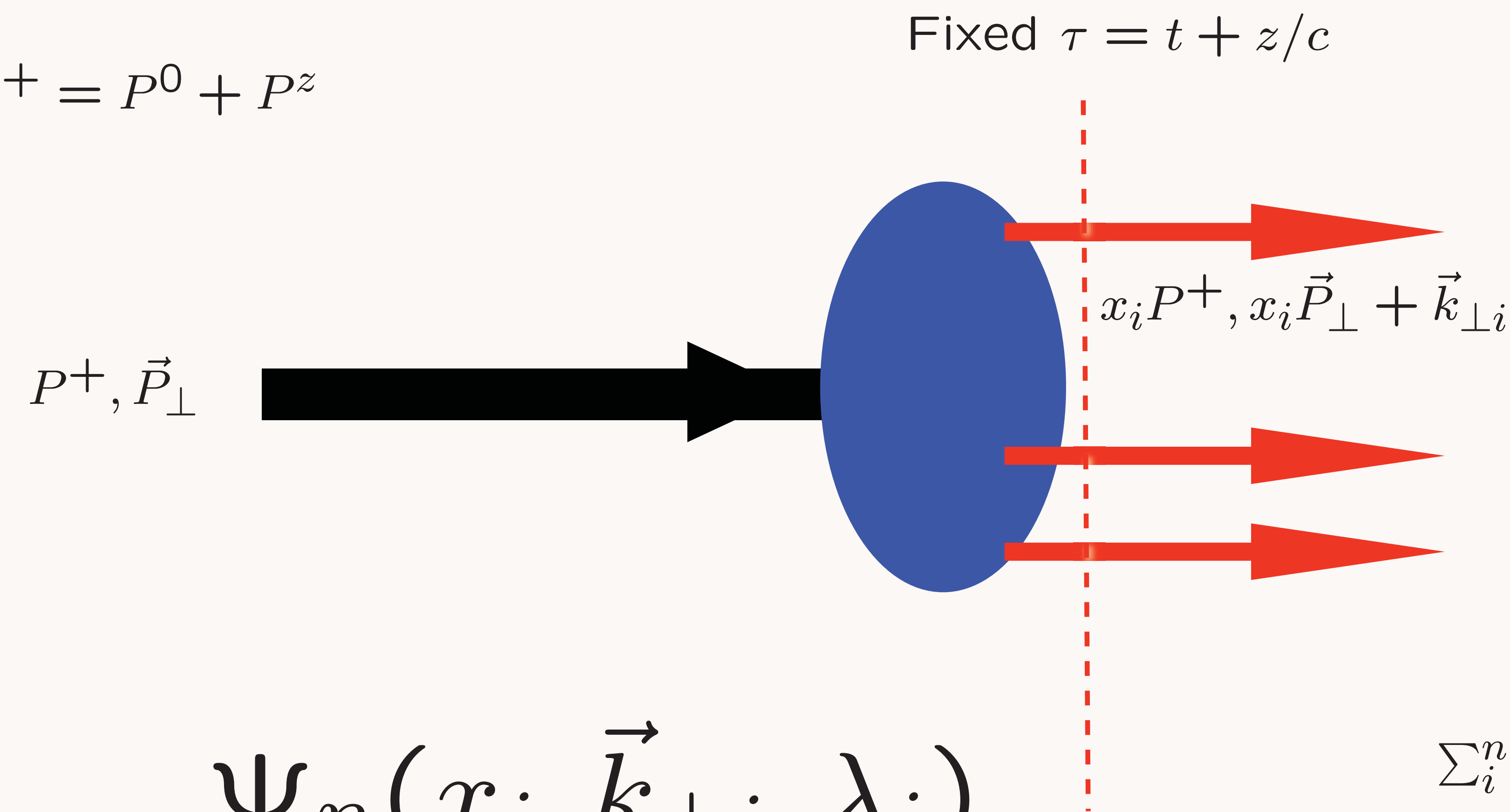
‘Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
‘tis an immortal thing

Montgomery

Light-Front Wavefunctions

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State!

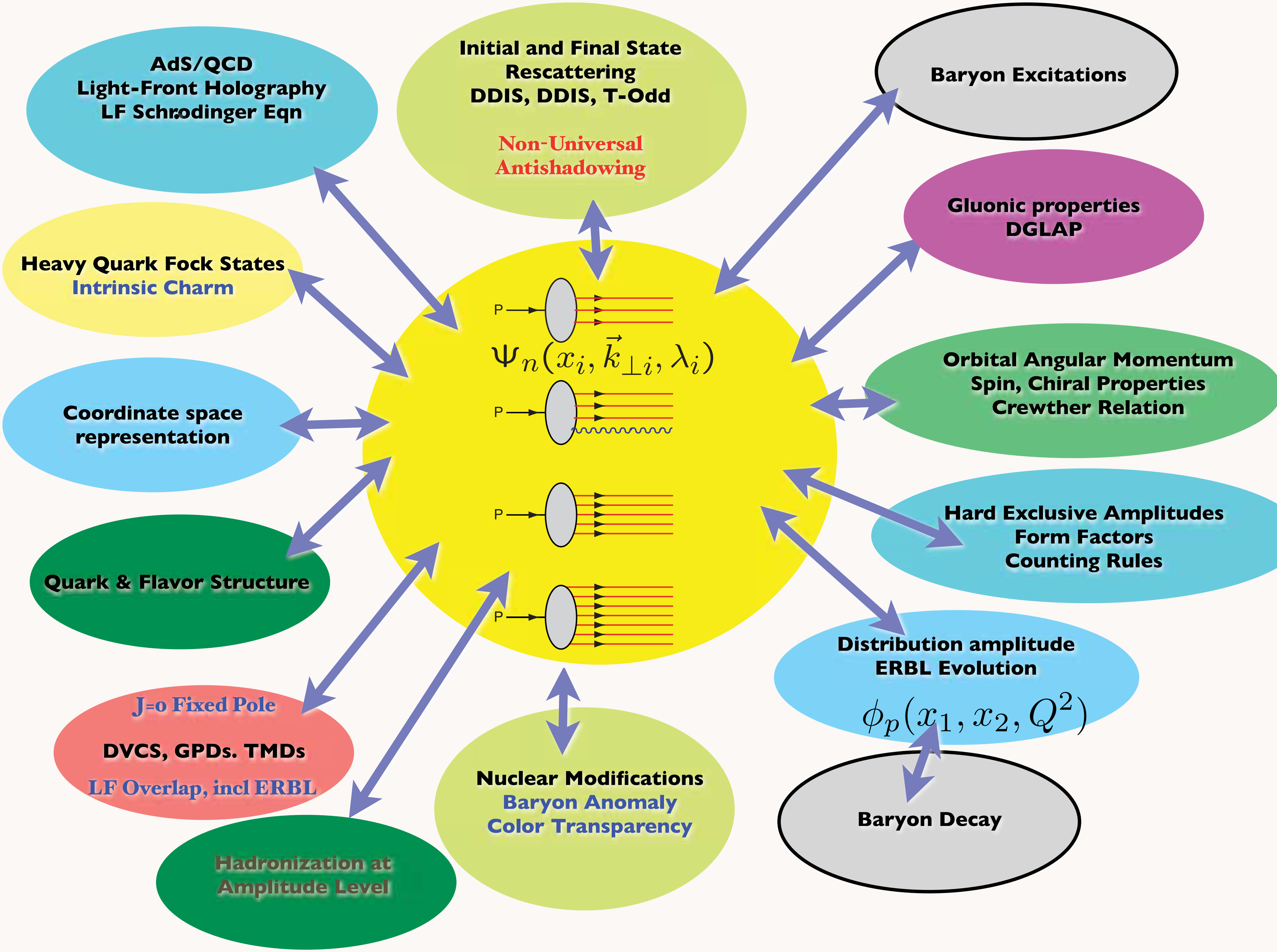
LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

QCD and the LF Hadron Wavefunctions



Light-Front QCD

Heisenberg Matrix Formulation

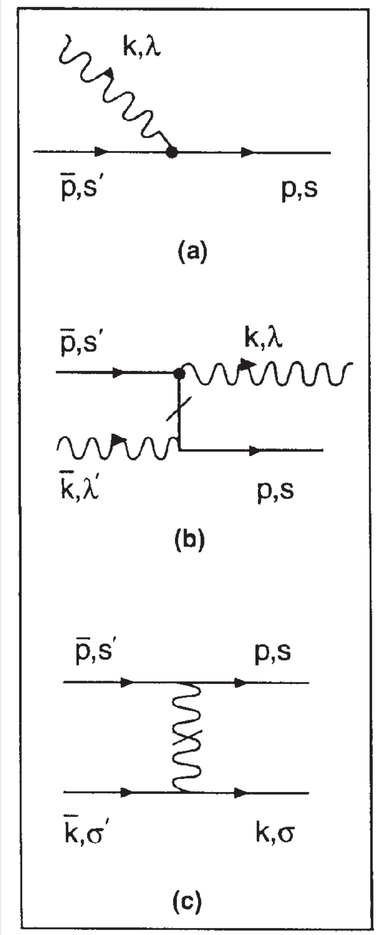
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h >= \mathcal{M}_h^2 |\Psi_h >$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

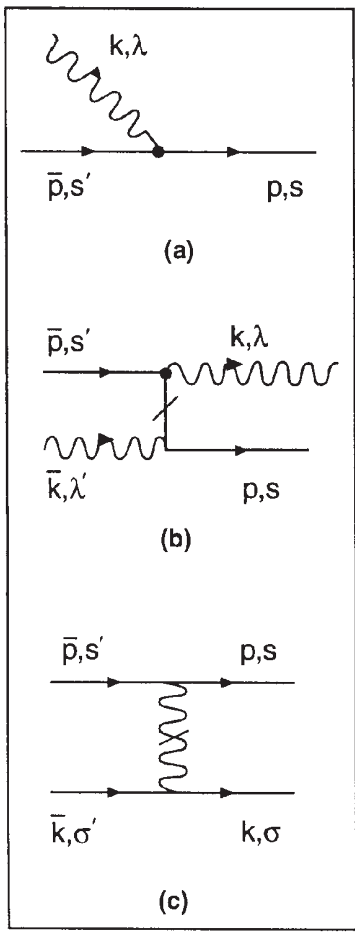
Light-Front QCD

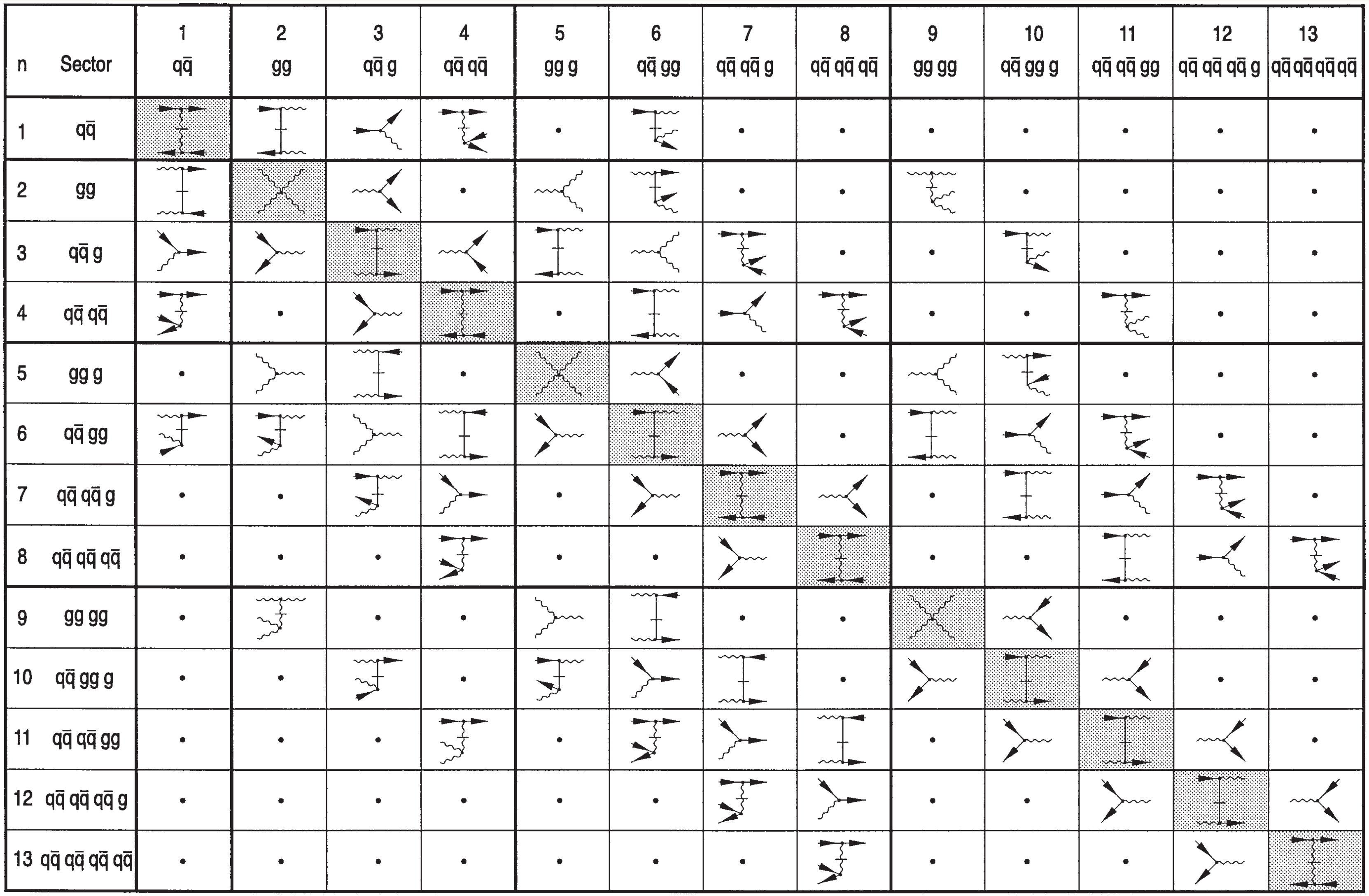
Heisenberg Matrix Formulation

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization



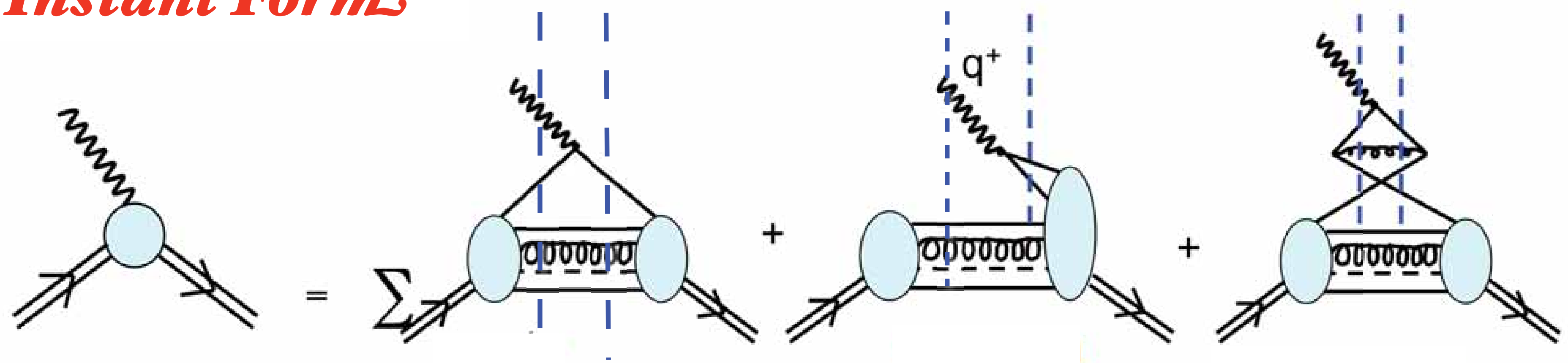


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Hans Christian Pauli & sjb

Calculation of Form Factors in Equal-Time Theory

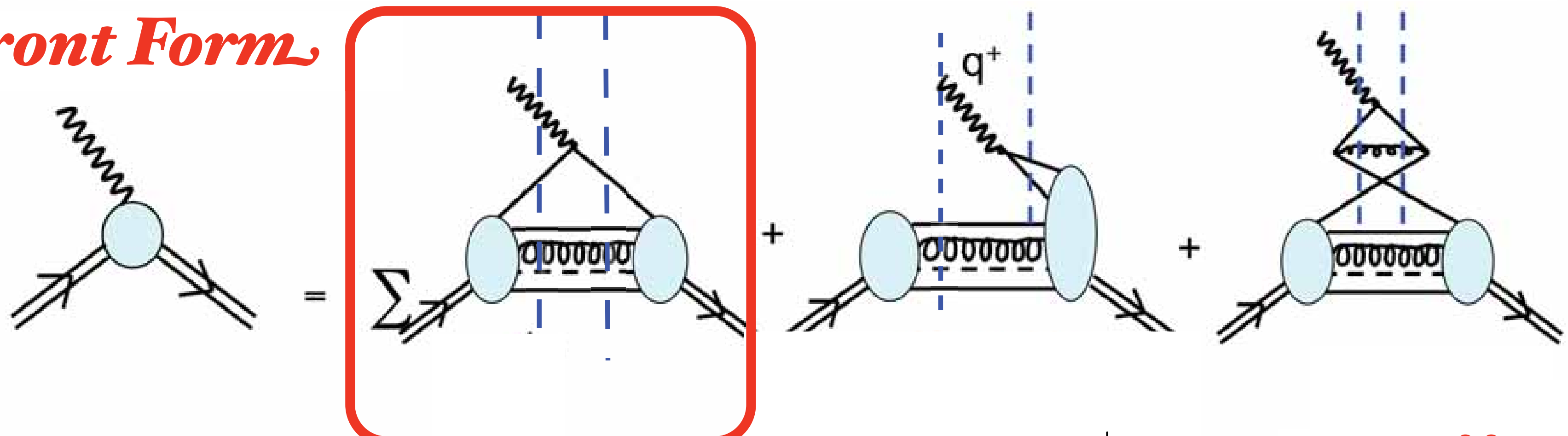
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



