Light-Front Quantization Approach to the Gauge/Gravity Correspondence and the Hadron Spectrum

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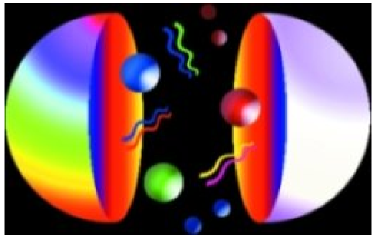

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GdT and Brodsky, PRL 102, 081601 (2009)

Outline

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1 Introduction: Gauge/Gravity Correspondence and QCD

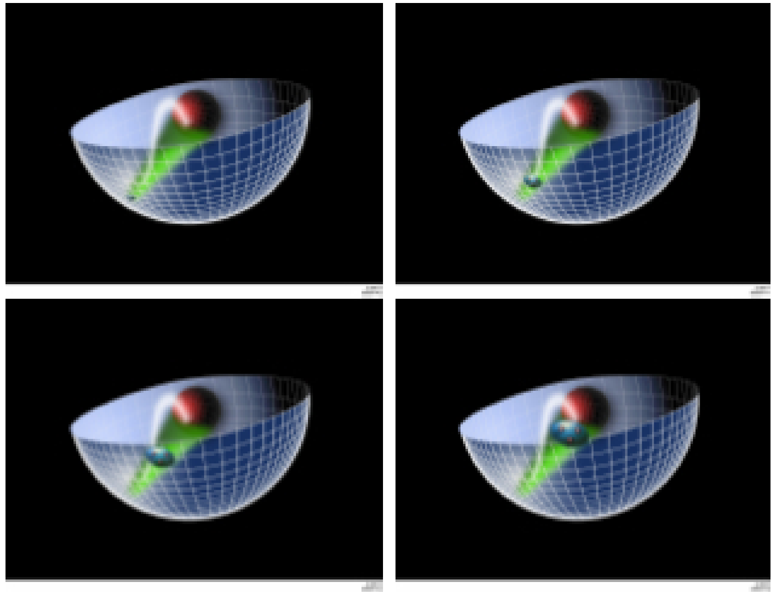
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom
- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between string states in AdS space and conformal field theories in physical space-time have led to analytical insights into the confining dynamics of QCD
- Description of strongly coupled gauge theory using a dual gravity description!
- Strings describe spin-J extended objects (no quarks). QCD degrees of freedom are pointlike particles and hadrons have orbital angular momentum: how can they be related?
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} \, L_{\rm AdS}$$



- Different values of z correspond to different scales at which the hadron is examined

- Since $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS$_5$ boundary $z \to 0$, which corresponds to the $Q \to \infty$ UV zero separation limit

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z at the IR boundary

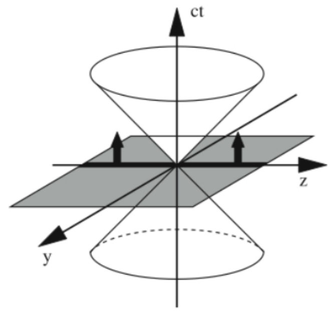
- Local operators like $\mathcal{O}$ and $\mathcal{L}_{\rm QCD}$ defined in terms of quark and gluon fields at the AdS$_5$ boundary

- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

2 Light-Front Quantization of QCD and AdS/CFT

- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...
- Frame-independent LF Hamiltonian equation $P_\mu P^\mu |P\rangle = \mathcal{M}^2 |P\rangle$ similar structure of AdS EOM
- First semiclassical approximation to the bound-state LF Hamiltonian equation in QCD is equivalent to equations of motion in AdS and can be systematically improved GdT and S. J. Brodsky, PRL **102**, 081601 (2009)

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



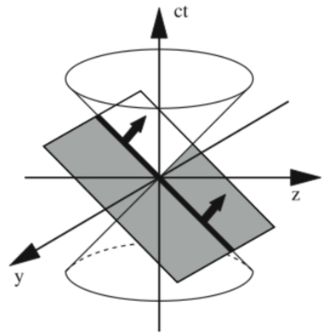
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum ($k^+ > 0$)

$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

- QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ, $\mathbf{A}_\perp$

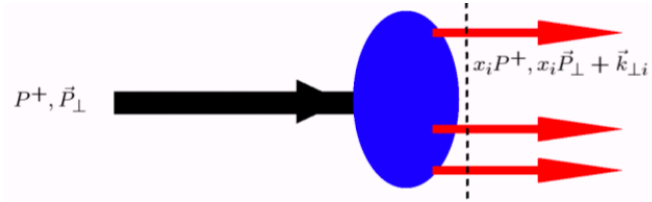
$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}\,\gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\partial^+\psi \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp\, \overline{\psi}\,\gamma^+ i\nabla_\perp\psi \end{aligned}$$

