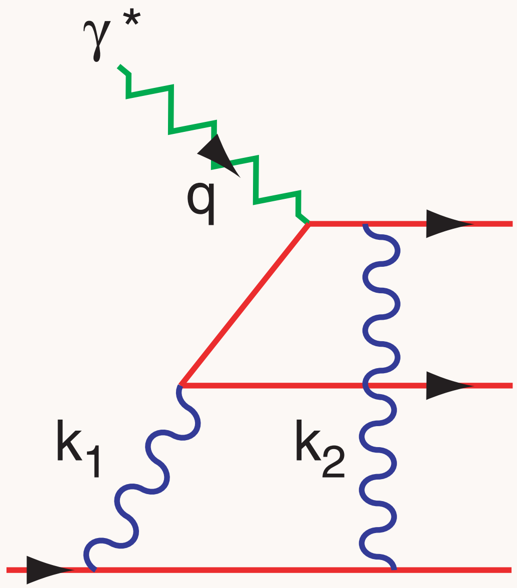
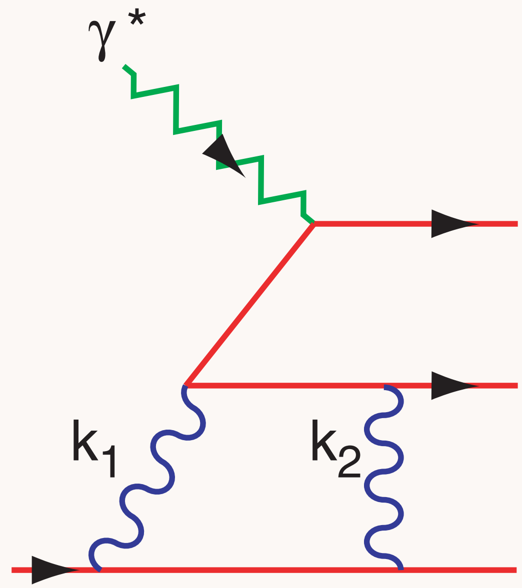
Final State Interactions in QCD

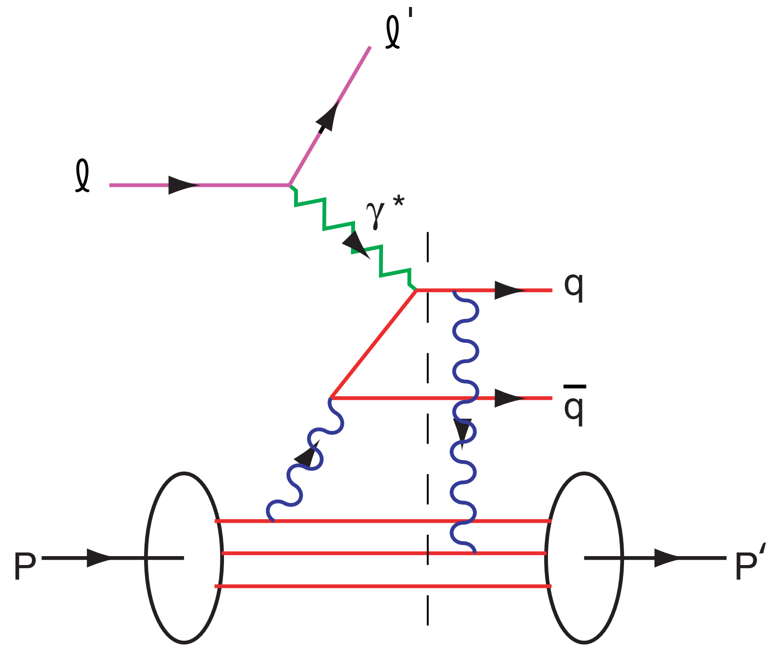


Feynman Gauge



Light-Cone Gauge

Result is Gauge Independent



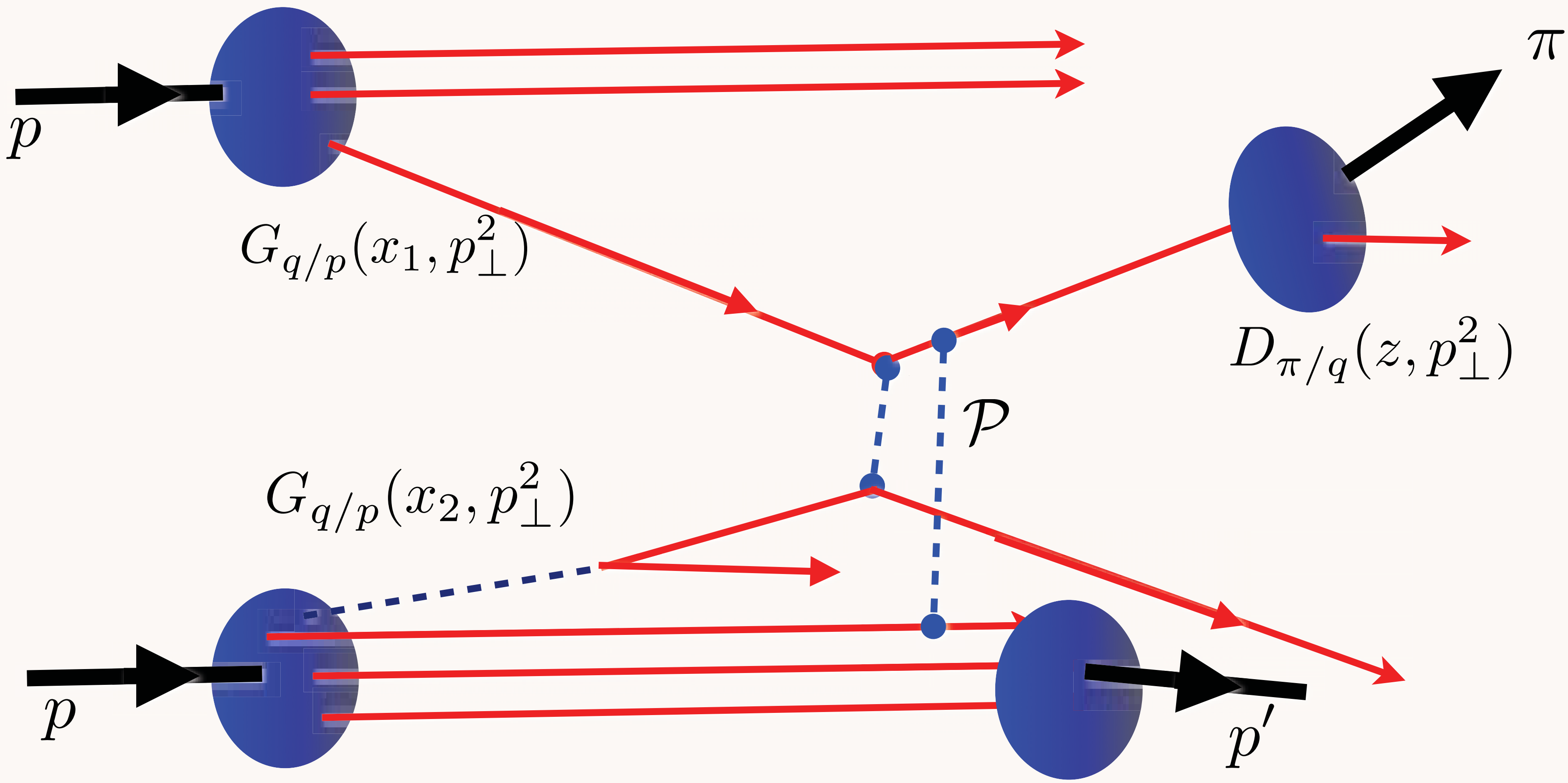
Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate
T-Odd Single-Spin Asymmetry

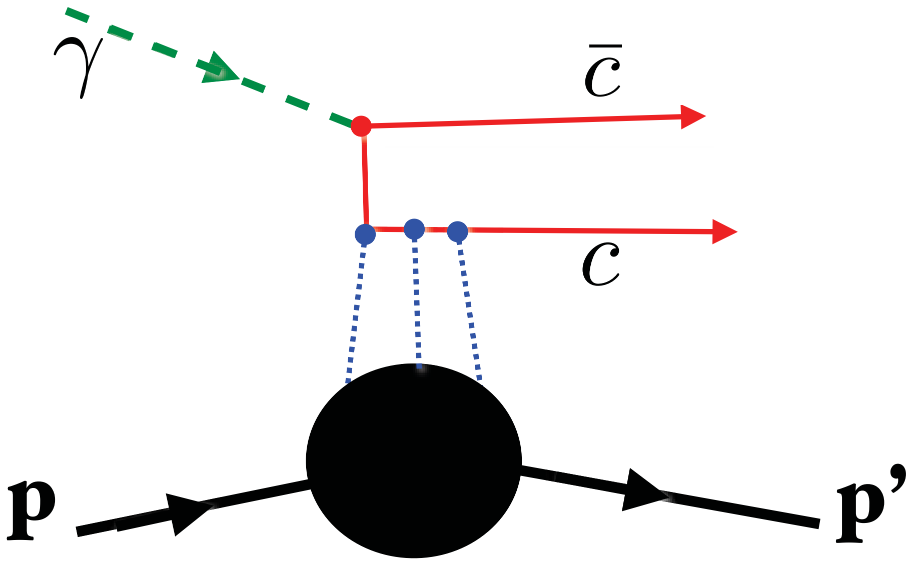
Physics of FSI not in Wavefunction of Target

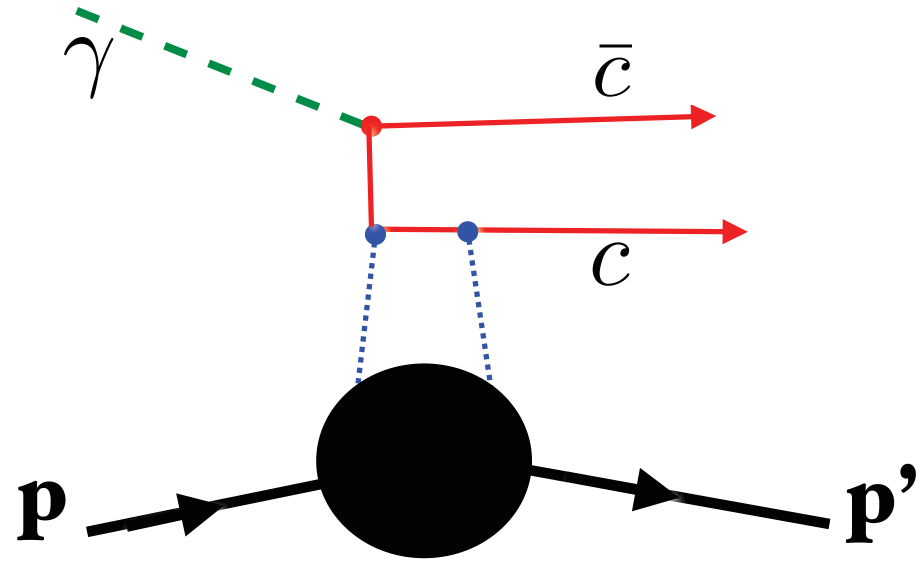
Leading-Twist Diffractive Contribution to High P_T Hadron Production



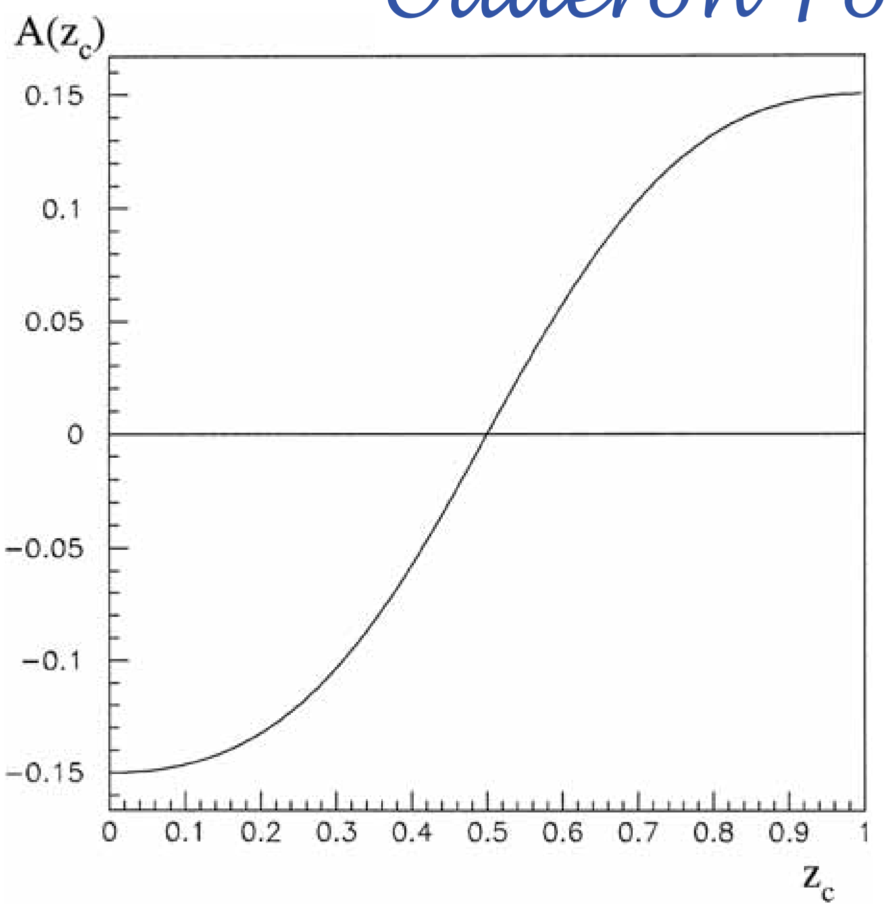
$\mathcal{P}$: **Color-Singlet Exchange**

$$\frac{d\sigma}{d^3p/E}(pp \to \pi p' X) = \alpha_s^2 \frac{F(x_\perp, y)}{p_\perp^4} \; \mathcal{P}$$





