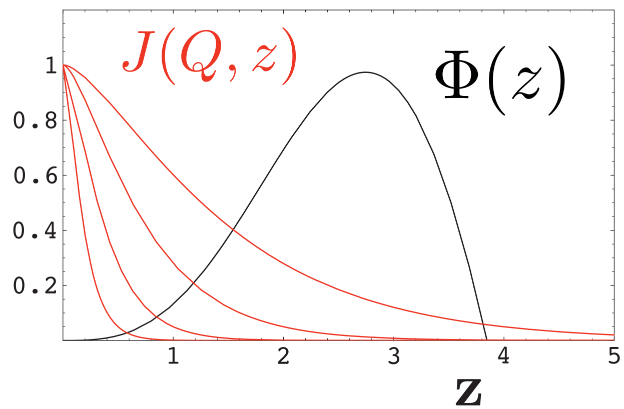
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

$$J(Q,z) = zQK_1(zQ)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

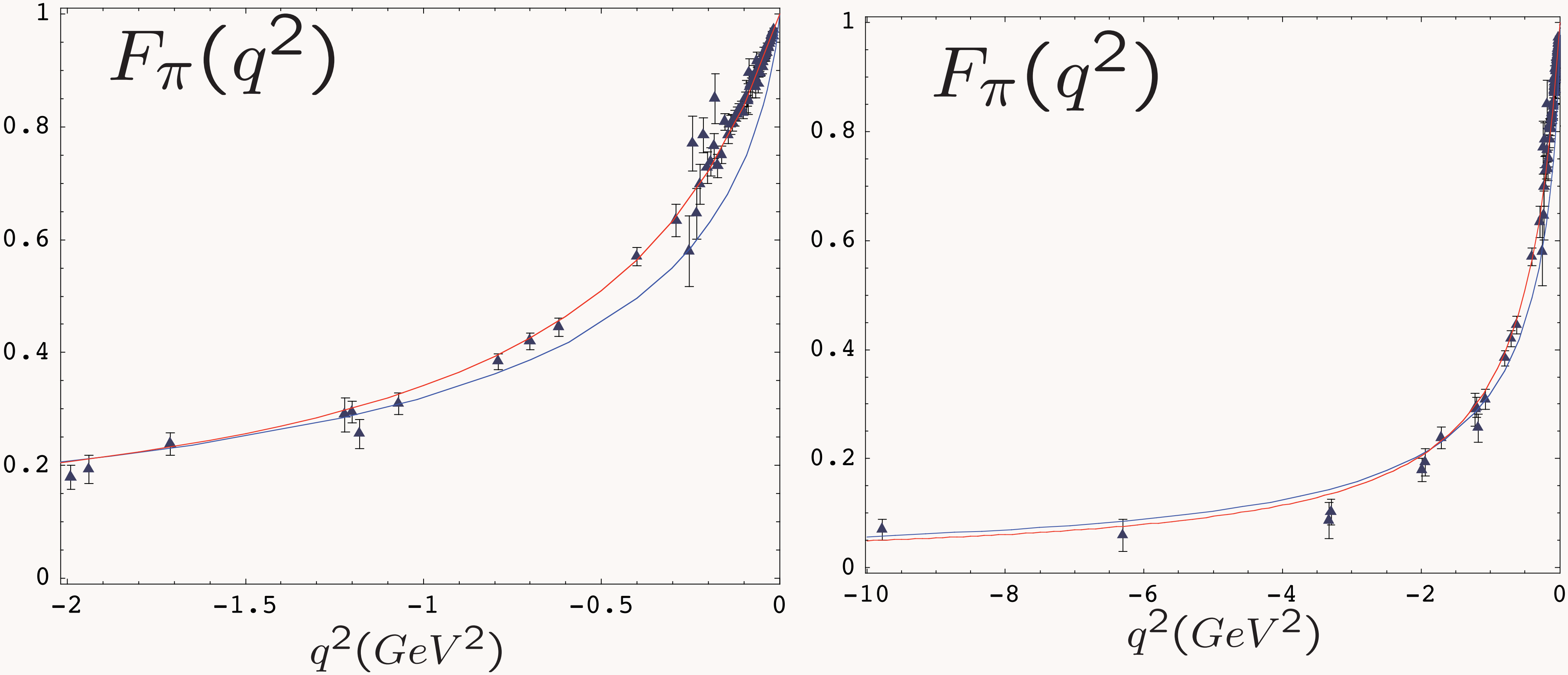
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ
scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

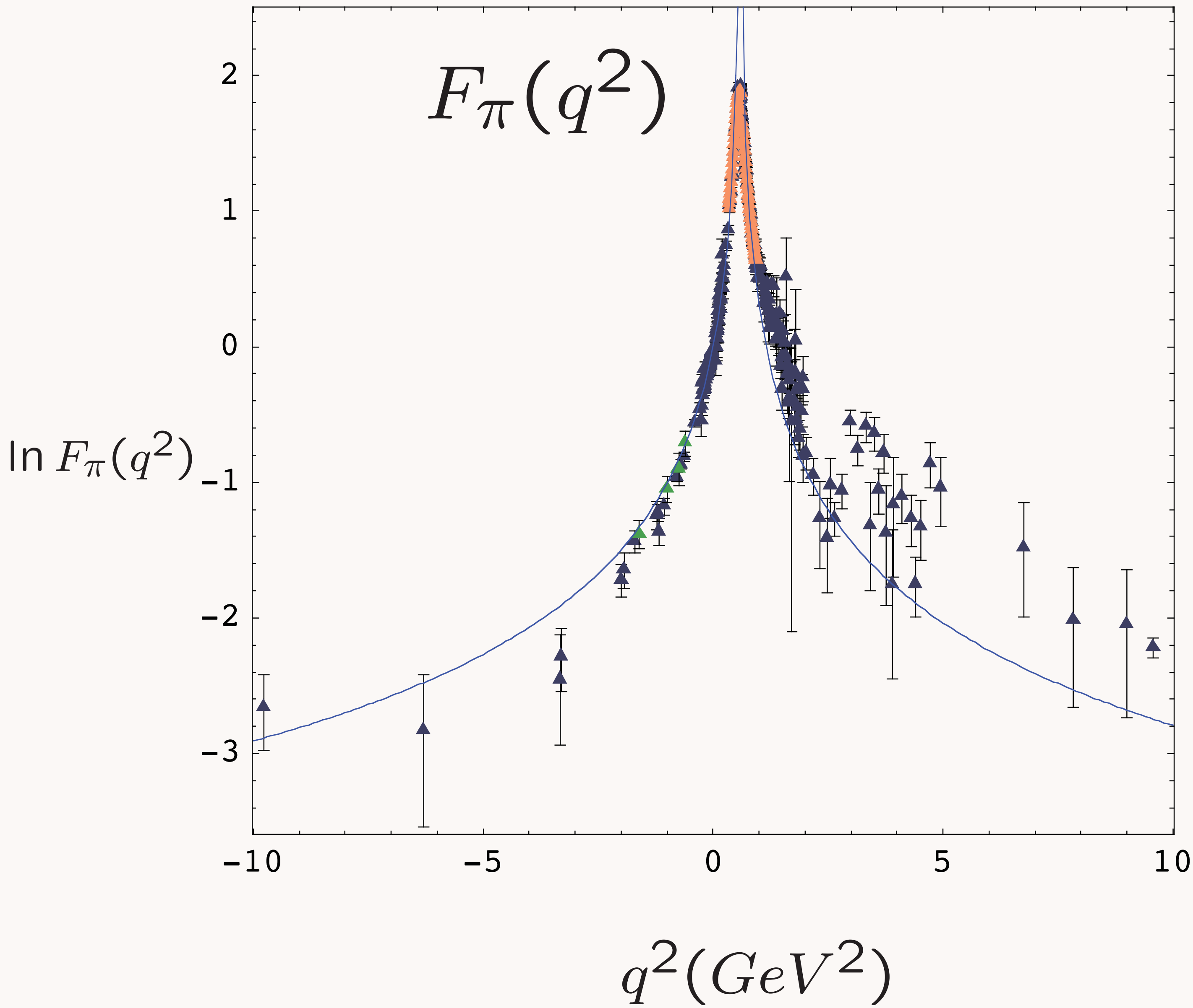
One parameter - set by pion decay constant

G. de Teramond, sjb

AdS/QCD

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$\begin{aligned} F_+(Q^2) &= g_+ R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_+(z)|^2, \\ F_-(Q^2) &= g_- R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_-(z)|^2, \end{aligned}$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

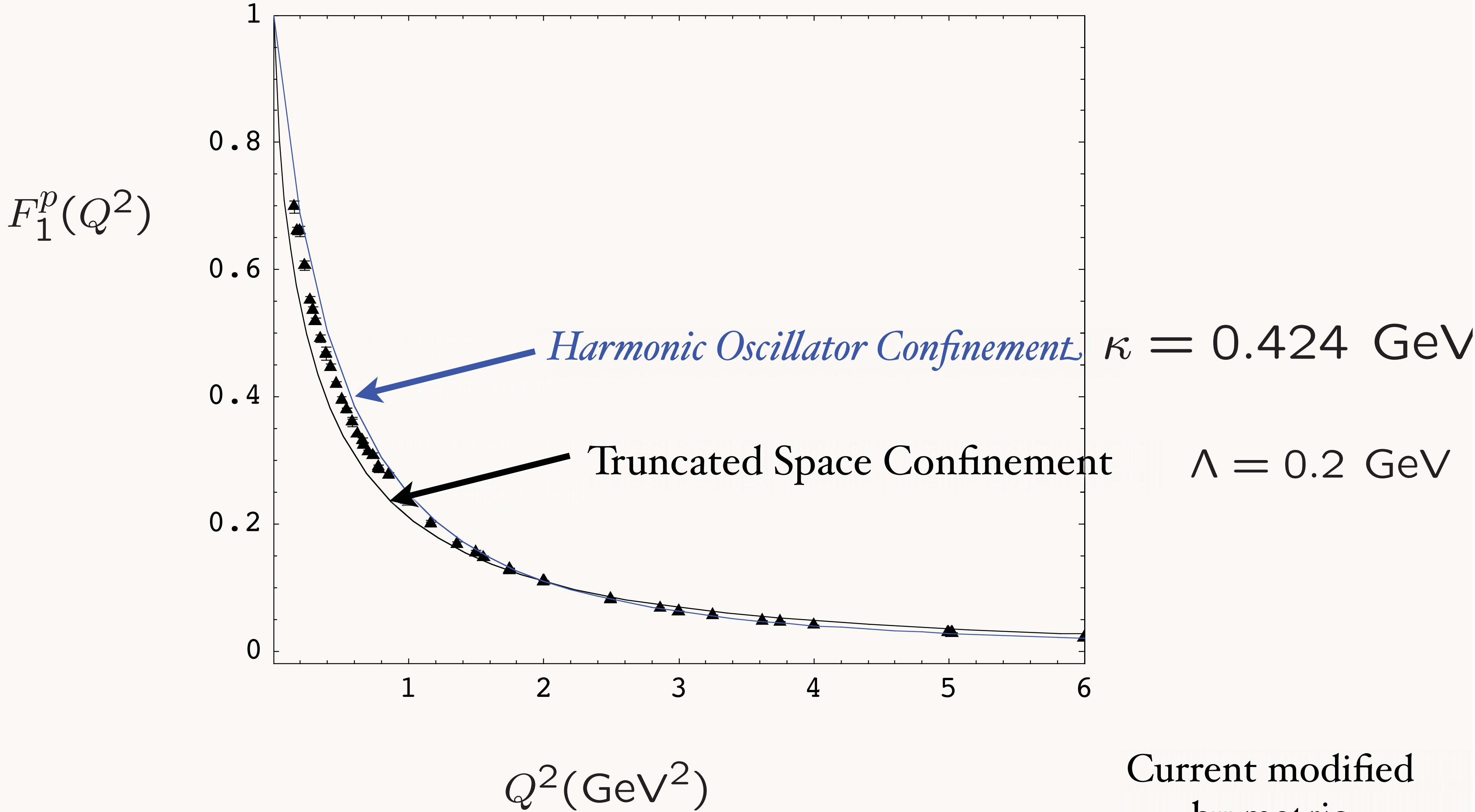
$$\begin{aligned} F_1^p(Q^2) &= R^3 \int \frac{dz}{z^3} \, J(Q,z) |\psi_+(z)|^2, \\ F_1^n(Q^2) &= -\frac{1}{3} R^3 \int \frac{dz}{z^3} \, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right], \end{aligned}$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb



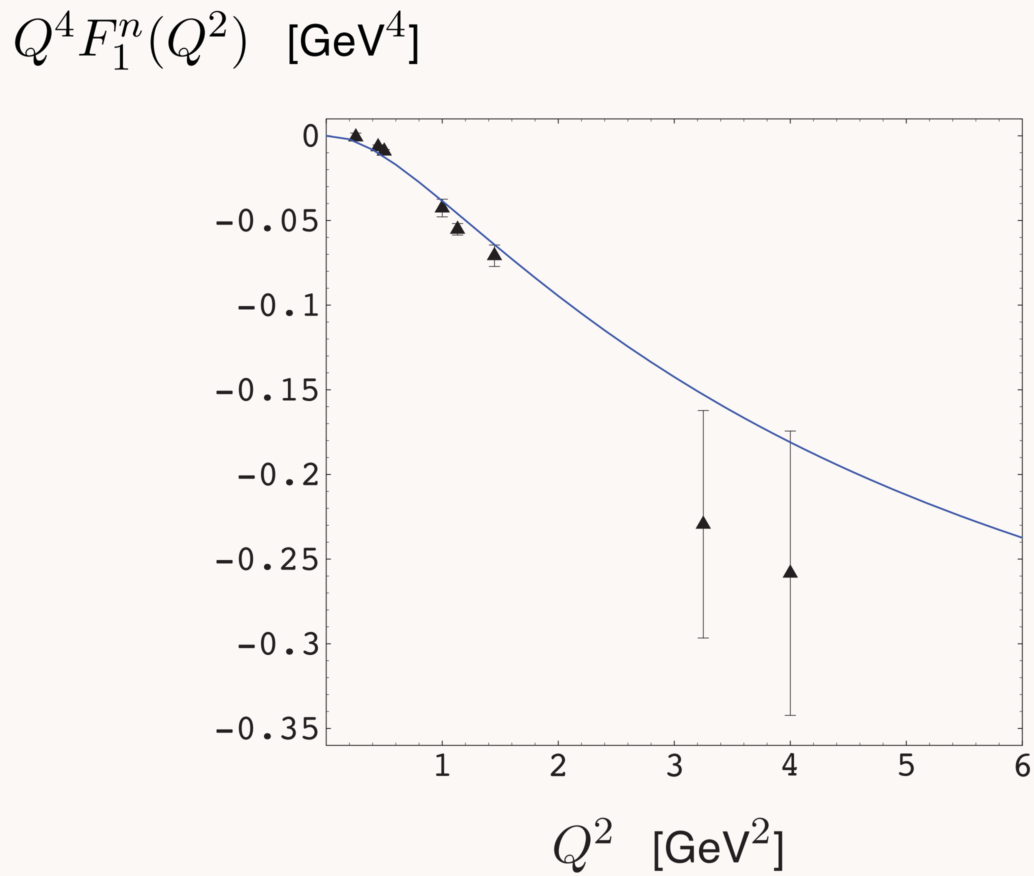
Current modified by metric

$$F_1(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F^{\uparrow}(z) J(Q,z)\Phi_I^{\uparrow}(z)$$

