frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



with $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$ and all indices along 3+1

Dual QCD Light-Front Wave Equation $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = \mathcal{M}^2\Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD

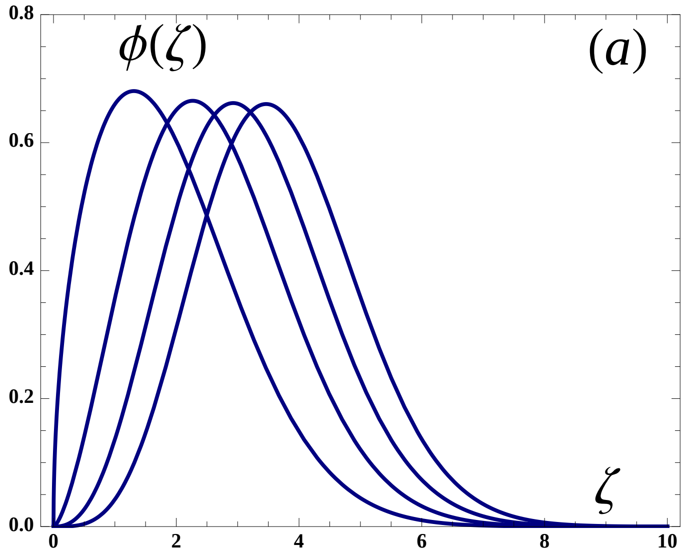
Bosonic Modes and Meson Spectrum

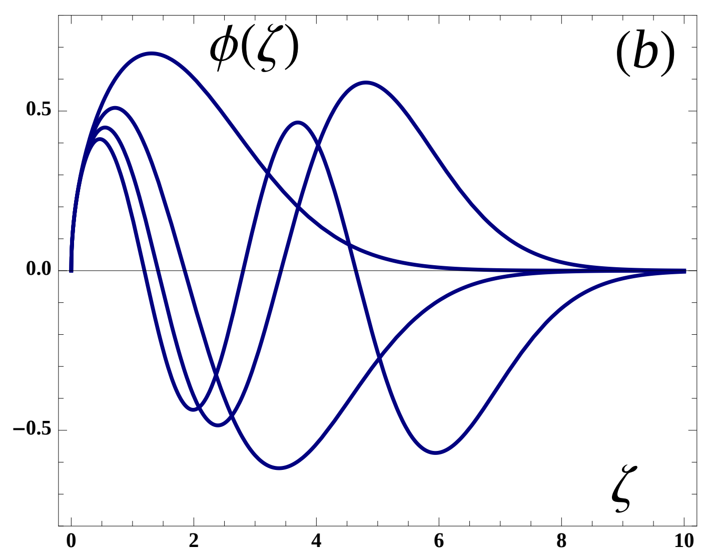
- Positive dilaton background $\varphi = \kappa^2 z^2$: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta \, |\phi(z)^2| = 1$

