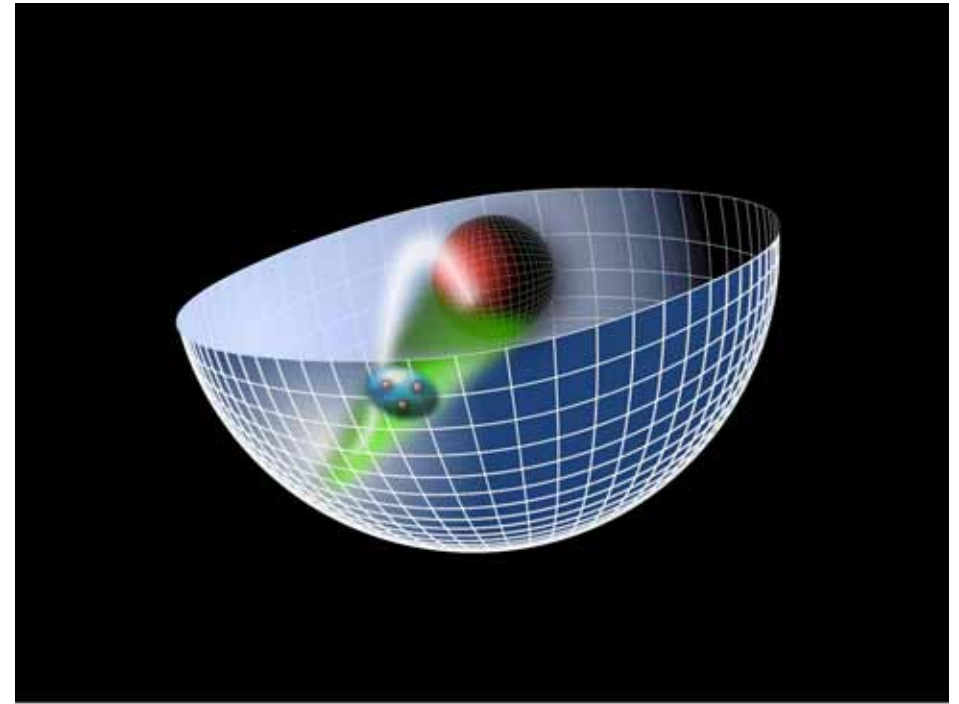
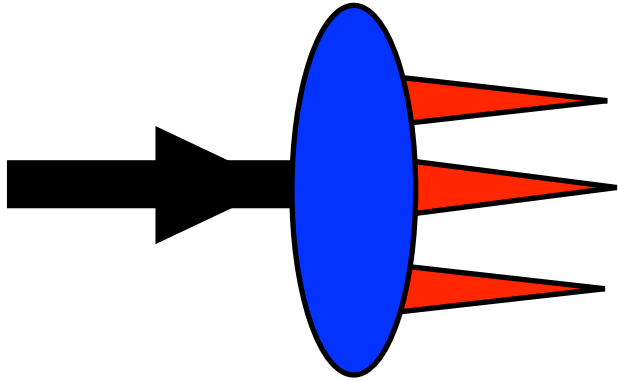


Applications of AdS/QCD and Light-Front Holography to Baryon Physics



Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY



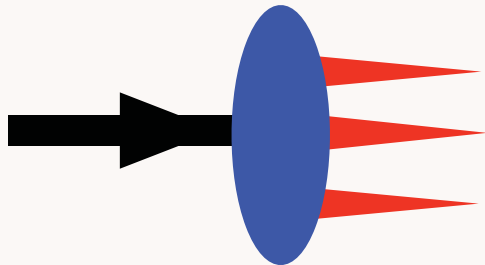
Baryon 2010
Osaka, December 7, 2010

Light-Front Holography and Non-Perturbative QCD

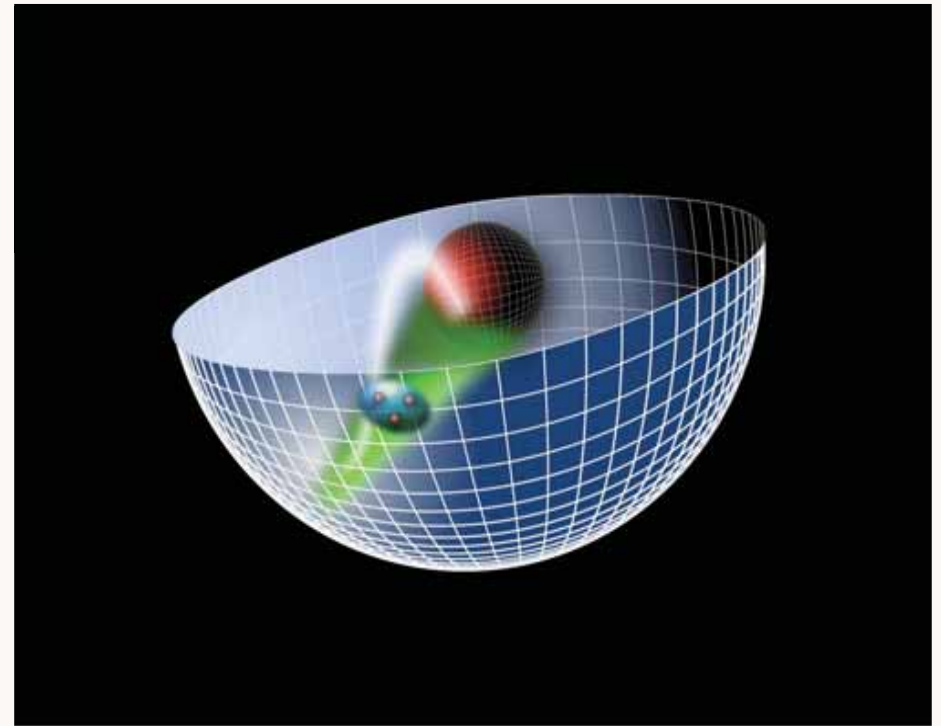
Goal:

Use AdS/QCD duality to construct
a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



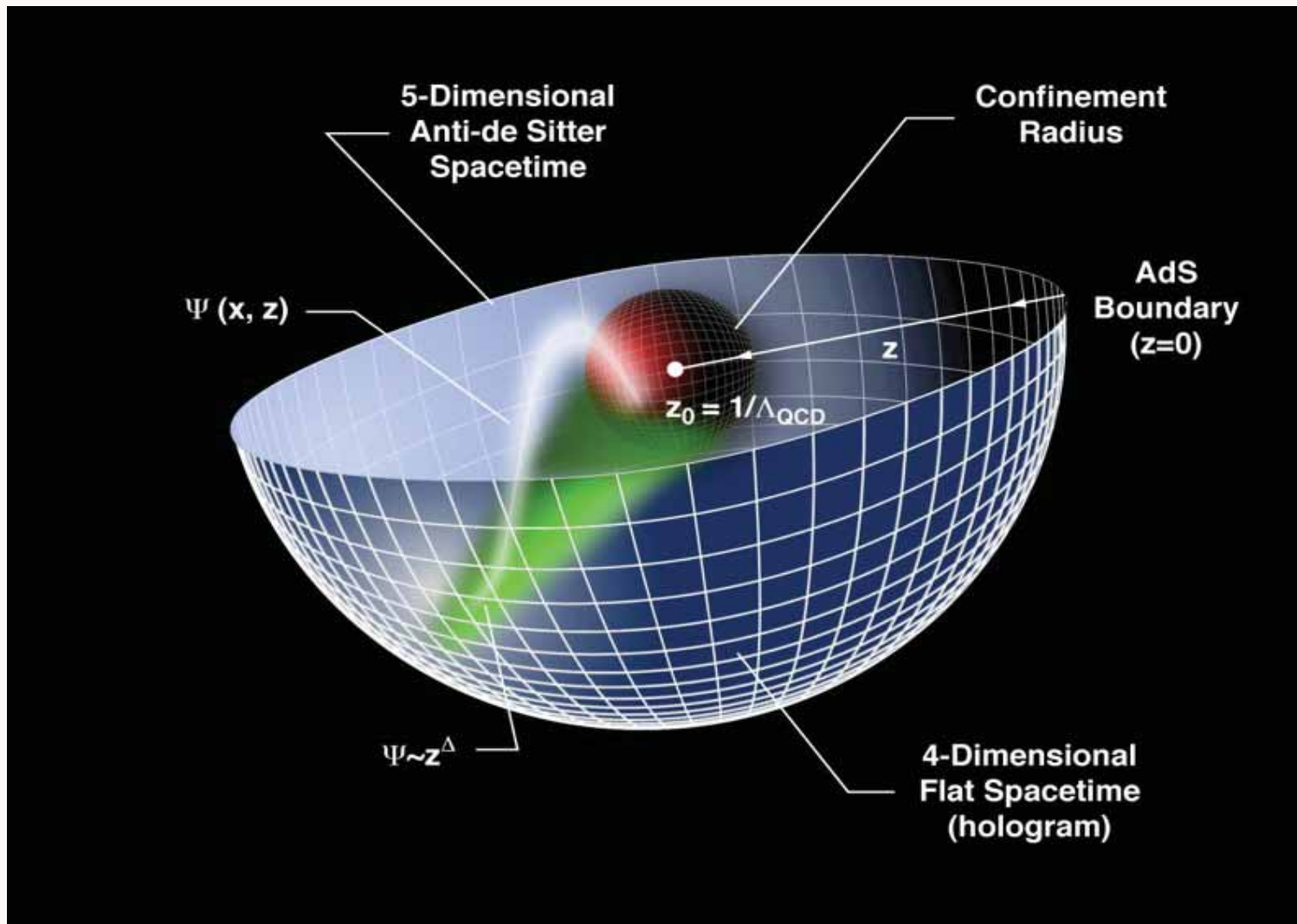
in collaboration with
Guy de Teramond

Central problem for strongly-coupled gauge theories

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Conformal Theories are invariant under the Poincare and conformal transformations with

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$

the generators of $SO(4,2)$

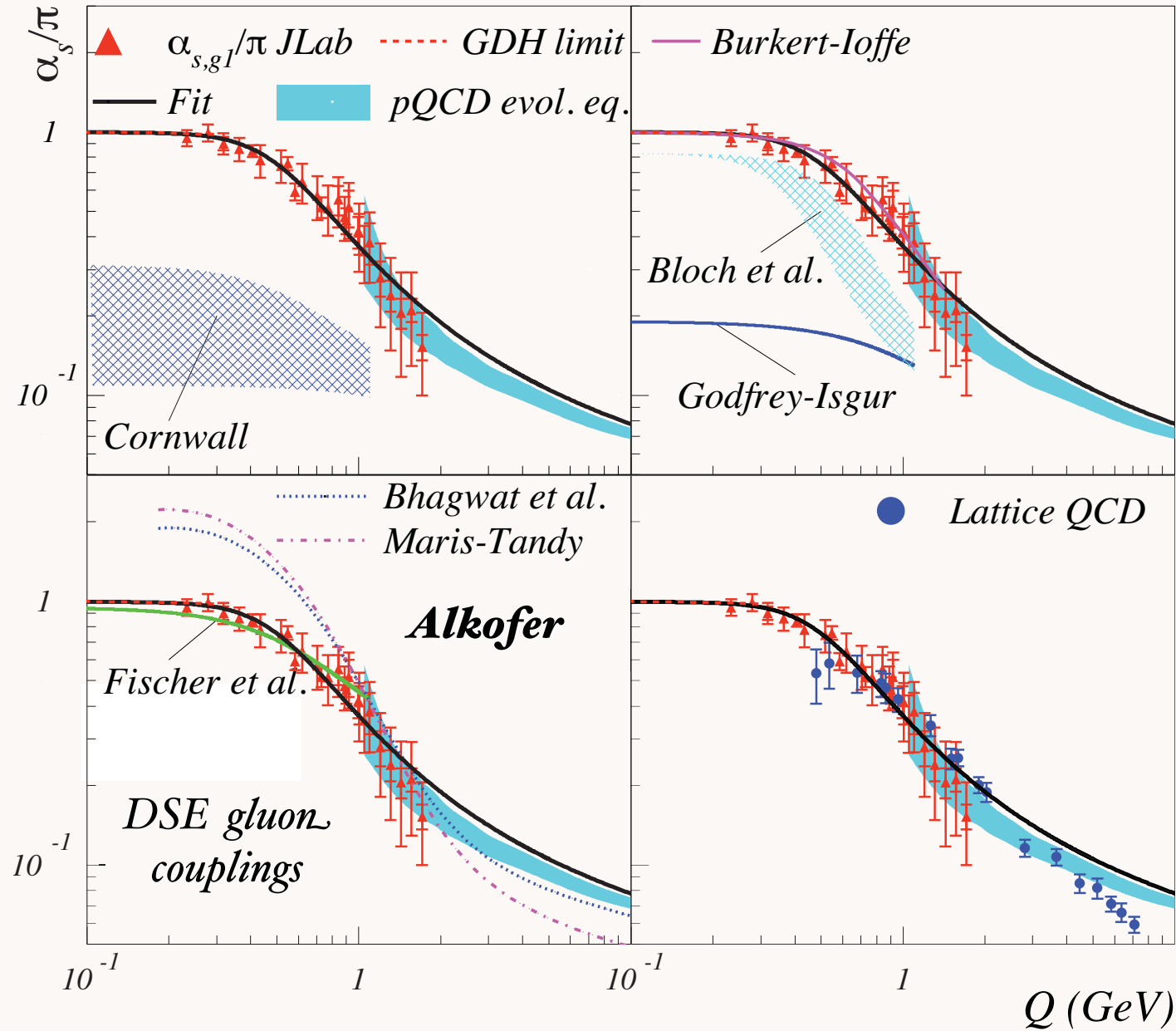
$SO(4,2)$ has a mathematical representation on AdS_5

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S^5$ to conformal $N=4$ SUSY

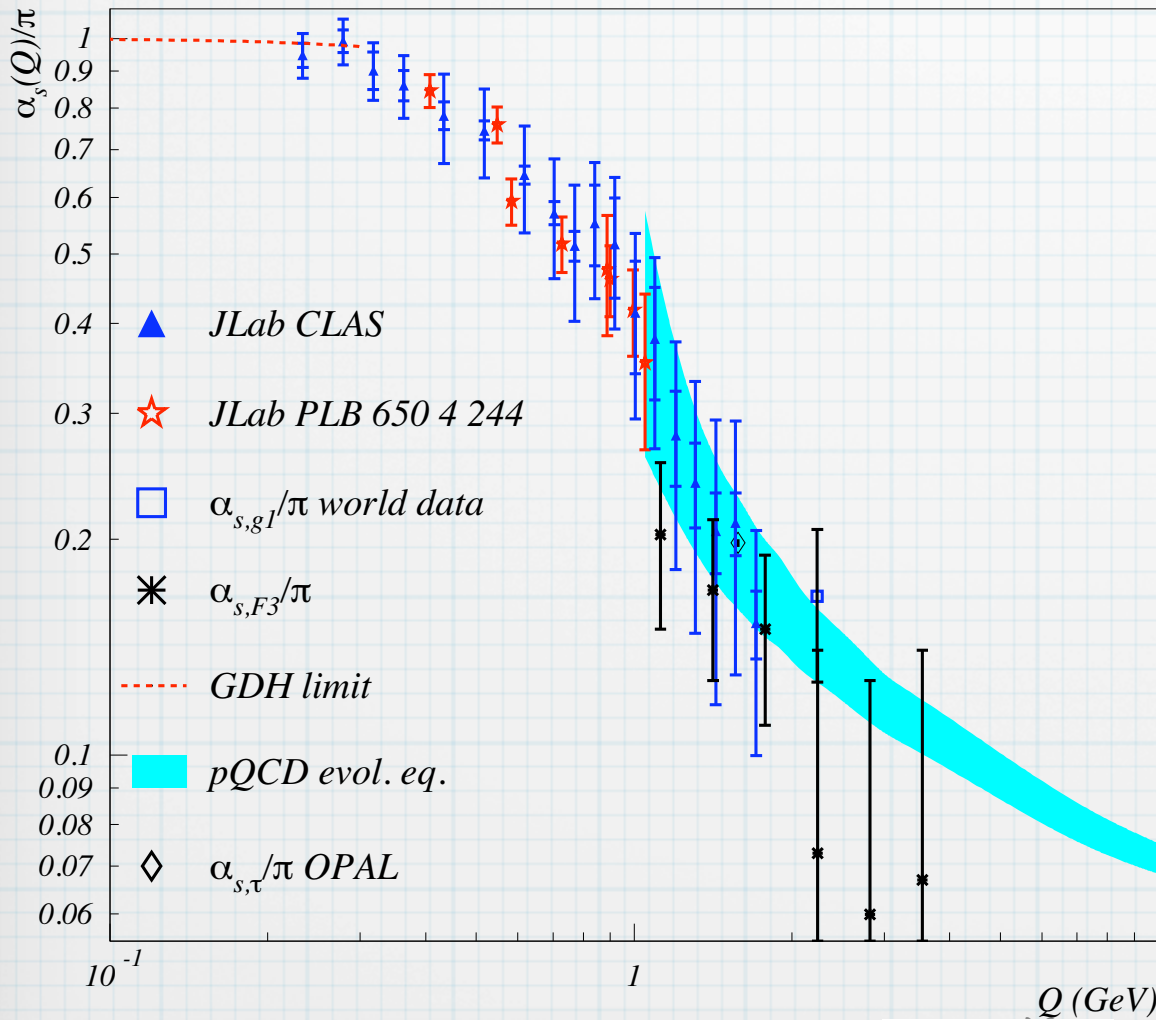
- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window**: $\alpha_s(Q^2) \simeq \text{const}$ at small Q^2
- Use mathematical mapping of the conformal group $SO(4,2)$ to AdS_5 space



Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left(g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi} \right)$$



g_1 = spin dependent structure function

Recent JLab data from E91(2008), CLAS, and Hall A

α_s runs only modestly at small Q^2

Gribov

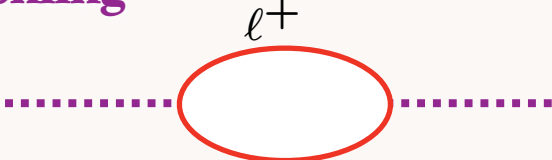
Fig. from 08034119, Duer et al.

Deur, de Teramond, sjb

IR Conformal Window for QCD

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**
- *Evidence from Lattice Gauge Theory* Furui, Nakajima
- *Stability of $\Upsilon \rightarrow ggg$* Shrock, sjb
- Define coupling from observable: **indications of IR fixed point for QCD effective charges** Deur, Chen, Burkert, Korsch,
- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$


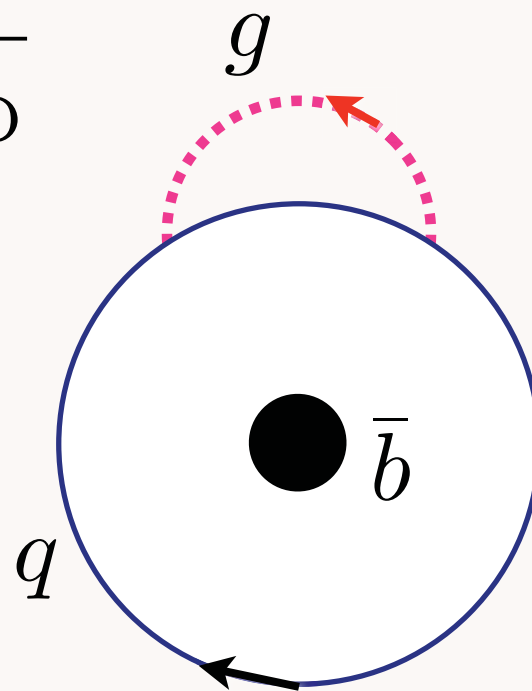
- **Justifies application of AdS/CFT in strong-coupling conformal window**

Confinement:

maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{\text{QCD}}}$$

$$\lambda < \Lambda_{\text{QCD}}$$



B-Meson

*gluon and quark propagators cutoff in IR
because of color confinement*

R. Shrock, sjb

Maximal Wavelength of Confined Fields

$$(x - y)^2 < \Lambda_{QCD}^{-2}$$

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

Shrock, sjb

*Quark and Gluon vacuum polarization insertions
decouple: IR fixed Point*

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

**Baryon 2010
December 7, 2010**

Applications of Light-Front Holography


II

**Stan Brodsky
SLAC**

Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

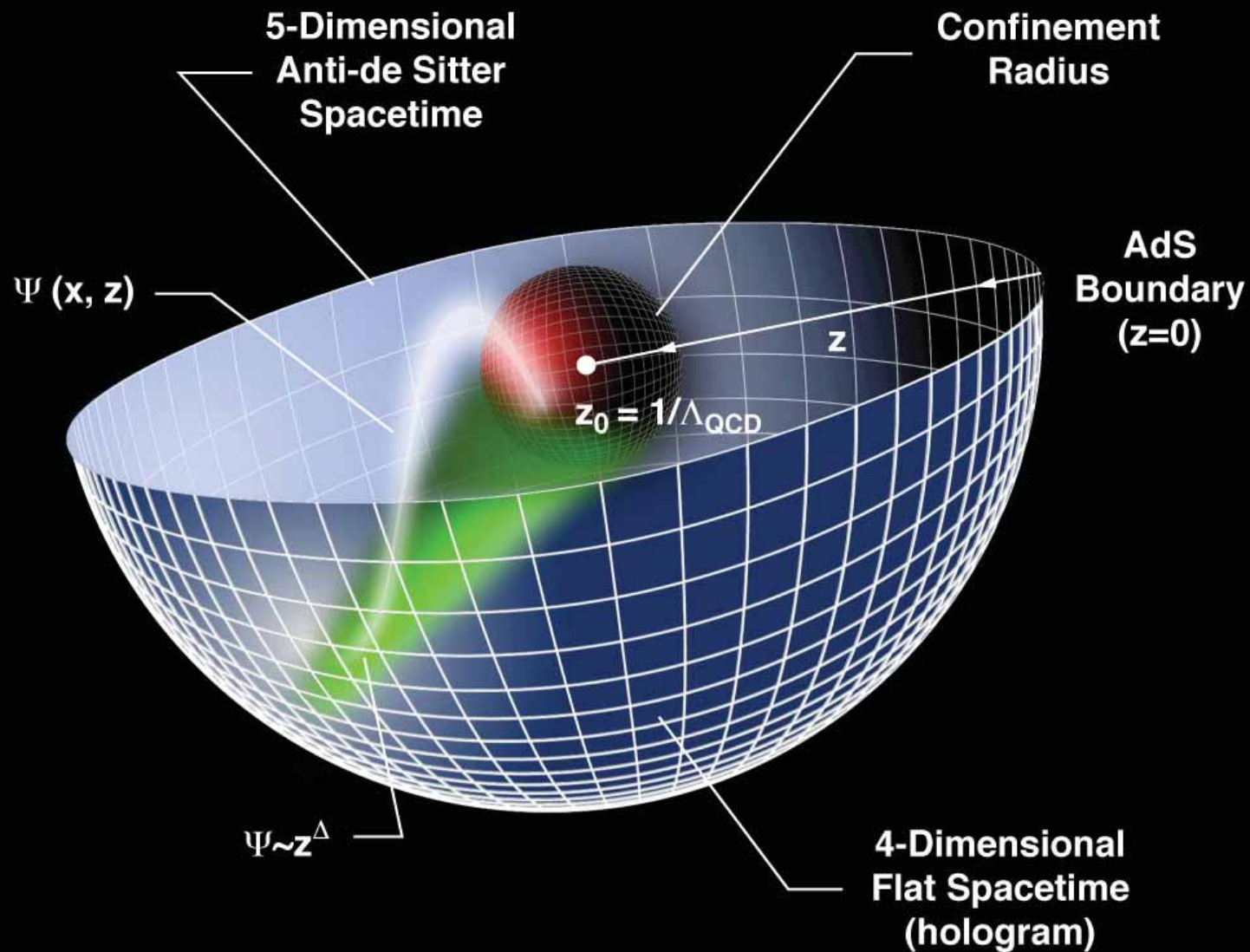
$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

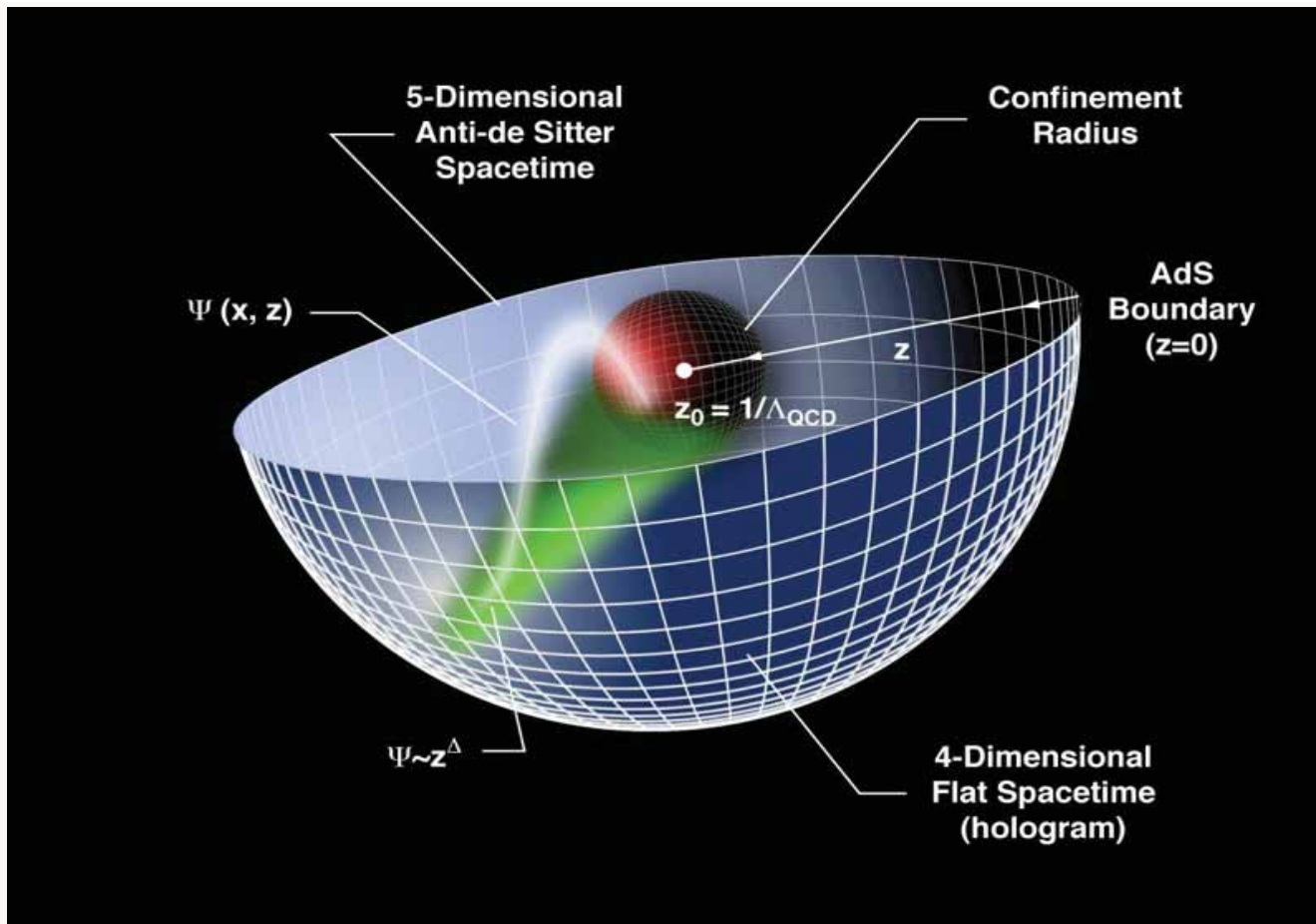
$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



8-2007
8685A14



8-2007
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- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) [Polchinski and Strassler \(2001\)](#).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) [Karch, Katz, Son and Stephanov \(2006\)](#).