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

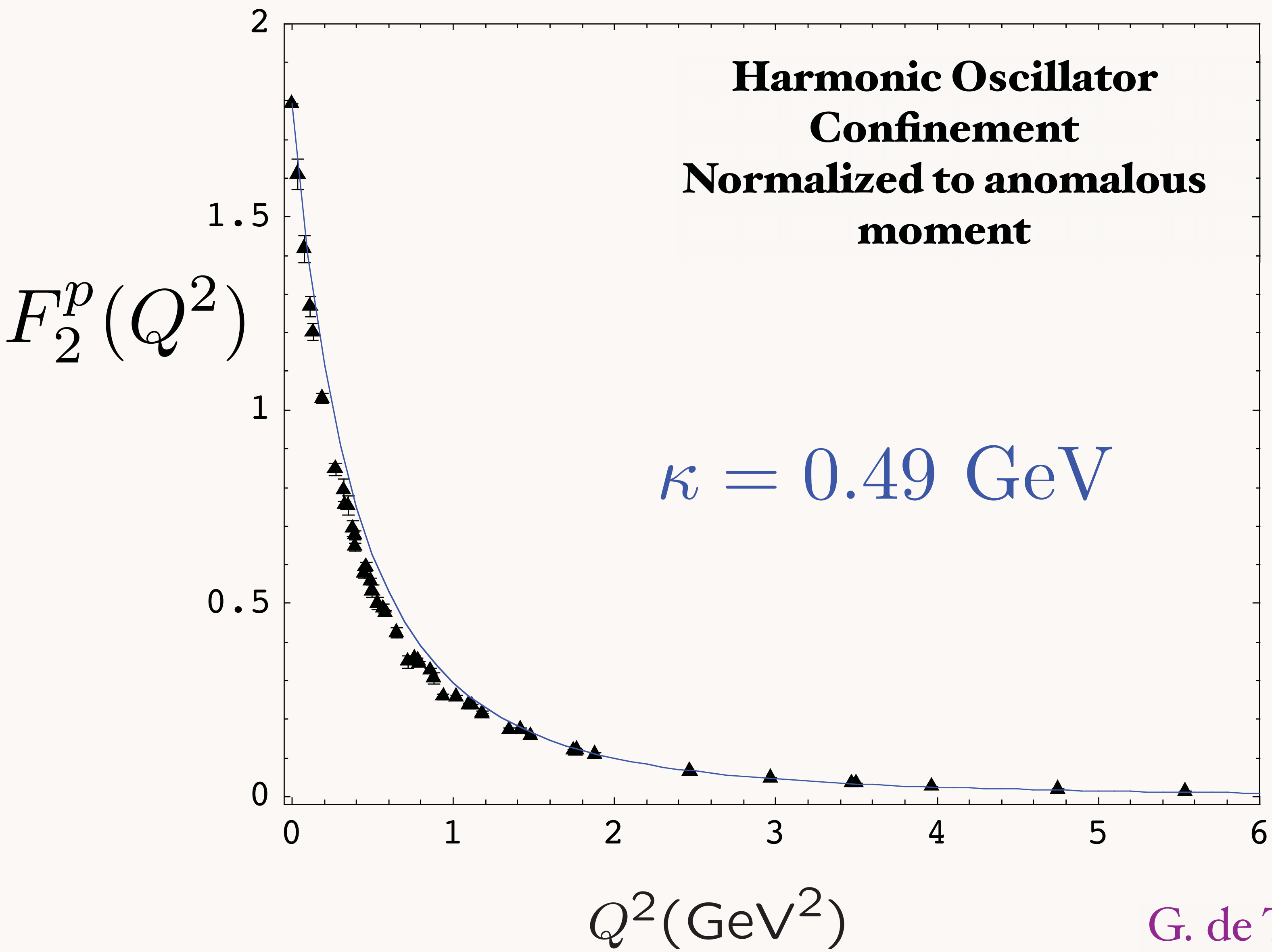


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

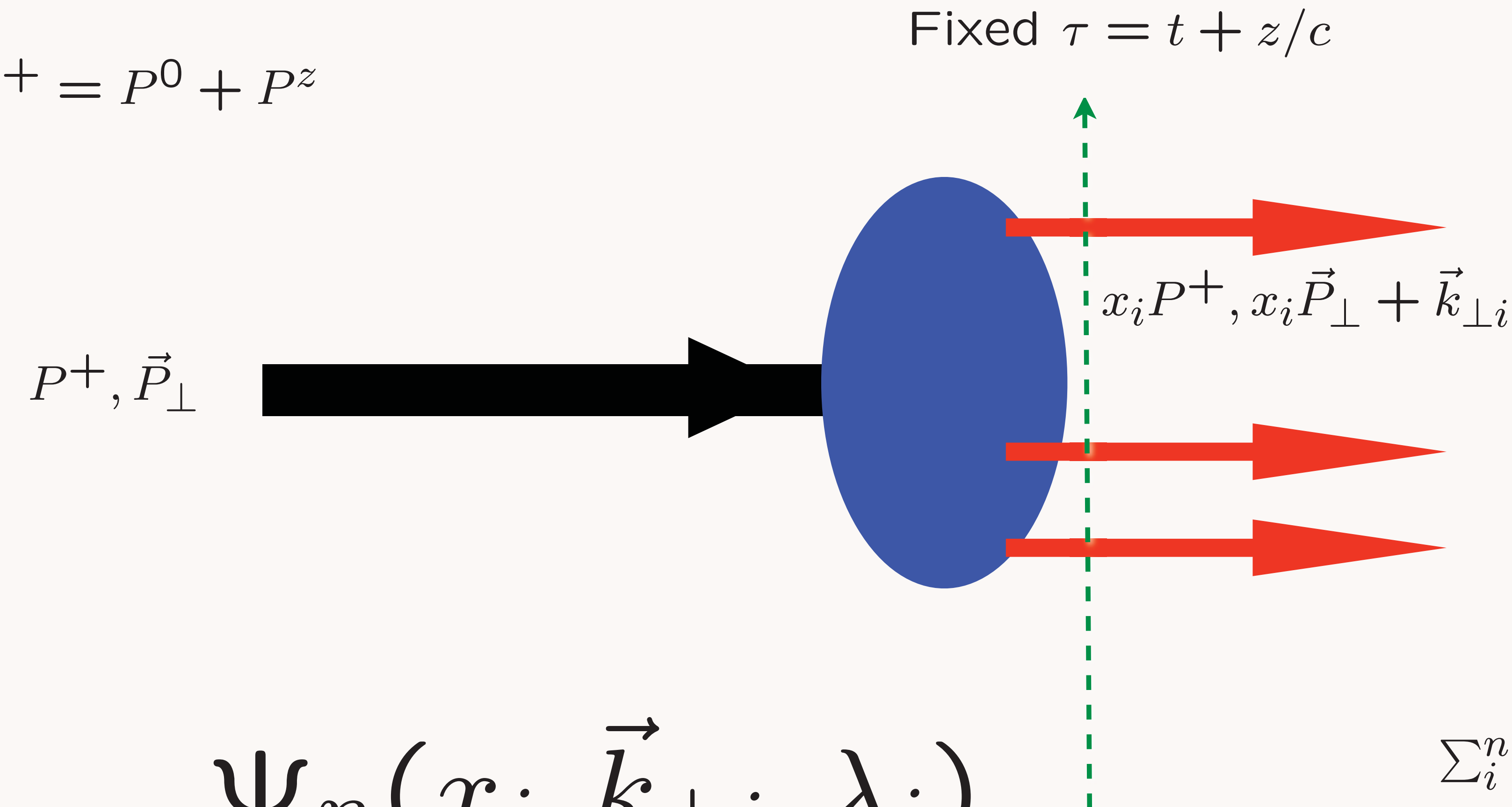
From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

Light-Front Wavefunctions

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

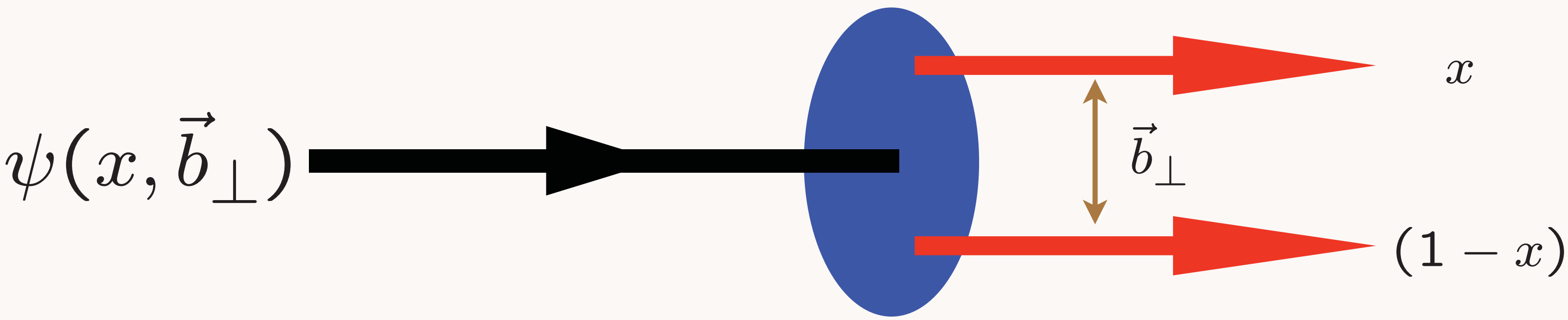
- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

LF(3+1) ↔ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

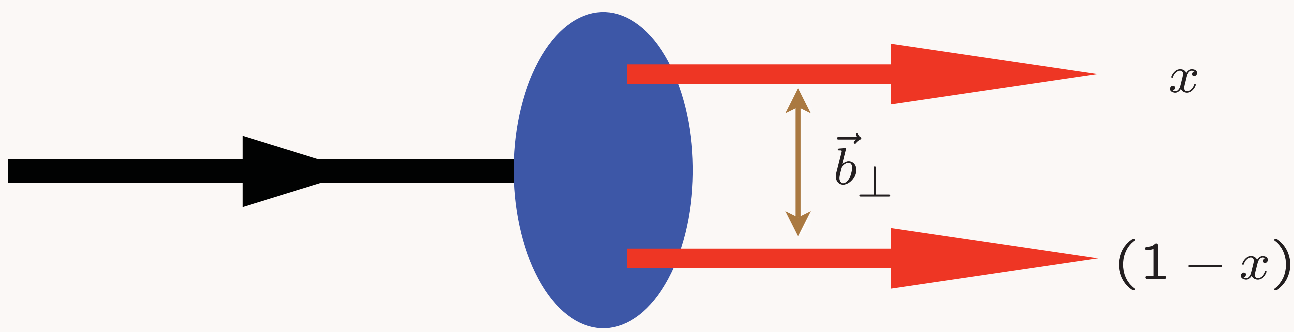
Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Holography: Map AdS/CFT to 3+1 LF Theory

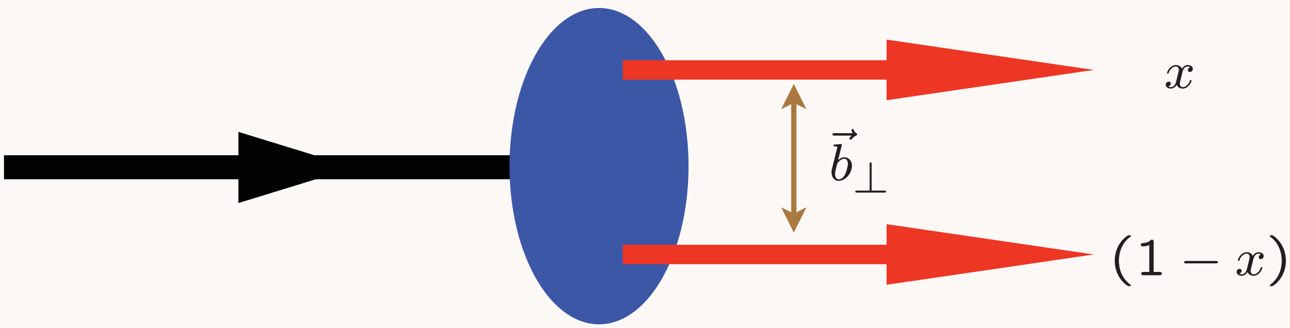
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

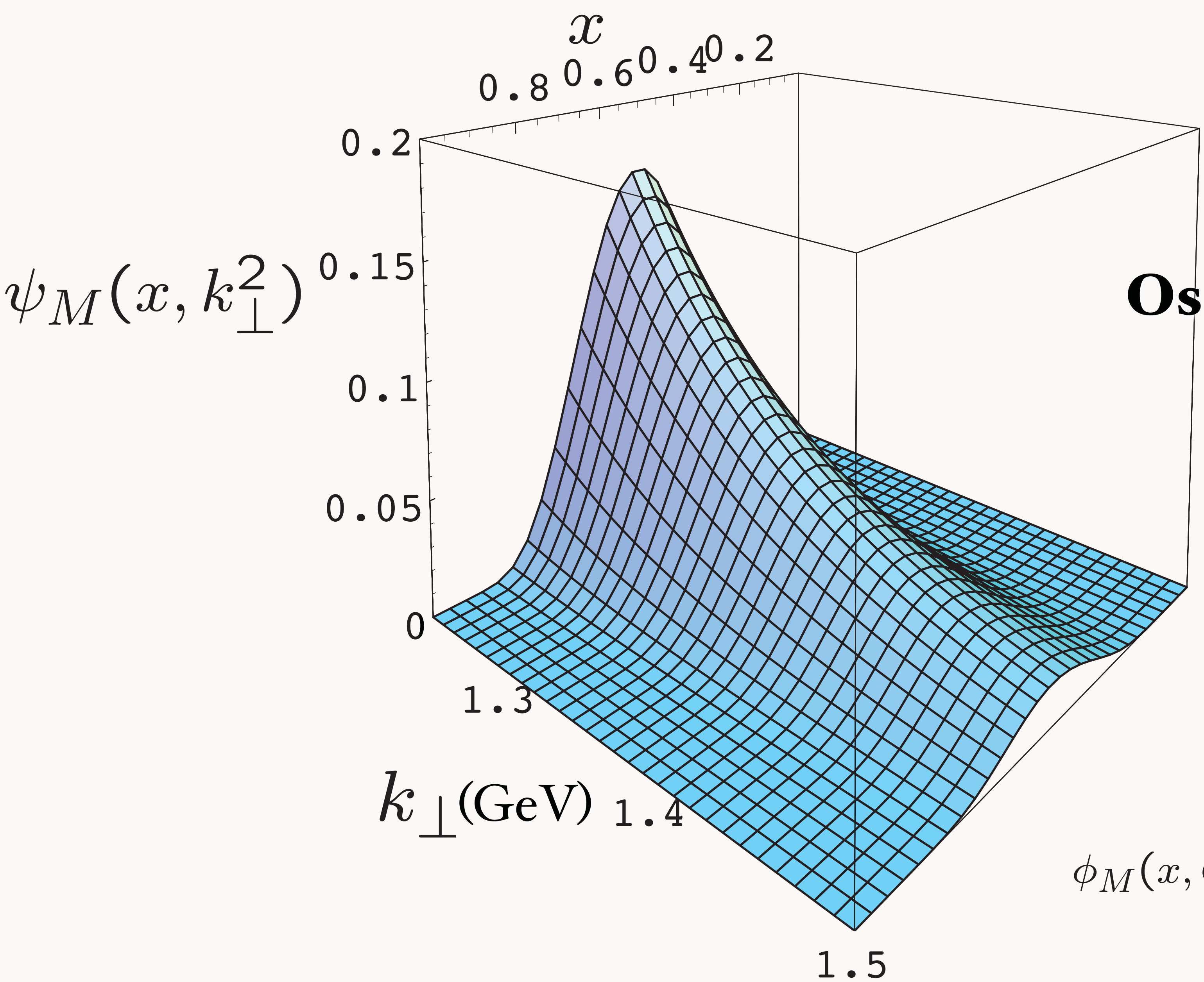
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

