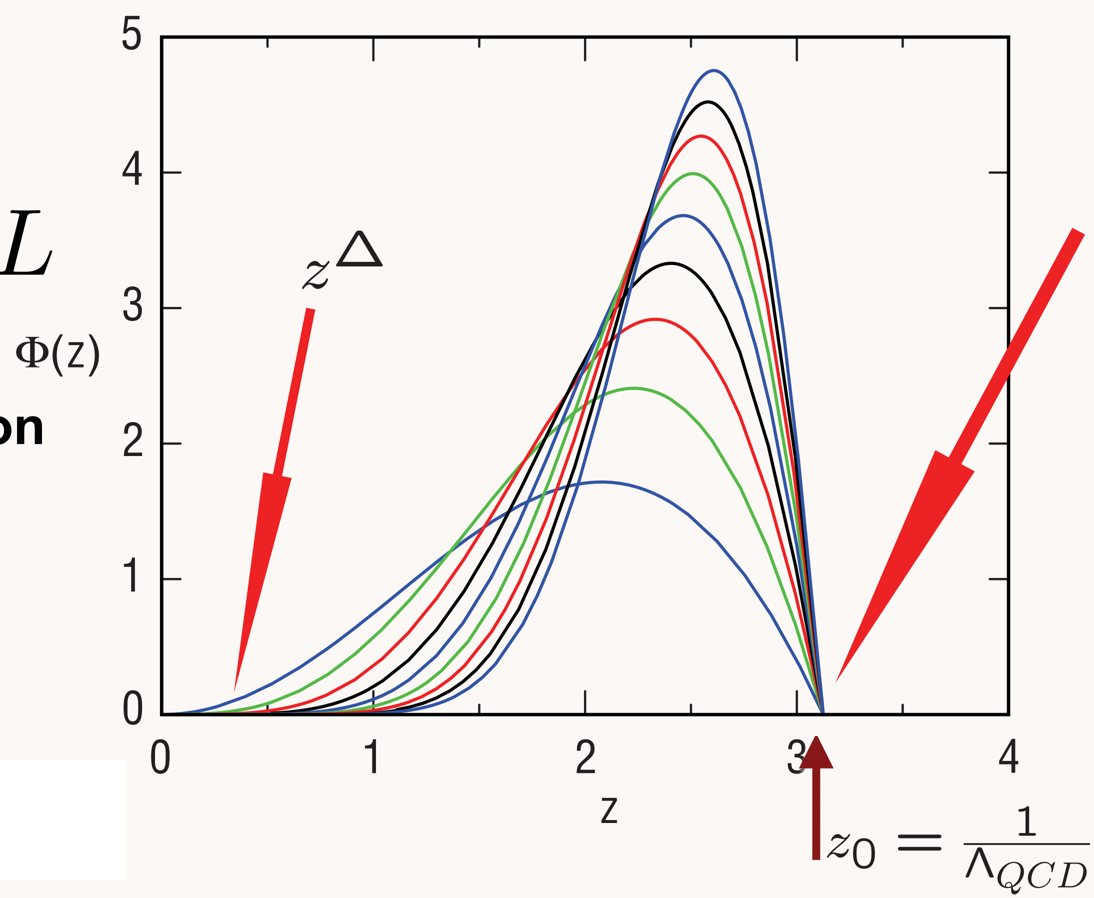
- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson



Confinement in the 5th dimension

Identify hadron by its interpolating operator at z -- > 0

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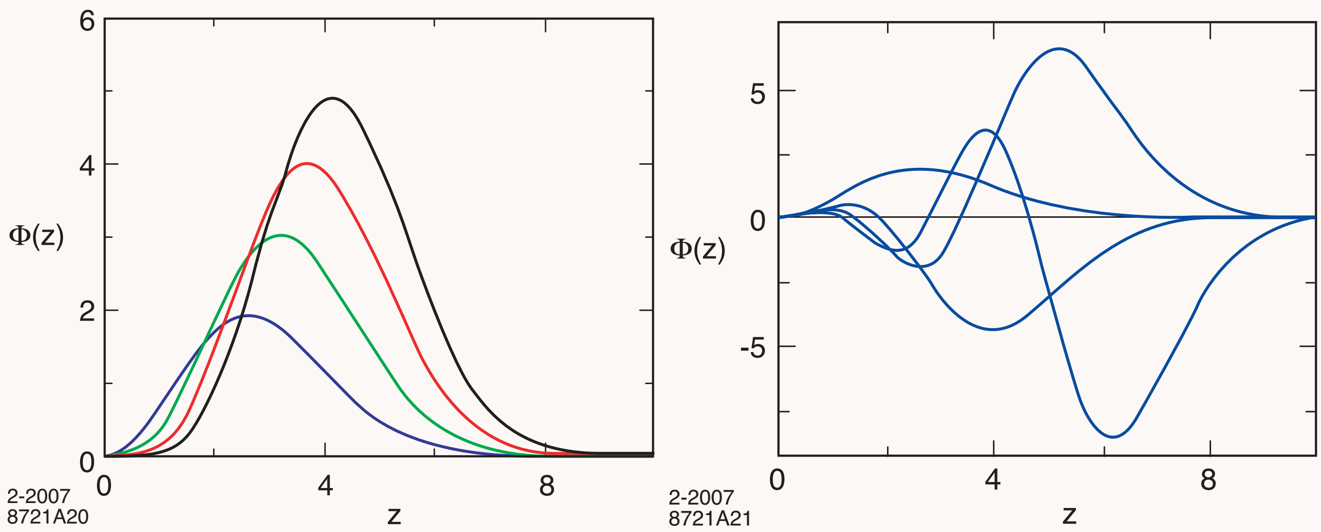
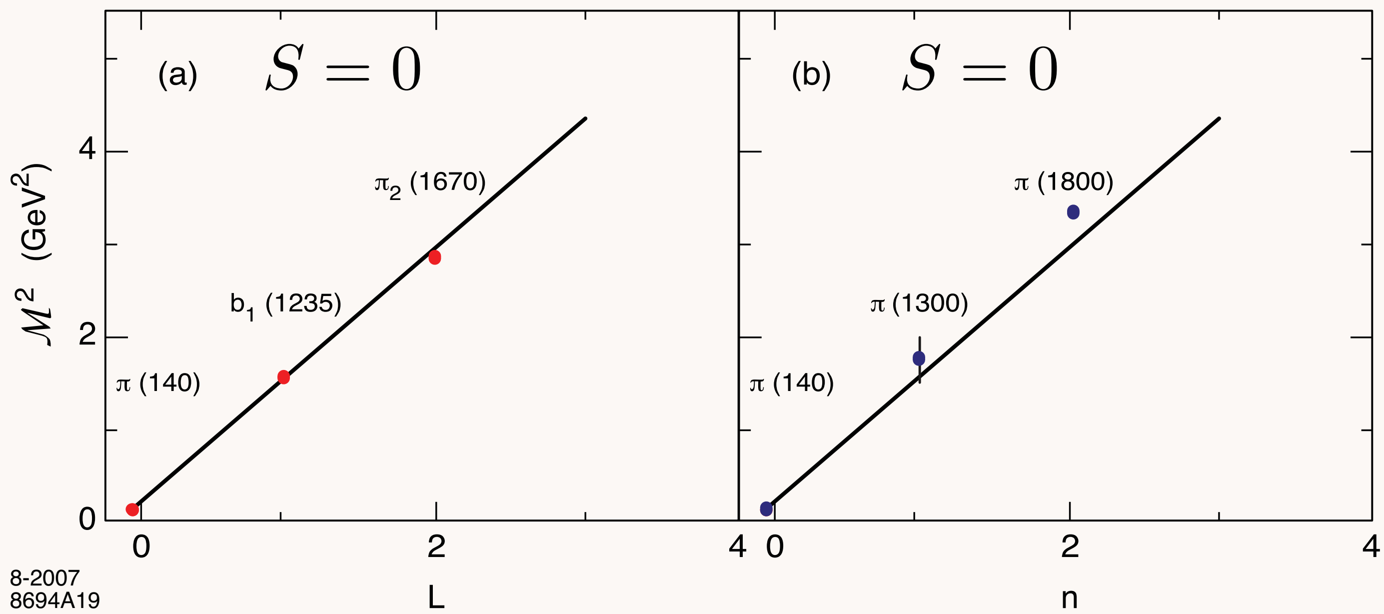


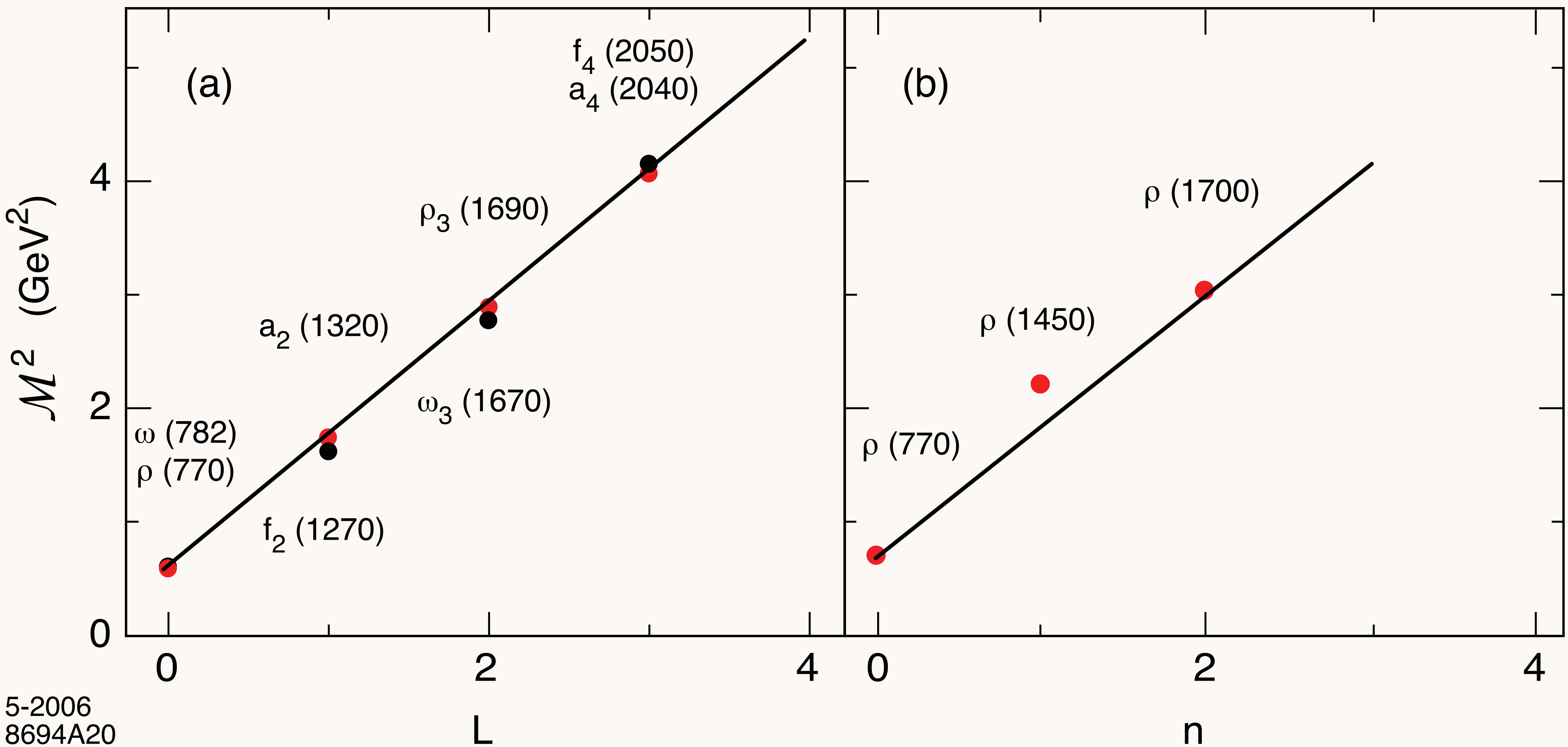
Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



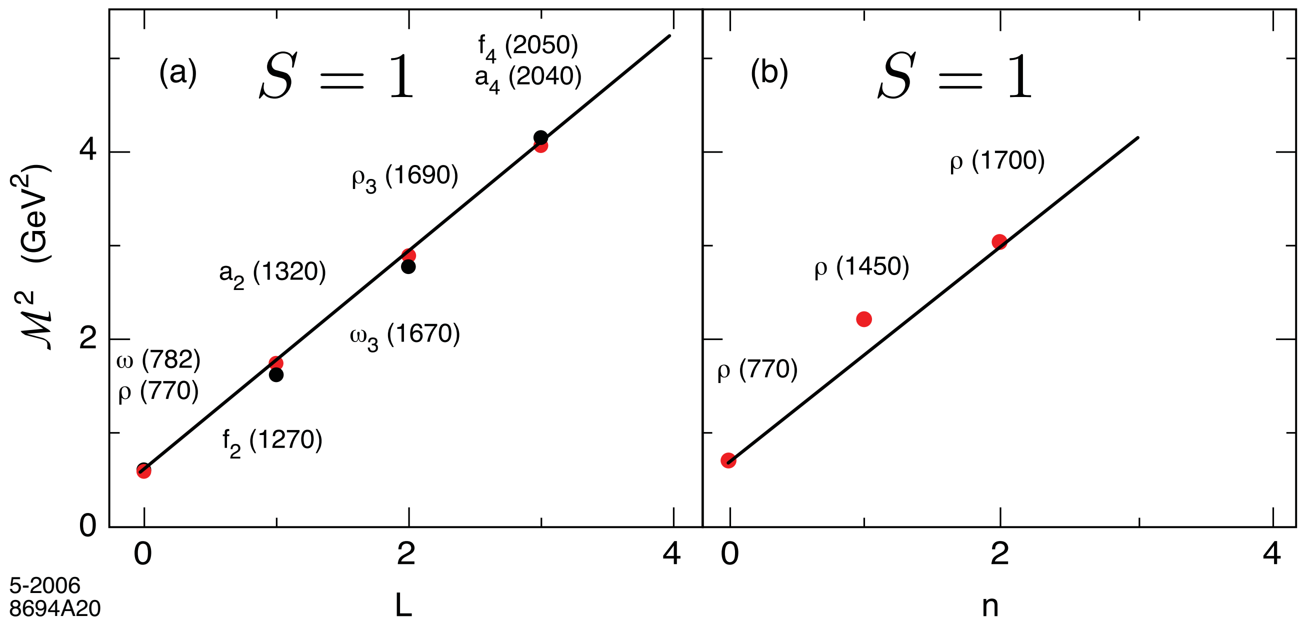
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

