

# Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space  $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ( $b = |\vec{b}_\perp|$ ):

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

**Soper**

## Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with  $\tilde{\rho}(x, \zeta)$  QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary  $Q$  using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for  $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$  !

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2,$$

where  $J(Q^2, z) = zQK_1(zQ)$ .

- Use integral representation for  $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary  $Q$

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4}$$

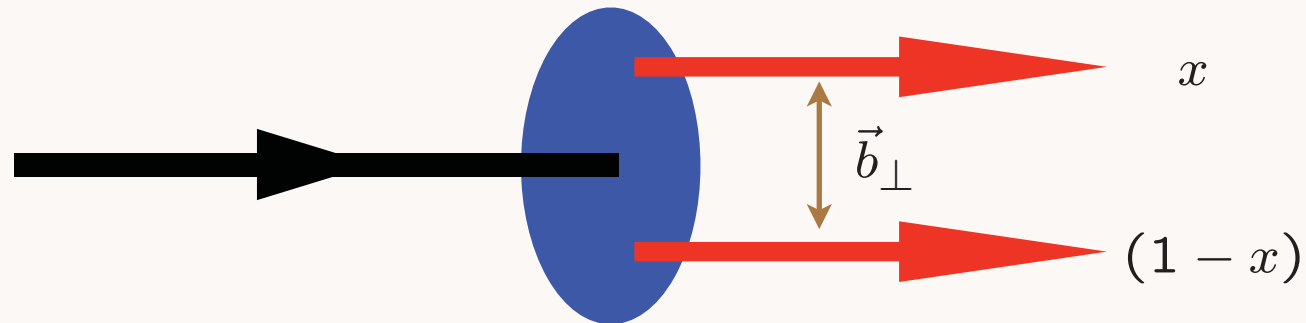
with  $\zeta = z$ ,  $0 \leq \zeta \leq \Lambda_{\text{QCD}}$

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

*Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

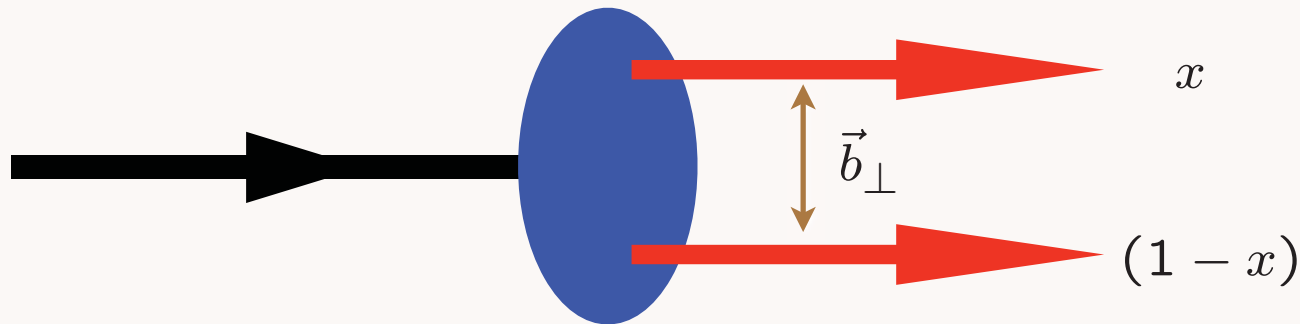
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*soft wall  
confining potential*

**G. de Teramond, sjb**

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[ z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where  $U(a, b, c)$  is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background  $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

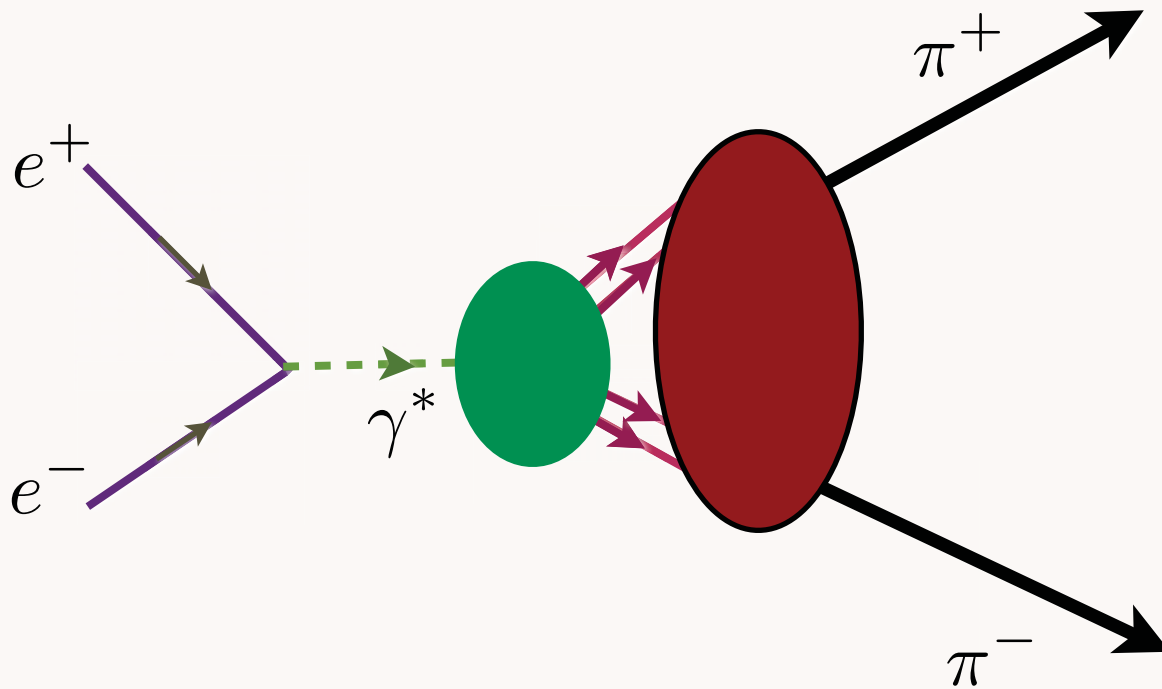
- For large  $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Soft Wall  
Model*

*Dressed soft-wall current bring in higher Fock states and more vector meson poles*



# Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

...

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \quad N,$$

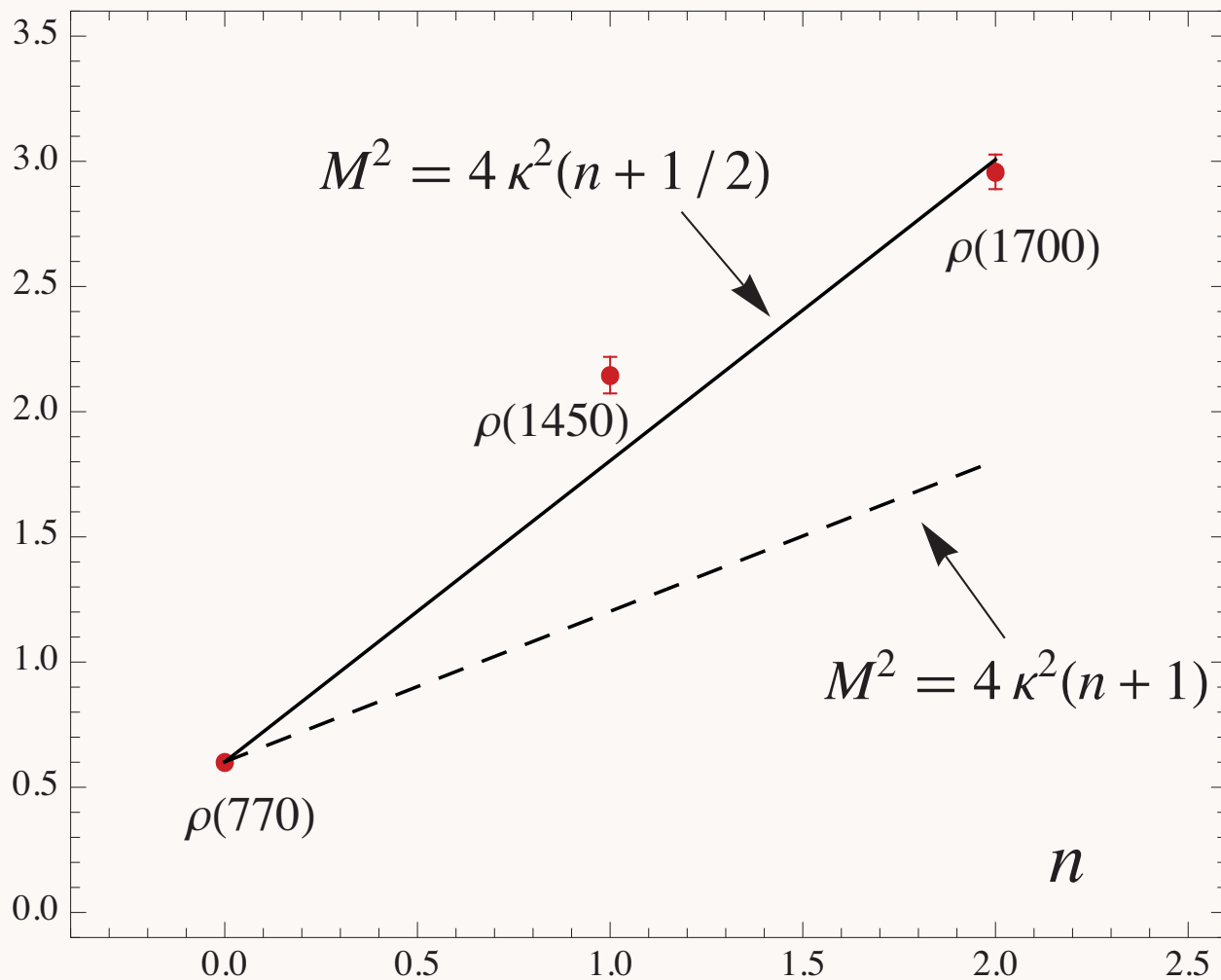
Positive Dilaton Background  $\exp(+\kappa^2 z^2)$        $\mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right)$

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}$$

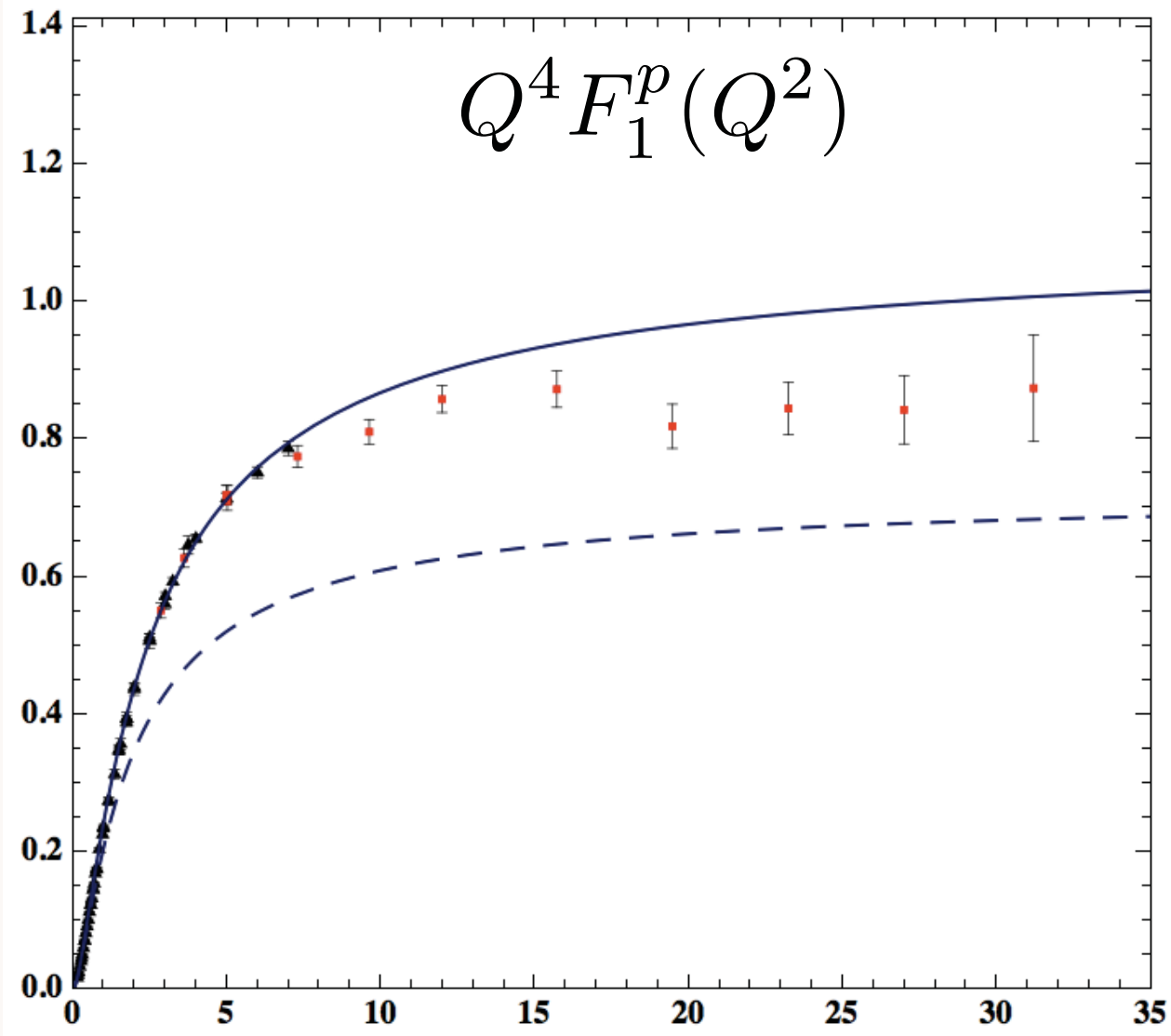
$$Q^2 \rightarrow \infty$$

*Constituent Counting*

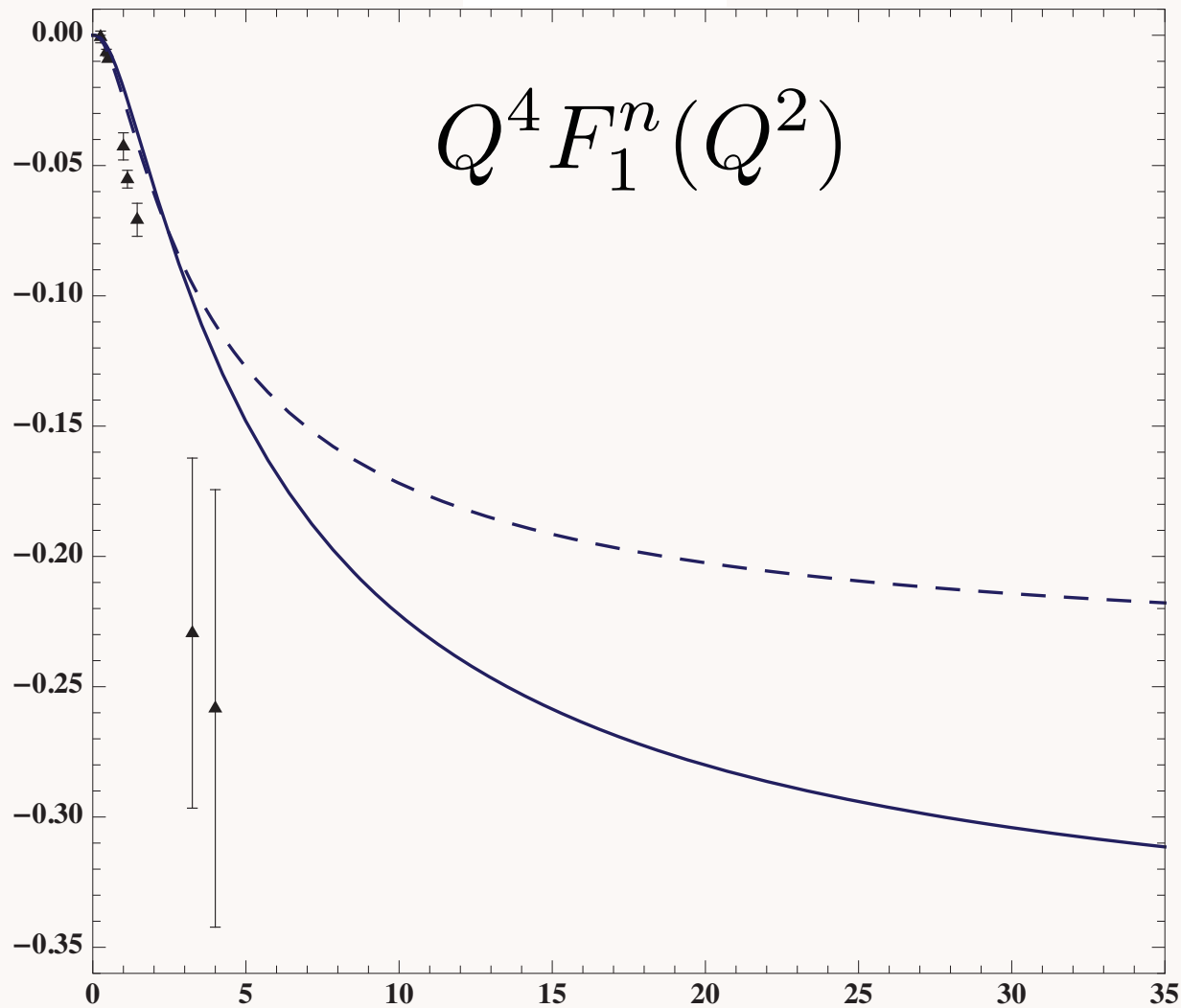




Vector meson radial trajectories in a negative (dashed line,  $\kappa = 0.3877$  GeV) and positive dilaton backgrounds (continuous line,  $\kappa = 0.5484$  GeV).



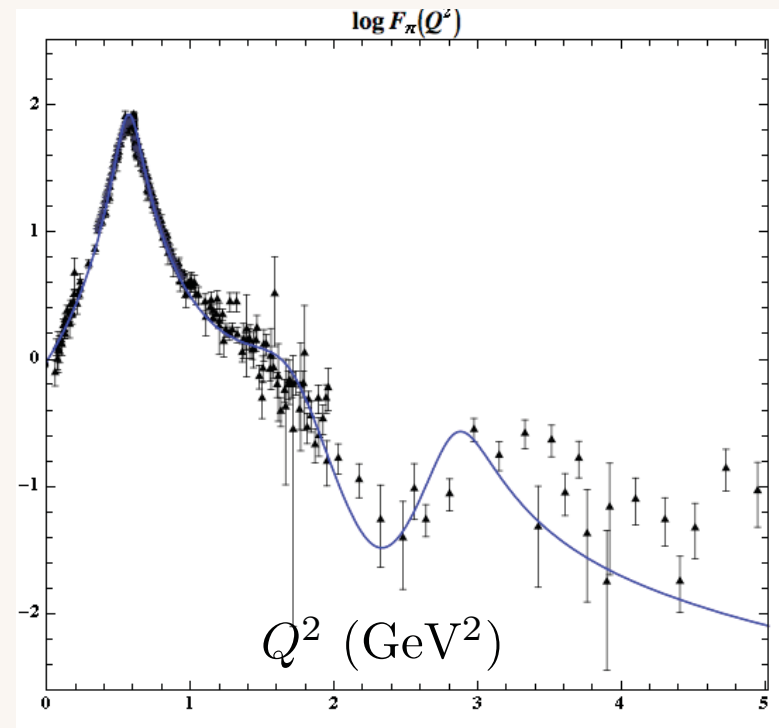
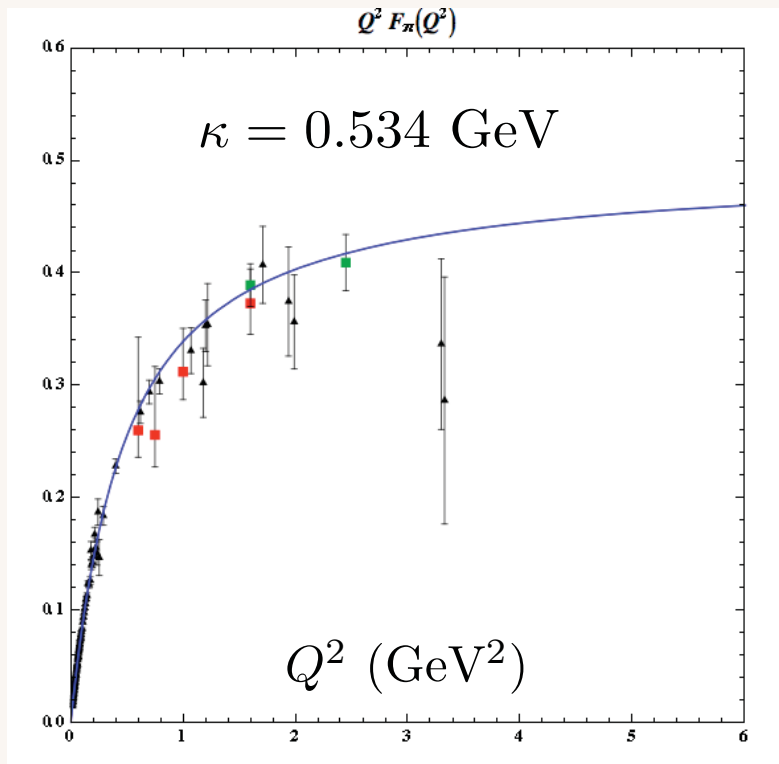
$Q^4 F_1^p(Q^2)$  in a negative (dashed line,  $\kappa = 0.3877$  GeV) and positive dilaton backgrounds (continuous line,  $\kappa = 0.5484$  GeV). The data compilation is from Diehl.



$Q^4 F_n^1(Q^2)$  in a negative (dashed line,  $\kappa = 0.3877$  GeV) and positive dilaton backgrounds (continuous line,  $\kappa = 0.5484$  GeV). The data compilation is from Diehl.

