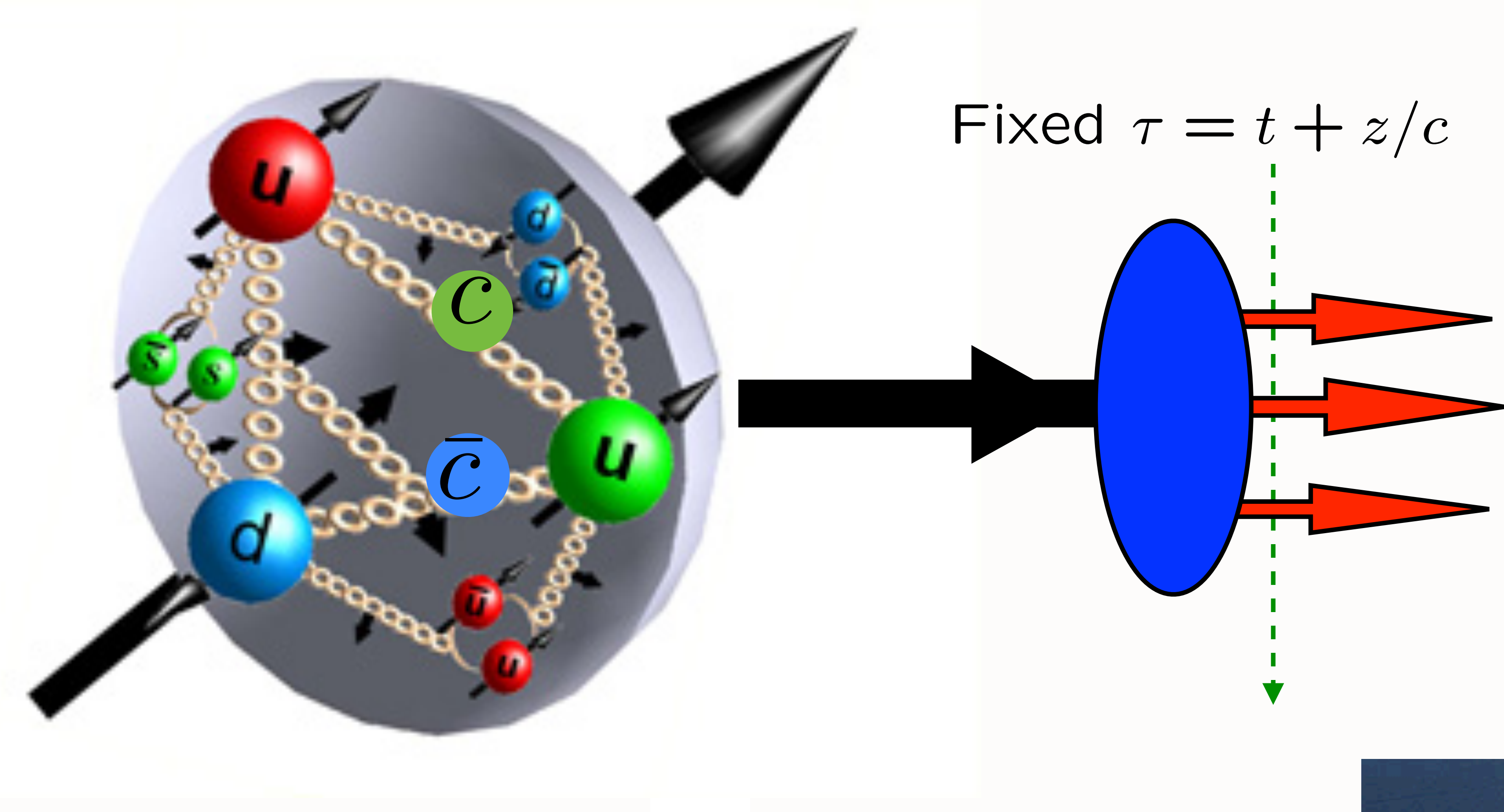
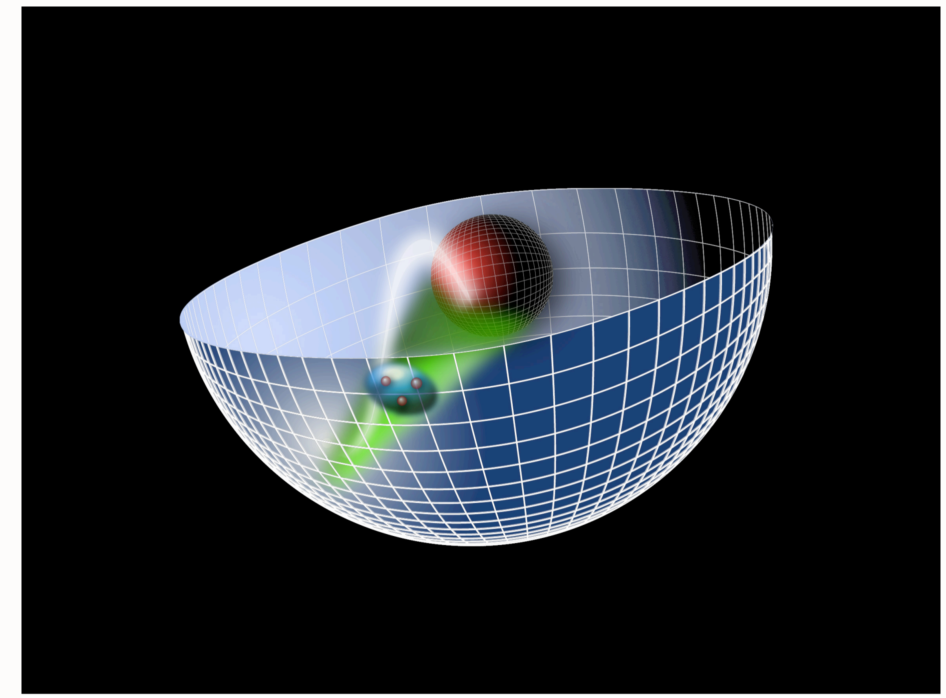
Light-Front Holography and Proton Transversity





TRANSVERSITY 2011

Third International Workshop on Transverse Polarization Phenomena in Hard Scattering

Veli Lošinj, Croatia, 29 August - 2 September 2011

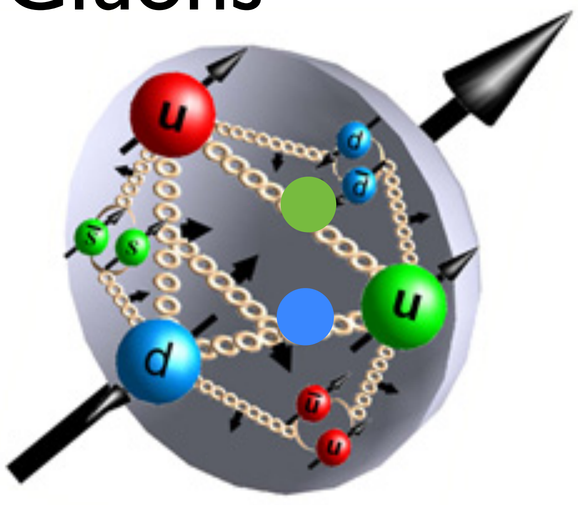
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Transversity

Angular Momentum Structure, and the Spin Dynamics of Hadrons

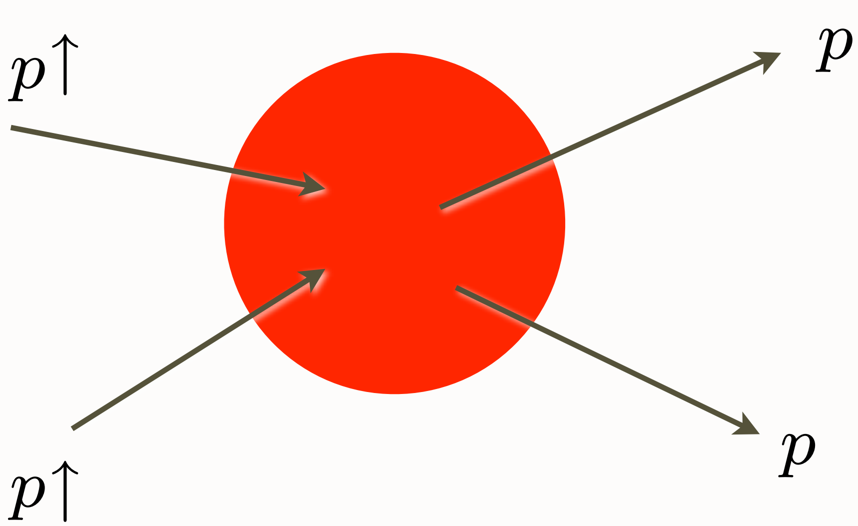
- Test Fundamentals of Gauge Structure of QCD
- Fundamental Measures of Hadron Structure
- Angular Momentum of Confined Quarks and Gluons
- Breakdown of Conventional Wisdom
- Breakdown of Factorization Ideas
- Crucial Experiment Tests, Measurements



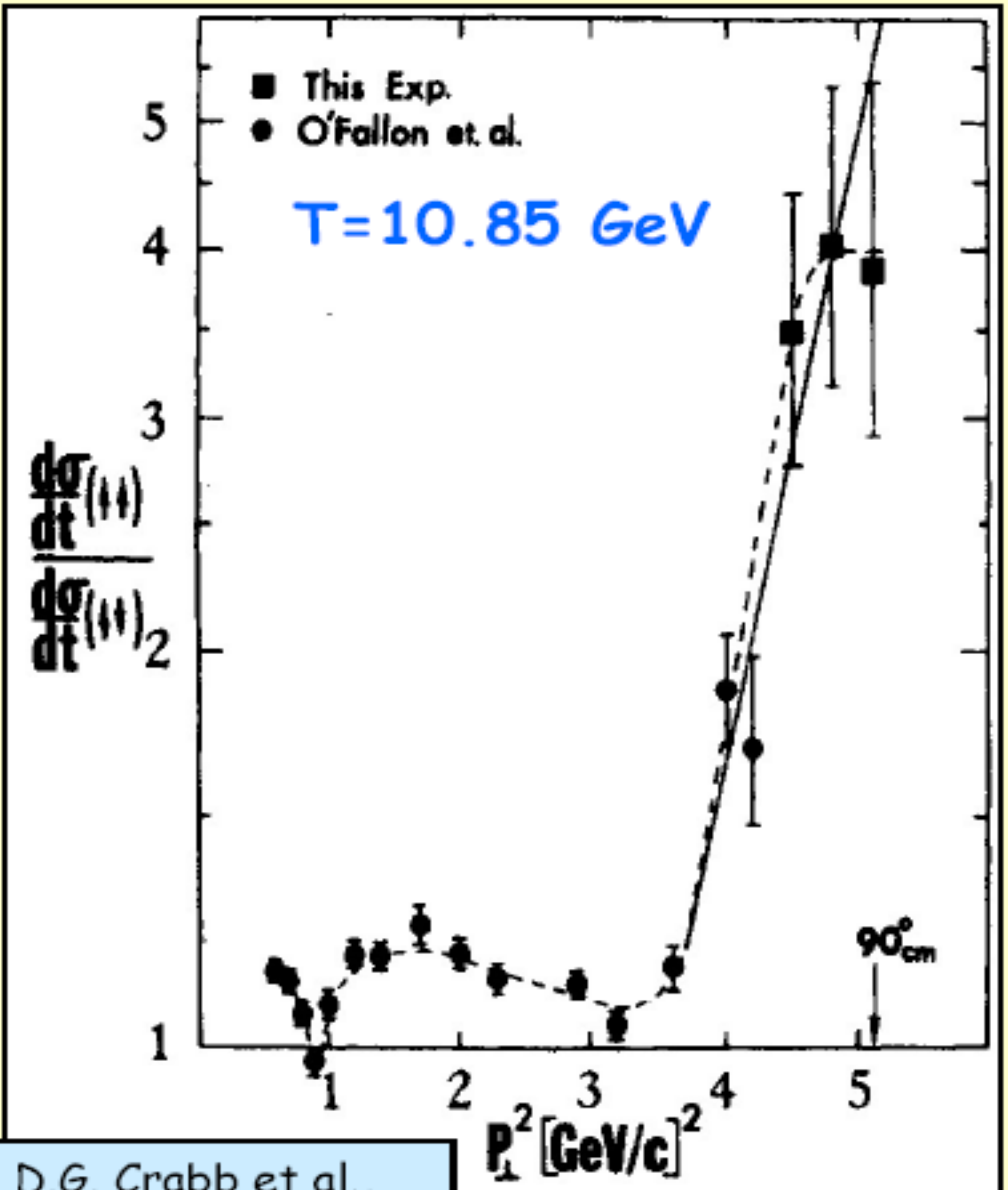
Remarkable array of theory and experimental talks

Krisch

Unexpected spin-spin correlation in pp elastic scattering

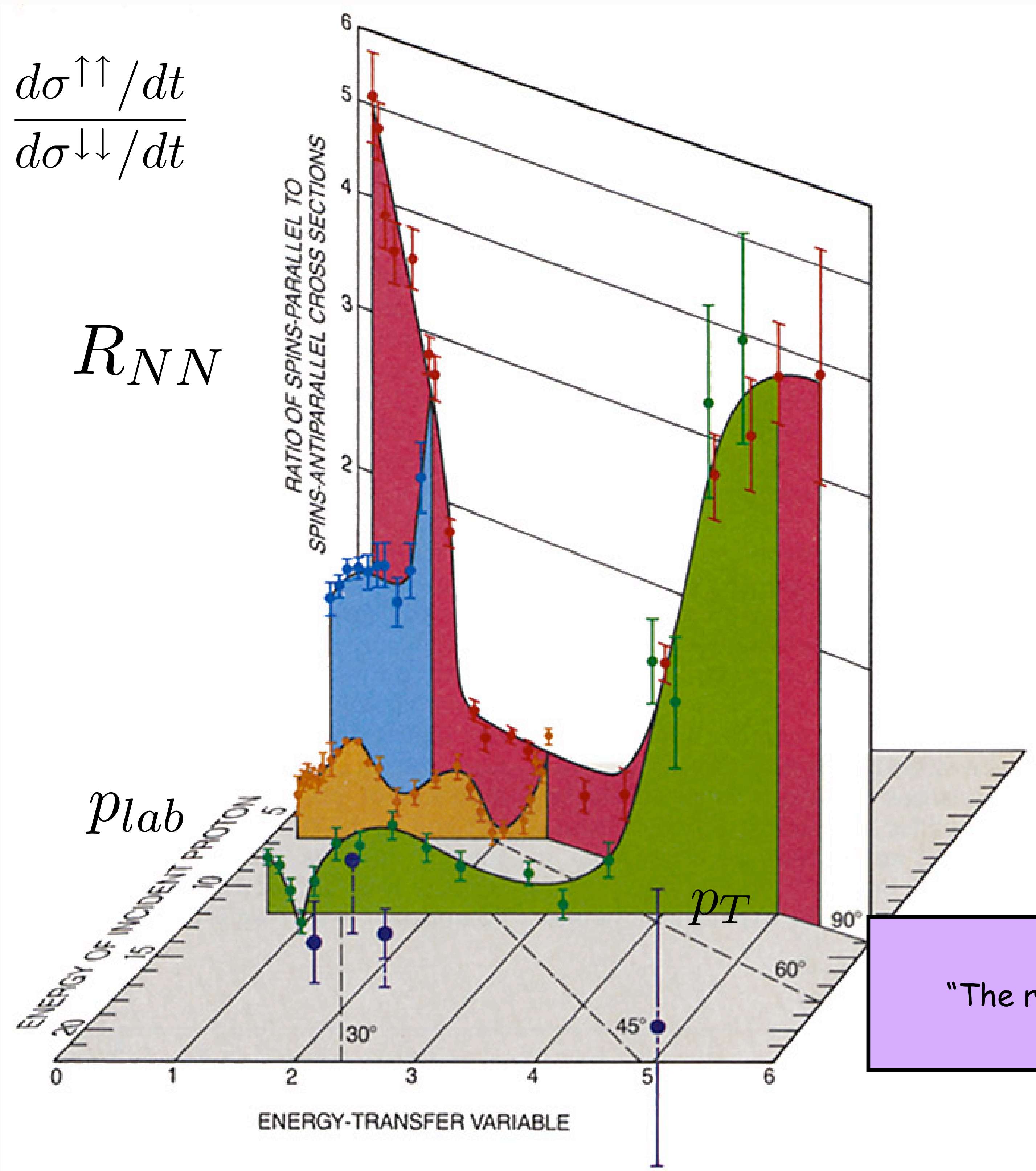


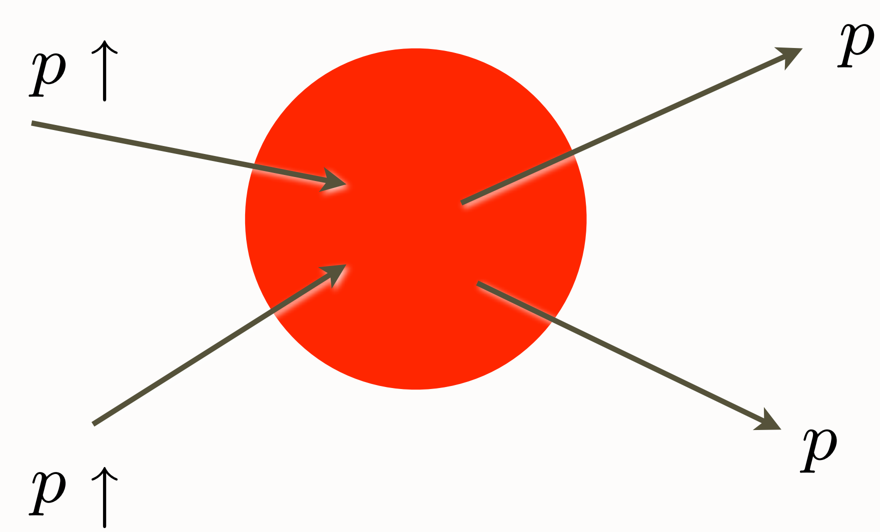
polarizations normal to scattering plane



D.G. Crabb et al., PRL 41, 1257 (1978)

Spin Correlations in Elastic $p-p$ Scattering

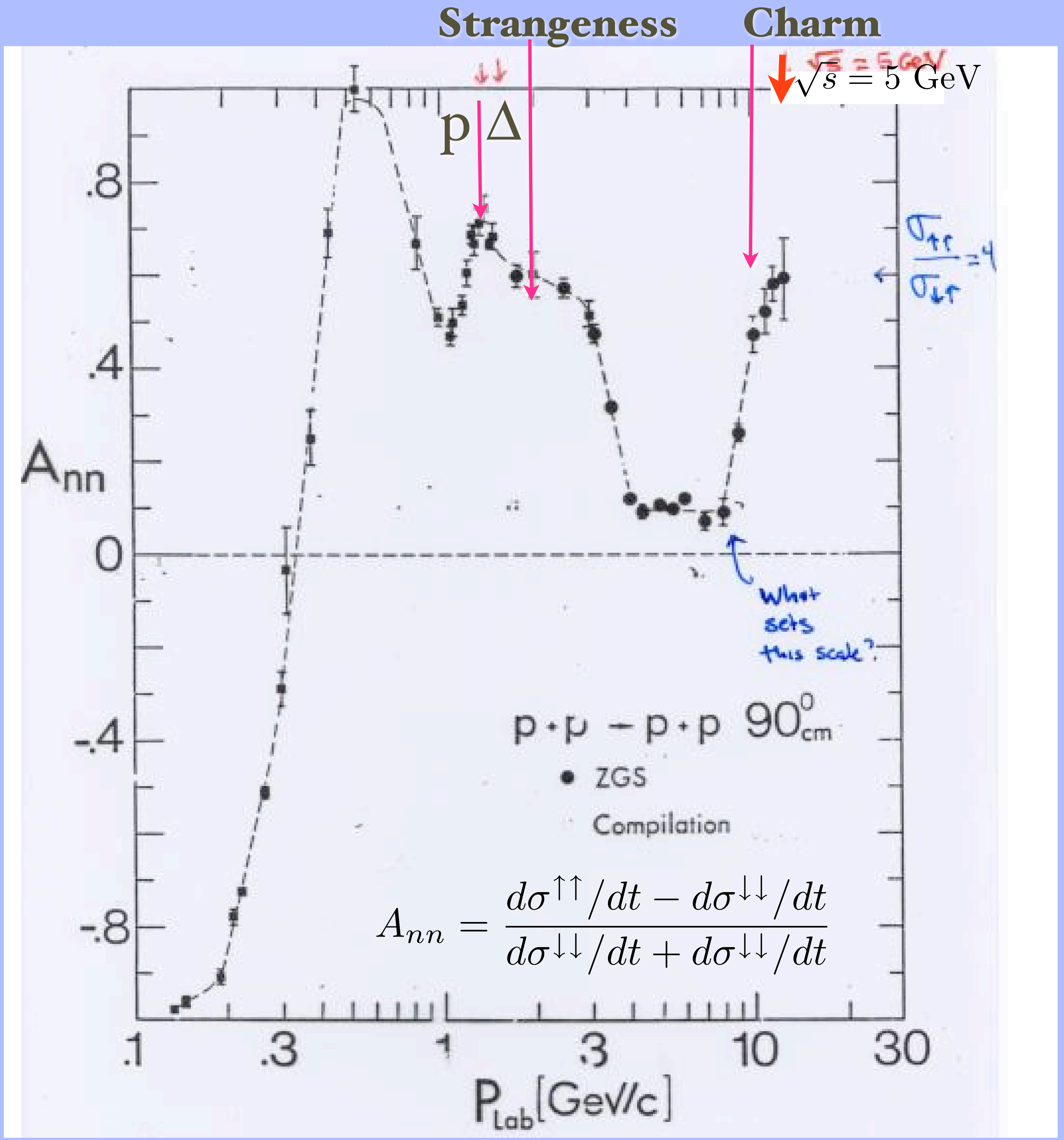




polarization normal to scattering plane

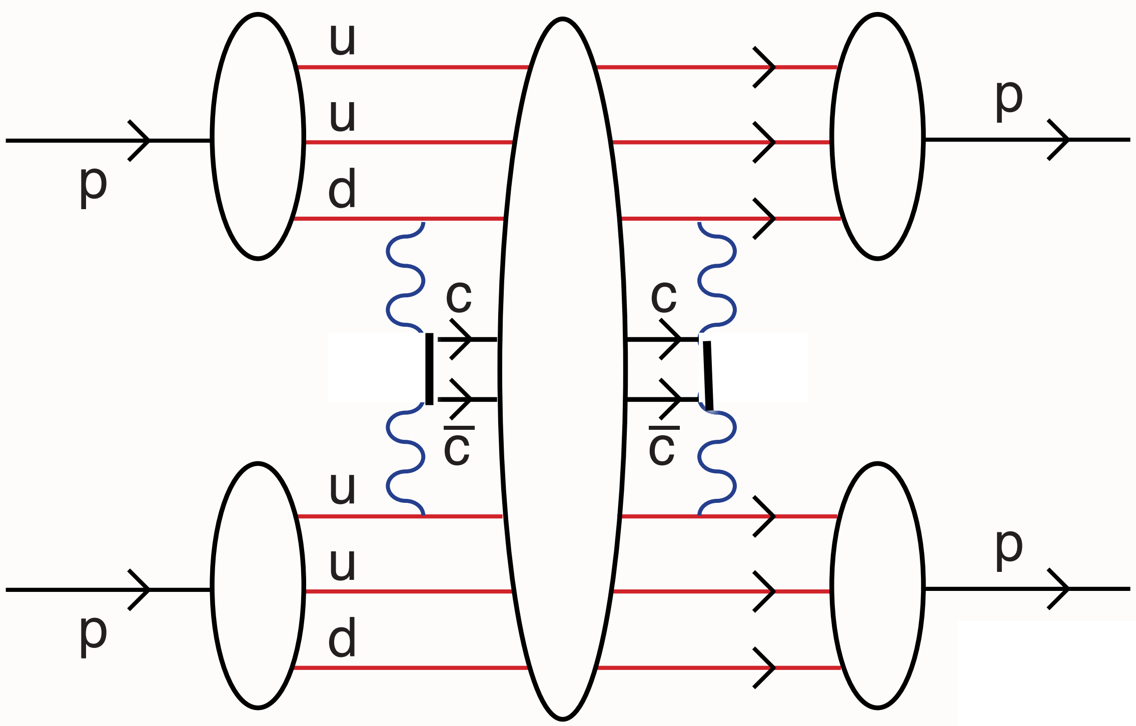
Ratio reaches 4:1 !