- LF Hamiltonian P^- generates LF time translations

$$\left[\psi(x), P^-\right] = i\frac{\partial}{\partial x^+}\psi(x)$$

and the generators P^+ and $\mathbf{P}_\perp$ are kinematical

Light-Front Partonic Representation



- Dirac field ψ, expanded in terms of ladder operators on the initial surface $x^+ = x^0 + x^3$

$$\psi(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x} \right]$$

- LF Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of constituents with momentum $q = (q^+, q^-, \mathbf{q}_\perp)$

$$\begin{aligned} P^- &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions} \\ P^+ &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \, q^+ \, b_\lambda^\dagger(q) b_\lambda(q) \\ \mathbf{P}_\perp &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \, \mathbf{q}_\perp \, b_\lambda^\dagger(q) b_\lambda(q) \end{aligned}$$

Light-Front Bound State Hamiltonian Equation

- Construct light-front invariant Hamiltonian for the composite system: $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$H_{LF} \, |\psi_H\rangle = \mathcal{M}_H^2 \, |\psi_H\rangle$$

- State $|\psi_H(P^+, \mathbf{P}_\perp, J_z)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian:

$$|\psi_H\rangle = \sum_n \psi_{n/H} |n\rangle, \qquad |n\rangle = \begin{cases} |uud\rangle \\ |uudg\rangle \\ |uud\bar{q}q\rangle \quad \cdots \end{cases}$$

  where $k_i^2 = m_i^2, \;\; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each component i

- Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

- Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle\psi_H(P')|H_{LF}|\psi_H(P)\rangle=\mathcal{M}_H^2\langle\psi_H(P')|\psi_H(P)\rangle$$

- Find

$$\mathcal{M}_H^2 = \sum_n \int \big[dx_i\big]\big[d^2\mathbf{k}_{\perp i}\big] \sum_\ell \left(\frac{\mathbf{k}_{\perp \ell}^2 + m_\ell^2}{x_q}\right) \big|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})\big|^2 + \text{interactions}$$

- Phase space normalization of LFWFs

$$\sum_n \int \big[dx_i\big]\ \big[d^2\mathbf{k}_{\perp i}\big]\ \big|\psi_{n/h}(x_i, \mathbf{k}_{\perp i})\big|^2 = 1$$

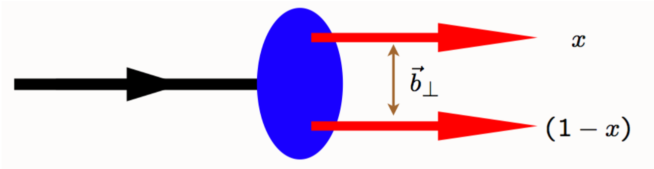
- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

$$\mathcal{M}_H^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi^*_{n/H}(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q}\right) \psi_{n/H}(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j}\, |\psi_n(x_j, \mathbf{b}_{\perp j})|^2 = 1$$

3 Semiclassical Approximation to QCD



- Consider a two-parton hadronic bound state in transverse impact space in the limit $m_q \to 0$

$$\mathcal{M}^2 = \int_0^1 \frac{dx}{1-x} \int d^2\mathbf{b}_\perp \, \psi^*(x, \mathbf{b}_\perp) \left(-\nabla^2_{\mathbf{b}_\perp}\right) \psi(x, \mathbf{b}_\perp) + \text{interactions}$$

- Functional dependence of Fock state $|n\rangle$ given by invariant mass

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^n k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}^2_{\perp a} + m_a^2}{x_a} \to \frac{\mathbf{k}^2_\perp}{x(1-x)}$$

  the off-energy shell of the bound state $\mathcal{M}^2 - \mathcal{M}_n^2$

- In impact space the relevant variable is $\zeta^2 = x(1-x)\mathbf{b}^2_\perp$

- To first approximation LF dynamics depend only on the invariant variable $\mathcal{M}_n$ or ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

  factoring out angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

- Find ($L = |M|$)

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Big) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

Hard-Wall Model

- Consider the potential (hard wall)

$$U(\zeta) = \begin{cases} 0 & \text{if} \quad \zeta \leq \frac{1}{\Lambda_{\rm QCD}} \\ \infty & \text{if} \quad \zeta > \frac{1}{\Lambda_{\rm QCD}} \end{cases}$$