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

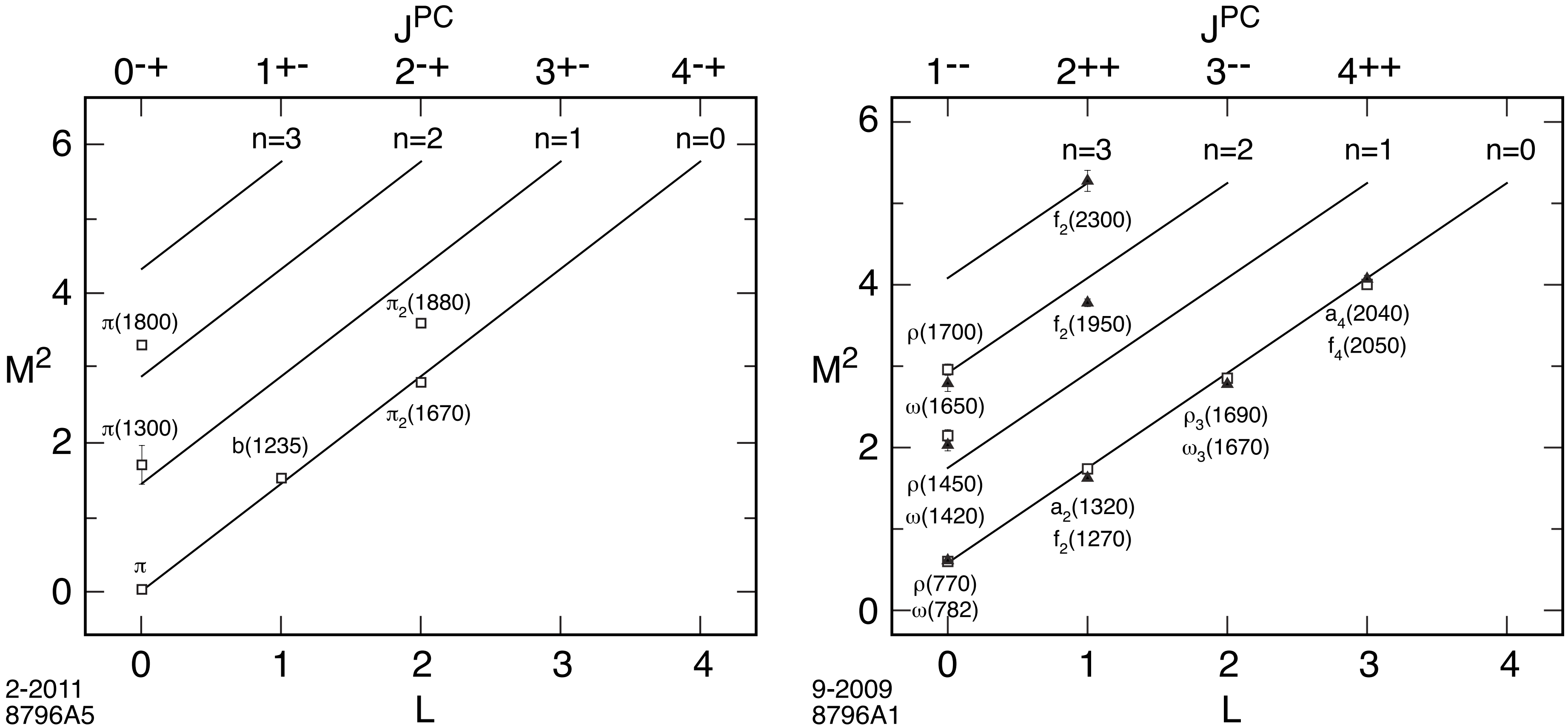
$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 \left(n + L + S/2\right)$$





LFWFs $\phi_{n,L}(\zeta)$ in physical spacetime for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

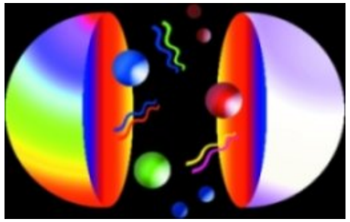
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I=1$ ρ-meson and $I=0$ ω-meson families ($\kappa = 0.54$ GeV)

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]



From Nick Evans

- For baryons LFWE equivalent to system of coupled linear equations $(\nu = L + 1)$

$$-\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - \kappa^2\zeta\psi_- + 2i\kappa\psi_+ = \mathcal{M}\psi_+$$

$$\frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - \kappa^2\zeta\psi_+ - 2i\kappa\psi_- = \mathcal{M}\psi_-$$

with eigenfunctions

$$\begin{aligned}\psi_+(\zeta) &\sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu}(\kappa^2\zeta^2)\\ \psi_-(\zeta) &\sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)\end{aligned}$$

