Integrability and Baryonic Modes in AdS/QCD

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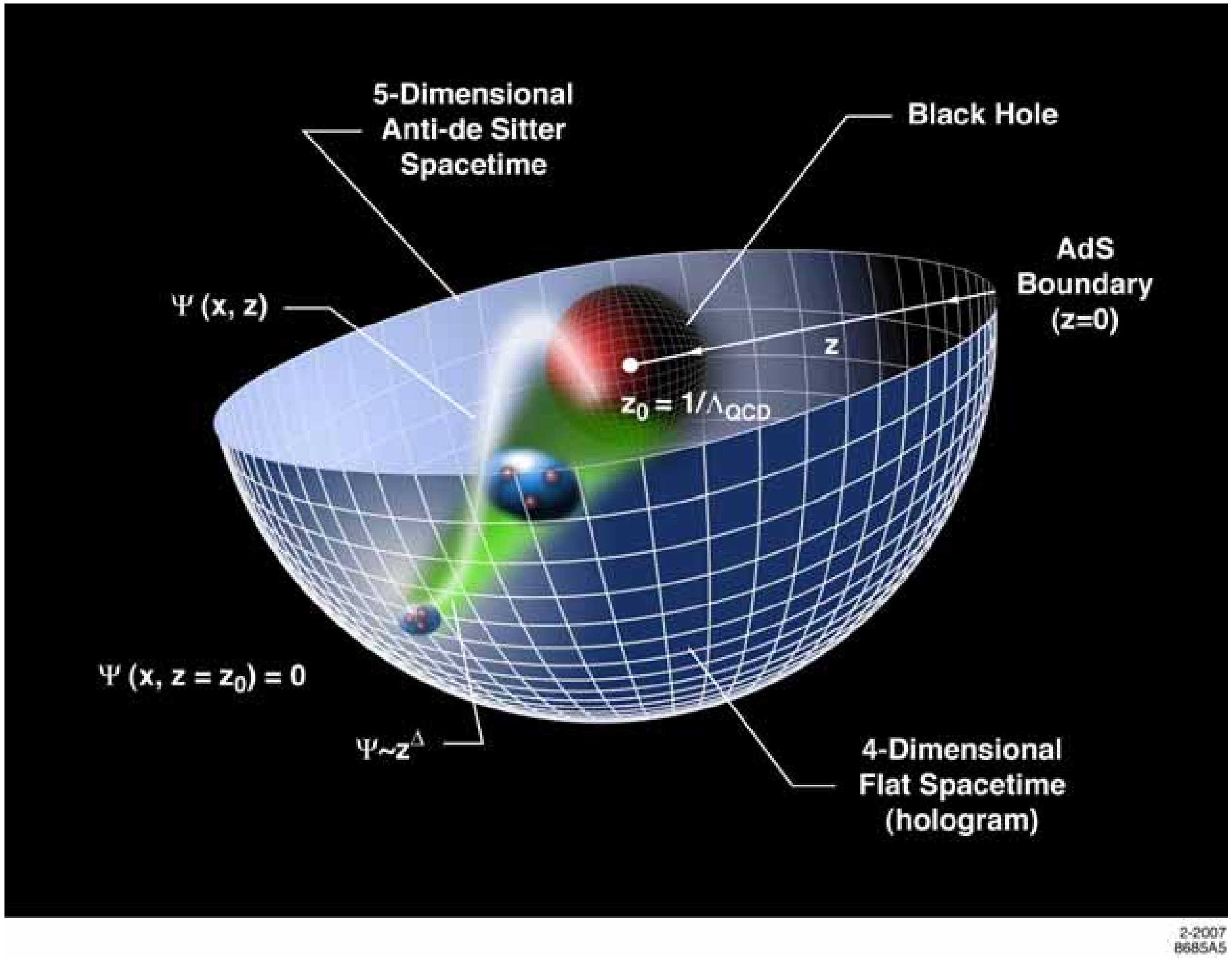
Motivation: Holographic QCD

- Strings describe extended objects (no quarks). QCD degrees of freedom are pointlike particles: how can they be related? How can we map string states into partons?
- Precise mapping of string amplitudes to light-front wavefuntions of hadrons in the light-front for strongly coupled QCD in the conformal limit $\longrightarrow$ effective gravity description (**Stan's talk**).
- Holographic duality requires a higher dimensional warped space. Space with negative curvature and a 4-dim boundary: AdS_5.
- Eigenvalues of normalizable modes inside AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for distribution of quarks at a given scale.
- To each state of the gauge theory should correspond a normalized mode in AdS. The lowest stable mode should correspond to the lowest state of the QCD Hamiltonian.
- Non-normalizable modes are related to external currents: they probe the cavity interior. Also couple to boundary QCD interpolating operators.