AdS/QCD

Prediction from AdS/CFT: Meson LFWF

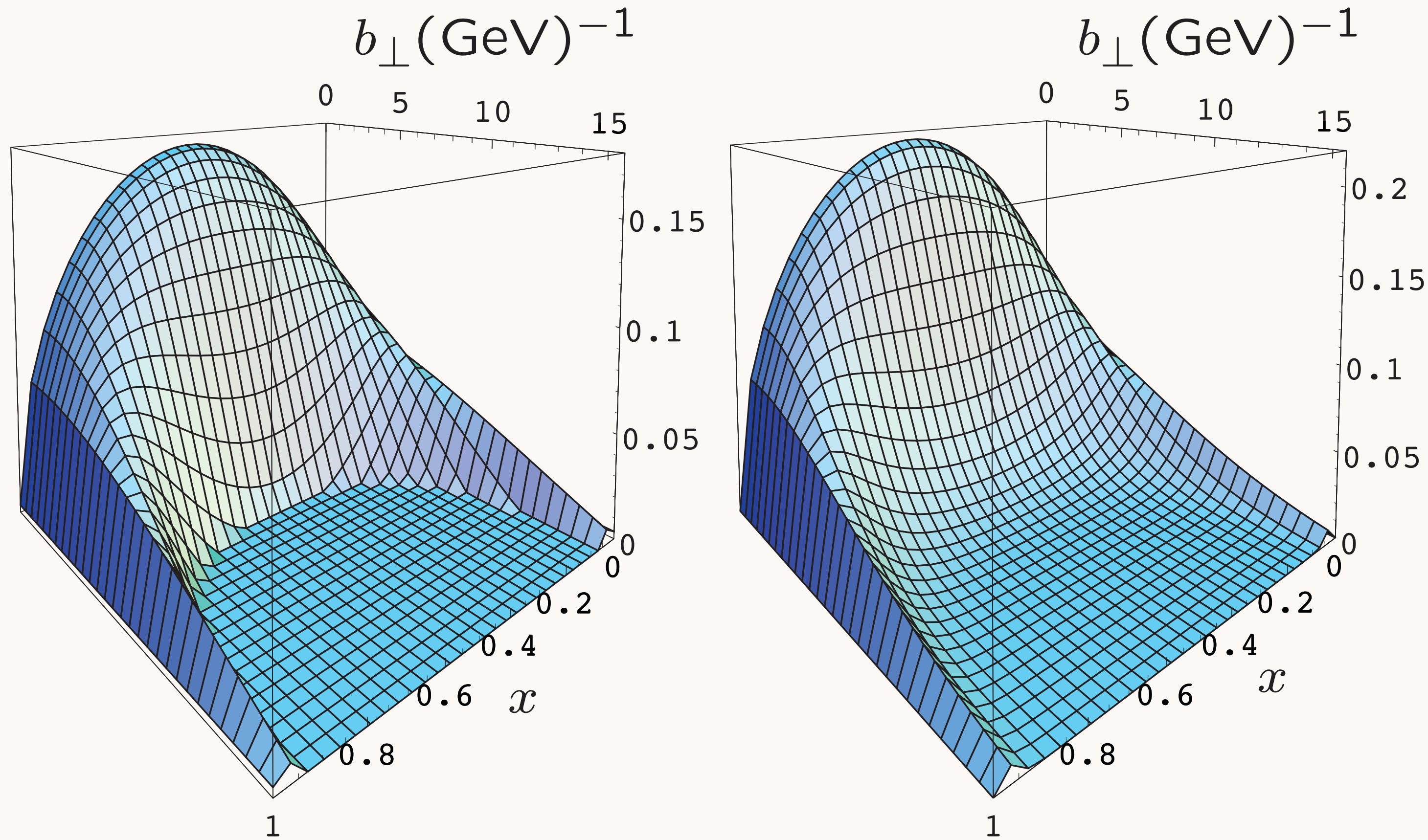


Harmonic Oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

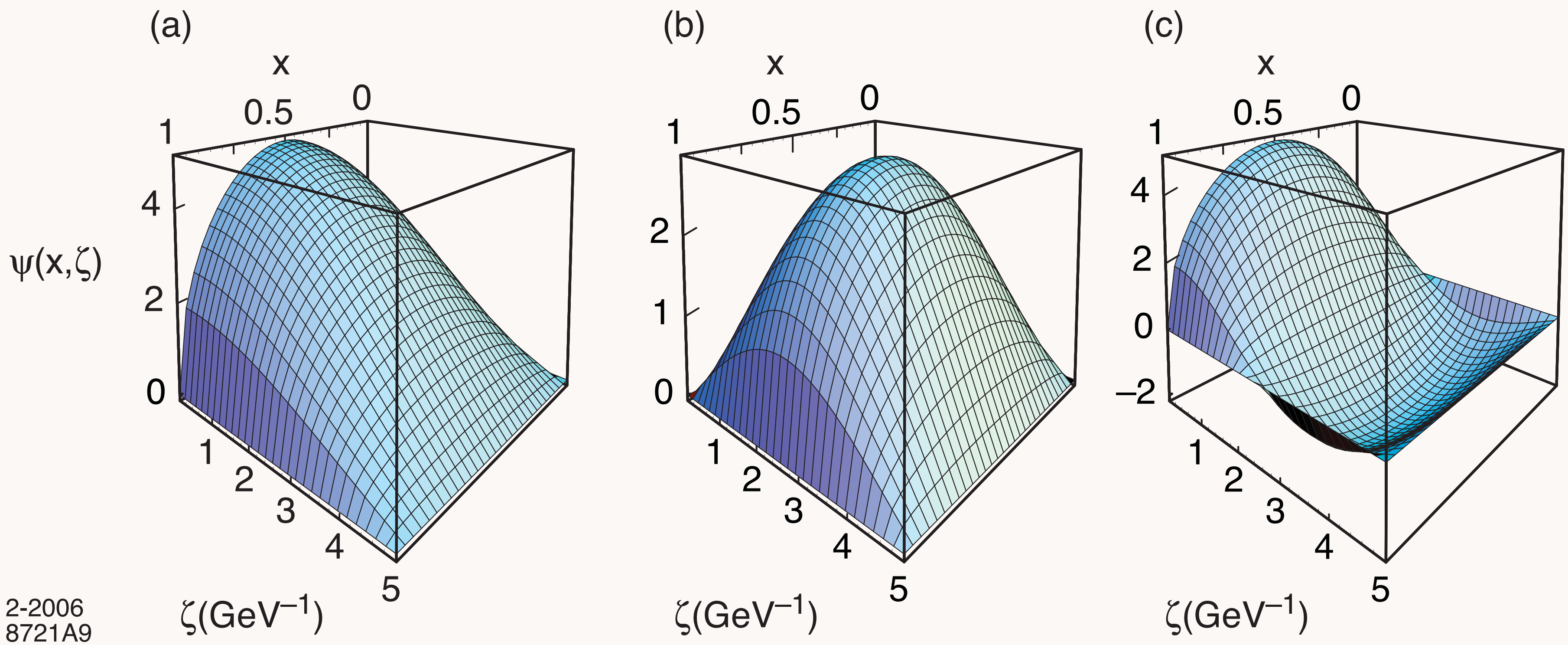
AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{\mathrm{QCD}} = 0.32\ \mathrm{GeV}$ $\kappa = 0.76\ \mathrm{GeV}$

Truncated Space **Harmonic Oscillator**

Guy de Teramond
SJB



Two-quark holographic LFWF in impact space $\psi(x,\zeta)$: $(a)\ \ell = 0,\ k = 1;\ (b)\ \ell = 1,\ k = 1;$ $(c)\ \ell = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

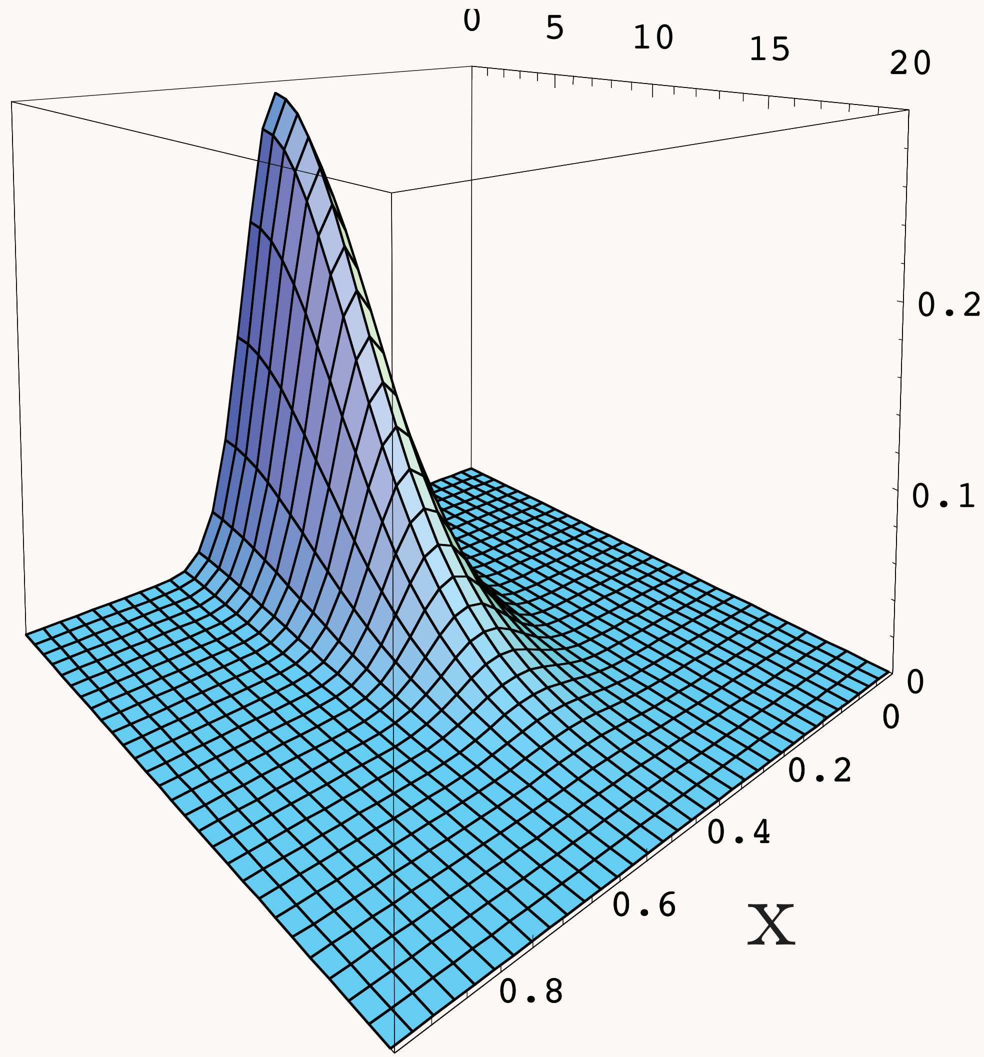
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$\psi_{J/\psi}(x, b)$$



$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d}>$

$m_u = 2\ MeV$
$m_d = 5\ MeV$

$|K^+ >= |u\bar{s}>$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d}>$

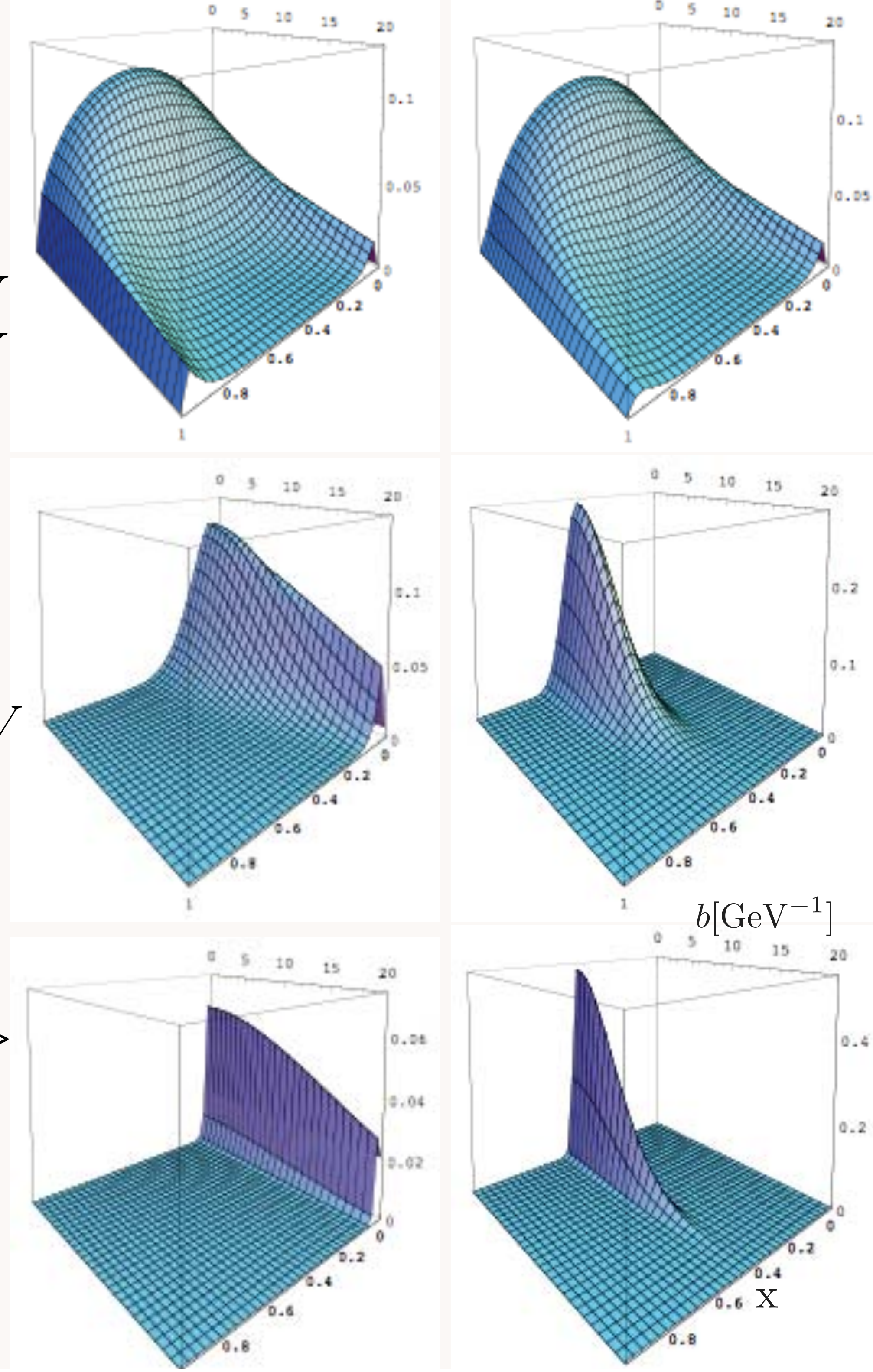
$m_c = 1.25\ GeV$

$|\eta_c >= |c\bar{c}>$

$|B^+ >= |u\bar{b}>$

$m_b = 4.2\ GeV$

$|\eta_b >= |b\bar{b}>$



$\kappa = 375\ MeV$

Meson LFWF (L=0) for massive quarks

$$\psi_{\bar{q}q/\pi}(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} \, e^{-\frac{\mathbf{k}_\perp^2 + m^2}{2\kappa^2 x(1-x)}},$$

