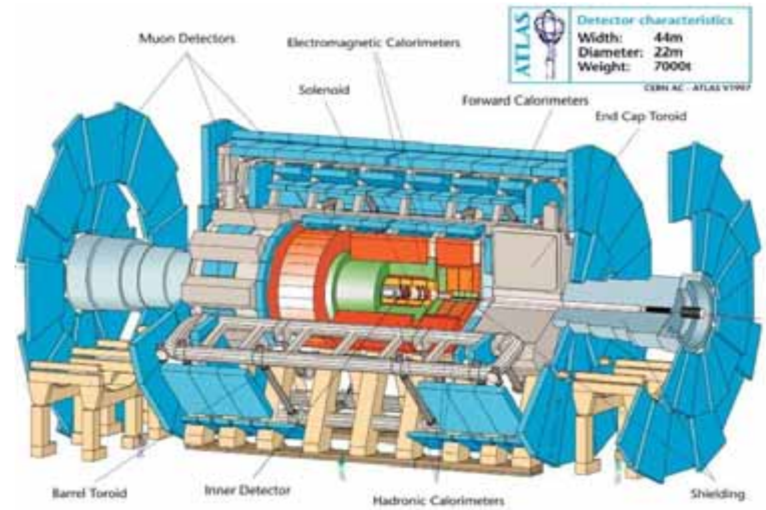
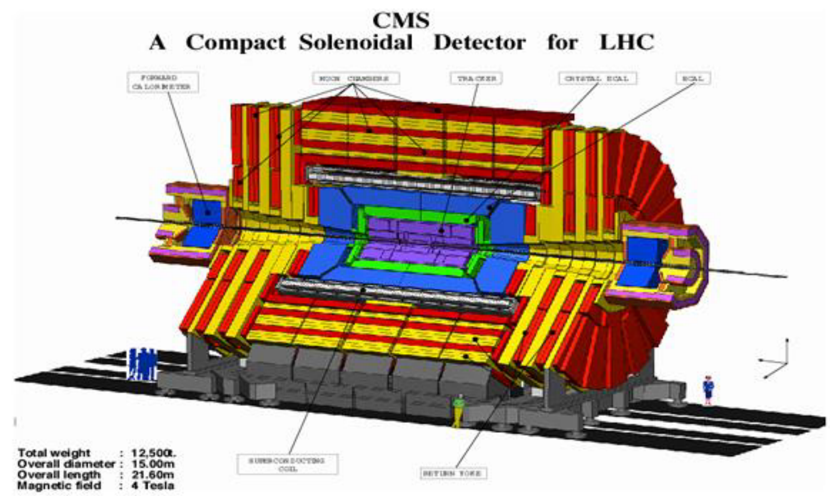
Collider Tests of Quantum Gravity





- In order to <u>directly</u> test any 'Theory of Quantum Gravity' at a collider such as the LHC, the effective Planck mass (where these QG effects potentially become important) must not be too far above ~ a few TeV. Otherwise …

- How is this possible given what we know?

Of course, anything we say here is HIGHLY speculative!

Either

(i) M_{Pl} runs rapidly via RGE at short distances to ~1 TeV due to many 'hidden' states in 4-d (Calmet, Dvali,..)

Or

(ii) There are extra spatial dimensions that we don't observe until we get to short distances ~ 1 TeV ←

Or

???????????????

→ Explore gravity in possible ED scenarios at short distances

GR , as described by the EH action in D-dimensions, is 'likely' only an effective field theory below the scale M_{Pl} since it is not a perturbatively renormalizable QFT like the SM. What paths can we take to understand this problem & how can we test them at colliders such as the LHC?

→Weinberg ('79) : There are only a few possibilities…

- The EH action is an incomplete description & new physics is missing which tames GR's poor UV behavior : strings, LQG,…
- Gravitons are composite objects: (Okui, Henty, Dvali,…)
- The conventional PT expansion used in quantizing GR must be re-ordered a la Yennie, Frautschi & Suura : (Ward,…)

OR

- The poor UV behavior is a perturbative artifact &GR is non-PT renormalizable being 'asymptotically safe' due to the existence of a non-Gaussian fixed point in the RGE. In such a scenario the running gravitational coupling becomes weaker as M_{Pl} is approached: (Lauscher, Reuter, Litim, Saueressig,Codello, Fischer, …)

→ How can we test ideas like these at the LHC??

- At the very least, we must first find TeV-scale ED & then search for 'non-GR-type physics effects' such as:
  - (i) TeV BH with 'QG-like' properties (Cavaglia)
  - (ii) String excitations
  - (iii) Gravity becoming weaker at hi-E (due to , e.g., a UV 'fixed point')
  - (iv) 'QG-like' higher curvature (e.g., R^2) effects
  - (v) Space-Time Non-Commutativity
  - (vi) TeV-scale Lorentz Violation
  - (vii) Causality Violation

etc.…

I will NOT advocate any of the ideas below but simply discuss how to test some of them at colliders.

TeV Scale ED @ the LHC & e^+e^- LC

Arkani-Hamed, Dimopoulos & Dvali
Randall & Sundrum

- After a dozen years there are essentially two 'toy' models of TeV-scale ED w/ gravity : ADD & RS, which have distinctive collider signatures. These were motivated to solve the gauge hierarchy problem & not to deal with GR issues.

- ADD: The SM lives on a 4-d hypersurface, $y^i = 0$, in a 4+n dimensional space where gravity lives & is described by GR. Then assuming ED compactified on a torus, $V_n = (2\pi R_c)^n$ &

$$S_{n+4} = \frac{M_*^{n+2}}{2} \int d^4x \, dy \sqrt{-g} R_{n+4} \,, \longrightarrow \overline{M}_{Pl}^2 = V_n M_*^{n+2}$$

- M_* ~ a few TeV is the TRUE Planck scale → R_c & the mass scale for the KK excitations :

$$m_n^2 = \vec{n}^{\,2} / R_c^2$$

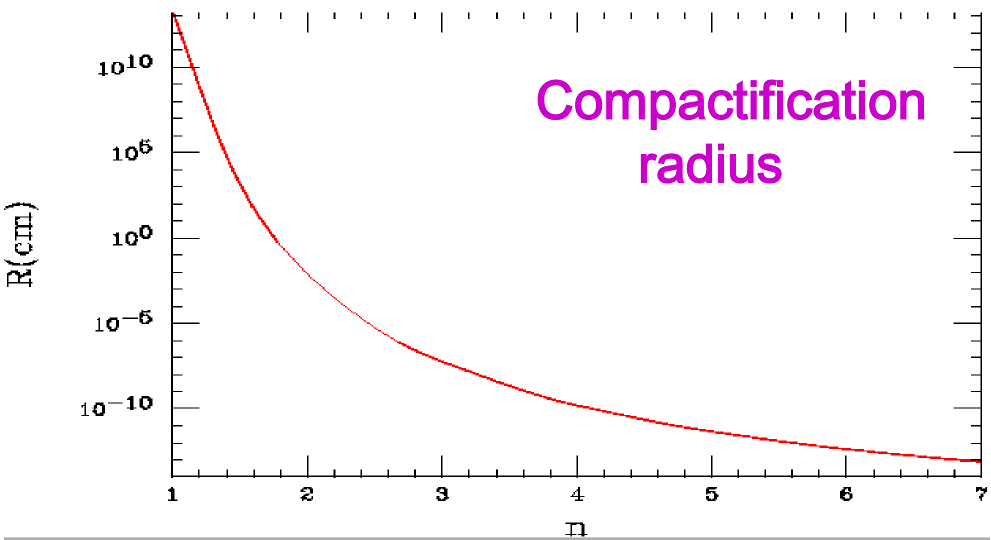
- $n \geq 2$ to satisfy existing constraints

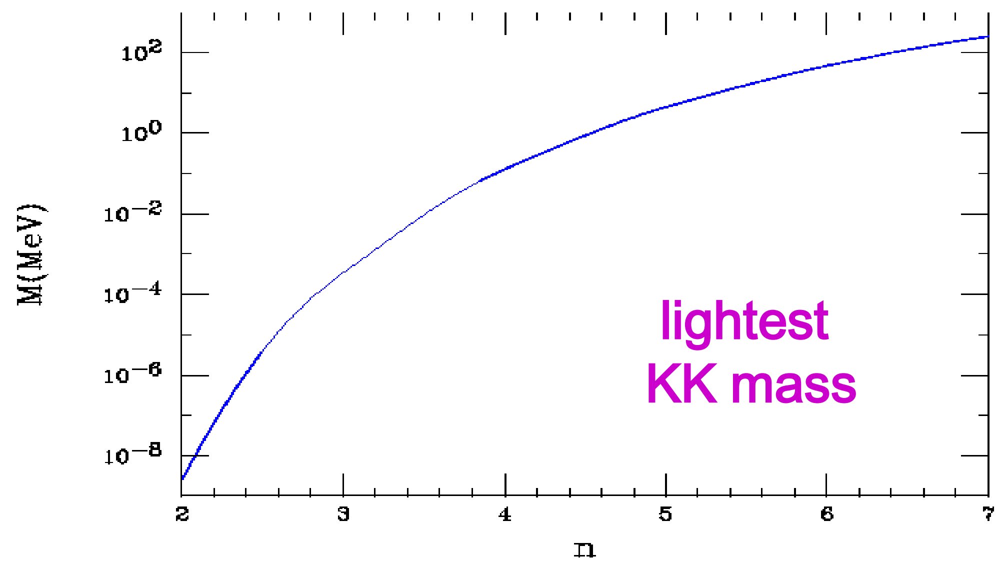
$$g_{AB} = \eta_{AB} + h_{AB}(x, y)/ M_*^{n/2+1}$$

$$S_{int} = -\frac{1}{M_*^{n/2+1}} \int \sqrt{-g}\, d^n y\, d^4 x\; h_{AB}(x,y)\, T^{AB}(x,y)$$

- Perform Graviton KK reduction
- Expand h_{AB} into KK tower
- Place SM on 3-brane …set $T_{AB} = \eta^{\mu}{}_{A}\eta^{\nu}{}_{B}\, T_{\mu\nu}\, \delta(y^a)$
- Pick the physical/unitary gauge(s)
- Integrate over $d^n y$

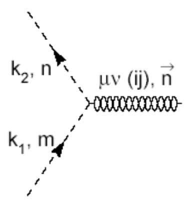
} do this





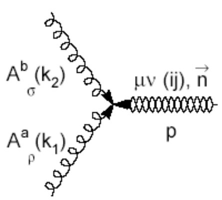
Feynman Rules: Graviton KK Tower

Massless 0-mode + all KK states have identical coupling to SM matter:



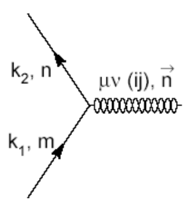
$\tilde h^{\vec n}_{\mu\nu}\,\Phi\Phi$: $-i\kappa/2\;\delta_{mn}\,(m_\Phi^2\,\eta_{\mu\nu} + C_{\mu\nu,\rho\sigma}\,k_1^\rho k_2^\sigma)$

$\tilde\phi^{\vec n}_{ij}\,\Phi\Phi$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta_{mn}\,(k_1\cdot k_2 - 2\,m_\Phi^2)$



