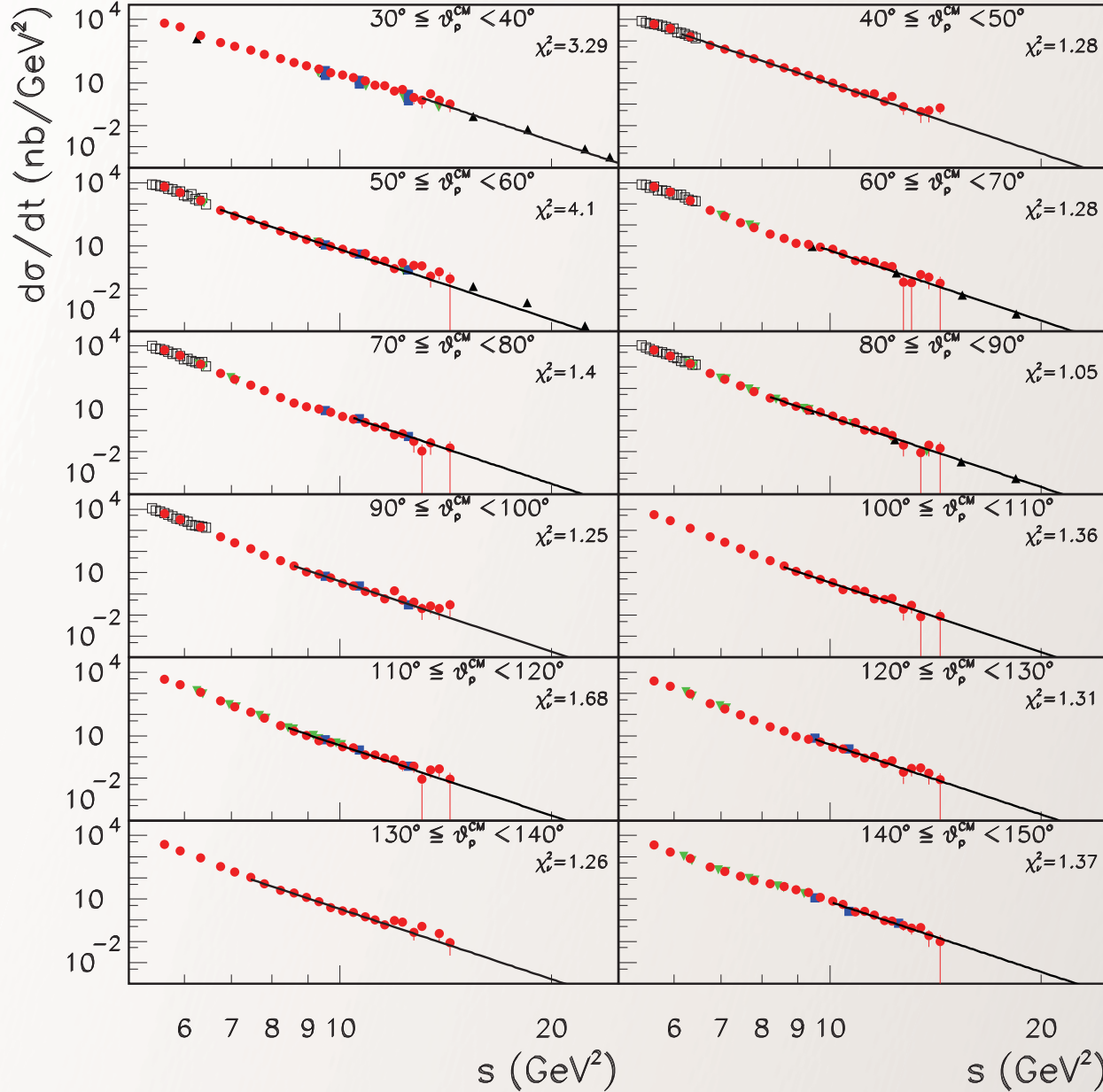


Deuteron Photodisintegration

J-Lab



PQCD and AdS/CFT:

$$s^{n_{tot}-2} \frac{d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B \rightarrow C+D}(\theta_{CM})$$

$$s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM})$$

$$n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11$$

Reflects conformal invariance

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AdS/QCD

- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \rightarrow np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

- $n_{tot} = 1 + 6 + 3 + 3 = 13$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$
at high p_T

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n^d - \gamma_m^d} \left[1 + \mathcal{O} \left(\alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-(2/5) C_F/\beta}$$

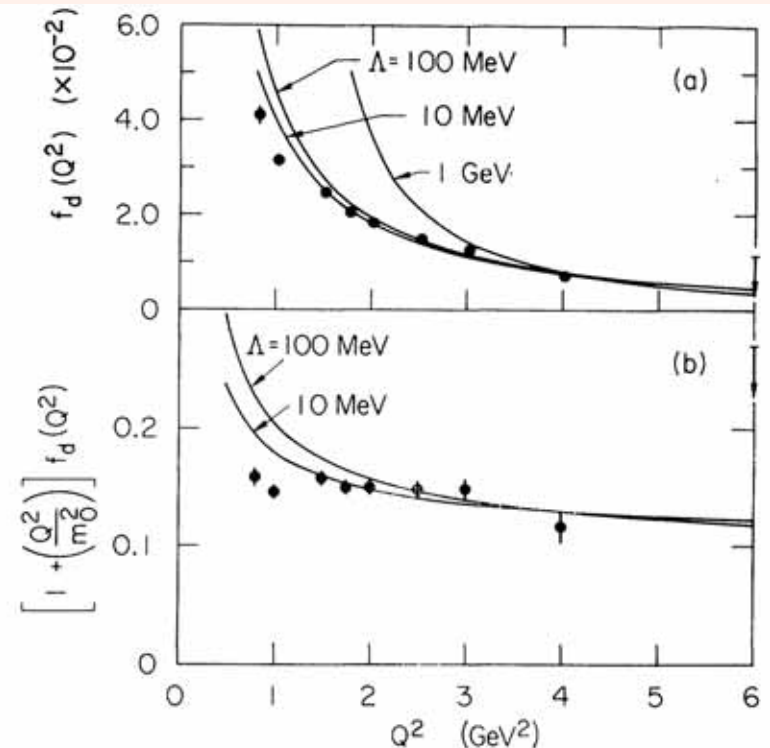
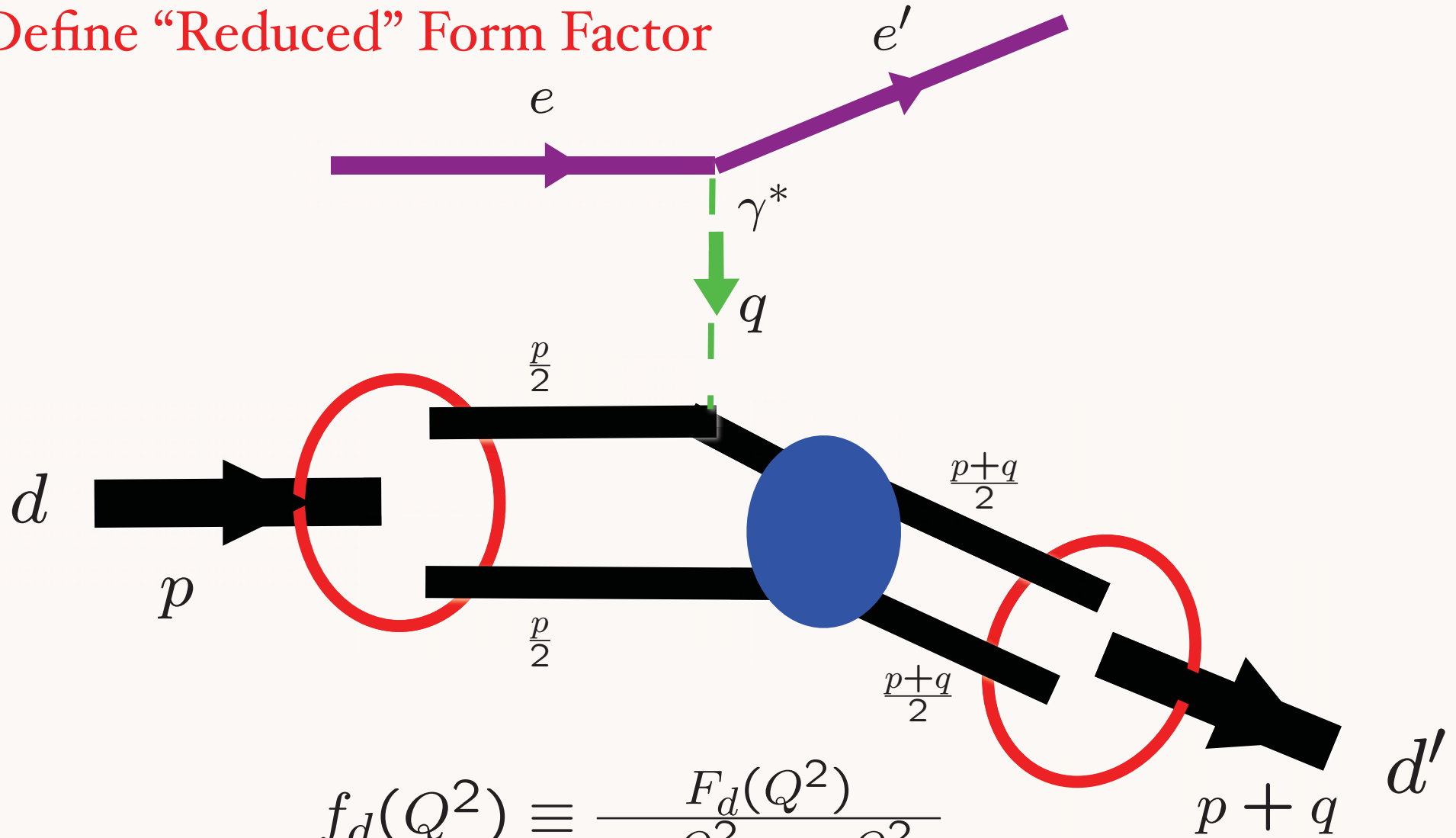


