Light-Front Quantization Approach to the Gauge/Gravity Correspondence and Strongly Coupled QCD

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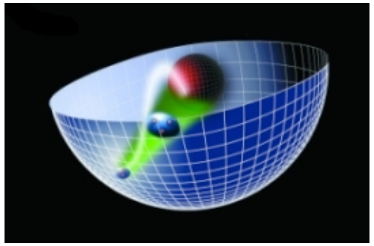

Institut de Physique Théorique

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GdT and Brodsky, PRL 102, 081601 (2009)

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1 Introduction: Gauge/Gravity Correspondence and QCD

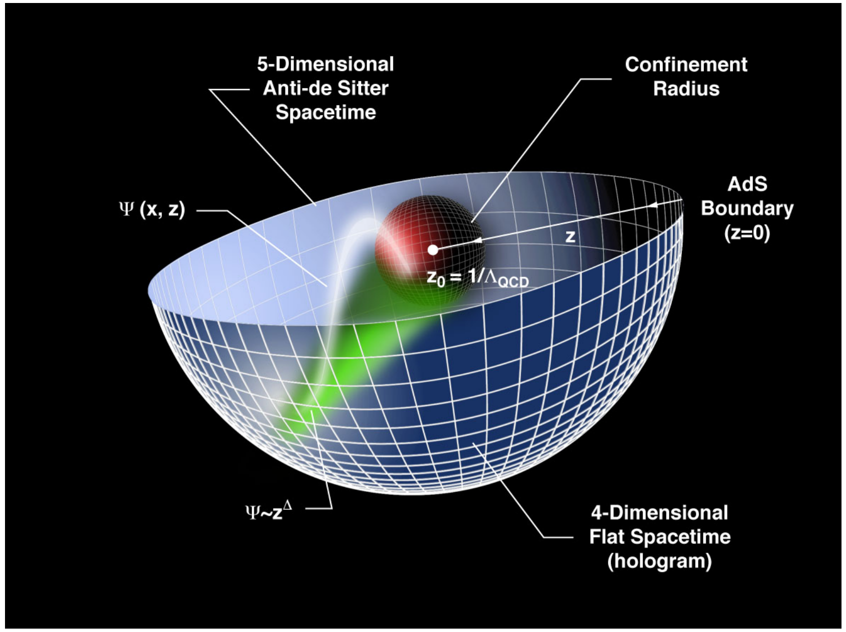
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom
- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between string states in AdS space and conformal field theories in physical space-time have led to analytical insights into the confining dynamics of QCD
- Description of strongly coupled gauge theory using a dual gravity description!
- Strings describe spin-J extended objects (no quarks). QCD degrees of freedom are pointlike particles and hadrons have orbital angular momentum: how can they be related?

- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} L_{\rm AdS}$$



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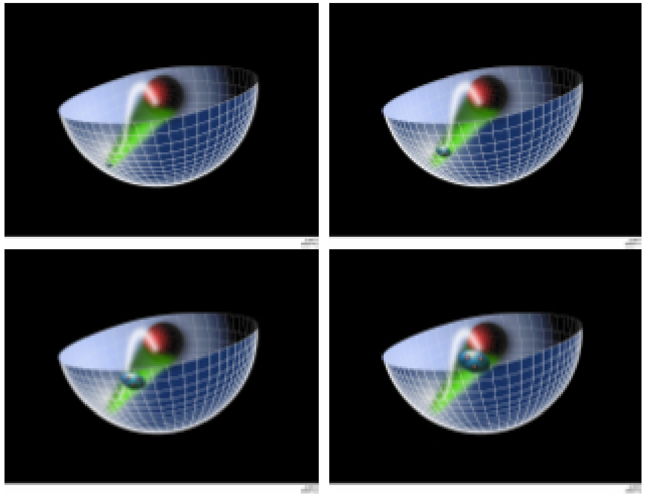
- Different values of z correspond to different scales at which the hadron is examined
- Since $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS$_5$ boundary $z \to 0$, which corresponds to the $Q \to \infty$ UV zero separation limit
- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z at the IR boundary
- Local operators like $\mathcal{O}$ and $\mathcal{L}_{\rm QCD}$ defined in terms of quark and gluon fields at the AdS$_5$ boundary
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

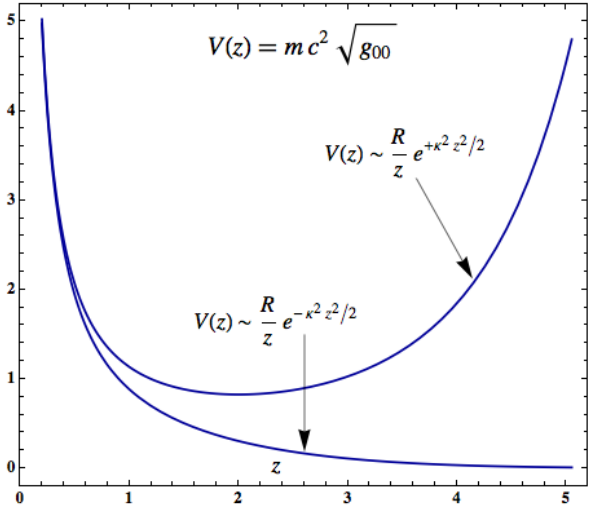
- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \; A(z \to 0) \to 0$$

- Non-conformal metric dual to a confining gauge theory
- Consider a warp factor $2A(z) = \pm\kappa^2 z^2$ and follow an object in AdS as it falls to the infrared region
- Gravitational potential energy in GR

$$V(z) = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{A(z)}}{z}$$





Confining solution:

Warp factor: $e^{+\kappa^2 z^2}$

$$z_0 = \frac{1}{\kappa}$$

- Consider the propagation of the gluon field in AdS in presence of a dilaton background $\varphi(z)$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

instead of modifying AdS metrics: $\sqrt{g} = (R/z)^5$

- Dilaton $\varphi(z) = \pm\kappa^2 z^2$ leads to Regge trajectories $\mathcal{M}^2 \sim J$ [Karch, Katz, Son and Stephanov]

- Define efective coupling $g_5(z)$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, \frac{1}{g_5^2(z)}\, G^2$$