# Spacelike and timelike pion form factor

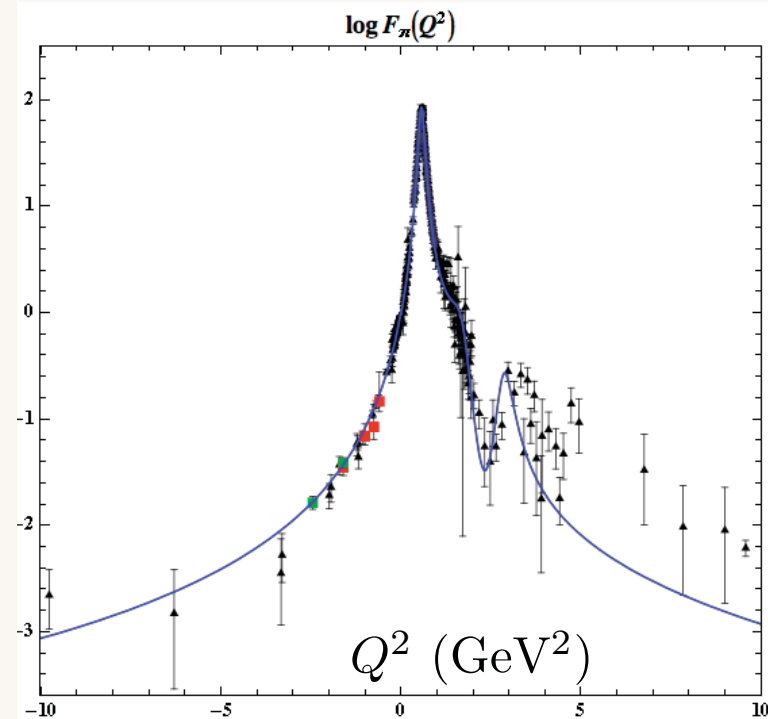
Preliminary



$$|\pi \rangle = \psi_{q\bar{q}} |q\bar{q} \rangle + \psi_{q\bar{q}q\bar{q}} |q\bar{q}q\bar{q} \rangle$$

$$\Gamma_\rho = 120 \text{ MeV}, \Gamma'_\rho = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 15\%$$



**Carl E. Carlson**

**Zainul Abidin**

AdS/CFT now extensive field---apologies for all omitted references  
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: "fancy"  
Start from string theory, develop QCD analogs  
on lower dimensional branes

**Sakai & Sugimoto**

"Bottom-up"  
Anticipate what 5D Lagrangian must be (guess),  
directly involving desired  $\rho$ ,  $\pi$ ,  $a_1$ , ... fields and  
connect to matching QCD structures

**Erlich et al.  
Da Rold & Pomarol**

EM form factors in "bottom-up" approach

**Brodsky & de Teramond  
Radyushkin & Grigoryan**

Gravitational form factors in bottom-up approach

**Zainul Abidin & me**

Soft-wall

**Karch, Katz, Son, and Stephanov  
Batell, Gherghetta, and Sword**

*LF(3+1)*

*AdS<sub>5</sub>*

$$\psi(x, \vec{b}_\perp)$$



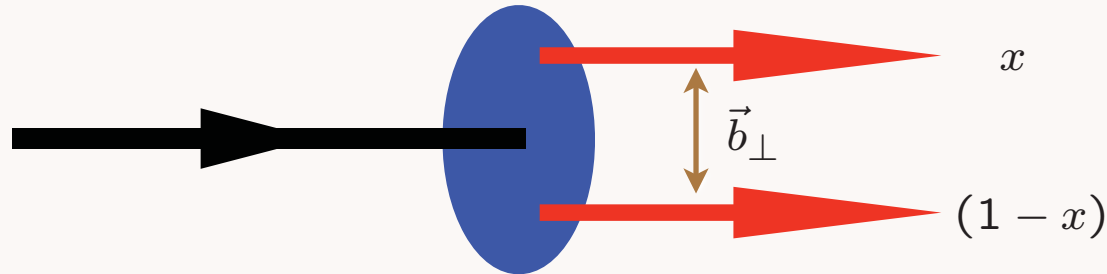
$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$



$$z$$

$$\psi(x, \vec{b}_\perp)$$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta)$$

*Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

# Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where  $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for  $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary  $Q$

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

*Identical to LF Holography obtained from electromagnetic current*

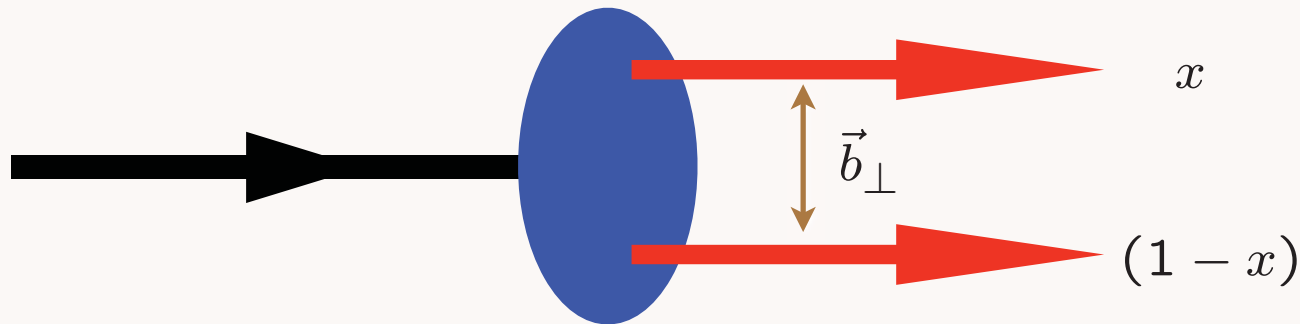
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Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

G. de Teramond, sjb

*soft wall  
confining potential:*



# Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left( -\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions.} \end{aligned}$$

**Change  
variables**

$$(\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

$H_{QED}$

*QED atoms: positronium  
and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

*Coupled Fock states*

$$\left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$\left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{l(l+1)}{r^2} + V_{\text{eff}}(r, S, l) \right] \psi(r) = E \psi(r)$$

*Spherical Basis  $r, \theta, \phi$*

$$V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

**Bohr Spectrum**

*Semiclassical first approximation to QED*

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

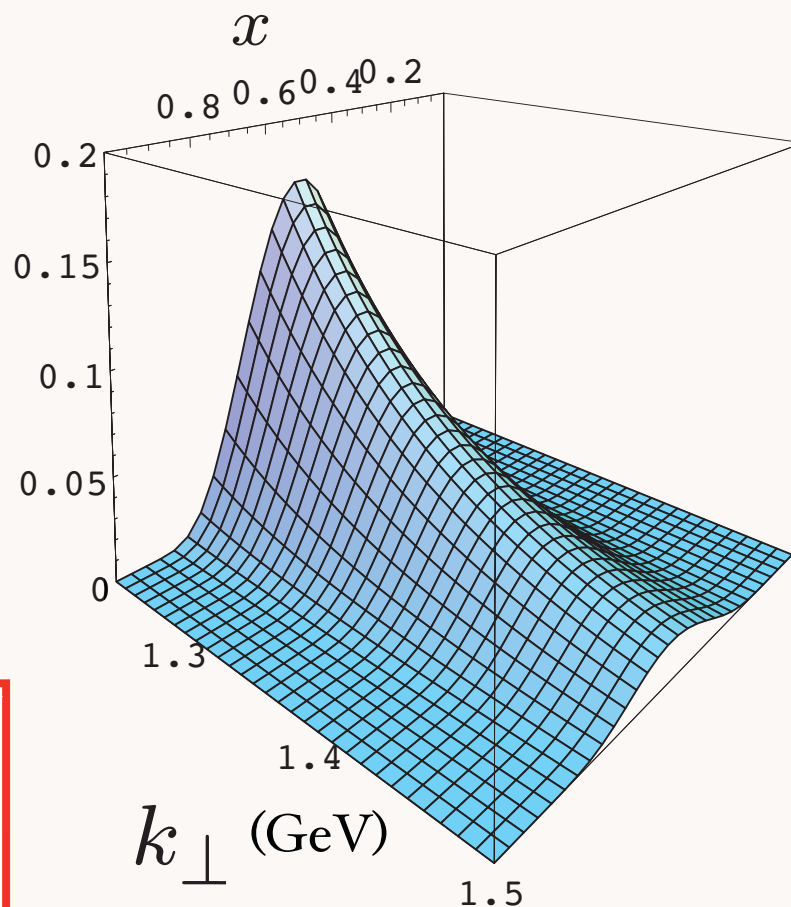
# Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

**“Soft Wall”  
model**

$\kappa = 0.375 \text{ GeV}$   
massless quarks

$$\psi_M(x, k_{\perp}^2)$$



**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

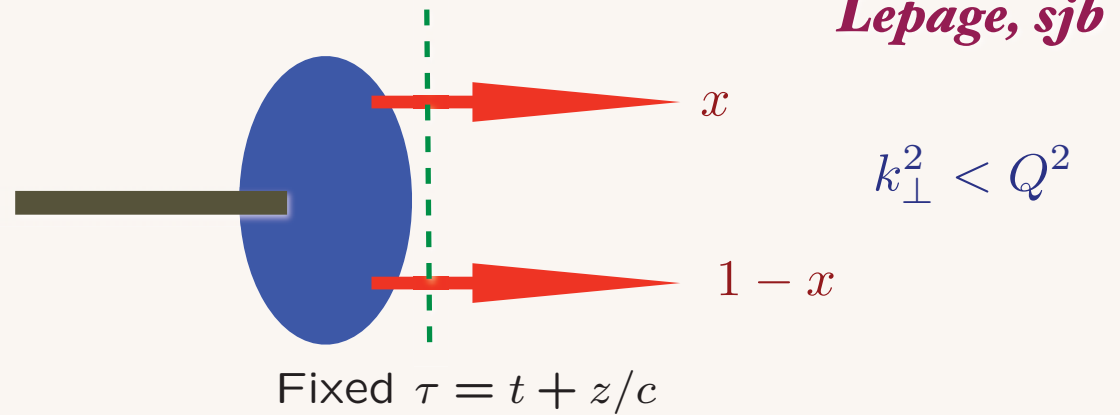
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

*Connection of Confinement to TMDs*

# Hadron Distribution Amplitudes

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

*Lepage, sjb*

*Efremov, Radyushkin*

*Sachrajda, Frishman Lepage, sjb*

*Braun, Gardi*

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_{\perp})$$

# Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

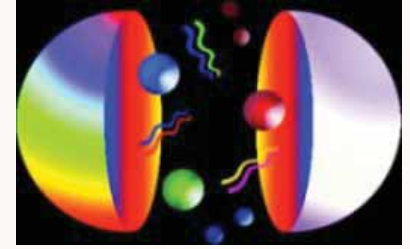
$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD  
GdT and Brodsky: hep-th/0409074, hep-th/0501022.



From Nick Evans

## *Baryons in AdS/CFT*

- Action for massive fermionic modes on AdS<sub>5</sub>:

$$S[\bar{\Psi}, \Psi] = \int d^4x dz \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion:  $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution ( $\mu R = \nu + 1/2$ )

$$\Psi(z) = C z^{5/2} [J_\nu(z\mathcal{M})u_+ + J_{\nu+1}(z\mathcal{M})u_-]$$

- Hadronic mass spectrum determined from IR boundary conditions  $\psi_\pm(z = 1/\Lambda_{\text{QCD}}) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}$$

with scale independent mass ratio

- Obtain spin- $J$  mode  $\Phi_{\mu_1 \dots \mu_{J-1/2}}$ ,  $J > \frac{1}{2}$ , with all indices along 3+1 from  $\Psi$  by shifting dimensions

# Baryons

## Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin- $\frac{1}{2}$  modes  $\psi(\zeta)$  and spin- $\frac{3}{2}$  modes  $\psi_\mu(\zeta)$  are **two-component spinor** solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where  $H_{LF} = \alpha\Pi$  and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint  $\Pi_L^\dagger(\zeta)$  satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$



- Note: in the Weyl representation ( $i\alpha = \gamma_5\beta$ )

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension  $3 + L$  ( $\nu = L + 1$ )

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_-].$$

Baryonic modes propagating in AdS space have two components: orbital  $L$  and  $L + 1$ .

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,$$

given by

$$\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},$$

with a scale independent mass ratio.

## Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[ i \left( z \eta^{\ell m} \Gamma_{\ell} \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^{\ell}) = 0.$$

- Solution ( $\mu R = \nu + 1/2$ ,  $d = 4$ )  
 $\nu = L + 1$

$$\Psi_+(z) \sim z^{\frac{5}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^{\nu}(\kappa^2 z^2)$$

$$\Psi_-(z) \sim z^{\frac{7}{2} + \nu} e^{-\kappa^2 z^2 / 2} L_n^{\nu+1}(\kappa^2 z^2)$$

**Hoyer's Linear  
Potential**

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin- $J$  mode  $\Phi_{\mu_1 \dots \mu_{J-1/2}}$ ,  $J > \frac{1}{2}$ , with all indices along 3+1 from  $\Psi$   
by shifting dimensions

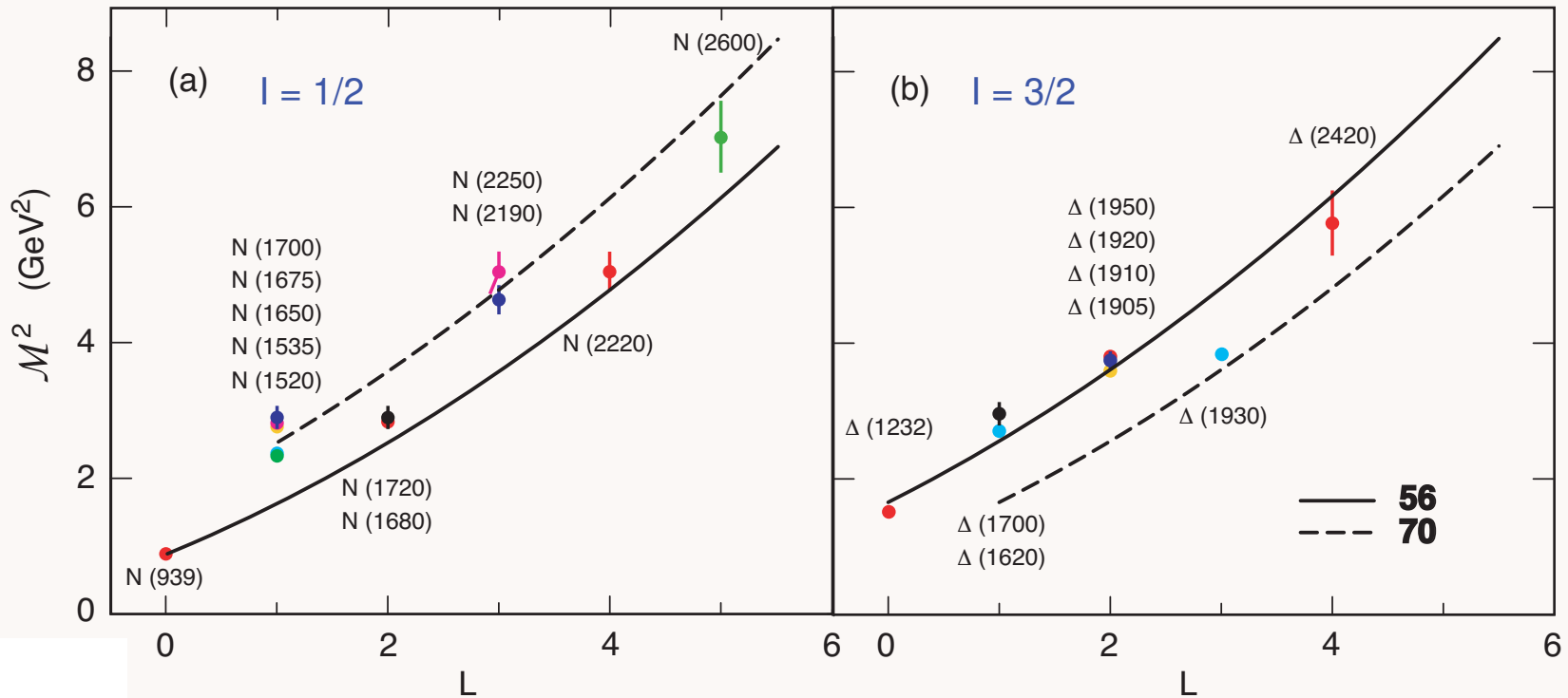


Fig: Light baryon orbital spectrum for  $\Lambda_{QCD} = 0.25$  GeV in the HW model. The **56** trajectory corresponds to  $L$  even  $P = +$  states, and the **70** to  $L$  odd  $P = -$  states.

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator  $\Pi$

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint  $\Pi^\dagger$ , with commutation relations

$$\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

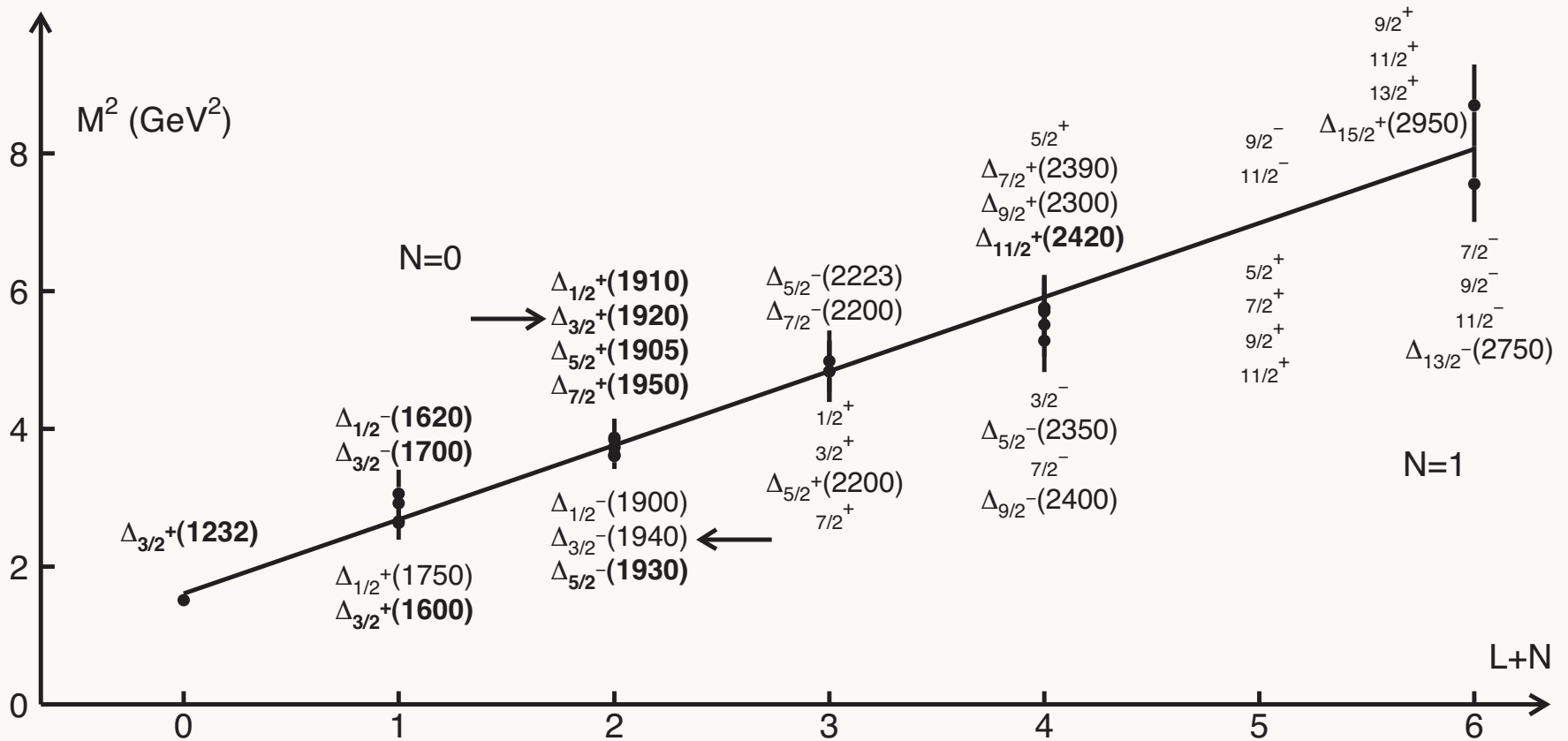
- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),$$

$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$



E. Klempt *et al.*:  $\Delta^*$  resonances, quark models, chiral symmetry and AdS/QCD

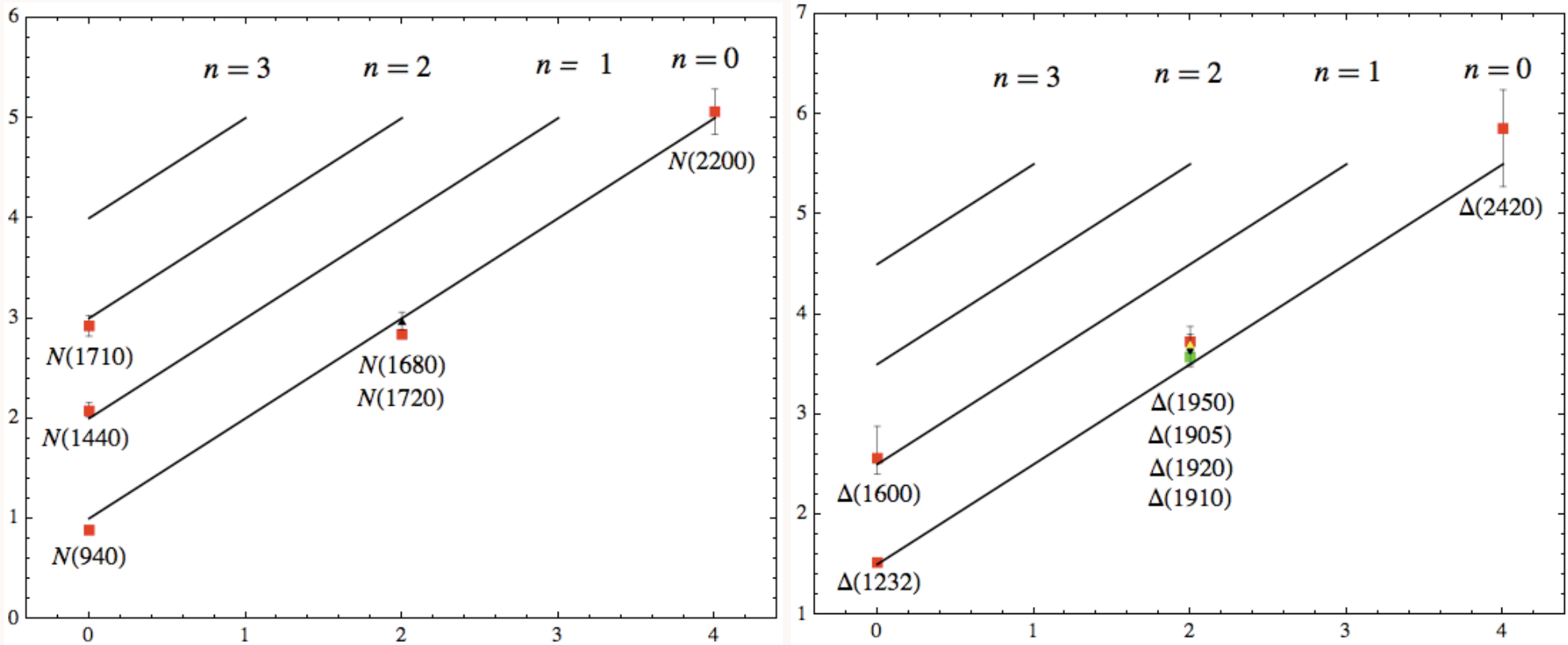
H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.

