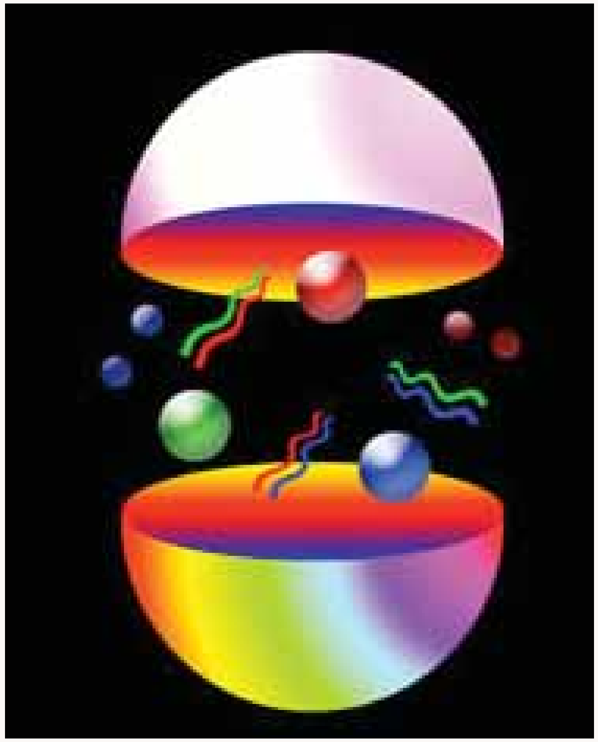
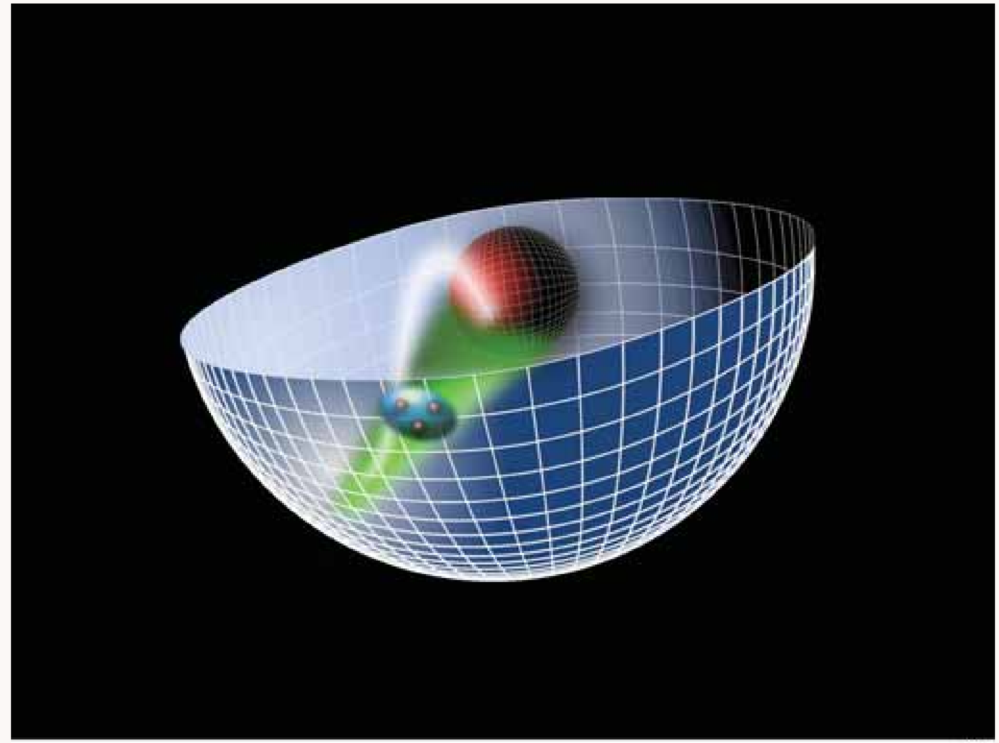
# New Insights for Hadron Physics from AdS/QCD


Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

$CP^3$ - Origins
Particle Physics & Origin of Mass



University of Oslo

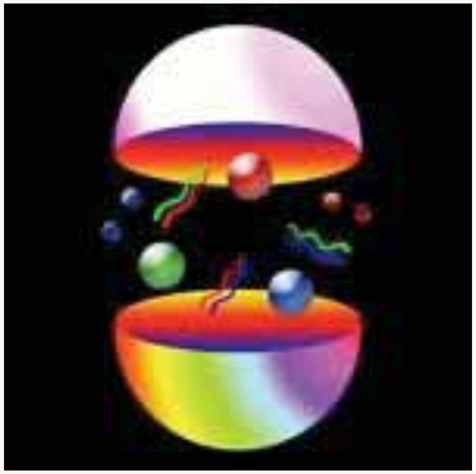
**May 21, 2010**

# *The World of Quarks and Gluons:*

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei

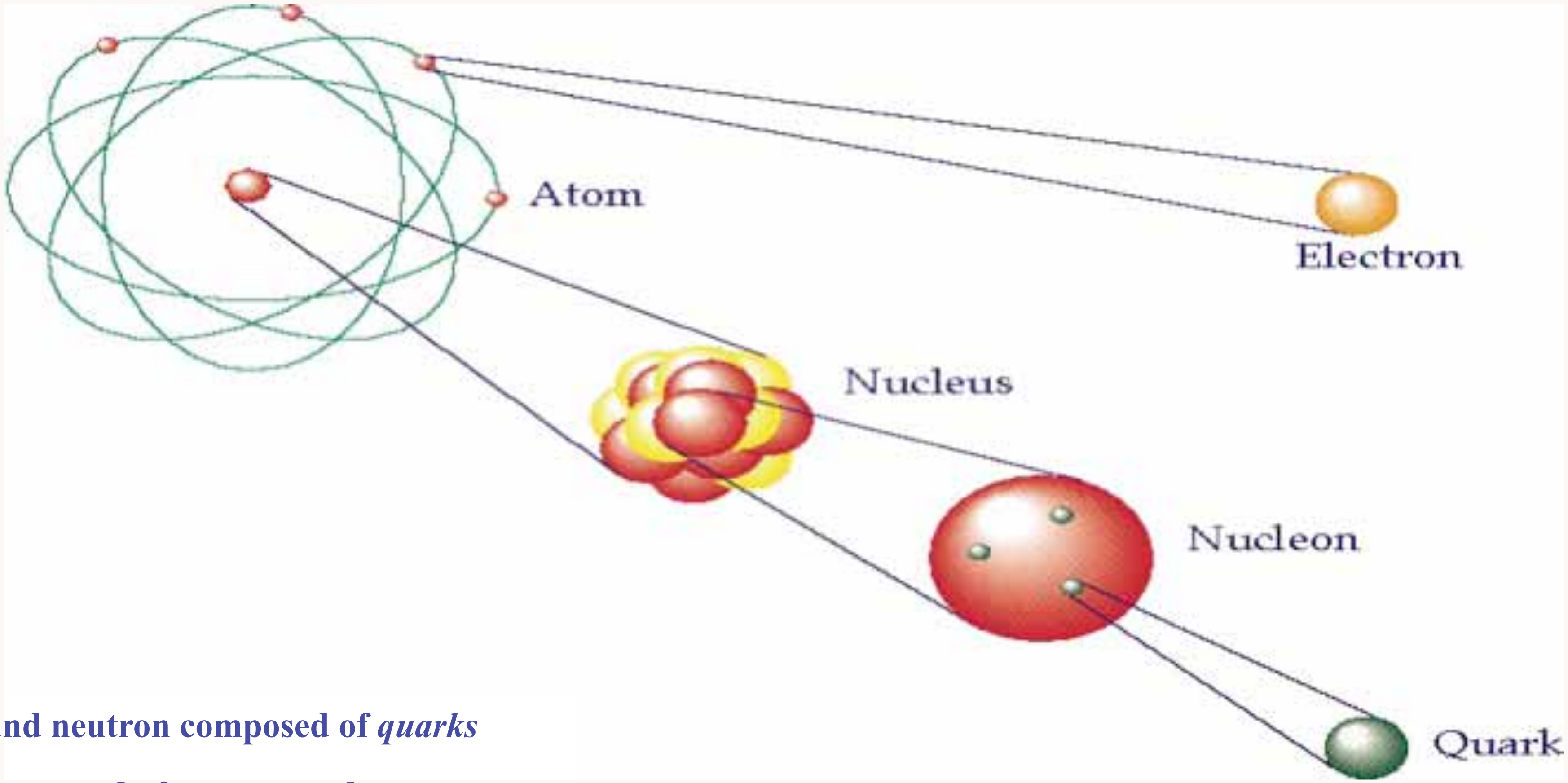
- Remarkable and novel properties of *Quantum Chromodynamics (QCD)*

- New Insights from higher space-time dimensions: Holography: AdS/CFT



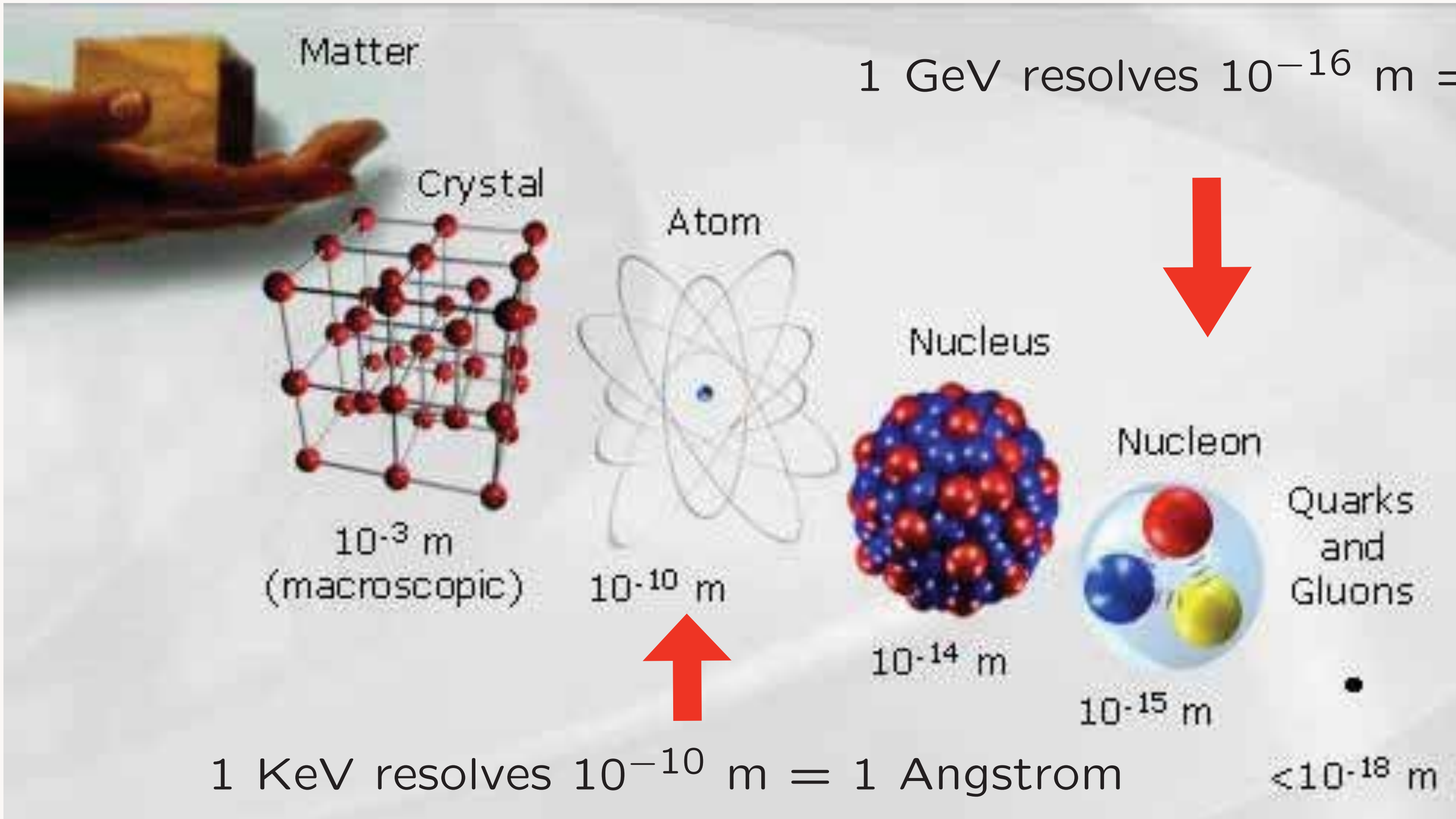
# Goal of Science:

## *To understand the laws of physics and the fundamental composition of matter at the shortest possible distances.*




- **Proton and neutron composed of *quarks***
- **Nuclei composed of protons and neutrons**
- **Atoms composed of nuclei and electrons ...**

# Searching for the Ultimate Constituents




1 GeV resolves $10^{-16}$ m = 0.1 fm

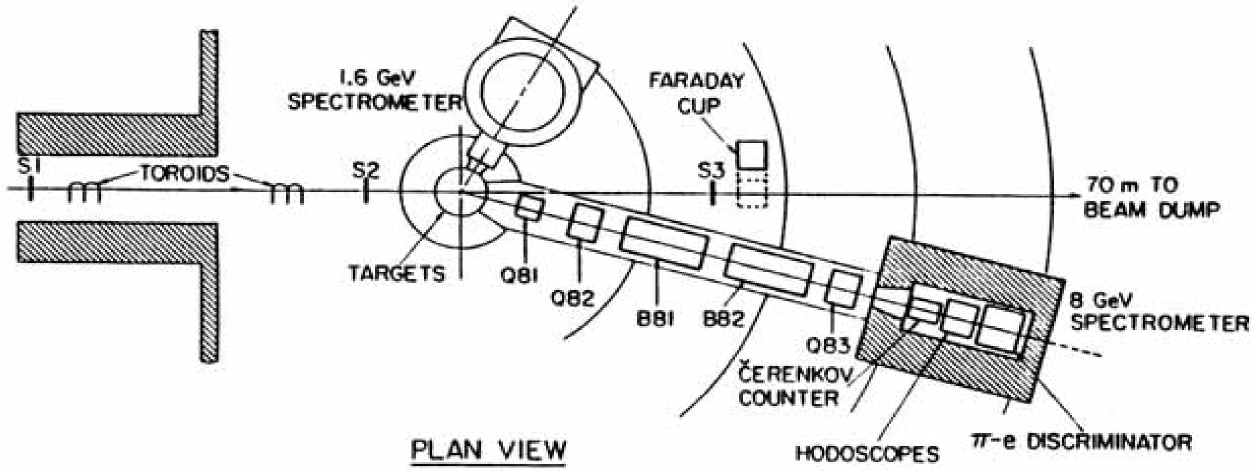
1 KeV resolves $10^{-10}$ m = 1 Angstrom

*Electrons, Quarks, and Gluons may be truly pointlike!*

1 TeV resolves $10^{-19}$ m = 0.0001 fm

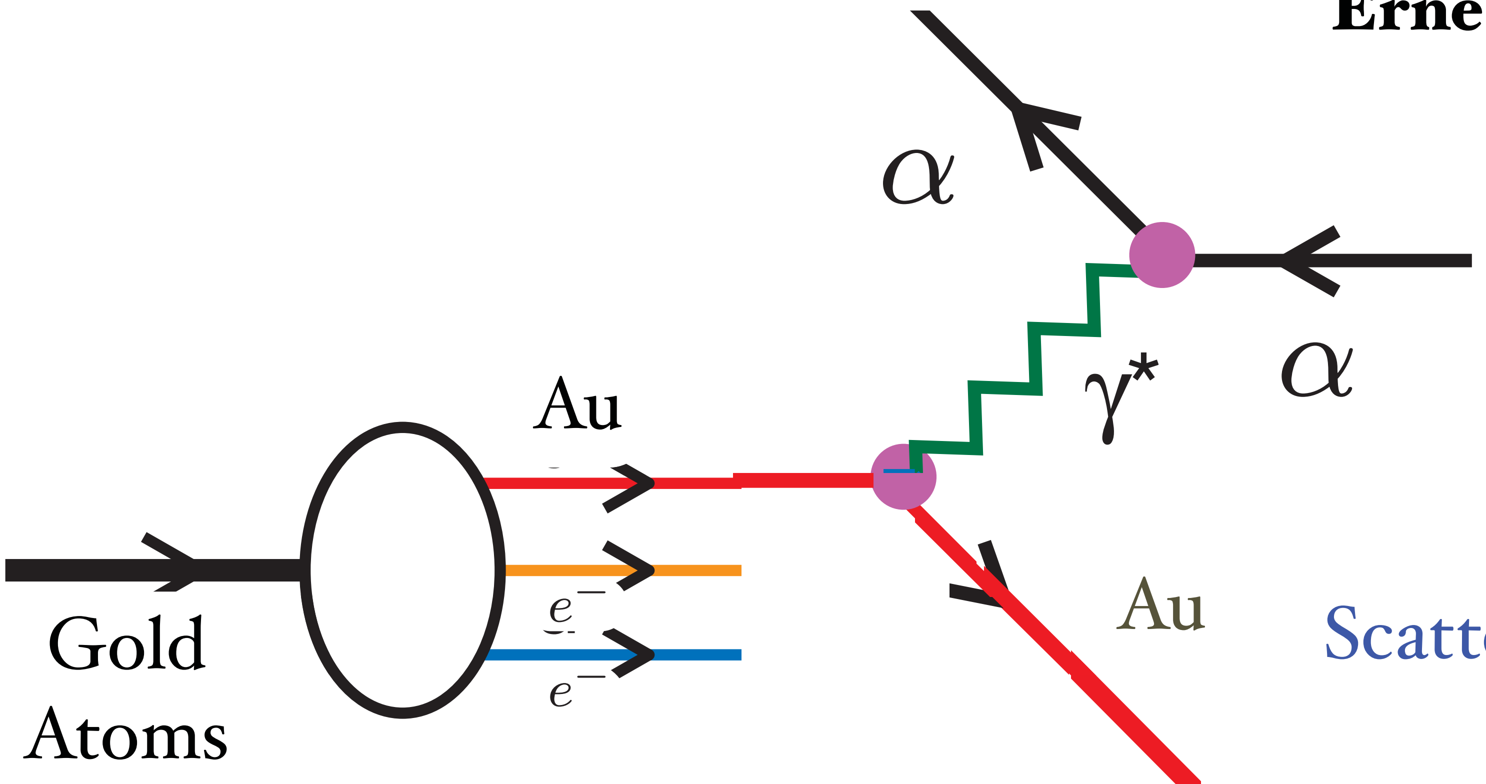
# SLAC Two-Mile Linear Accelerator

**Pief**



# First Evidence for Nuclear Structure of Atoms

**Ernest Rutherford**
**1911**

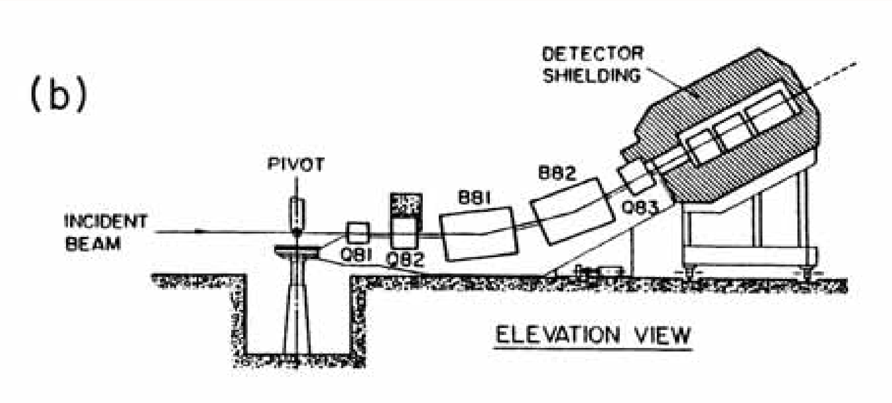


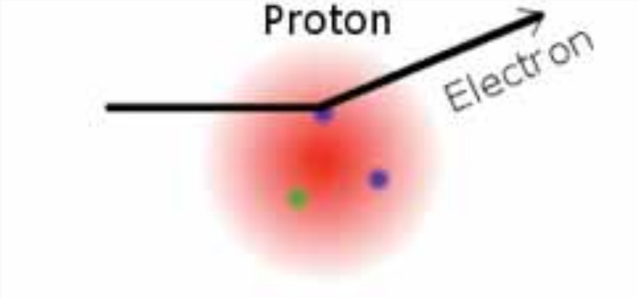

Scattering at Large Angles!
"Point-like" Nucleus

*Rutherford Scattering*

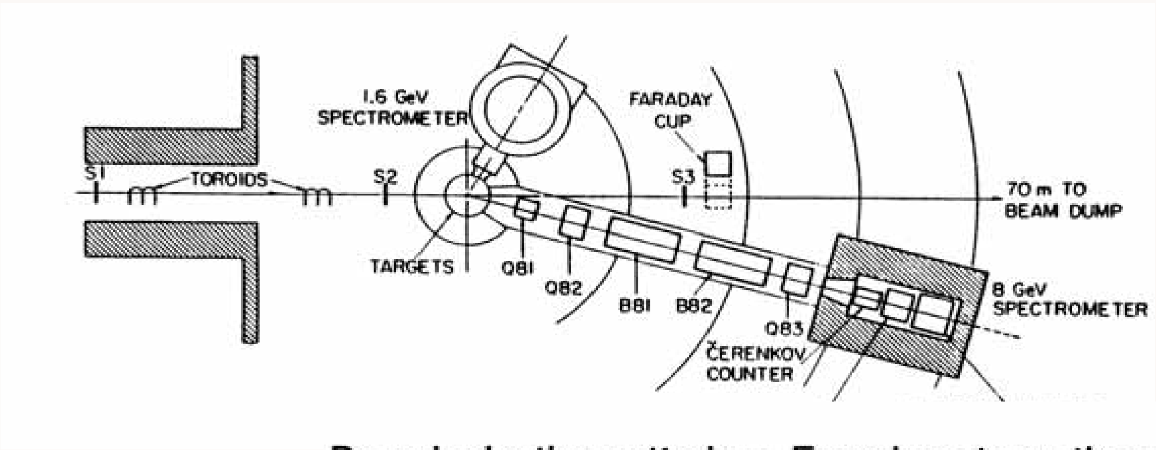
# 1967 SLAC Experiment:
## Scatter 20 GeV/c Electrons on protons in a Hydrogen Target
## Discovery of the Quark Structure of Matter

$$ep \to e'X$$

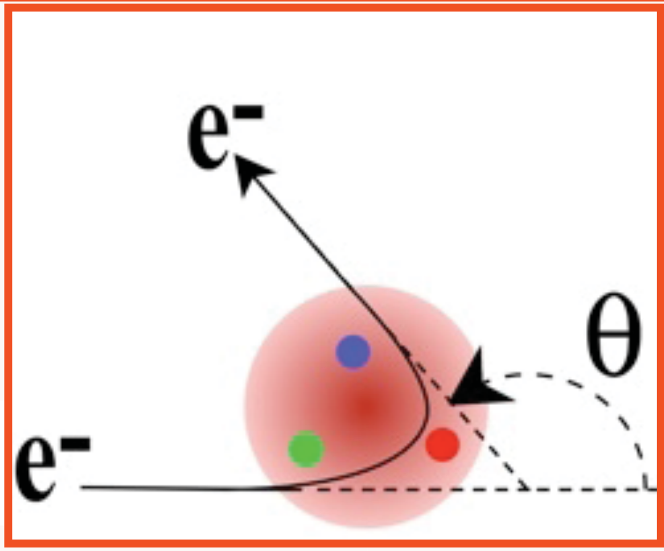



**Discovery of quarks!**

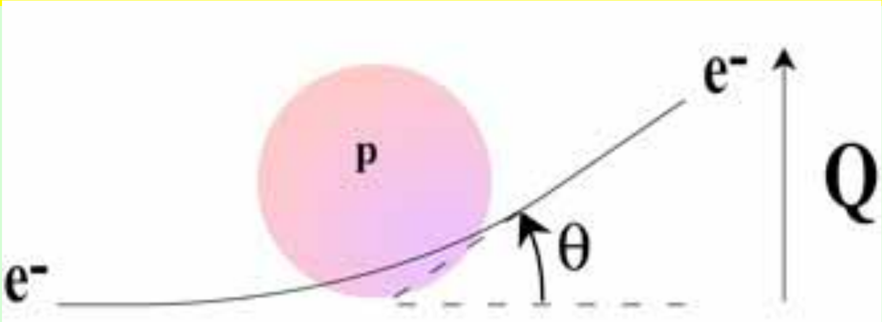



Deep inelastic scattering: Experiments on the proton and the observation of scaling