5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
AdS
Boundary
(z=0)
Ψ (x, z)
z
$z_0 = 1/\Lambda_{QCD}$
Ψ (x, z = z_0) = 0
$\Psi \sim z^\Delta$
4-Dimensional
Flat Spacetime
(hologram)
2-2007
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Outline

1. The Holographic Correspondence

   Conformal QCD Window in Exclusive Processes

   Semi-Classical Correspondence and Interpolating Operators

2. Bosonic Modes

   Holographic Light-Front Representation (Hard Wall Model)

   Algebraic Structure and Stability Conditions

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   Higher Spin Bosonic Modes and Ultraviolet Matching of Twist Operators

   Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

   Space and Time-Like Pion Form Factor

3. Fermionic Modes

   Holographic Light-Front Integrable Form and Spectrum (Hard Wall Model)

   Non-Conformal Extension of Algebraic Structure and Linear Confinement (Soft Wall Model)

   Space-Like Dirac Proton Form Factor

   Linear Holographic Confinement

1 The Holographic Correspondence

- In the semi-classical approximation to QCD with massless quarks and no quantum loops the β function is zero and the approximate theory is scale and conformal invariant.
- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

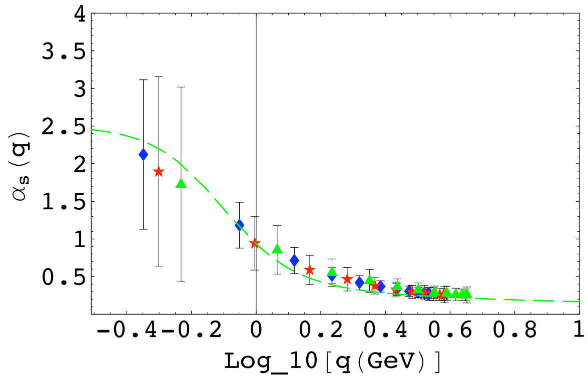
$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

- Semi-classical correspondence as a first approximation to QCD (strongly coupled at all scales).
- $x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.
- Different values of z correspond to different scales at which the hadron is examined: AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.
- There is a maximum separation of quarks and a maximum value of z at the IR boundary
- Truncated AdS/CFT model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard Wall Model)

Conformal QCD Window in Exclusive Processes

- Does α_s develop an IR fixed point? D-S Equation Alkofer, Fischer, LLanes-Estrada, Deur ...
- Recent lattice simulations: evidence that α_s becomes constant and not small in the infrared

  Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE)



- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

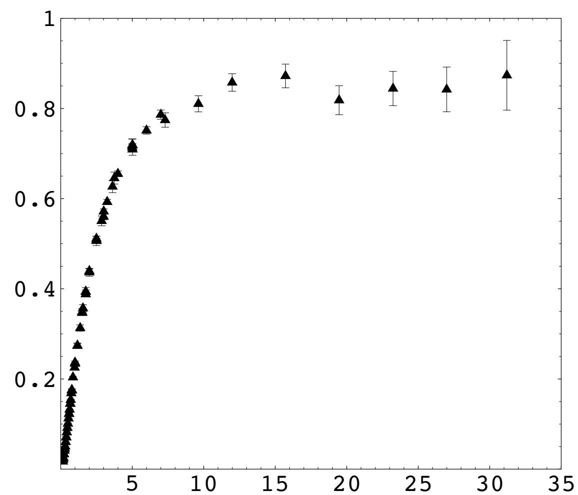
  implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

  Brodsky and Farrar (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

- Example: Dirac proton form factor: $F_1(Q^2) \sim \left[1/Q^2\right]^{n-1}, \; n = 3$

$Q^4 F_1^p(Q^2)$ [GeV^4]



Q^2 [GeV^2]

From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Semi-Classical Correspondence and Interpolating Operators

Precise statement of duality between a gravity theory in AdS$_{d+1}$ and the strong coupling limit of a conformal field theory at the $z = 0$ boundary Gubser, Klebanov and Polyakov (1998); Witten (1998) :

- $d + 1$-dim gravity partition function for scalar field in AdS$_{d+1}$: $\Phi(x, z)$

$$Z_{grav}[\Phi(x, z)] = e^{iS_{eff}[\Phi]} = \int \mathcal{D}[\Phi] e^{iS[\Phi]} \, .$$

- d-dim generating functional in presence of external source Φ_0

$$Z_{CFT}[\Phi_0(x)] = e^{iW_{CFT}[\Phi_0]} = \left\langle \exp\left(i \int d^d x \Phi_0(x) \mathcal{O}(x) \right) \right\rangle .$$

  with $\mathcal{O}$ a hadronic interpolating operator ($\mathcal{O} = G^a_{\mu\nu} G^{a\mu\nu}, \cdots$)

- Boundary condition:

$$Z_{grav}\left[\Phi(x, z = 0) = \Phi_0(x)\right] = Z_{QCD}\left[\Phi_0\right] .$$

- Semi-Classical Approximation

$$W_{CFT}\left[\phi_0\right] = S_{eff}\Big[\Phi(x, z)|_{z=0} = \Phi_0(x)\Big]_{\text{on}-\text{shell}}.$$

