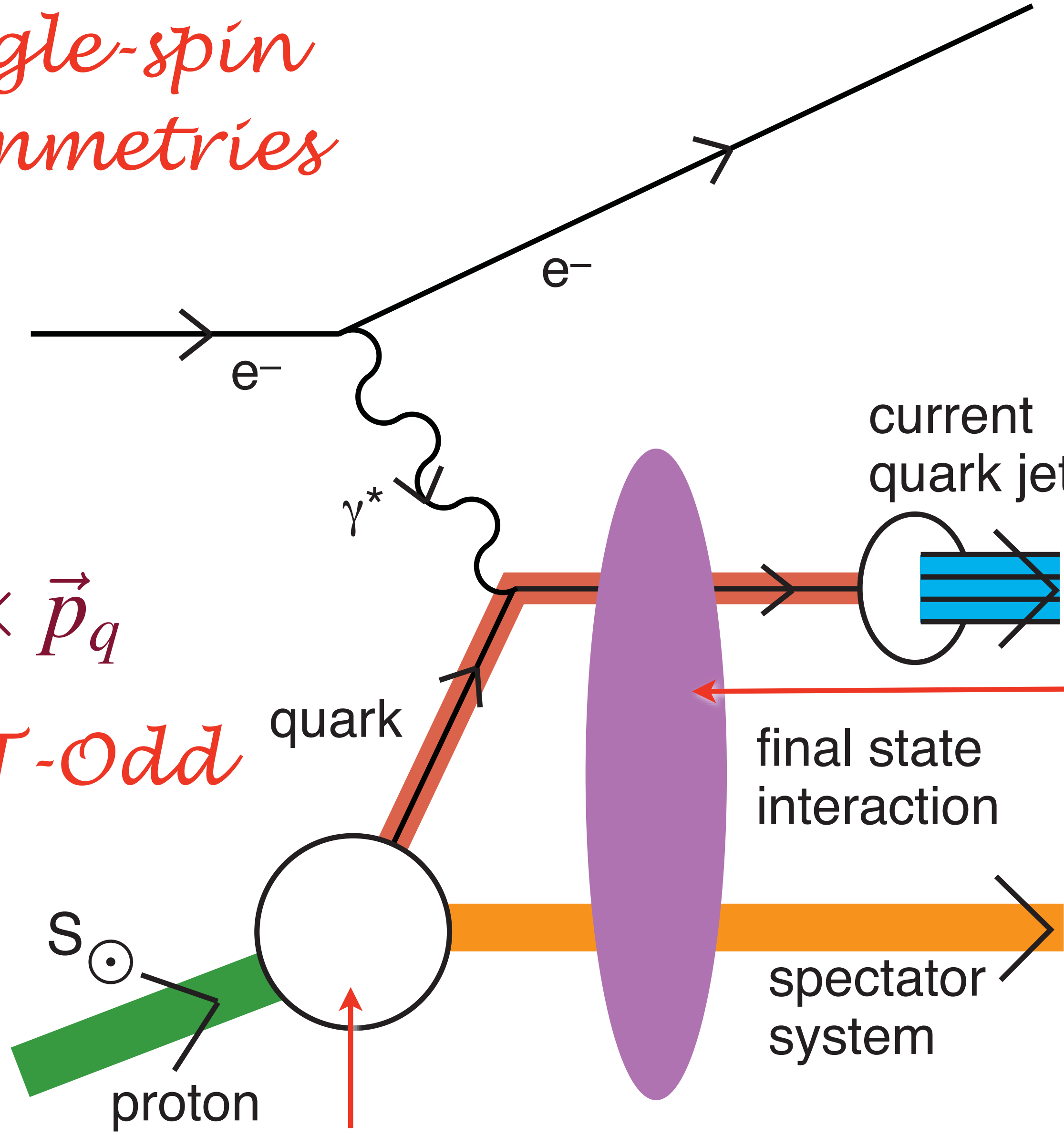
Single-spin asymmetries

Leading-Twist Sivers Effect



i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD}; variational methods

AdS/CFT

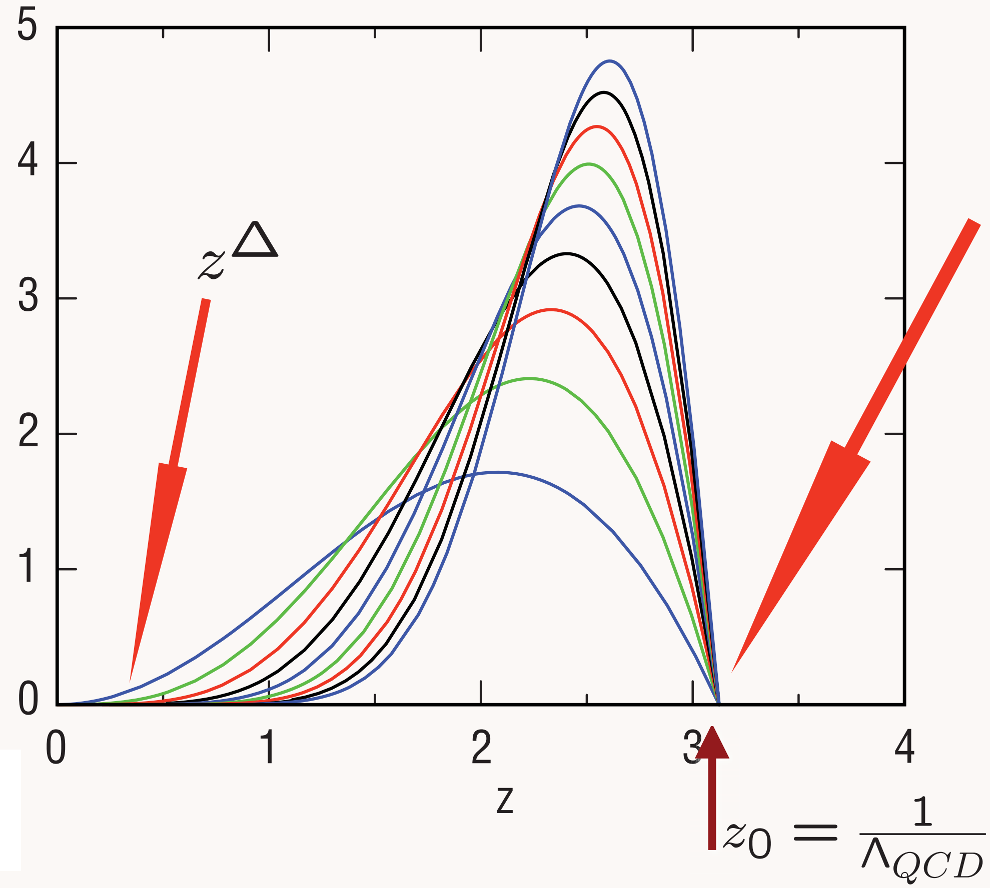
- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2$ $z \to \lambda z$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates "bag" boundary conditions

$$0 < z < z_0 \qquad \psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson



Confinement in the 5th dimension

de Teramond, sjb

Identify hadron by its interpolating operator at z --> 0

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1} x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$\text{Let } \Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} + V(z)\right]\phi(z) = M^2\phi(z)$$

$$V(z) = -\frac{1-4L^2}{4z^2}$$

Interpret L as orbital angular momentum

Derived from variation of Action in AdS_5

Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

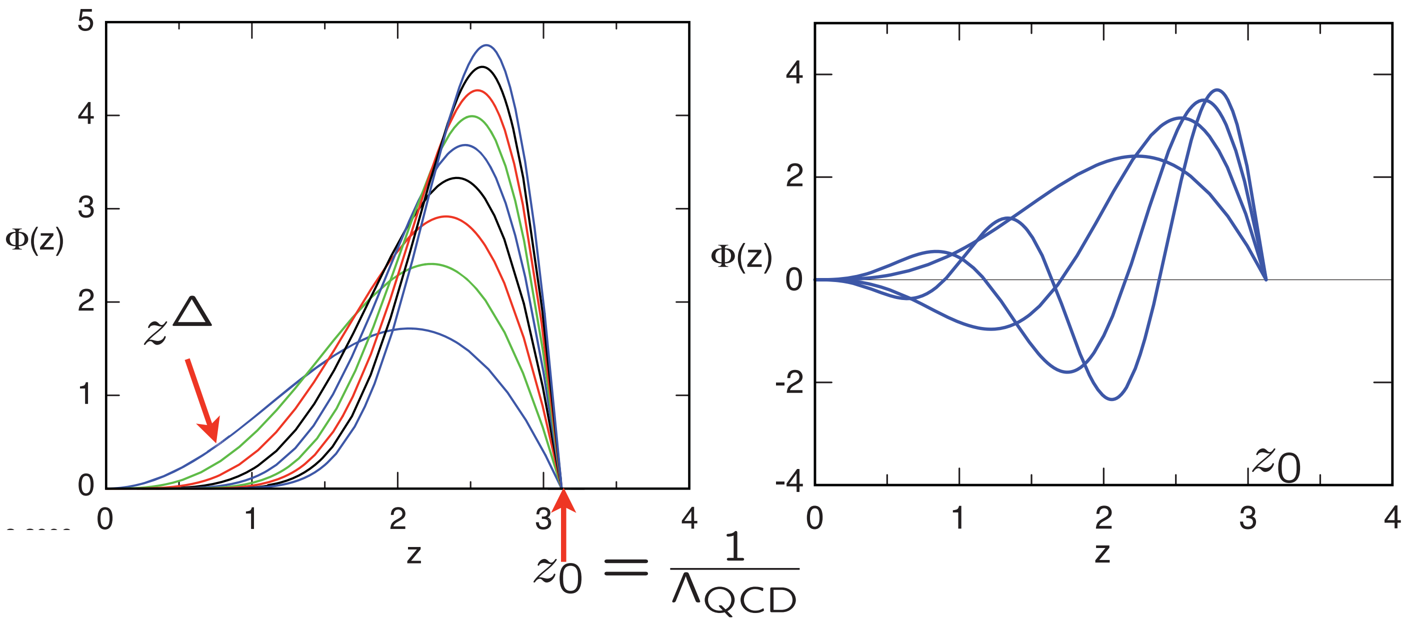
Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

