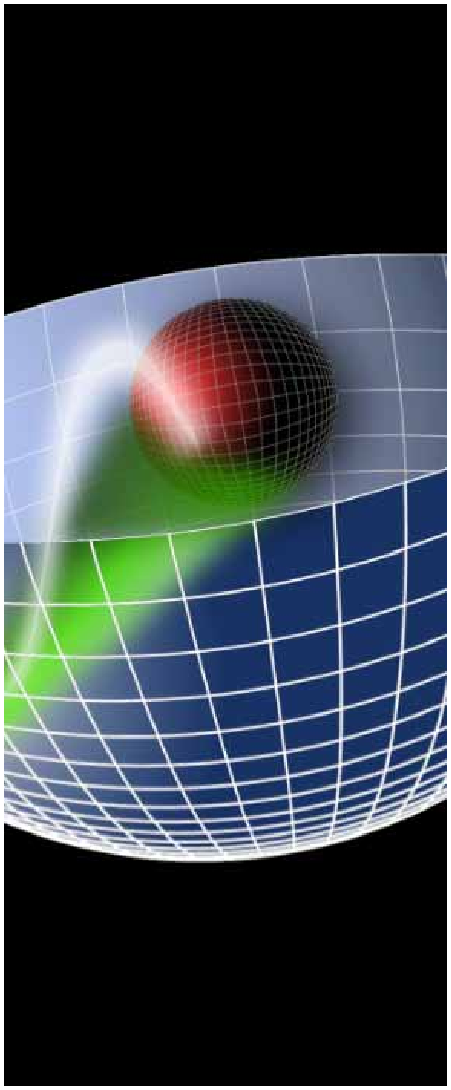
# Light-Front Quantization Approach to the Gauge/Gravity Correspondence and the Bound State Structure of Hadrons in QCD



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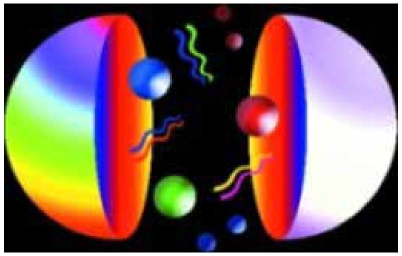


# 1 Introduction

## Gauge Gravity Correspondence and Light-Front QCD

- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in physical spacetime has led to a semiclassical approximation for strongly-coupled QCD, which provides physical insights on non-perturbative QCD dynamics

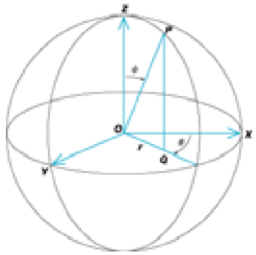
- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...

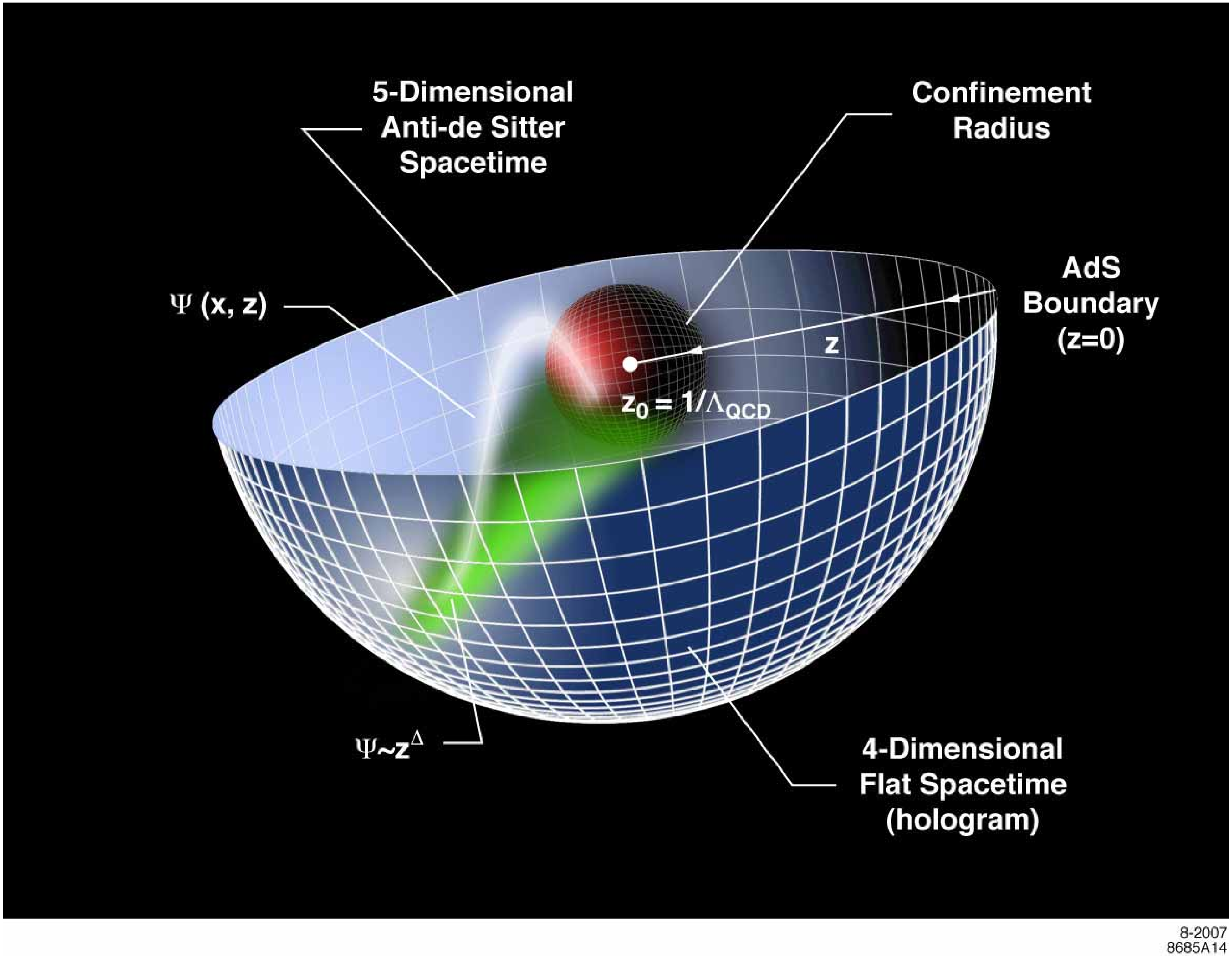
- Light-front holography provides a remarkable connection between the equations of motion in AdS and the bound-state LF Hamiltonian equation in QCD [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