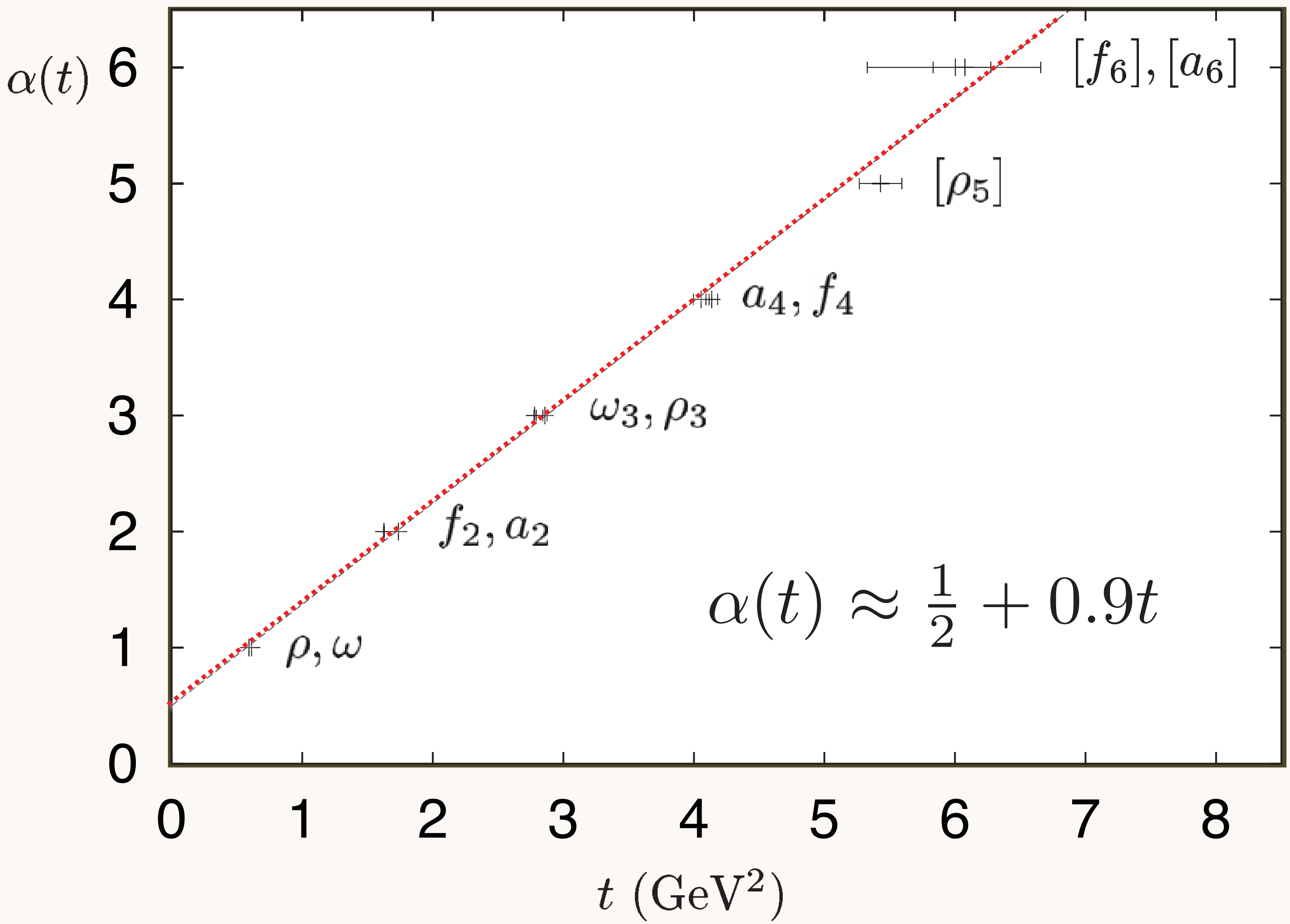
with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$.

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\left(n+L+1/2\right)$.

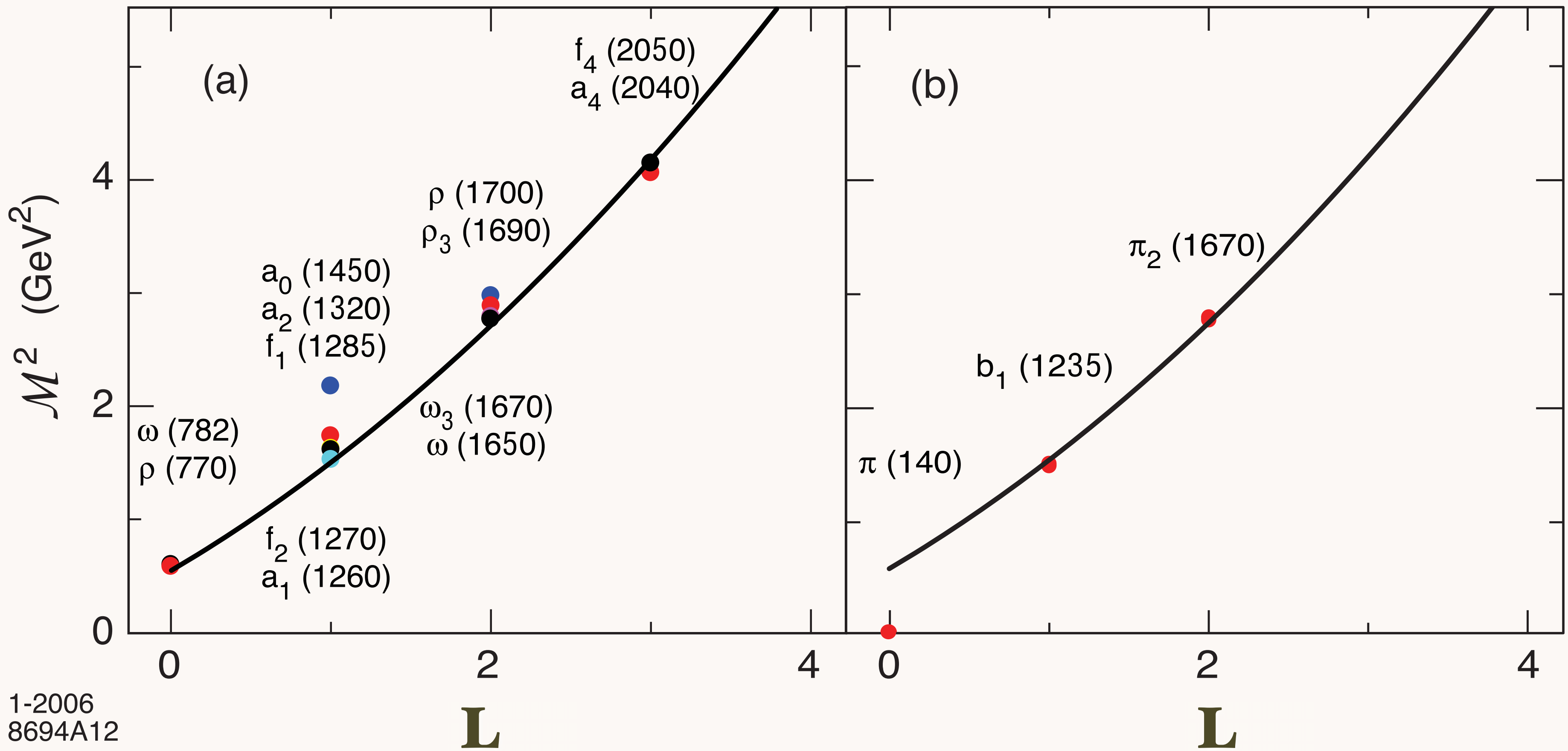


Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

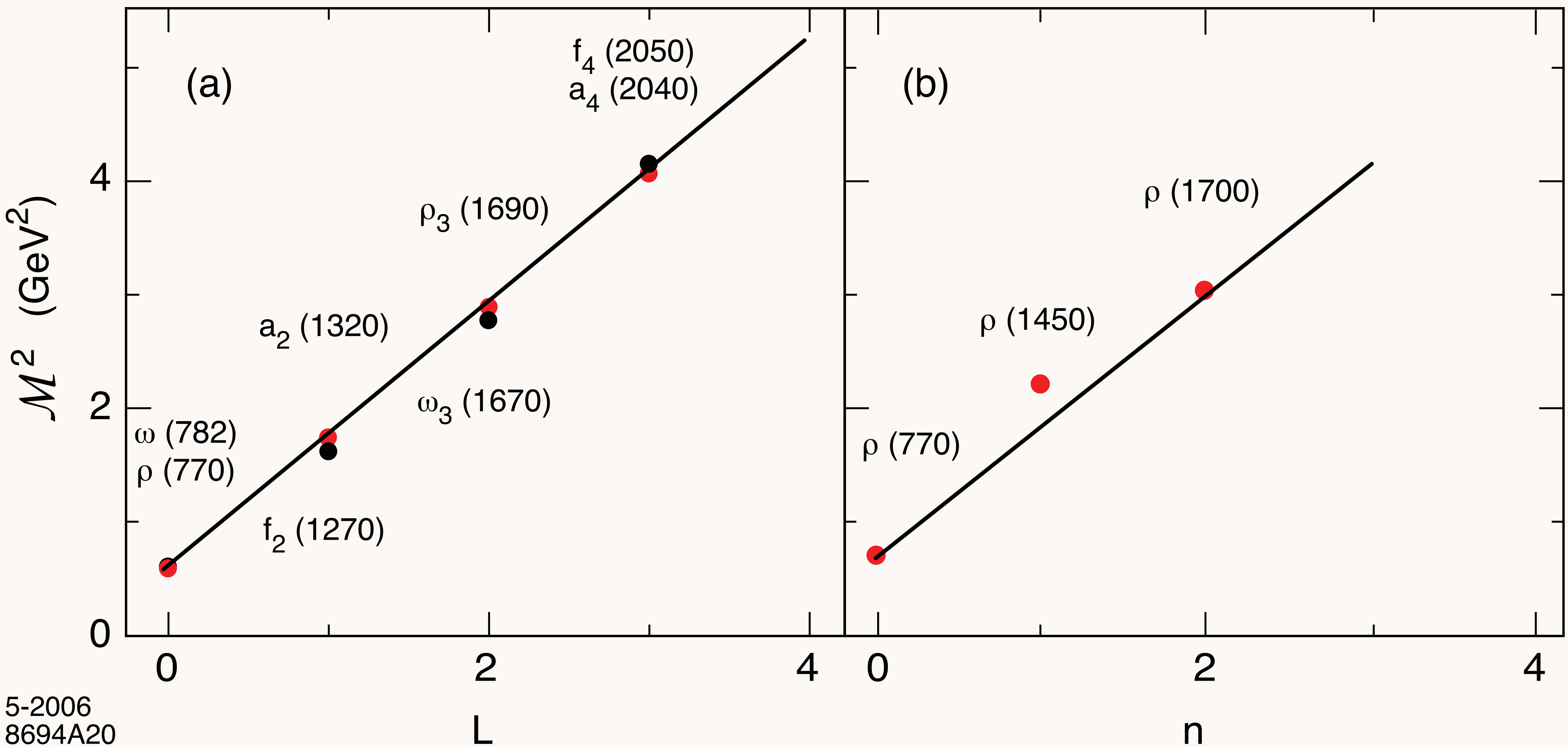


Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Guy de Teramond
SJB

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation: $\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \left[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P) \right]$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{QCD}, \qquad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Predictions of AdS/CFT

Entire light quark baryon spectrum

Only one parameter!

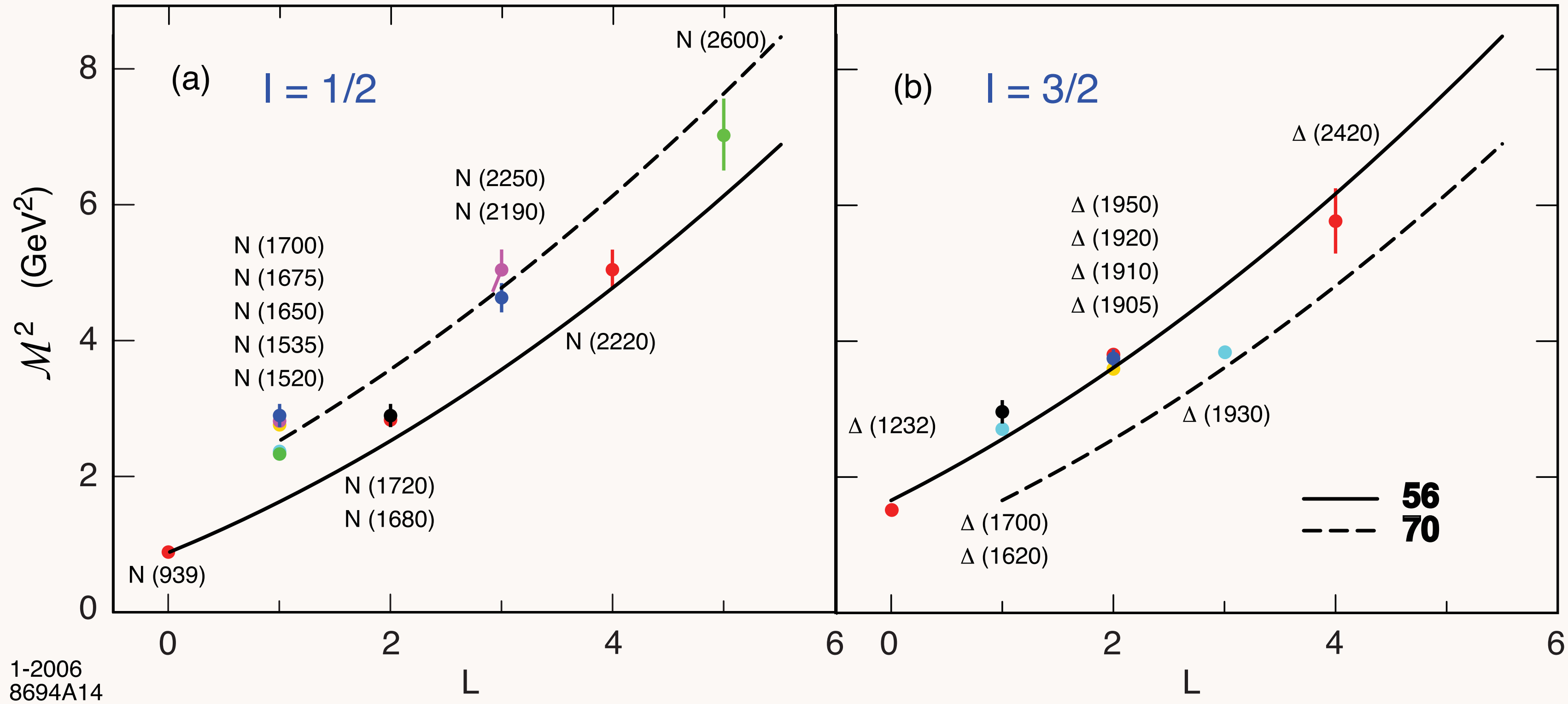


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

de Teramond, sjb

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^{+}(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^{+}(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^{-}(1535)$ $N\frac{3}{2}^{-}(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^{-}(1650)$ $N\frac{3}{2}^{-}(1700)$ $N\frac{5}{2}^{-}(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^{-}(1620)$ $\Delta\frac{3}{2}^{-}(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^{+}(1720)$ $N\frac{5}{2}^{+}(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^{+}(1910)$ $\Delta\frac{3}{2}^{+}(1920)$ $\Delta\frac{5}{2}^{+}(1905)$ $\Delta\frac{7}{2}^{+}(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}$
	$\frac{3}{2}$	3	$N\frac{3}{2}^{-}$ $N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}(2190)$ $N\frac{9}{2}^{-}(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^{-}(1930)$ $\Delta\frac{7}{2}^{-}$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^{+}$ $N\frac{9}{2}^{+}(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^{+}$ $\Delta\frac{7}{2}^{+}$ $\Delta\frac{9}{2}^{+}$ $\Delta\frac{11}{2}^{+}(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}$
	$\frac{3}{2}$	5	$N\frac{7}{2}^{-}$ $N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}(2600)$ $N\frac{13}{2}^{-}$