- Thus the flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

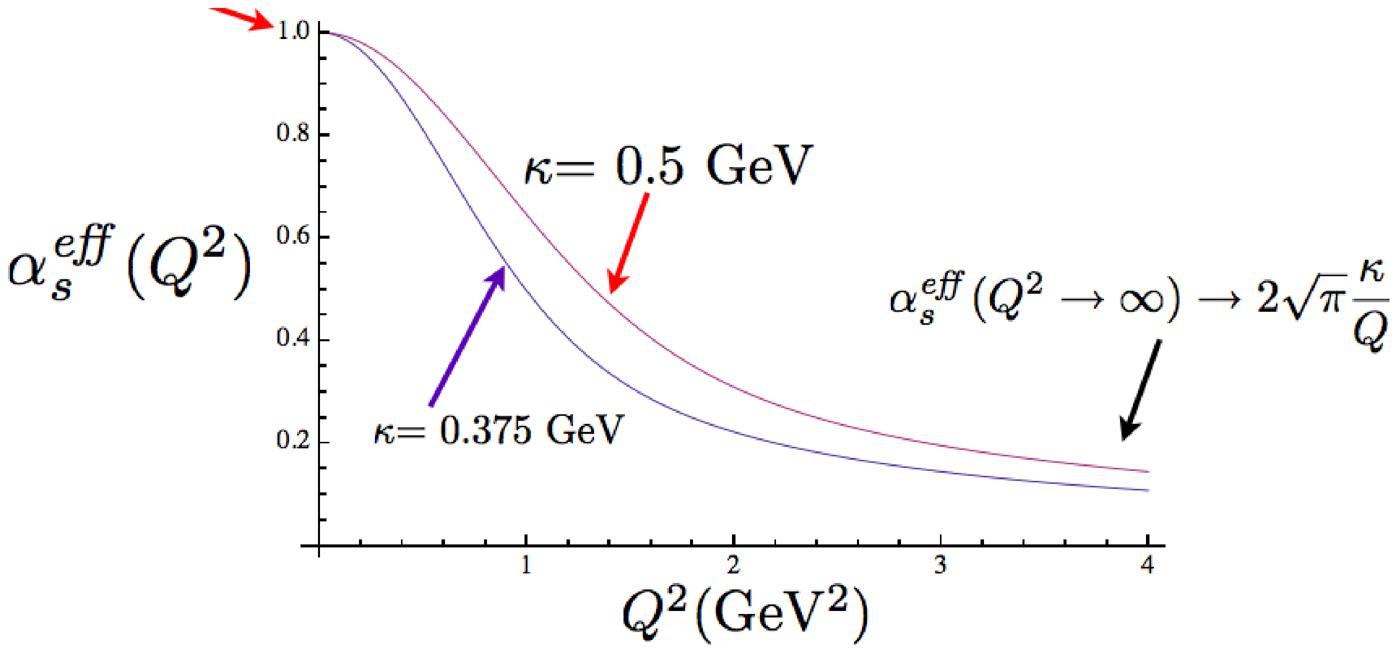
- Coupling measured at momentum scale Q

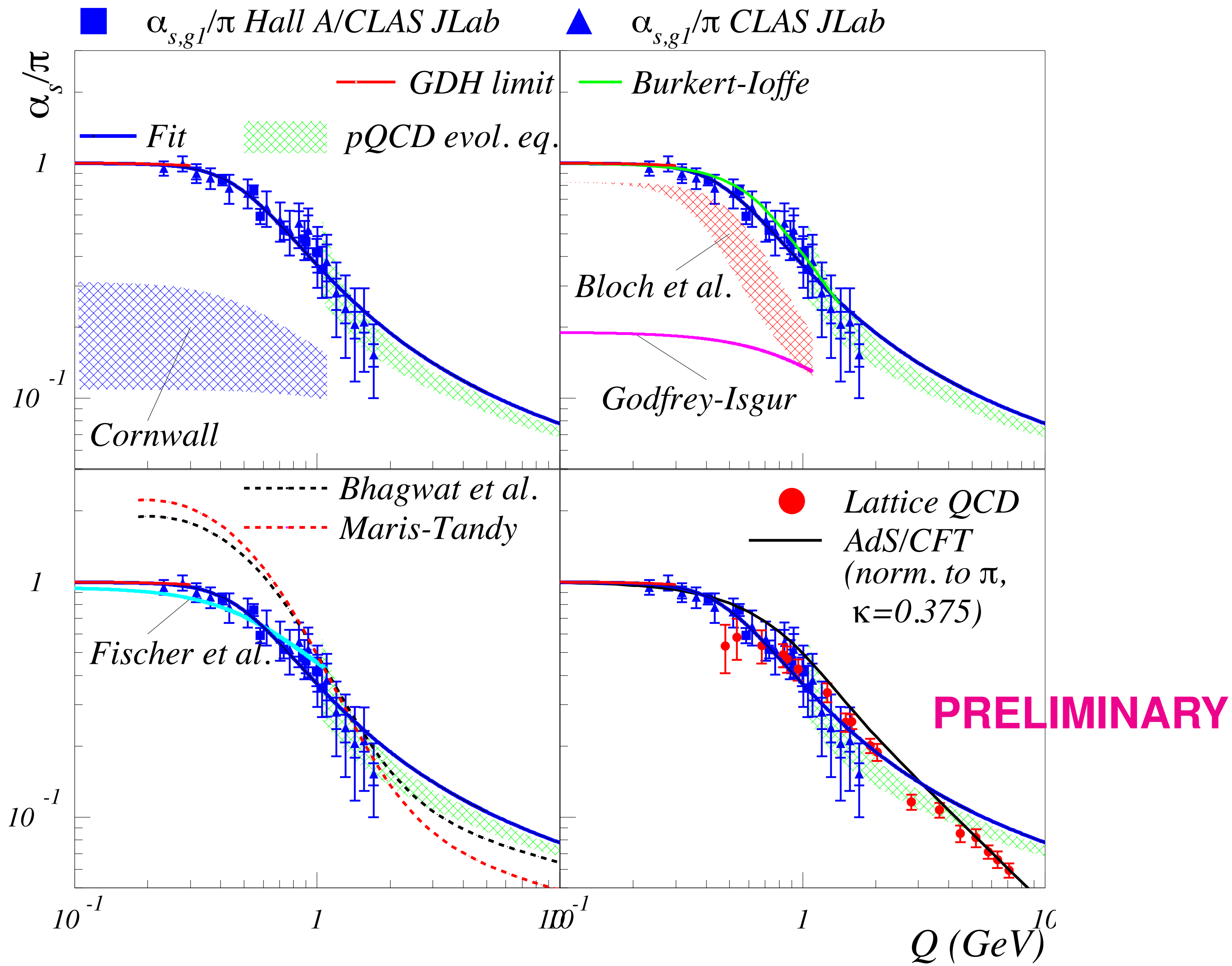
$$\alpha_s(Q) \sim \int_0^\infty dz J_0(zQ)\, \alpha_s(z)$$

where $\alpha_s(z) = e^{-\kappa^2 z^2}\alpha_s(0) \sim g_5^2(z)$

- Define normalization $\alpha_s^{eff}(Q^2)/\pi \to 1$ as $Q^2 \to 0$ (BJ sum rule)

$$\frac{\alpha_s^{eff}(Q)}{\pi} = e^{-Q^2/8\kappa^2} I_0\left(\frac{Q^2}{8\kappa^2}\right)$$