H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

- $\Delta$  spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

$4\kappa^2$  for  $\Delta n = 1$   
 $4\kappa^2$  for  $\Delta L = 1$   
 $2\kappa^2$  for  $\Delta S = 1$

$$\mathcal{M}^2$$



$L$

Parent and daughter **56** Regge trajectories for the  $N$  and  $\Delta$  baryon families for  $\kappa = 0.5$  GeV

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory.

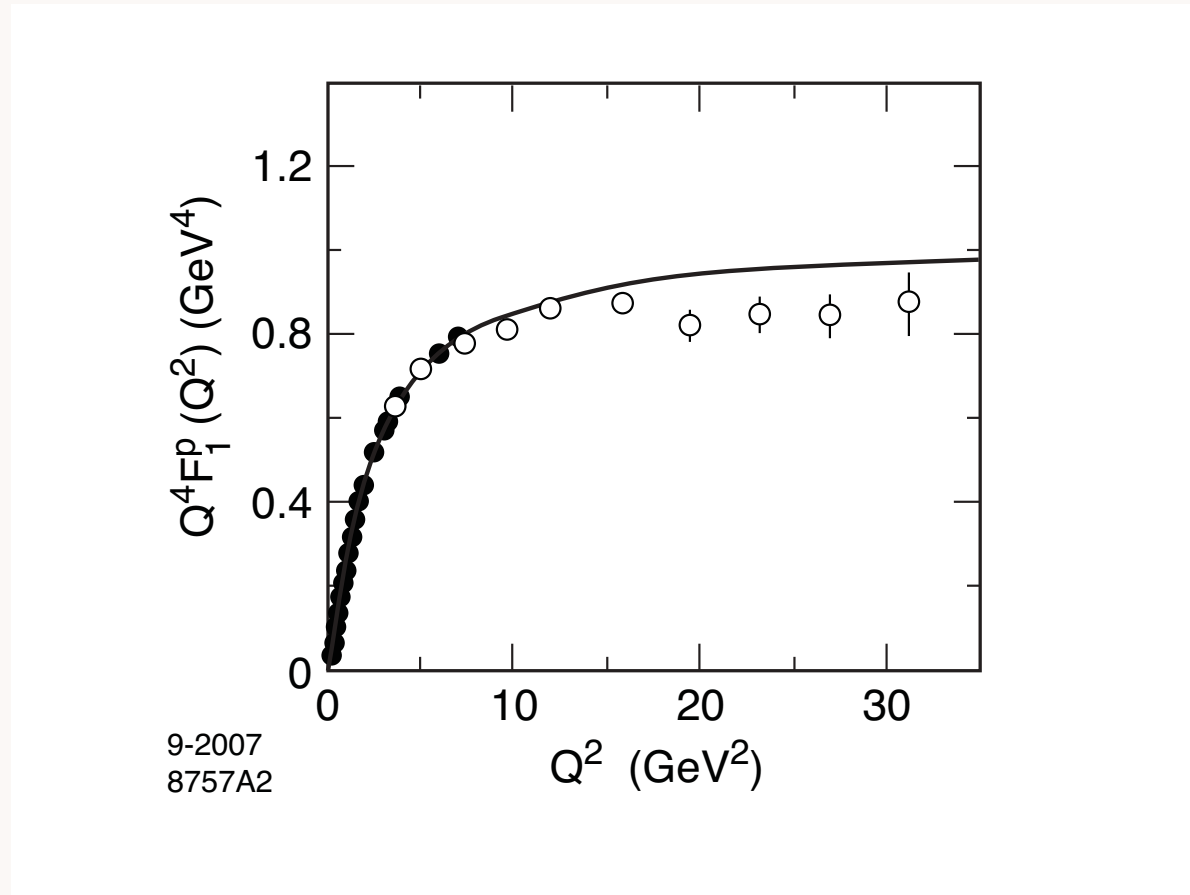
- Choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\psi_+(\zeta)$  and  $\psi_-(\zeta)$  correspond to nucleons with  $J^z = +1/2$  and  $-1/2$ .
- For  $SU(6)$  spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ .

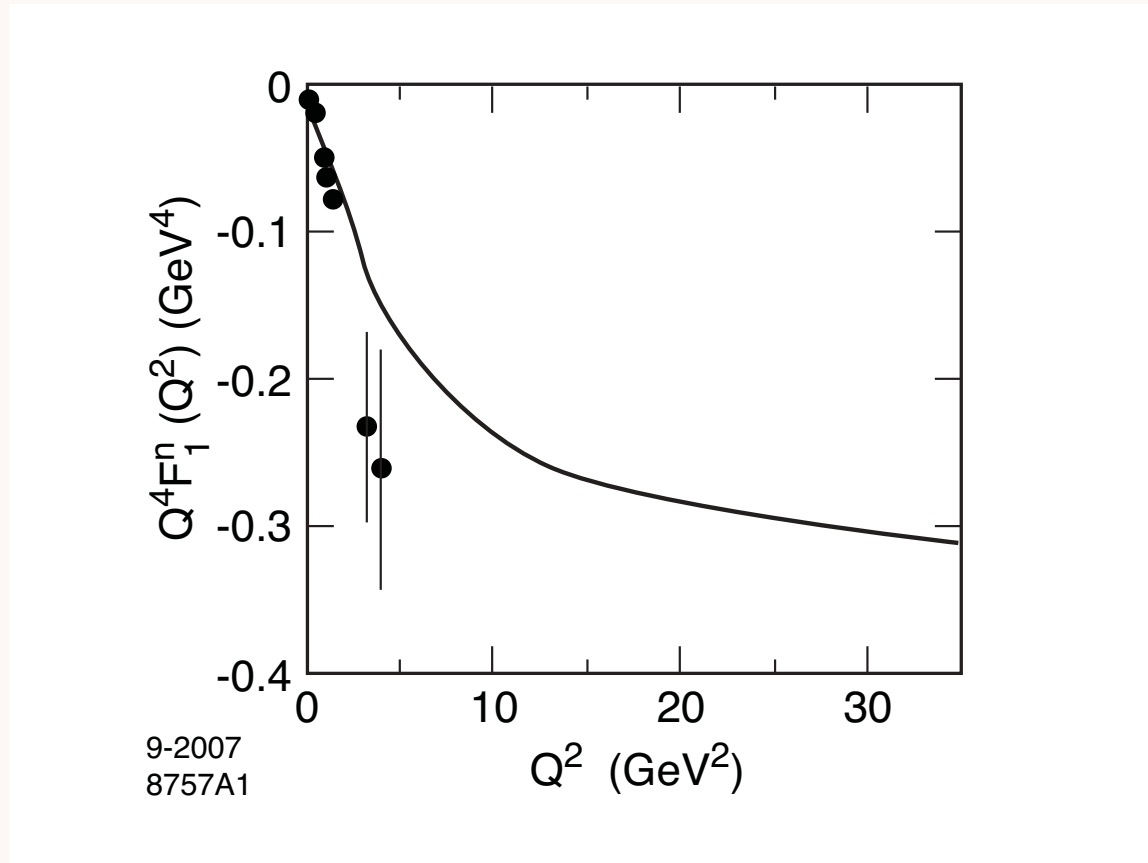
- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  Proton  $\tau = 3$



SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).



- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$  Neutron  $\tau = 3$

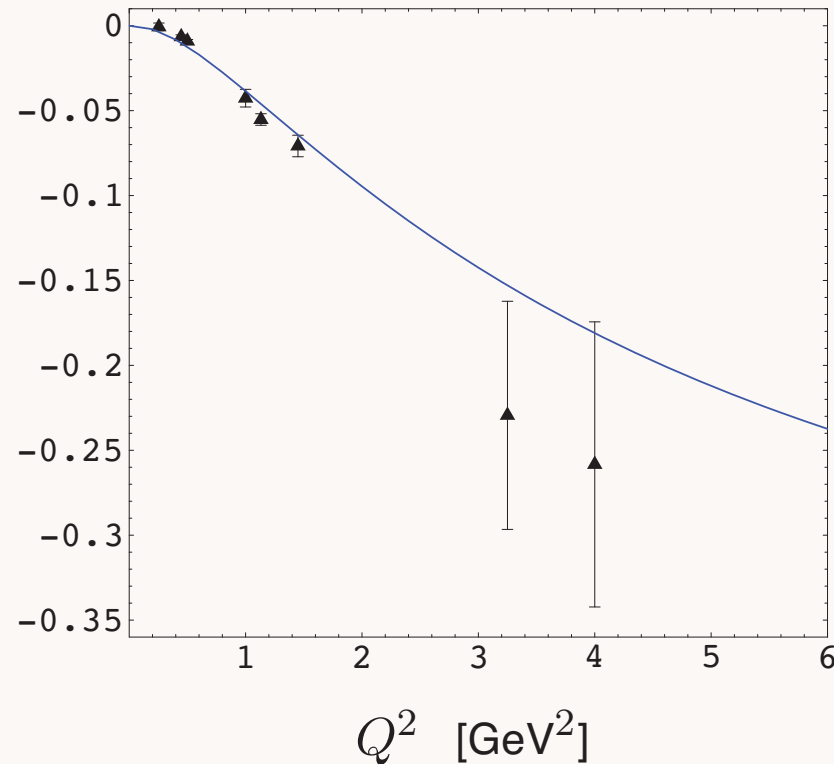


SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

# Dirac Neutron Form Factor (Valence Approximation)

Truncated Space Confinement

$$Q^4 F_1^n(Q^2) \text{ [GeV}^4\text{]}$$

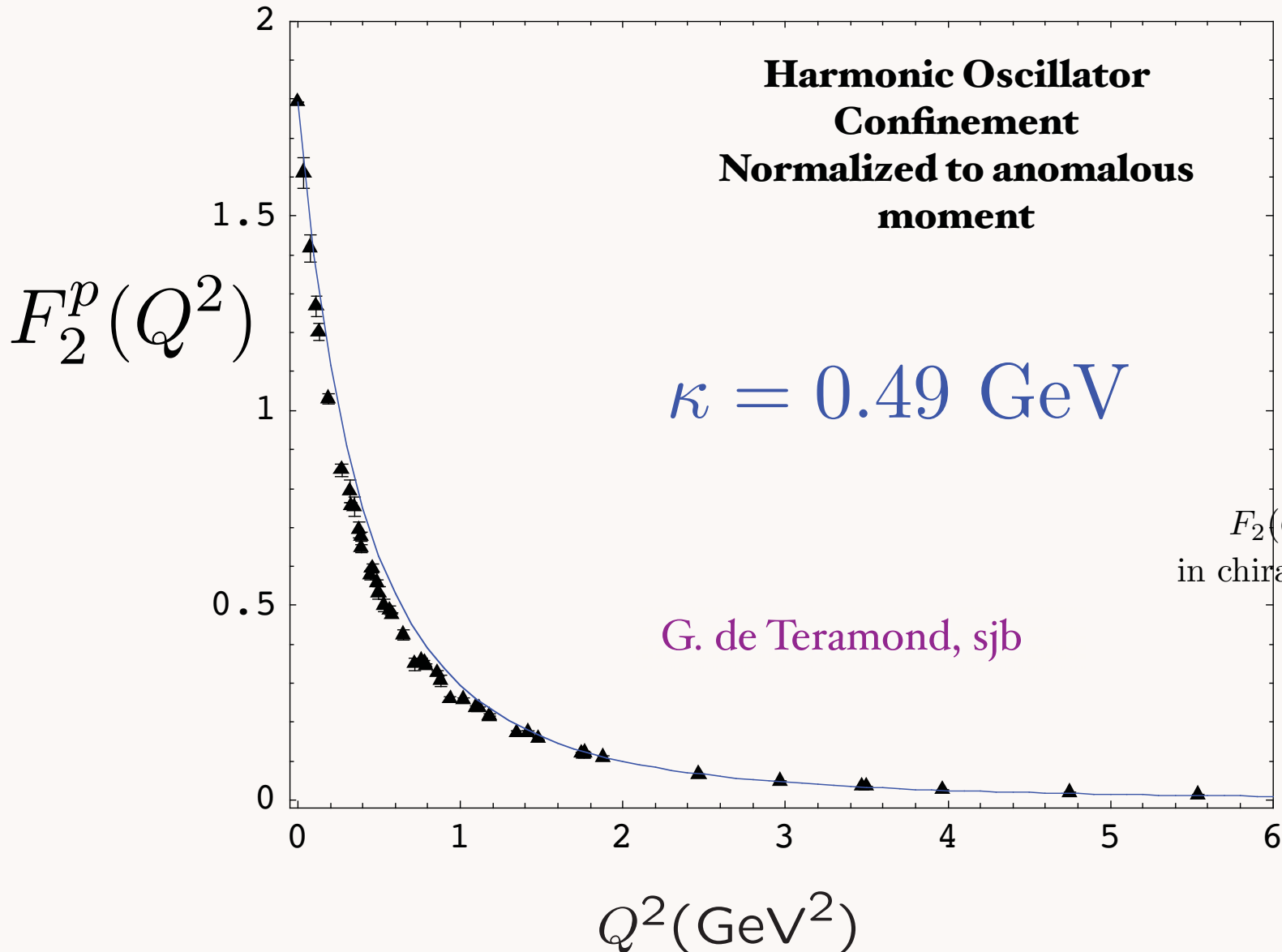


Prediction for  $Q^4 F_1^n(Q^2)$  for  $\Lambda_{\text{QCD}} = 0.21$  GeV in the hard wall approximation. Data analysis from Diehl (2005).

# Spacelike Pauli Form Factor

Preliminary

From overlap of  $L = 1$  and  $L = 0$  LFWFs



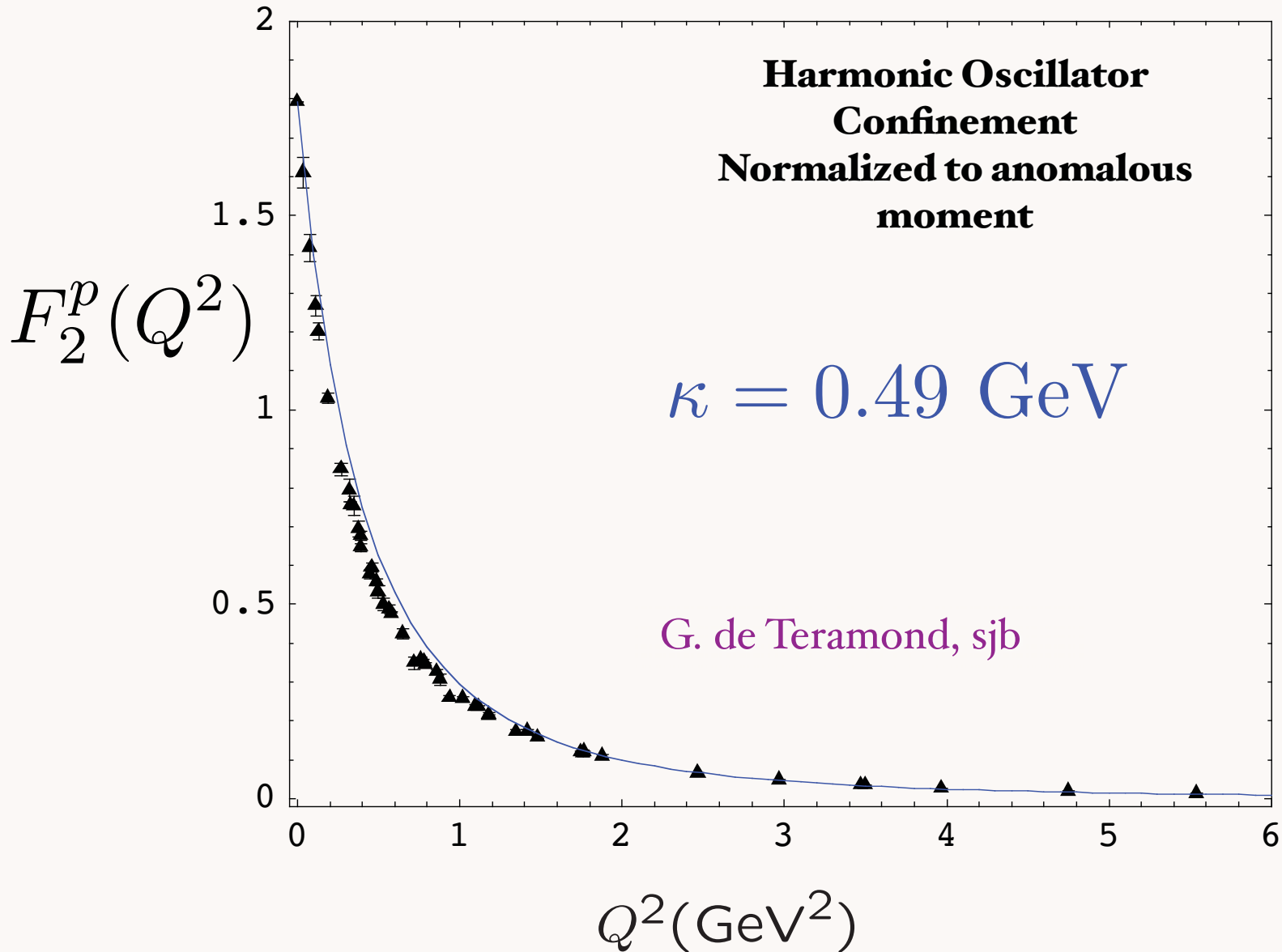
*AdS/QCD No  
chiral  
divergence!*

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$   
in chiral perturbation theory

# Spacelike Pauli Form Factor

Preliminary

From overlap of  $L = 1$  and  $L = 0$  LFWFs



String Theory



AdS/CFT

Mapping of Poincare' and Conformal  $SO(4,2)$  symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering  
Regge Trajectories

Conformal behavior at short distances  
+ Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

$J=0, 1, 1/2, 3/2$  plus  $L$

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

# Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS<sub>5</sub> space in dilaton background  $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling  $g_5(z)$  incorporates the non-conformal dynamics of confinement

- YM coupling  $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$  is the five dim coupling up to a factor:  $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale  $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

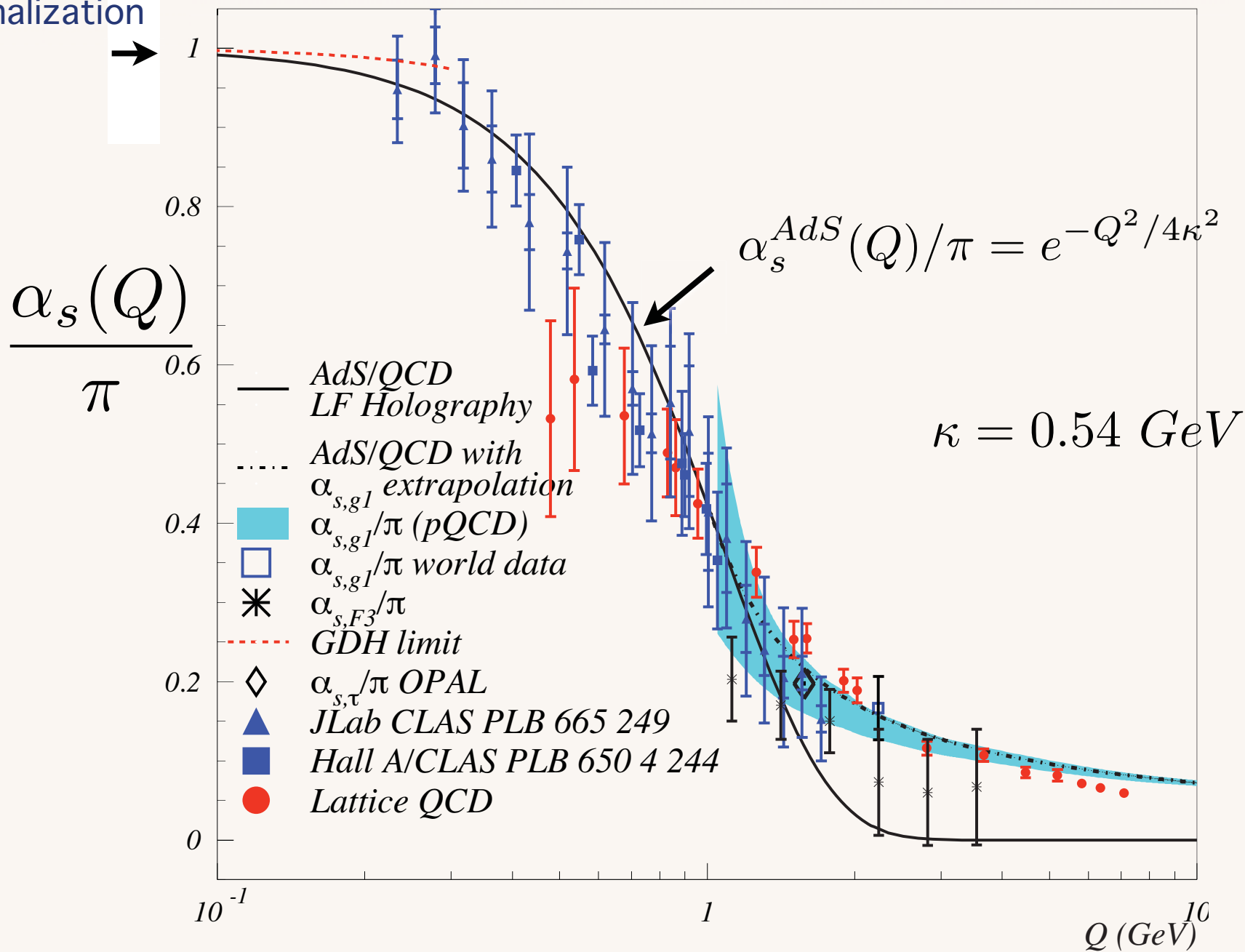
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling  $\alpha_s^{AdS}$  incorporates the non-conformal dynamics of confinement