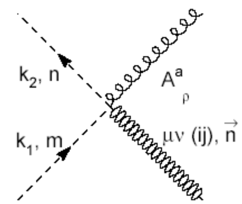
$\tilde h^{\vec n}_{\mu\nu}\,AA$: $-i\kappa/2\;\delta^{ab}\,(\,(m_A^2 + k_1\cdot k_2)\,C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(k_1,k_2) + \xi^{-1}\,E_{\mu\nu,\rho\sigma}(k_1,k_2)\,)$

$\tilde\phi^{\vec n}_{ij}\,AA$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta^{ab}\,(\eta_{\rho\sigma}\,m_A^2 + \xi^{-1}\,(k_{1\rho}\,p_\sigma + k_{2\sigma}\,p_\rho))$



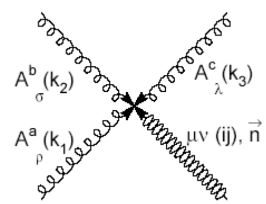
$\tilde h^{\vec n}_{\mu\nu}\,\psi\psi$: $-i\kappa/8\;\delta_{mn}\,(\gamma_\mu\,(k_{1\nu}+k_{2\nu}) + \gamma_\nu\,(k_{1\mu}+k_{2\mu}) - 2\,\eta_{\mu\nu}\,(\not k_1 + \not k_2 - 2\,m_\psi))$

$\tilde\phi^{\vec n}_{ij}\,\psi\psi$: $i\,\omega\,\kappa\,\delta_{ij}\,\delta_{mn}\,(3/4\,\not k_1 + 3/4\,\not k_2 - 2\,m_\psi)$



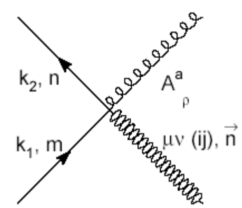
$\tilde h^{\vec n}_{\mu\nu}\,\Phi\Phi A$: $i\,g\,\kappa/2\;C_{\mu\nu,\rho\sigma}\,(k_1^\sigma + k_2^\sigma)\,T^a_{nm}$

$\tilde\phi^{\vec n}_{ij}\,\Phi\Phi A$: $-i\,\omega\,g\,\kappa\,\delta_{ij}\,(k_{1\rho} + k_{2\rho})\,T^a_{nm}$



$\tilde h^{\vec n}_{\mu\nu}\,AAA$: $g\,\kappa/2\;f^{abc}\,(\,C_{\mu\nu,\rho\sigma}\,(k_{1\lambda} - k_{2\lambda}) + C_{\mu\nu,\rho\lambda}\,(k_{3\sigma} - k_{1\sigma}) + C_{\mu\nu,\sigma\lambda}\,(k_{2\rho} - k_{3\rho}) + F_{\mu\nu,\rho\sigma\lambda}\,(k_1,k_2,k_3)\,)$

$\tilde\phi^{\vec n}_{ij}\,AAA$: 0



$\tilde h^{\vec n}_{\mu\nu}\,\psi\psi A$: $i\,g\,\kappa/4\;T^a_{nm}\,(\,C_{\mu\nu,\rho\sigma} - \eta_{\mu\nu}\,\eta_{\rho\sigma}\,)\,\gamma^\sigma$

$\tilde\phi^{\vec n}_{ij}\,\psi\psi A$: $-i\;3/2\;\omega\,g\,\kappa\,\delta_{ij}\,T^a_{nm}\,\gamma_\rho$

etc.

- Han, Lykken, Zhang
- Giudice, Rattazzi, Wells

How do we test a theory like this at a collider ??

- Graviton KK states can be emitted during the collision of 2 SM particles.

- Any given KK couples so weakly it will traverse the entire detector without any further interactions. KK graviton emission will appear as missing energy/momentum.

→ 'Monojet' signature

$$q\bar{q} \to g\, G_n$$

$$g\overset{(-)}{q} \to \overset{(-)}{q}\, G_n$$

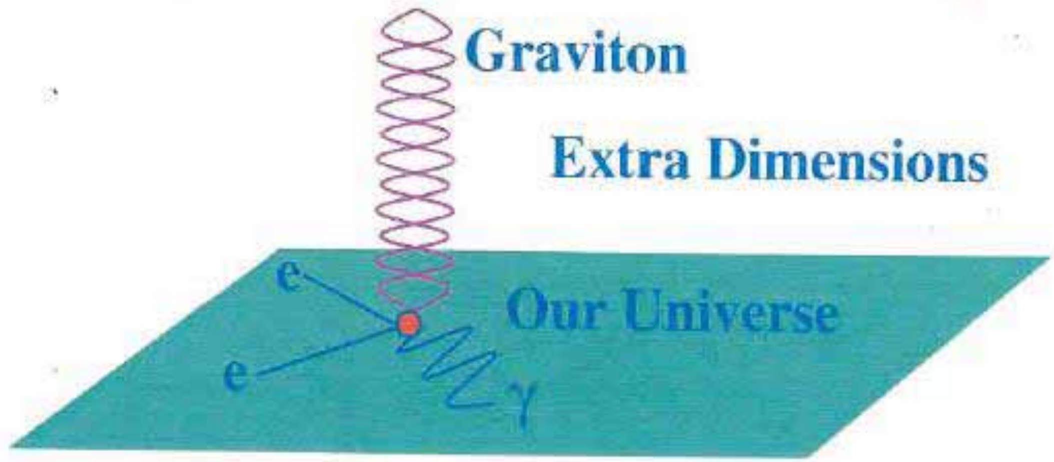
$$gg \to g\, G_n$$

- A second possibility is to exchange an entire KK tower of gravitons between pairs of SM particles in, e.g., dilepton pair production at the LHC or at a LC. This causes an interference between SM & graviton exchange amplitudes .

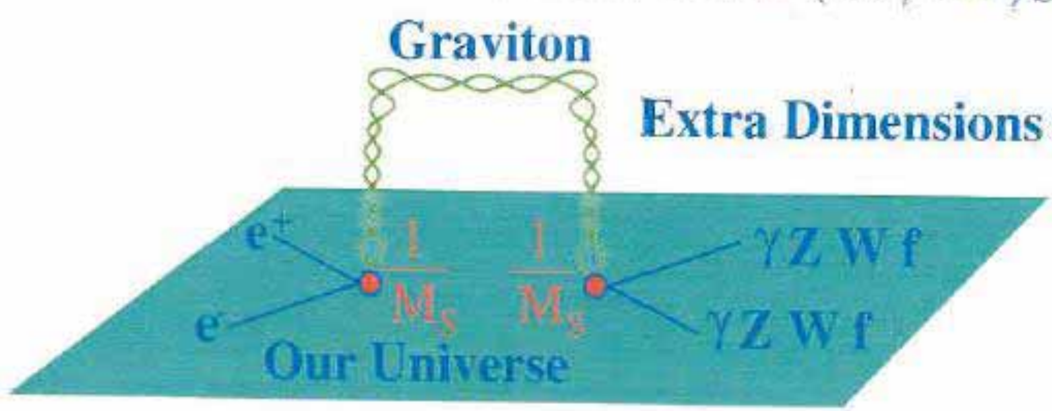
Collider Tests

Search Strategy

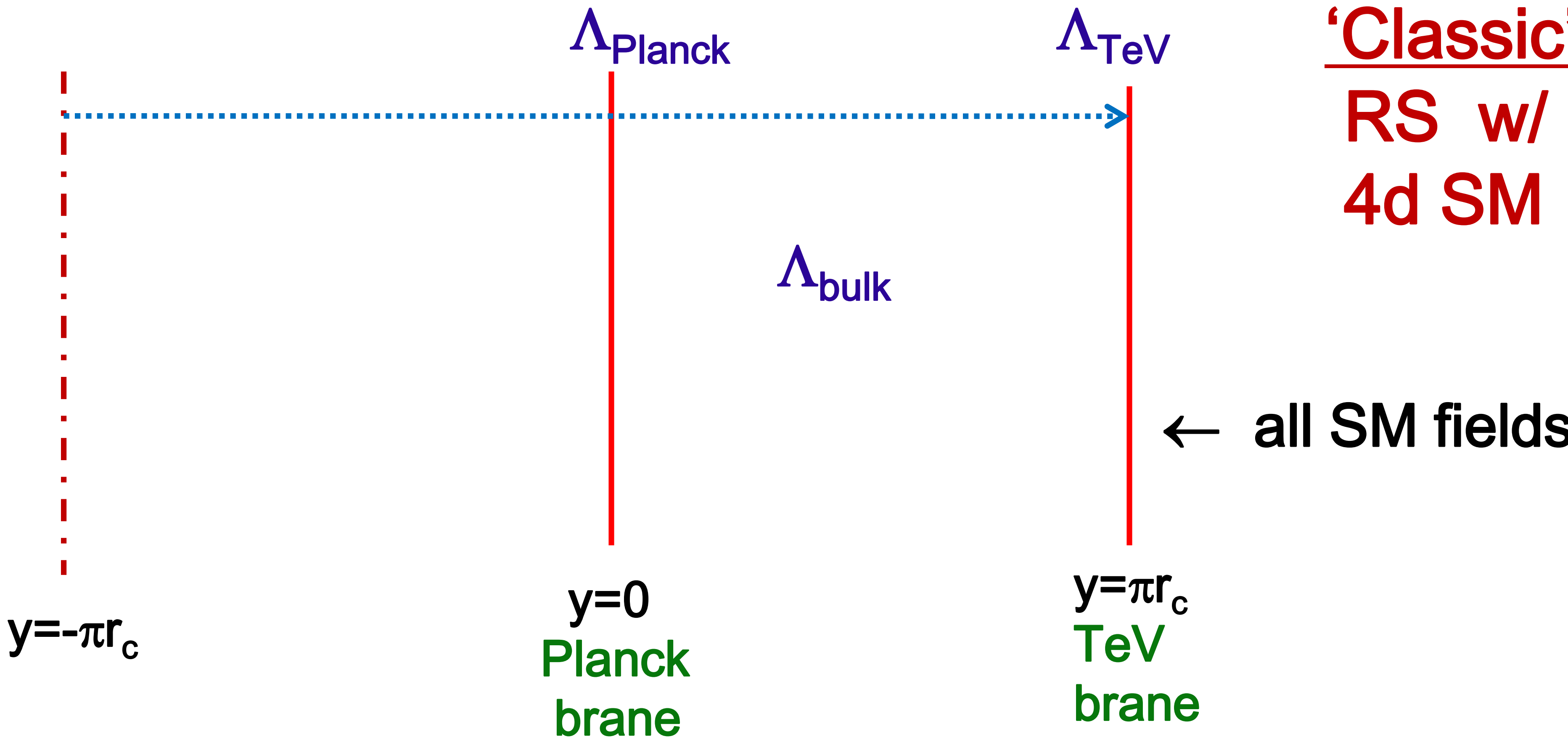
Direct Search: 1 photon or 1 Z boson + missing energy.



Indirect Search: Look for deviations from $(d\sigma/d\Omega)_{SM}$.