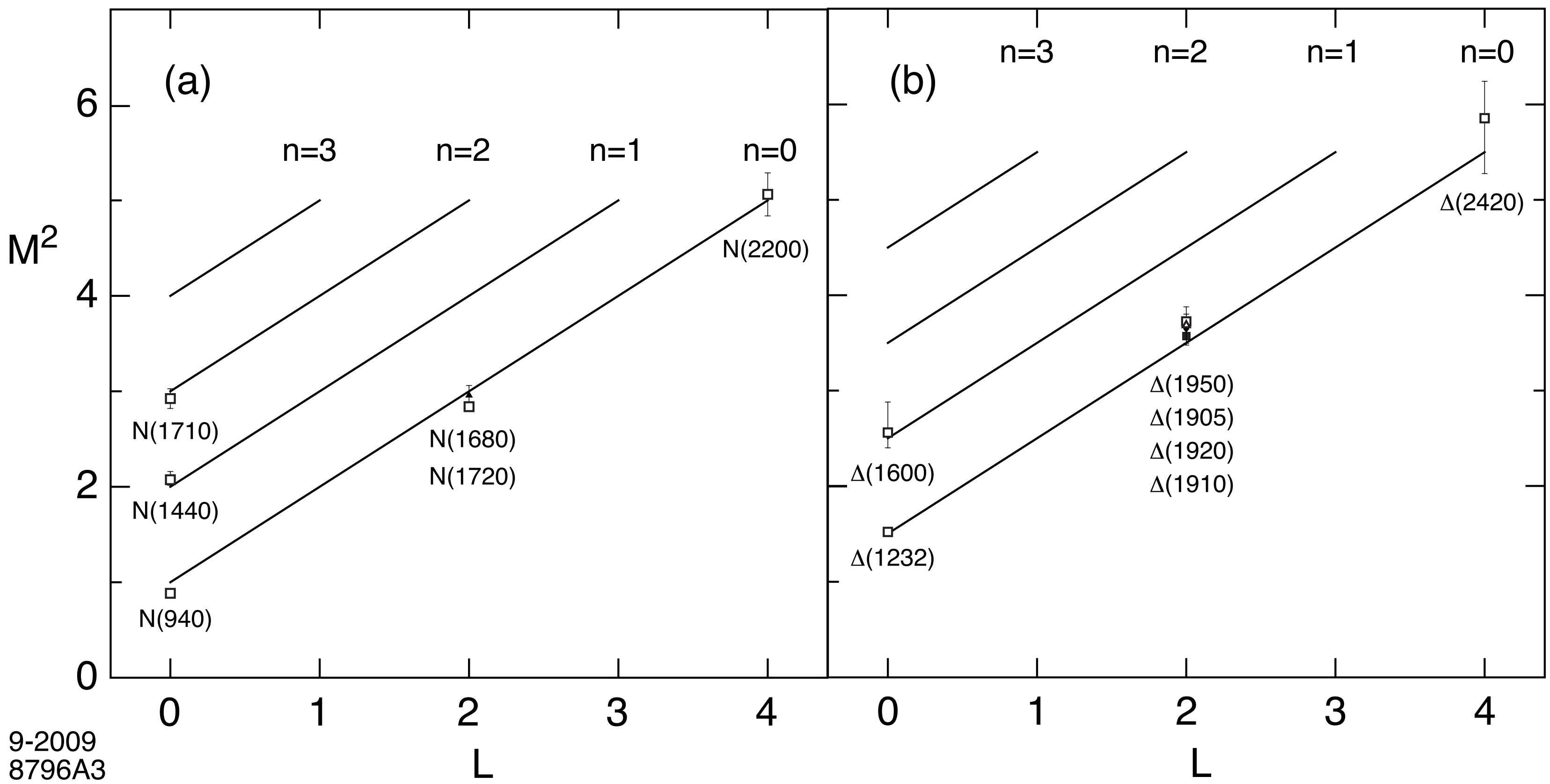
and eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu)$$

- Large N_C: $\quad \mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \quad \Longrightarrow \quad \mathcal{M} \sim \sqrt{N_C}\,\Lambda_{\rm QCD}$

Same multiplicity of states for mesons and baryons!

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Regge trajectories for positive parity N and Δ baryon families ($\kappa = 0.5$ GeV)

4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)], PRD **77**, 056007 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle\psi(P')|J^\mu|\psi(P)\rangle = (P+P')F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \overline{q}\gamma^\mu q$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode $\Phi(x,z)$

$$\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)} A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z)$$

- Are the transition amplitudes related ?

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

  [Polchinski and Strassler (2002)]

- Mapping of J^+ elements at fixed light-front time: $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} V(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the 'free' AdS wave equation

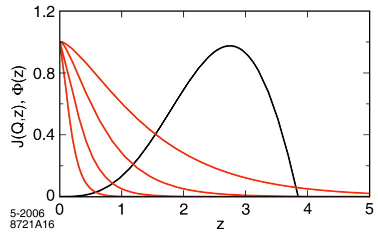
$$\left[z^2 \partial_z^2 - z\,\partial_z - z^2 Q^2\right] V(Q,z) = 0$$

- Solution $\;V(Q,z) = zQK_1(zQ)$
- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \qquad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{\varphi(z)} V(Q,z)\, \Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau - 1}$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