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.

- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates, $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2} \phi(z)$$

*AdS Schrodinger Equation for bound state
of two scalar constituents:*

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

**L: light-front orbital angular
momentum**

Derived from variation of Action in AdS₅

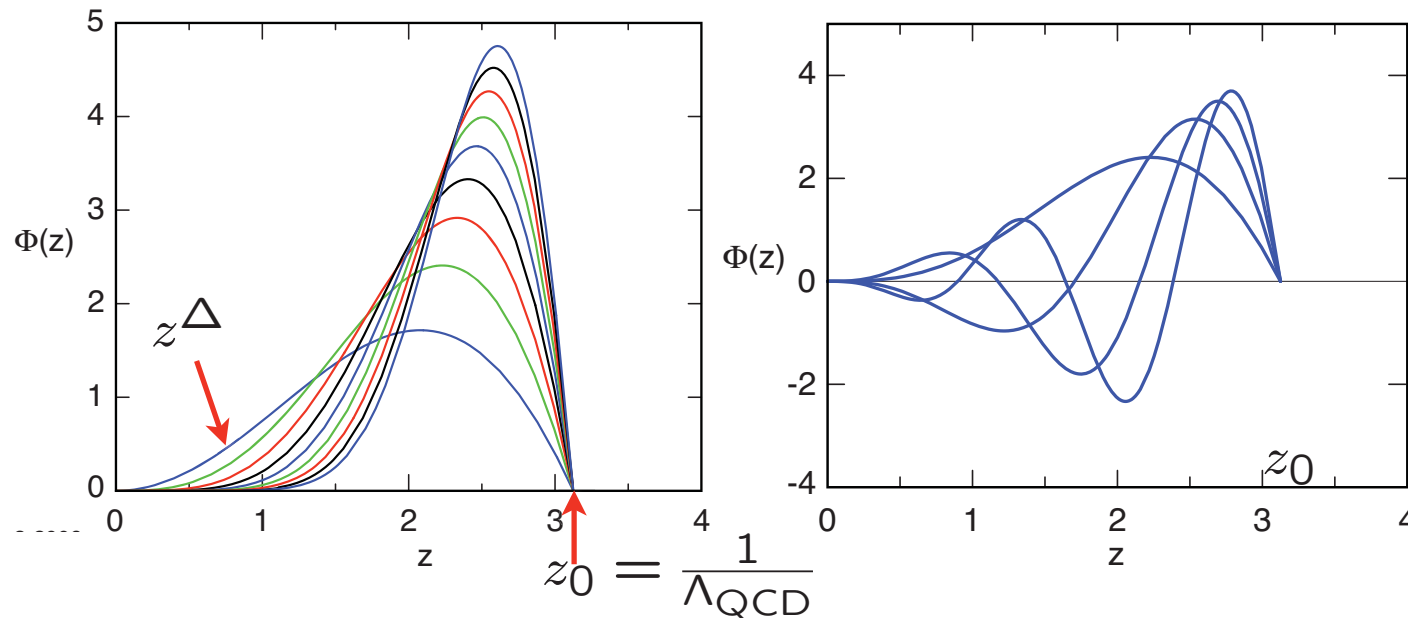
Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

***Match fall-off at small z to conformal twist-dimension
at short distances***

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1 \dots \ell_m\}} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4- d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

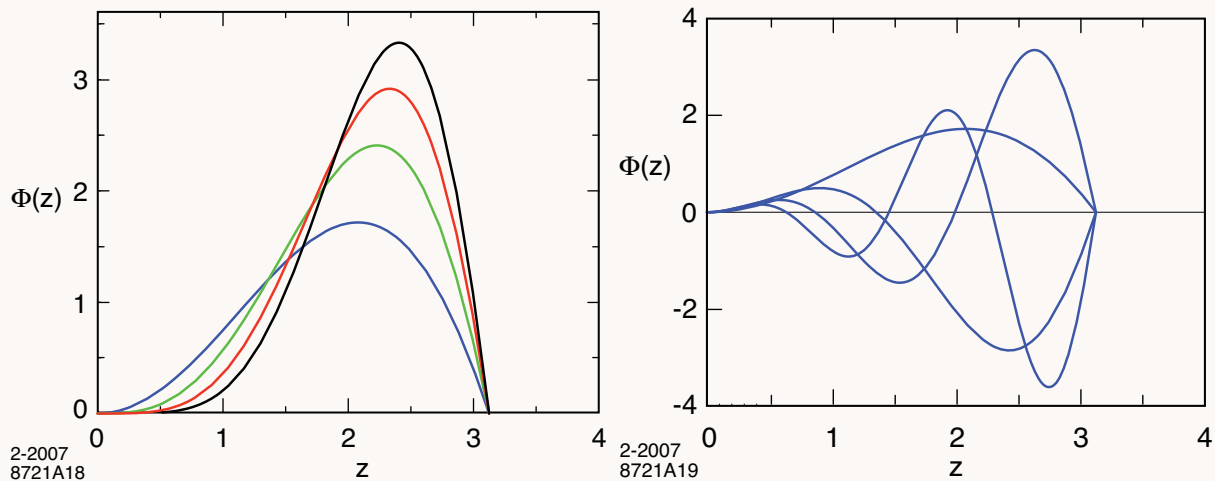


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV .

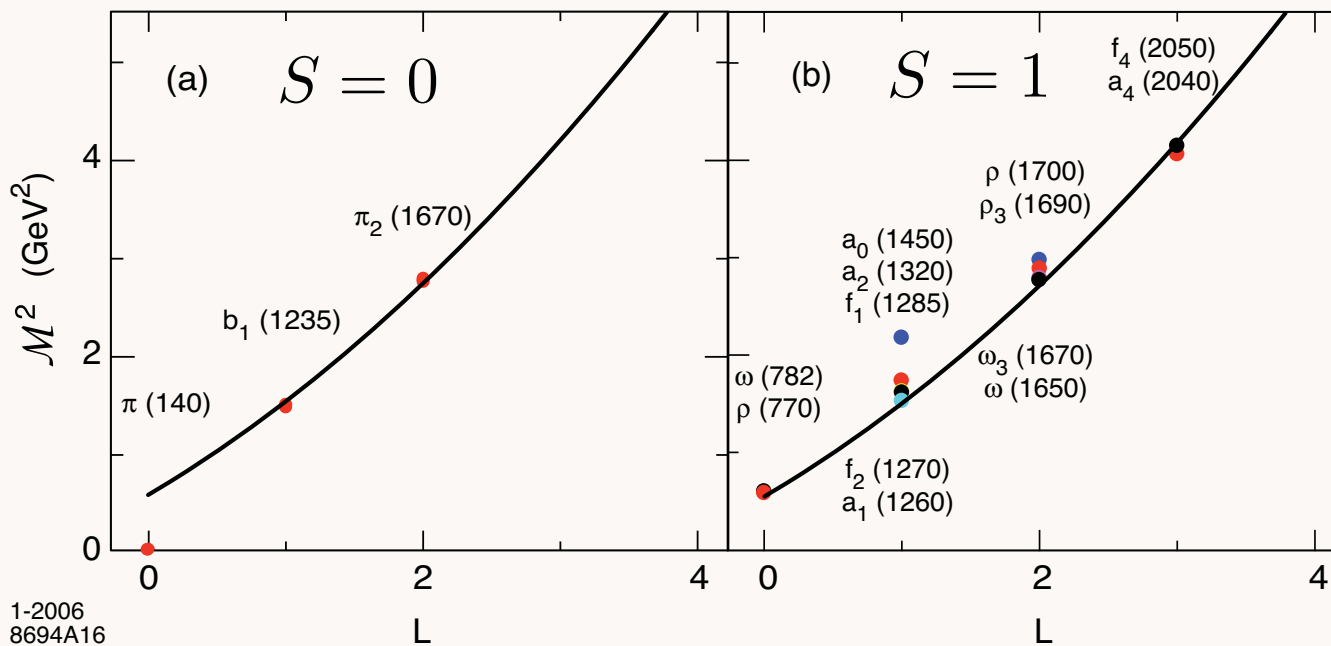


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Soft-Wall Model

$$S = \int d^4x dz \sqrt{g} e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2$$

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion!

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action
Dilaton-Modified AdS₅*

Quark separation increases with L

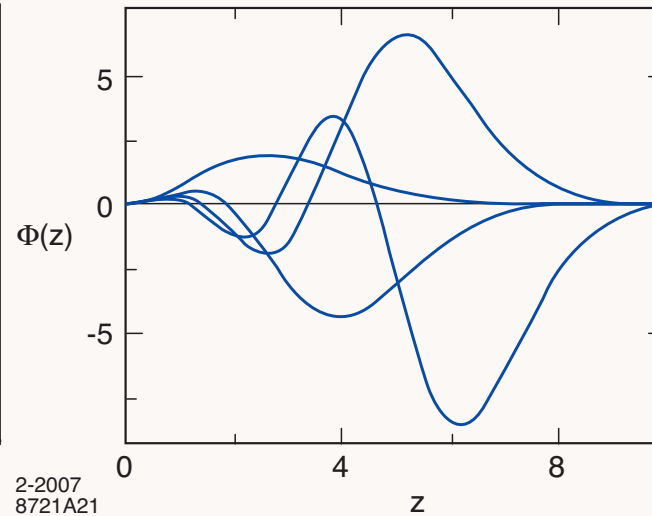
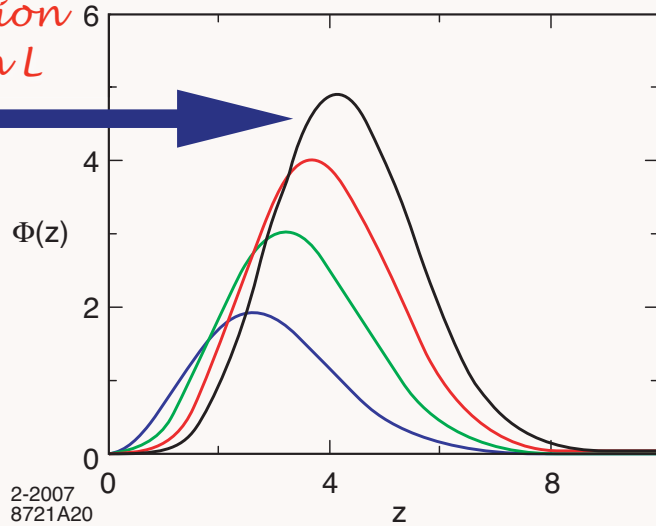
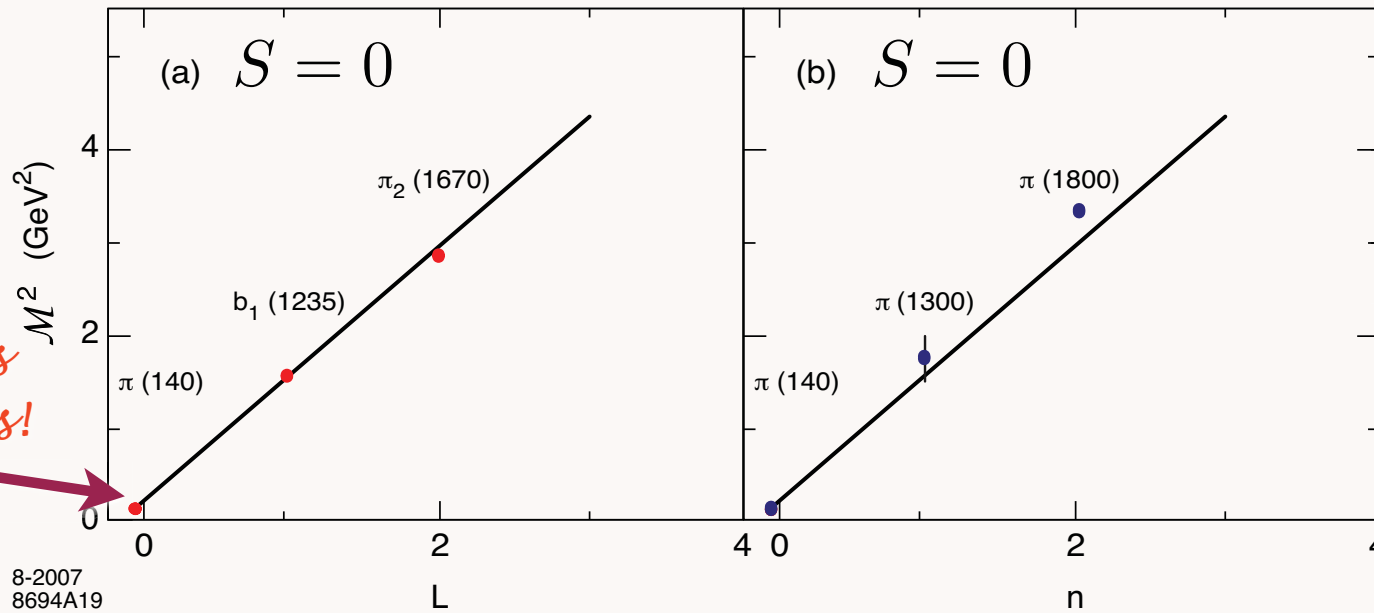


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model

Pion mass automatically zero!

$$m_q = 0$$

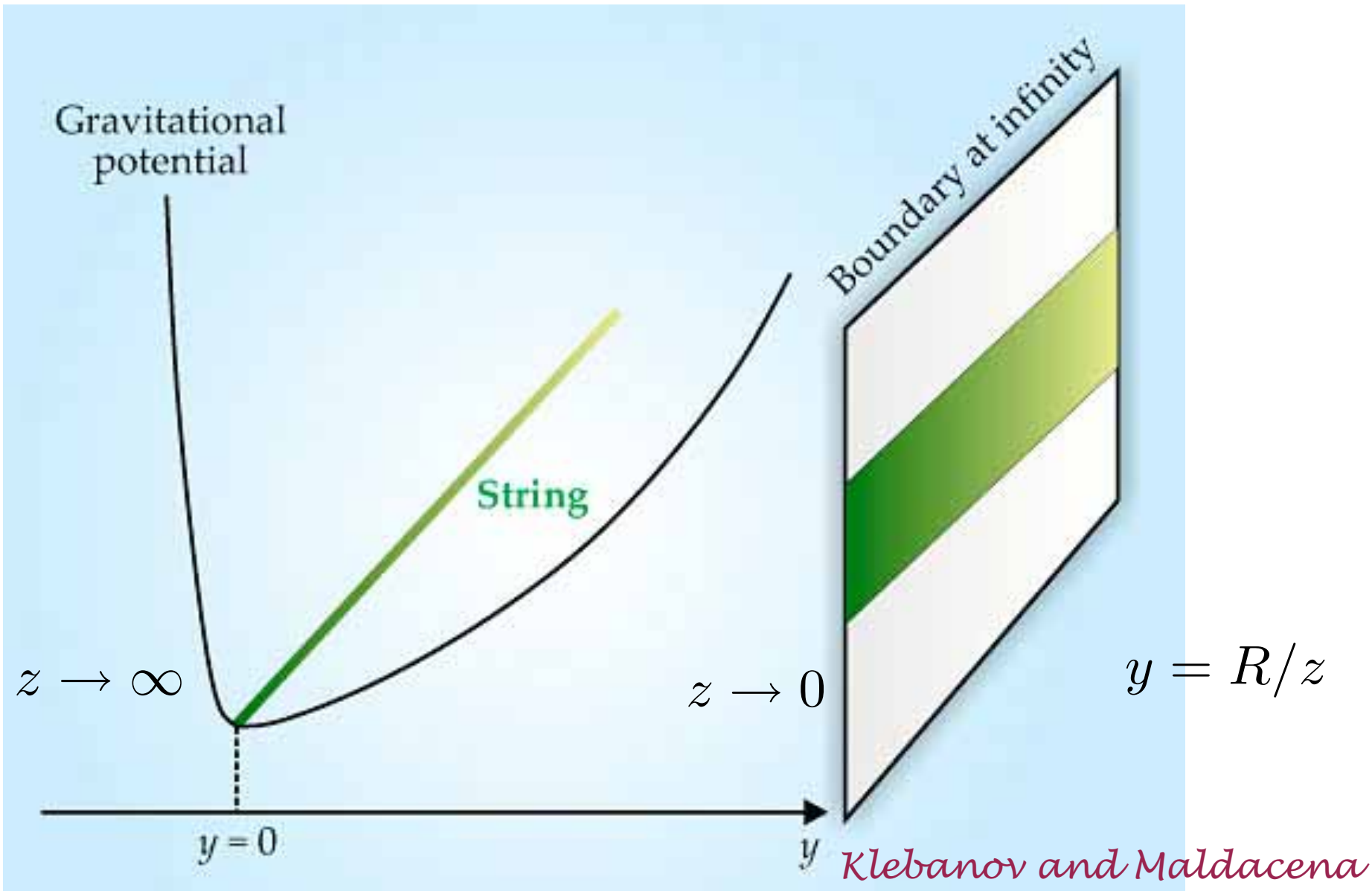


Pion has zero mass!



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2)$$



$$ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2) + dy^2$$

- Nonconformal metric dual to a confining gauge theory

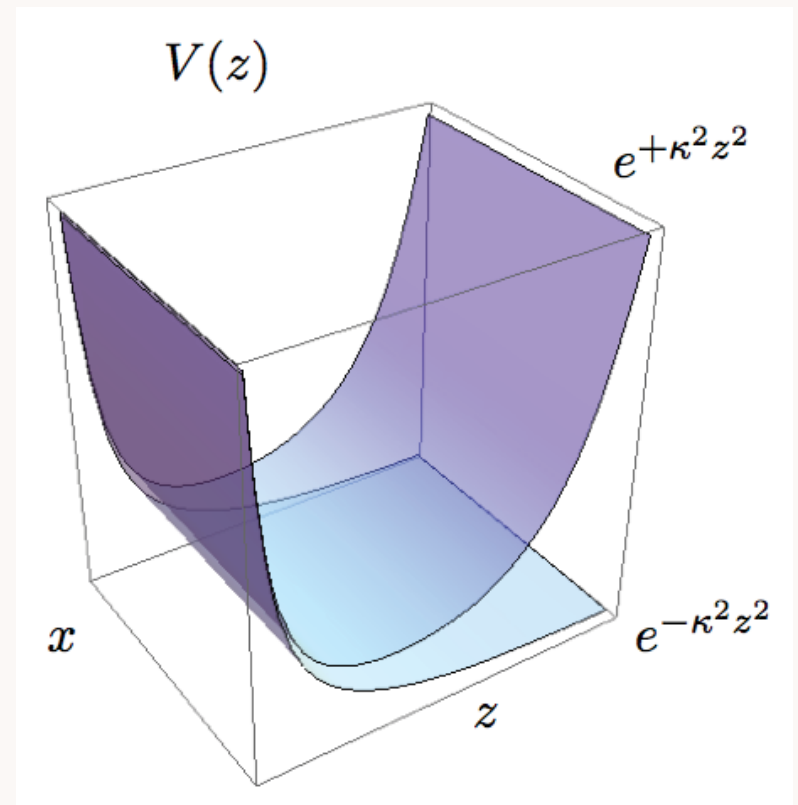
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS₅

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Higher-Spin Hadrons

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

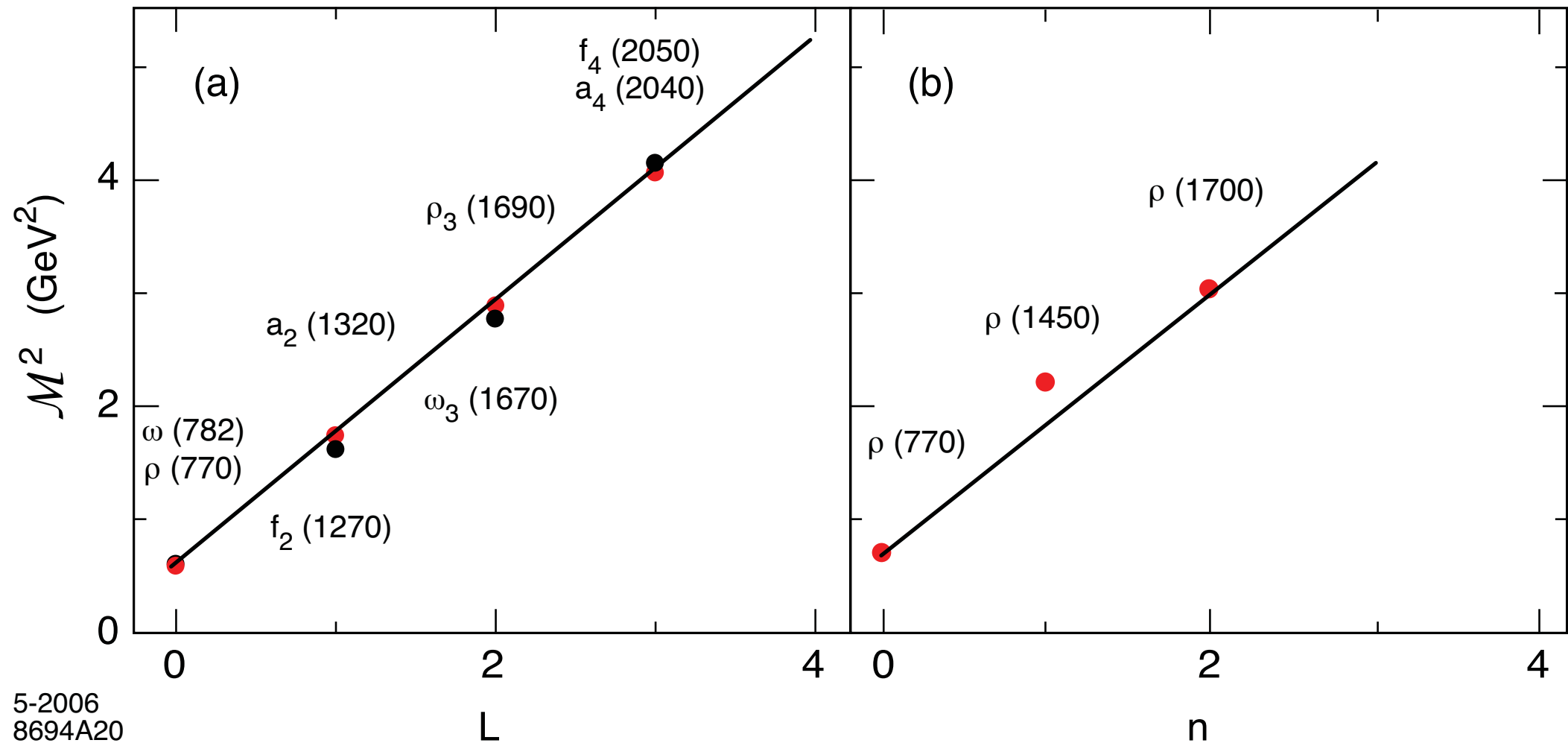
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$