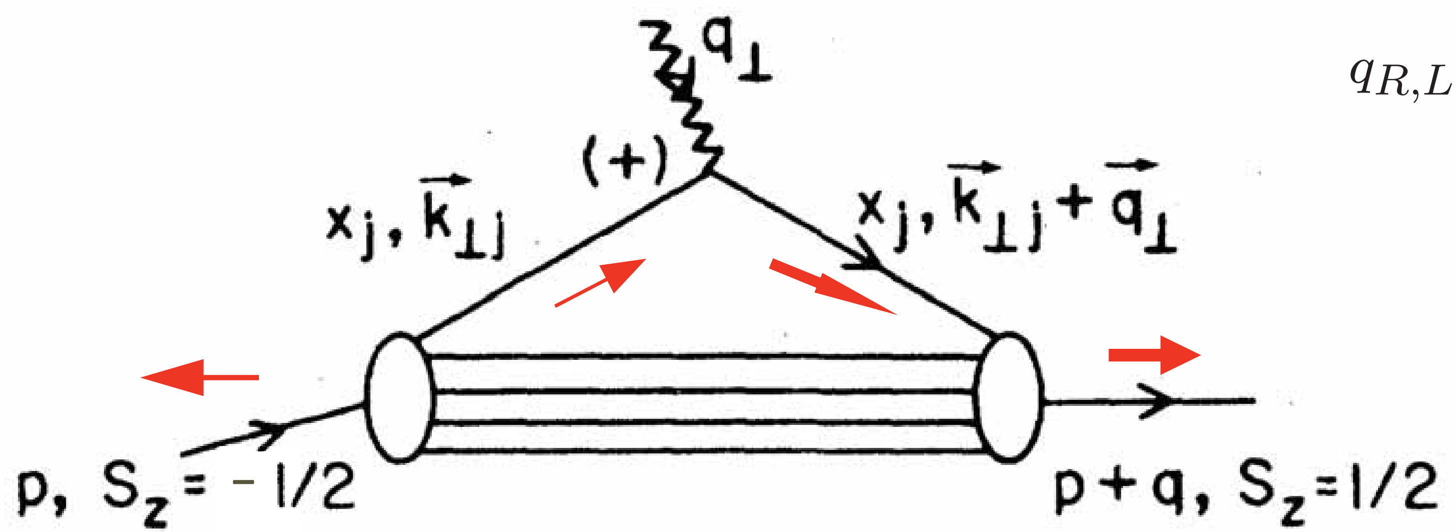
Absent for $q^+ = 0$ **zero !!**

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



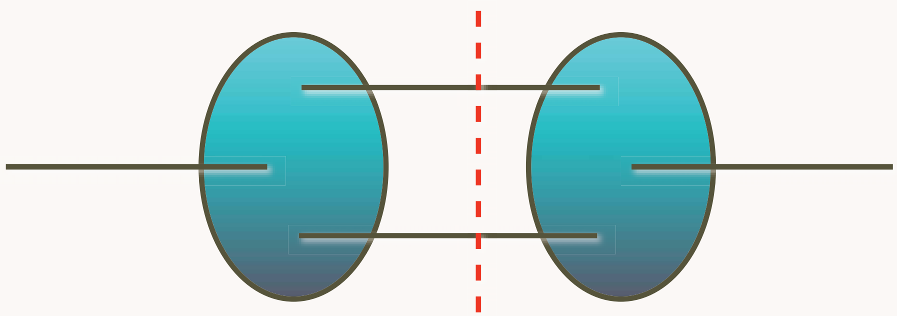
$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

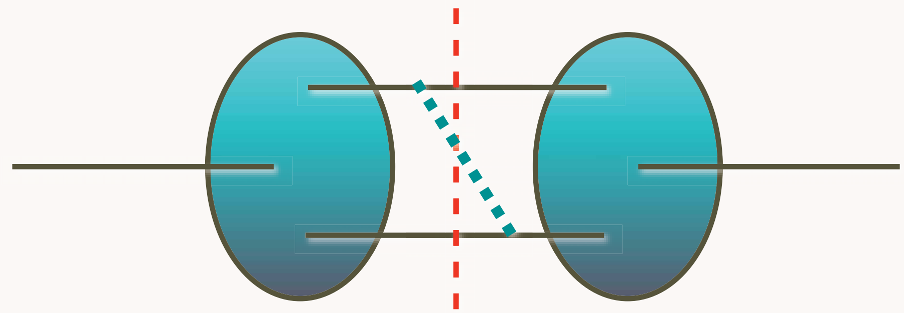
Quantum Mechanics: Uncertainty in p, x, spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



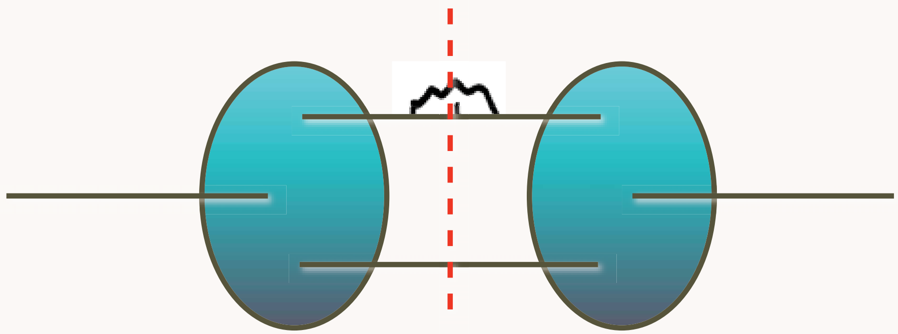
Positronium n=2

e^+e^-



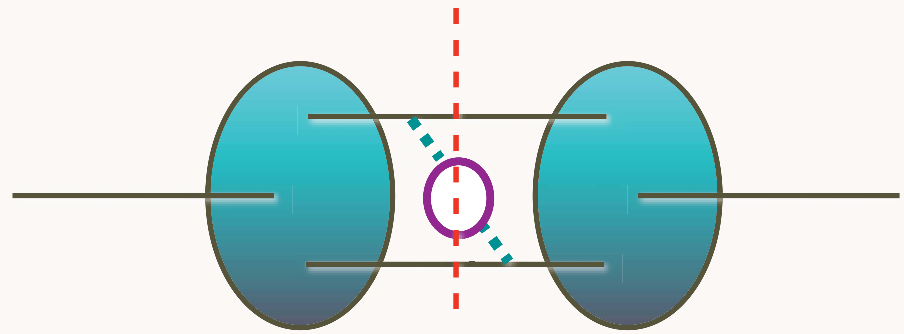
Hyperfine splitting n=3

$e^+e^-\gamma$



Lamb Shift n=3

$e^+e^-\gamma$



Vacuum Polarization n=4

$e^+e^-e^+e^-$

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

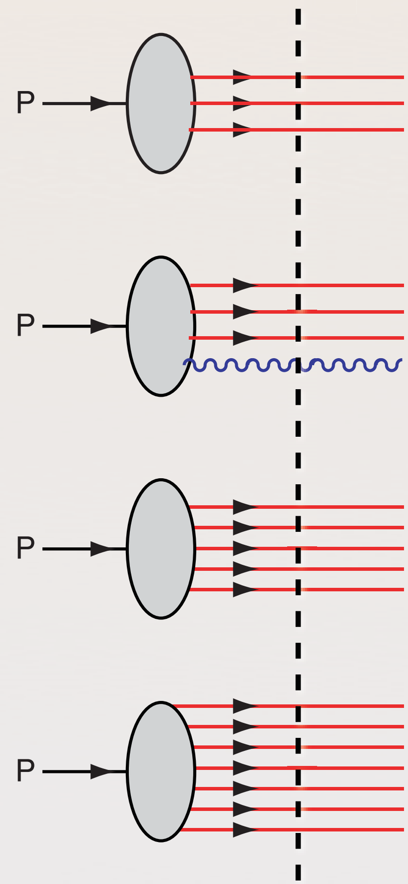
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \; \sum_i^n x_i = 1, \; \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks

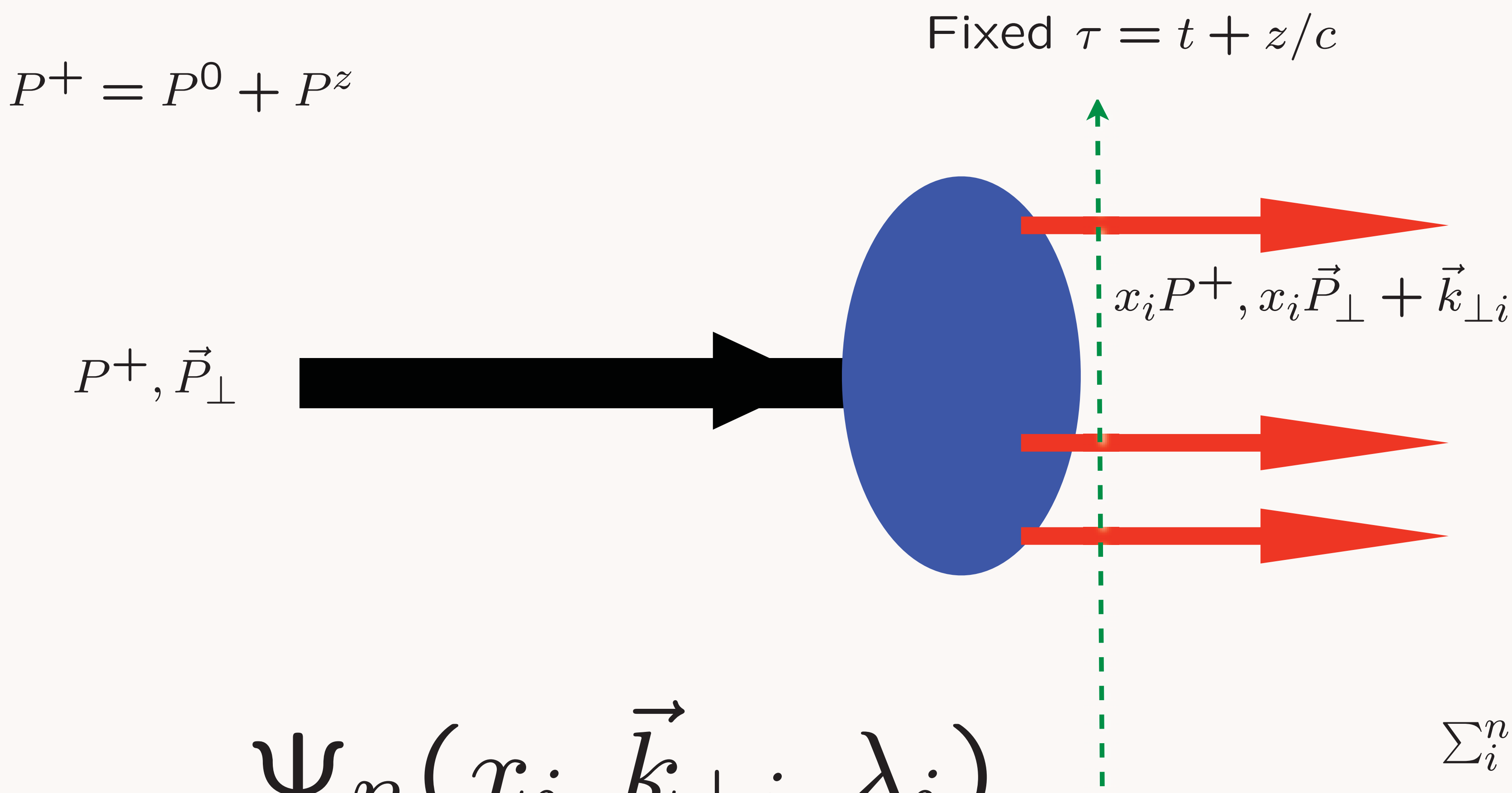
$c(x), b(x)$ at high x

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

AdS/QCD

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Single-spin asymmetries

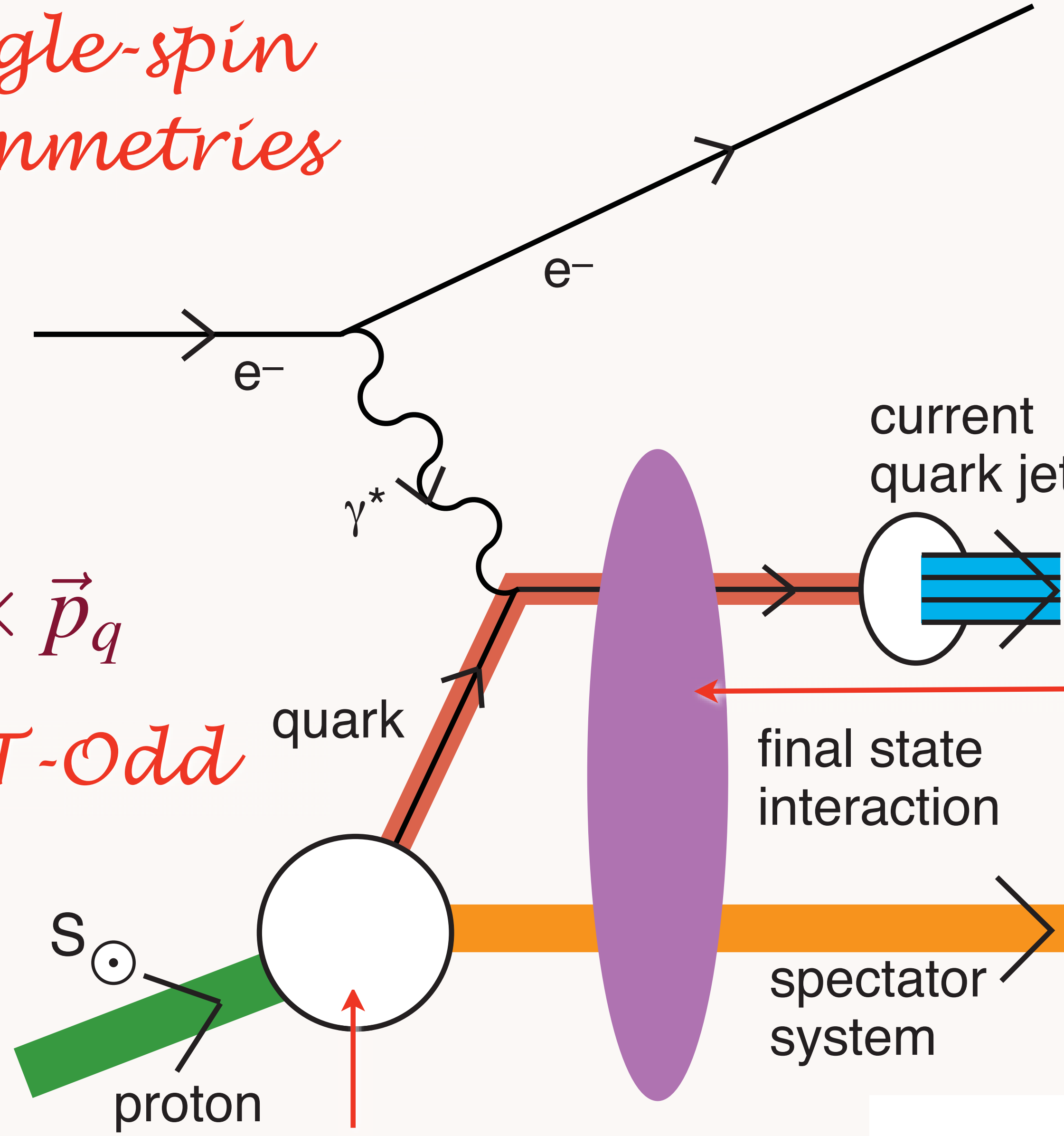
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$\mathrm{i}\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