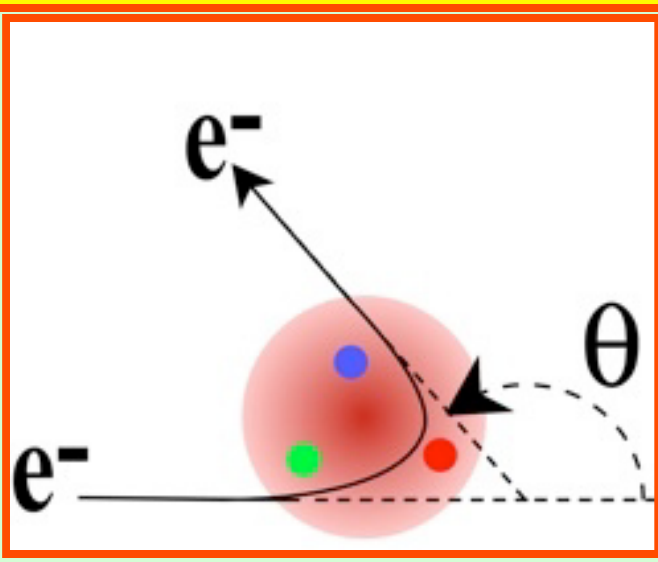

Friedman, Kendall, Taylor: Nobel Prize

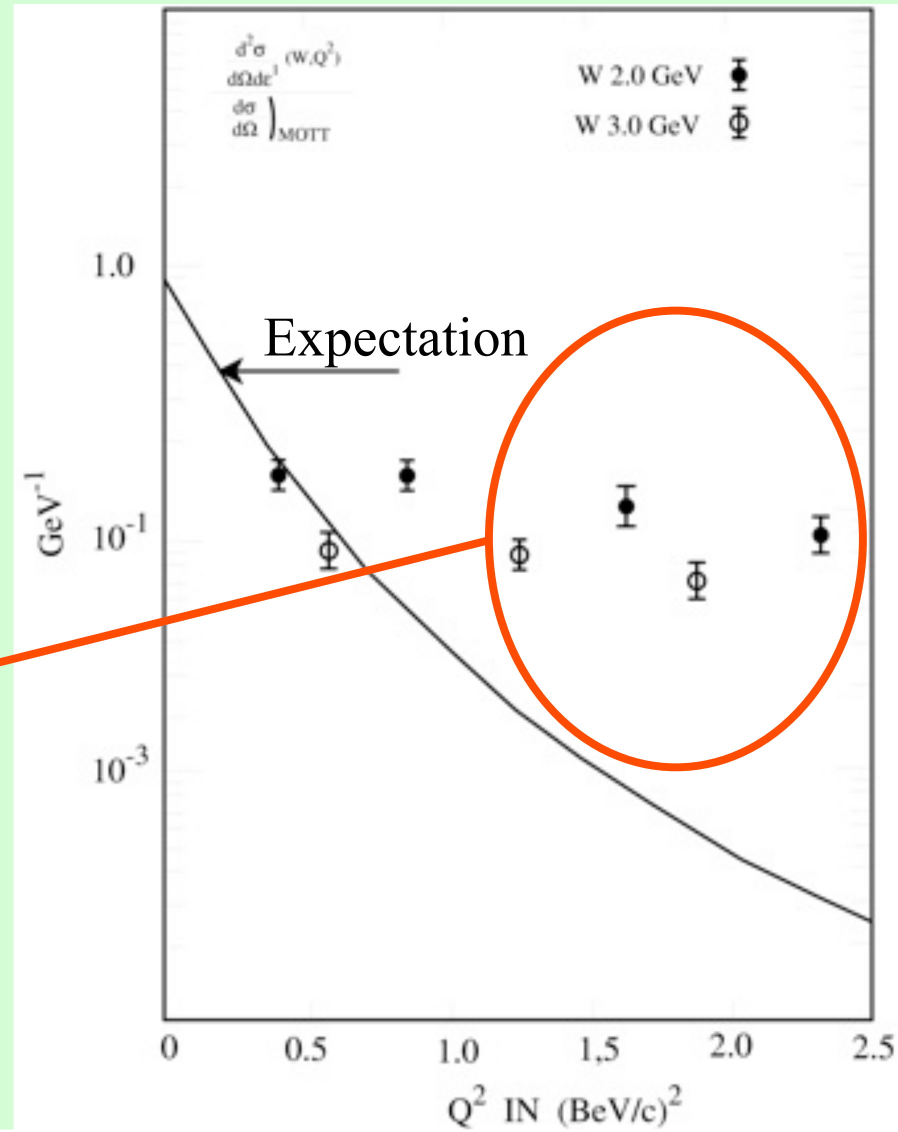
# Deep inelastic electron-proton scattering



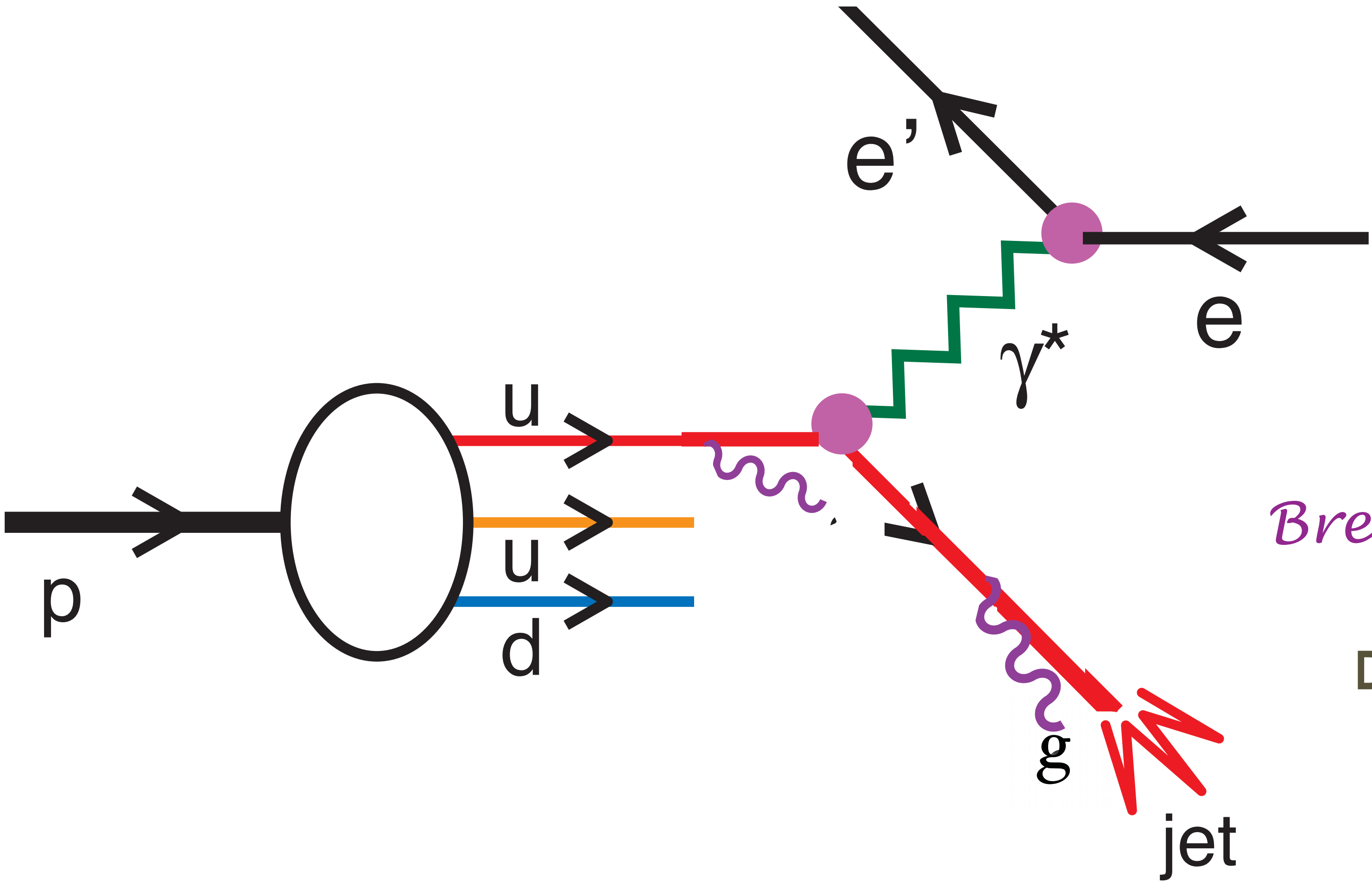

- Rutherford scattering using *very* high-energy electrons striking protons

**Discovery of quarks!**



# First Evidence for Quark Structure of Matter

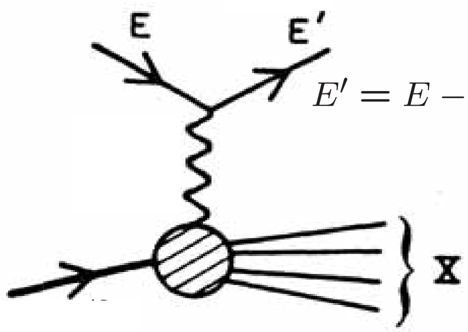



Gluonic Bremmstrahlung

DGLAP Evolution

Deep Inelastic Electron-Proton Scattering

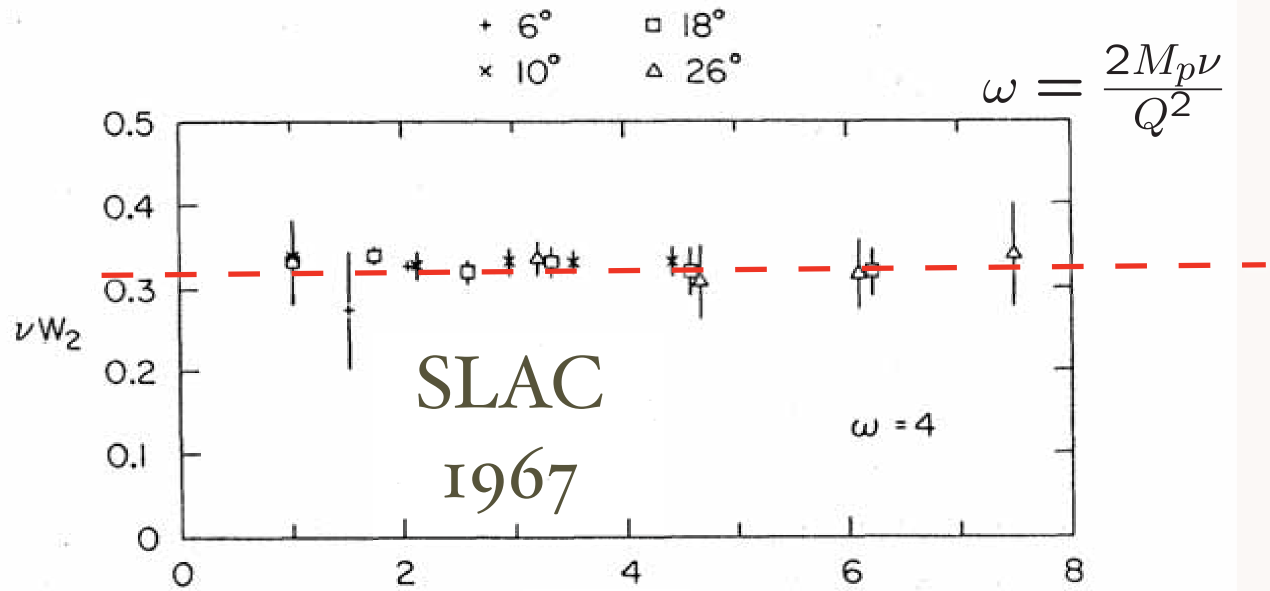
$$ep \to e'X$$




$E' = E - \nu, \vec{q}$

p

$$Q^2 = \vec{q}^2 - \nu^2$$




$Q^2$

$$\omega = \frac{2M_p\nu}{Q^2}$$

*No intrinsic length scale !*

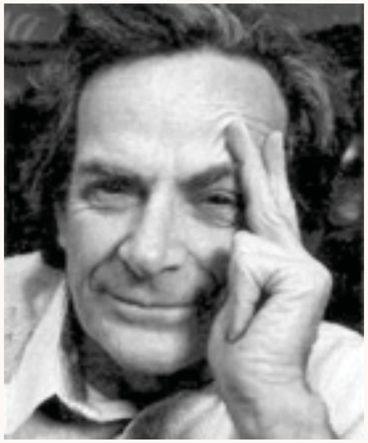
Measure rate as a function of energy loss $\nu$ and momentum transfer $Q$

Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

*Discovery of Bjorken Scaling*

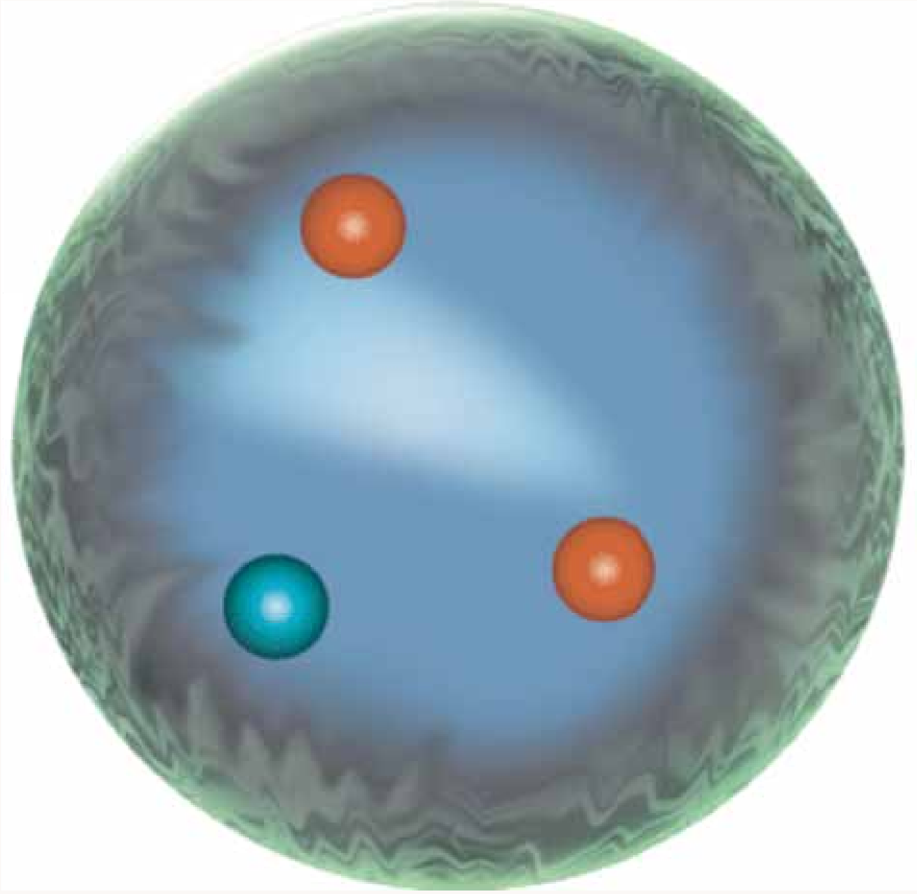
*Electron scatters on point-like quarks!*

# *Quarks in the Proton*



Feynman & Bjorken: "Parton" model

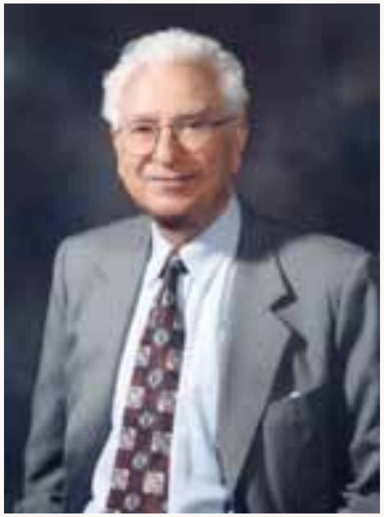
p = (u u d)



$1\,fm$

$10^{-15}m = 10^{-13}cm$

Zweig: "Aces, Deuces, Treys"



Gell Mann: "Three Quarks for Mr. Mark"

# The charges of the matter particles

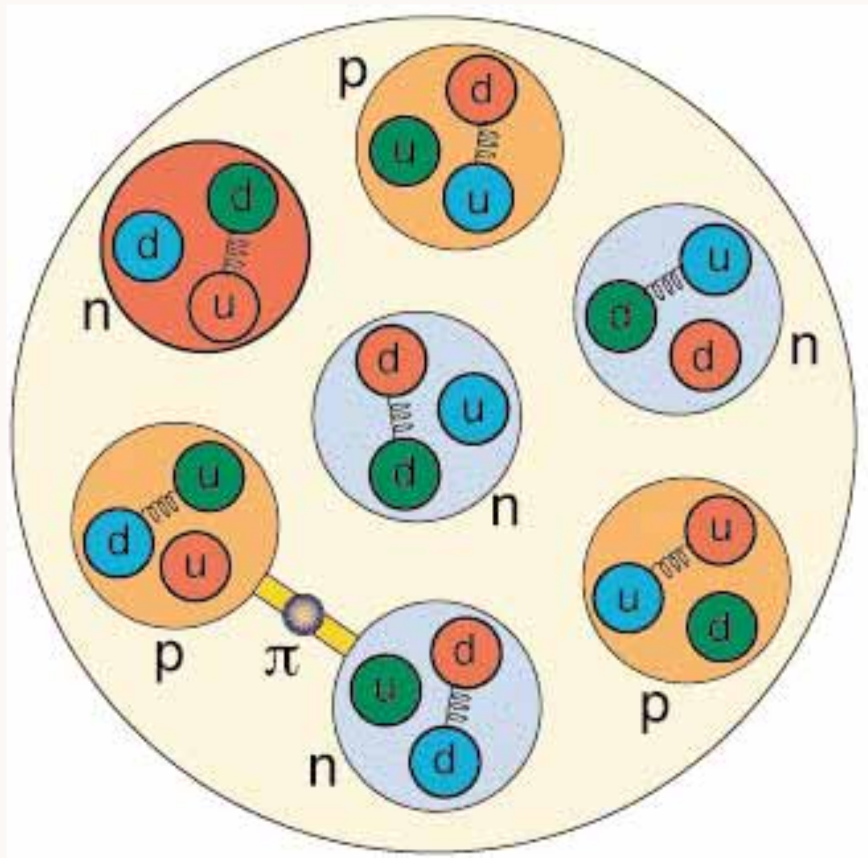
| Charge | -1 | 0 | +2/3 | -1/3 |
|---|---|---|---|---|
| 1st Generation | $e^-$ | $\nu_e$ | u u u | d d d |
| 2nd Generation | $\mu^-$ | $\nu_\mu$ | c c c | s s s |
| 3rd Generation | $\tau^-$ | $\nu_\tau$ | t t t | b b b |

# The Quark Structure of the Nucleus

$$e_u = +\frac{2}{3} \quad e_d = -\frac{1}{3}$$

$$p = (uud)$$

$$n = (ddu)$$




$$2e_u + e_d = e_p$$

$$2e_d + e_u = e_n$$

$$2 \times (+\tfrac{2}{3}) \; + \; 1 \times (-\tfrac{1}{3}) = 1$$

$$2 \times (-\tfrac{1}{3}) \; + \; 1 \times (+\tfrac{2}{3}) = 0$$

# THE PERIODIC TABLE

**Particles like the electron (fermions, spin 1/2)**

| Leptons | | Quarks (each in 3 "colors") | |
|---|---|---|---|
| $e$ 0.511 MeV | $\nu_e$ $< 0.000003$ | $d$ 7 | $u$ 3 |
| $\mu$ 106 | $\nu_\mu$ $< 0.2$ | $s$ 120 | $c$ 1200 |
| $\tau$ 1777 | $\nu_\tau$ $< 20$ | $b$ 4300 | $t$ 175,000 |
| $-1$ | $0$ | $-1/3$ | $2/3$ ← charge |

**Particles like the photon (bosons, spin 1)**

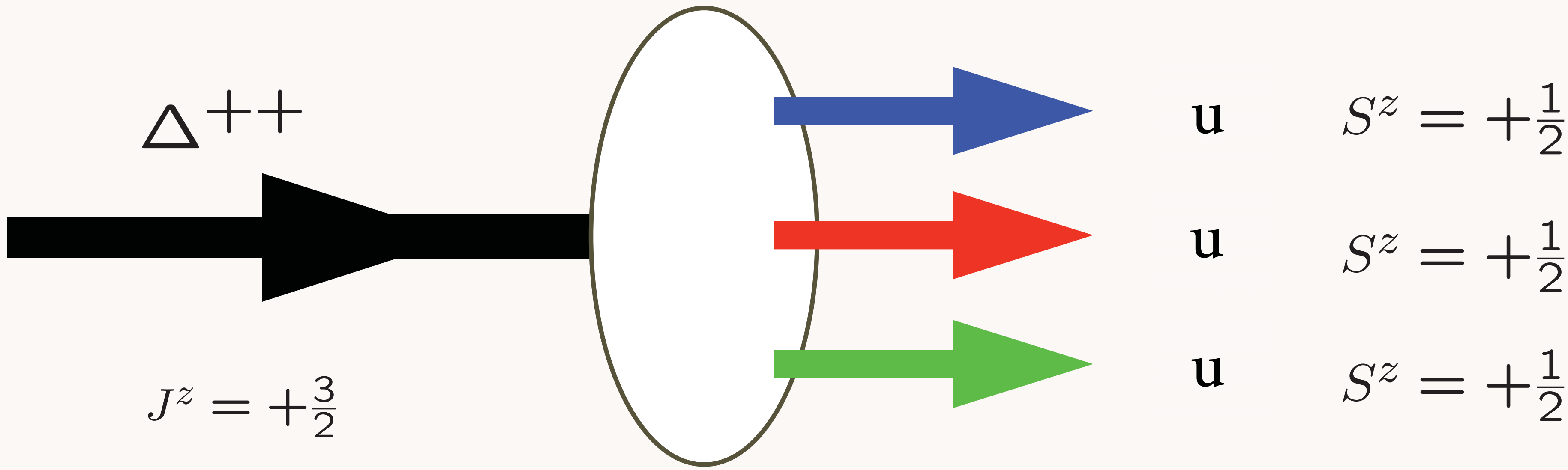
| | | |
|---|---|---|
| $\gamma$ 0 | photon | "electromagnetism" |
| $g$ 0 | gluon (8 "colors") | "strong interaction" |
| $W^{\pm}$ 80,420 | $Z^0$ 91,188 | "weak interaction" |

Why are there three colors of quarks?

**Greenberg**

*Pauli Exclusion Principle!*

spin-half quarks cannot be in same quantum state !

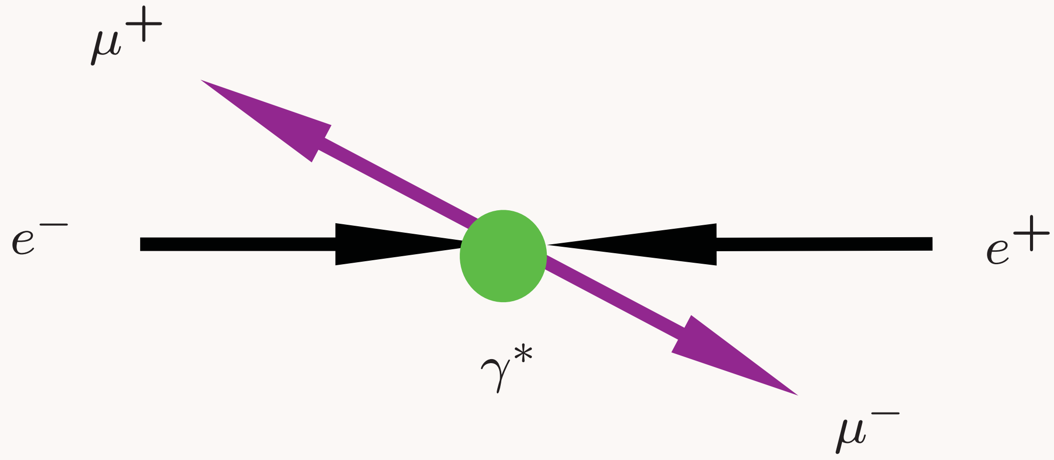



*Three Colors (Parastatistics) Solves Paradox*

**3 Colors Combine : WHITE** $SU(N_C), N_C = 3$

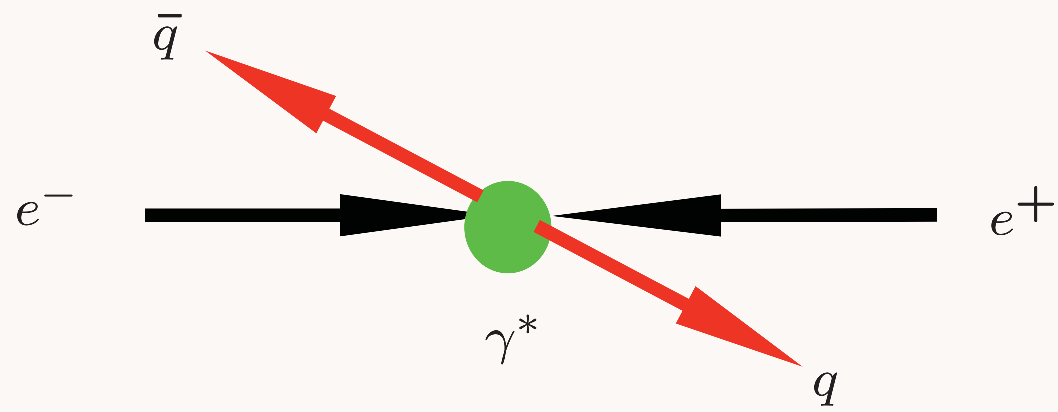
# SPEAR Electron-Positron Collider

# Electron-Positron Annihilation




$$e^+e^- \to \gamma^* \to \mu^+\mu^-$$

# Electron-Positron Annihilation

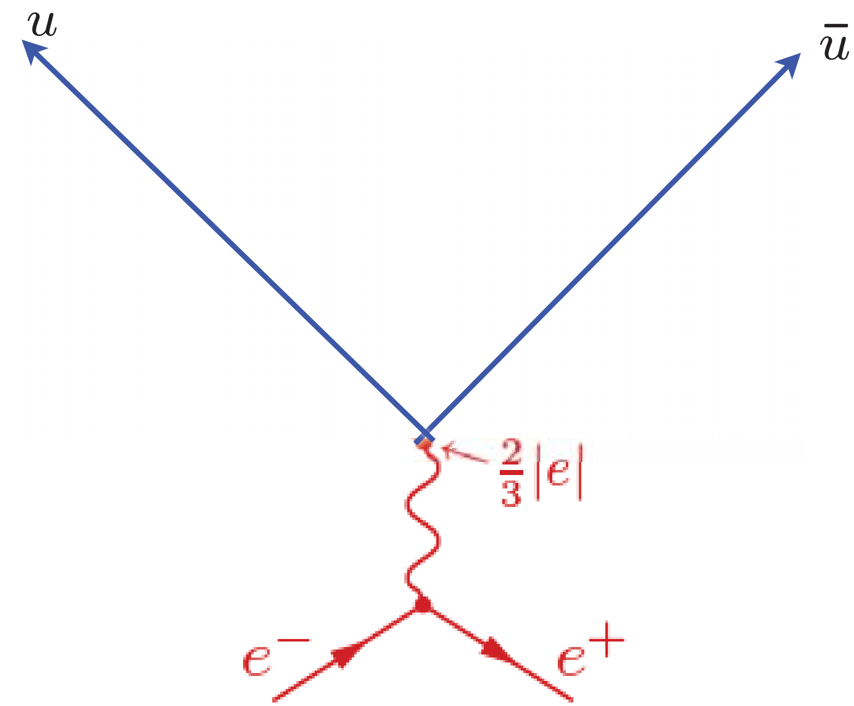



$$e^+e^- \to \gamma^* \to q\bar{q}$$

Rate proportional to quark charge squared and number of colors

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

# How to Count Quarks




*Color-triplet quark representation*

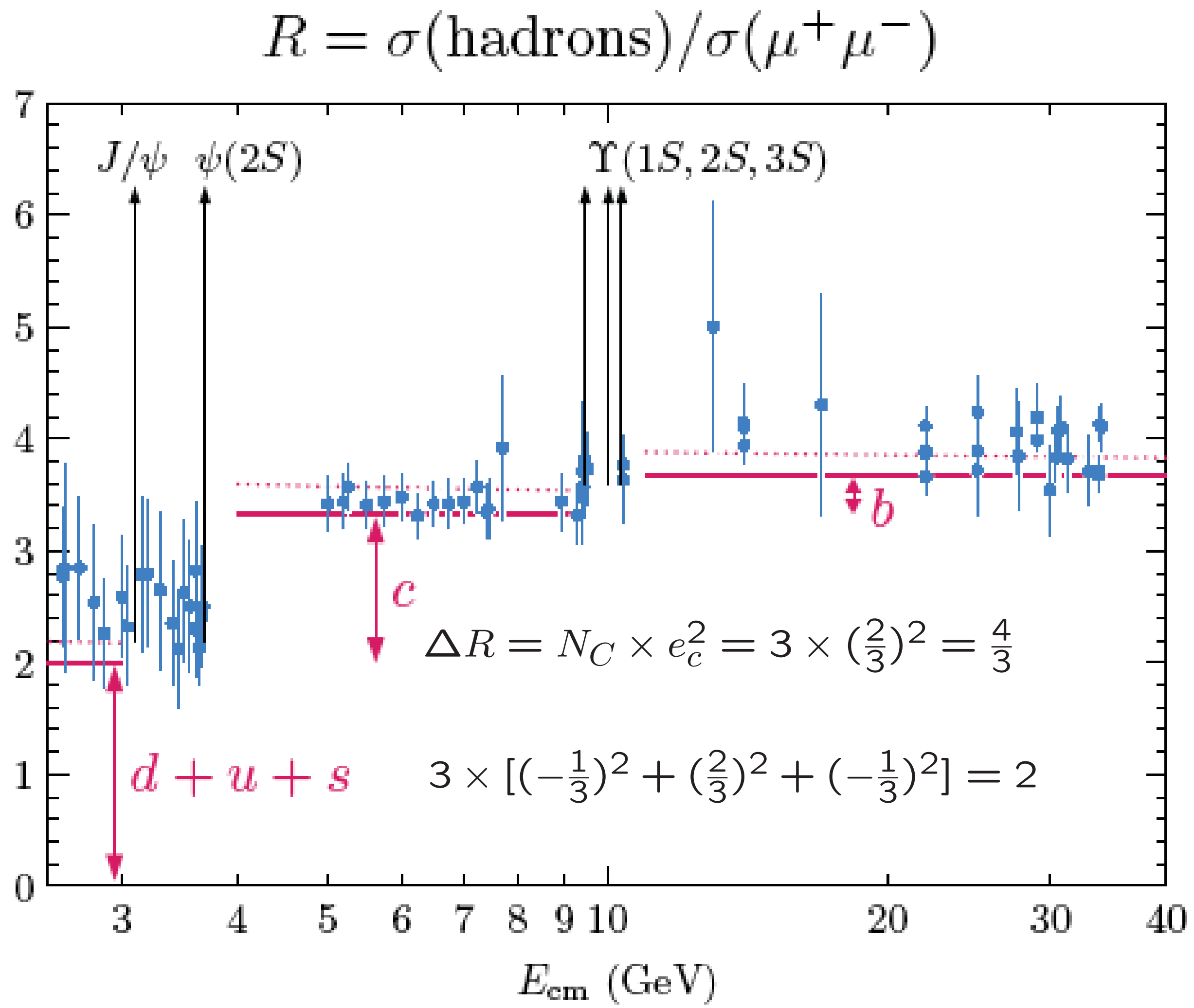
For 10 GeV $< E_{\rm cm} <$ 40 GeV,

$$\frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = \underset{\substack{\uparrow \\ \text{colors}}}{3} \times \left[ \underset{\substack{\uparrow \\ d}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ u}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ s}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ c}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ b}}{\left(\frac{1}{3}\right)^2} \right]$$