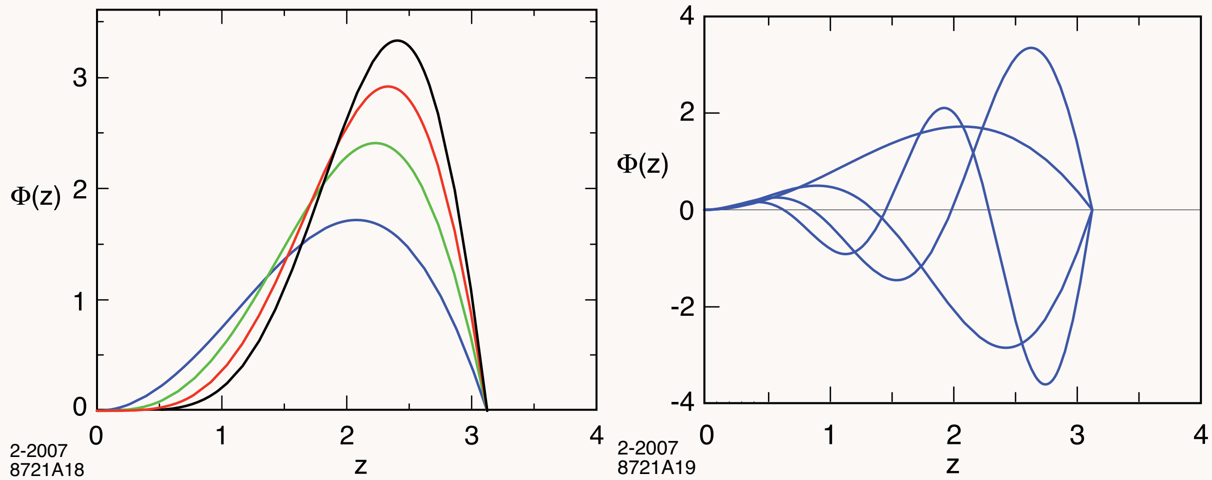


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .

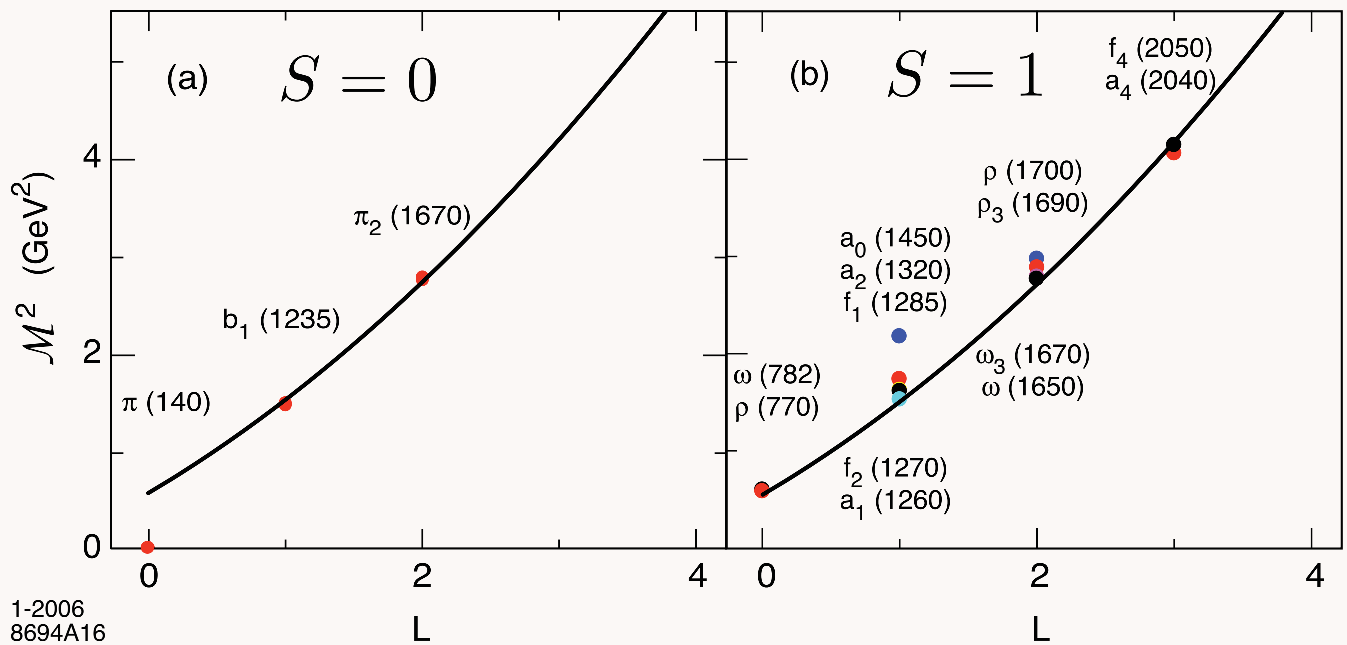


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32 \ \mathrm{GeV}$

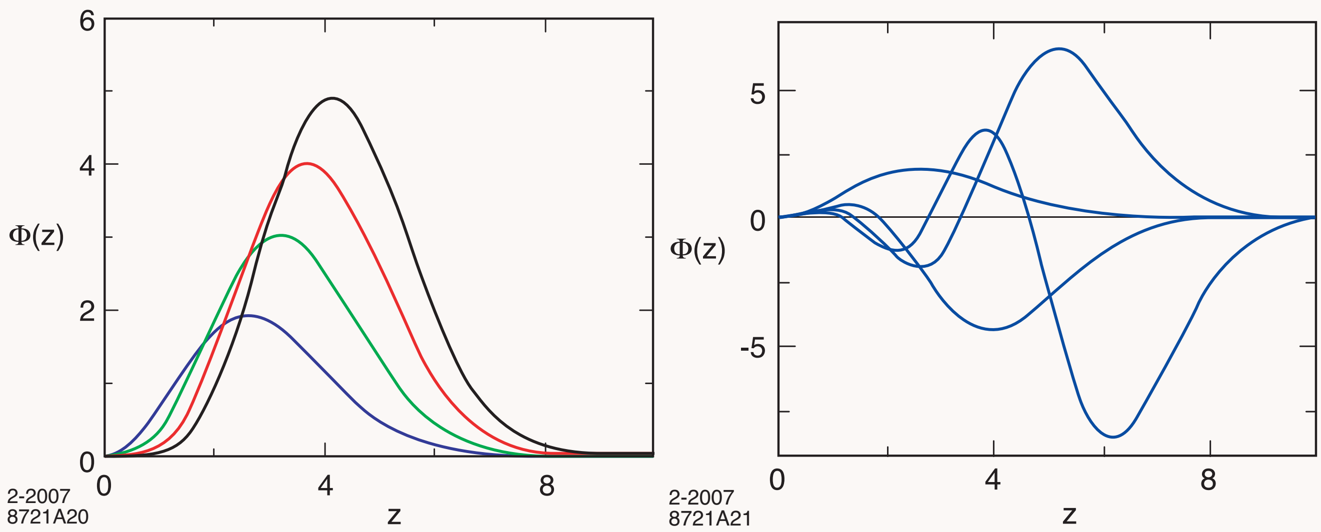
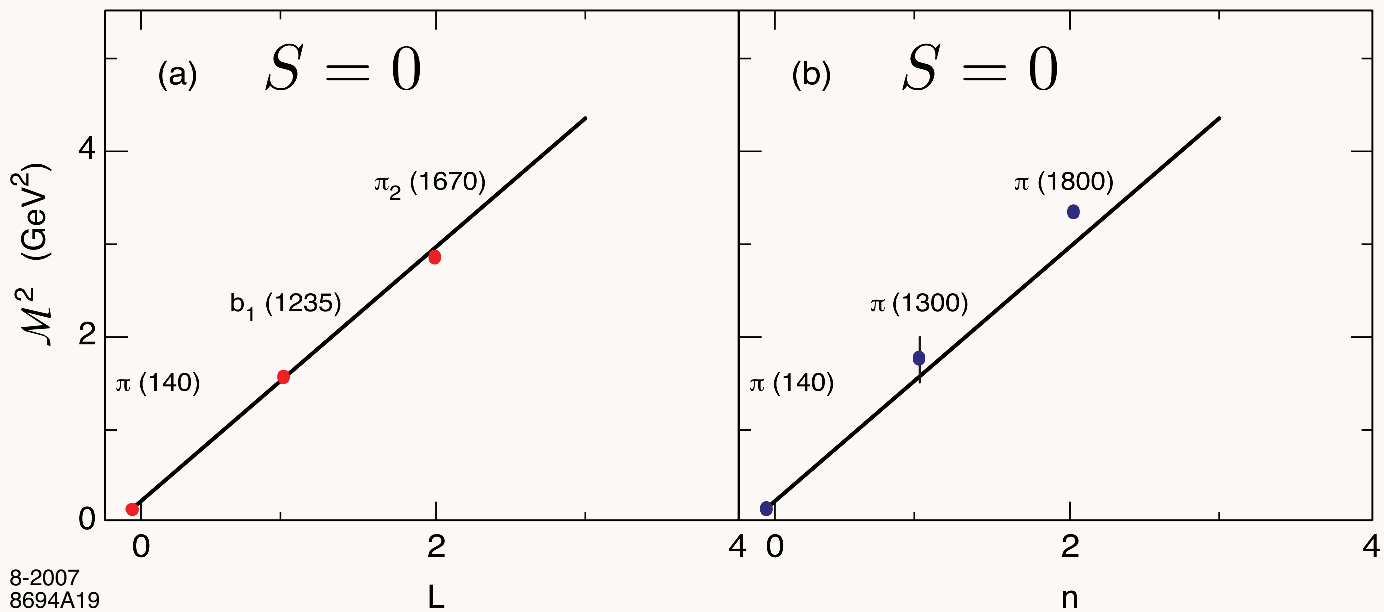