Odderon-Pomeron Interference!



$$\frac{d\sigma}{dz_c}(\gamma p \to c\bar{c}p')$$

$$\mathscr{A}\left(t \simeq 0, M_X^2, z_c\right) \simeq 0.45 \left(\frac{s_{\gamma p}}{M_X^2}\right)^{-0.25} \frac{2\,z_c - 1}{z_c^2 + (1 - z_c)^2}$$

Measure charm momentum asymmetry in photon fragmentation region

Only one charm quark needs to be measured

Merino, Rathsman, sjb

Single-spin asymmetries

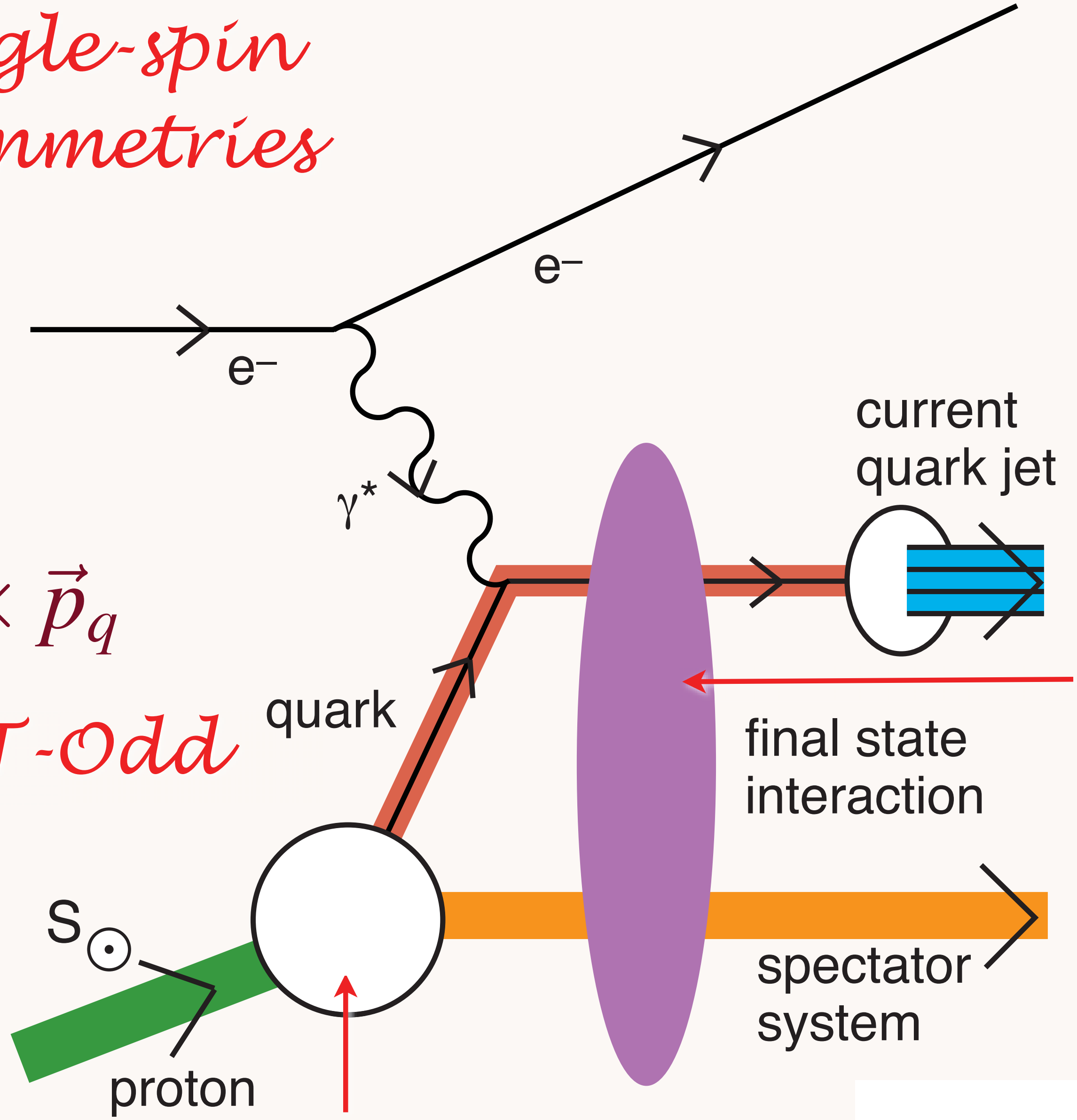
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

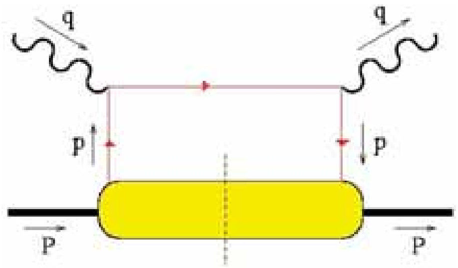
$i \; \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

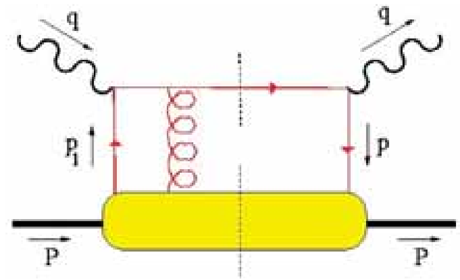


QCD S- and P-Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves



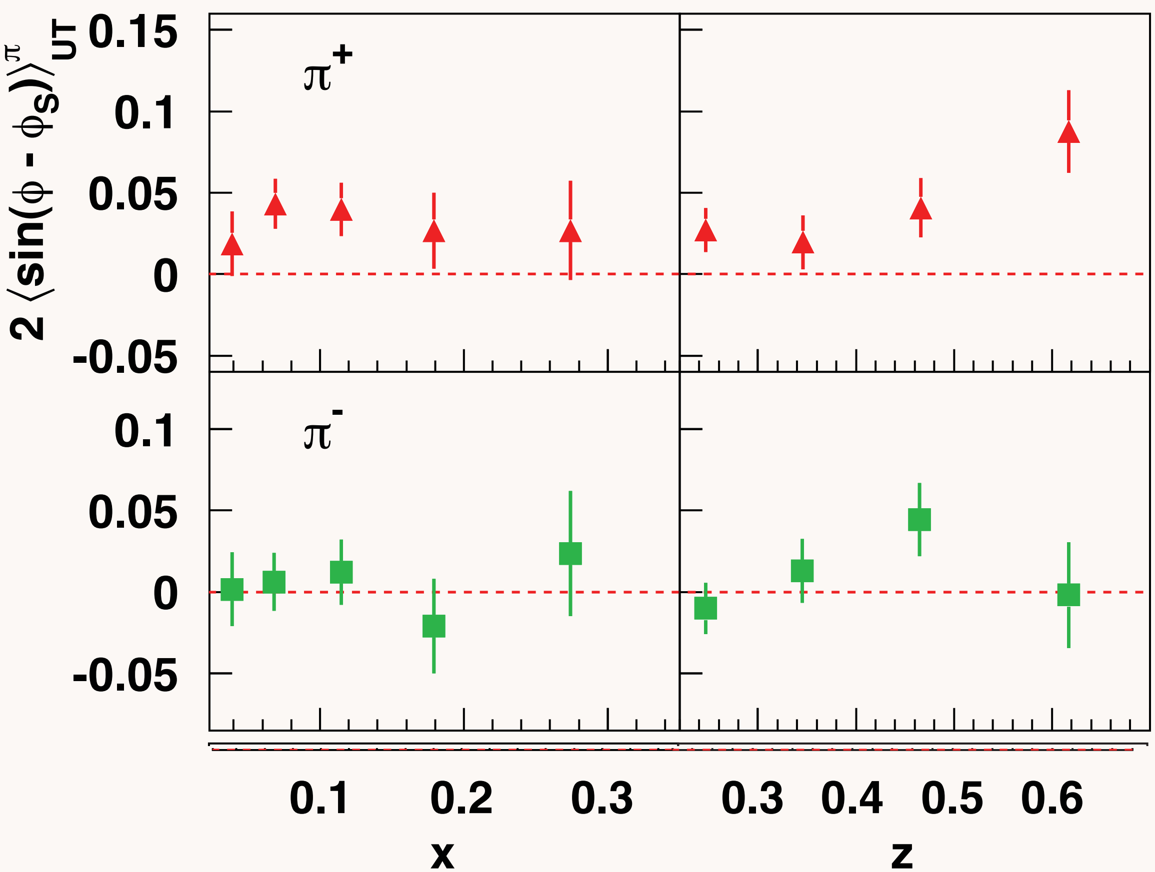
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

Sivers asymmetry from HERMES



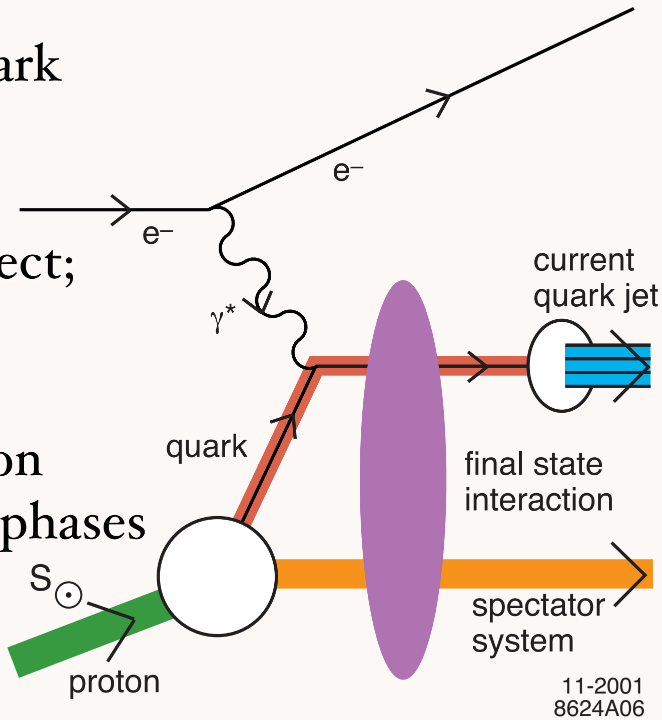
- First evidence for non-zero Sivers function!
- ⇒ presence of non-zero **quark orbital angular momentum**!
- Positive for **π+**... Consistent with zero for **π−**...

Gamberg: Hermes data compatible with BHS model

Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment

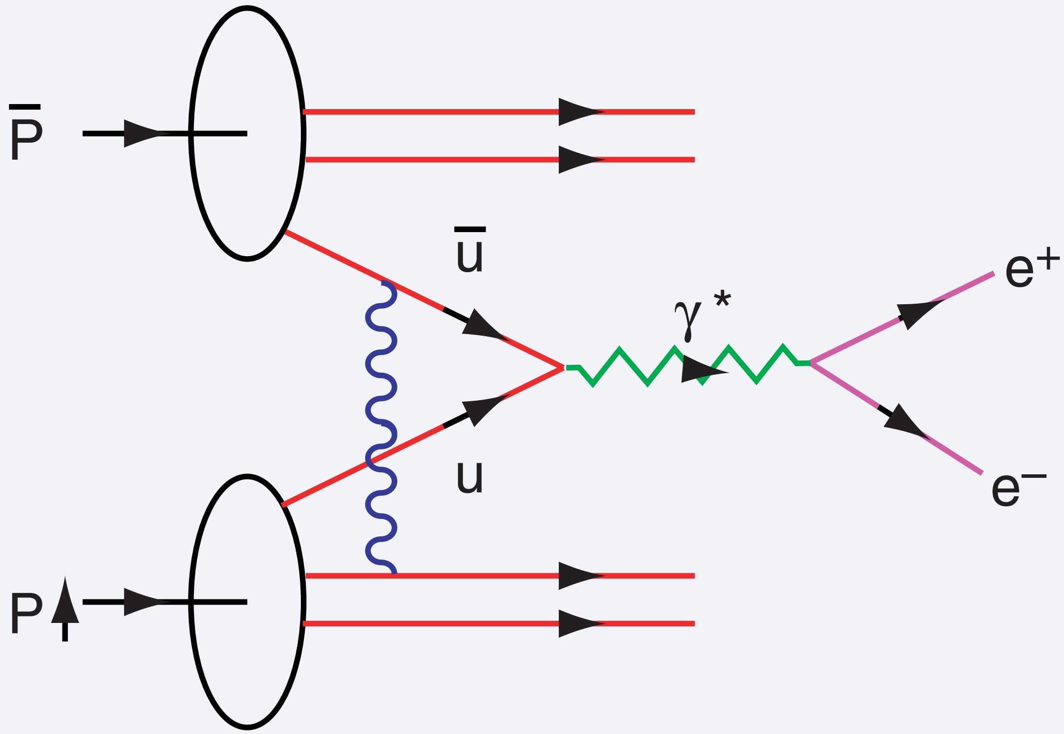
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$



- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

Predict Opposite Sign SSA in DY !