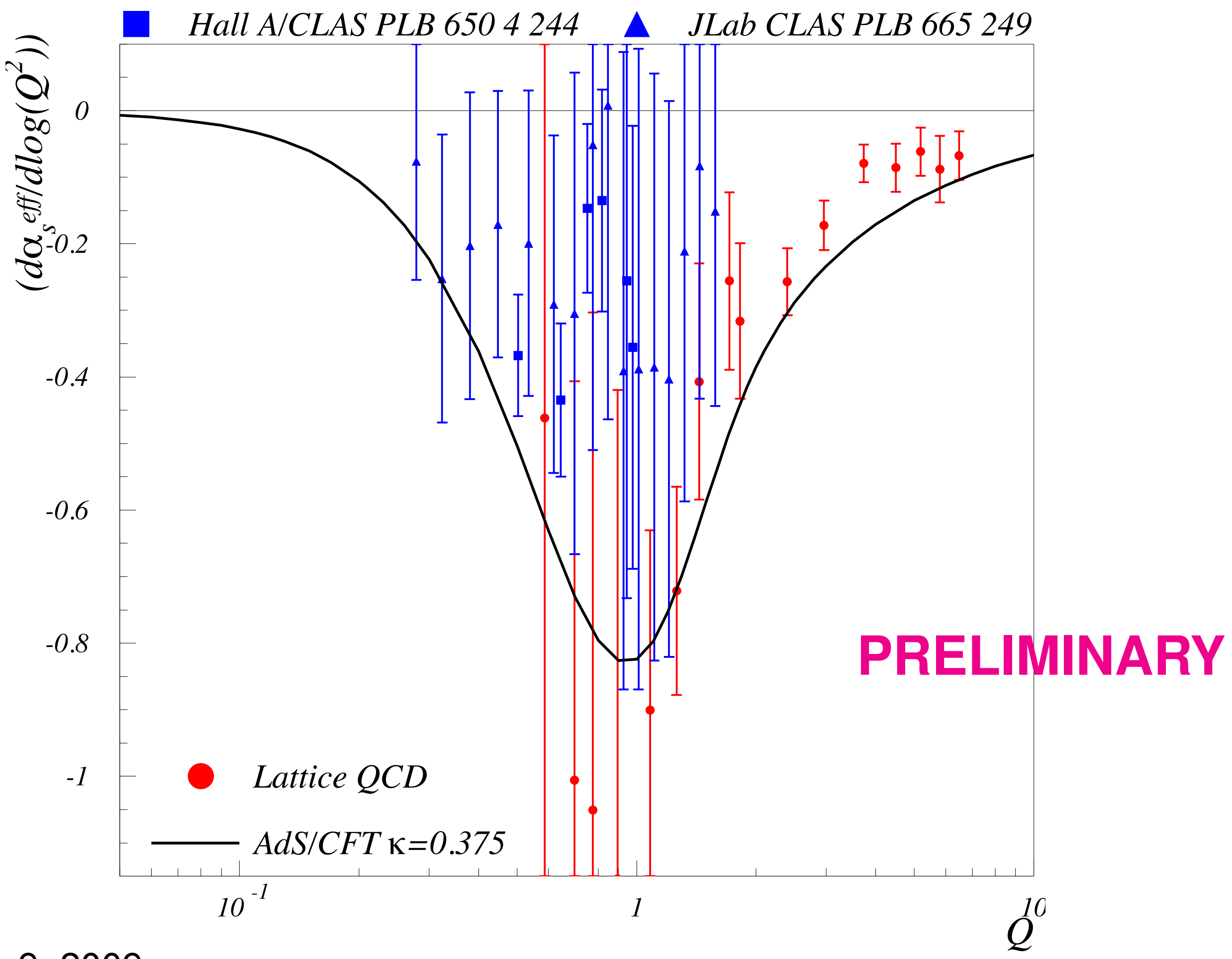


$\alpha_{s,g1}/\pi$ Hall A/CLAS JLab
$\alpha_{s,g1}/\pi$ CLAS JLab
α_s/π
GDH limit
Burkert-Ioffe
Fit
pQCD evol. eq.
Cornwall
Bloch et al.
Godfrey-Isgur
Bhagwat et al.
Maris-Tandy
Fischer et al.
Lattice QCD
AdS/CFT
(norm. to π, κ=0.375)
PRELIMINARY
Q (GeV)

- Beta function

With A. Deur and S. J. Brodsky

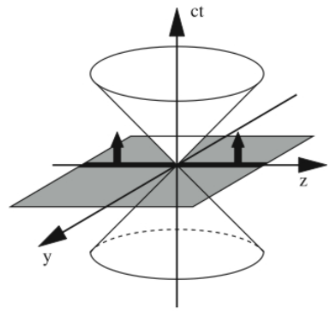
$$\beta^{eff}(Q^2) = \frac{d}{d\,log(Q^2)}\alpha_s^{eff}(Q^2) = -\frac{\pi Q^2}{8\kappa^2}e^{-Q^2/8\kappa^2}\left[I_0\left(\frac{Q^2}{8\kappa^2}\right) - I_1\left(\frac{Q^2}{8\kappa^2}\right)\right]$$



2 Light-Front Quantization of QCD and AdS/CFT

- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...
- Frame-independent LF Hamiltonian equation $P_\mu P^\mu |P\rangle = \mathcal{M}^2 |P\rangle$ similar structure of AdS EOM
- First semiclassical approximation to the bound-state LF Hamiltonian equation in QCD is equivalent to equations of motion in AdS and can be systematically improved GdT and S. J. Brodsky, PRL **102**, 081601 (2009)

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



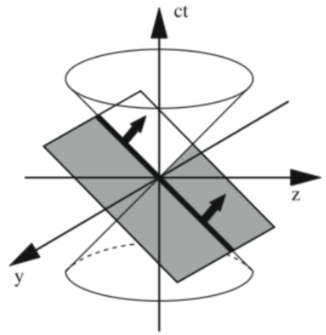
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum ($k^+ > 0$)

$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\,{\rm Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ, $\mathbf{A}_\perp$

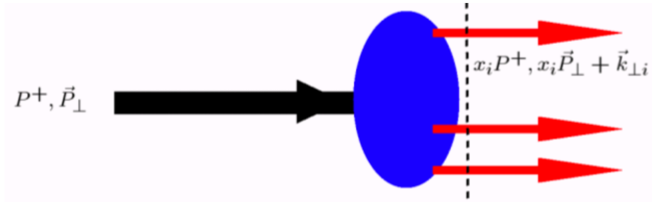
$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}\,\gamma^+ \,\frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \,\overline{\psi}\,\gamma^+ i\partial^+\psi \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \,\overline{\psi}\,\gamma^+ i\nabla_\perp\psi \end{aligned}$$

- LF Hamiltonian P^- generates LF time translations

$$\left[\psi(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi(x)$$

and the generators P^+ and $\mathbf{P}_\perp$ are kinematical

Light-Front Partonic Representation



- Dirac field ψ, expanded in terms of ladder operators on the initial surface $x^+ = x^0 + x^3$

$$\psi(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x} \right]$$

- LF Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of constituents with momentum $q = (q^+, q^-, \mathbf{q}_\perp)$

$$P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

$$P^+ = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \, q^+ \, b_\lambda^\dagger(q) b_\lambda(q)$$

$$\mathbf{P}_\perp = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \, \mathbf{q}_\perp \, b_\lambda^\dagger(q) b_\lambda(q)$$

Light-Front Bound State Hamiltonian Equation

- Construct light-front invariant Hamiltonian for the composite system: $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$H_{LF} \, |\psi_H\rangle = \mathcal{M}_H^2 \, |\psi_H\rangle$$