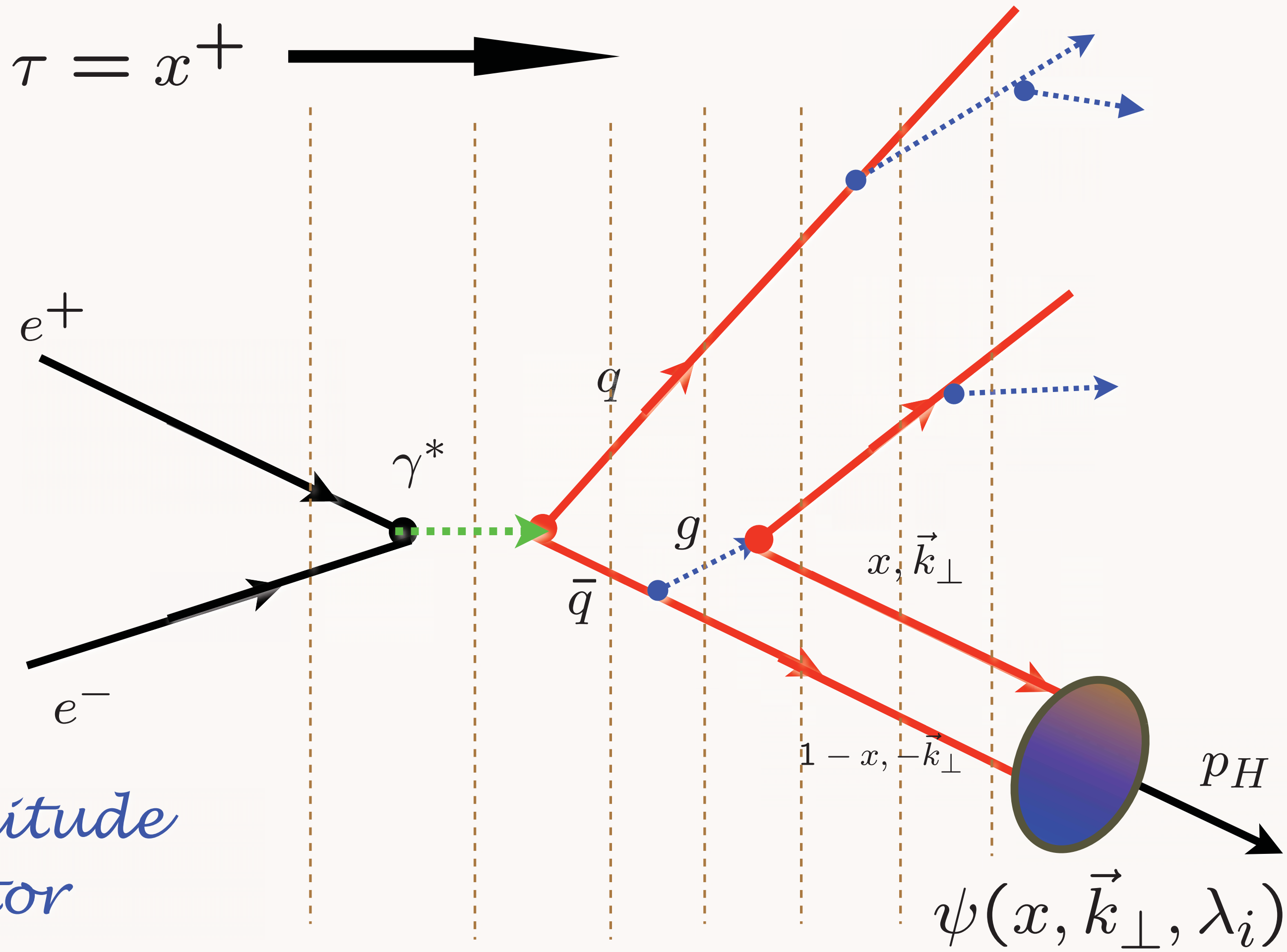
$$\widetilde{\psi}_{q\bar{q}/\pi}(x, \mathbf{b}_\perp) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)} \, \exp\left(-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{m^2}{2\kappa^2 x(1-x)}\right)$$

Key variable for n-parton LFWF with massive quarks:

$$\boxed{\chi^2 = \zeta^2 + \frac{1}{\kappa^4}\sum_{i=1}^{n}\frac{m_i^2}{x_i},}$$

$$\zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right|$$

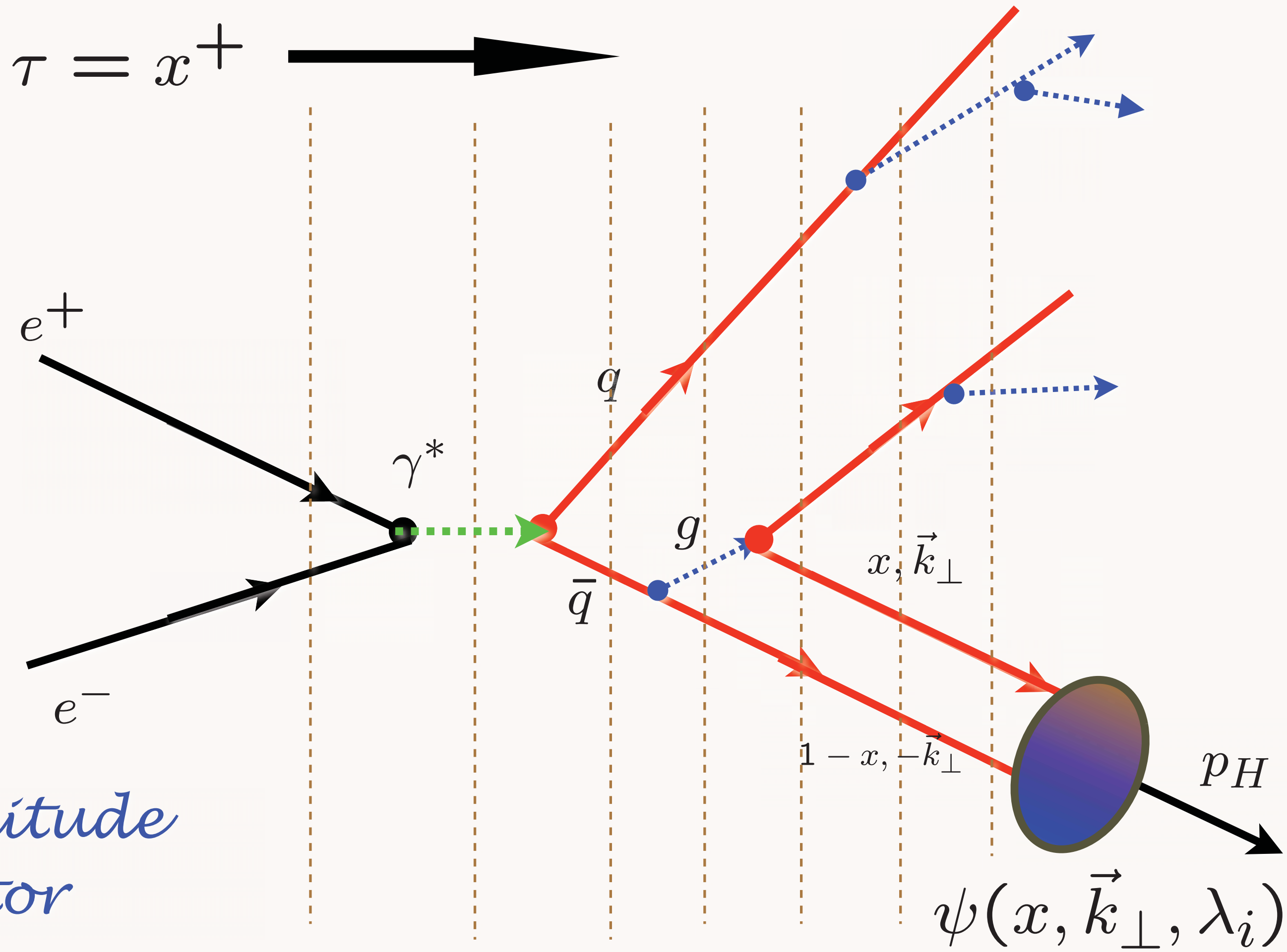
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



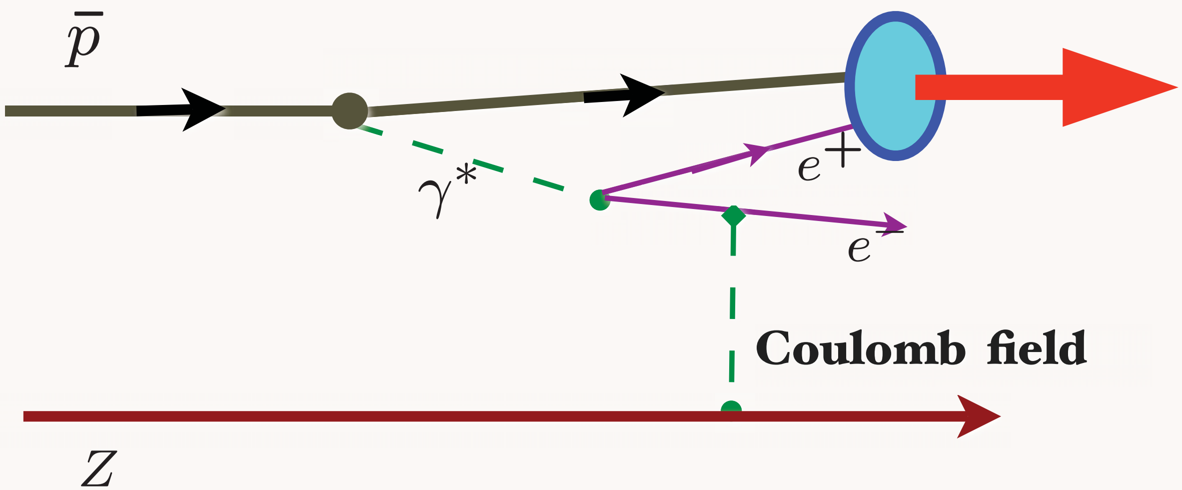
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

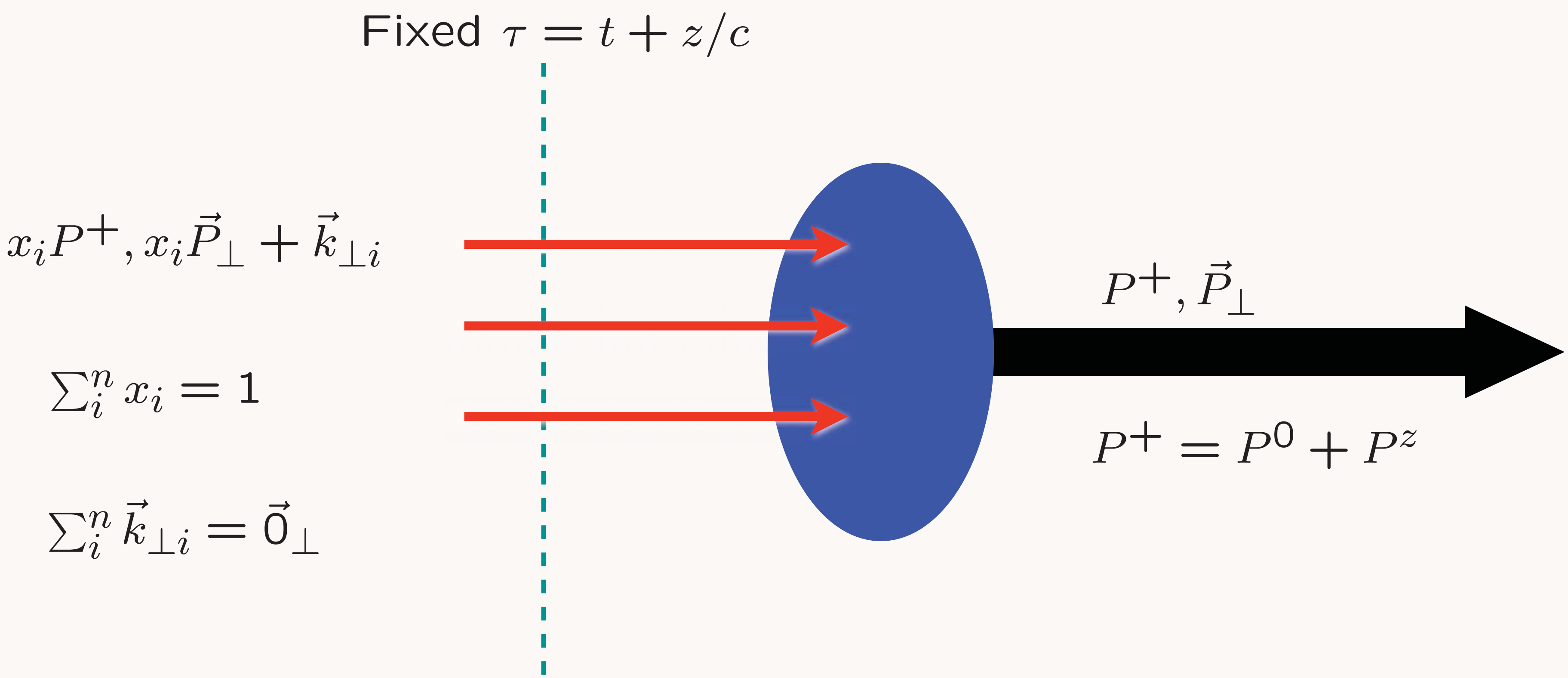
Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

AdS/QCD

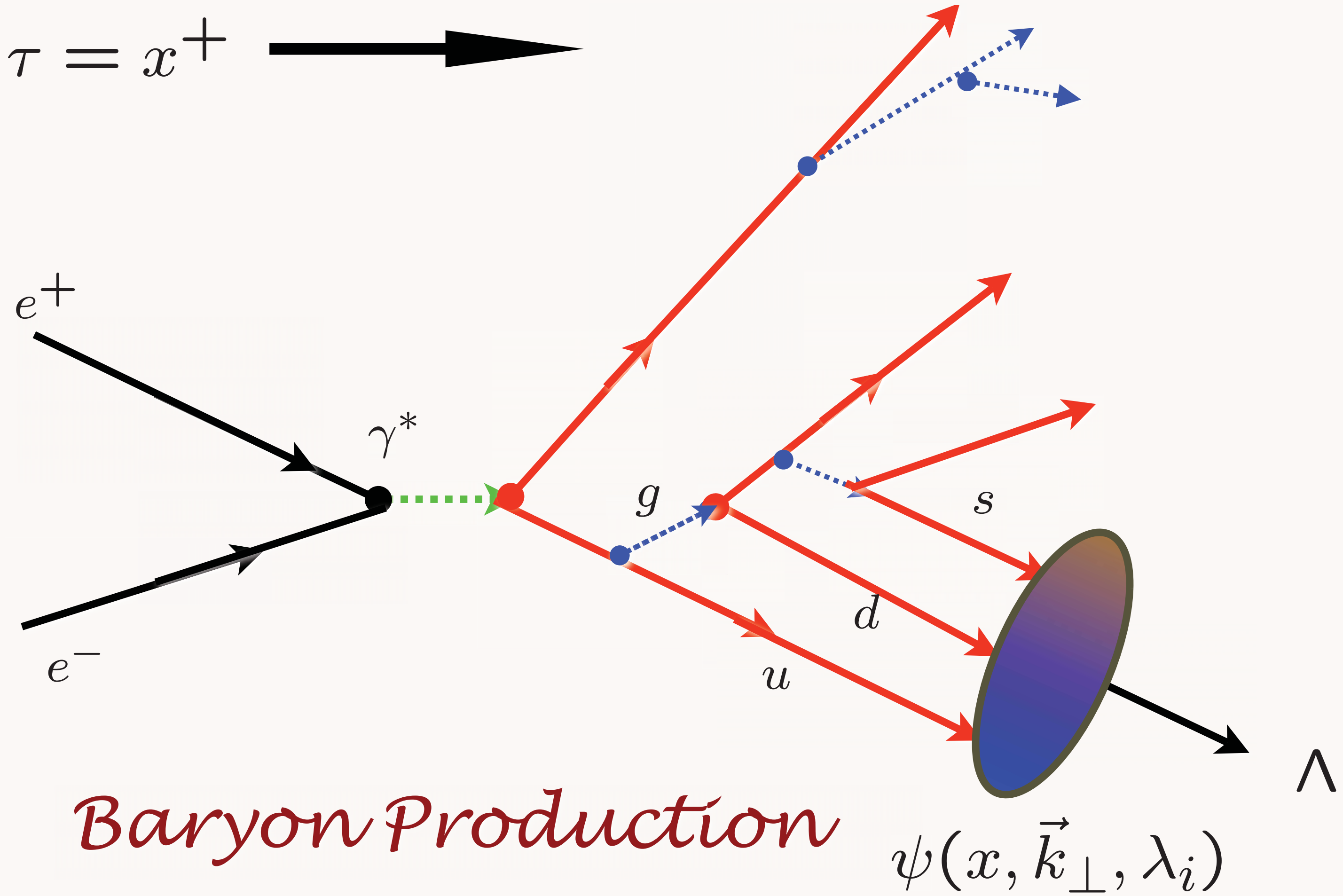
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