Electromagnetic Form-Factor

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

- Consider a two-quark π^+ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q\!=\!0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space

$$F(Q^2) = R^3 \int \frac{dz}{z^3} \, V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) \Phi^2_{\pi^+}(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

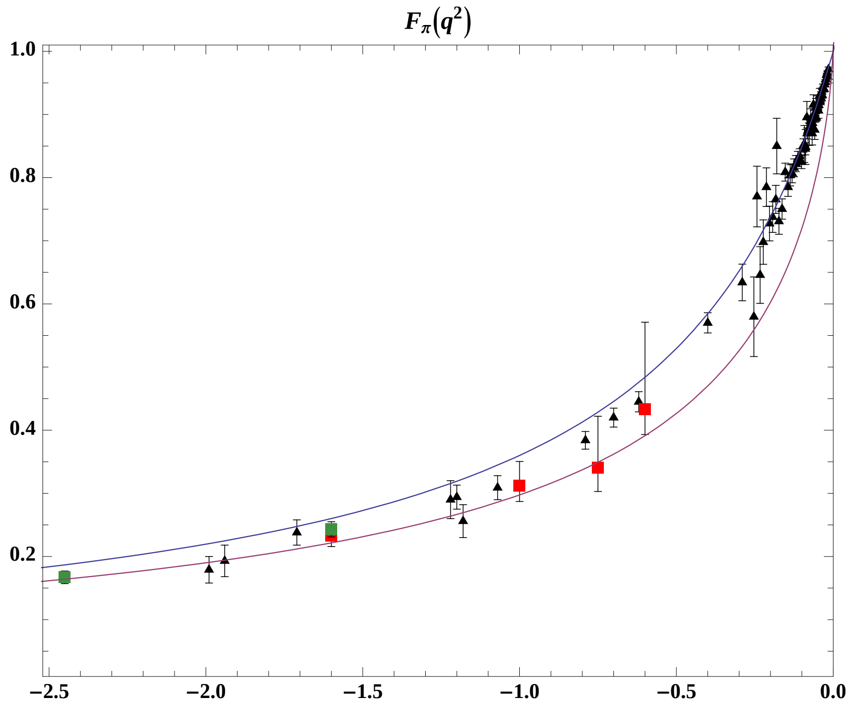
$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

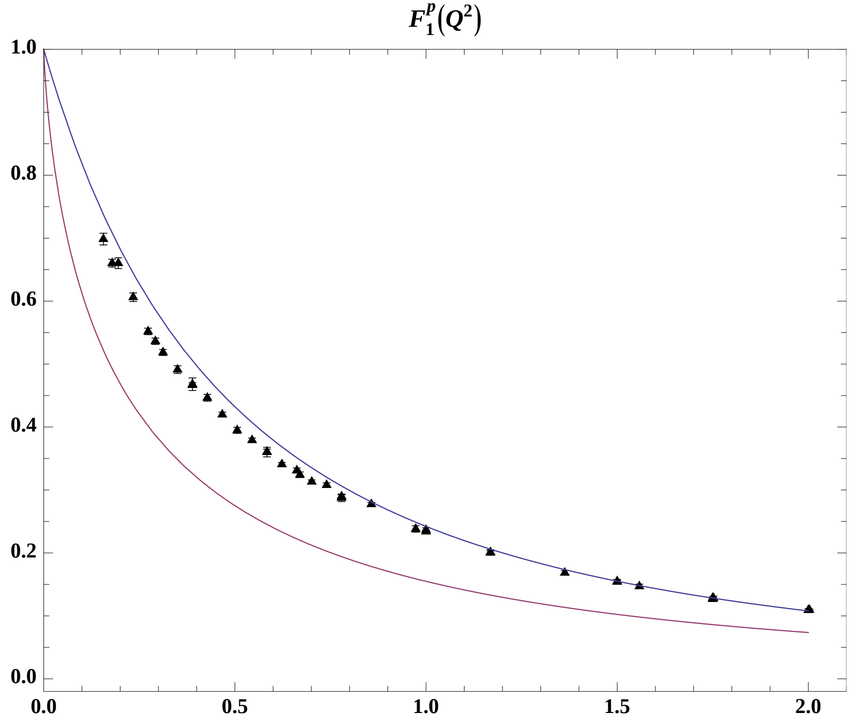
$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} e^{\varphi(z)/2} \, \Phi(\zeta), \qquad z \to \zeta$$