A. Krisch, Sci. Am. 257 (1987)
"The results challenge the prevailing theory that describes the proton's structure and forces"



Strangeness
Charm
$\sqrt{s} = 5$ GeV
p
Δ
.8
.4
0
-.4
-.8
A_{nn}
What sets this scale?
p+p → p+p 90°cm
● ZGS
Compilation
$$A_{nn} = \frac{d\sigma^{\uparrow\uparrow}/dt - d\sigma^{\downarrow\downarrow}/dt}{d\sigma^{\downarrow\downarrow}/dt + d\sigma^{\downarrow\downarrow}/dt}$$
.1
.3
1
3
10
30
P_{Lab} [GeV/c]

$$A_{nn} = 1!$$



QCD Schwinger-Sommerfeld Enhancement at Heavy Quark Threshold

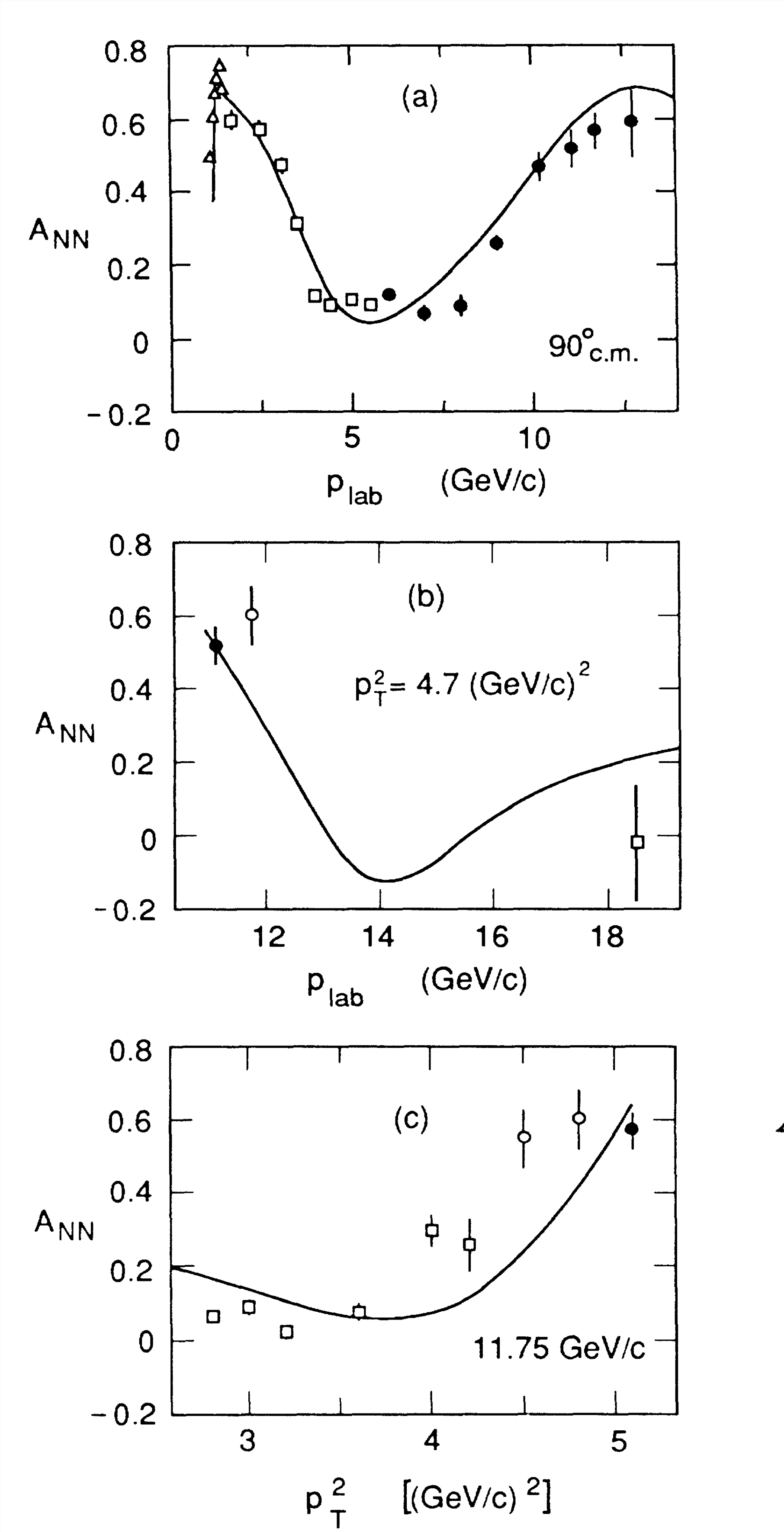
Hebecker, Kuhn, sjb

S. J. Brodsky and G. F. de Teramond, "Spin Correlations, QCD Color Transparency And Heavy Quark Thresholds In Proton Proton Scattering," Phys. Rev. Lett. **60**, 1924 (1988).

Production of $uud\,\bar{c}\,c\,uud$ octoquark resonance

J=L=S=1, C=-, P=- state

8 quarks in S-wave: odd parity



S. J. Brodsky and G. F. de Teramond, "Spin Correlations, QCD Color Transparency And Heavy Quark Thresholds In Proton Proton Scattering," Phys. Rev. Lett. **60**, 1924 (1988).

Quark Interchange + 8-Quark Resonance

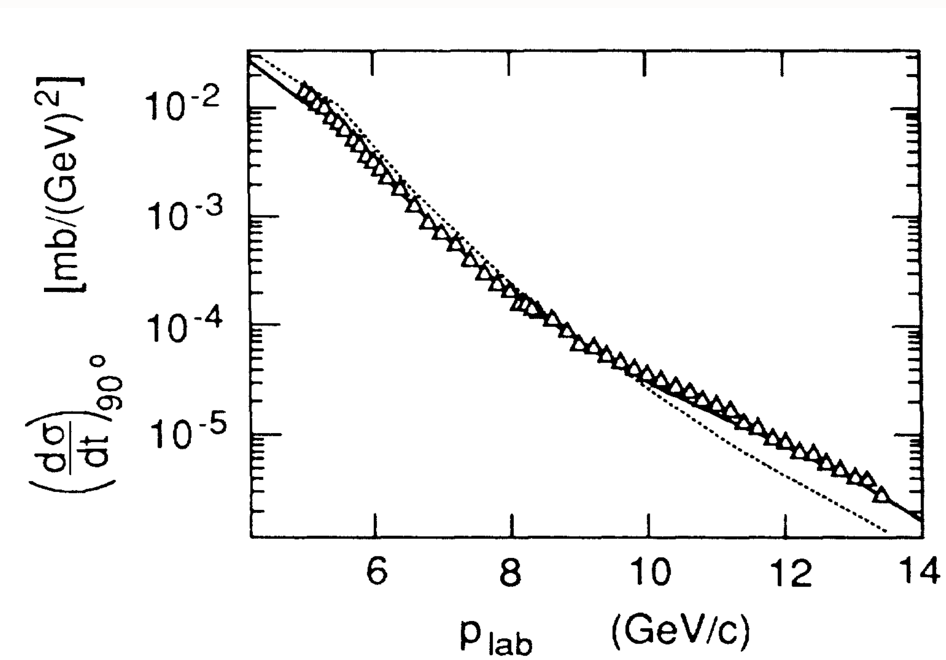
$$|uuduudss\bar{s}> \quad |uuduudc\bar{c}>$$

Strange and Charm Octoquark!

$M = 3$ GeV, $M = 5$ GeV.

$J = L = S = 1,\ B = 2$

$$A_{NN} = \frac{d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)}$$



"Exclusive Transversity"

A. Krisch, Sci. Am. 257 (1987)
"The results challenge the prevailing theory that describes the proton's structure and forces"

Spin-dependence at large-P_T ($90°_{cm}$):

Hard scattering takes place with spins ↑↑

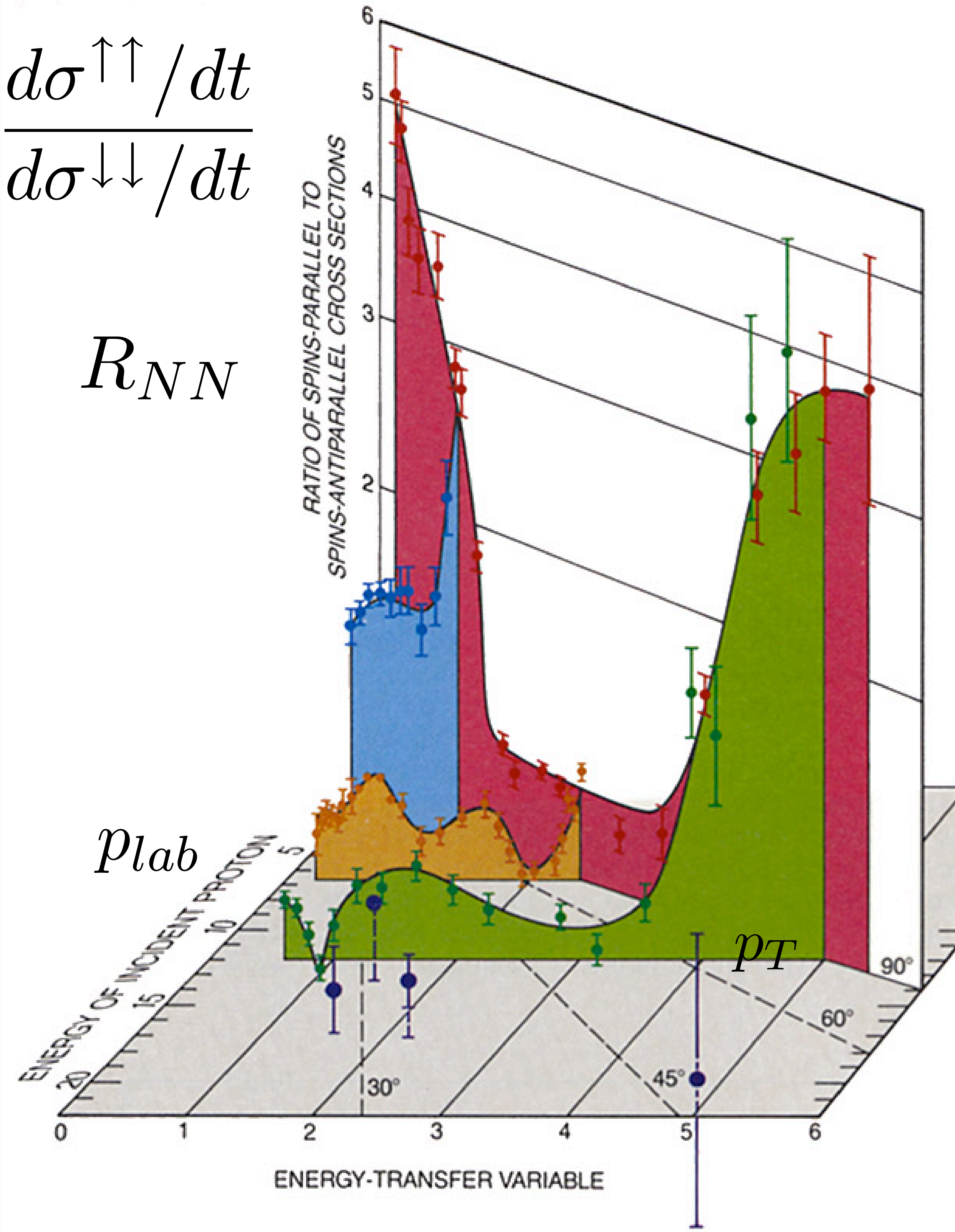
Charm and Strangeness Thresholds

Heppelmann et al: Quenching of Color Transparency

B=2 Octoquark Resonances?

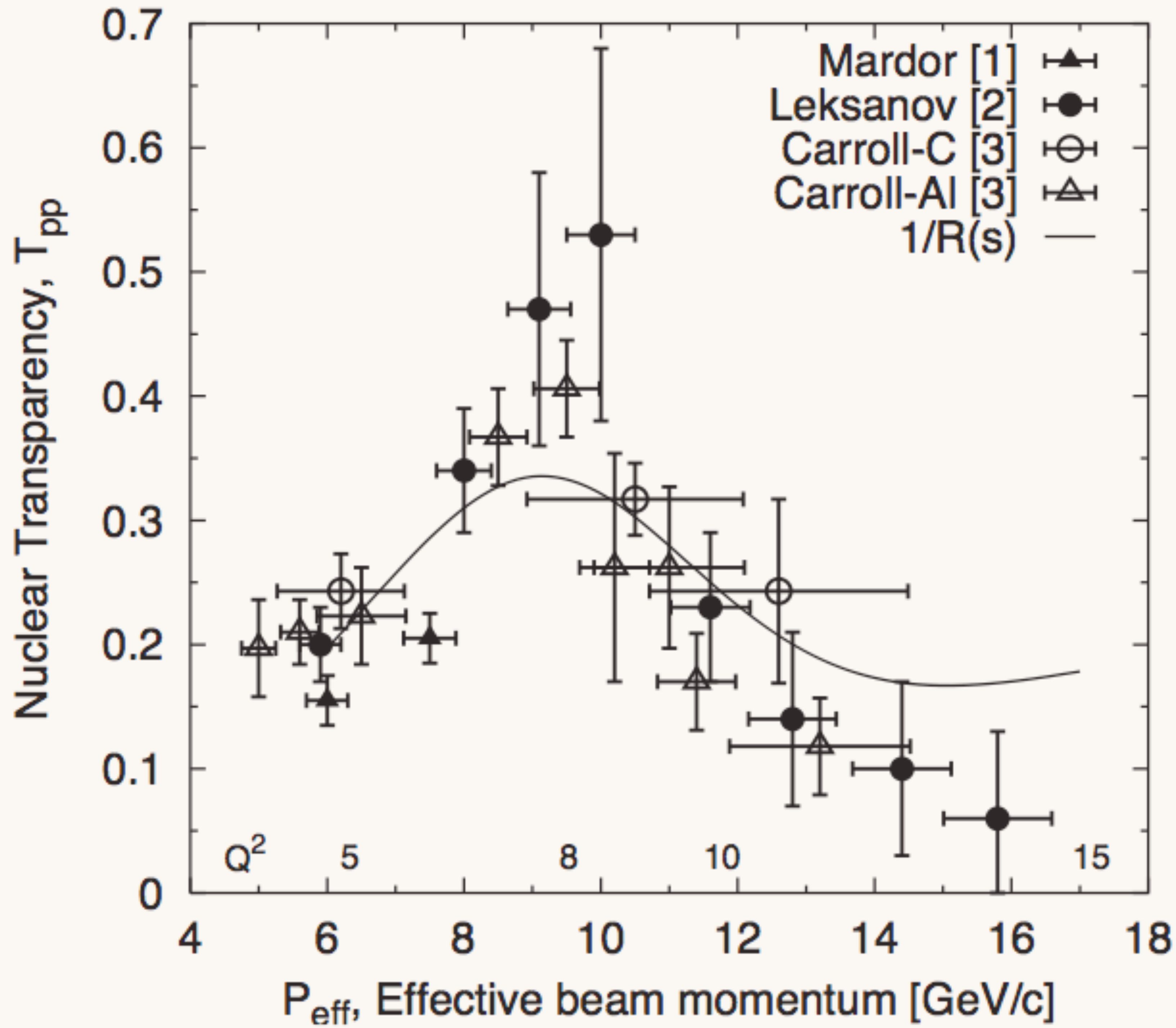
$$\frac{d\sigma^{\uparrow\uparrow}/dt}{d\sigma^{\downarrow\downarrow}/dt}$$

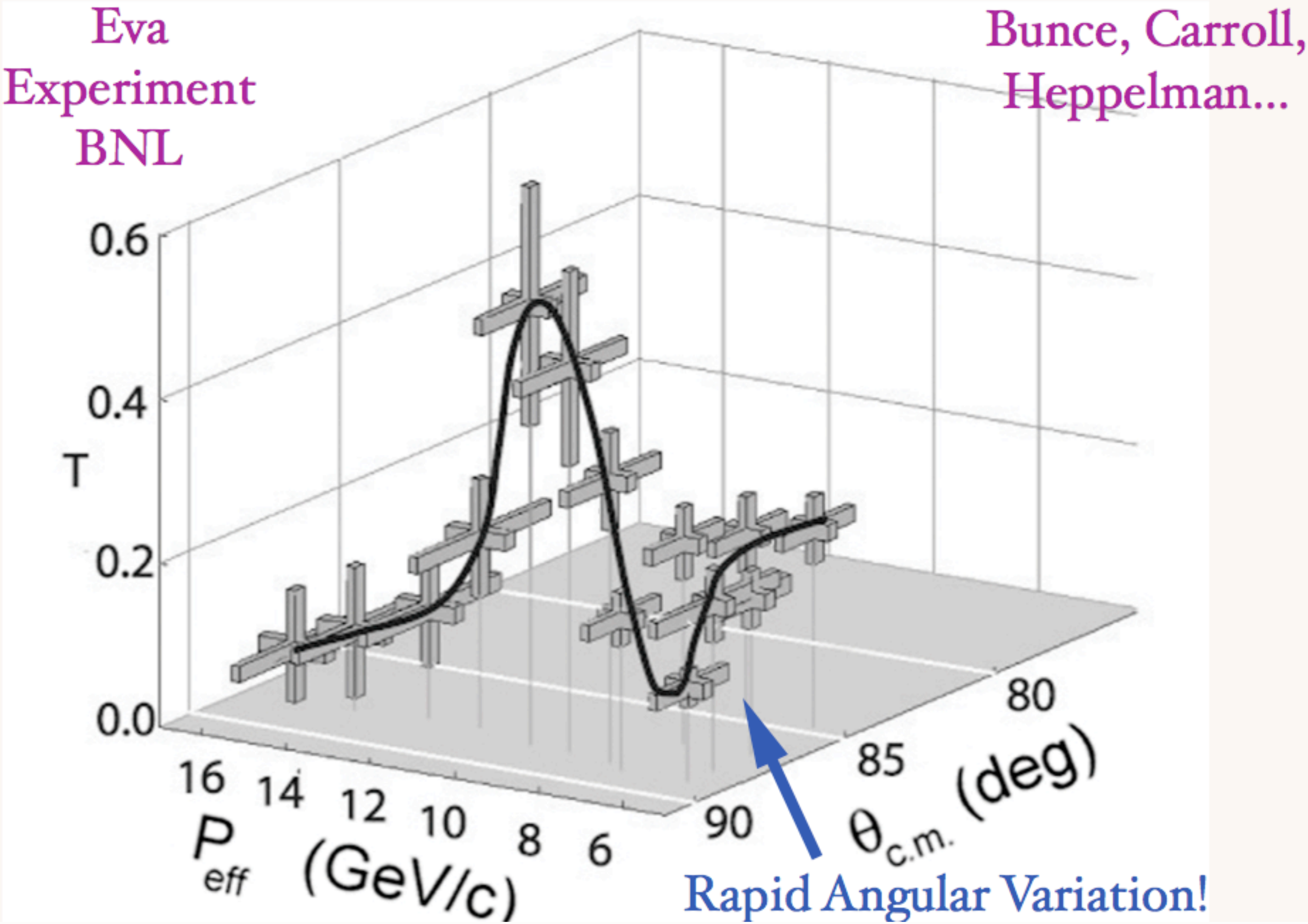
$$R_{NN}$$



Color Transparency fails when A_{nn} is large

Mueller, sjb




Eva
Experiment
BNL
Bunce, Carroll,
Heppelman...
0.6
0.4
0.2
0.0
T
16 14 12 10 8 6
P eff (GeV/c)
80
85
90
θ c.m. (deg)
Rapid Angular Variation!

- New QCD physics in proton-proton elastic scattering at the charm threshold
- Anomalously large charm production at threshold!!?
- Octoquark resonances?
- Color Transparency disappears at charm threshold
- Key physics at GSI: second charm threshold

$$\bar{p}p \to \bar{p}pJ/\psi$$

$$\bar{p}p \to \bar{p}\Lambda_c D$$

Key QCD Experiment at GSI

Total open charm cross section at threshold

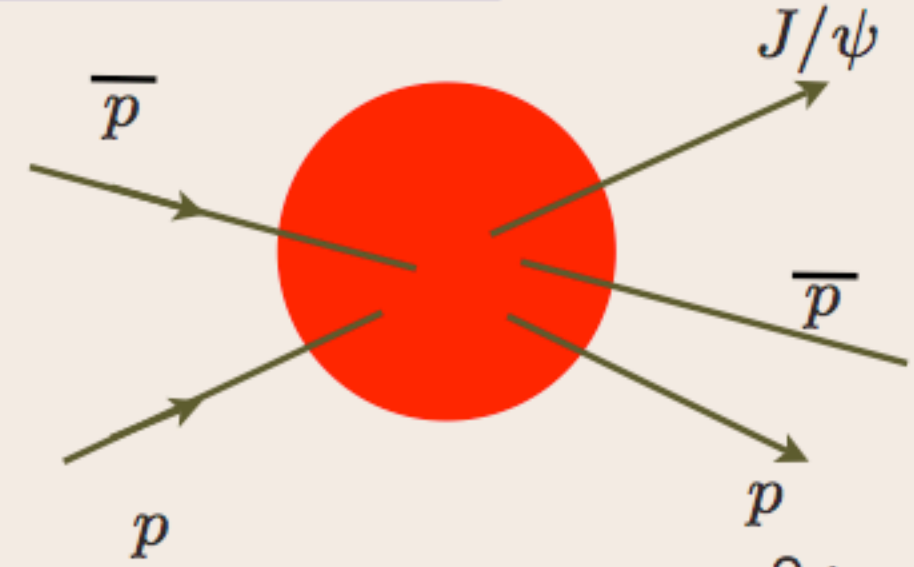
$\sigma(\overline{p}p \to cX) \simeq 1 \mu b$

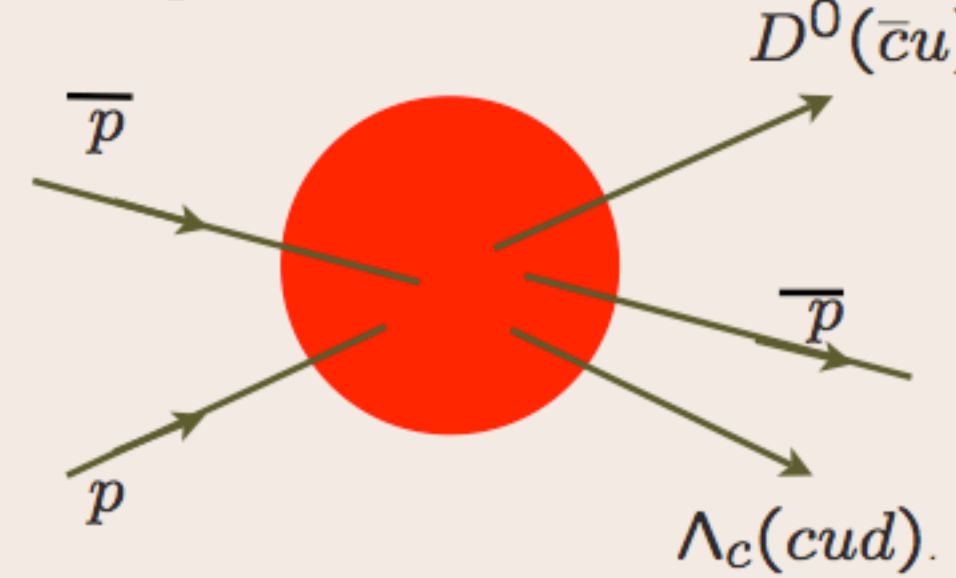
needed to explain Krisch A_{NN}

$\overline{p}p \to \overline{p} + J/\psi + p$

$\overline{p}p \to \overline{p} + \eta_c + p$

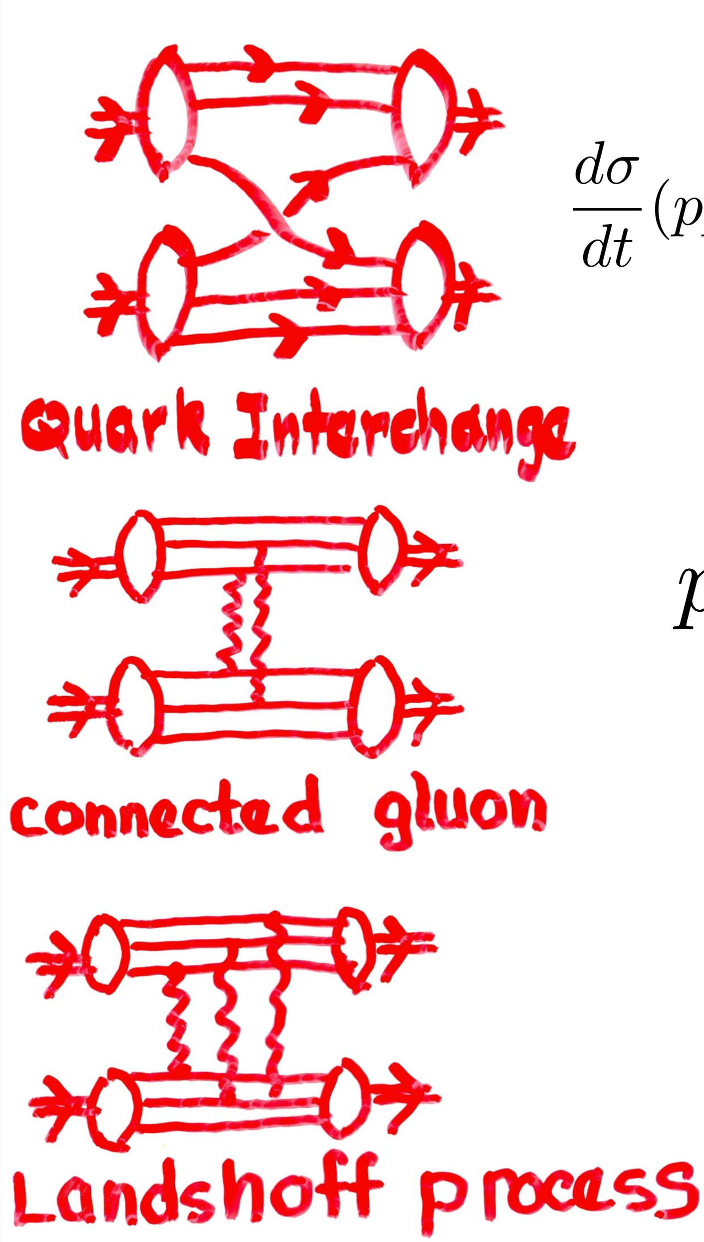
$\overline{p}p \to \overline{\Lambda}_c(\overline{cud}) D^0(\overline{c}u) p$