$J/\psi = (c\bar{c})_{1S}$

# How to Count Quarks

$\Upsilon = (b\bar{b})_{1S}$

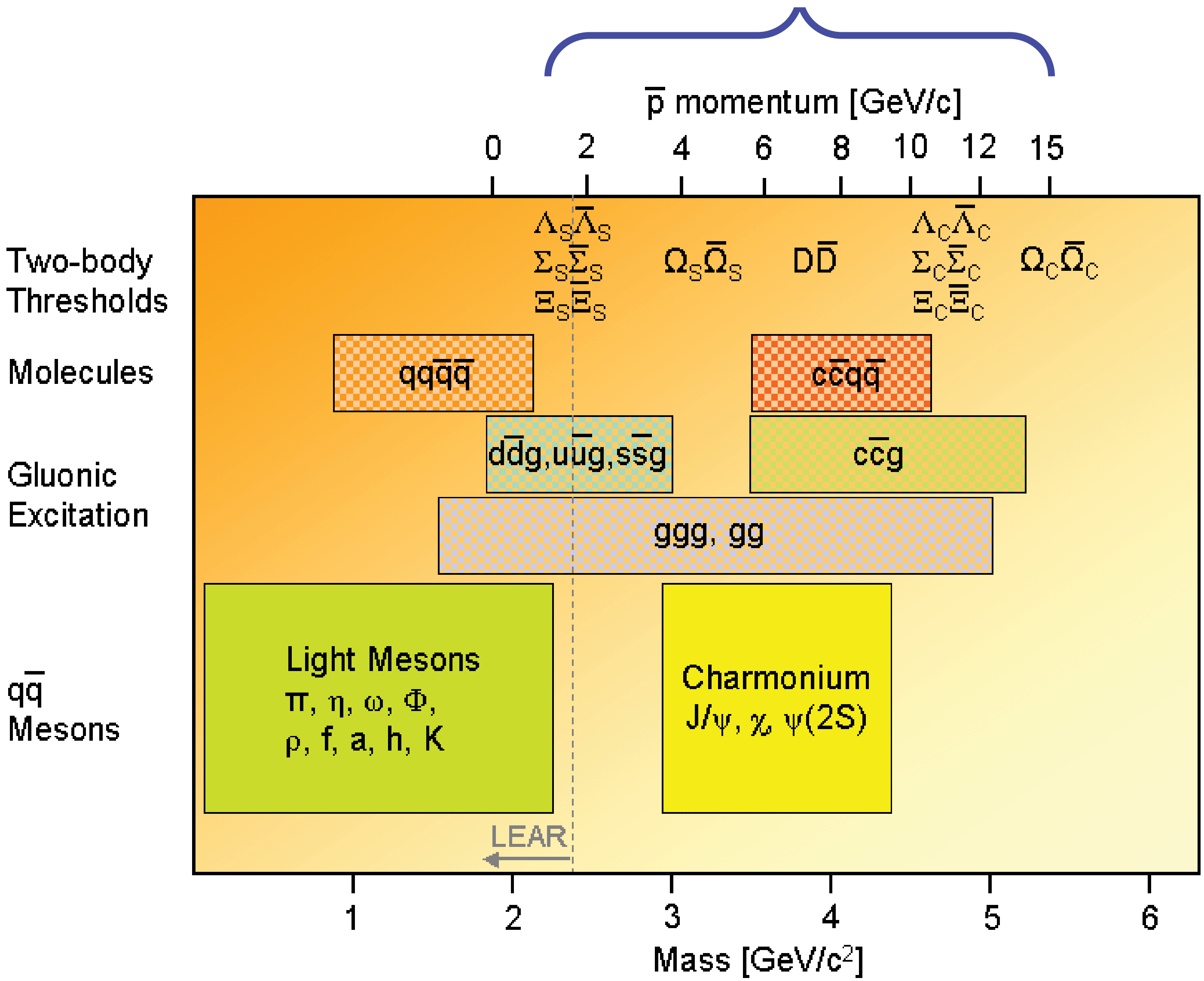



$3 \times (-\frac{1}{3})^2 = \frac{1}{3}$

$N_C = 3$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

# *Mass and Anti-Proton Momentum Range at FAIR*




- Production of open charm
- Charmed hybrids
- Glueballs
- Charmonium

*Michael Düren*

# QCD Lagrangian

gluon dynamics | quark kinetic energy + quark-gluon dynamics | mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

# QED Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4}Tr(F^{\mu\nu}F_{\mu\nu}) + \sum_{\ell=1}^{n_\ell} i\bar{\Psi}_\ell D_\mu \gamma^\mu \Psi_\ell + \sum_{\ell=1}^{n_\ell} m_\ell \bar{\Psi}_\ell \Psi_\ell$$

$$iD^\mu = i\partial^\mu - eA^\mu \qquad F^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu$$

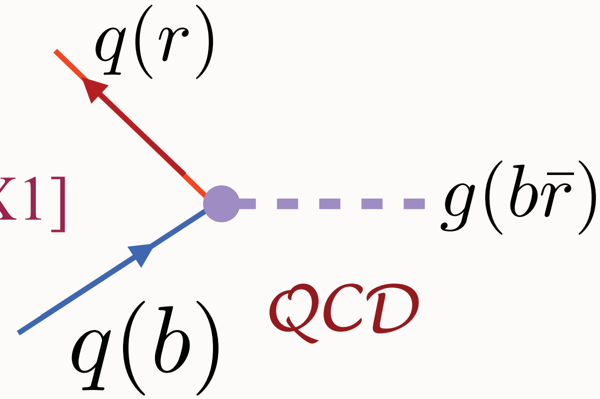
Yang Mills Gauge Principle:
Phase Invariance at Every
Point of Space and Time

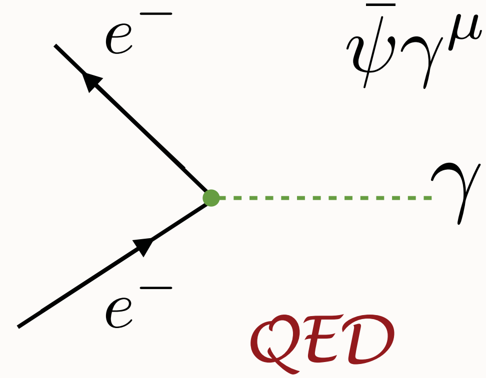
Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Landau Pole

# Fundamental Couplings of QCD

$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

[1X3] [3X3] [3X1]



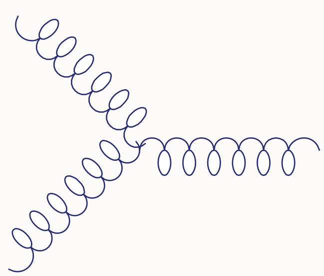



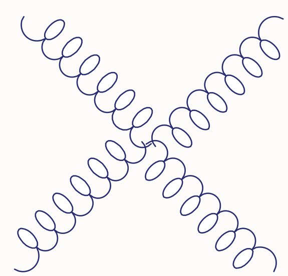

$\bar{\psi}\gamma^{\mu}A^{\mu}\bar{\psi}$

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$G^{\mu\nu} = \partial^{\mu}A^{\mu} - \partial^{\nu}A^{\mu} - g[A^{\mu}, A^{\nu}]$$

Gluon vertices





$G^{\mu\nu}G_{\mu\nu}$

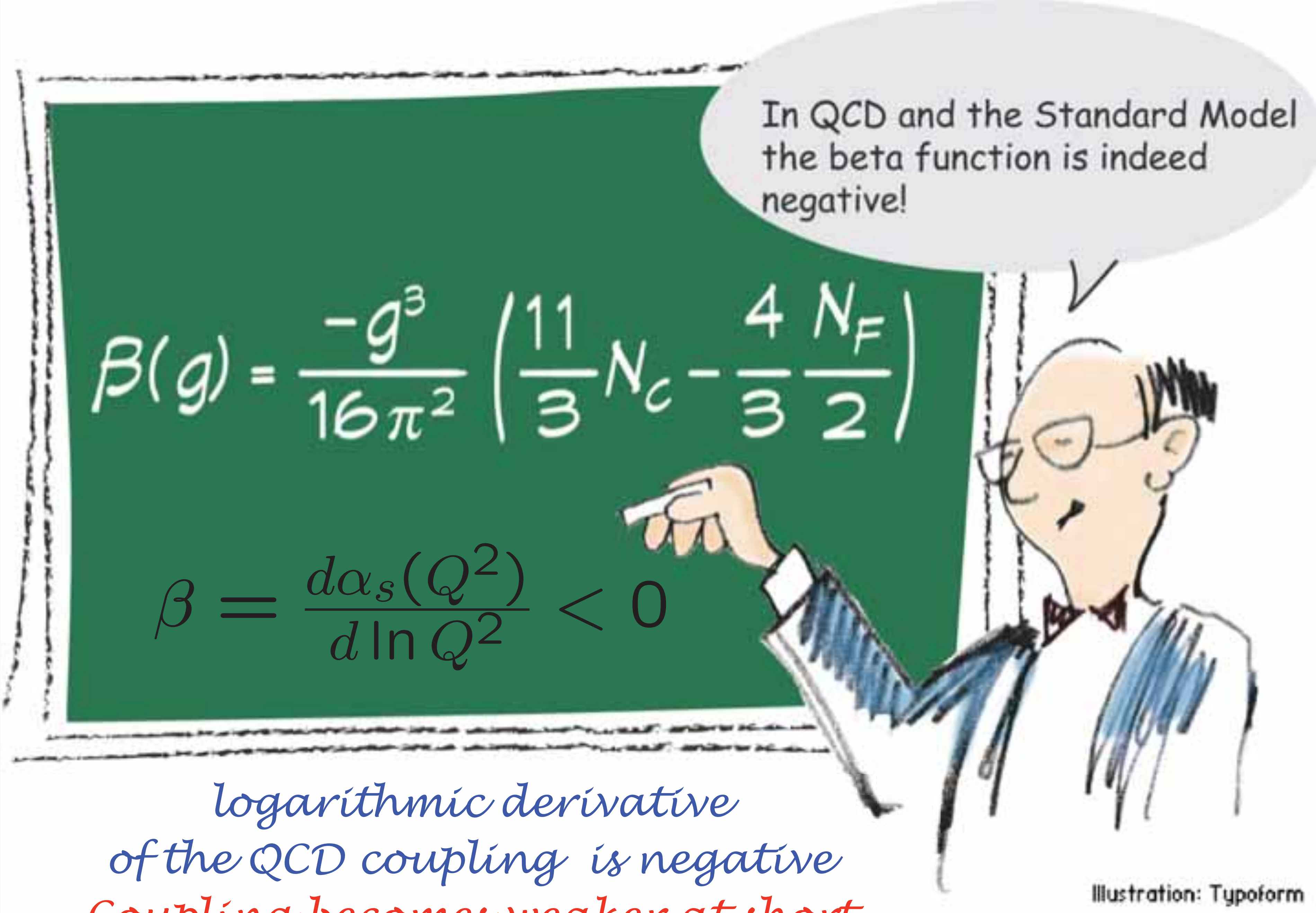
gluon self couplings

*QED: Underlies Atomic Physics, Molecular Physics, Chemistry, Electromagnetic Interactions ...*

*QCD: Underlies Hadron Physics, Nuclear Physics, Strong Interactions, Jets*

## *Theoretical Tools*

- Feynman diagrams and perturbation theory
- Bethe Salpeter Equation, Dyson-Schwinger Equations
- Lattice Gauge Theory,
- Discretized Light-Front Quantization
- AdS/CFT !



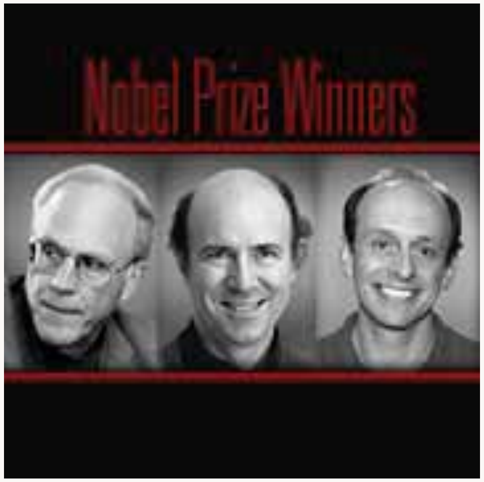

$$\beta = \frac{d\alpha_s(Q^2)}{d\ln Q^2} < 0$$

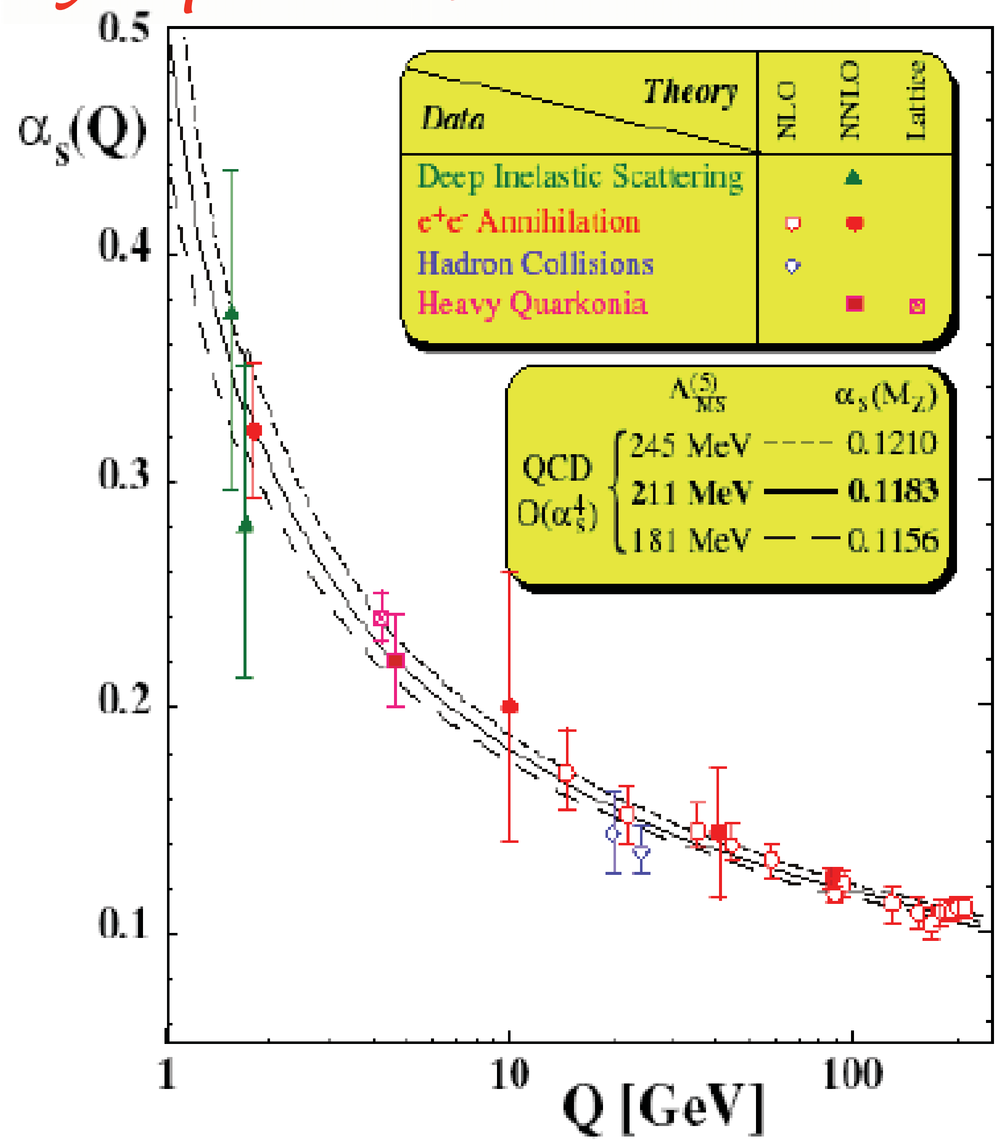
*logarithmic derivative of the QCD coupling is negative*

*Coupling becomes weaker at short distances or high momentum transfer*

# Verification of Asymptotic Freedom

$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$

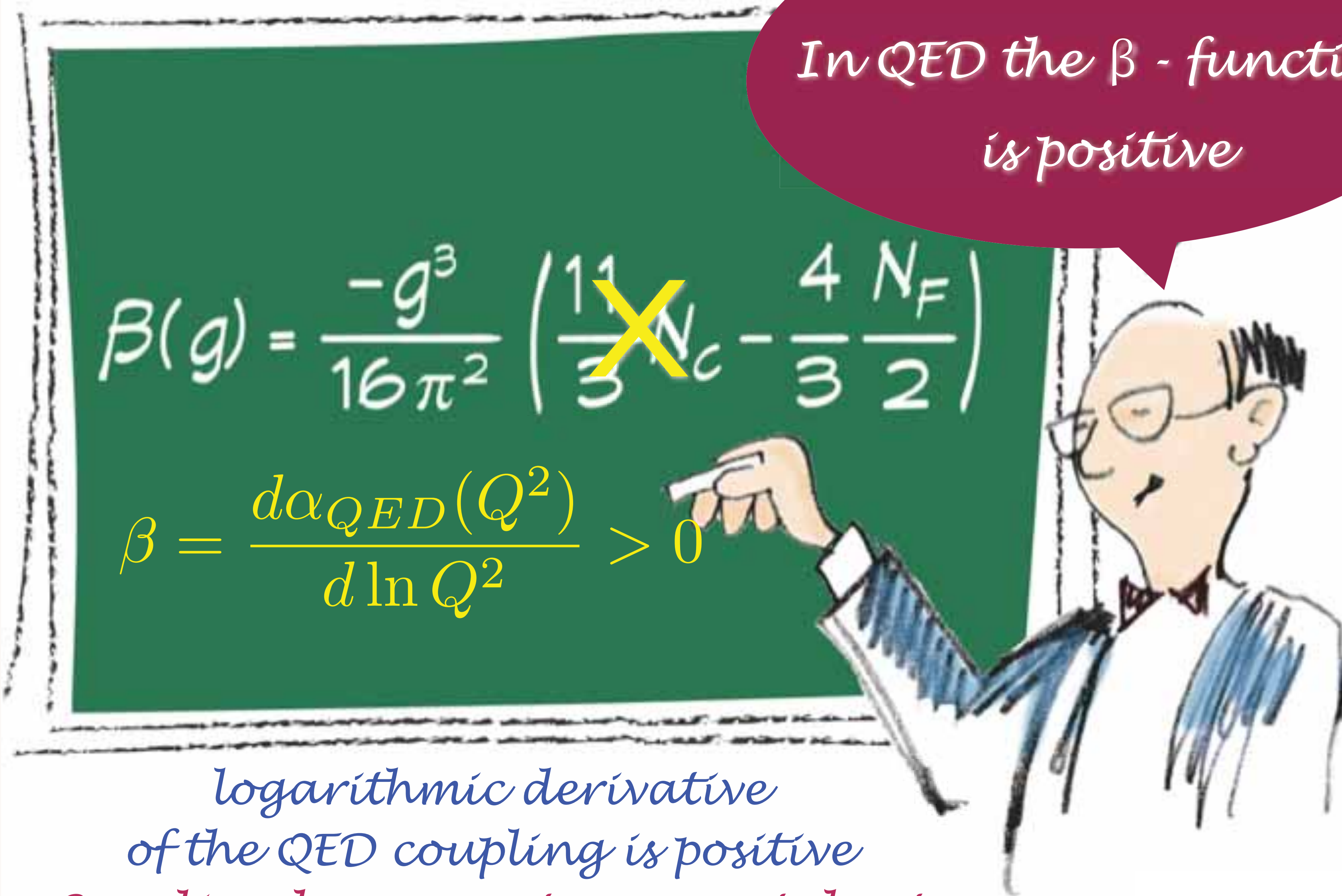



$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

proportional to $\alpha_s(Q)$

Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$




In QED the β - function is positive

$$\beta = \frac{d\alpha_{QED}(Q^2)}{d\ln Q^2} > 0$$

logarithmic derivative
of the QED coupling is positive

Coupling becomes stronger at short distances or high momentum transfer

**Landau Pole!**

# QCD Lagrangian

gluon dynamics

quark kinetic energy + quark-gluon dynamics

mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad [D^\mu, D^\nu] = igG^{\mu\nu}$$

$\lim N_C \to 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

## Analytic limit of QCD: Abelian Gauge Theory

**QCD → QED**

P. Huet, sjb

Given the elementary gauge theory interactions, all fundamental processes described in principle!

## *Example from QED:*

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

$\frac{1}{2}g_e = 1.001\ 159\ 652\ 201(30)$ QED prediction (Kinoshita, et al.)

$\frac{1}{2}g_e = 1.001\ 159\ 652\ 193(10)$ Measurement (Dehmelt, et al.)

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 180\ 85\ [0.76\ ppt]$$

*Dirac:* $g_e \equiv 2$ Measurement (Gabrielse, et al.)