- Near the boundary of AdS_{d+1} space $z \to 0$:

$$\Phi(x,z) \to z^{\Delta}\Phi_+(x) + z^{d-\Delta}\Phi_-(x).$$

- $\Phi_-(x)$ is the boundary limit of non-normalizable mode (source): $\Phi_- = \Phi_0$
- $\Phi_+(x)$ is the boundary limit of the normalizable mode (physical states)
- Using the equations of motion AdS action reduces to a UV surface term

$$S_{eff} = \frac{R^{d-1}}{4} \lim_{z \to 0} \int d^d x \, \frac{1}{z^{d-1}} \, \Phi \partial_z \Phi,$$

- S_{eff} is identified with the boundary functional W_{CFT}

$$\langle \mathcal{O} \rangle_{\Phi_0} = \frac{\delta W_{CFT}}{\delta \Phi_0} = \frac{\delta S_{\rm eff}}{\delta \Phi_0} \sim \Phi_+(x),$$

Balasubramanian *et. al.* (1998), Klebanov and Witten (1999).

- Physical AdS modes $\Phi(x,z) \sim e^{-iP\cdot x}\Phi^P(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

2 Bosonic Modes

$$ds^2 = g_{\ell m} dx^\ell dx^m, \;\; x^\ell = (x^\mu, z), \;\; g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m})$$

- Action for massive scalar modes on AdS_{d+1}:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi(x,z) = e^{-iP \cdot x} \Phi^P(z), \;\; P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution:

$$\Phi(x,z) = e^{-iP \cdot x} z^{\frac{d}{2}} C J_{\Delta - \frac{d}{2}} \left(z \mathcal{M} \right), \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4 \mu^2 R^2} \, \right).$$

- Normalization

$$R^{d-1} \int_0^{\Lambda_{\mathrm{QCD}}^{-1}} \frac{dz}{z^{d-1}} \, \Phi^2_{S=0}(z) = 1.$$

Holographic Light-Front Representation (Hard Wall Model)

- We can represent the EOM in AdS space in light-cone Hamiltonian form in physical 3+1 space-time

$$H_{LC}|\phi_h\rangle = \mathcal{M}^2|\phi_h\rangle,$$

- We can identify a light-cone variable ζ in 3+1 space with the fifth dimension z of AdS space: $\zeta = z$.
- ζ represents the invariant transverse separation between pointlike constituents

  SJB and GdT, PRL **96**, 201601 (2006) [hep-ph/0602252]
- Substitute in AdS EOM: $\phi(\zeta) = (\zeta/R)^{-3/2}\Phi(\zeta)$
- Result :

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2}\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta),$$

- AdS/CFT equation as effective Schrödinger equation: relativistic, covariant and analytically tractable. Its eigenmodes $\phi_h(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-partons at transverse impact separation $\zeta = z$.
- Impact ζ-representation lfwf $\phi_h(\zeta) = \langle\zeta|\phi_h\rangle$ normalized by

$$\langle\phi_h|\phi_h\rangle = \int d\zeta\, |\langle\zeta|\phi_h\rangle|^2 = 1,$$

Algebraic Structure and Stability Conditions

- If $\nu^2 > 0$ the Hamiltonian is written as a bilinear form

$$H_{LC}^{\nu}(\zeta) = \Pi_\nu^\dagger(\zeta)\Pi_\nu(\zeta), \quad \nu^2 \geq 0,$$

in terms of the operator

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\right),$$

and its adjoint

$$\Pi_\nu^\dagger(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta}\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \frac{2\nu + 1}{\zeta^2}.$$

- For $\nu^2 \geq 0$ the Hamiltonian is positive definite

$$\langle\phi\,|H_{LC}^{\nu}|\,\phi\rangle = \int d\zeta\,|\Pi_\nu \phi(z)|^2 \geq 0$$

and thus $\mathcal{M}^2 \geq 0$.

- For $\nu^2 < 0$

$$\langle \phi \,| H^{\nu}_{LC} |\, \phi \rangle \geq 2\nu^2 \int d\zeta \frac{|\phi|^2}{\zeta^2}.$$

  and the Hamiltonian is not bounded from below ("Fall-to-the-center" problem in Q.M.)

- Critical value of the potential corresponds to $\nu = 0$ with potential

$$V_{crit}(\zeta) = \frac{1}{4\zeta^2}.$$

- The Q.M. stability conditions are equivalent to the Breitenlohner-Freedman stability conditions

$$(\mu R)^2 \geq \frac{d^4}{4}.$$

- For $d = 4$

$$(\mu R)^2 = -4 + \nu^2,$$

  thus $\nu = 0$ correspond to the lowest stable solution.

Ladder Construction of Orbital States

- Orbital excitations are constructed by the L-th application of the raising raising operator $a^\dagger_L = -i\Pi_L$:

$$a^\dagger|L\rangle = c_L|L+1\rangle,$$

  on the ground state.