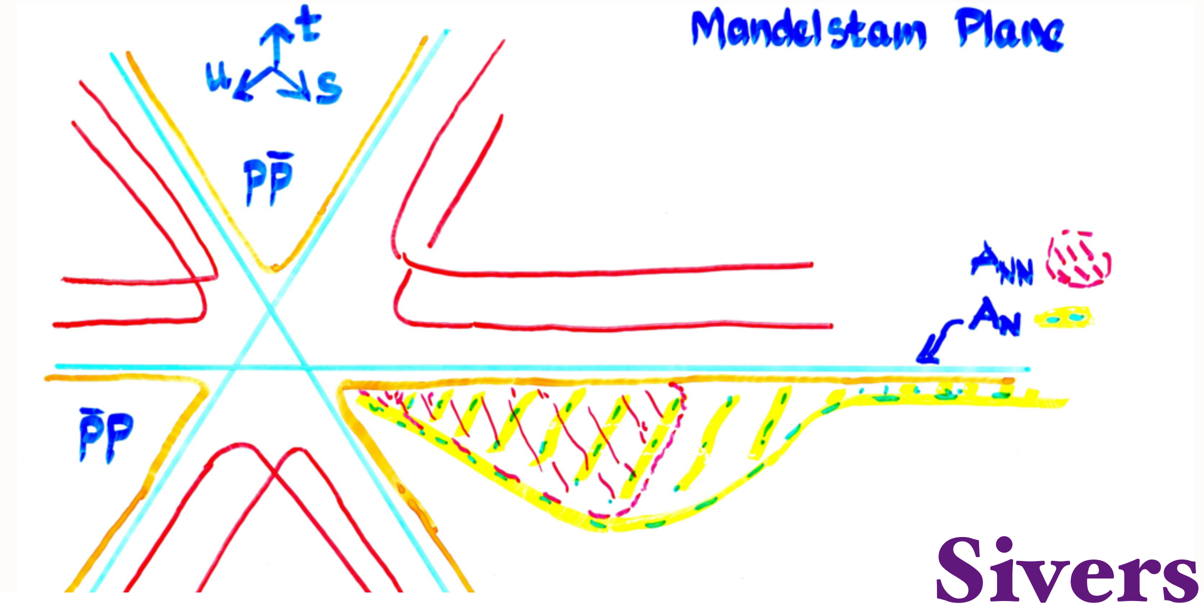
Octoquark: $|\overline{uud}c\overline{c}uud>$

Huge Number of Tests of Transversity in Exclusive Reactions

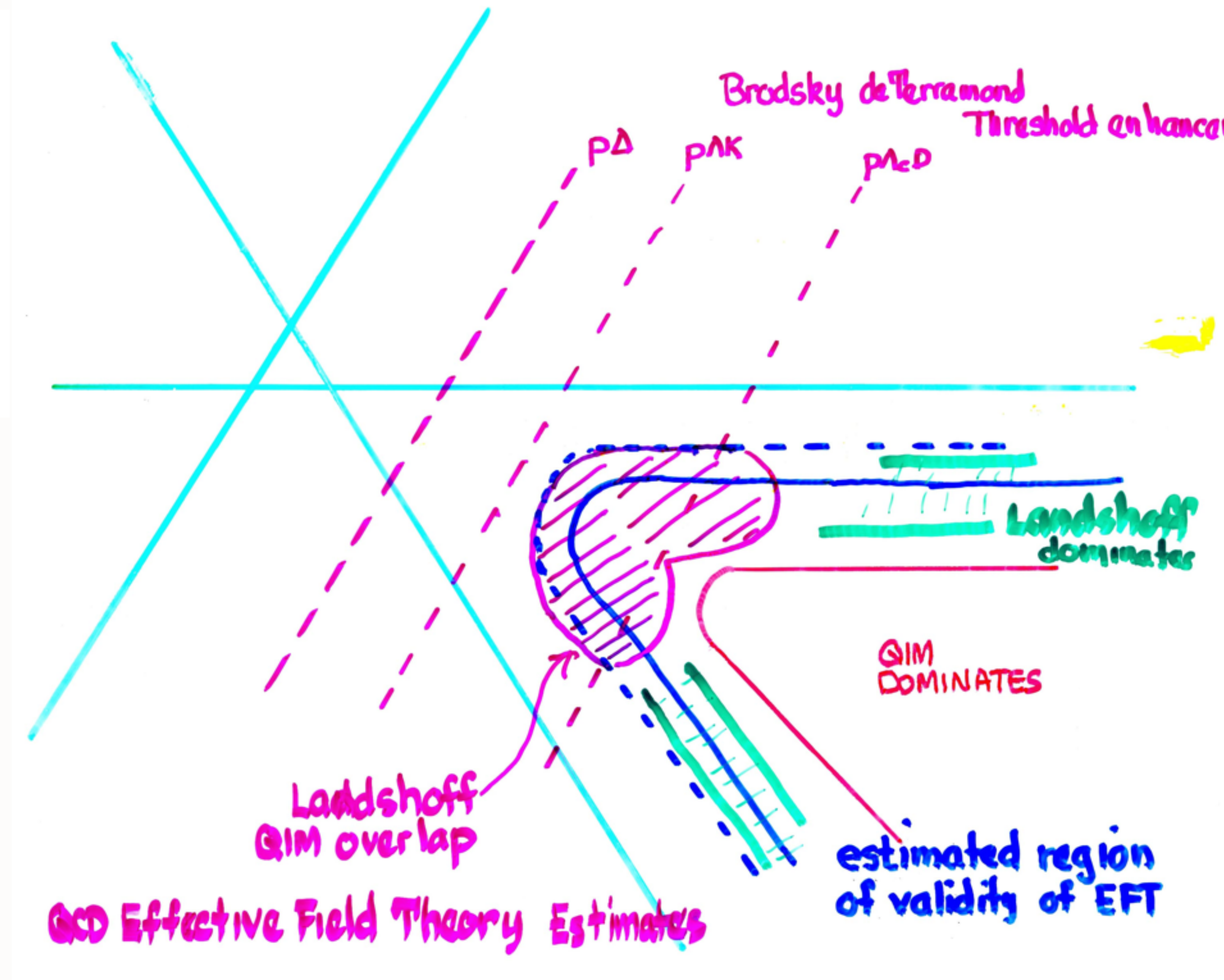


$$\frac{d\sigma}{dt}(pp \to pp) \simeq \frac{f(\theta_{cm})}{s^{10}}$$

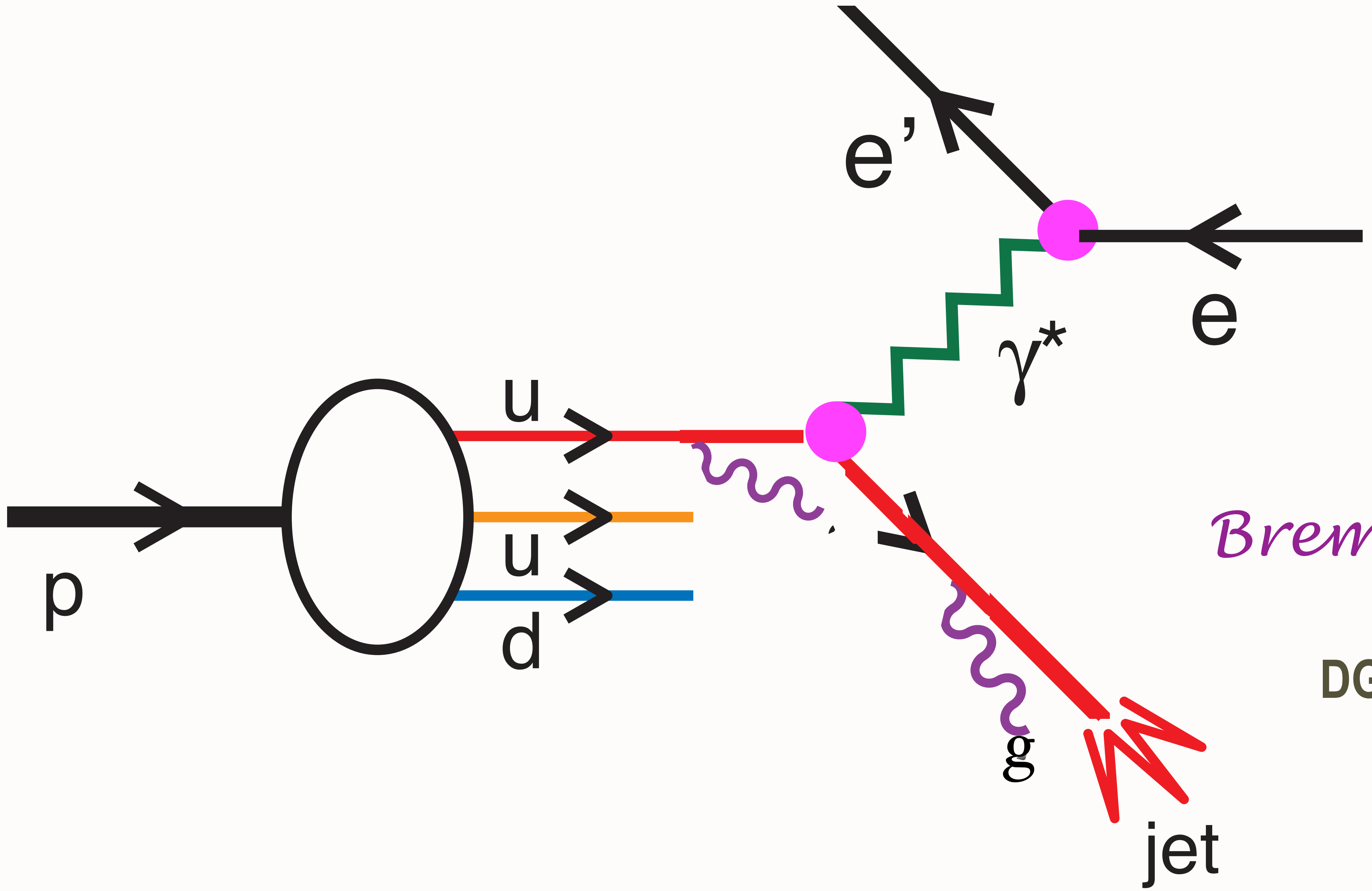
$$pp \to pp$$



Sivers



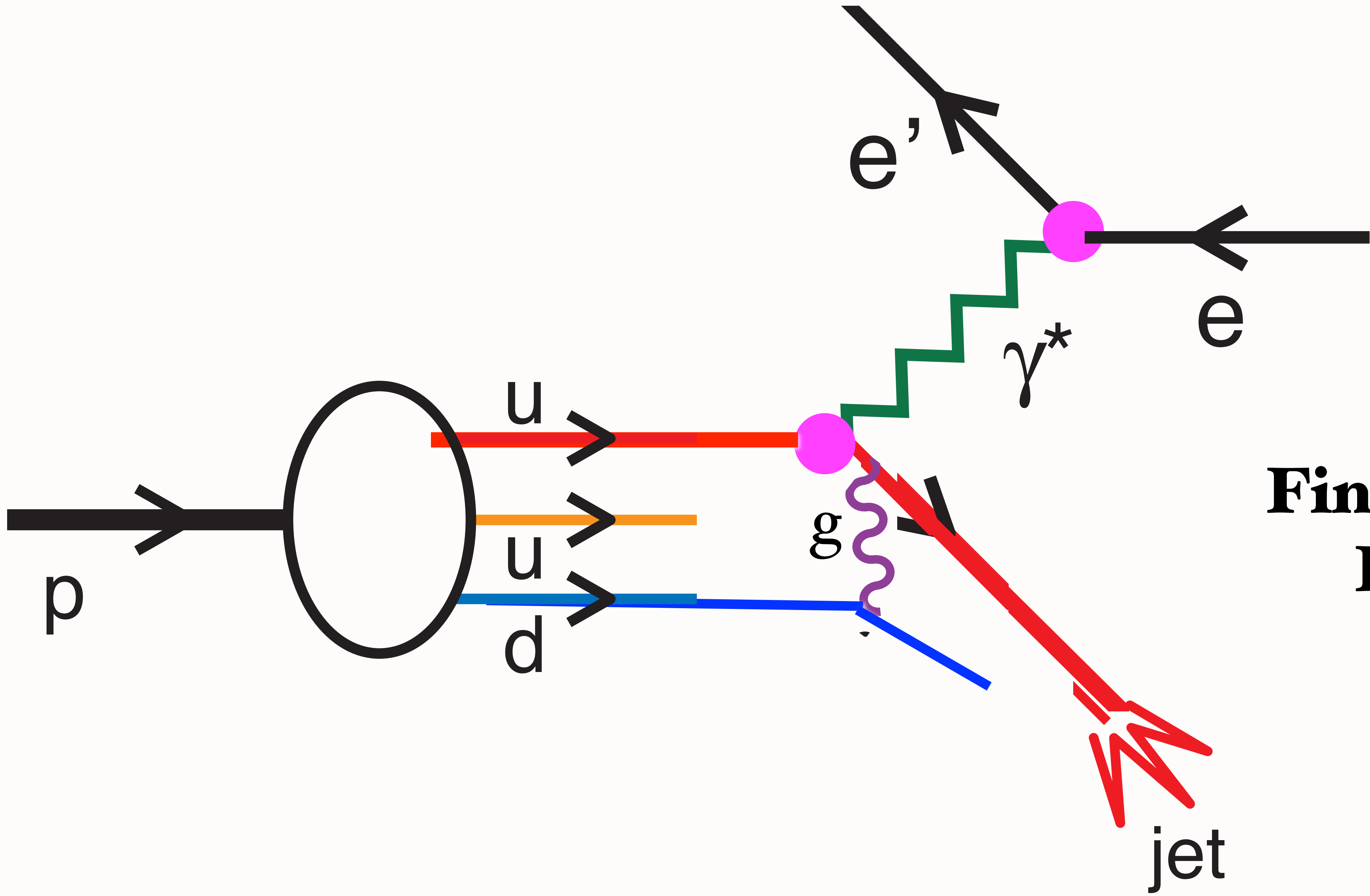
Deep Inelastic Electron-Proton Scattering



Gluonic Bremmstrahlung

DGLAP Evolution

Deep Inelastic Electron-Proton Scattering



Final-State QCD Interaction

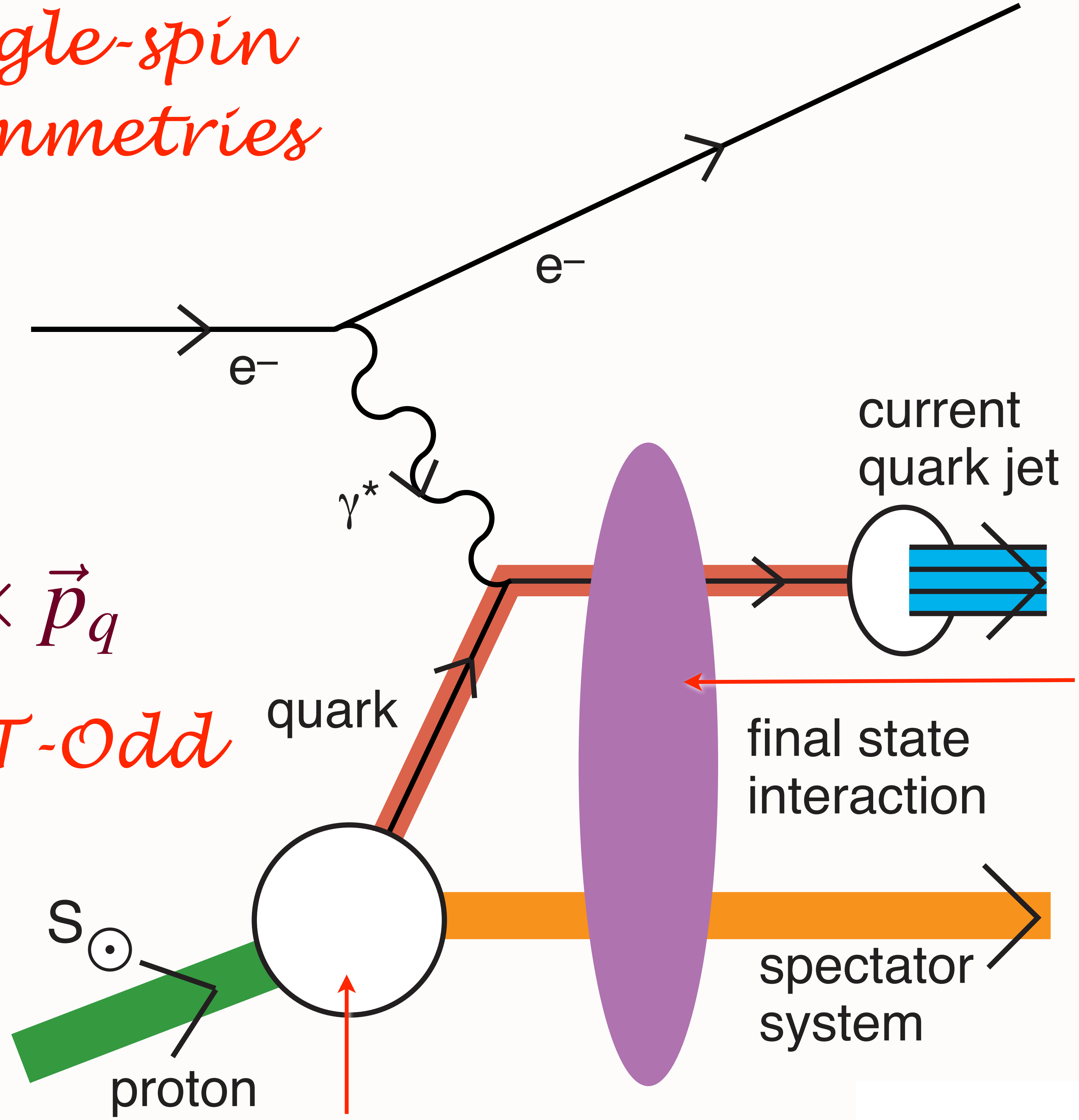
Conventional wisdom:
Final-state interactions of struck quark can be neglected

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...



QCD S- and P- Coulomb Phases --Wilson Line

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

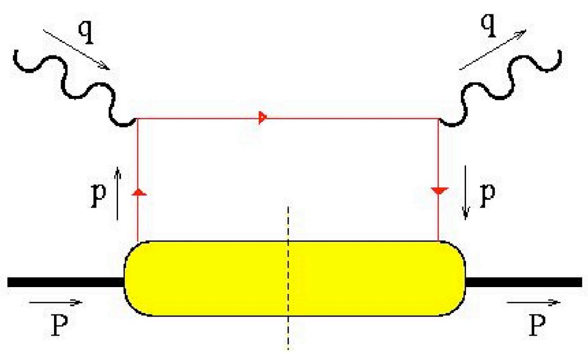
Pseudo- T-Odd

"Lensing Effect"

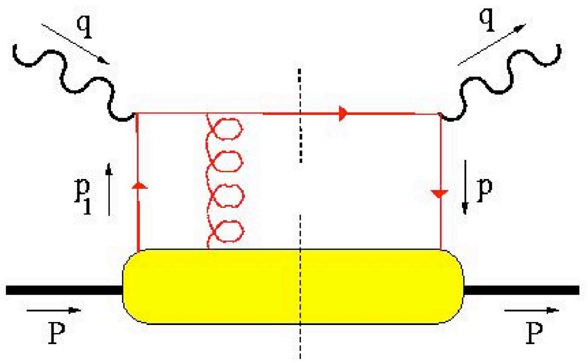
Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!



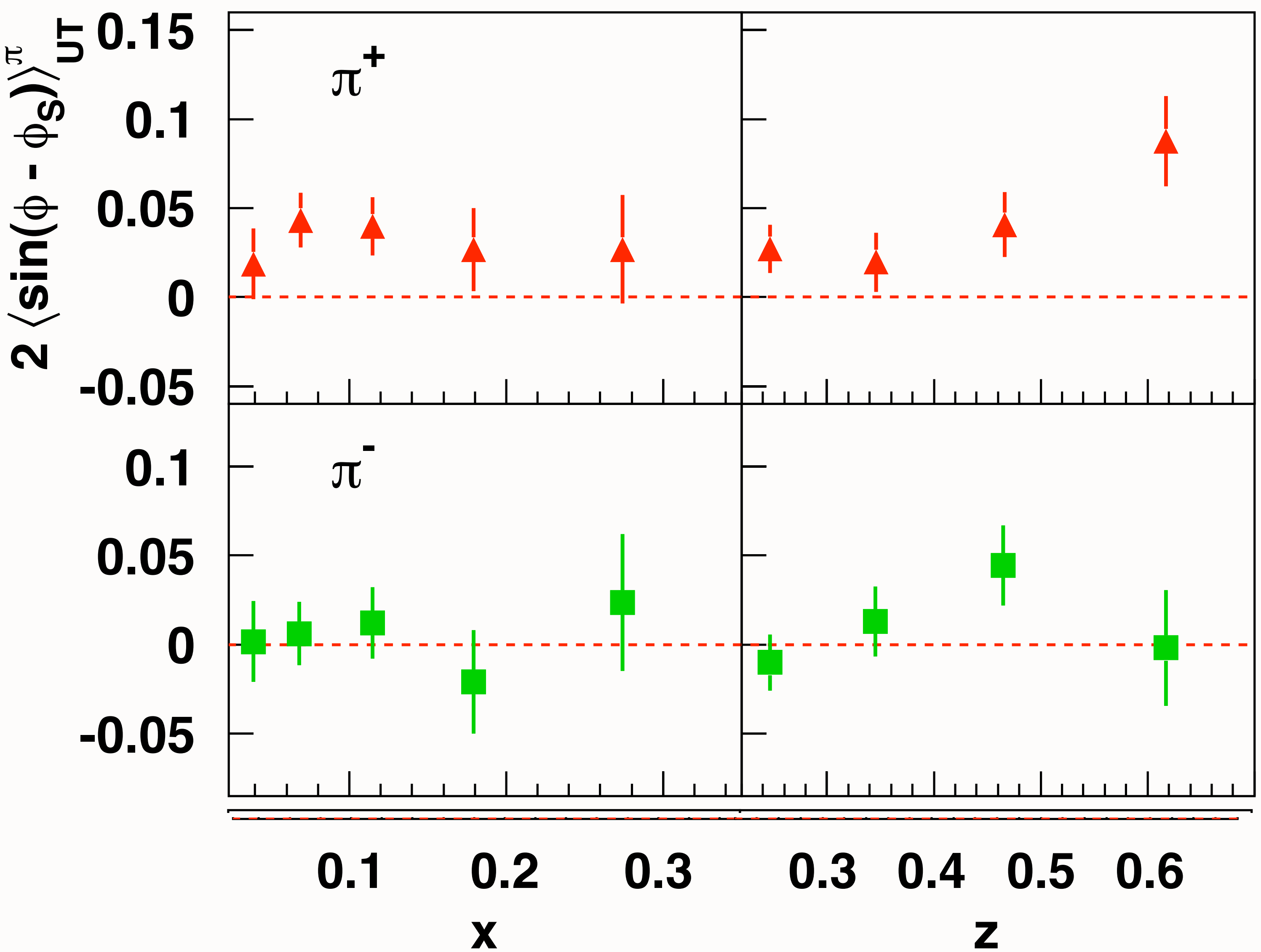
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

Sivers asymmetry from HERMES



- First evidence for non-zero Sivers function!
- ⇒ presence of non-zero **quark orbital angular momentum**!
- Positive for **π^+**... Consistent with zero for **π^-**...