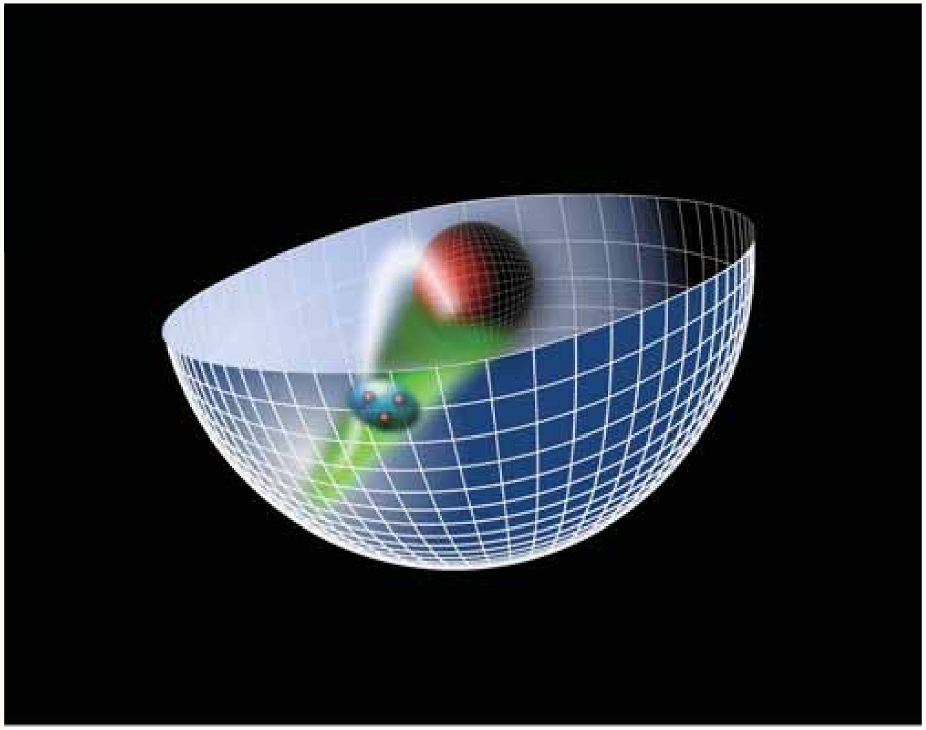
# Goal: an analytic first approximation to QCD

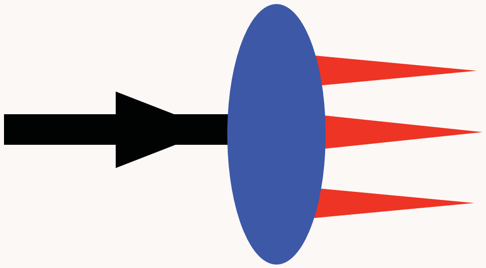
- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

# Light-Front Holography and Non-Perturbative QCD

**Goal:**
**Use AdS/QCD duality to construct a first approximation to QCD**

*Hadron Spectrum*
*Light-Front Wavefunctions,*
*Form Factors, DVCS, etc*





$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

**in collaboration with**
**Guy de Teramond and Alexandre Deur**

***Central problem for strongly-coupled gauge theories***

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

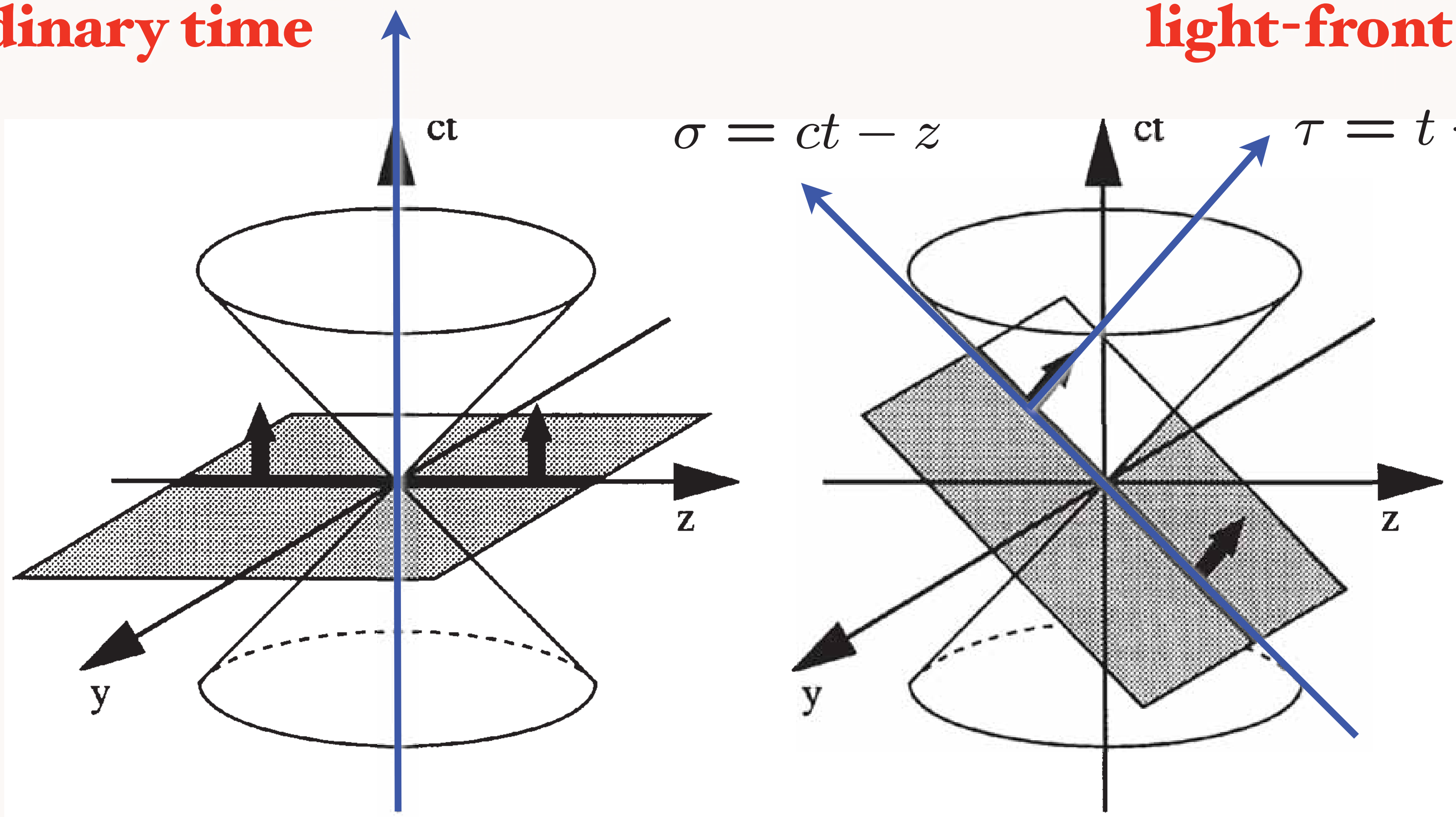
# Dirac's Amazing Idea: The Front Form

**Evolve in ordinary time**

**Evolve in light-front time!**

$\sigma = ct - z$

$\tau = t + z/c$




**Instant Form**

**Front Form**

Each element of flash photograph illuminated at same Light Front time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

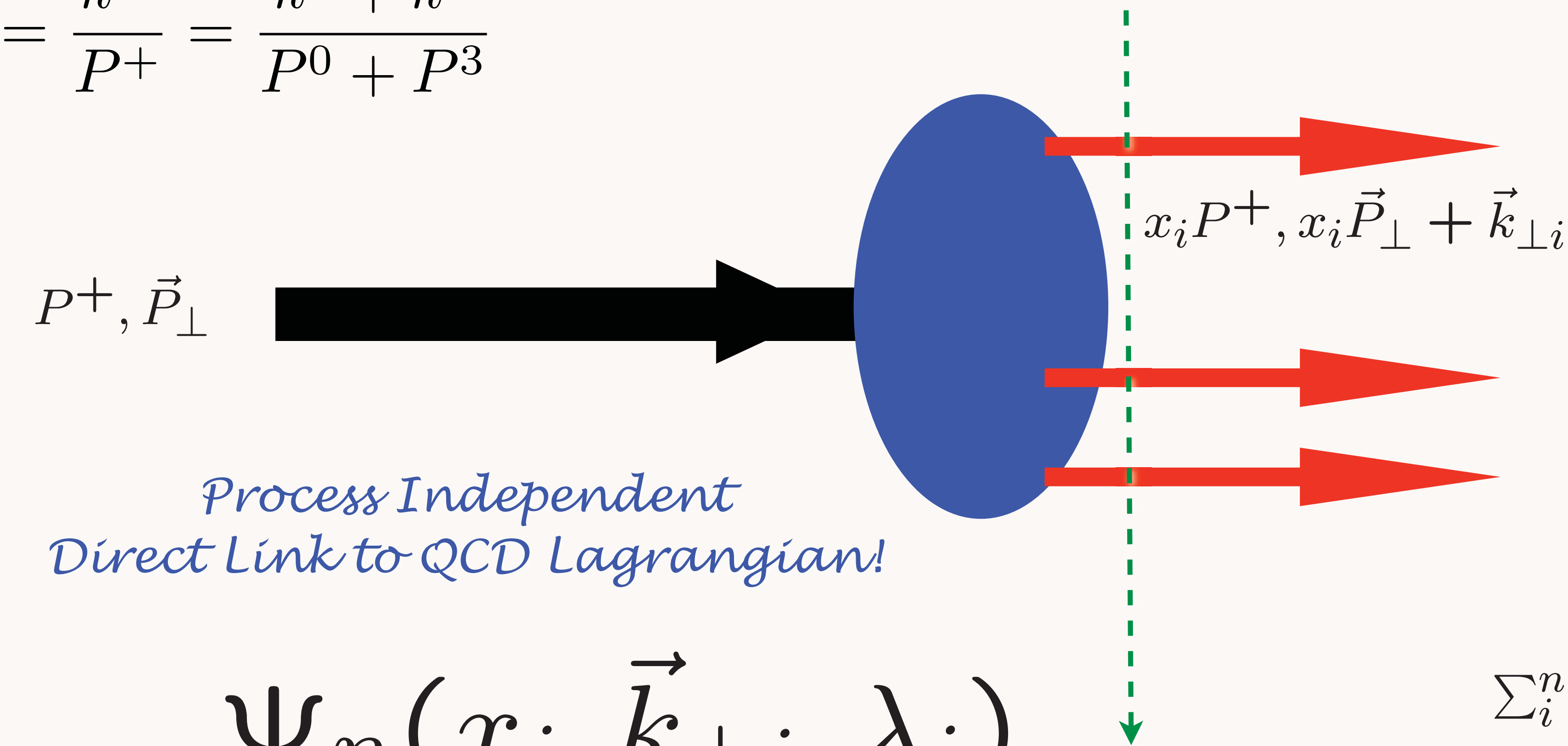
**Causal, Trivial Vacuum**

DIS, Form Factors, DVCS, etc. measure proton WF at fixed

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$




*Process Independent*
*Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

*Invariant under boosts! Independent of $P^\mu$*

# Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved at every vertex
and
LF Fock state by Fock State!

LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

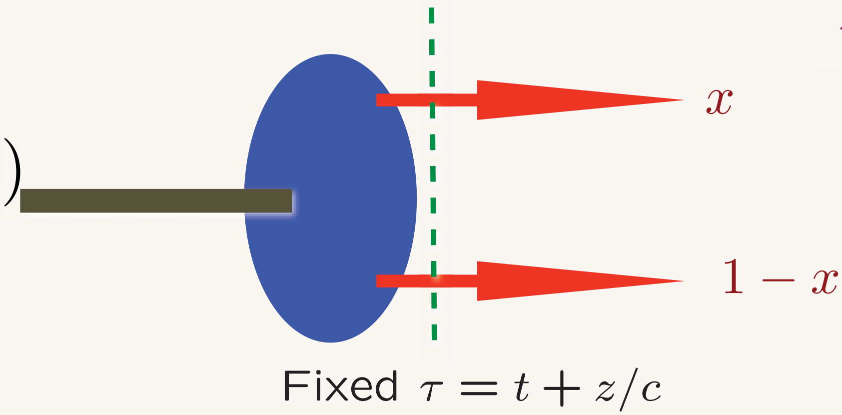
n-1 orbital angular momenta

*Nonzero Anomalous Moment -->*
*Nonzero quark orbital angular momentum!*

# Hadron Distribution Amplitudes

**Lepage, sjb**

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$




$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE,

- Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

**Lepage, sjb**
**Efremov, Radyushkin**
**Sachrajda, Frishman Lepage, sjb**
**Braun, Gardi**

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

*sum over states with n=3, 4, ...constituents*

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

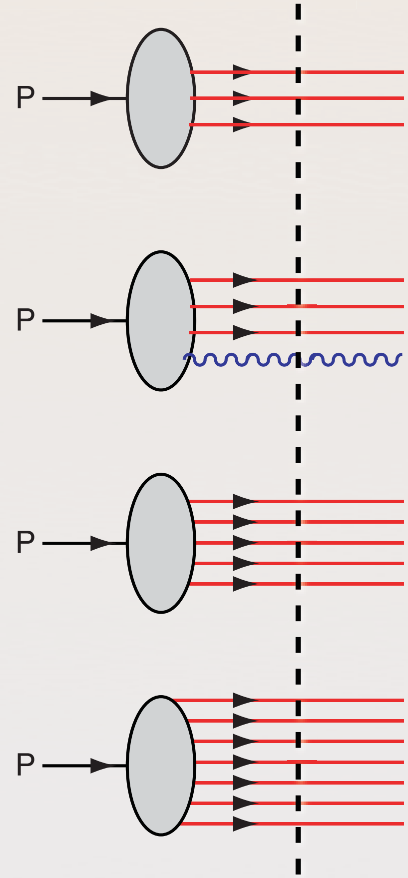
are boost invariant; they are independent of the hadron's energy and momentum $P^\mu$.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$




**Intrinsic heavy quarks**

$c(x), b(x)$ at high $x$

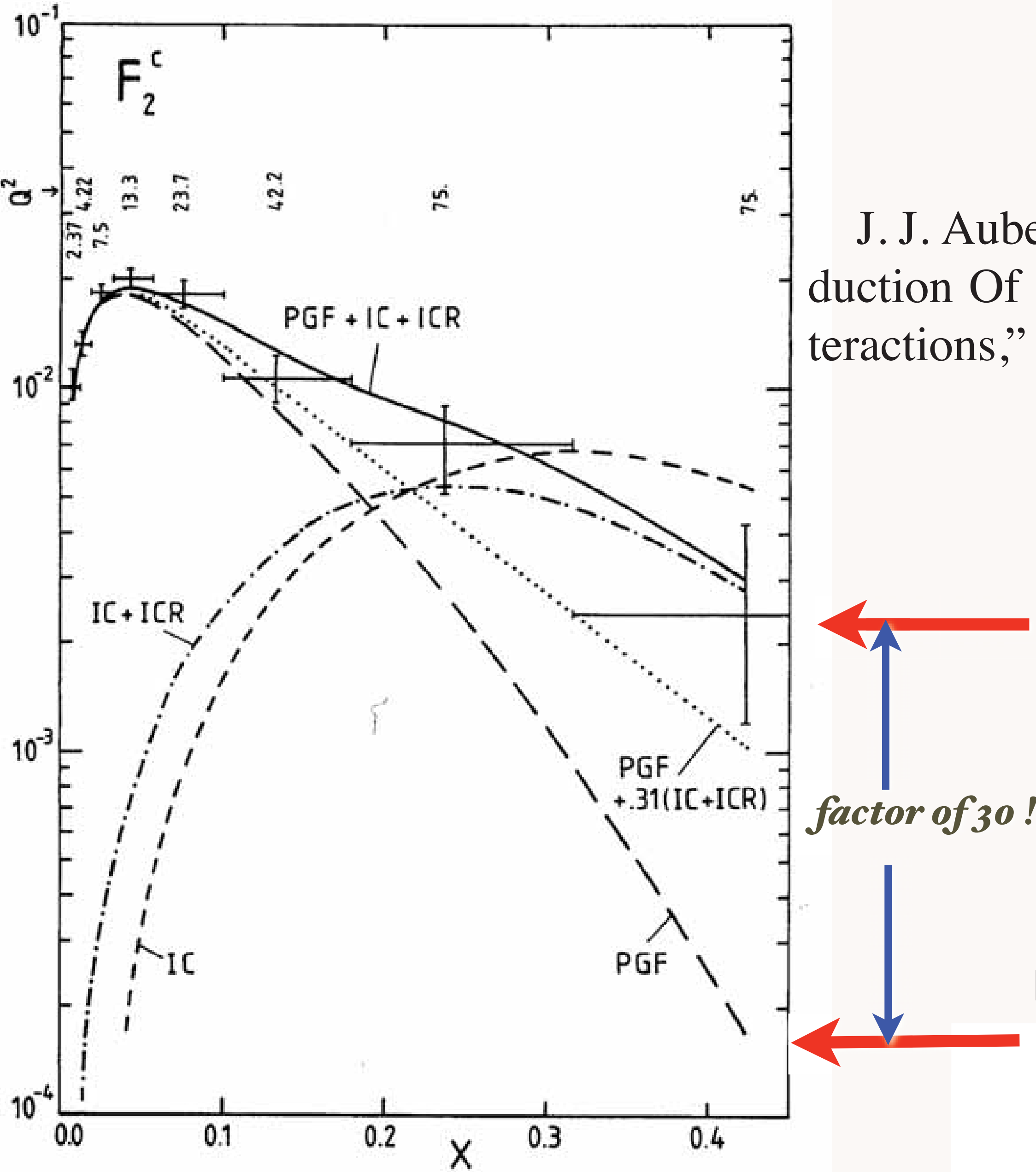
$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

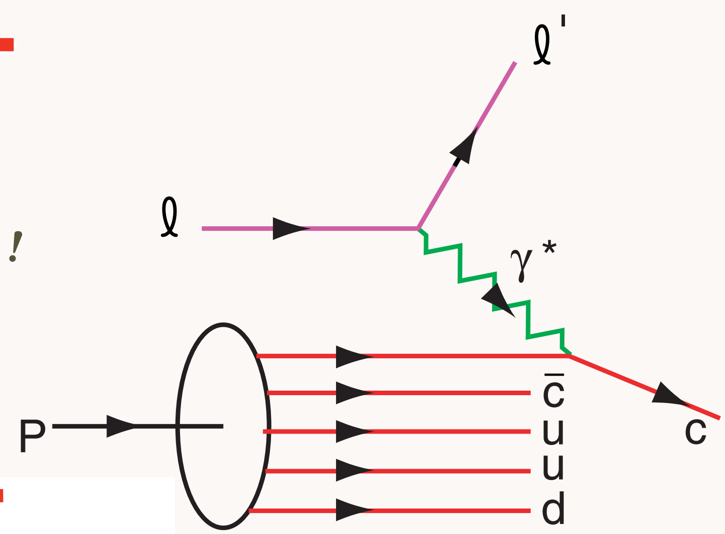
*Fixed LF time*

# Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

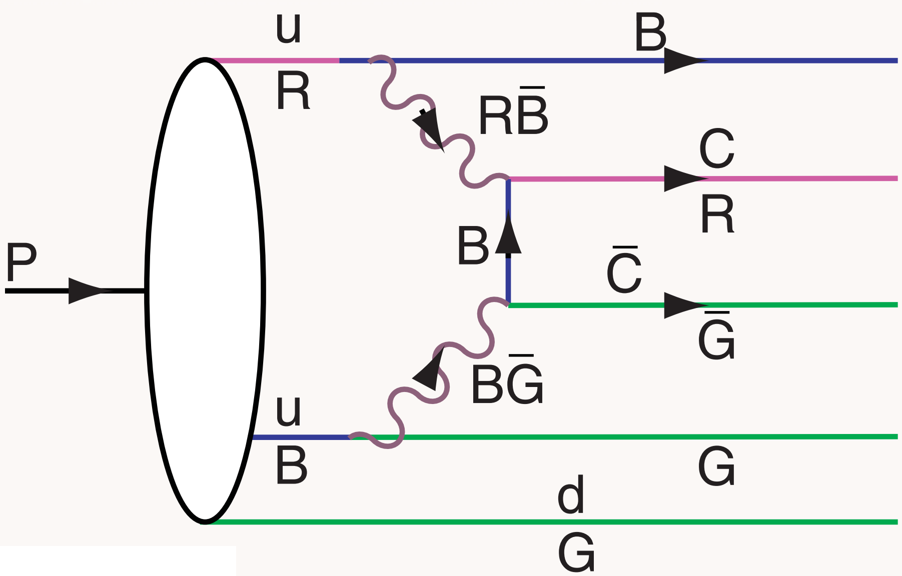



## First Evidence for Intrinsic Charm




***DGLAP / Photon-Gluon Fusion: factor of 30 too small***

Hoyer, Peterson, Sakai, sjb




$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

*High x charm!* *Charm at Threshold*

- EMC data: $c(x,Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV$^2$, $x = 0.42$

- High $x_F$ $pp \to J/\psi X$

- High $x_F$ $pp \to J/\psi J/\psi X$

- High $x_F$ $pp \to \Lambda_c X$

- High $x_F$ $pp \to \Lambda_b X$

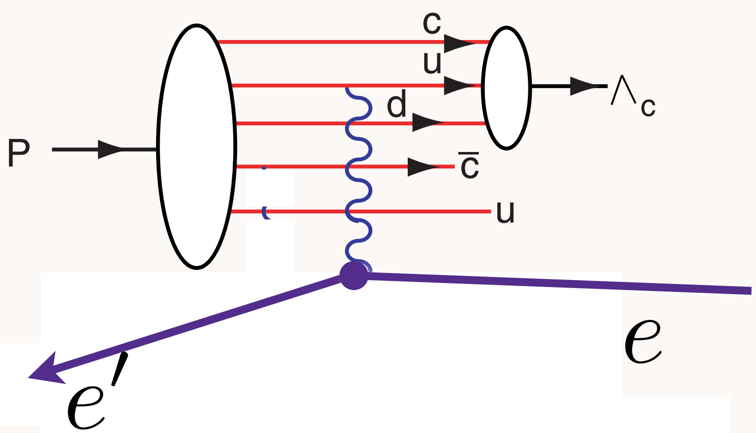
- High $x_F$ $pp \to \Xi(ccd) X$ (SELEX)

## IC Structure Function: Critical Measurement for EIC

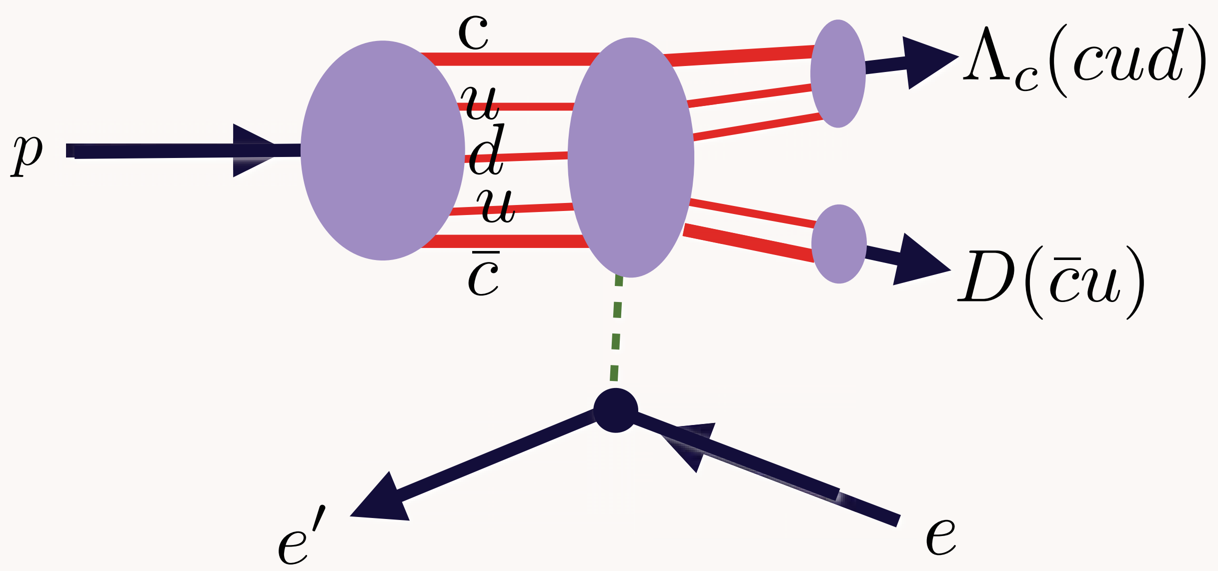
# Leading charm production in proton fragmentation region at the EIC

Intrinsic charm and bottom quarks have same rapidity as valence quarks

Produce $\Xi(ccd), B(\bar{b}u), \Lambda(cbu), \Xi(bbu)$




Coalescence of Comoving Charm and Valence Quarks
Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$

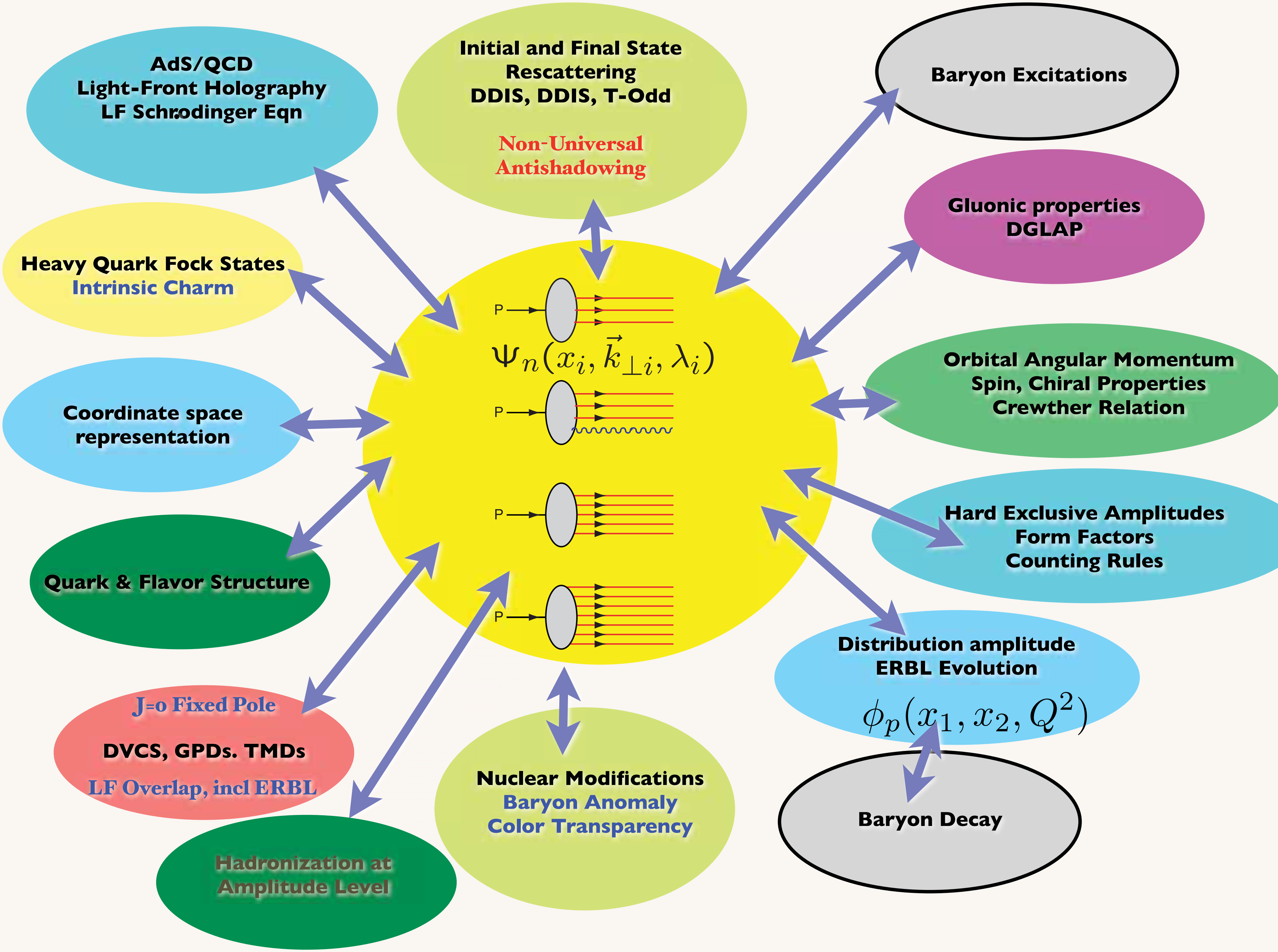



Dissociate proton to high $x_F$ heavy-quark pair

$$\gamma^* p \to \Lambda_c(cdd) + D(\bar{c}u), \gamma^* p \to \Lambda_b(bud)B^+(\bar{b}u)$$

*Test intrinsic charm, bottom*

# QCD and the LF Hadron Wavefunctions



# Light-Front QCD

**Heisenberg Matrix Formulation**

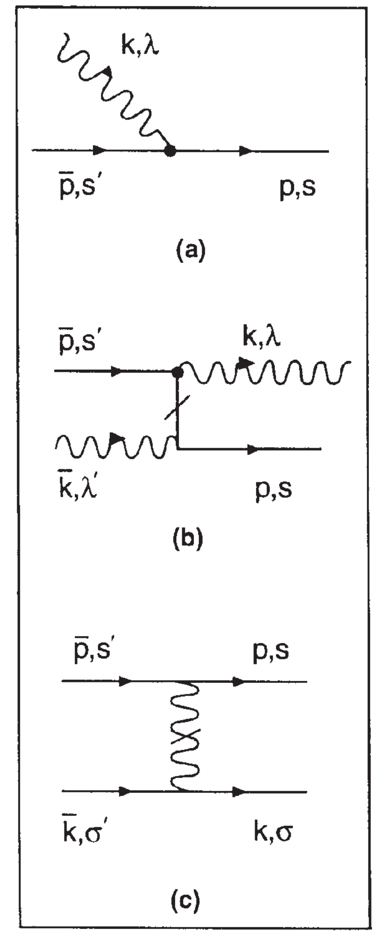
*Physical gauge:* $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