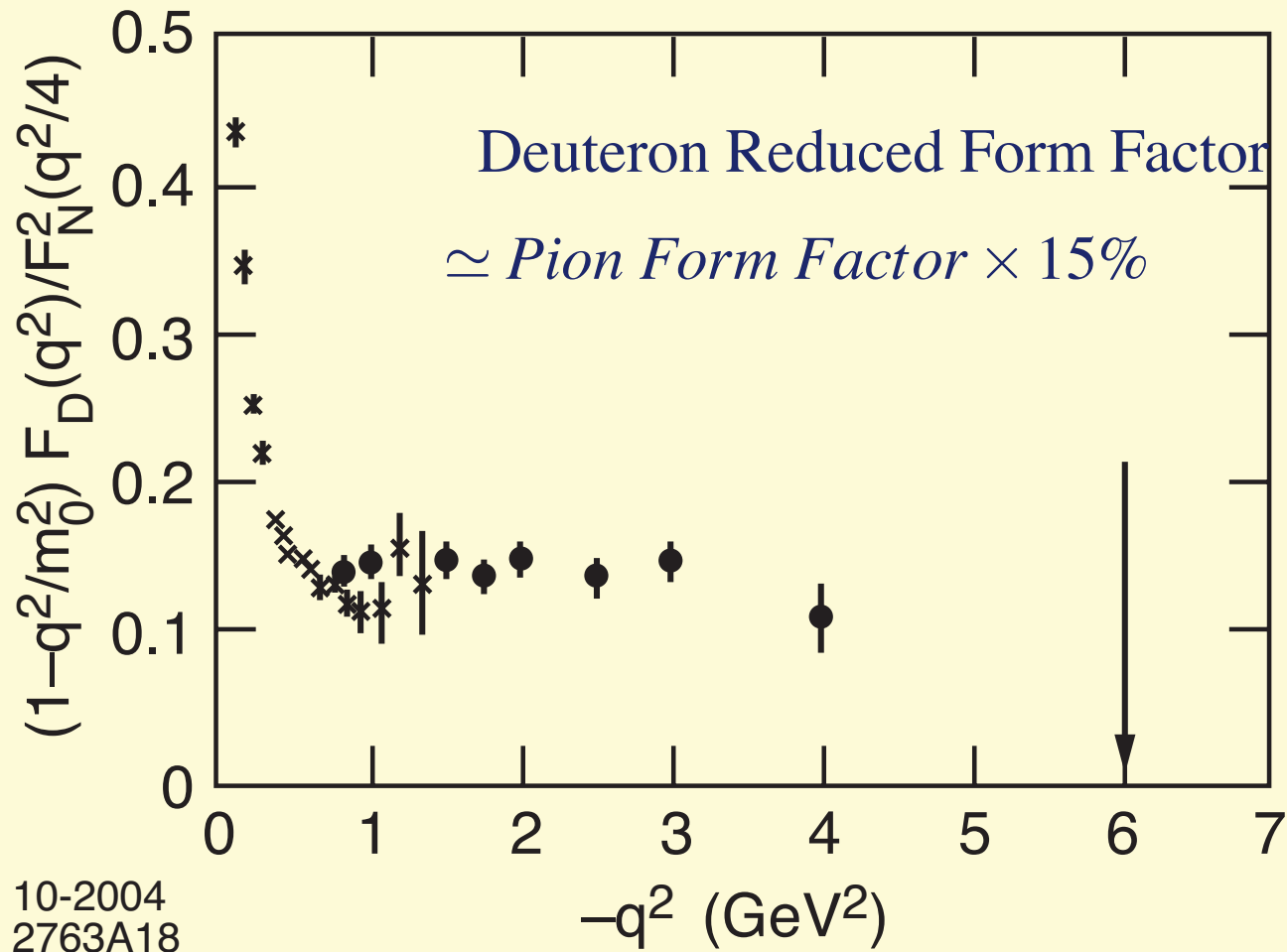
FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2) [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71 \text{ GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4 \text{ GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28 \text{ GeV}^2$ is used (Ref. 8).

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

Elastic electron-deuteron scattering



- 15% Hidden Color in the Deuteron

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is $|n\ p\rangle$
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- **Predict** $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$ at high Q^2

QCD Lagrangian

Generalization of QED

The diagram shows the QCD Lagrangian L_{QCD} enclosed in a red box. Above the box, three labels with arrows point to parts of the equation: 'gluon dynamics' points to the first term, 'quark kinetic energy + quark-gluon dynamics' points to the second term, and 'mass term' points to the third term. Below the box, four labels with arrows point to specific parts: 'QCD color charge' points to the $4g^2$ denominator, 'field strength tensor' points to $G_{\mu\nu}$, 'covariant derivative' points to D_μ , and 'quark field' points to ψ_f .

$$L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \bar{\psi}_f \psi_f$$

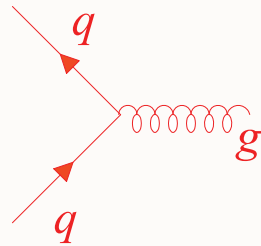
Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

QCD

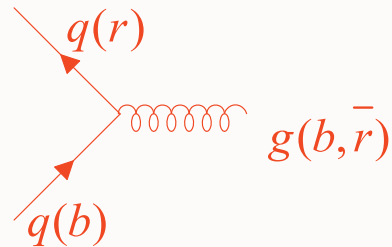
Fundamental Couplings

Only quarks and gluons involve basic vertices: Quark-gluon vertex



Similar to QED

More exactly



Gluon vertices



colored particles couple to gluons

- Although we know the QCD Lagrangian, we have only begun to understand its remarkable properties and features.
- Novel QCD Phenomena: hidden color, color transparency, strangeness asymmetry, intrinsic charm, anomalous heavy quark phenomena, anomalous spin effects, single-spin asymmetries, odderon, diffractive deep inelastic scattering, dangling gluons, shadowing, antishadowing, QGP, CGC, ...

Truth is stranger than fiction, but it is because Fiction is obliged to stick to possibilities. *—Mark Twain*

In QCD and the Standard Model
the beta function is indeed
negative!

$$\beta(g) = \frac{-g^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{4}{3} \frac{N_F}{2} \right)$$

$$\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} < 0$$

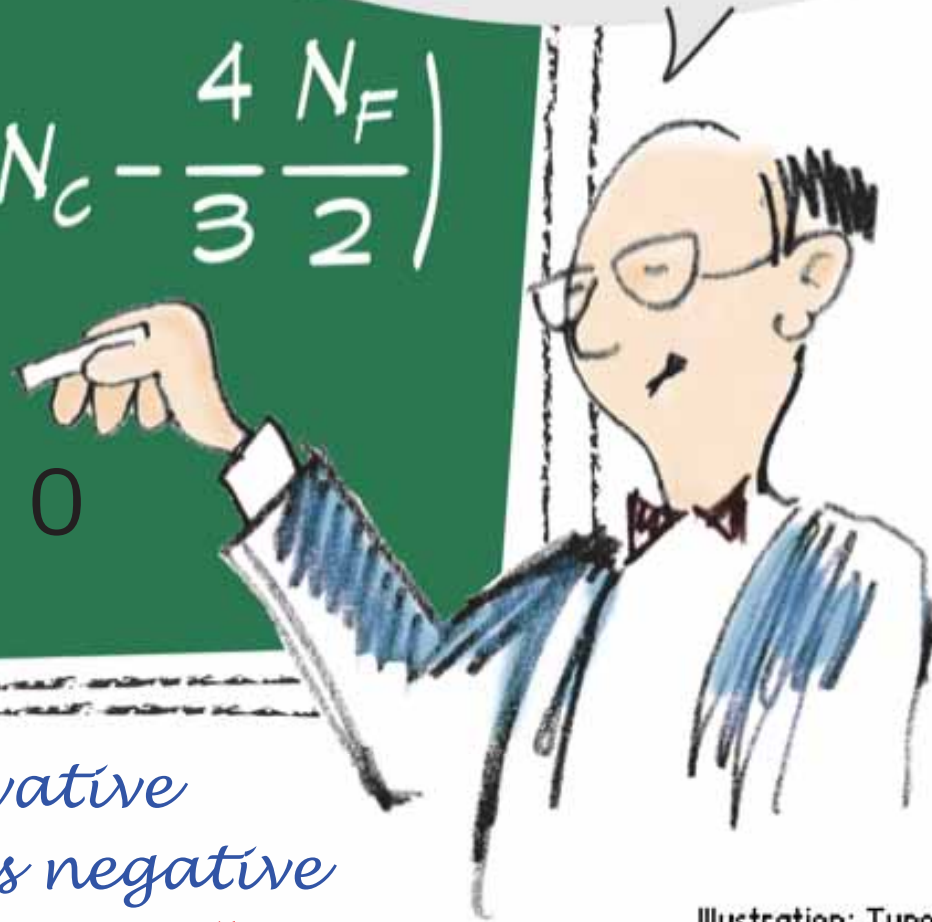


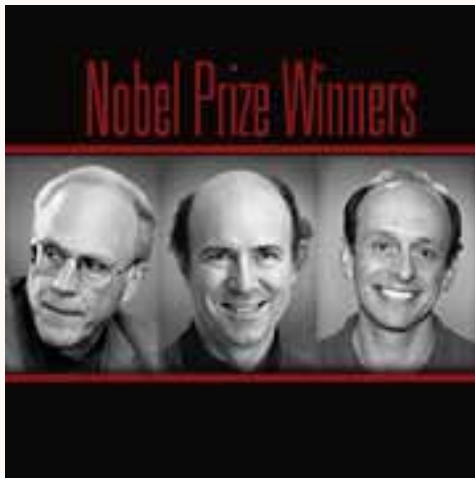
Illustration: Typoform

*logarithmic derivative
of the QCD coupling is negative
Coupling becomes weaker at short
distances or high momentum transfer
AdS/QCD*

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Verification of Asymptotic Freedom