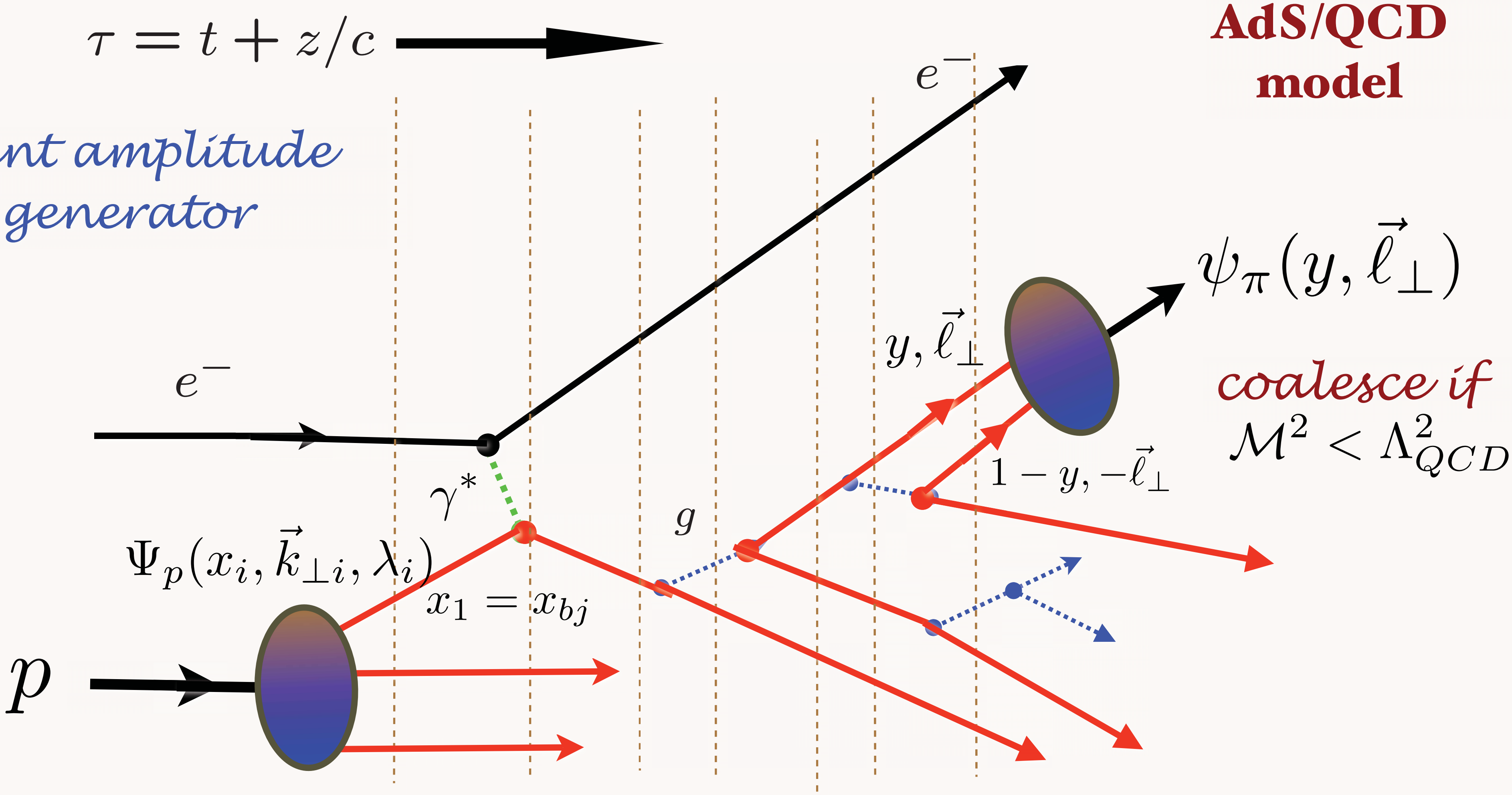
Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Jet Hadronization at the Amplitude Level

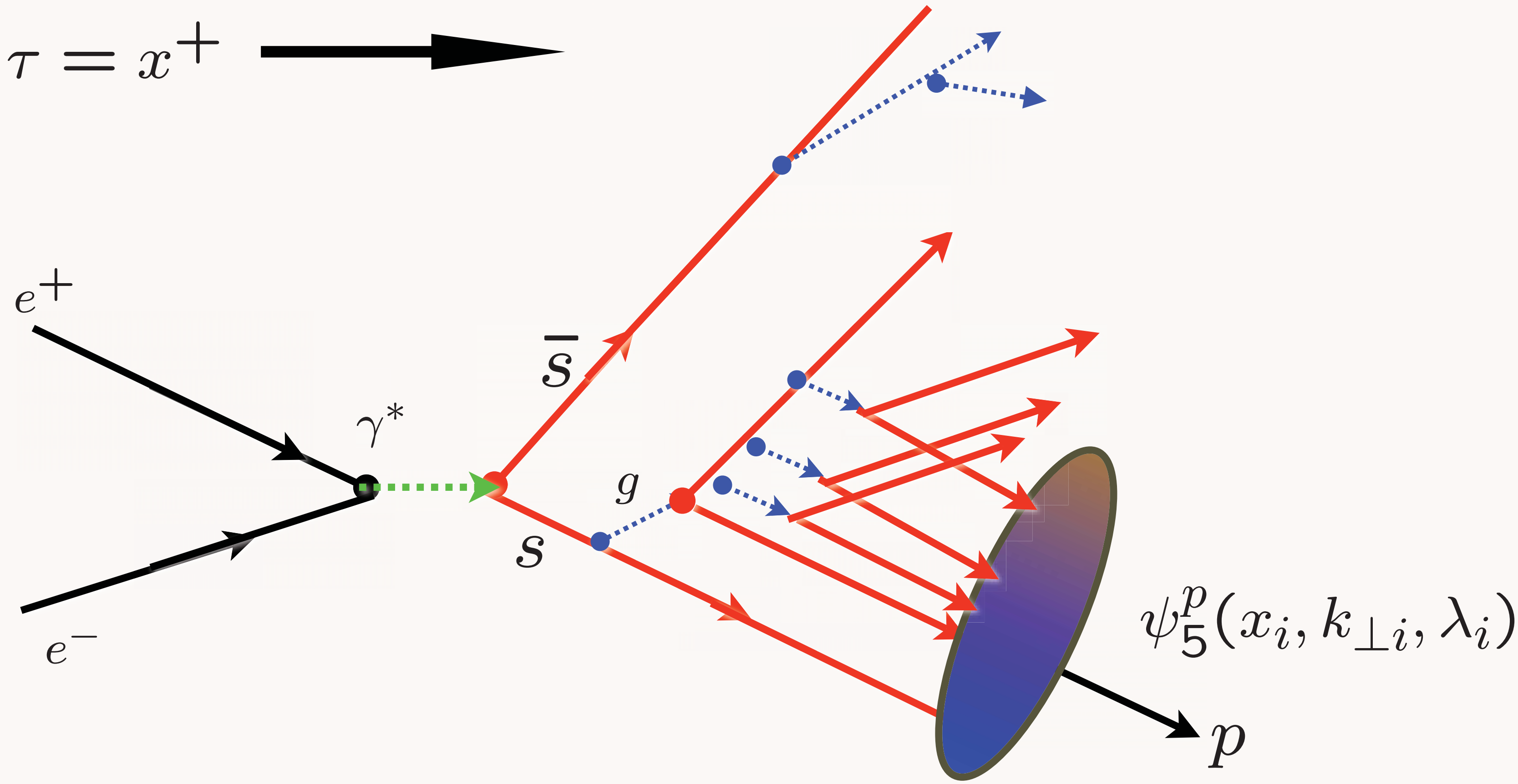
AdS/QCD model

Event amplitude generator



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via Light-Front Wavefunctions

Hadronization at the Amplitude Level



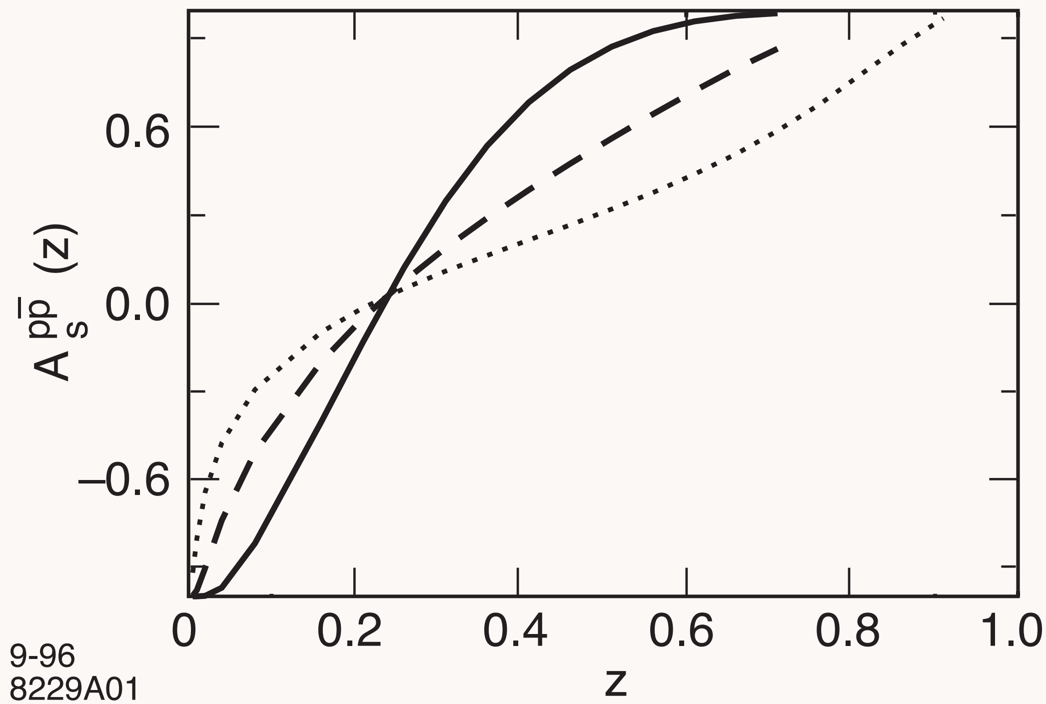
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s \to p}(z) \neq D_{s \to \bar{p}}(z)$

B-Q Ma, sjb

$D_{s\to p}(z) \neq D_{s\to \bar{p}}(z)$

B-Q Ma, sjb



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to \bar{p}}(z)}{D_{s\to p}(z) + D_{s\to \bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momenta
- Hadron Spectra, Regge Trajectories

Features of Holographic Model

de Teramond sjb

- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.

- Scale Λ_{QCD} determines hadron spectrum (slightly different for mesons and baryons)

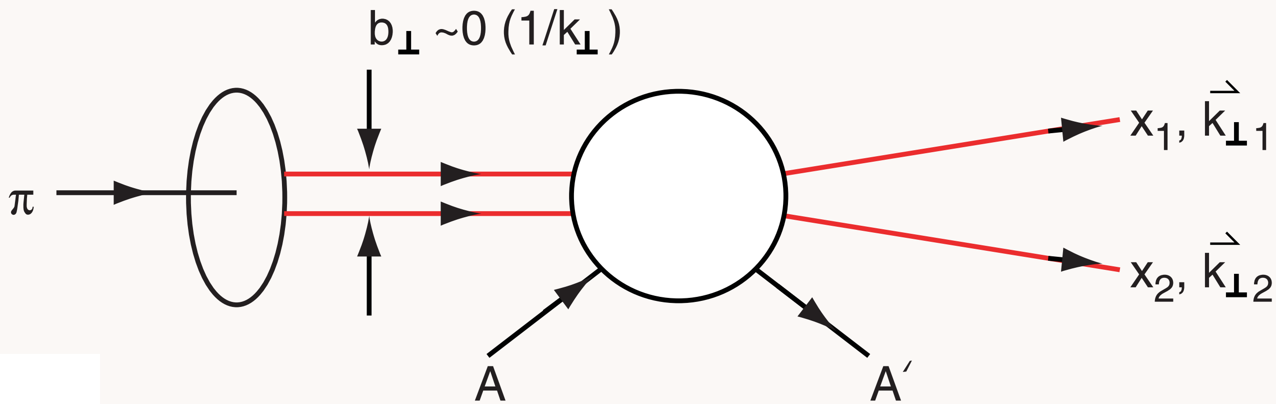
- Covariant version of bag model: confinement +conformal symmetry

- Pion decay constant

- Dominance of Quark Interchange

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



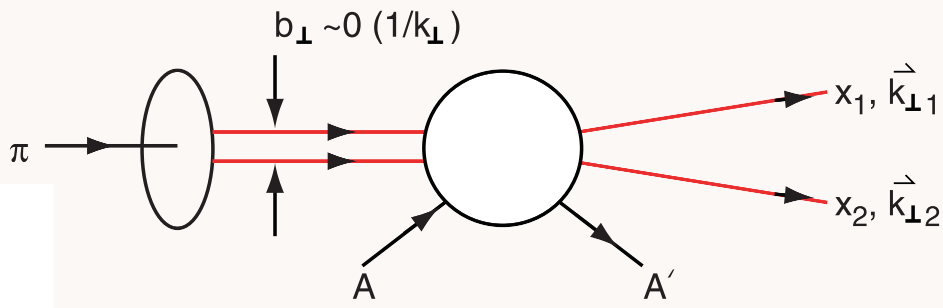
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