$H^{int}_{LF}$: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h >= \mathcal{M}^2_h|\Psi_h >$$




*Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions*

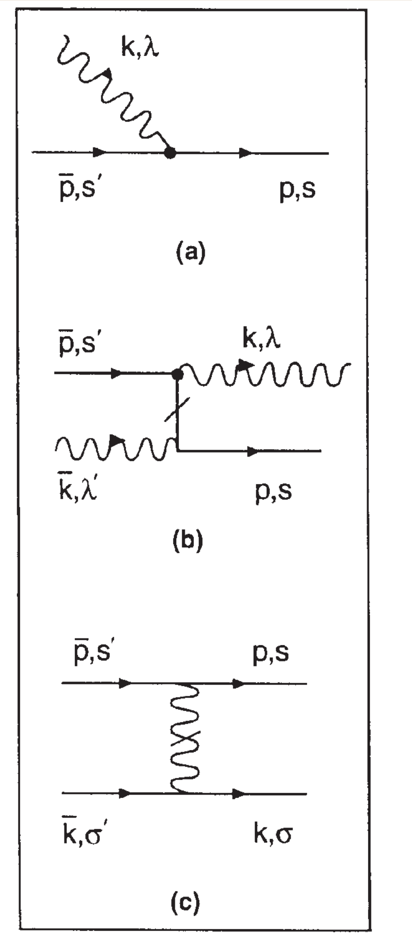
# Light-Front QCD

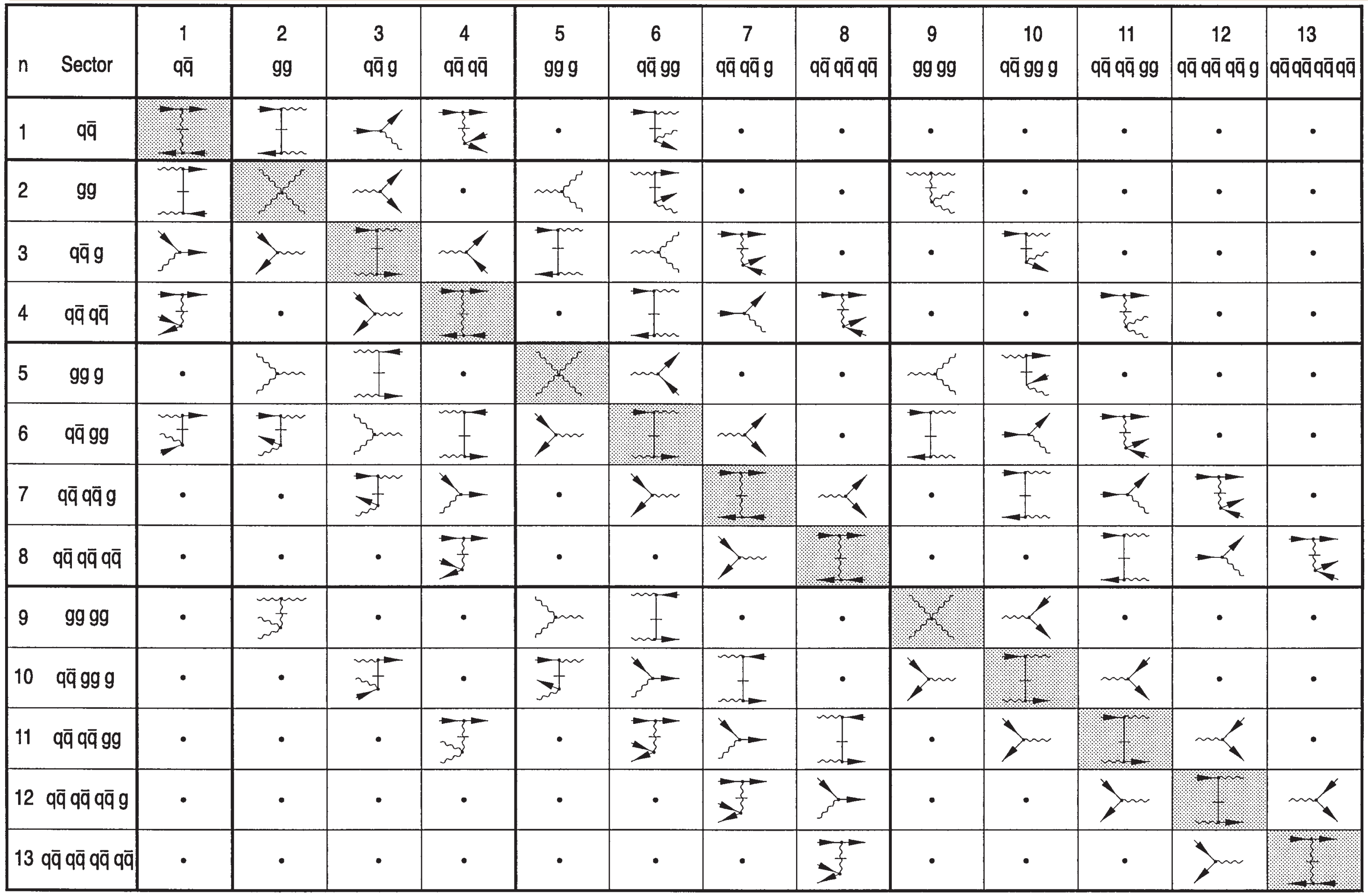
**Heisenberg Matrix Formulation**

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

**H.C. Pauli & sjb**

**Discretized Light-Cone Quantization**




**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**

*DLCQ: Frame-independent, No fermion doubling; Minkowski Space*

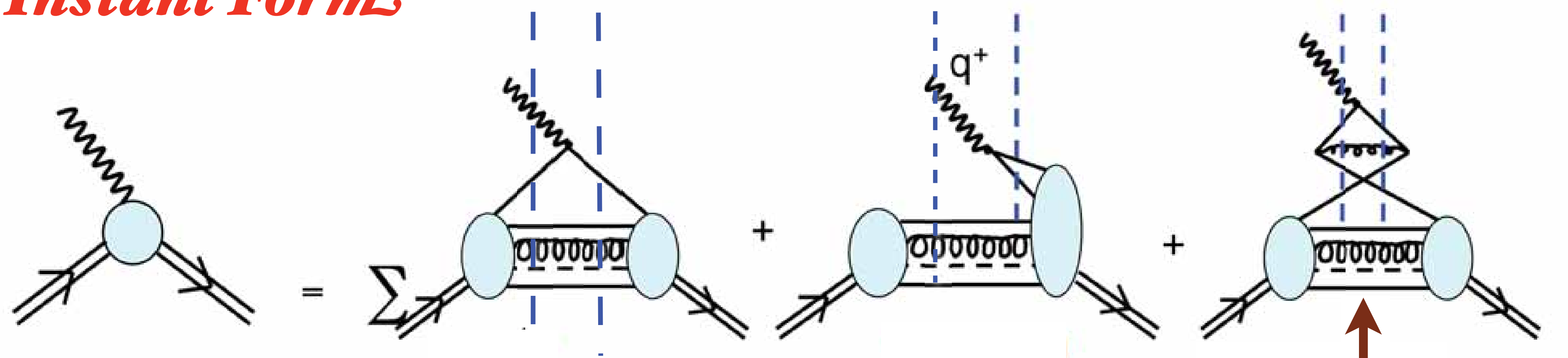
DLCQ: Periodic BC in $x^-$. Discrete $k^+$; frame-independent truncation

# *Light-Front QCD Features and Phenomenology*

- **Trivial Vacuum**
- **Hidden color, Intrinsic glue, sea, Color Transparency**
- **Physics of spin, orbital angular momentum**
- **Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints**
- **Vanishing anomalous gravitomagnetic moment**
- **Relation between edm and anomalous magnetic moment**
- **Cluster Decomposition Theorem for relativistic systems**
- **OPE: DGLAP, ERBL evolution; invariant mass scheme**

# Calculation of Form Factors in Equal-Time Theory

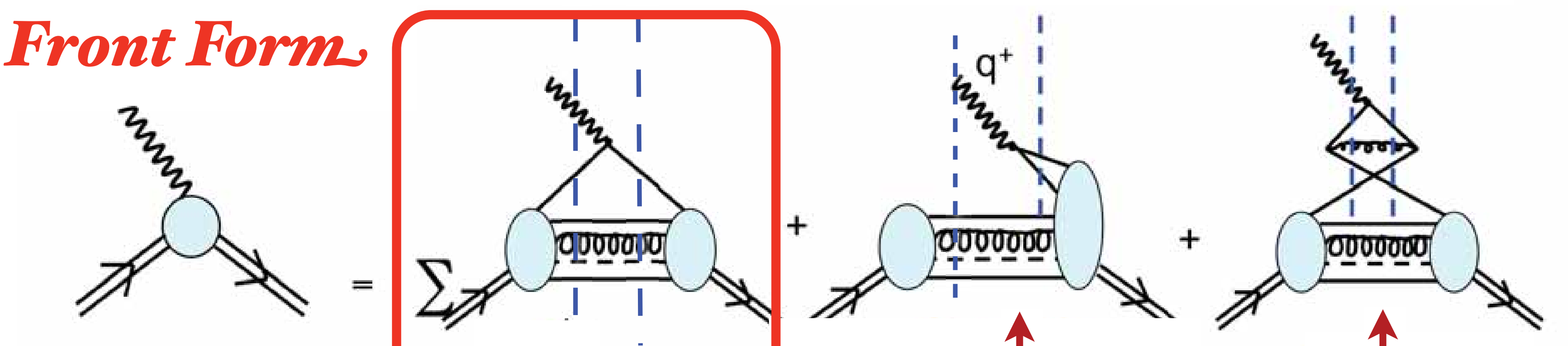
**Instant Form**




**Need vacuum-induced currents**

# Calculation of Form Factors in Light-Front Theory

**Front Form**




Complete Answer

Absent for $q^+ = 0$

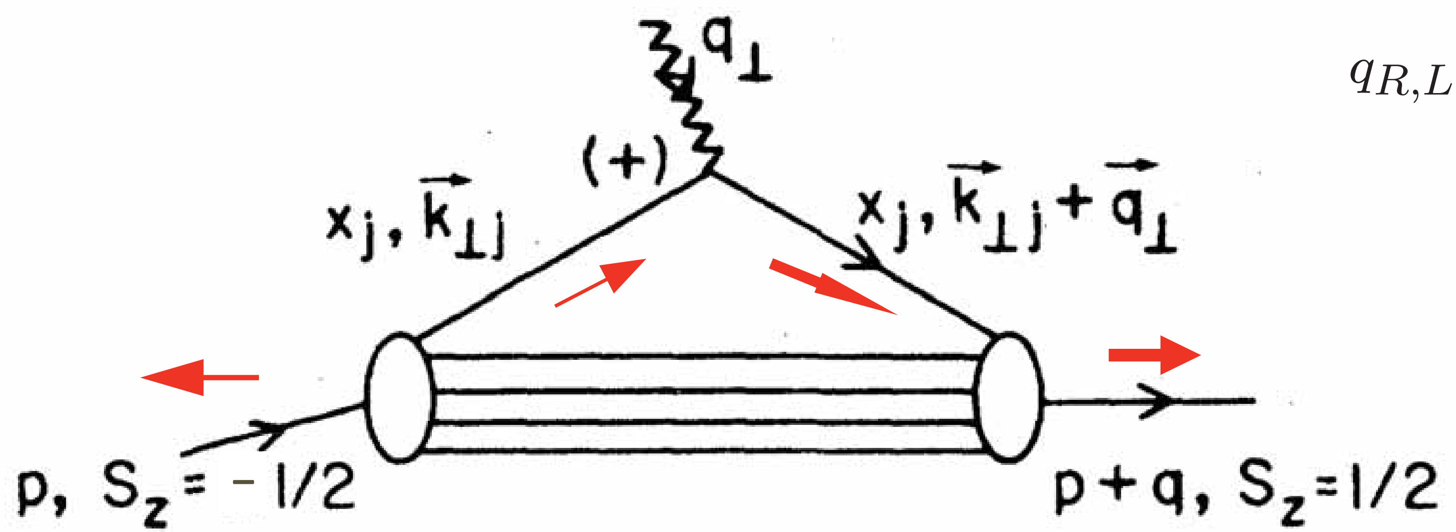
**zero !!**

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[ -\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



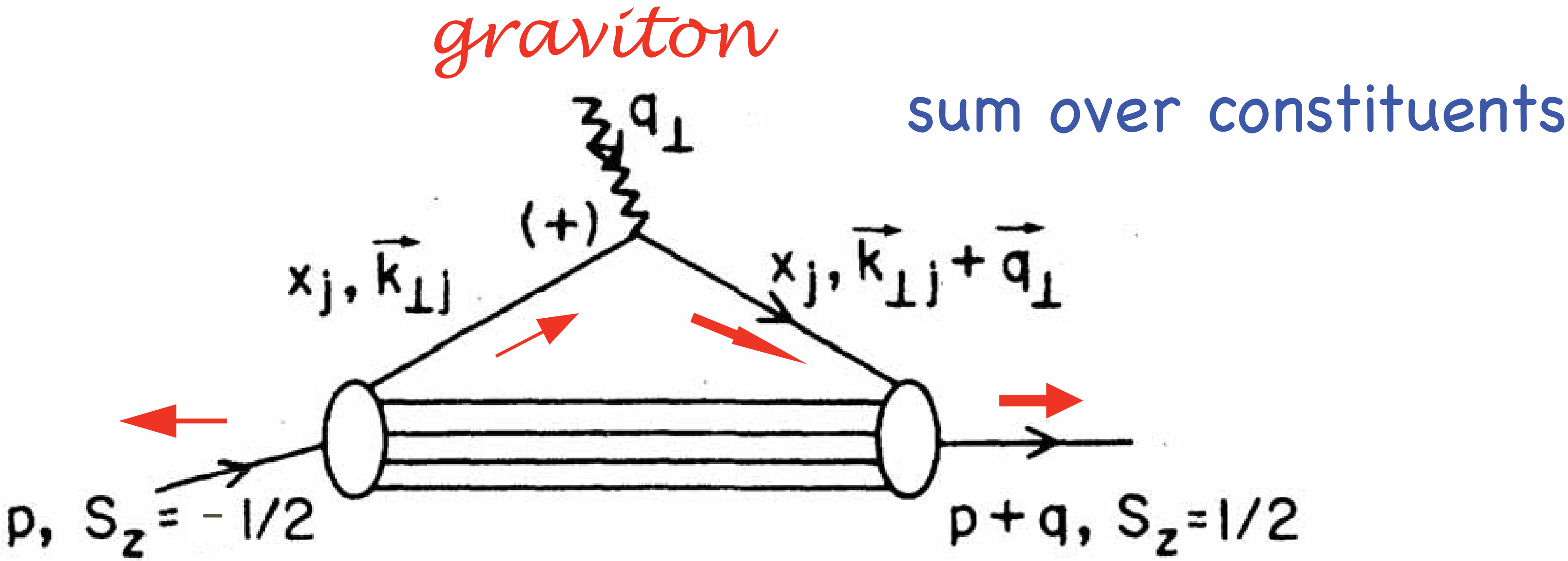

$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

*Same matrix elements appear in Sivers effect*
*-- connection to quark anomalous moments*

# Anomalous gravitomagnetic moment B(0)

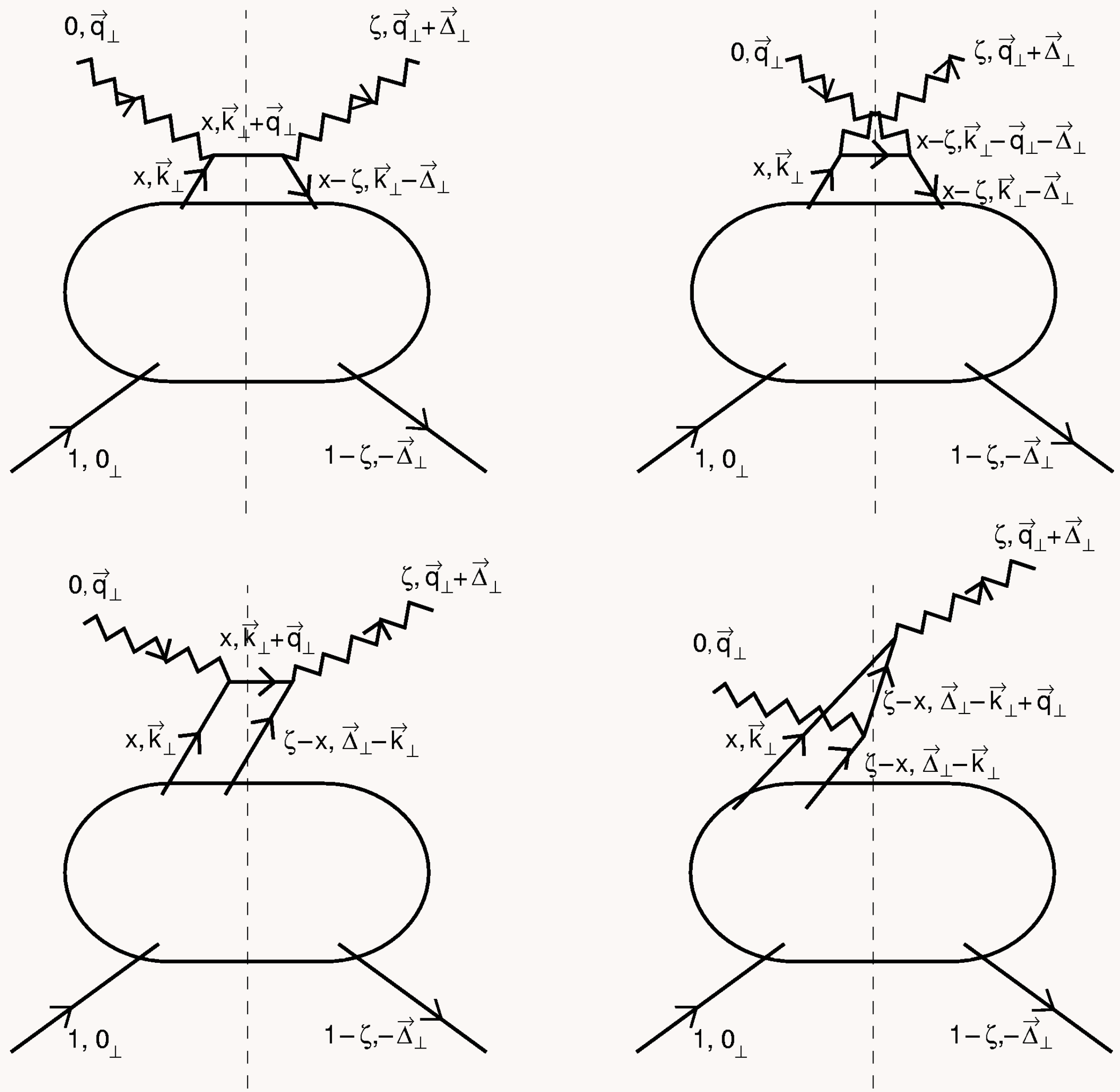
**Terayev, Okun, et al:** *B(0) Must vanish because of Equivalence Theorem*




**Hwang, Schmidt, sjb; Holstein et al**

$B(0) = 0$

*Each Fock State*




# Light-cone wavefunction representation of deeply virtual Compton scattering ☆


Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

# Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i \, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times \delta(x - x_1) \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i) \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1 - \zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

# Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^1 dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^1 dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

Quark angular momentum (Ji's sum rule)

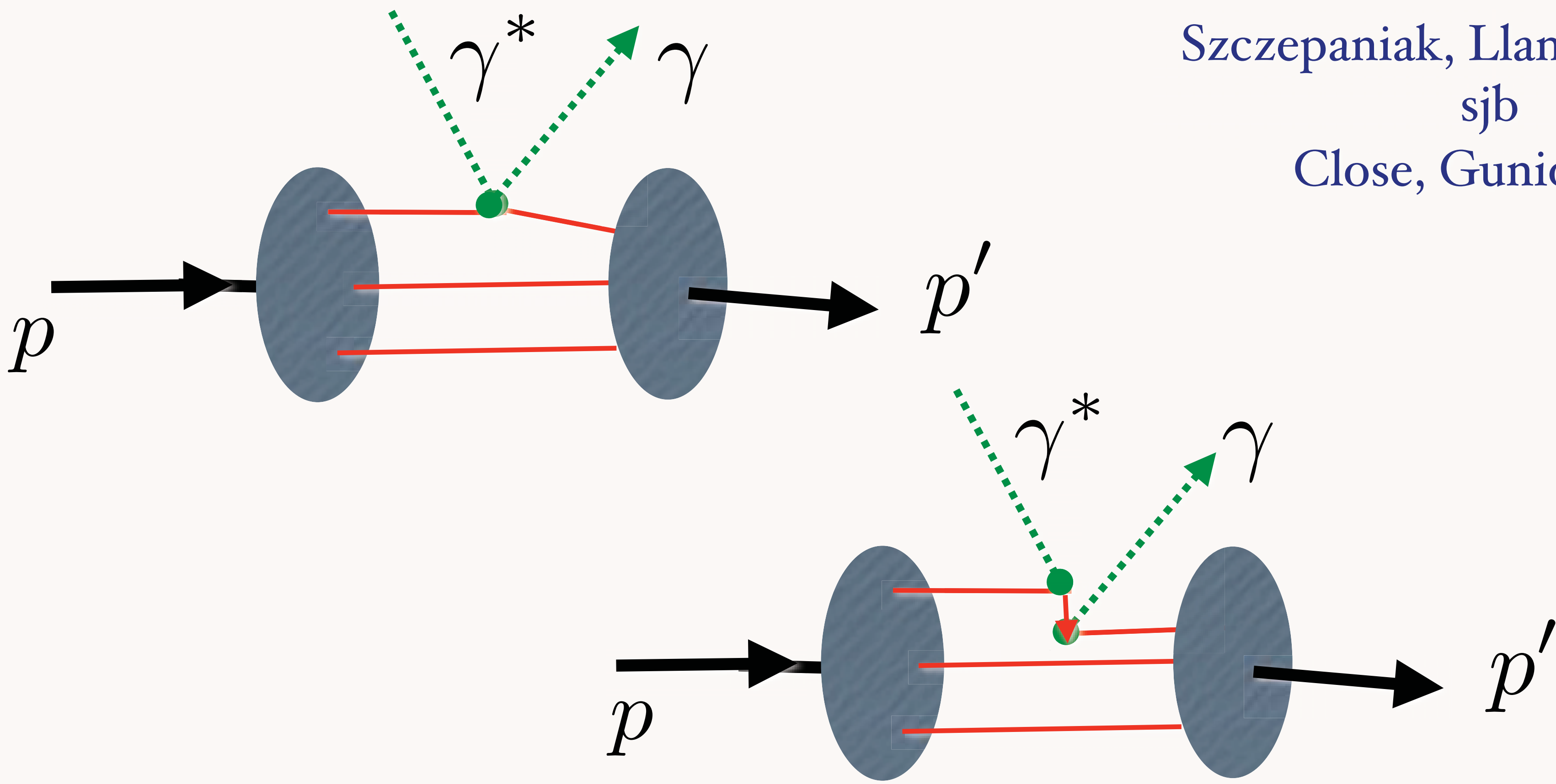
$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

# J=0 Fixed Pole Contribution to DVCS

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

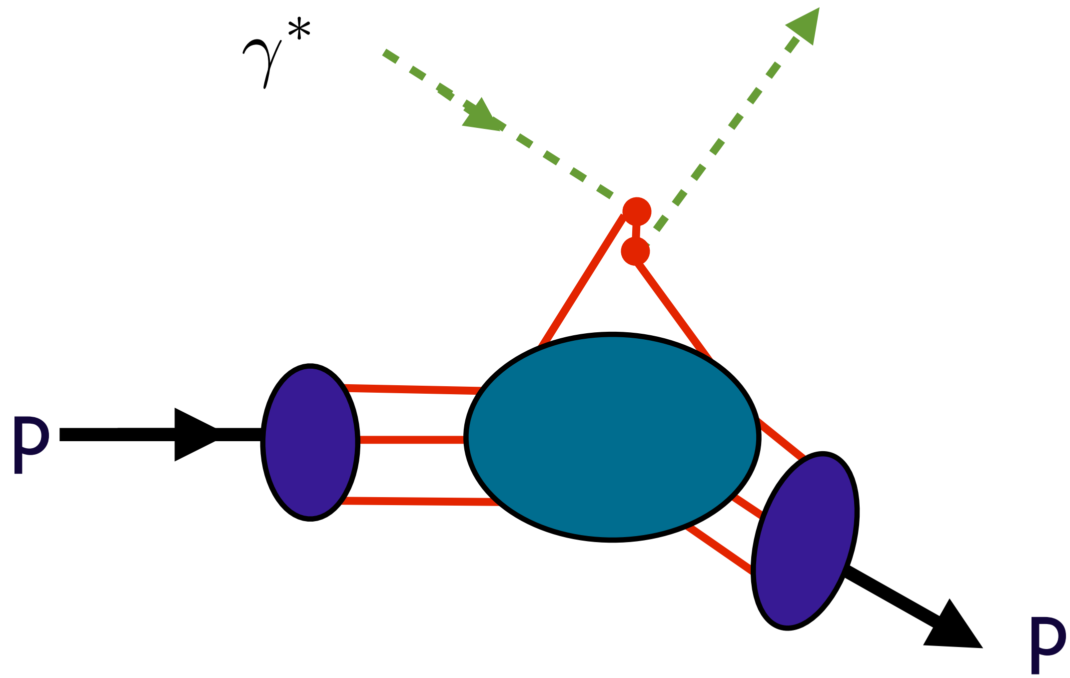
Szczepaniak, Llanes-Estrada, sjb
Close, Gunion, sjb




Real amplitude, independent of $Q^2$ at fixed $t$

# Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$




*Seagull interaction (instantaneous quark exchange or Z-graph)*

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

Hard Reggeon Domain

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^{\alpha}(t)\beta_R(t)$$

$$\alpha_R(t) \to 0$$

*Reflects elementary coupling of two photons to quarks*

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

# J=0 Fixed pole in real and virtual Compton scattering

Damashek, Gilman
Close, Gunion, sjb
Llanes-Estrada,
Szczepaniak, sjb

Effective two-photon contact term

Seagull for scalar quarks

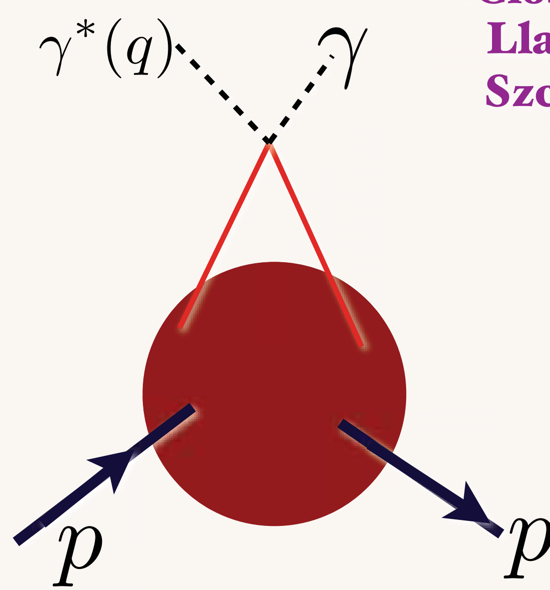
Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of Q² at fixed t