- State $|\psi_H(P^+, \mathbf{P}_\perp, J_z)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian:

$$|\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle, \qquad |n\rangle = \begin{cases} |uud\rangle \\ |uudg\rangle \\ |uud\bar{q}q\rangle \quad \cdots \end{cases}$$

where $k_i^2 = m_i^2, \;\; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each component i

- Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

- Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle\psi_H(P')|H_{LF}|\psi_H(P)\rangle = \mathcal{M}_H^2\langle\psi_H(P')|\psi_H(P)\rangle$$

- Find

$$\mathcal{M}_H^2 = \sum_n \int \big[dx_i\big]\big[d^2\mathbf{k}_{\perp i}\big] \sum_\ell \left(\frac{\mathbf{k}_{\perp\ell}^2 + m_\ell^2}{x_q}\right) \big|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})\big|^2 + \mathrm{interactions}$$

- Phase space normalization of LFWFs

$$\sum_n \int \big[dx_i\big]\;\big[d^2\mathbf{k}_{\perp i}\big]\;\big|\psi_{n/h}(x_i, \mathbf{k}_{\perp i})\big|^2 = 1$$

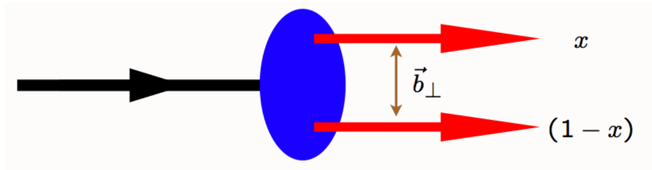
- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

$$\mathcal{M}_H^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_{n/H}^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla_{\mathbf{b}_{\perp\ell}}^2 + m_\ell^2}{x_q}\right) \psi_{n/H}(x_i, \mathbf{b}_{\perp i}) + \mathrm{interactions}$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j}\,|\psi_n(x_j, \mathbf{b}_{\perp j})|^2 = 1$$

3 Semiclassical Approximation to QCD



- Consider a two-parton hadronic bound state in transverse impact space in the limit $m_q \to 0$

$$\mathcal{M}^2 = \int_0^1 \frac{dx}{1-x} \int d^2\mathbf{b}_\perp \, \psi^*(x, \mathbf{b}_\perp) \left(-\nabla^2_{\mathbf{b}_\perp}\right) \psi(x, \mathbf{b}_\perp) + \text{interactions}$$

- Functional dependence of Fock state $|n\rangle$ given by invariant mass

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^{n} k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}^2_{\perp a} + m_a^2}{x_a} \to \frac{\mathbf{k}^2_\perp}{x(1-x)} \qquad (1)$$

the off-energy shell of the bound state $\mathcal{M}^2 - \mathcal{M}_n^2$

- In impact space the relevant variable is $\zeta^2 = x(1-x)\mathbf{b}^2_\perp$

- To first approximation LF dynamics depend only on the invariant variable $\mathcal{M}_n$ or ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} \qquad (2)$$

factoring out angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

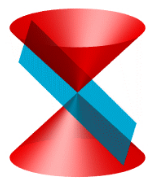
- Find ($L = |M|$)

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

Hard-Wall Model

- Consider the potential (hard wall)

$$U(\zeta) = \begin{cases} 0 & \text{if} \quad \zeta \leq \frac{1}{\Lambda_{\rm QCD}} \\ \infty & \text{if} \quad \zeta > \frac{1}{\Lambda_{\rm QCD}} \end{cases}$$

- If $L^2 \geq 0$ the Hamiltonian is positive definite $\langle \phi \left| H_{LF}^{L} \right| \phi \rangle \geq 0$ and thus $\mathcal{M}^2 \geq 0$
- If $L^2 < 0$ the Hamiltonian is not bounded from below (“Fall-to-the-center” problem in Q.M.)
- Critical value of the potential corresponds to $L = 0$, the lowest possible stable state
- Solutions:

$$\phi_L(\zeta) = C_L \sqrt{\zeta} J_L \left(\zeta \mathcal{M}\right)$$

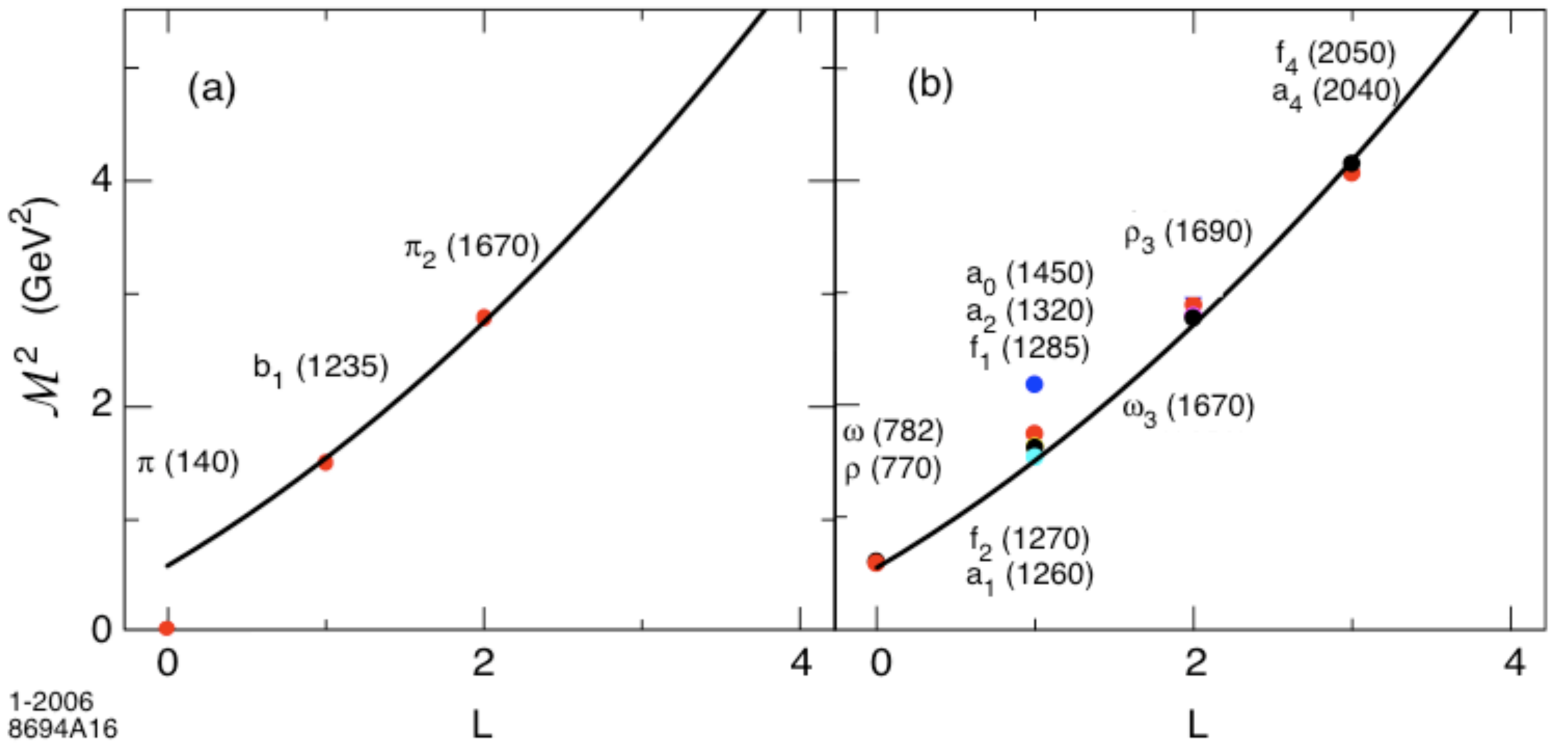
- Mode spectrum from boundary conditions

$$\phi\left(\zeta = \frac{1}{\Lambda_{\rm QCD}}\right) = 0$$

Thus

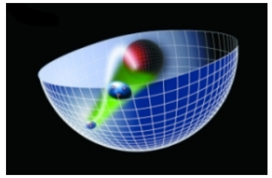
$$\mathcal{M}^2 = \beta_{Lk} \Lambda_{\rm QCD}$$