'Classic' RS w/ 4d SM



S^1/Z_2 symmetry imposed on the extra dimension y

$$ds^2 = e^{-2\sigma(y)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu - dy^2 \qquad \sigma = k|y|$$

No large hierarchy : $k \sim M_5 \sim \bar{M}_{Pl}$

Some Predictions

- KK gravitons have discrete masses : $m_n / m_1 = x_n / x_1$ with $J_1(x_n) = 0$ & $m_1 \sim$ TeV, $x_n = 3.83, 7.02, 10.17, \ldots$

- KK states couple universally with an (inverse) strength of $\Lambda_\pi = M_{Pl}\, \varepsilon \sim$ TeV

- Spin-2 graviton resonance structures with ~EWK couplings will appear in all $2 \rightarrow 2$ SM processes once collision energies are sufficiently large to make them , i.e., ~TeV

$\rightarrow$ But, for us, RS & ADD predictions will CHANGE in QG scenarios !

Asymptotically Safe ED @ Colliders

Plehn, Litim, Gerwick
Hewett, TGR

- Wilsonian RGE analysis of a truncated gravity action, e.g., the EH one with $G_D = (8\pi M^{D-2})^{-1}$. Qualitative results not very sensitive to D or to the operator basis truncation in the action. (Bonanno, Reuter, Wetterich, Litim, ….)

- Study the flow of dimensionless coupling $g(\mu) = G_D \mu^{D-2}$

⟶ toy models

$$\frac{dg}{dt} = [d - 2 + \eta]g \qquad \eta = \frac{gB_1}{1 - gB_2}$$

IR Gaussian FP, $g_{IR} = 0$, & a UV non-Gaussian FP, $g_{UV} = 1/\omega'$

Idea is supported by 'lattice' GR results (Hamber)

$$B_1 = \frac{1}{3}\,(4\pi)^{1-D/2}\Big[D(D-3)\phi_1 - [6D(D-1)+24]\phi_2\Big]$$

$$B_2 = -\frac{1}{6}\,(4\pi)^{1-D/2}\Big[D(D+1)\tilde{\phi}_1 - 6D(D-1)\tilde{\phi}_2\Big]\,,$$

$$\omega = -B_1/2 \text{ and } \omega' = \omega + B_2$$

$$\phi_1 = \frac{1}{\Gamma(D/2-1)}\int_0^\infty dz\; z^{D/2-2}\,\frac{R - zR'}{z+R}$$

$$\phi_2 = \frac{1}{\Gamma(D/2)}\int_0^\infty dz\; z^{D/2-1}\,\frac{R - zR'}{(z+R)^2}$$

$$\tilde{\phi}_1 = \frac{1}{\Gamma(D/2-1)}\int_0^\infty dz\; z^{D/2-2}\,\frac{R}{z+R}$$

$$\tilde{\phi}_1 = \frac{1}{\Gamma(D/2)}\int_0^\infty dz\; z^{D/2-1}\,\frac{R}{(z+R)^2}\,.$$

R is an arbitrary, smooth ‘cutoff function ‘ w/ R(0)=1 & R(∞)=0 .. so $B_{1,2}$ show mild function choice dependence

→ $$\frac{g}{(1-\omega' g)^{\omega/\omega'}} = \frac{g(\mu_0)}{[1-\omega' g(\mu_0)]^{\omega/\omega'}}\left(\frac{\mu}{\mu_0}\right)^{D-2}$$

$\omega \simeq \omega'$

numerically

→ $$g \simeq \frac{g_0(\mu/\mu_0)^{D-2}}{[1+\omega g_0((\mu/\mu_0)^{D-2}-1)]}$$

or in terms of M

→ $$\frac{1}{M^{D-2}} \to \frac{1}{M_{eff}^{D-2}} = \frac{1}{M^{D-2}}\left[1 + \frac{\omega}{8\pi}\left(\frac{\mu^2}{M^2}\right)^{D/2-1}\right]^{-1}$$

$$\mathcal{L} = -T_{AB}H^{AB}/M_{eff}^{D-2}$$

Matter coupling

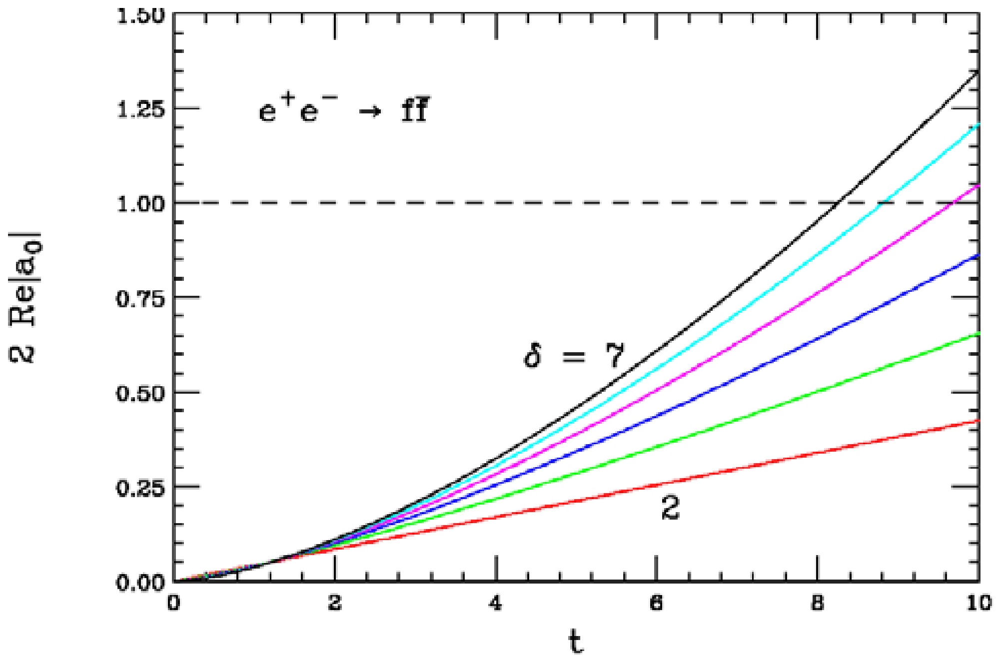
$$\frac{1}{M^{D-2}} \to \frac{1}{M^{D-2}}F(\mu^2) \qquad F = \left[1 + \left(\frac{\mu^2}{t^2M^2}\right)^{\delta/2+1}\right]^{-1} \qquad \delta = D - 4$$

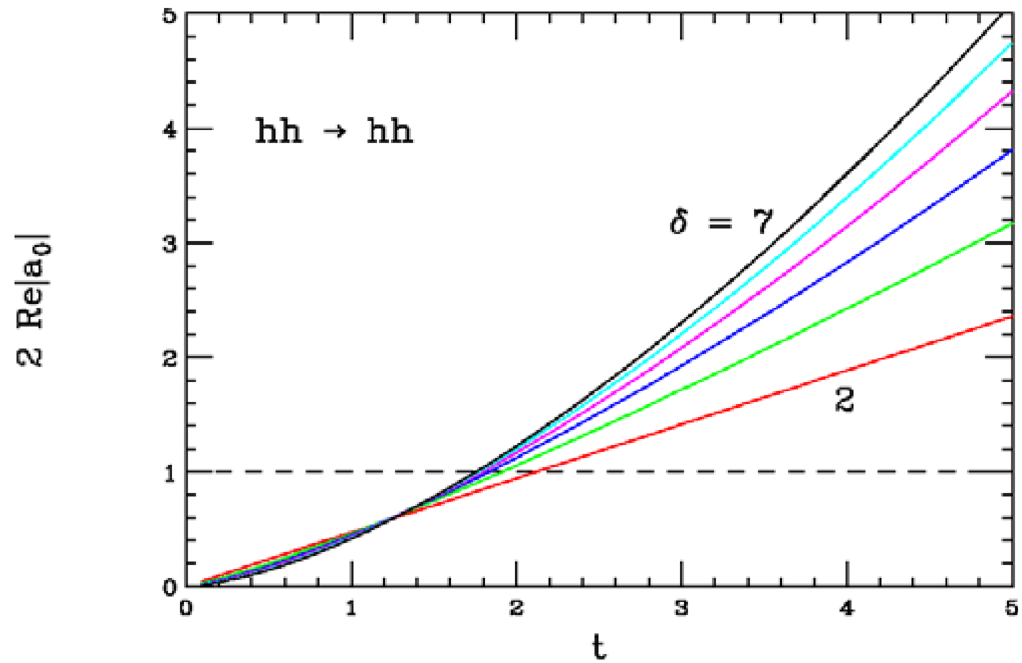
- Here t is treated as a free O(1) parameter used to accommodate the ‘slop’ from the approximations employed above (& below !).

- F acts like a Form Factor that cuts off gravitational interactions w/ matter as the Planck energy scale is approached

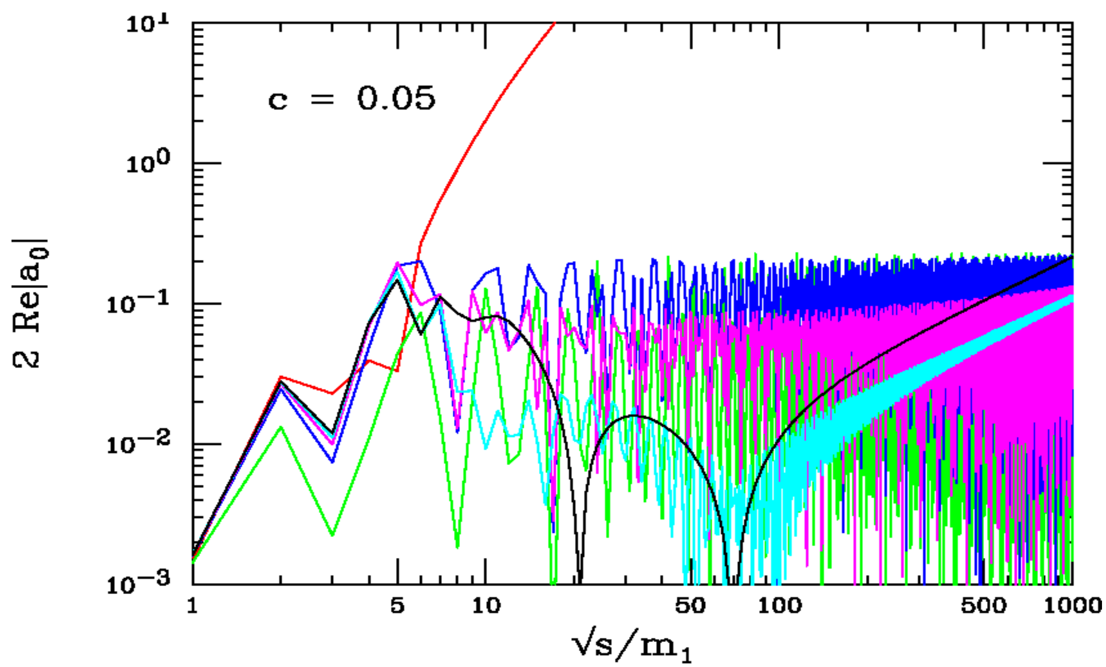
- For s-channel collider processes it is natural to take μ^2 =s with the further slop rolled into t