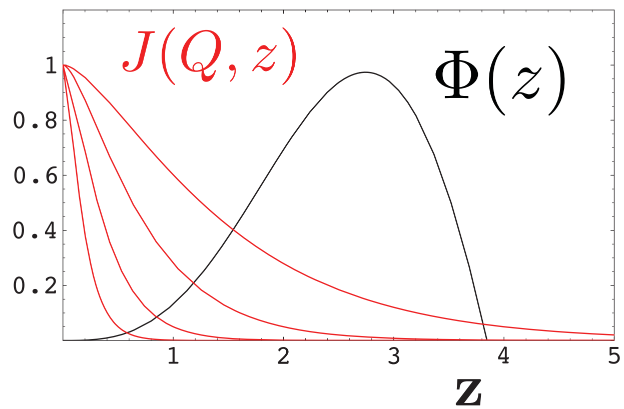
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

$$J(Q,z) = zQK_1(zQ)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

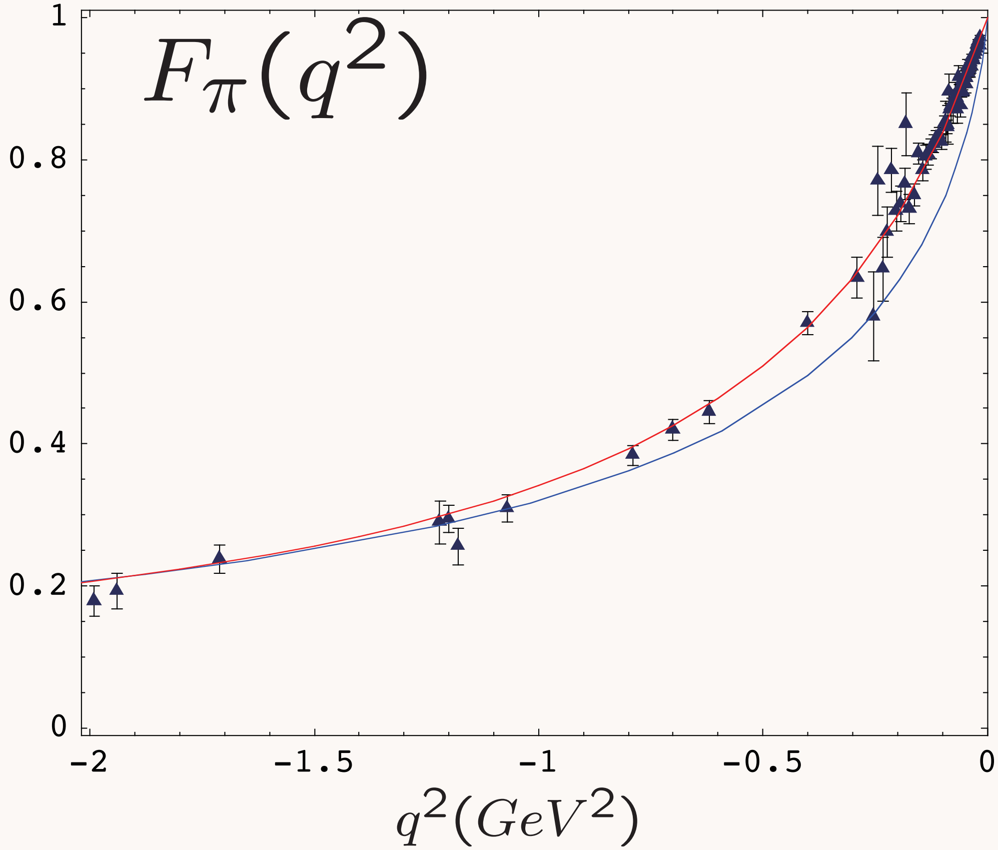
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ
scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

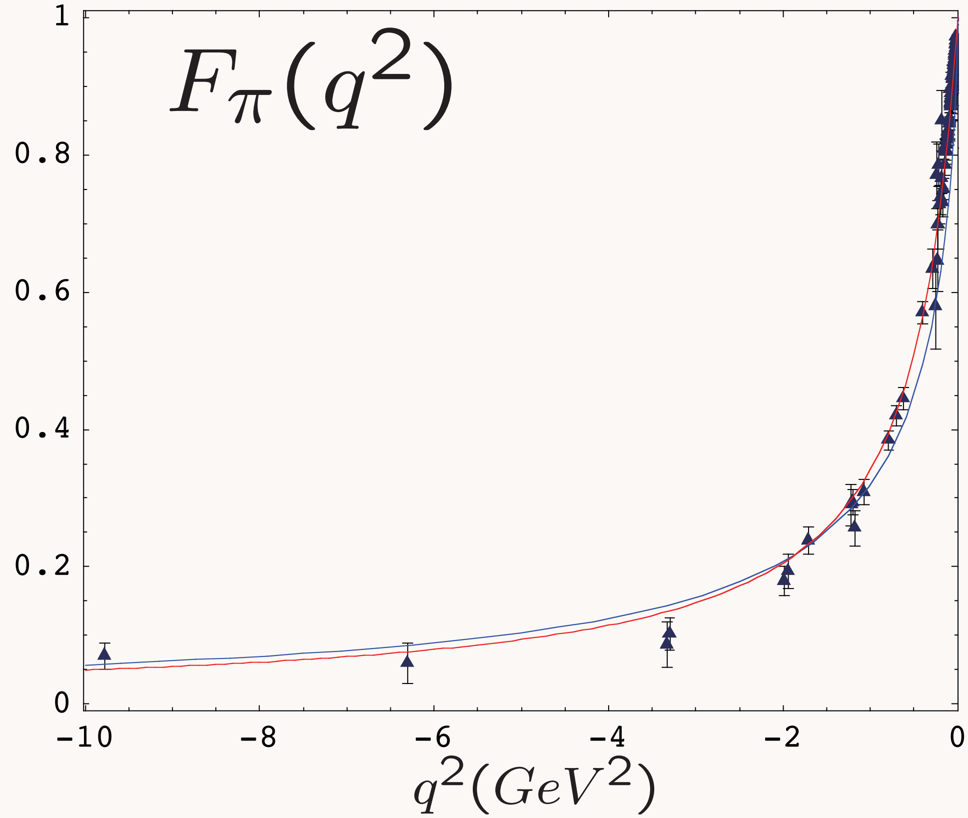
$$F(Q^2) \rightarrow \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT





Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

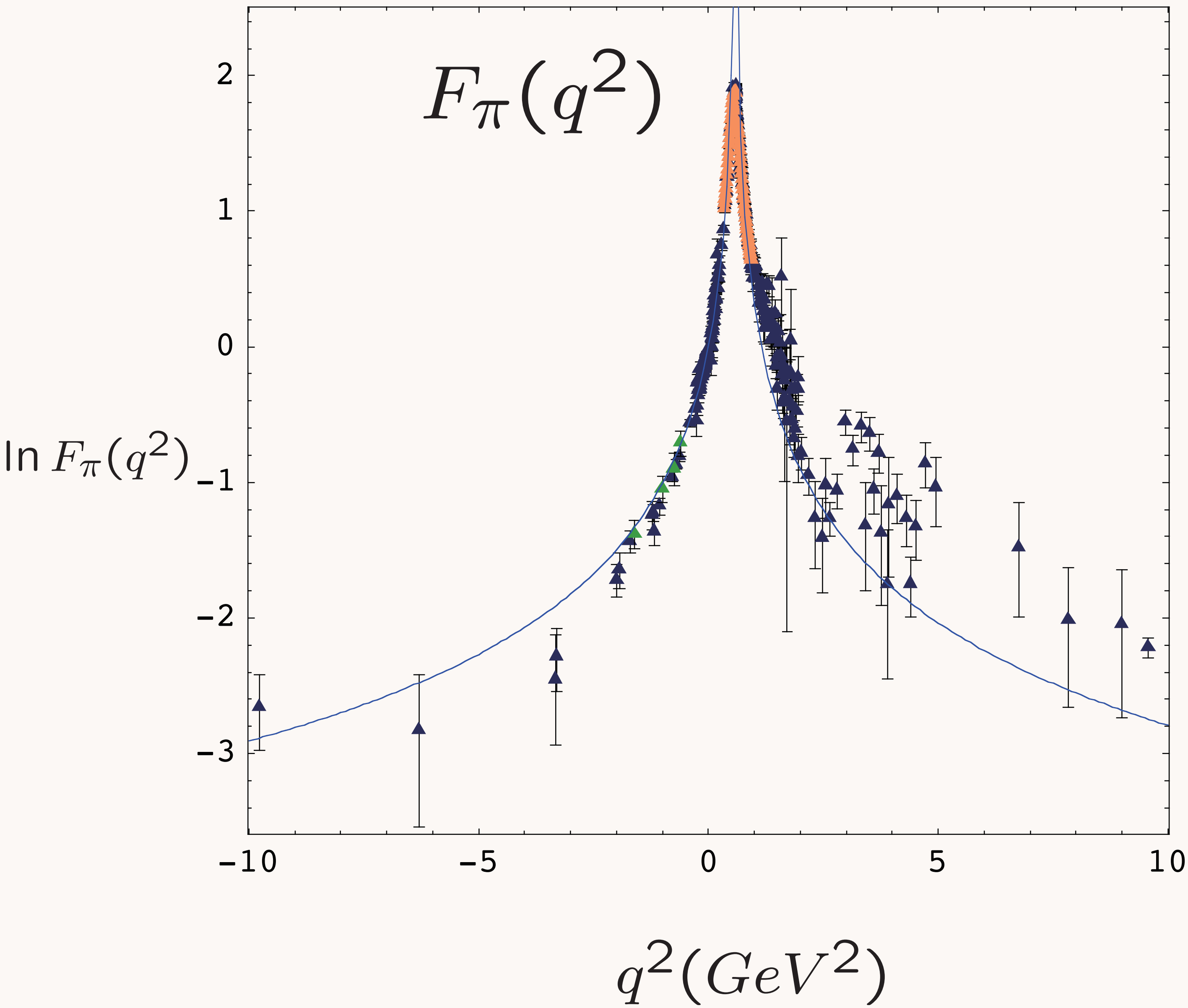
Truncated Space Confinement

One parameter - set by pion decay constant

G. de Teramond, sjb

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} \, J(Q,z) \, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

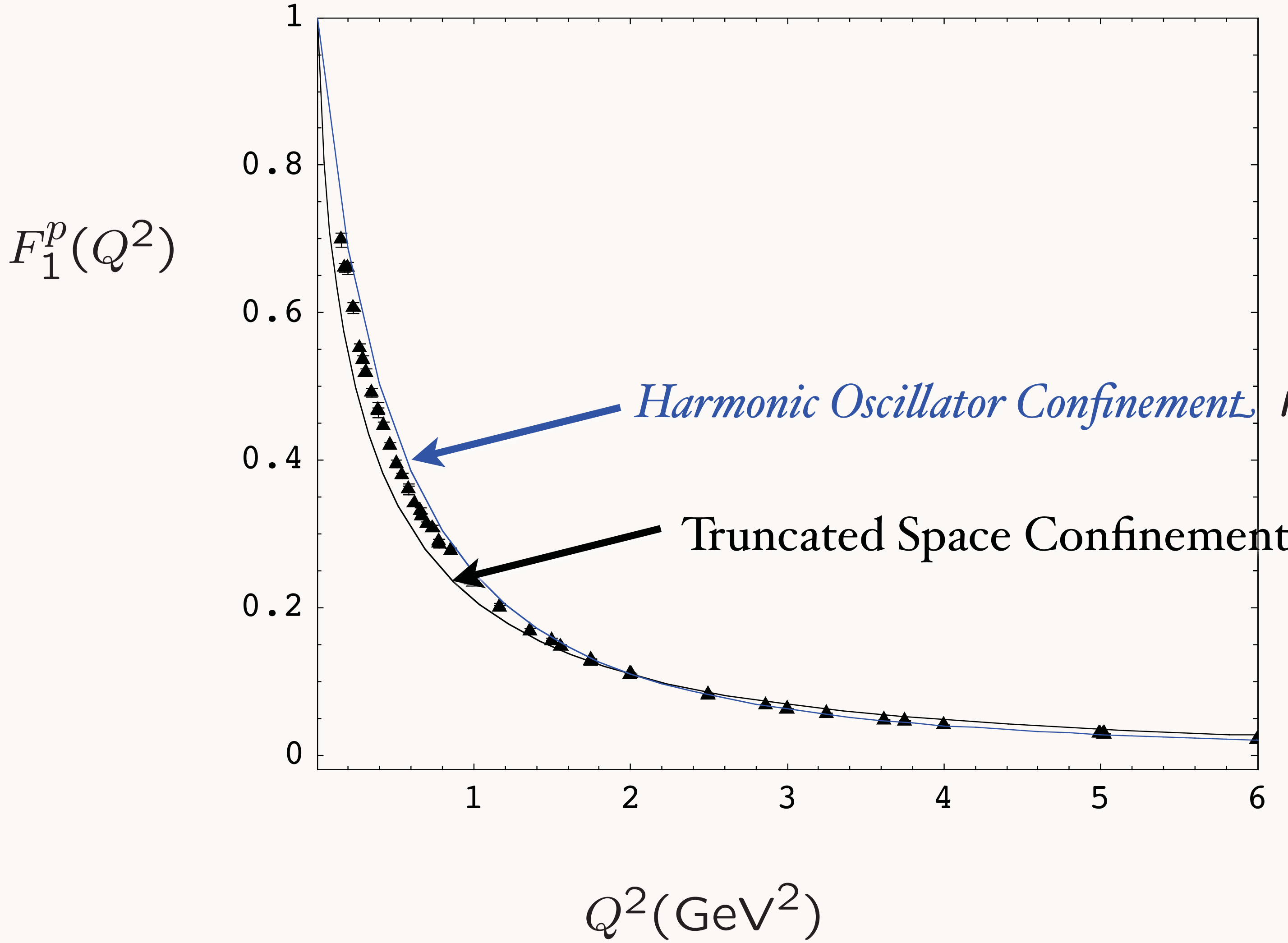
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} \, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb



$\kappa = 0.424$ GeV

$\Lambda = 0.2$ GeV

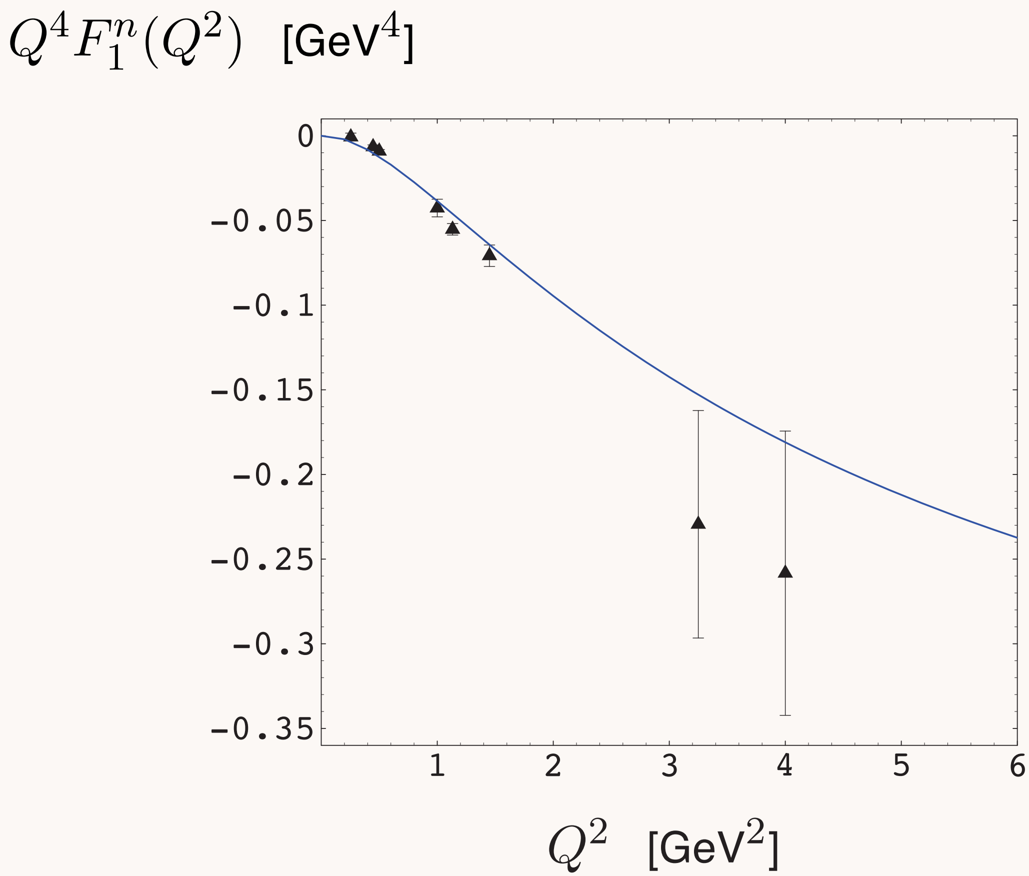
Current modified by metric

$$F_1(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F^{\uparrow}(z) J(Q,z) \Phi_I^{\uparrow}(z)$$