- If $L^2 \geq 0$ the Hamiltonian is positive definite $\langle\phi \left| H^L_{LF} \right| \phi\rangle \geq 0$ and thus $\mathcal{M}^2 \geq 0$
- If $L^2 < 0$ the Hamiltonian is not bounded from below (“Fall-to-the-center” problem in Q.M.)
- Critical value of the potential corresponds to $L = 0$, the lowest possible stable state
- Solutions:

$$\phi_L(\zeta) = C_L \sqrt{\zeta} J_L\left(\zeta \mathcal{M}\right)$$

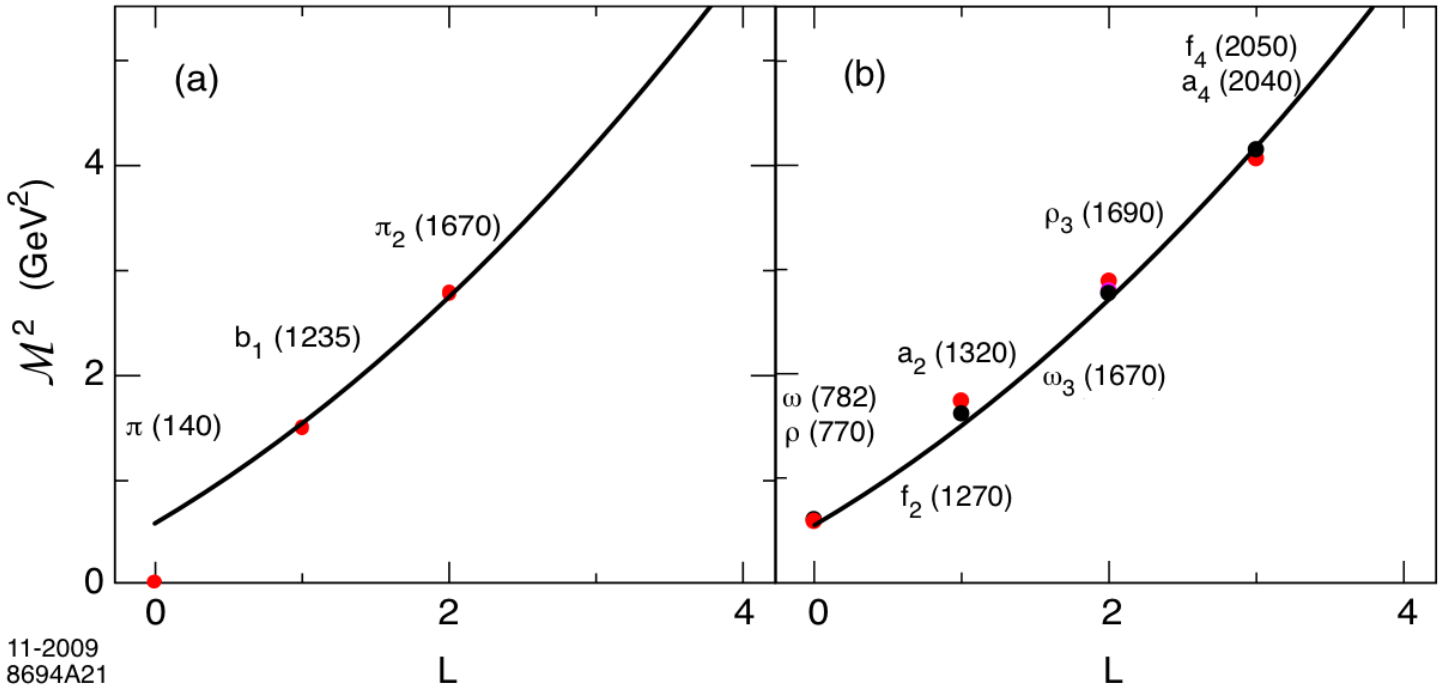
- Mode spectrum from boundary conditions

$$\phi\left(\zeta = \frac{1}{\Lambda_{\rm QCD}}\right) = 0$$

Thus

$$\mathcal{M}^2 = \beta_{Lk} \Lambda_{\rm QCD}$$

- Excitation spectrum hard-wall model: $\mathcal{M}_{n,L} \sim L + 2n$



Light-meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Holographic Mapping

- Holographic mapping found originally by matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Substitute $\Phi(\zeta) \sim \zeta^{3/2}\phi(\zeta), \quad \zeta \to z$ in the conformal LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

- Find:

$$\boxed{\left[z^2\partial_z^2 - 3z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0}$$

with $(\mu R)^2 = -4 + L^2$, the wave equation of string mode in AdS_5 !

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Conformal dimension Δ of AdS mode Φ given in terms of 5-dim mass by $(\mu R)^2 = \Delta(\Delta - 4)$. Thus $\Delta = 2 + L$ in agreement with the twist scaling dimension of a two parton object in QCD

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3-2J) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

with $(\mu R)^2 = (\Delta - J)(\Delta - 4 + J)$

- Upon substitution $z \to \zeta$ and

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} \Phi_J(\zeta)$$

we recover the QCD LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$

- J-decoupling in the HW model

Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff wich depends on the profile of a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(3 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$.