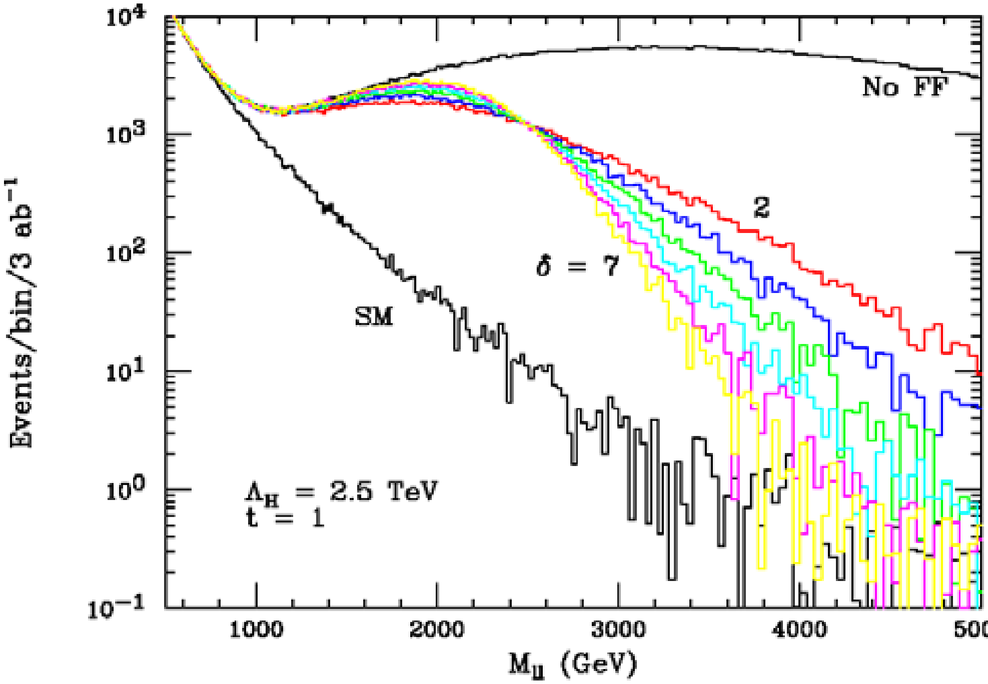
- F significantly modifies both ADD & RS predictions

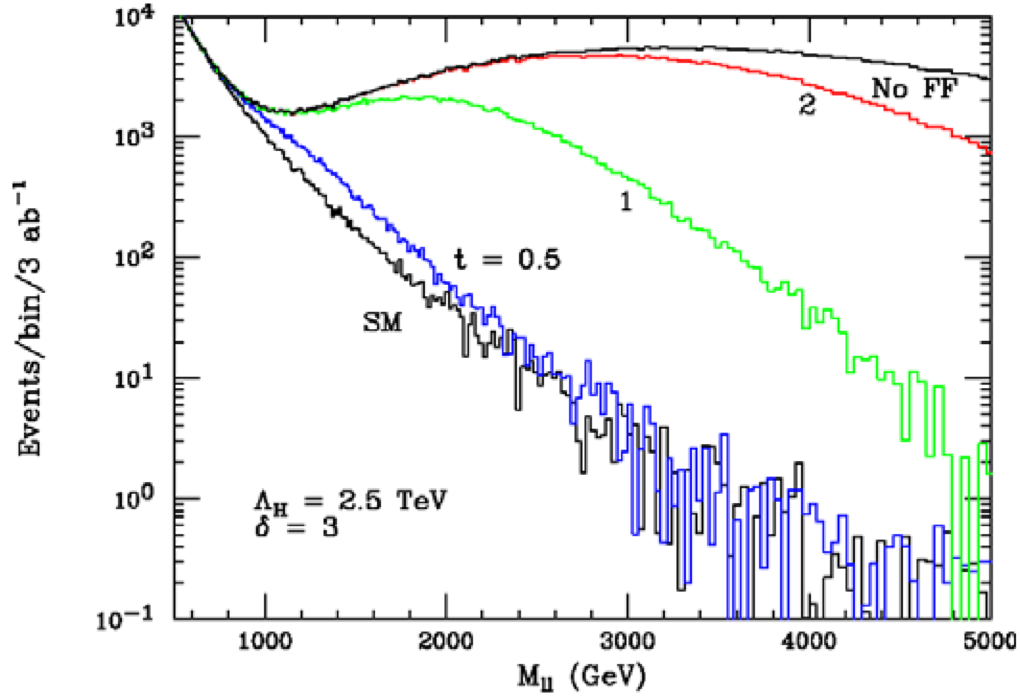




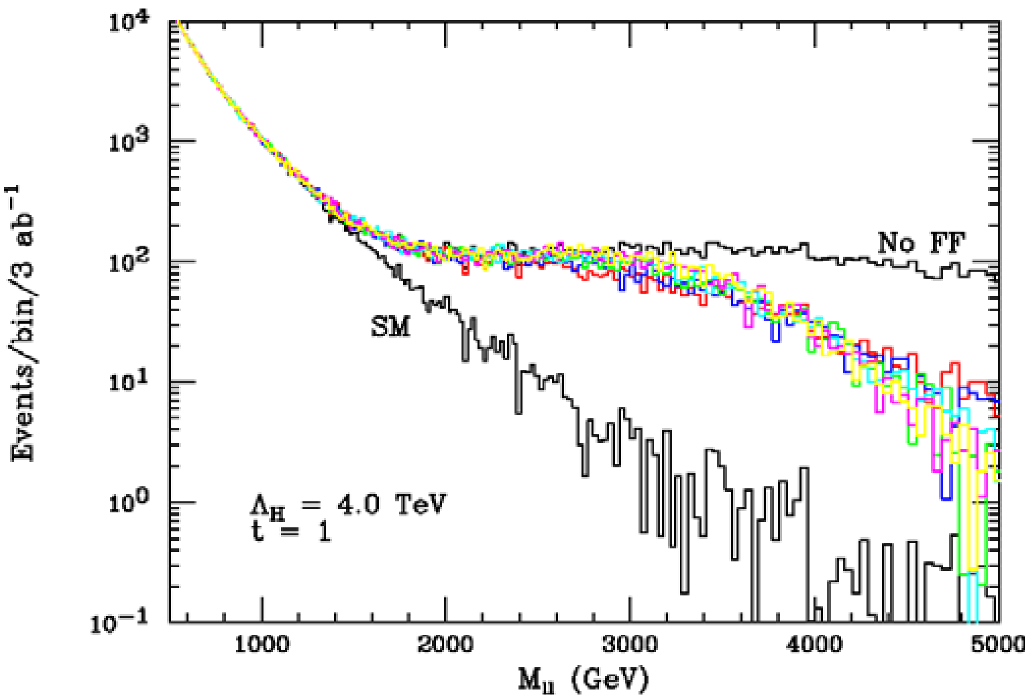
In both ADD & RS t < ~2 restores perturbative unitarity to graviton exchange cross sections !