- Excitation spectrum hard-wall model: $\mathcal{M}_{n,L} \sim L + 2n$



Light-meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Holographic Mapping



- Holographic mapping found originally by matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Substitute $\Phi(\zeta) \sim \zeta^{3/2}\phi(\zeta), \quad \zeta \to z$ in the conformal LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

- Find:

$$\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0$$

with $(\mu R)^2 = -4 + L^2$, the wave equation of string mode in AdS$_5$!

- Isomorphism of $SO(4,2)$ group of conformal QCD with generators $P^\mu, M^{\mu\nu}, D, K^\mu$ with the group of isometries of AdS$_5$ space: $x^\mu \to \lambda x^\mu, \ z \to \lambda z$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Conformal dimension Δ of AdS mode Φ given in terms of 5-dim mass by $(\mu R)^2 = \Delta(\Delta - 4)$. Thus $\Delta = 2 + L$ in agreement with the twist scaling dimension of a two parton object in QCD

4 Higher-Spin Bosonic Modes

Hard-Wall Model

- AdS$_{d+1}$ metric $x^\ell = (x^\mu, z)$:

$$ds^2 = g_{\ell m} dx^\ell dx^m = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- Action for gravity coupled to scalar field in AdS$_{d+1}$

$$S = \int d^{d+1}x \sqrt{g} \Big(\underbrace{\frac{1}{\kappa^2}(\mathcal{R} - 2\Lambda)}_{S_G} + \underbrace{\frac{1}{2}\big(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\big)}_{S_M} \Big)$$

- Equations of motion for S_M

$$z^3 \partial_z \left(\frac{1}{z^3} \partial_z \Phi \right) - \partial_\rho \partial^\rho \Phi - \left(\frac{\mu R}{z} \right)^2 \Phi = 0$$

- Physical AdS modes $\Phi_P(x, z) \sim e^{-iP\cdot x}\, \Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$

- Factoring out dependence of string mode $\Phi_P(x, z)$ along x^μ-coordinates

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0$$

- Spin J-field on AdS represented by rank-J totally symmetric tensor field $\Phi(x,z)_{\ell_1\cdots\ell_J}$ [Fronsdal; Fradkin and Vasiliev]

- Action in AdS$_{d+1}$ for spin-J field

$$S_M = \frac{1}{2}\int d^{d+1}x\sqrt{g}\,\left(\partial_\ell \Phi_{\ell_1\cdots\ell_J}\partial^\ell \Phi^{\ell_1\cdots\ell_J} - \mu^2 \Phi_{\ell_1\cdots\ell_J}\Phi^{\ell_1\cdots\ell_J} + \ldots\right)$$

- Each hadronic state of total spin J is dual to a normalizable string mode

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{-iP\cdot x}\,\Phi(z)_{\mu_1\cdots\mu_J}$$

  with four-momentum P_μ, spin polarization indices along the 3+1 physical coordinates and hadronic invariant mass $P_\mu P^\mu = \mathcal{M}^2$

- For string modes with all indices along Poincaré coordinates, $\Phi_{z\mu_2\cdots\mu_J} = \Phi_{\mu_1 z\cdots\mu_J} = \cdots = 0$ and appropriate subsidiary conditions system of coupled differential equations from S_M reduce to a homogeneous wave equation for $\Phi(z)_{\mu_1\cdots\mu_J}$

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Normalization [Hong, Yoon and Strassler (2006)]

$$R^{d-2J-1} \int_0^{z_{max}} \frac{dz}{z^{d-2J-1}} \, \Phi_J^2(z) = 1$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (d-1-2J) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

upon fifth-dimensional mass rescaling $(\mu R)^2 \to (\mu R)^2 - J(d - J)$

- Conformal dimension of J-mode

$$\Delta = \frac{1}{2} \left(d + \sqrt{(d - 2J)^2 + 4\mu^2 R^2}\right)$$

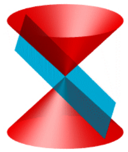
and thus $(\mu R)^2 = (\Delta - J)(\Delta - d + J)$

- Upon substitution $z \to \zeta$ and

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} \Phi_J(\zeta)$$

we recover the QCD LF wave equation ($d = 4$)

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$



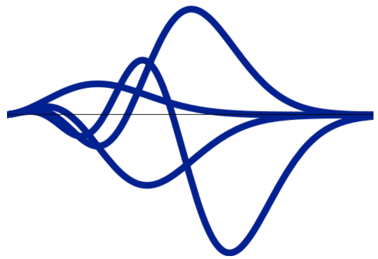
with $(\mu R)^2 = -(2 - J)^2 + L^2$

- J-decoupling in the HW model

- For $L^2 \geq 0$ the LF Hamiltonian is positive definite $\langle \phi_J | H_{LF} | \phi_J \rangle \geq 0$ and we find the stability bound $(\mu R)^2 \geq -(2 - J)^2$

- The scaling dimensions are $\Delta = 2 + L$ independent of J in agreement with the twist scaling dimension of a two parton bound state in QCD

Soft-Wall Model



- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff wich depends on the profile of a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(d - 1 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

- Action in AdS$_{d+1}$ for spin J-field

$$S_M = \frac{1}{2}\int d^d x\, dz\, \sqrt{g}\, e^{\kappa^2 z^2} \left(\partial_\ell \Phi_{\ell_1 \cdots \ell_J} \partial^\ell \Phi^{\ell_1 \cdots \ell_J} - \mu^2 \Phi_{\ell_1 \cdots \ell_J} \Phi^{\ell_1 \cdots \ell_J} + \dots \right)$$

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Normalization

$$R^{d-2J-1} \int_0^\infty \frac{dz}{z^{d-2J-1}}\, e^{\kappa^2 z^2} \Phi_J^2(z) = 1.$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(d-1-2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

upon mass rescaling $(\mu R)^2 \to (\mu R)^2 - J(d-J)$ and $\mathcal{M}^2 \to \mathcal{M}^2 - 2J\kappa^2$

- Upon substitution $z \to \zeta \quad (J_z = L_z + S_z)$ we find for $d = 4$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2\zeta^2/2}\, \Phi_J(\zeta), \qquad (\mu R)^2 = -(2-J)^2 + L^2$$