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

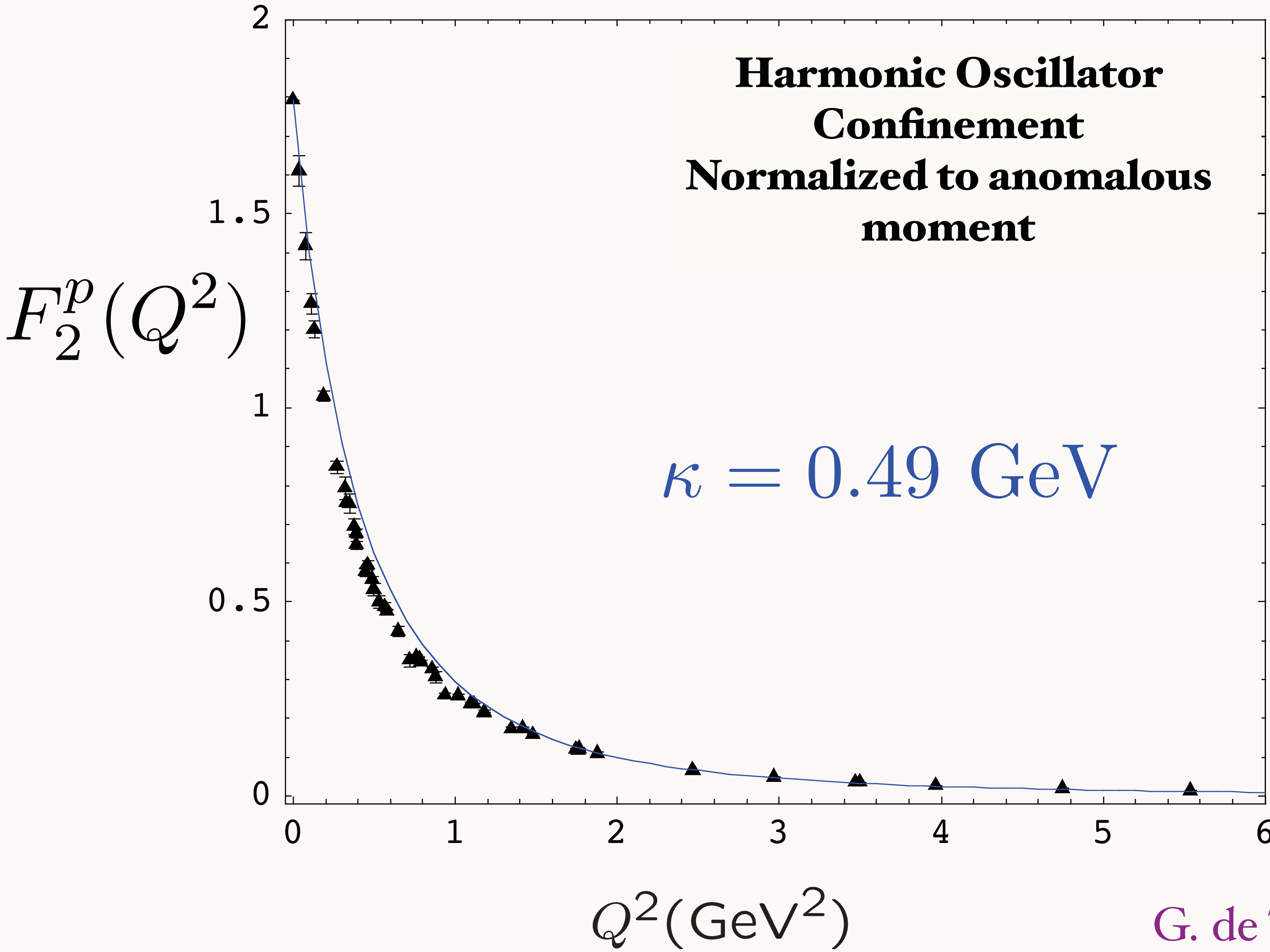


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



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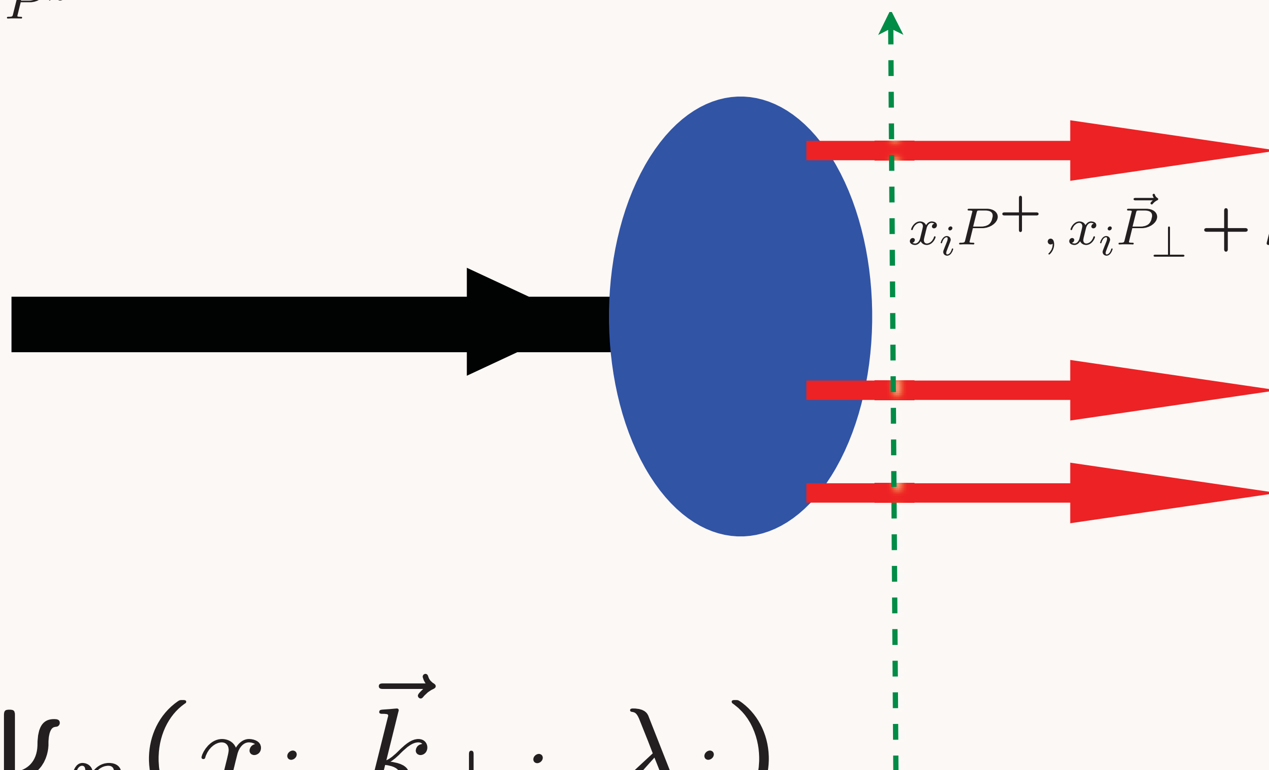
Light-Front Wavefunctions

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

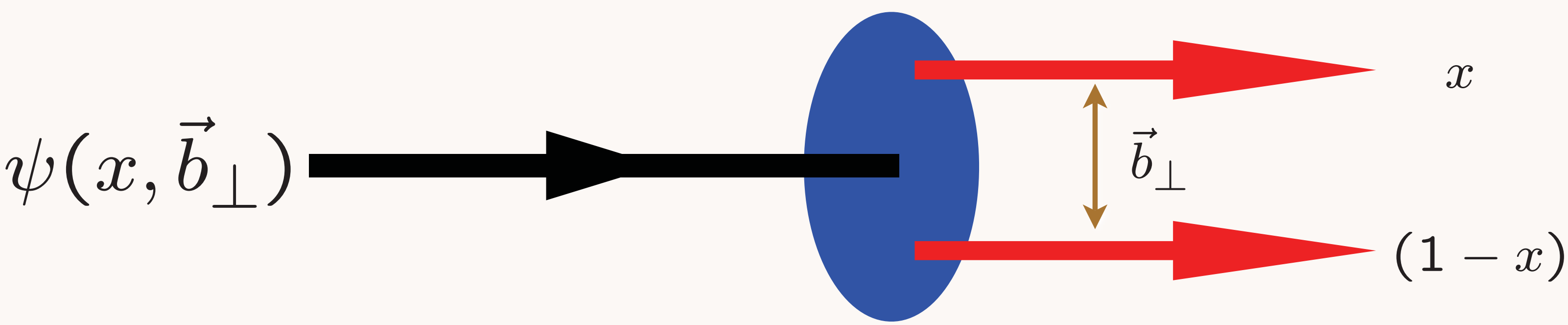
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

LF(3+1) $\quad\longleftrightarrow\quad$ AdS_5

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

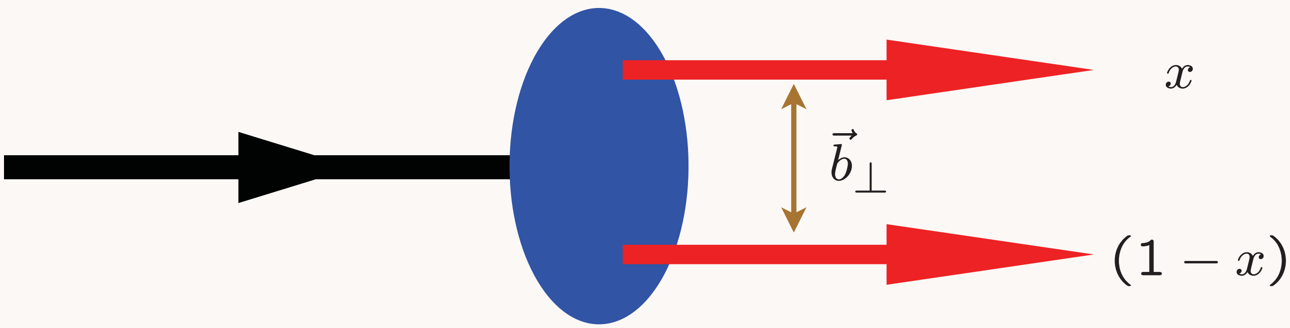
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Holography:
Map AdS/CFT to 3+1 LF Theory

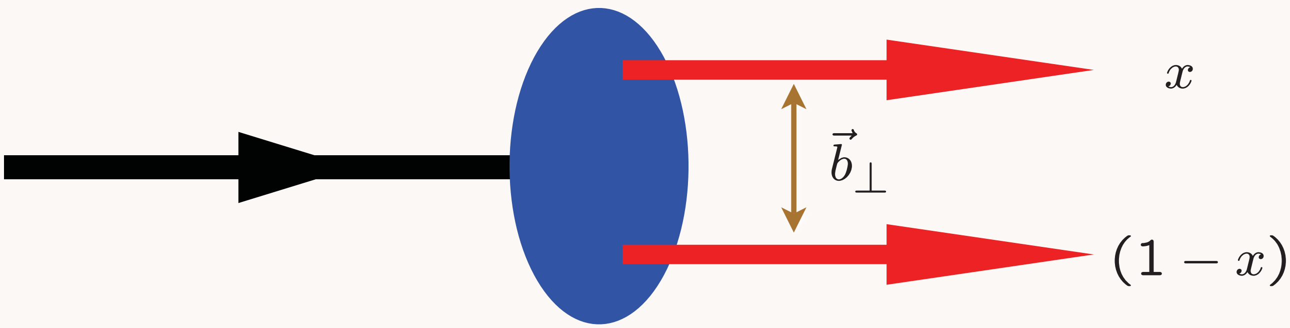
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

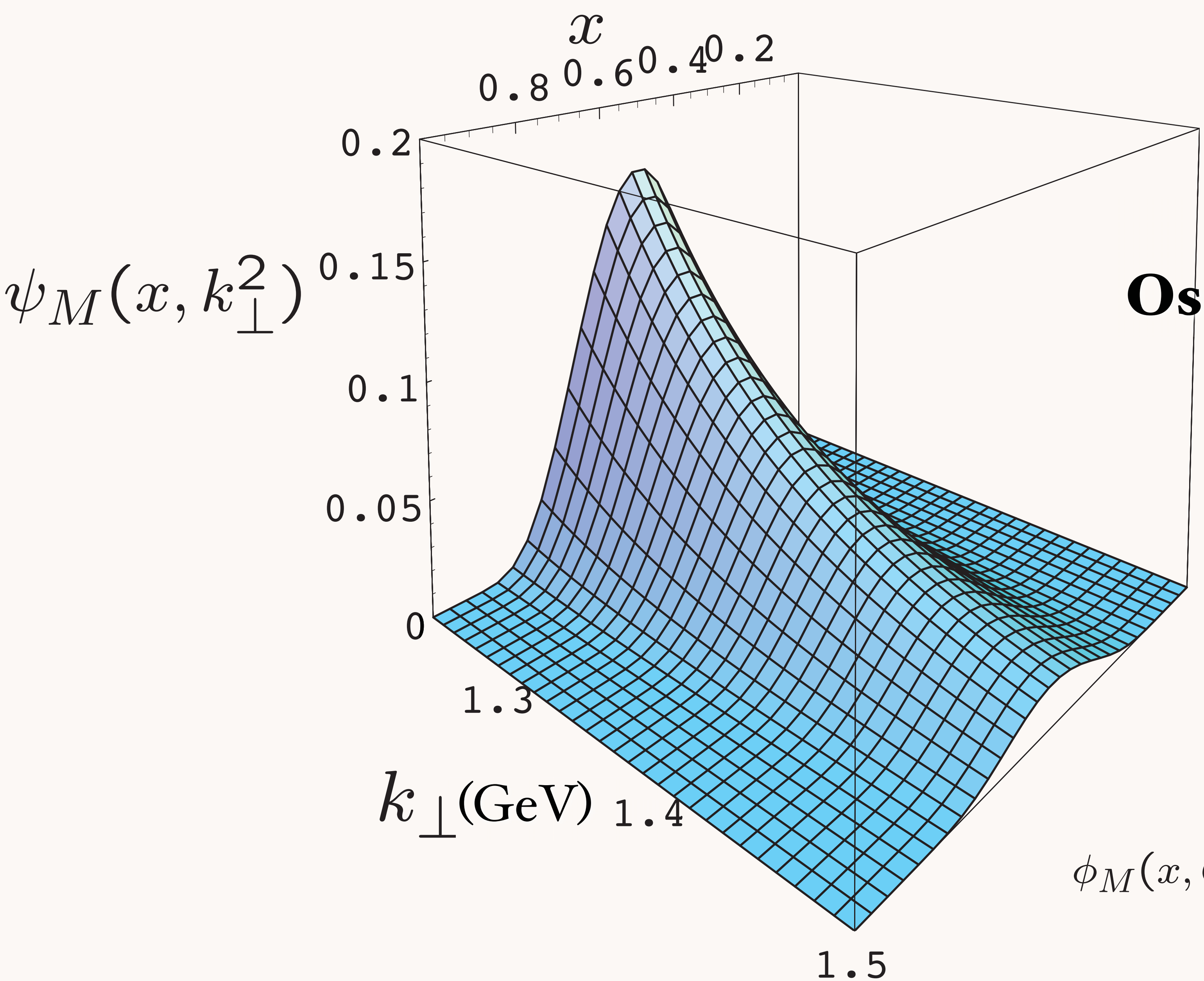
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