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$

- Eigenfunctions

$$\phi_{nL}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

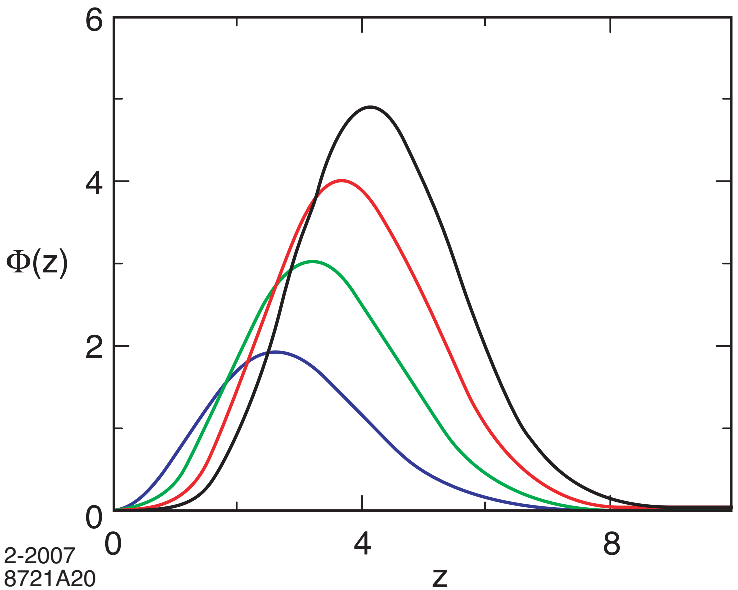
- Eigenvalues

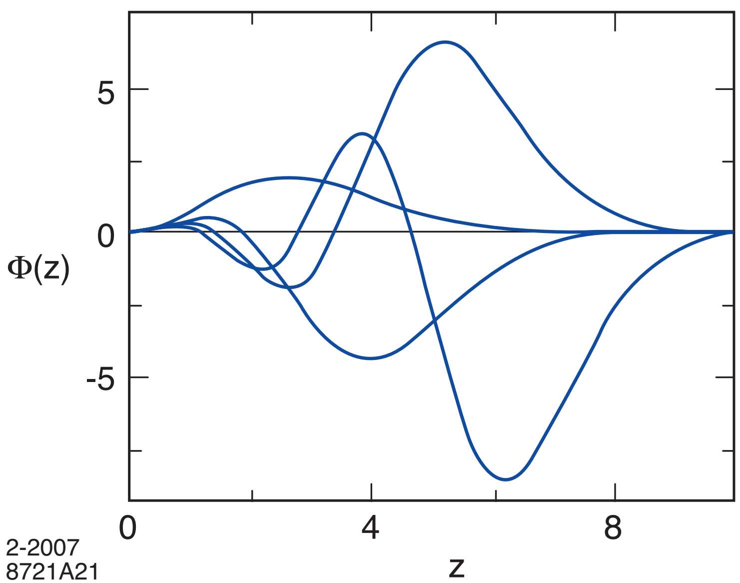
$$\mathcal{M}^2_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2}\right)$$