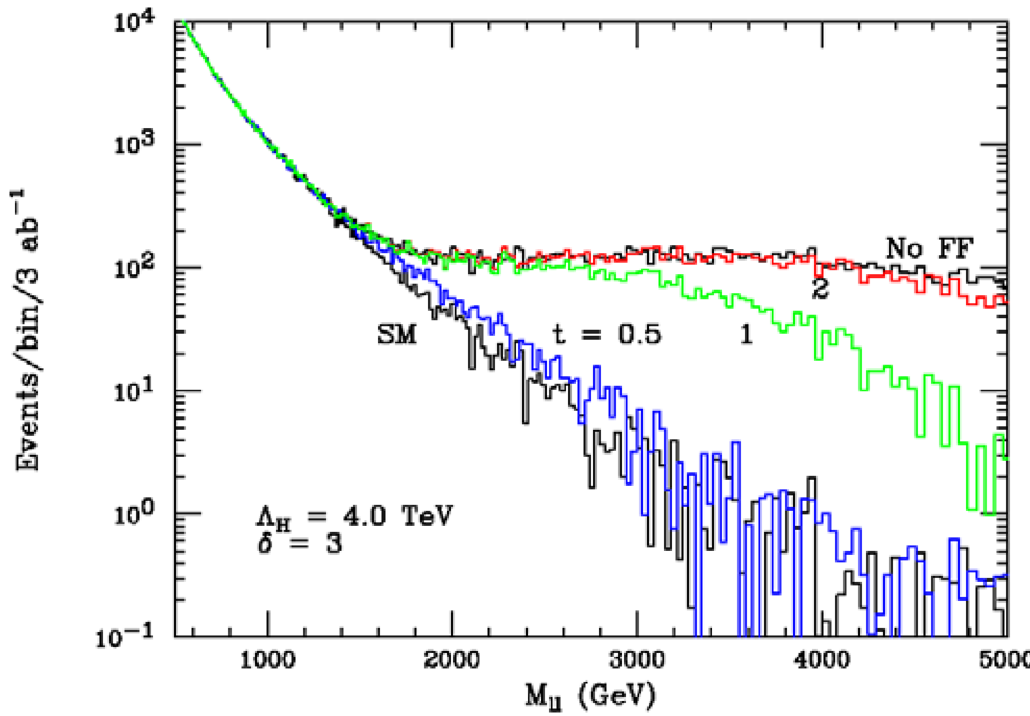


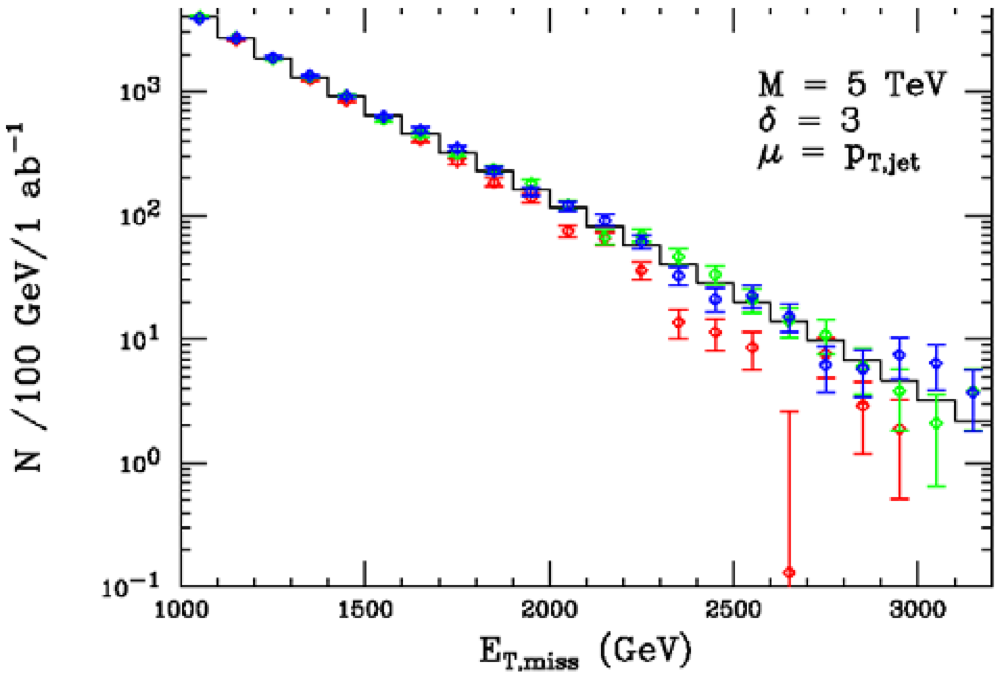


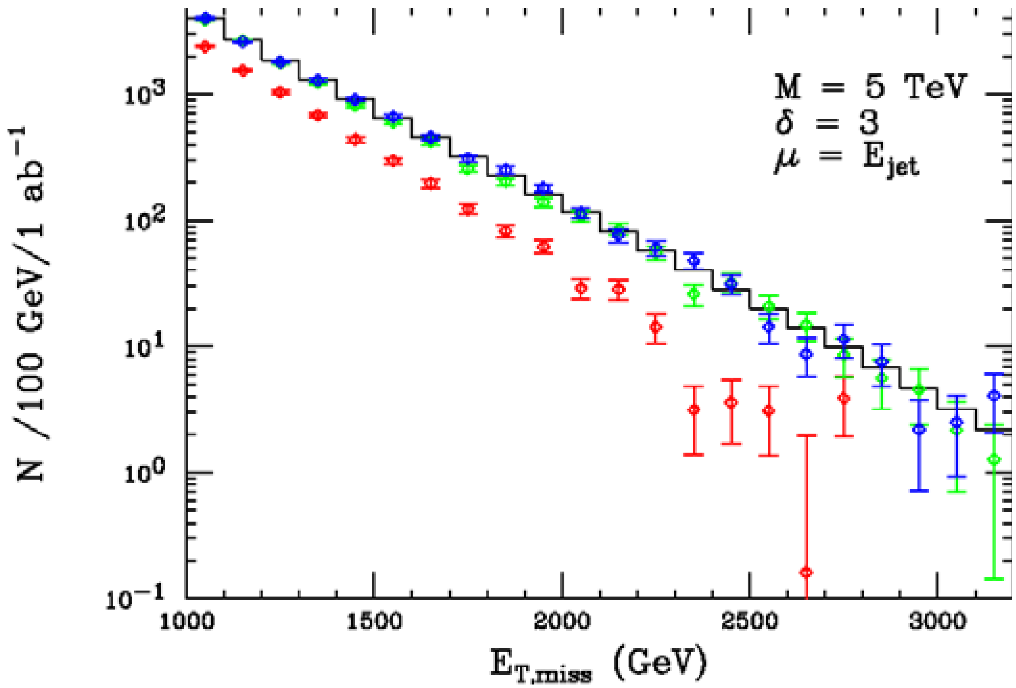


Graviton exchange in the Drell-Yan Channel in A.S. ADD

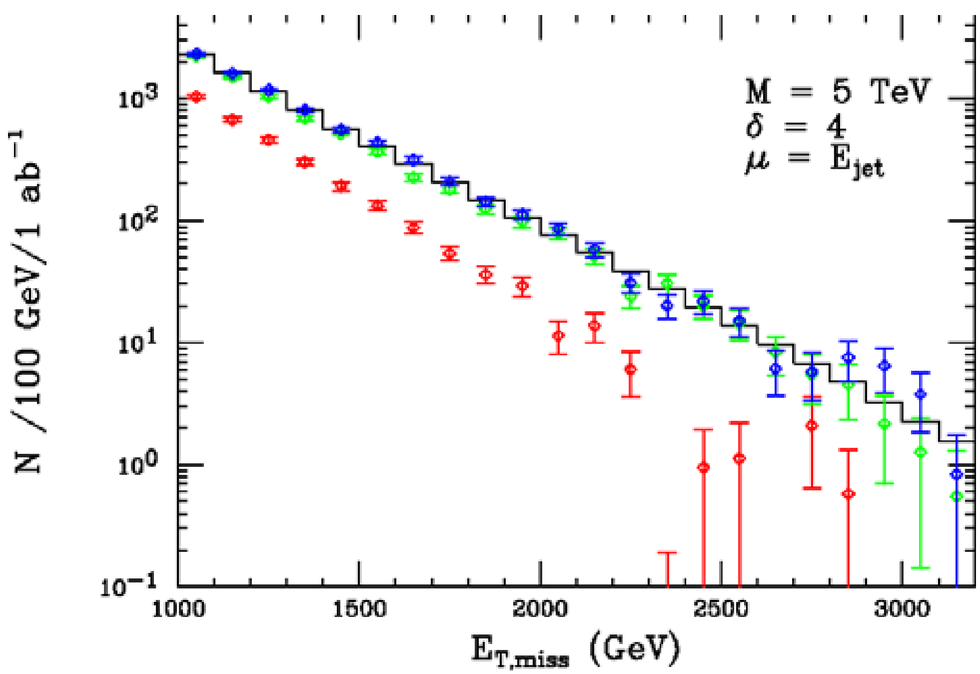


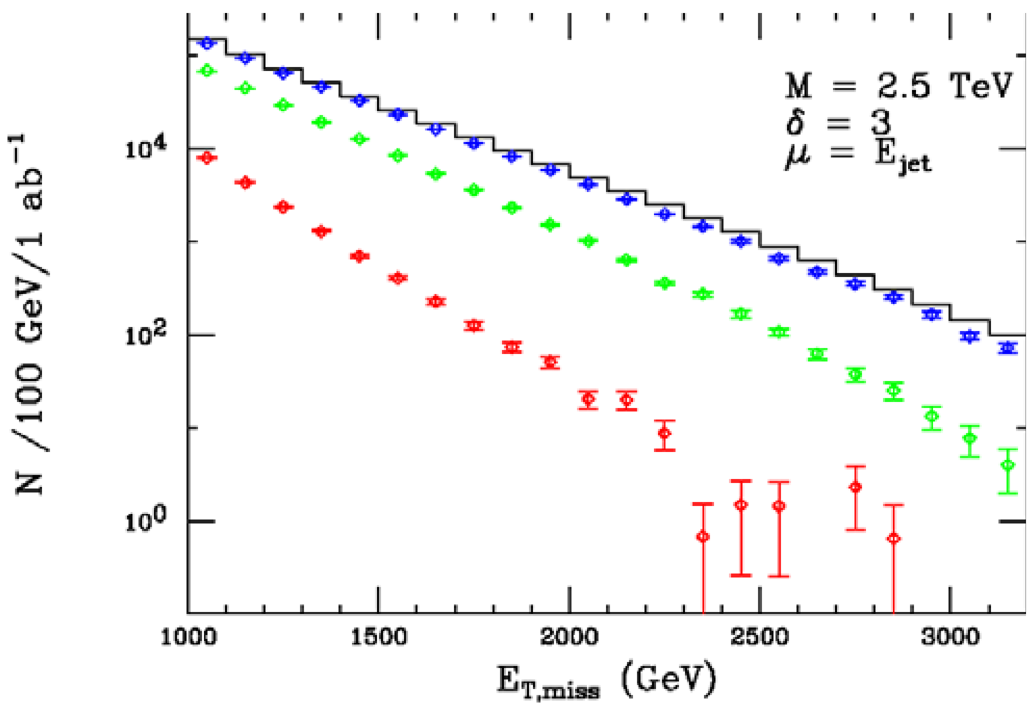






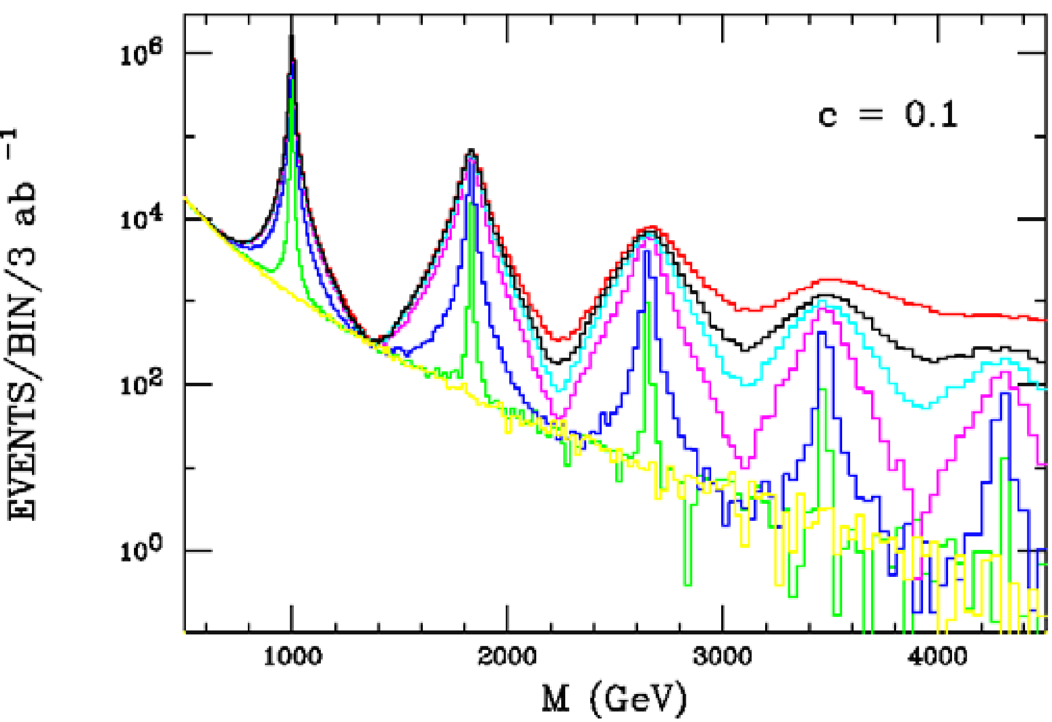
Modified monojet event rate distributions at the LHC

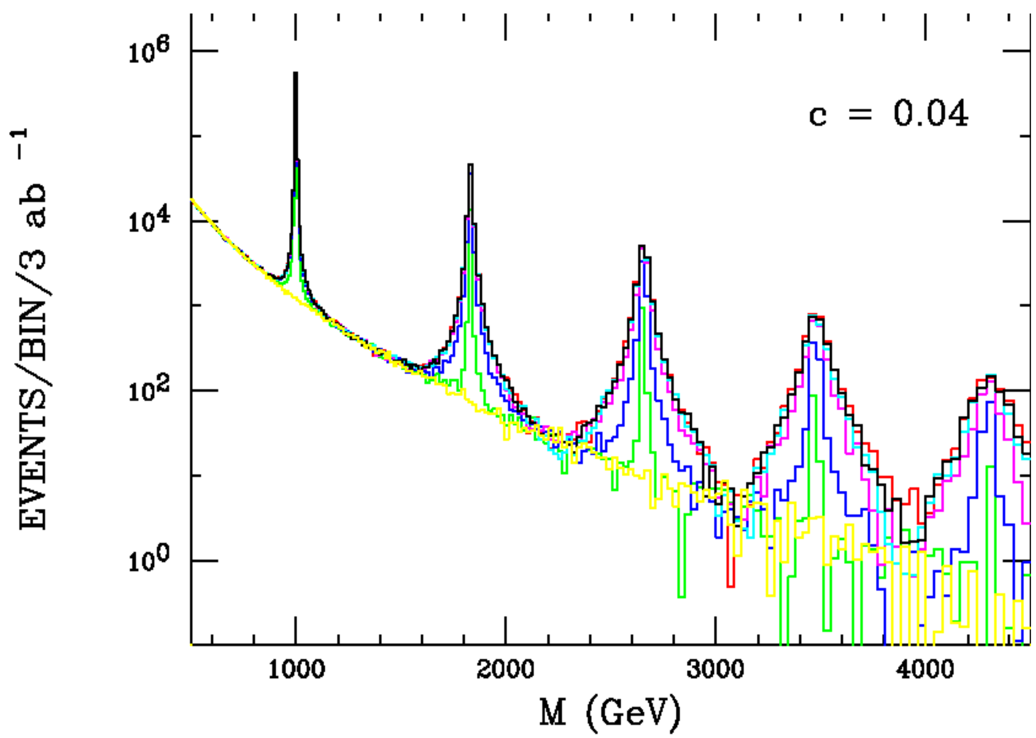




Asymptotic Safety is found to prevent the RS KK graviton resonances' decay widths from becoming too large so as to be smeared out at the LHC by restoring unitarity ! Furthermore, the cross sections themselves at large s are also well-behaved as in the ADD model case.