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L+S-1)\right)\phi_{\mu_1\cdots\mu_J} = \mathcal{M}^2\phi_{\mu_1\cdots\mu_J}}$$

- Eigenfunctions

$$\phi_{nL}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

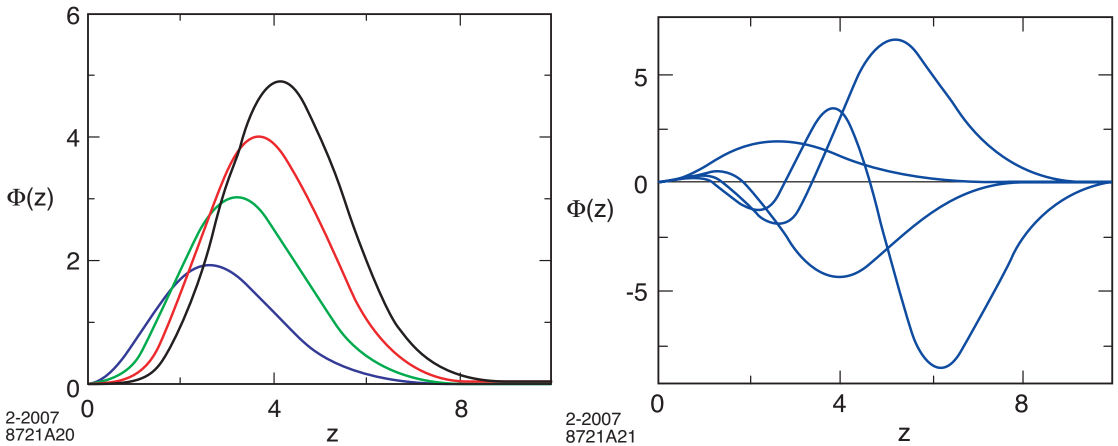
- Eigenvalues

$$\mathcal{M}^2_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2}\right)$$

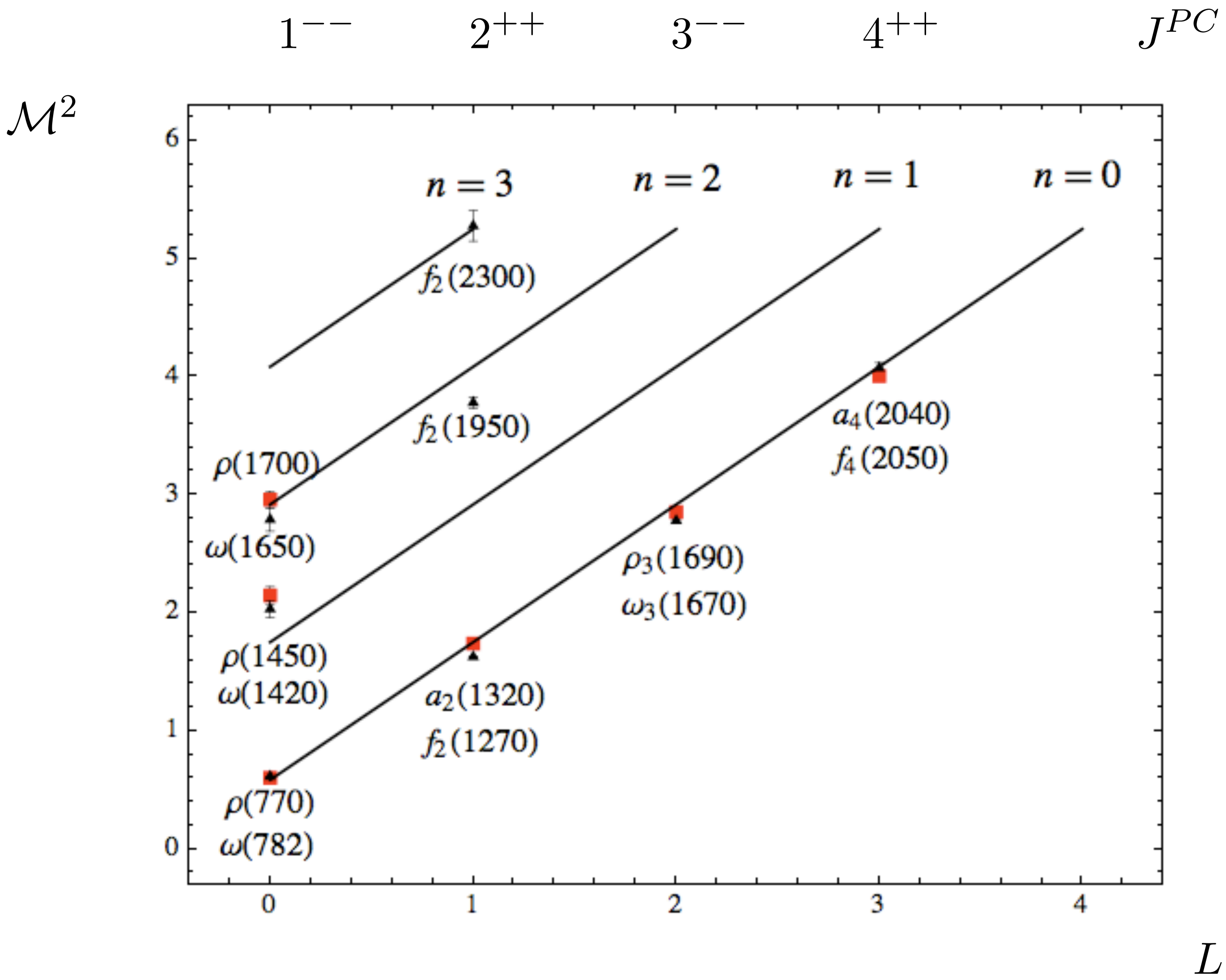
$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$



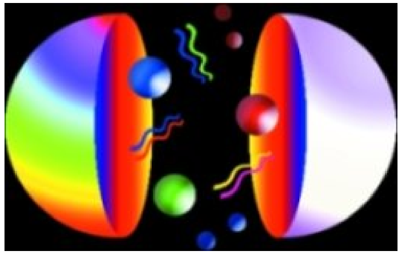
Orbital and radial states: $\langle\zeta\rangle$ increase with L and n



Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red)

and the $I = 0$ ω-meson family (black) for $\kappa = 0.54$ GeV

5 Higher-Spin Fermionic Modes



From Nick Evans

Hard-Wall Model

- Action for massive fermionic modes on AdS_{d+1}:

$$S[\overline{\Psi}, \Psi] = \int d^d x \, dz \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution $(\mu R = \nu + 1/2, \, d = 4)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{\rm QCD} \right) = 0$