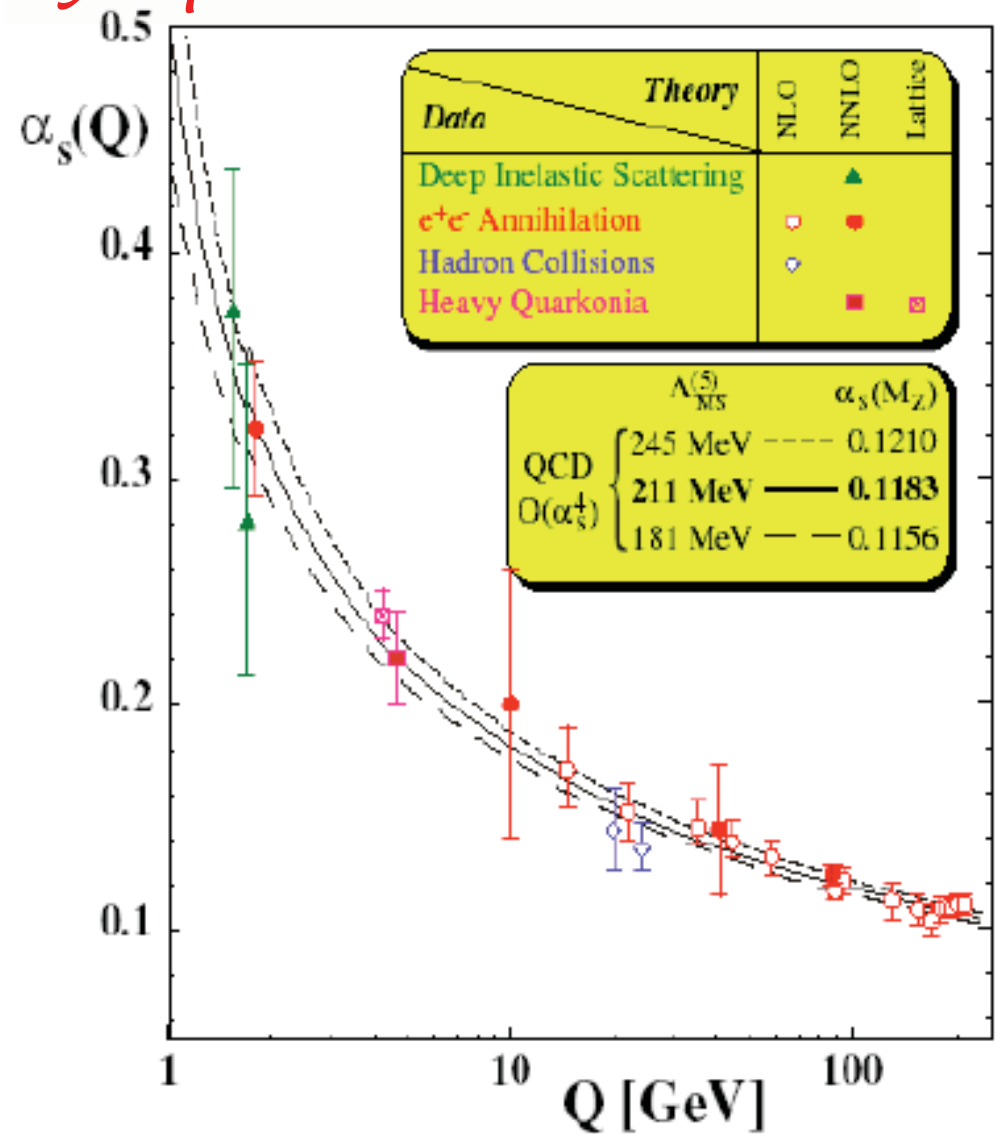
$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$



$$\frac{\sigma(e^+e^- \rightarrow \text{three jets})}{\sigma(e^+e^- \rightarrow \text{two jets})}$$

proportional to $\alpha_s(Q)$

Ratio of rate for $e^+e^- \rightarrow q\bar{q}g$ to $e^+e^- \rightarrow q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$



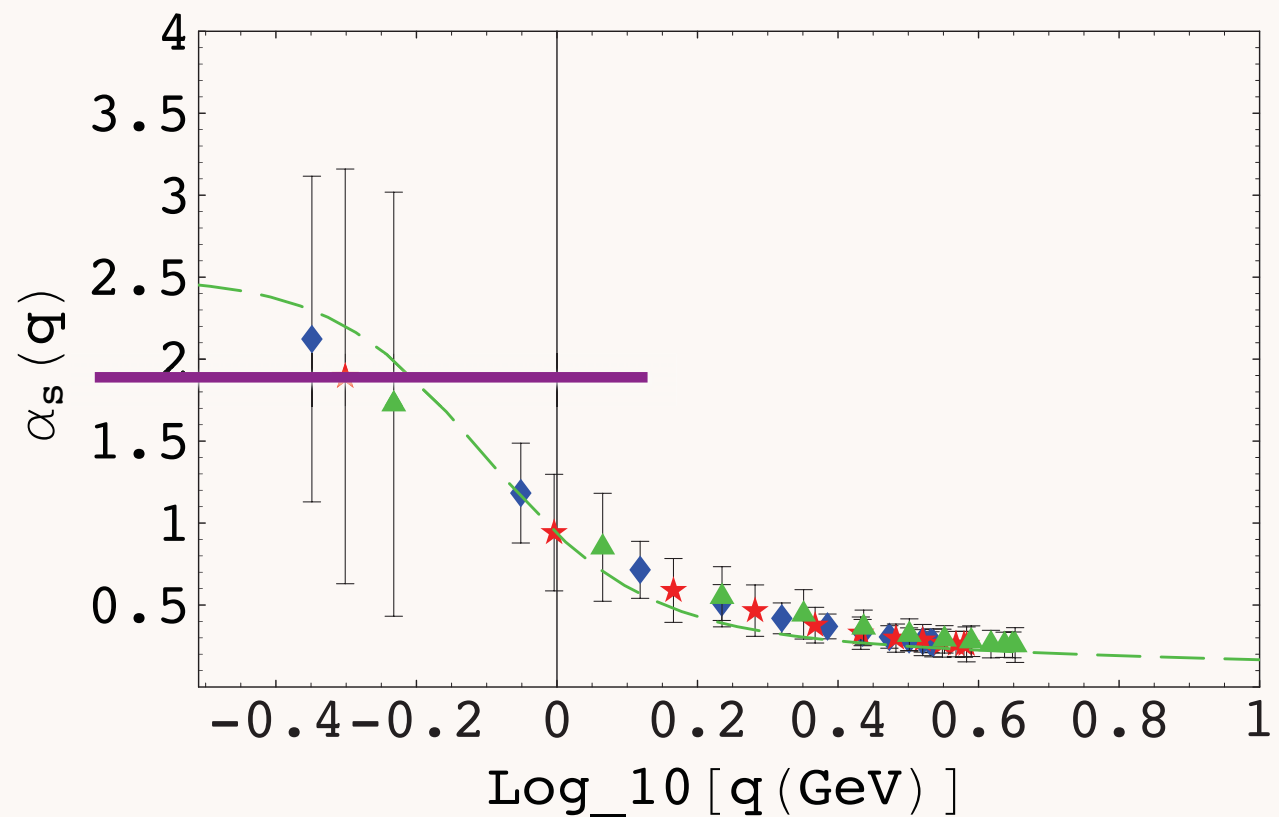
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AdS/QCD

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Conformal QCD Window in Exclusive Processes

- Does α_s develop an IR fixed point? Dyson–Schwinger Equation [Alkofer, Fischer, LLanes-Estrada, Deur ...](#)
- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared [Furui and Nakajima, hep-lat/0612009](#) (Green dashed curve: DSE).



AdS/QCD

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s , logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin; Landshoff; Mueller, Duncan
- **DSE: QCD ggg coupling (mom scheme) has IR Fixed point** Alkofer, Fischer, von Smekal et al.
- **Lattice results show similar flat behavior** Furui, Nakajima
- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Fundamental Interactions of QED, QCD, and Weak Interactions –

All derived from the Yang-Mills Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a}$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

$$D_\mu = \partial_\mu + igT^a A_\mu^a$$

$$[T^a, T^b] = f^{abc}T^c$$

QED : $T = 1, f = 0$

QCD : $T^a = 3 \times 3$ traceless matrices

ψ : charged leptons

ψ : quarks – color triplets

Electroweak : $T^a = 2 \times 2$ traceless matrices

ψ : chiral fermion doublets

QCD Lagrangian

$$L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{nf} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \bar{\psi}_f \psi_f$$

gluon dynamics (points to $-\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu})$)
 quark kinetic energy + quark-gluon dynamics (points to $\sum_{f=1}^{nf} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f$)
 mass term (points to $\sum_{f=1}^{nf} m_f \bar{\psi}_f \psi_f$)

QCD color charge (points to g)
 field strength tensor (points to $G_{\mu\nu}$)
 covariant derivative (points to D_μ)
 quark field (points to ψ_f)

$$\lim N_C \rightarrow 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

Analytic limit of QCD: Abelian Gauge Theory

QCD \rightarrow **QED**

P. Huet, sjb

QED: Underlies Atomic Physics, Molecular Physics,
Chemistry, Electromagnetic Interactions ...

QCD: Underlies Hadron Physics, Nuclear Physics,
Strong Interactions, Jets

Theoretical Tools

- Feynman diagrams and perturbation theory
- Bethe Salpeter Equation, Dyson-Schwinger Equations
- Lattice Gauge Theory, Discretized Light-Front Quantization
- AdS/CFT !

Given the elementary gauge theory interactions, all fundamental processes described in principle!

Example from QED:

Electron gyromagnetic moment - ratio of spin precession frequency to Larmor frequency in a magnetic field

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 201(30) \quad \text{QED prediction (Kinoshita, et al.)}$$

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 193(10) \quad \text{Measurement (Dehmelt, et al.)}$$

$$\frac{1}{2}g_e = 1.001\ 159\ 652\ 180\ 85 [0.76 \text{ ppt}]$$

$$\textit{Dirac: } g_e \equiv 2 \quad \text{Measurement (Gabrielse, et al.)}$$

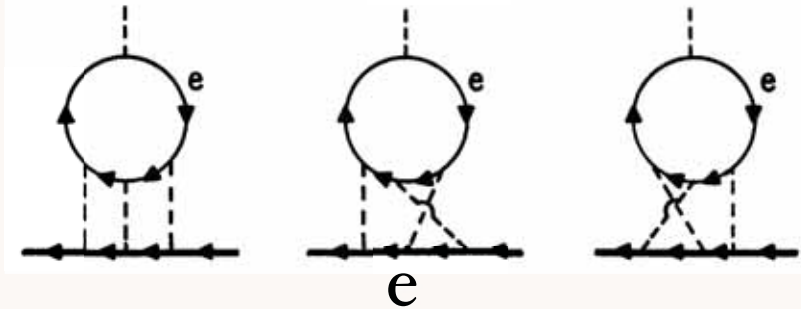
QED provides an asymptotic series relating g and α ,

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \dots$$

$$+ a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

B

Light-by-Light Scattering Contribution to C_6



$$\alpha^{-1} = 137.035\,999\,710\,(90)\,(33)\,[0.66\text{ ppb}][0.24\text{ ppb}],$$

$$= 137.035\,999\,710\,(96)\,[0.70\text{ ppb}].$$

| G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, Phys. Rev. Lett. **97**, 030802 (2006).