- In the light-front ζ-representation

$$\begin{aligned}\phi_L(\zeta) &= \langle\zeta|L\rangle = C_L\sqrt{\zeta}\,(-\zeta)^L\left(\frac{1}{\zeta}\frac{d}{d\zeta}\right)^L J_0(\zeta\mathcal{M}) \\ &= C_L\sqrt{\zeta}J_L\left(\zeta\mathcal{M}\right)\end{aligned}$$

- The solutions ϕ_L are solutions of the light-front equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-L^2}{4\zeta^2}\right]\phi(z) = \mathcal{M}^2\phi(\zeta).$$

  with $L = 0, \pm1, \pm2, \cdots$.

- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ group of rotations. (The Casimir for $SO(n) \sim S^{n-1}$ is $L(L+n-2)$).

Higher Spin Bosonic Modes and ultraviolet Matching and Twist Operators

- Each hadronic state of integer spin $S \le 2$ is dual to a normalizable string mode

$$\Phi(x,z)_{\mu_1\mu_2\cdots\mu_S} = \epsilon_{\mu_1\mu_2\cdots\mu_S}\, e^{-iP\cdot x}\, \Phi_S(z).$$

  with four-momentum P_μ and spin polarization indices along the 3+1 physical coordinates. The hadronic invariant mass is $P_\mu P^\mu = \mathcal{M}^2$.

- Wave equation for spin S-mode W. S. l'Yi, Phys. Lett. B **448**, 218 (1999)

$$\left[z^2\partial_z^2 - (d{+}1{-}2S) z\, \partial_z + z^2\mathcal{M}^2 {-} (\mu R)^2\right]\Phi_S(z) = 0,$$

- Solution

$$\widetilde\Phi(z)_{\mu_1\mu_2\cdots\mu_S} = \left(\frac{z}{R}\right)^S \Phi(z)_{\mu_1\mu_2\cdots\mu_S} = C e^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}(z\mathcal{M})\,\epsilon(P)_{\mu_1\mu_2\cdots\mu_S},$$

- Conformal dimension:

$$\Delta = \frac{1}{2}\left(d + \sqrt{(d-2S)^2 + 4\mu^2R^2}\right).$$

- Normalization:

$$R^{d-2S-1}\int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^{d-2S-1}}\,\Phi_S^2(z) = 1.$$

- Stable solutions satisfy a generalized B-F bound

$$(\mu R)^2 \geq -\frac{(d-2S)^2}{4}.$$

For the ground state $\Delta = 2$, independent of S.

- Upon the substitution in the spin-S AdS wave equation ($d = 4$)

$$\phi(\zeta)_{\mu_1\mu_2\cdots\mu_S} = \left(\frac{\zeta}{R}\right)^{-3/2+S} \Phi(\zeta)_{\mu_1\mu_2\cdots\mu_S}$$

- Find light-front equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right]\phi_{\mu_1\mu_2\cdots\mu_S} = \mathcal{M}^2\phi_{\mu_1\mu_2\cdots\mu_S},$$

where $(\mu R)^2 = -(2-S)^2 + L^2$.

- Solution

$$\phi(\zeta)_{\mu_1\mu_2\cdots\mu_S} = \epsilon_{\mu_1\mu_2\cdots\mu_S}\phi(\zeta),$$

where the profile function $\phi(z)$ is the solution for the scalar mode !

- The lowest stable solution corresponds to $L = 0$ for every spin mode.

- Consider the matrix element for the external source $J(Q,z) = zQK_1(zQ)$:

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, \widetilde{\Phi}_S(z) J(Q,z) \widetilde{\Phi}_S(z).$$

- Since the external source is suppressed inside AdS for large Q, the important contribution is from $z \sim 1/Q$, where $\widetilde{\Phi} \sim z^{\Delta}$,
- For large Q^2: $F(Q^2) \to \left[1/Q^2\right]^{\Delta-1}$ (Dimensional counting!)
- Shifted field couples to the interpolating operator $\mathcal{O}^{i_1 i_2 \cdots i_S}$ in the generating functional with scaling dimensions

$$\left[\mathcal{O}^{i_1 i_2 \cdots i_S}\right] = d - \left[\widetilde{\Phi}_{i_1 i_2 \cdots i_S}\right] = 2 + L, \qquad (1)$$

thus the twist-dimension is $2 + L$.

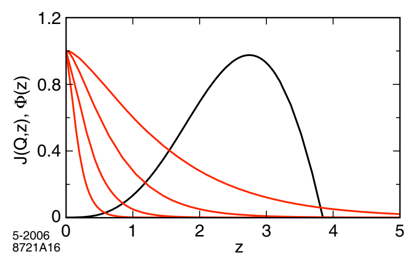


Fig: Suppression of external modes for large Q inside AdS. Red curves: $J(Q,z)$, black: $\Phi(z)$.