AdS/QCD

Prediction from AdS/CFT: Meson LFWF

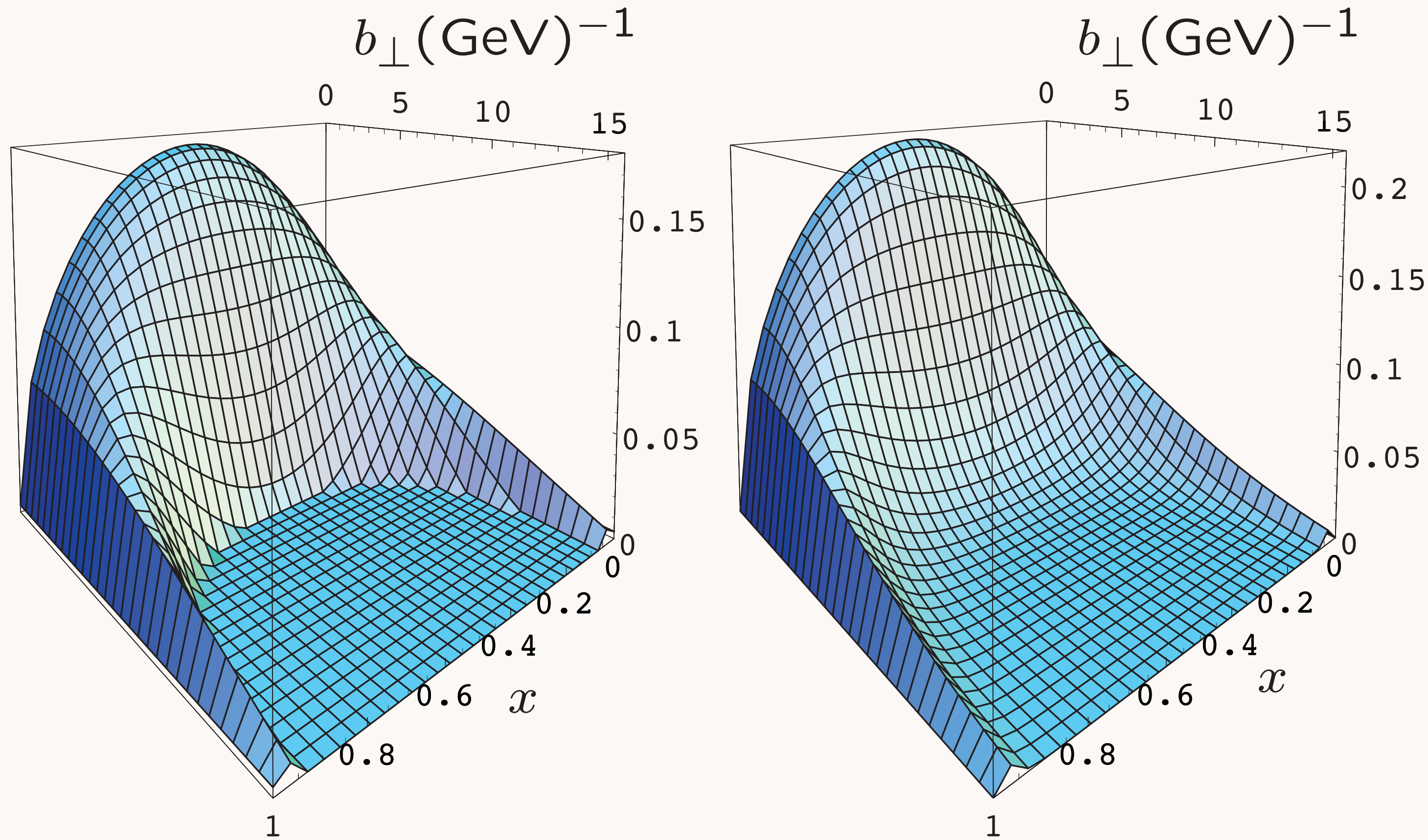


Harmonic Oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

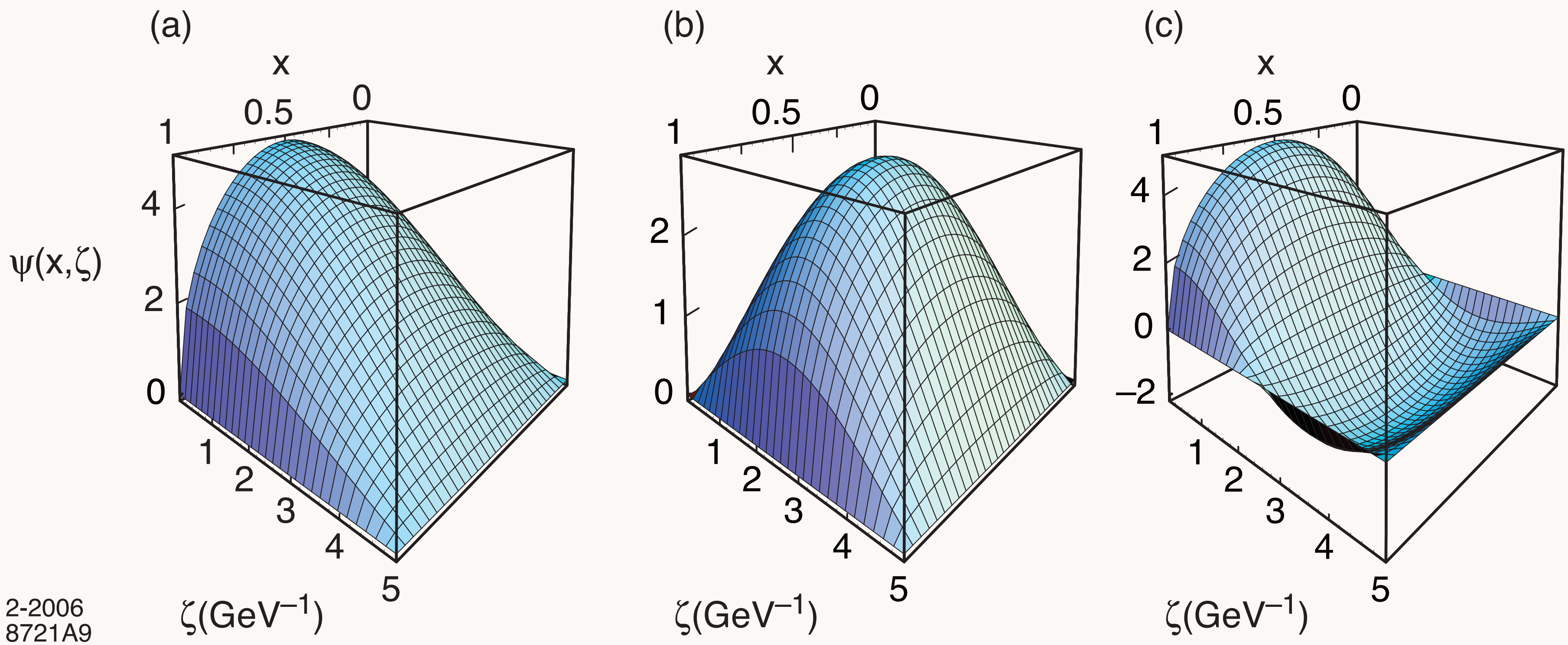
AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{\text{QCD}} = 0.32\ \text{GeV}$ $\kappa = 0.76\ \text{GeV}$

Truncated Space ***Harmonic Oscillator***

Guy de Teramond
SJB



Two-quark holographic LFWF in impact space $\psi(x,\zeta)$: $(a)\;\ell = 0,\; k = 1;\; (b)\;\ell = 1,\; k = 1;$ $(c)\;\ell = 0,\; k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

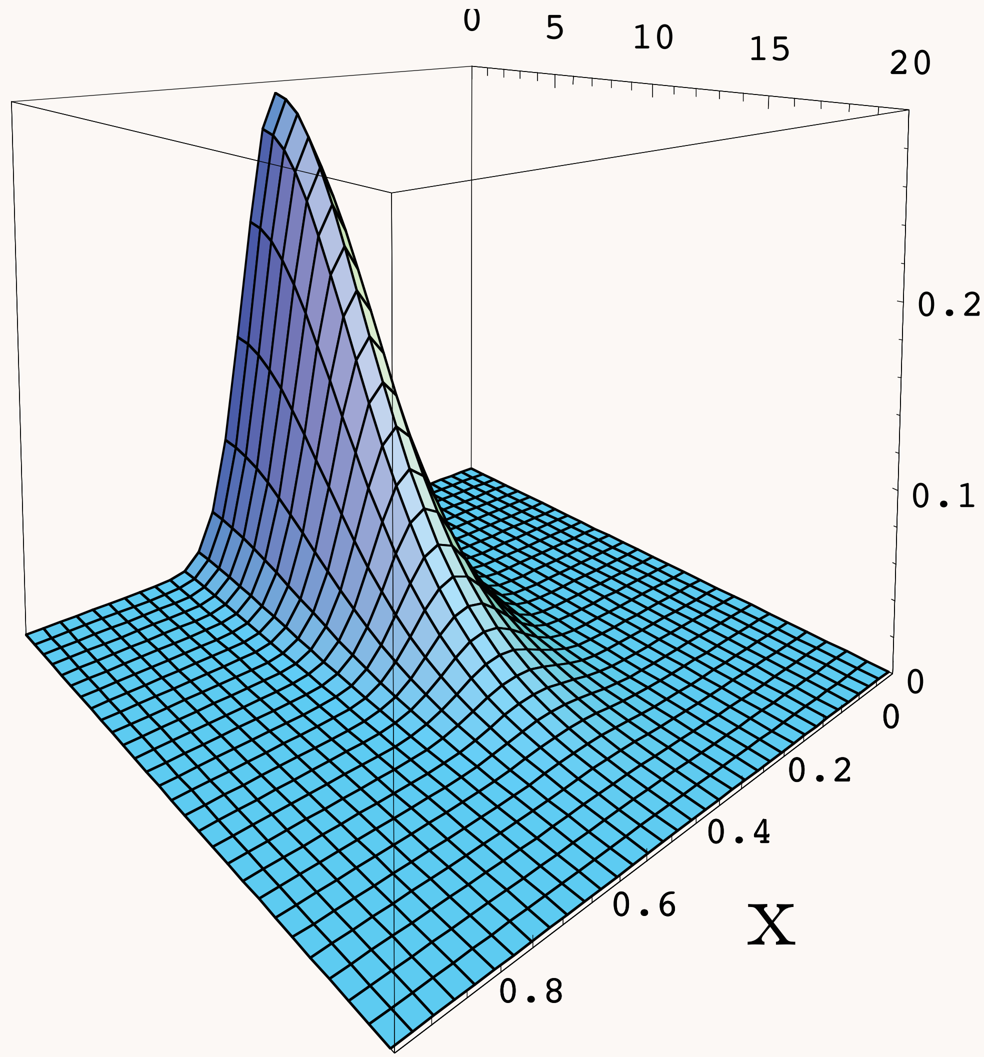
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$\psi_{J/\psi}(x, b)$$

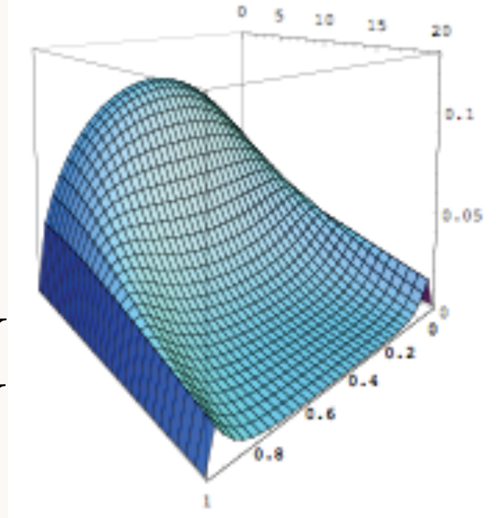


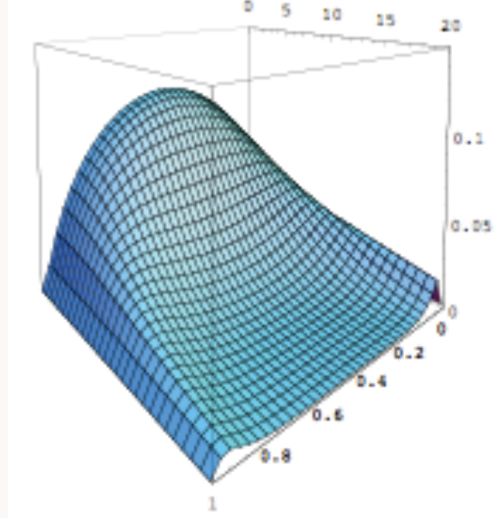
$$m_a = m_b = 1.25 \text{ GeV}$$

AdS/QCD

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



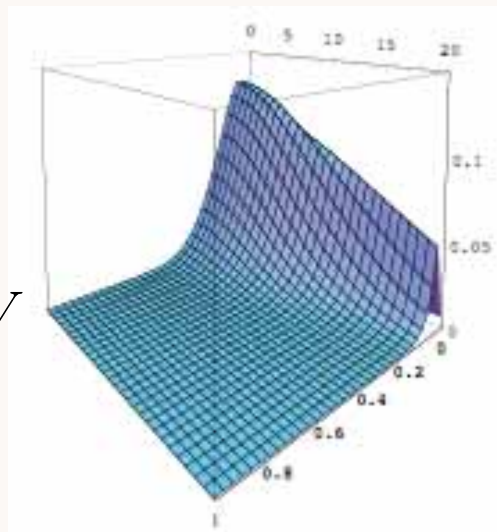


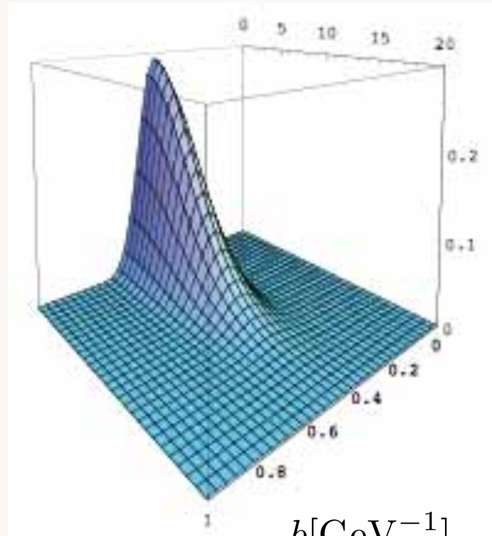
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

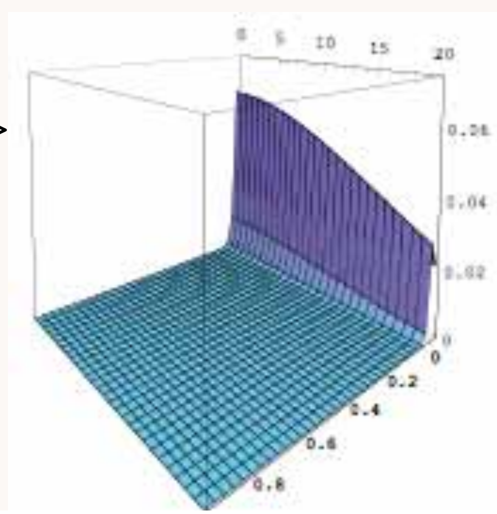


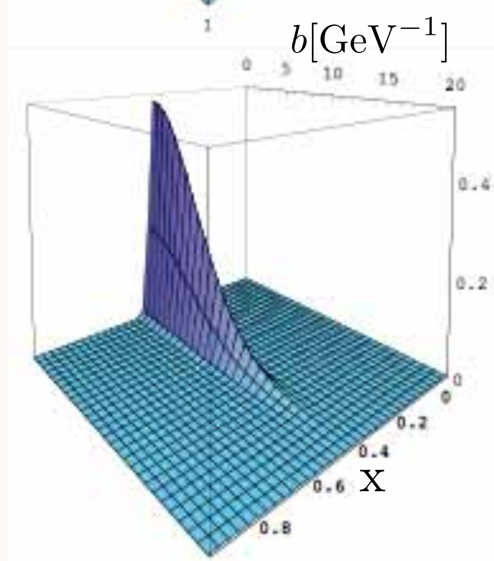


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2 \, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0 \left[b \, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