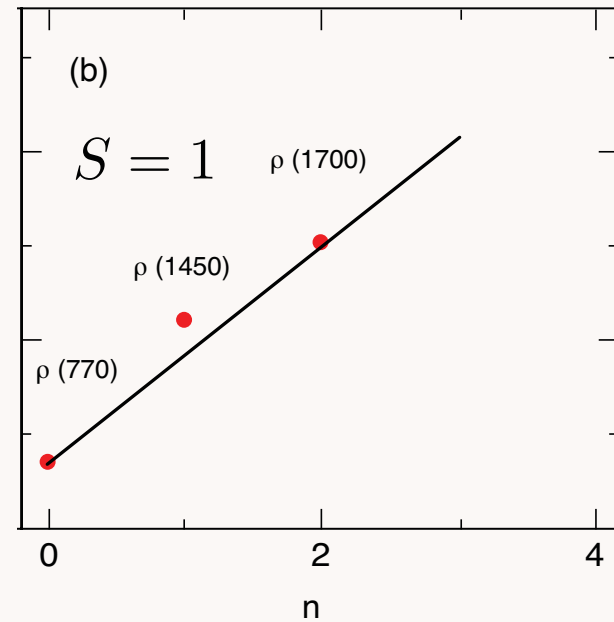
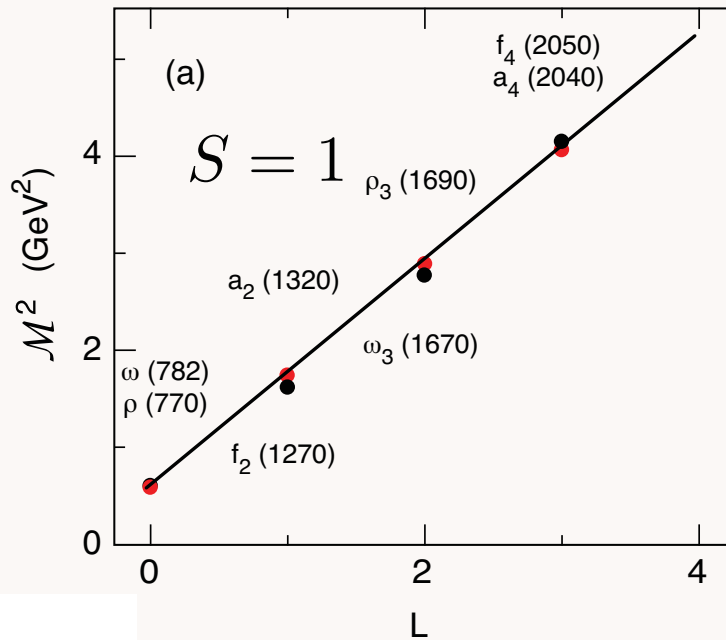
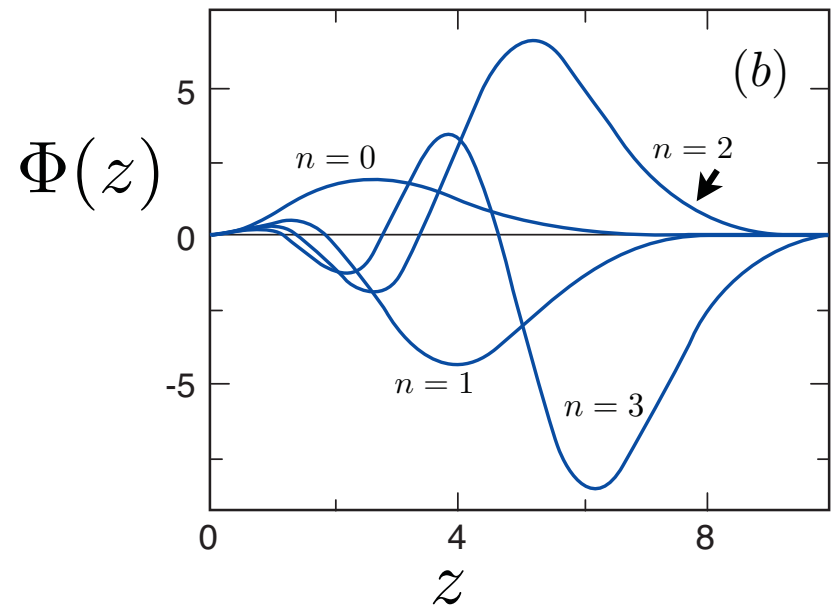
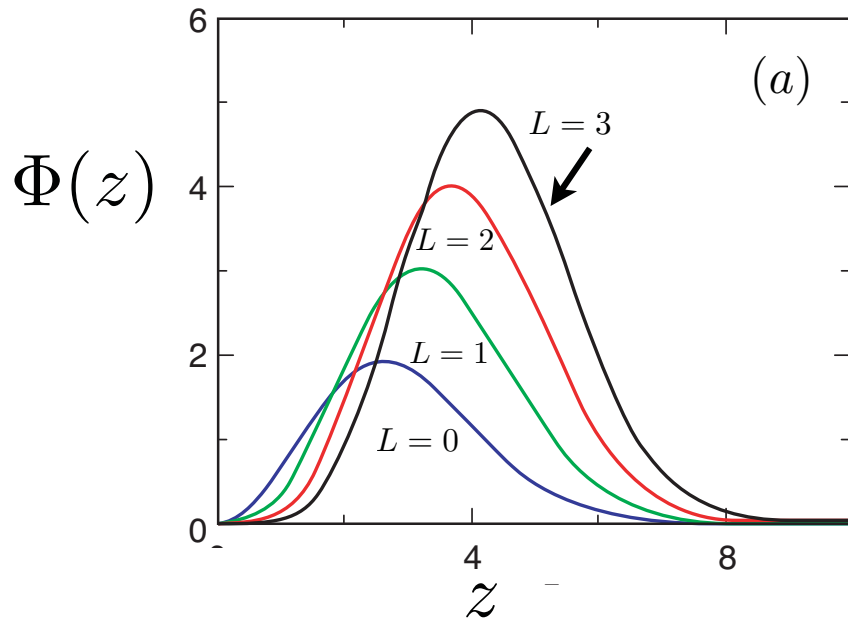


with $(\mu R)^2 = -(2 - J)^2 + L^2$

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$

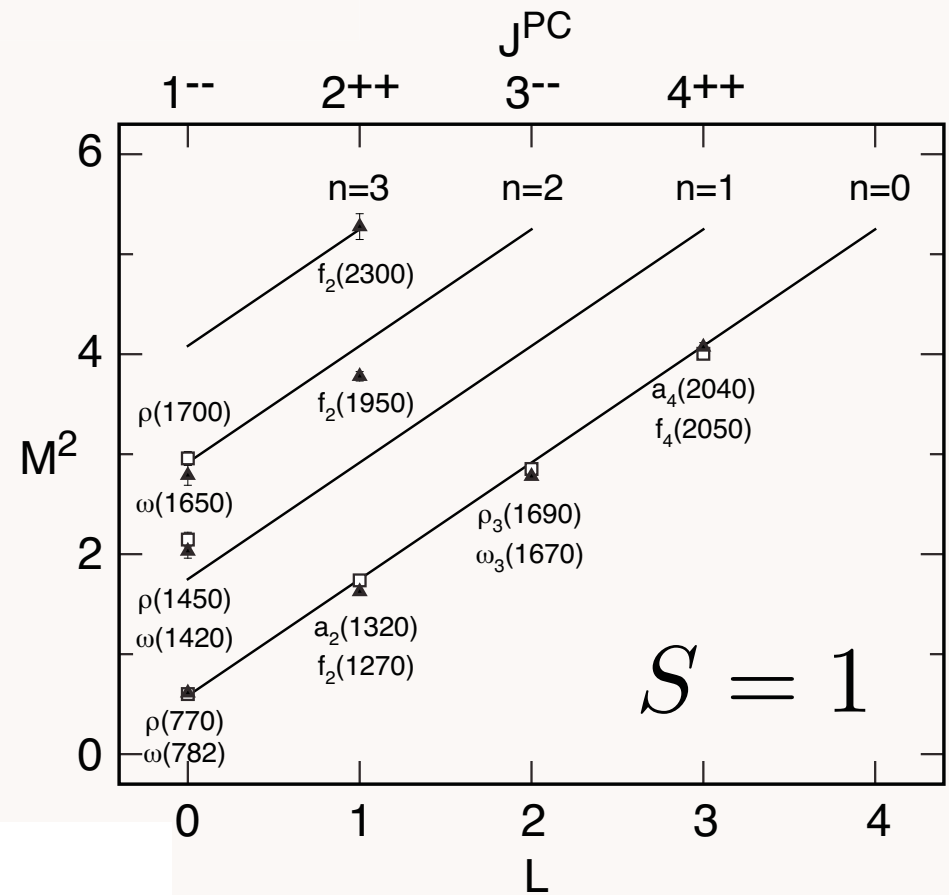
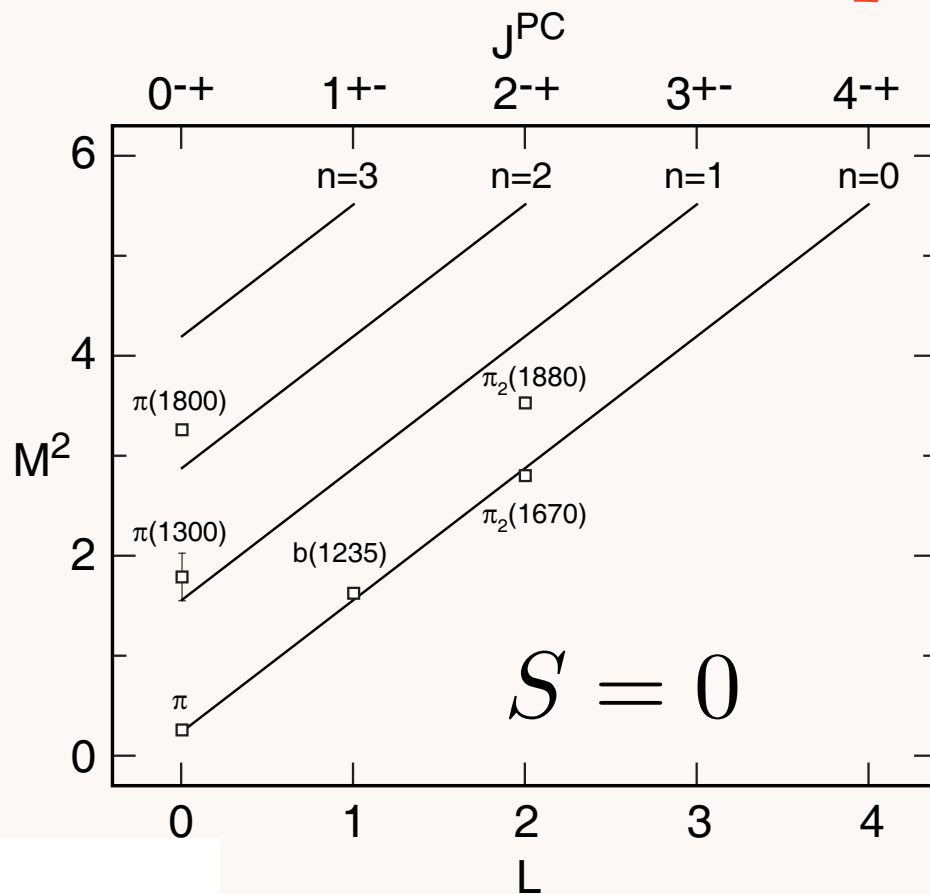




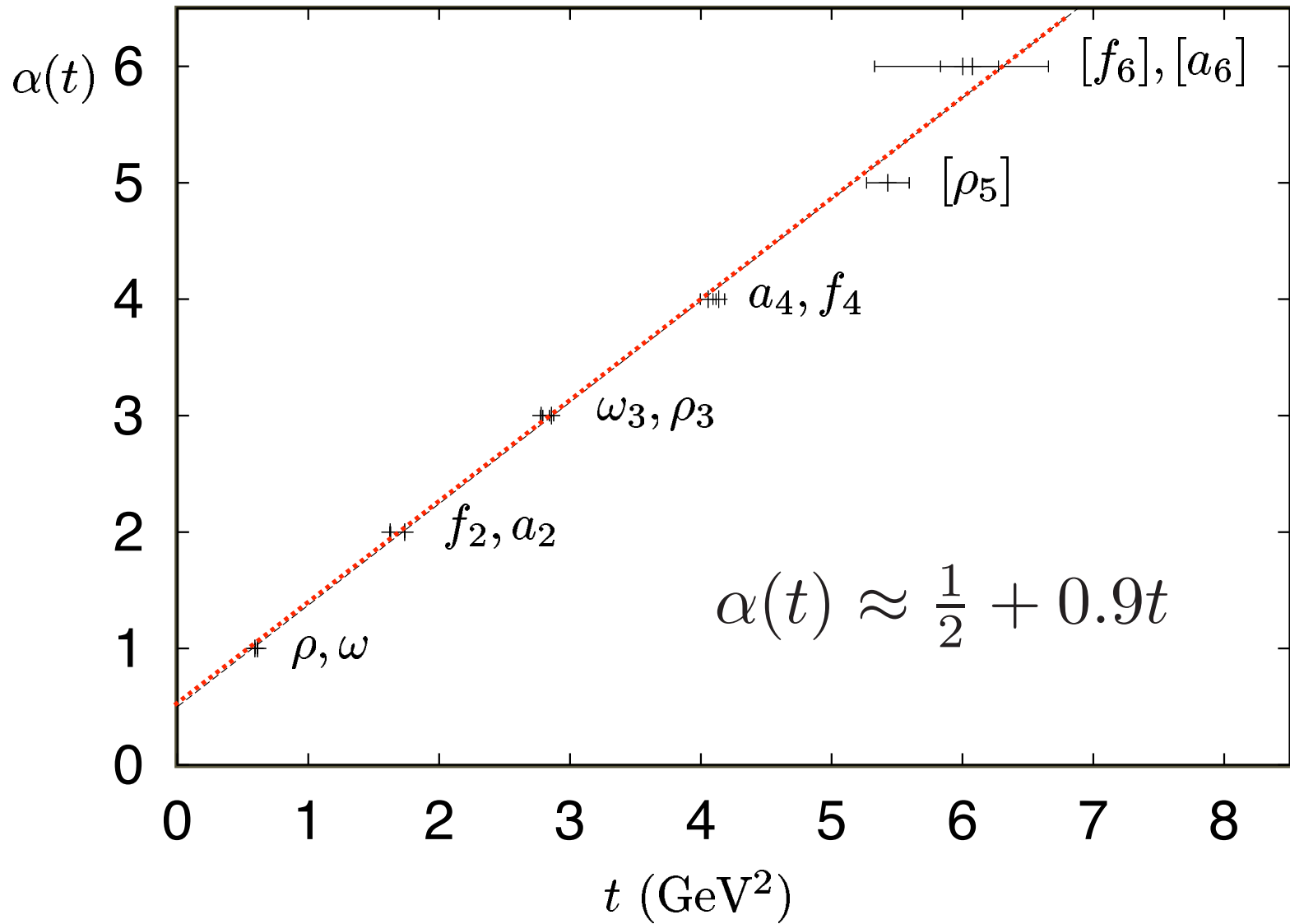
Bosonic Modes and Meson Spectrum $M^2 = 4\kappa^2(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
 $4\kappa^2$ for $\Delta L = 1$
 $2\kappa^2$ for $\Delta S = 1$

Same slope in n and L

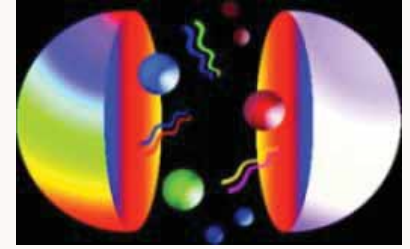


Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I=1$ ρ -meson and $I=0$ ω -meson families ($\kappa = 0.54$ GeV)



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

- Baryons Spectrum in "bottom-up" holographic QCD
GdT and Brodsky: hep-th/0409074, hep-th/0501022.



From Nick Evans

Baryons in AdS/CFT

- Action for massive fermionic modes on AdS₅:

$$S[\bar{\Psi}, \Psi] = \int d^4x dz \sqrt{g} \bar{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion: $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution ($\mu R = \nu + 1/2$)

$$\Psi(z) = C z^{5/2} [J_\nu(z\mathcal{M})u_+ + J_{\nu+1}(z\mathcal{M})u_-]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm(z = 1/\Lambda_{\text{QCD}}) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}$$

with scale independent mass ratio

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin- $\frac{1}{2}$ modes $\psi(\zeta)$ and spin- $\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are **two-component spinor** solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$

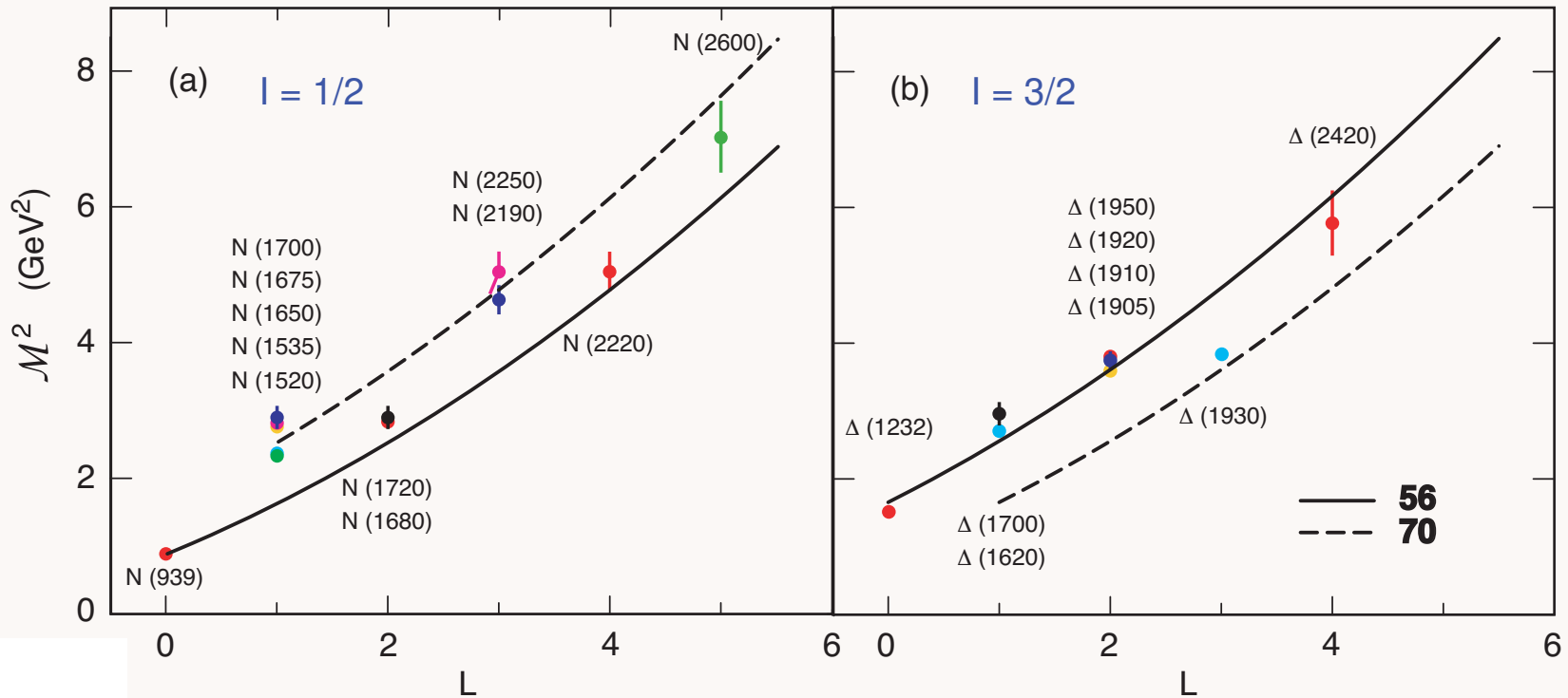
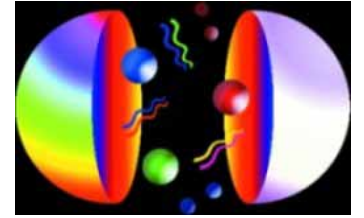


Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)



From Nick Evans

- Action for Dirac field in AdS_{d+1} in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g} e^{\varphi} (i \bar{\Psi} e_A^M \Gamma^A D_M \Psi + h.c. + \varphi(z) \bar{\Psi} \Psi - \mu \bar{\Psi} \Psi)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP \cdot x} \Psi(z)$

$$\left[i \left(z \eta^{\ell m} \Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution ($\nu = \mu R - \frac{1}{2}$, $\nu = L + 1$)

$$\Psi_+(z) \sim z^{\frac{5}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

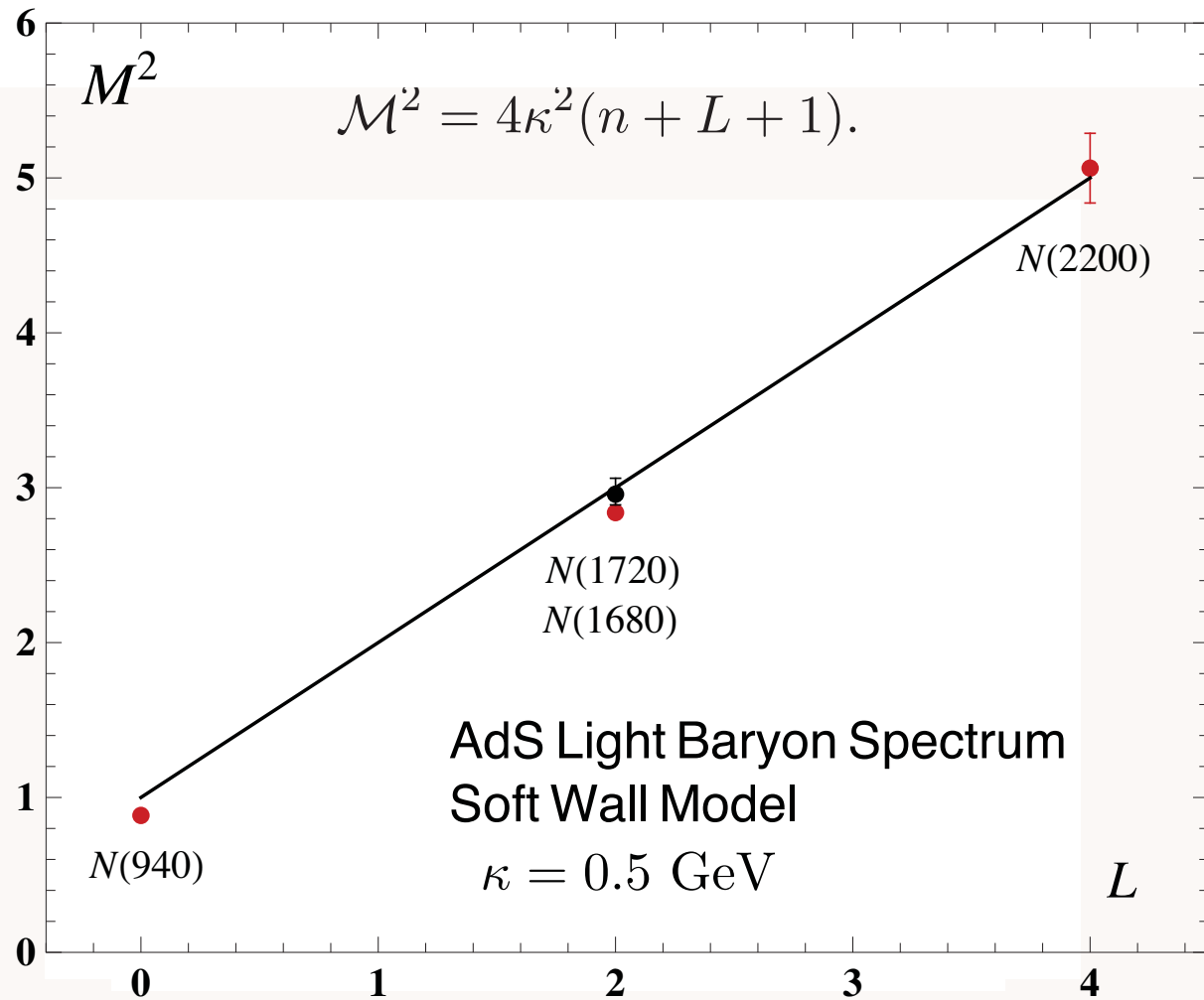
$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

- Large N_C : $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C} \Lambda_{\text{QCD}}$

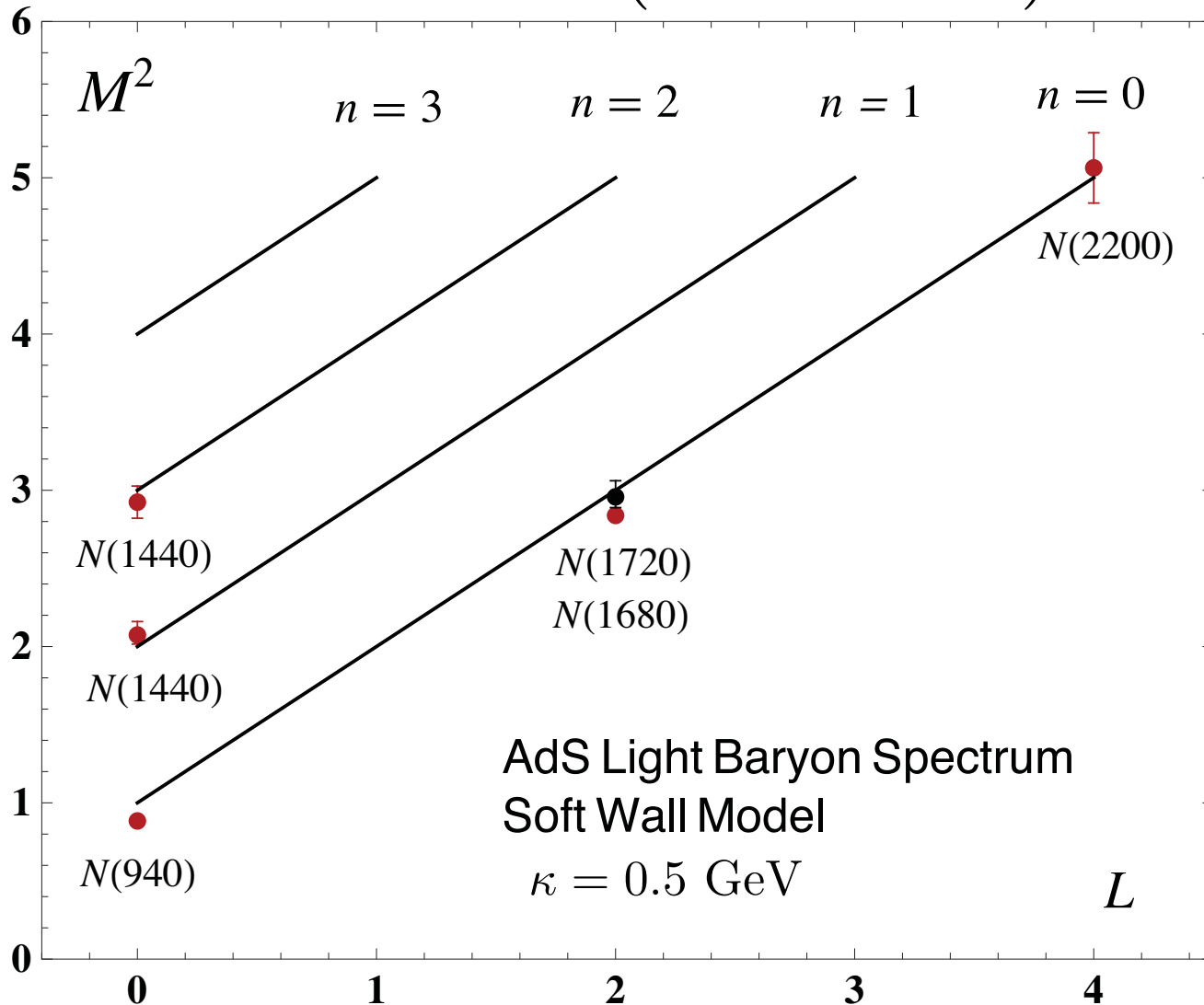
- Baryon: twist-dimension $3 + L$ ($\nu = L + 1$)