Gamberg: Hermes data compatible with BHS model

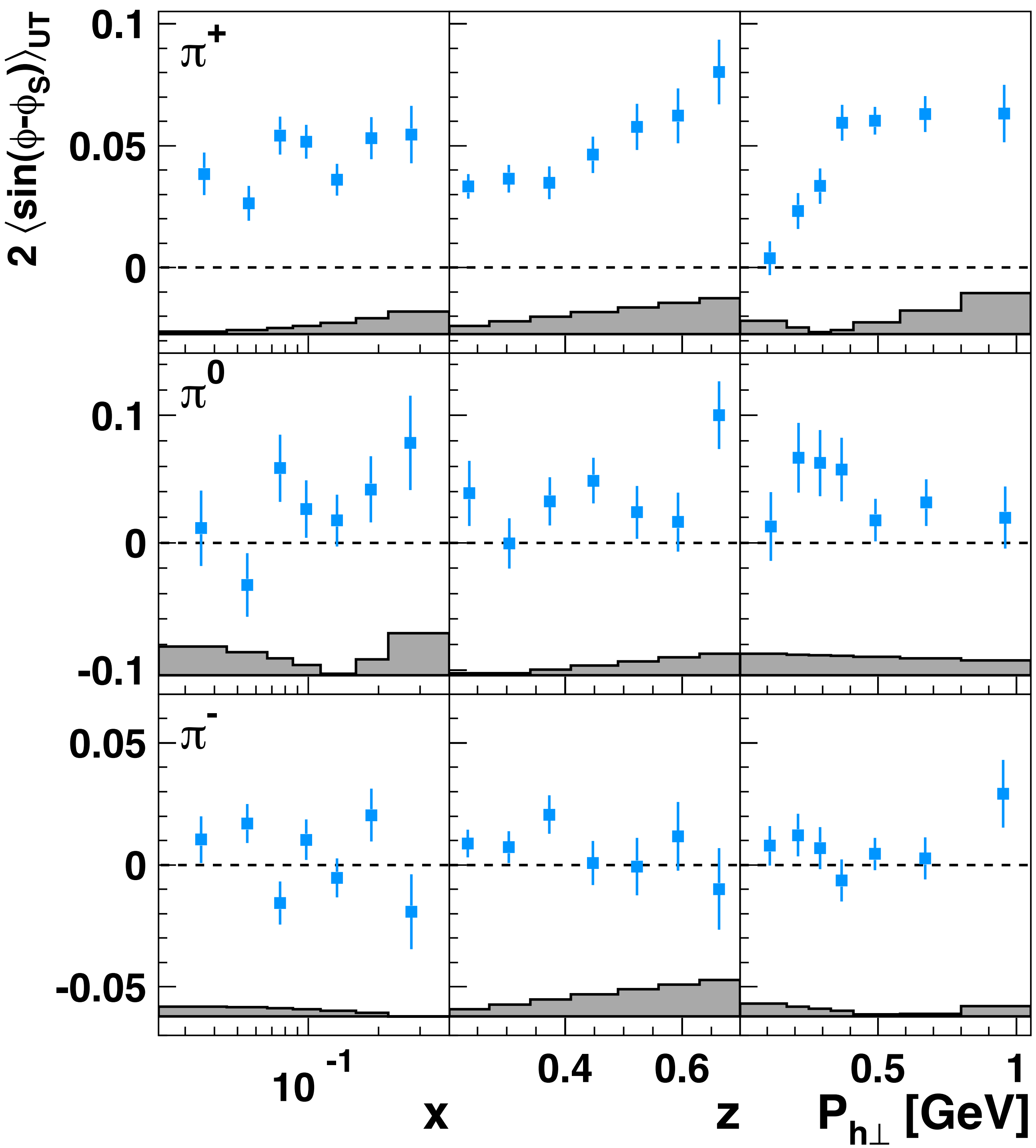
Schmidt, Lu:
Asymmetry ratios should follow quark contributions to anomalous moment

Sivers amplitudes for pions

Phys. Rev. Lett. 103 (2009) 152002

Ami Rostomyan

$$2\langle\sin(\phi-\phi_s)\rangle_{UT} = -\frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x,p_T^2)\otimes_w D_1^q(z,k_T^2)}{\sum_q e_q^2 f_1^q(x,p_T^2)\otimes D_1^q(z,k_T^2)}$$



π^+

- significantly positive
- clear rise with z
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$
- dominated by scattering off u-quark:

$$\simeq -\frac{f_{1T}^{\perp,u}(x,p_T^2)\otimes_w D_1^{u\to\pi^+}(z,k_T^2)}{f_1^u(x,p_T^2)\otimes D_1^{u\to\pi^+}(z,k_T^2)}$$

- u-quark Sivers DF<0
- non-zero orbital angular momentum

π^0

- slightly positive

π^-

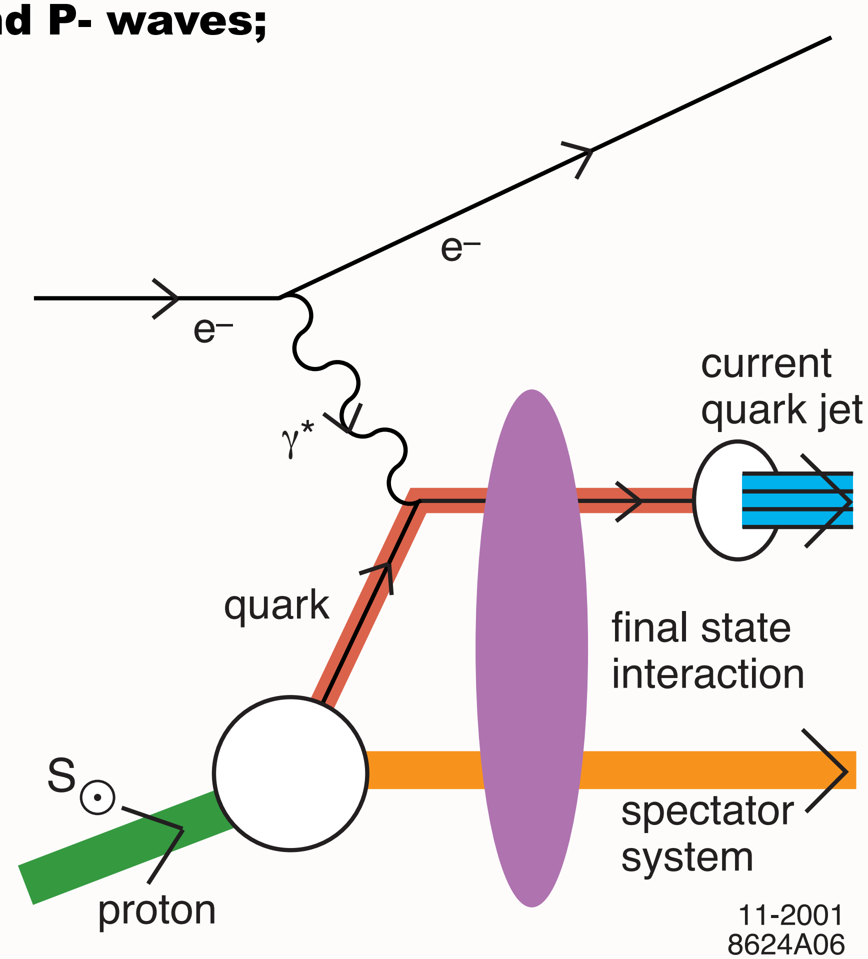
- consistent with 0
- u- and d-quark cancellation
- d-quark Sivers DF>0

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

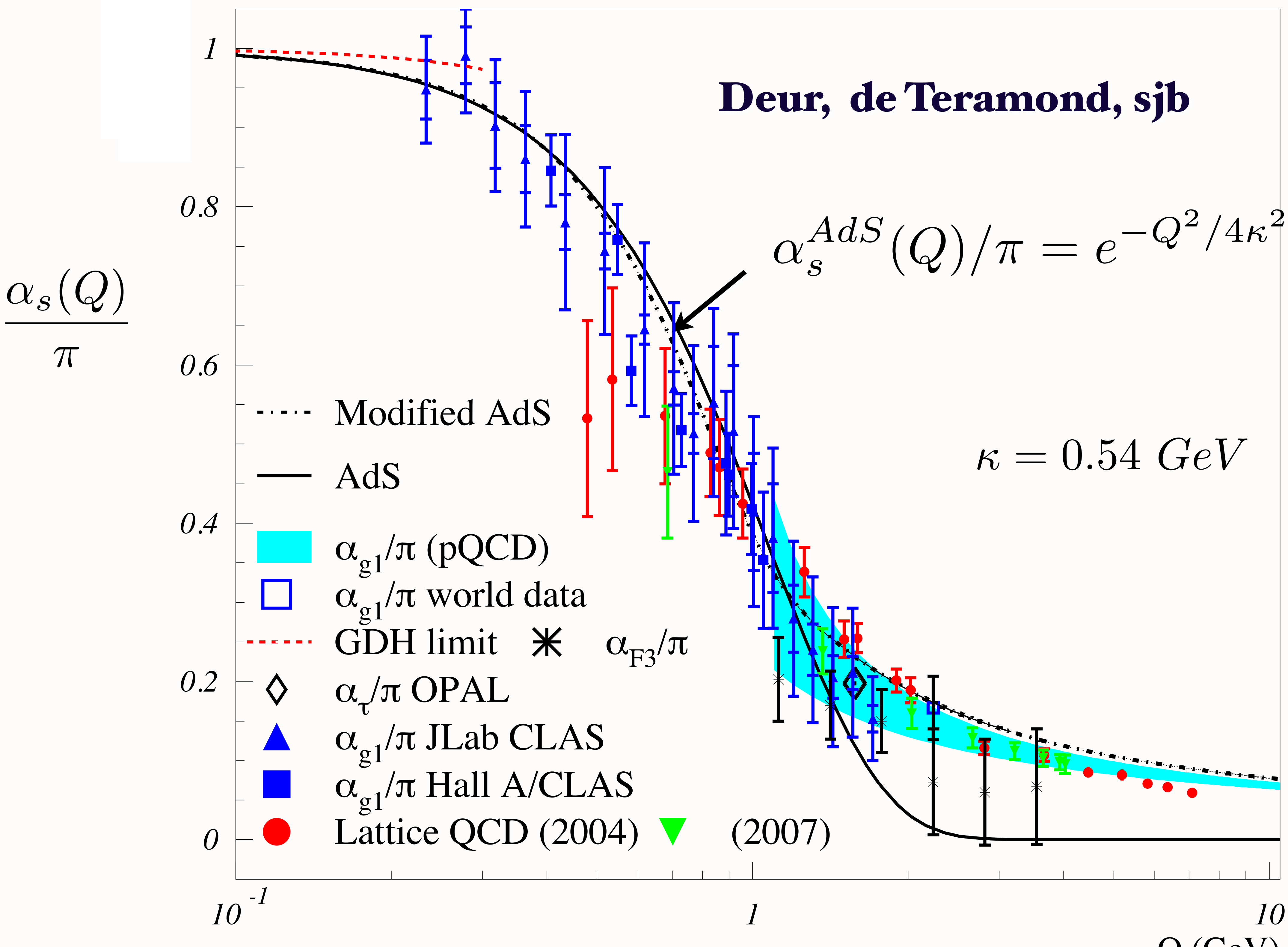
- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;**
- Burkardt: *"Lens Effect"*
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases!**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**



Pasquini, Xiao, Yuan, sjb
Mulders, Boer   Qiu, Sterman

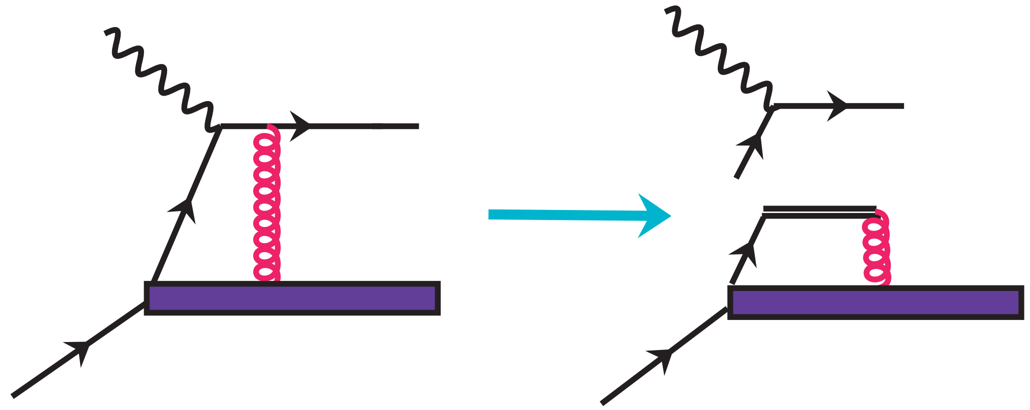
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



FSI phases in TSSAs unsuppressed

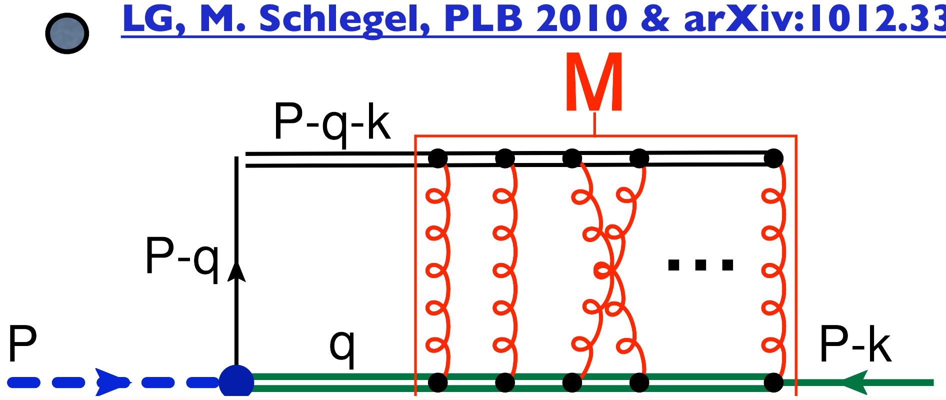
Gamberg



$$\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp}) \; f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

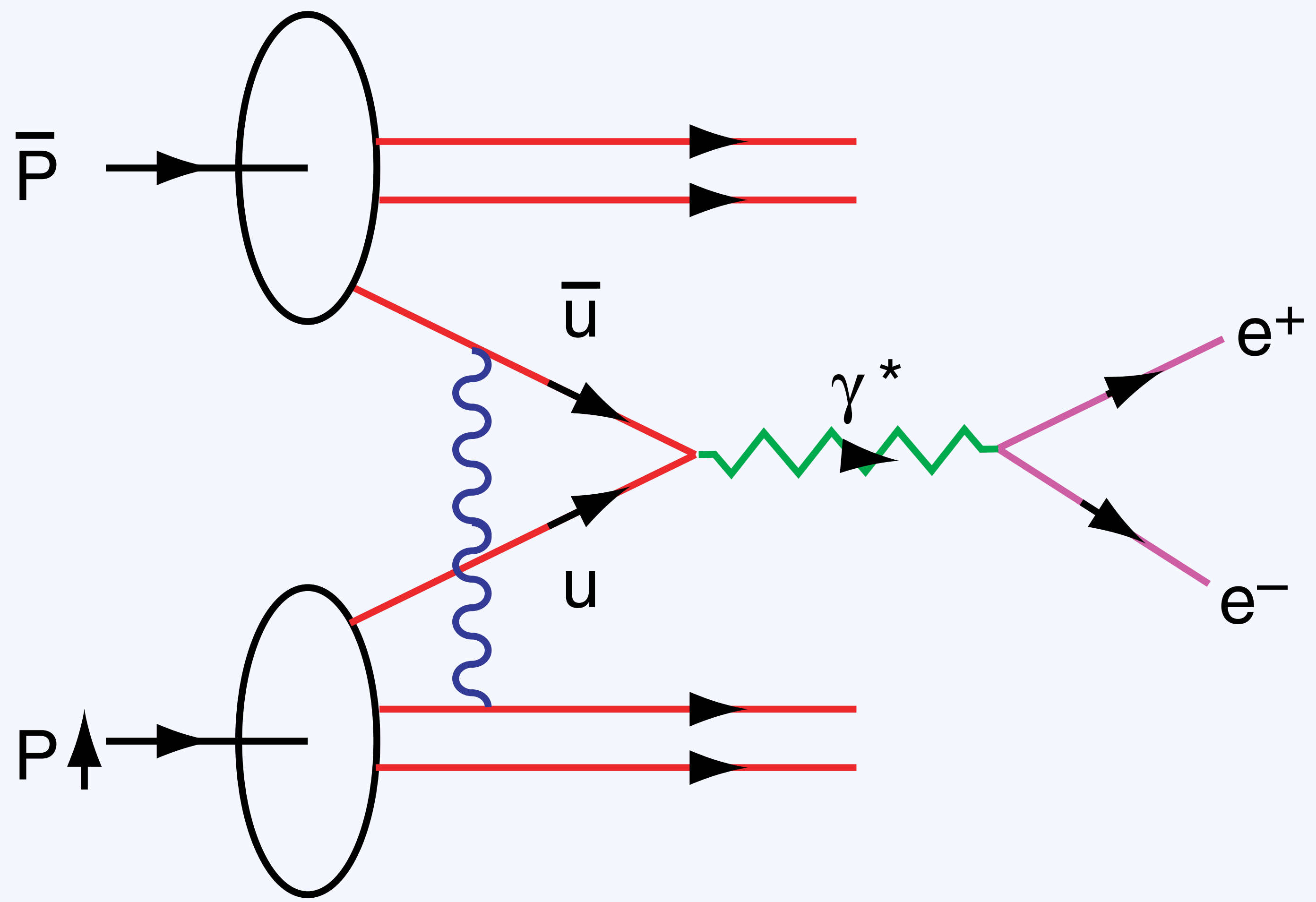
"Handbag" diagram invalid!

- **Unsuppressed reaction mech. Boer PRD 1999 context of DY @ RHIC**
- ***Brodsky Hwang Schmidt PLB 2002- SIDIS w/ transverse polarized target***
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***
- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- **LG, Goldstein, Oganessyan, Schlegel 2002, 2003 2008** Boer-Mulders Fnct, and Sivers -spectator model
- **Burkardt** Sivers chromdynamic lensing NPA 2004
- **Bacchetta, Schaefer, Yang, PLB 2004, Bacchetta Conti Radici ... 2008,2010,2011 PRD**
- **LG, M. Schlegel, PLB 2010 & arXiv:1012.3395** B-M, Sivers sum FSIs w/color Chromo Lensing M. Schegel



Many more model calcs. talk of A. Bacchetta

Predict Opposite Sign SSA in DY !



Collins

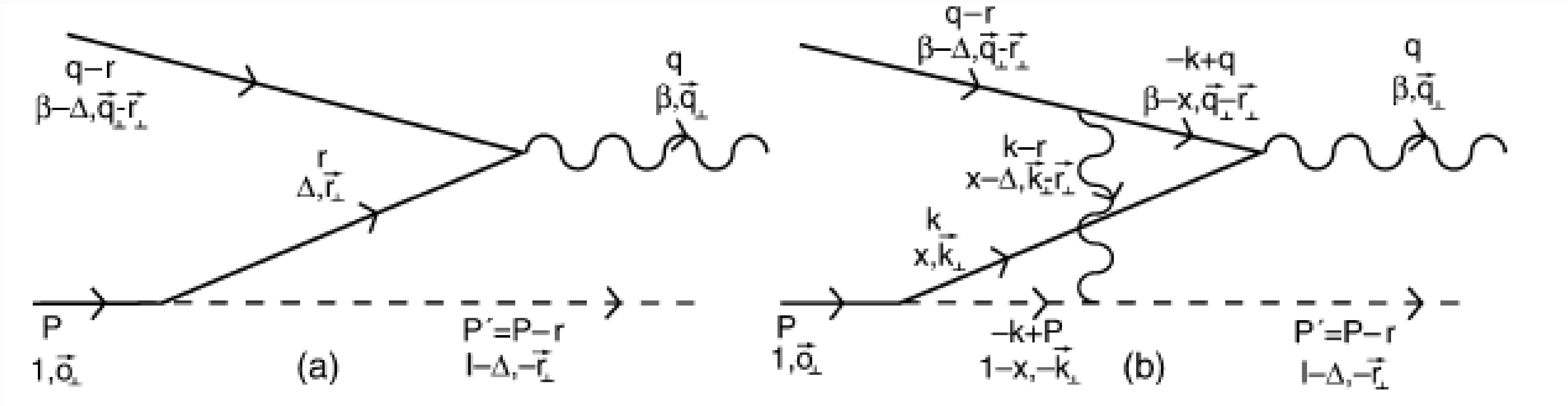
Hwang
Schmidt
sjb

Single Spin Asymmetry In the Drell Yan Process
$\vec{S}_p \cdot \vec{\bar{p}} \times \vec{q}_{\gamma^*}$
Quarks Interact in the Initial State
Interference of Coulomb Phases for *S* and *P* states
Produce Single Spin Asymmetry [Siver's Effect]Proportional to the Proton Anomalous Moment and α_s.
Opposite Sign to DIS! No Factorization

Initial-state interactions and single-spin asymmetries in Drell–Yan processes ☆

Stanley J. Brodsky [a], Dae Sung Hwang [a,b], Ivan Schmidt [c]

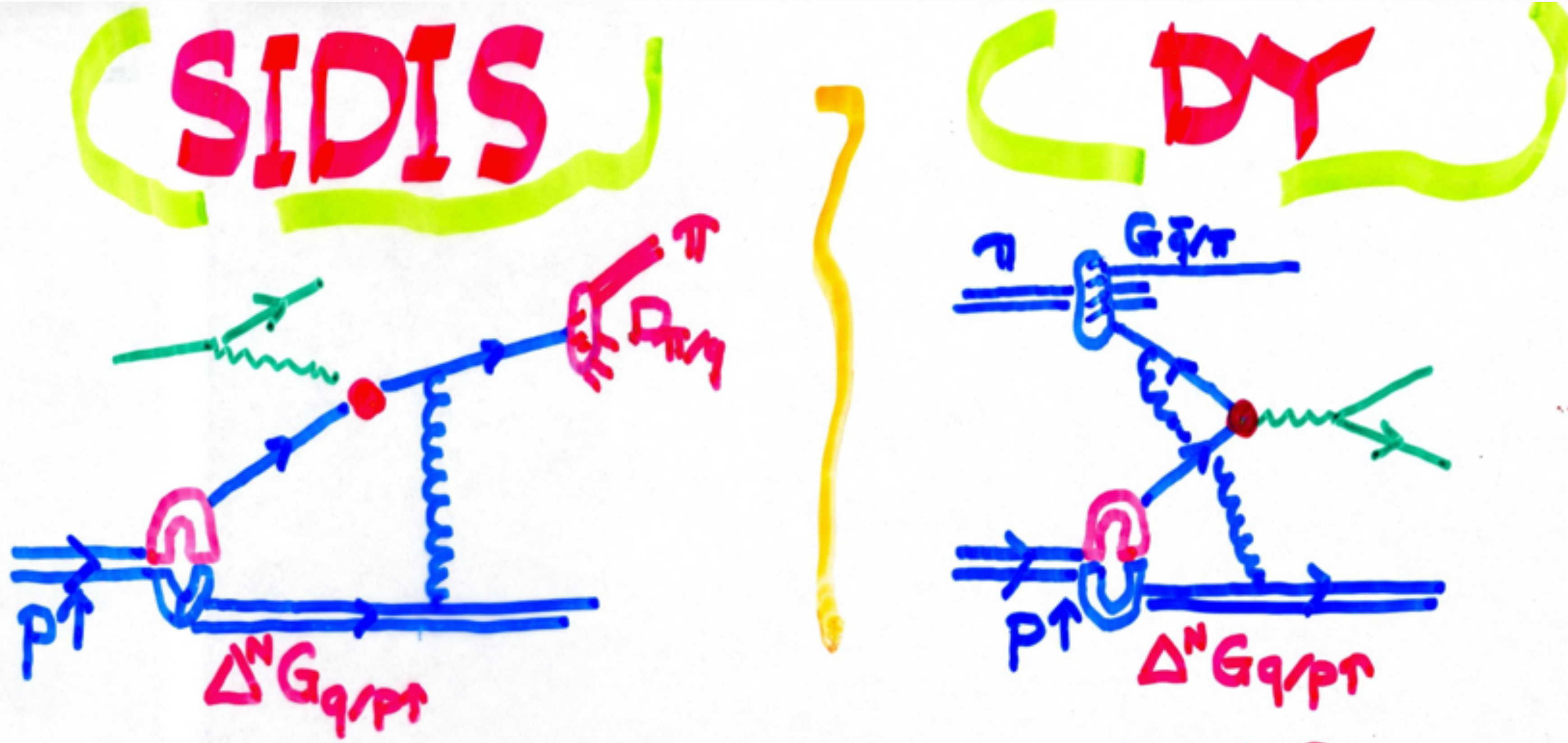
Nuclear Physics B 642 (2002) 344–356



$$\mathcal{P}_y = -\frac{e_1 e_2}{8\pi} \frac{2(\Delta M + m) r^1}{[(\Delta M + m)^2 + \vec{r}_\perp^{\,2}]} \left[\vec{r}_\perp^{\,2} + \Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta} \right) \right]$$
$$\times \frac{1}{\vec{r}_\perp^{\,2}} \ln \frac{\vec{r}_\perp^{\,2} + \Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})}{\Delta(1-\Delta)(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta})}.$$

Here $\Delta = \frac{q^2}{2P\cdot q} = \frac{q^2}{2M\nu}$ where ν is the energy of the lepton pair in the target rest frame.