Collins;
Hwang, Schmidt.
sjb

Single Spin Asymmetry In the Drell Yan Process

$\vec{S}_p \cdot \vec{\overline{p}} \times \vec{q}_{\gamma^*}$

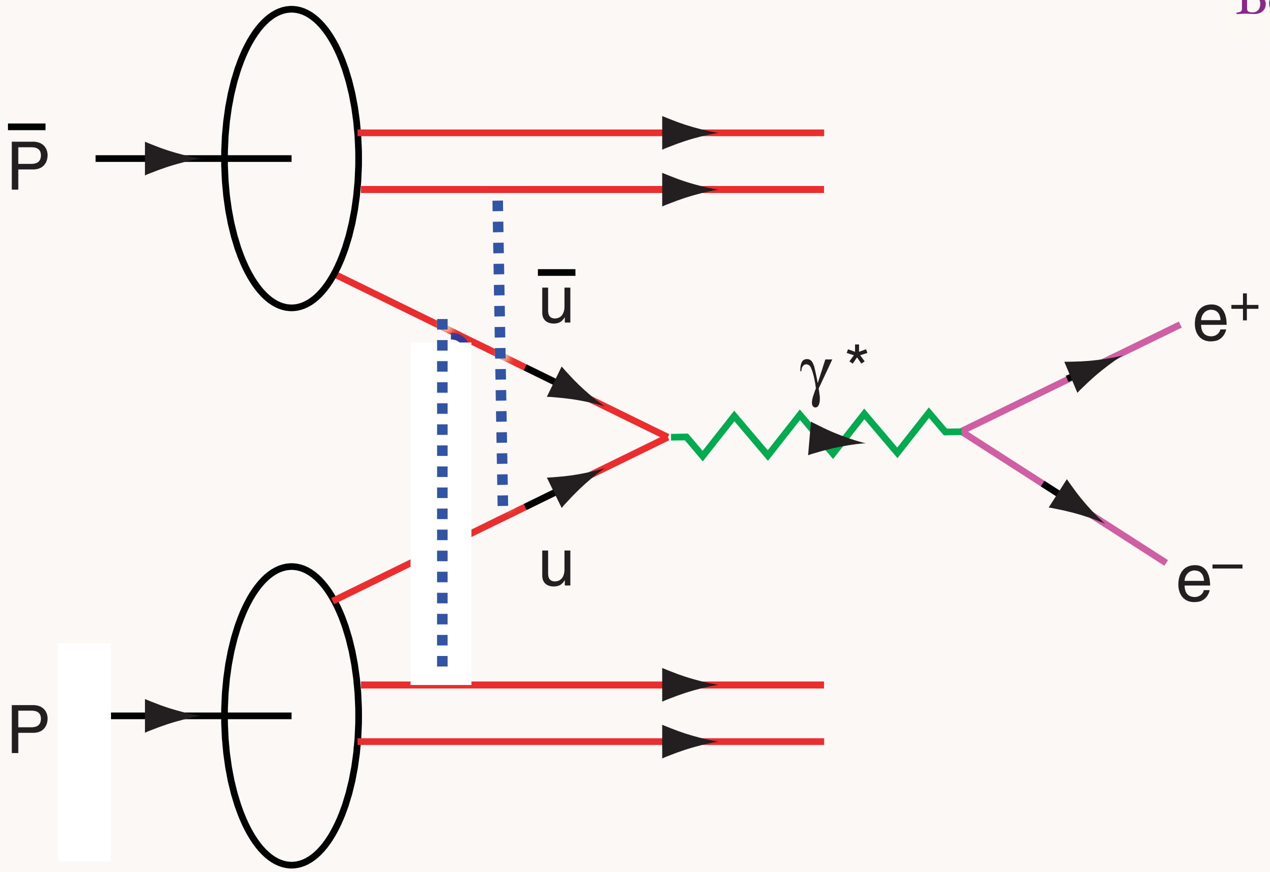
Quarks Interact in the Initial State

Interference of Coulomb Phases for S and P states

Produce Single Spin Asymmetry [Siver's Effect]Proportional to the Proton Anomalous Moment and α_s.

Opposite Sign to DIS! No Factorization

Boer, Hwang, sjb

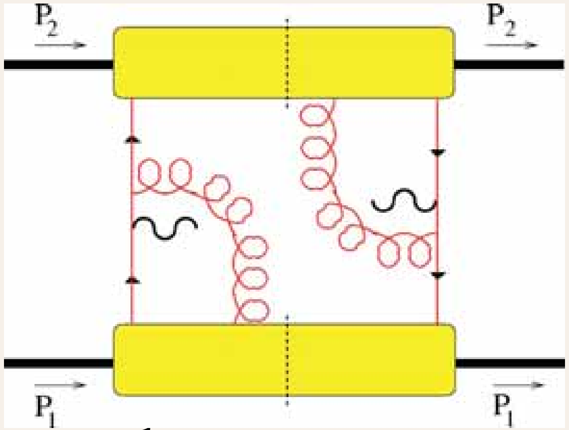


DY $\cos 2\phi$ correlation at leading twist from double ISI

Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation



- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

Double Initial-State Interactions generate anomalous $\cos 2\phi$

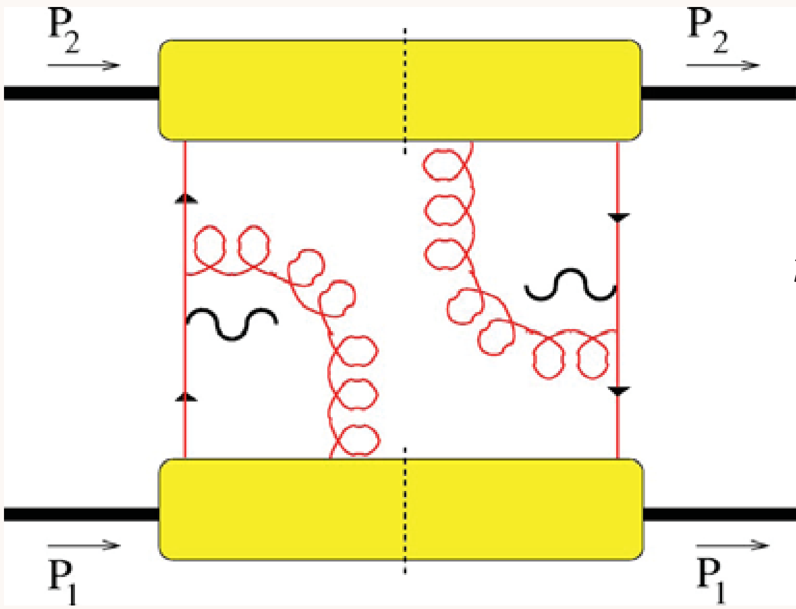
Boer, Hwang, sjb

Drell-Yan planar correlations

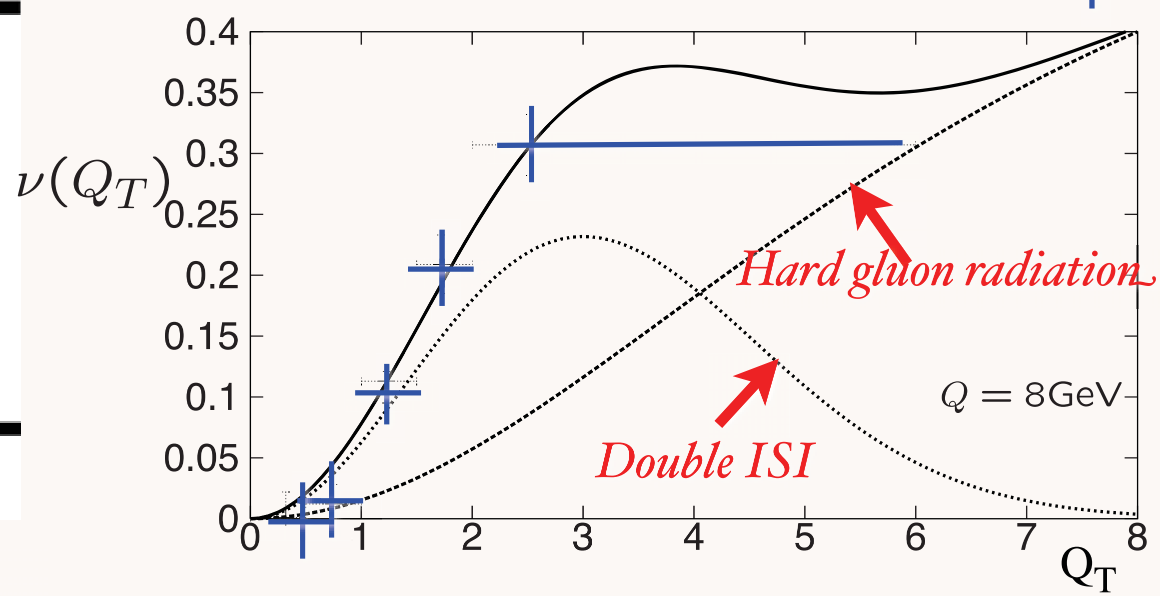
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^{\perp}(\pi)\, h_1^{\perp}(N)$$

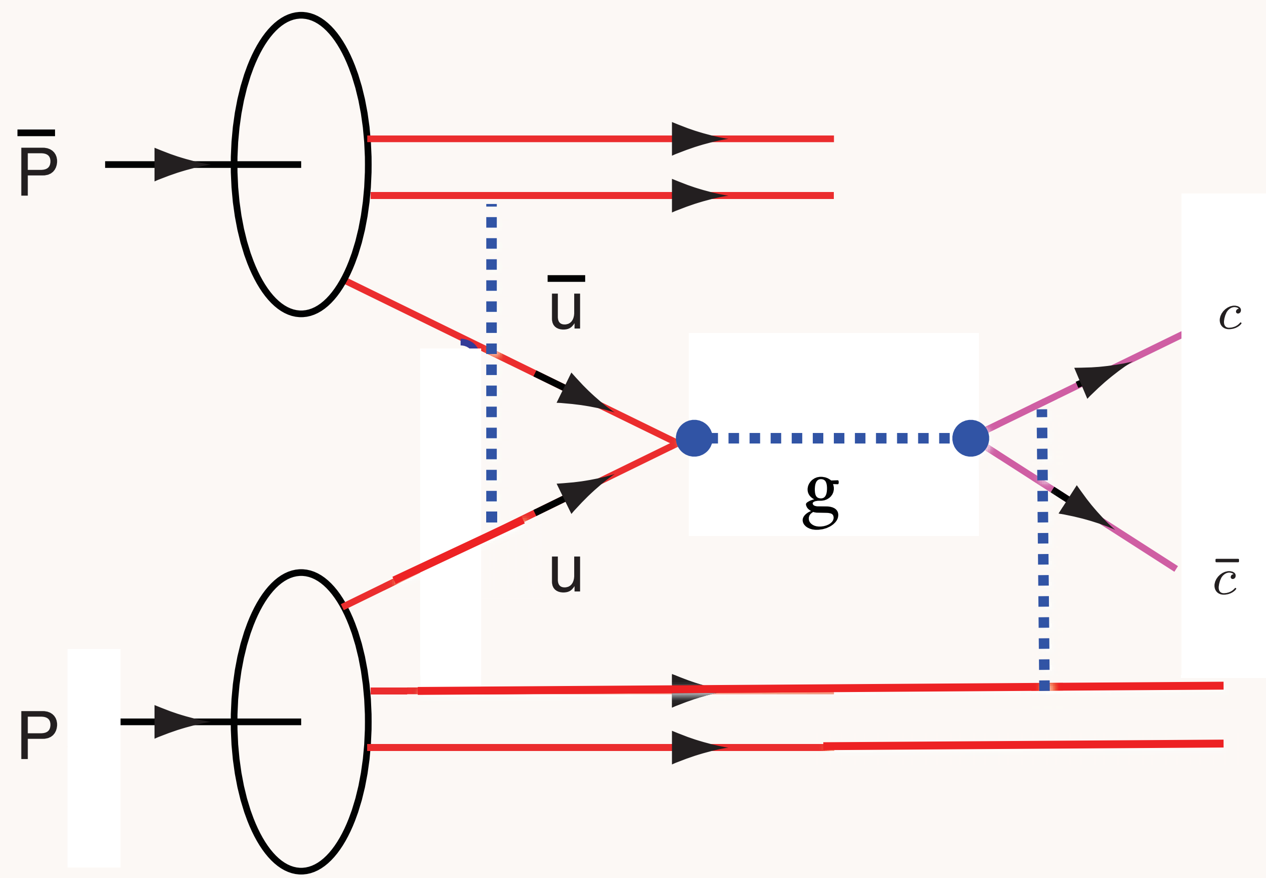


$\pi N \to \mu^+\mu^- X$ NA10



Violates Lam-Tung relation!