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$



Positive Parity Nucleons

$$M^2 = 4\kappa^2(n + L + 1)$$

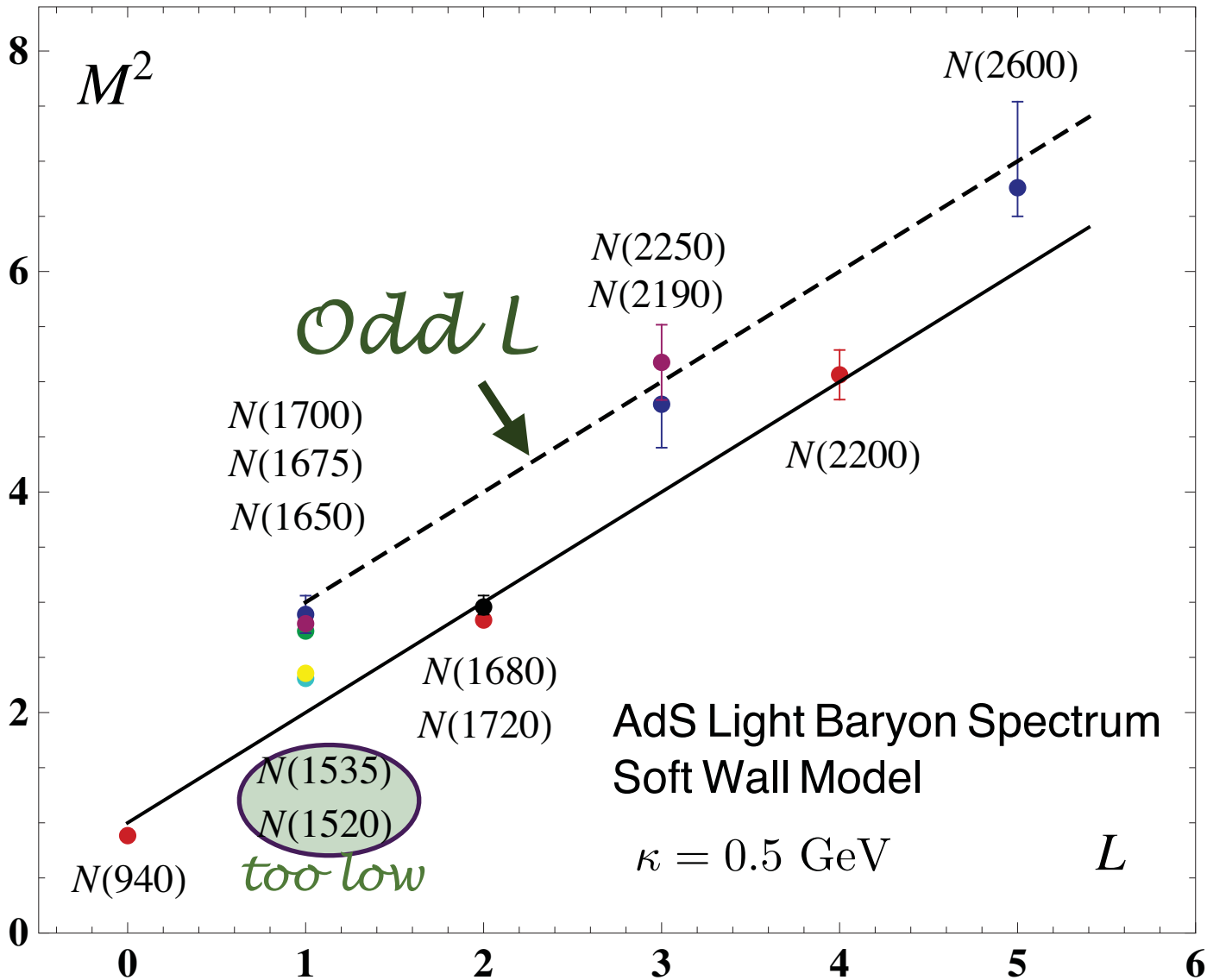


Positive Parity Nucleons

Negative Parity Nucleons

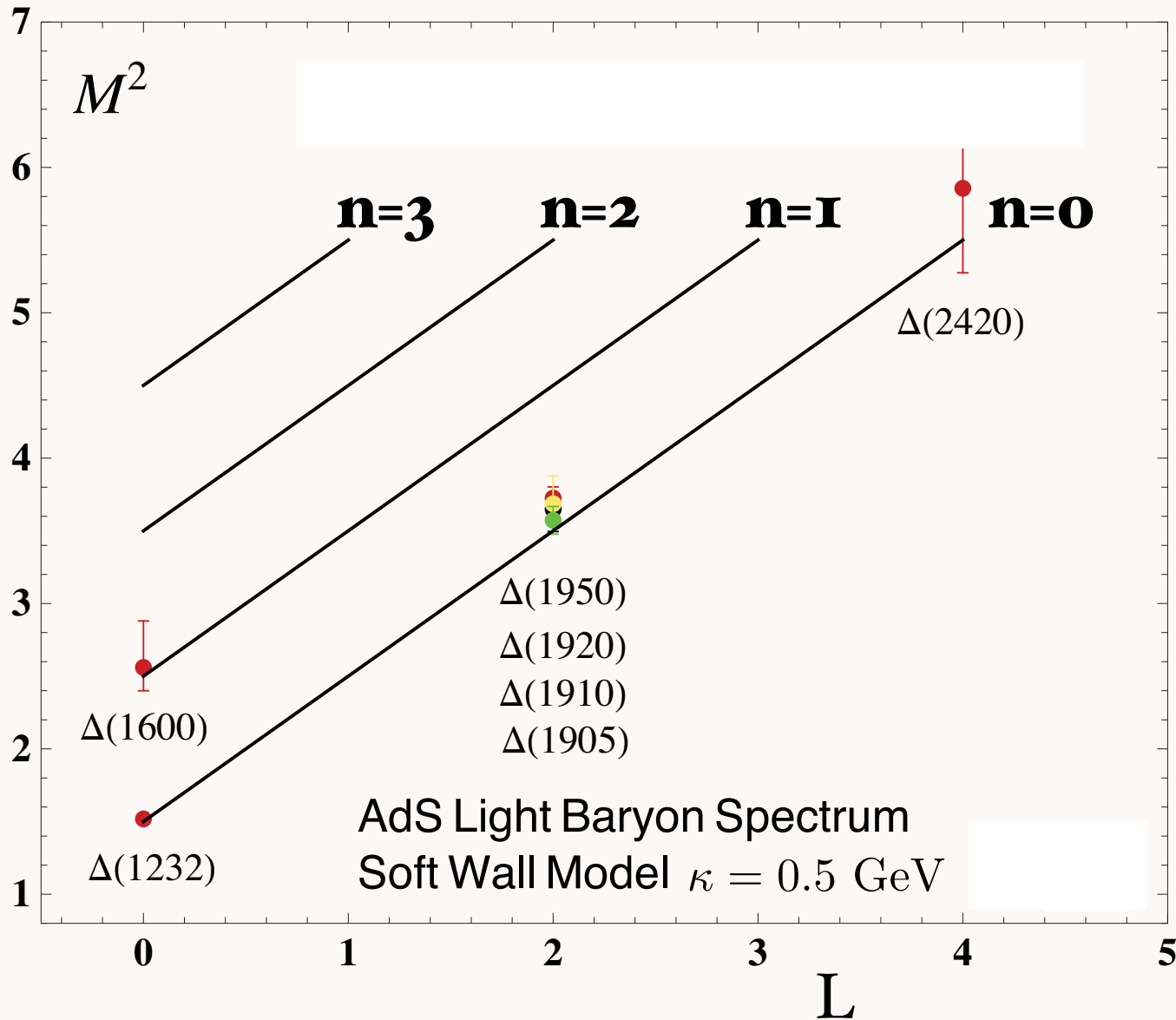
$$M^2 = 4\kappa^2(n + L + 1)$$

$$M^2 = 4\kappa^2(n + L + 2)$$



Positive Parity Deltas

$$M^2 = 4\kappa^2(n + L + 3/2)$$



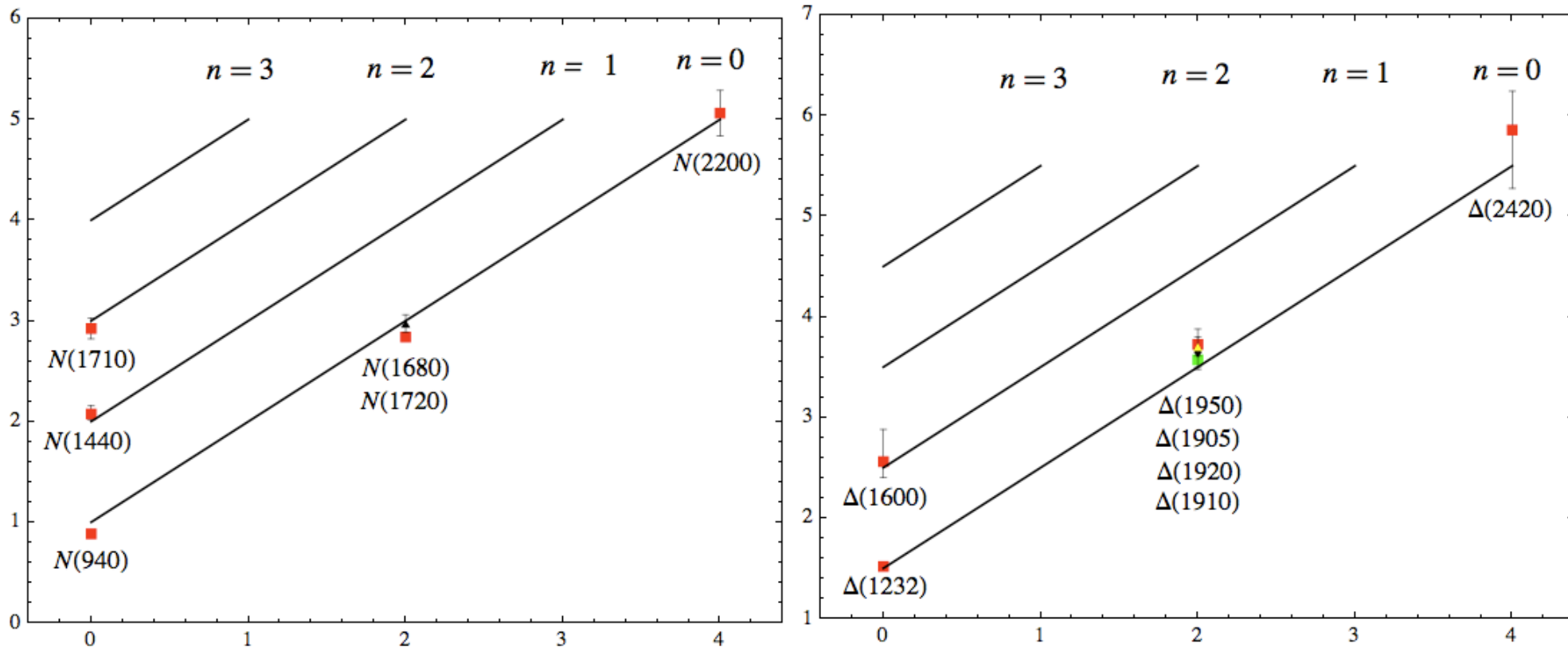
- Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$

$$\mathcal{M}^2$$



$$L$$

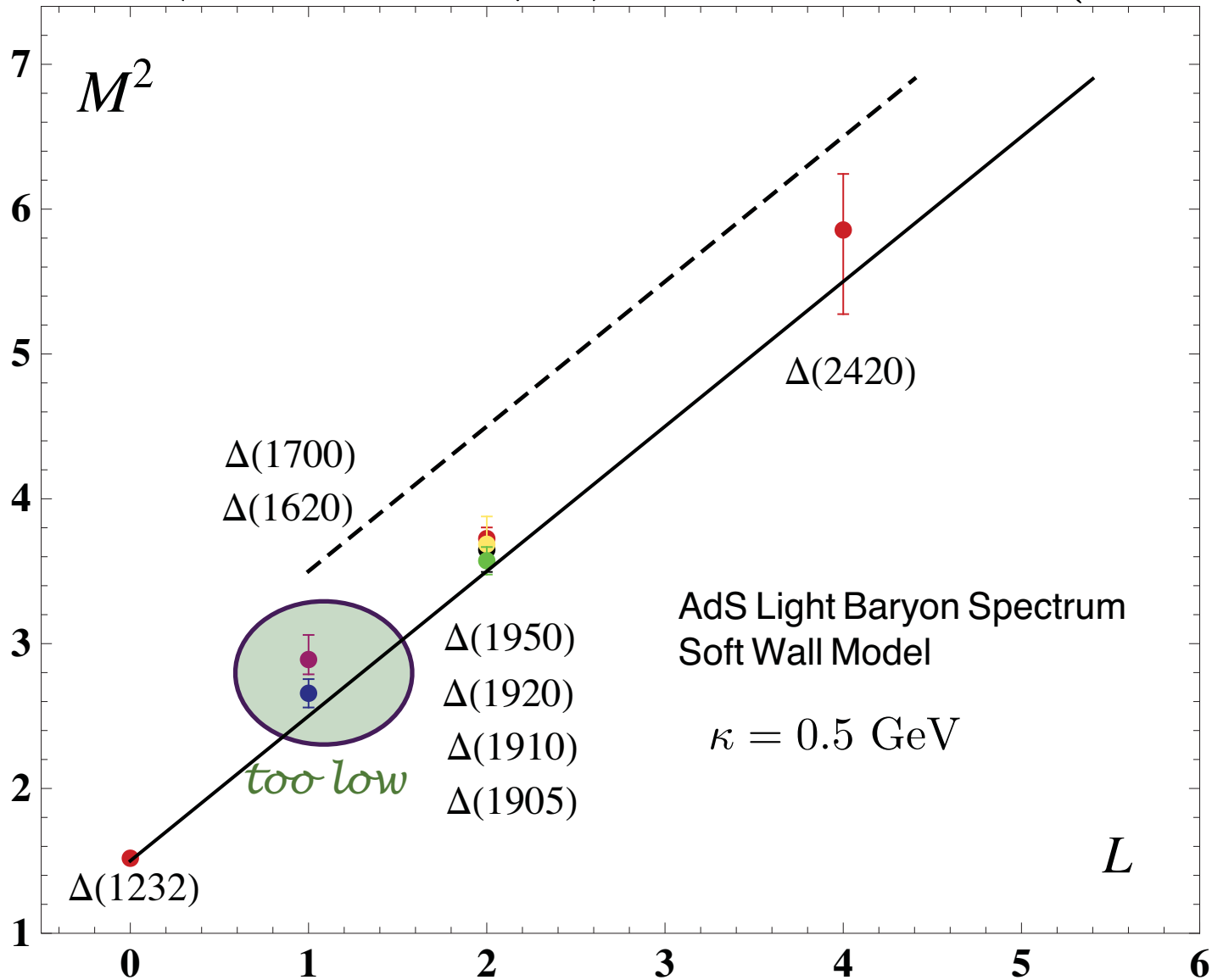
Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

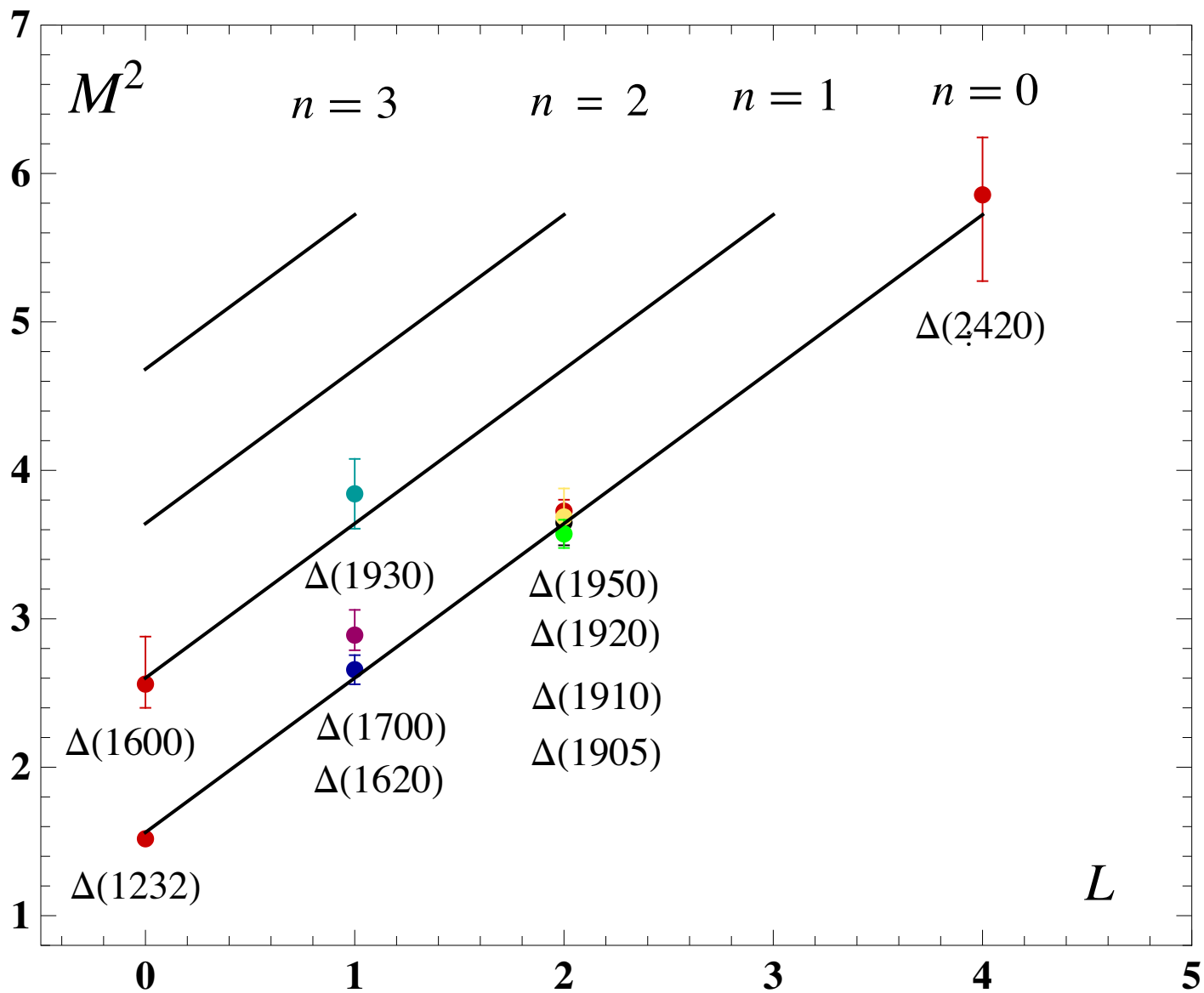
Positive Parity Deltas

$$M^2 = 4\kappa^2(n + L + 3/2)$$

Negative Parity Deltas

$$M^2 = 4\kappa^2(n + L + 5/2)$$





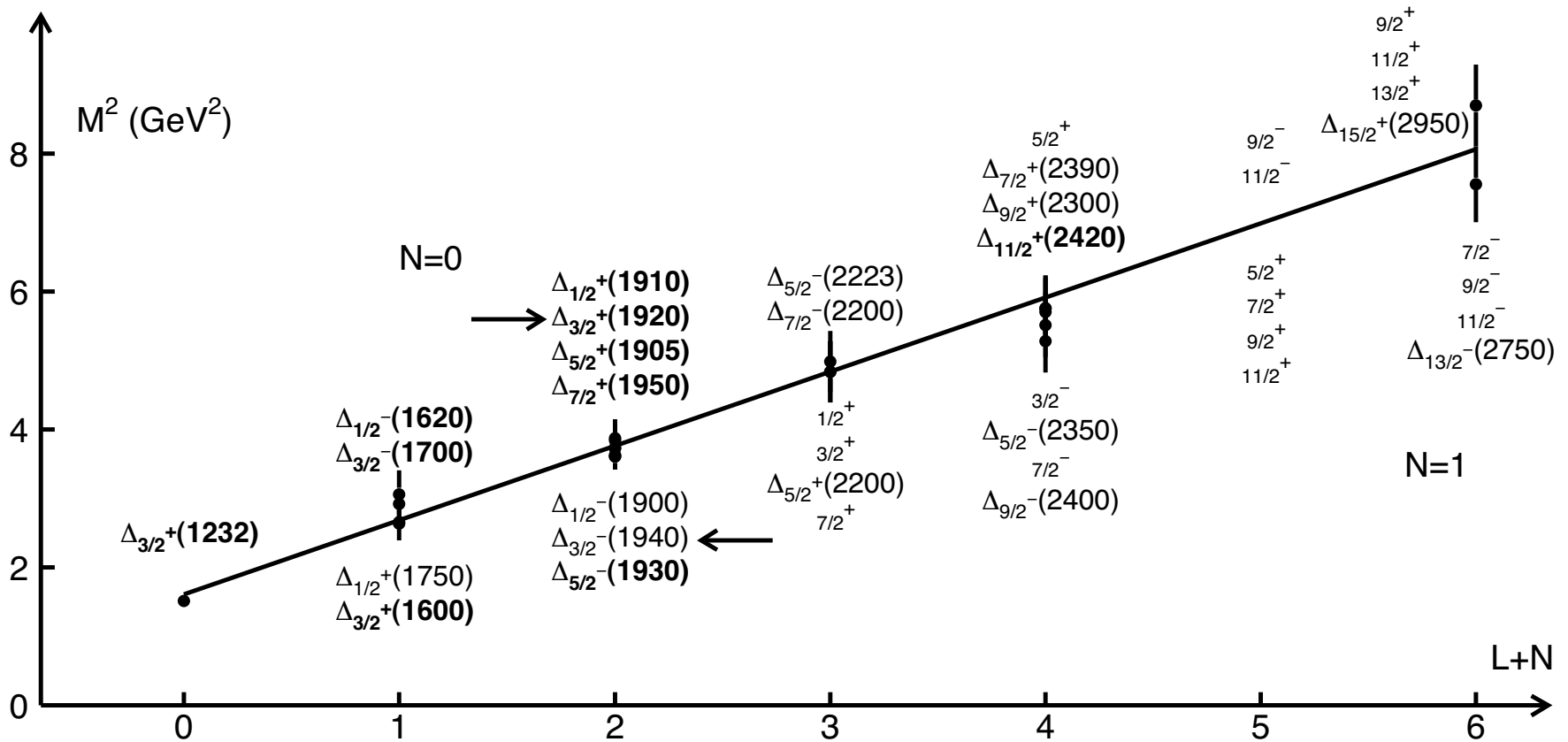
To compare with
E. Klempt and J. M. Richard

Assume single Δ trajectory

$\Delta(1930) : N = 1, L = 1, S = 3/2,$

not $N = 0, L = 1, S = 1/2.$

bold: pdg confirmed



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.

H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

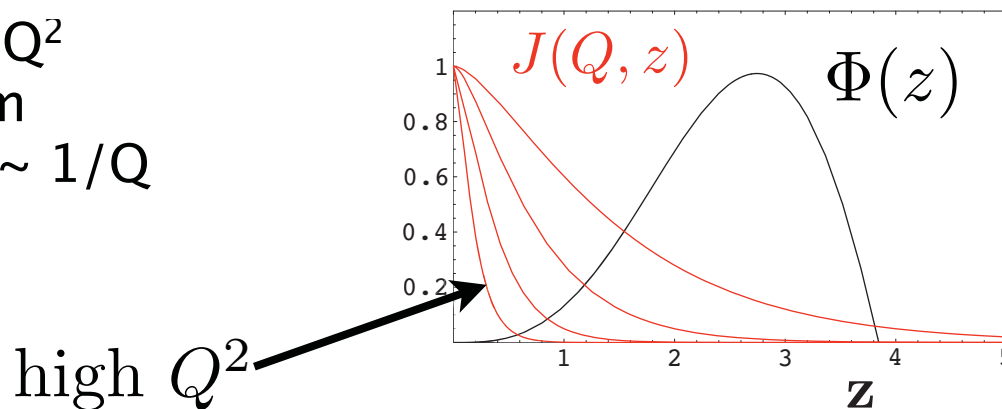
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



Polchinski, Strassler
de Teramond, sjb

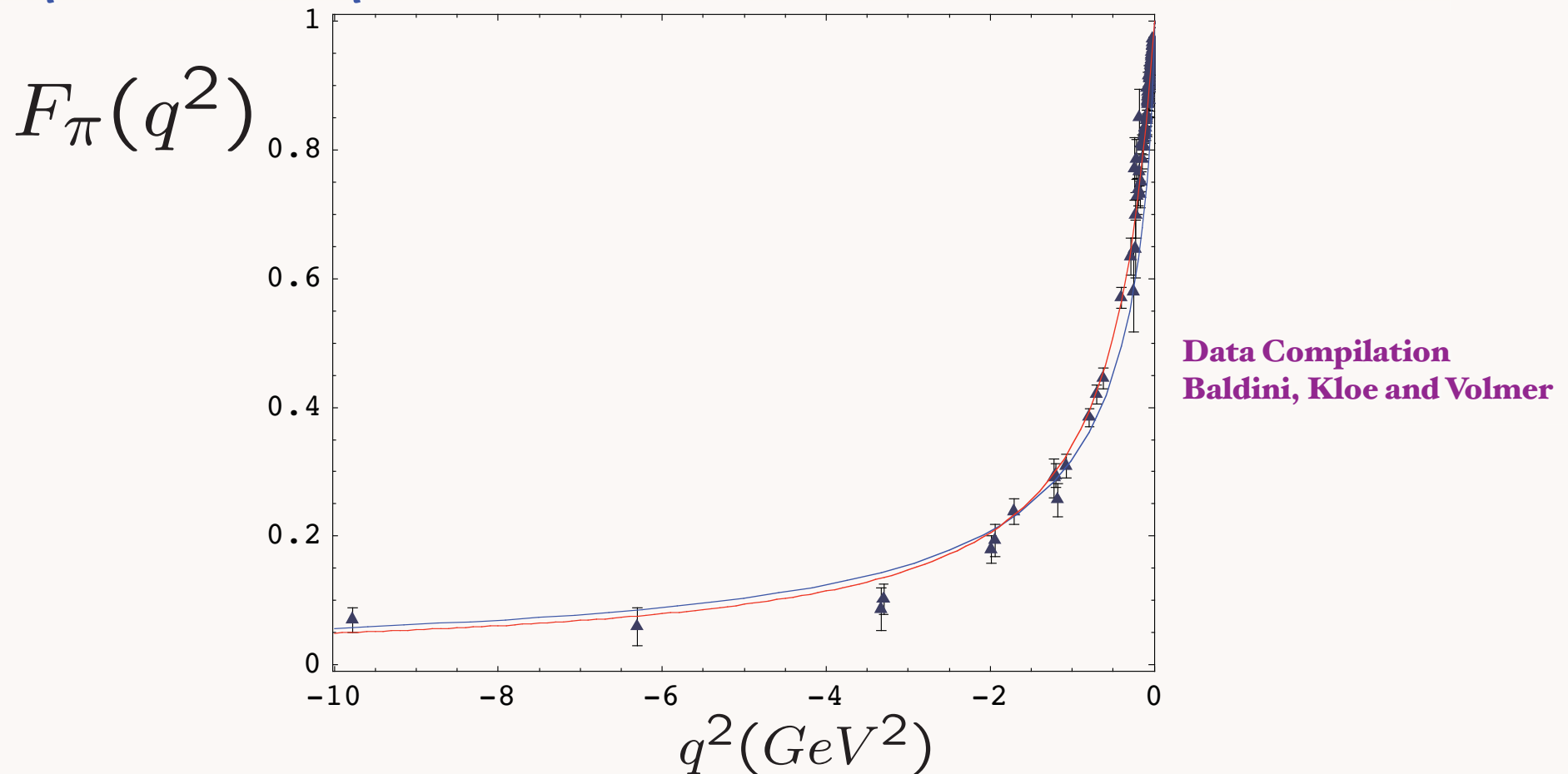
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

de Teramond, sjb
See also: Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Soft Wall
Model*

Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

...