It's implications for RS models with bulk SM fields still remains unexplored..





Stringy Excitations @ Colliders

Accomando, Antoniadis, Benakli, Cullen, Perelstein, Peskin, Burikham, Han, Hussain, McKay, Lust, Dong, Stieberger, Taylor, Anchordoqui, Goldberg, Nawata, Huang, Shiu, Tye, Shrock, Chialva, Iengo, Russo, Rosa, March-Russell, Hassanain,….

- IF ST is operative near the TeV scale as a candidate theory of QG, THEN we might expect to see Regge excitations of the SM particles with various spins at these energies in a number of different scattering processes. N.B. : Excitations of a set of degenerate resonances with different spins is a rather unique collider signature.

- This requires modeling how to embed the SM into ST by, e.g., using D-brane constructions. Some of the features of this procedure are fairly ‘universal’ to LO in the gauge couplings for boson resonances, e.g., gluons. Examining QCD-induced processes may have the greatest reach though the DY process has much less background.

- The dominant effect in these string models is the ‘rescaling ‘ of SM amplitudes in the various s,t,u exchange channels by Veneziano factors:

$$V(s,t,u) = \frac{s\,u}{tM_s^2} B(-s/M_s^2, -u/M_s^2) = \frac{\Gamma(1-s/M_s^2)\,\Gamma(1-u/M_s^2)}{\Gamma(1+t/M_s^2)}.$$

- These can develop poles near the Regge resonances:

$$B(-s/M_s^2, -u/M_s^2) = -\sum_{n=0}^{\infty} \frac{M_s^{2-2n}}{n!} \frac{1}{s-nM_s^2} \left[\prod_{J=1}^{n} (u + M_s^2 J) \right];$$

$$V(s,t,u) \approx \frac{1}{s-nM_s^2} \times \frac{M_s^{2-2n}}{(n-1)!} \prod_{J=0}^{n-1} (u + M_s^2 J)$$

Spin sum

Resonances with mass = $\sqrt{n}\, M_s$ where M_s is the string scale

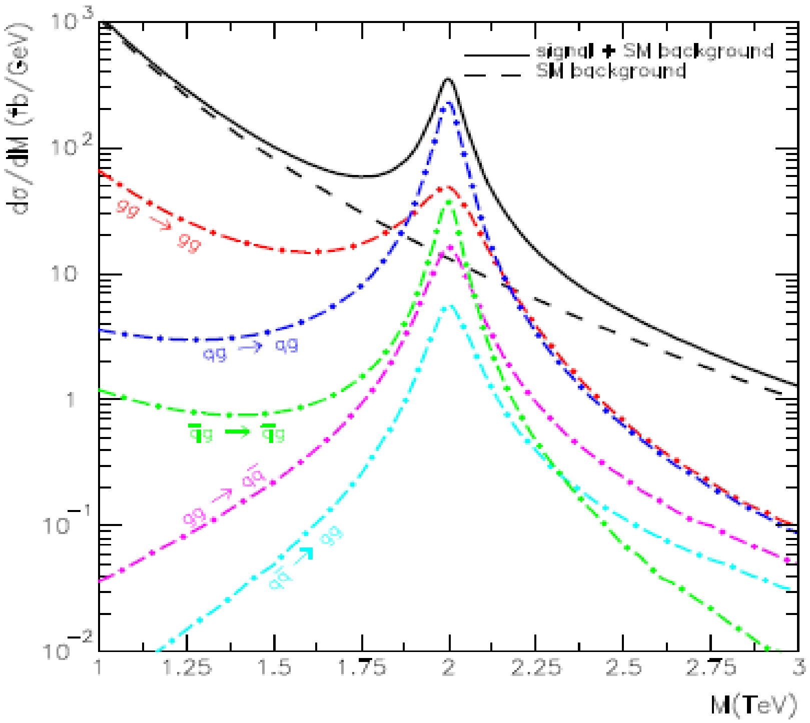
Finite resonance widths need to be included !

- If the Regge resonances are heavy, far below M_s ,V reduces to

$$V(s,t,u) \approx 1 - \frac{\pi^2}{6} su/M_s^4$$

so that we recover a dim-8 'contact interaction' operator limit…

- Note that in these 'stacks of D-branes' constructions, e.g., the SU(3) of QCD is necessarily extended to U(3) so that an extra U(1) field , C, is also present.



A signal in the dijet channel is one of the 'easiest' possibilities to consider as Γ/M for resonances is ~5-15%.

All of the relevant quantities are calculable :

Anchordoqui etal

$$|\mathcal{M}(gg \to gg)|^2 = \frac{19}{12}\,\frac{g^4}{M_s^4}\left\{W_{g^*}^{gg\to gg}\left[\frac{M_s^8}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=0}\ M_s)^2}+\frac{t^4+u^4}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2}\right]\right.$$
$$\left.+\ W_{C^*}^{gg\to gg}\left[\frac{M_s^8}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=0}\ M_s)^2}+\frac{t^4+u^4}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right]\right\}, \qquad (4)$$

Anchordoqui etal

$$|\mathcal{M}(gg \to q\bar{q})|^2 = \frac{7}{24}\frac{g^4}{M_s^4}\,N_f\left[W_{g^*}^{gg\to q\bar{q}}\,\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2}\right.$$
$$\left.+\ W_{C^*}^{gg\to q\bar{q}}\,\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right] \qquad (5)$$

$$|\mathcal{M}(q\bar{q} \to gg)|^2 = \frac{56}{27}\frac{g^4}{M_s^4}\left[W_{g^*}^{q\bar{q}\to gg}\,\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2}\right.$$
$$\left.+\ W_{C^*}^{q\bar{q}\to gg}\,\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right], \qquad (6)$$

$$|\mathcal{M}(qg \to qg)|^2 = -\frac{4}{9}\frac{g^4}{M_s^2}\left[\frac{M_s^4 u}{(s-M_s^2)^2+(\Gamma_{q^*}^{J=1/2}\ M_s)^2}+\frac{u^3}{(s-M_s^2)^2+(\Gamma_{q^*}^{J=3/2}\ M_s)^2}\right], \qquad (7)$$

where g is the QCD coupling constant ($\alpha_{\rm QCD} = \frac{g^2}{4\pi} \approx 0.1$) and $\Gamma_{g^*}^{J=0} = 75\,(M_s/{\rm TeV})$ GeV, $\Gamma_{C^*}^{J=0} = 150\,(M_s/{\rm TeV})$ GeV, $\Gamma_{g^*}^{J=2} = 45\,(M_s/{\rm TeV})$ GeV, $\Gamma_{C^*}^{J=2} = 75\,(M_s/{\rm TeV})$ GeV, $\Gamma_{q^*}^{J=1/2} = \Gamma_{q^*}^{J=3/2} = 37\,(M_s/{\rm TeV})$ GeV are the total decay widths for intermediate states g^*, C^*, and q^* (with angular momentum J) [3]. The associated weights of these intermediate states are given in terms of the probabilities for the various entrance and exit channels

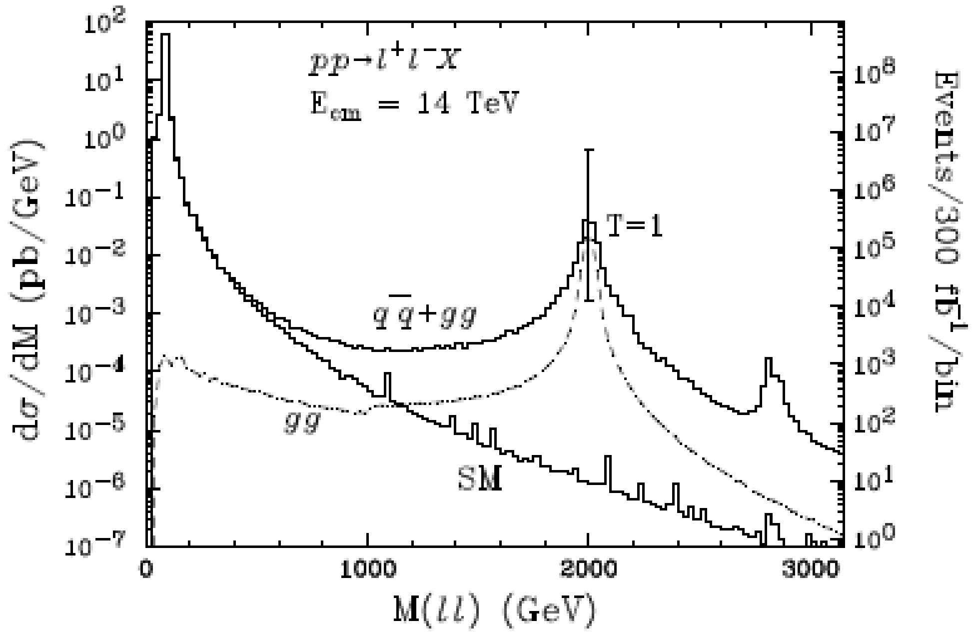
$$W_{g^*}^{gg\to gg} = \frac{(\Gamma_{g^*\to gg})^2}{(\Gamma_{g^*\to gg})^2+(\Gamma_{C^*\to gg})^2} = 0.09\,, \qquad (8)$$