Model: Boer,

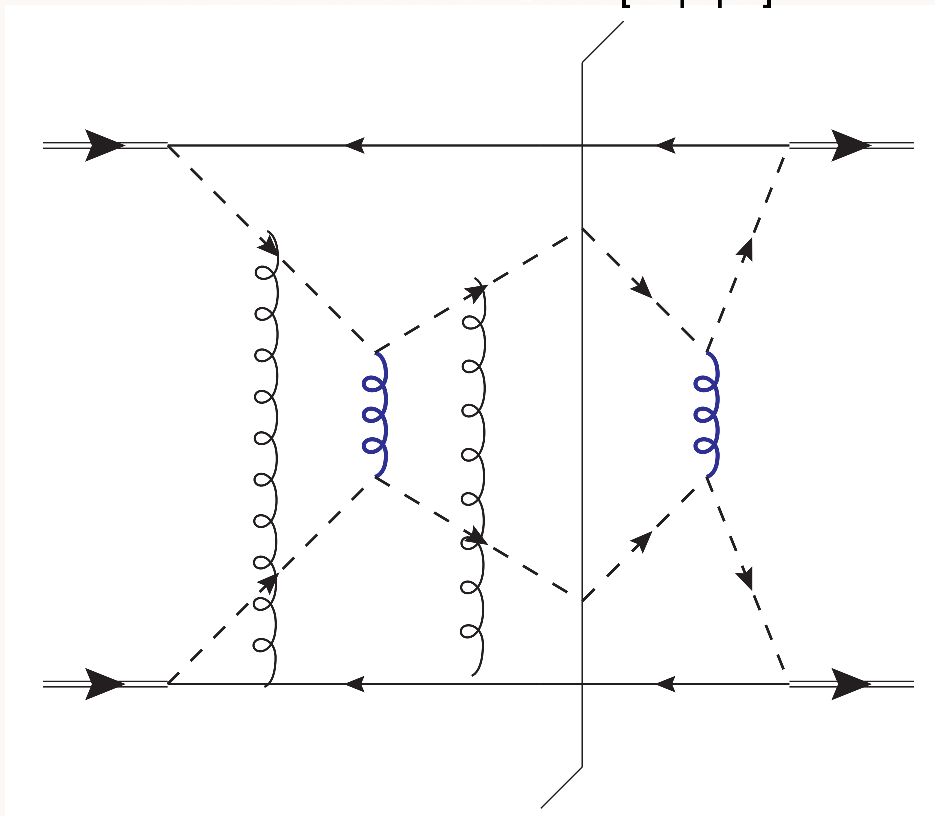


Problem for factorization when both ISI and FSI occur

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

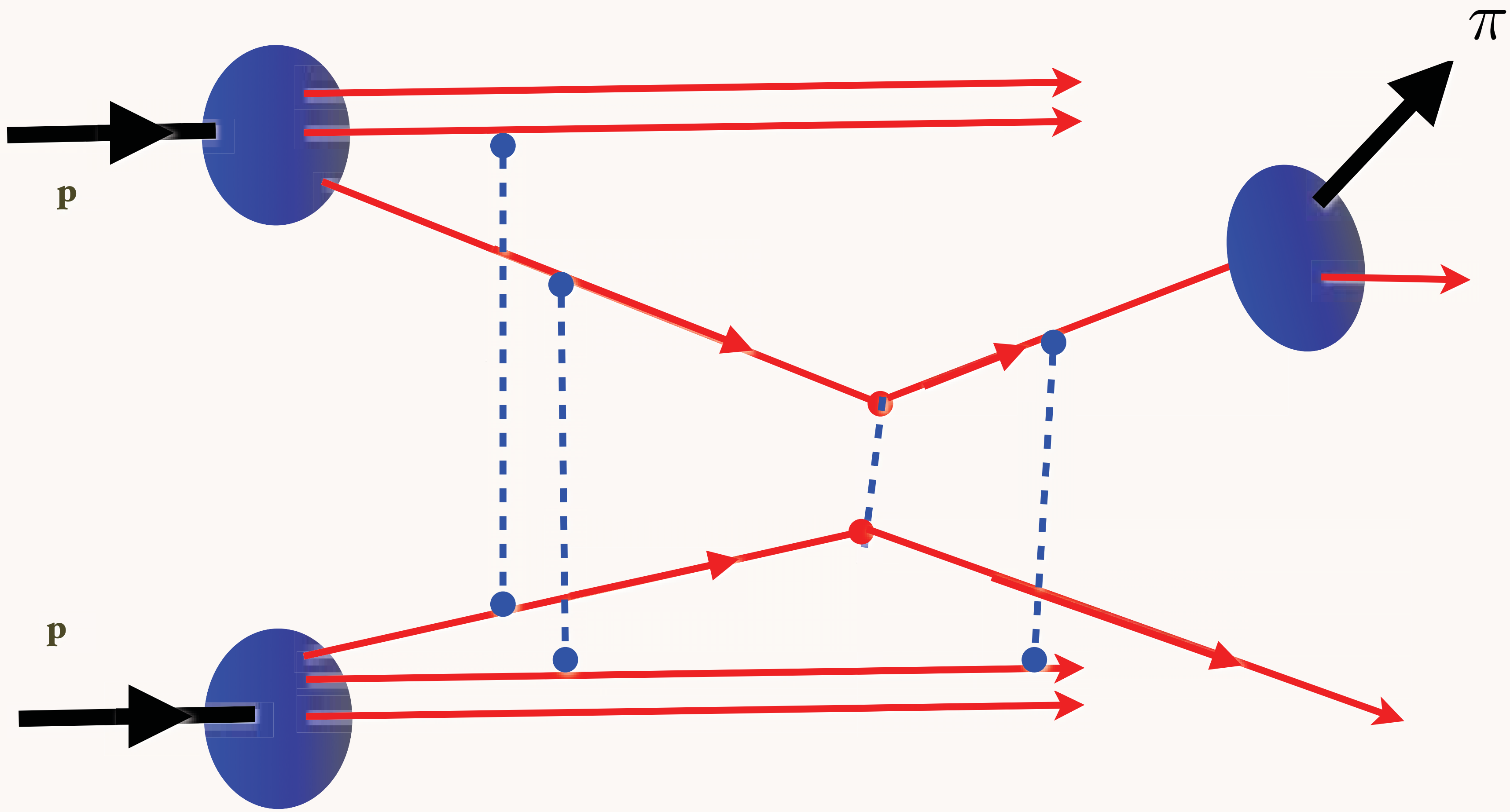
John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

e-Print: **arXiv:0705.2141** [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Important Corrections from Initial and Final State Corrections

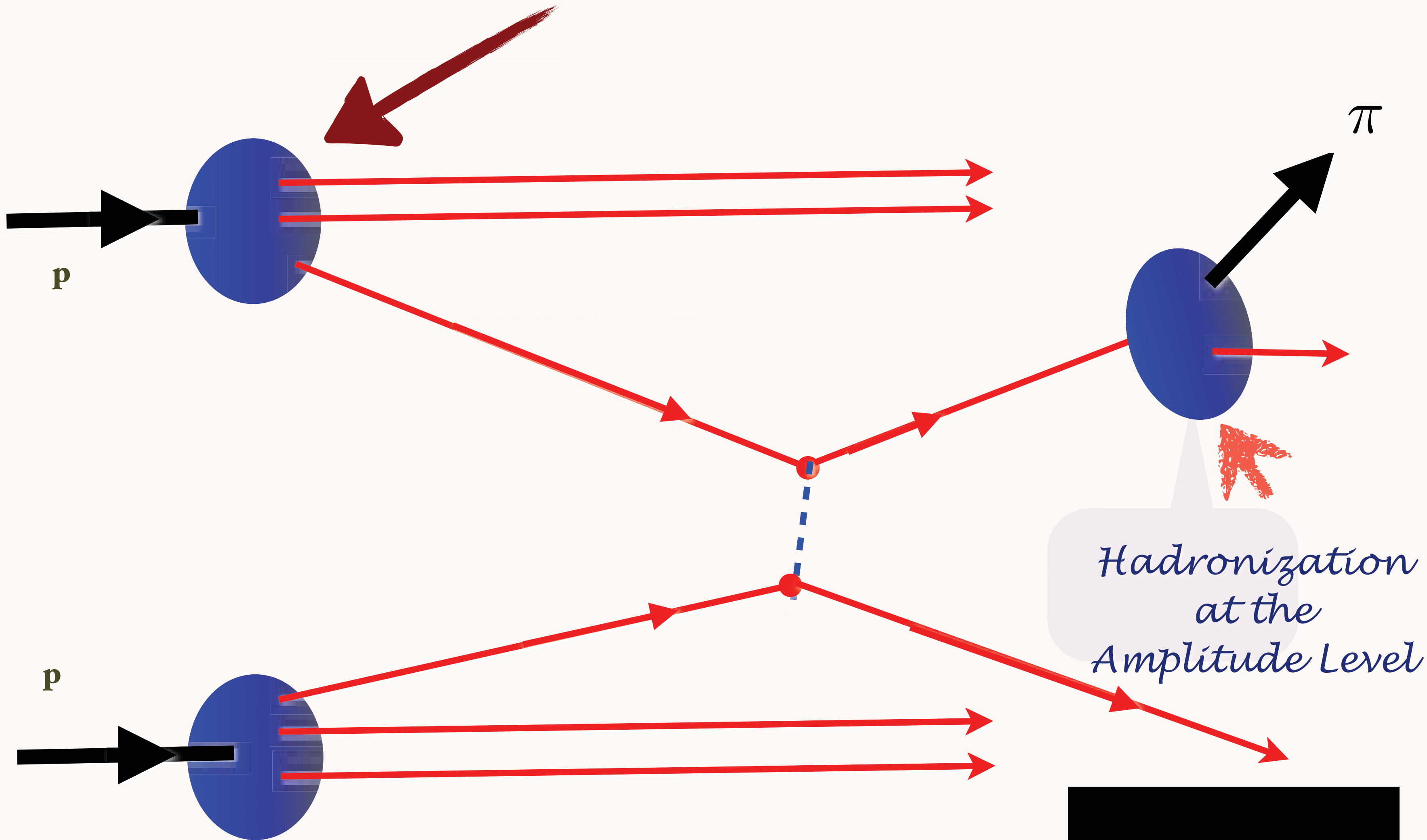


Sivers & Collins Odd-T Spin Effects, Co-planarity Correlations

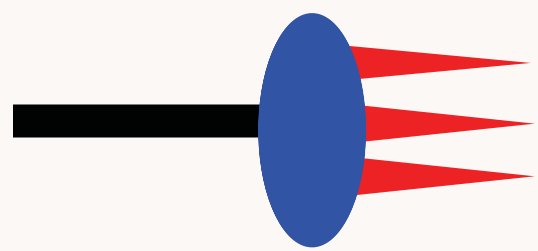
Physics of Rescattering

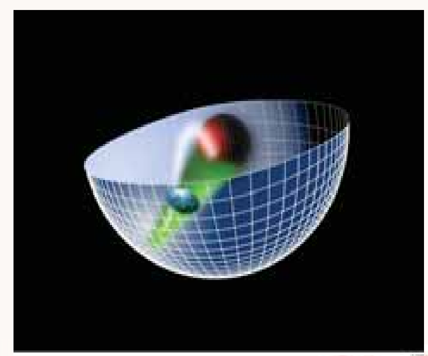
- Sivers Asymmetry and Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions! *Not square of LFWFs*
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon

Light-Front Wavefunctions from AdS/CFT



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$





Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

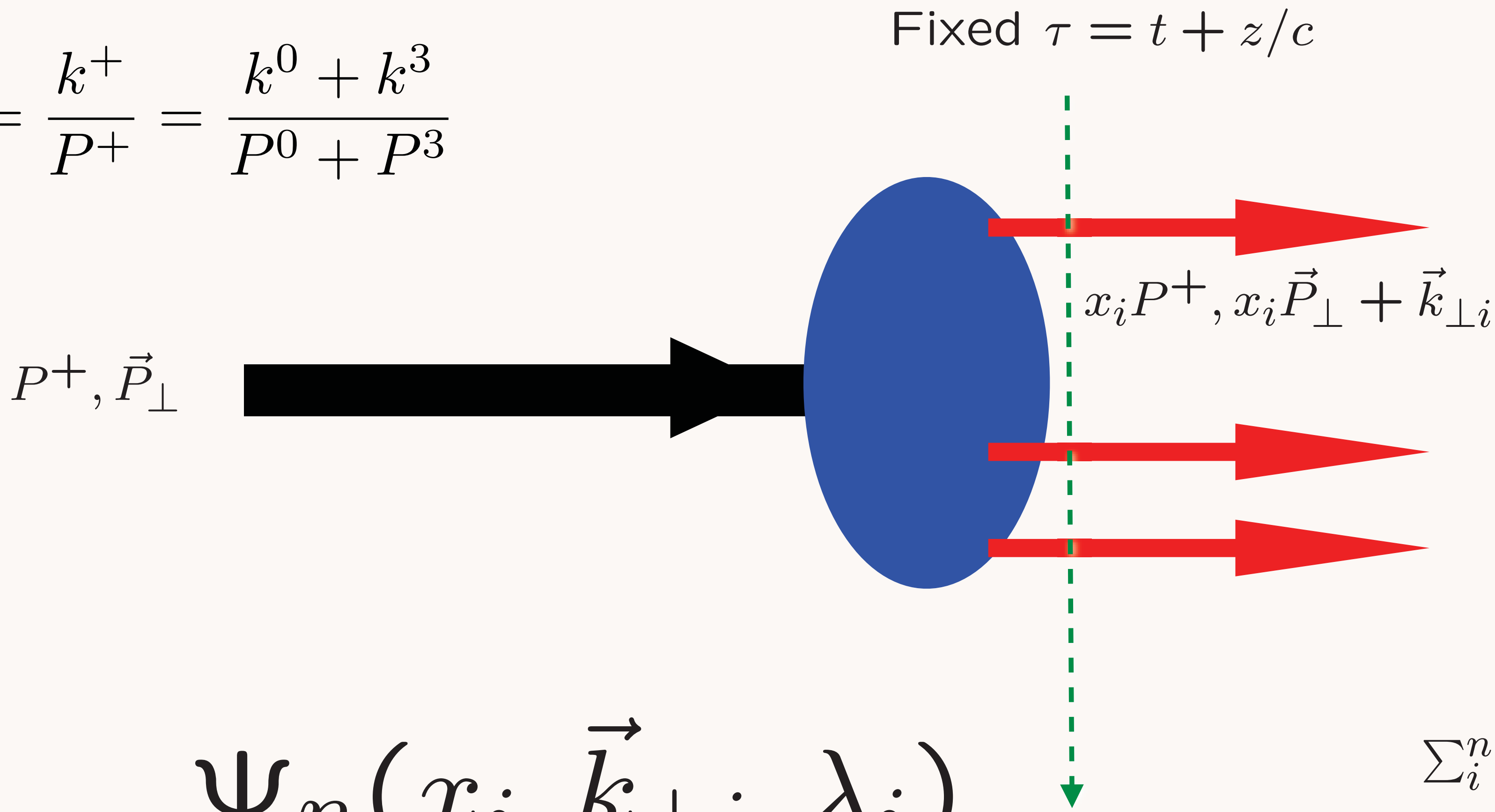
$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