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \quad N,$$

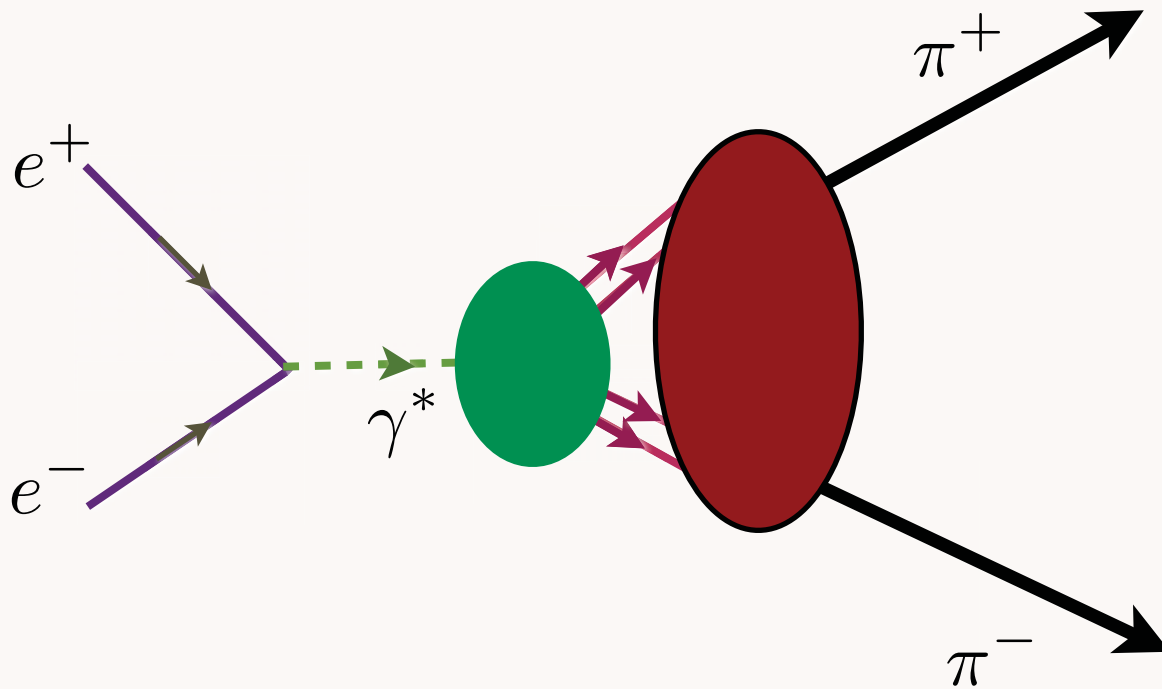
Positive Dilaton Background $\exp(+\kappa^2 z^2)$ $\mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right)$

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}$$

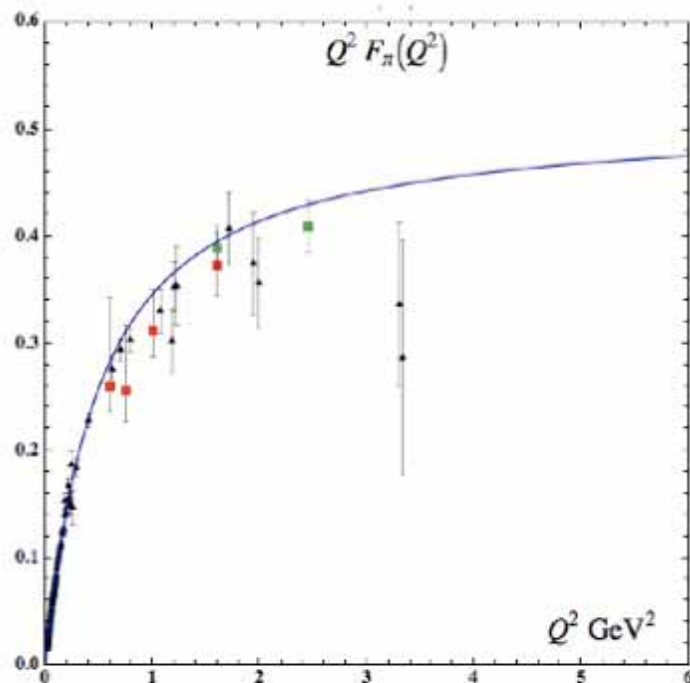
$$Q^2 \rightarrow \infty$$

Constituent Counting

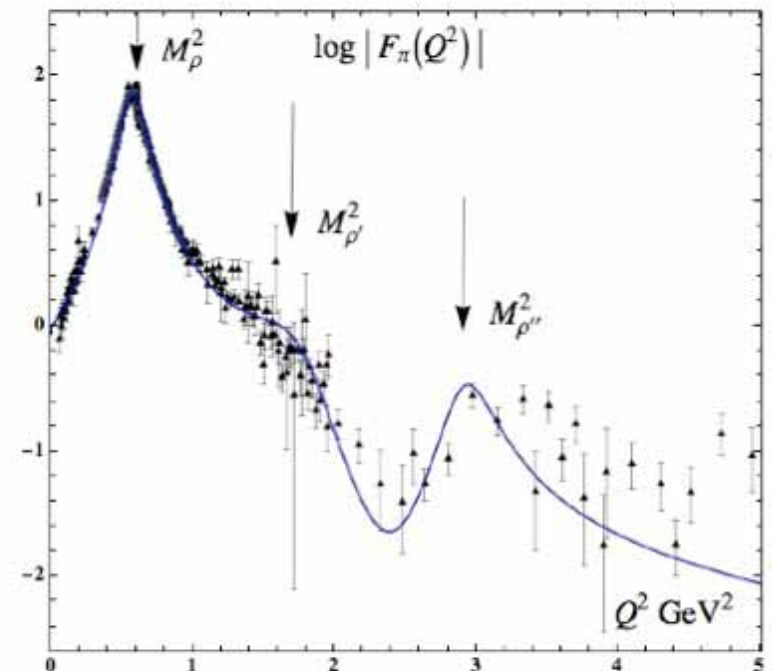
Dressed soft-wall current brings in higher Fock states and more vector meson poles



Space- and Time Like Pion Form-Factor (HFS)



PRELIMINARY



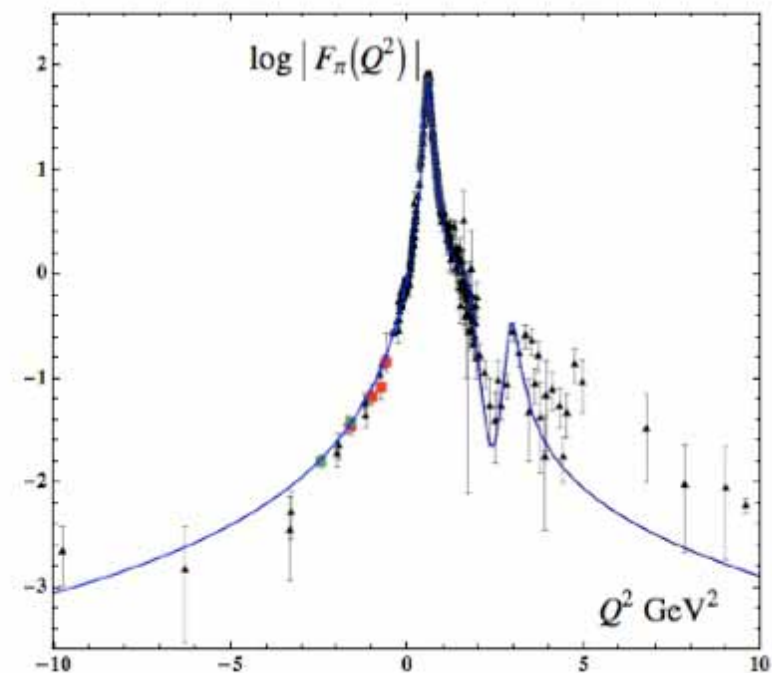
$$|\pi\rangle = \psi_{q\bar{q}/\pi} |q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}/\pi} |q\bar{q}q\bar{q}\rangle$$

$$\mathcal{M}^2 \rightarrow 4\kappa^2(n + 1/2)$$

$$\kappa = 0.54 \text{ GeV}$$

$$\Gamma_\rho = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 13\%$$



Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

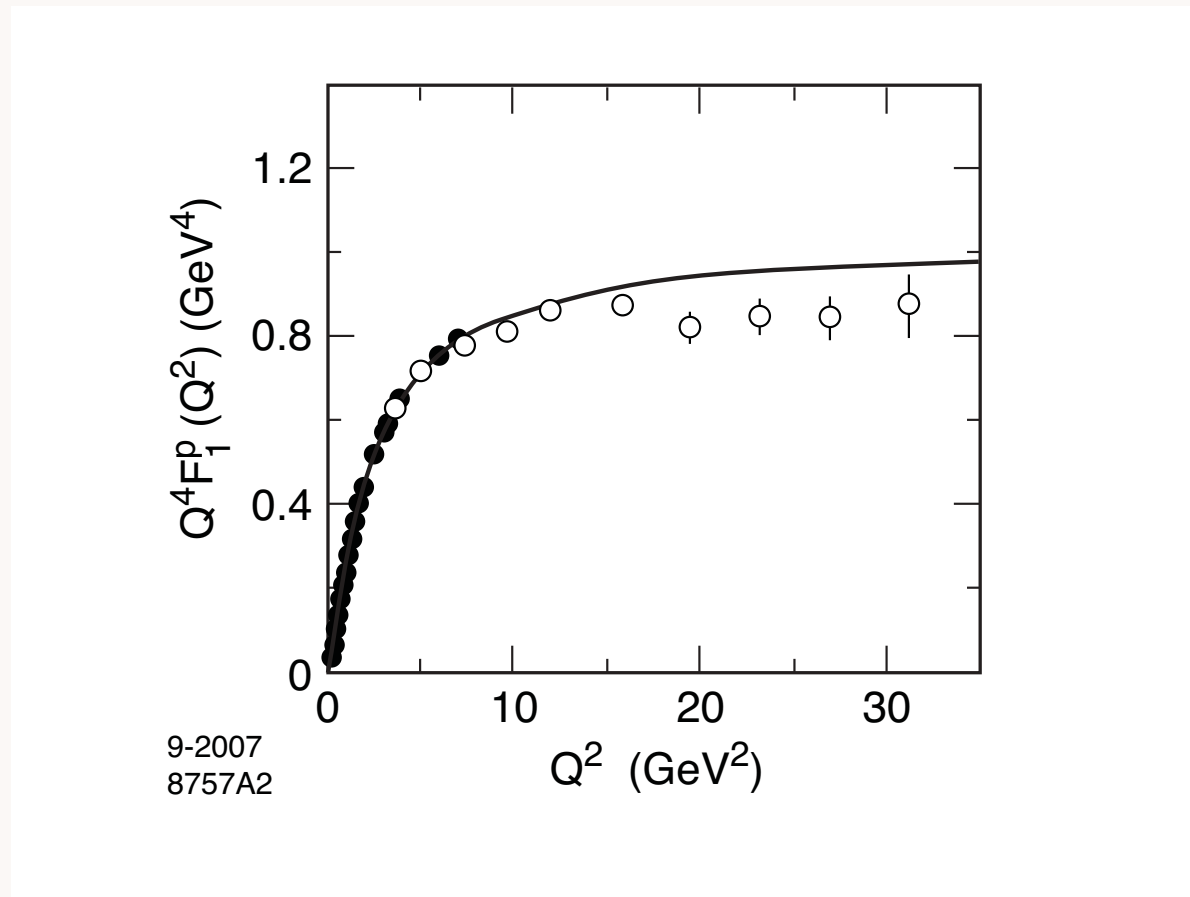
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

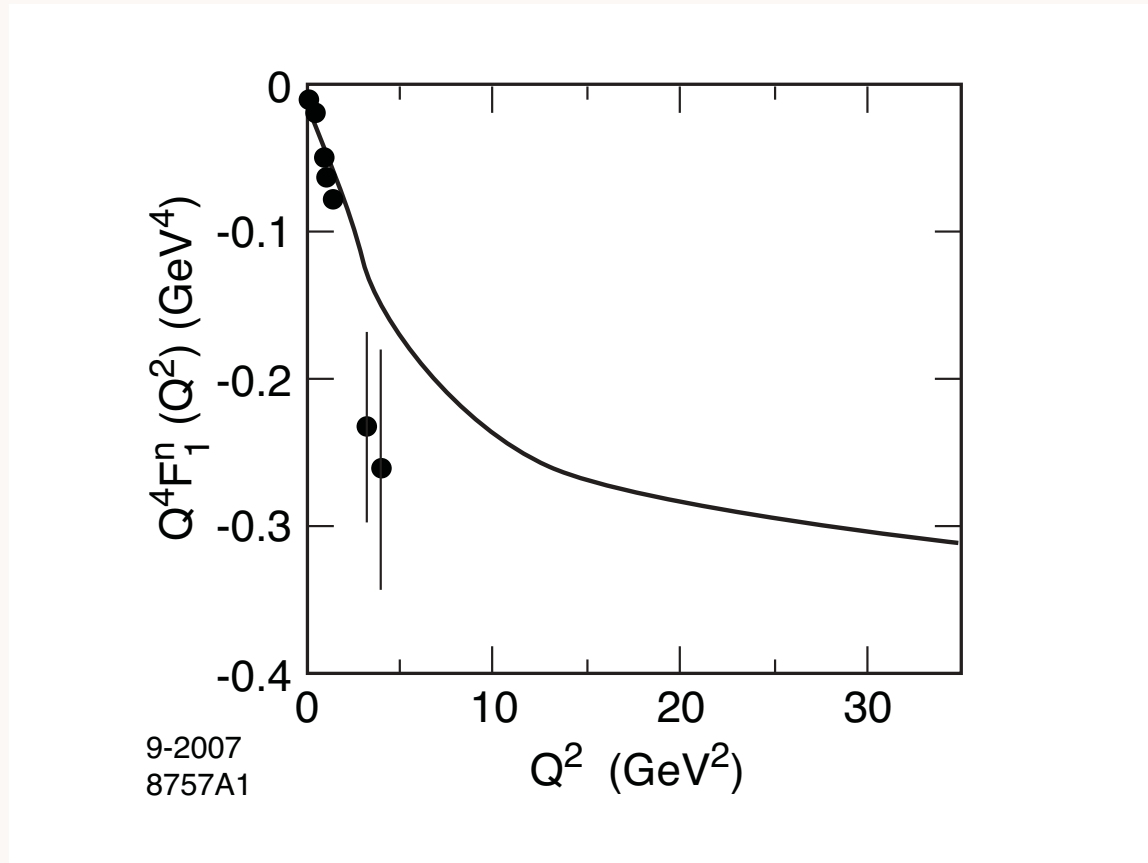
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

- Scaling behavior for large Q^2 : $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ Proton $\tau = 3$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2 : $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ Neutron $\tau = 3$

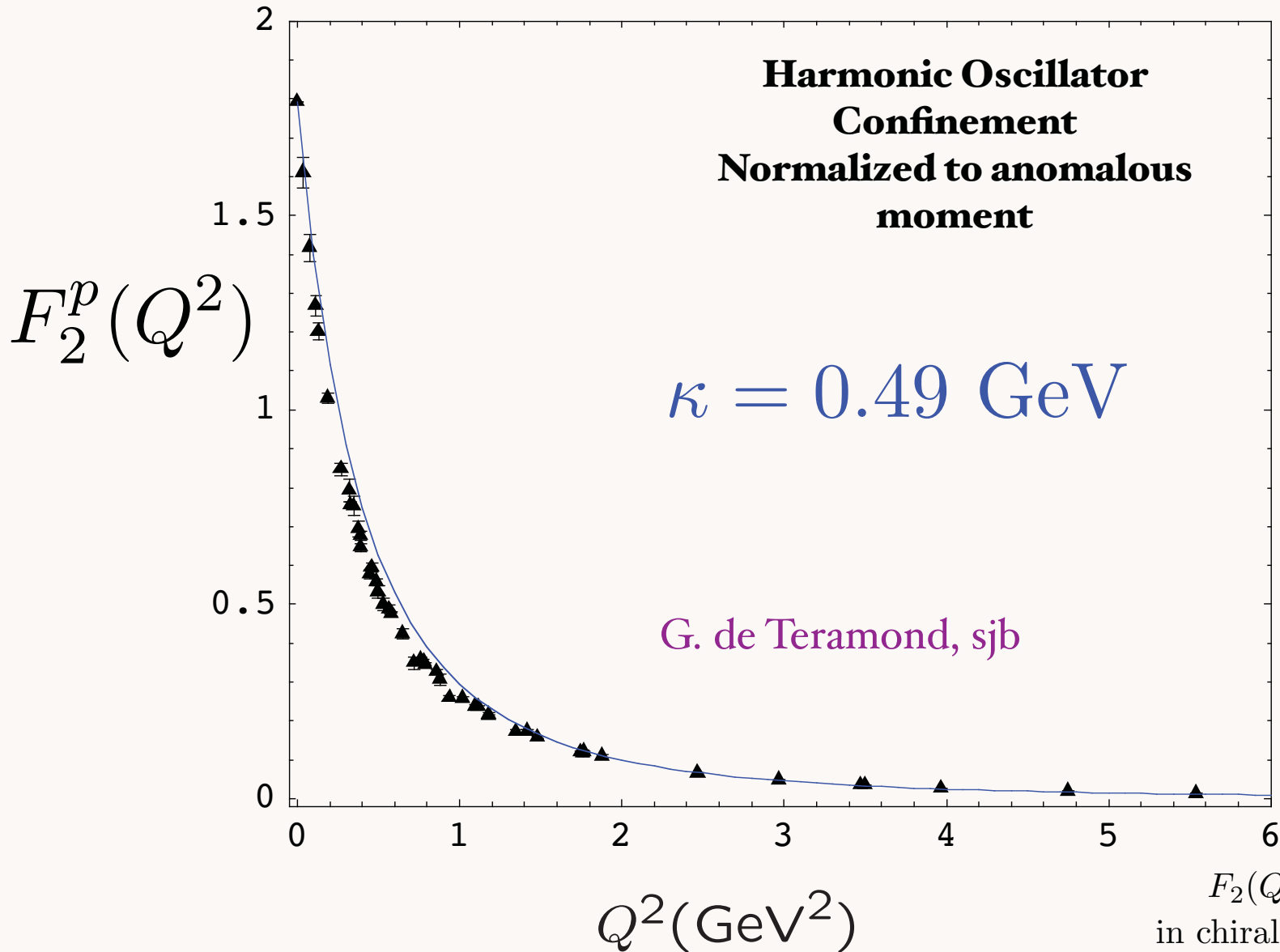


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

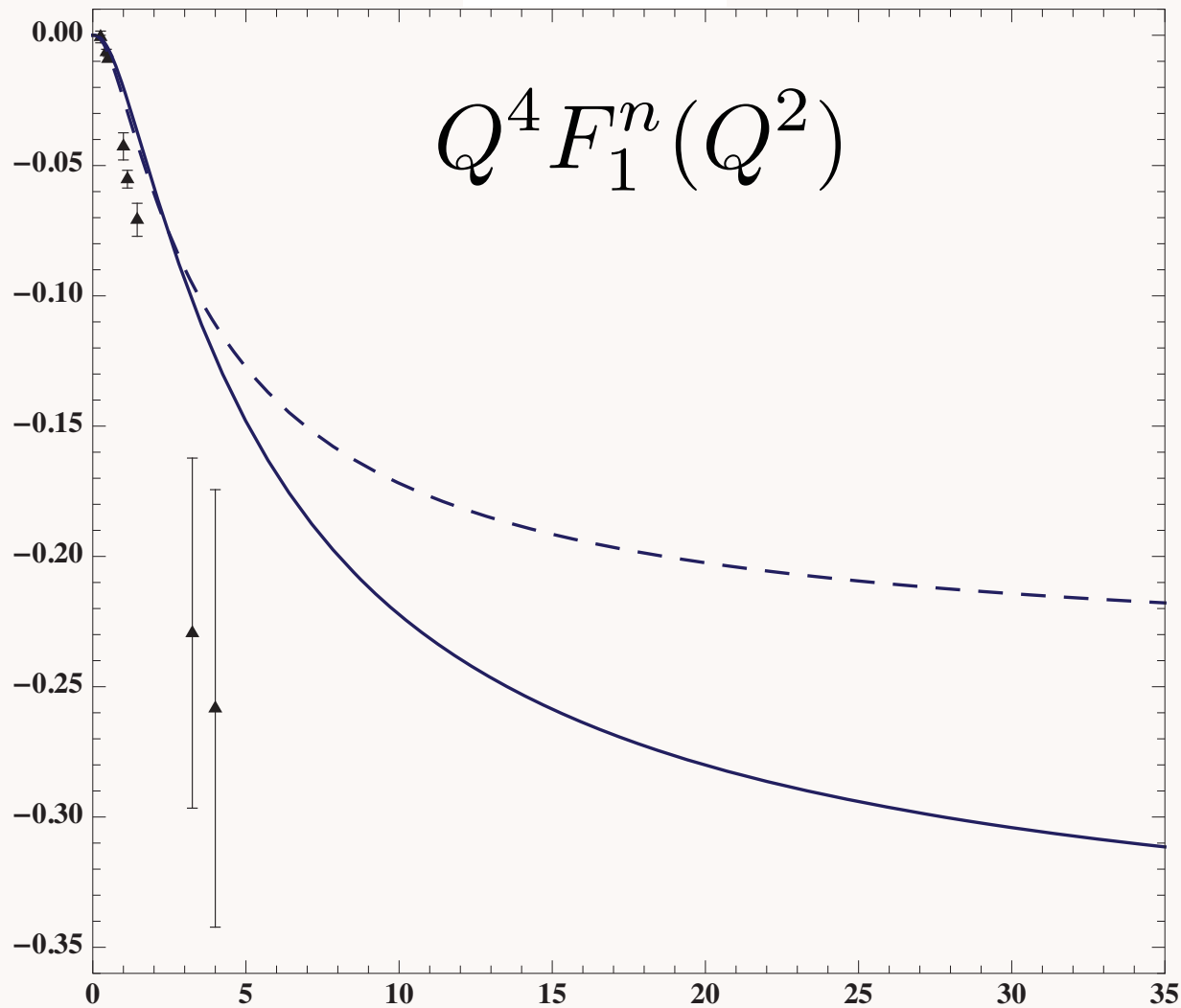
Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



*AdS/QCD No
chiral
divergence!*

$F_2(Q^2) = 1 + \mathcal{O}\left(\frac{Q^2}{m_\pi m_p}\right)$
in chiral perturbation theory



$Q^4 F_n^1(Q^2)$ in a negative (dashed line, $\kappa = 0.3877$ GeV) and positive dilaton backgrounds (continuous line, $\kappa = 0.5484$ GeV). The data compilation is from Diehl.

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi(x, k_{\perp}) \quad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^{μ}

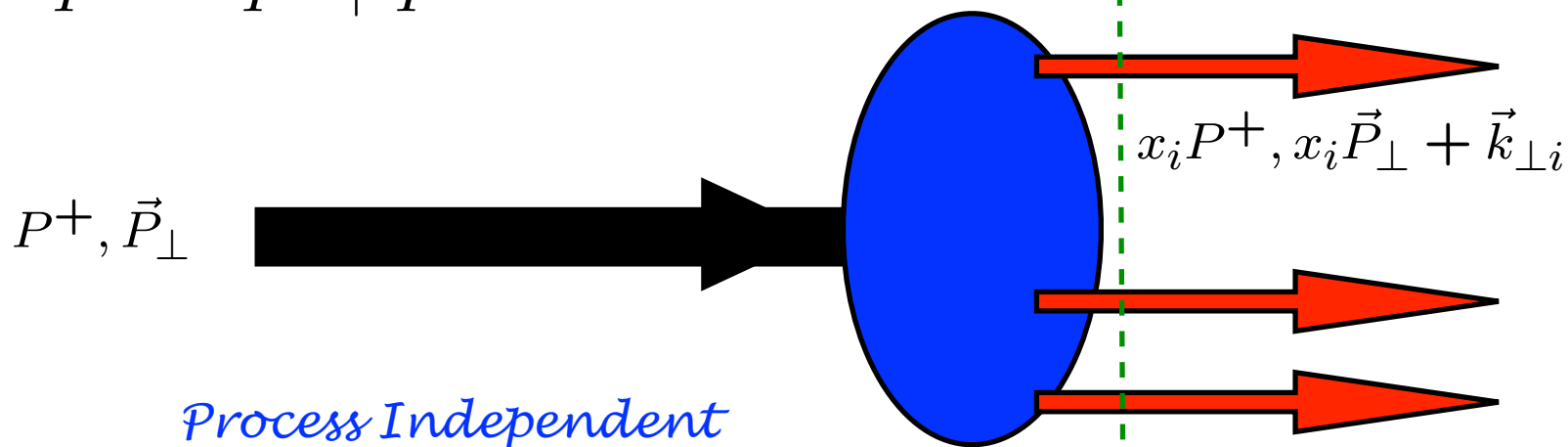
$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



*Process Independent
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Plus momenta conserved; all $k^+ \geq 0$

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

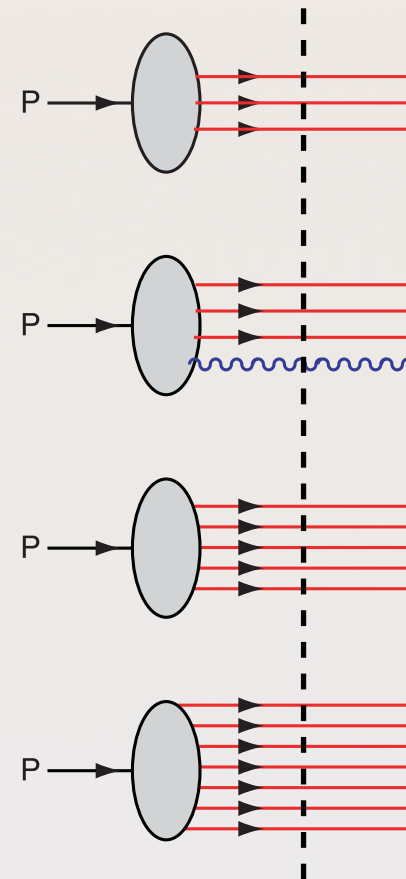
are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks
 $c(x), b(x)$ at high x !