Sivers



BHS approach

SPECTATOR MODELS

gauge-link formalism plus "time-reversal"

non-zero result requires $L \neq 0$ in "wave function"

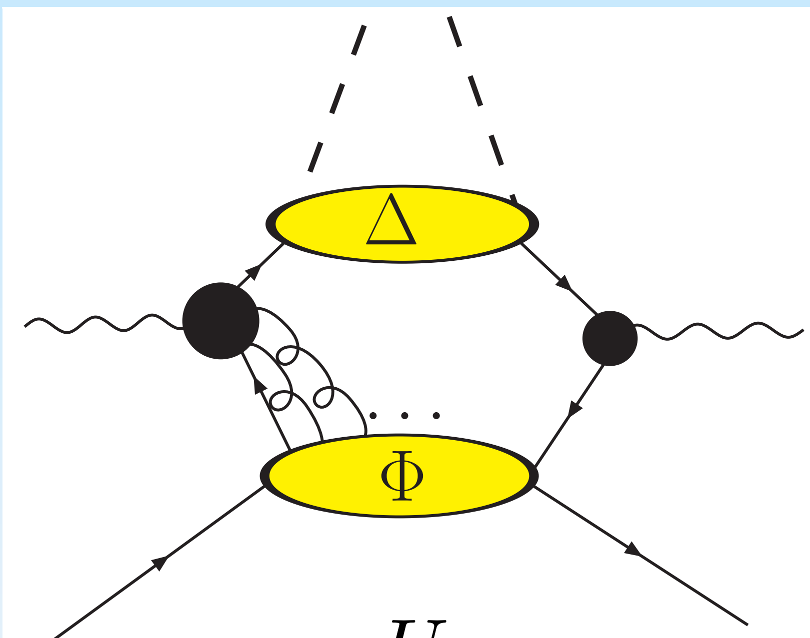
change in sign DY · SIDIS insensitive to details of bound system -- reduces to geometrical argument (ISI · FSI)

"Generalized Universality" Fund. Prediction of QCD Factorization

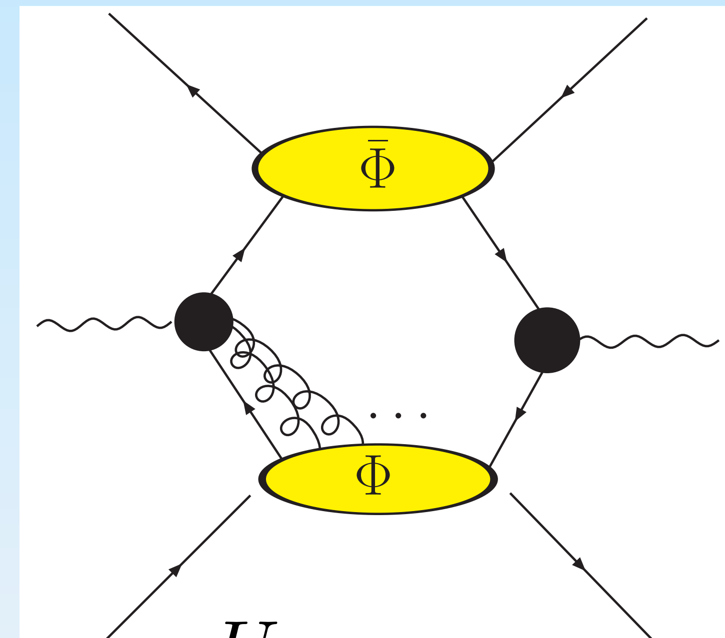
$$f_{1T\,sidis}^{\perp}(x, k_T) = -f_{1T\,DY}^{\perp}(x, k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

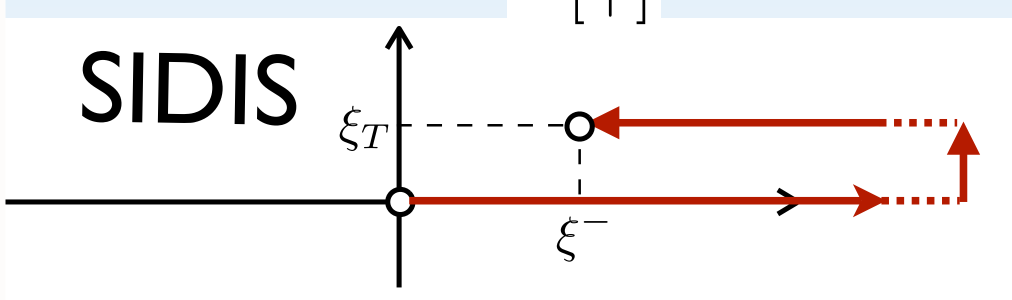
Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



$d\sigma = L_{\mu\nu} \mathcal{W}^{\mu\nu} \Rightarrow$

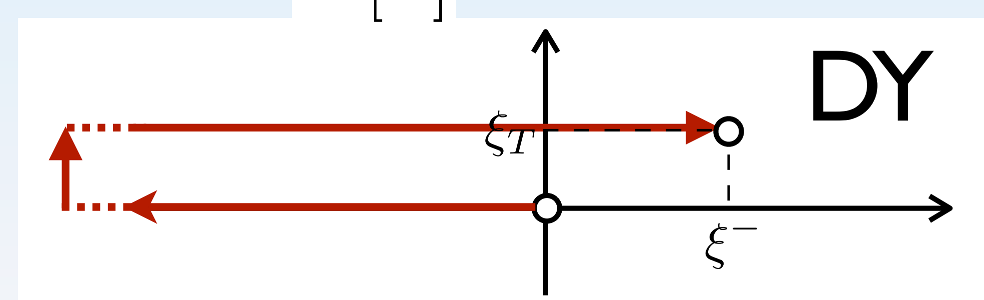


$U_{[+]}$



P&T ⟷

$U_{[-]}$



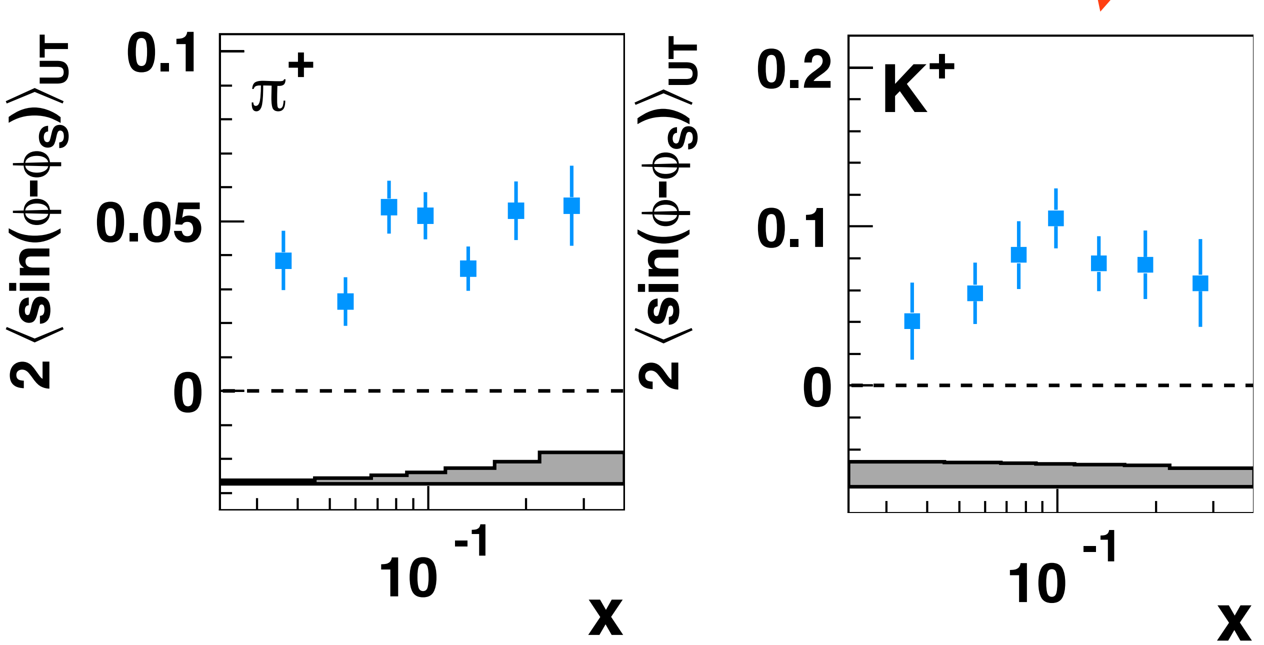
$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T)i\gamma^1\gamma^3$$

Gamberg

Sivers amplitudes for kaons

Ami Rostomyan

Factor of 2!



- **K+** amplitudes are larger in size than the π^+ amplitudes
  - non-trivial role of sea quarks

$$\pi^+ \equiv |u\bar{d}\rangle \qquad K^+ \equiv |u\bar{s}\rangle$$

K^+

- significantly positive
- clear rise with z
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$

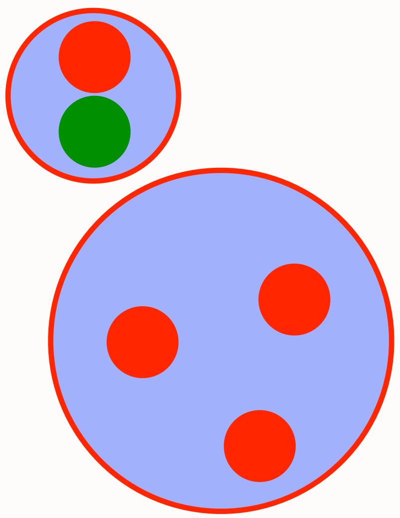
K^-

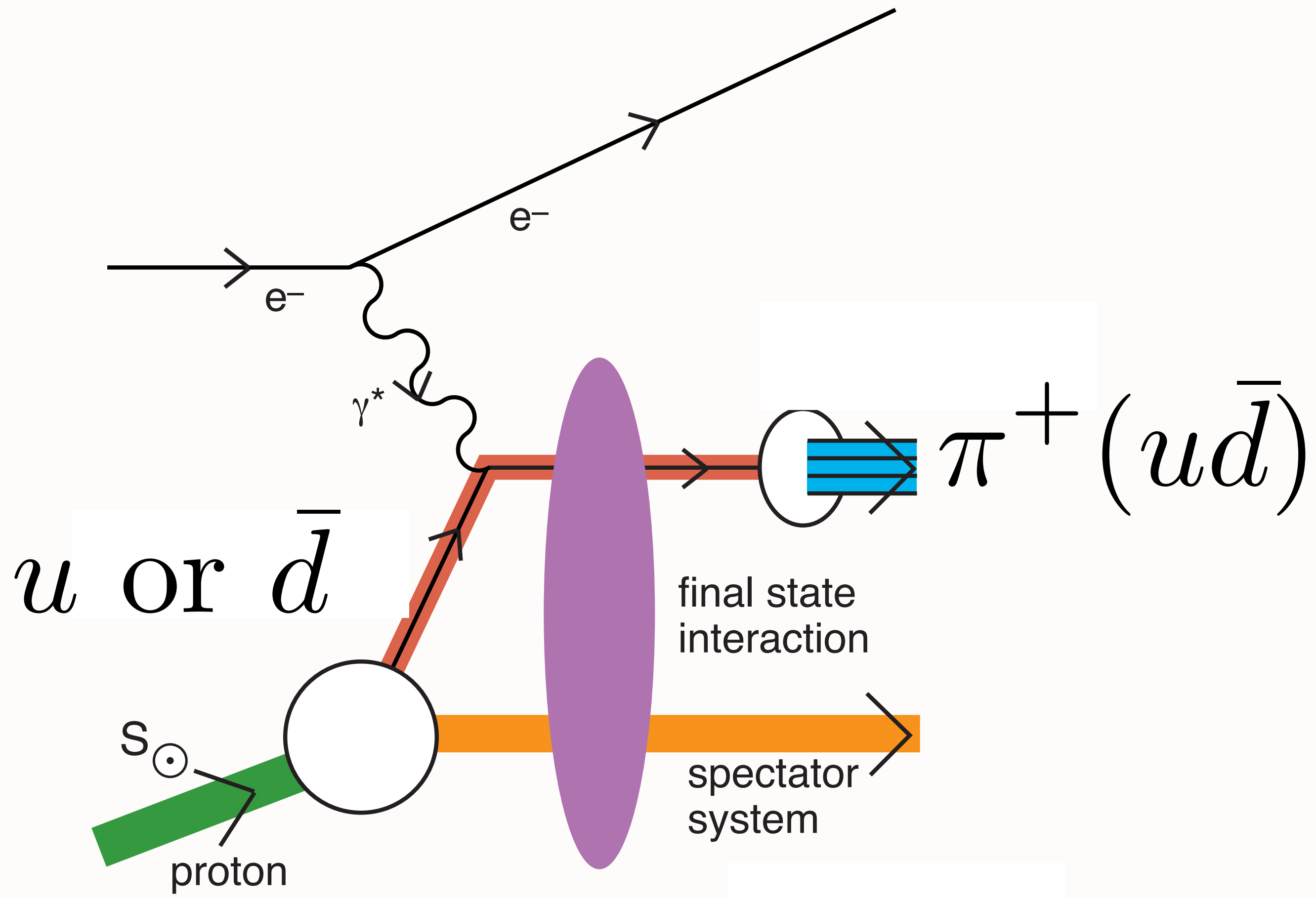
- slightly positive

- similar to π^+, **K+** dominated by scattering off u-quarks:

$$\propto -\frac{f_{1T}^{\perp,u}(x,p_T^2) \otimes_w D_1^{u\to\pi^+/K^+}(z,k_T^2)}{f_1^u(x,p_T^2) \otimes D_1^{u\to\pi^+/K^+}(z,k_T^2)}$$

Sea quarks carry orbital angular momentum





Sivers effect for $\pi^+(u\bar{d})$ reduced by $L_{\bar{d}}$ at low x

Sivers effect for $\pi^-(d\bar{u})$ reduced by $L_{\bar{u}}$ at low x

Sivers effect for $K^+(u\bar{s})$ increased by $L_{\bar{s}}$!

Estimate of $< L_q >$

Orbital functions	Song parameters	This paper
u quark	0.150	0.197 ± 0.02
d quark	0.025	-0.012 ± 0.01
s quark	0.025	0.015 ± 0.005
Sum of quarks	0.200	0.200 ± 0.02

Orbital functions	Song parameters	This paper
$\bar{u}$ antiquark	0.017	0.015 ± 0.002
$\bar{d}$ antiquark	0.058	0.053 ± 0.006
$\bar{s}$ antiquark	0.025	0.022 ± 0.002
Sum of antiquarks	0.100	0.090 ± 0.01

Chiral Mechanisms Leading to Orbital Quantum Structures in the Nucleon.
Dennis Sivers (Portland Phys. Inst. & Michigan U.) . Apr 2007. 28pp.
e-Print: **arXiv:0704.1791** [hep-ph]

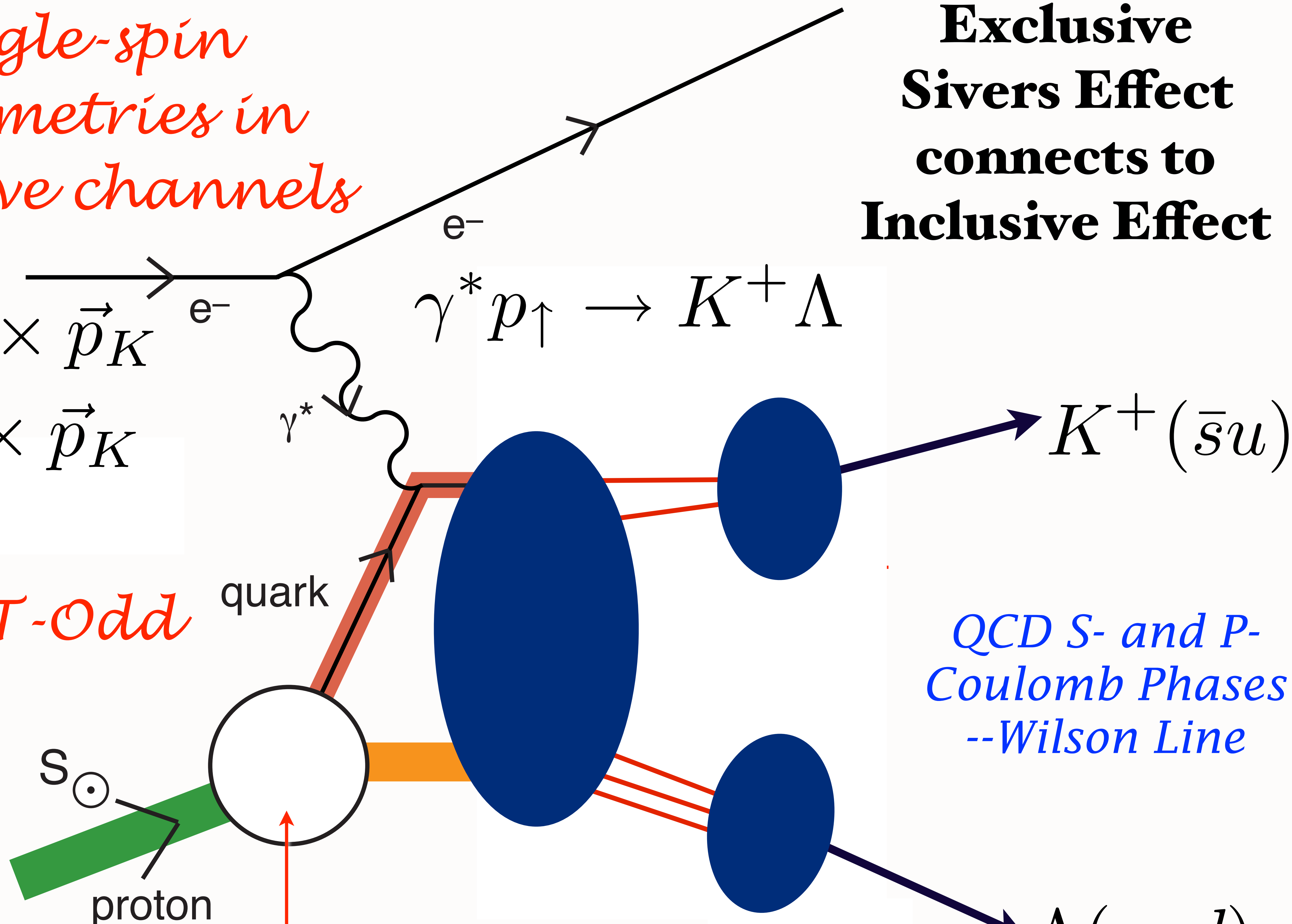
Single-spin asymmetries in exclusive channels

Exclusive Sivers Effect connects to Inclusive Effect

$i\vec{S}_\Lambda \cdot \vec{q} \times \vec{p}_K$

$i\vec{S}_p \cdot \vec{q} \times \vec{p}_K$

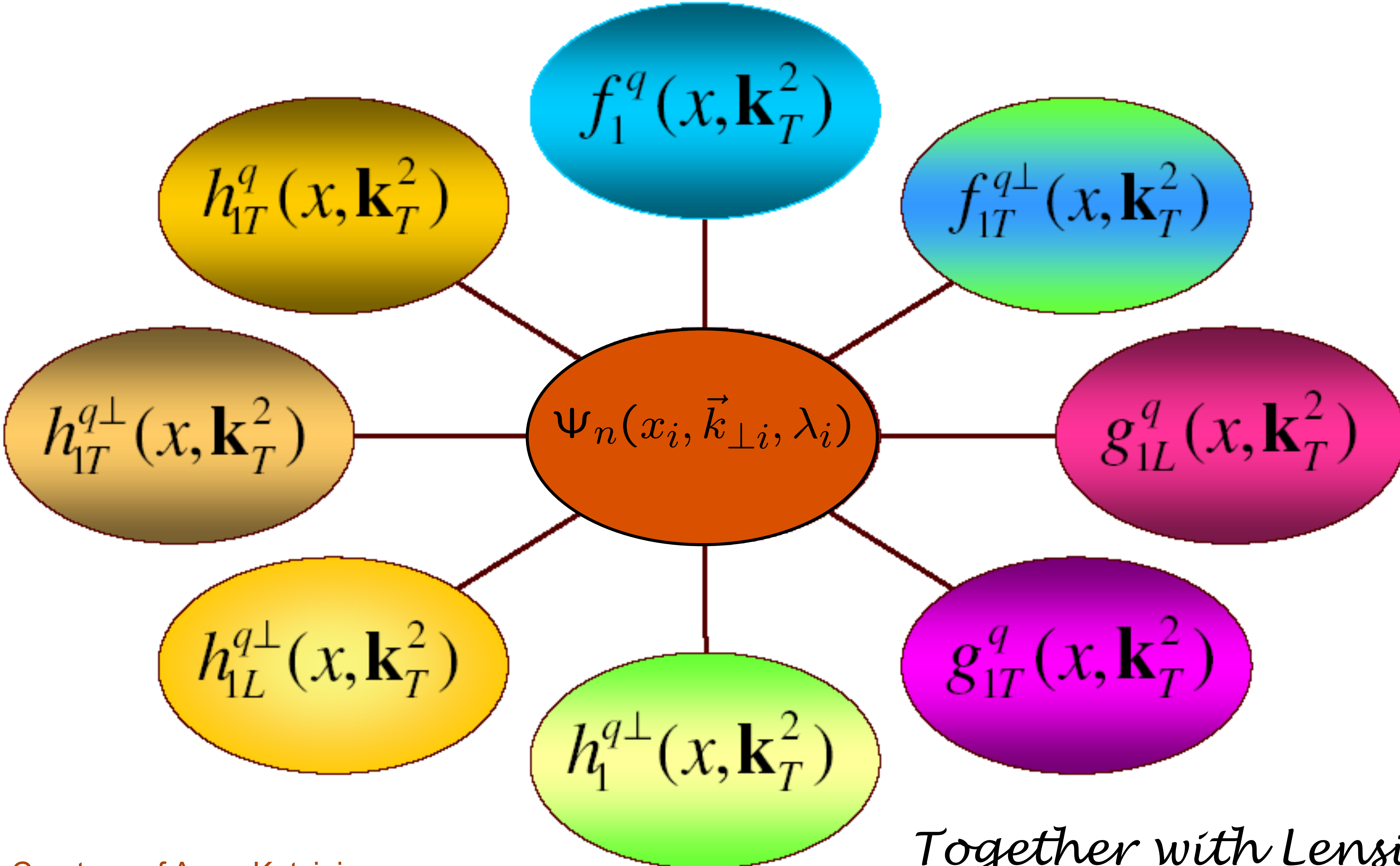
Pseudo- T-Odd



QCD S- and P- Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

8 leading-twist spin-$k_\perp$ dependent distribution functions

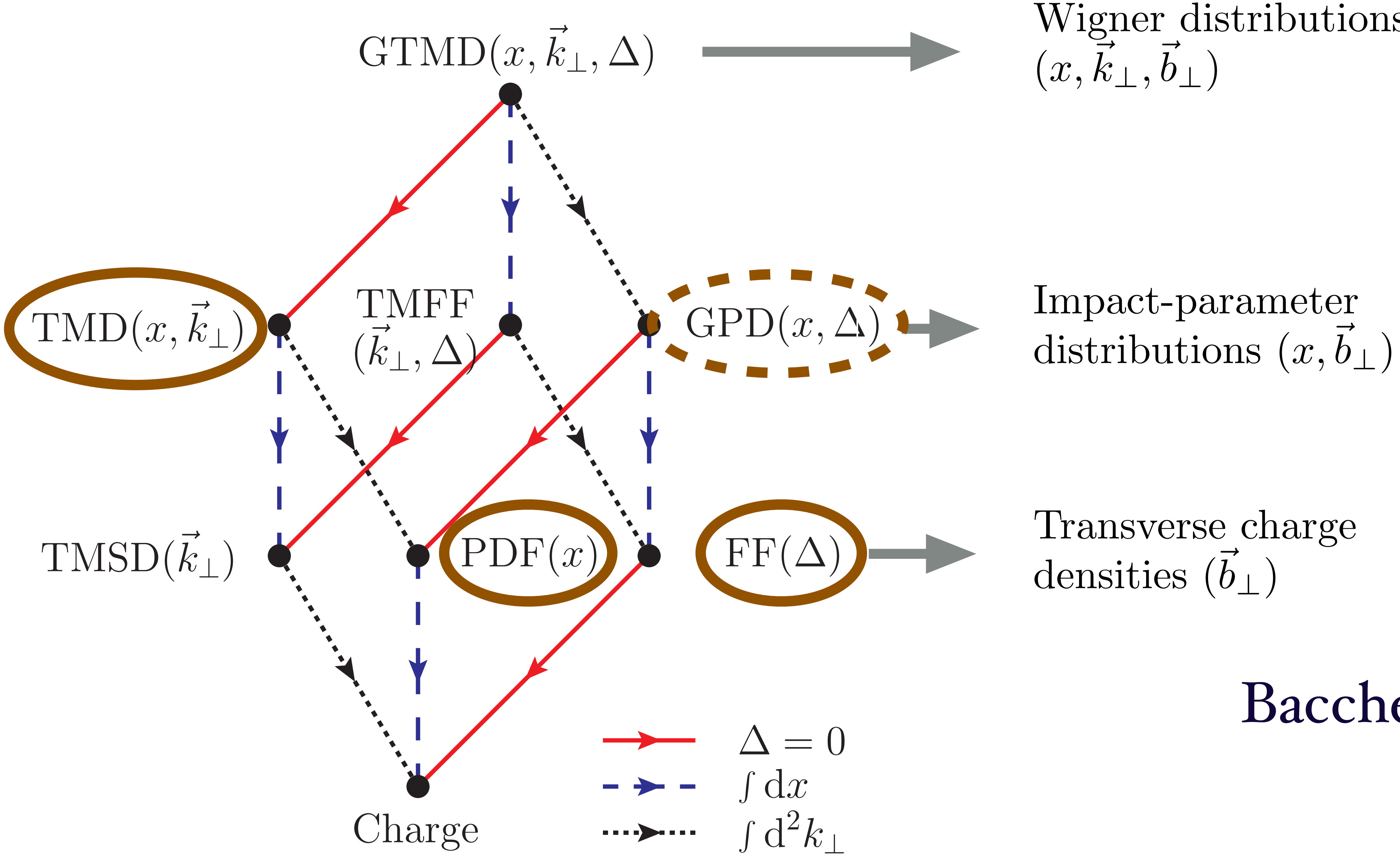


Courtesy of Aram Kotzinian

Together with Lensing

Light-Front Wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Hadron Eigenstates of QCD Hamiltonian!