<1/x> Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

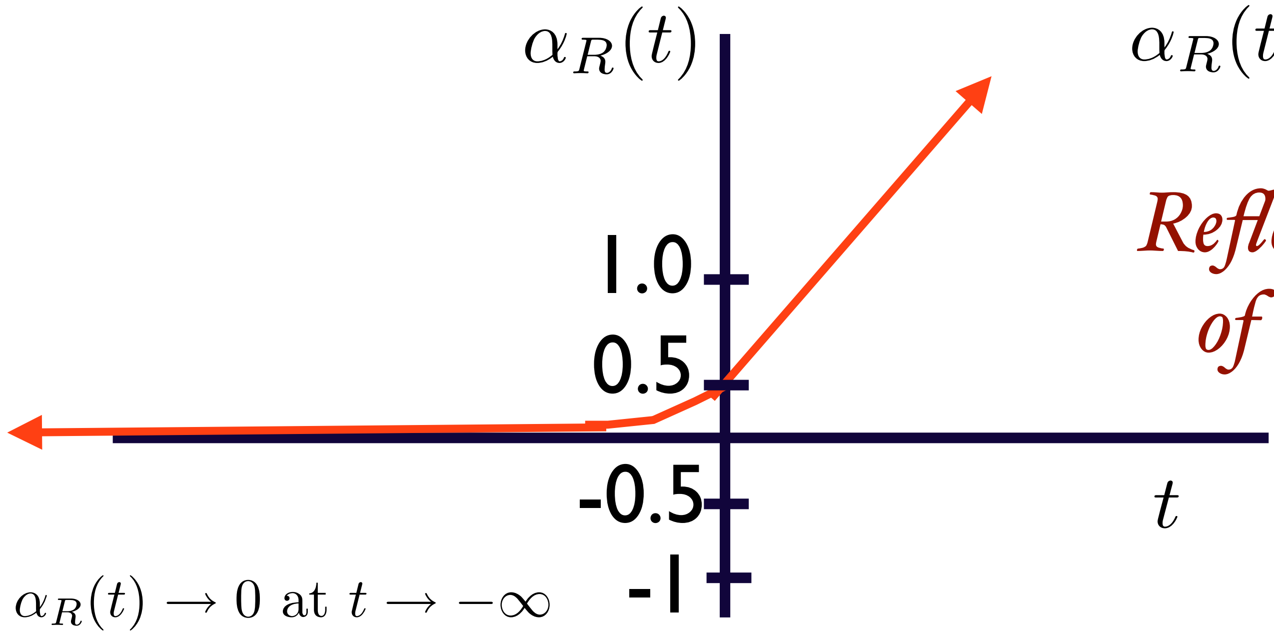



$Q^2$-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt}(\gamma^* p \to \gamma p) = F^2(t)$$

# Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t)\beta_R(t) \qquad s >> -t, Q^2$$




$\alpha_R(t) \to 0$ at $t \to -\infty$

J=0 fixed pole

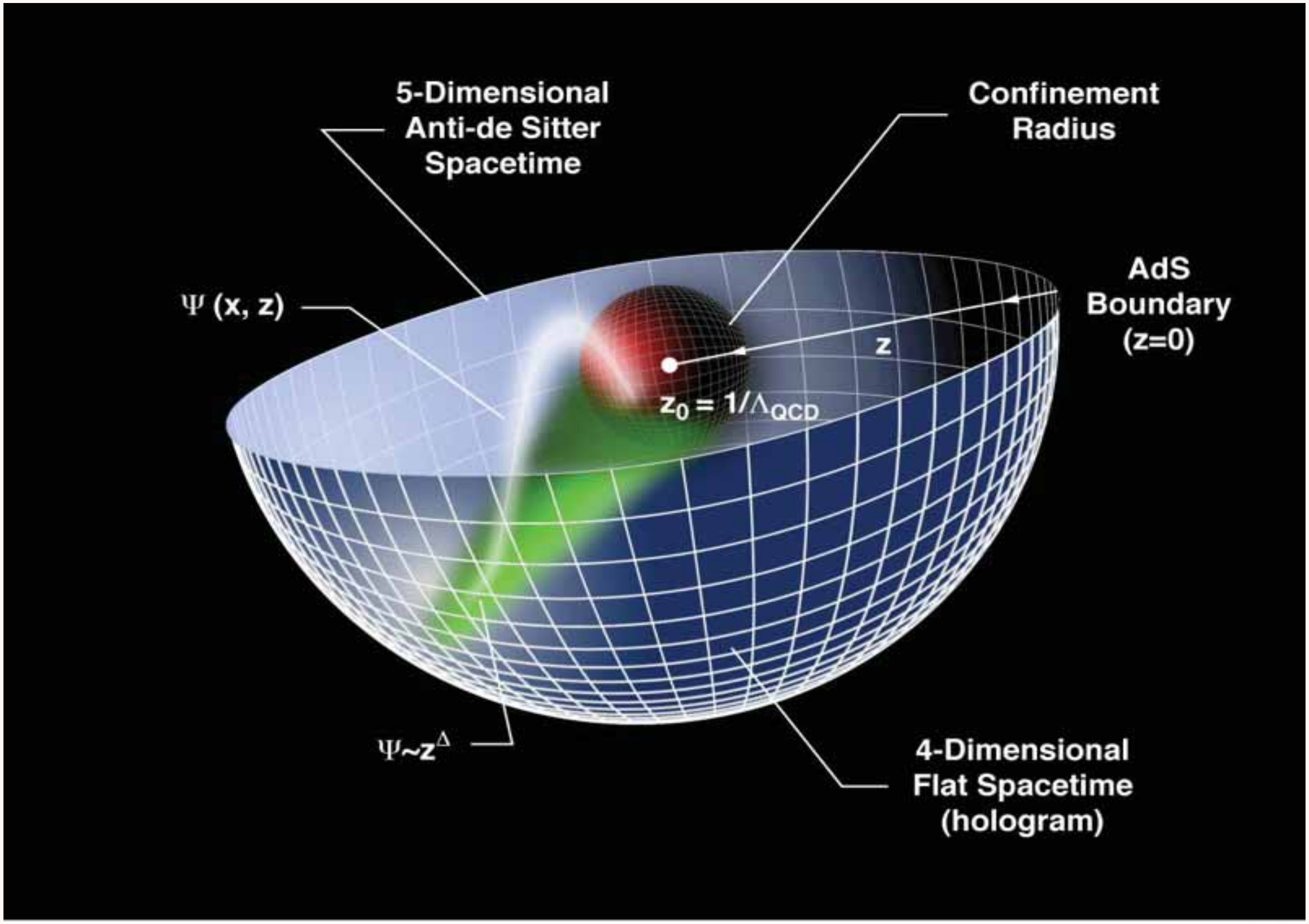
*Reflects elementary coupling of two photons to quarks*

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \gamma p) \to \frac{1}{s^2}\beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

# Fundamental test of QCD

# Applications of AdS/CFT to QCD




*Changes in physical length scale mapped to evolution in the 5th dimension z*

**in collaboration with Guy de Teramond**

# *Goal:*

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrödinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

*Conformal Theories are invariant under the Poincare and conformal transformations with*

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

*the generators of SO(4,2)*

**SO(4,2) has a mathematical representation on AdS5**

# AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

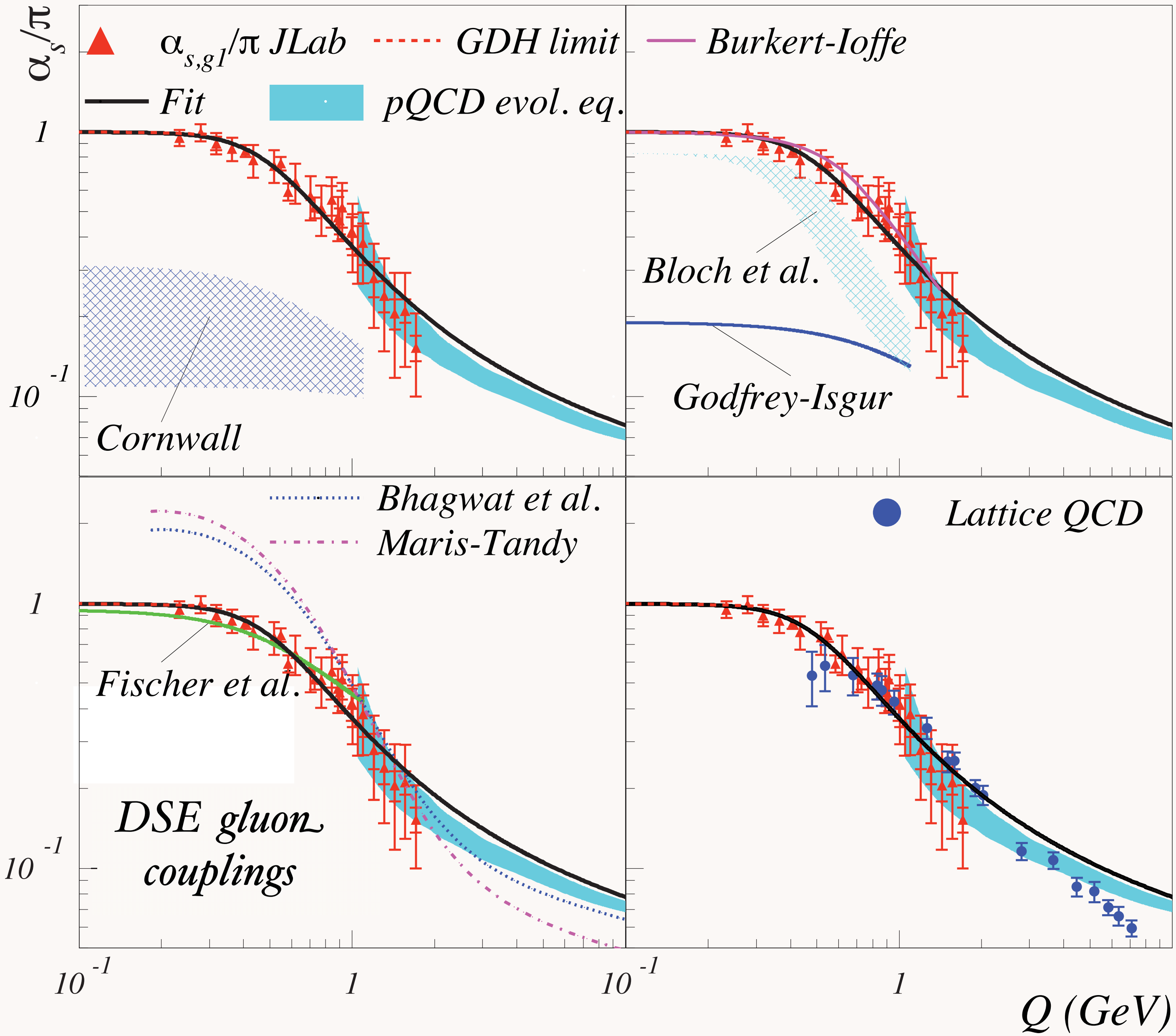
Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

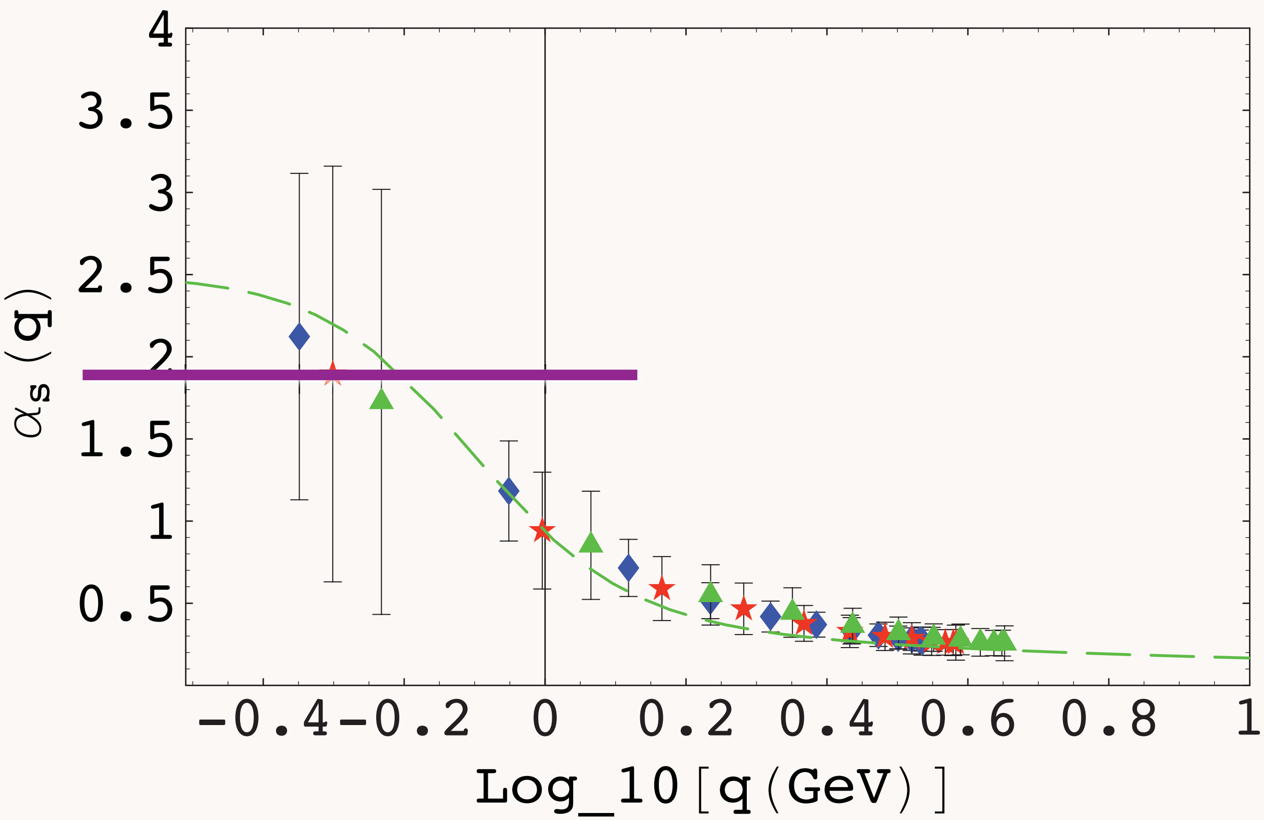
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

# Deur, Korsch, et al.



# Conformal Behavior of QCD in Infrared

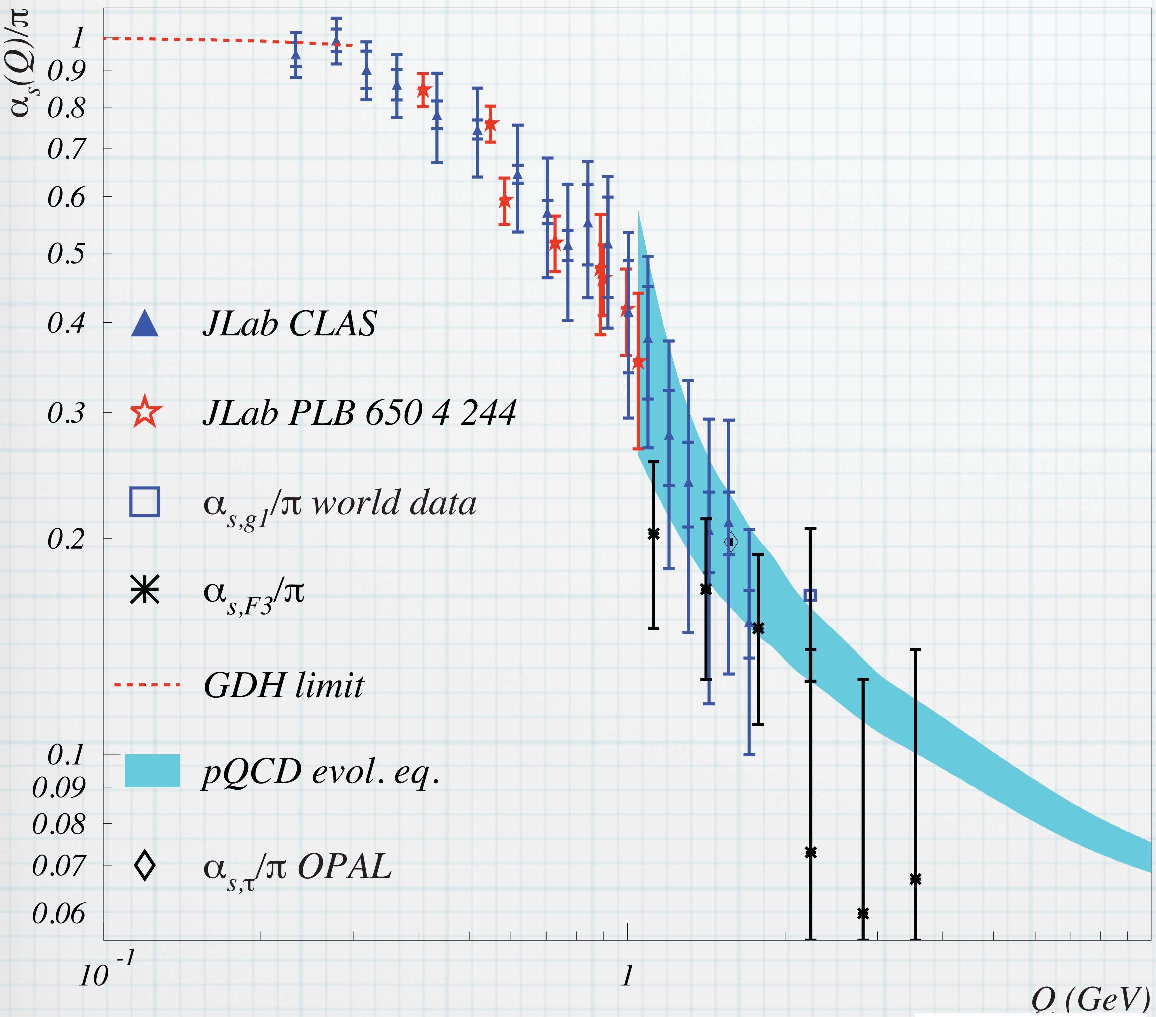
- Does $\alpha_s$ develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur . . .
- Recent lattice simulations: evidence that $\alpha_s$ becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).



# Nearly conformal QCD?

Define $\alpha_s$ from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left( g_1^p(x,Q^2) - g_1^n(x,Q^2) \right) = \frac{1}{6} g_A \left( 1 - \frac{\alpha_{s,g_1}}{\pi} \right)$$




g1 = spin dependent structure function (from inelastic ep scattering

Data from EG1 exp., at JLab CLAS (2008)

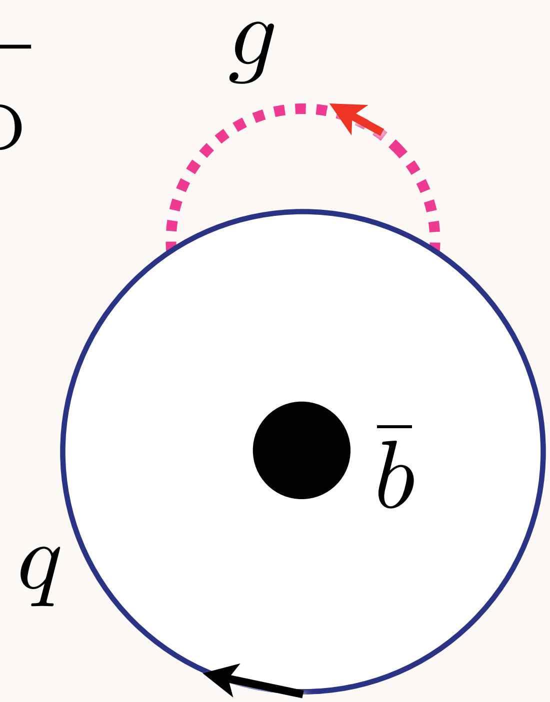
$\alpha_s$ runs only modestly at small $Q^2$

Fig. from 0803.4119, Duer et al.

# Lesson from QED and Lamb Shift:

*maximum wavelength of bound quarks and gluons*

$$k > \frac{1}{\Lambda_{QCD}} \qquad \lambda < \Lambda_{QCD}$$




**B-Meson**

*gluon and quark propagators cutoff in IR*
*because of color confinement*

**R. Shrock, sjb**

# Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain** $(x-y)^2 < \Lambda^{-2}_{QCD}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

*Quark and Gluon vacuum polarization insertions decouple: IR fixed Point*

**Shrock, sjb**

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

***A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).***

# Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

*invariant measure*

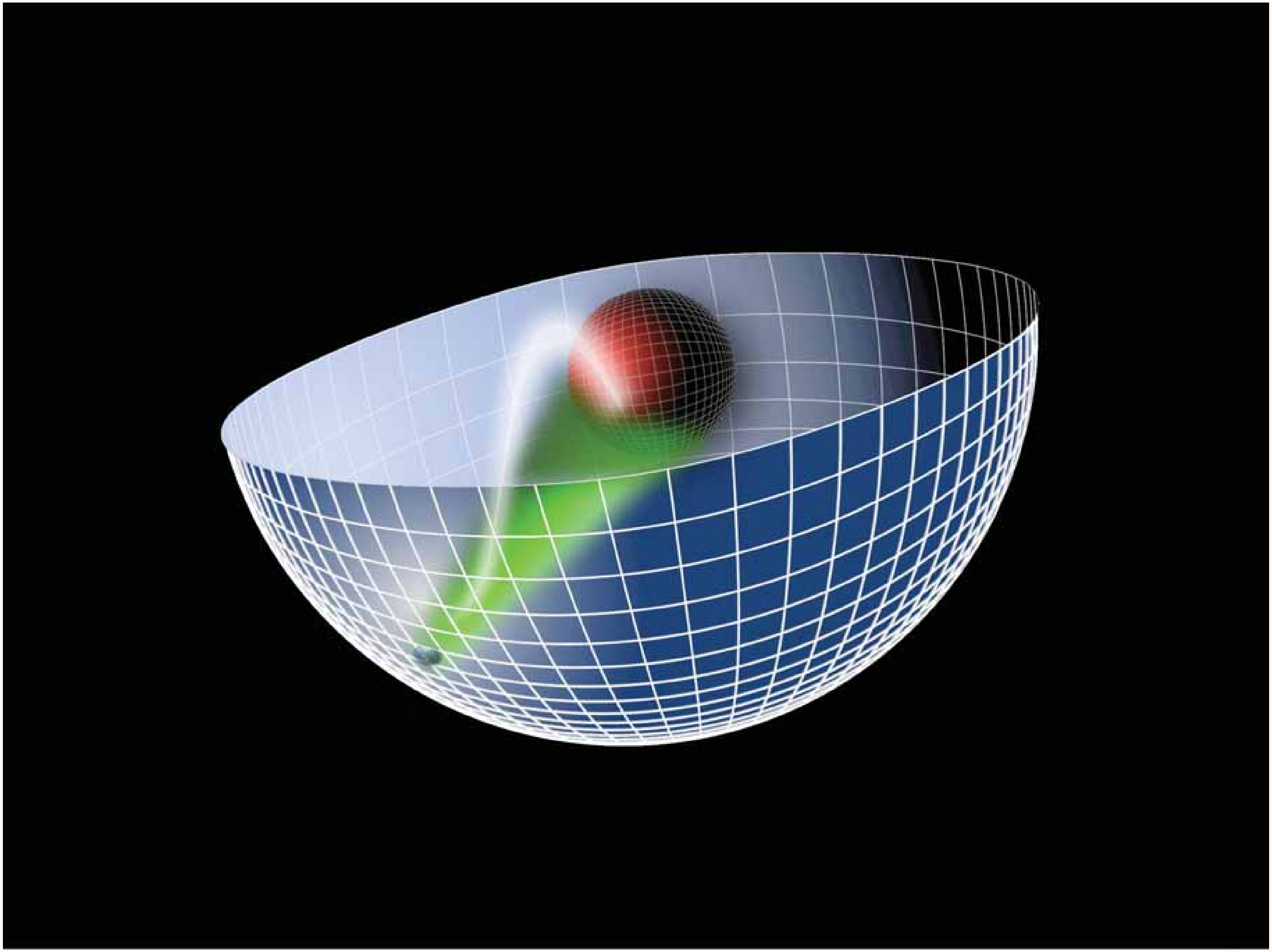
$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

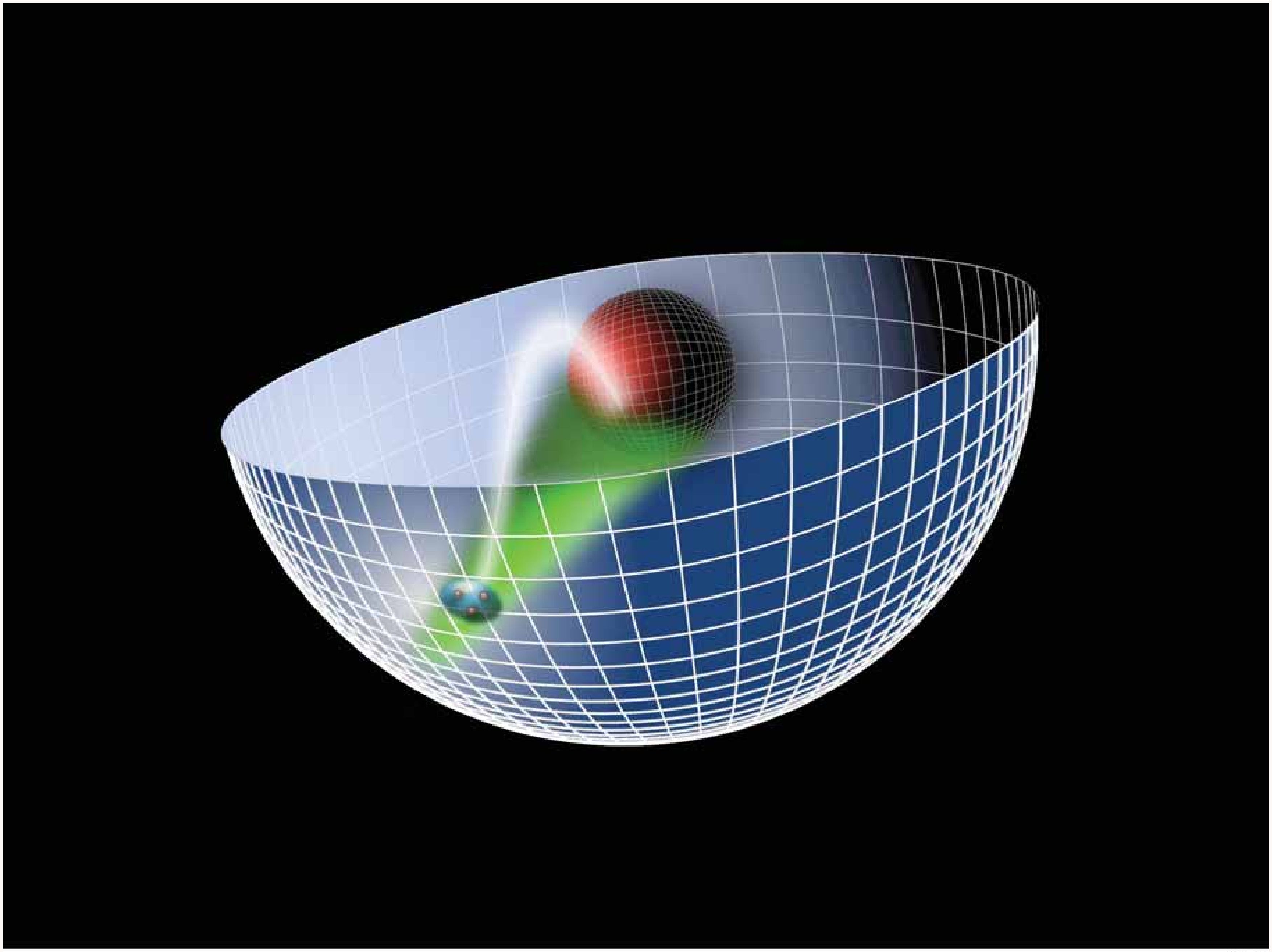
- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.
- Different values of $z$ correspond to different scales at which the hadron is examined.