$4\kappa^2$ for $\Delta n = 1$

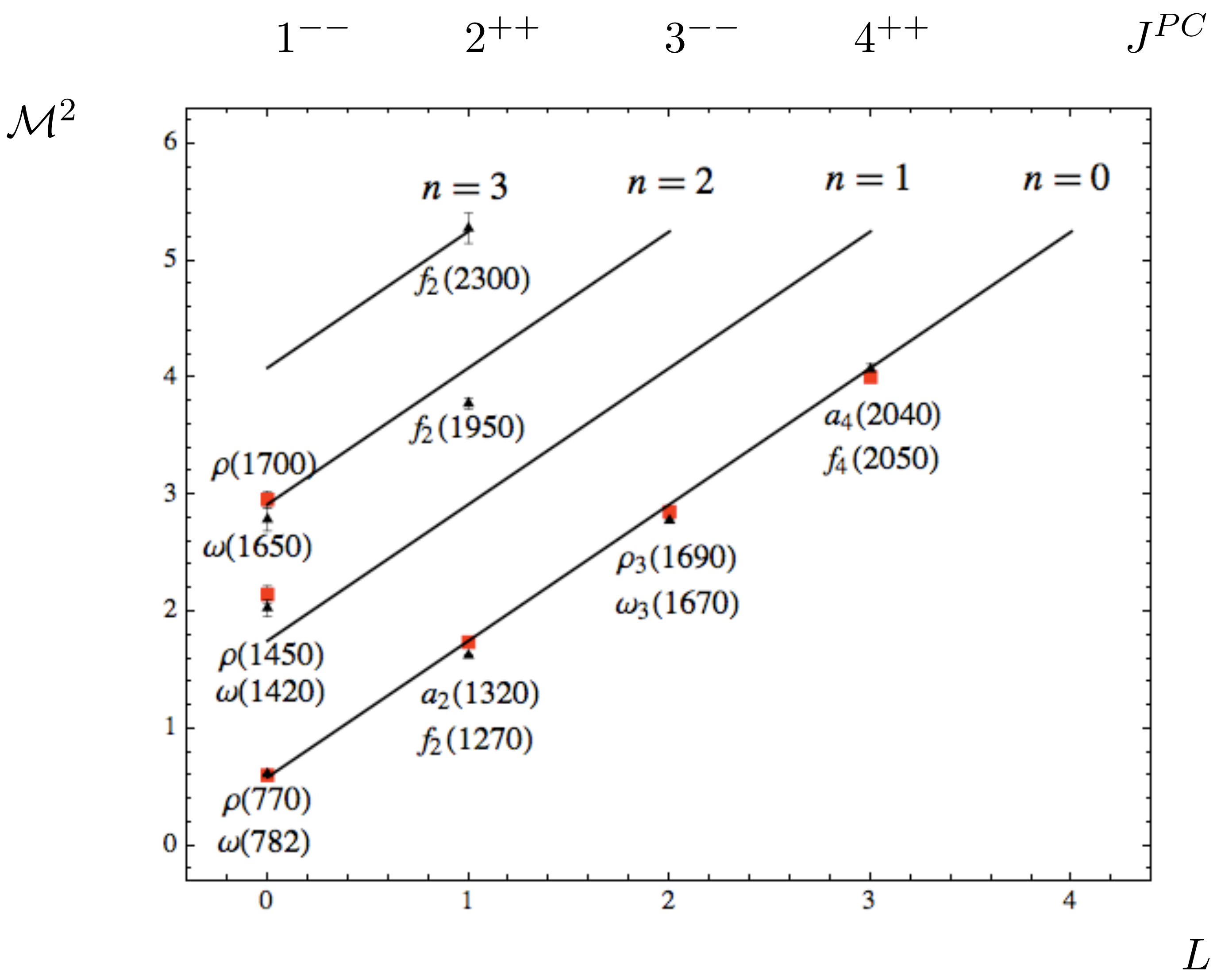
$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$





Orbital and radial states: $\langle\zeta\rangle$ increase with L and n

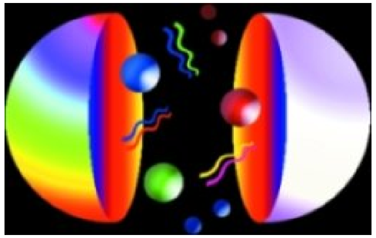


Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red)

and the $I = 0$ ω-meson family (black) for $\kappa = 0.54$ GeV

4 Fermionic Modes

Hard-Wall Model



From Nick Evans

- Action for massive fermionic modes on AdS$_5$:

$$S[\overline{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{\rm QCD}\right) = 0$

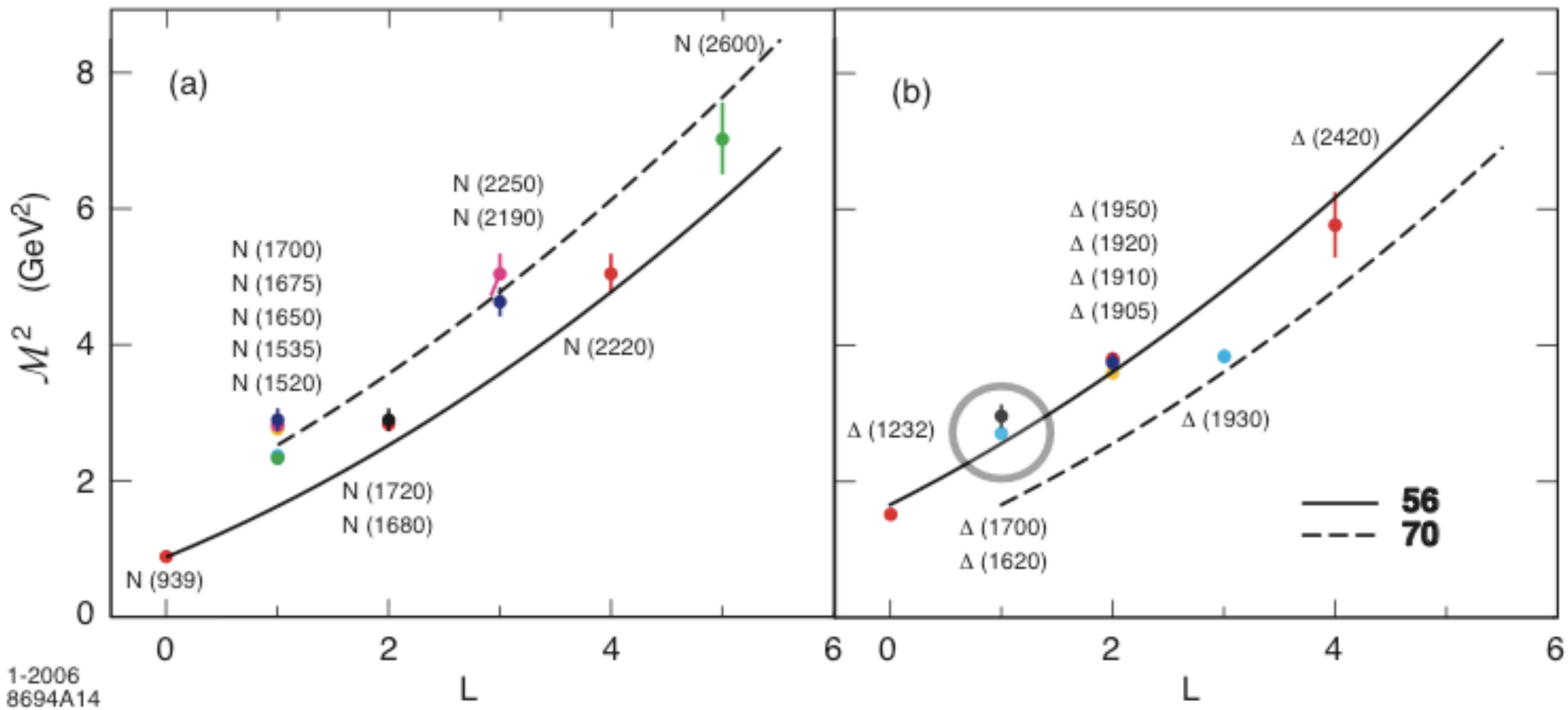
$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{\rm QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{\rm QCD}$$