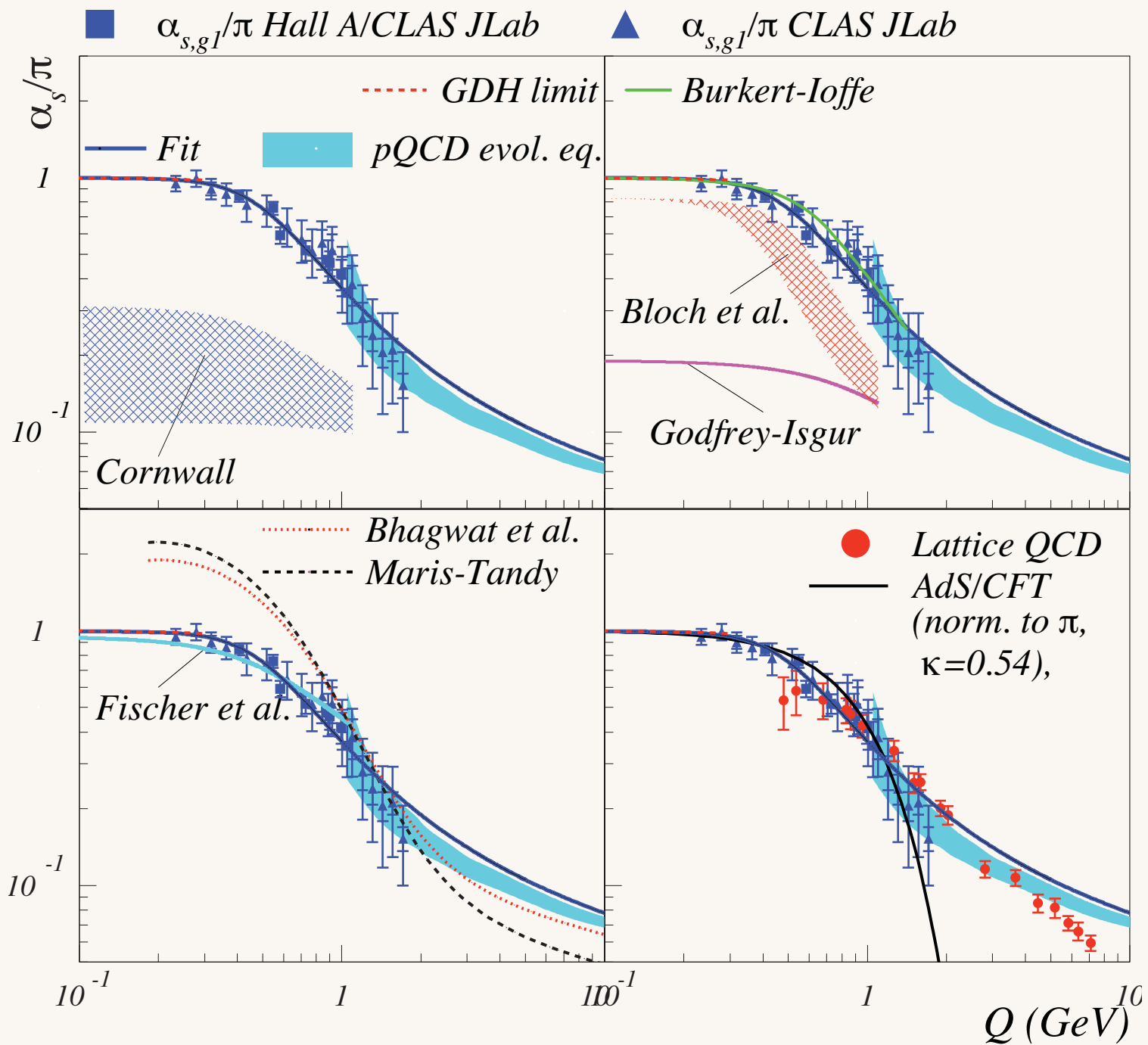
# Running Coupling from Light-Front Holography and AdS/QCD

## Analytic, defined at all scales, IR Fixed Point

normalization  
→



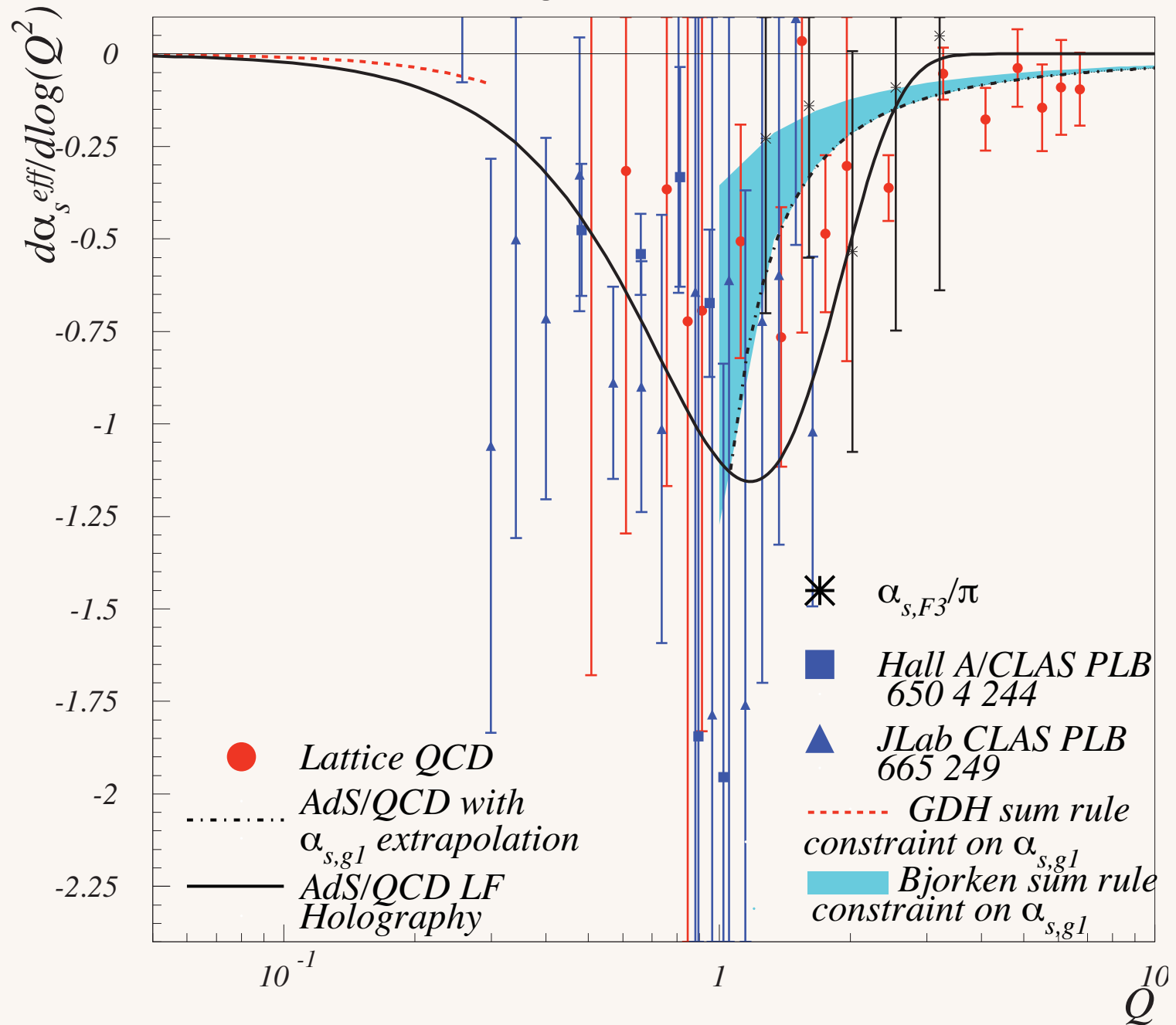
Deur, de Teramond, sjb, (preliminary)



Deur, de Teramond, sjb, (preliminary)



$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb, (preliminary)

# *Applications of Nonperturbative Running Coupling from AdS/QCD*

- **Sivers Effect in SIDIS, Drell-Yan**
- **Double Boer-Mulders Effect in DY**
- **Diffraction DIS**
- **Heavy Quark Production at Threshold**

*All involve gluon exchange at small  
momentum transfer*

*Single-spin asymmetries*

**Leading Twist  
Sivers Effect**

Hwang,  
Schmidt, sjb

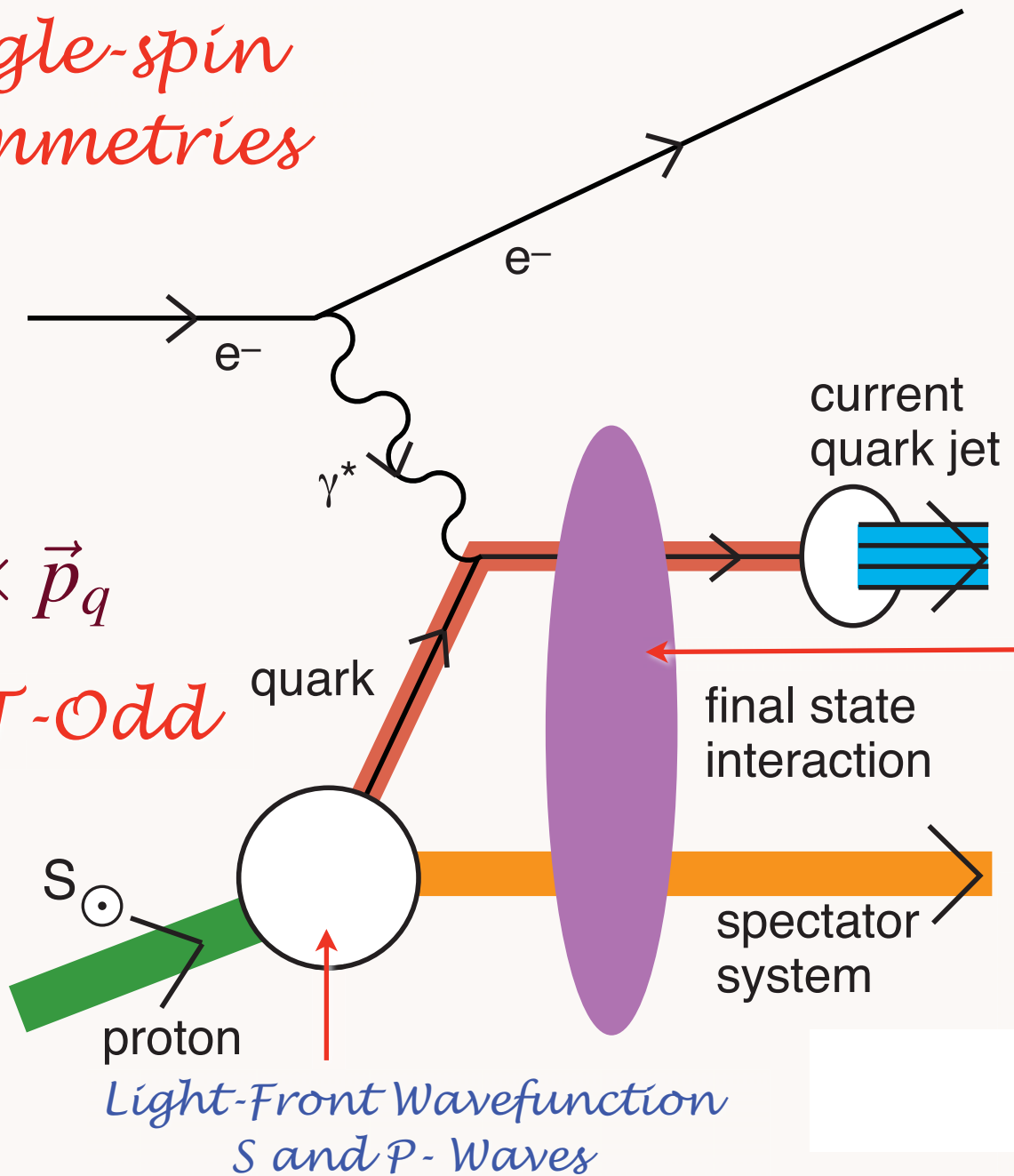
Collins, Burkardt  
Ji, Yuan

*QCD S- and P-  
Coulomb Phases  
--Wilson Line*

*Leading-Twist  
Rescattering  
Violates pQCD  
Factorization!*

*Pseudo-T-Odd*

$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$



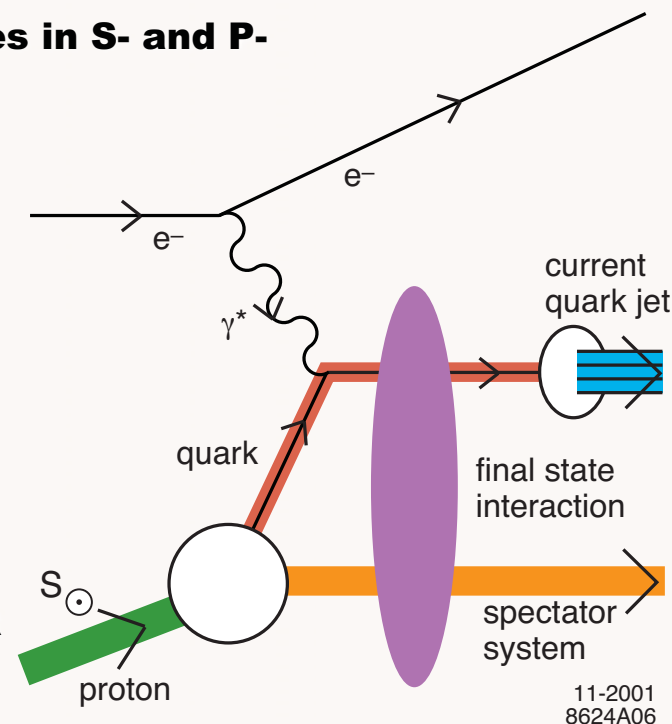
*Light-Front Wavefunction  
S and P- Waves*

# Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb  
Collins

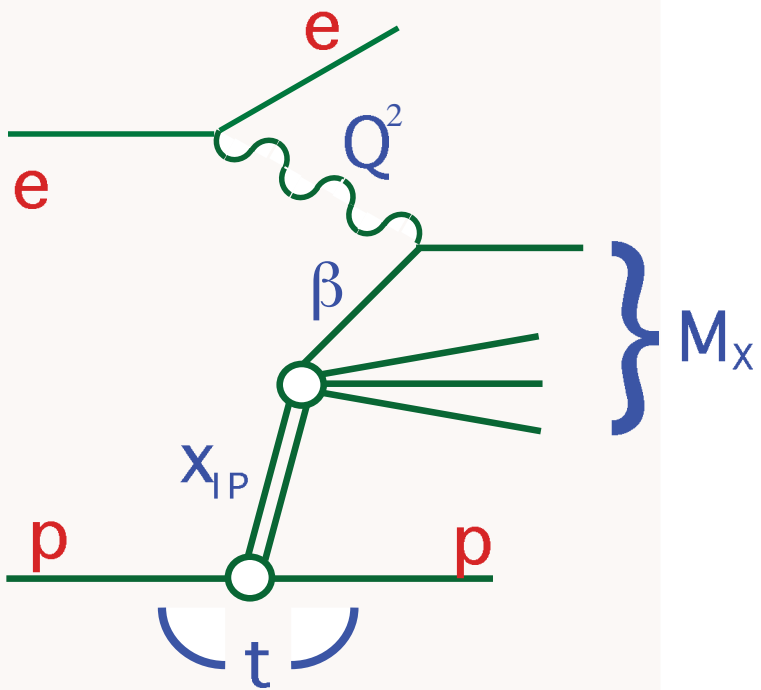
- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**

$$i \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

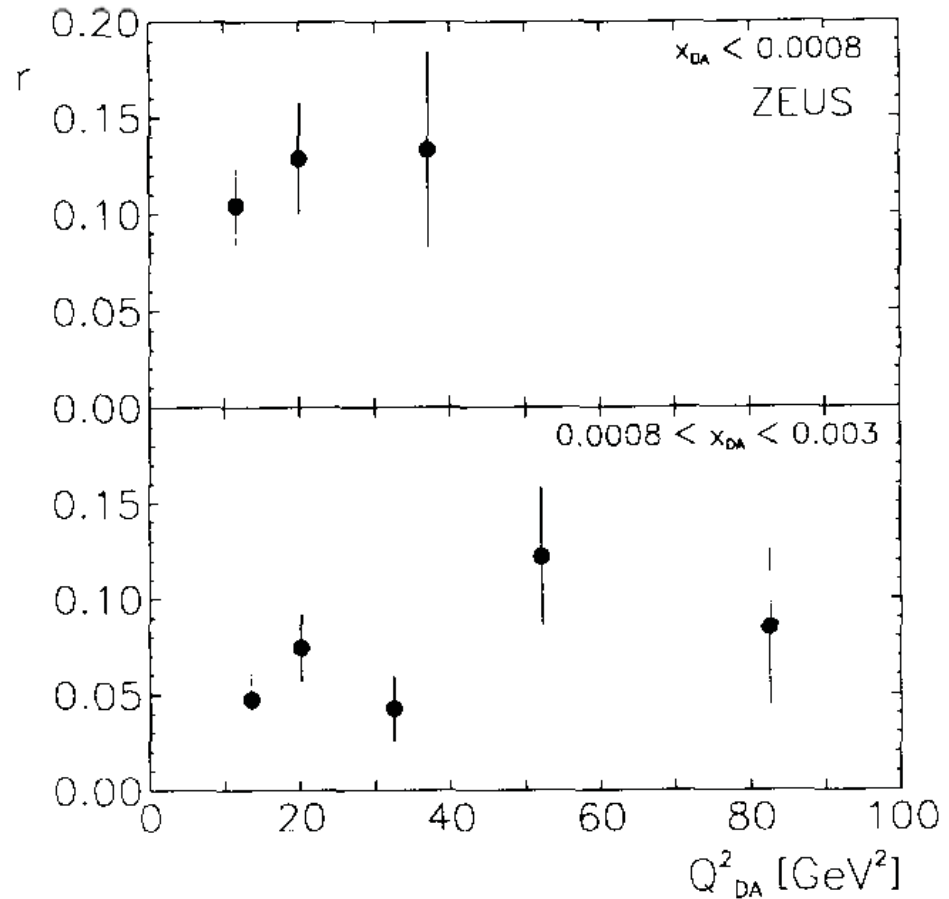


Pasquini, Xiao, Yuan, sjb  
Mulders, Boer Qiu, Sterman

# Remarkable observation at HERA



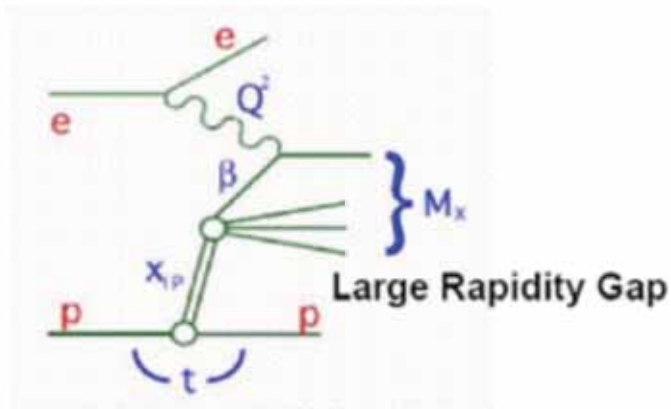
10% to 15%  
of DIS events  
are  
diffractive!



Fraction  $r$  of events with a large rapidity gap,  $\eta_{\max} < 1.5$ , as a function of  $Q_{DA}^2$  for two ranges of  $x_{DA}$ . No acceptance corrections have been applied.