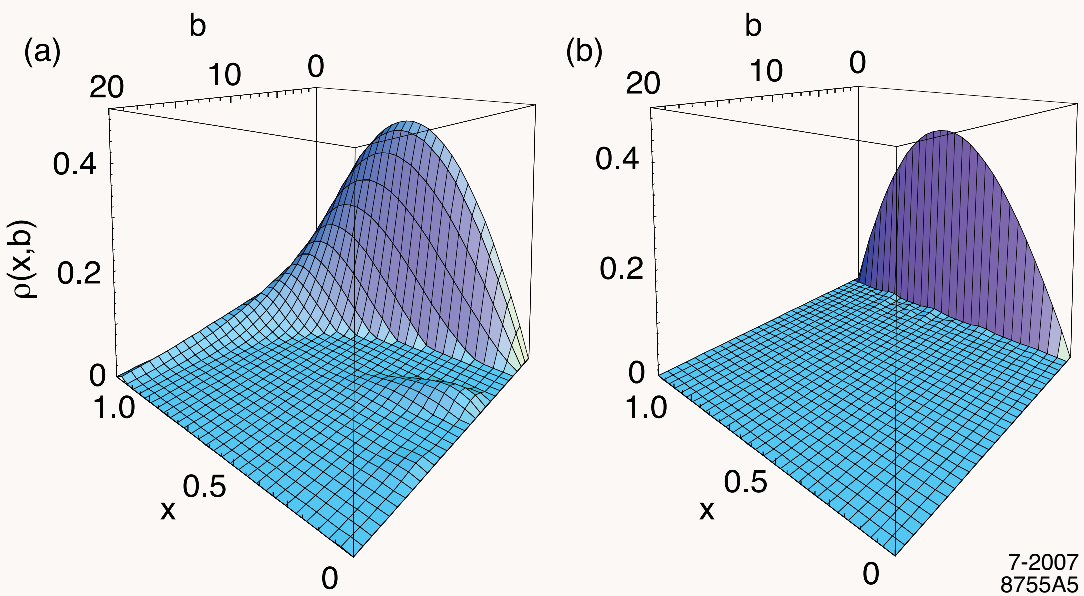
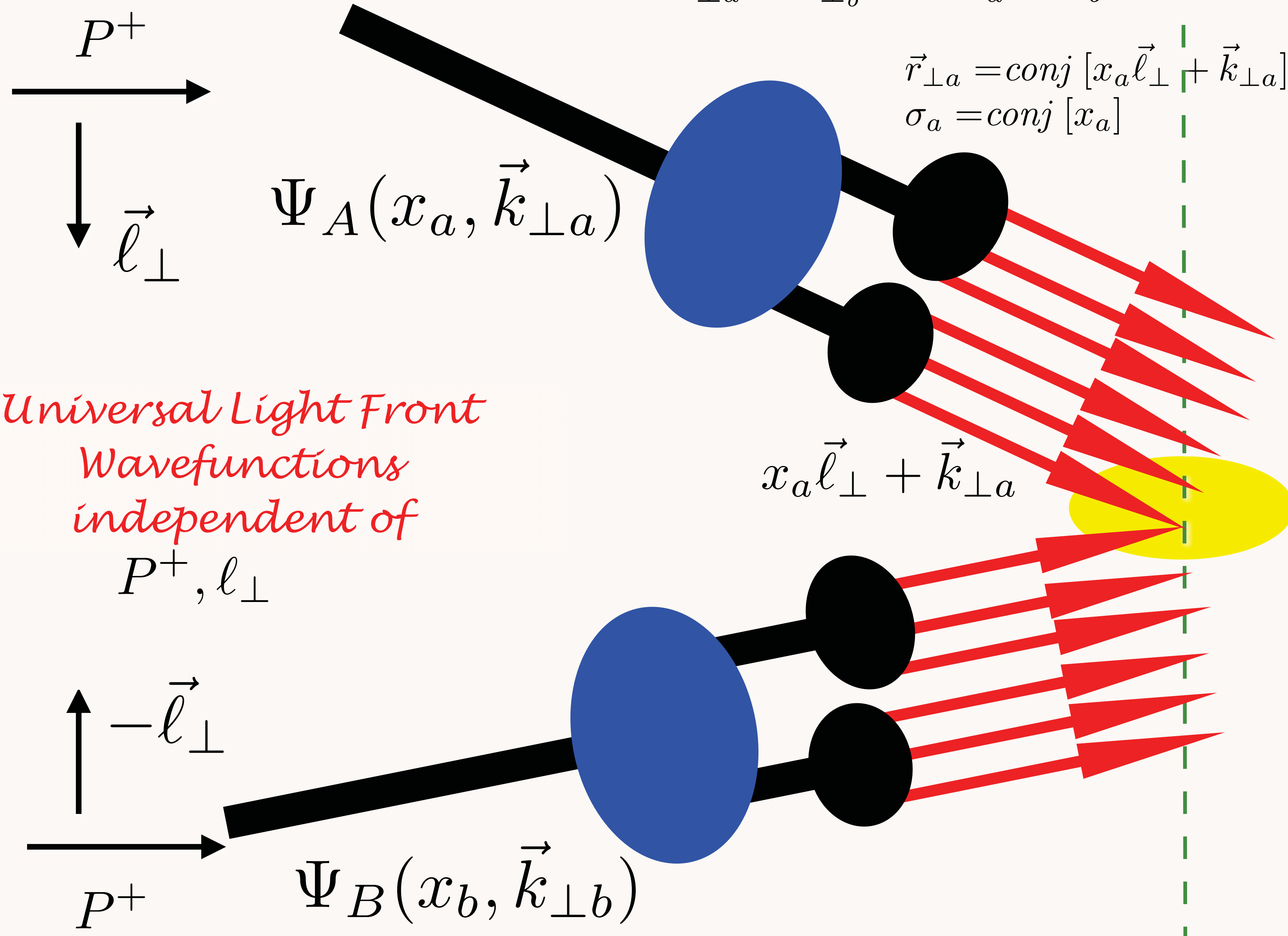
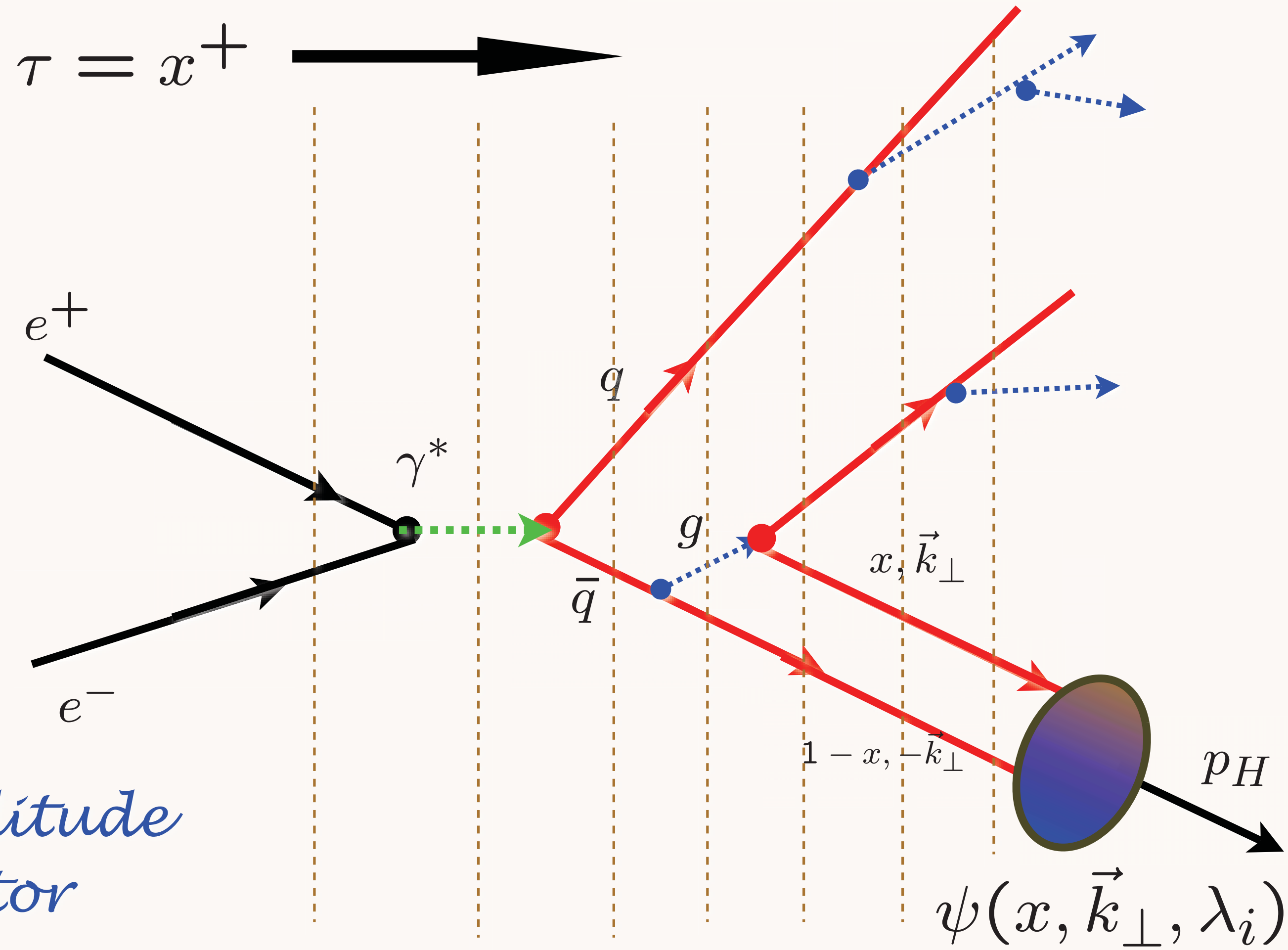


Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

Interaction of a and b when $\vec{r}_{\perp a} \simeq \vec{r}_{\perp b}$ and $\sigma_a \simeq \sigma_b$



Hadronization at the Amplitude Level



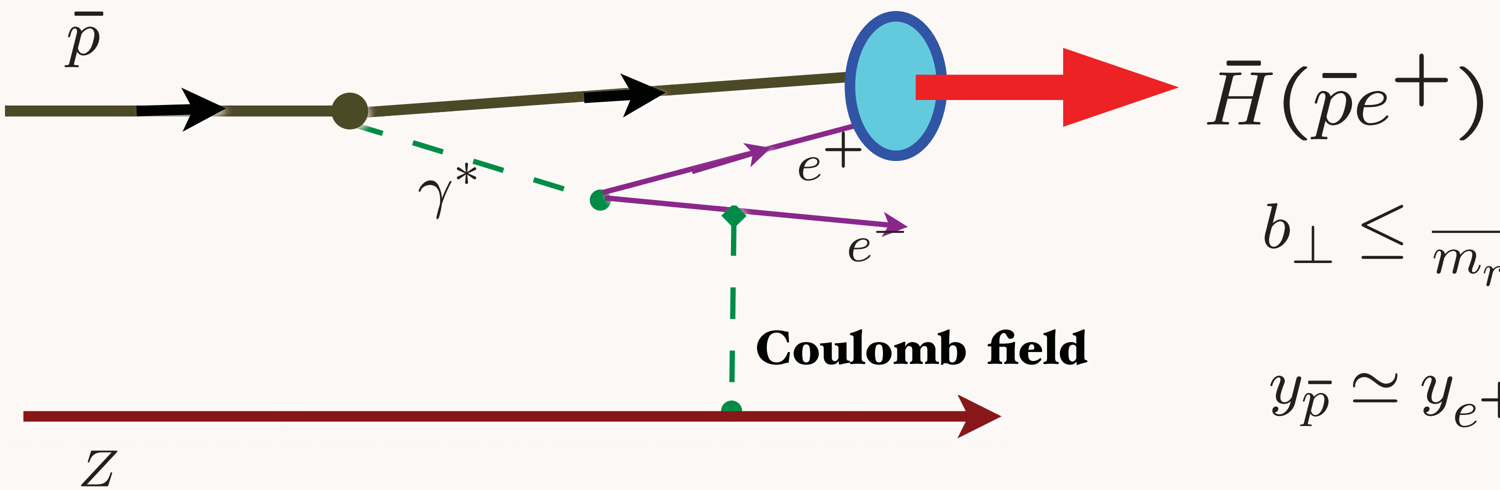
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

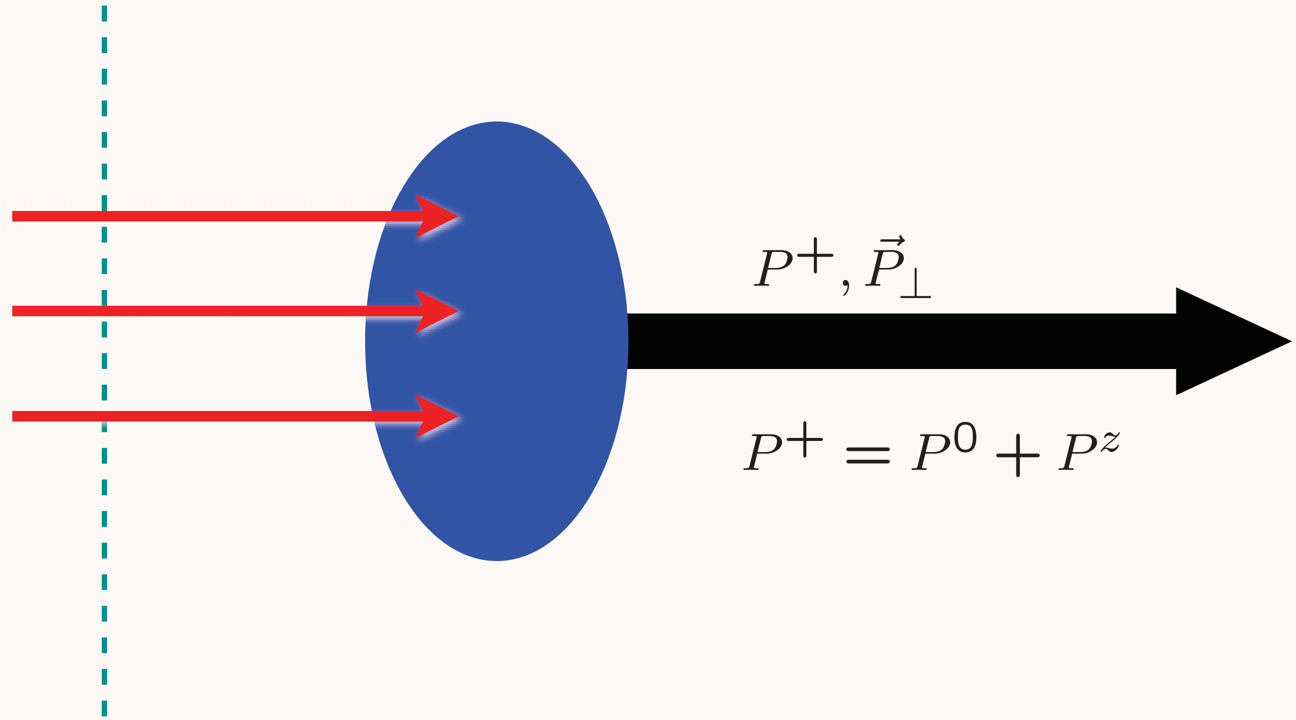
Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$