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Soft-wall model

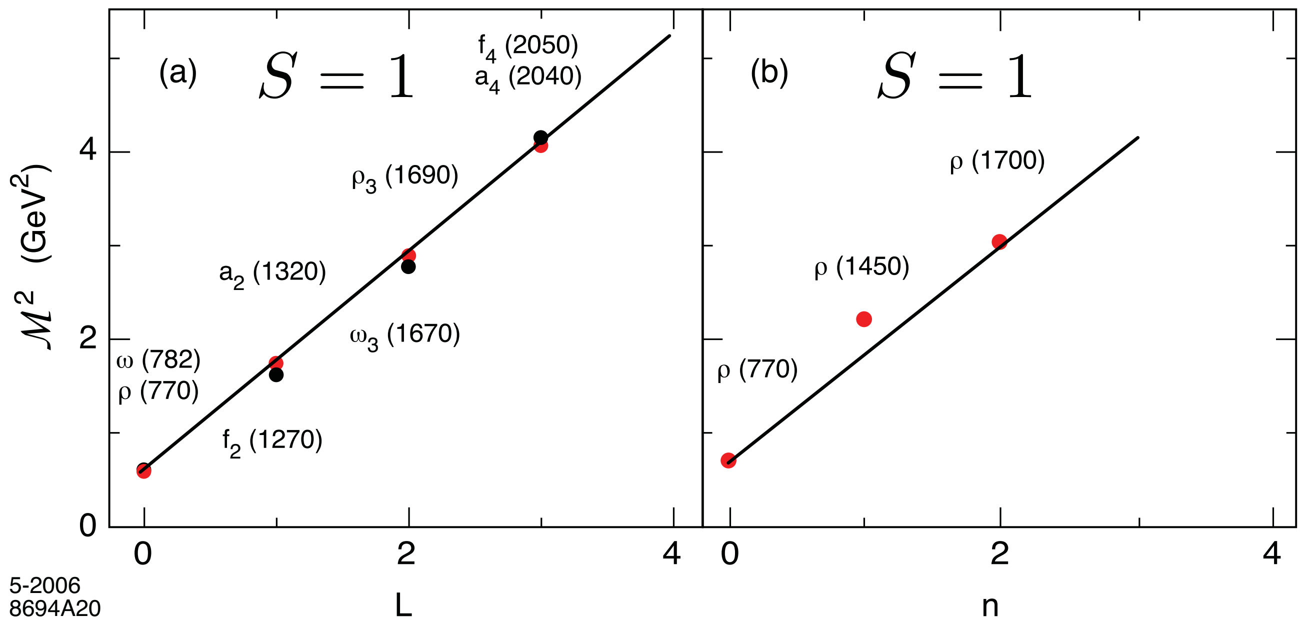
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2\phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n+2L+S)$.

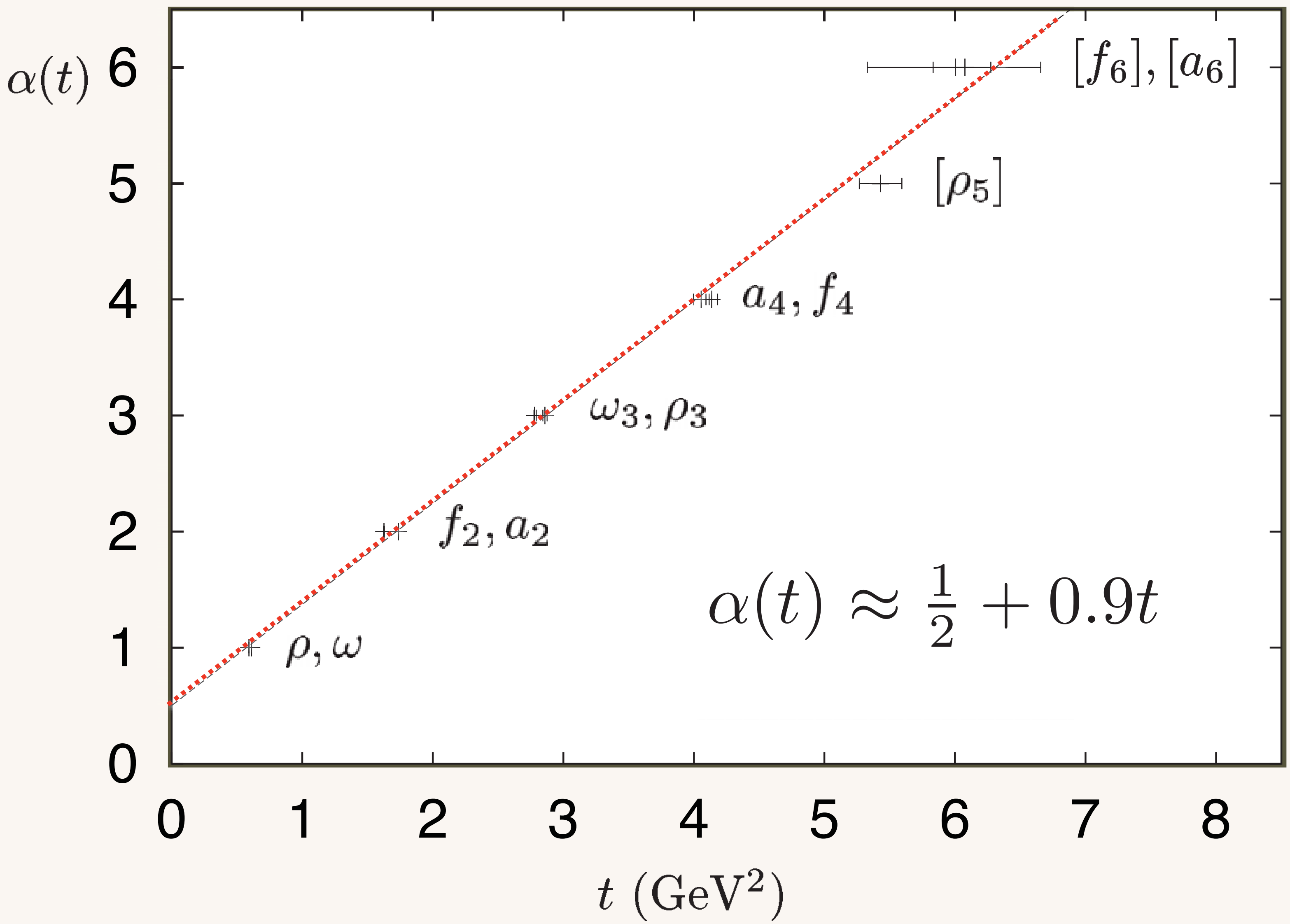
Same slope in n and L

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n+L+1/2)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri(2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

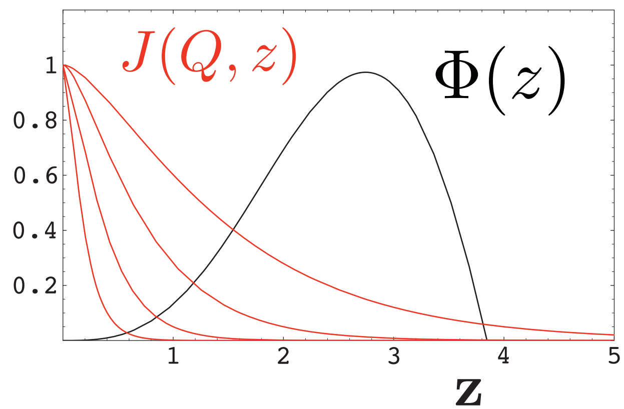
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \; x^\ell = (x^\mu, z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0.$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z \, \partial_z - z^2 Q^2\right] J(Q,z) = 0,$$

subject to boundary conditions $J(Q=0,z) = J(Q, z=0) = 1.$

- Solution

$$J(Q,z) = zQK_1(zQ).$$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z), \quad \Phi(z) \to z^\Delta, \;\; z \to 0.$$

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1+2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\,\Phi(z) J_\kappa(Q,z)\Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Space and Time-Like Pion Form Factor

- Hadronic string modes $\Phi_\pi(z) \to z^2$ as $z \to 0$ (twist $\tau = 2$)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} J_1(\beta_{0,1})} z^2 J_0\left(z\beta_{0,1}\Lambda_{QCD}\right),$$

$$\Phi_\pi^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2.$$

- F_π has analytical solution in the SW model $F_\pi(Q^2) = \frac{4\kappa^2}{4\kappa^2+Q^2}$.