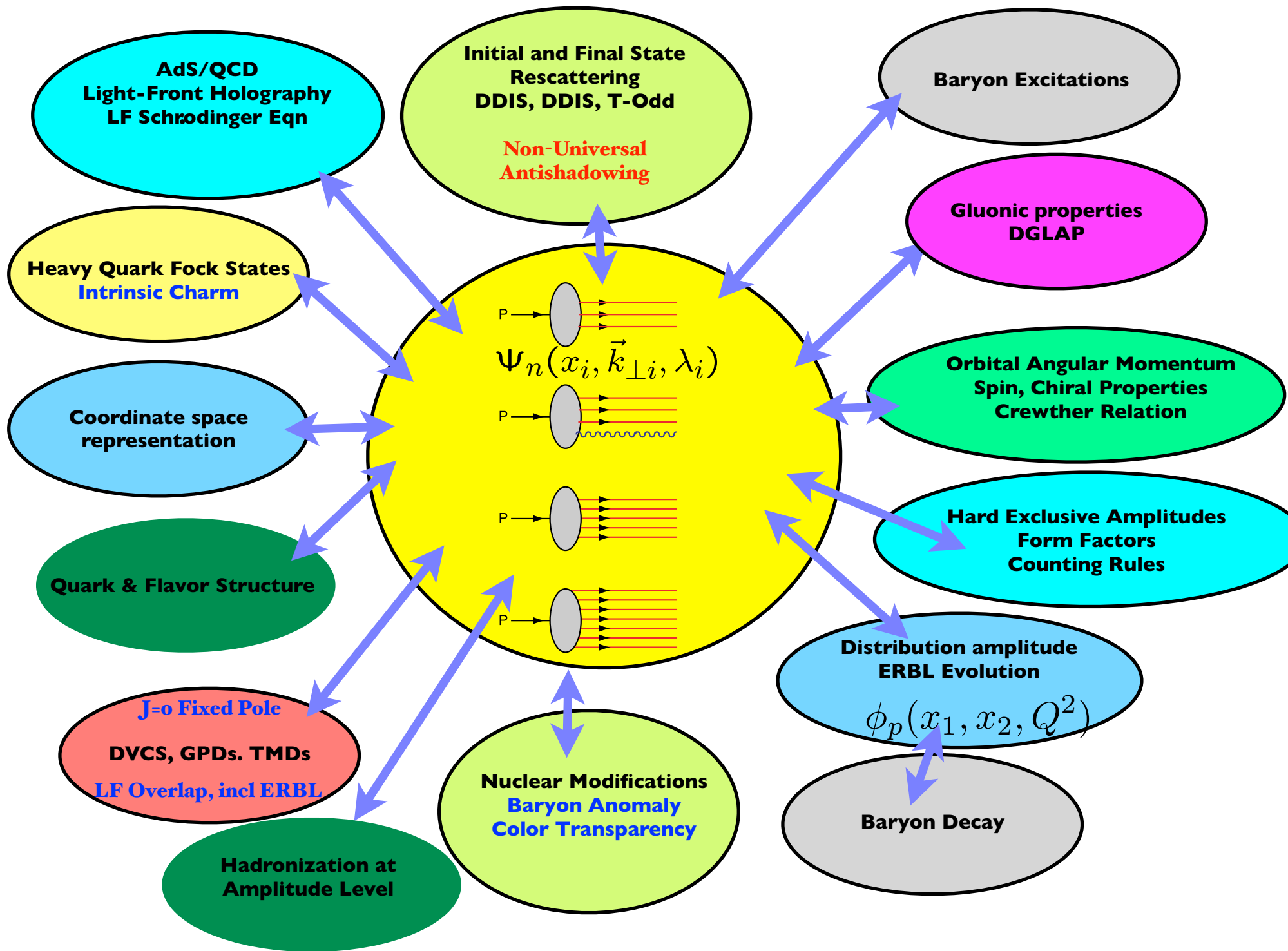
$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$



Fixed LF time

QCD and the LF Hadron Wavefunctions



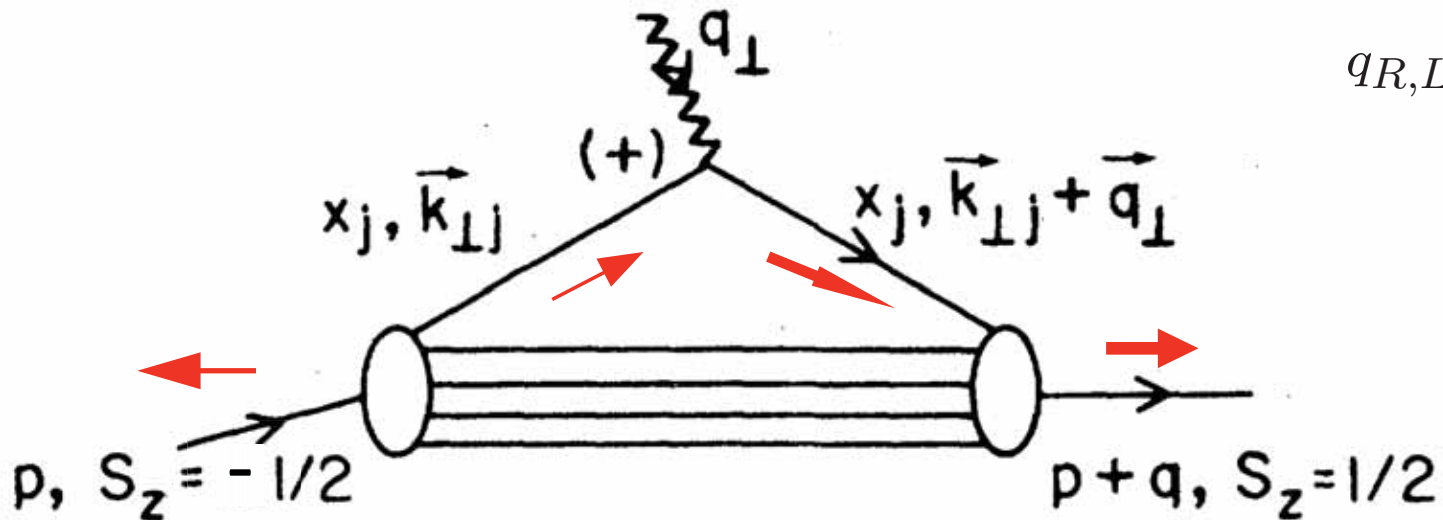
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$



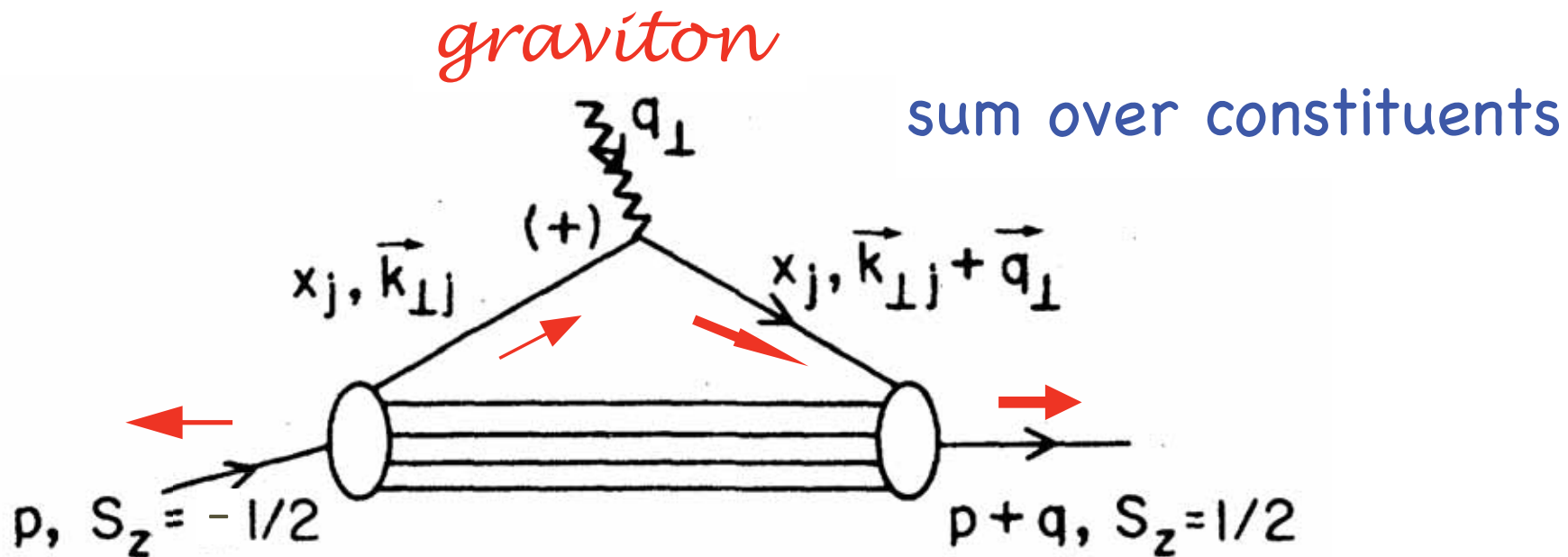
$$q_{R,L} = q^x \pm iq^y$$

Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum*

Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem



**Hwang, Schmidt, sjb;
Holstein et al**

$B(0) = 0$

Each Fock State

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space \vec{b}_\perp

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$):

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4}$$

with $\zeta = z$, $0 \leq \zeta \leq \Lambda_{\text{QCD}}$

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

Identical to LF Holography obtained from electromagnetic current

$LF(3+1)$

AdS_5

$$\psi(x, \vec{b}_\perp)$$



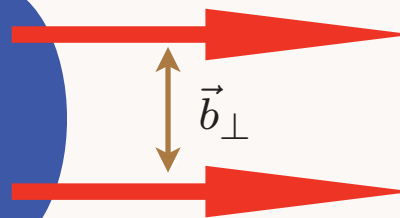
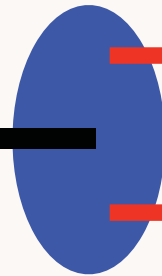
$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$



$$z$$

$$\psi(x, \vec{b}_\perp)$$



x

$(1-x)$

$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

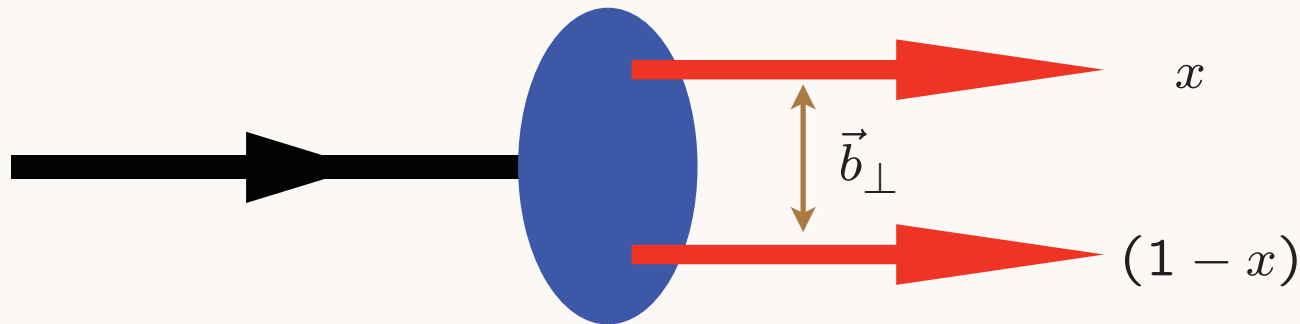
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

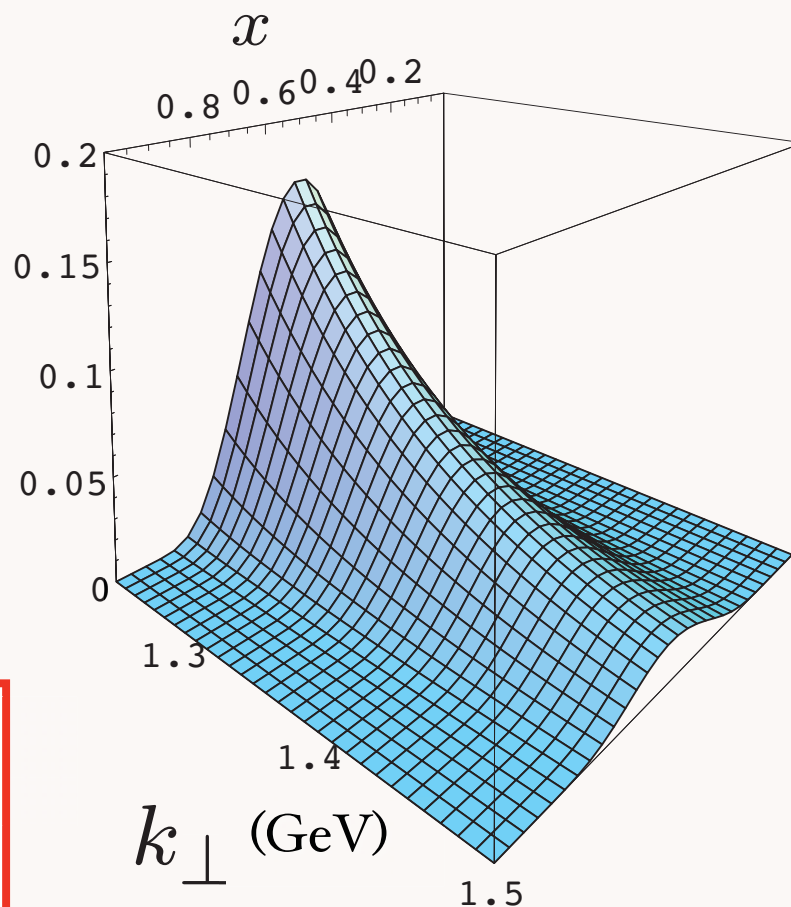
*soft wall
confining potential:*

G. de Teramond, sjb

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_{\perp}^2)$$



**“Soft Wall”
model**

$\kappa = 0.375$ GeV
massless quarks

Note coupling

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions.} \end{aligned}$$

**Change
variables**

$$(\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

$$H_{QED}$$

*QED atoms: positronium
and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, l)\right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

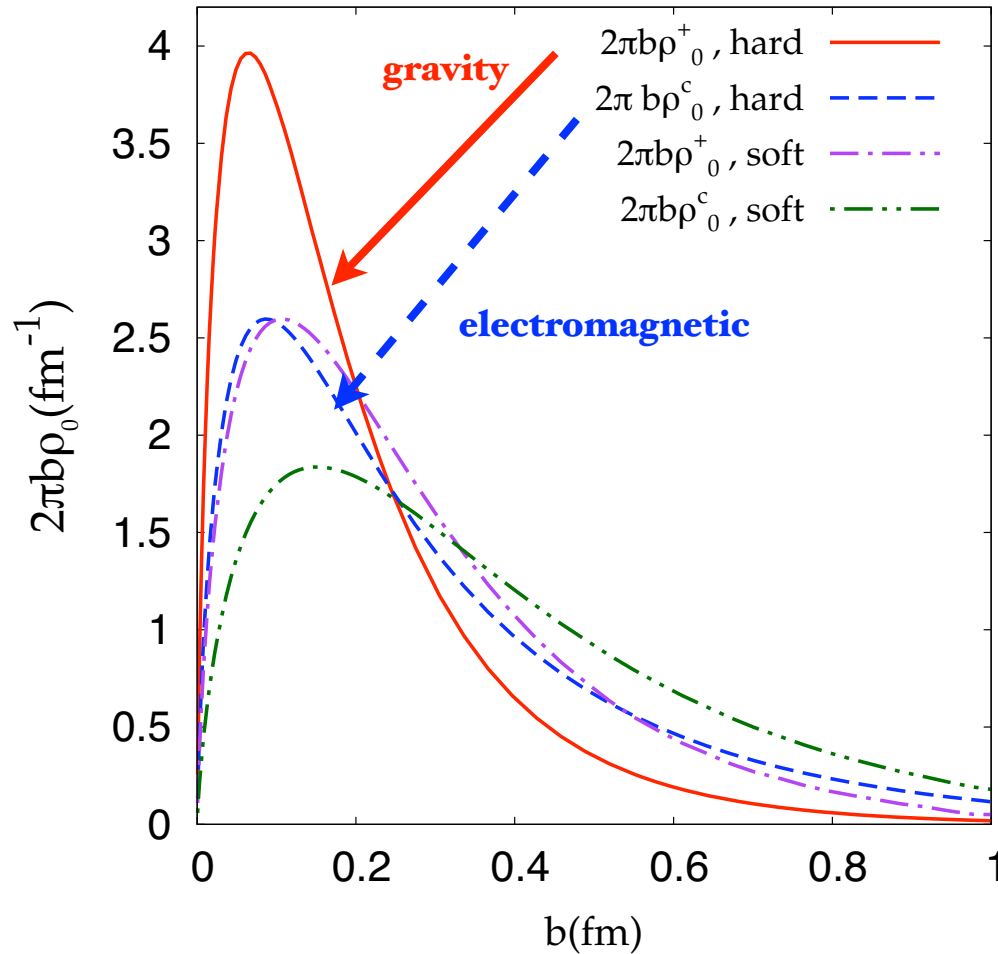
Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

Momentum Density more Compact than Charge Density



Z. Abidin and C. E. Carlson,
 “Hadronic Momentum Densities in the Transverse Plane”
 arXiv:0808.3097 [hep-ph].

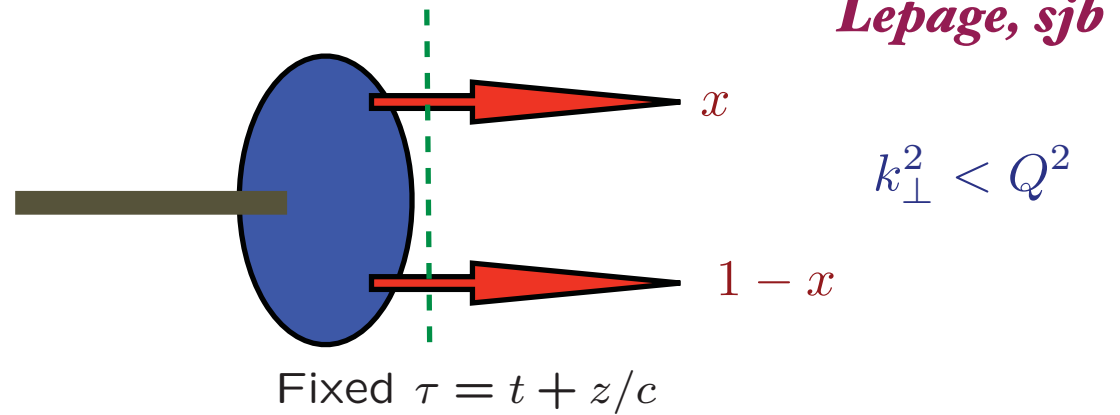
Immediate property of LF Holography

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} F\left(\frac{k_\perp^2}{2\kappa^2 x(1-x)}\right)$$

Hadron Distribution Amplitudes

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb

Efremov, Radyushkin

Sachrajda, Frishman Lepage, sjb

- Compute from valence light-front wavefunction in light-cone gauge

Braun, Gardi

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_{\perp})$$

Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

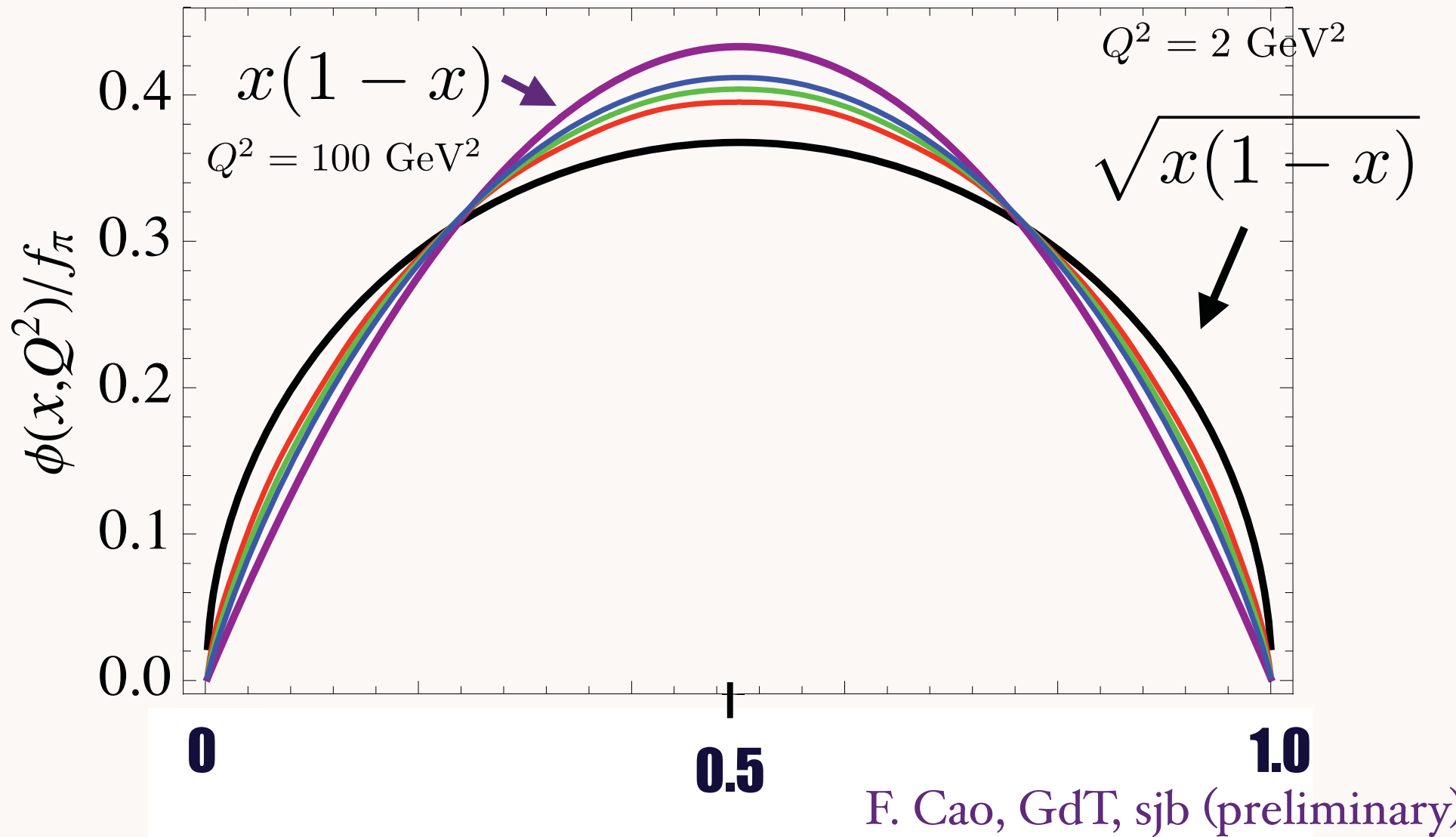
$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

ERBL Evolution of Pion Distribution Amplitude



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

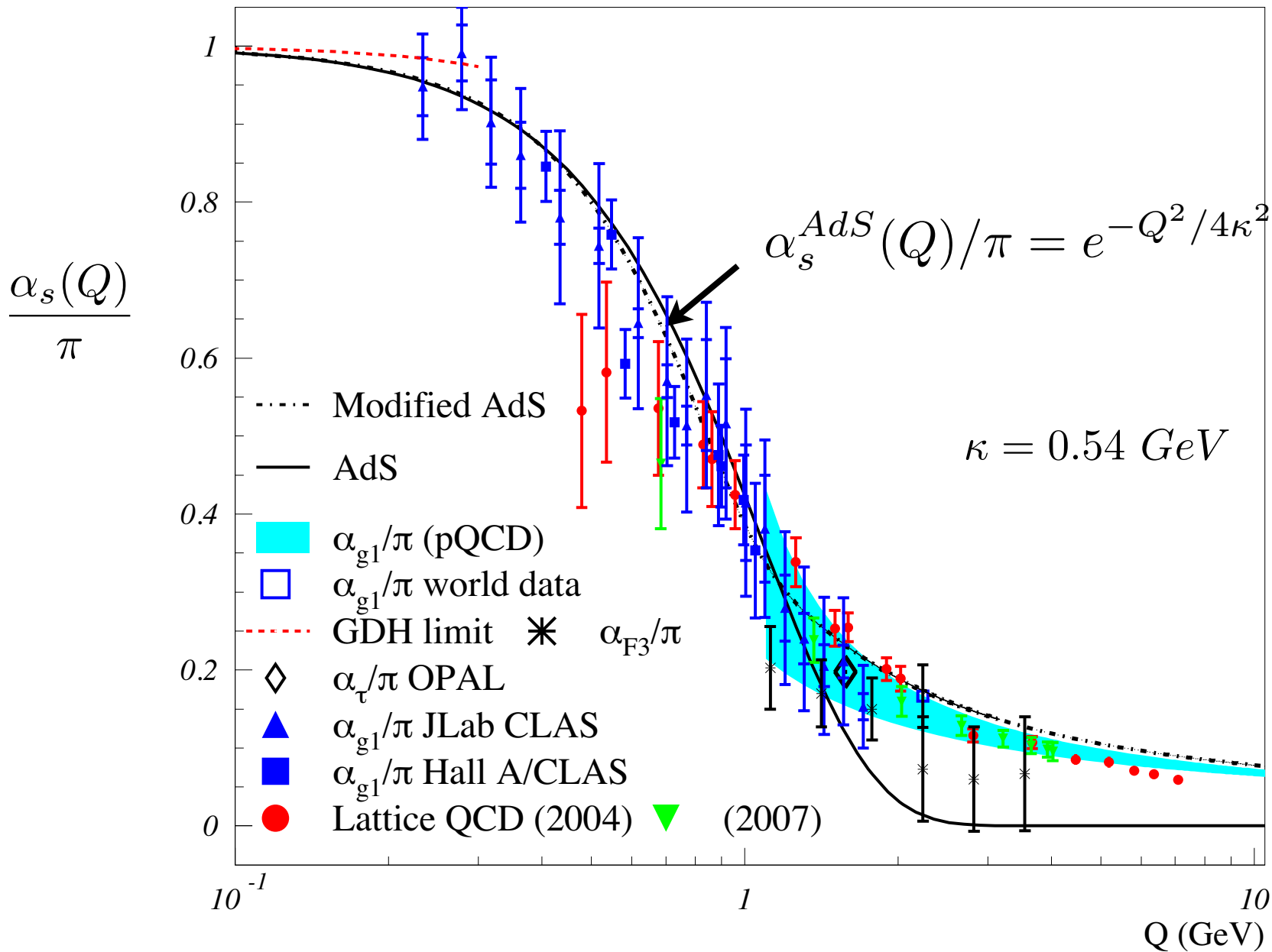
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