  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

SU(6)	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

- Excitation spectrum for baryons in the hard-wall model: $\mathcal{M} \sim L + 2n$



Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states: (a) $I = 1/2$ and (b) $I = 3/2$

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

- Solution $(\mu R = \nu + 1/2,\, d = 4)$

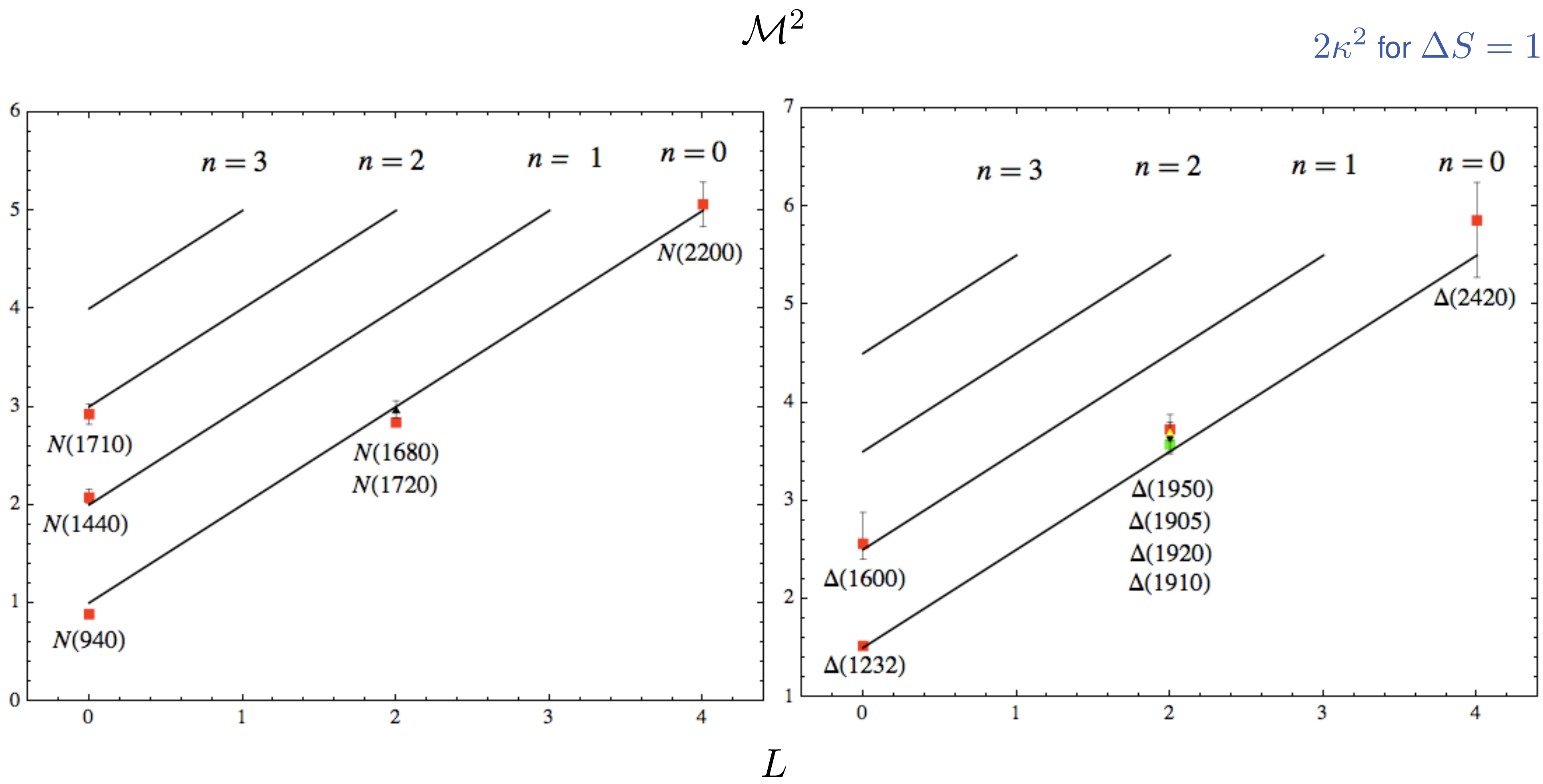
$$\begin{aligned} \Psi_+(z) &\sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2) \\ \Psi_-(z) &\sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2) \end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$$