- “Free current” $V(Q,z) = zQK_1(zQ) \longrightarrow$ infinite hadron radius (mauve)
- “Dressed current” non-perturbative sum of an infinite number of terms $\longrightarrow$ finite radius (blue)
- Form factor in soft-wall model expressed as $N-1$ product of poles along vector radial trajectory [Brodsky and GdT (2008)] $\left(\mathcal{M}_\rho{}^2 \to 4\kappa^2(n+1/2)\right)$

$$F(Q^2) = \left[\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1+\frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)\right]^{-1}$$



Pion form factor (lowest mode)



Proton form factor (lowest mode)

Gravitational or Energy-Momentum Form-Factor

[S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]

- Gravitational form factor of composite hadrons in QCD: local coupling to pointlike constituents

$$\left\langle P' \left| \Theta_\mu^{\;\nu} \right| P \right\rangle = \left(P^\nu P'_\mu + P_\mu P'^\nu \right) A(Q^2)$$

  where $Q = P' - P$ and

$$\Theta_{\mu\nu} = \tfrac{1}{2} \overline{q} i (\gamma_\mu D_\nu + \gamma_\nu D_\mu)\, q - g_{\mu\nu} \overline{q}\, (i \not{D} - m)\, q - G^a_{\mu\lambda} G^{a\,\lambda}_\nu + \tfrac{1}{4} g_{\mu\nu} G^a_{\lambda\sigma} G^{a\,\lambda\sigma}$$

- Hadronic matrix element of energy-momentum tensor from perturbing the static AdS metric: non-local coupling of external graviton field propagating in AdS with extended mode $\Phi(x,z)$

$$\int d^4x\, dz \sqrt{g}\, h_{\ell m} \left(\partial^\ell \Phi^*_{P'} \partial^m \Phi_P + \partial^m \Phi^*_{P'} \partial^\ell \Phi_P \right)$$

- Are the transition amplitudes related ?

- Mapping of Θ^{++} elements at fixed LF time: Identical mapping $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$ as EM FF

5 Beyond the Lowest Order Approximation

Higher Fock states

[GdT and S. J Brodsky, arXiv:1010.1204 [hep-ph]]

- Only interaction in LF holographic semiclassical approx is the confinement potential: create Fock states with extra quark-antiquark pairs, no dynamical gluons

- Explain the dominance of quark interchange in large angle elastic scattering

- Form factor in soft-wall model expressed as $N-1$ product of poles along vector radial trajectory [Brodsky and GdT (2008)] $\left(\mathcal{M}_\rho{}^2 \to 4\kappa^2(n+1/2)\right)$

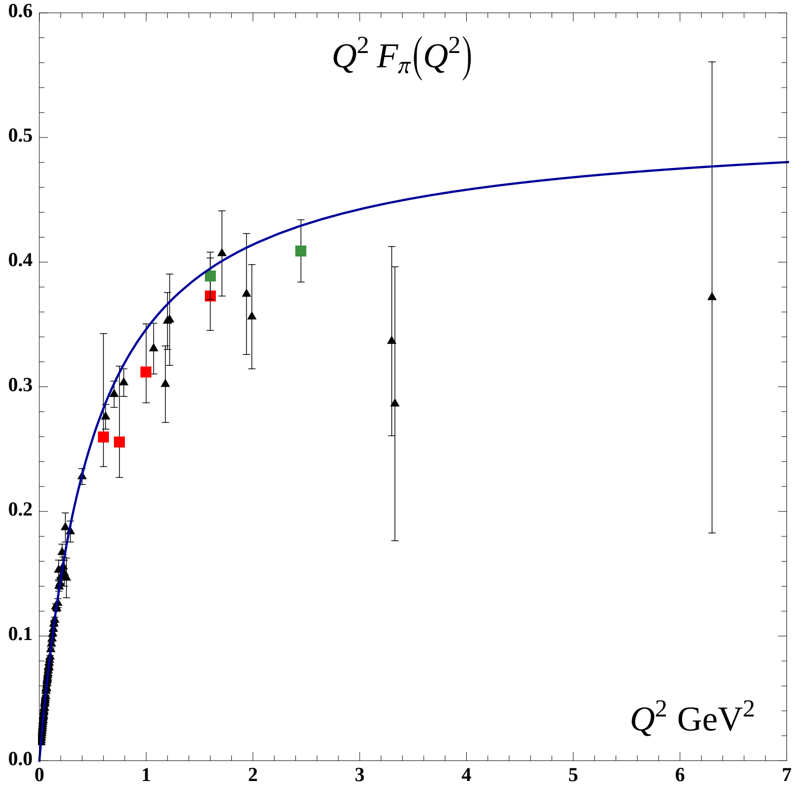
$$F(Q^2) = \left[\left(1 + \frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho'}}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)\right]^{-1}$$