Bacchetta

C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

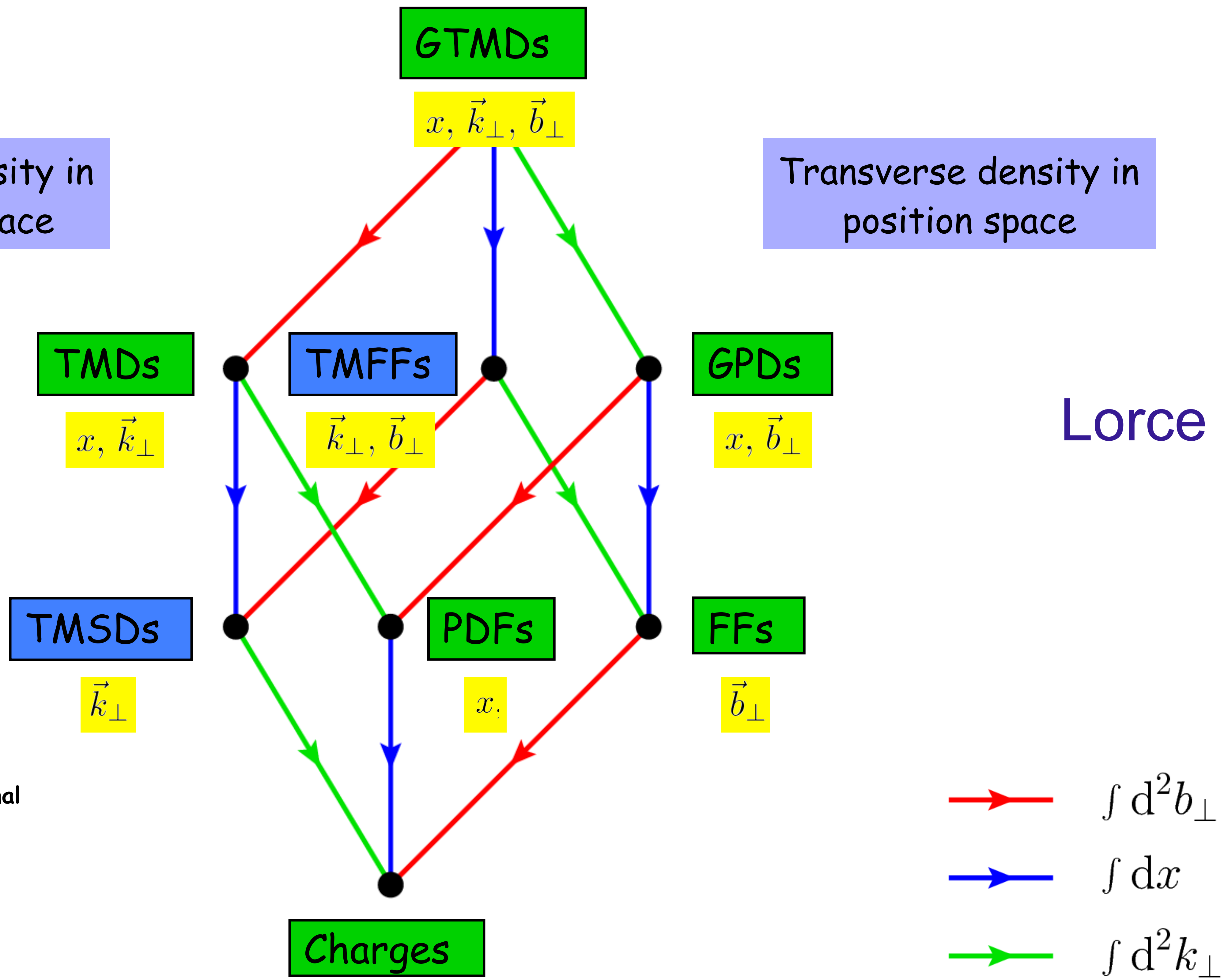
C

Complete picture @ $\xi = 0$

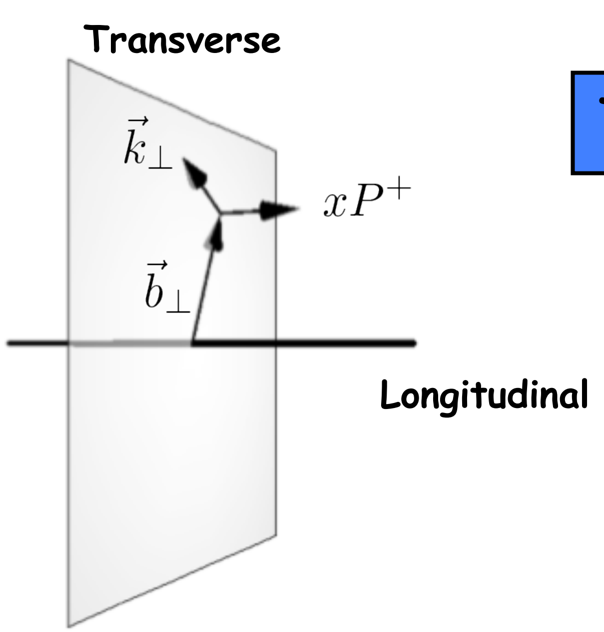
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

Transverse density in position space

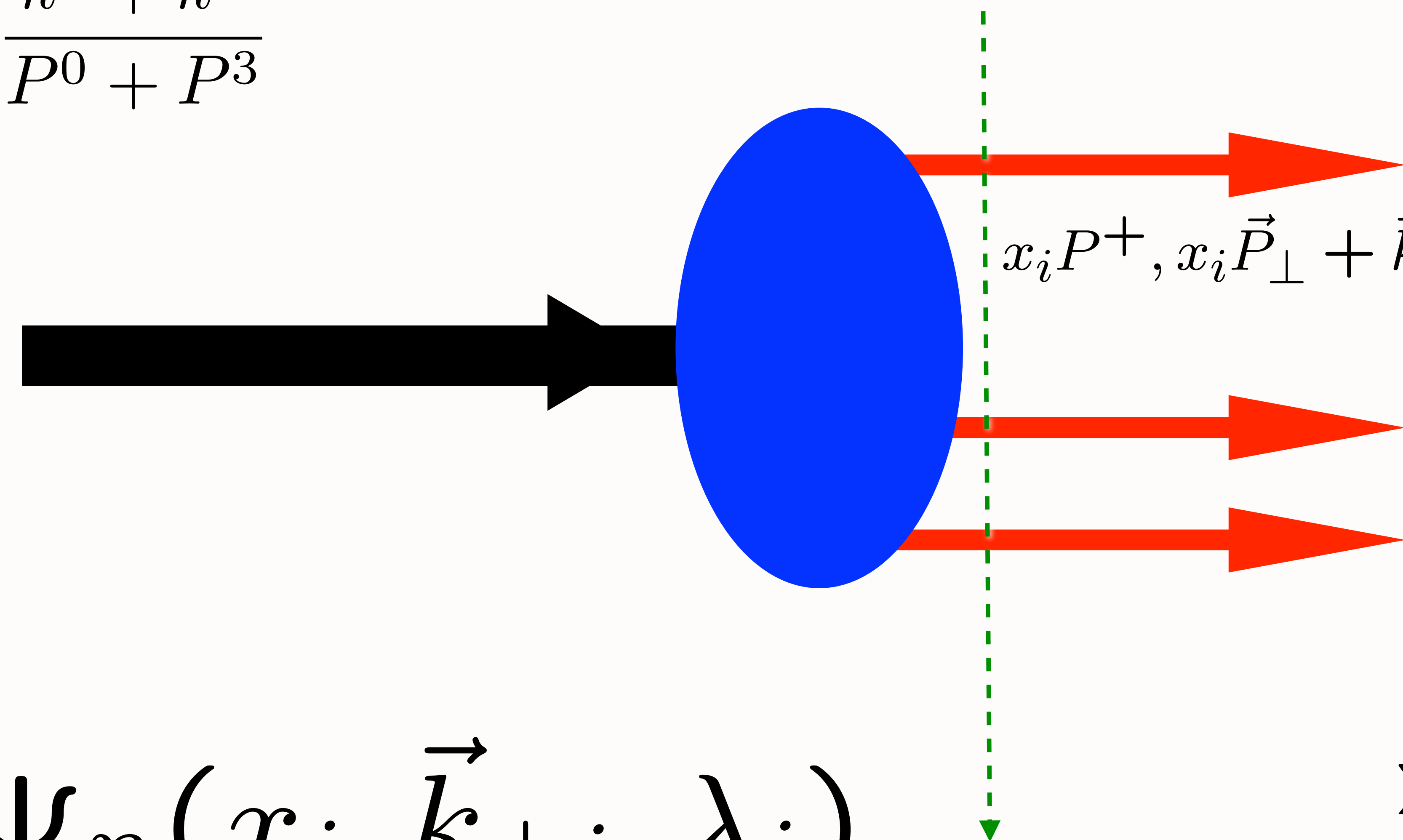


Lorce



Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

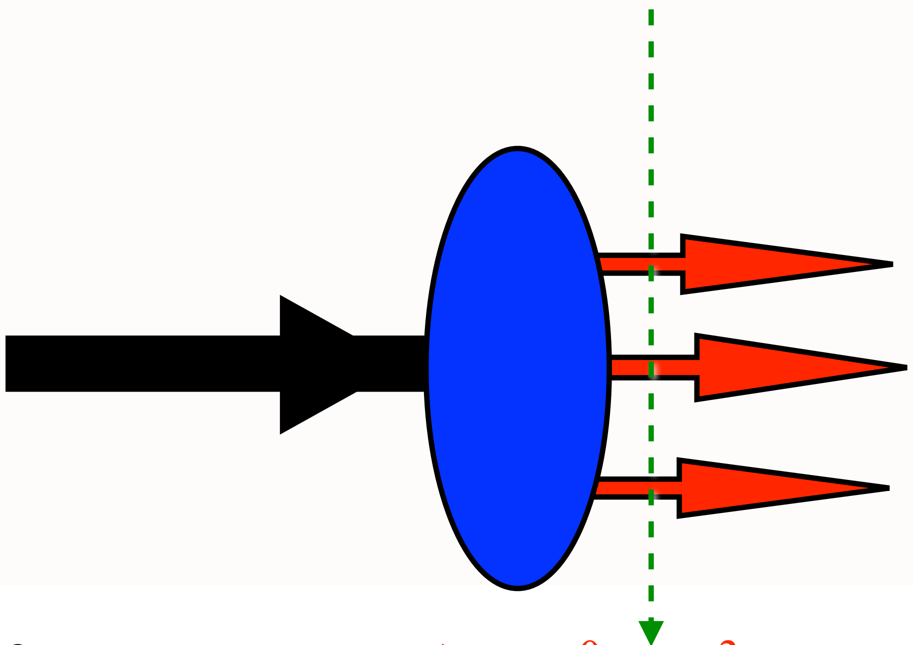


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ



Hoyer

Fixed $\tau = t + z/c$

A hadron state of momentum $P^+ = P^0 + P^3$ can at fixed $x^+ = x^0 + x^3$ be expanded in terms its quark and gluon Fock states as

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i) |n;\, x_i P^+, x_i \boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

The LF wave functions $\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)$ are independent of P^+, $P_\perp$.
Hadrons can be (trivially) boosted.

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

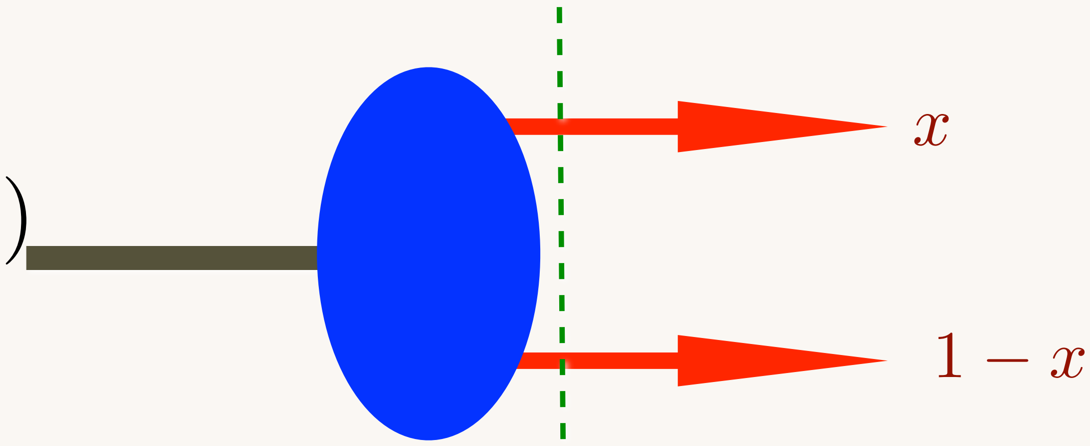
Direct connection to QCD Lagrangian!

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\; \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

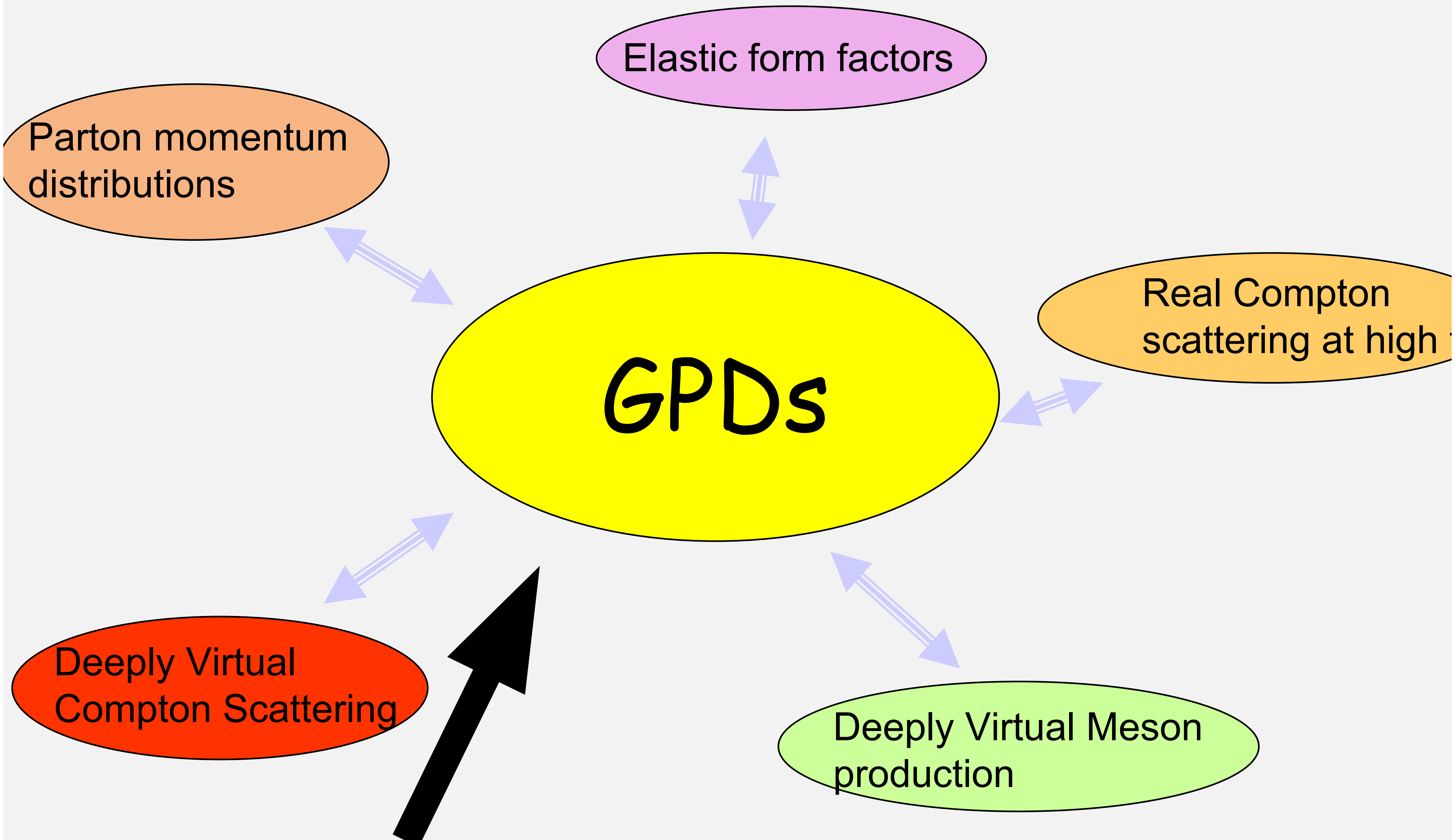
$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons
- Evolution Equations from PQCD, OPE
- Conformal Invariance
- Compute from valence light-front wavefunction in light-cone gauge

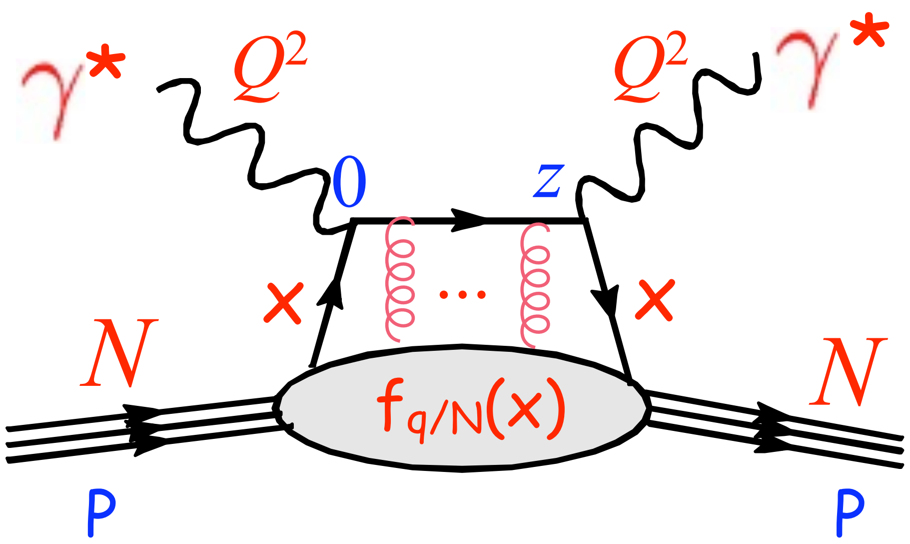
Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

A Unified Description of Hadron Structure



Light Front Wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

The probability interpretation of PDF's is expressed in terms of LF wave functions:



$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1-\sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x - x_k) |\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

Note: 1. Parton distributions factorize at leading twist ($Q^2 \to \infty$).

2. The above expression is approximate, since rescattering of the struck parton (the Wilson line) is neglected.

Diffractive DIS Shadowing

Hoyer

ELSEVIER

Nuclear Physics B441 (1995) 197–214

PHYSICS B

QCD constraints on the shape of polarized quark and gluon distributions ☆

Stanley J. Brodsky [a], Matthias Burkardt [b,1], Ivan Schmidt [c]

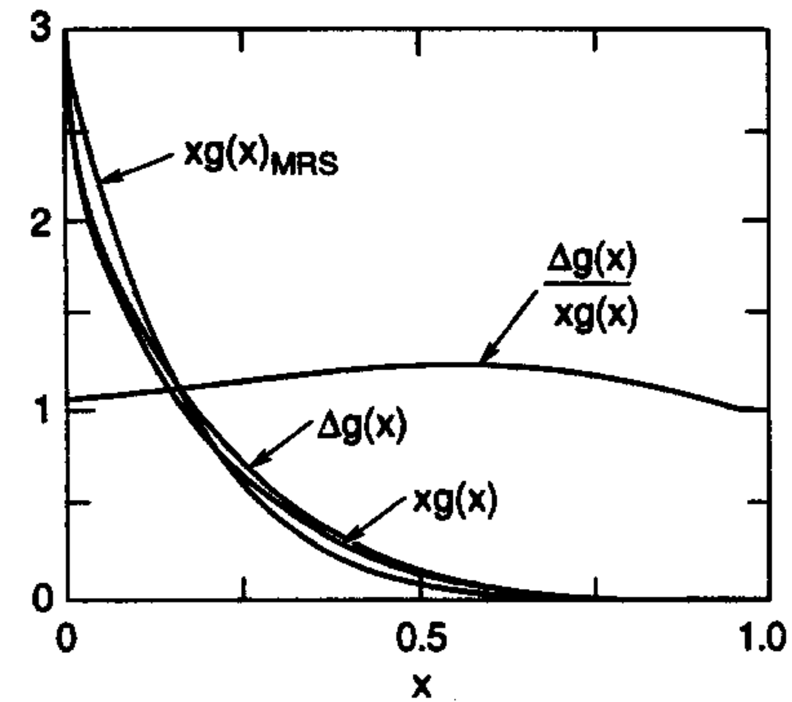
[a] Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA
[b] Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[c] Universidad Federico Santa María, Casilla 110-V, Valparaiso, Chile

The limiting power-law behavior at $x \to 1$ of the helicity-dependent distribution derived from the minimally connected graphs is