- Pseudoscalar meson interpolating operator $\mathcal{O}_{2+L} = \bar{q}\gamma_5 D_{\{\ell_1} \cdots D_{\ell_m\}} q, \;\; L = \sum_{i=1}^{m} \ell_i$.
- Vector-meson interpolating operator: $\mathcal{O}_{2+L}^{\mu} = \bar{q}\gamma^{\mu} D_{\{\ell_1} \cdots D_{\ell_m\}} q, \;\; L = \sum_{i=1}^{m} \ell_i$.
- Mode spectrum from boundary conditions : $\phi\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0$

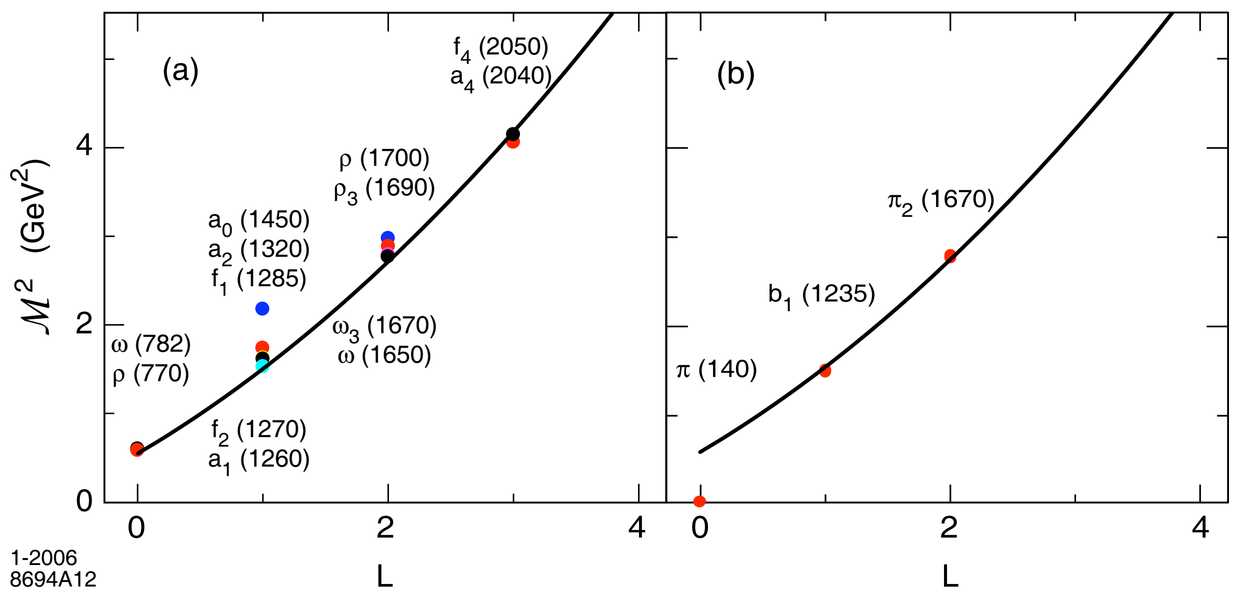


Fig: Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \; \mathrm{GeV}$

Non-Conformal Extension of Algebraic Integrability

- Consider the generator (short-distance Coulombic and long-distance linear potential)

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

  and its adjoint $\Pi_\nu^\dagger$ with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

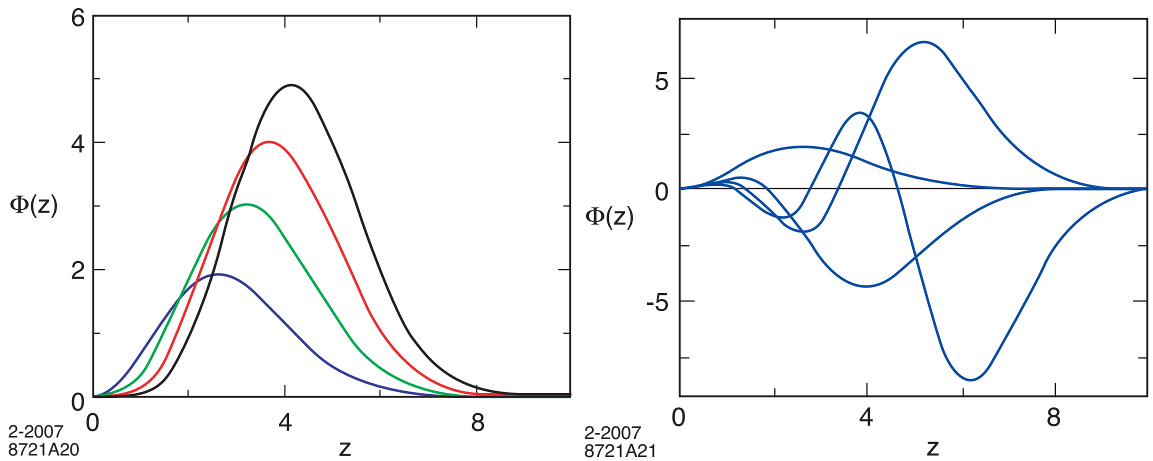
- Light-cone hamiltonian Hamiltonian $H_{LC} = \Pi_\nu^\dagger \Pi_\nu + C$ is positive definite $\langle\phi|H_{LC}|\phi \geq 0$ for $\nu^2 \geq 0$, and $C \geq -4\kappa^2$.