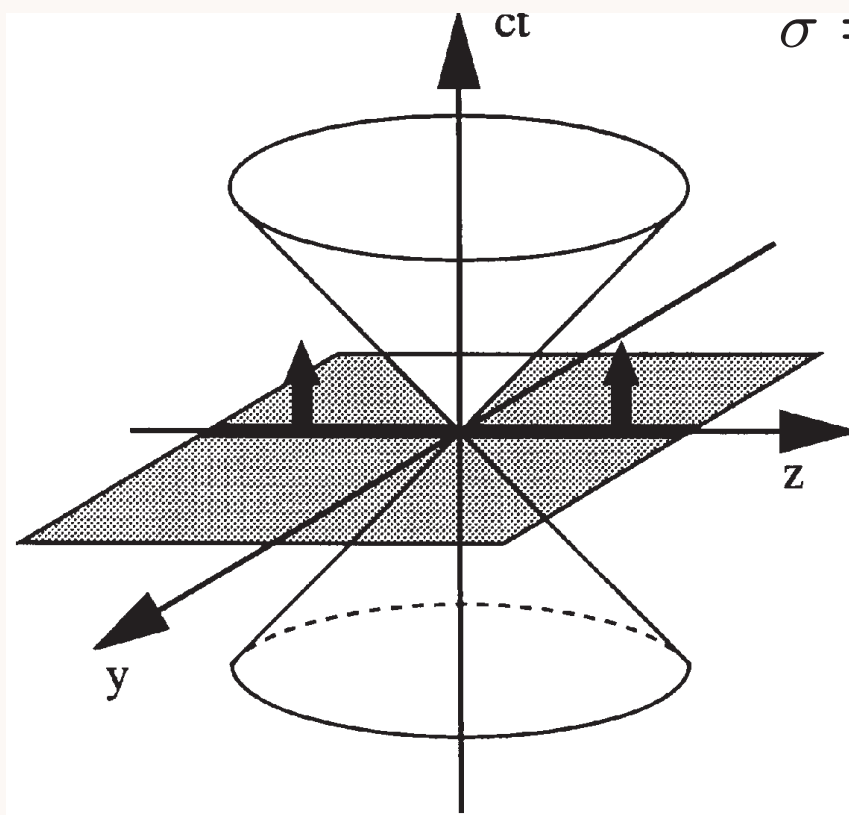
Other High-Precision Atomic Physics Tests of QED

- Lamb Shift in Hydrogen
- Hyperfine splitting of muonium and hydrogen
- Muonic Atom spectroscopy
- Positronium Lifetime

All Accurate to subppm

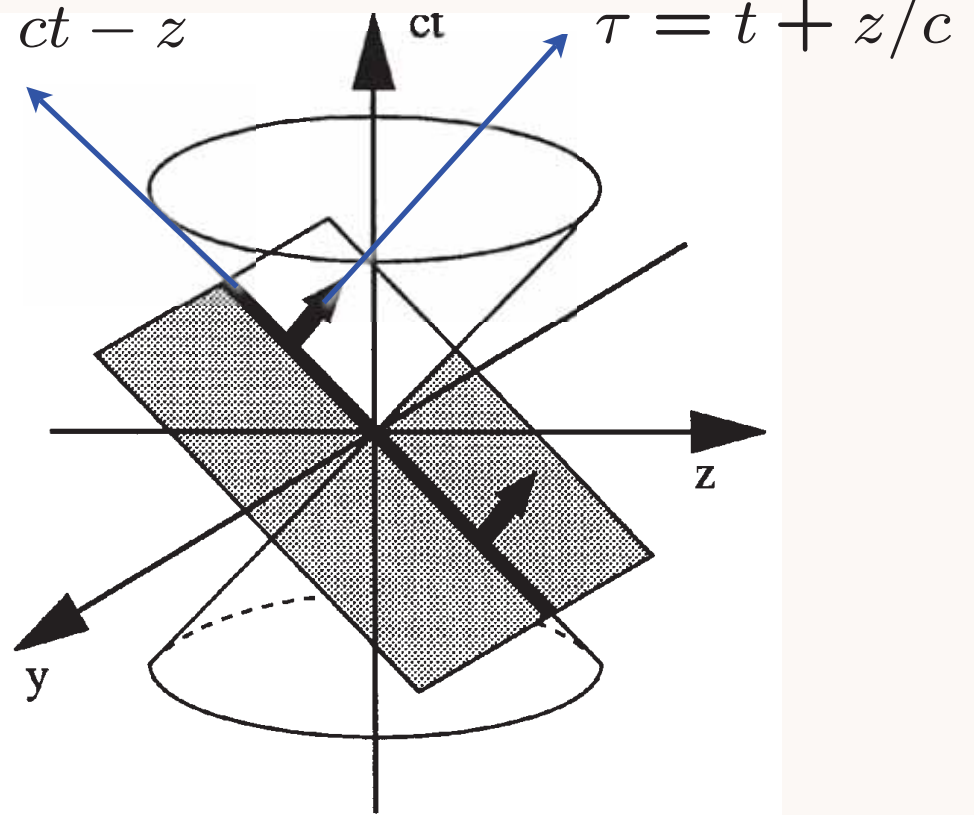
Dirac's Amazing Idea: The "Front Form"

Evolve in
light-cone time



Instant Form

$$\sigma = ct - z$$

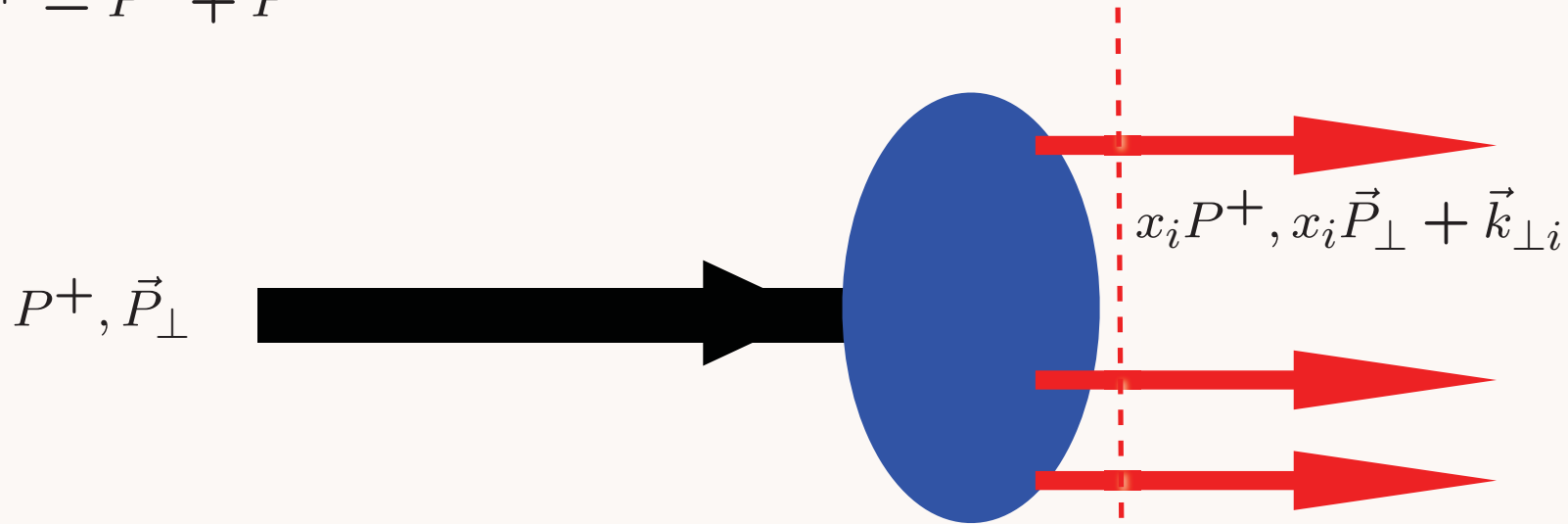


Front Form

Light-Front Wavefunctions

$$P^+ = P^0 + P^z$$

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

*'Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
'tis an immortal thing*

Montgomery

Angular Momentum on the Light-Front

$A^+ = 0$ gauge:

No unphysical degrees of freedom

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

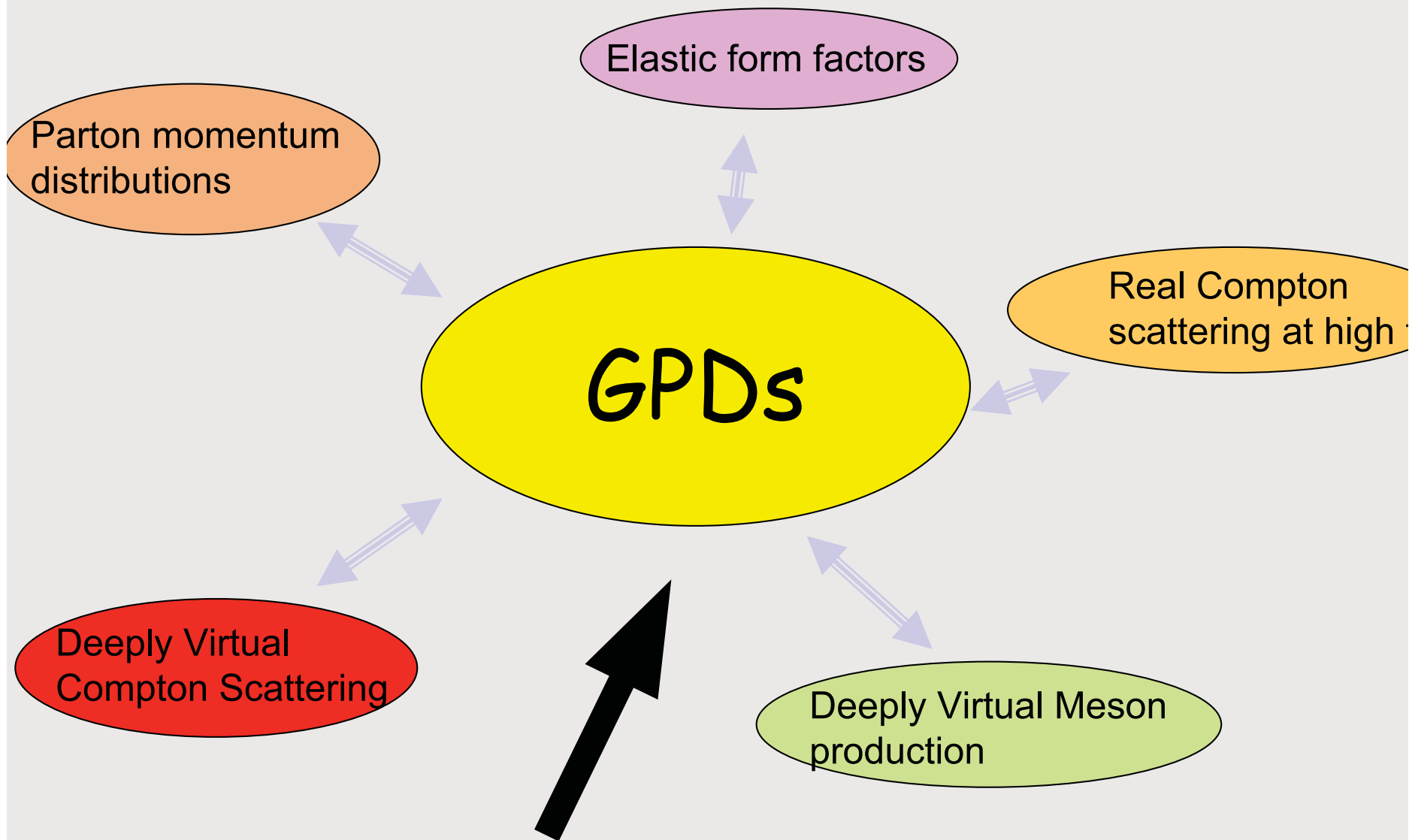
Conserved
LF Fock state by Fock State

$$l_j^z = -i \left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)$$

$n-1$ orbital angular momenta

***Nonzero Anomalous Moment requires
Nonzero orbital angular momentum.***

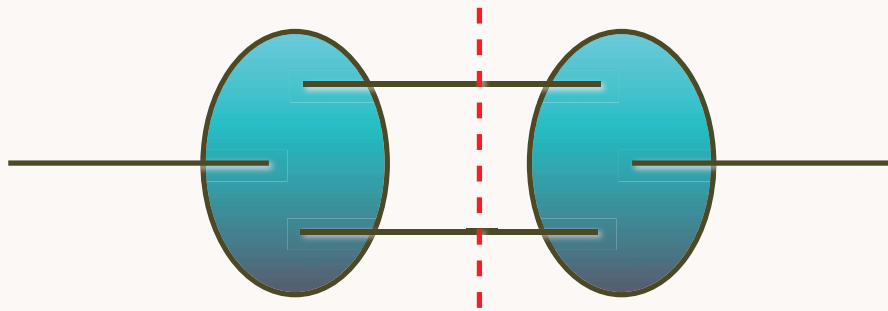
A Unified Description of Hadron Structure



Light Front Wavefunctions

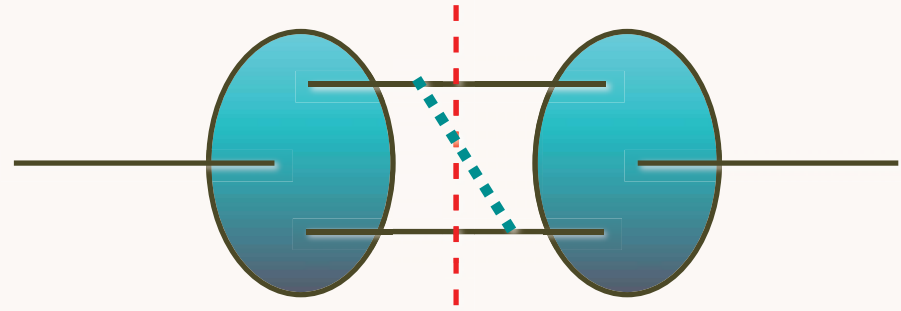
Quantum Mechanics: Uncertainty in p , x , spin

Relativistic Quantum Field Theory: Uncertainty in particle number n



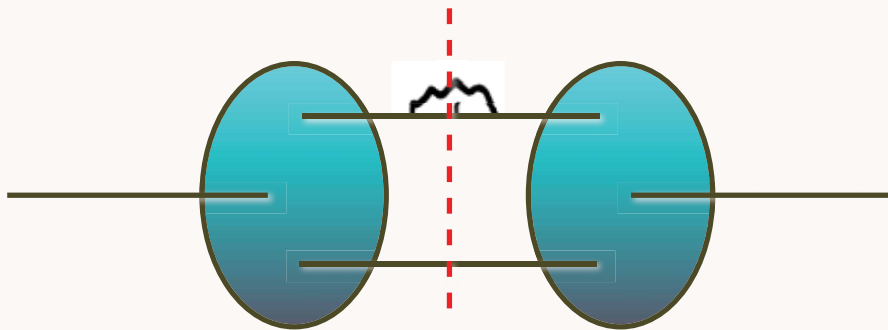
Positronium $n=2$

$$e^+e^-$$



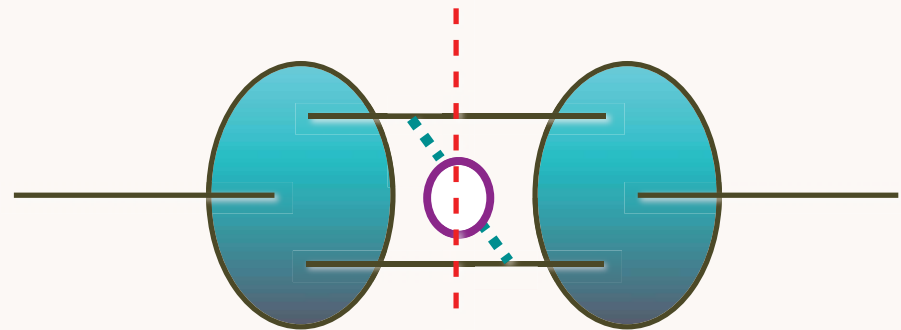
Hyperfine splitting $n=3$

$$e^+e^-\gamma$$



Lamb Shift $n=3$

$$e^+e^-\gamma$$



Vacuum Polarization $n=4$

$$e^+e^-e^+e^-$$

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

sum over states with n=3, 4, ... constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

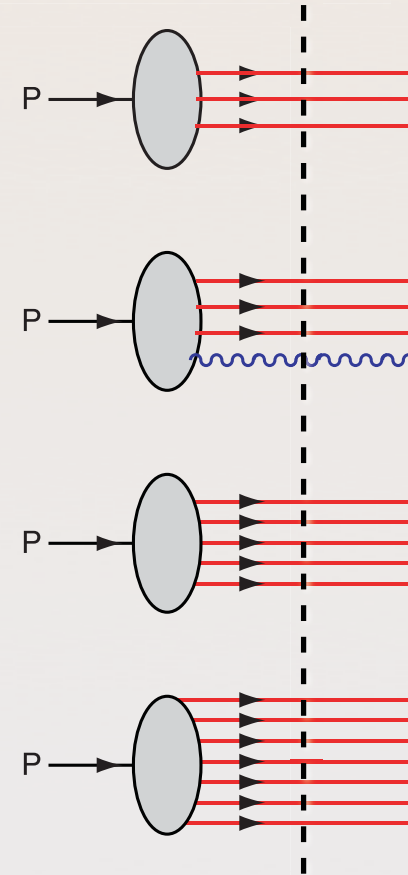
$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

Intrinsic heavy quarks,

$$\bar{s}(x) \neq s(x)$$

$$\bar{u}(x) \neq \bar{d}(x)$$

AdS/QCD

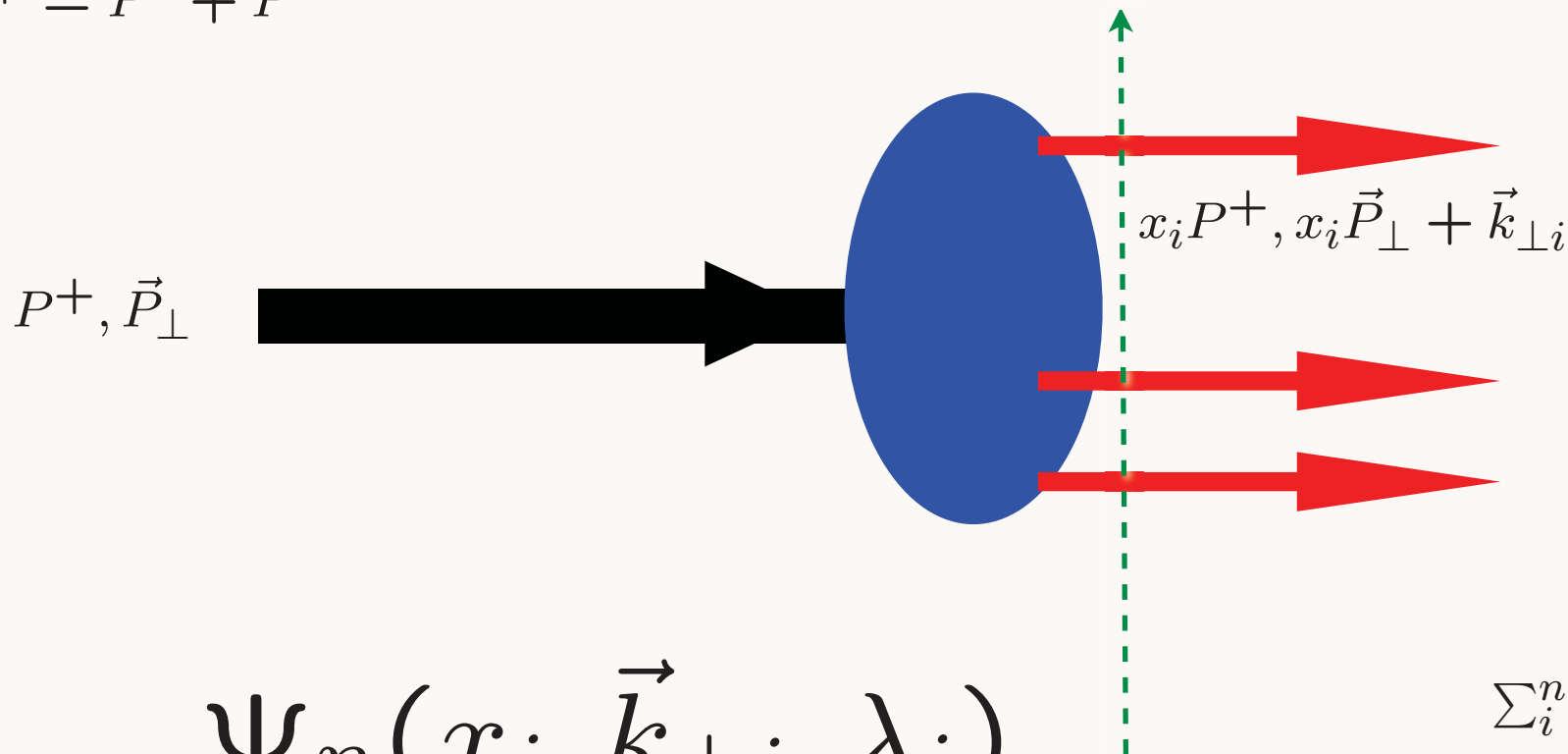


Fixed LF time

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$P^+ = P^0 + P^z$$

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi(x, k_{\perp}) \quad x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of p^{μ}