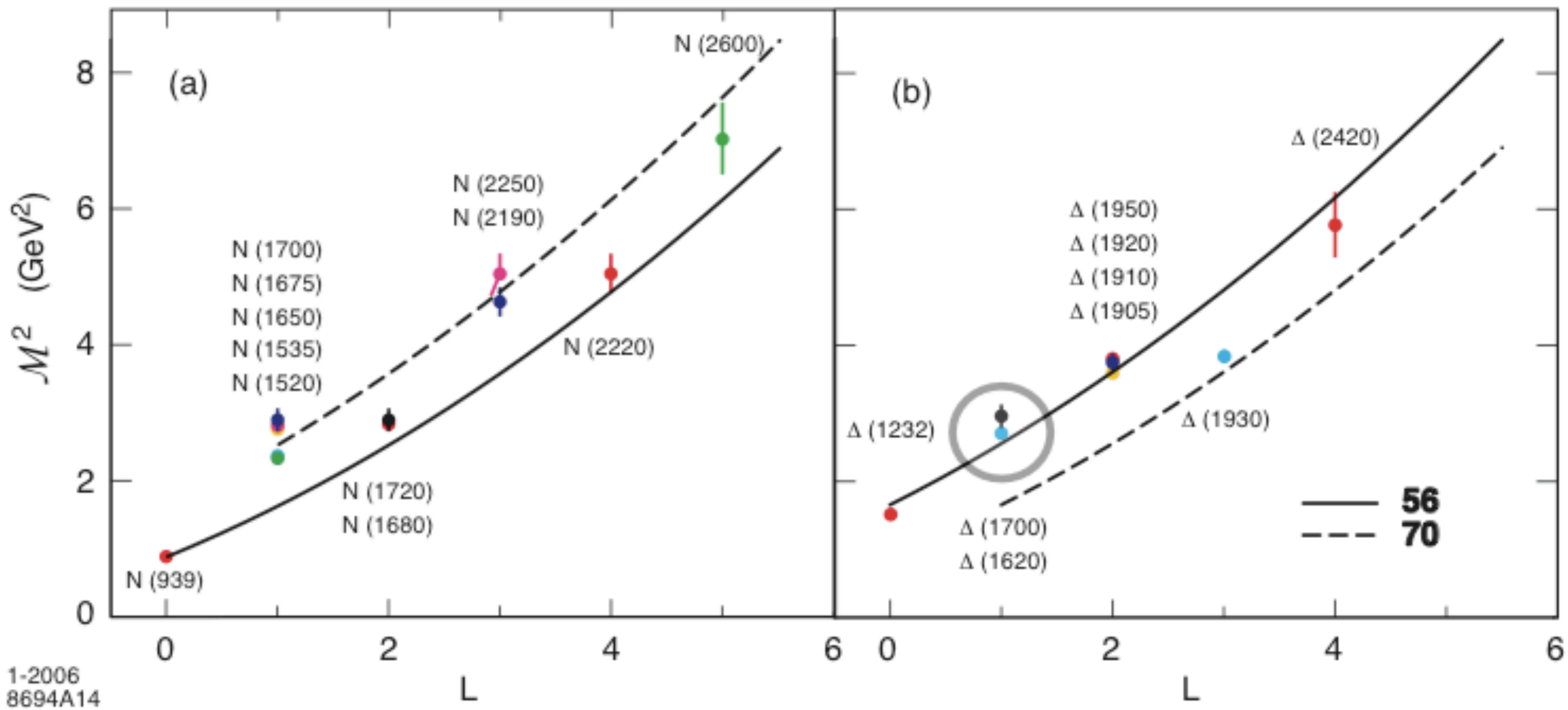
$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{\rm QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{\rm QCD}$$

  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

SU(6)	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^{+}(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^{+}(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^{-}(1535)$ $N\frac{3}{2}^{-}(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^{-}(1650)$ $N\frac{3}{2}^{-}(1700)$ $N\frac{5}{2}^{-}(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^{-}(1620)$ $\Delta\frac{3}{2}^{-}(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^{+}(1720)$ $N\frac{5}{2}^{+}(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^{+}(1910)$ $\Delta\frac{3}{2}^{+}(1920)$ $\Delta\frac{5}{2}^{+}(1905)$ $\Delta\frac{7}{2}^{+}(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}$
	$\frac{3}{2}$	3	$N\frac{3}{2}^{-}$ $N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}(2190)$ $N\frac{9}{2}^{-}(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^{-}(1930)$ $\Delta\frac{7}{2}^{-}$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^{+}$ $N\frac{9}{2}^{+}(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^{+}$ $\Delta\frac{7}{2}^{+}$ $\Delta\frac{9}{2}^{+}$ $\Delta\frac{11}{2}^{+}(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^{-}$ $N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}$ $N\frac{13}{2}^{-}$

- Excitation spectrum for baryons in the hard-wall model: $\mathcal{M} \sim L + 2n$



Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states: (a) $I = 1/2$ and (b) $I = 3/2$

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

- Solution $(\mu R = \nu + 1/2,\, d = 4)$

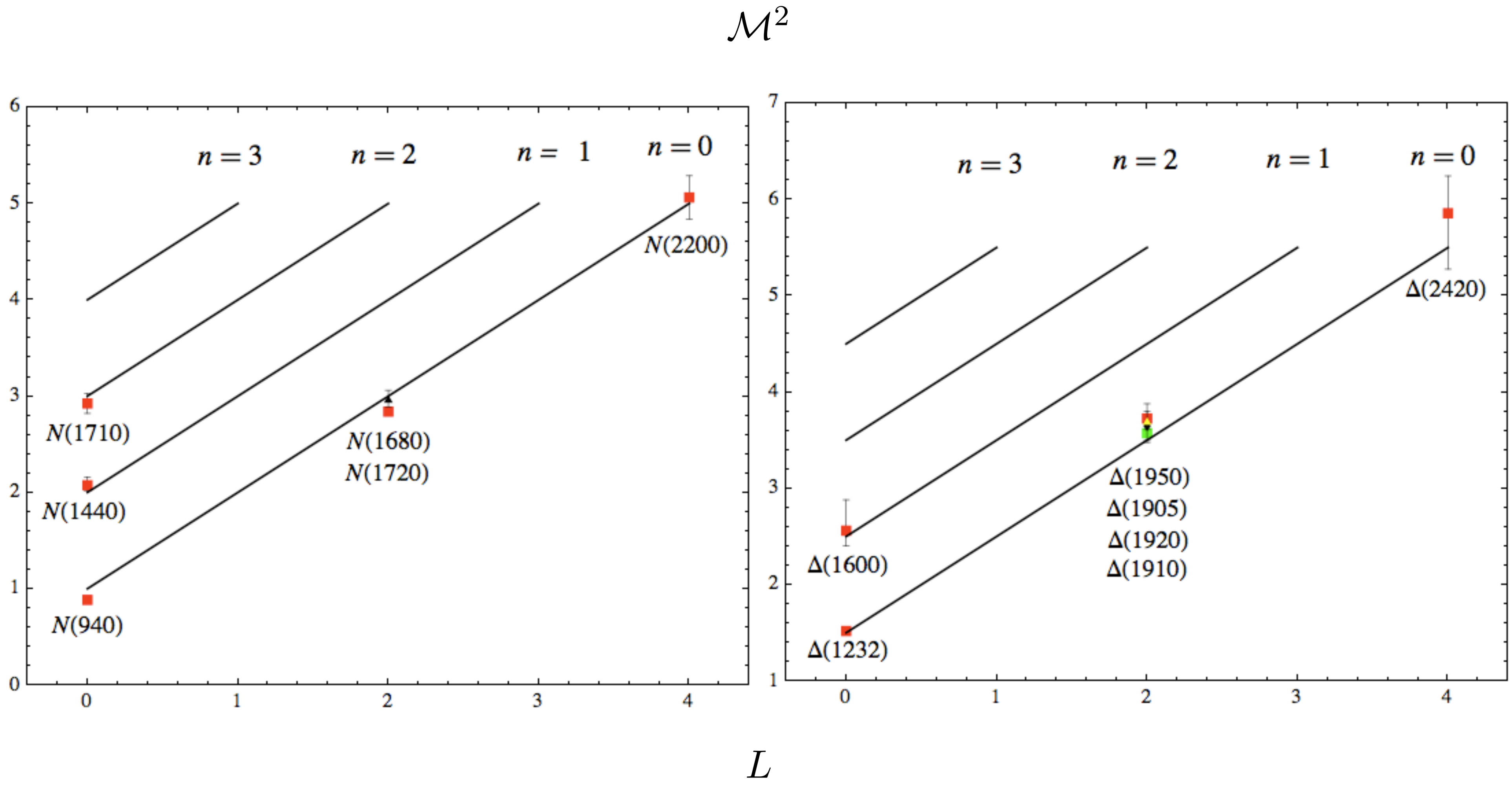
$$\begin{aligned}\Psi_+(z) &\sim z^{\frac{5}{2}+\nu}e^{-\kappa^2 z^2/2}L_n^\nu(\kappa^2 z^2)\\ \Psi_-(z) &\sim z^{\frac{7}{2}+\nu}e^{-\kappa^2 z^2/2}L_n^{\nu+1}(\kappa^2 z^2)\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