- Orbital and radial excited states are constructed from the ladder operators from $\nu = 0$ state

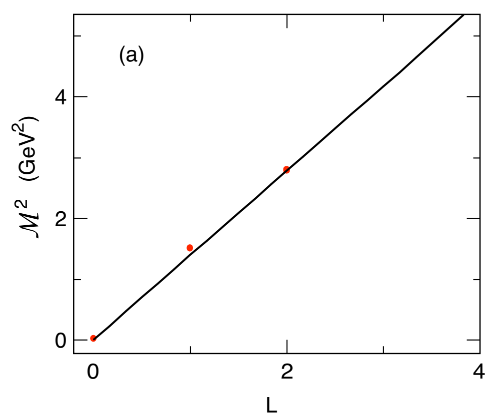
$$\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L\left(\kappa^2\zeta^2\right).$$

- Identify the zero mode ($C = -4\kappa^2$) with the pion $\mathcal{M}^2 = 4\kappa^2(n+L)$.

- Similar model with background dilaton: Karch, Katz, Son and Stephanov (2006).



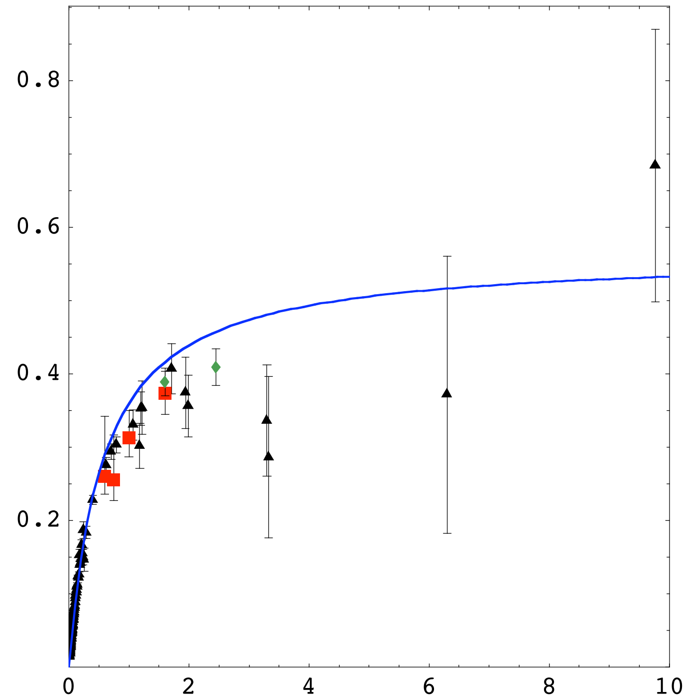
Pion orbital and radial modes in a soft wall model.