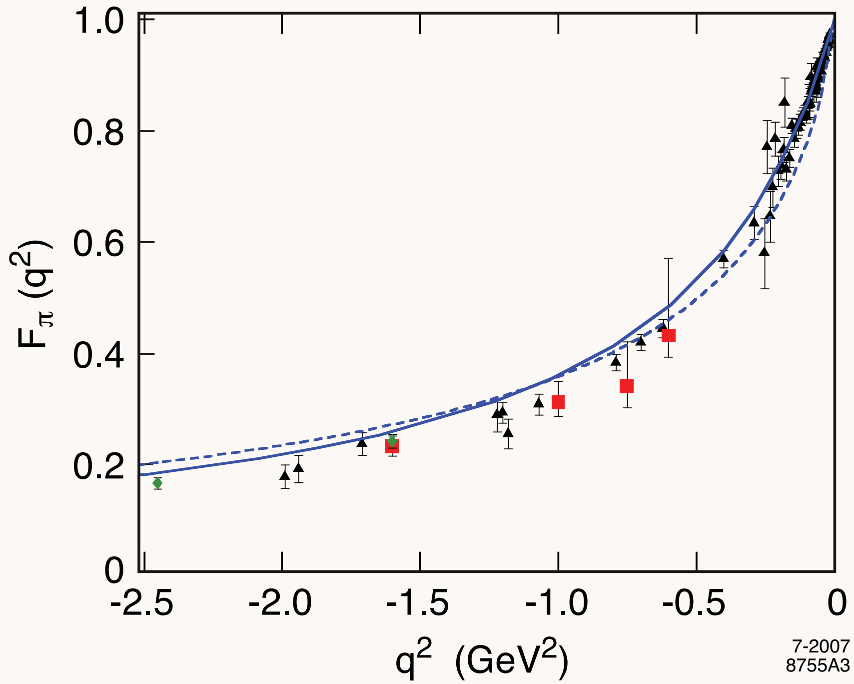


Fig: $F_\pi(q^2)$ for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: SW, dashed line: HW.

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N+z) = (N-1+z)(N-2+z)\ldots(1+z)\Gamma(1+z).$

- Form factor expressed as $N-1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1+\frac{Q^2}{4\kappa^2}}, \quad N=2, \\
F(Q^2) &= \frac{2}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)}, \quad N=3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

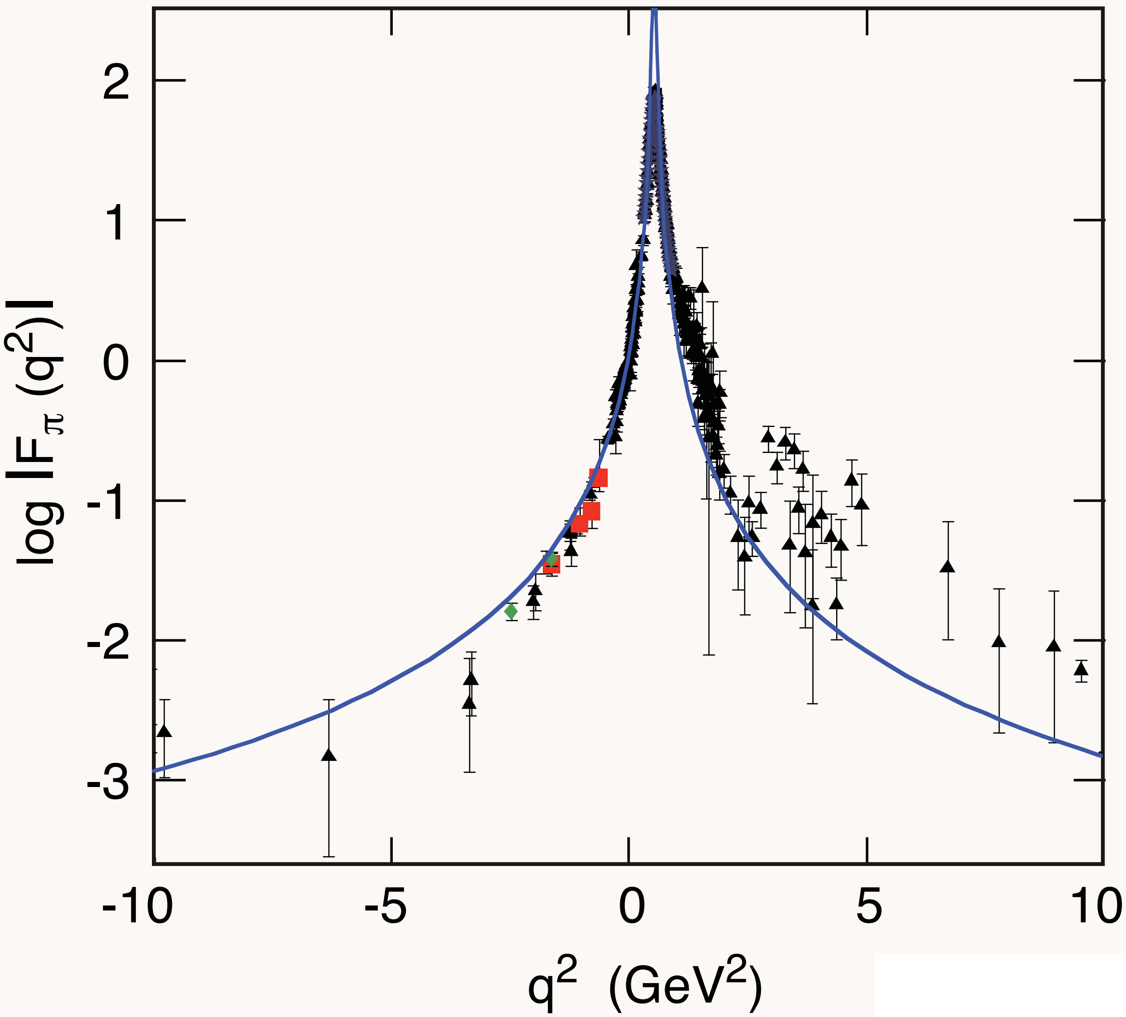
- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

- Analytical continuation to time-like region $q^2 \to -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\,(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2,z) \left|\Phi_{\pi^+}(z)\right|^2,$$

where $J(Q^2,z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2,z)$

$$J(Q^2,z) = \int_0^1 dx\, J_0\!\left(\zeta Q\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\!\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_{\pi^+}(z)\right|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

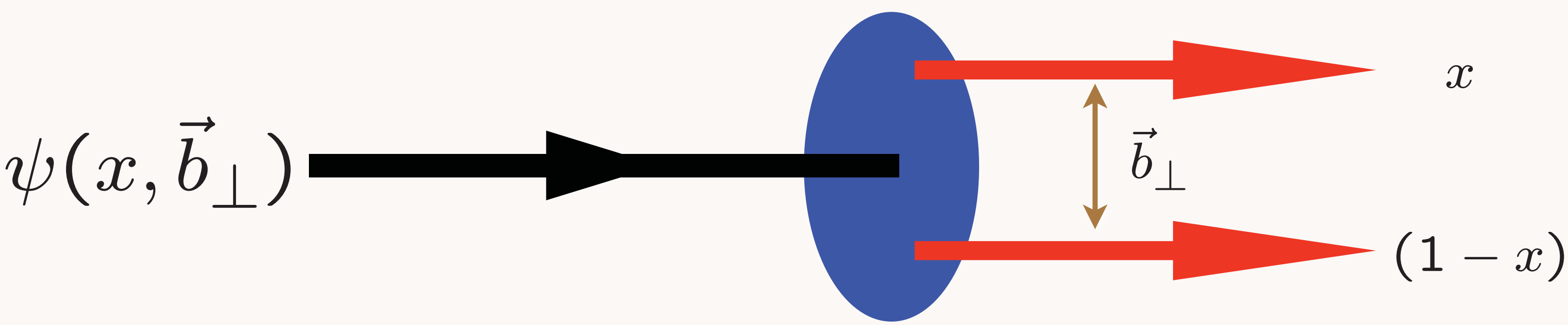
$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4}}$$

with $\zeta = z,\ 0 \le \zeta \le \Lambda_{\rm QCD}$

LF(3+1) AdS$_5$

$$\psi(x, \vec{b}_\perp) \qquad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \qquad z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\left\langle P' \left| \Theta^{++}(0) \right| P \right\rangle = 2\left(P^+\right)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $S_{\rm QCD} = \int d^4x \sqrt{g}\,\mathcal{L}_{\rm QCD}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta S_{\rm QCD}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \tfrac{1}{2}\overline{\psi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\,\psi - g^{\mu\nu}\overline{\psi}\left(i\not{D} - m\right)\psi - G^{a\,\mu\lambda} G^{a\,\nu}{}_{\lambda} + \tfrac{1}{4} g^{\mu\nu} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0,\ g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2} \sum_f \overline{\psi}^f(x) \gamma^+ \overleftrightarrow{\partial}^+ \psi^f(x)$$

Gravitational Form Factor on the LF

$$A_f(q^2) = \int_0^1 x\, dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp\cdot\vec{q}_\perp}\tilde{\rho}_f(x,\vec{\eta}_\perp),$$

where

$$\tilde{\rho}_f(x,\vec{\eta}_\perp) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{\eta}_\perp\cdot\vec{q}_\perp}\rho_f(x,\vec{q}_\perp)$$

$$= \sum_n \prod_{j=1}^{n-1}\int dx_j\, d^2\vec{b}_{\perp j}\,\delta\Big(1-x-\sum_{j=1}^{n-1}x_j\Big)$$

$$\times\delta^{(2)}\Big(\sum_{j=1}^{n-1}x_j\vec{b}_{\perp j}-\vec{\eta}_\perp\Big)\left|\tilde{\psi}_n(x_j,\vec{b}_{\perp j})\right|^2.$$