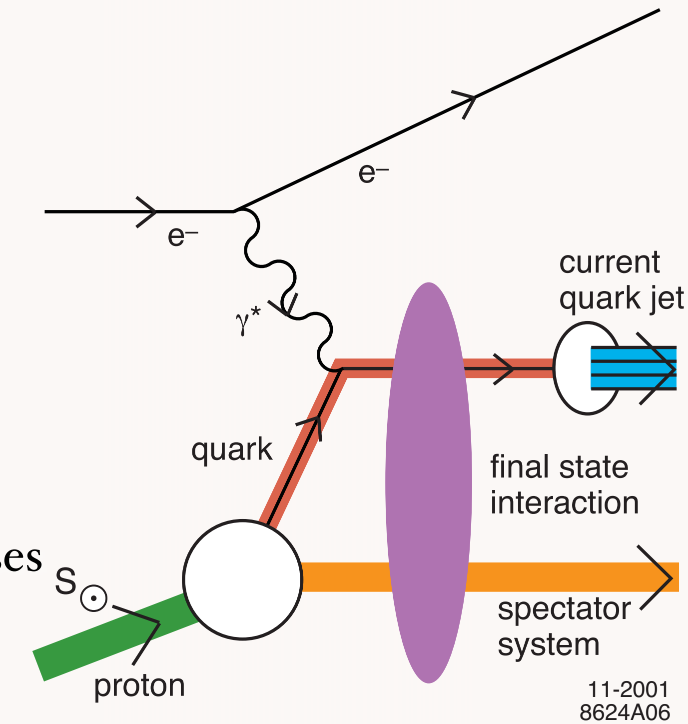


QCD S- and P-Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

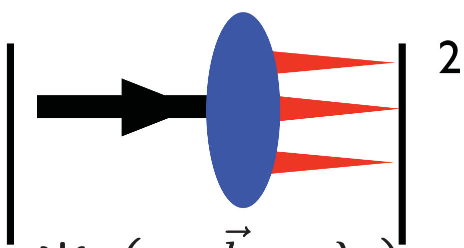
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

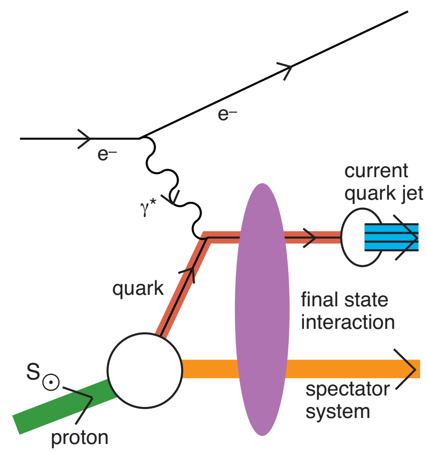


- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!

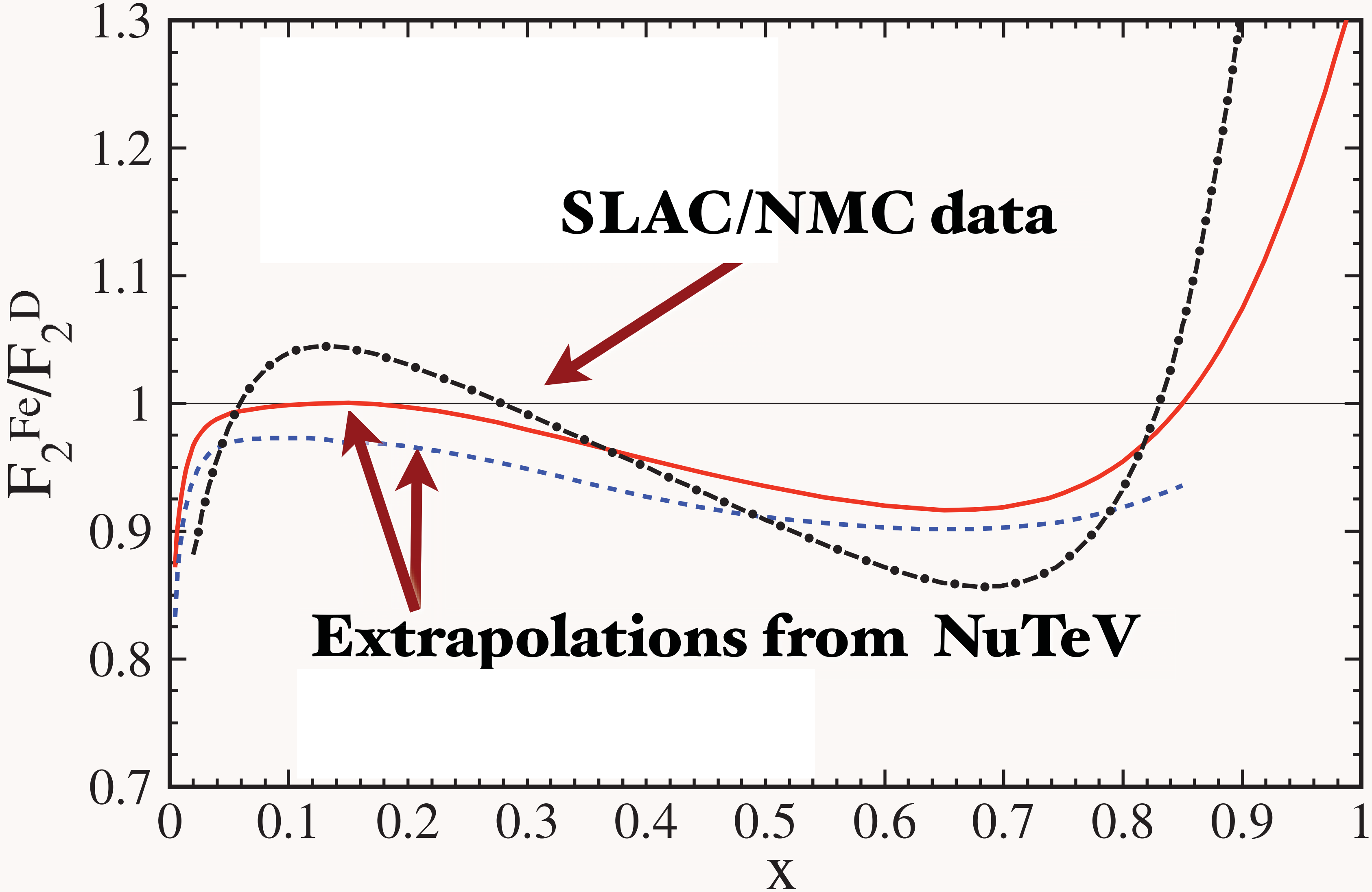
Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS



$\left|\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$



$Q^2 = 5 \text{ GeV}^2$



Scheinbein, Yu, Keppel, Morfin, Olness, Owens

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Heisenberg Matrix Equation for QCD

New Perspectives in QCD from AdS/CFT

- Need to understand QCD at the Amplitude Level: Hadron wavefunctions!
- Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

New Way to Solve QCD: AdS/CFT

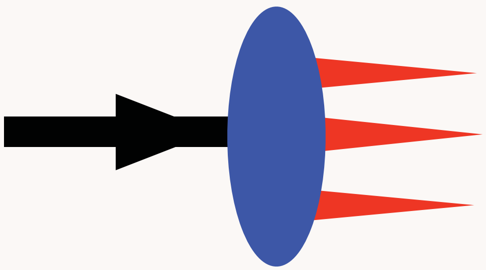
- Maldacena Correspondence
- Mathematical Representation of Lorentz Invariant and Conformal (Scale-Free) Theories
- Add new 5th space dimension to 3+1 space-time
- Holographic Model with Color Confinement and Quark Counting Rules

de Teramond, sjb

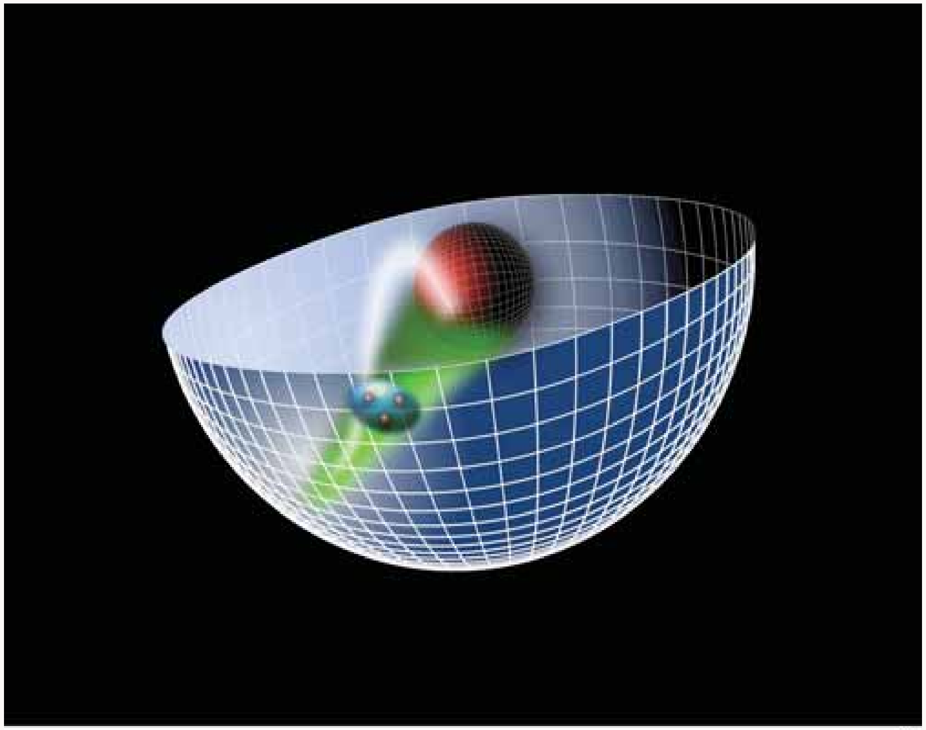
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

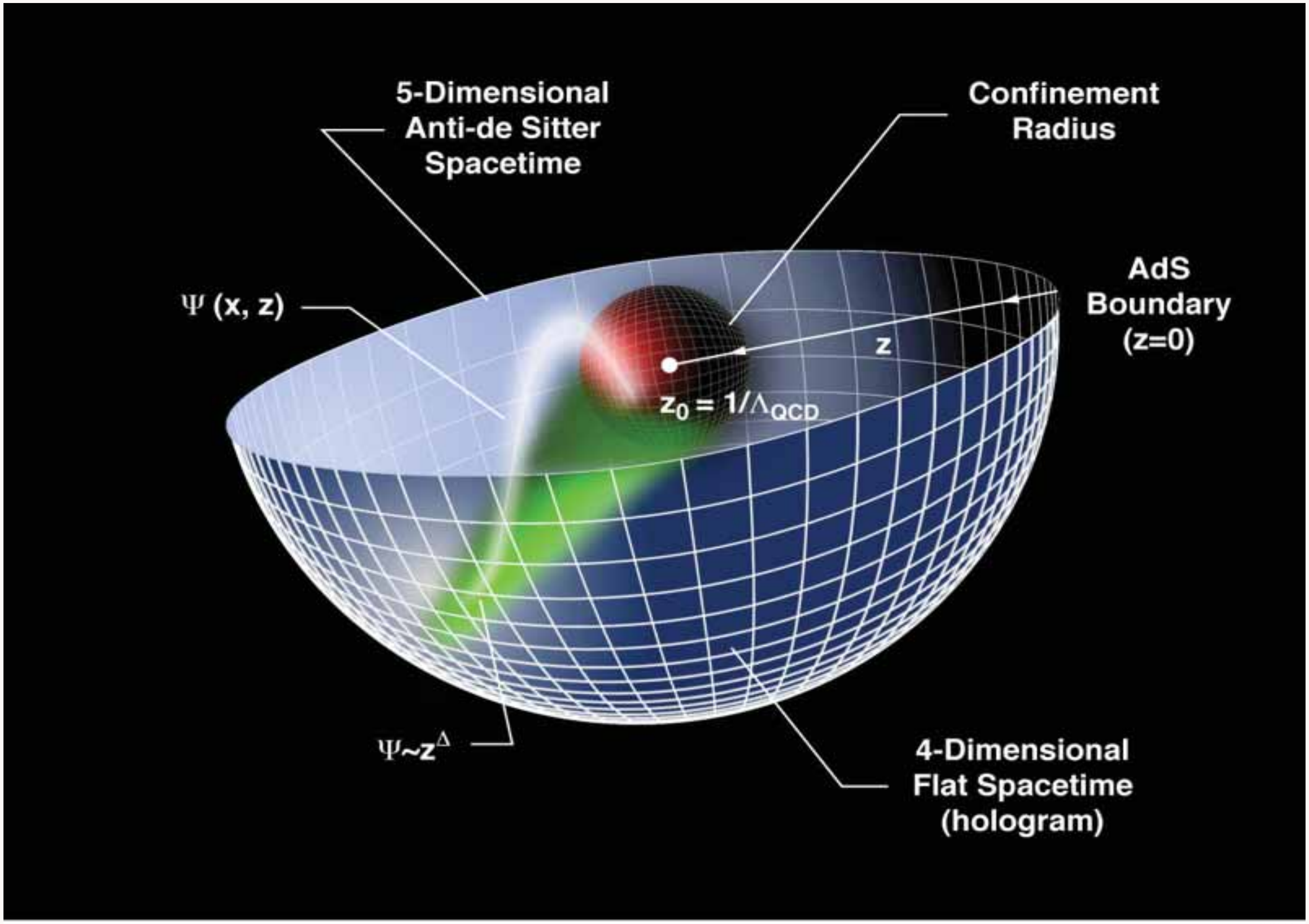
Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

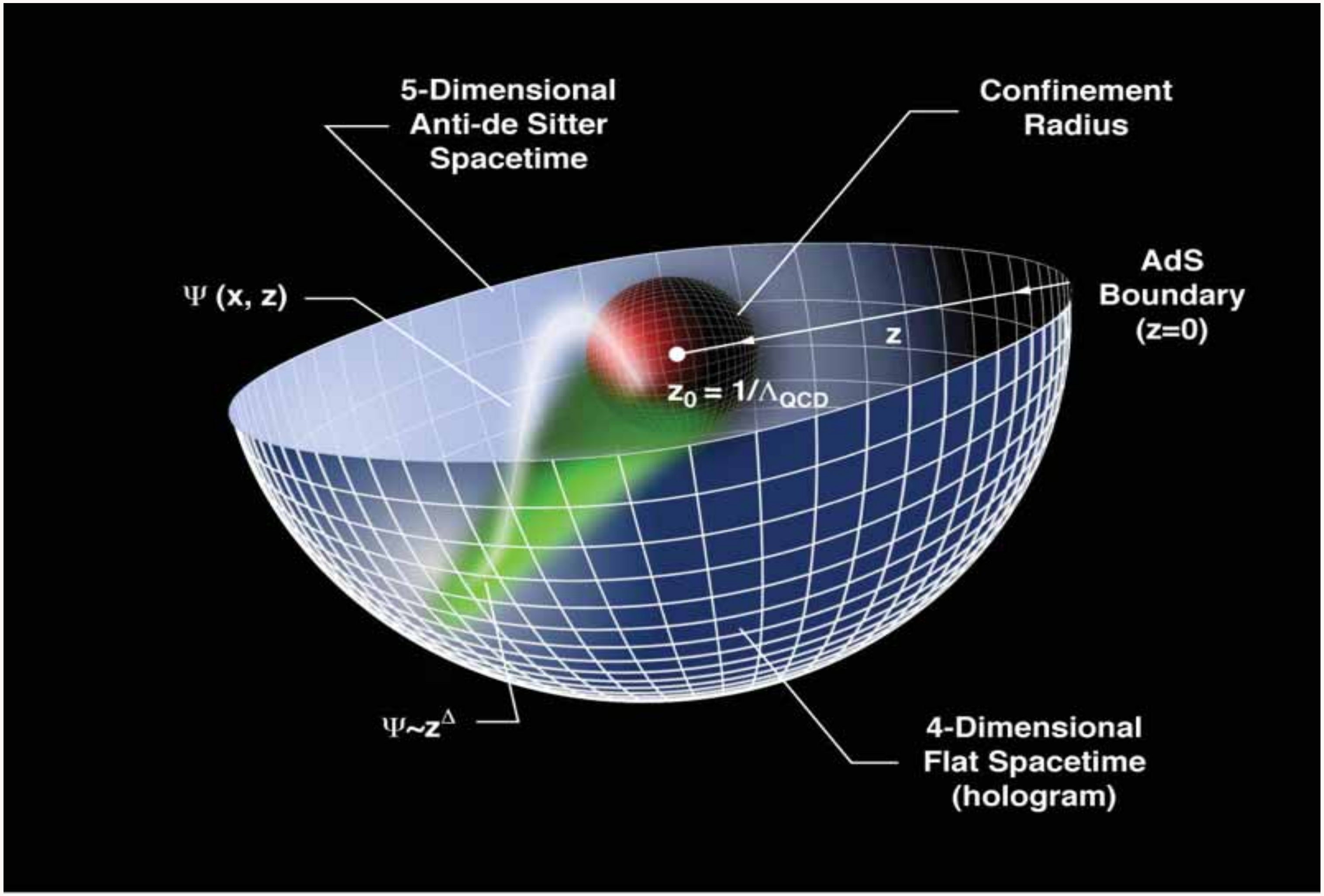
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Applications of AdS/CFT to QCD