Pion Regge Trajectory $\kappa = 0.59$ GeV

Space-Like Pion Form Factor

$Q^2 F_\pi(Q^2)$

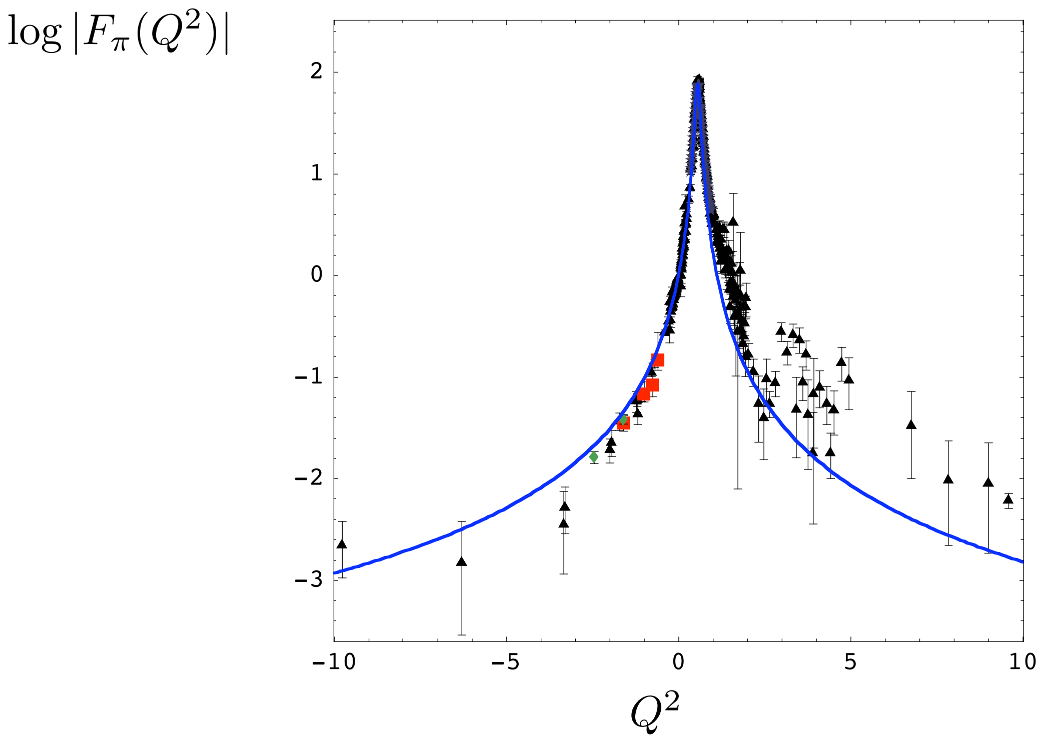


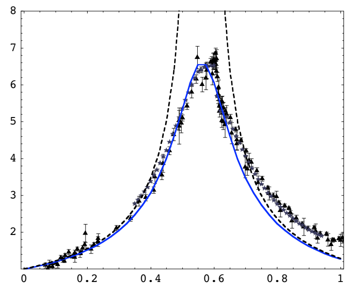
Q^2

- Bulk-to-boundary propagator $J_\kappa(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) \rightarrow zQK_1(zQ)$, for large Q^2 $(\kappa = 0.375$ GeV).

Space and Time-Like Pion Form Factor

Analytic continuation $Q^2 \to -Q^2$





$\kappa = 0.375$ GeV, $q^2 \to q^2 + iM\Gamma$, Γ_ρ= 110 MeV **PRELIMINARY**

3 Fermionic Modes

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(z)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(z)$ of are solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LC} = \alpha\Pi$ and the operator

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_\nu^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \frac{2\nu + 1}{\zeta^2}\gamma_5.$$

- In the Weyl representation ($i\alpha = \gamma_5\beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3+L$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

  Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

  given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

  with a scale independent mass ratio.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

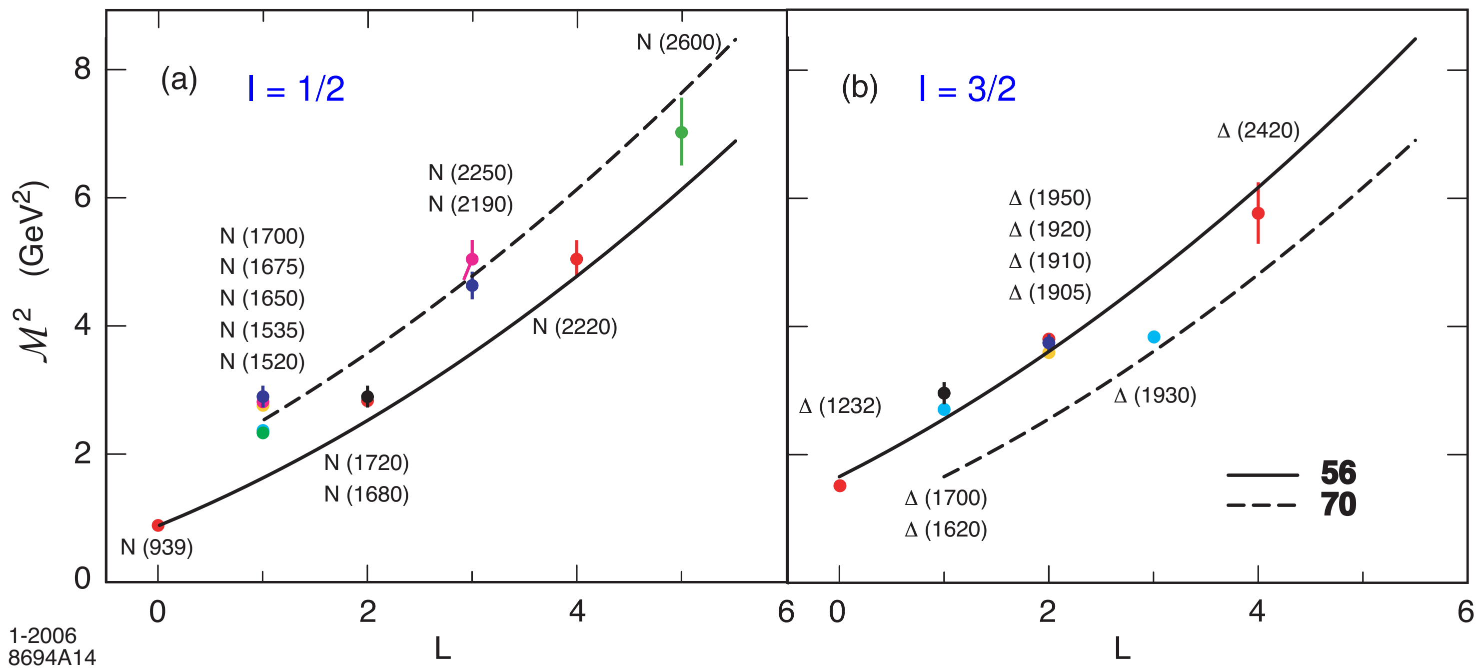


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure and Linear Confinement (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2),.\end{aligned}$$