- Higher Fock components in pion LFWF

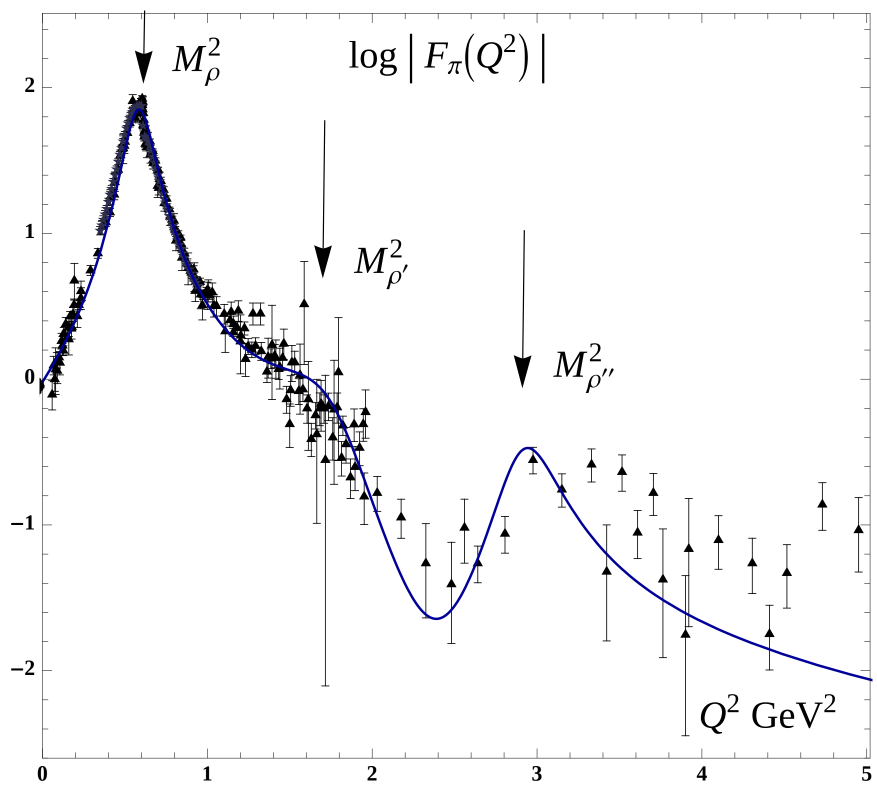
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Expansion of LFWF up to twist 4 (monopole + tripole)

$\kappa = 0.54\,\mathrm{GeV}, \Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300\,\mathrm{MeV}, P_{q\bar{q}q\bar{q}} = 13\%$



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7



M_ρ^2
$\log|F_\pi(Q^2)|$
$M_{\rho'}^2$
$M_{\rho''}^2$
Q^2 GeV^2
2
1
0
−1
−2
0
1
2
3
4
5