  Isometry group: most general group of transformations which leave invariant the distance between two points. Ej: $S^N \sim O(N+1)$ Dim: $N(N+1)/2$

  Dim isometry group of AdS$_{d+1}$ is $(d+1)(d+2)/2$




5-Dimensional
Anti-de Sitter
Spacetime
Confinement
Radius
AdS
Boundary
(z=0)
Ψ (x, z)
z
$z_0 = 1/\Lambda_{QCD}$
$\Psi \sim z^{\Delta}$
4-Dimensional
Flat Spacetime
(hologram)
8-2007
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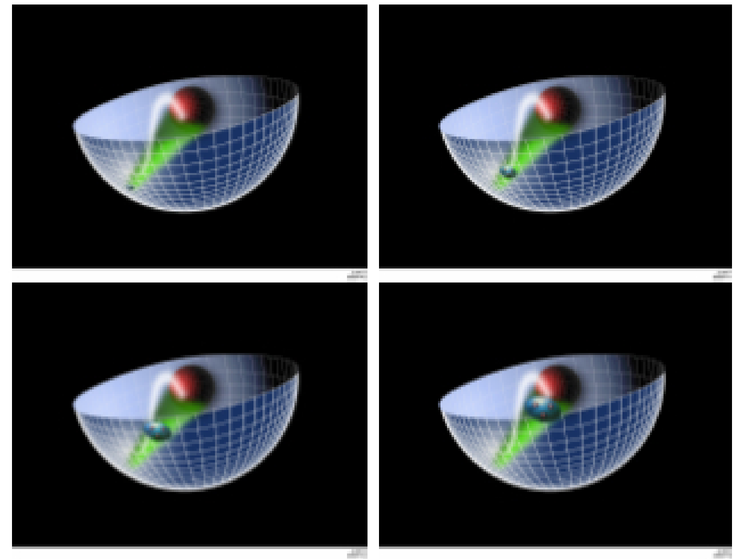
- $\mathrm{AdS}_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\Big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\Big)$$



- A distance $L_{\rm AdS}$ shrinks by a warp factor $z/R$ as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable $z$ acts like a scaling variable in Minkowski space

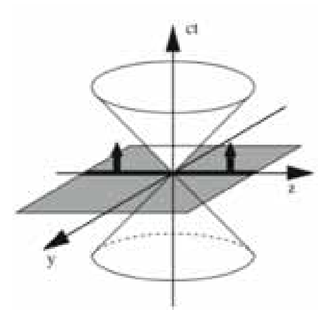
- Short distances $x_\mu x^\mu \to 0$ maps to UV conformal $\mathrm{AdS}_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of $\mathrm{AdS}_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of $z$

- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

## 2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



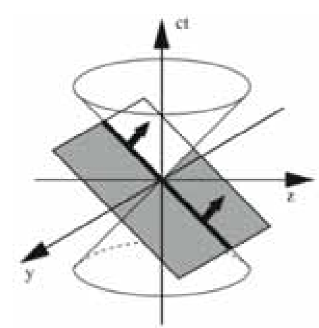

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

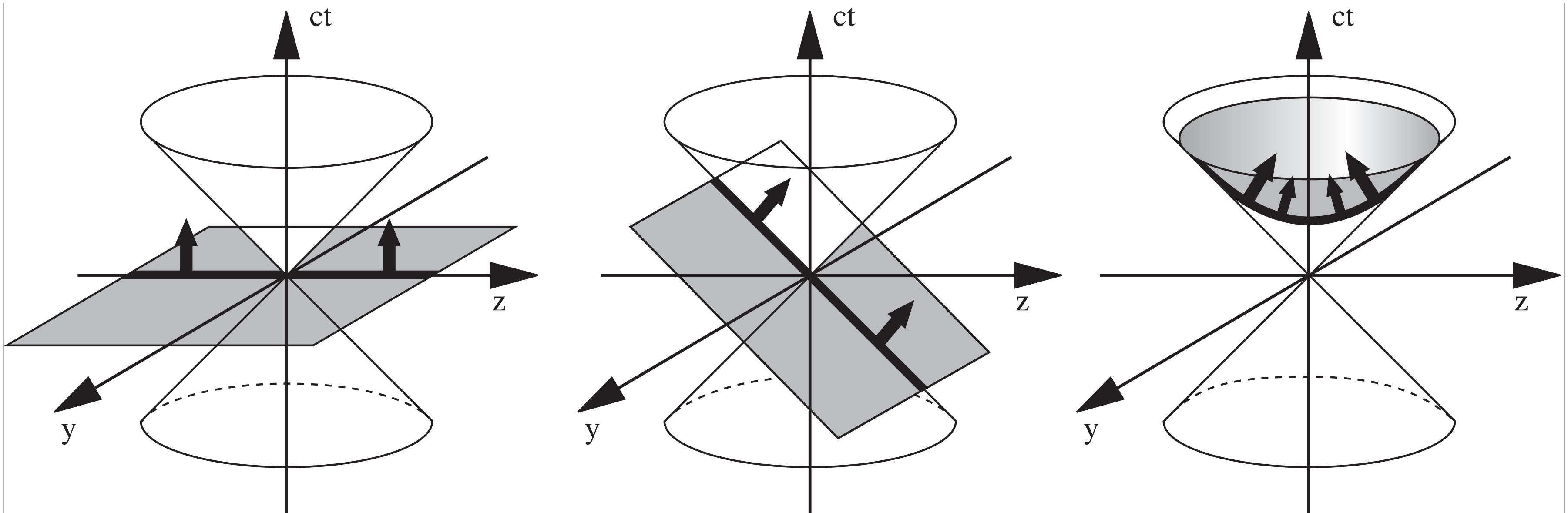
$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy




$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$




**The instant form**

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

**The front form**

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

**The point form**

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau\cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau\sinh\omega\,\sin\theta\,\cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau\sinh\omega\,\sin\theta\,\sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau\sinh\omega\,\cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2\sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2\sinh^2\omega\,\sin^2\theta \end{pmatrix}$$

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2} \mathrm{Tr}\left(G^{\mu\nu} G_{\mu\nu}\right) + i\overline{\psi} D_\mu \gamma^\mu \psi + m \overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields $\psi$, $\mathbf{A}_\perp$

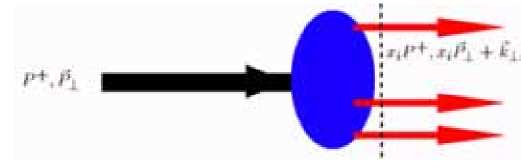
$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}\,\gamma^+ \frac{\left(i\nabla_\perp\right)^2 + m^2}{i\partial^+}\psi + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\partial^+ \psi \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\nabla_\perp \psi \end{aligned}$$

- LF Hamiltonian $P^-$ generates LF time translations

$$\left[\psi(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi(x)$$

and the generators $P^+$ and $\mathbf{P}_\perp$ are kinematical

## Light-Front Fock Representation




- Dirac field $\psi$, expanded in terms of ladder operators on the initial surface

$$P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+}\right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \qquad\qquad |n\rangle = \{\, |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \;\cdots\}$$

  with $k_i^2 = m_i^2, \;\; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each constituent $i$ in state $n$

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ independent of $P^+$ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin $\lambda_i^z$