Extra factor of x relative to charge form factor

For each quark and

Integrate over angle

$$A_f(q^2) = 2\pi\int_0^1 dx\,(1-x)\int\zeta d\zeta\, J_0\left(\zeta q\sqrt{\frac{1-x}{x}}\right)\tilde{\rho}_f(x,\zeta)$$

$$\zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1}x_j\mathbf{b}_{\perp j}\right|$$

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2,z) \left|\Phi_\pi(z)\right|^2,$$

Abidin & Carlson

where $H(Q^2,z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2,z)$

$$H(Q^2,z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

$\boxed{\text{Identical to LF Holography obtained from electromagnetic current}}$

$$H(Q^2, z) = 2\int_0^1 x\,dx\,J_0\left(zQ\sqrt{\frac{1-x}{x}}\right).$$

$$A(Q^2) = 2\,R^3 \int x\,dx \int \frac{dz}{z^3}\,J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)\left|\Phi(z)\right|^2.$$ **AdS**

Compare with gravitational form factor from LF

$$A(Q^2) = 2\pi \int_0^1 dx\,(1-x) \int \zeta d\zeta\,J_0\left(\zeta Q\sqrt{\frac{1-x}{x}}\right)\tilde{\rho}(x,\zeta)$$ **LF**

Holography: identify AdS and LF density for all Q

$$\tilde{\rho}(x,\zeta) = 2\,\frac{R^3}{2\pi}\,\frac{x}{1-x}\,\frac{\left|\Phi(\zeta)\right|^2}{\zeta^4}.$$

with

$$\zeta \equiv z \qquad\qquad \zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right|$$

Holographic result for LFWF identical for electroweak and gravity couplings! Highly nontrivial consistency test

AdS/QCD can predict

- Momentum fractions for each quark flavor and the gluons $A_f(0) = < x_f >, \sum_f A_f(0) = A(0) = 1$
- Orbital Angular Momentum for each quark flavor and the gluons $B_f(0) = < L^3_f >, \sum_f B_f(0) = B(0) = 0$
- Vanishing Anomalous Gravitomagnetic Moment
- Shape and Asymptotic Behavior of $A_f(Q^2), B_f(Q^2)$

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \rightarrow \lambda x^\mu,\ z \rightarrow \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu}dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+dx^- - dx_\perp{}^2$

and

$ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2)$ for $x^+ = 0$.

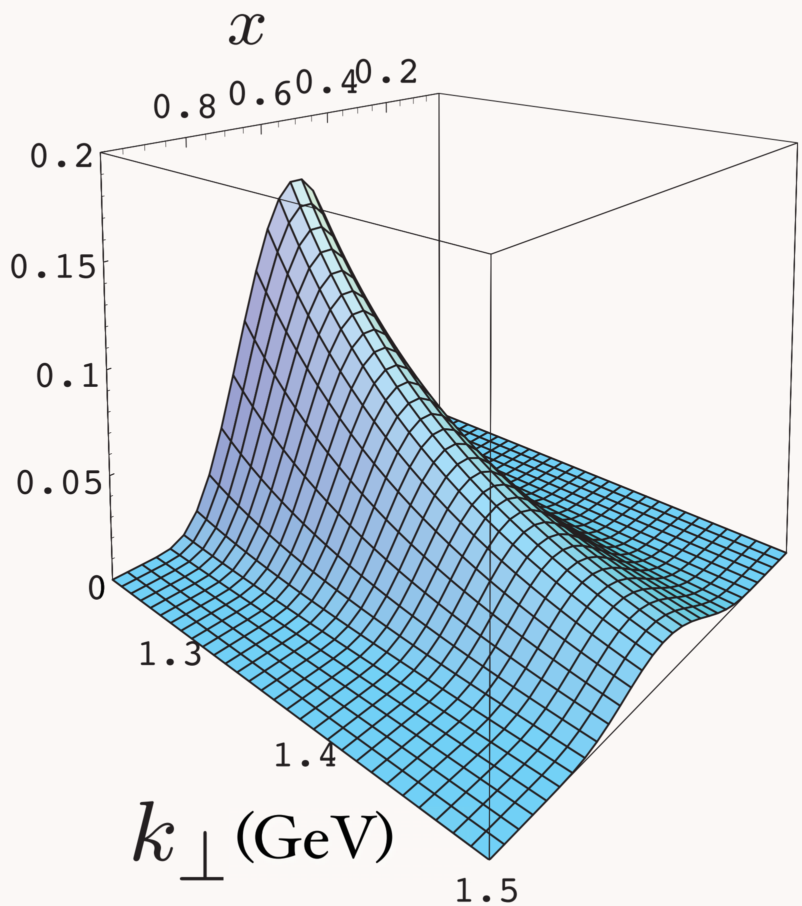
Light-Front AdS_5 Duality

- ds^2 is invariant if $dx_\perp{}^2 \rightarrow \lambda^2 dx_\perp{}^2$, and $z \rightarrow \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \qquad \phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

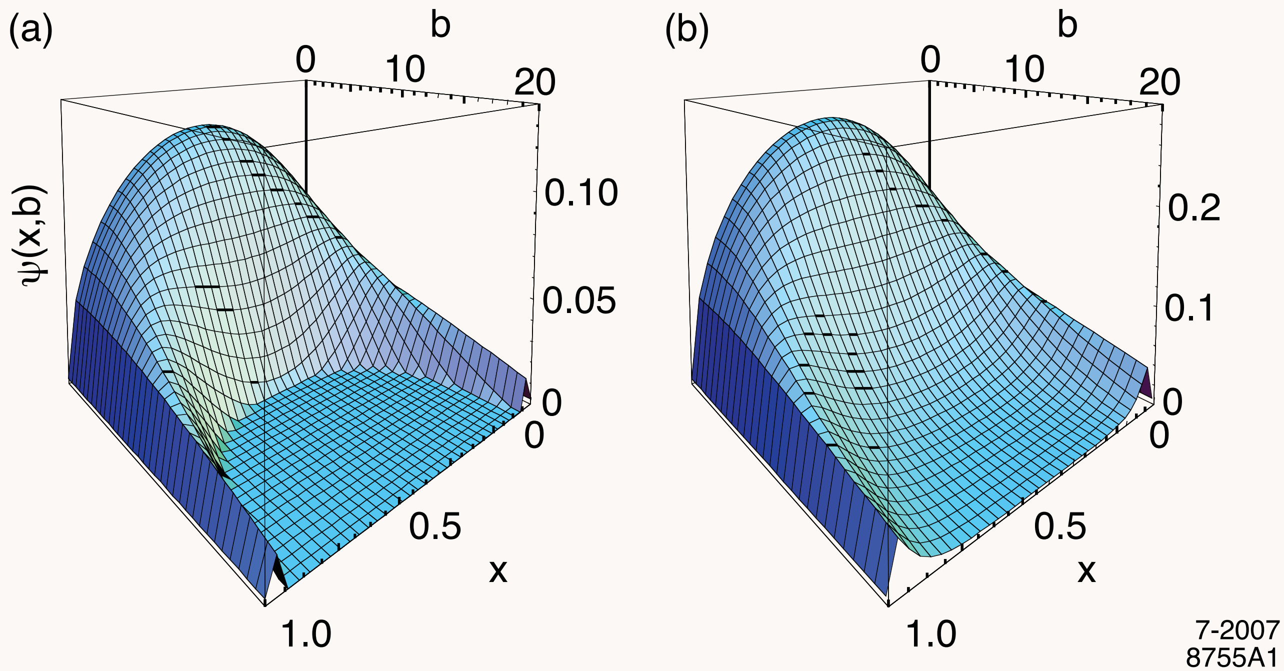


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\,d\downarrow} d^\dagger_{c\,u\uparrow} - b^\dagger_{c\,d\uparrow} d^\dagger_{c\,u\downarrow} \right) |0\rangle .$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}}\, R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{\rm QCD} = 91.7 \ {\rm MeV}, \quad f_\pi^{SW} = \frac{\sqrt{3}}{8} \kappa = 81.2 \ {\rm MeV},$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

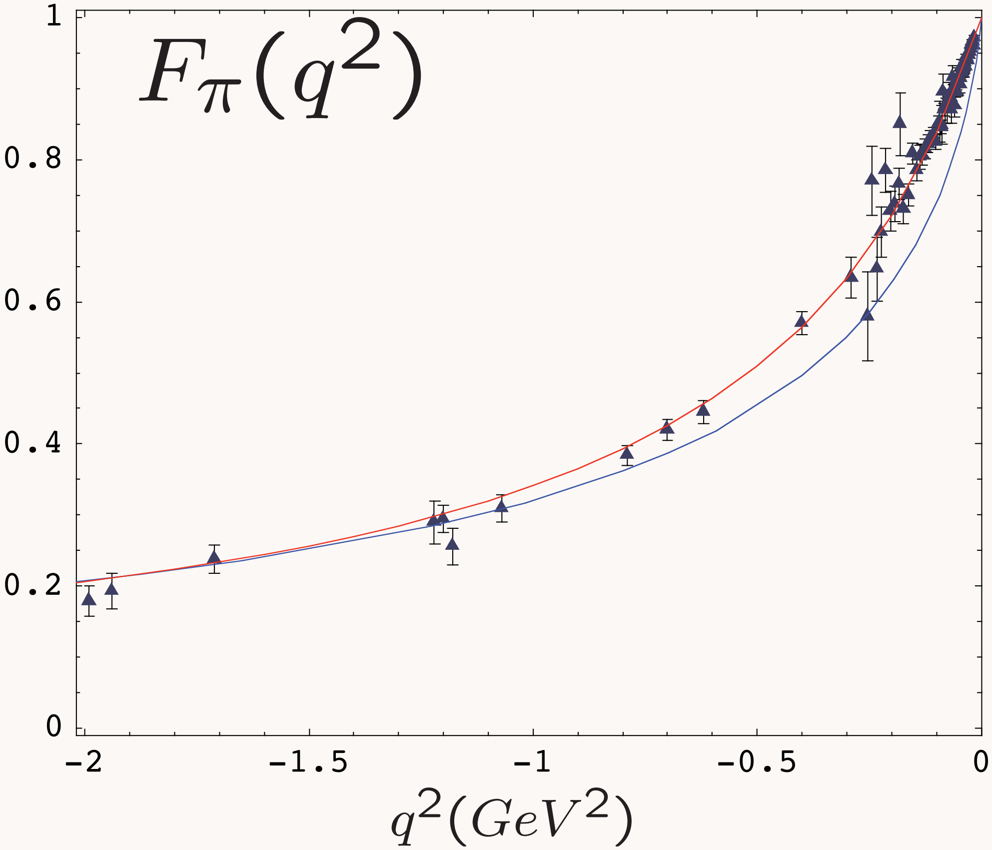
$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