M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993)

# Diffractive Structure Function $F_2^D$



Diffractive inclusive cross section

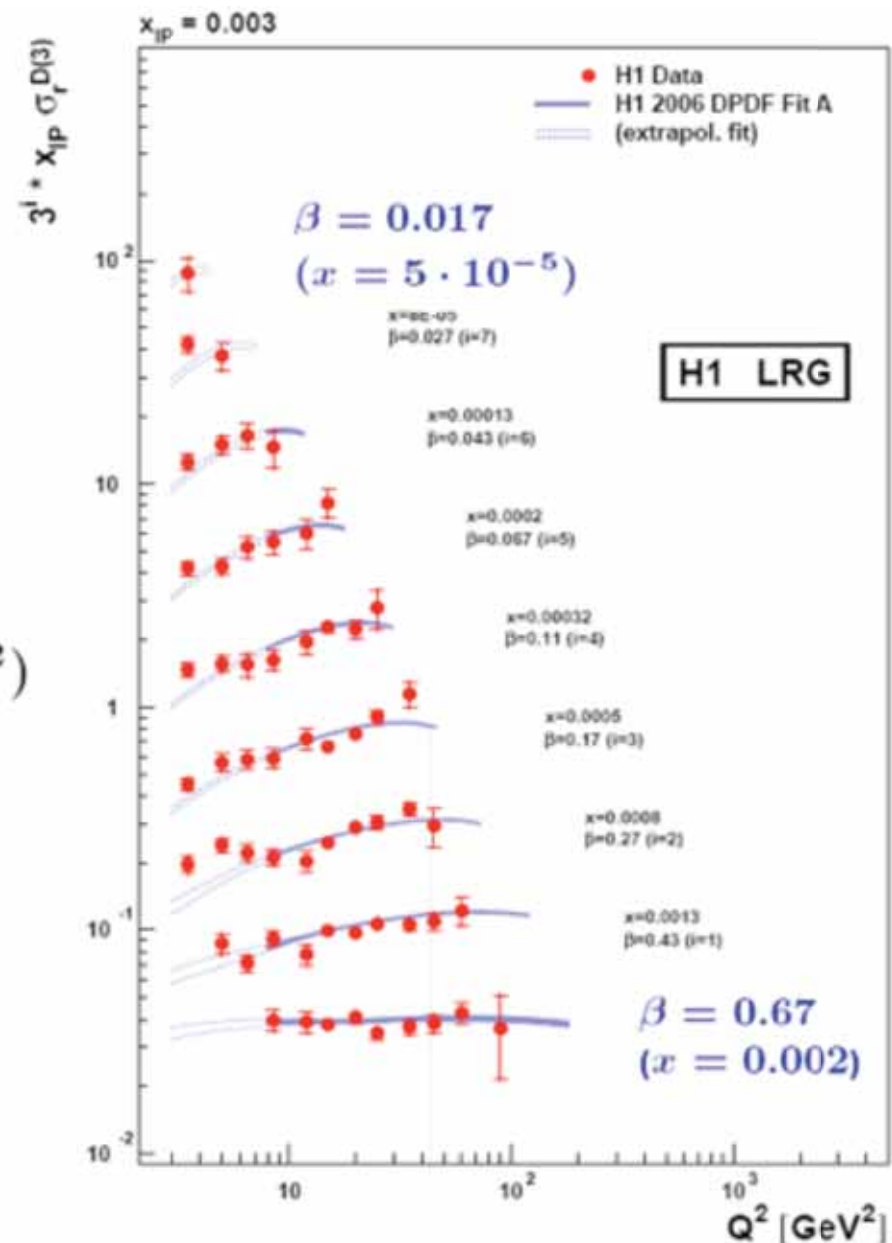
$$\frac{d^3 \sigma_{NC}^{diff}}{dx_{IP} d\beta dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{IP}, \beta, Q^2)$$

$$F_2^D(x_{IP}, \beta, Q^2) = f(x_{IP}) \cdot F_2^{IP}(\beta, Q^2)$$

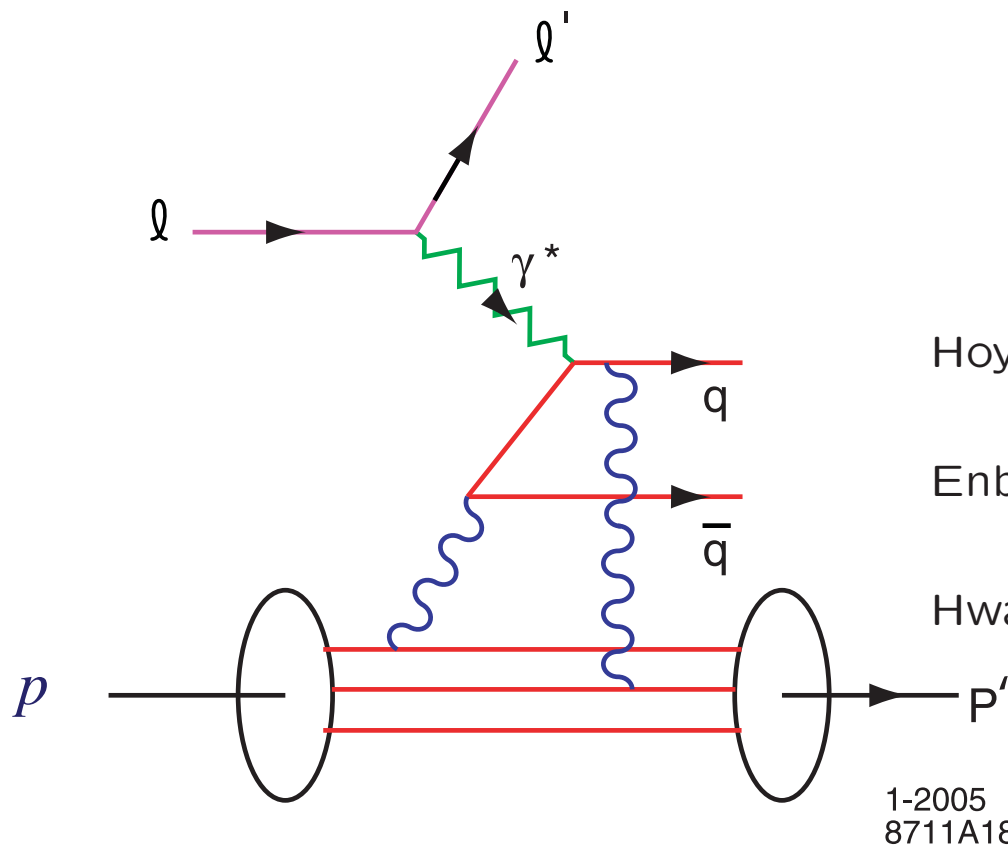
extract DPDF and  $xg(x)$  from scaling violation

Large kinematic domain  $3 < Q^2 < 1600 \text{ GeV}^2$

Precise measurements sys 5%, stat 5–20%



# Final-State Interaction Produces Diffractive DIS



## Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM)

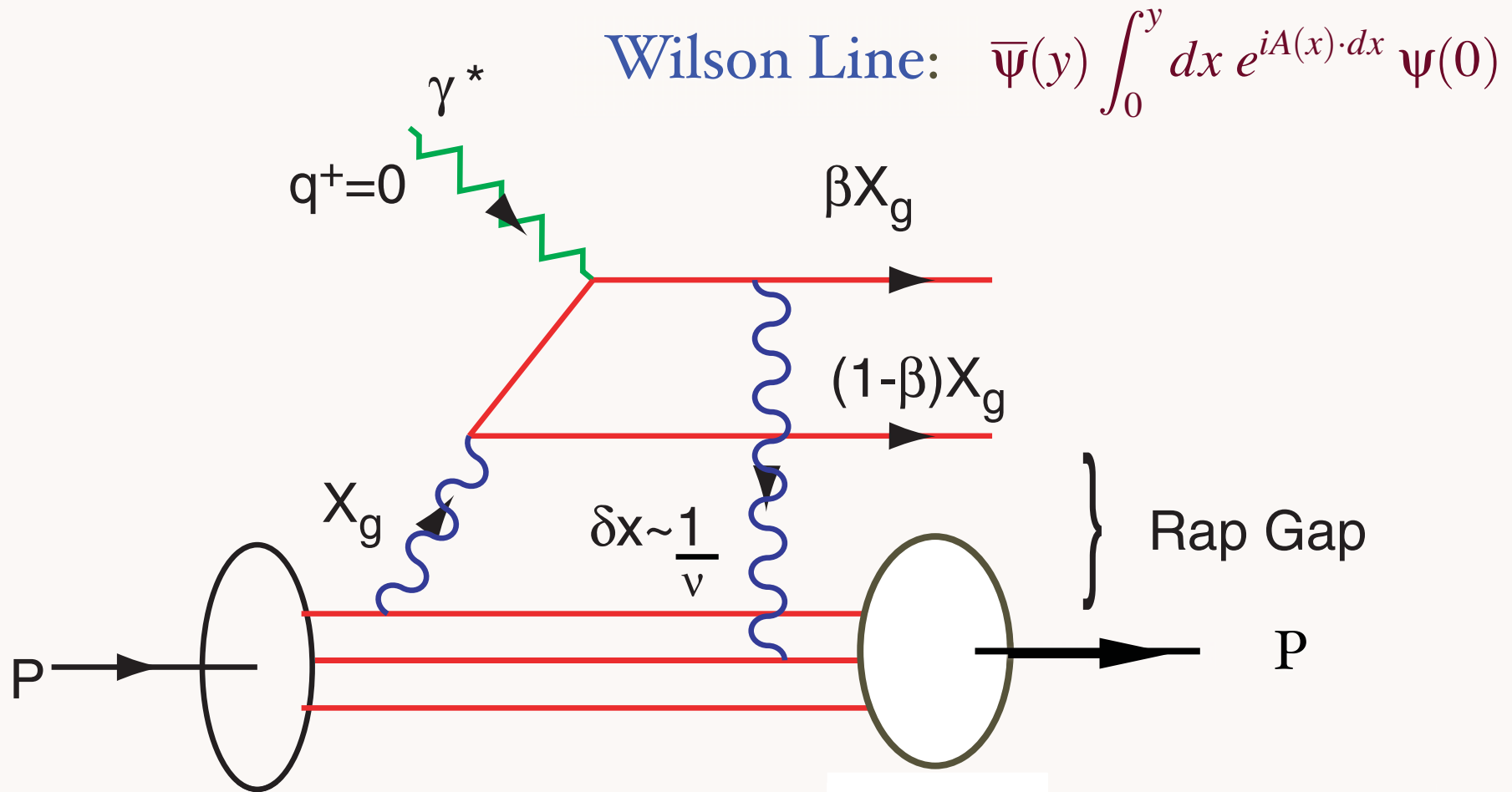
Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

1-2005  
8711A18

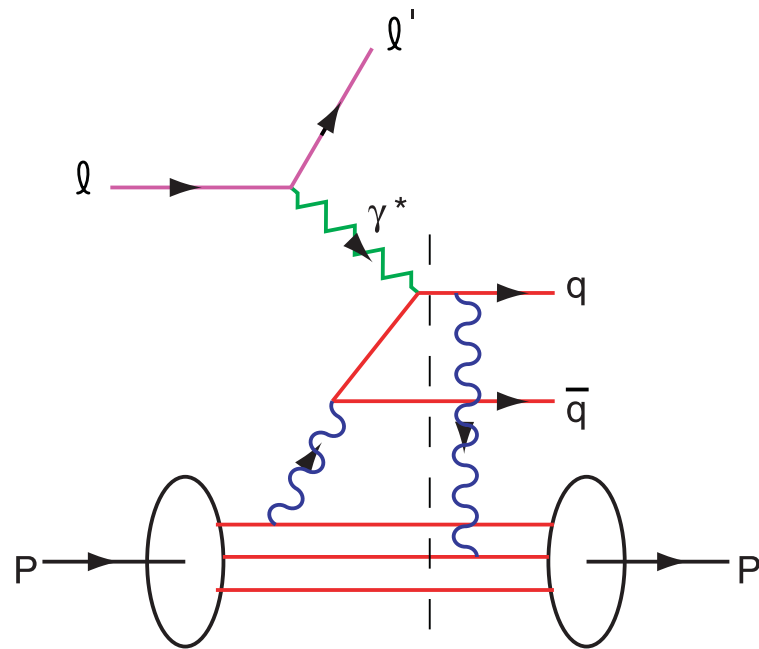
## Low-Nussinov model of Pomeron

# QCD Mechanism for Rapidity Gaps



**Reproduces lab-frame color dipole approach**



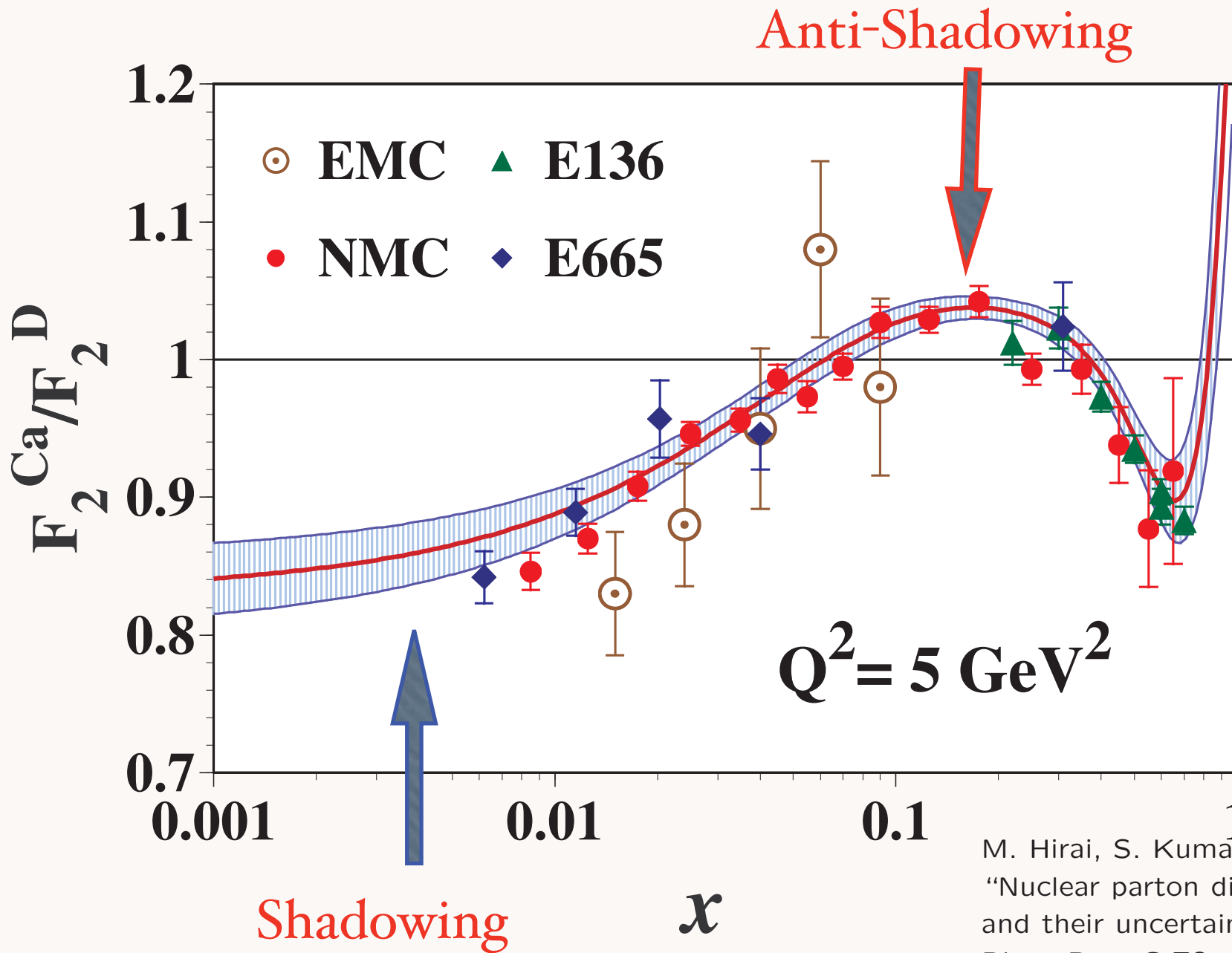


Integration over on-shell domain produces phase  $i$

Need Imaginary Phase to Generate Pomeron

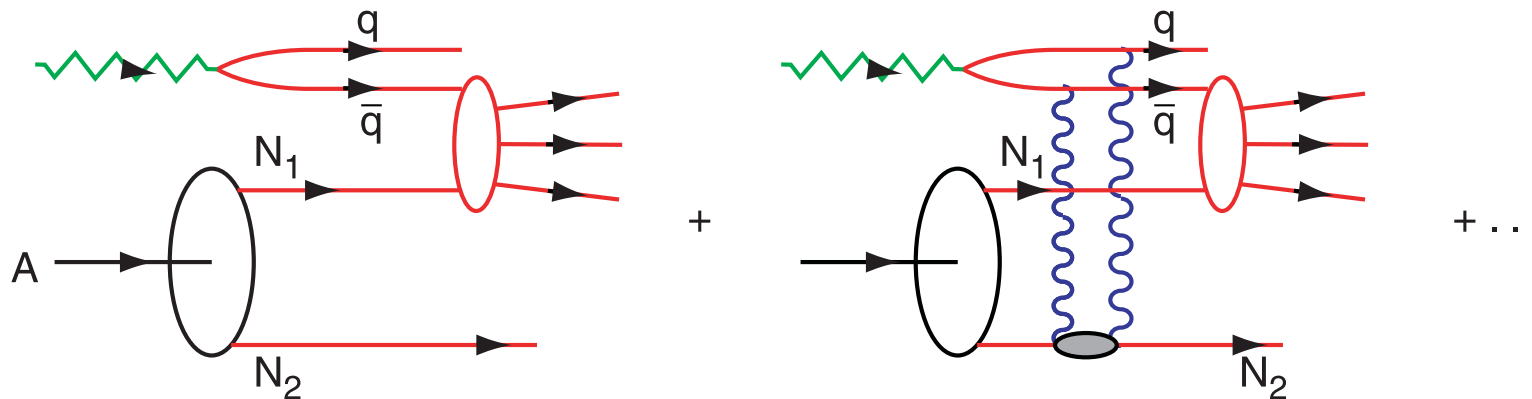
Need Imaginary Phase to Generate  
T-Odd Single-Spin Asymmetry

*Physics of FSI not in Wavefunction of Target*



M. Hirai, S. Kumano and T. H. Nagai,  
 "Nuclear parton distribution functions  
 and their uncertainties,"  
 Phys. Rev. C **70**, 044905 (2004)  
 [arXiv:hep-ph/0404093].

## Nuclear Shadowing in QCD



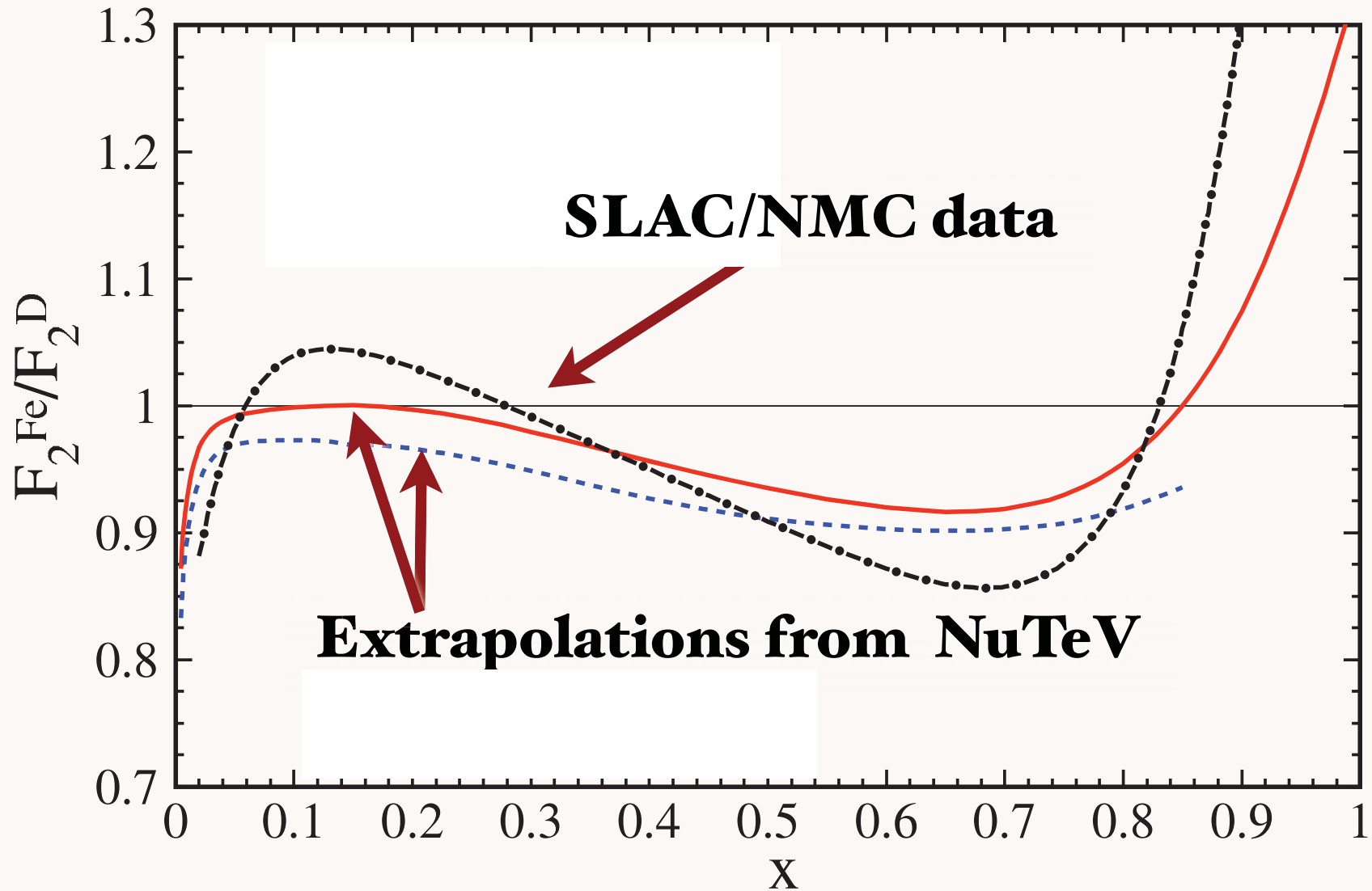
*Shadowing depends on understanding leading twist-diffraction in DIS*