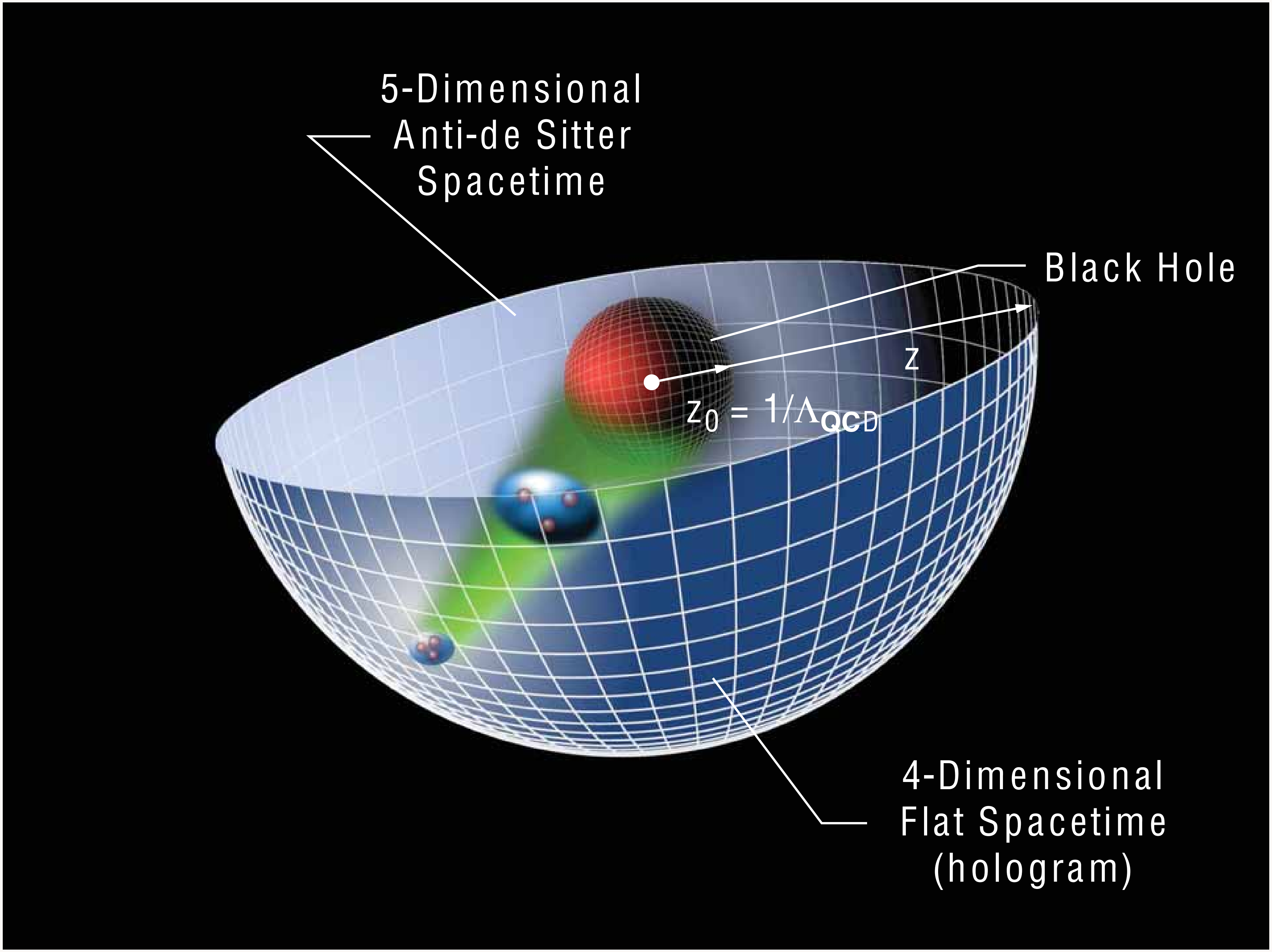


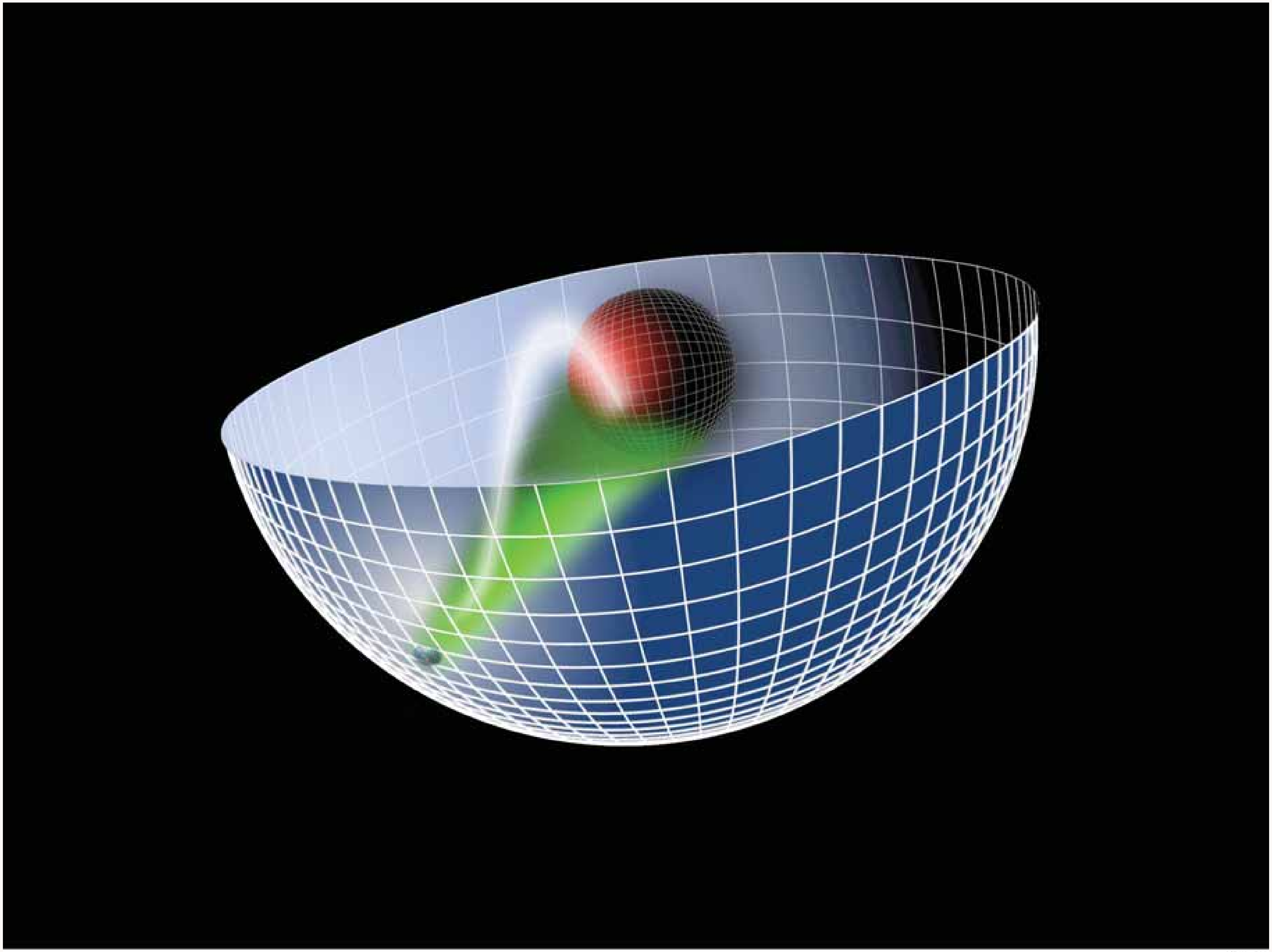
Changes in physical length scale mapped to evolution in the 5th dimension z

String Theory

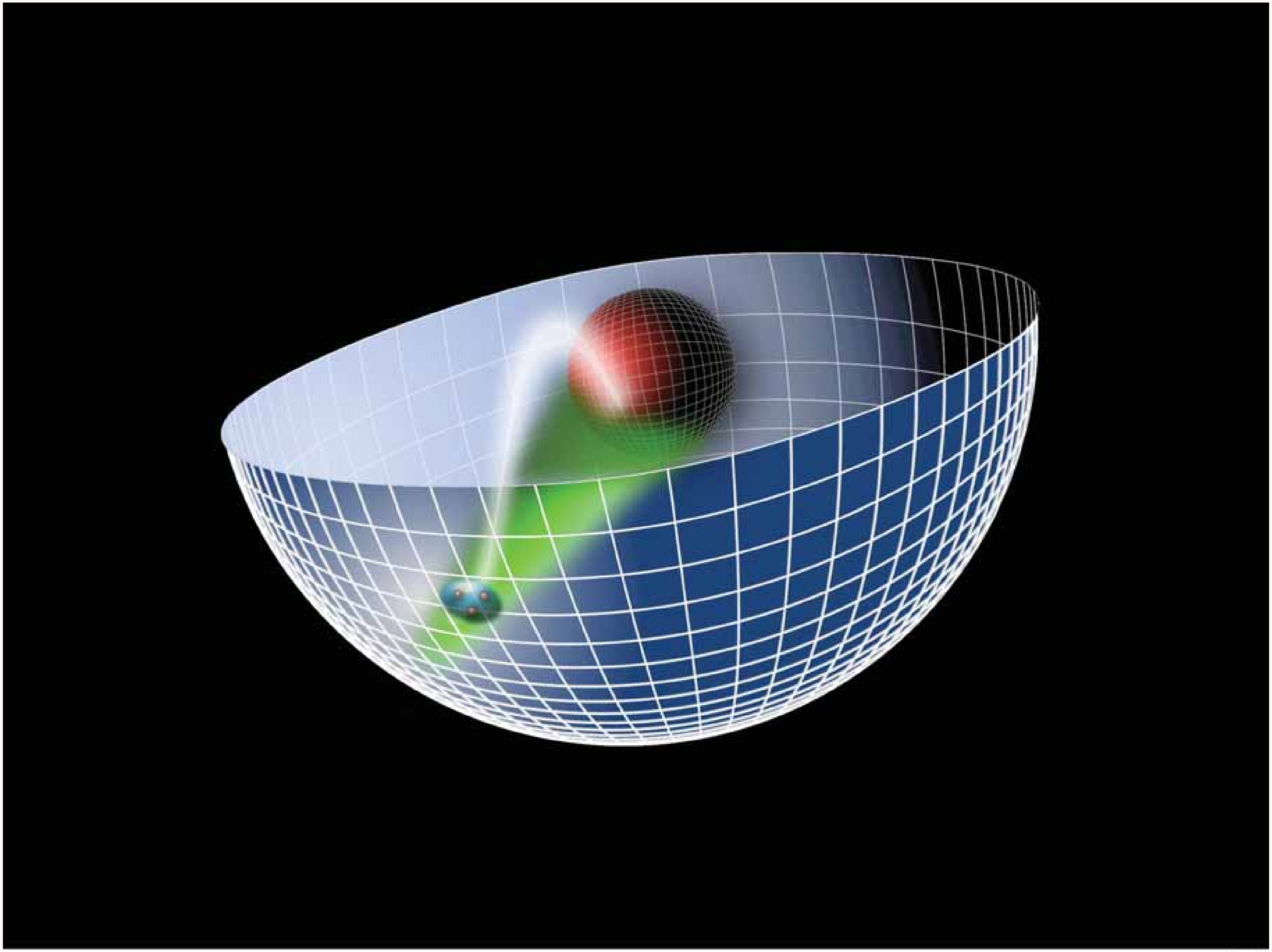
Bottom-Up

Top-Down

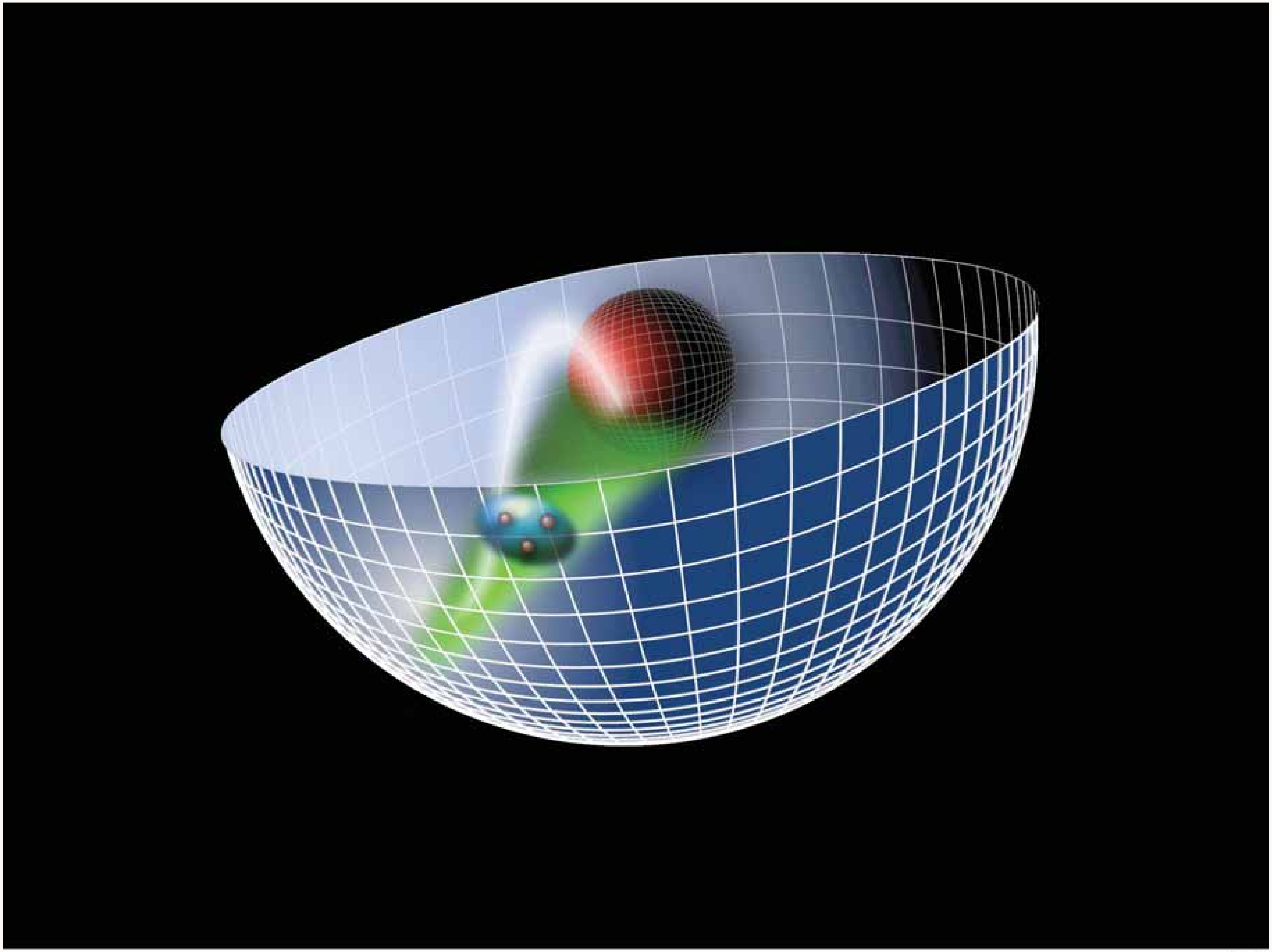

5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)