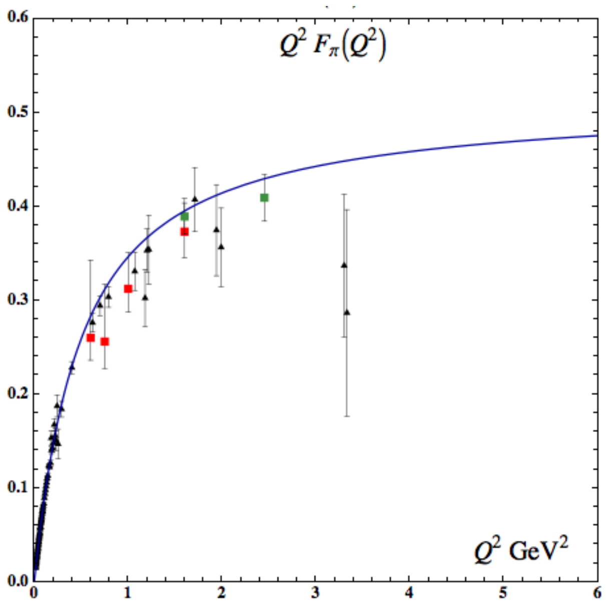
Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

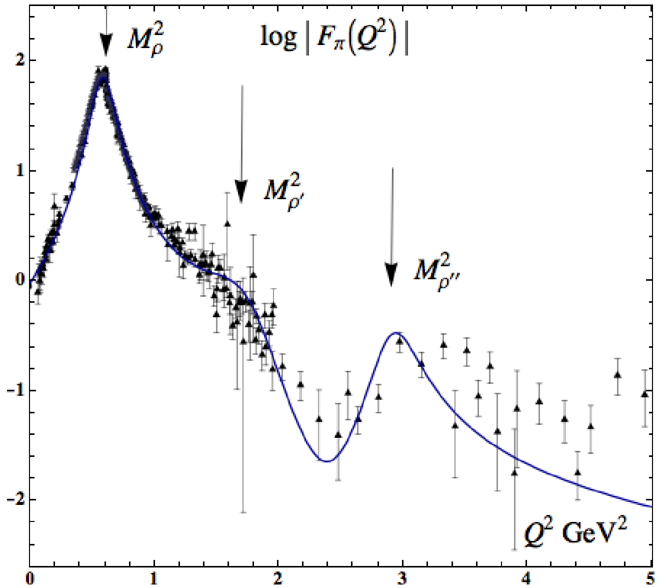
6 New and Future Applications of Light-Front Holography

- Introduction of massive quarks (heavy and heavy-light quark systems)
- Systematic improvement of LF semiclassical approximation: QCD Coulomb forces, higher Fock states (HFS) from Lippmann-Schwinger equation . . .
- Quantum effects and evolution equations
- Derivation of effective effective potential $U(z)$ from higher Fock gluonic states (dynamical quarks in stochastically averaged gluon medium from HFS)
- Pauli Form Factor
- Transition form factor in AdS
- Multicomponent vector meson state in AdS (DKP equation)
- Connect dilaton to string physics in AdS, non-perturbative derivation of α_s
- . . .

Contribution of HFS to Pion Form-Factor $\mathrm{F}_\pi(\mathrm{Q}^2)$

With S. J. Brodsky





PRELIMINARY

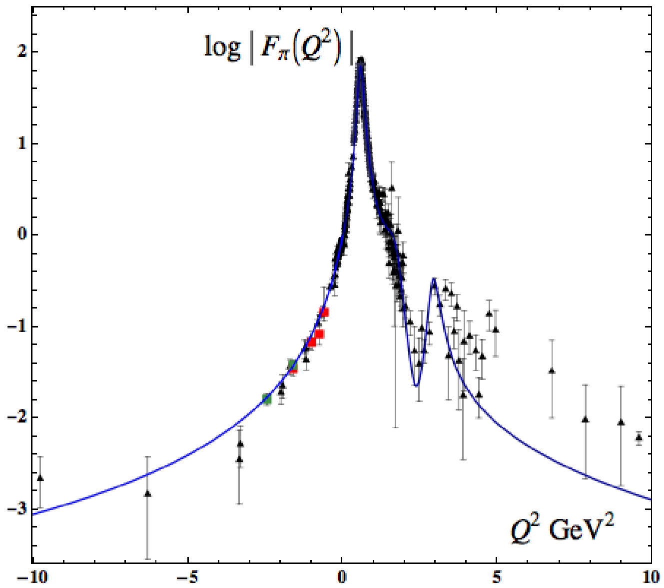
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle$$

$$\mathcal{M}_\rho{}^2 \to 4\kappa^2(n+1/2)$$

$$\kappa = \sqrt{2} \times 0.375 \simeq 0.54 \text{ GeV}$$

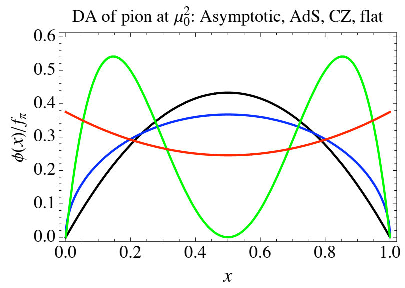
$$\Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 13\ \%$$



Pion Transition Form-Factor $F_{\pi \to \gamma\gamma*}(Q^2)$

With F-G Cao and S. J. Brodsky



pion-gamma transition form factor, Q2Fpigamma