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

## Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$
- Find

$$\mathcal{M}^2 = \sum_n \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right] \sum_\ell \left(\frac{\mathbf{k}_{\perp\ell}^2 + m_\ell^2}{x_q}\right) \left|\psi_n(x_i, \mathbf{k}_{\perp i})\right|^2 + \text{interactions}$$

- LFWF $\psi_n$ represents a bound state which is off the LF energy shell $\mathcal{M}^2 - \mathcal{M}_n^2$

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^n k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a}$$

with $k_a^2 = m_a^2$ for each constituent

- Invariant mass $M_n^2$ key variable which controls the bound state: LFWF peaks at the minimum $\mathcal{M}_n^2$
- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \ldots, k_n) \to \phi_n\big(\underbrace{(k_1 + k_2 + \cdots + k_n)^2}_{\mathcal{M}_n^2}\big), \quad m_q \to 0$$

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

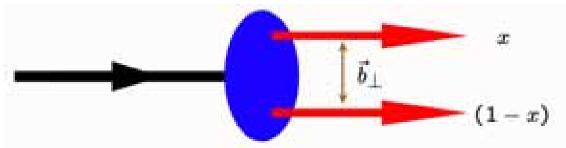
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left( \frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big|$$

the $x$-weighted transverse impact coordinate of the spectator system (${x}$ active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$




- To first approximation LF dynamics depend only on the invariant variable $\zeta$, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular $\varphi$, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

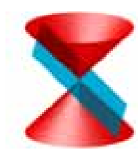
- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = L^z)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for $\phi$

$$\boxed{\Big( \underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Big) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find $n$-massless partons at transverse impact separation $\zeta$ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

# 3 Light-Front Holographic Mapping

## Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Action for spin-$J$ field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\quad \left( x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big( g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where $D_M$ is the covariant derivative which includes parallel transport

$$[D_N, D_K] \Phi_{M_1 \cdots M_J} = - R^L_{\ M_1 N K} \Phi_{L \cdots M_J} - \cdots - R^L_{\ M_J N K} \Phi_{M_1 \cdots L}$$

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu \! = \! M^2$

- Construct effective action in terms of spin-$J$ modes $\Phi_J$ with only physical degrees of freedom

- Lagrangian for scalar field in AdS$_{d+1}$

$$S = \int d^d x\, dz\, \sqrt{g}\, e^{\varphi(z)} \left(g^{MN} \partial_M \Phi^* \partial_N \Phi - \mu^2 \Phi^* \Phi\right)$$

- Factor out plane waves along 3+1: $\Phi_P(x^\mu, z) = e^{-iP\cdot x}\Phi(z)$

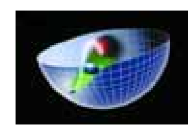
$$\left[-\frac{z^{d-1}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

where $P_\mu P^\mu = \mathcal{M}^2$ invariant mass of physical hadron with four-momentum $P_\mu$

- Spin-$J$ mode $\Phi_{\mu_1\cdots\mu_J}$ with all indices along 3+1 and shifted dimensions $\Phi_J(z) \sim z^{-J}\Phi(z)$

- Find AdS wave equation

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = \mathcal{M}^2\Phi_J(z)}$$



## Dual QCD Light-Front Wave Equation $\quad \Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=\mathcal{M}^2\Phi_J(z)$$

find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$

with

$$U(\zeta)=\frac{1}{2}\varphi''(z)+\frac{1}{4}\varphi'(z)^2+\frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2=-(2-J)^2+L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension $\tau$ of AdS mode $\Phi_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD

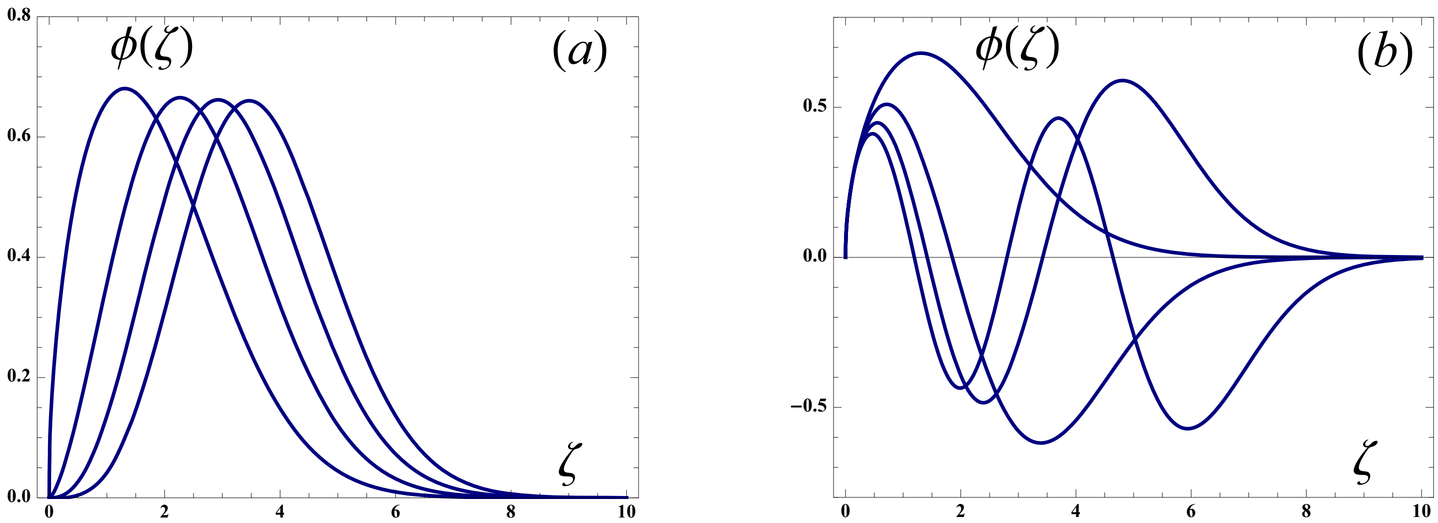
## Bosonic Modes and Meson Spectrum

- Positive dilaton background $\varphi = \kappa^2 z^2$ : $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta \, |\phi(z)^2| = 1$