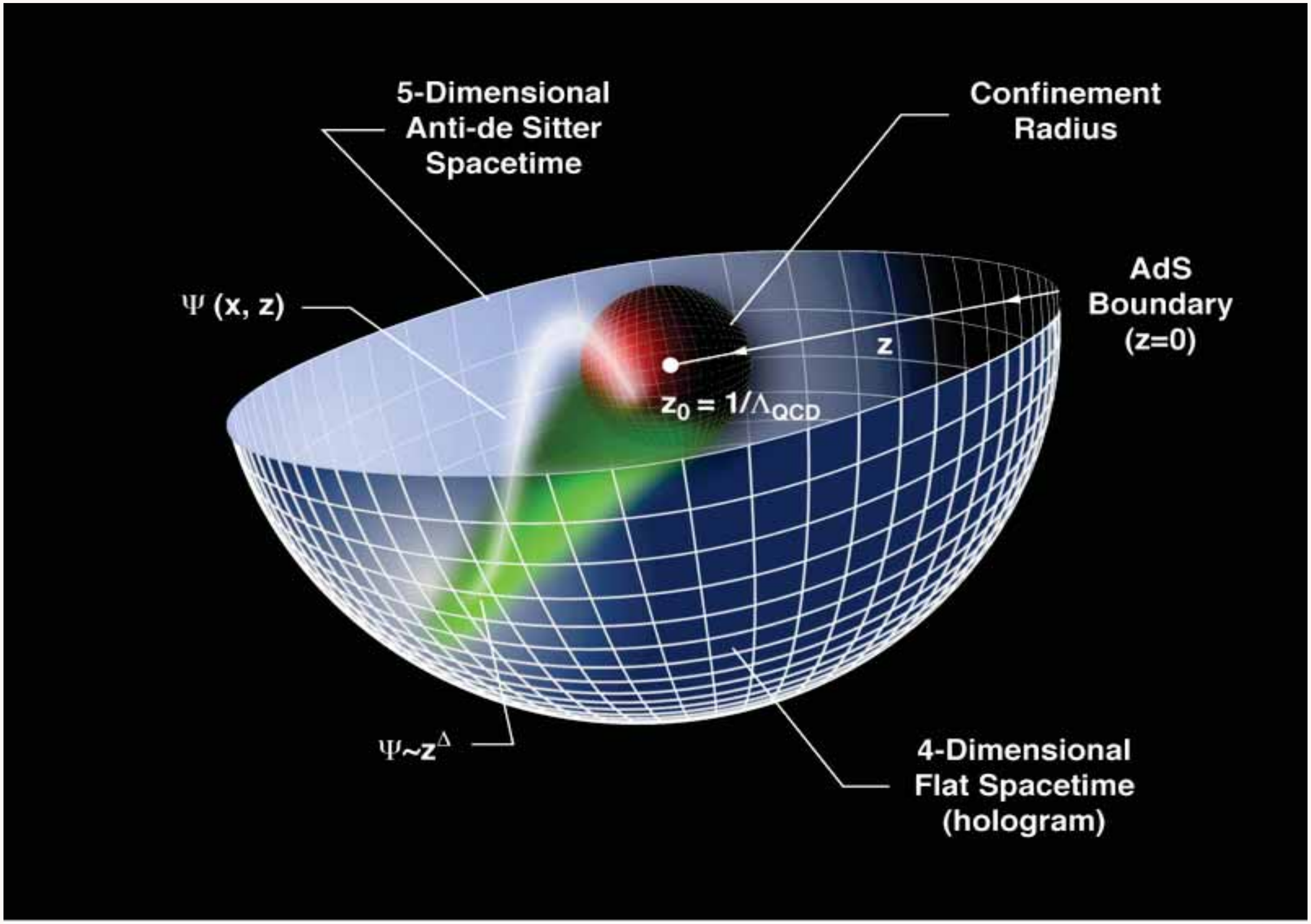
Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

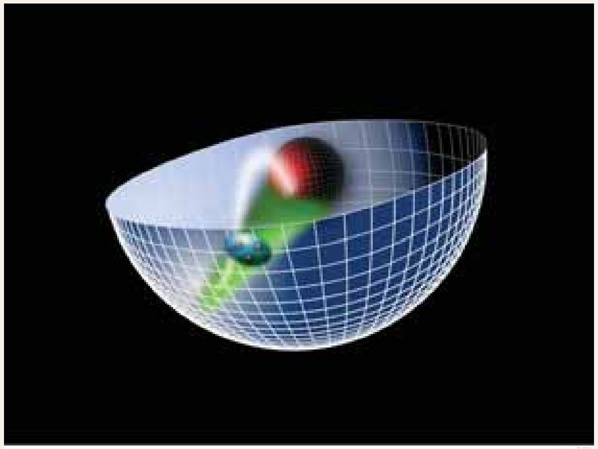
Applications of AdS/CFT to QCD



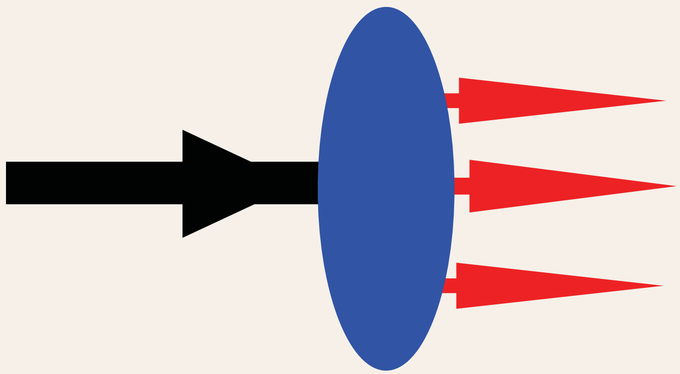
Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

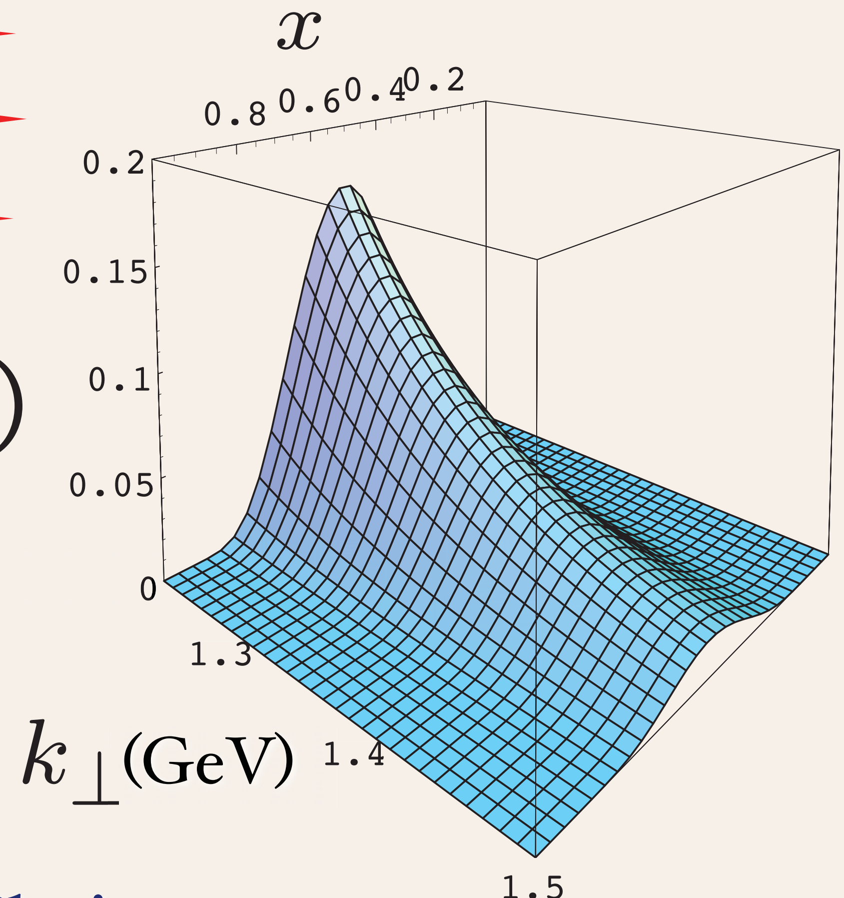
$\phi(z)$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



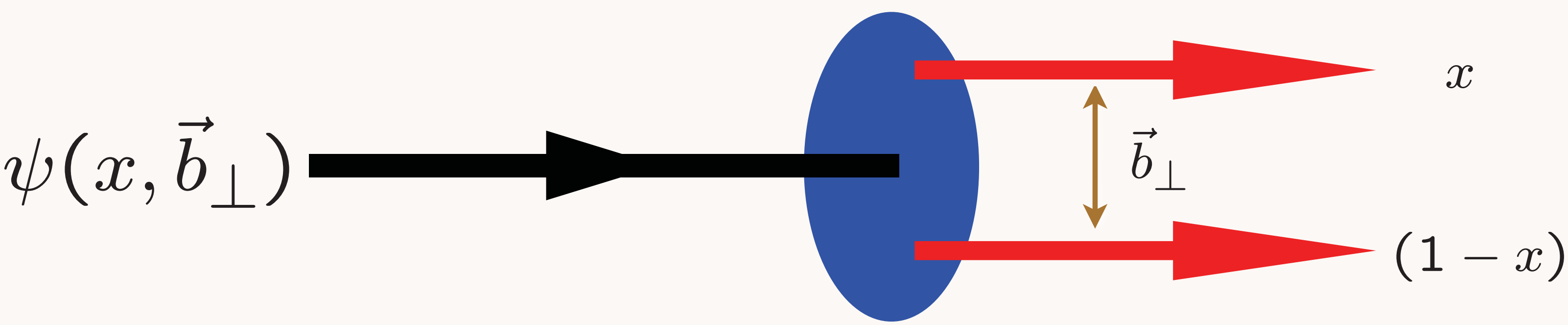
- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

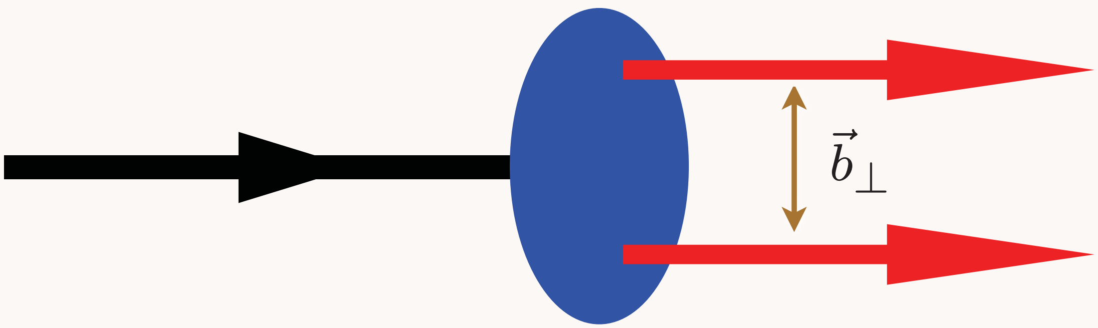
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



$$U(\zeta) = \kappa^4 \zeta^2$$

soft wall confining potential:

G. de Teramond, sjb

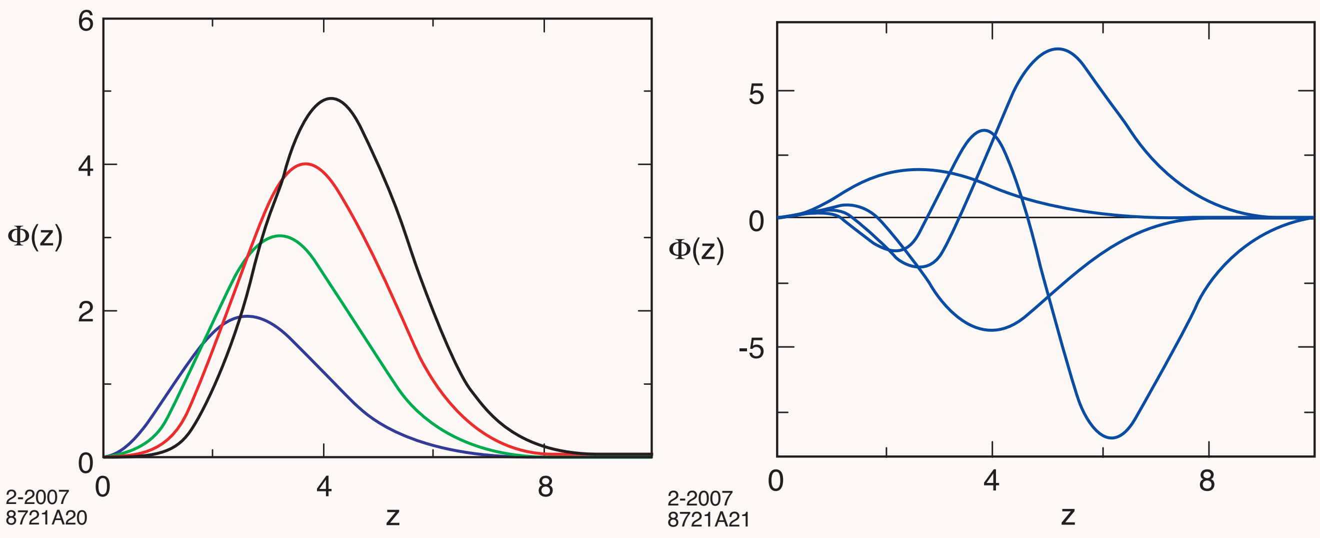
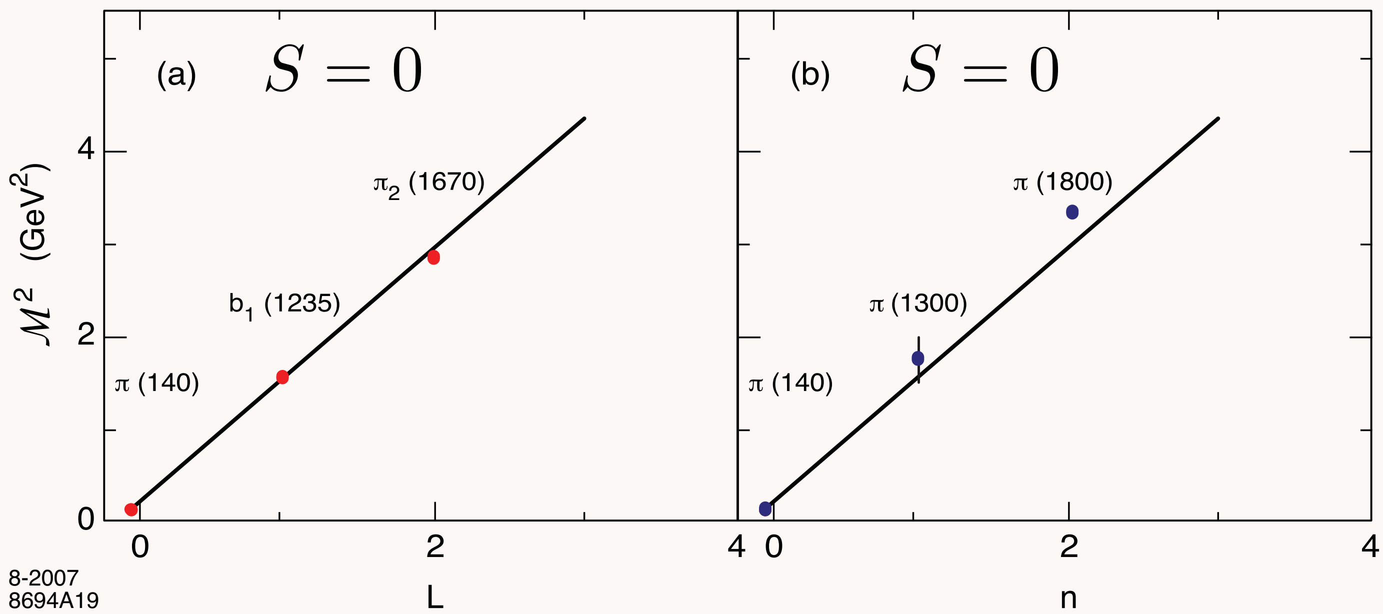