**Nuclear Shadowing not included in nuclear LFWF !**

**Dynamical effect due to virtual photon interacting in nucleus**

*Antishadowing (Reggeon exchange) is not universal!*

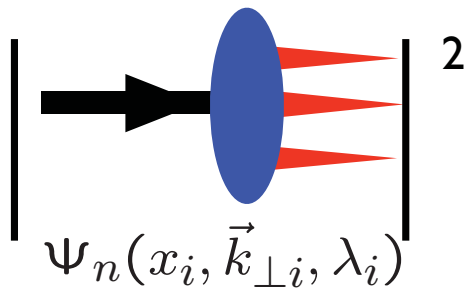
$$Q^2 = 5 \text{ GeV}^2$$



*Scheinbein, Yu, Keppel, Morfin, Olness, Owens*

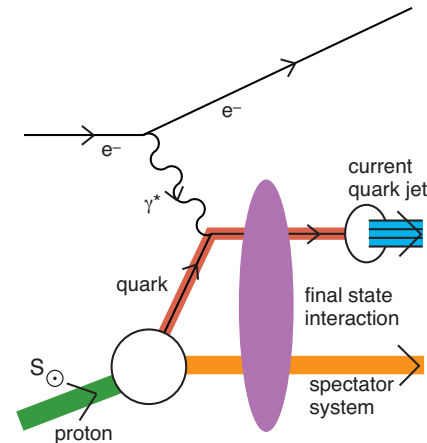
# Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and  $J^z$
- DGLAP Evolution; mod. at large  $x$
- No Diffractive DIS



# Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
  - No Probabilistic Interpretation
  - Process-Dependent - From Collision
  - T-Odd (Sivers, Boer-Mulders, etc.)
  - Shadowing, Anti-Shadowing, Saturation
  - Sum Rules Not Proven
  - DGLAP Evolution
  - Hard Pomeron and Odderon Diffractive DIS



**Hwang,  
Schmidt, sjb,  
Mulders, Boer  
Qiu, Sterman  
Collins, Qiu  
Pasquini, Xiao,  
Yuan, sjb**

# *Key QCD Questions for Phenomenology*

- **Quark and Gluon Confinement**
- **Hadronic Spectrum**
- **Light Front Wavefunctions, Distribution Amplitudes**
- **Running Coupling at Low Scales**
- **Rescattering Phenomena**
- **Hadronization at Amplitude Level**
- **Charm at Threshold**

# Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion ( $m_q = 0$ )**
- **Regge Trajectories: universal slope in  $n$  and  $L$**
- **Valid for all integer  $J$  &  $S$ . Spectrum is independent of  $S$**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large  $N_c$  limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize  $H_{LF}$  on AdS basis**

# *Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian*

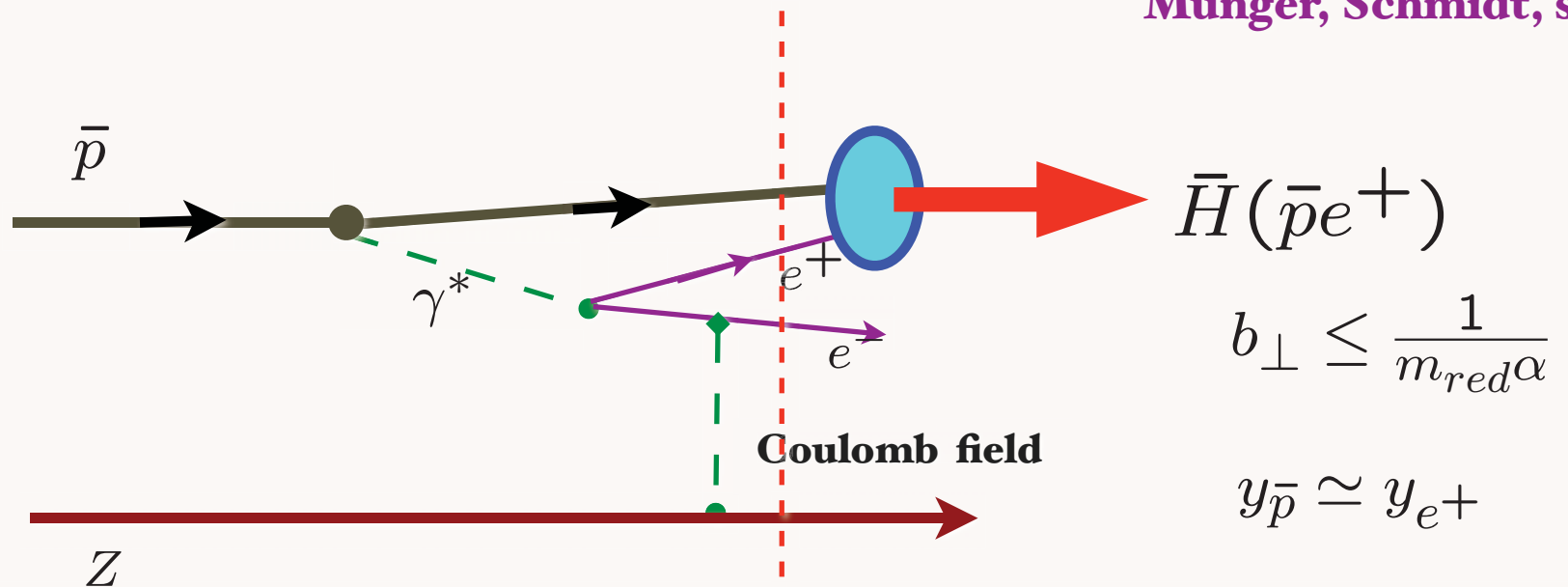
- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful **I+P Pauli, Hornbostel, Hiller, McCartor, sjb**
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- Hamiltonian light-front field theory within an AdS/QCD basis. **J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,**



# Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

Munger, Schmidt, sjb

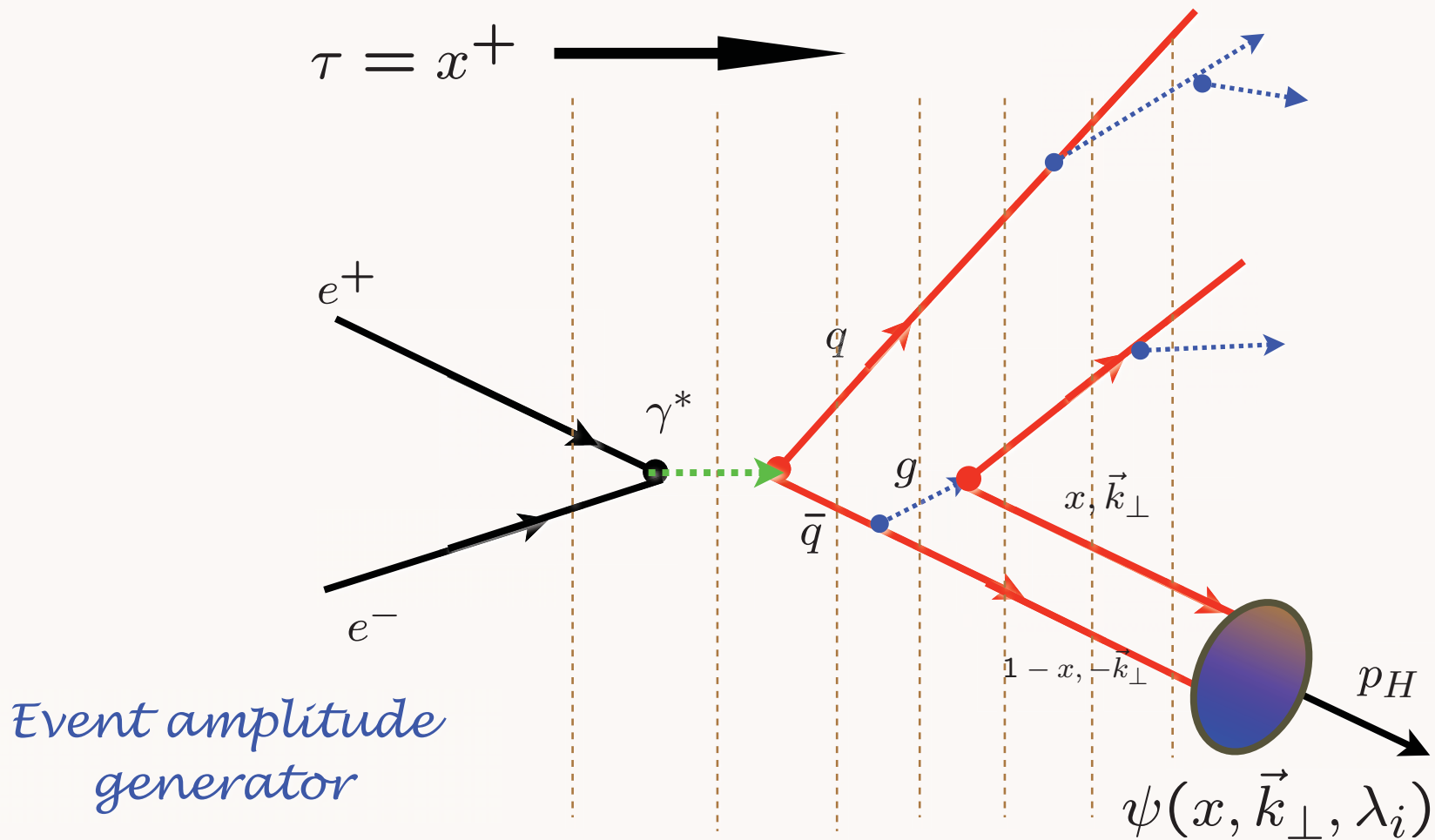


**Coalescence of off-shell co-moving positron and antiproton**

*Wavefunction maximal at small impact separation and equal rapidity*

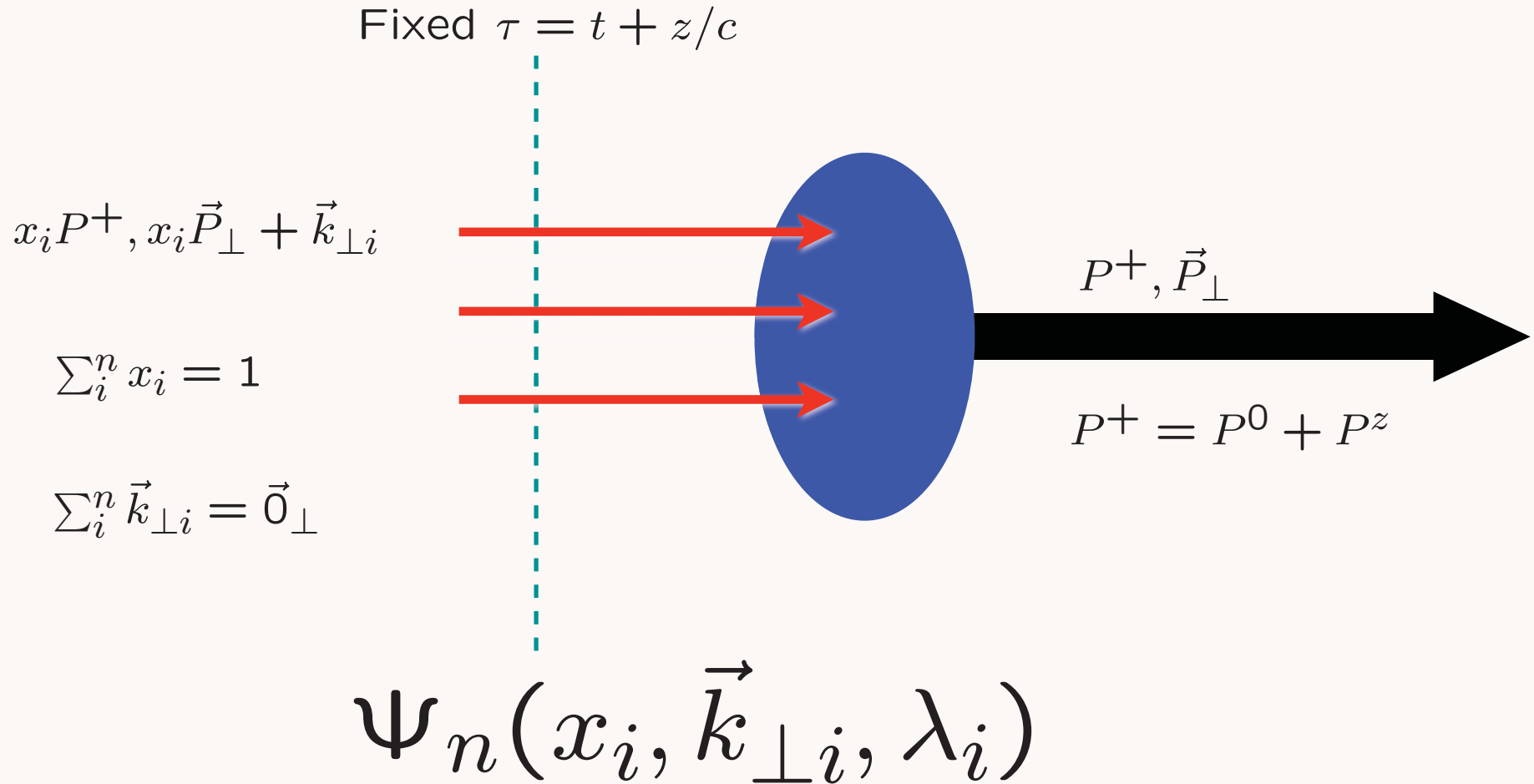
“Hadronization” at the Amplitude Level

# Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front  
Perturbation theory; coalesce quarks via LFWFs**

# Light-Front Wavefunctions



*Invariant under boosts! Independent of  $P^\mu$*

# Features of LF T-Matrix Formalism

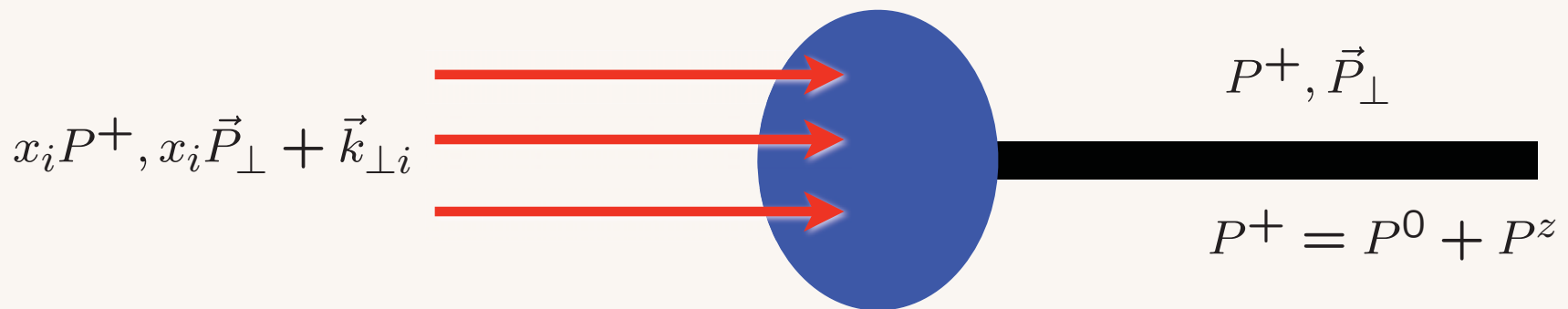
## “Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^n \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

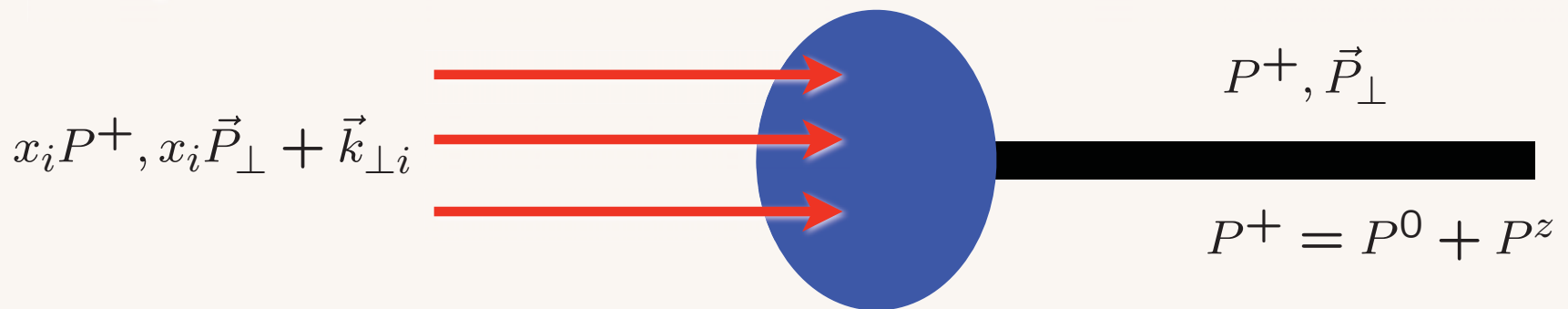
- No IR divergences: Maximal gluon and quark wavelength from confinement



# Features of LF T-Matrix Formalism

## “Event Amplitude Generator”

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large  $z$
- hadron helicity conservation if hadron LFWF has  $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



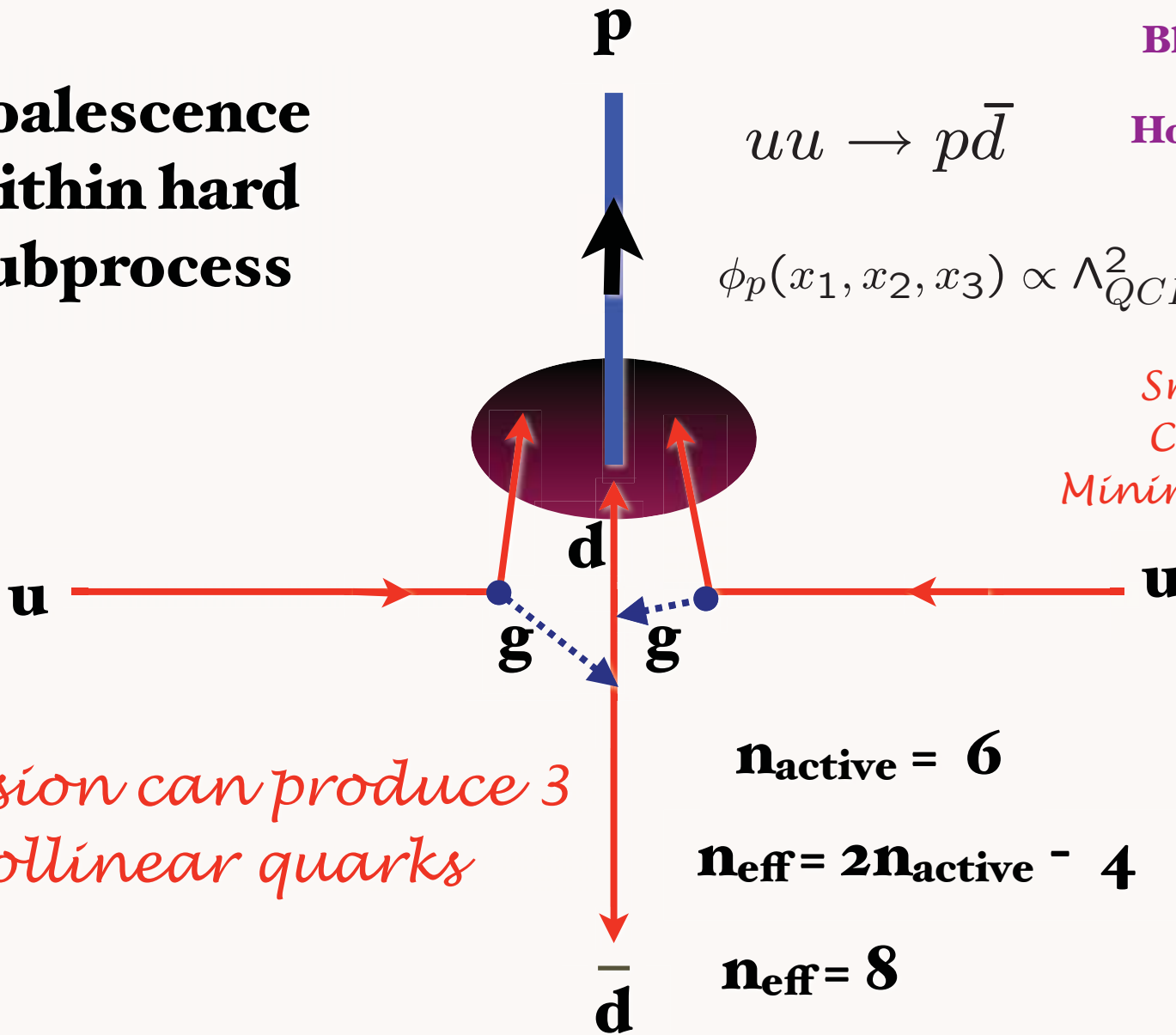
*Baryon can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