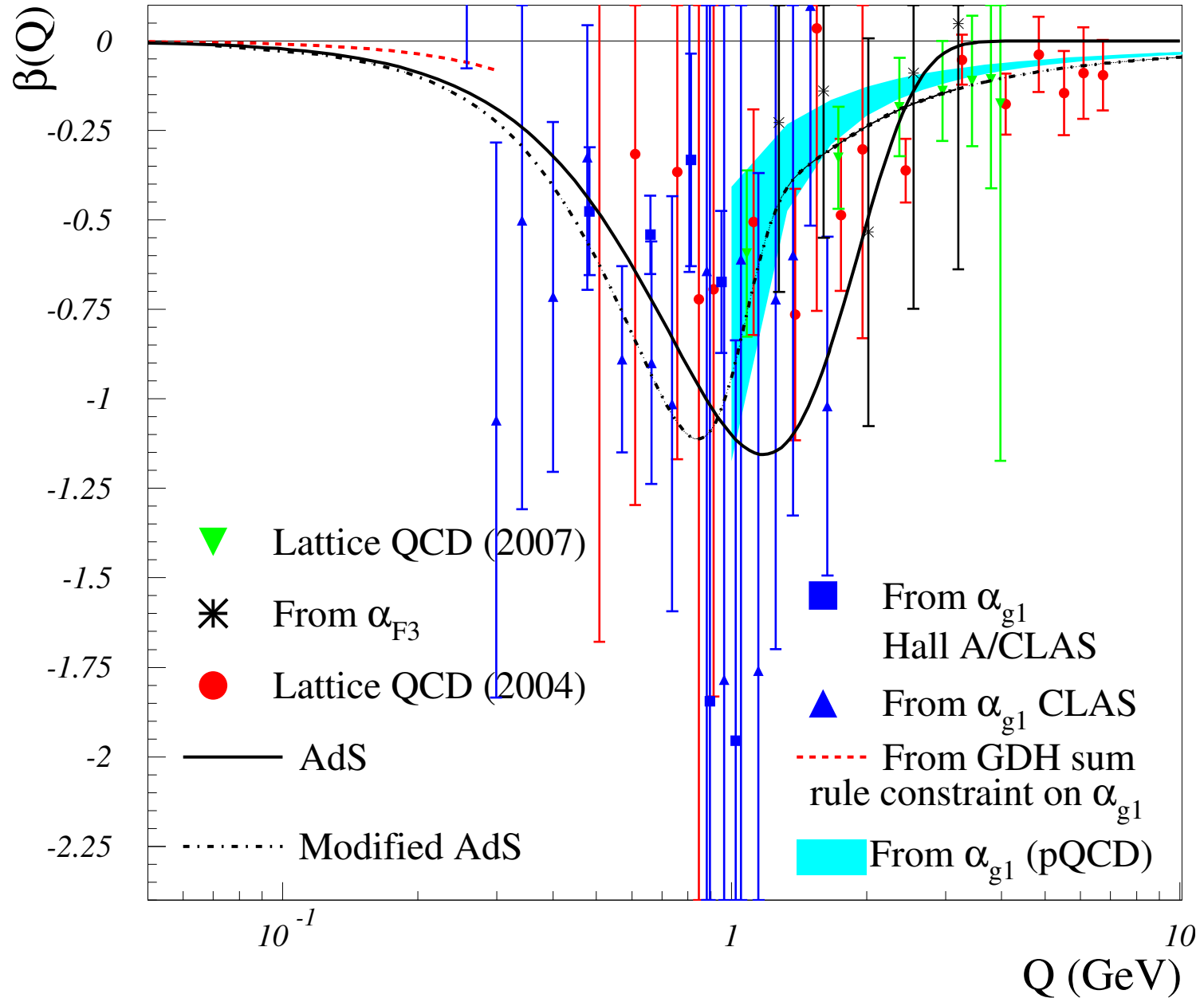
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

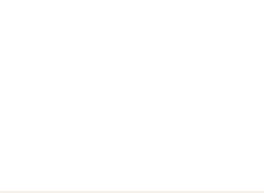
Analytic, defined at all scales, IR Fixed Point



$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



Features of Soft-Wall AdS/QCD Model

- Boost Invariant
- Trivial LF vacuum.
- Massless Pion
- Hadron Eigenstates have LF Fock components of different L^z
- Proton: equal probability $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
- Self-Dual Massive Eigenstates: Proton is its own chiral partner.
- Label State by minimum L as in Atomic Physics 
- Minimum L dominates at short distances
- AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z \rightarrow 0$

Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion ($m_q = 0$)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S . Spectrum is independent of S**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large N_c limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Light-Front QCD
Heisenberg Equation

$$H_{LC}^{QCD} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle$$

Use AdS/QCD
basis functions

n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	gg g
6	q \bar{q} gg						
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gg gg
10	q \bar{q} gg g
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g				
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}		

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful I + I
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- Hamiltonian light-front field theory within an AdS/QCD basis.
J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

**Pauli, Hornbostel,
Hiller, McCartor, sjb**

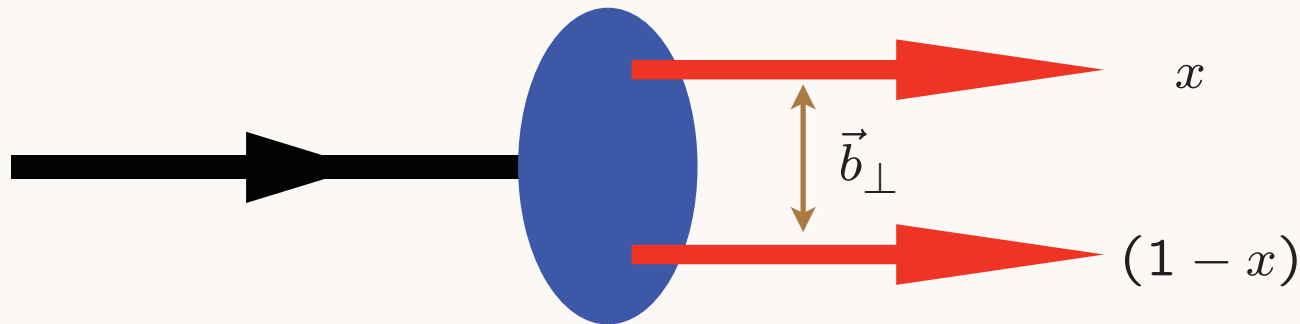
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*soft wall
confining potential:*

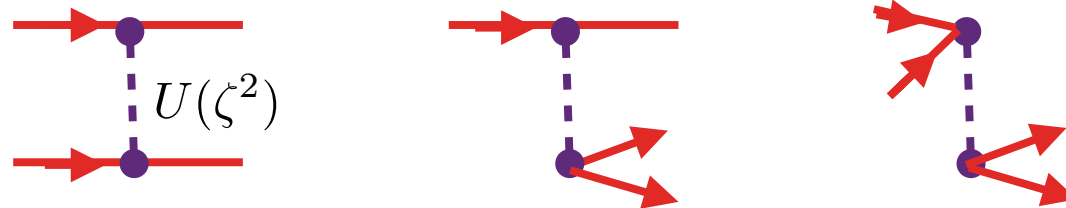
G. de Teramond, sjb

Higher Fock States

- Exposed by timelike form factor through dressed current.
- Created by confining interaction

$$H_I = \bar{\psi}\psi U(\zeta^2)\bar{\psi}\psi$$

- Similar to QCD(1+1) in lcg



AdS/QCD and Light-Front Holography

- Hadrons are composites of quark and anti-quark constituents
- Explicit gluons absent!
- Higher Fock states with extra quark/anti-quark pairs created by confining potential
- Dominance of Quark Interchange in Hard Exclusive Reactions
- Short-distance behavior matches twist of interpolating operator at short distance -- guarantees dimensional counting rules --

Comparison of 20 exclusive reactions at large t

C. White,^{4,*} R. Appel,^{1,5,†} D. S. Barton,¹ G. Bunce,¹ A. S. Carroll,¹
 H. Courant,⁴ G. Fang,^{4,‡} S. Gushue,¹ K. J. Heller,⁴ S. Heppelmann,²
 K. Johns,^{4,§} M. Kmit,^{1,||} D. I. Lowenstein,¹ X. Ma,³ Y. I. Makdisi,¹
 M. L. Marshak,⁴ J. J. Russell,³
 and M. Shupe^{4,§}

¹Brookhaven National Laboratory, Upton, New York 11973

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³University of Massachusetts Dartmouth, N. Dartmouth, Massachusetts 02747

⁴University of Minnesota, Minneapolis, Minnesota 55455

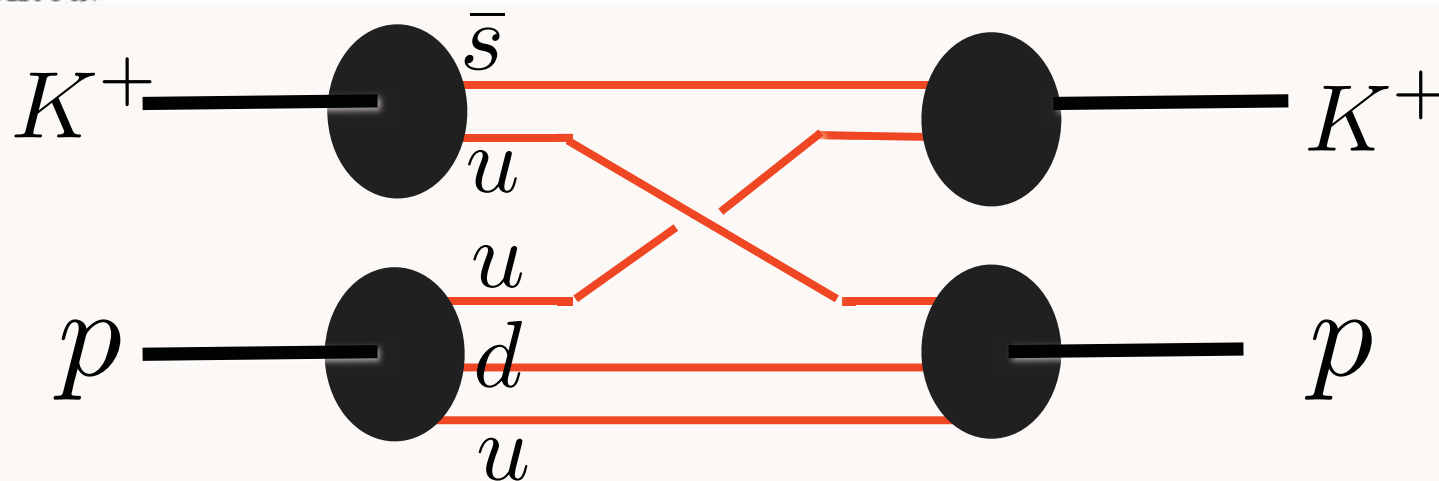
⁵New York University, New York, New York 10003

(Received 28 May 1993)

We report a study of 20 exclusive reactions measured at the AGS at 5.9 GeV/ c incident momentum, 90° center of mass. This experiment confirms the strong quark flow dependence of two-body hadron-hadron scattering at large angle. At 9.9 GeV/ c an upper limit had been set for the ratio of cross sections for $(\bar{p}p \rightarrow \bar{p}p)/(pp \rightarrow pp)$ at 90° c.m., with the ratio less than 4%. The present experiment was performed at lower energy to gain sensitivity, but was still within the fixed angle scaling region. A ratio $R(\bar{p}p/pp) \approx 1/40$ was measured at 5.9 GeV/ c , 90° c.m. in comparison to a ratio near 1.7 for small angle scattering. In addition, many other reactions were measured, often for the first time at 90° c.m. in the scaling region, using beams of π^\pm , K^\pm , p , and \bar{p} on a hydrogen target. There are similar large differences in cross sections for other reactions: $R(K^-p \rightarrow \pi^+\Sigma^-/K^-p \rightarrow \pi^-\Sigma^+) \approx 1/12$, for example. The relative magnitudes of the different cross sections are consistent with the dominance of quark interchange in these 90° reactions, and indicate that pure gluon exchange and quark-antiquark annihilation diagrams are much less important. The angular dependence of several elastic cross sections and the energy dependence at a fixed angle of many of the reactions are also presented.

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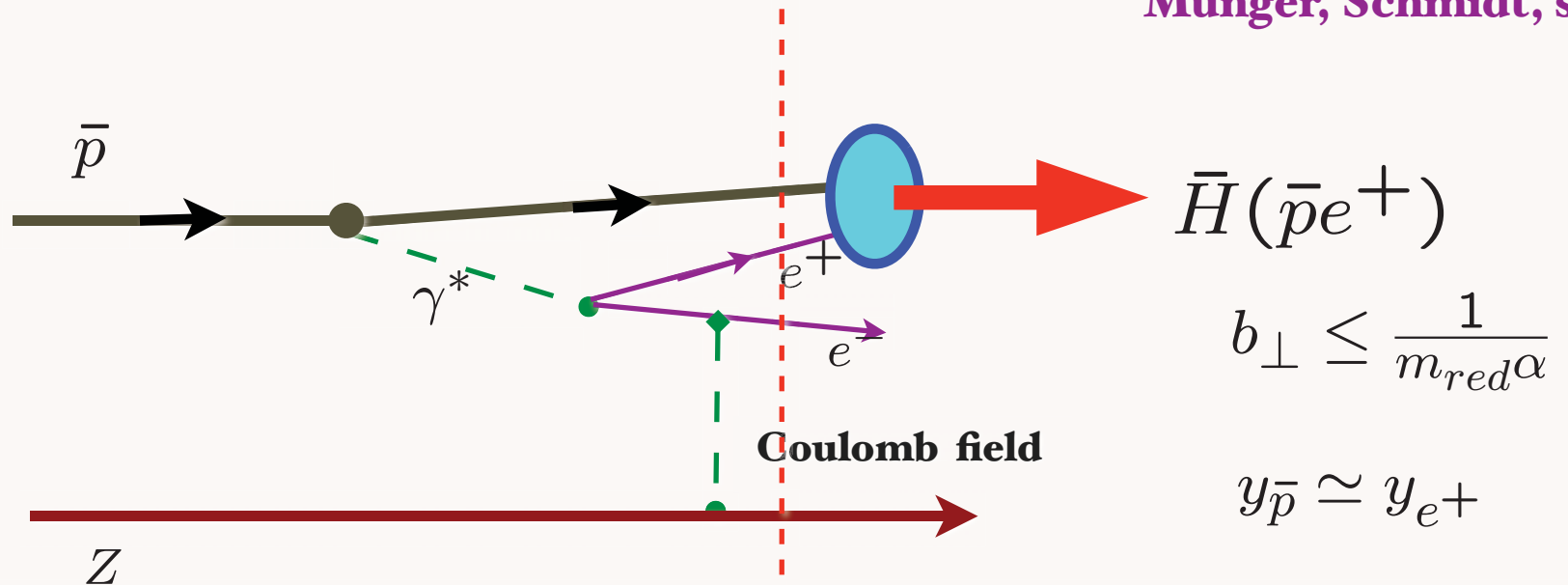
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Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

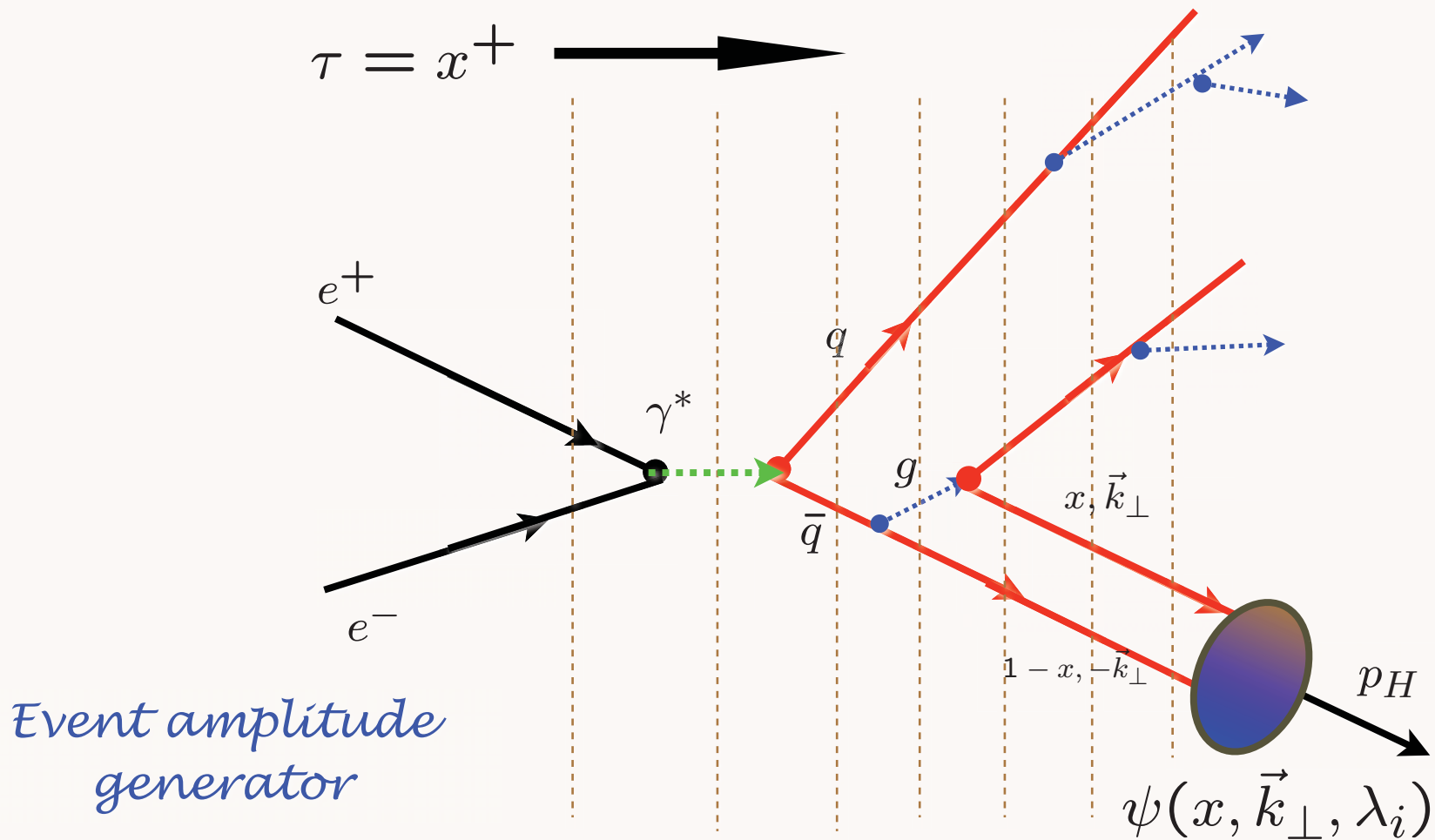


Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

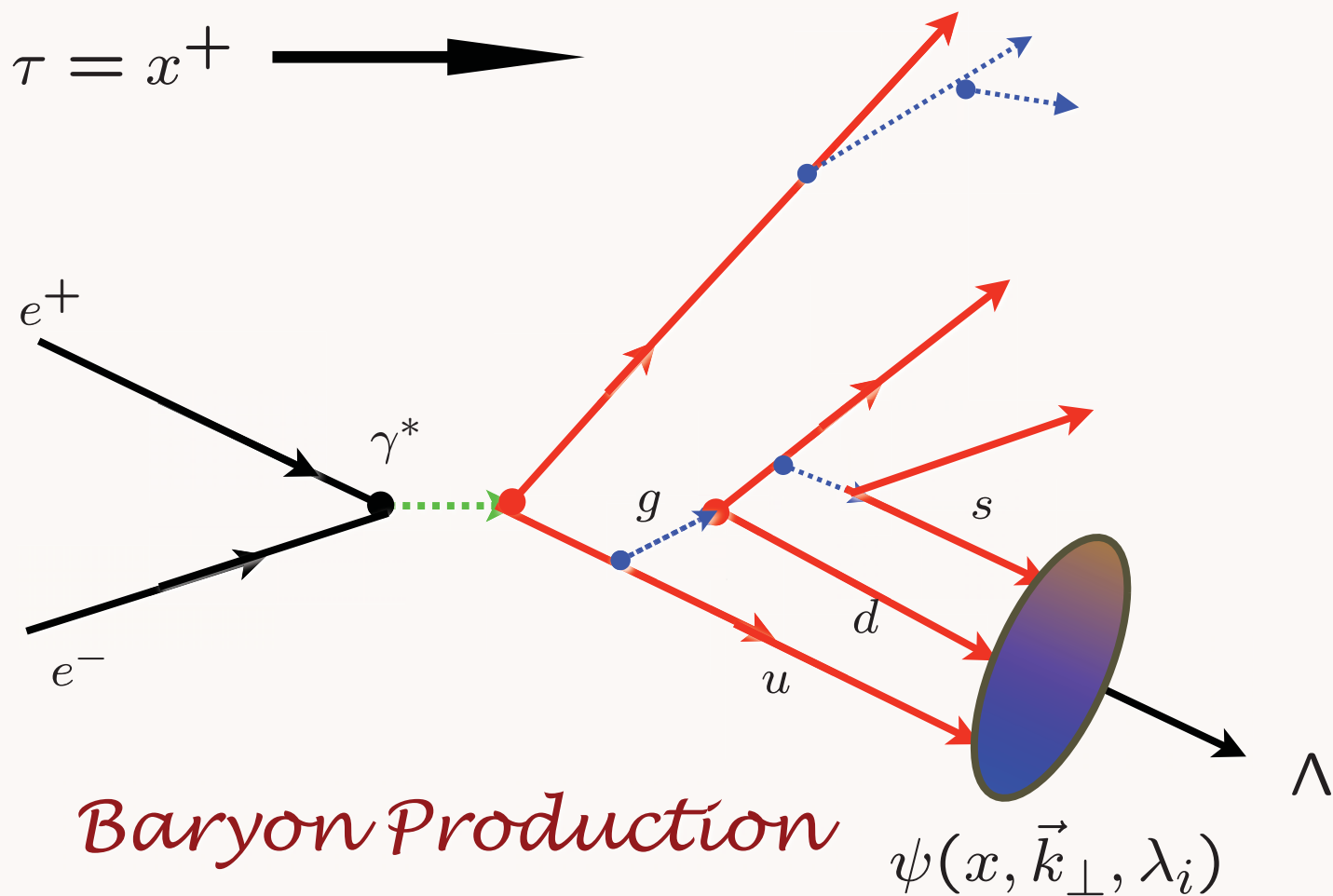
“Hadronization” at the Amplitude Level

Hadronization at the Amplitude Level



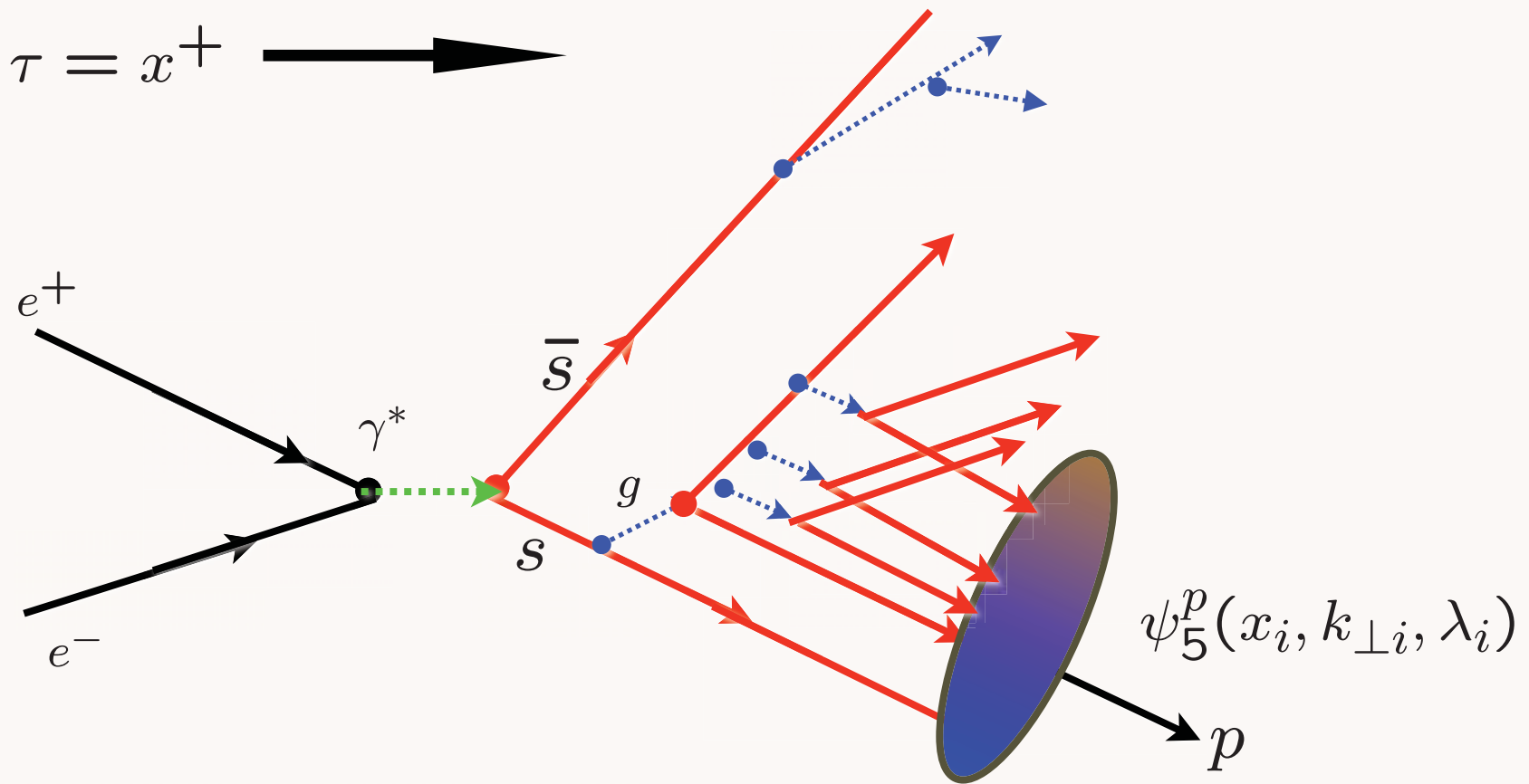
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Higher Fock State Coalescence $|uuds\bar{s}\rangle$