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

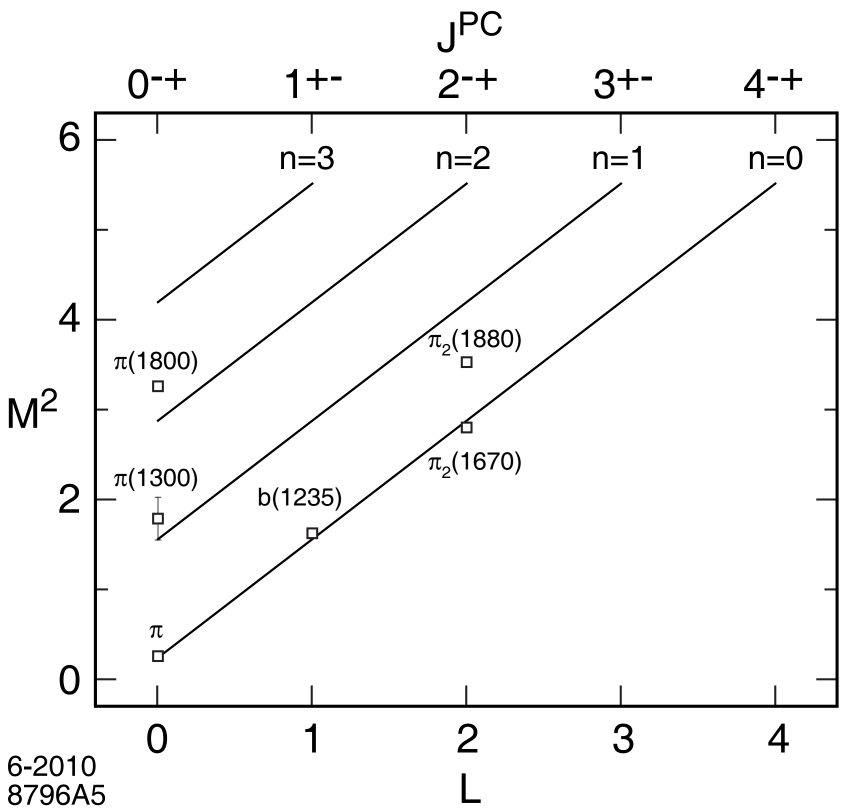
- Eigenvalues

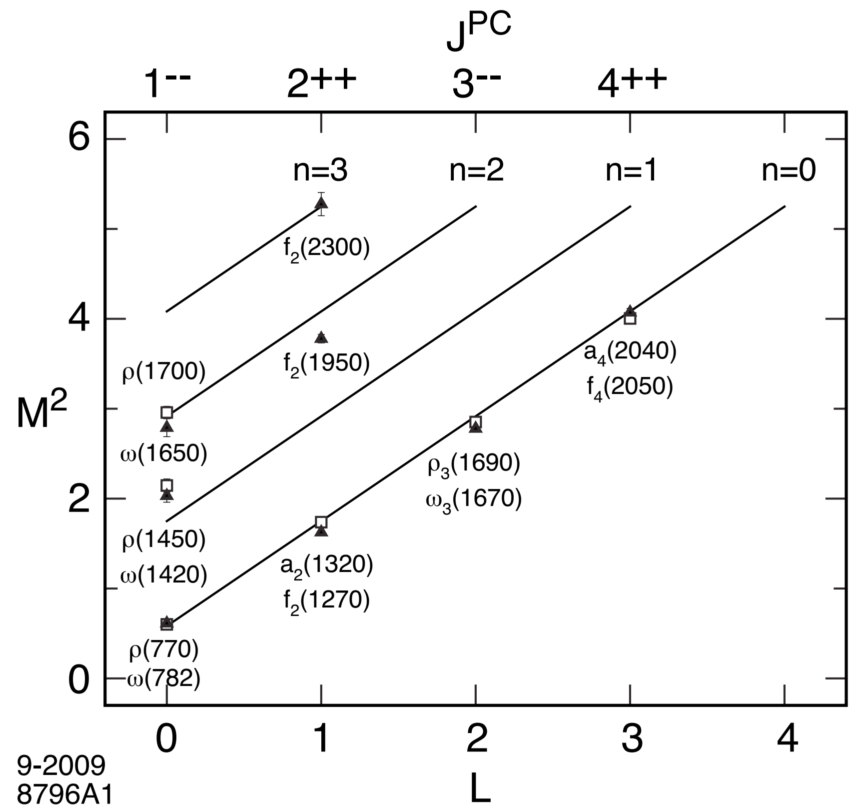
$$\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$$




LFWFs $\phi_{n,L}(\zeta)$ in physical spacetime for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

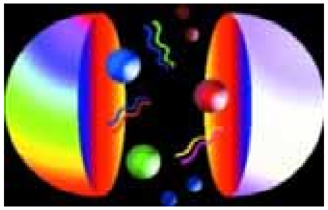



Regge trajectories for the $\pi$ ($\kappa = 0.6$ GeV) and the $I\!=\!1$ $\rho$-meson and $I\!=\!0$ $\omega$-meson families ($\kappa = 0.54$ GeV)

## Fermionic Modes and Baryon Spectrum

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]



From Nick Evans

- For baryons LFWE equivalent to system of coupled linear equations $(\nu = L + 1)$

$$-\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - \kappa^2\zeta\psi_- + 2i\kappa\psi_+ = \mathcal{M}\psi_+$$

$$\frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - \kappa^2\zeta\psi_+ - 2i\kappa\psi_- = \mathcal{M}\psi_-$$

with eigenfunctions

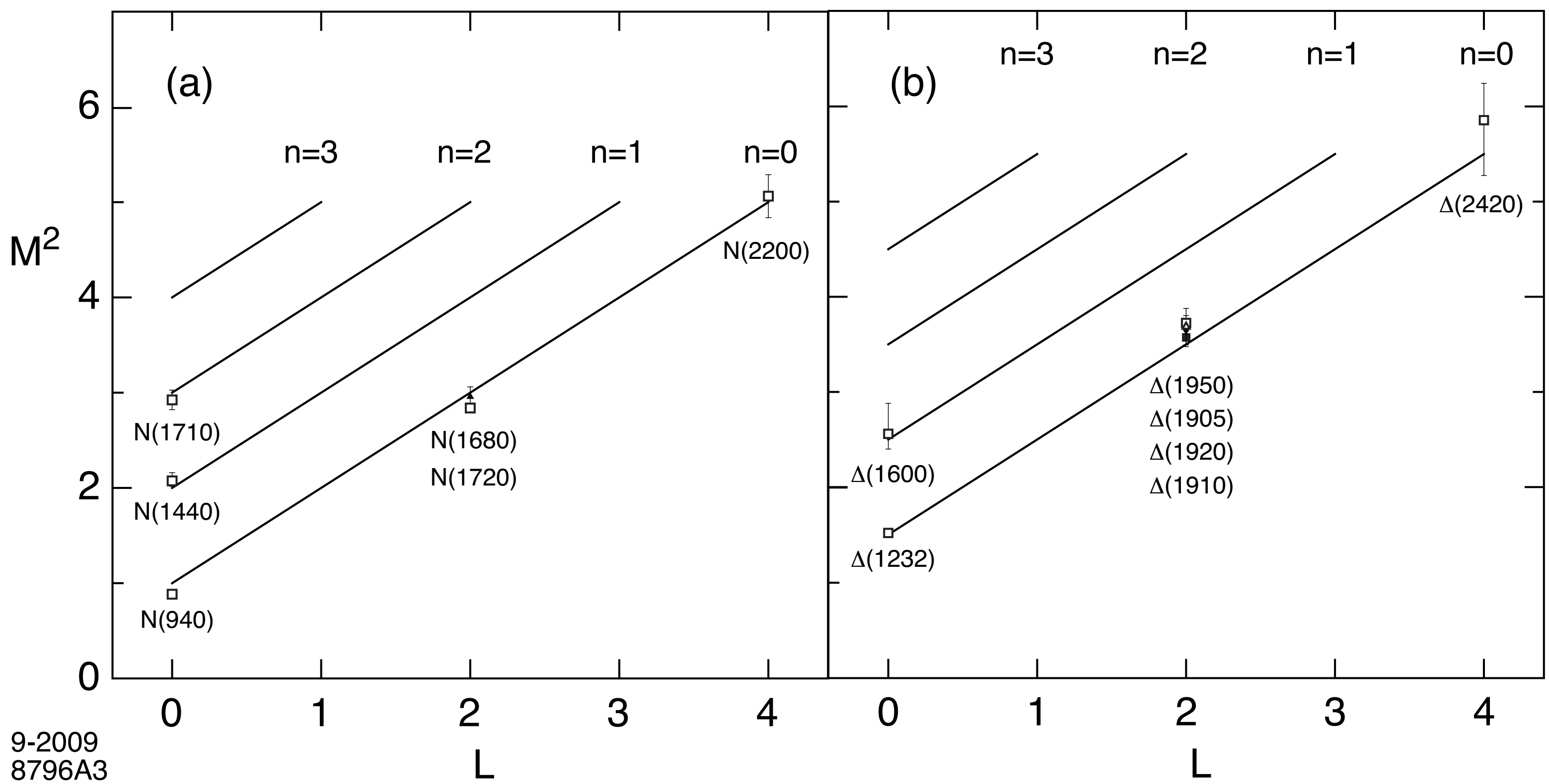
$$\begin{aligned}\psi_+(\zeta) &\sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2)\\ \psi_-(\zeta) &\sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)\end{aligned}$$

and eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu)$$

Same multiplicity of states for mesons and baryons!

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$




Regge trajectories for positive parity $N$ and $\Delta$ baryon families ($\kappa = 0.5$ GeV)

## 4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)], PRD **77**, 056007 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P') F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} A^\ell(x, z) \Phi^*_{P'}(x, z) \overleftrightarrow{\partial}_\ell \Phi_P(x, z)$$

- Are the transition amplitudes related ?

- How to recover hard pointlike scattering at large $Q$ out of soft collision of extended objects?

  [Polchinski and Strassler (2002)]

- Mapping of $J^+$ elements at fixed light-front time: $\Phi_P(z) \;\; \Leftrightarrow \;\; |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} V(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the 'free' AdS wave equation

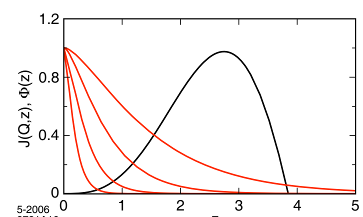
$$\left[z^2\partial_z^2 - z\,\partial_z - z^2Q^2\right] V(Q,z) = 0$$

- Solution $\quad V(Q,z) = zQK_1(zQ)$
- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \qquad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons $\Phi_P$ and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{\varphi(z)} V(Q,z)\, \Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$




At large $Q$ important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

## Electromagnetic Form-Factor

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\Big) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Consider a two-quark $\pi^+$ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q\!=\!0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space

$$F(Q^2) = R^3 \int \frac{dz}{z^3} \, V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \Phi^2_{\pi^+}(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary $Q$. Expressions can be matched only if LFWF is factorized

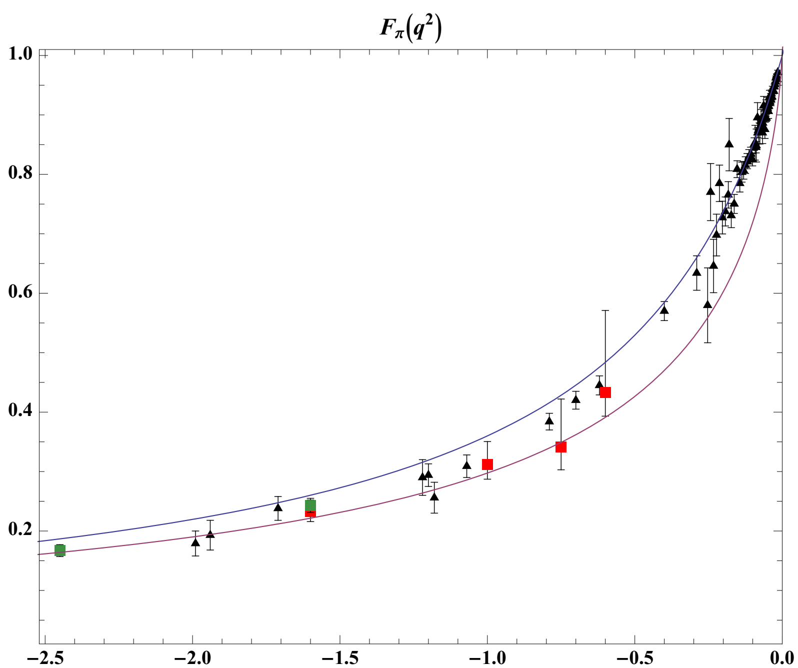
$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