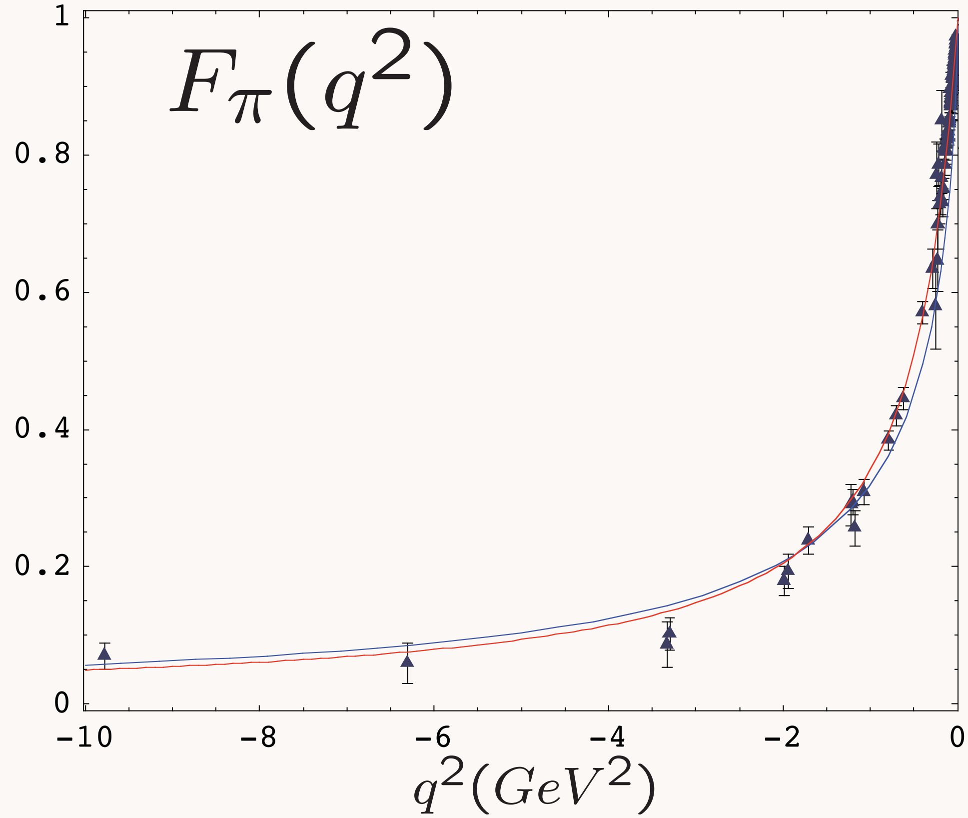
$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

Spacelike pion form factor from AdS/CFT





Data Compilation from Baldini, Kloe and Volmer

SW: Harmonic Oscillator Confinement

HW: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space $\mathbf{b}_\perp$

$$F_\pi(q^2) = \int_0^1 dx \int db^2\, \widetilde{\rho}(x, b, Q),$$

with $\widetilde{\rho}(x, b, Q) = \pi J_0\left[b\, Q(1-x)\right] |\widetilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \to 1$ as Q increases.

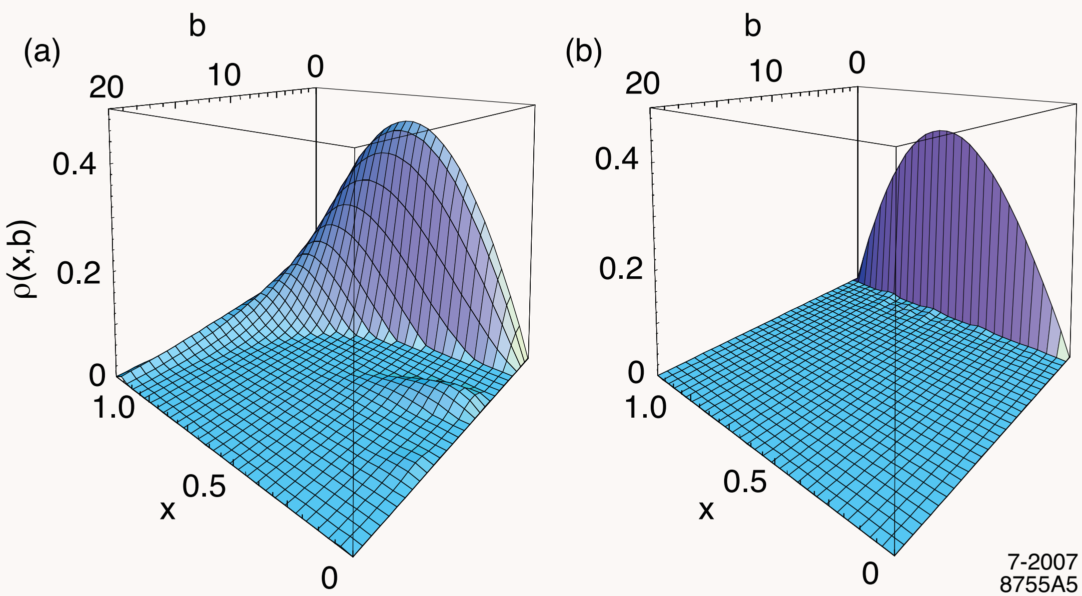


Fig: LF partonic density $\rho(x, b, Q)$: (a) Q = 1 GeV/c, (b) very large Q.

Holography: Map AdS/CFT to 3+1 LF Theory

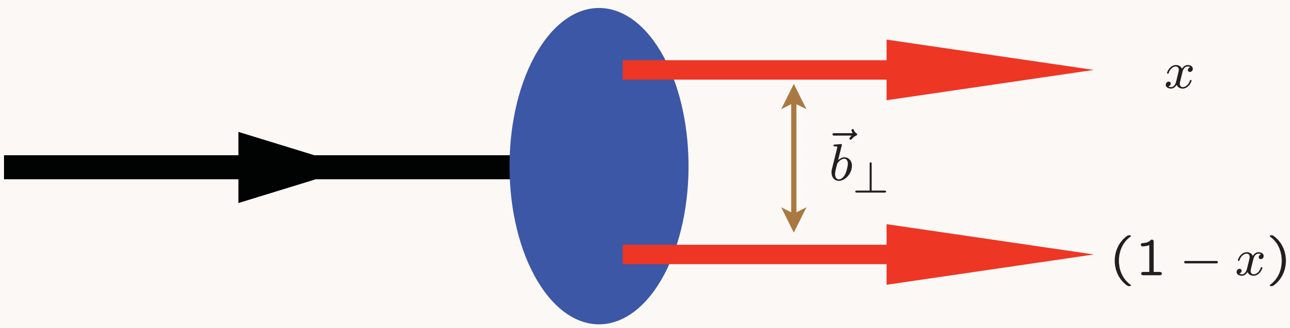
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

- Light-front Hamiltonian equation

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle,$$

leads to effective LF Schrödinger wave equation (KKSS)

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L-1)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

with eigenvalues $\mathcal{M}^2 = 4\kappa^2(n+L)$ and eigenfunctions

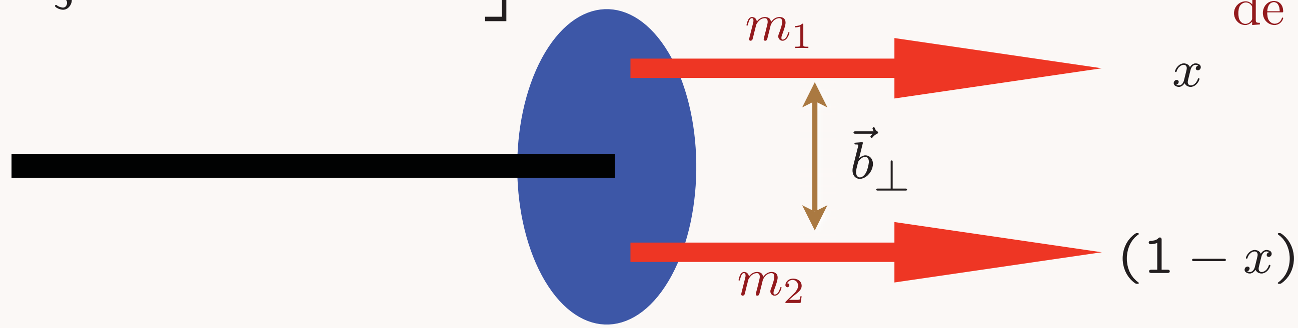
$$\phi_L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L\left(\kappa^2\zeta^2\right).$$

- Transverse oscillator in the LF plane with $SO(2)$ rotation subgroup has Casimir L^2 representing rotations for the transverse coordinates $\mathbf{b}_\perp$ in the LF.