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

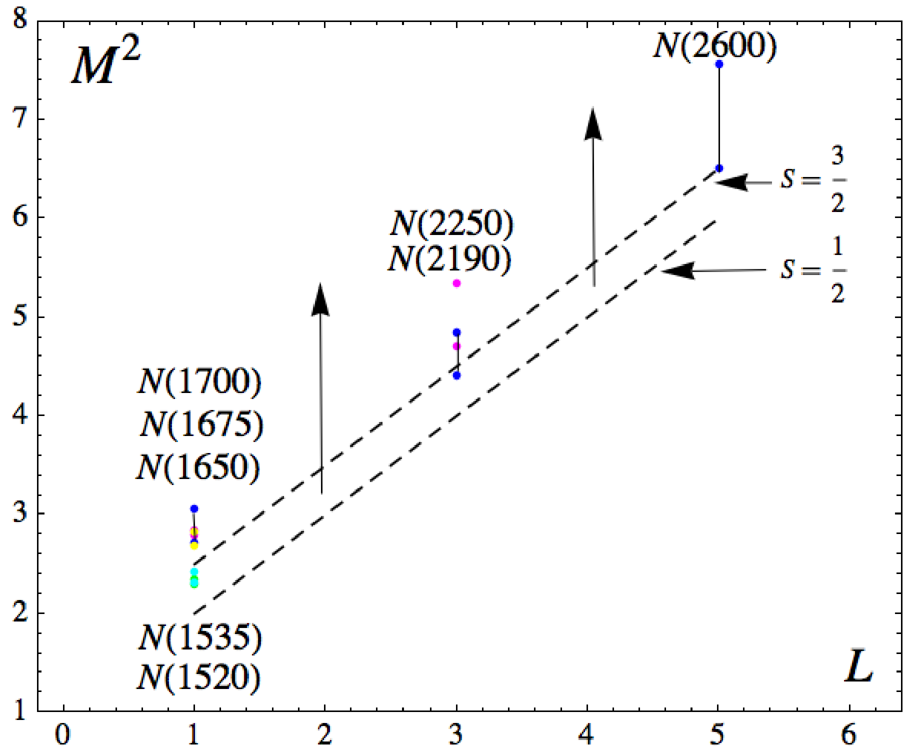
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

- Δ spectrum identical to Forkel and Klempt, Phys. Lett. B **679**, 77 (2009)

$\mathcal{M}^2$



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Parent **70** Regge trajectories for the N family for $\kappa = 0.5$ GeV

- Not so well described by one parameter model: requires additional attraction (smaller hadronic size)

  Forkel and Klempt, Phys. Lett. B **679**, 77 (2009)