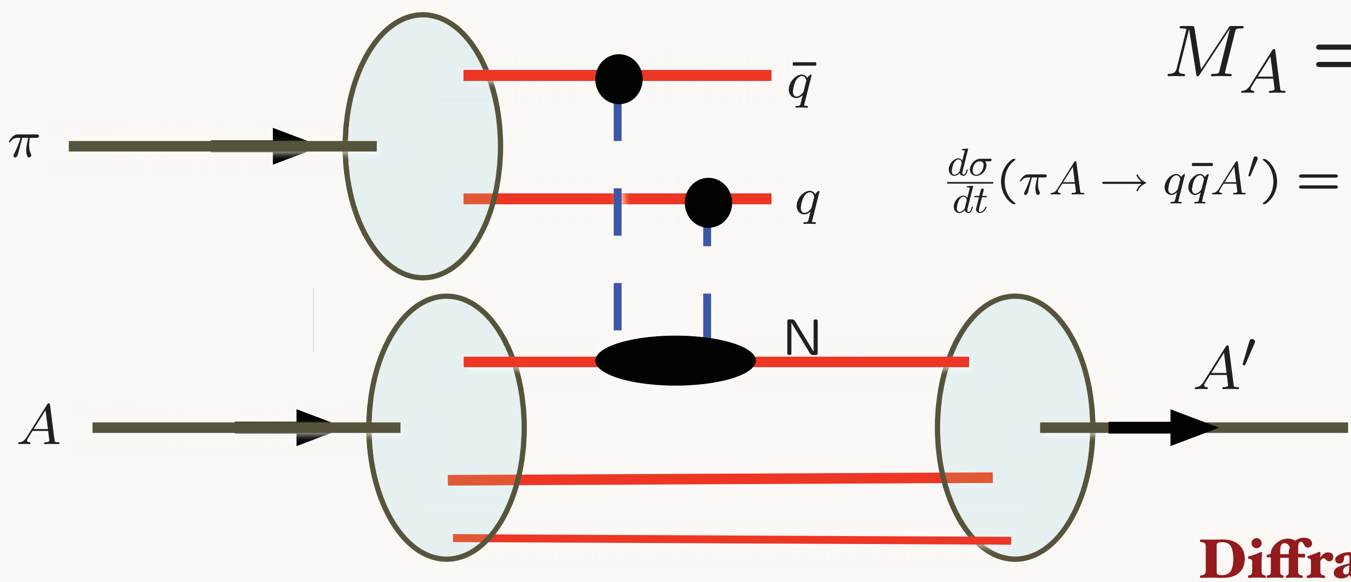
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

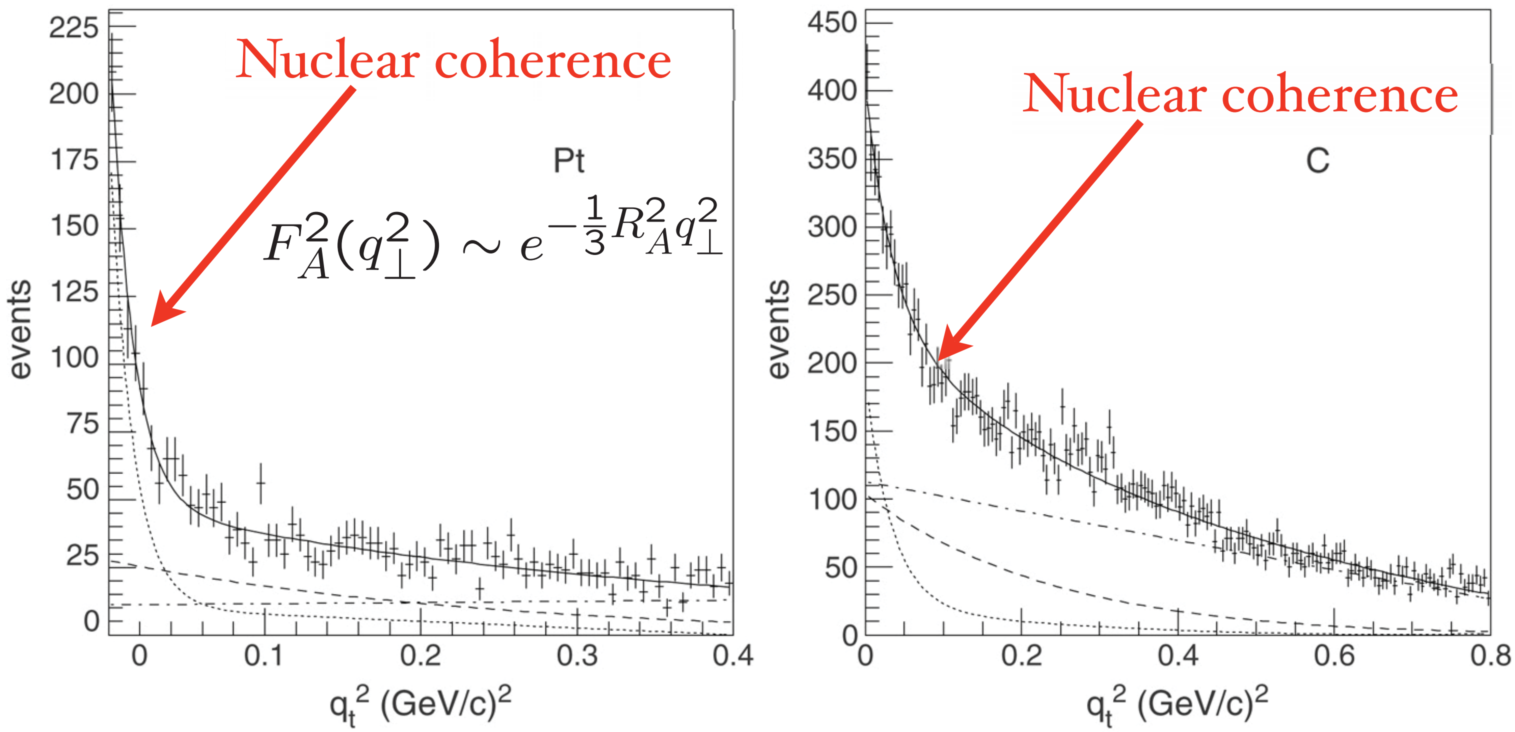
Diffraction, Rapidity gap

- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

Ashery E791

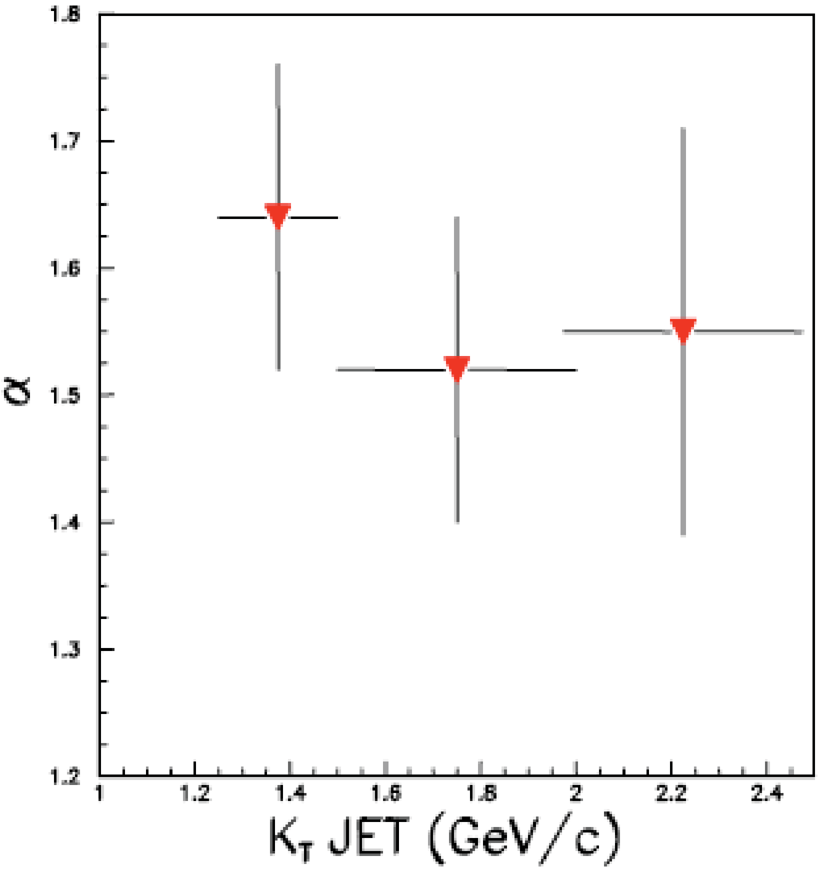
α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

Factor of 7

AdS/QCD

A(π,dijet) data from FNAL



Coherent π^+ diffractive dissociation with 500 GeV/c pions on Pt and C.

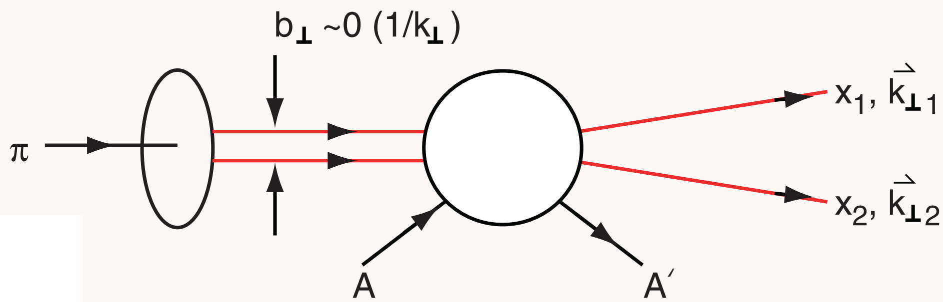
Fit to $\sigma = \sigma_0 A^\alpha$

α = 0.76 from pion-nucleus total cross-section.

Aitala et al., PRL 86 4773 (2001)

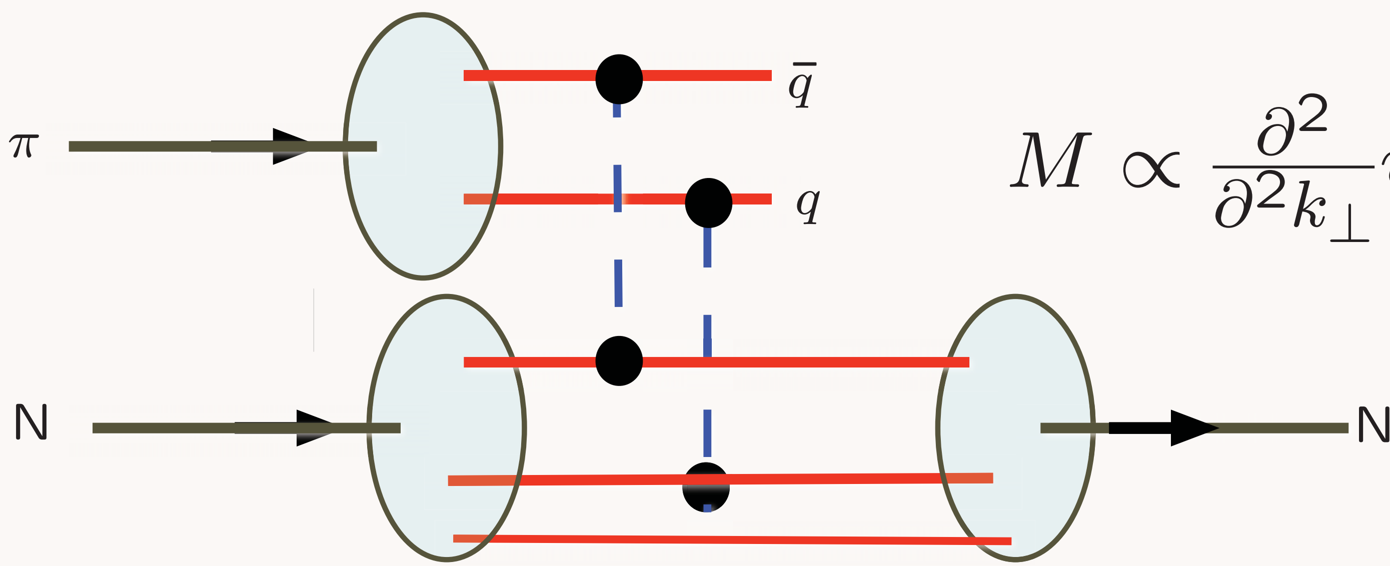
L. L. Frankfurt, G. A. Miller, and M. Strikman, Found. Of Phys. 30 (2000) 533

E791 FNAL Diffractive DiJet



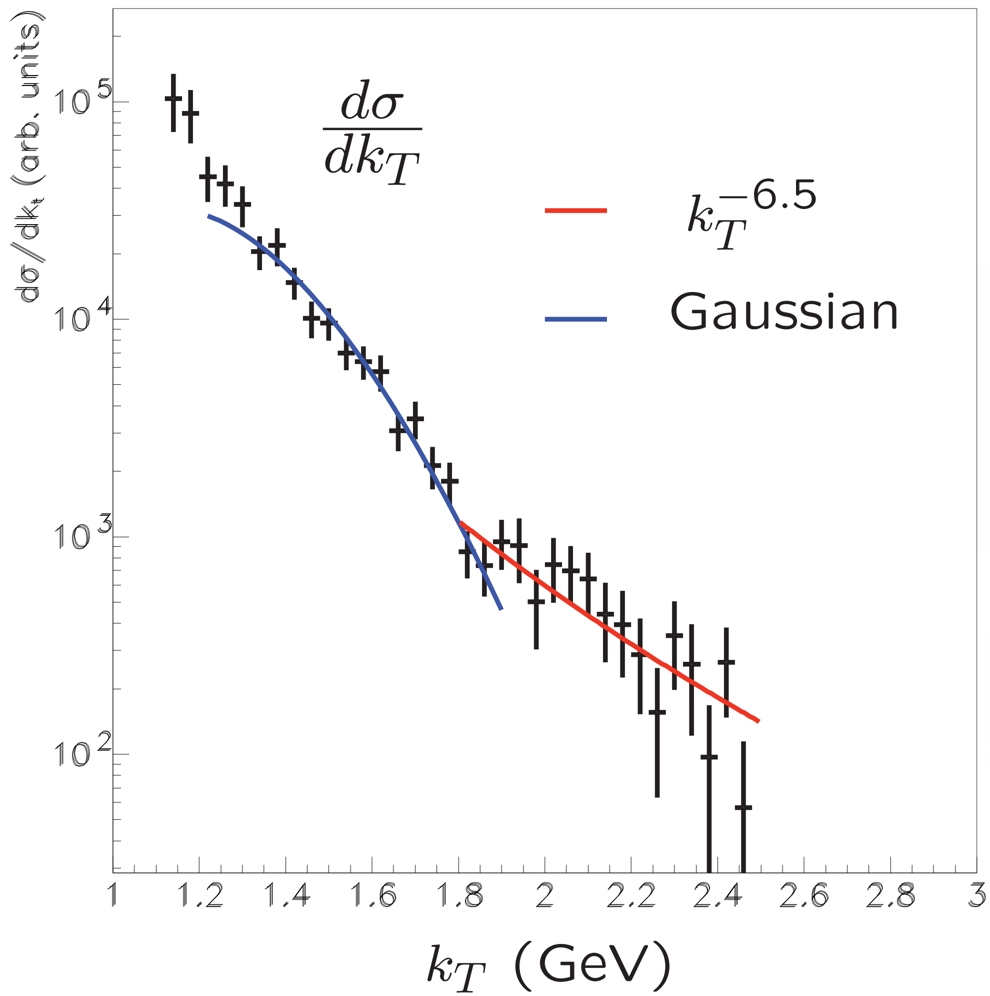
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution



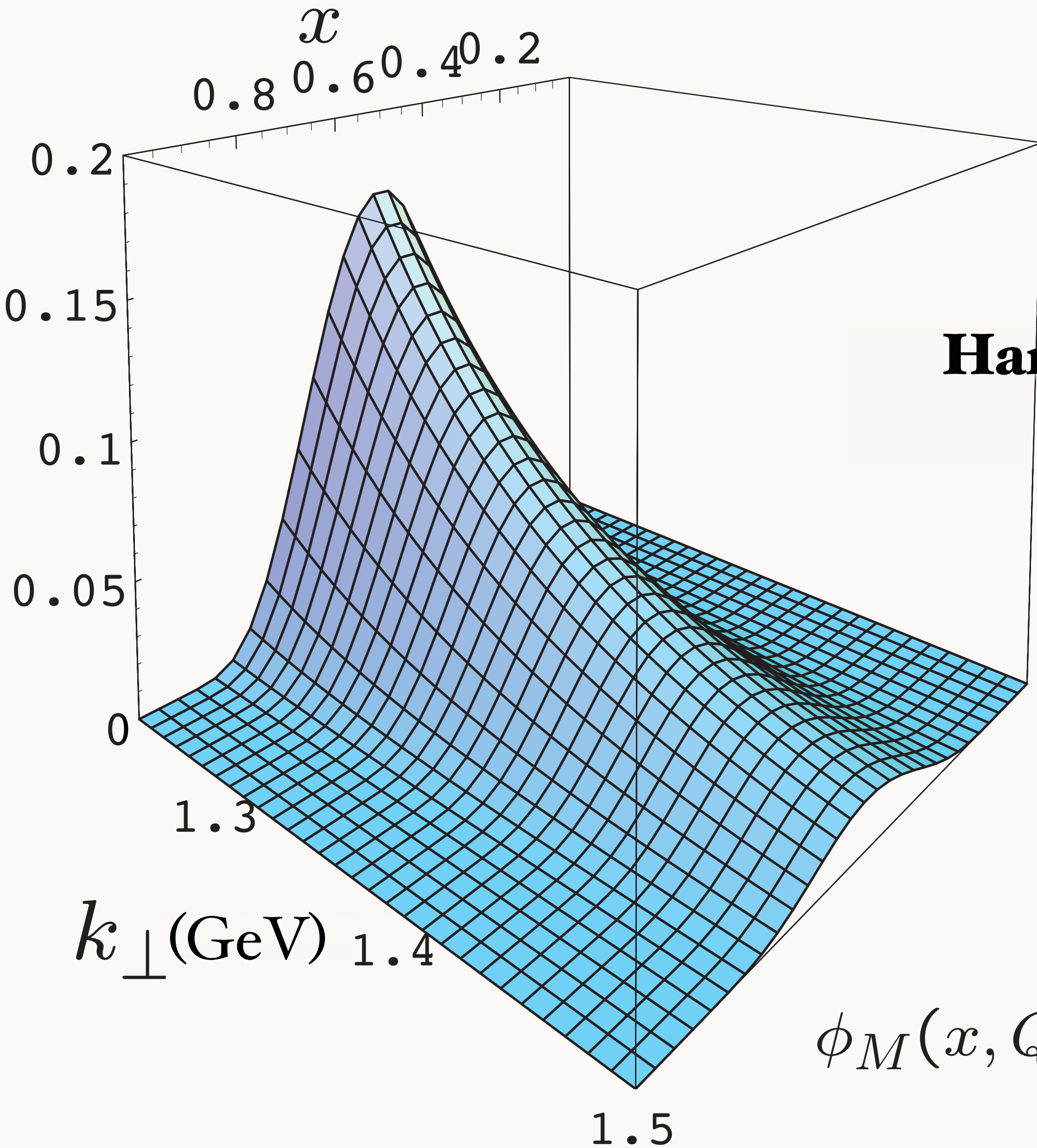
Two Components

High Transverse momentum component $k_T^{-6.5}$ consistent with PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT HO LFWF

Shuryak:
Transition reflects domain walls

Prediction from AdS/CFT: Meson LFWF



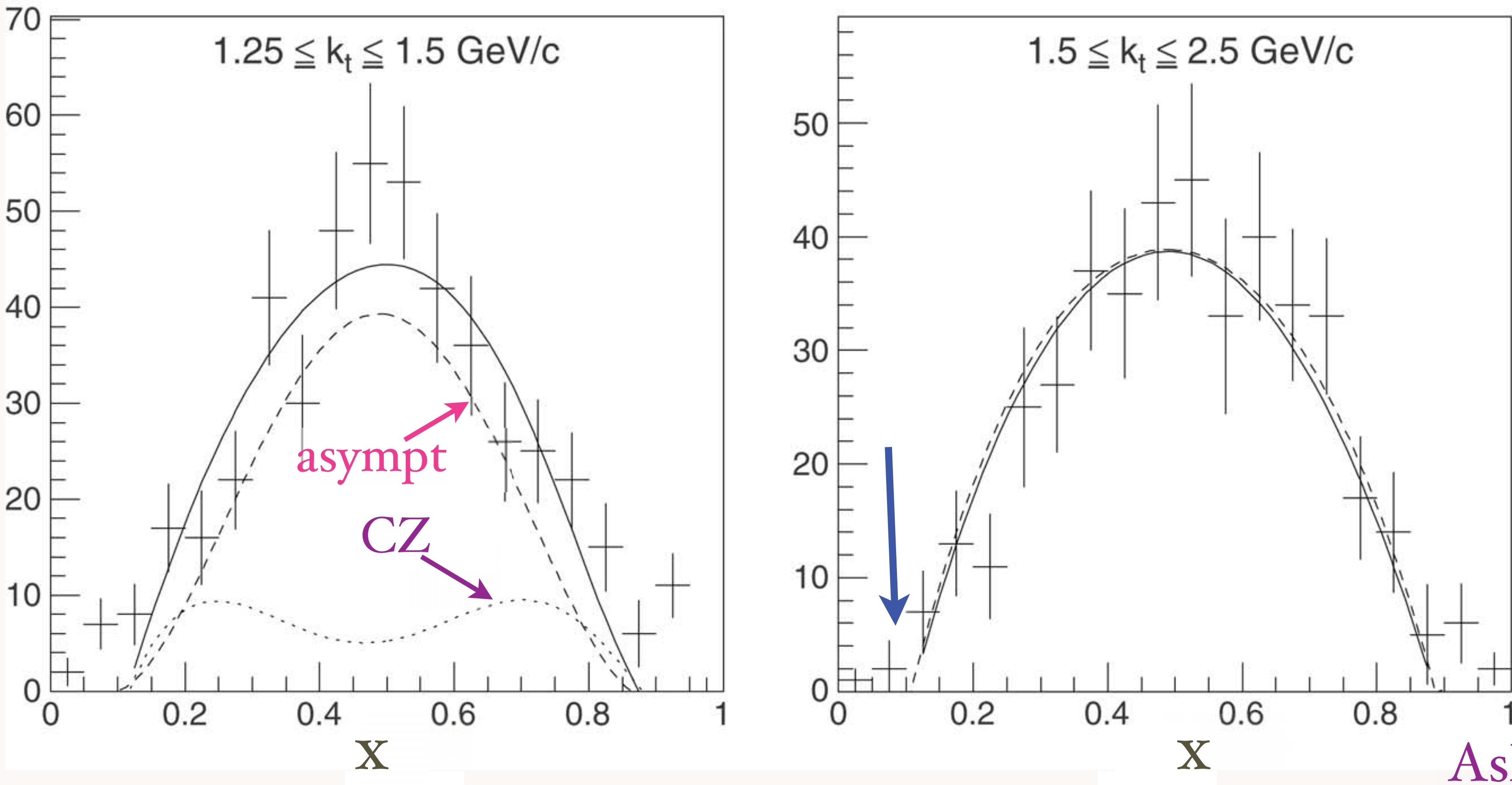
$\psi_M(x, k_\perp^2)$

$k_\perp$(GeV)

Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$



Ashery E791

Narrowing of x distribution at higher jet transverse momentum

x distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$

AdS/QCD

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

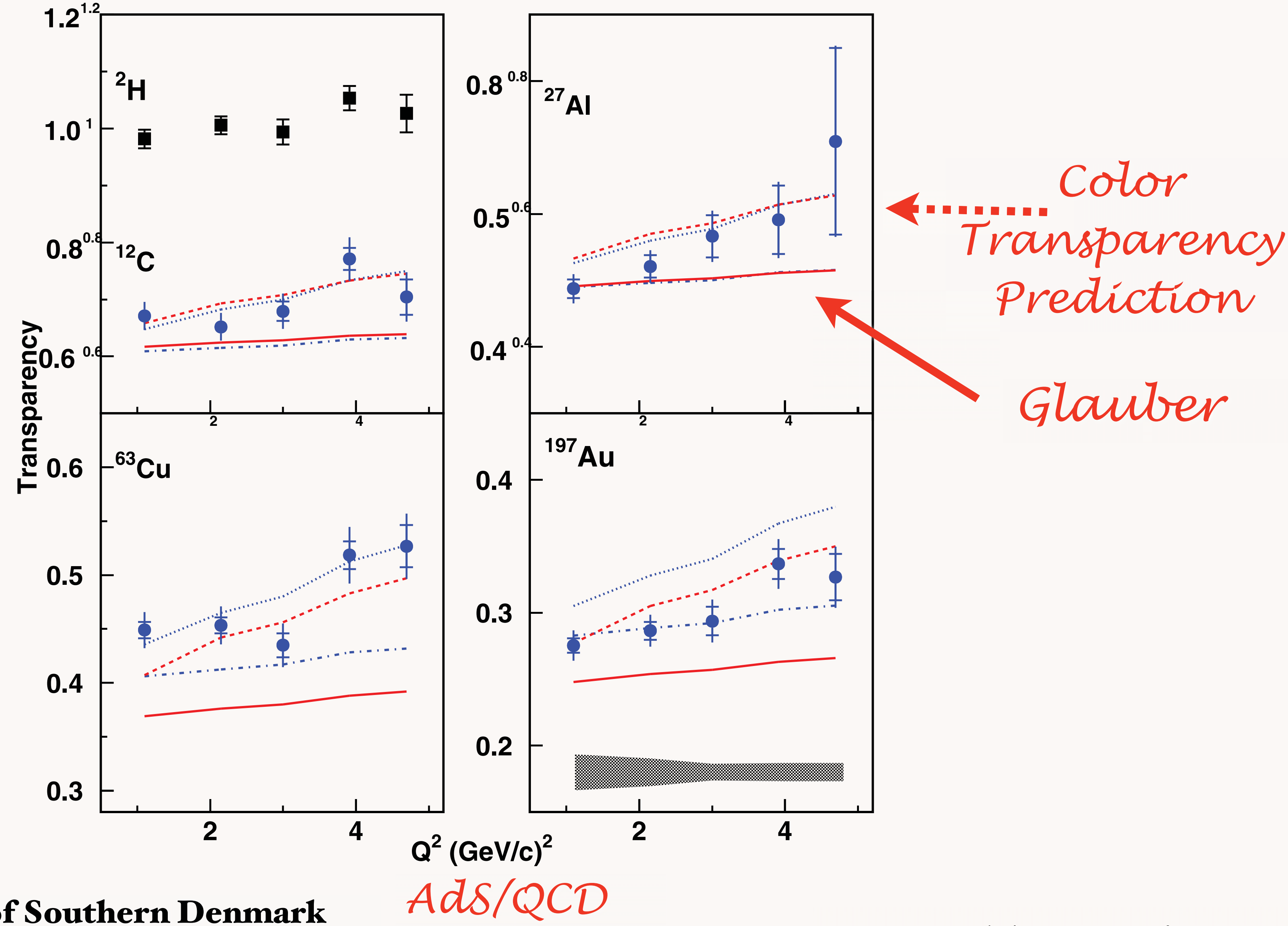
- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Measurement of Nuclear Transparency for the $A(e, e'\pi^+)$ Reaction

$$eA \rightarrow e'\pi^+ X$$

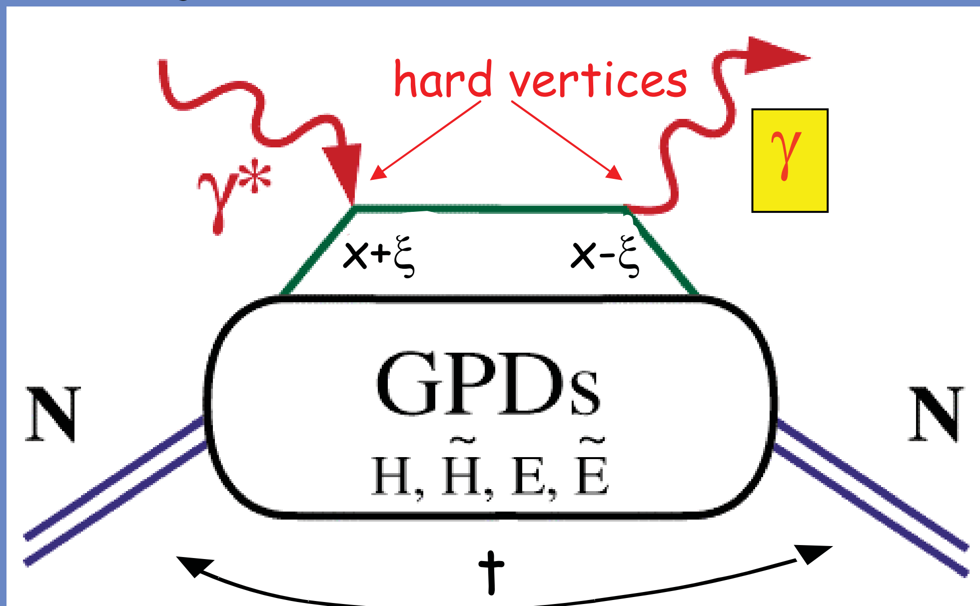
B. Clasie, et al , Jlab

PRL **99,** 242502 (2007)



GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)



x – quark momentum fraction

ξ– longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

H(x,ξ,t), E(x,ξ,t), . . . "Generalized Parton Distributions"

Quark angular momentum (Ji sum rule)

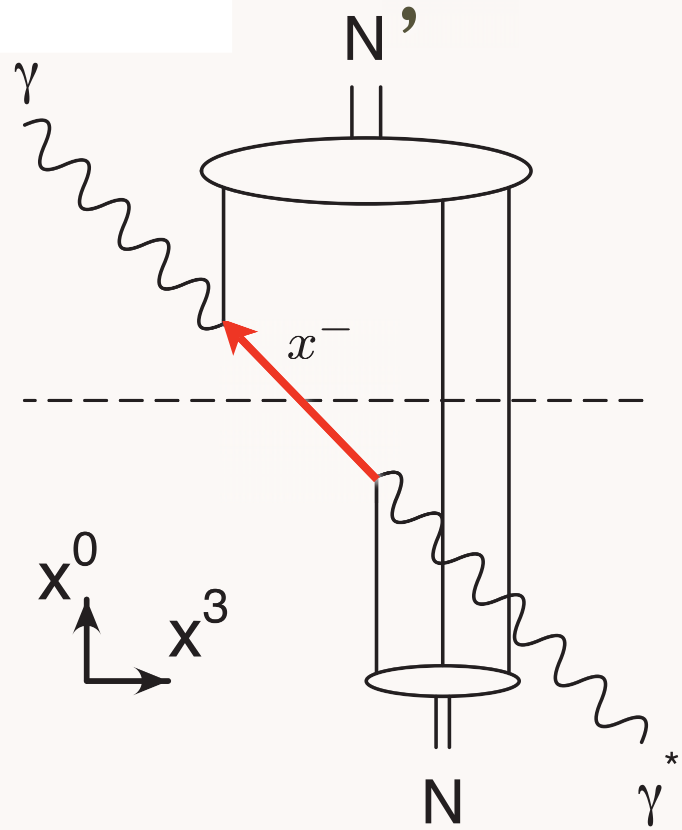
$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x dx \left[H^q(x,\xi,0) + E^q(x,\xi,0) \right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