**Bjorken  
Blankenbecler, Gunion, sjb  
Berger, sjb  
Hoyer, et al: Semi-Exclusive**

**Sickles; sjb**

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

$$n_{\text{active}} = 6$$

$$qq \rightarrow B\bar{q}$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

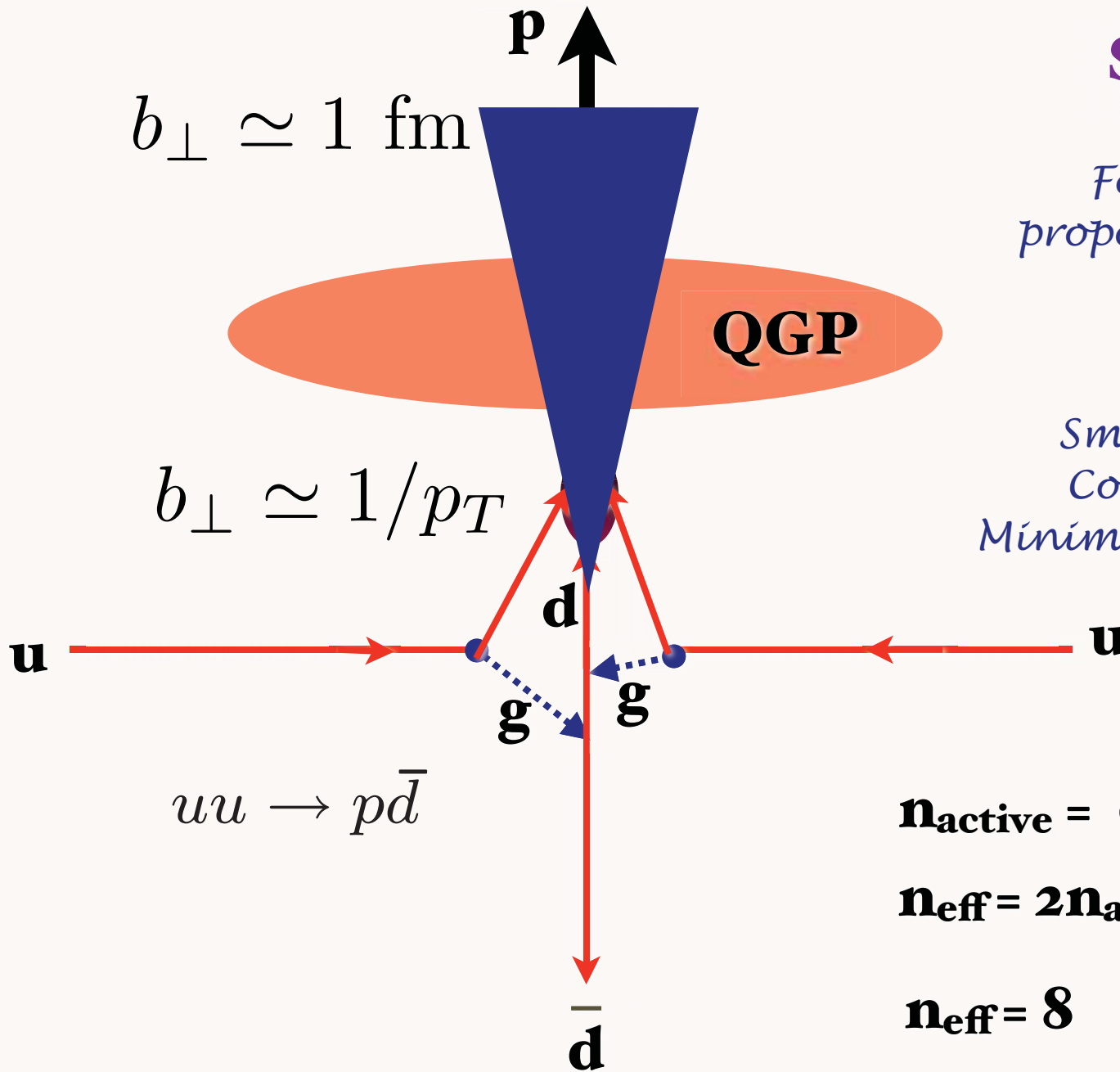
*Collision can produce 3  
collinear quarks*

*Baryon made directly within hard subprocess*

**Sickles; sjb**

*Formation Time  
proportional to Energy*

*Small color-singlet  
Color Transparent  
Minimal same-side energy*



$$b_{\perp} \simeq 1 \text{ fm}$$

$$b_{\perp} \simeq 1/p_T$$

**QGP**

**d**

**gg**

**gg**

$$uu \rightarrow p\bar{d}$$

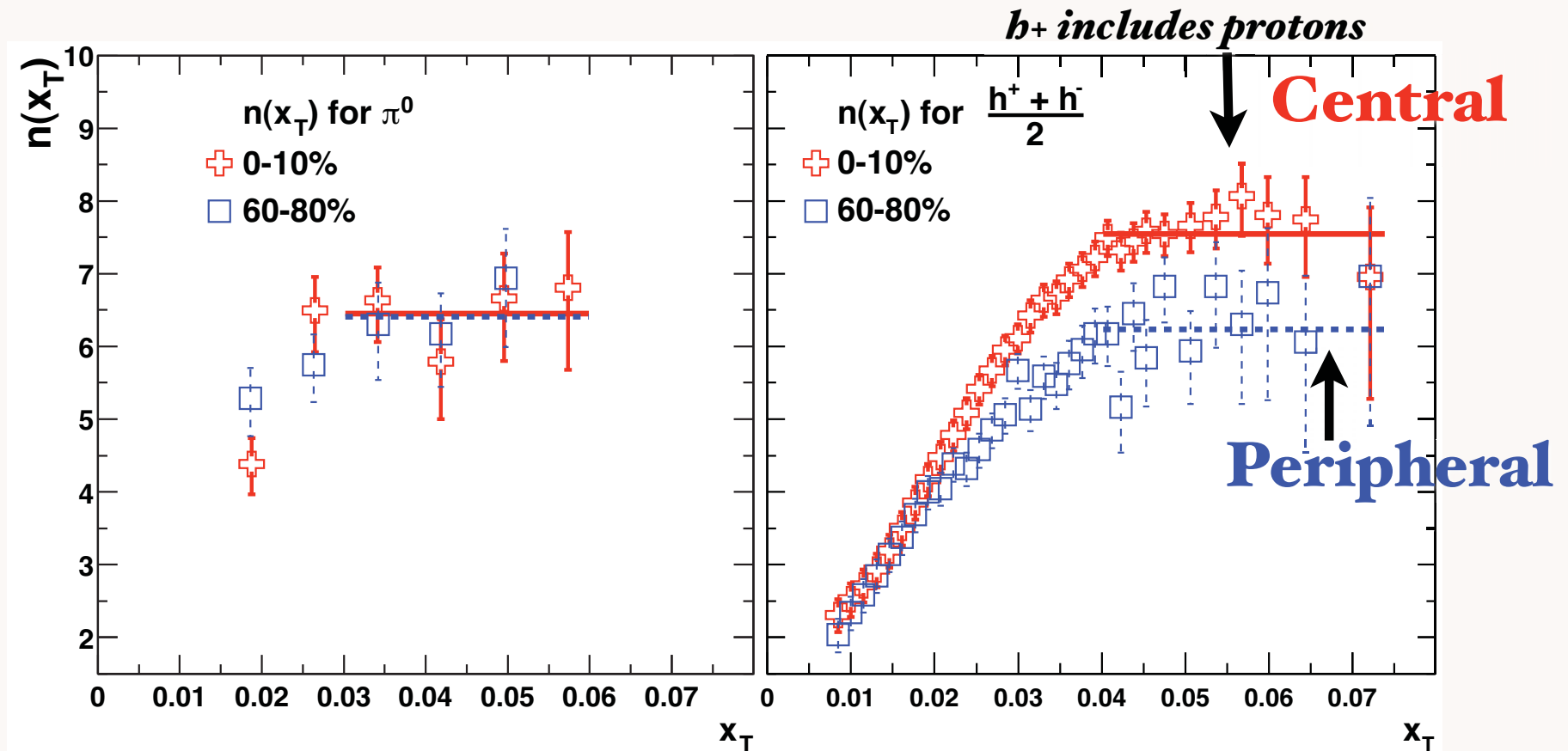
$$\mathbf{n}_{\text{active}} = 6$$

$$\mathbf{n}_{\text{eff}} = 2\mathbf{n}_{\text{active}} - 4$$

$$\mathbf{n}_{\text{eff}} = 8$$

Power-law exponent  $n(x_T)$  for  $\pi^0$  and  $h$  spectra in central and peripheral Au+Au collisions at  $\sqrt{s_{NN}} = 130$  and 200 GeV

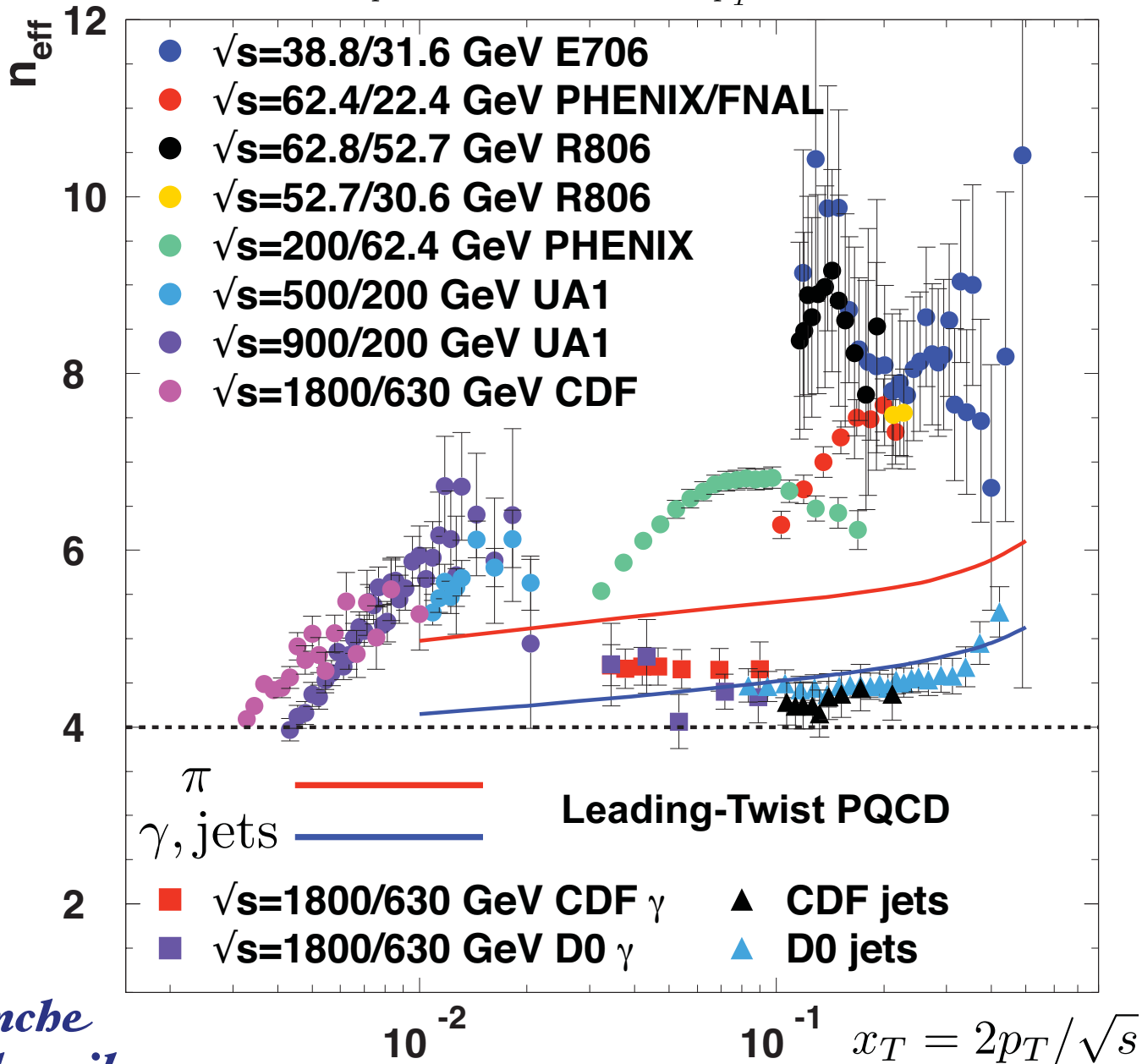
S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev. C* **69**, 034910 (2004) [nucl-ex/0308006].



*Proton production dominated by color-transparent direct high  $n_{eff}$  subprocesses*



$$E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\text{eff}}}}$$

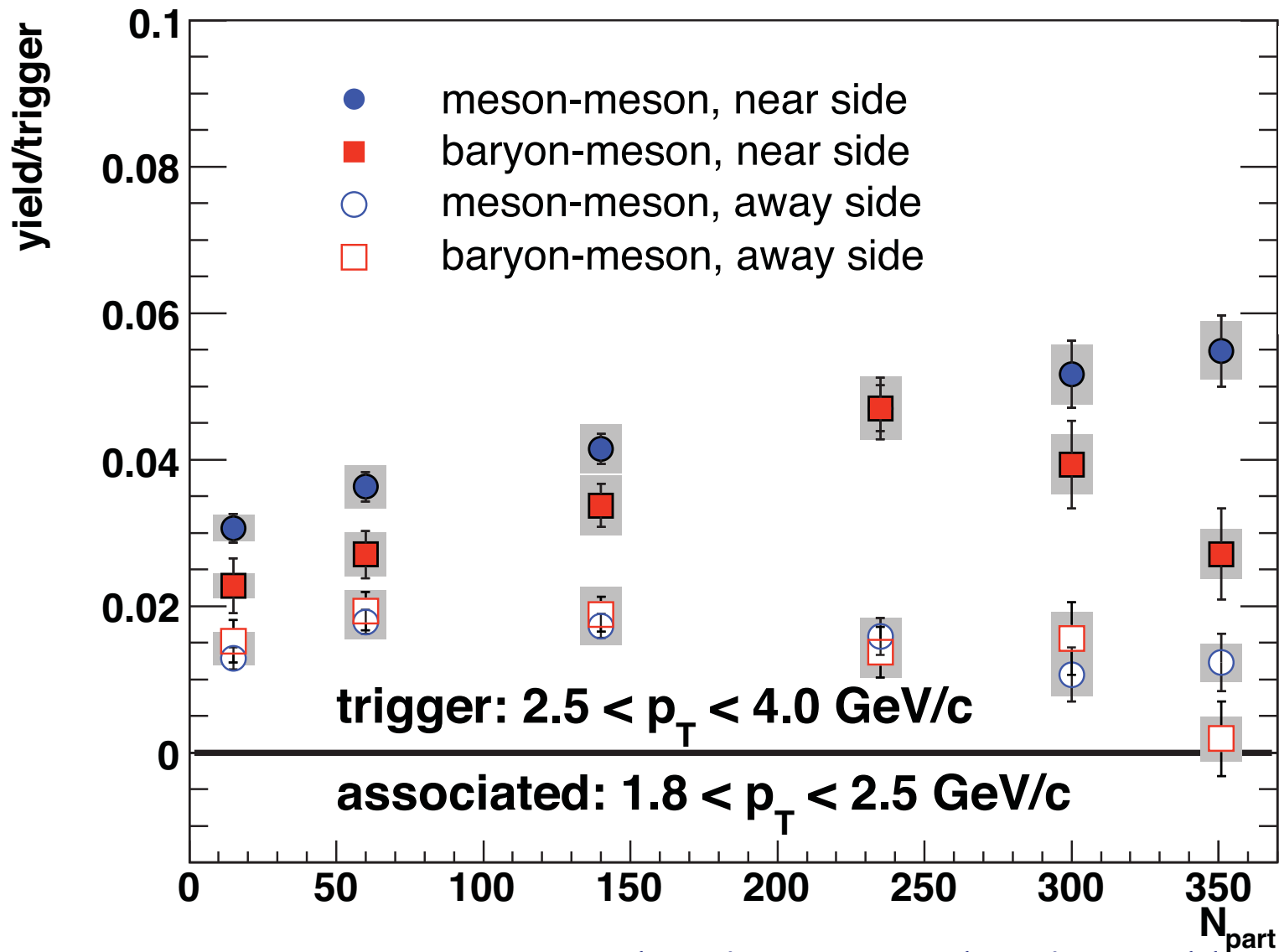


Arleo, Aurenche  
Hwang, Sickles, sjb

Oslo May 21, 2010

AdS/QCD and Hadron Physics

Stan Brodsky, SLAC & CP<sup>3</sup>



proton trigger:  
# same-side  
particles  
*decreases* with  
centrality



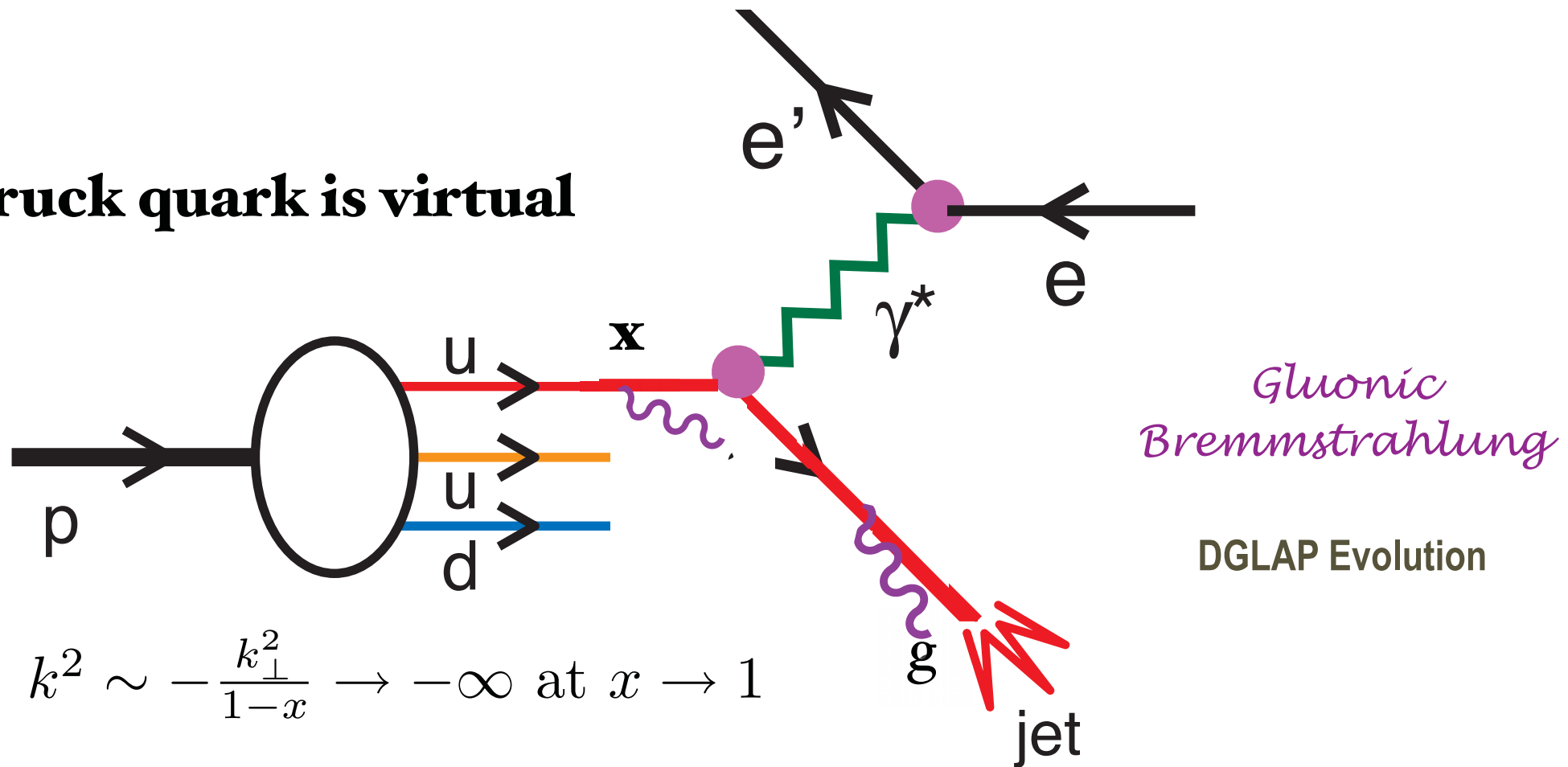
**Proton production more dominated by  
color-transparent direct high- $n_{eff}$  subprocesses**

# Features of LF T-Matrix Formalism

- **Only positive + momenta; no backward time-ordered diagrams**
- **Frame-independent! Independent of  $P^+$  and  $P^z$**
- **LC gauge: No ghosts; physical helicity**
- **$J^z = L^z + S^z$  conservation at every vertex**
- **Sum all amplitudes with same initial-and final-state helicity, then square to get rate**
- **Renormalize each UV-divergent amplitude using “alternating denominator” method**
- **Multiple renormalization scales (BLM)**
- **Cluster Decomposition; Unitary Cuts; ‘History’**

# Deep Inelastic Electron-Proton Scattering

**Struck quark is virtual**

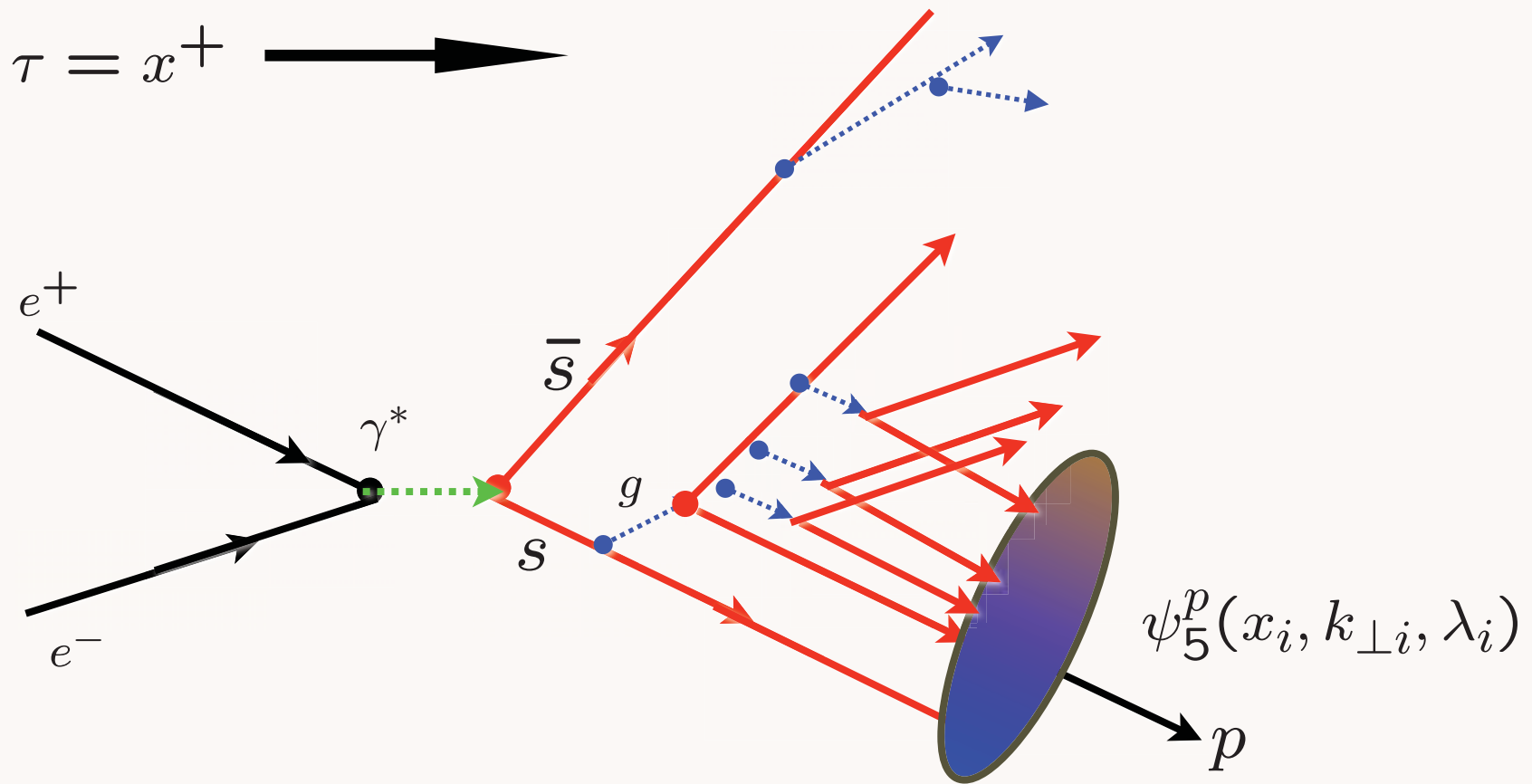


$$k^2 \sim -\frac{k_{\perp}^2}{1-x} \rightarrow -\infty \text{ at } x \rightarrow 1$$