Asymmetric Hadronization! $D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z)$

B-Q Ma, sjb

Features of LF T-Matrix Formalism

“Event Amplitude Generator”

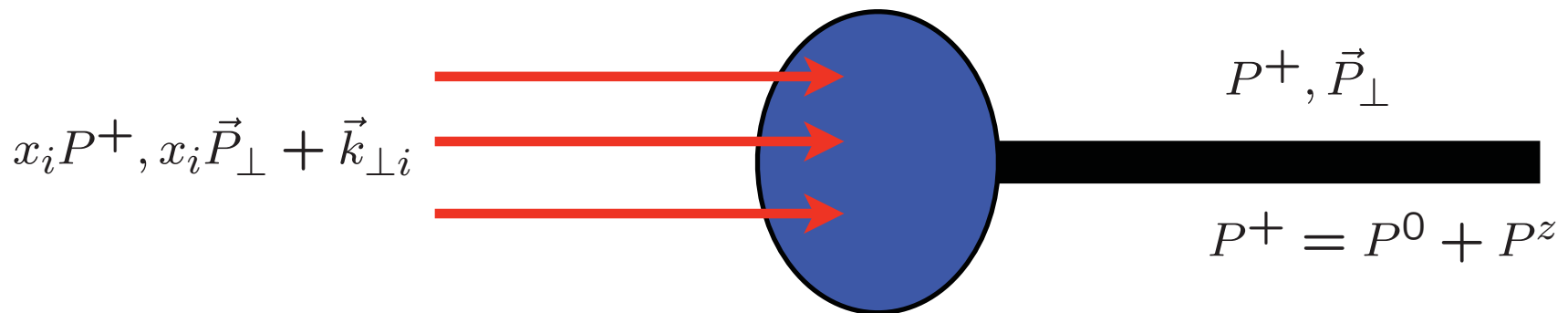
- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^n \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction

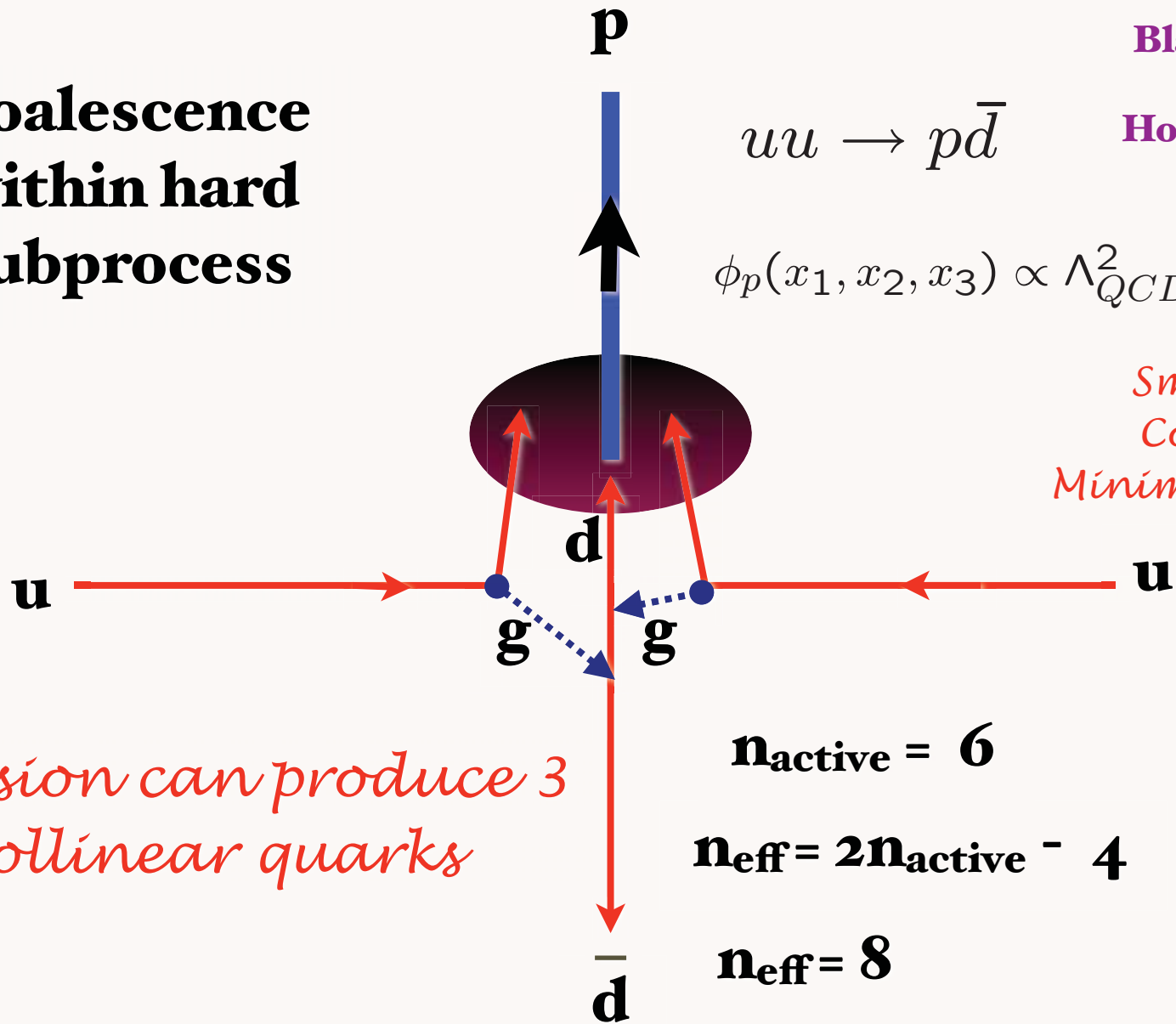
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- No IR divergences: Maximal gluon and quark wavelength from confinement



Baryon can be made directly within hard subprocess

**Coalescence
within hard
subprocess**



$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

**Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive**

Sickles; sjb

*Small color-singlet
Color Transparent
Minimal same-side energy*

*Collision can produce 3
collinear quarks*

$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

*Explains
Baryon
anomaly
 $qq \rightarrow B\bar{q}$*

Direct Subprocesses

- Explains Drell Yan polarization at high x_F
- Hadrons produced without jet hadronization
- Explains power-laws at fixed x_T
- Energy efficient; minimal x_1, x_2 ; large rate
- Color Transparent; Explains Baryon-Anomaly in Heavy-Ion collisions; change of power with centrality; depletion of same-side yield

*Crucial Test of Leading -Twist QCD:
Scaling at fixed x_T*

$$x_T = \frac{2p_T}{\sqrt{s}}$$

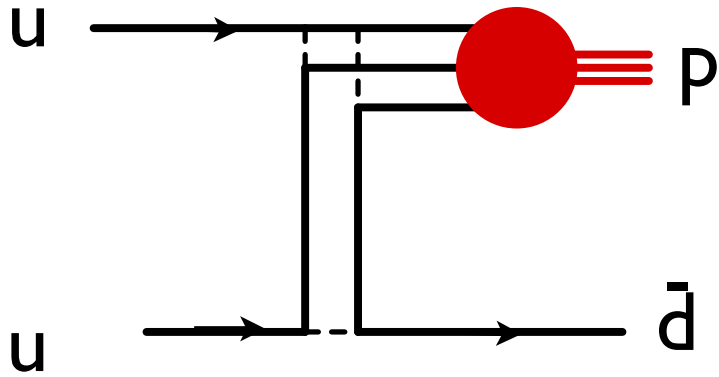
$$E \frac{d\sigma}{d^3p} (pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2 n_{active} - 4$

Direct Proton Production

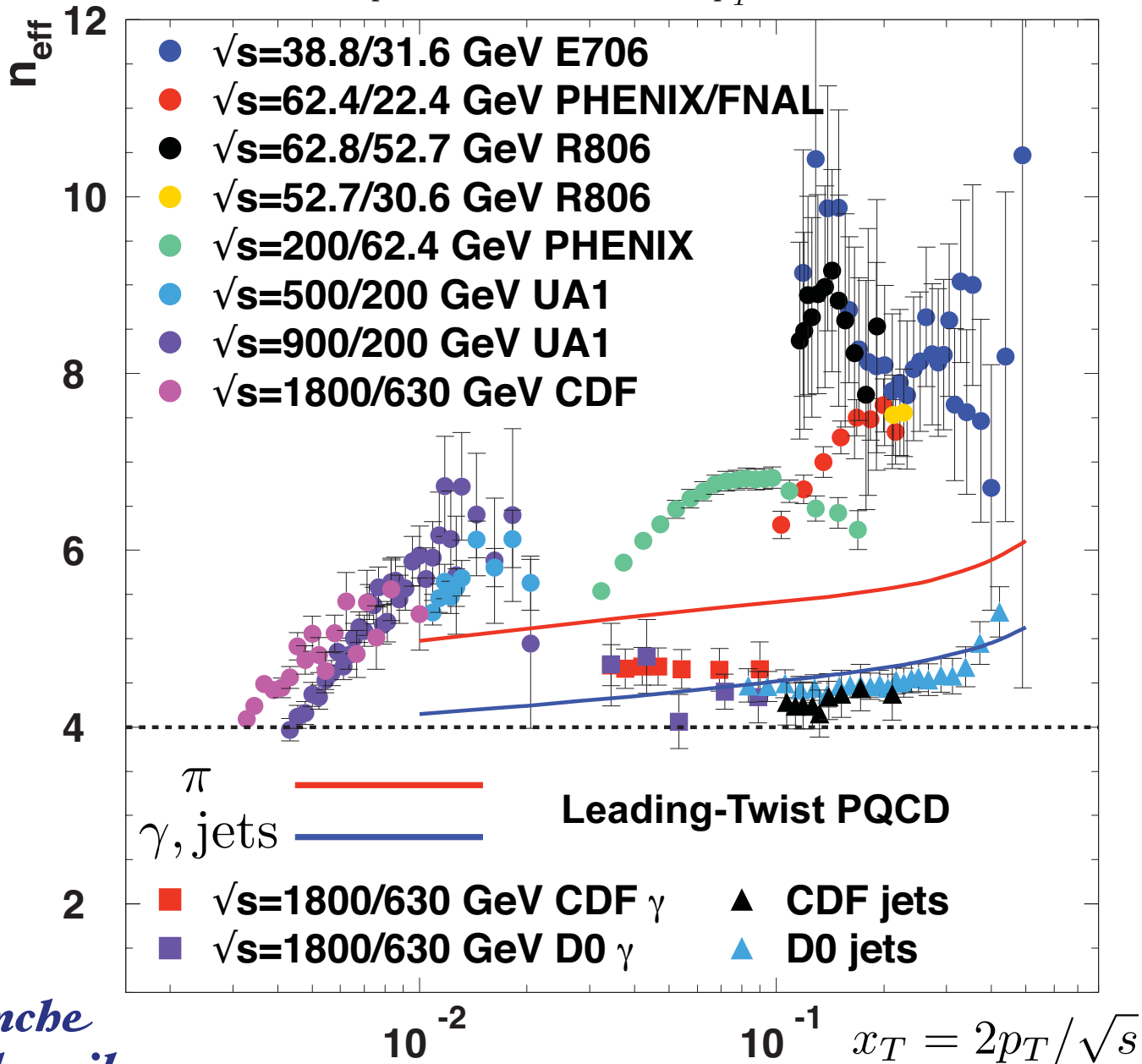


$$n_{\text{active}} = 6$$

$$E \frac{d\sigma}{d^3p} (p p \rightarrow p X) \sim \frac{F(x_{\perp}, v^{\text{cm}})}{p_{\perp}^8}$$

Explains “Baryon anomaly” at RHIC

$$E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\text{eff}}}}$$



*Arleo, Aurenche
Hwang, Sickles, sjb*

Baryon 2010
December 7, 2010

Applications of Light-Front Holography

94

Stan Brodsky
SLAC

Other Baryon Topics

- In-hadron condensates: property of higher Fock states; GMOR satisfied; eliminates 10^{46} conflict with dark energy measurements
- Intrinsic heavy quarks: high- x_F charm and bottom; solves D0 anomaly; solves nuclear dependence anomaly; new mechanism for high- x_F Higgs production
- hadronization at amplitude level
- direct subprocesses; solves RHIC baryon anomaly
- rescattering: Sivers effect; breakdown of factorization of transverse-momentum sensitive observables
- antishadowing is nonuniversal; flavor specific; solves NuTeV and Drell-Yan anomalies
- Reggeon saturation at large t ; $J=0$ fixed pole in Compton scattering
$$\alpha_R(t) \rightarrow -n \text{ at } t \rightarrow -\infty$$

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(\text{expt})$$

$$(\Omega_\Lambda)_{QCD} \propto \langle 0|q\bar{q}|0 \rangle^4$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb

arXiv:0905.1151 [hep- th], Proc. Nat'l. Acad. Sci., (in press);
“Condensates in Quantum Chromodynamics and the Cosmological Constant.”

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,^{1,2} Craig D. Roberts,^{3,4} Robert Shrock,⁵ and Peter C. Tandy⁶

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²*Centre for Particle Physics Phenomenology: CP³-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

³*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

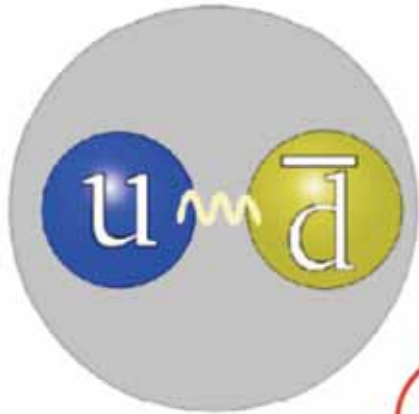
⁴*Department of Physics, Peking University, Beijing 100871, China*

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⁶*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.



Gell-Mann - Oakes - Renner Relation (1968)

$$f_{\pi}^2 m_{\pi}^2 = -2 m(\zeta) \langle \bar{q}q \rangle_0^{\zeta}$$

- ❑ Pion's leptonic decay constant, mass-dimensioned observable which describes rate of process $\pi^+ \rightarrow \mu^+ \nu$
- ❑ *Vacuum quark condensate*

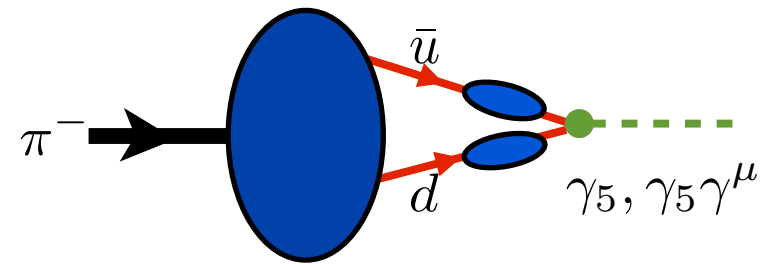
How is this expression modified and interpreted in a theory with confinement?

Bethe-Salpeter Analysis

Maris,
Roberts, Tandy

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

f_H Meson Decay Constant
 T_H flavor projection operator,
 $Z_2(\Lambda), Z_4(\Lambda)$ renormalization constants
 $\mathcal{S}(p)$ dressed quark propagator
 $\Gamma_H(q; P) = F.T. \langle H | \psi(x_a) \bar{\psi}(x_b) | 0 \rangle$
 Bethe-Salpeter bound-state vertex amplitude.



$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q} \rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

In-Hadron Condensate!

$$i\rho^\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$

$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q$$

$$m_\pi^2 \propto (m_q + m_{\bar{q}}) / f_\pi \quad \text{G-MOR}$$

$$f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta$$

In-meson condensate

Maris & Roberts

[nucl-th/9708029](https://arxiv.org/abs/nucl-th/9708029)

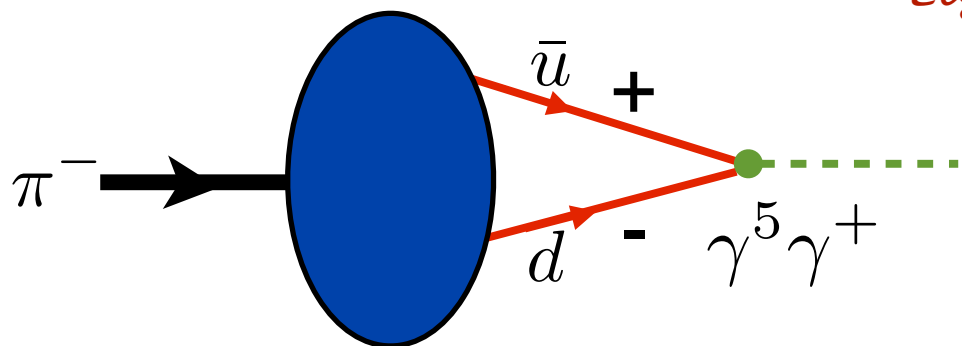
- Pseudoscalar projection of pion's Bethe-Salpeter wavefunction onto the origin in configuration space
 - or the pseudoscalar pion-to-vacuum matrix element

$$\begin{aligned} i\rho_\pi &= -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle \\ &= Z_4(\zeta, \Lambda) \text{tr}_{\text{CD}} \int \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+) \Gamma_\pi(q; P) S(q_-) \end{aligned}$$

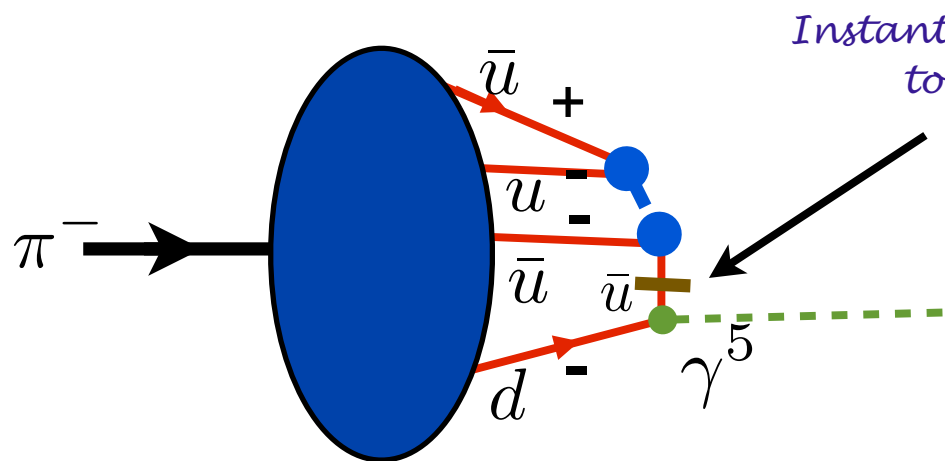
- Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.
 - For arbitrary current-quark masses
 - For any pseudoscalar meson

Higher Light-Front Fock State of Pion Simulates DCSB

Light Front Fock state Analysis

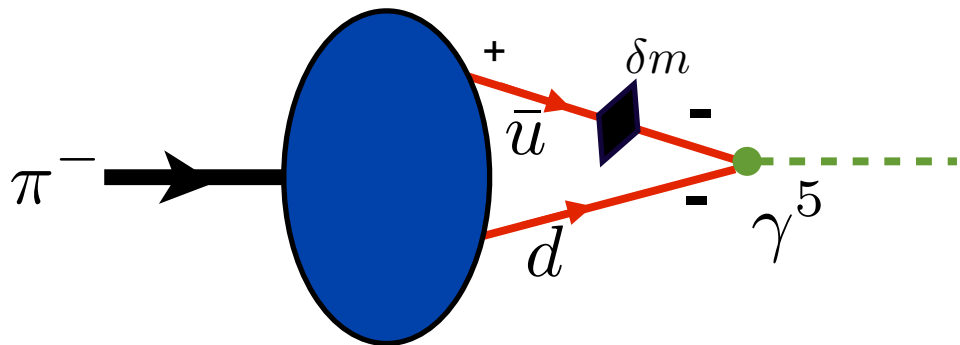


$$f_\pi P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle$$



Instantaneous quark propagator contribution to π derived from higher Fock state

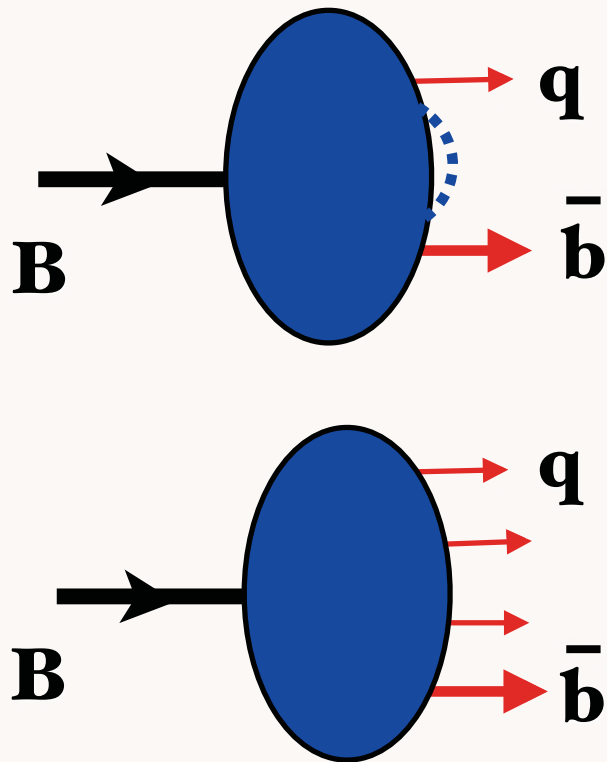
$$i\rho_\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$



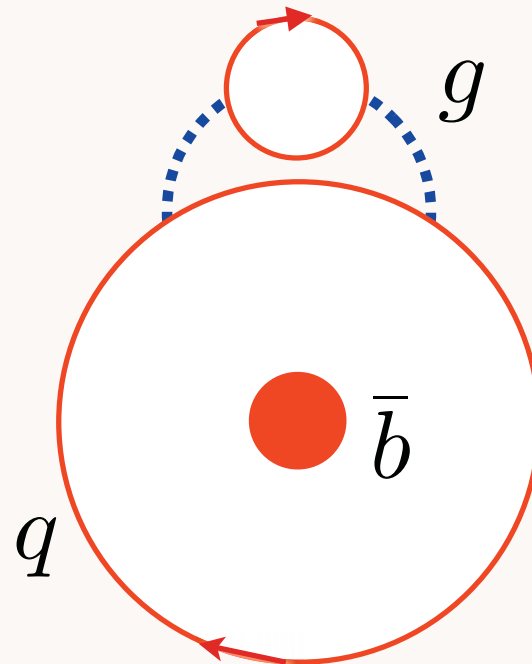
Higher Fock state acts like mass insertion

*Simple physical argument for
"in-hadron" condensate*

Roberts, Shrock, Tandy, sjb



Gribov pairs

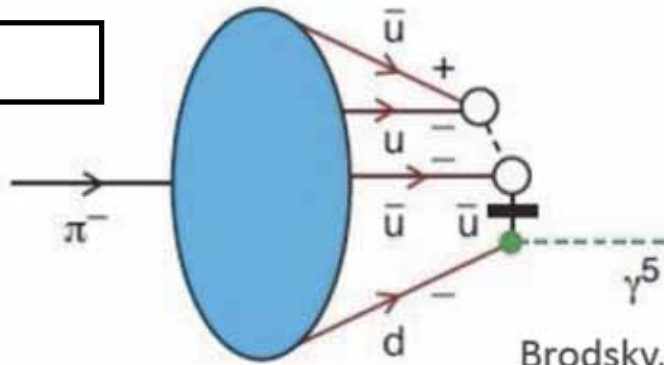


B-Meson

*Use Dyson-Schwinger Equation for bound-state quark propagator: find
confined condensate*

$$\langle B | \bar{q}q | B \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle$$

Paradigm shift: In-Hadron Condensates



Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C82 (Rapid Comm.) (2010) 022201
 Brodsky and Shrock, arXiv:0905.1151 [hep-th], to appear in PNAS

Resolution

- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.
- *So-called vacuum condensates* can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
- No qualitative difference between f_π and ρ_π

$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H$$

$$i\rho^\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$

Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

-0.005 ± 0.003 from τ decay.

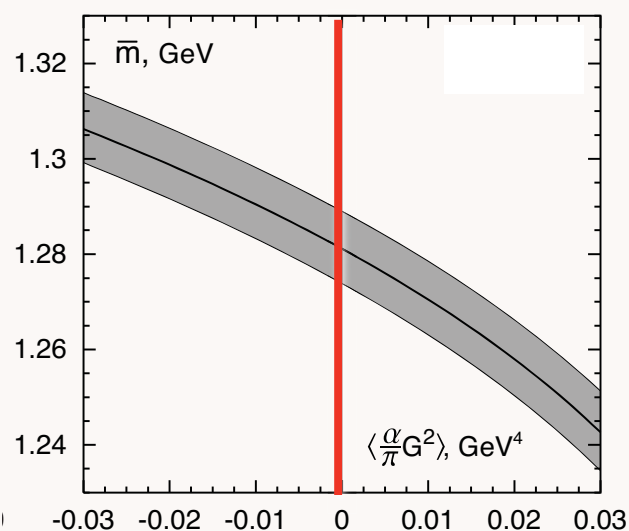
Davier et al.

$+0.006 \pm 0.012$ from τ decay.

Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



*Consistent with zero
vacuum condensate*

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

String Theory



AdS/CFT

Mapping of Poincare' and Conformal $SO(4,2)$ symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

Conformal behavior at short distances
+ Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$ plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS₅**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite $N_c = 3$: Baryons built on 3 quarks -- Large N_c limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter κ**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**