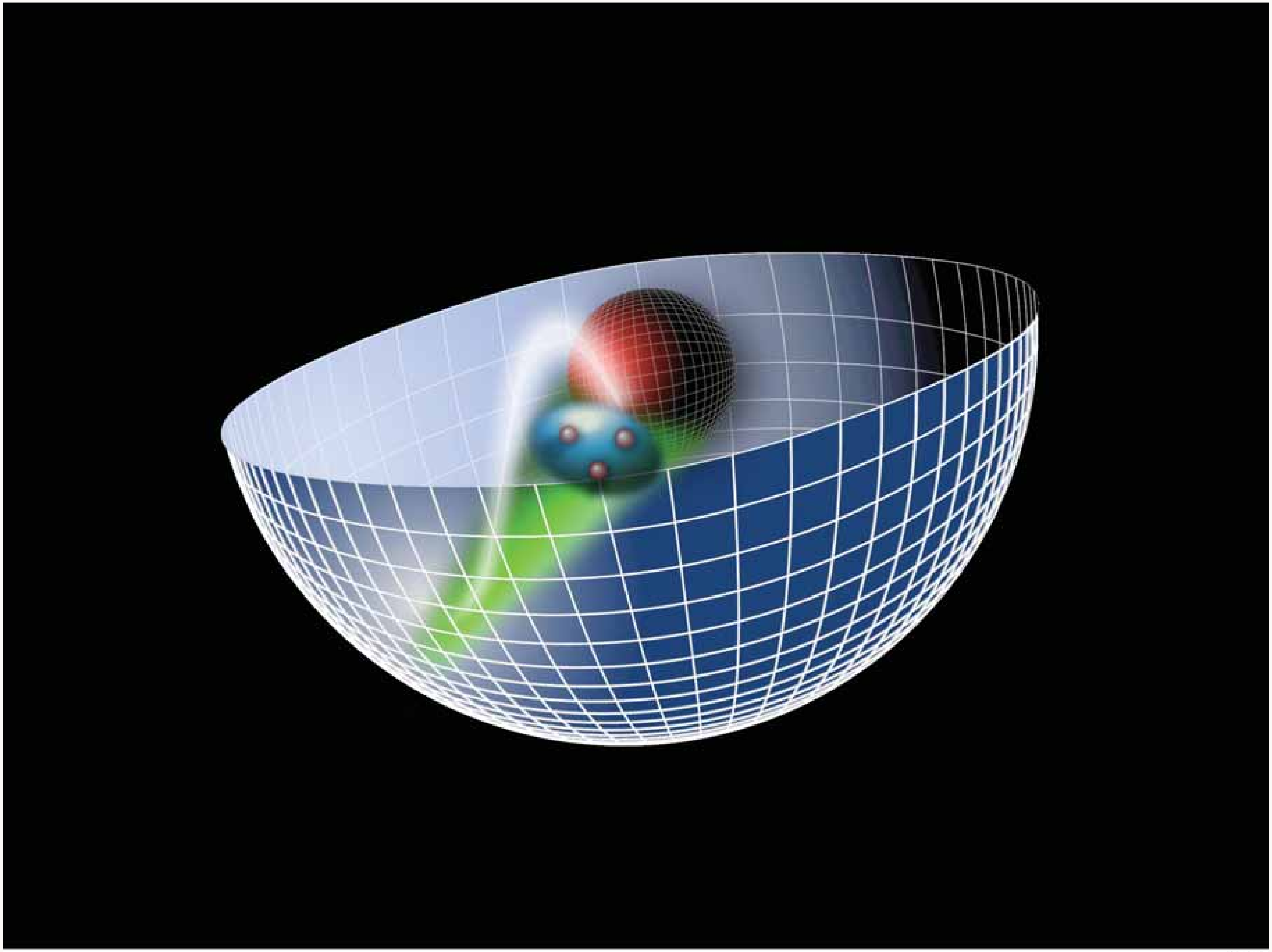
AdS/QCD

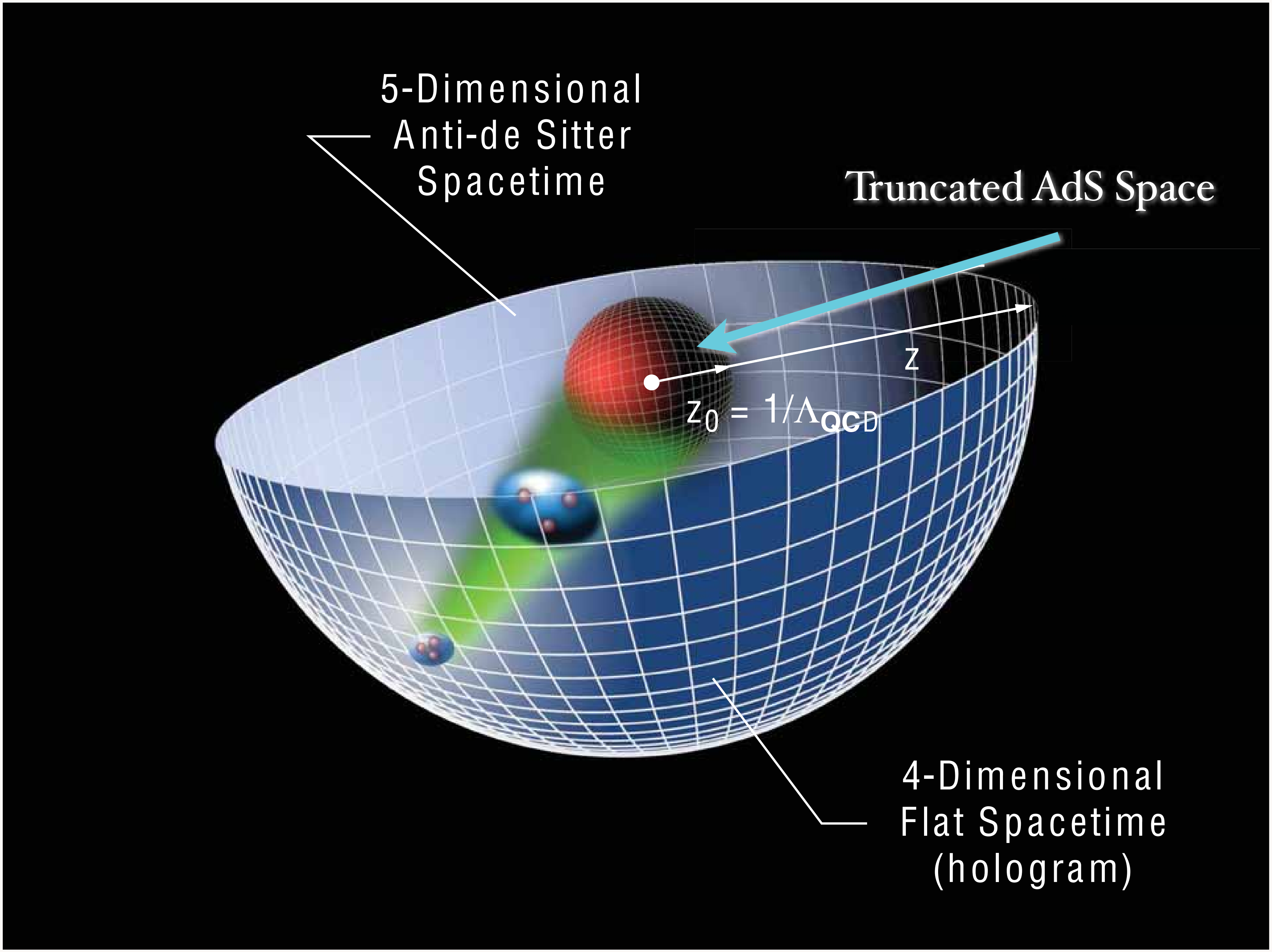


AdS/QCD



AdS/QCD




5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial "classical" approximation to QCD: Remarkable agreement with light hadron spectroscopy

Guy de Teramond, sjb

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \rightarrow \lambda x^\mu, \; z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z.

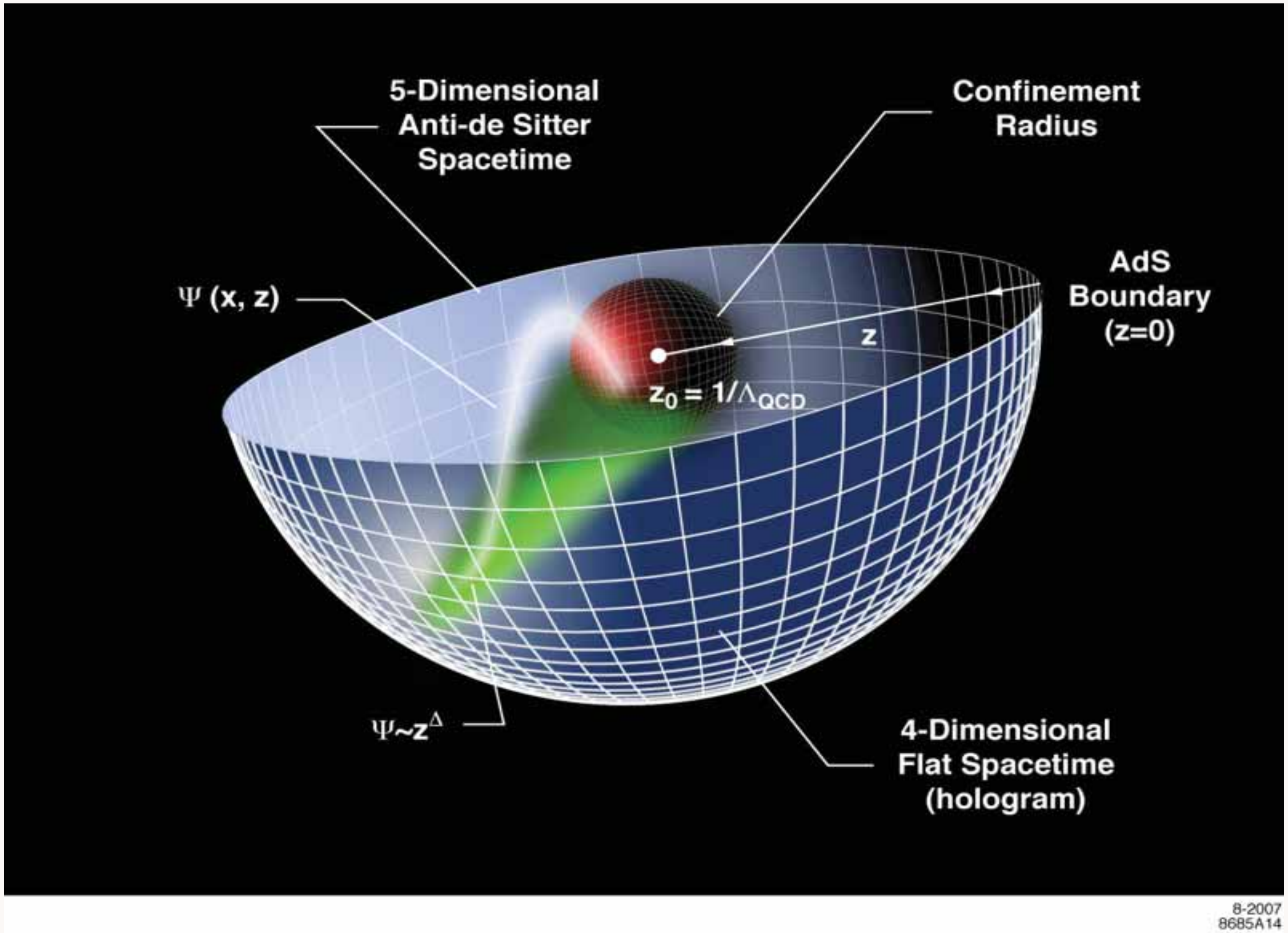
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

We will consider both holographic models

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

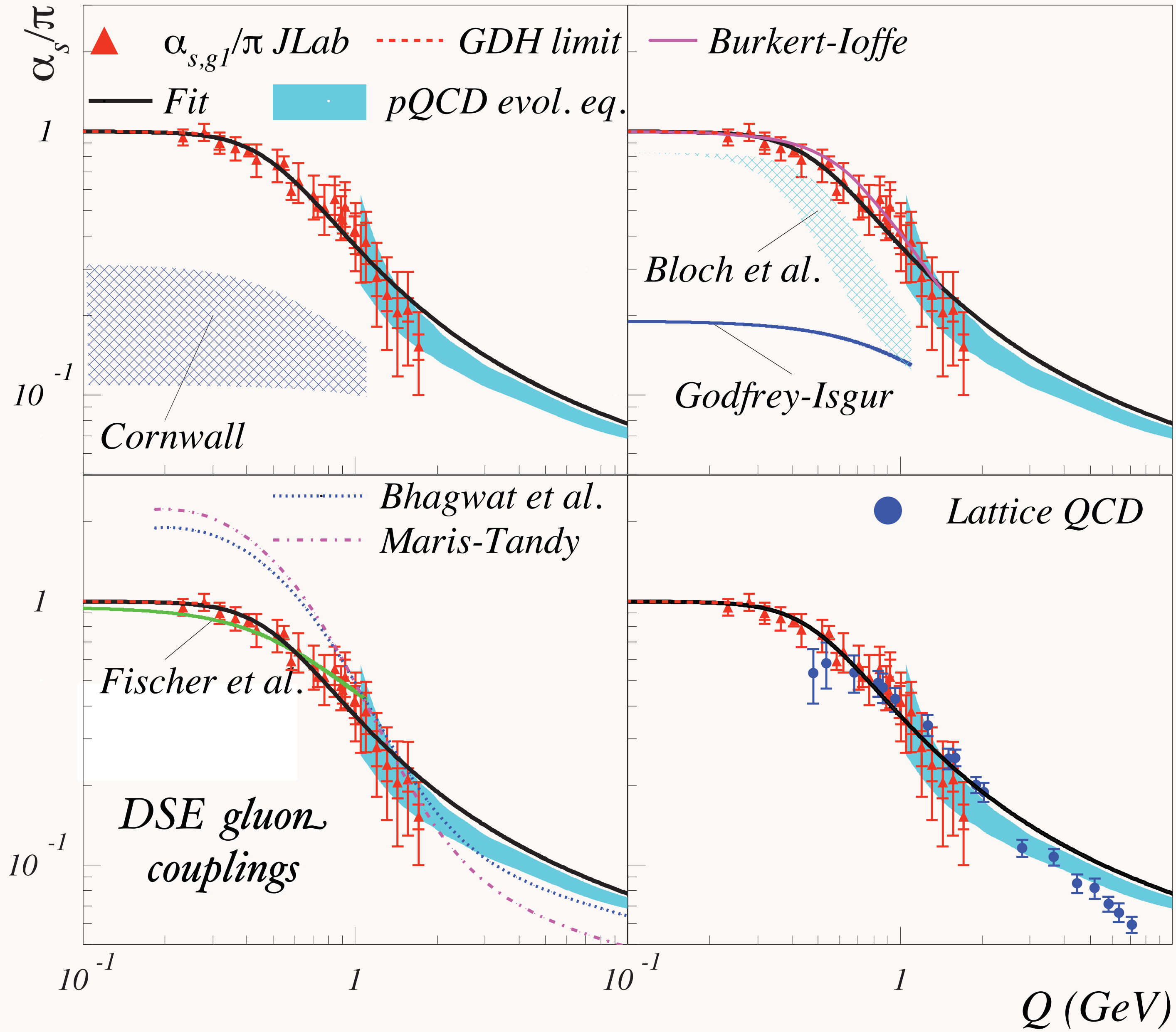
Map $AdS_5 \times S_5$ to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Deur, Korsch, et al.



Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain** $(x-y)^2 < \Lambda^{-2}_{QCD}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

Shrock, sjb

J. D. Bjorken, SLAC-PUB 1053 Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

Guy de Teramond
SJB

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

L: light-front orbital angular momentum

Derived from variation of Action in AdS_5

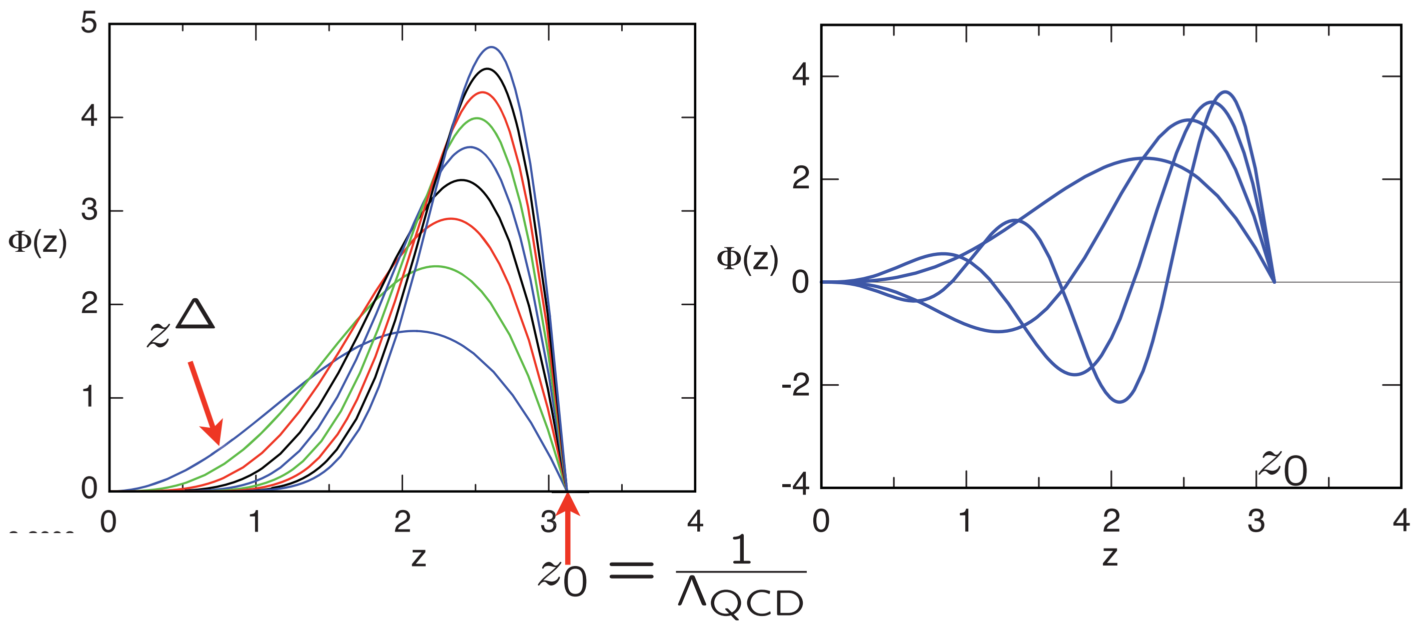
Hard wall model: truncated space

$$\phi(\mathbf{z} = \mathbf{z_0} = \frac{1}{\Lambda_c}) = 0.$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

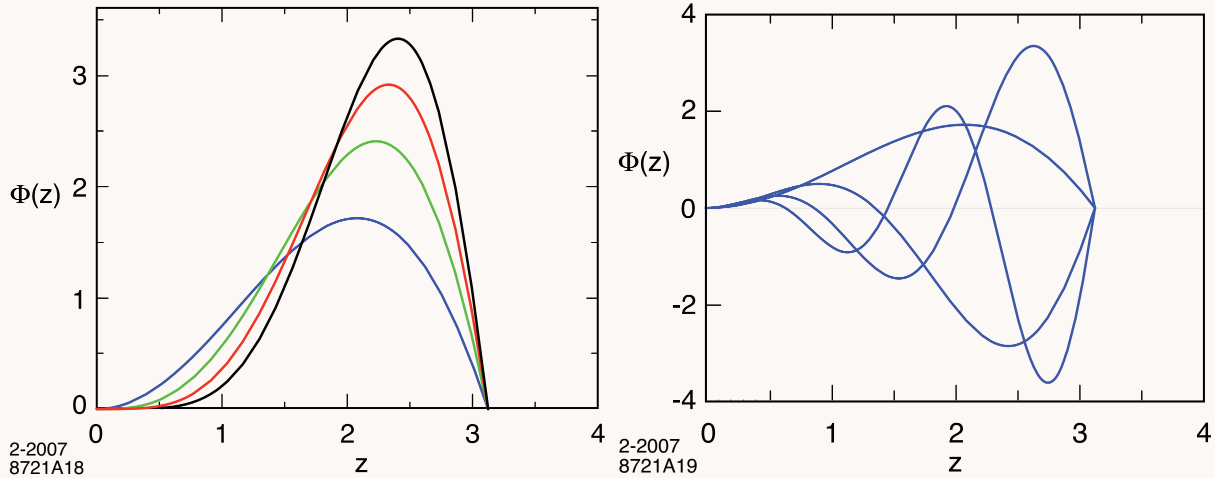


Fig: Orbital and radial AdS modes in the hard wall model for Λ_{QCD} = 0.32 GeV .

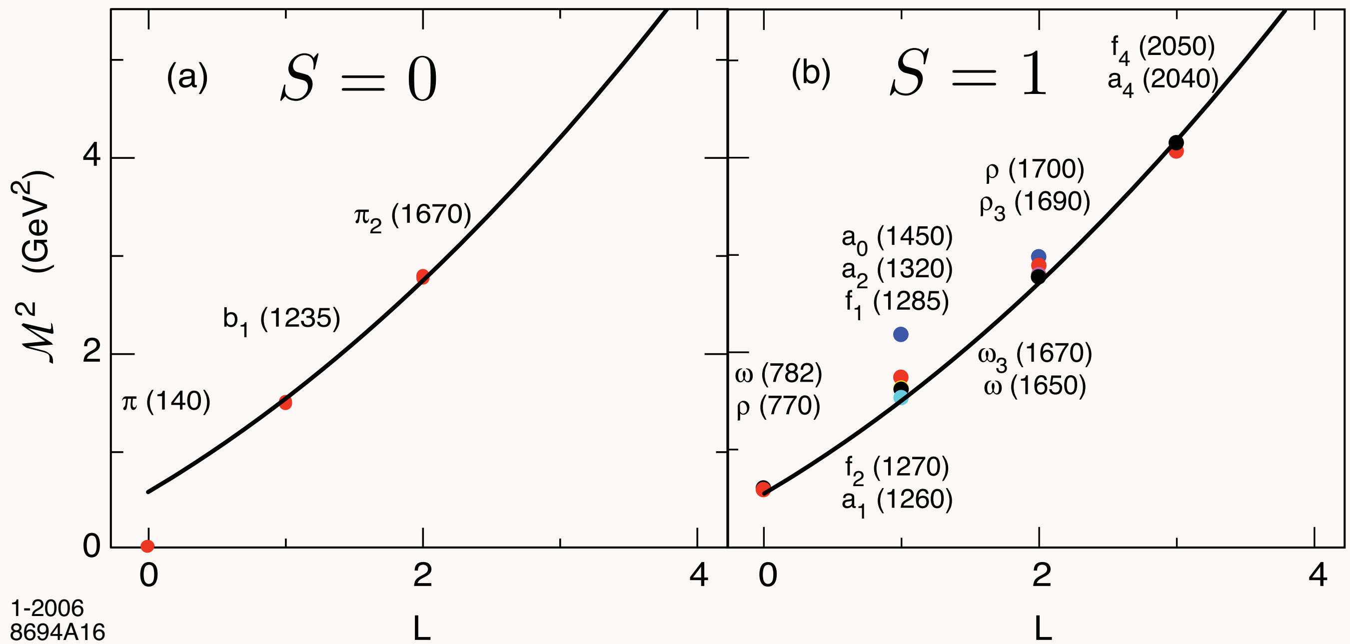


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff wich depends on the profile of a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(d - 1 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$. See also [Metsaev (2002), Andreev (2006)] **+ sign: Fen Zuo(2009**

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$ a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

Massless pion

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

$$\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$$

Same slope in n and L

Derived from variation of Action Dilaton-Modified AdS_5

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Quark separation increases with L

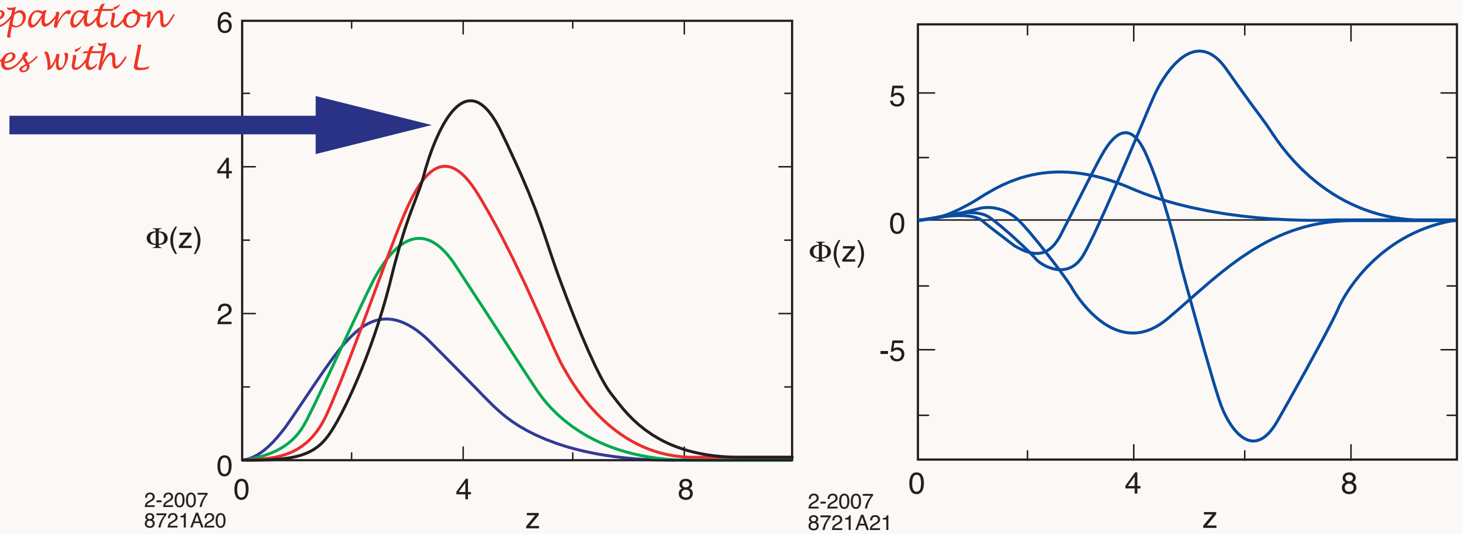
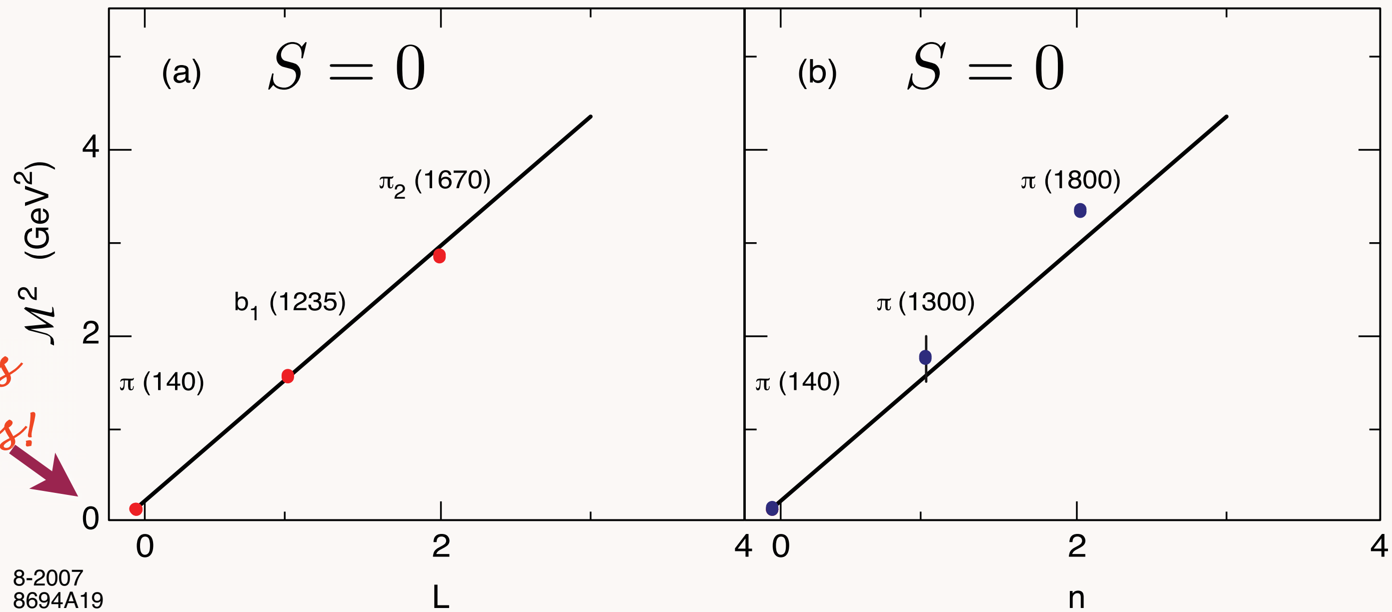


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model



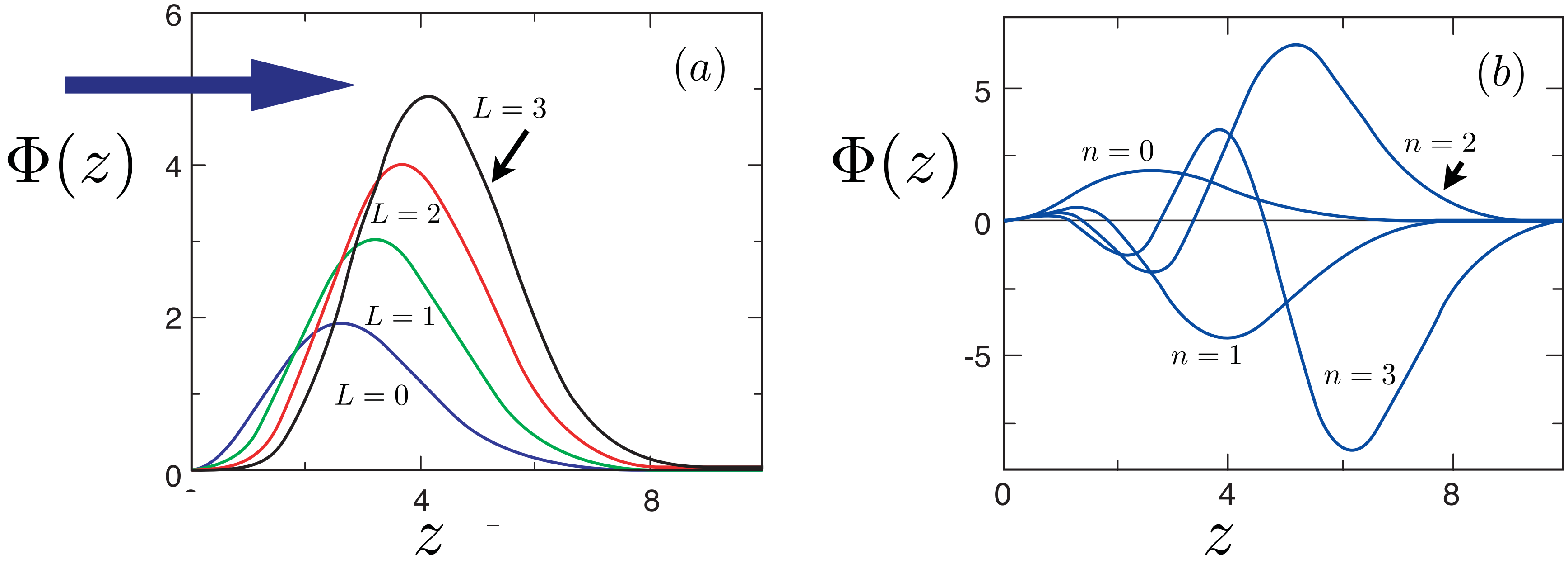
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

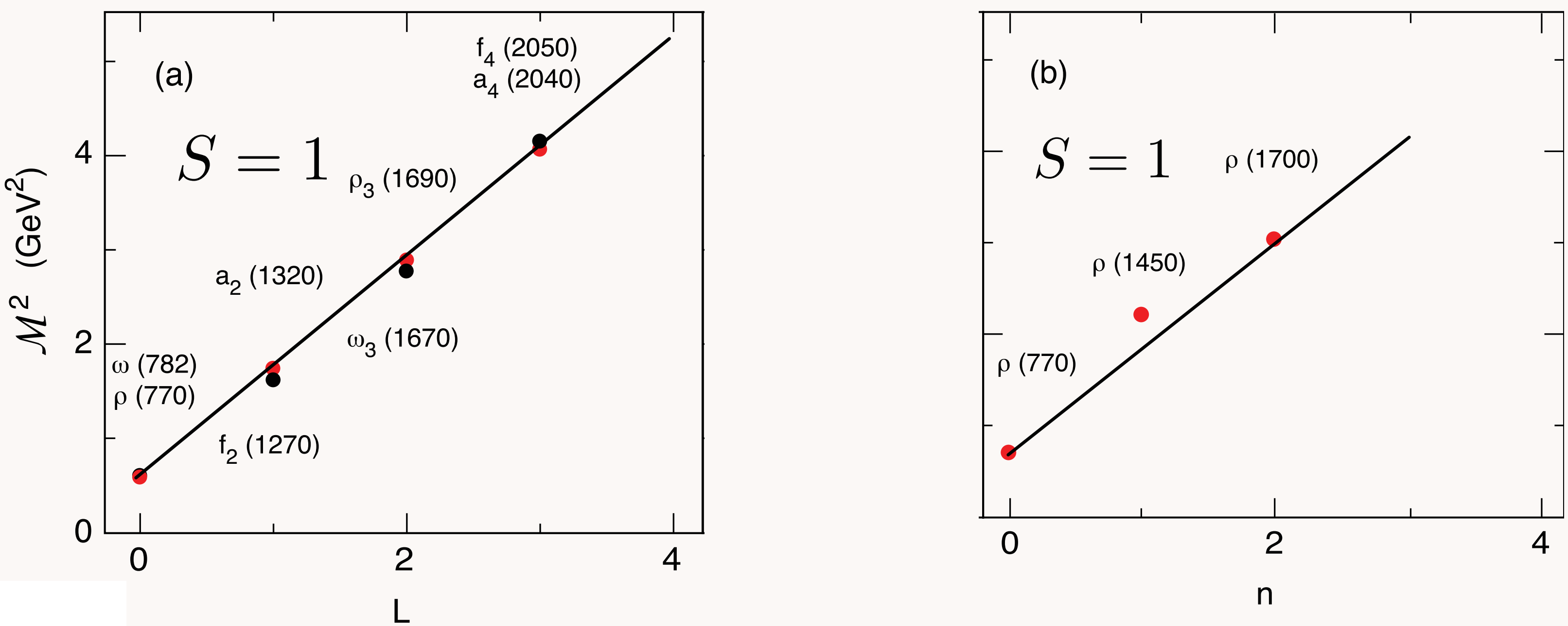
Pion has zero mass!