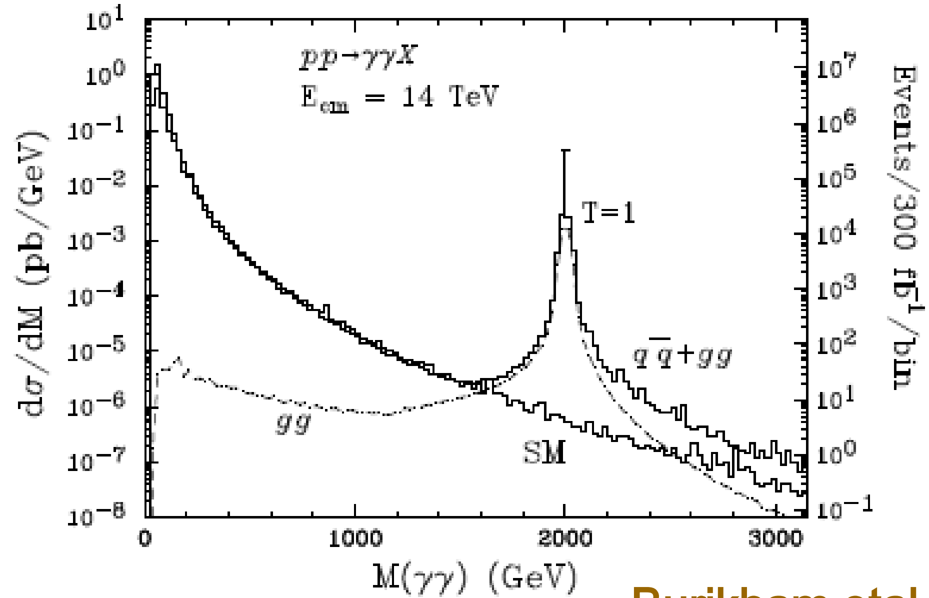
$$W_{C^*}^{gg\to gg} = \frac{(\Gamma_{C^*\to gg})^2}{(\Gamma_{g^*\to gg})^2+(\Gamma_{C^*\to gg})^2} = 0.91\,, \qquad (9)$$

$$W_{g^*}^{gg\to q\bar{q}} = W_{g^*}^{q\bar{q}\to gg} = \frac{\Gamma_{g^*\to gg}\,\Gamma_{g^*\to q\bar{q}}}{\Gamma_{g^*\to gg}\,\Gamma_{g^*\to q\bar{q}}+\Gamma_{C^*\to gg}\,\Gamma_{C^*\to q\bar{q}}} = 0.24\,, \qquad (10)$$

$$W_{C^*}^{gg\to q\bar{q}} = W_{C^*}^{q\bar{q}\to gg} = \frac{\Gamma_{C^*\to gg}\,\Gamma_{C^*\to q\bar{q}}}{\Gamma_{g^*\to gg}\,\Gamma_{g^*\to q\bar{q}}+\Gamma_{C^*\to gg}\,\Gamma_{C^*\to q\bar{q}}} = 0.76\,. \qquad (11)$$

Other Channels



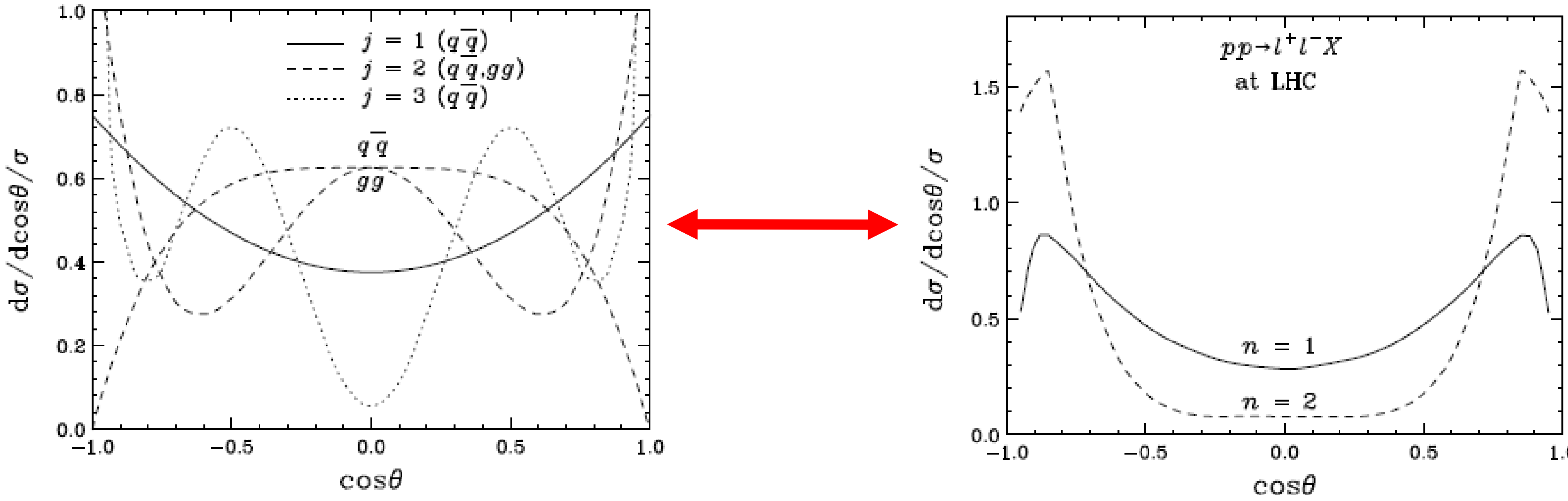


Burikham etal

$$A(s,t) = 16\pi \sum_{j=M}^{\infty} (2j+1) a_j(s) d^j_{mm'}(\cos\theta)$$

process		angular dependence	
$q\bar{q} \to \ell\bar{\ell}$			
$n=1,$	$j=1$	$(d^1_{1,-1})^2 + (d^1_{1,1})^2 \propto$	$1+\cos^2\theta$
	$j=2$	$(d^2_{1,-1})^2 + (d^2_{1,1})^2 \propto$	$1-3\cos^2\theta+4\cos^4\theta$
$n=2,$	$j=1$	$(d^1_{1,-1})^2 + (d^1_{1,1})^2 \propto$	$1+\cos^2\theta$
	$j=2$	$(d^2_{1,-1})^2 + (d^2_{1,1})^2 \propto$	$1-3\cos^2\theta+4\cos^4\theta$
	$j=3$	$(d^3_{1,-1})^2 + (d^3_{1,1})^2 \propto$	$1+111\cos^2\theta$ $-305\cos^4\theta+225\cos^6\theta$
$gg \to \ell\bar{\ell}$			
$n=1,$	$j=2$	$(d^2_{2,-1})^2 + (d^2_{2,1})^2 \propto$	$1-\cos^4\theta$
$q\bar{q} \to \gamma\gamma$			
$n=1,$	$j=2$	$(d^2_{2,-1})^2 + (d^2_{2,1})^2 \propto$	$1-\cos^4\theta$
$gg \to \gamma\gamma$			
$n=1,$	$j=2$	$(d^2_{2,-2})^2 + (d^2_{2,2})^2 \propto$	$1+6\cos^2\theta+\cos^4\theta$

Observing a resonance is NOT enough as many New Physics scenarios can produce them…the key here is to make measurements of angular distributions which would tell us that MORE than a single state is being produced within a given resonance structure. Significant integrated luminosity will be required for this type of analysis.



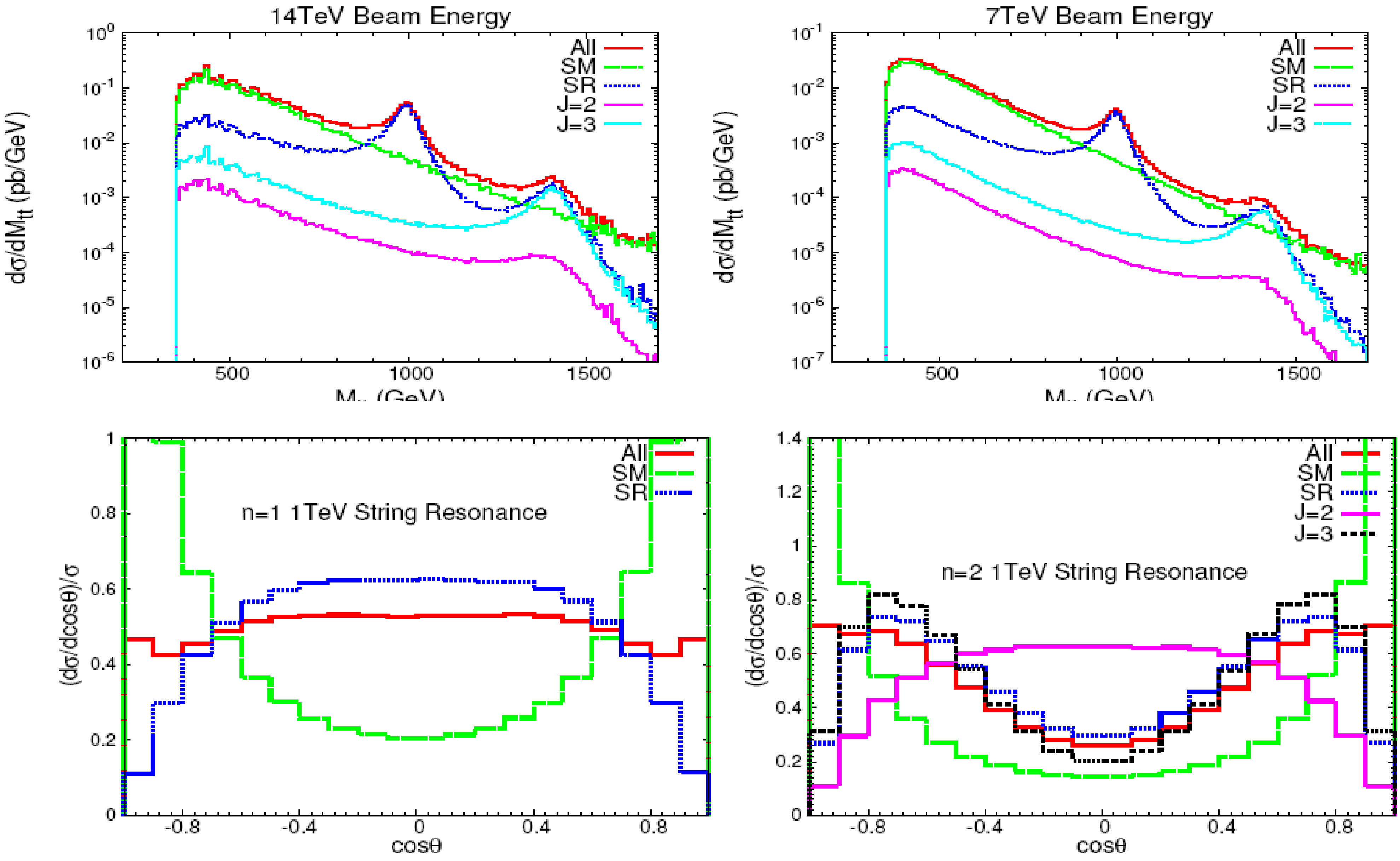
Burikham etal

• Taking the actual measured distribution, which is subject to detector acceptance cuts, & obtaining the 'spin content' of the resonance structure may not be that easy without high lumi especially if the string scale is high

• This will need to be done in several (or even many) channels before we can 'believe' in the underlying model , e.g., top quark pairs…

String Resonances in the Top Pair Channel

Dong etal



Higher Curvature Gravity @ Colliders

Demir, Tanyildizi, Aslan, TGR

- Mostly this has concentrated on BH physics (Cavaglia) but here we are interested in changes to ADD & RS physics

- Many forms of higher curvature terms are possible

- In PT, adding higher curvature terms to the D-dimensional action leads to extra, (non-graviton) degrees of freedom with spin-0,2 & with non-zero bulk masses (before compactification) which will have their own KK towers. The spin-2 pieces, if they are allowed to be present, will generally be ghosts & must be removed from the spectrum

- How are ADD/RS pheno changed by these terms ?

$$S_g = \frac{M^{D-2}}{2} \int d^D x \sqrt{g}\, F(R,P,Q) \qquad \begin{cases} P = R_{AB}R^{AB} \\ Q = R_{ABCD}R^{ABCD} \end{cases}$$

is a good example

- To determine the couplings to matter & the field props., it is sufficient to expand this action to 2nd order around in the background metric which for ADD/RS is a space of constant curvature :

$$F = F_0 + (R - R_0)F_R + (P - P_0)F_P + (Q - Q_0)F_Q + \text{quadratic terms}$$

$F_X = \partial F / \partial X|_0$ are derivatives of F evaluated in the fixed background

- For ADD, R_0 ,etc., are all zero, so things simplify a lot…
- For RS, $R_0 = -20k^2$, $P_0 = R_0^2/5$, etc. are all known

→ Graviton KK exchange amplitude between brane-localized pairs of 4-d sources

Accioly, Azeredo & Mukai

$$\mathcal{A} \sim \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)}{k^2 - m_n^2} - \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)}{k^2 - (m_2^2 + m_n^2)} + \frac{T^2}{(n+2)(n+3)[k^2 - (m_S^2 + m_n^2)]}$$

ADD

Ghost!

$$m_S^2 = \frac{(n+2)m_0^2}{2} = \frac{(n+2)F_R}{4(n+3)(\beta F_P + \epsilon F_Q + F_{RR}/2)}$$

NO massless scalar KK modes !

$$m_2^2 = \frac{-F_R}{(n+2)(F_P + 4F_Q)}, \rightarrow \infty$$

F(R,Q-4P)

- Non-tachyonic scalar → $F_{RR} > 2F_Q$
- New scalar KKs couple to the trace of the stress tensor… weaker by ~ $(M_{SM}^2/s)^2$ /100 compared w/ gravitons!