Higher Loop Effects

S. J. Brodsky, F.-G. Cao and GdT (in preparation)

- Pion distribution amplitude

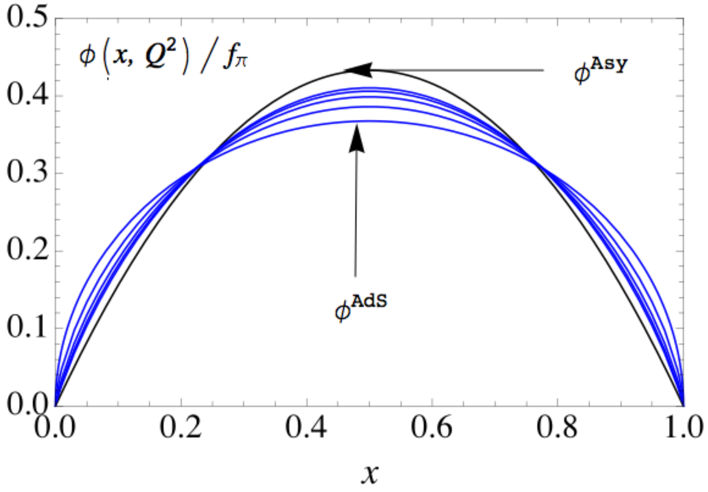
$$\begin{aligned}\phi(x,Q) &= \int \frac{dz^-}{2\pi} e^{i(2x-1)z^-/2} \left\langle 0 \left| \overline{\psi}(-z) \frac{\gamma^+\gamma_5}{2\sqrt{2}} \Omega\, \psi(z) \right| \pi \right\rangle^{(Q)}_{z^+=z_\perp=0} \\ &= \int_0^{Q^2} \frac{\mathrm{d}k_\perp^2}{16\pi^2} \psi(x,k_\perp)\end{aligned}$$

- Normalization $\int_0^1 \mathrm{d}x\, \phi(x,\mu_0) = \frac{f_\pi}{2\sqrt{3}}$
- Evolution of pion DA given by the ERBL equation.

$$\phi(x,Q^2) = x(1-x) \sum_{n=0,2,4,\cdots}^{\infty} a_n(Q^2) C_n^{3/2}(2x-1)$$

- Meson transition form factor $(\overline{x} = 1-x)$

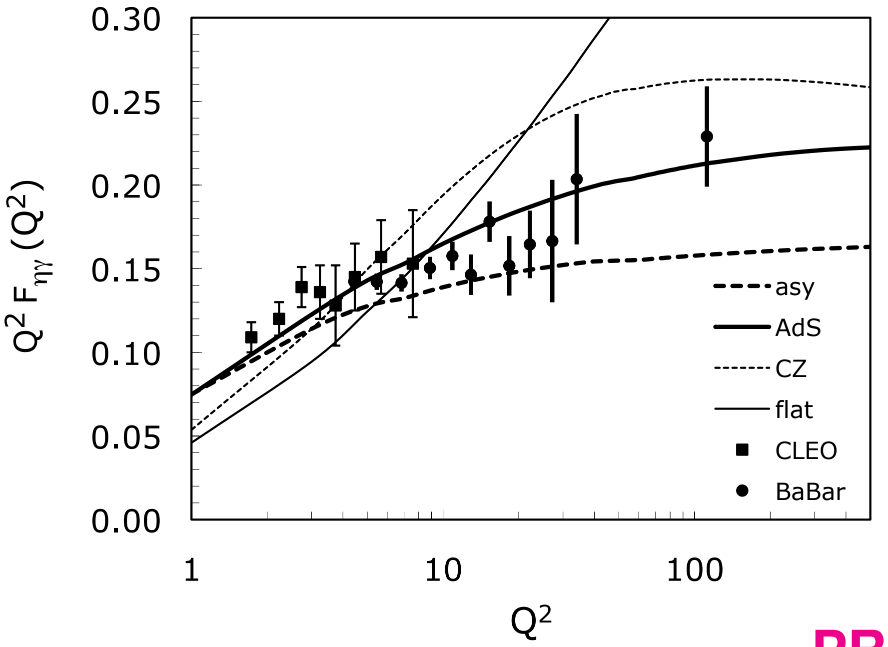
$$Q^2 F_{M\gamma}(Q^2) = c_M \frac{4}{\sqrt{3}} \int_0^1 \mathrm{d}x \frac{\phi(x,\overline{x}Q)}{\overline{x}} \left[1 - \exp\left(-\frac{\overline{x}Q^2}{2\kappa^2 x}\right)\right]$$