- SW model is a remarkable example of integrability to a non-conformal extension of AdS/CFT [Chim and Zamolodchikov (1992) - Potts Model.]

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

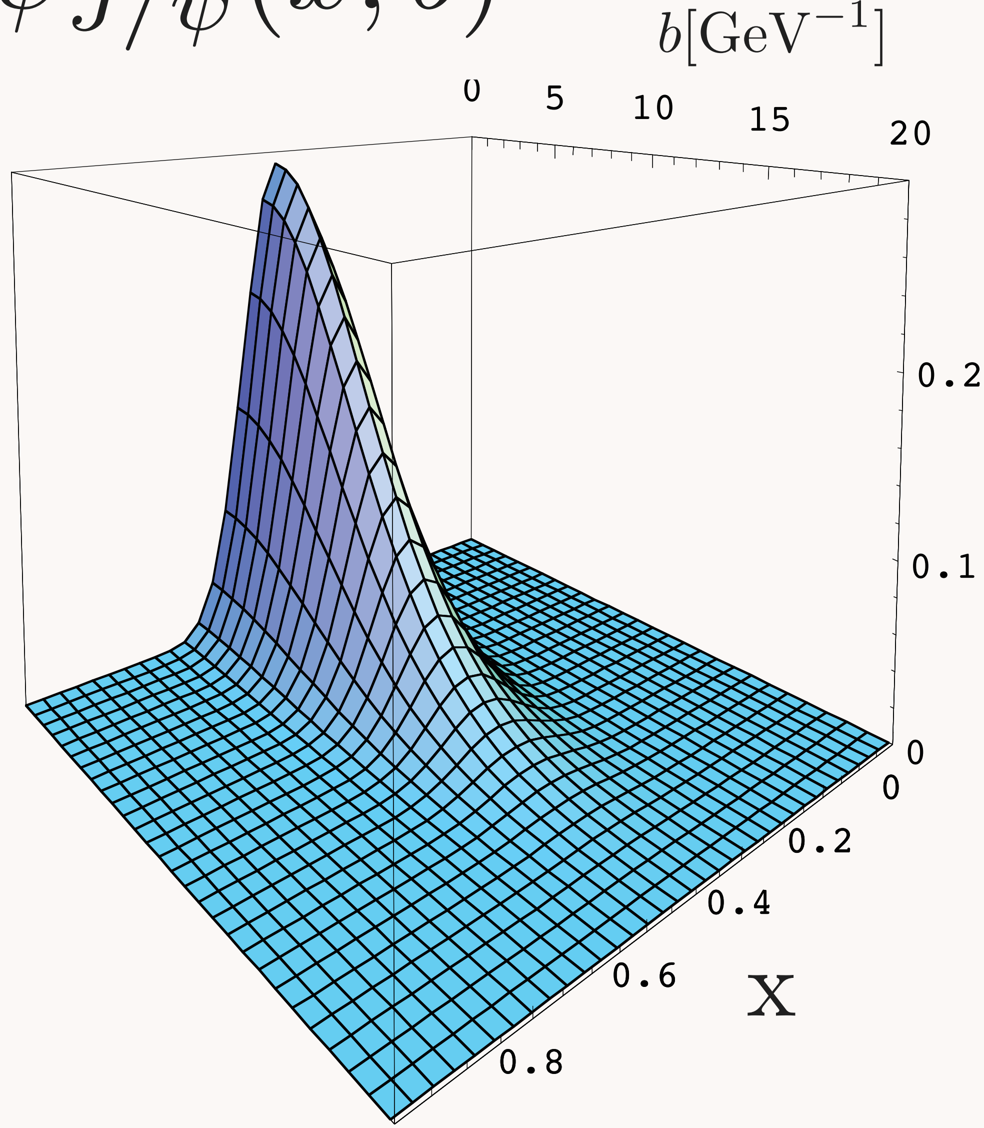
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$\psi_{J/\psi}(x, b)$$