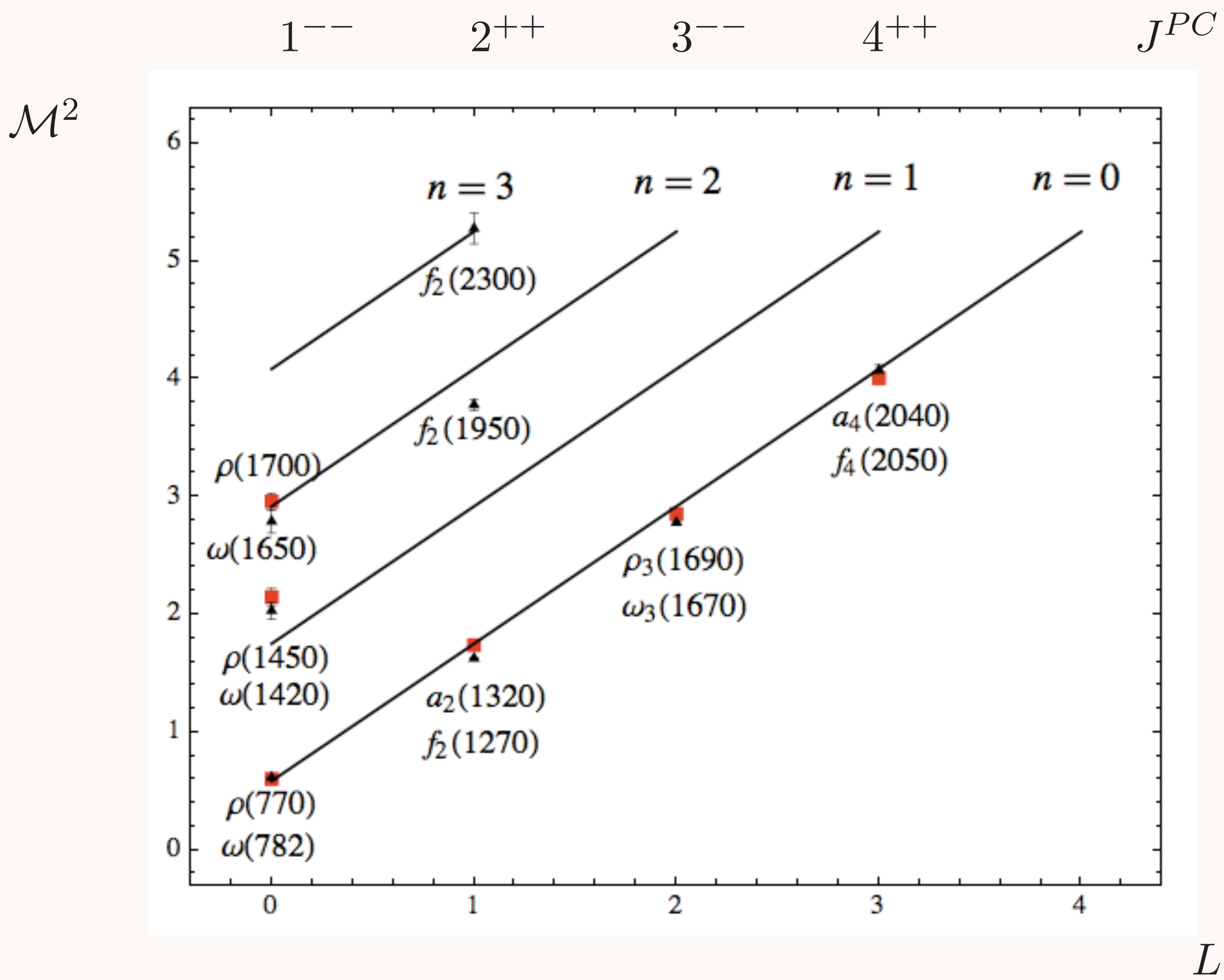
Pion mass automatically zero!

$$m_q = 0$$

Quark separation increases with L







Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red) and the $I = 0$ ω-meson family (black) for $\kappa = 0.54$ GeV

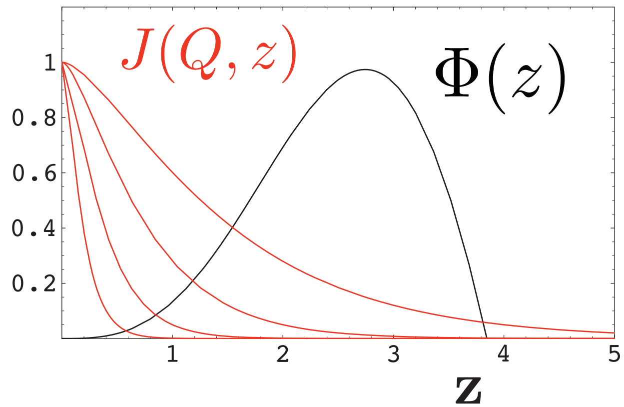
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

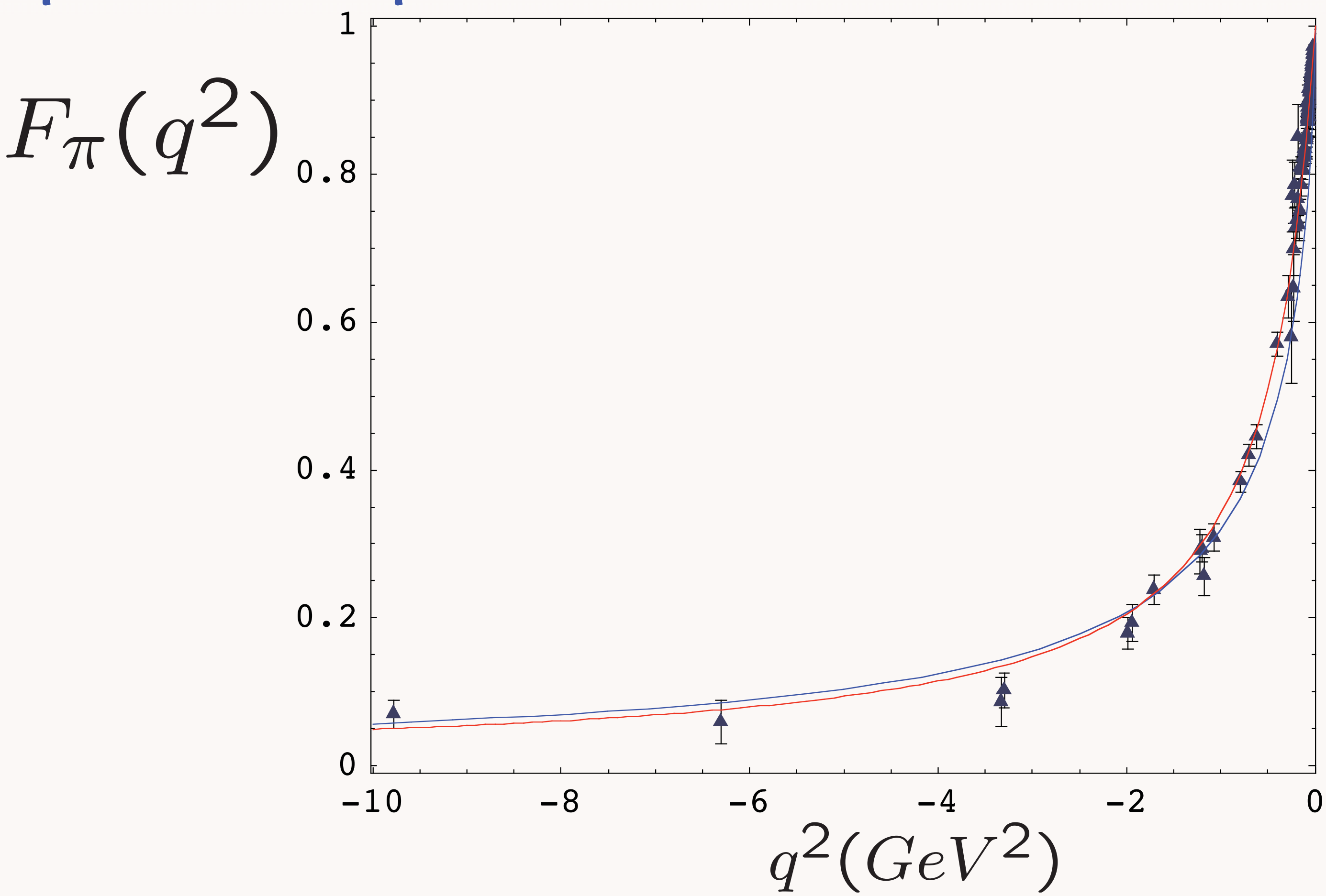
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

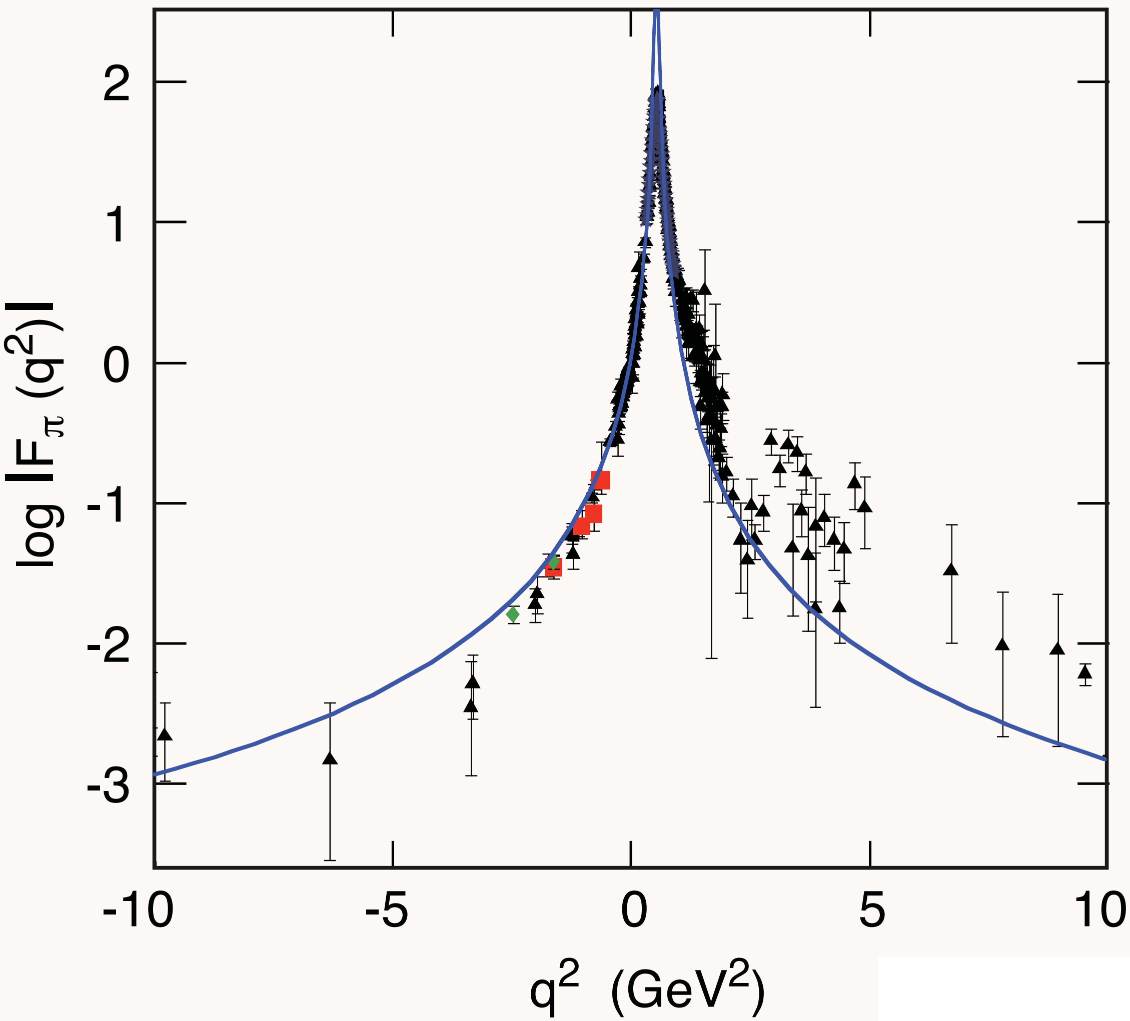
de Teramond, sjb
See also: Radyushkin

AdS/QCD

- Analytical continuation to time-like region $q^2 \to -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp) \, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp \, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp \, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \, |\Phi_{\pi^+}(z)|^2 \, ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

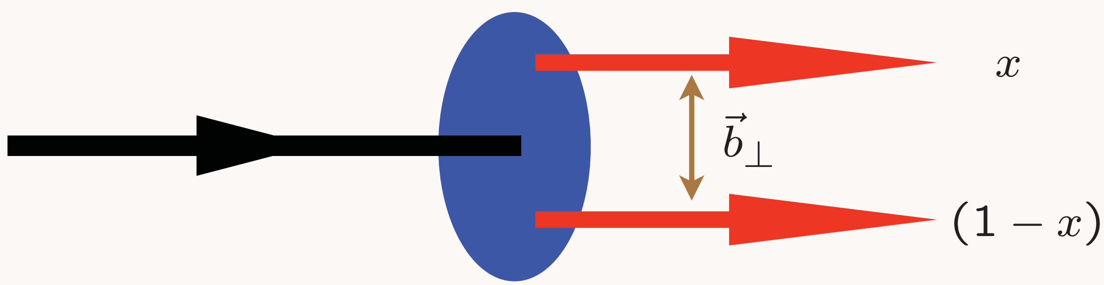
with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \qquad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \qquad z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

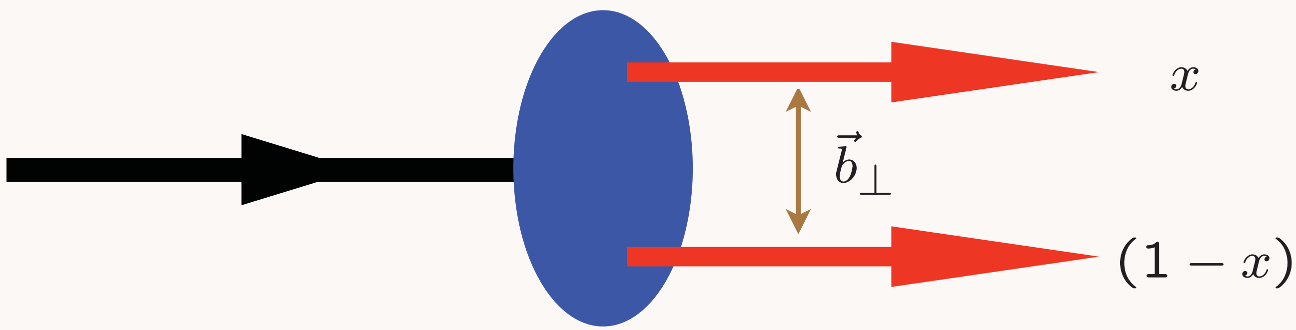
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables

$$(\vec{\zeta},\varphi), \ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

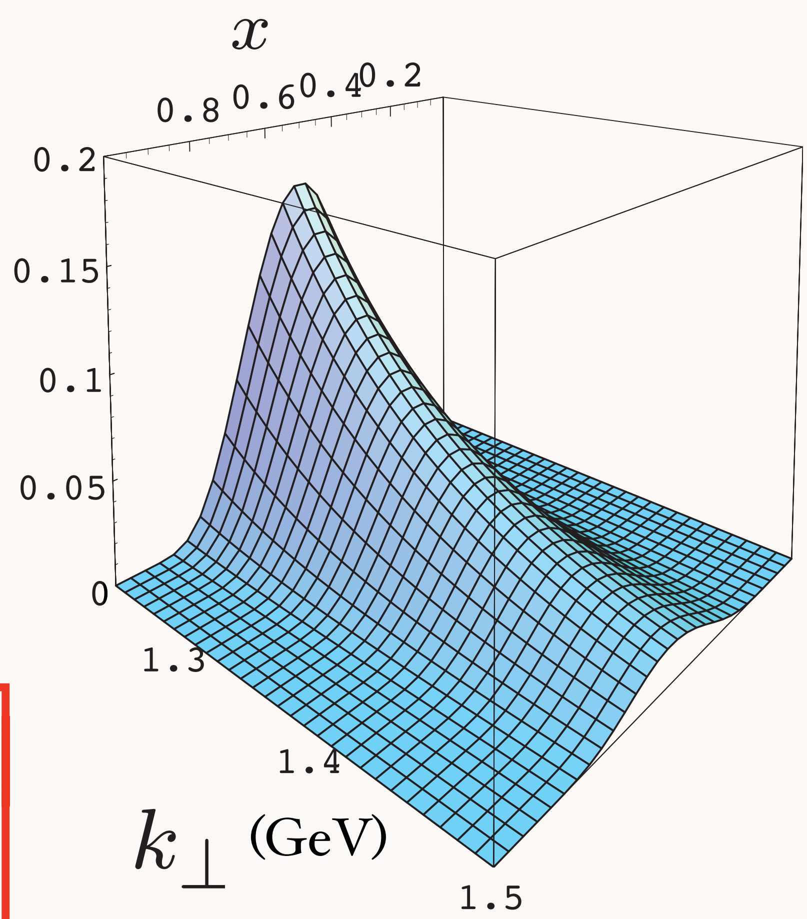
Confining AdS/QCD potential

Semiclassical first approximation to QCD

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling $k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