Measure x- distribution from DVCS:
Take Fourier transform of skewness,
the longitudinal momentum transfer

$$\zeta = \frac{Q^2}{2p \cdot q}$$

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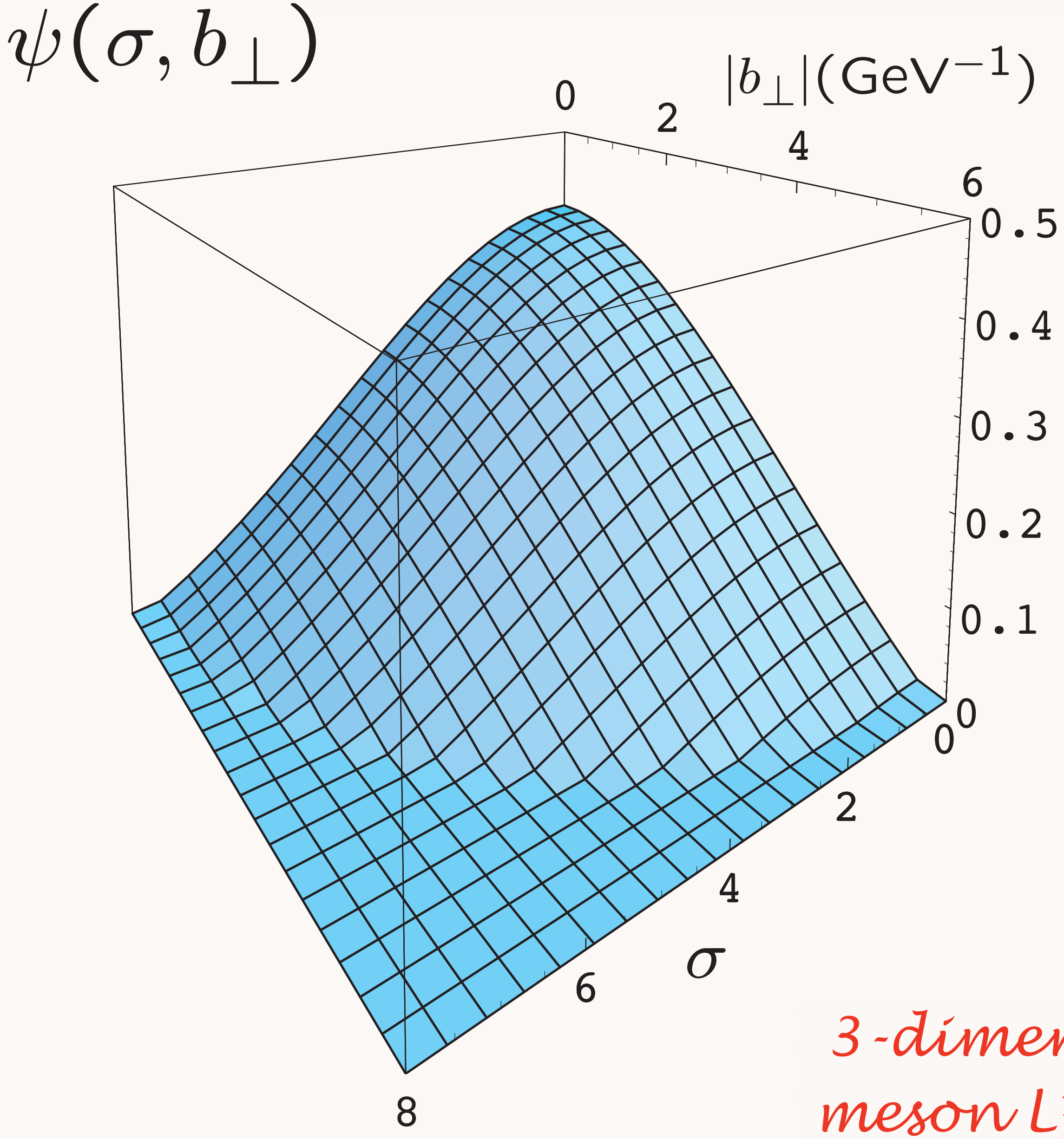
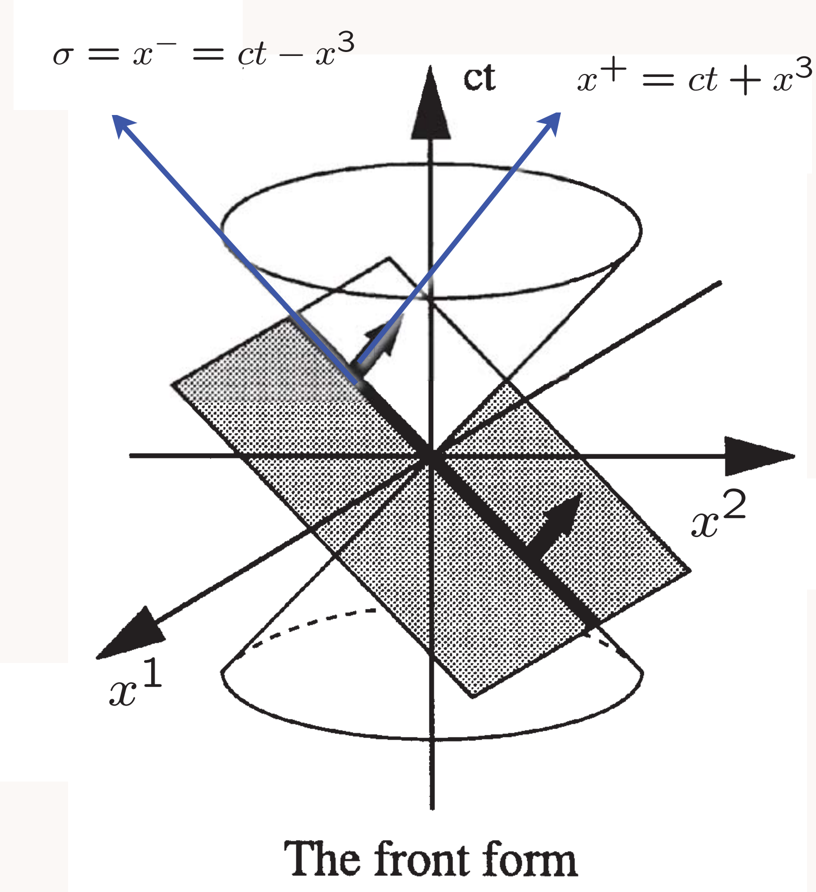
AdS/CFT Holographic Model

G. de Teramond
SJB





3-dimensional photograph: meson LFWF at fixed LF Time

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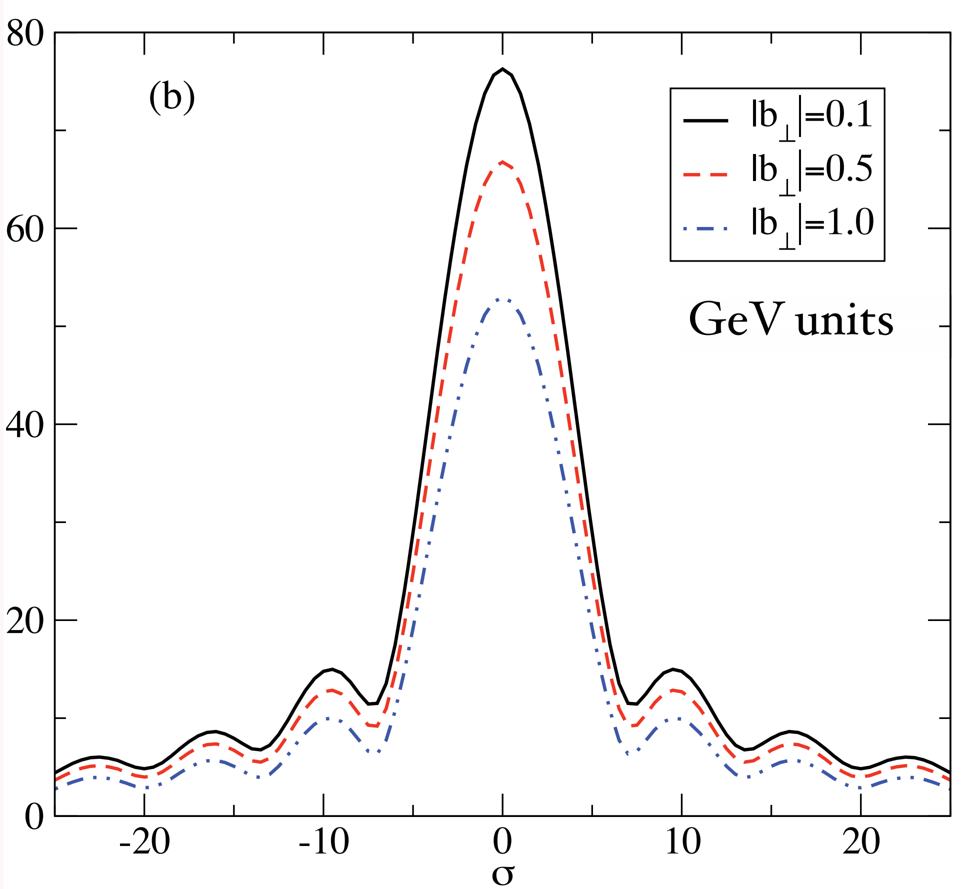
Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi} \int d\zeta e^{i\sigma\zeta} \tilde{A}(b_\perp, \zeta)$$

$\sigma = \frac{1}{2} x^- P^+ \qquad \zeta = \frac{Q^2}{2p\cdot q}$



The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

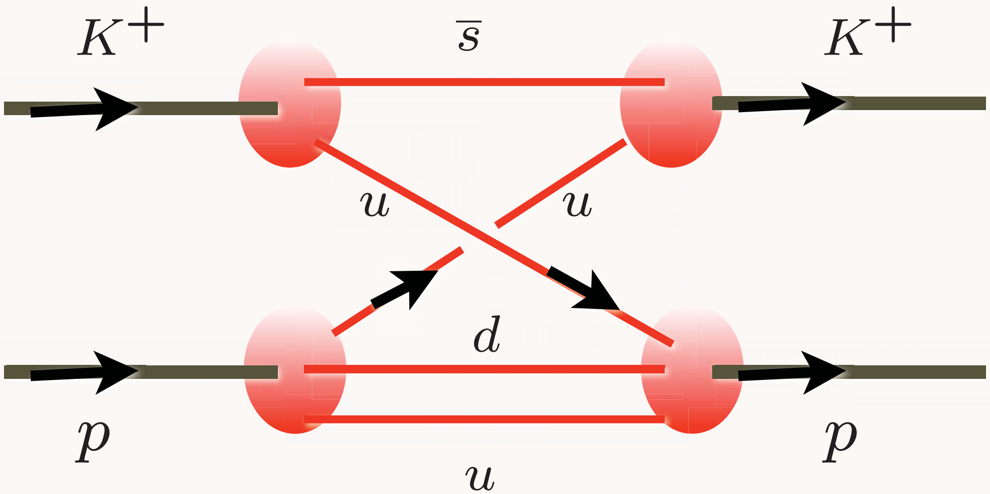
DVCS Amplitude using holographic QCD meson LFWF

$\Lambda_{QCD} = 0.32$

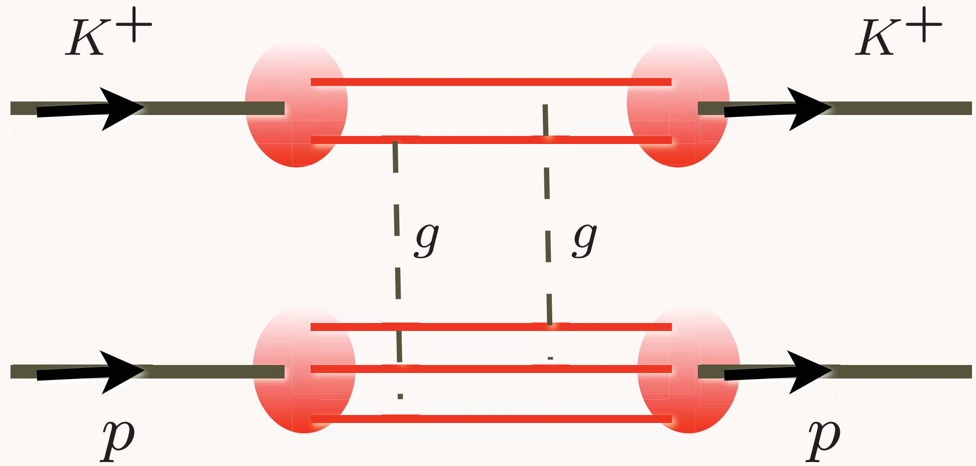
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)



Gluon Exchange
(Van der Waal -- Landshoff)

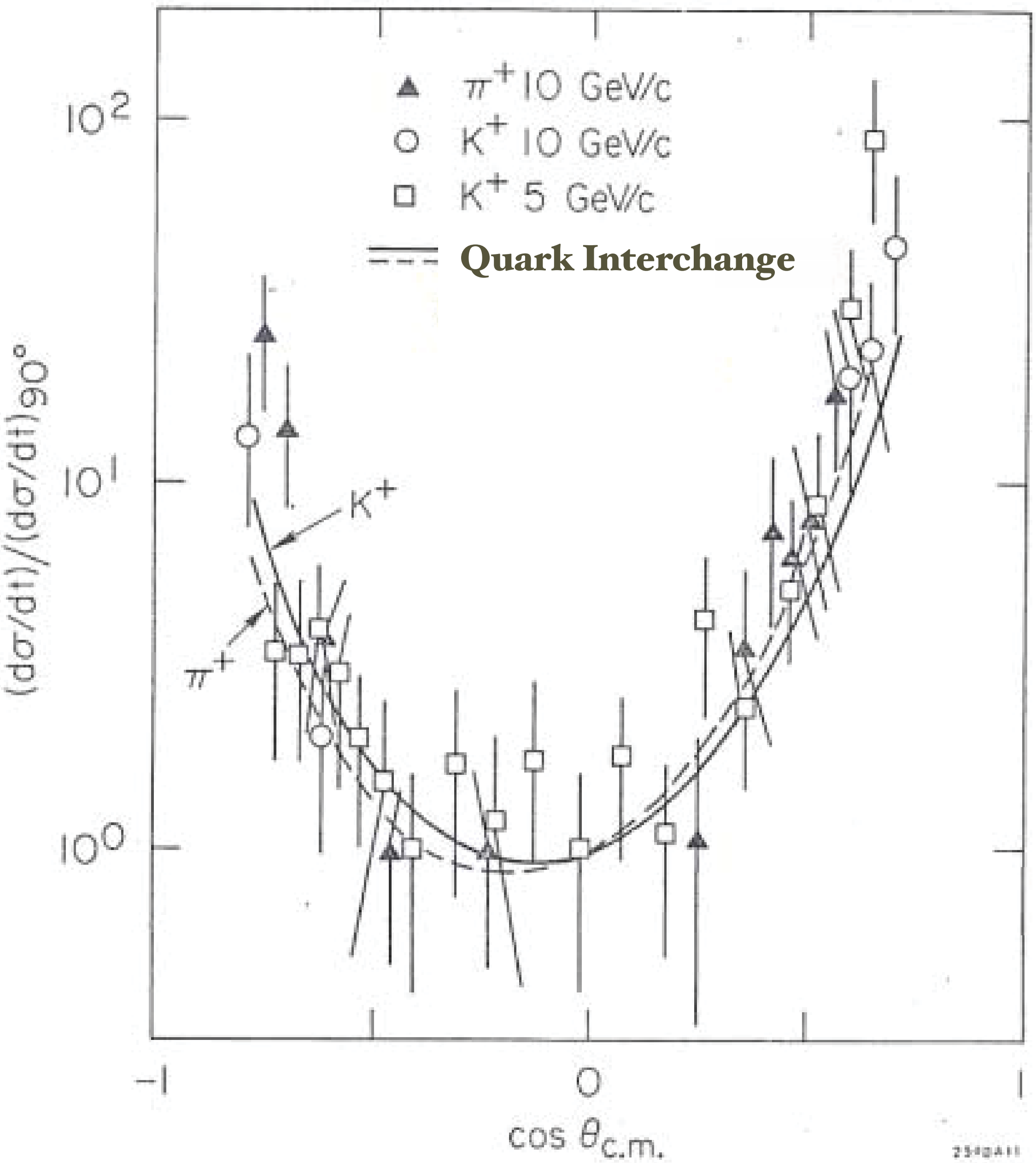
$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$M(s,t)_{\text{gluonexchange}} \propto sF(t)$$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:

AdS/QCD



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$