5 Non-Perturbative QCD Coupling From LF Holography

With A. Deur and S. J. Brodsky

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

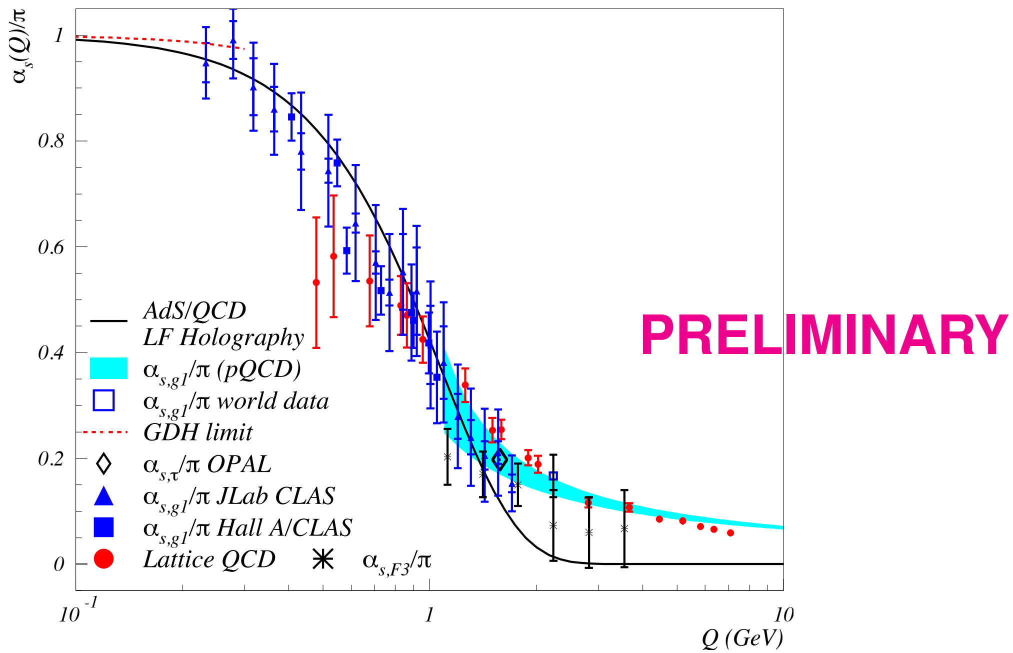
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

- Non-perturbative AdS/QCD running coupling

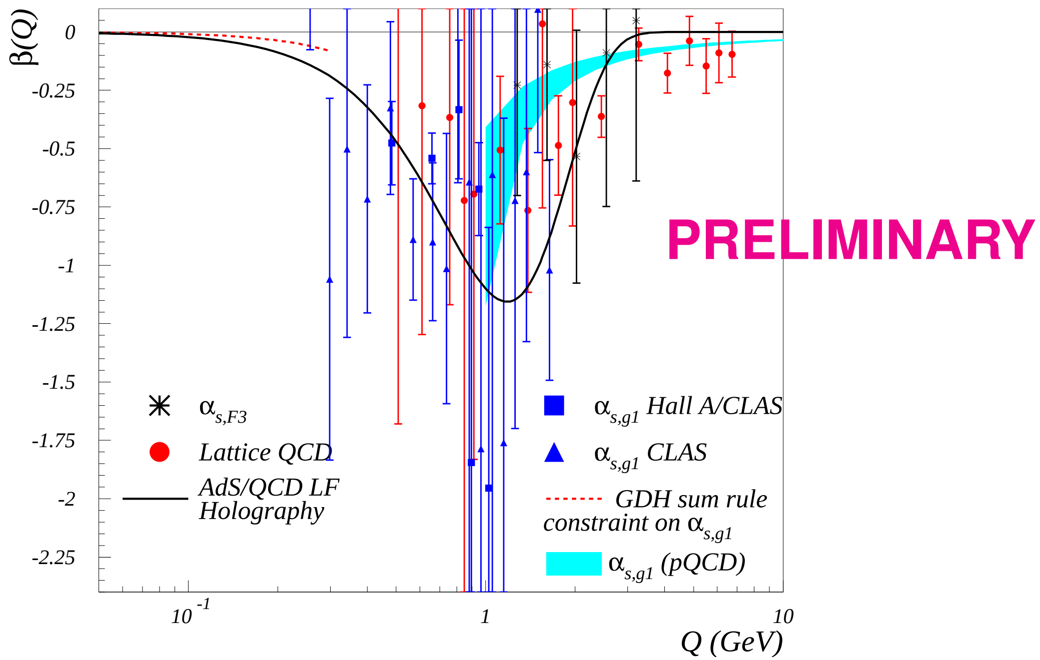
$$\frac{\alpha_s^{AdS}(Q)}{\pi} = e^{-Q^2/4\kappa^2}.$$



Effective coupling from LF holography for $\kappa = 0.54\ \mathrm{GeV}$

- β-function

$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}.$$



Effective coupling from LF holography for $\kappa = 0.54 \text{ GeV}$

Conjectured behavior of the full β-function of QCD

$$\beta(Q \to 0) = \beta(Q \to \infty) = 0, \tag{1}$$

$$\beta(Q) < 0, \text{ for } Q > 0, \tag{2}$$

$$\frac{d\beta}{dQ}\Big|_{Q=Q_0} = 0, \tag{3}$$

$$\frac{d\beta}{dQ} < 0, \text{ for } Q < Q_0, \ \frac{d\beta}{dQ} > 0, \text{ for } Q > Q_0. \tag{4}$$

1. QCD is conformal in the far UV and deep IR
2. Anti-screening behavior of QCD which leads to asymptotic freedom
3. Hadronic-partonic transition: the minimum is an absolute minimum
4. Since there is only one transition (4) follows from the above