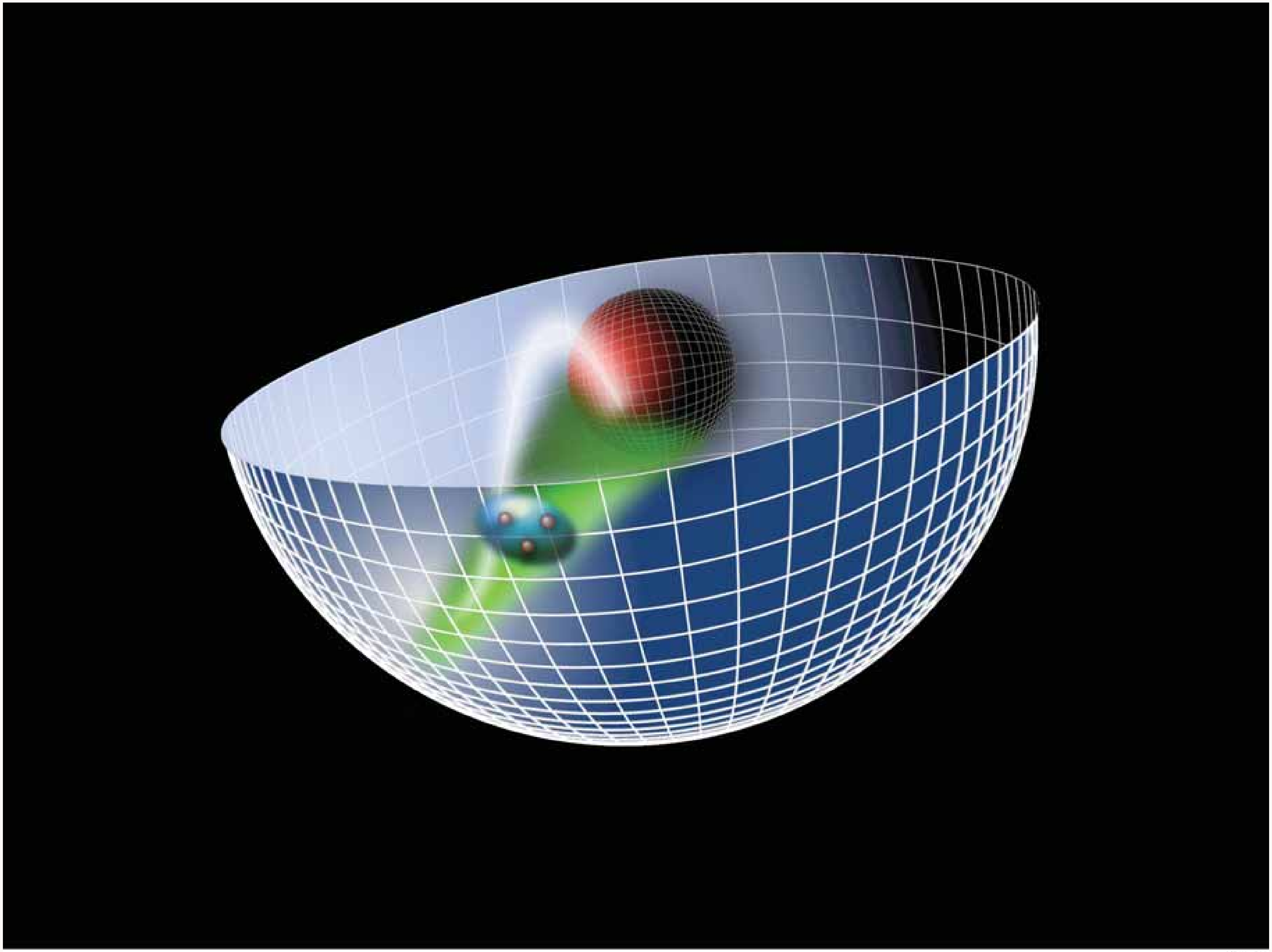
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

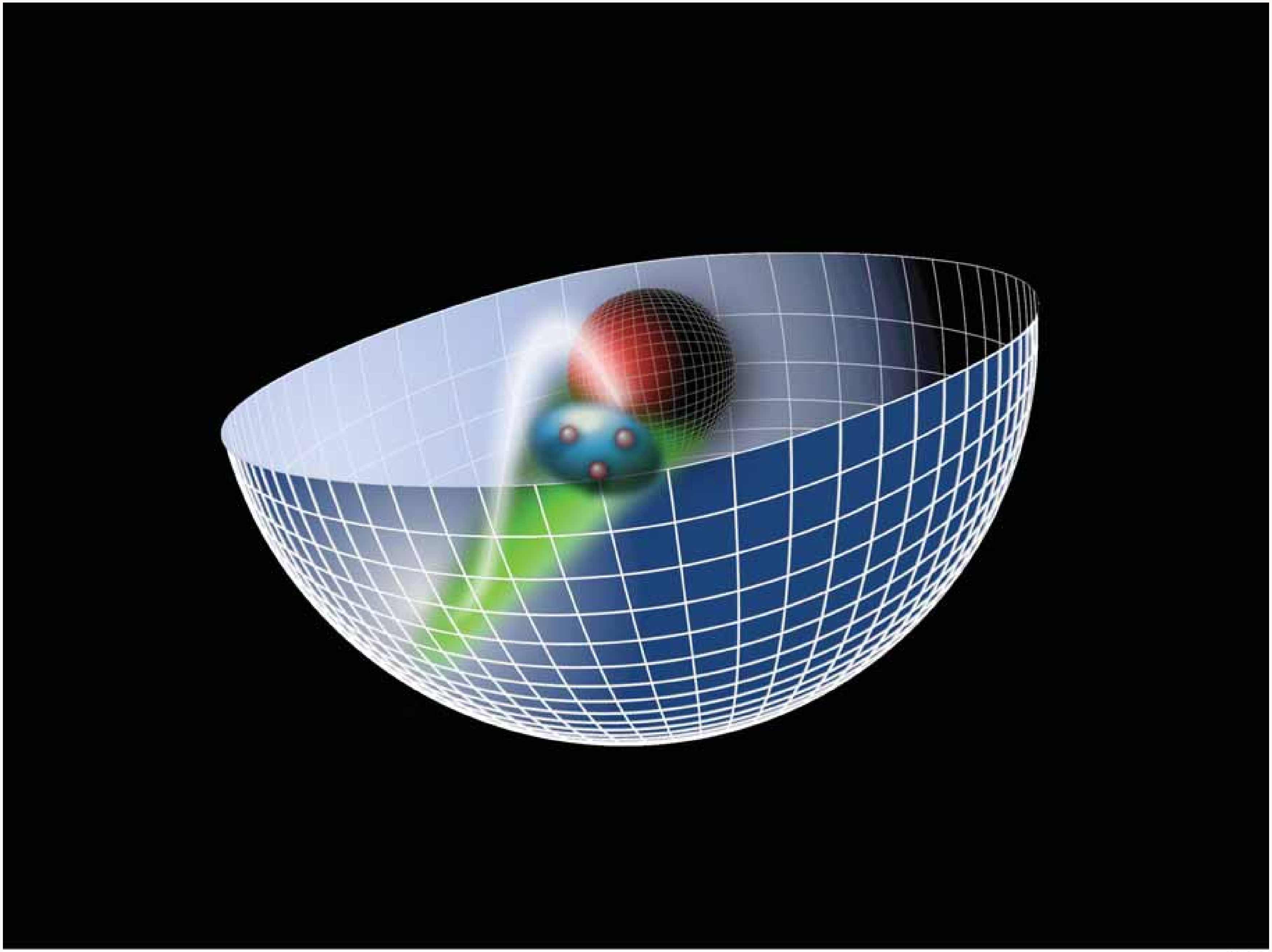
$x^2 = x_\mu x^\mu$: invariant separation between quarks

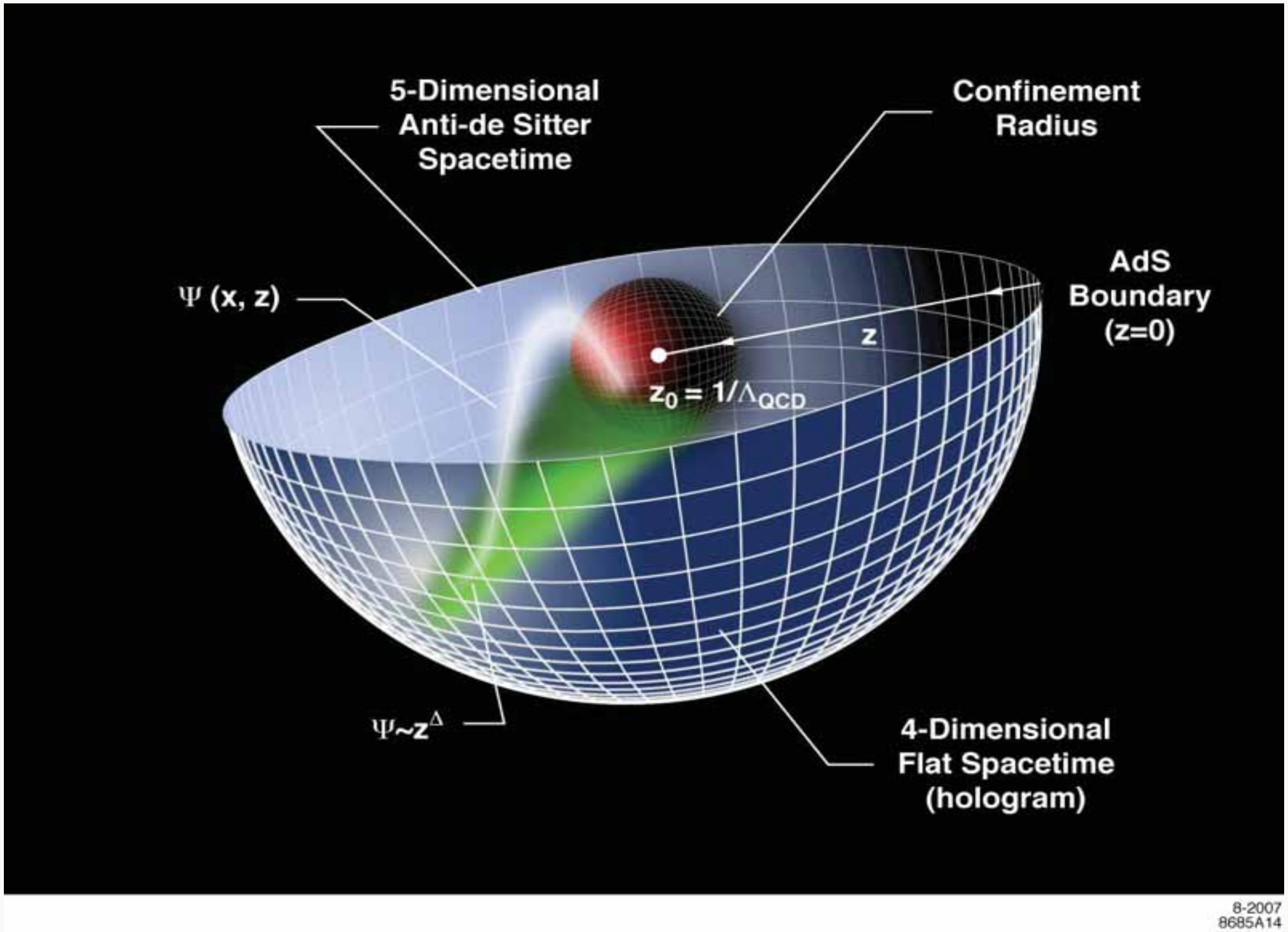
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.












- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

# AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2 \qquad z \to \lambda z$
- Match solutions at small z to conformal twist dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

# Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$ .
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along $x^\mu$-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2}\phi(z)$$

## AdS Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}]\phi(z) = \mathcal{M}^2\phi(z)$$

**L: light-front orbital angular momentum**

## Derived from variation of Action in $AdS_5$

## Hard wall model: truncated space

$$\phi(\mathrm{z} = \mathrm{z}_0 = \frac{1}{\Lambda_c}) = 0.$$

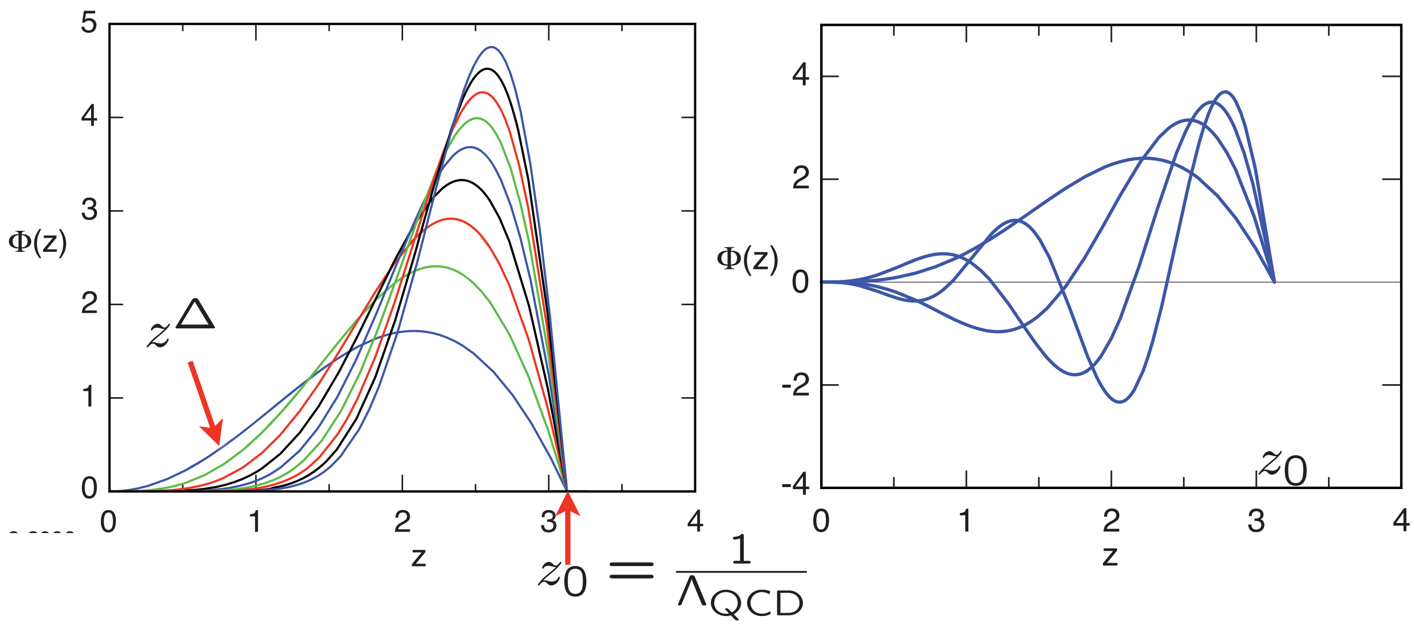
## *Match fall-off at small z to conformal twist-dimension at short distances*

*twist*

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$




$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

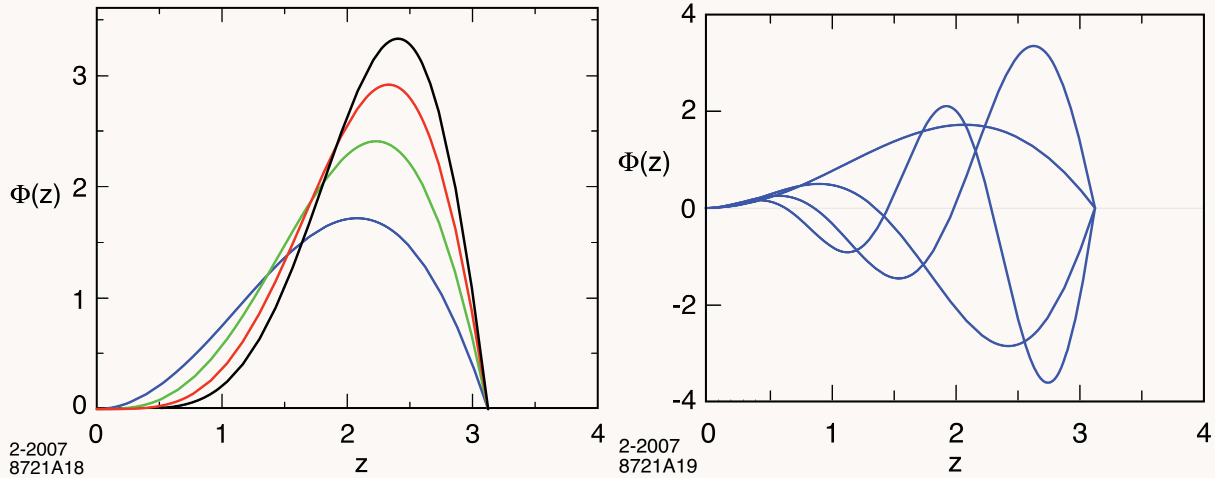



Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

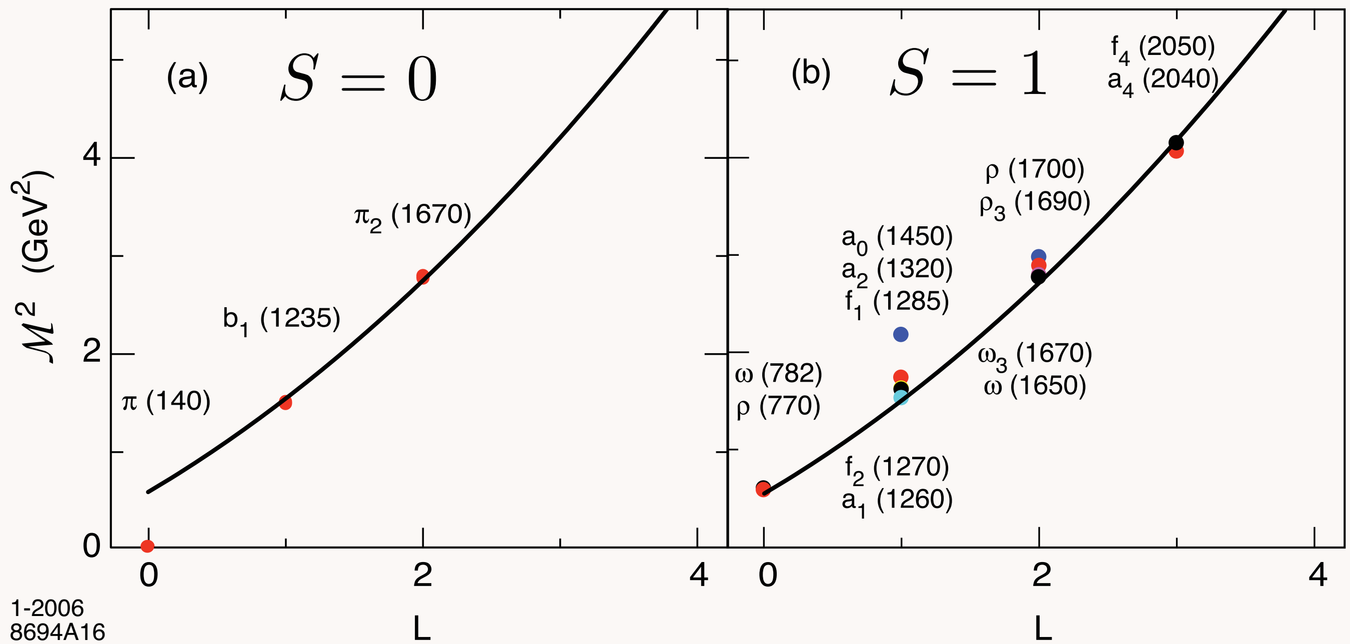



Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

# Soft-Wall Model

$$S = \int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)} \mathcal{L}, \qquad \varphi(z) = \pm\kappa^2 z^2$$

***Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field***

**Karch, Katz, Son and Stephanov (2006)]**

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2\partial_z^2 - \left(3 \mp 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

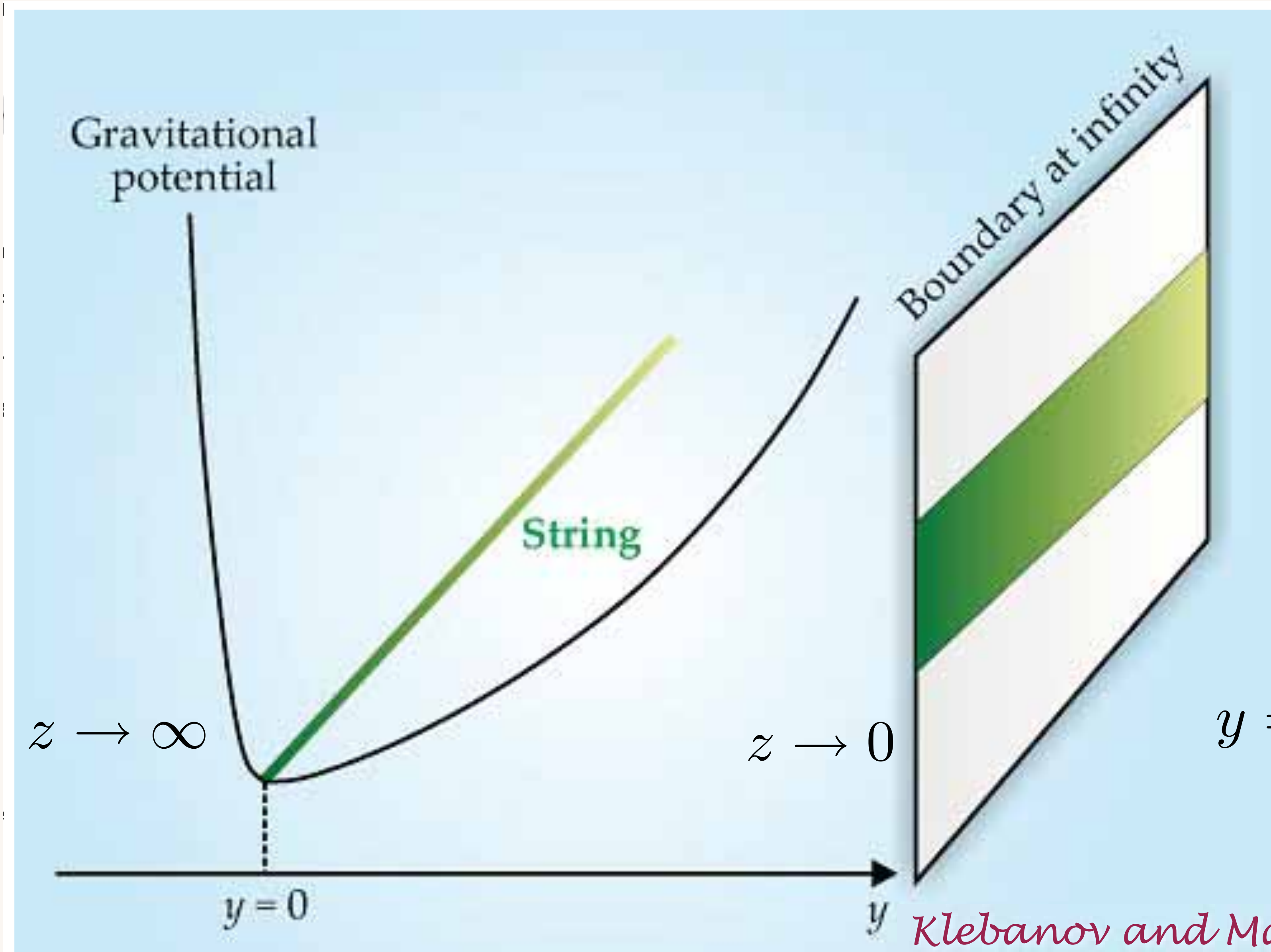
- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

# Massless pion

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2}(dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$