*Off-shell Effect: Breakdown of DGLAP at  $x \sim 1$  !*

*Off-shell Effect: Breakdown of DGLAP at  $z \sim 1$  !*

# Hadronization at the Amplitude Level

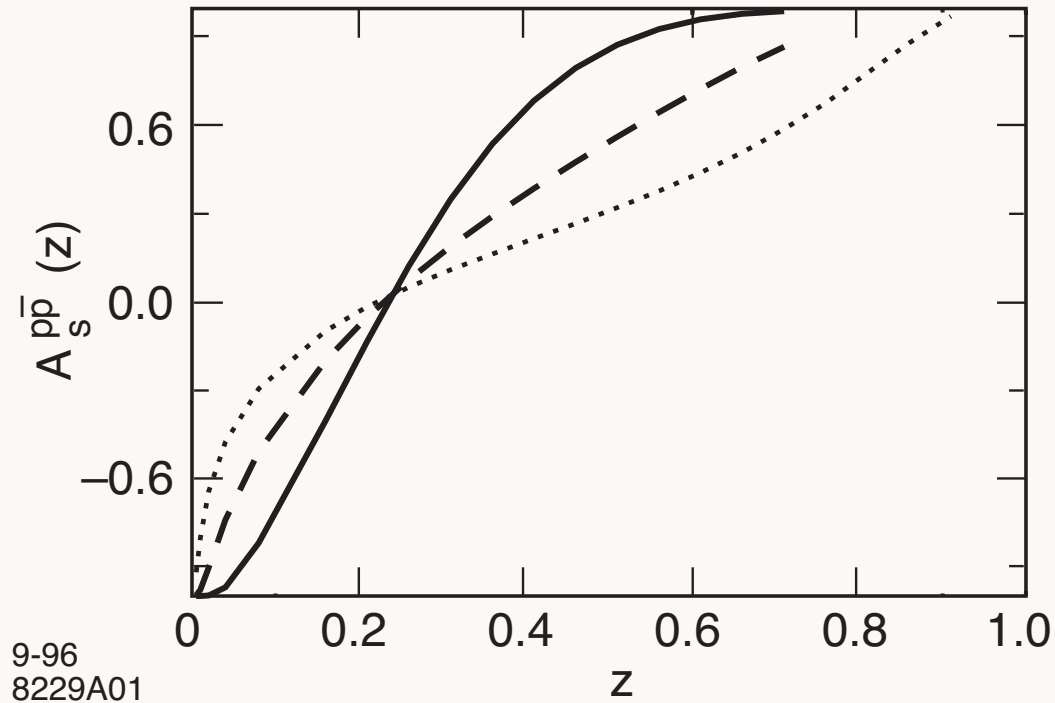


**Higher Fock State Coalescence**  $|uuds\bar{s}\rangle$

**Asymmetric Hadronization!**  $D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z)$

B-Q Ma, sjb

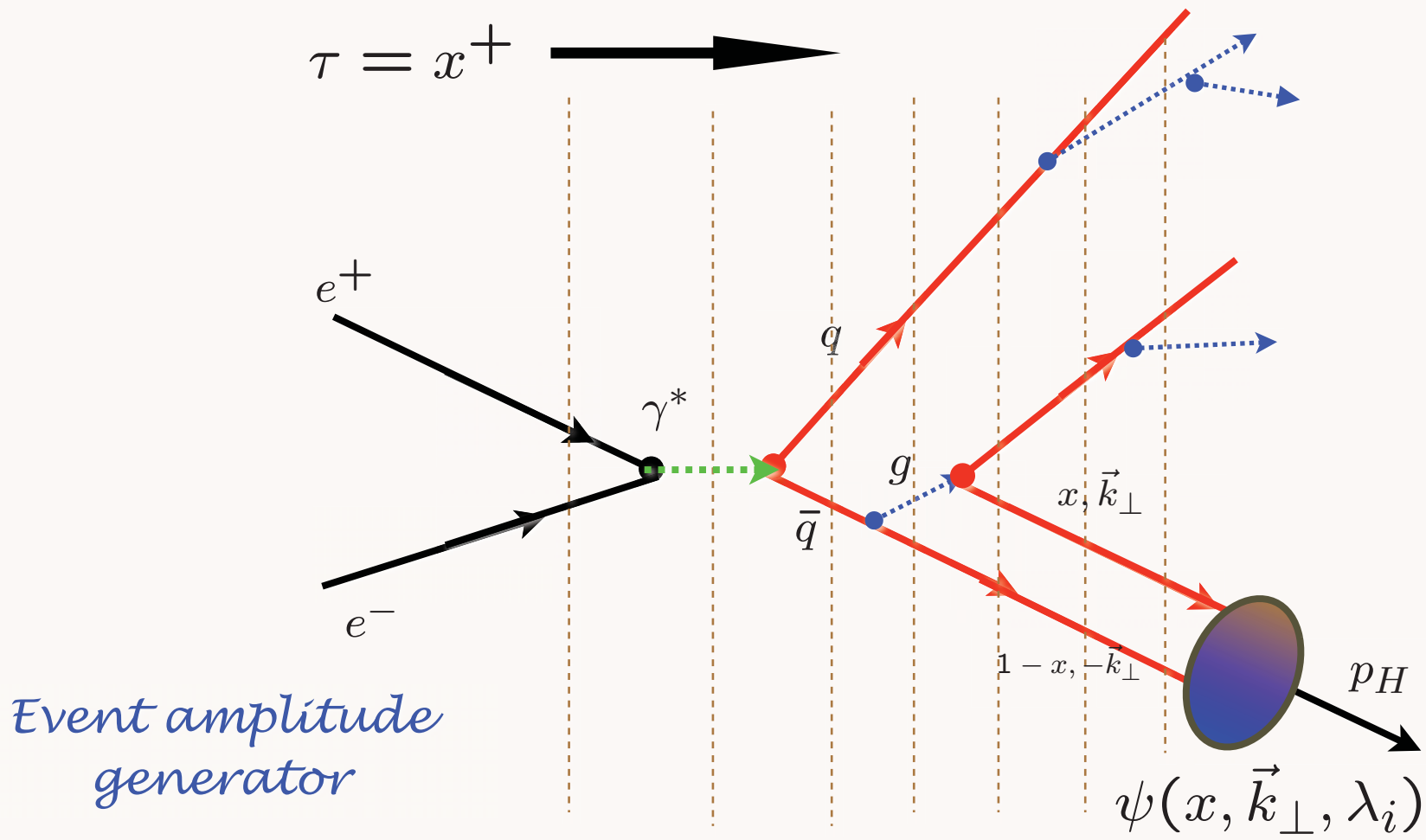
$$D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s \rightarrow p}(z) - D_{s \rightarrow \bar{p}}(z)}{D_{s \rightarrow p}(z) + D_{s \rightarrow \bar{p}}(z)}$$

Consequence of  $s_p(x) \neq \bar{s}_p(x)$        $|uuds\bar{s}\rangle \simeq |K^+\Lambda\rangle$

# Hadronization at the Amplitude Level



**Construct helicity amplitude using Light-Front  
Perturbation theory; coalesce quarks via LFWFs**

# Chiral Symmetry Breaking in AdS/QCD

Erlich et al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted  $z^2$  distribution, not constant condensate**

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \phi^2(z) z^2$$

- **$z^2$  weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD: confined condensate**
- **Suggests “In-Hadron” Condensates**

de Teramond, Shrock, sjb



In presence of quark masses the Holographic LF wave equation is ( $\zeta = z$ )

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2} \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (1)$$

and thus

$$\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle. \quad (2)$$

The parameter  $a$  is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3, \quad (3)$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle, \quad (4)$$

where we have used the sum over fractional longitudinal momentum  $\sum_i x_i = 1$ .

*Mass shift from dynamics inside hadronic boundary*

## Chiral magnetism (or magnetohydrochironics)

Aharon Casher and Leonard Susskind

*Tel Aviv University Ramat Aviv, Tel-Aviv, Israel*

(Received 20 March 1973)

### I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.<sup>1</sup> Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.<sup>2</sup> A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.<sup>3</sup>

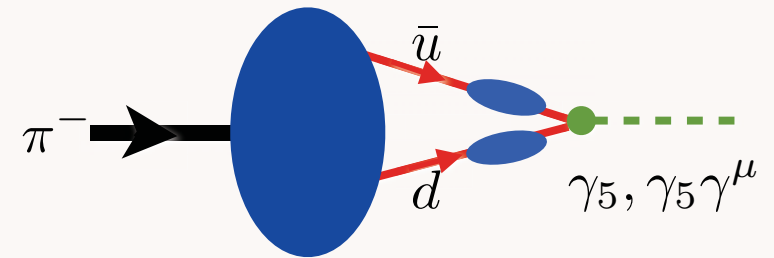
*Light-Front  
Formalism*

# Bethe-Salpeter Analysis

Maris,  
Roberts, Tandy

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

$f_H$  Meson Decay Constant  
 $T_H$  flavor projection operator,  
 $Z_2(\Lambda)$ ,  $Z_4(\Lambda)$  renormalization constants  
 $\mathcal{S}(p)$  dressed quark propagator  
 $\Gamma_H(q; P) = F.T. \langle H | \psi(x_a) \bar{\psi}(x_b) | 0 \rangle$   
 Bethe-Salpeter bound-state vertex amplitude.



$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q} \rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

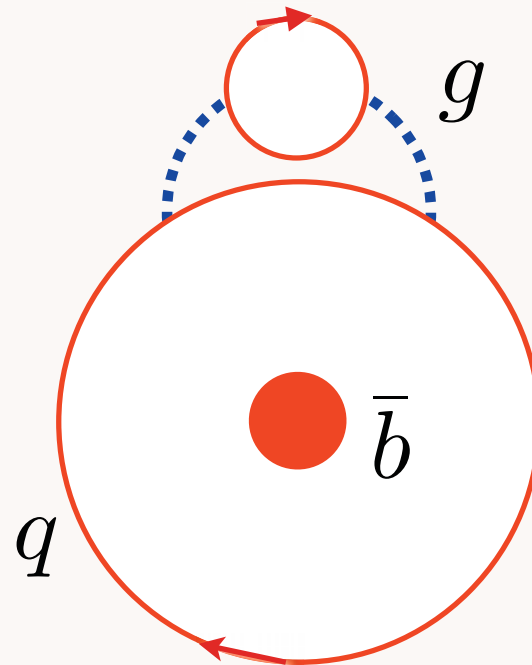
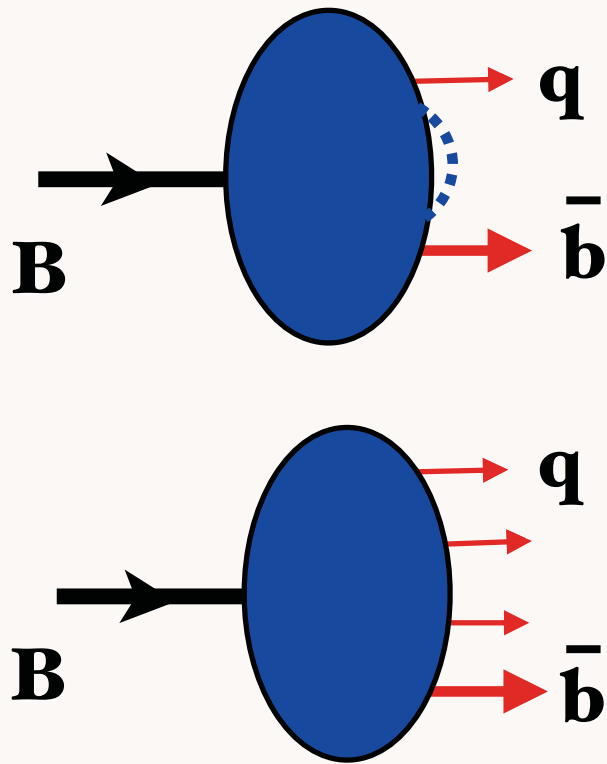
*In-Hadron Condensate!*

$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q$$

$$m_\pi^2 \propto (m_q + m_{\bar{q}}) / f_\pi \quad \text{GMOR}$$

# Simple physical argument for "in-hadron" condensate

Roberts, Shrock, Tandy, sjb

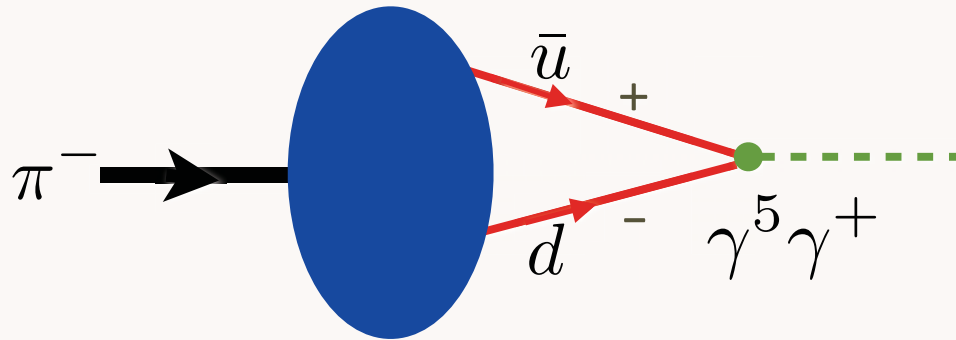


*B-Meson*

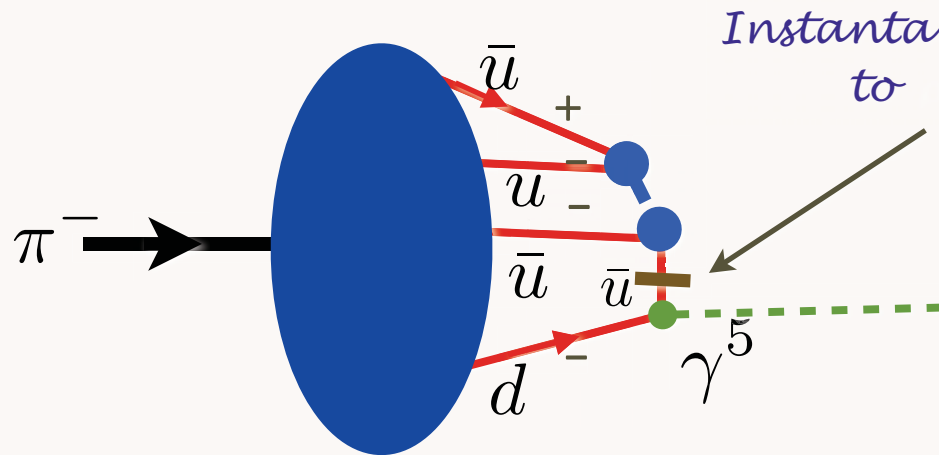
*Use Dyson-Schwinger Equation for bound-state quark propagator:  
find confined condensate*

$$\langle B | \bar{q}q | B \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle$$

# Higher Light-Front Fock State of Pion Simulates DCSB

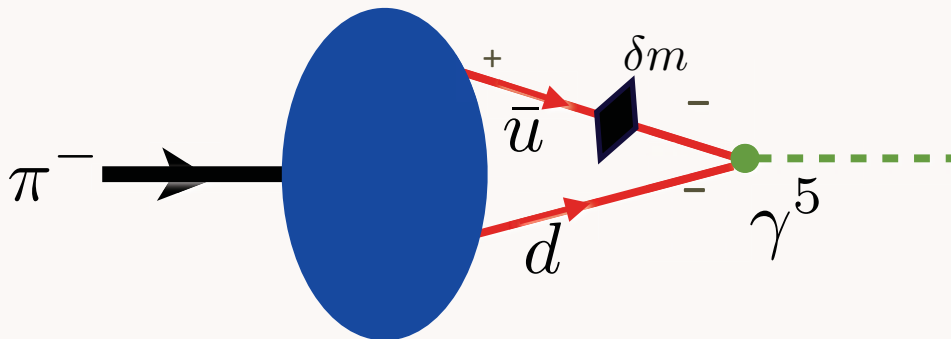


$$f_\pi P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle$$



*Instantaneous quark propagator contribution to  $\pi$  derived from higher Fock state*

$$i\rho_\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$



*Higher Fock state acts like mass insertion*

# Essence of the vacuum quark condensate

Stanley J. Brodsky,<sup>1,2</sup> Craig D. Roberts,<sup>3,4</sup> Robert Shrock,<sup>5</sup> and Peter C. Tandy<sup>6</sup>

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<sup>2</sup>*Centre for Particle Physics Phenomenology: CP<sup>3</sup>-Origins,  
University of Southern Denmark, Odense 5230 M, Denmark*

<sup>3</sup>*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>4</sup>*Department of Physics, Peking University, Beijing 100871, China*

<sup>5</sup>*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794*

<sup>6</sup>*Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA*

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

# *Quark and Gluon condensates reside within hadrons, not vacuum*

Casher and Susskind

Maris, Roberts, Tandy

Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant --  
Eliminates 45 orders of magnitude conflict**

R. Shrock, sjb

*“One of the gravest puzzles of  
theoretical physics”*

**DARK ENERGY AND  
THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

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$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

*QCD Problem Solved if Quark and Gluon condensates reside  
within hadrons, not vacuum!*

**R. Shrock, sjb**

arXiv:0905.1151 [hep- th], Proc. Nat'l. Acad. Sci., (in press);  
“Condensates in Quantum Chromodynamics and the Cosmological Constant.”



*Quark and Gluon condensates reside within  
hadrons, not LF vacuum*

- **Bound-State Dyson-Schwinger Equations**
- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**
- **Finite size phase transition - infinite # Fock constituents**
- **AdS/QCD Description -- CSB is in-hadron Effect**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant**

**Maris, Roberts,  
Tandy**

**Casher  
Susskind**

**Shrock, sjb**

*“Confined QCD Condensates”*

# Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

$-0.005 \pm 0.003$  from  $\tau$  decay.

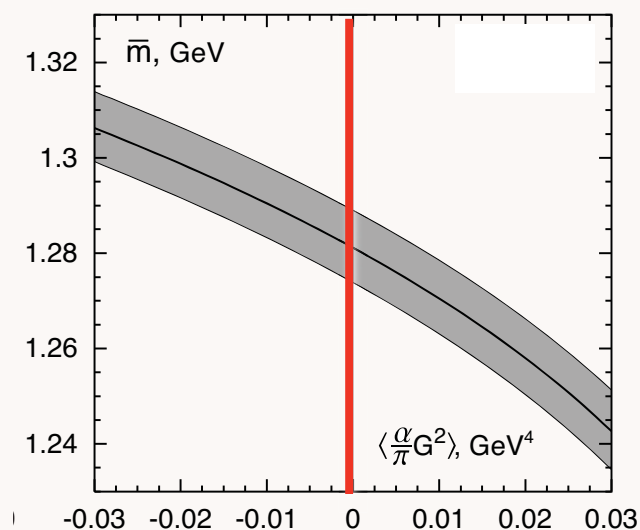
Davier et al.

$+0.006 \pm 0.012$  from  $\tau$  decay.

Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$  from charmonium sum rules

Ioffe, Zyablyuk



*Consistent with zero  
vacuum condensate*

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, ...)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

**Shrock and sjb**

# Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS<sub>5</sub>**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite  $N_c = 3$ : Baryons built on 3 quarks -- Large  $N_c$  limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

# Conformal Template

- **BLM scale-setting: Retain conformal series; nonzero  $\beta$ -terms set multiple renormalization scales. No renormalization scale ambiguity. Result is scheme-independent.**
- **Commensurate Scale Relations** between observables based on conformal template -- prime tests of QCD independent of theory conventions
- **BLM scales for three-gluon coupling; multi-scale problems**
- **Analytic scheme for coupling unification**
- **Direct high  $p_T$  processes, baryon anomaly, intrinsic heavy quarks**
- **IR Fixed point -- conformal symmetry motivation for AdS/CFT**
- **Light-Front Schrödinger Equation: analytic first approximation to QCD**
- **Dilaton-modified AdS<sub>5</sub>: Predict Hadron Spectra, Light-Front Wavefunctions, Form Factors, Hadronization at Amplitude Level**
- **Non-Perturbative running QCD coupling -- new range of QCD phenomena such as Sivers and Diffractive DIS fraction calculable; IR fixed point**

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

# *An analytic first approximation to QCD*

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable  $\zeta$  conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter  $\kappa$**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

# A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

## Frank and Ernest



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