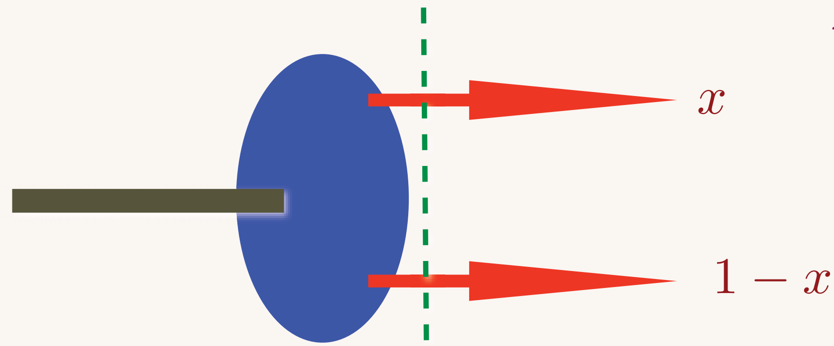
Connection of Confinement to TMDs

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



Fixed $\tau = t + z/c$

$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

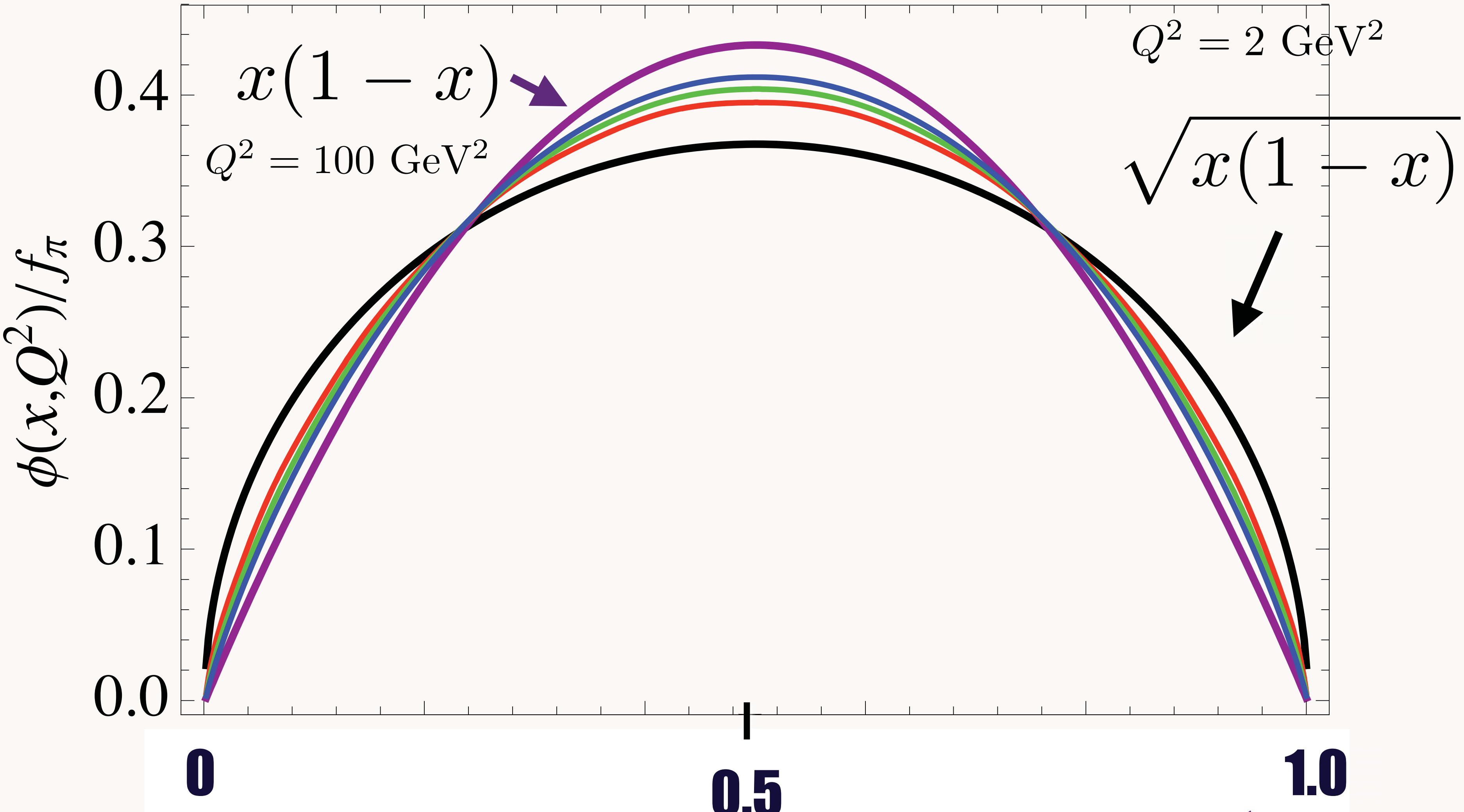
$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ — Donnellan et al.

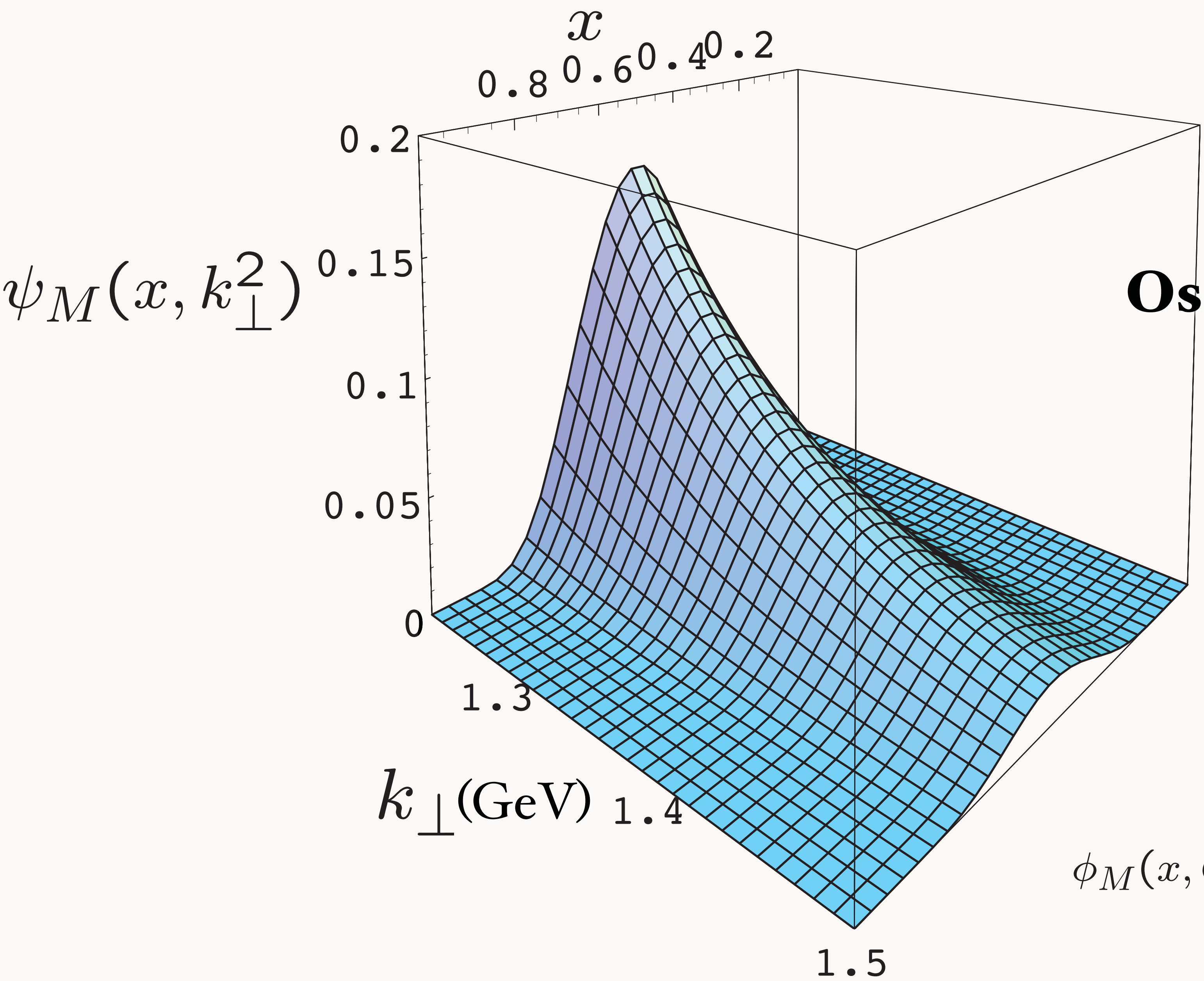
Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ — Braun et al.

ERBL Evolution of Pion Distribution Amplitude



F. Cao, GdT, sjb (preliminary)

Prediction from AdS/CFT: Meson LFWF

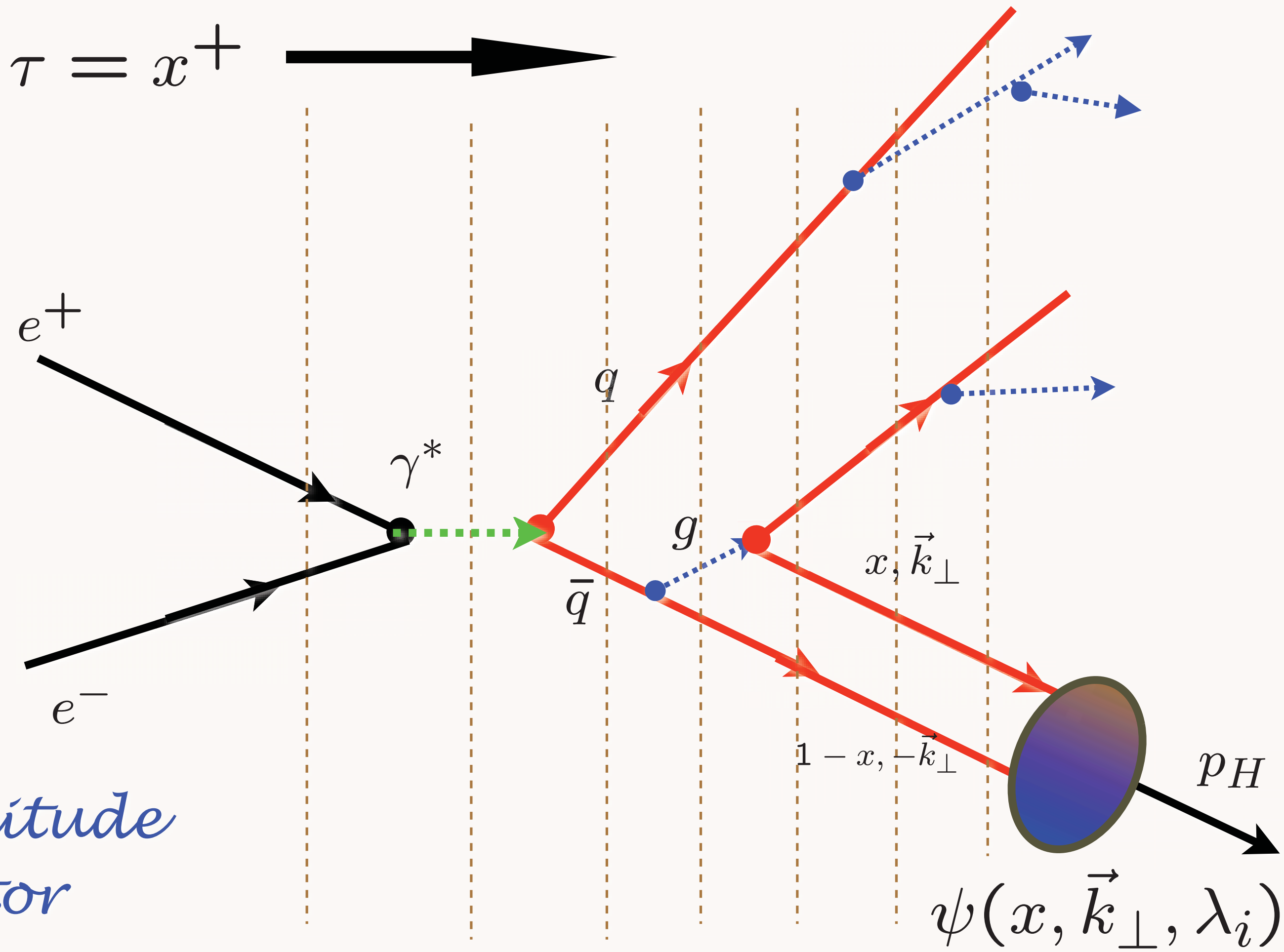


Harmonic Oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Hadronization at the Amplitude Level



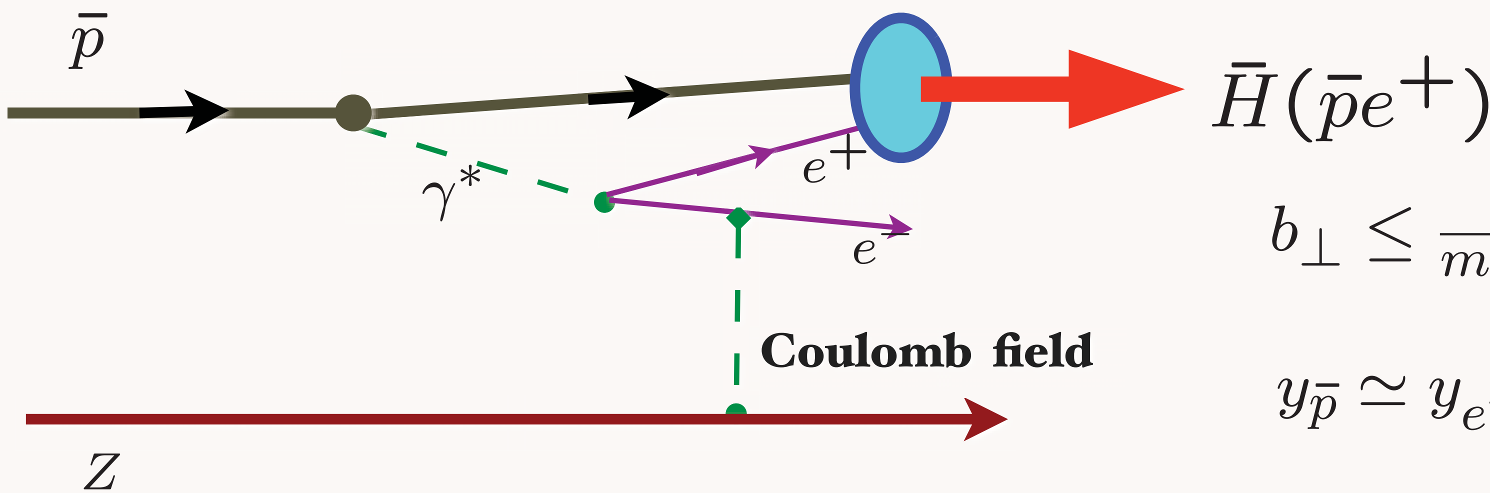
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momenta
- Hadron Spectra, Regge Trajectories

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$