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Soft-wall model

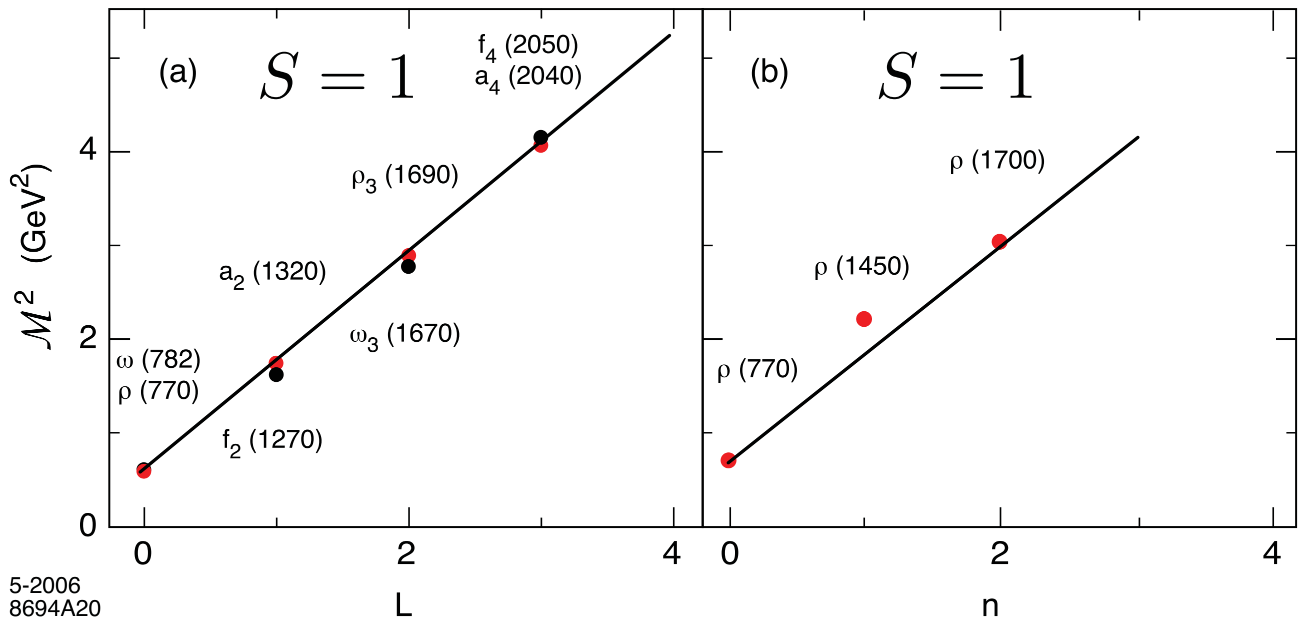
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$.

Same slope in n and L

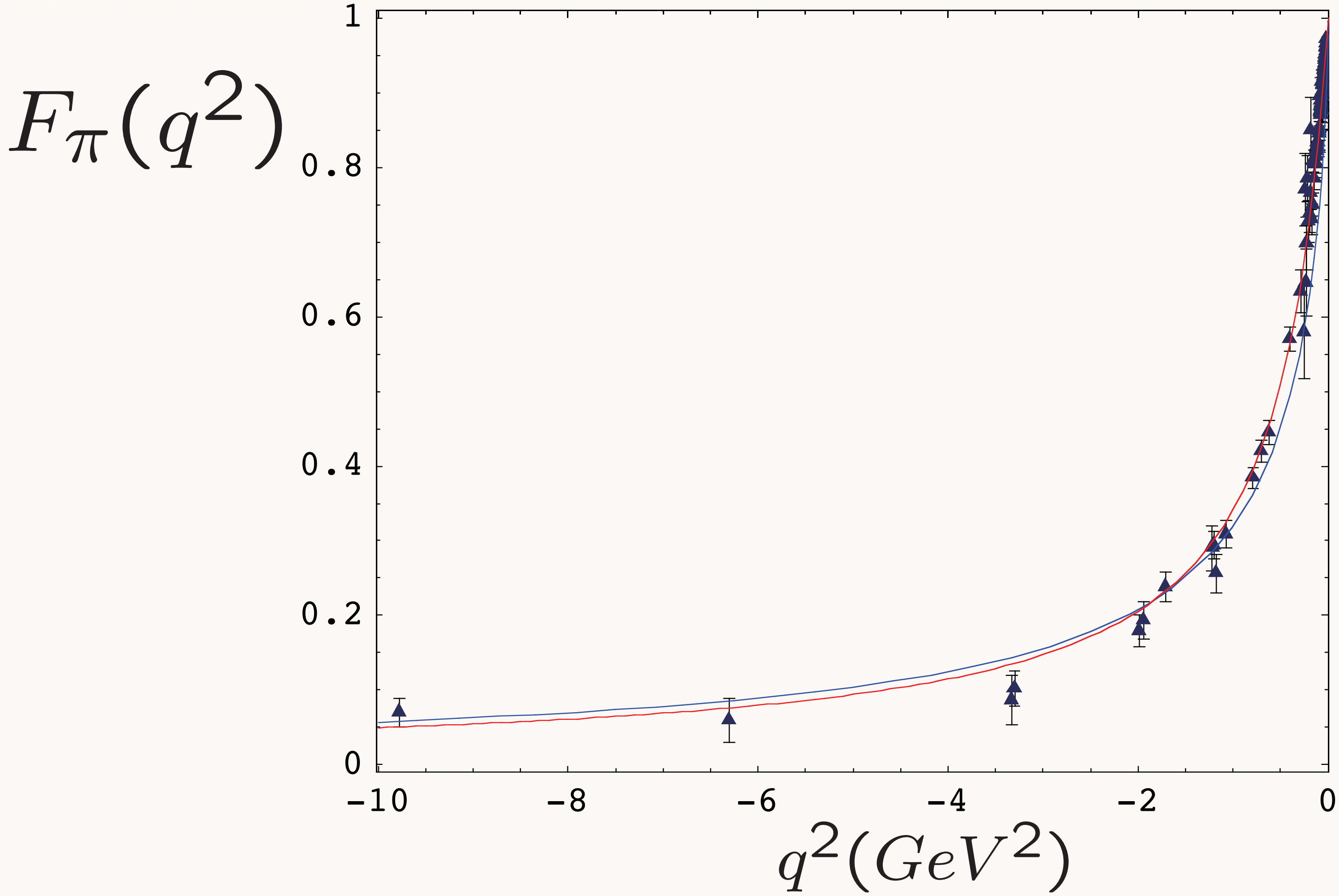
- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\left(n+L+1/2\right)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

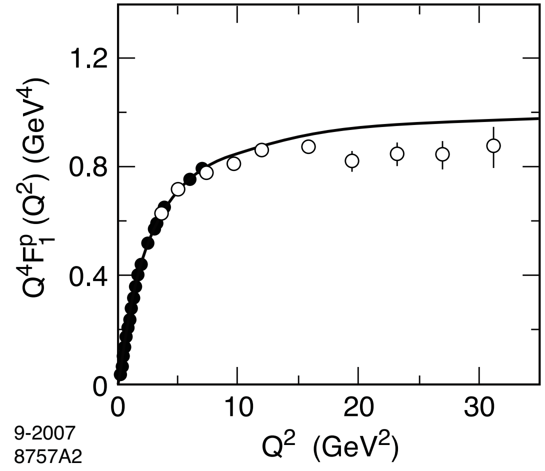
Hard Wall: Truncated Space Confinement

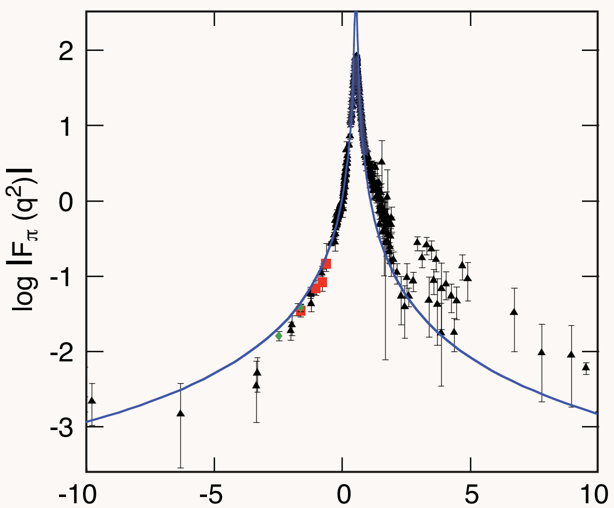
One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Other Applications of Light-Front Holography

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form factors of composite hadronss
- n-parton holographic mapping
- Heavy flavor mesons





hep-th/0501022

hep-ph/0602252

arXiv:0707.3859

arXiv:0802.0514

arXiv:0804.0452

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

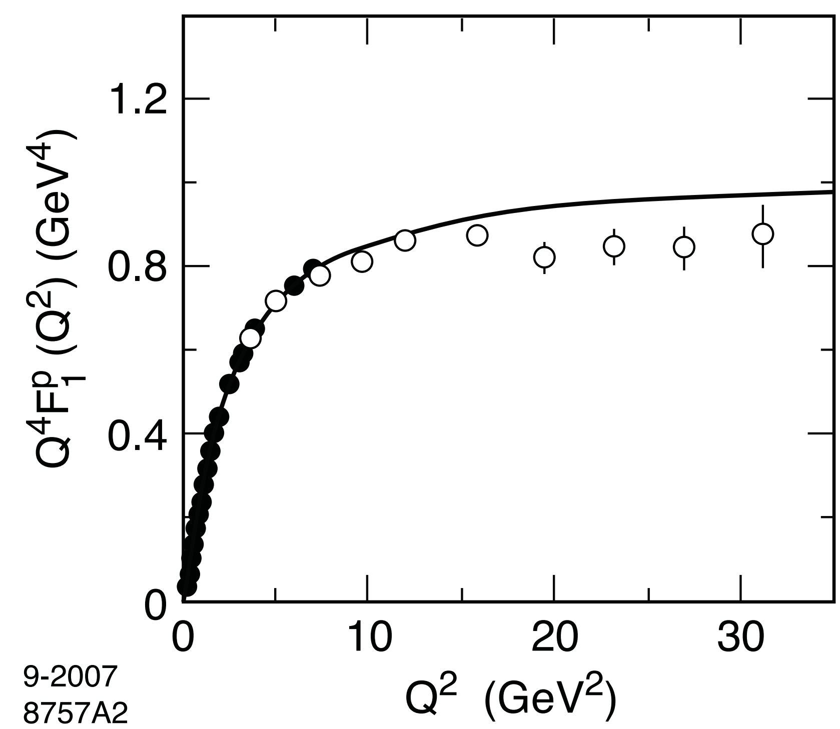
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