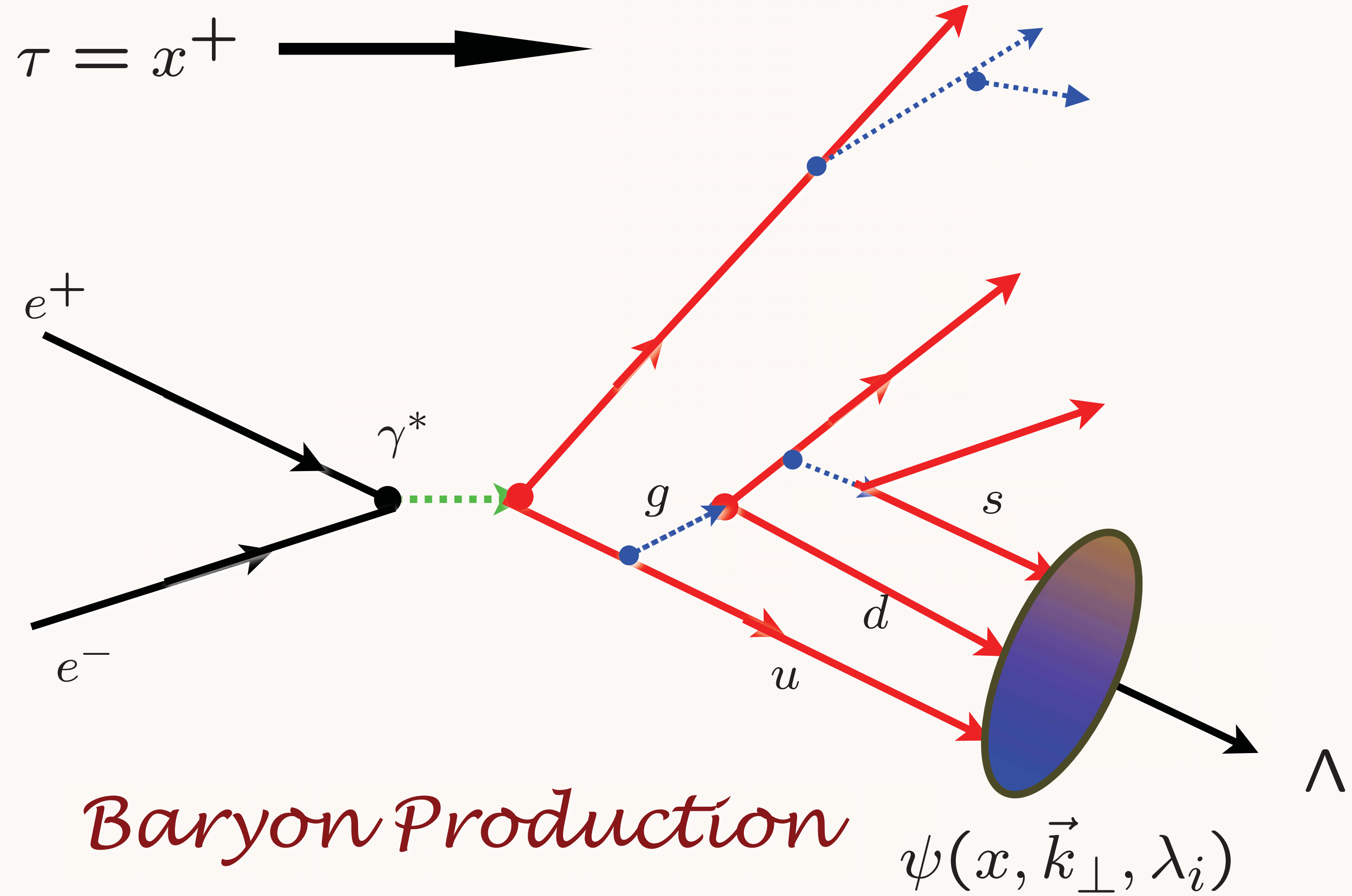
$P^+, \vec{P}_\perp$

$P^+ = P^0 + P^z$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

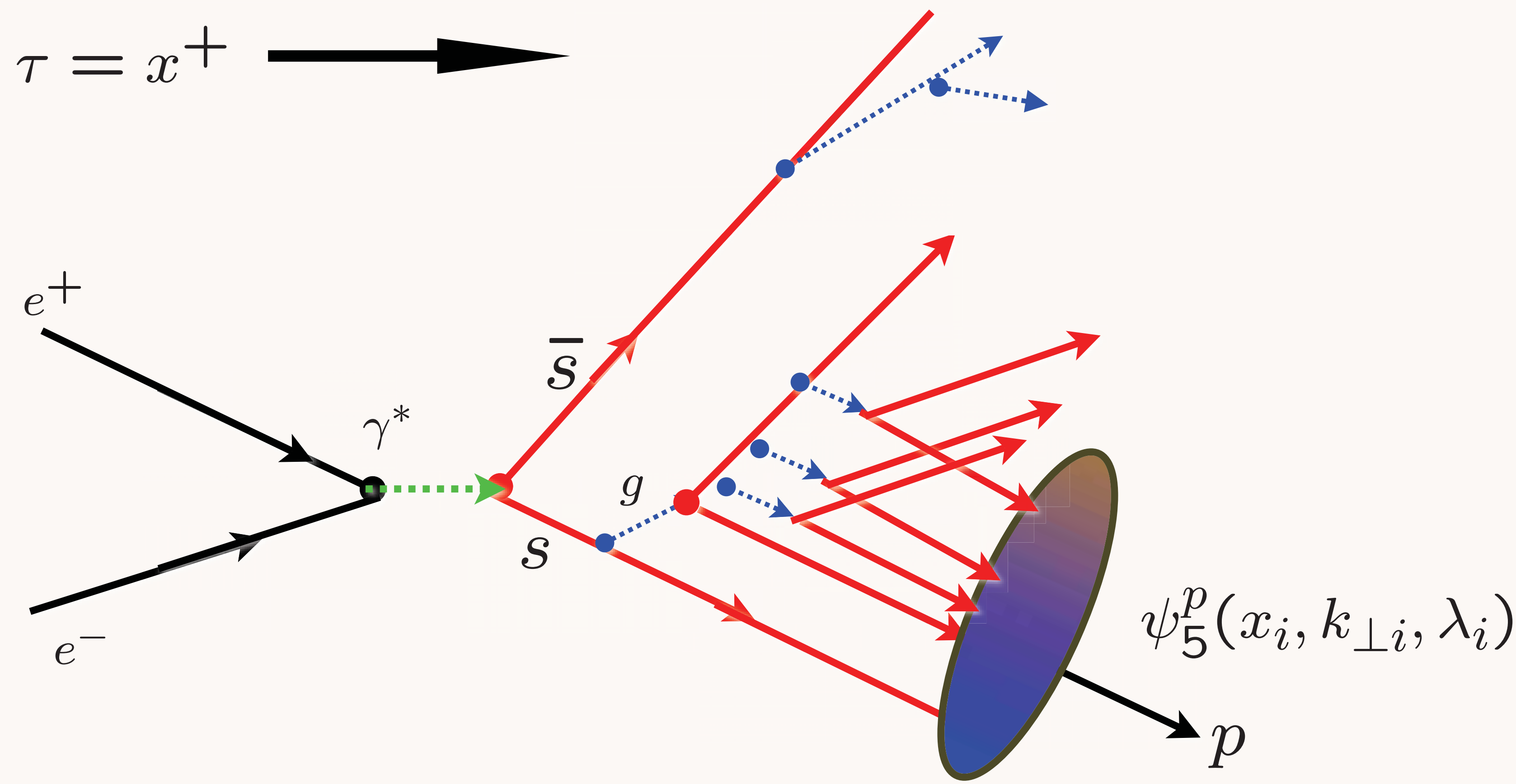
Invariant under boosts! Independent of P^μ

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



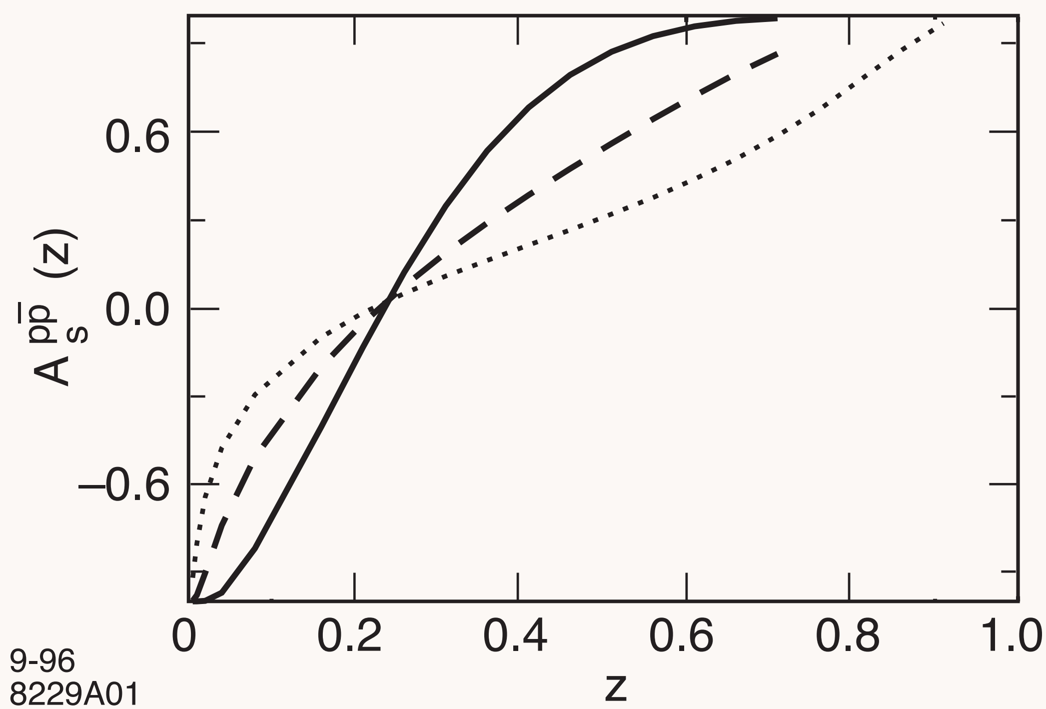
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$

B-Q Ma, sjb

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to\bar{p}}(z)}{D_{s\to p}(z) + D_{s\to\bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s} > \simeq |K^+\Lambda >$

AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momenta
- Hadron Spectra, Regge Trajectories

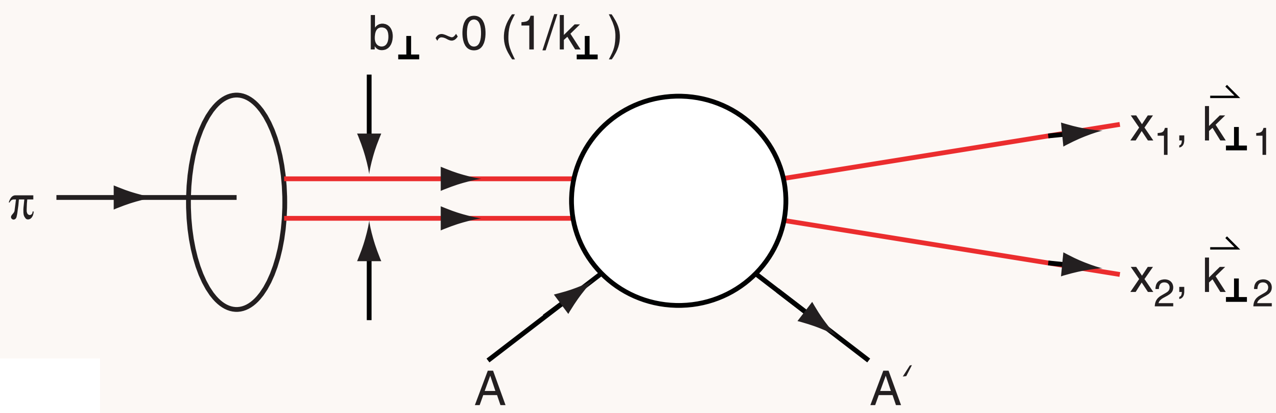
Features of HolographicModel

de Teramond sjb

- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.
- Scale Λ_{QCD} determines hadron spectrum (slightly different for mesons and baryons)
- Covariant version of bag model: confinement +conformal symmetry
- Pion decay constant
- Dominance of Quark Interchange

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



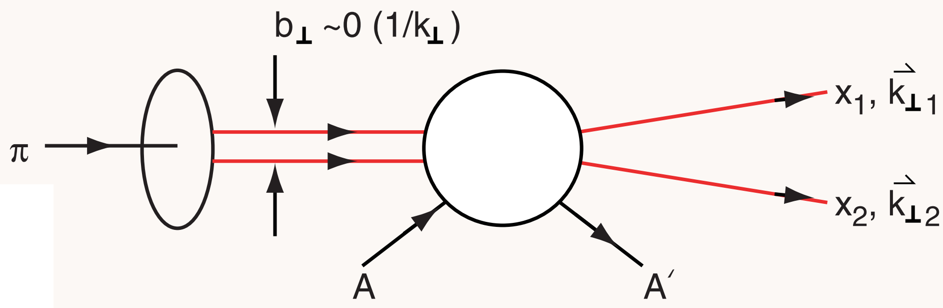
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

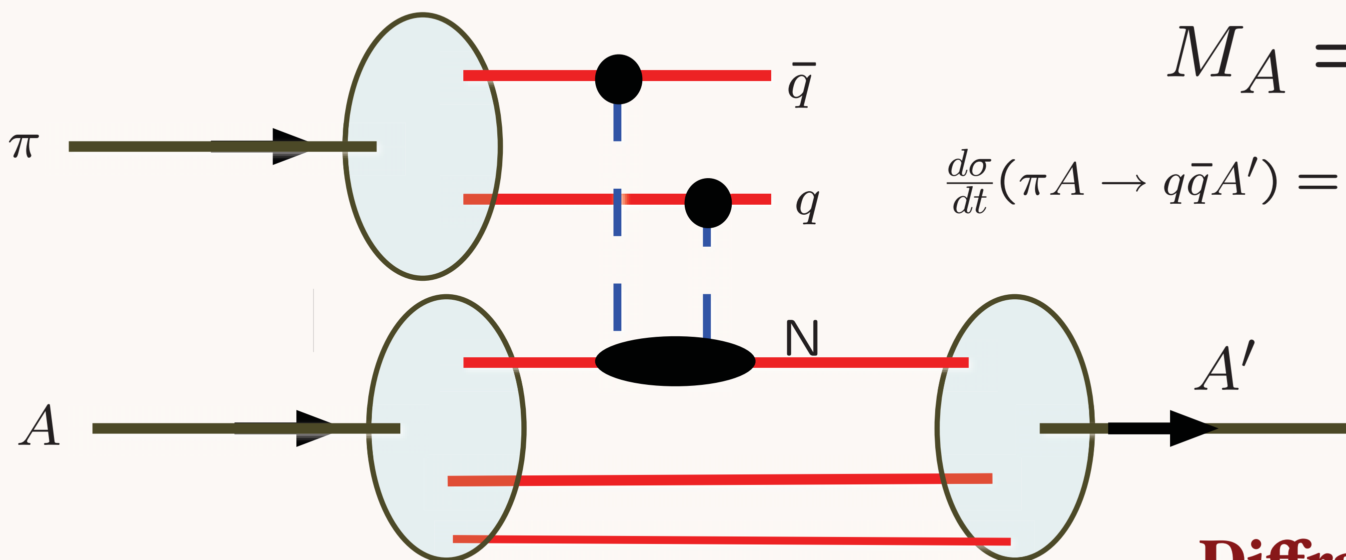
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with <u>each</u> nucleon coherently