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

**Heisenberg Matrix Equation
for QCD**

AdS/QCD

Light-Front QCD

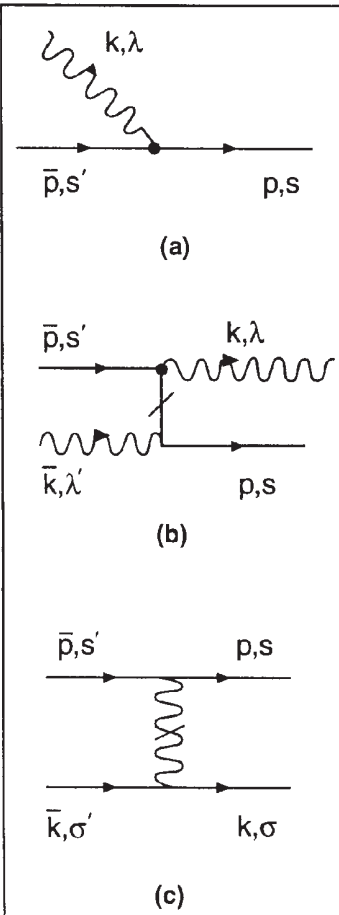
Heisenberg Matrix Formulation

$$H_{LC}^{QCD} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle$$

DLCQ

Discretized Light-Cone Quantization

n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	gg g
6	q \bar{q} gg						
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gg gg
10	q \bar{q} gg g
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g			
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}			



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Hans Christian Pauli & sjb

New Perspectives in QCD from AdS/CFT

- Need to understand QCD at the Amplitude Level: Hadron wavefunctions!
- Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Equation for Atomic Physics**
- *AdS/QCD Holographic Model*

New Way to Solve QCD: AdS/CFT

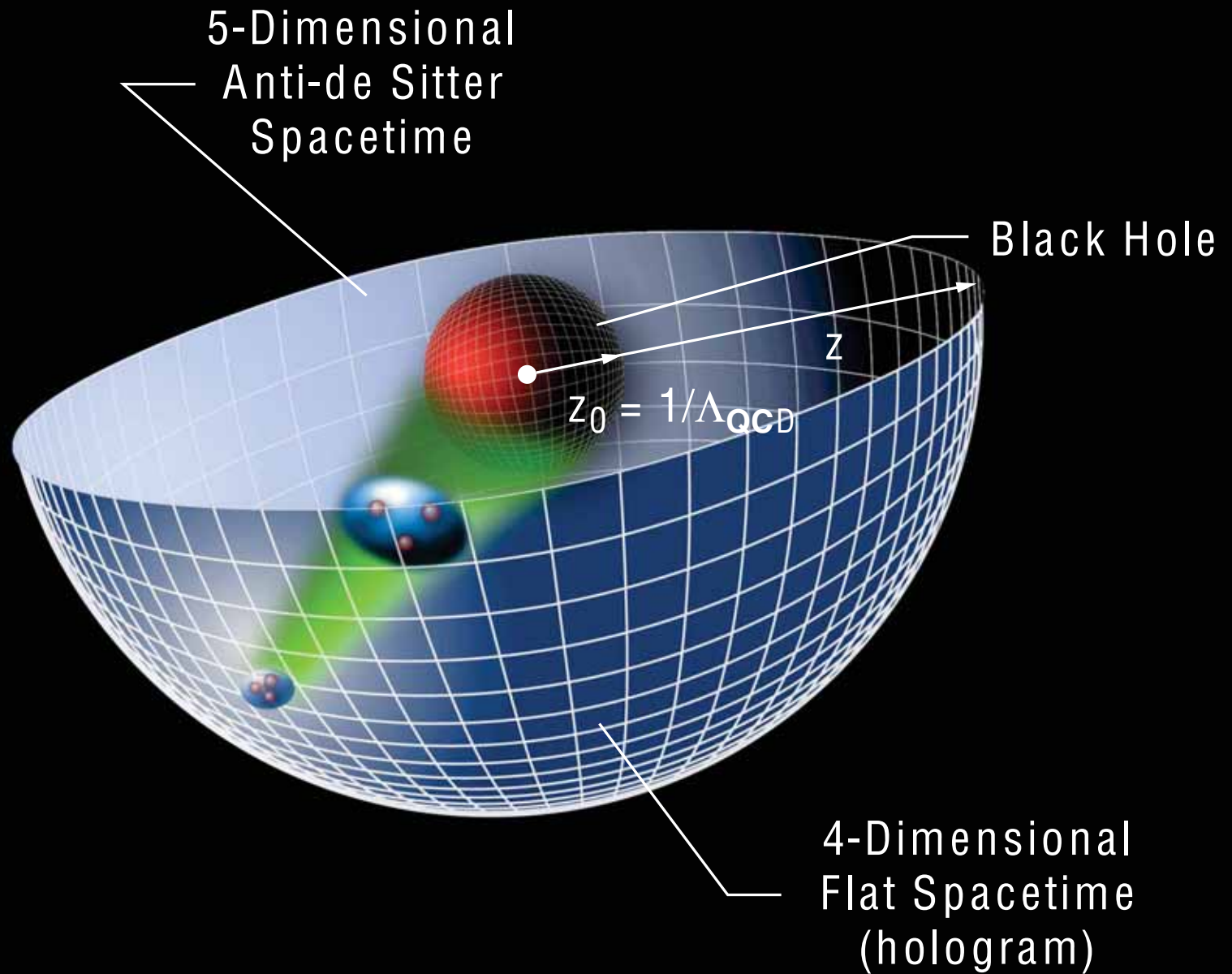
- Maldacena Correspondence
- Mathematical Representation of Lorentz Invariant and Conformal (Scale-Free) Theories
- Add new 5th space dimension to 3+1 space-time
- Holographic Model with Color Confinement and Quark Counting Rules de Teramond, sjb

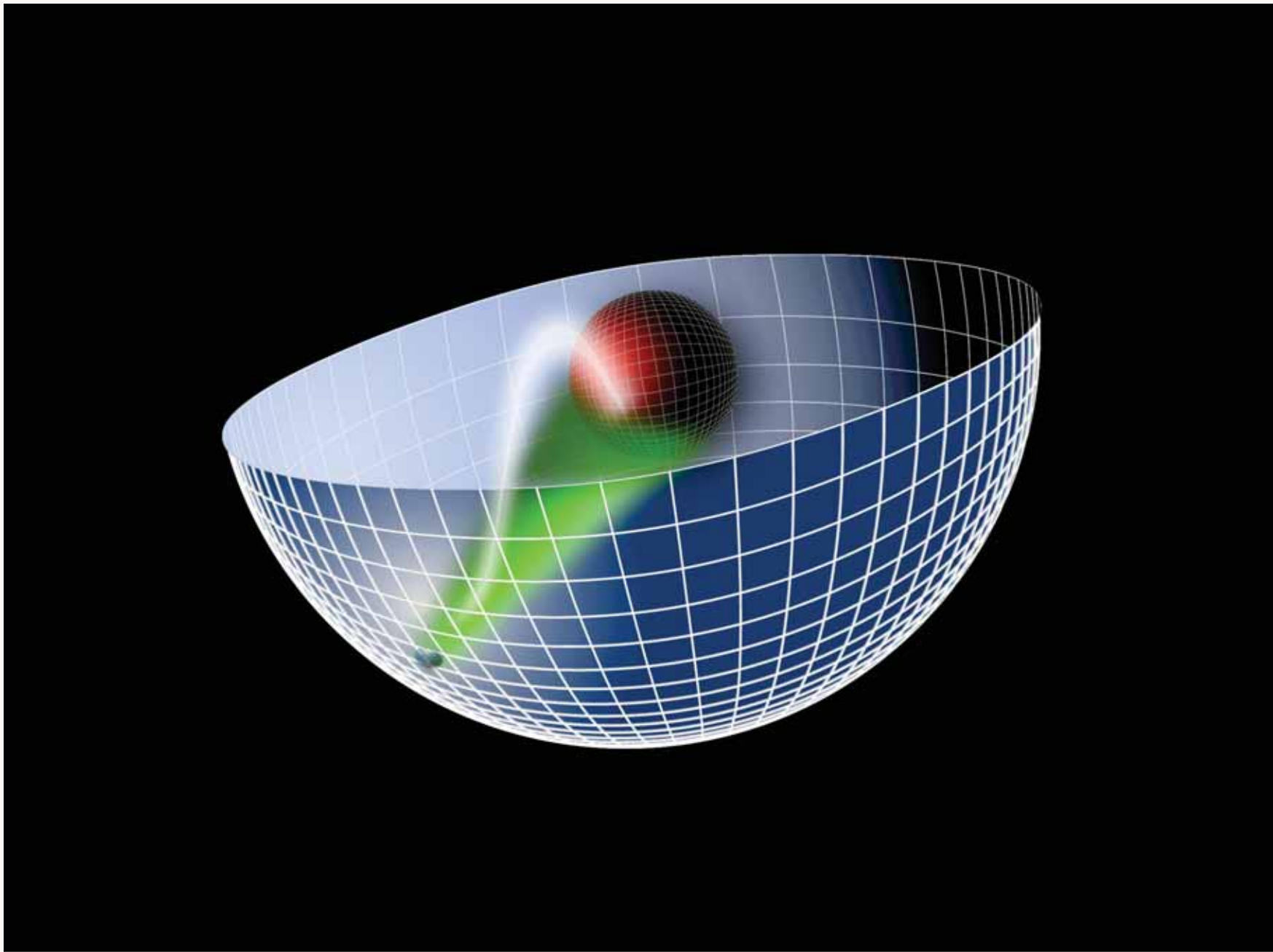
Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of $SO(4,2)$