$z \to \infty$

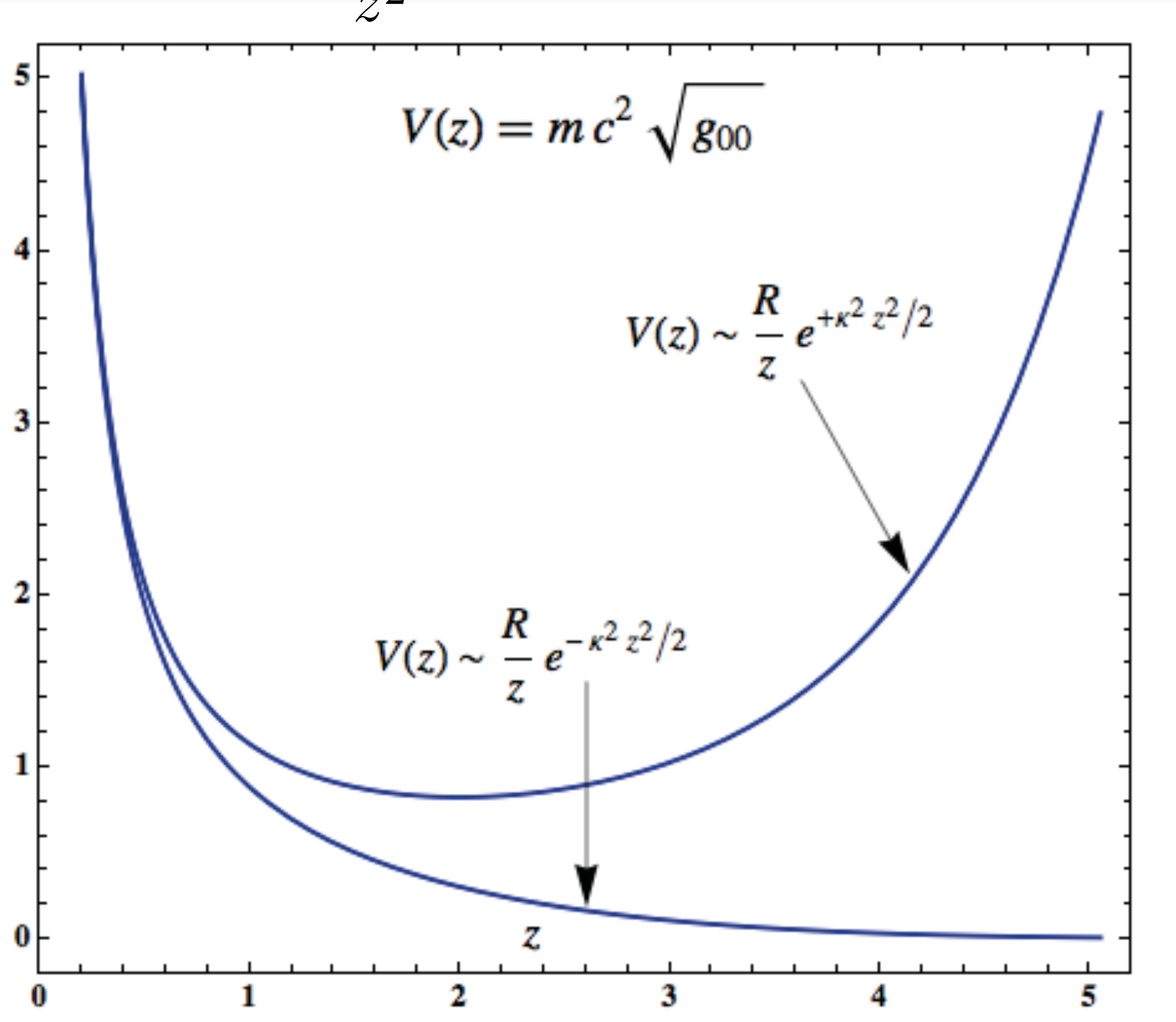
$z \to 0$

$y = R/z$

Klebanov and Maldacena

$$ds^2 = e^{A(y)}(-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2$$

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2}(dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$




Agrees with Klebanov and Maldacena for positive-sign exponent of dilaton

- Erlich, Karch, Katz, Son, Stephanov
- de Teramond, sjb

# AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action Dilaton-Modified $AdS_5$

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

*Quark separation increases with L*

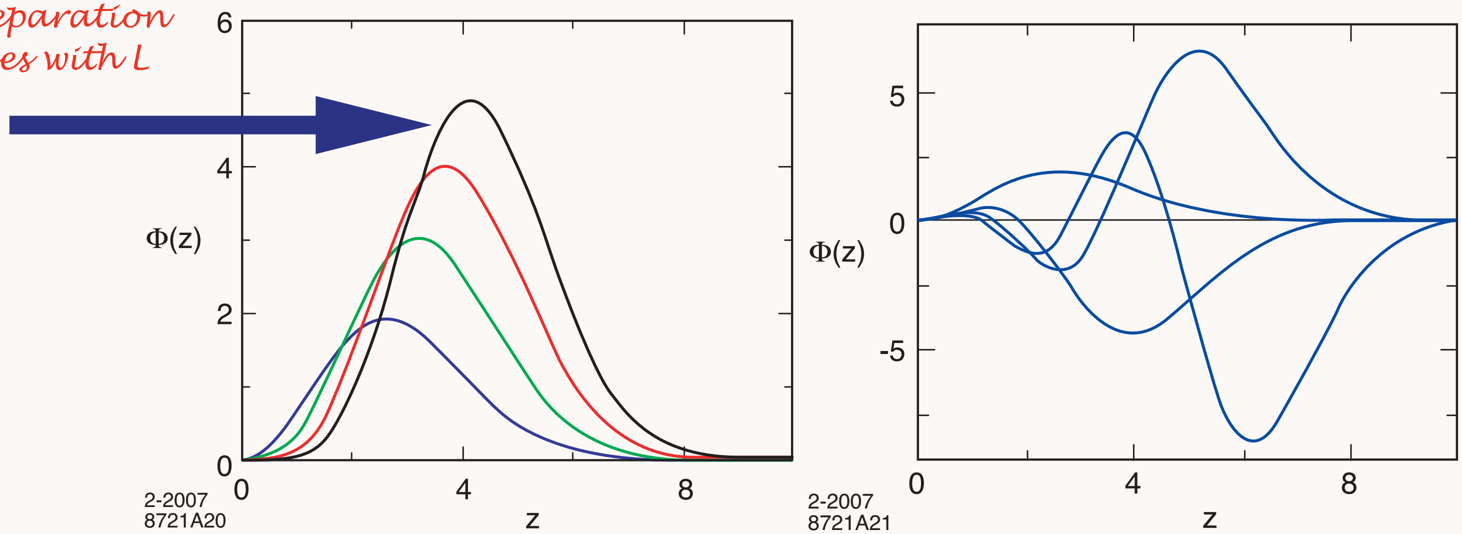
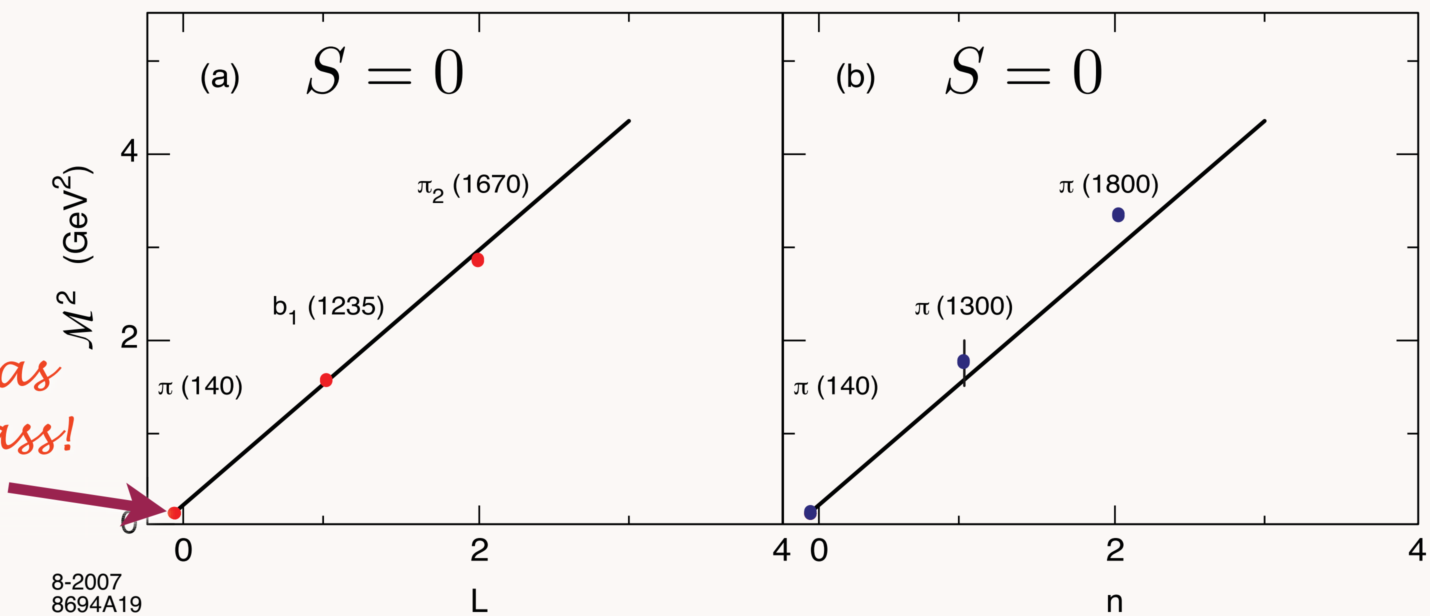



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

# *Soft Wall Model*

*Pion has zero mass!*




Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

**Pion mass automatically zero!**

$$m_q = 0$$

# Higher-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

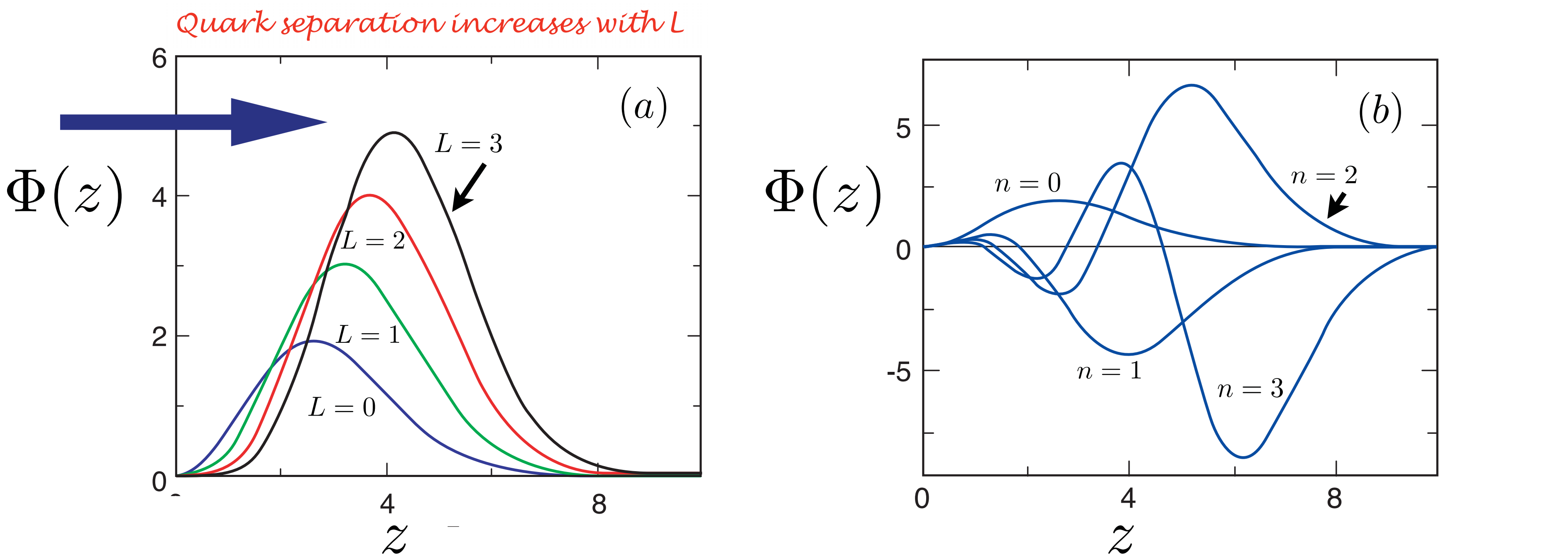
- Upon substitution $z \rightarrow \zeta$

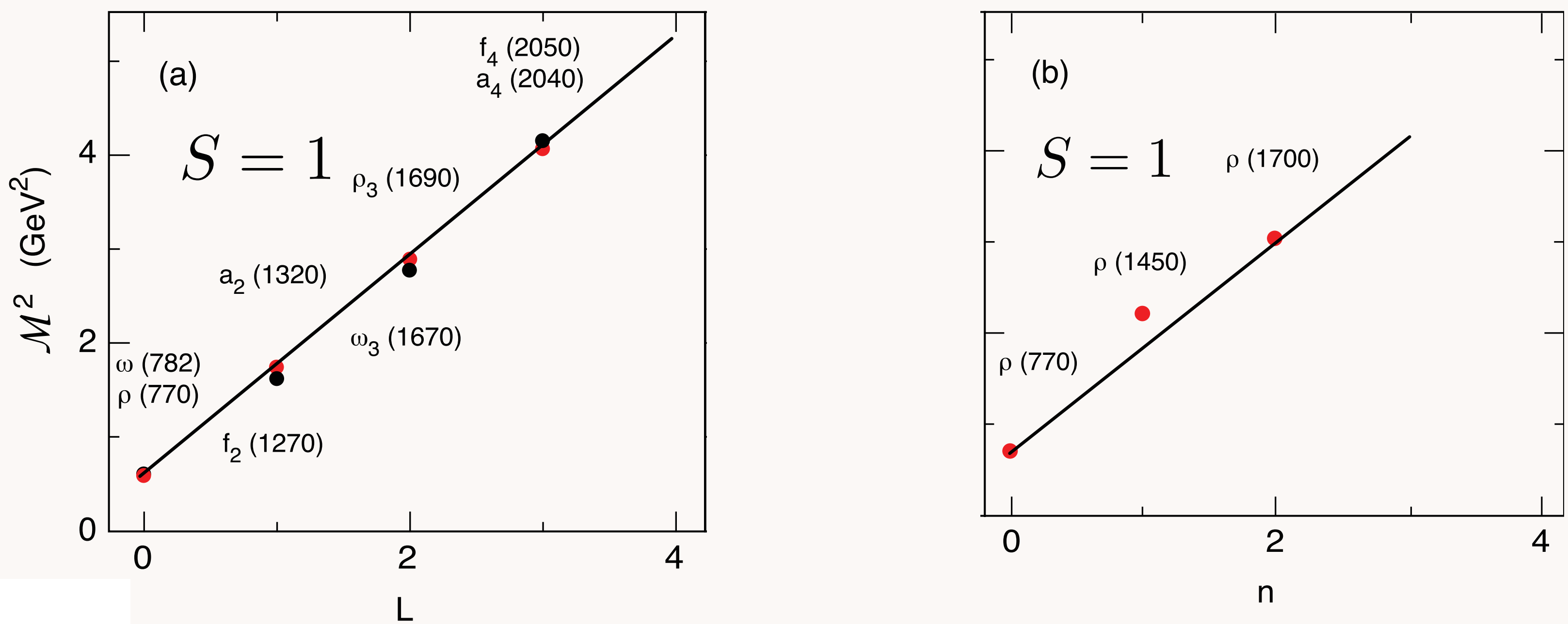
$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

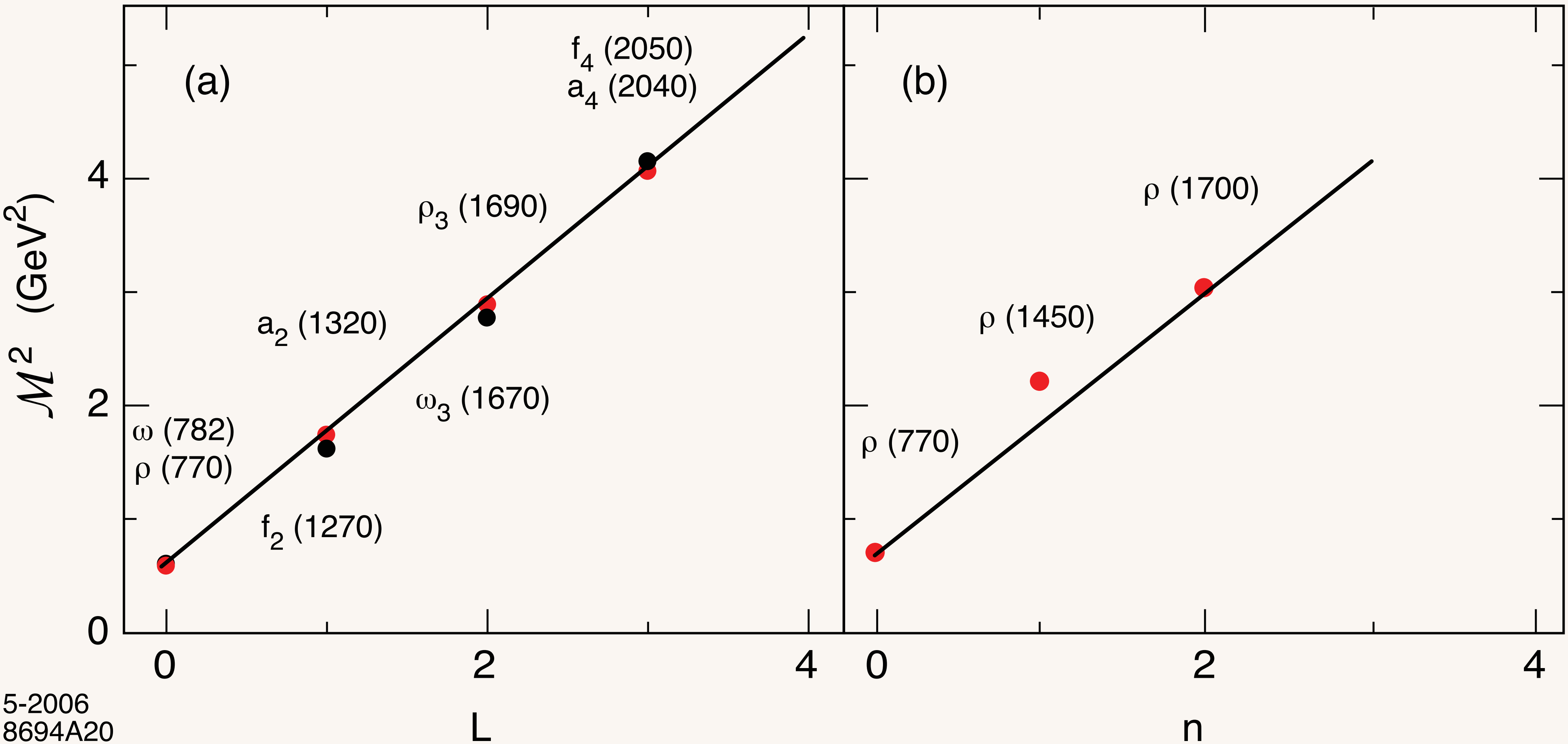
with $(\mu R)^2 = -(2-J)^2 + L^2$

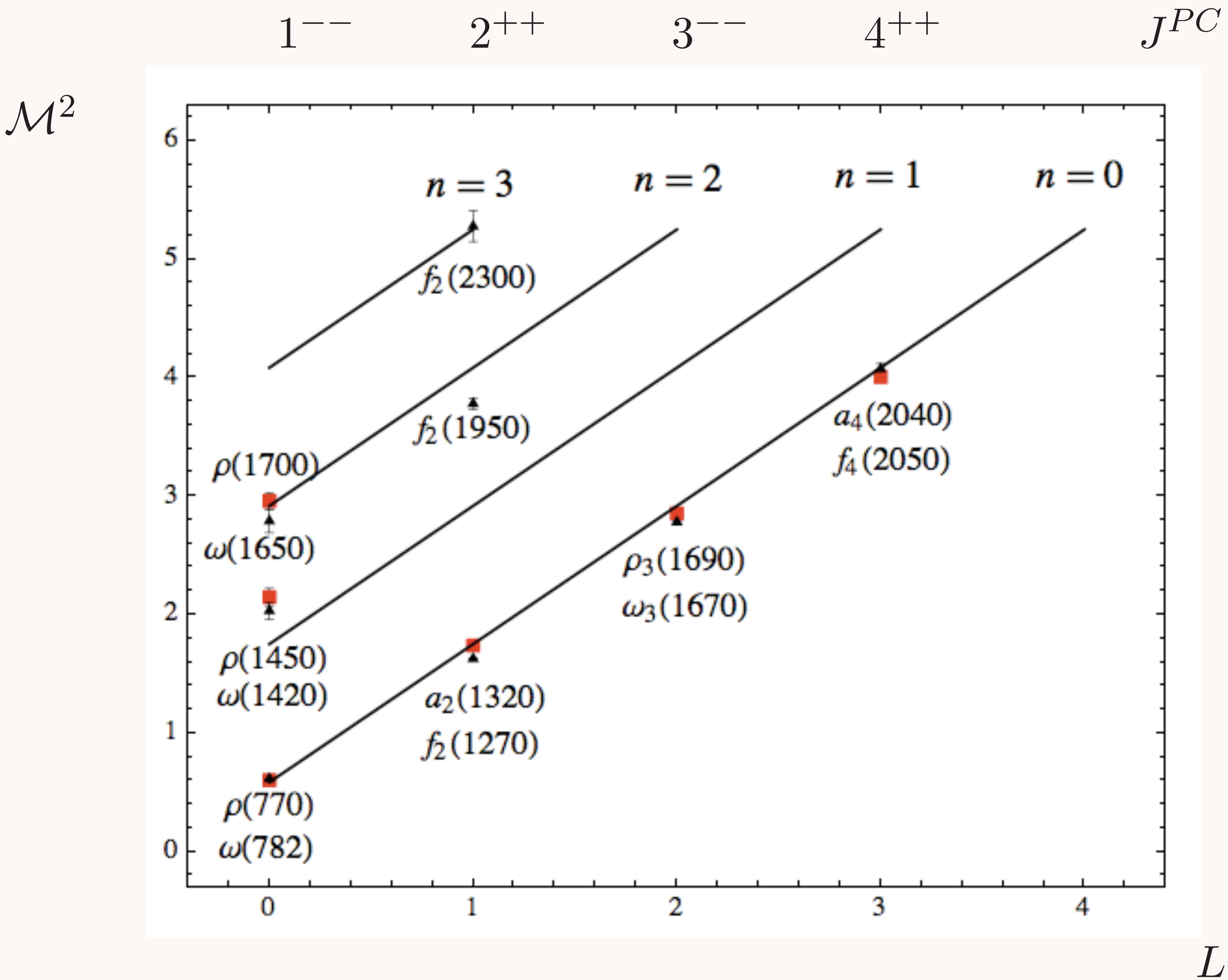

Quark separation increases with L
(a)
Φ(z)
L = 3
L = 2
L = 1
L = 0
z
(b)
n = 0
n = 1
n = 2
n = 3


(a)
S = 1
M² (GeV²)
L
f₄ (2050)
a₄ (2040)
ρ₃ (1690)
a₂ (1320)
ω₃ (1670)
ω (782)
ρ (770)
f₂ (1270)
(b)
ρ (1700)
ρ (1450)
ρ (770)
n

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$






Parent and daughter Regge trajectories for the $I=1$ $\rho$-meson family (red) and the $I=0$ $\omega$-meson family (black) for $\kappa = 0.54$ GeV

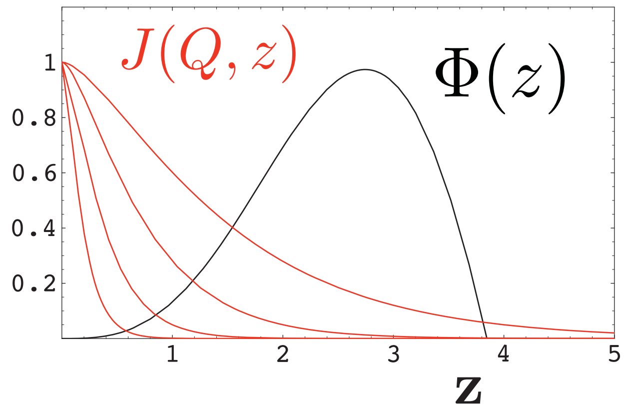
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High $Q^2$
from
small z ~ 1/Q




**Polchinski, Strassler**
**de Teramond, sjb**

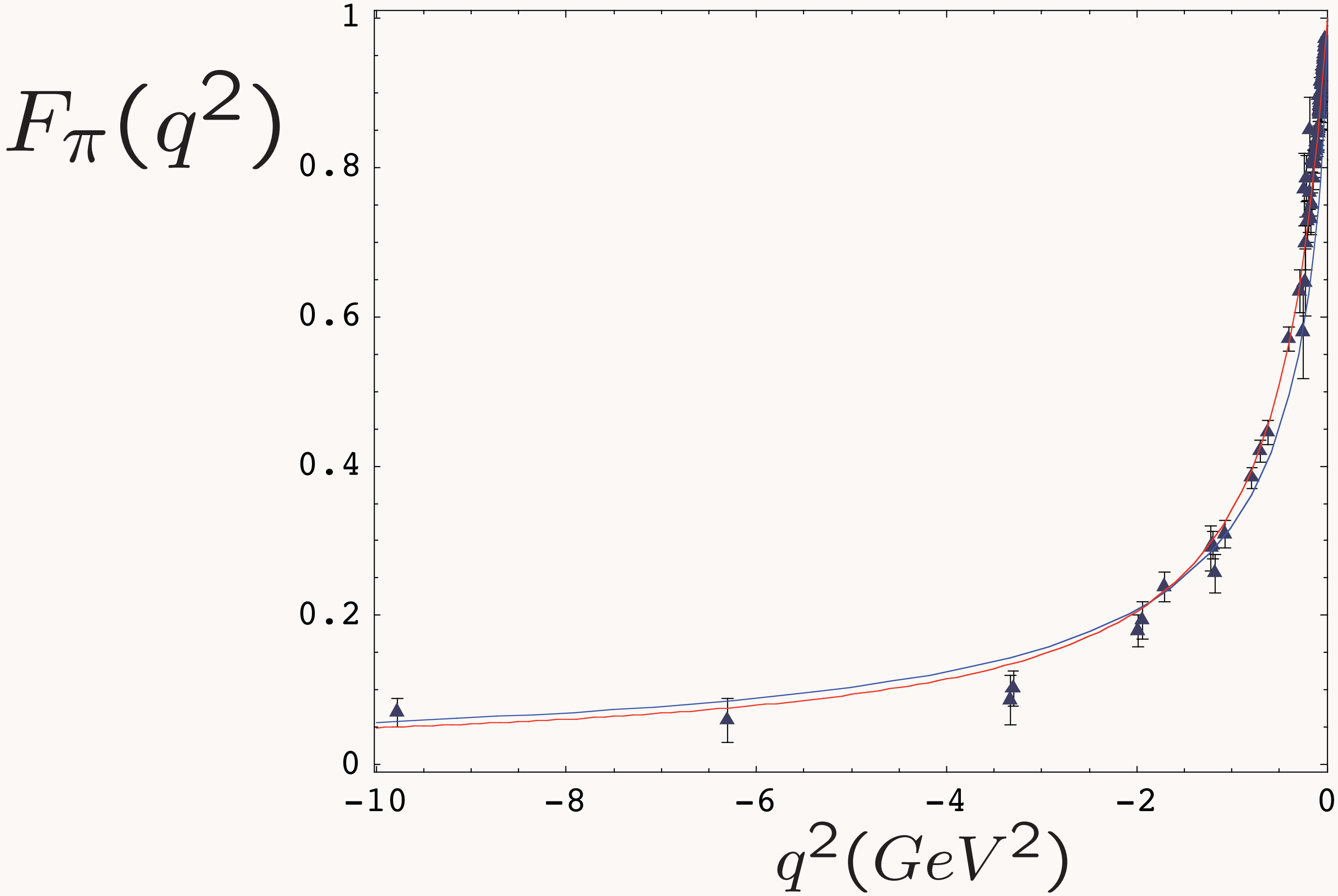
Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

**Dimensional Quark Counting Rules:**
**General result from**
**AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

# Spacelike pion form factor from AdS/CFT




**Data Compilation**
**Baldini, Kloe and Volmer**

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

**de Teramond, sjb**
**See also: Radyushkin**