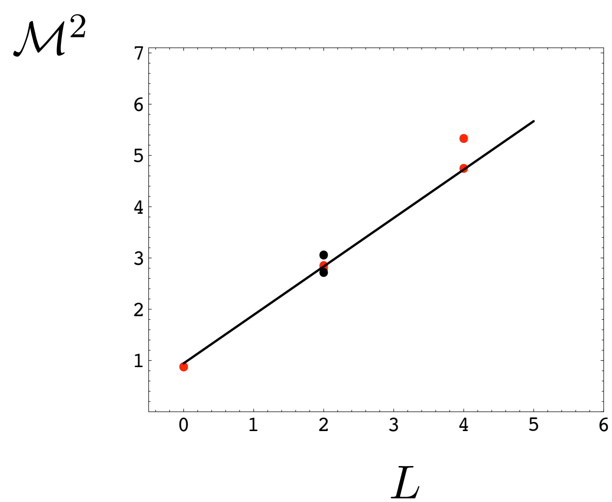
- Eigenvalues $\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

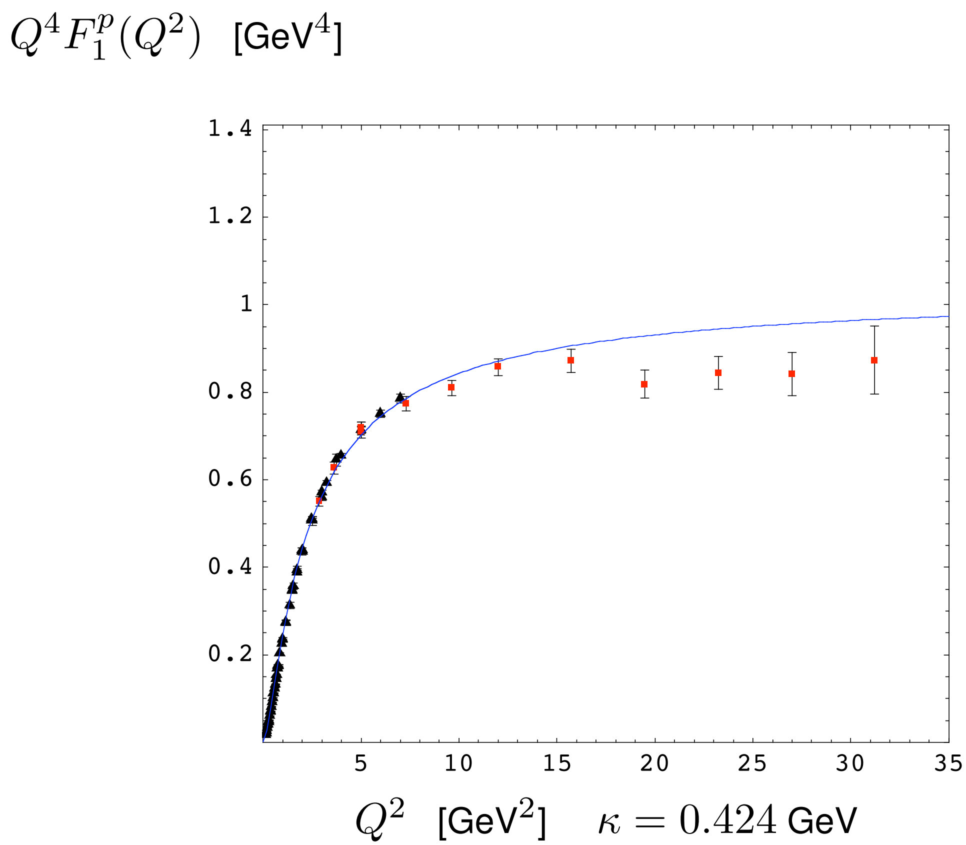
$$\mathcal{M}^2 = 4\kappa^2 (n + L + 1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV

PRELIMINARY: stability conditions of fermionic modes

Space-Like Dirac Proton Form Factor



Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Linear Holographic Confinement

- Dirac equation in AdS space In presence of a potential $V(z) \quad \left(x^\ell = (x^\mu, z)\right)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + V(z)\right]\Psi(x^\ell) = 0. \tag{2}$$

- We consider the linear confining potential

$$V(z) = \kappa^2 z. \tag{3}$$

- Writing the solution in the form

$$\Psi(x, z) = e^{-iP\cdot x} z^2 \psi(z), \tag{4}$$

we find

$$\alpha\Pi(z)\psi(z) = \mathcal{M}\psi(z), \tag{5}$$

with

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right). \tag{6}$$

We identify $\mu R = \nu + \frac{1}{2}$, to recover our previous results.