$SO(4,2)$ has a mathematical representation on AdS_5



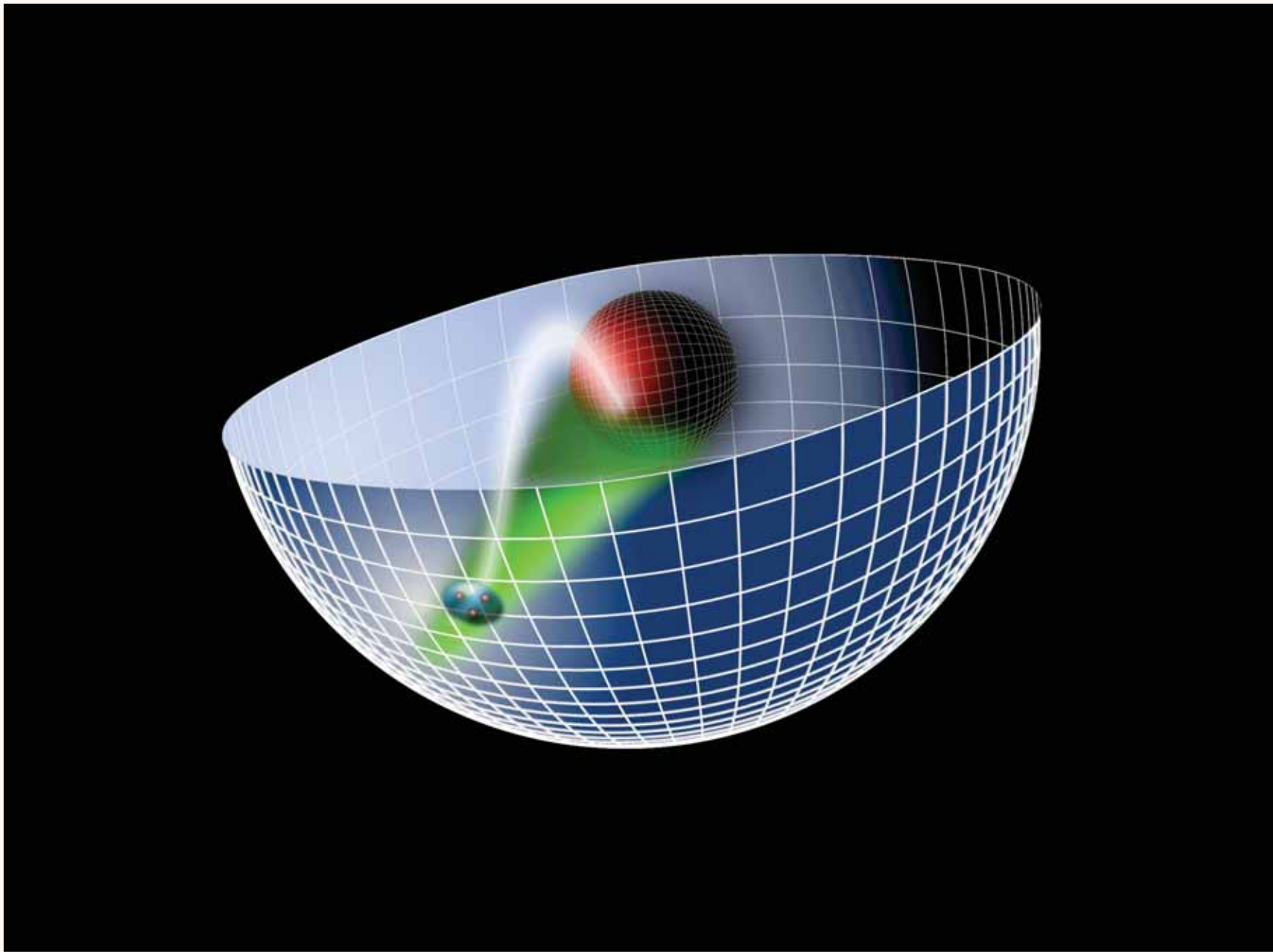


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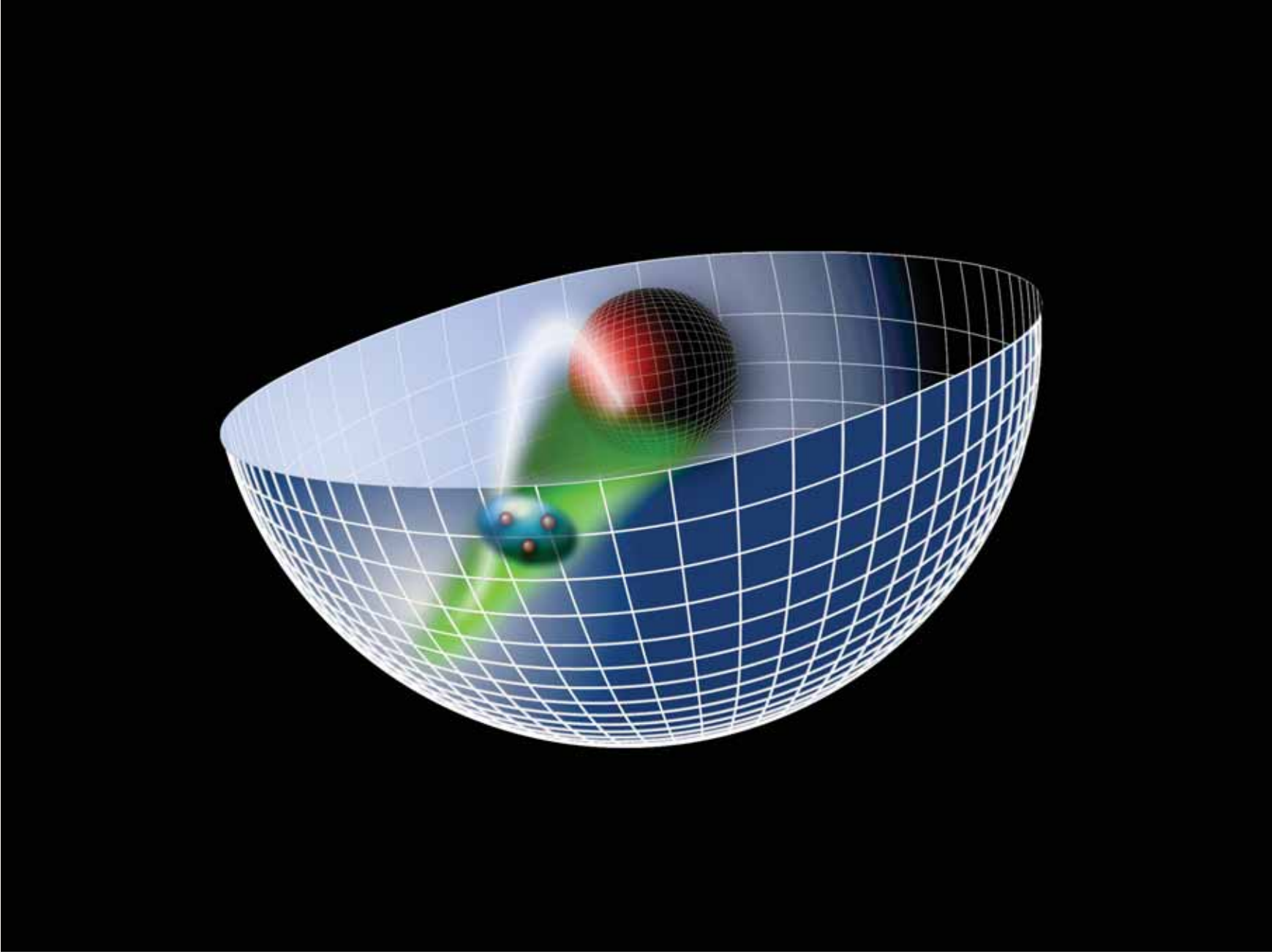