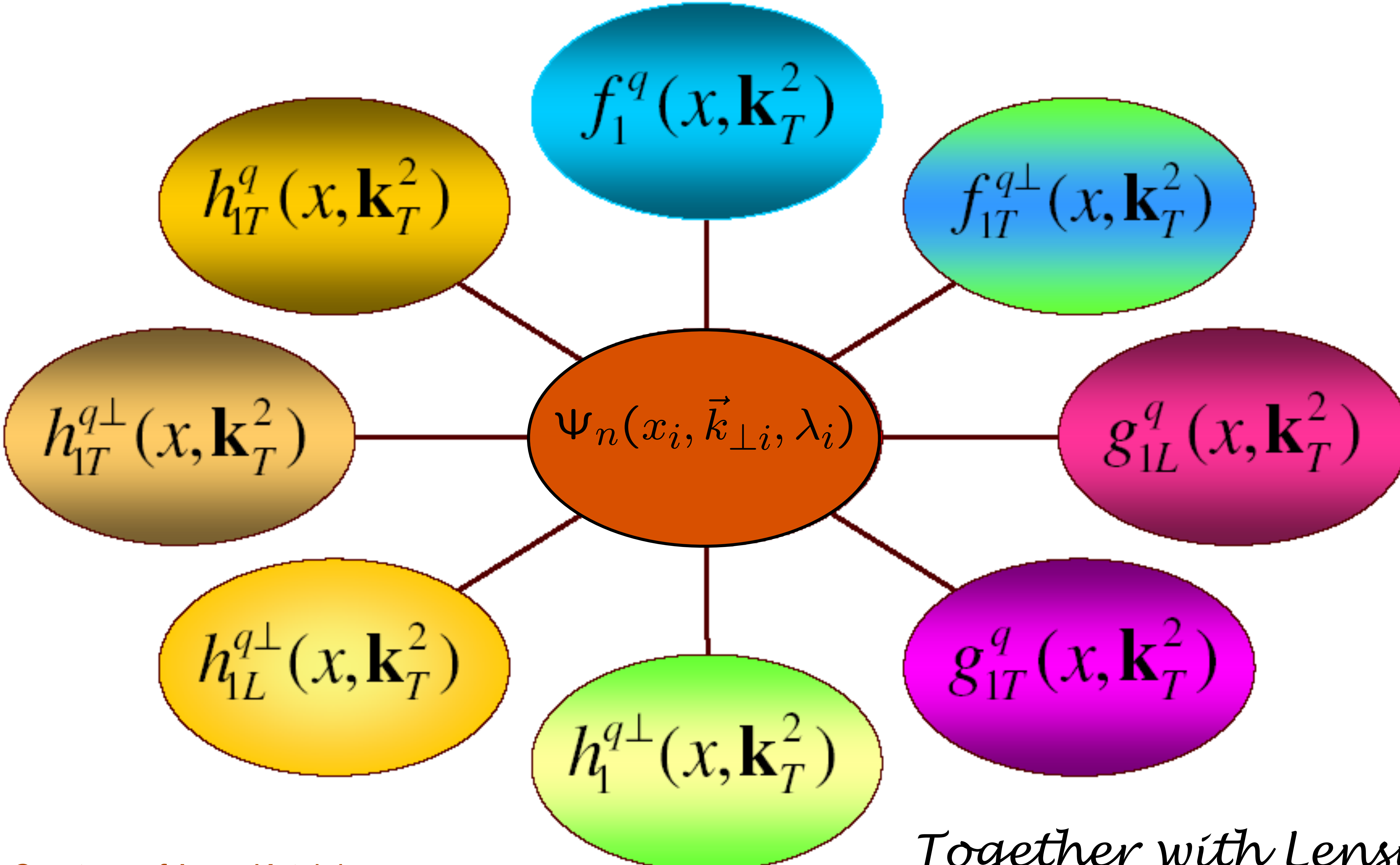
$$G_{q/H} \sim (1-x)^p,$$

where

$$p = 2n - 1 + 2\Delta S_z .$$



8 leading-twist spin-$k_\perp$ dependent distribution functions

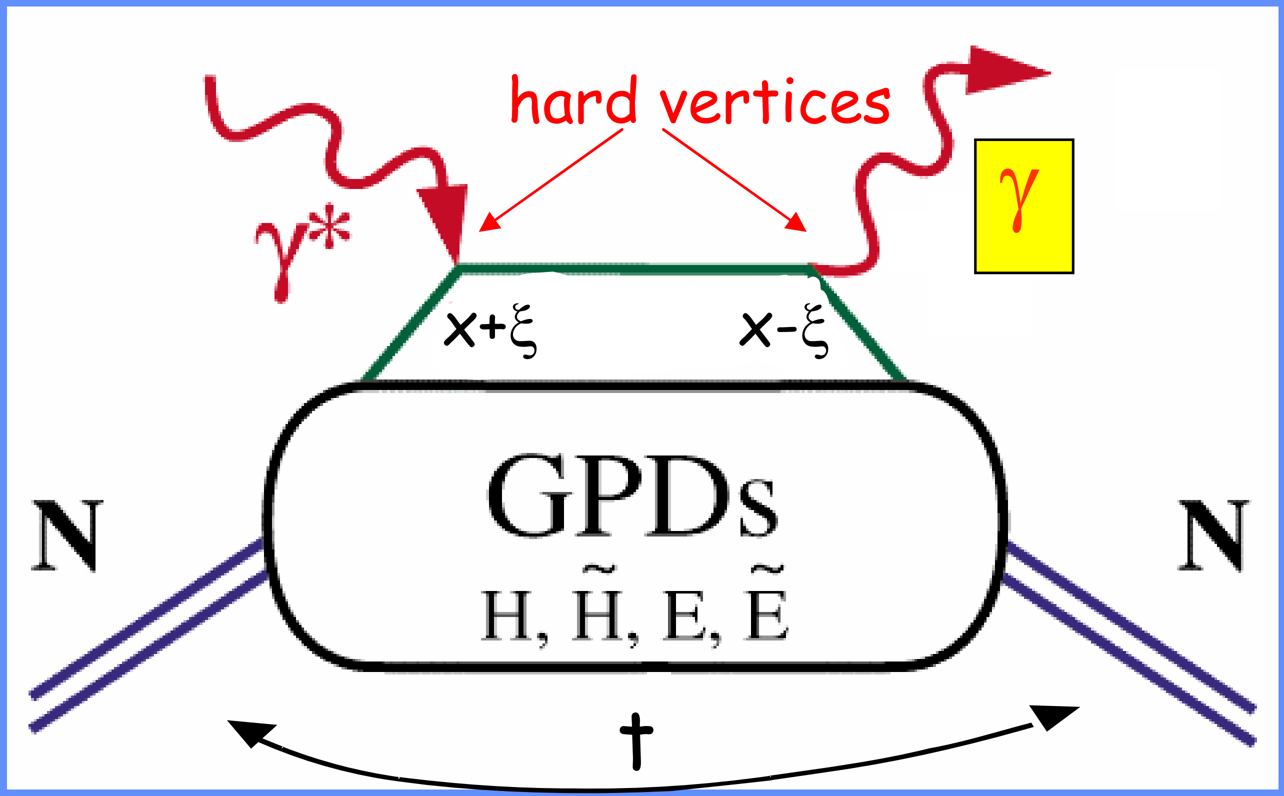


Courtesy of Aram Kotzinian

Together with Lensing

GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)



x – quark momentum fraction

ξ– longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

H(x,ξ,t), E(x,ξ,t), . . . "Generalized Parton Distributions"

- Generalized Parton Distributions in gauge/gravity duals

  [Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

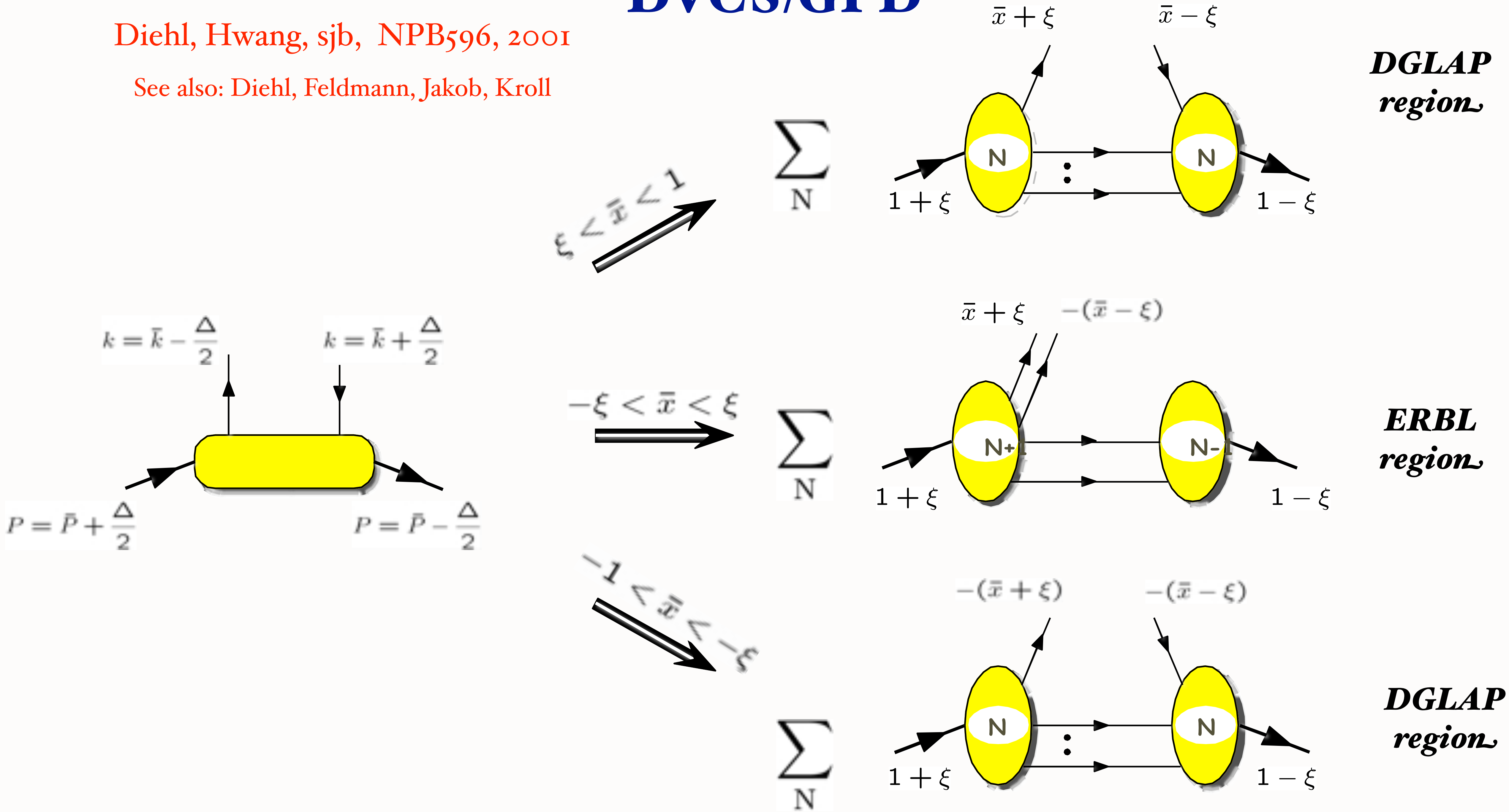
  [Nishio and Watari, arXiv:1105.290]

Light-Front Wave Function Overlap Representation

DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll



Bakker & JI
Lorce

Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x - x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1 - \zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1 - x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^1 dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^1 dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

Verified using LFWFs

Diehl, Hwang, sjb

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^1 x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

Close, Gunion, sjb (1972, 1973)

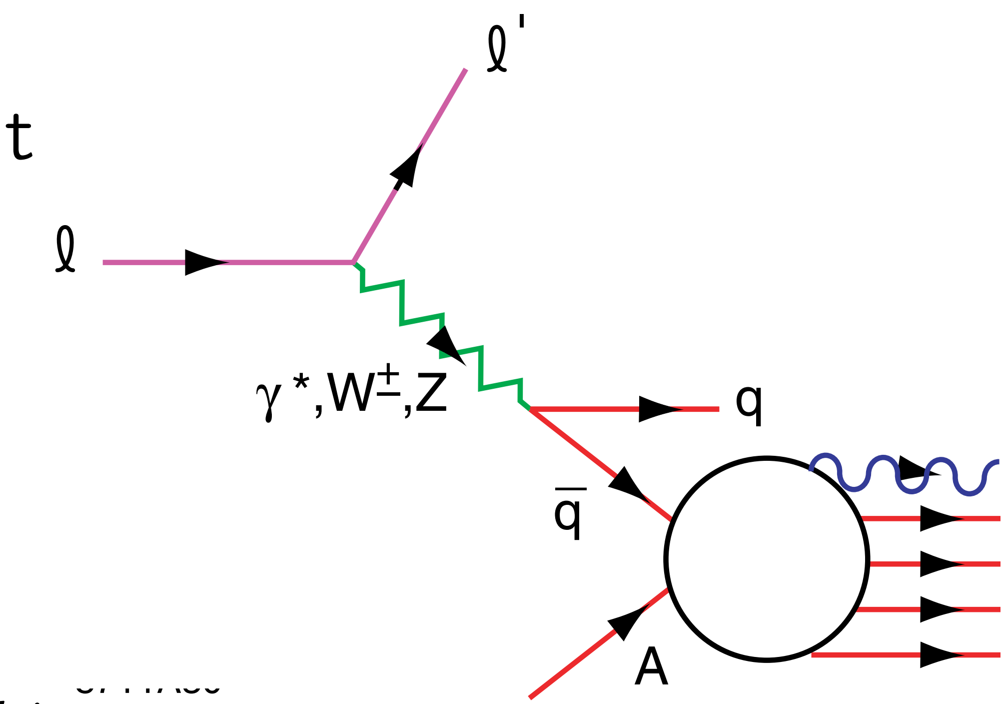
Features of DVCS

- Imaginary part constrained by unitarity: DIS!
- Reggeon Exchange determined by small x DIS
- Phase from C=+ Reggeon Signature Factor
- J=0 Fixed Pole
- Interference with Bethe-Heitler

Origin of Regge Behavior of Deep Inelastic Structure Functions

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$

Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$



Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

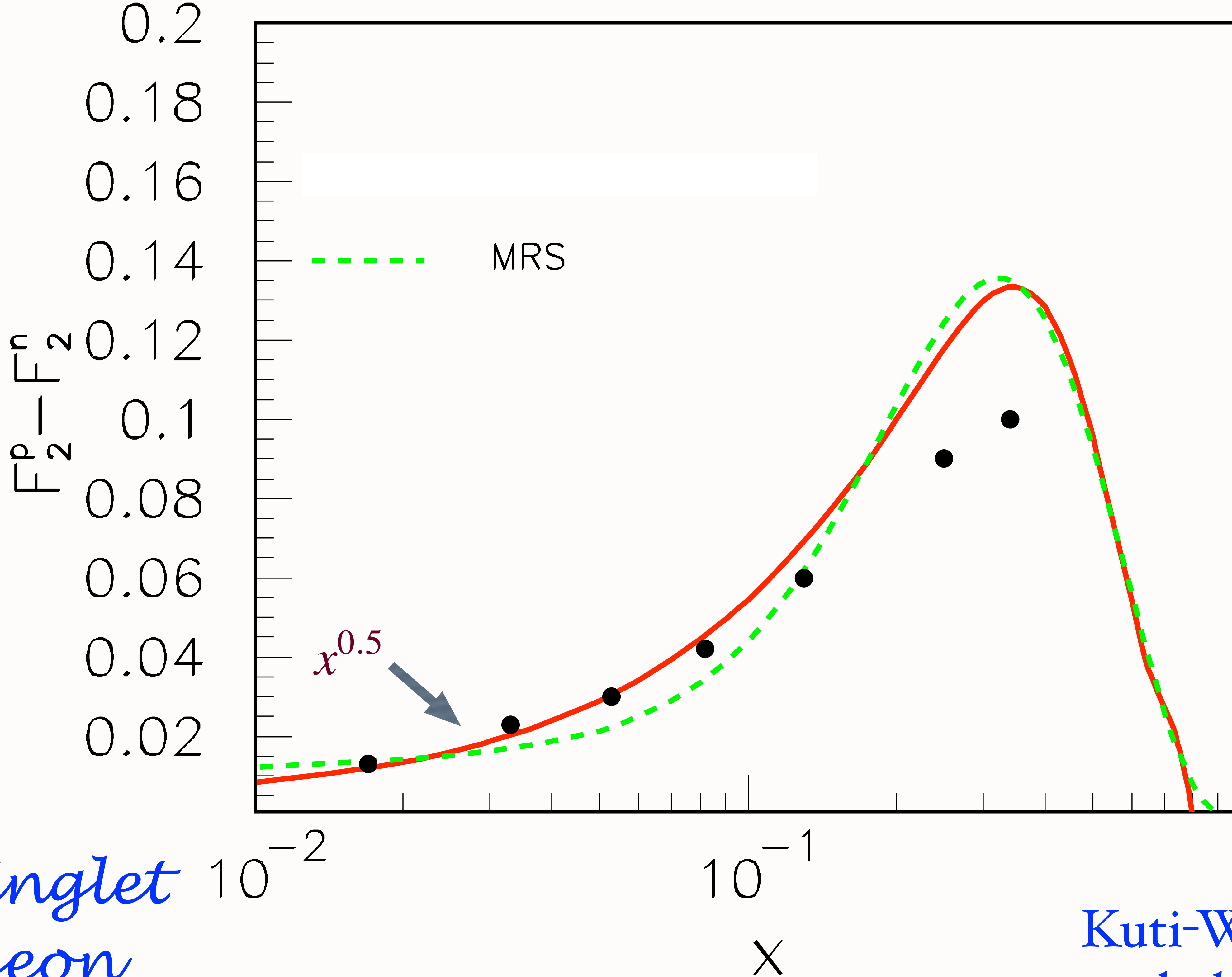
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$ at small x_{bj}.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Schmidt, Yang, Lu, sjb

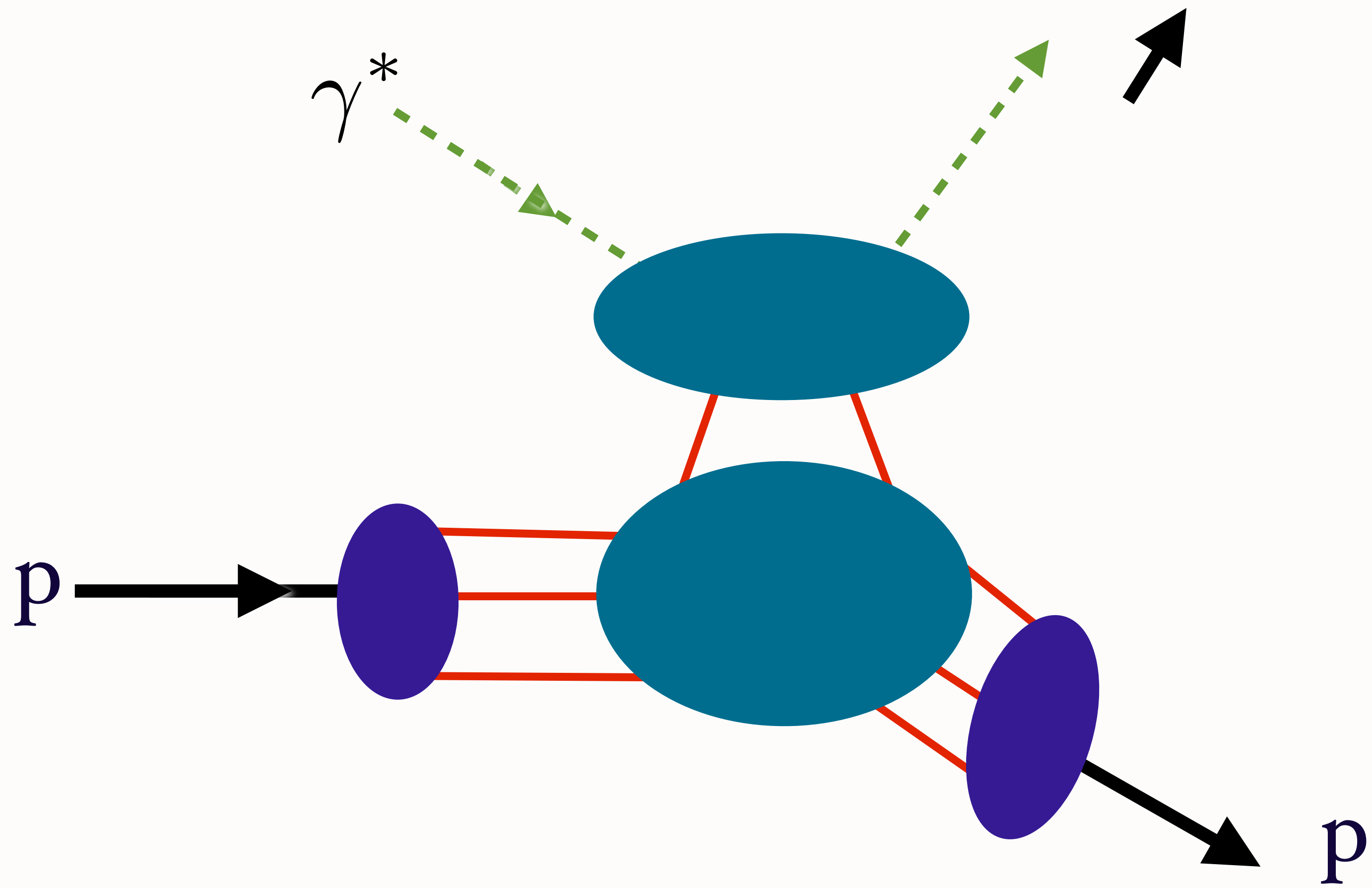


Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Deeply Virtual Compton Scattering

$$\gamma^* p \to \gamma p$$



Hard Reggeon Domain

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

$$T(\gamma^*(q)p \to \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^{\alpha}(t)\beta_R(t)$$

$$\alpha_R(t) \to 0$$

Reflects elementary coupling of two photons to quarks

$$\beta_R(t) \sim \frac{1}{t^2}$$

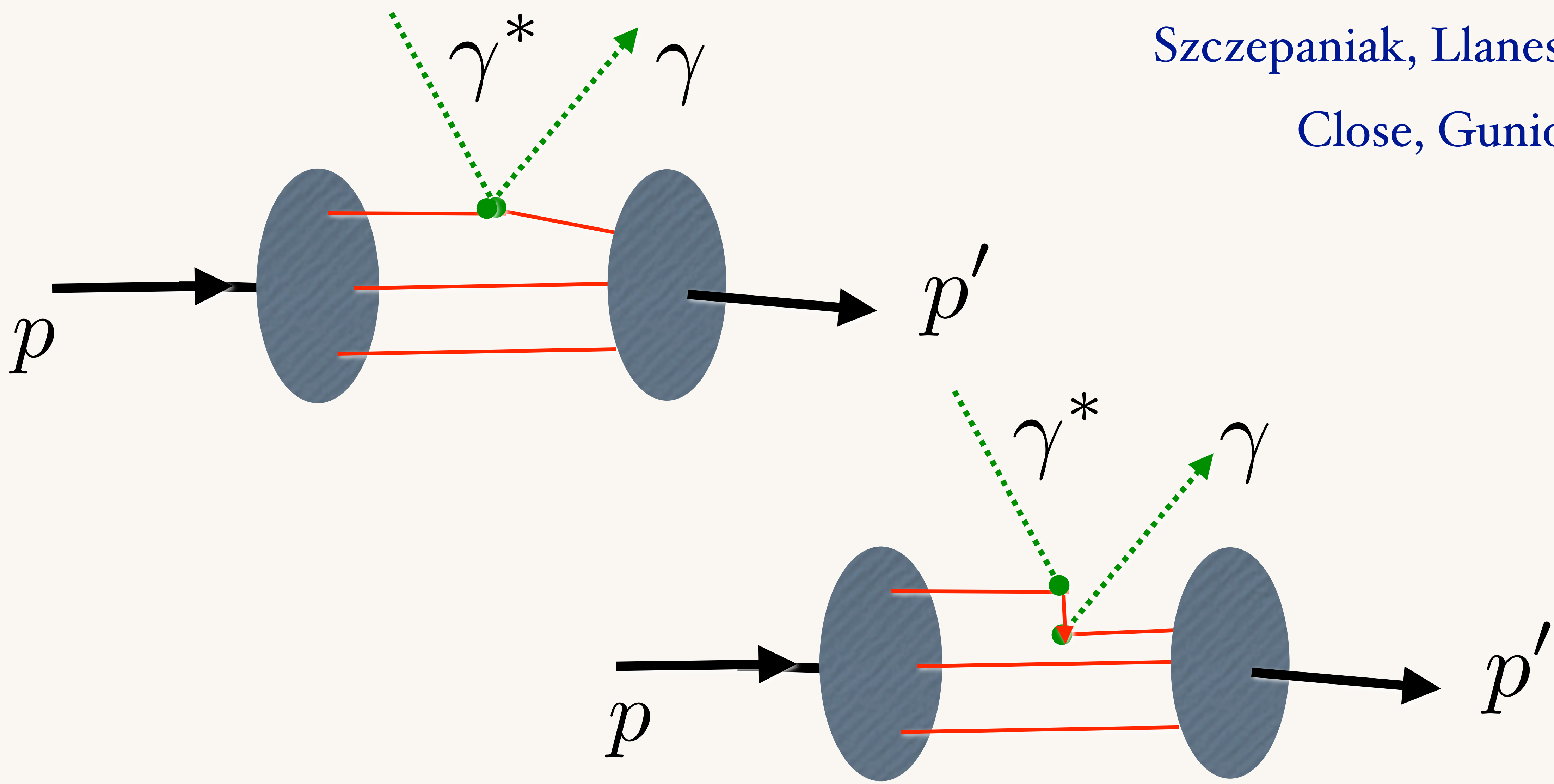
$$\frac{d\sigma}{dt} \sim \frac{1}{s^2}\frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