<u>QCD COLOR Transparency</u>



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

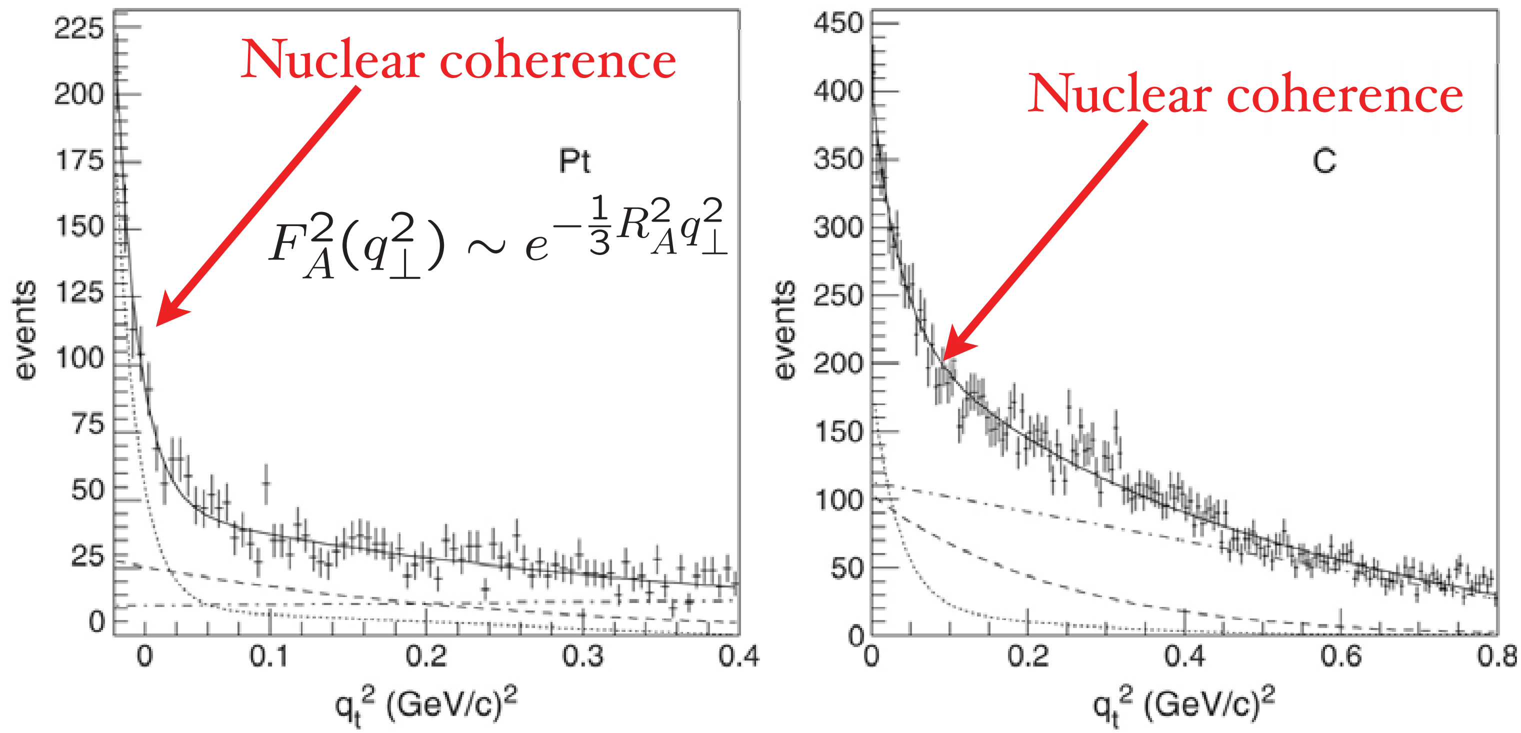
Diffraction, Rapidity gap

- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

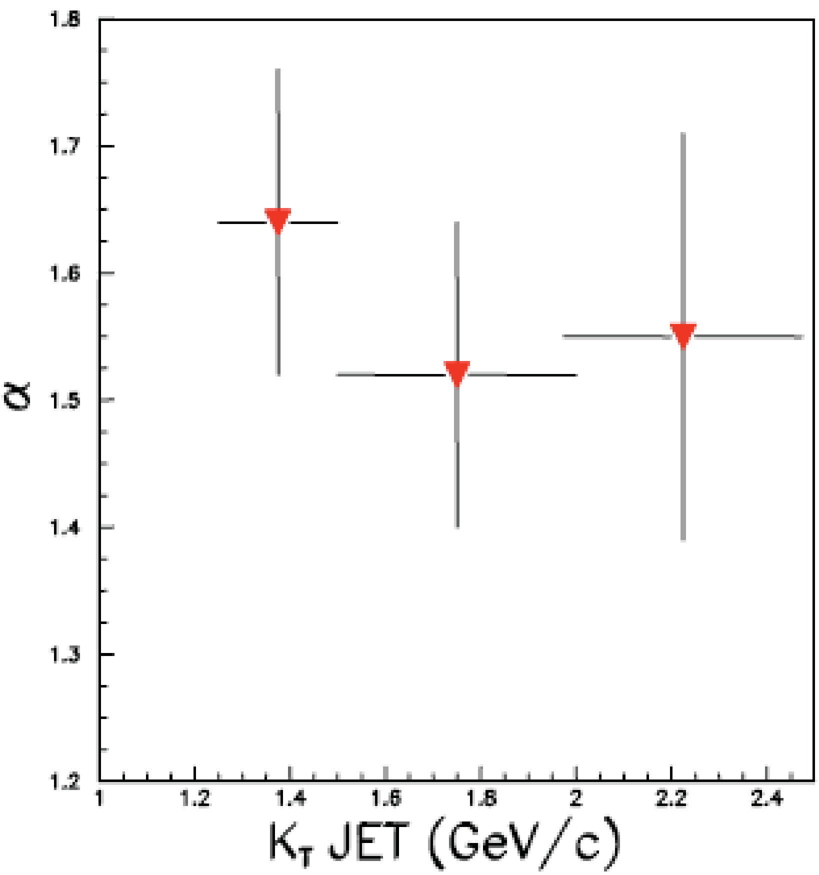
Ashery E791

α (Incoh.) $= 0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

Factor of 7

A(π,dijet) data from FNAL



Coherent π^+ diffractive dissociation with 500 GeV/c pions on Pt and C.

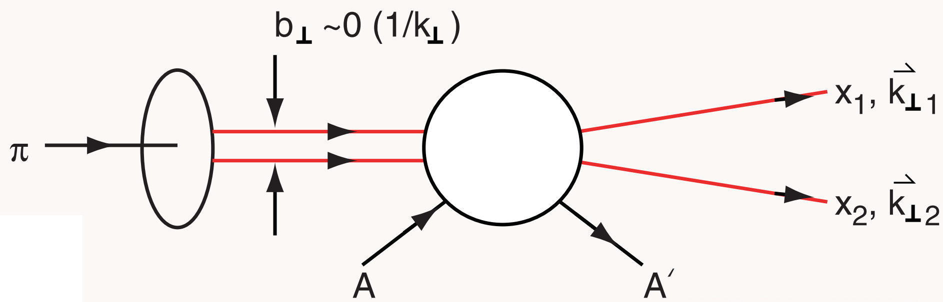
Fit to $\sigma = \sigma_0 A^\alpha$

α = 0.76 from pion-nucleus total cross-section.

Aitala et al., PRL 86 4773 (2001)

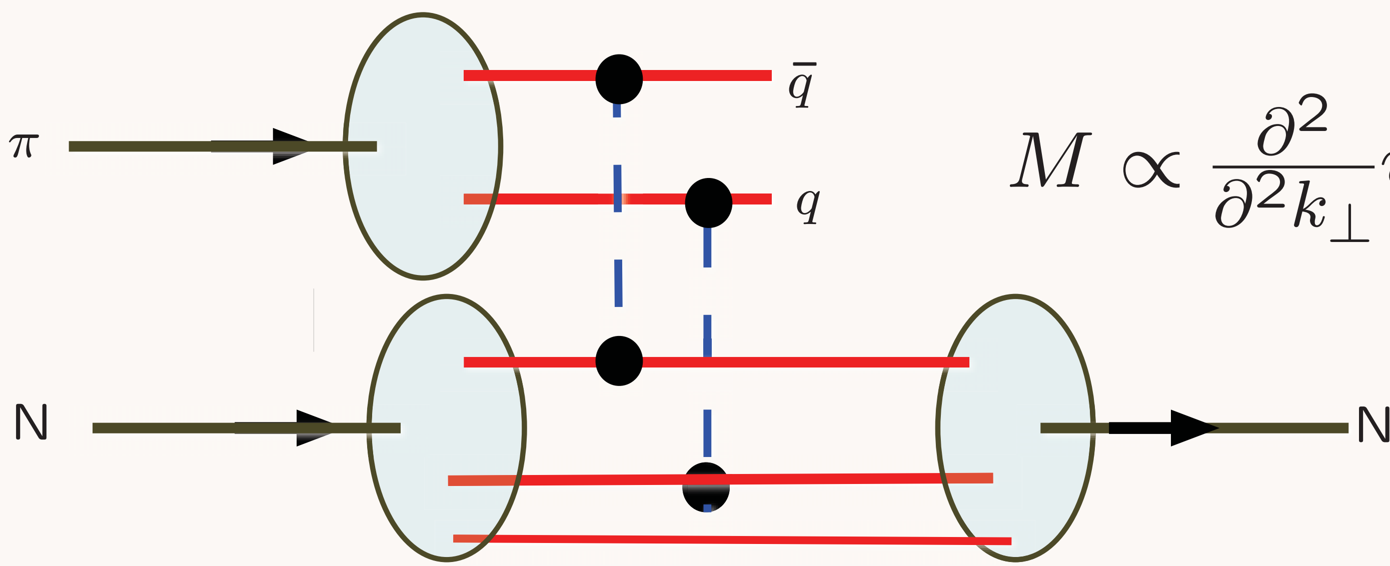
L. L. Frankfurt, G. A. Miller, and M. Strikman, Found. Of Phys. 30 (2000) 533

E791 FNAL Diffractive DiJet



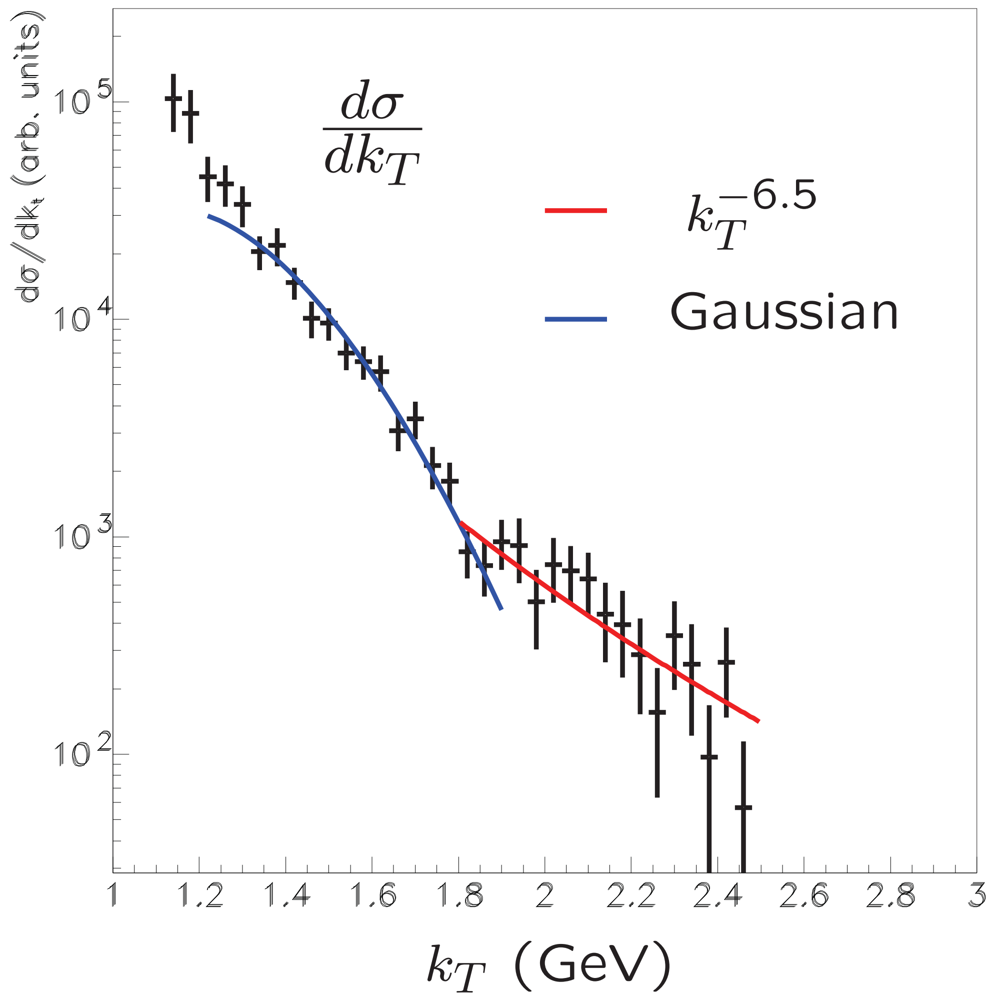
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution



Two Components

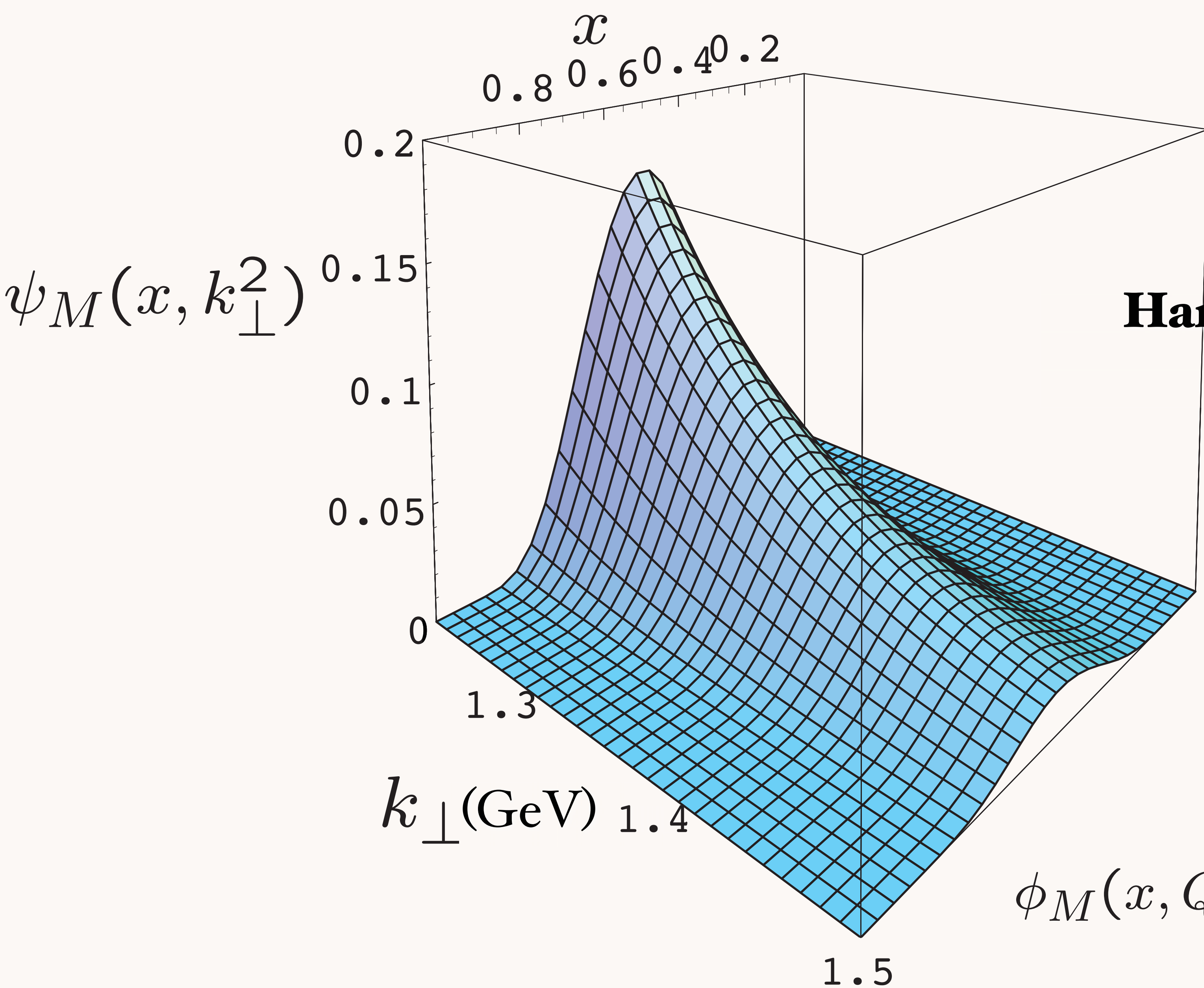
High Transverse momentum component consistent with PQCD, ERBL Evolution $k_T^{-6.5}$

Gaussian component similar to AdS/CFT HO LFWF

Shuryak:
Transition reflects domain walls

AdS/QCD

Prediction from AdS/CFT: Meson LFWF



Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$