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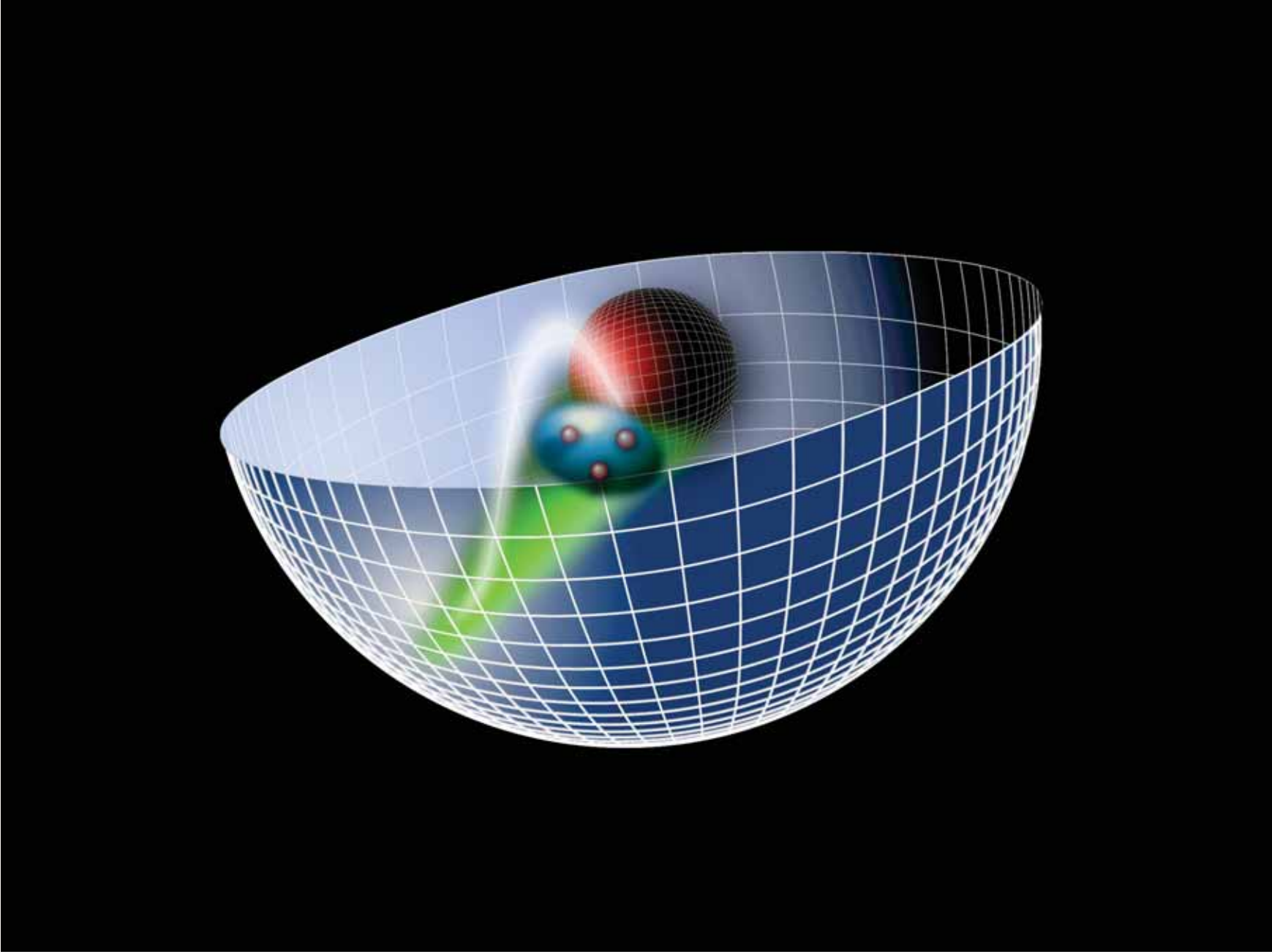
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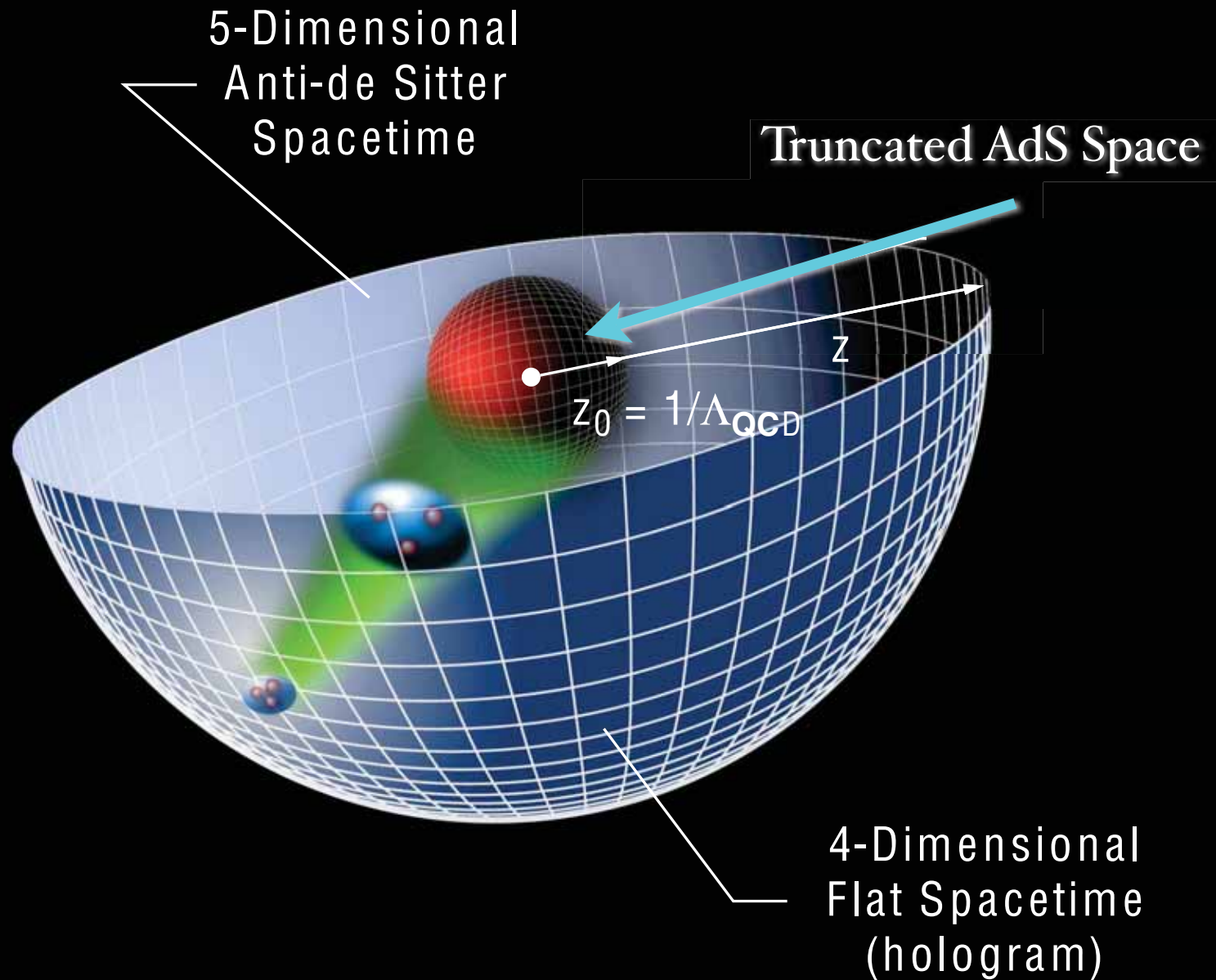
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Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure 

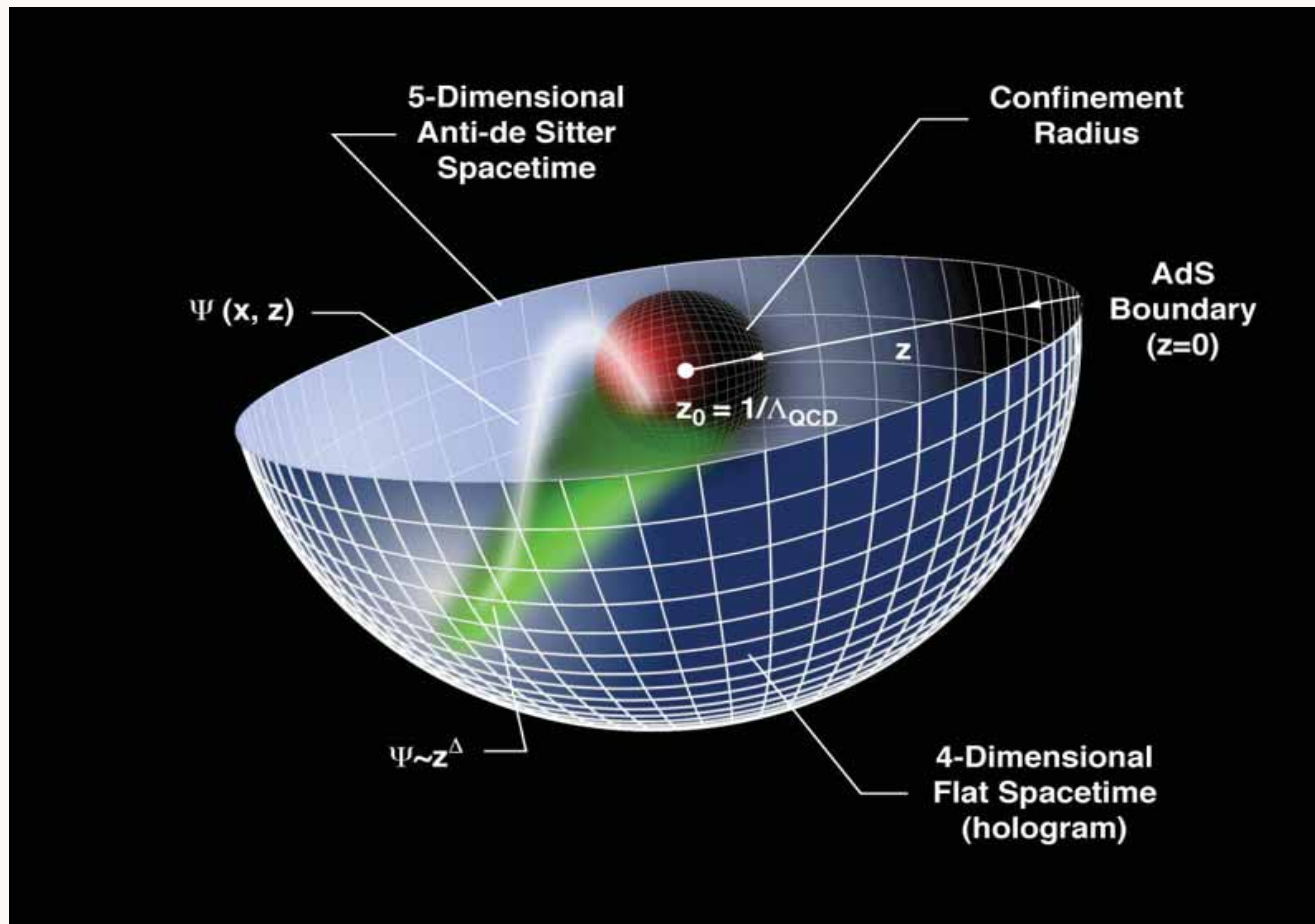
$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



8-2007
8685A14

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

We will consider both holographic models

- **Polchinski & Strassler:** AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation
- **Goal:** Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances
- **Holographic Model:** Initial “classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy Guy de Teramond, sjb
- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{\text{LF}}_{\text{QCD}}$; variational methods

AdS/CFT

- Use mapping of conformal group $SO(4,2)$ to AdS_5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension

$$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$
- Truncated space simulates “bag” boundary conditions

$$\psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$

$$\Phi(z) = z^{3/2}\phi(z)$$

*AdS Schrodinger Equation for bound state
of two scalar constituents*

$$\left[-\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)$$

Truncated space

$$V(z) = -\frac{1-4L^2}{4z^2} \quad \phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Alternative: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2 \quad \text{Karch, et al.}$$

Derived from variation of Action in AdS5