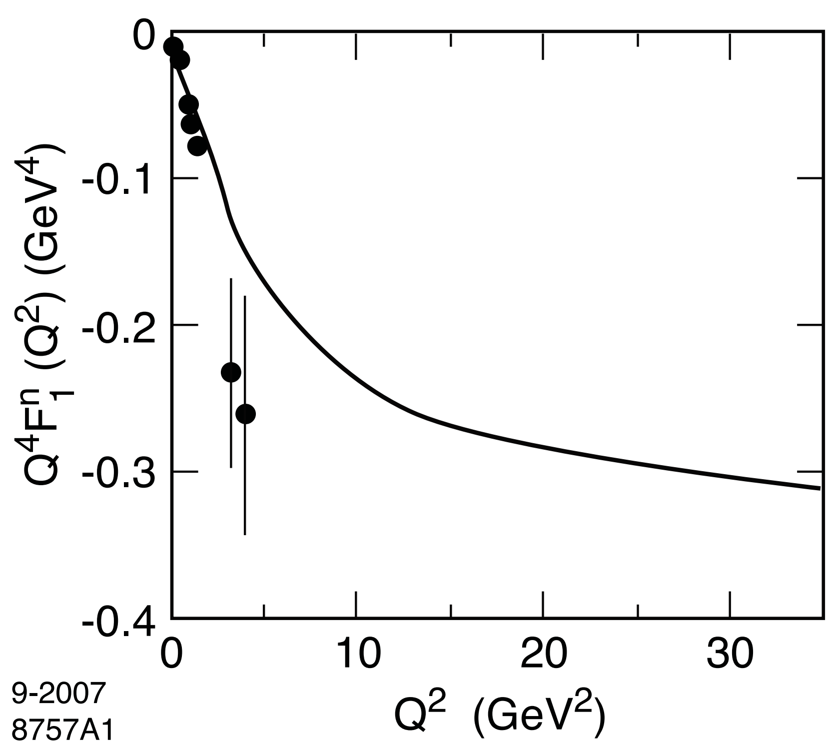
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



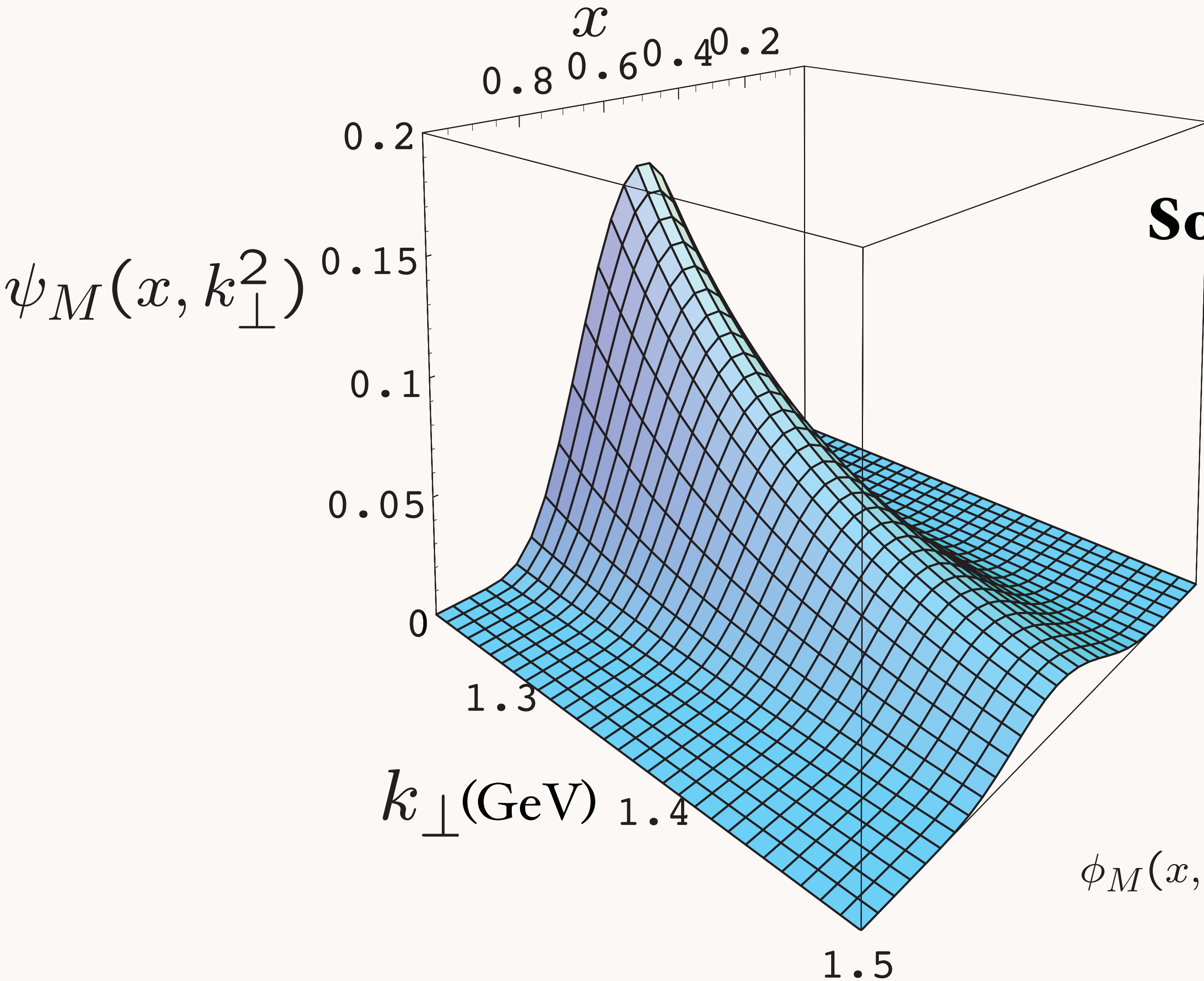
SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Pauli, Hornbostel, Hiller, McCartor, sjb

Vary, Harinandrath, Maris, sjb

$|\pi^+> = |u\bar{d}>$

$m_u = 2\ MeV$
$m_d = 5\ MeV$

$|K^+> = |u\bar{s}>$

$m_s = 95\ MeV$

$|D^+> = |c\bar{d}>$

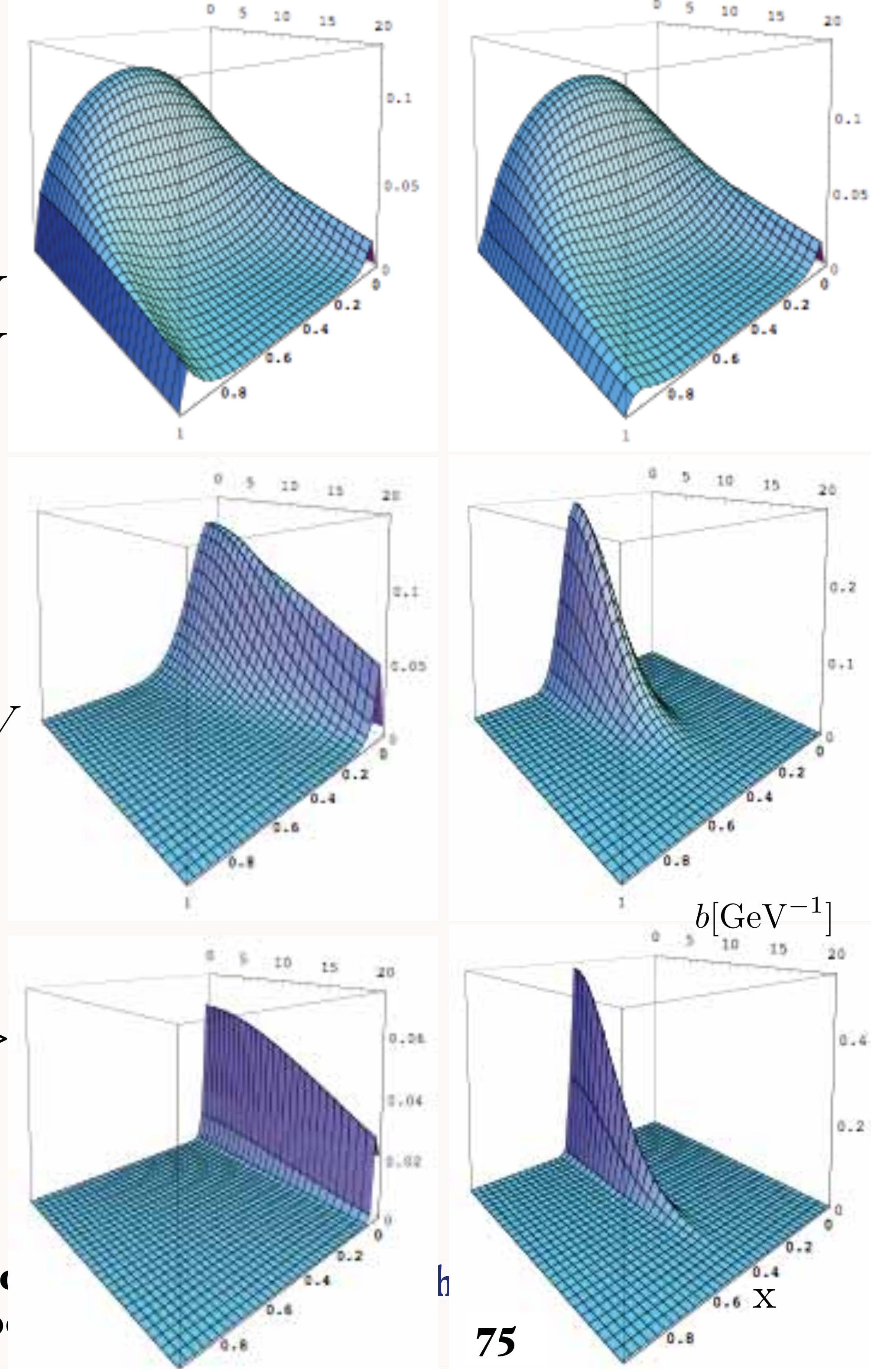
$m_c = 1.25\ GeV$

$|\eta_c> = |c\bar{c}>$

$|B^+> = |u\bar{b}>$

$m_b = 4.2\ GeV$

$|\eta_b> = |b\bar{b}>$

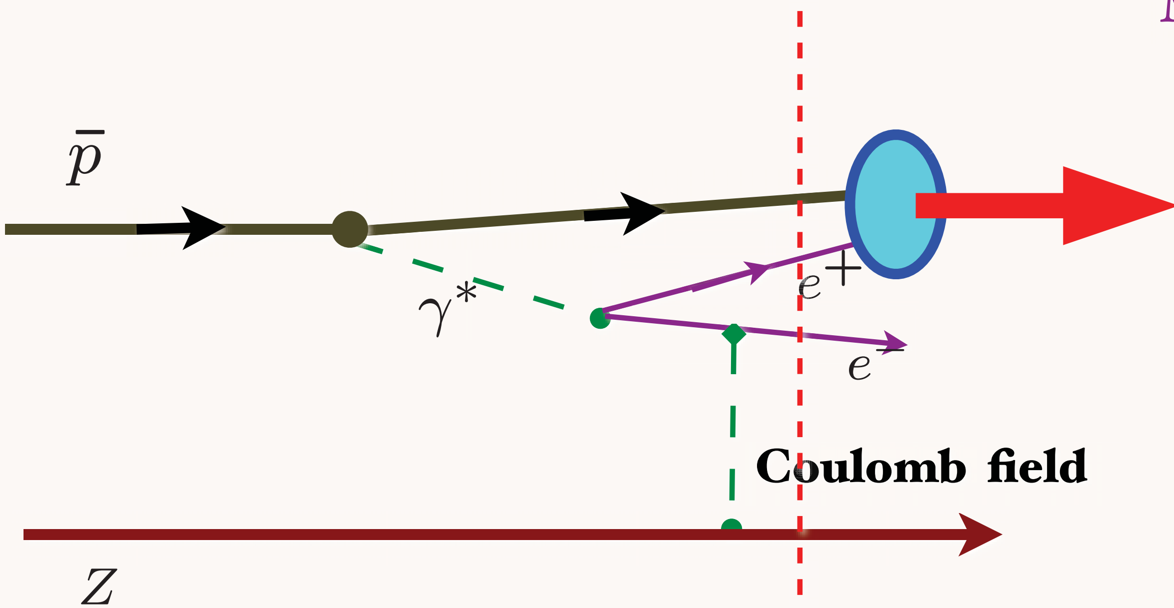


$\kappa = 375\ MeV$

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

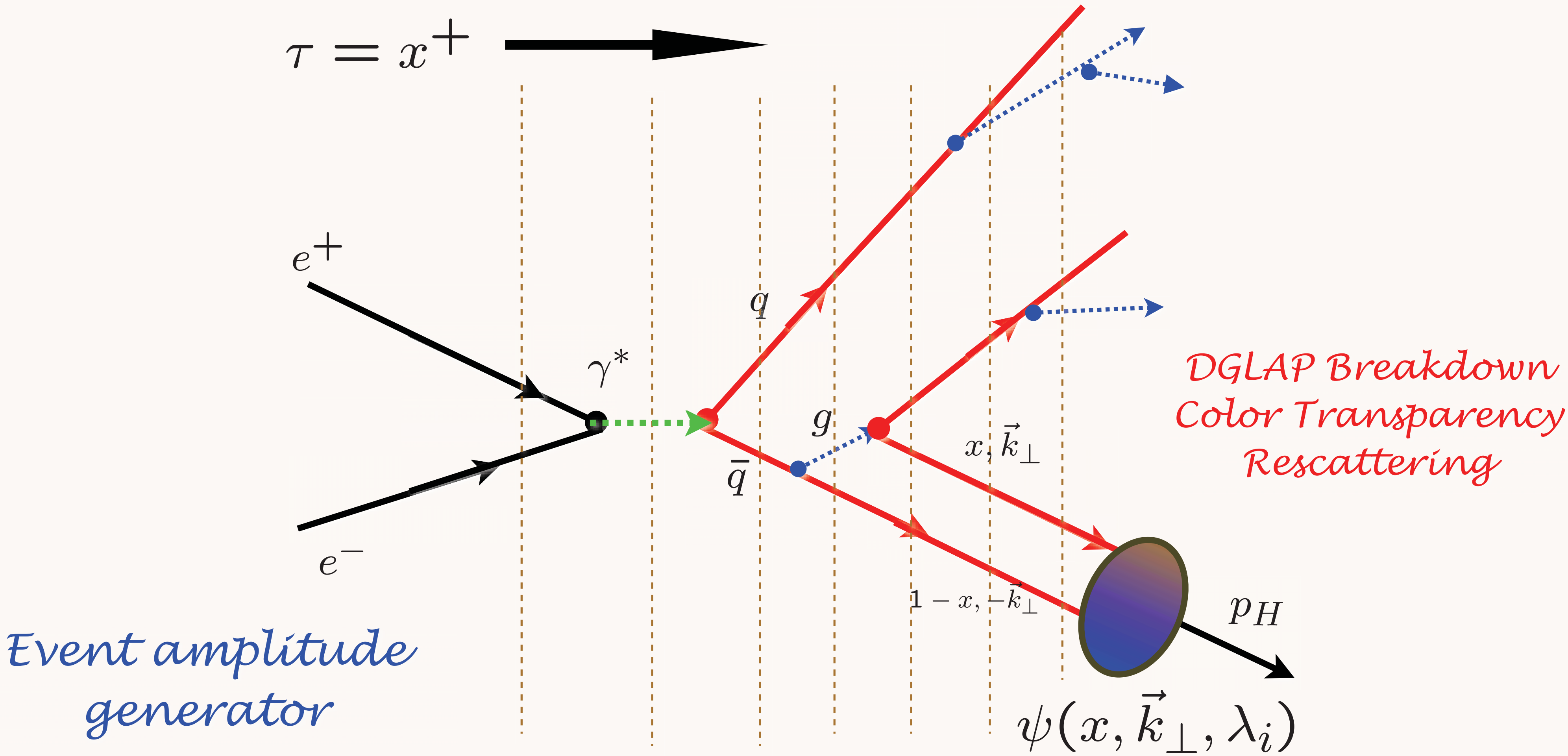
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