$$m_a = m_b = 1.25 \text{ GeV}$$

$$|\pi^+ >= |u\bar{d}>$$

$$m_u = 2\ MeV$$
$$m_d = 5\ MeV$$

$$|K^+ >= |u\bar{s}>$$

$$m_s = 95\ MeV$$

$$|D^+ >= |c\bar{d}>$$

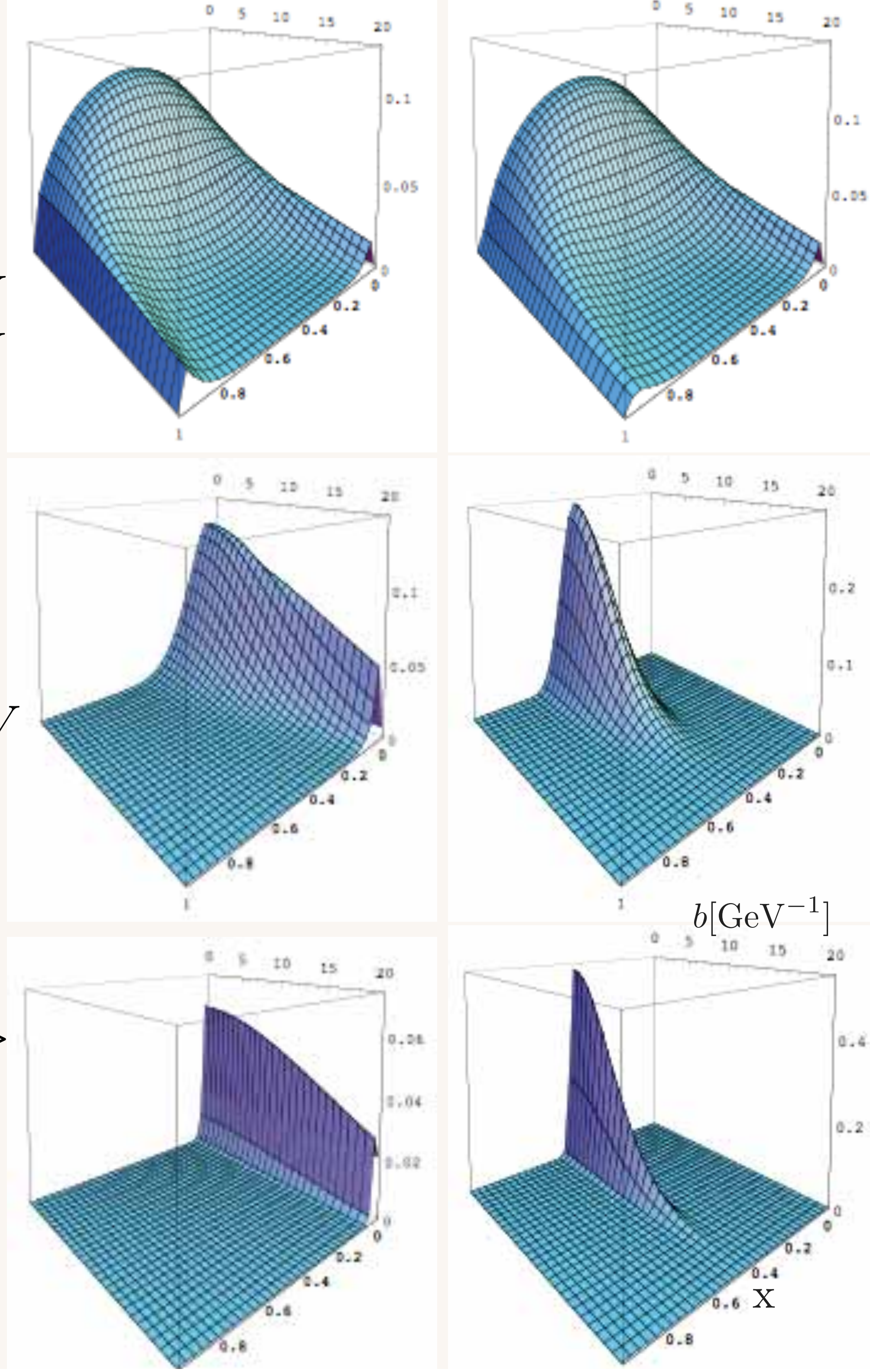
$$m_c = 1.25\ GeV$$

$$|\eta_c >= |c\bar{c}>$$

$$|B^+ >= |u\bar{b}>$$

$$m_b = 4.2\ GeV$$

$$|\eta_b >= |b\bar{b}>$$

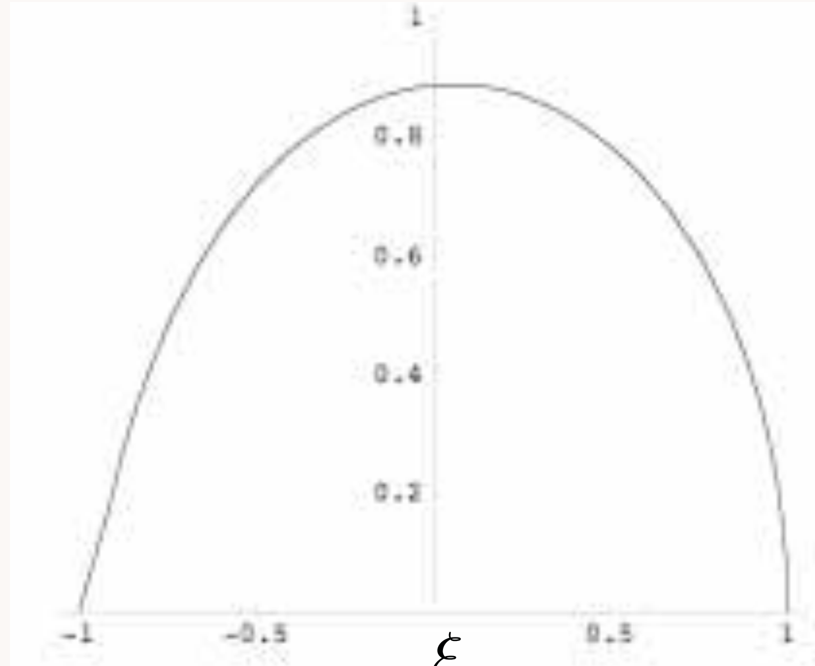


$$\kappa = 375\ MeV$$

First Moment of Kaon Distribution Amplitude

$$< \xi >= \int_{-1}^{1} d\xi \; \xi \; \phi(\xi)$$

$$\xi = 1 - 2x$$



$$< \xi >_K = 0.04 \pm 0.02$$

$\kappa = 375 \; MeV$

Range from $m_s = 65 \pm 25$ MeV (PDG)

$< \xi >_K = 0.029 \;\; \pm 0.002$ Donnellan et al.

$< \xi >_K = 0.0272 \pm 0.0005$ Braun et al.

AdS/QCD $m_s = 65 \pm 25$ MeV (PDG)

M	$\langle\xi\rangle_M$		$\langle\xi^2\rangle_M$
π			0.25
K	$0.04 \pm 0.02\,^a$		0.235 ± 0.005^a
D	0.71	AdS/QCD	0.54
η_c			0.02
B	0.96		0.91
η_b			0.002
π			0.28 ± 0.03^b
K	$0.029 \pm 0.002\,^b$		$0.27 \pm 0.02\,^b$
π		Lattice	0.269 ± 0.039^c
K	$0.0272 \pm 0.0005\,^c$		$0.260 \pm 0.006\,^c$

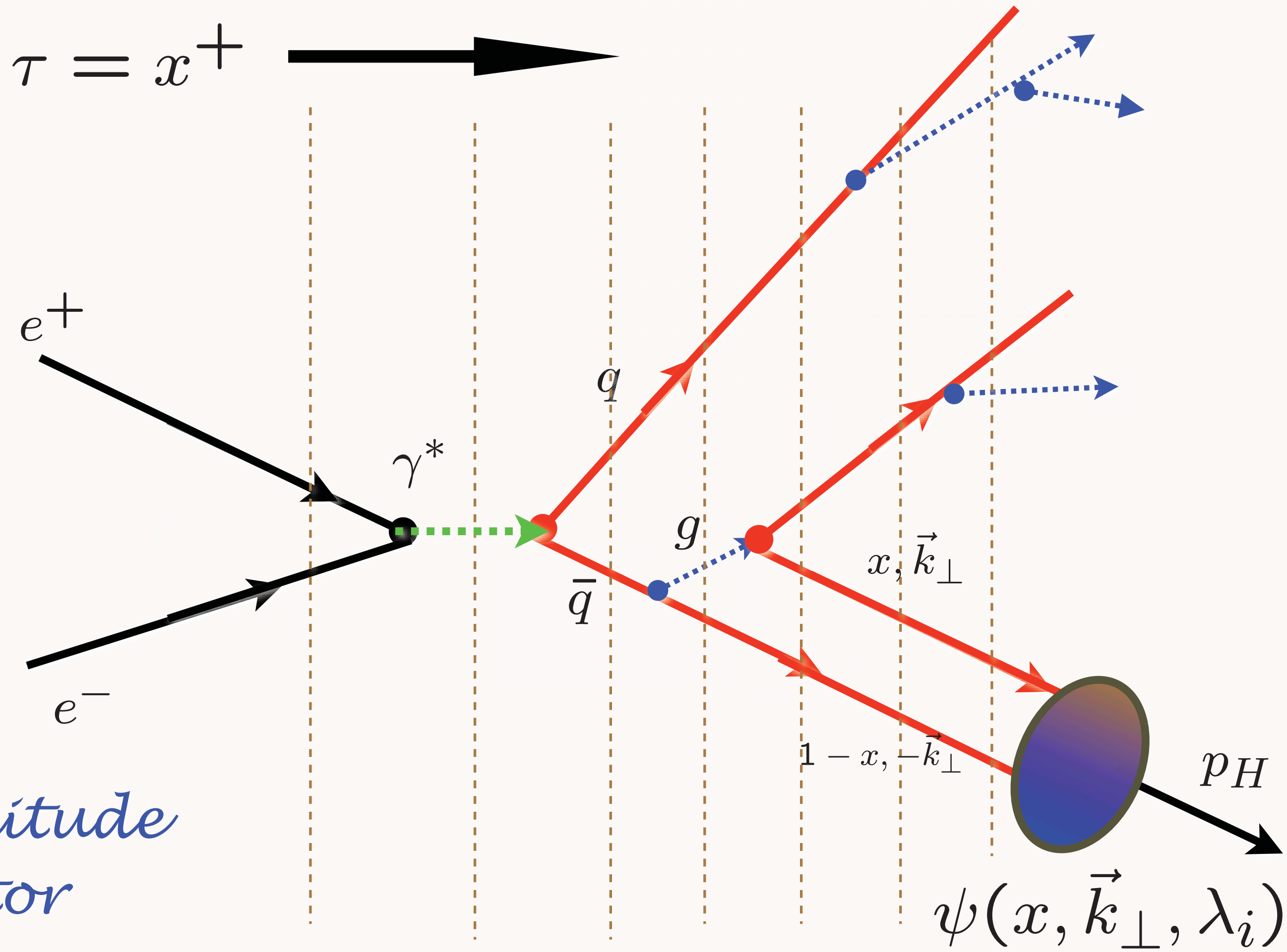
M. A. Donnellan *et al.*, "Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes," arXiv:0710.0869 [hep-lat].

b: Lattice

V. M. Braun *et al.*, "Moments of pseudoscalar meson distribution amplitudes from the lattice," Phys. Rev. D **74**, 074501 (2006) [arXiv:hep-lat/0606012].

c: Lattice

Hadronization at the Amplitude Level

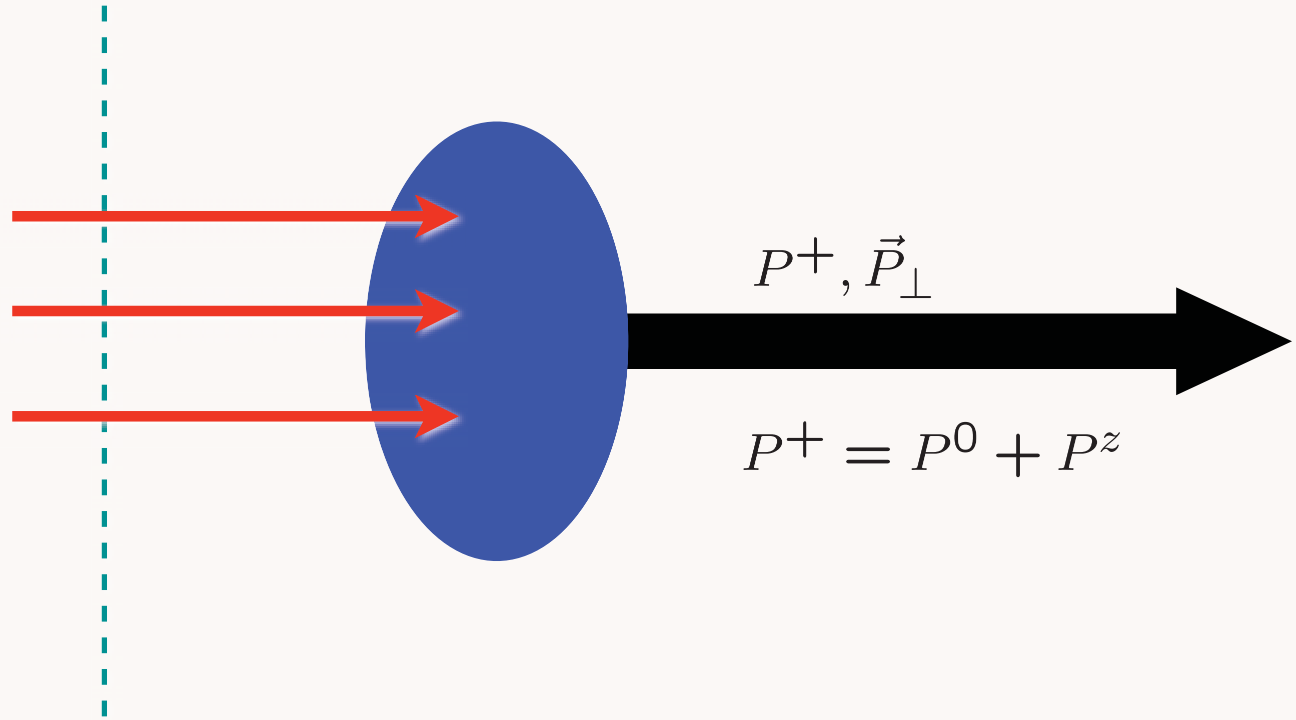


Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions

Fixed $\tau = t + z/c$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