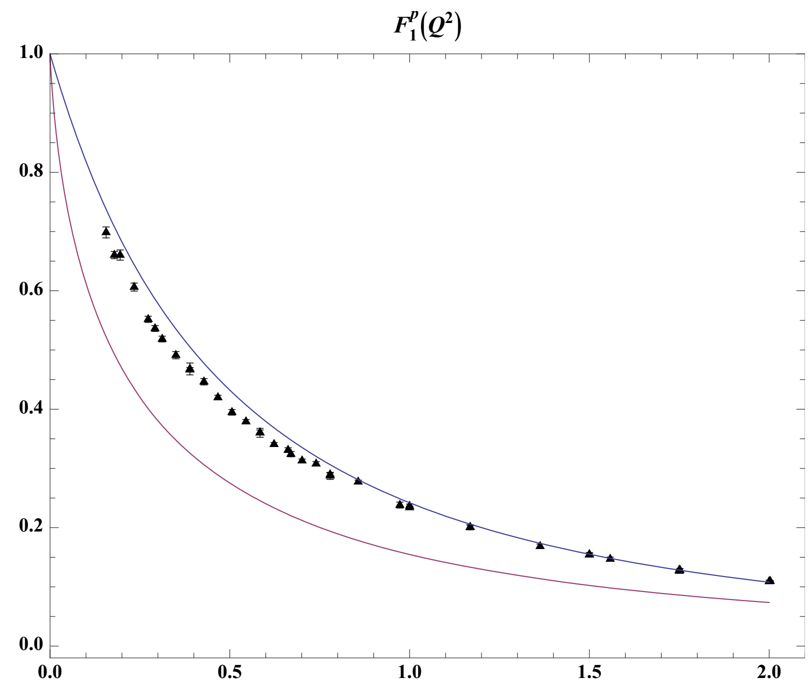
$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} e^{\varphi(z)/2} \, \Phi(\zeta), \qquad z \to \zeta$$

- "Free current" $V(Q,z) = zQK_1(zQ) \rightarrow$ infinite hadron radius (mauve)
- "Dressed current" non-perturbative sum of an infinite number of terms $\rightarrow$ finite radius (blue)
- Form factor in soft-wall model expressed as $N-1$ product of poles along vector radial trajectory [Brodsky and GdT (2008)] $\left(\mathcal{M}_\rho{}^2 \rightarrow 4\kappa^2(n+1/2)\right)$

$$F(Q^2) = \left[\left(1 + \frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)\right]^{-1}$$



Pion form factor (lowest mode)



Proton form factor (lowest mode)

## Gravitational or Energy-Momentum Form-Factor

[S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]

- Gravitational form factor of composite hadrons in QCD: local coupling to pointlike constituents

$$\left\langle P' \left| \Theta_\mu^{~\nu} \right| P \right\rangle = \left( P^\nu P'_\mu + P_\mu P'^\nu \right) A(Q^2)$$

where $Q = P' - P$ and

$$\Theta_{\mu\nu} = \tfrac{1}{2} \overline{\psi} i (\gamma_\mu D_\nu + \gamma_\nu D_\mu) \, \psi - g_{\mu\nu} \overline{\psi} \, (i \not{D} - m) \, \psi - G^a_{\mu\lambda} G^{a\,\lambda}_{\nu} + \tfrac{1}{4} g_{\mu\nu} G^a_{\lambda\sigma} G^{a\,\lambda\sigma}$$

- Hadronic matrix element of energy-momentum tensor from perturbing the static AdS metric: non-local coupling of external graviton field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \sqrt{g} \, h_{\ell m} \left( \partial^\ell \Phi^*_{P'} \partial^m \Phi_P + \partial^m \Phi^*_{P'} \partial^\ell \Phi_P \right)$$

- Are the transition amplitudes related ?
- Mapping of $\Theta^{++}$ elements at fixed LF time: Identical mapping $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$ as EM FF

*“ Working with a front is a process that is unfamiliar to physicists.*
*But still I feel that the mathematical simplification that it introduces*
*is all-important. I consider the method to be promising and have recently*
*been making an extensive study of it. It offers new opportunities,*
*while the familiar instant form seems to be played out ”*

P.A.M. Dirac (1977)