- Furthermore, the scalar KK sums begin at $m_S \sim M$, thus minimizing their contributions to any cross section in ADD → very difficult to see this at colliders

In RS, however, the situation is different for several reasons

- The usual RS parameter relationships are altered, e.g., expanding to 2nd order again we get :

$$S^{(3)}_{eff} = \int d^5x\sqrt{g}\ [-\Lambda_b + a_1\frac{M^3}{2}R + \frac{\alpha M}{2}G + \frac{\beta M}{2}R^2]$$

$$\longrightarrow \quad a_1 - \frac{2k^2}{M^2}\alpha - \frac{20k^2}{3M^2}\beta = -\frac{\Lambda_b}{6k^2M^3}$$

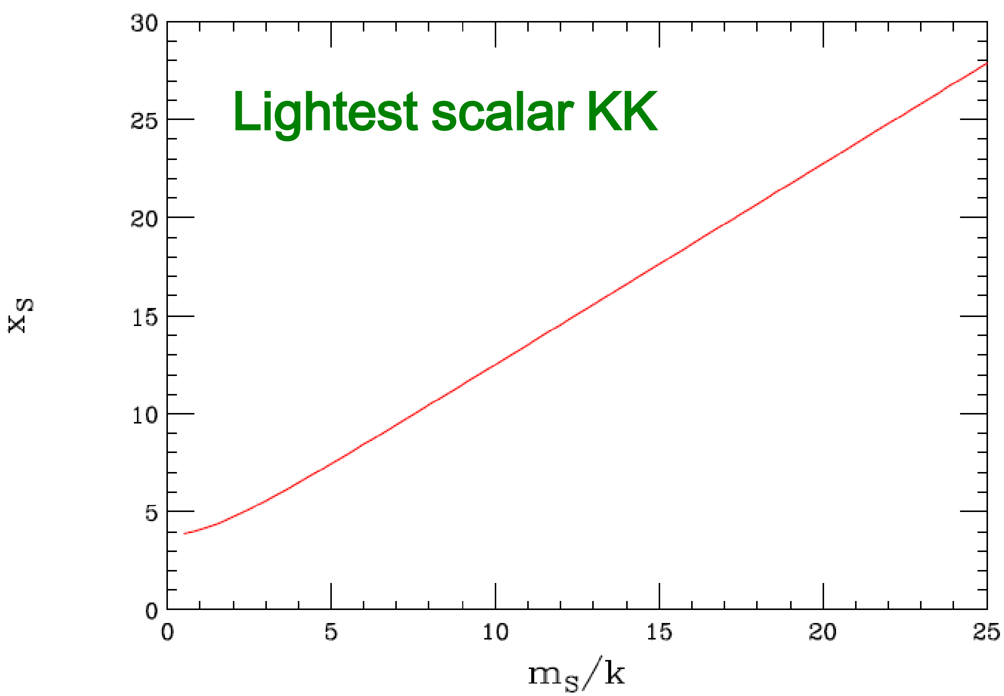
$$\longrightarrow \quad H(k)\frac{M^3}{k} = \overline{M}^2_{Pl} \qquad H = a_1 - \frac{4k^2}{M^2}\alpha - \frac{40k^2}{M^2}\beta$$

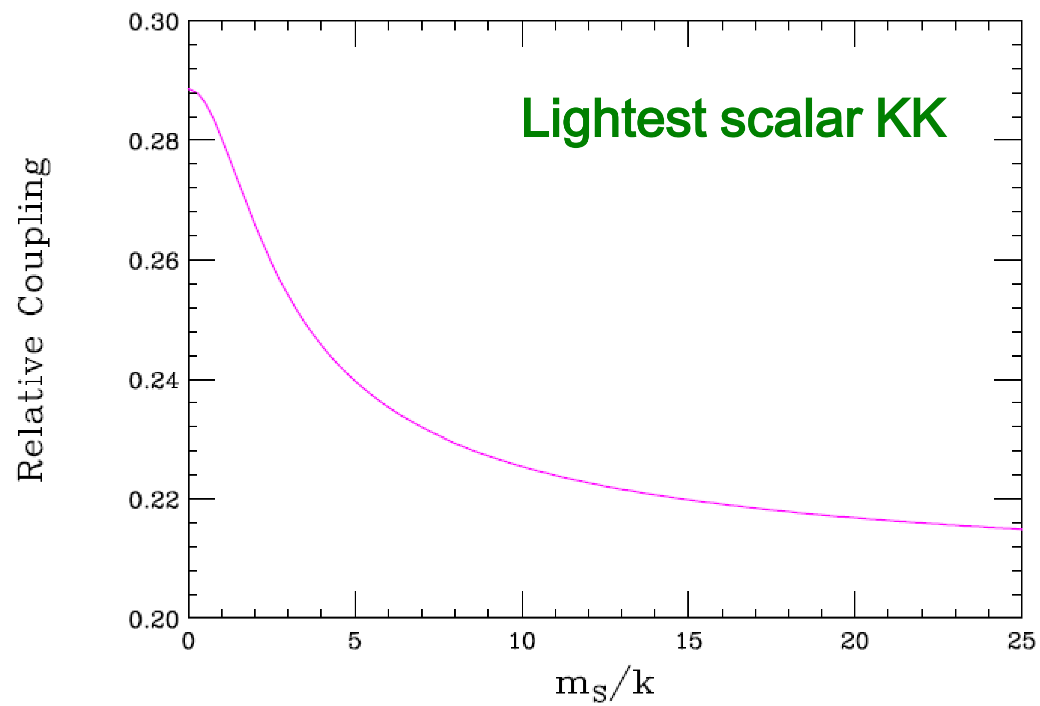
- The bulk scalar mass modifies the KK spectrum wrt gravitons

$$m_S^2 = \frac{3a_1}{16\beta} M^2$$ Bulk scalar mass

$$(2-\nu)J_\nu(x_S) + x_S J_{\nu-1}(x_S) = 0 \qquad \nu^2 = 4 + m_S^2/k^2$$

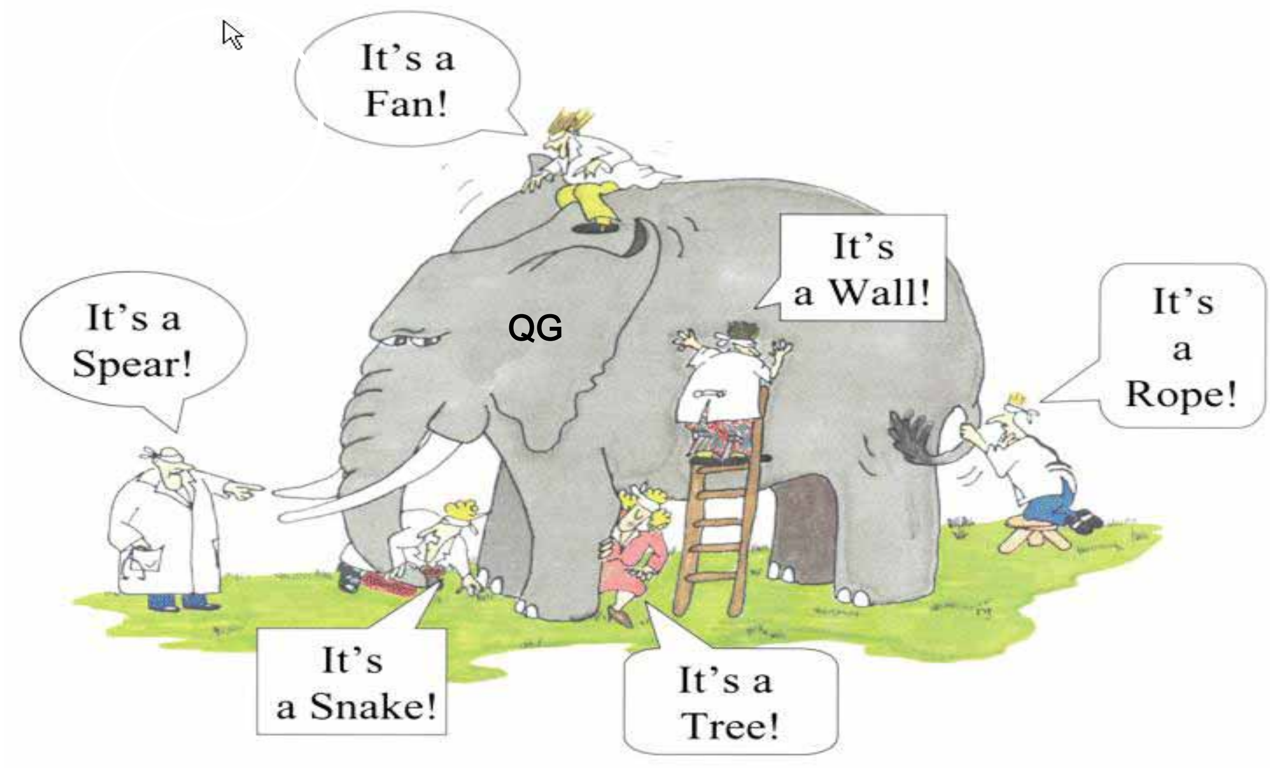
→ However, even though they are resonances, these new scalar KKs will, unfortunately, be almost impossible to see





Summary & Conclusions

- IF the scale of QG is only a few TeV then it may be possible to see some ‘unusual’ direct effects at the LHC and/or the LC
- Unfortunately, since we have no real idea of what this theory will be like we can only speculate what the signal might be



- There are many ‘models’ with a wide variety of predictions

- Clearly many current scenarios can be tested at some level but they are based on certain model frameworks which could be completely wrong so it’s a gamble. The LHC’s ability to see a very wide range of phenomena gives us good reasons to hope that QG can be observable whatever it looks like!

- Of course if QG is ‘nearby’ then the LHC might provide the necessary experimental input to help us to reconstruct the underlying theory. This may be the best possible outcome.

- We look forward to great discoveries at the LHC!