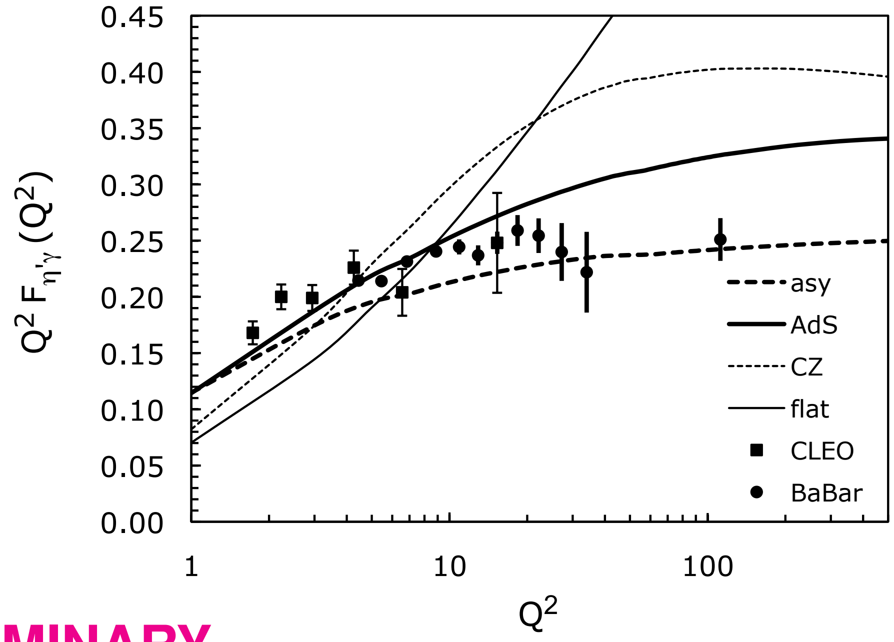


Asymptotic form: $\phi^{\rm asy}(x, \mu_0) = \sqrt{3} f_\pi x(1-x)$

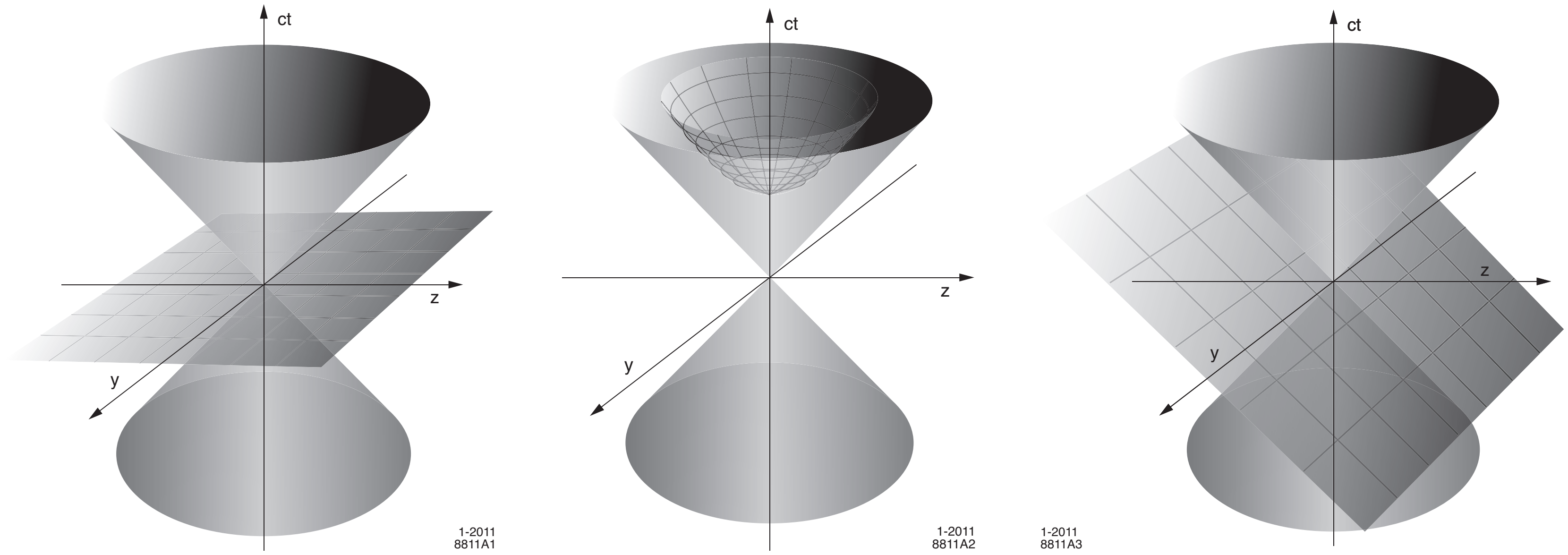
AdS/QCD form: $\phi^{\rm AdS}(x, \mu_0) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$

DA evolution Q^2 = 0.5, 1, 10, 100, 1000 GeV^2





PRELIMINARY



" Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out "

P.A.M. Dirac (1977)