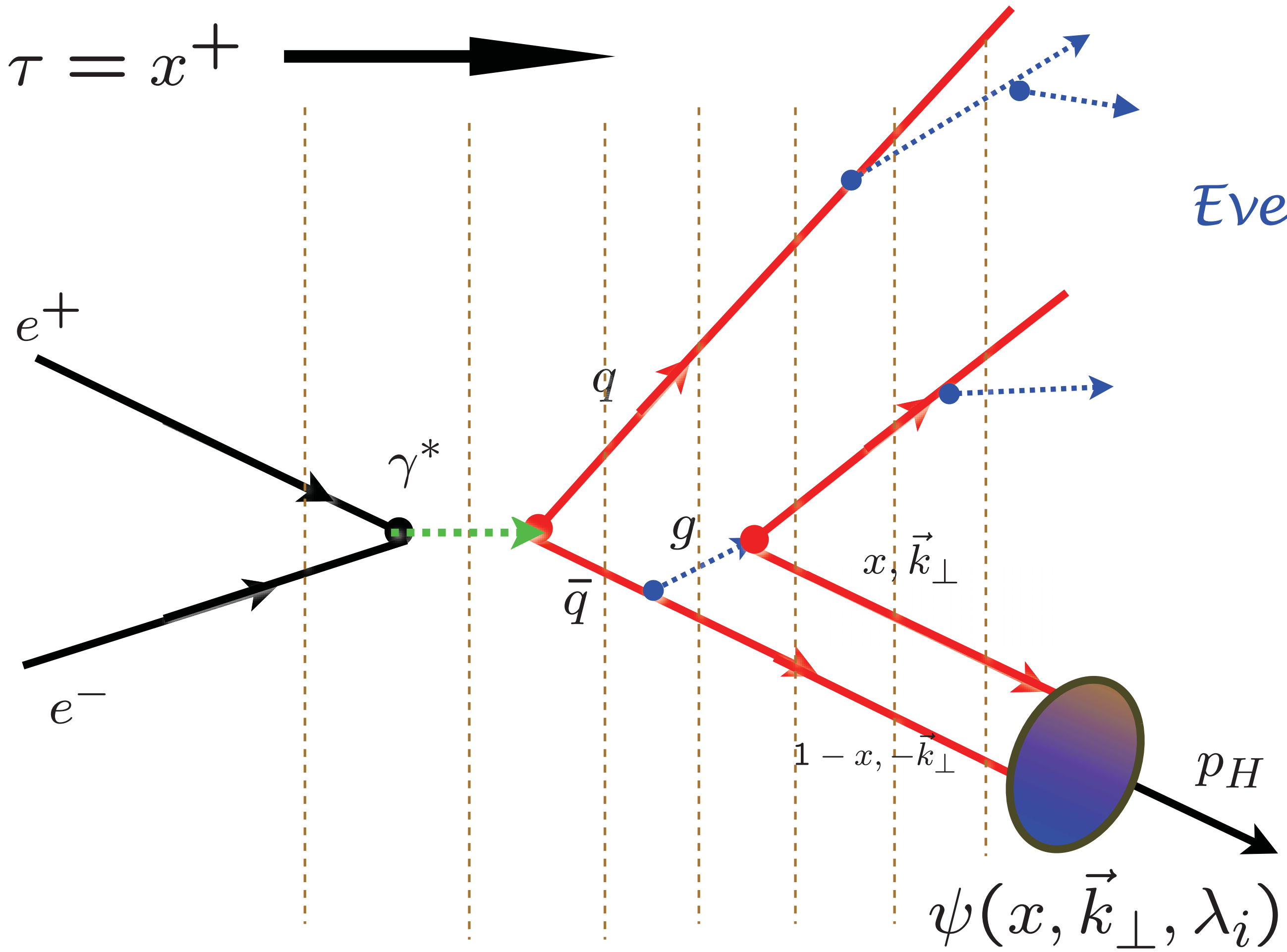
"Hadronization" at the Amplitude Level

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Event amplitude generator

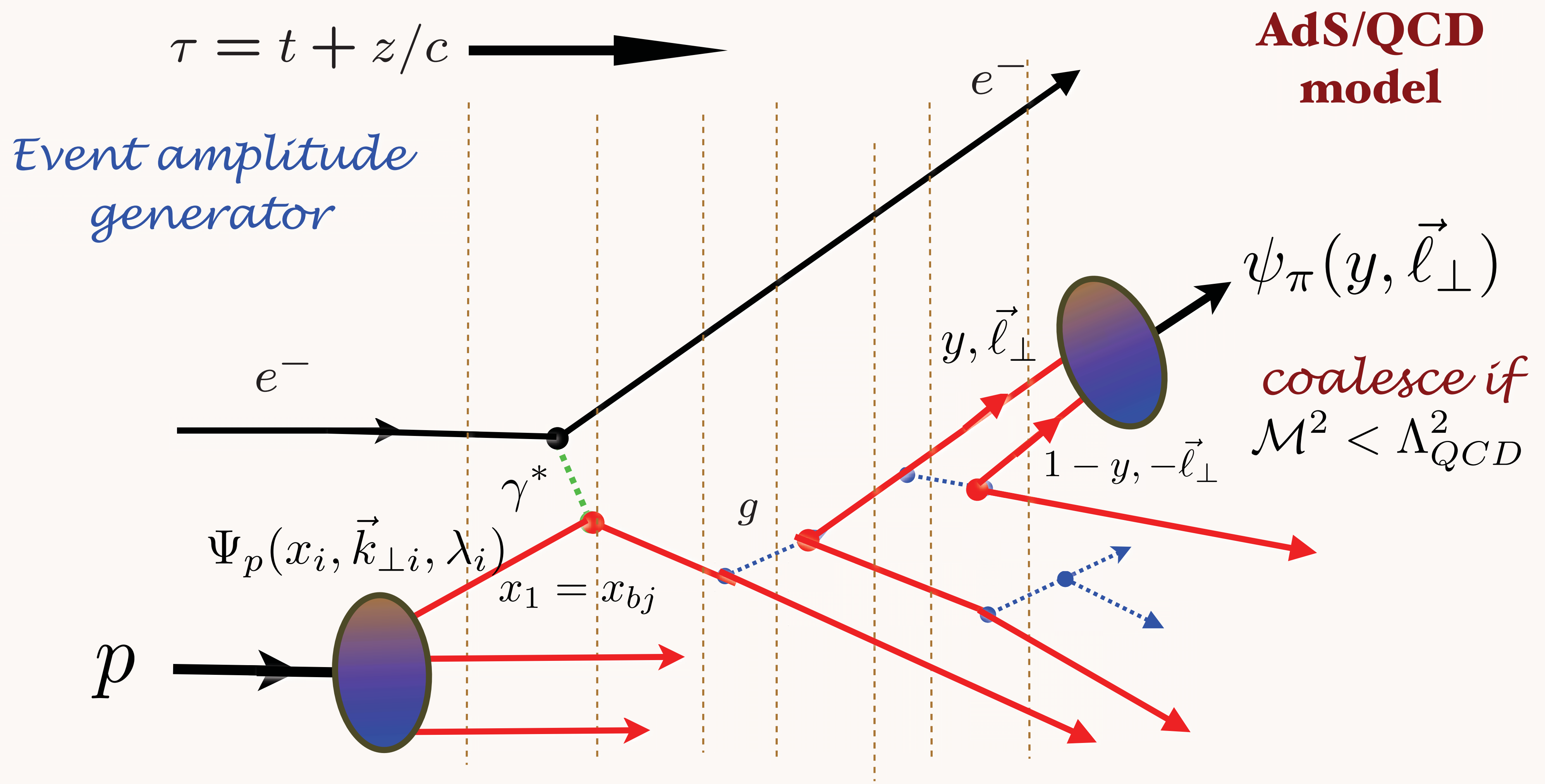
Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda_{QCD}^2}$

AdS/QCD Hard Wall Confinement:

i.e.,

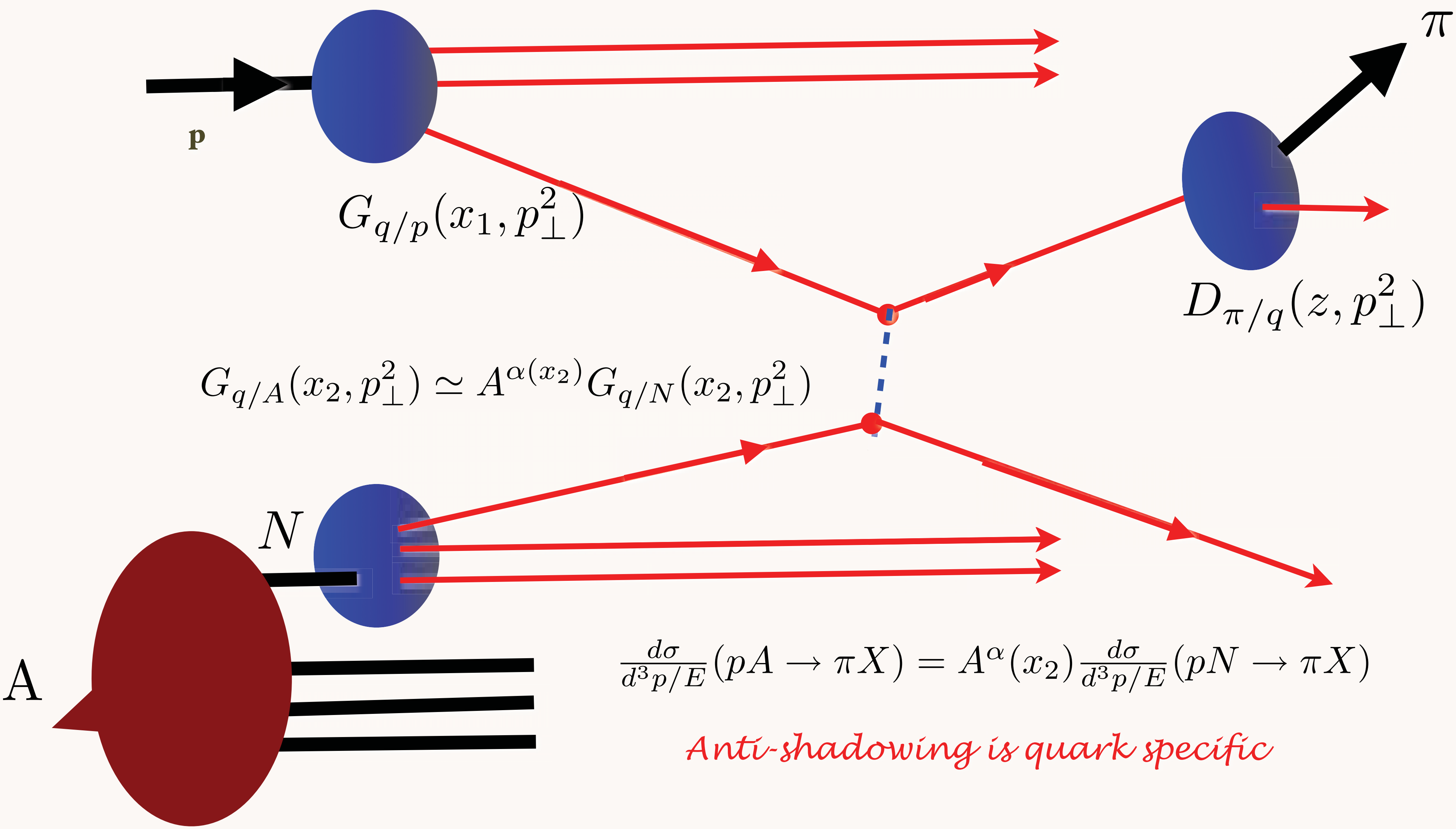
$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda_{QCD}^2$

Jet Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via Light-Front Wavefunctions

Leading-Twist Contribution to Hadron Production on Nuclei



$$\frac{d\sigma}{d^3p/E}(pA \to \pi X) = A^{\alpha}(x_2)\frac{d\sigma}{d^3p/E}(pN \to \pi X)$$

Anti-shadowing is quark specific