J=0 Fixed Pole Contribution to DVCS

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

Szczepaniak, Llanes-Estrada, sjb

Close, Gunion, sjb



Real amplitude, independent of Q^2 at fixed t

J=0 Fixed pole in real and virtual Compton scattering

Damashek, Gilman; Close, Gunion, sjb
Llanes-Estrada, Szczepaniak, sjb

Effective two-photon contact term

Seagull for scalar quarks

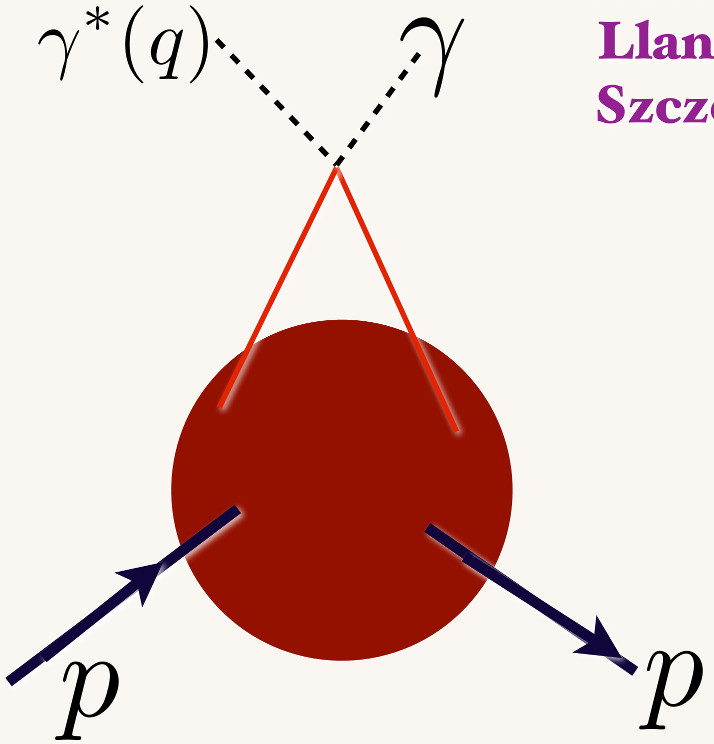
Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of Q^2 at fixed t

<1/x> Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory **No ambiguity in D-term**

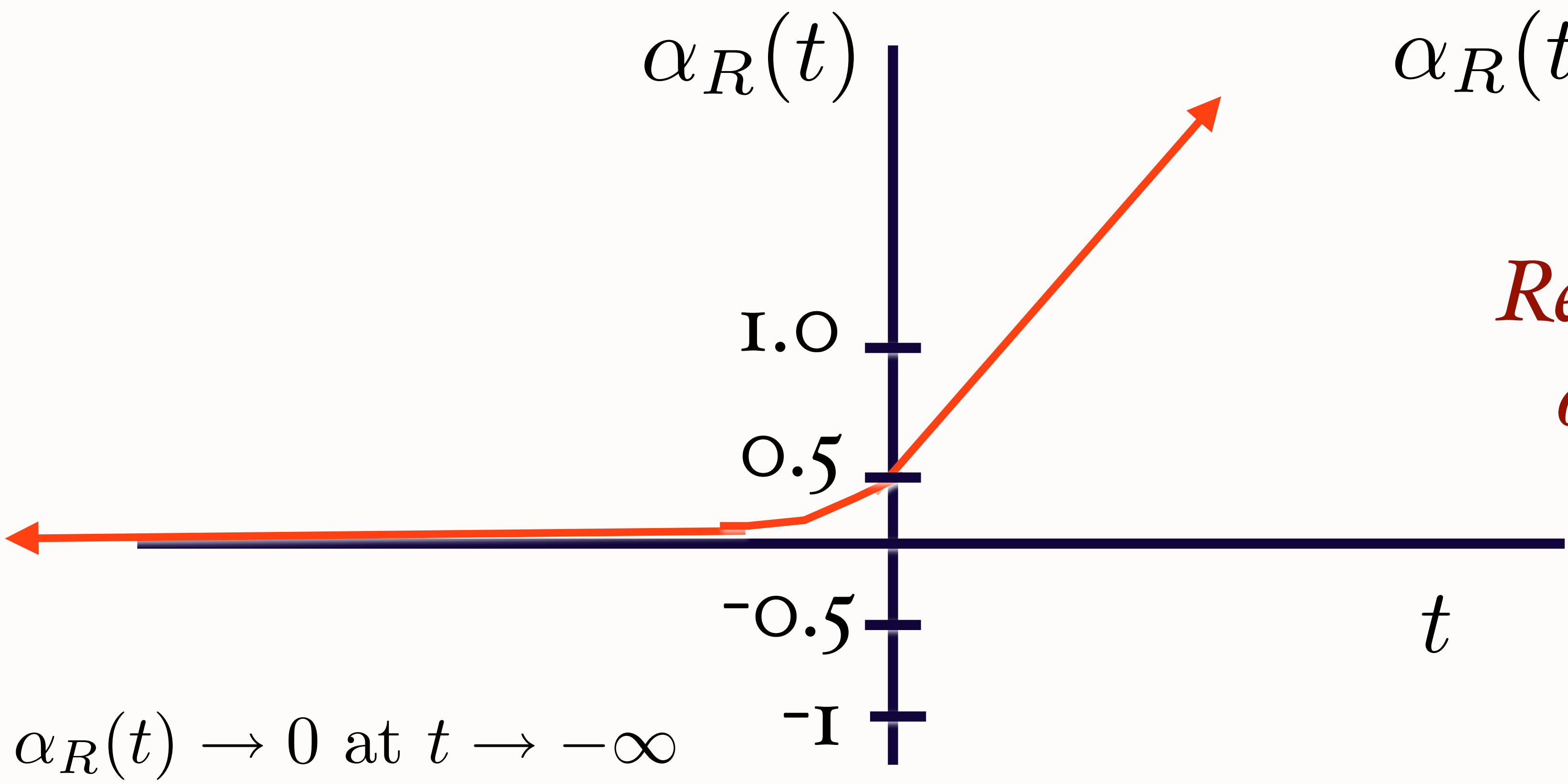


Q^2-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt}(\gamma^* p \to \gamma p) = F^2(t)$$

Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t) \beta_R(t) \qquad s >> -t, Q^2$$



$\alpha_R(t) \to 0$ at $t \to -\infty$

J=0 fixed pole

Reflects elementary coupling of two photons to quarks

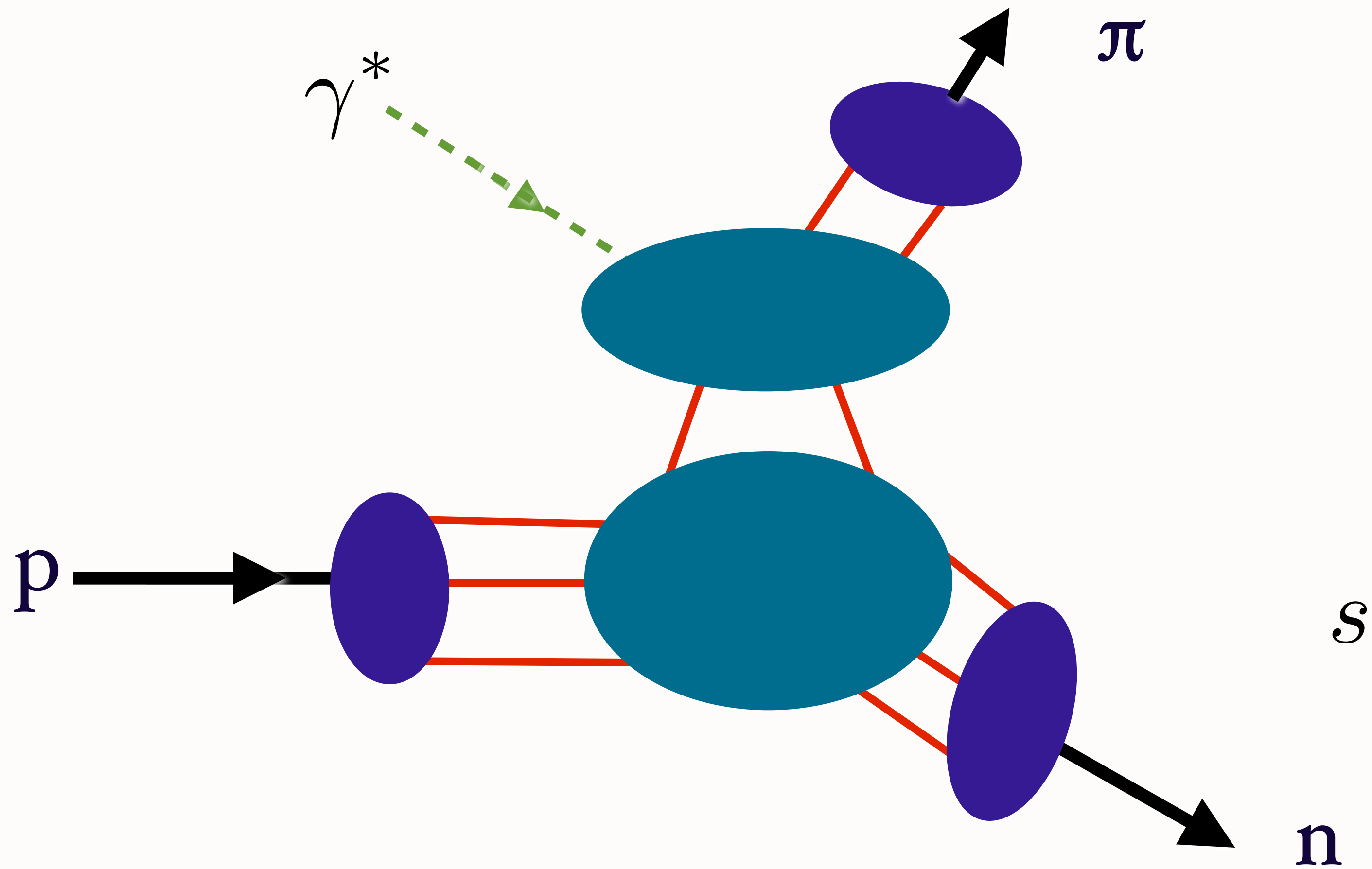
$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \gamma p) \to \frac{1}{s^2} \beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

Fundamental test of QCD

Exclusive Electroproduction

$$ep \to e'\pi^+ n$$



Hard Reggeon Domain

$$s >> -t, Q^2 >> \Lambda^2_{QCD}$$

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s^{\alpha}_R(t)\beta_R(t)$$

$$\alpha_R(t) \to -1$$

Reflects elementary exchange of quarks in t-channel

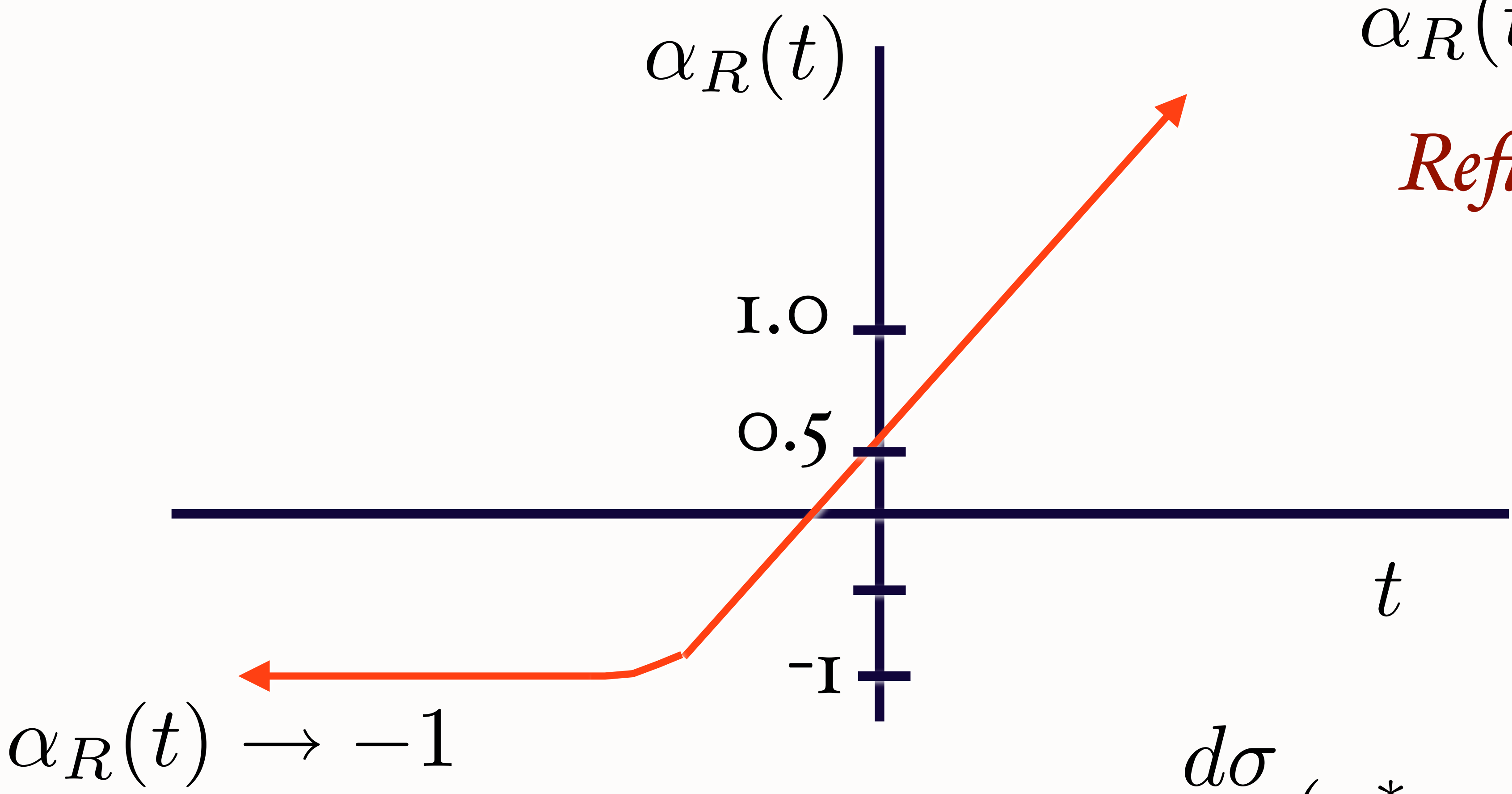
$$\beta_R(t) \sim \frac{1}{t^2}$$

$\frac{d\sigma}{dt} \sim \frac{1}{s^7}$ at fixed $\frac{Q^2}{s}, \frac{t}{s}$

Gunion, Blankenbecler, Savit, sjb

Regge domain

$$T(\gamma^* p \to \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t)\beta_R(t) \qquad s >> -t, Q^2$$



$\alpha_R(t) \to -1$ at $t \to -\infty$

Reflects elementary exchange of quarks in t-channel

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \to \pi^+ n) \to \frac{1}{s^3}\beta_R^2(t)$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^3}\frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

Fundamental test of QCD

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

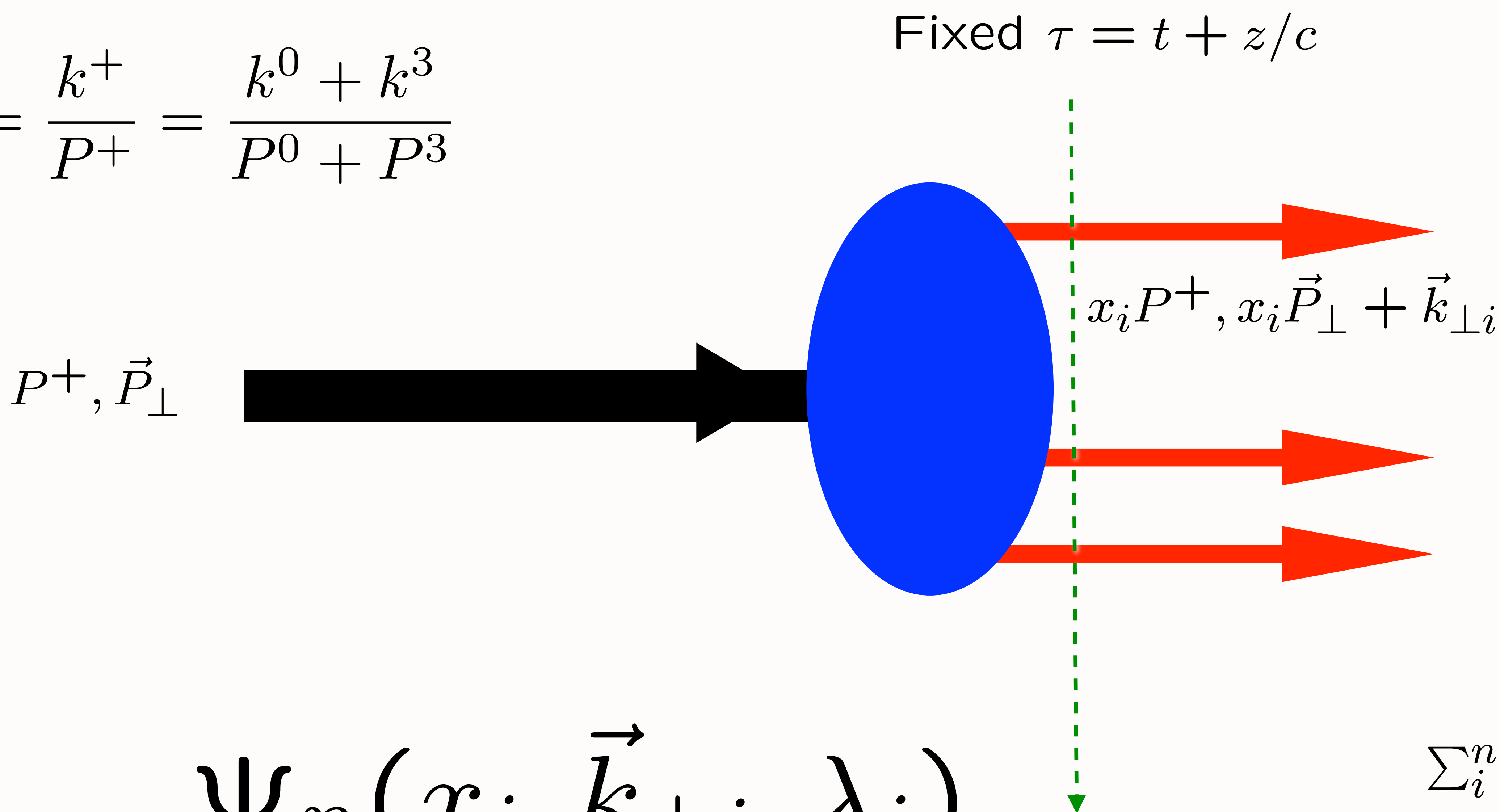
$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

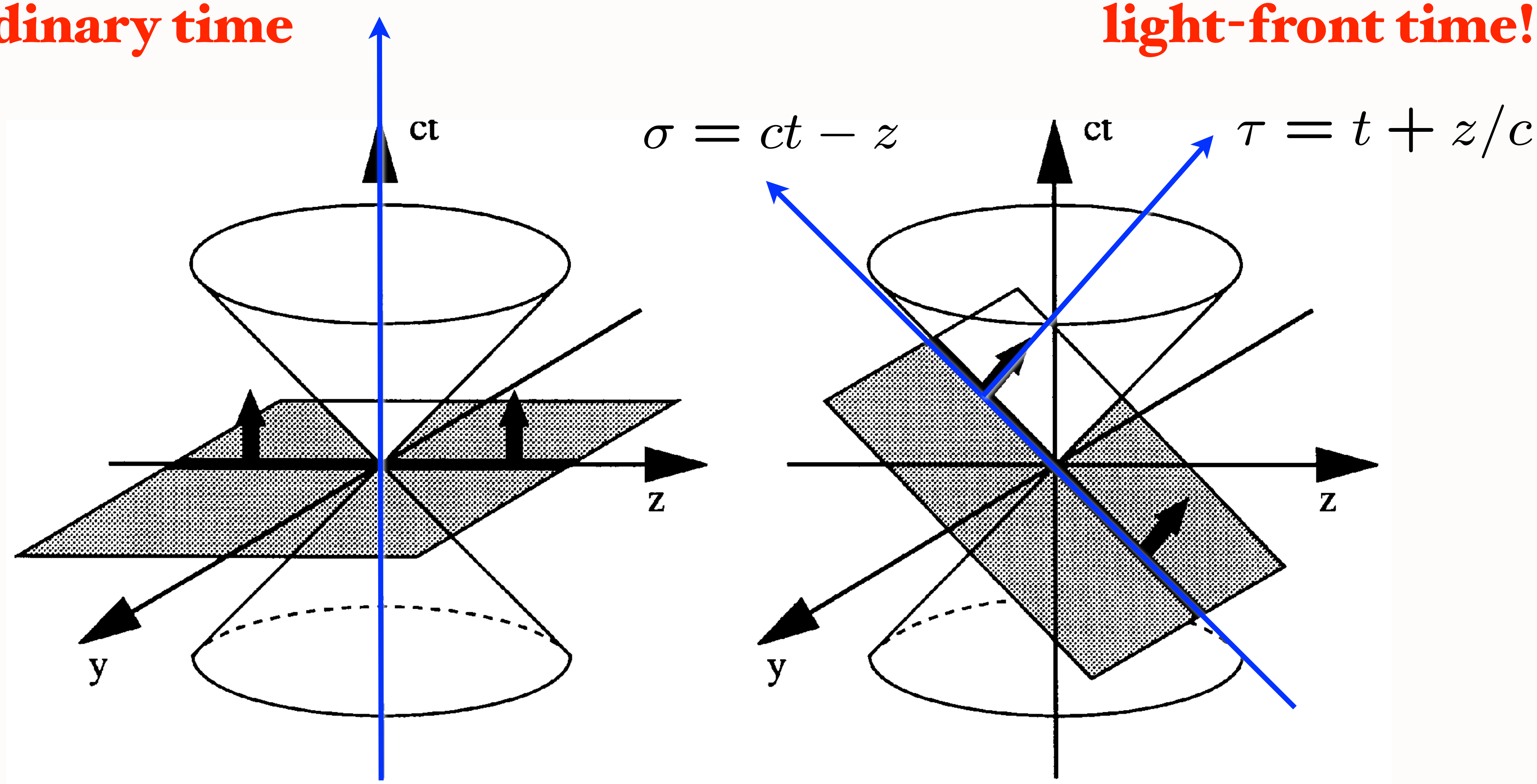
Invariant under boosts! Independent of P^μ

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

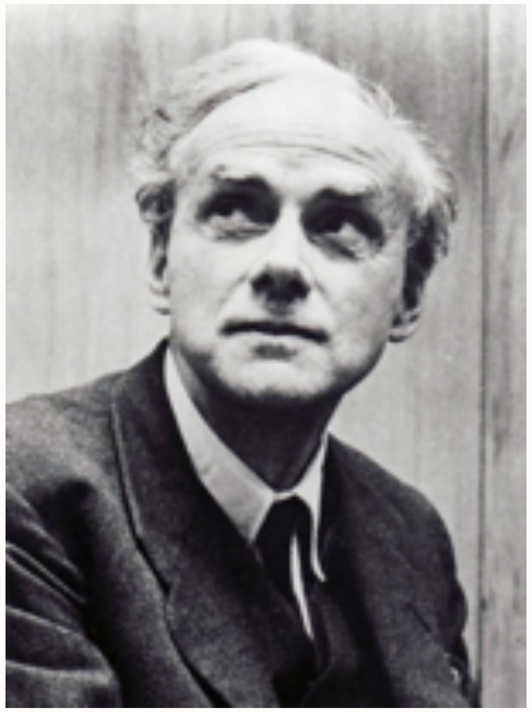
Dirac's Amazing Idea: The Front Form

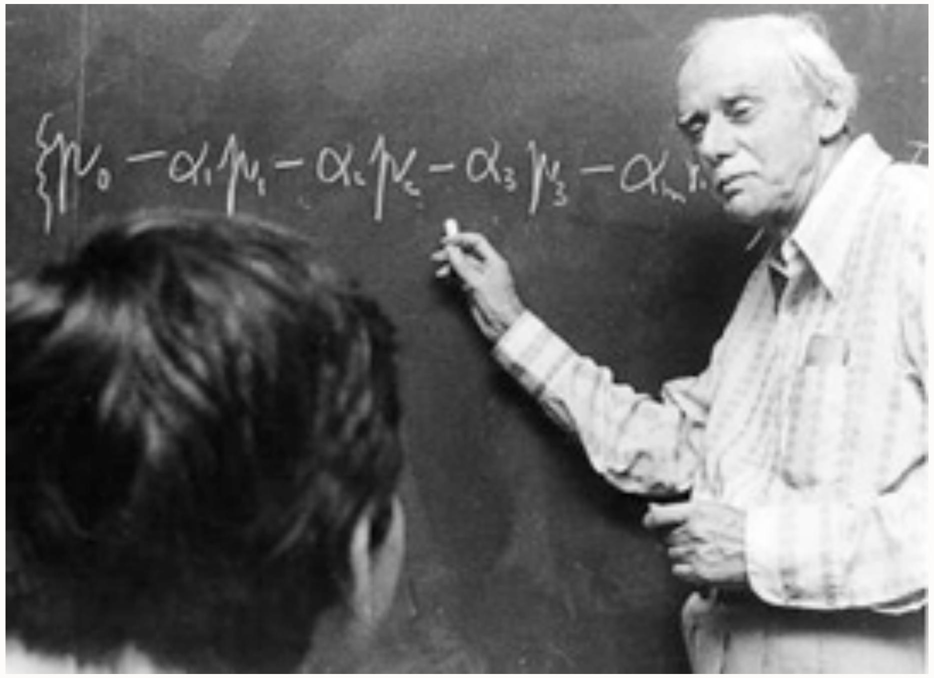
Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**





"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out." -

P.A.M. Dirac (1977)

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

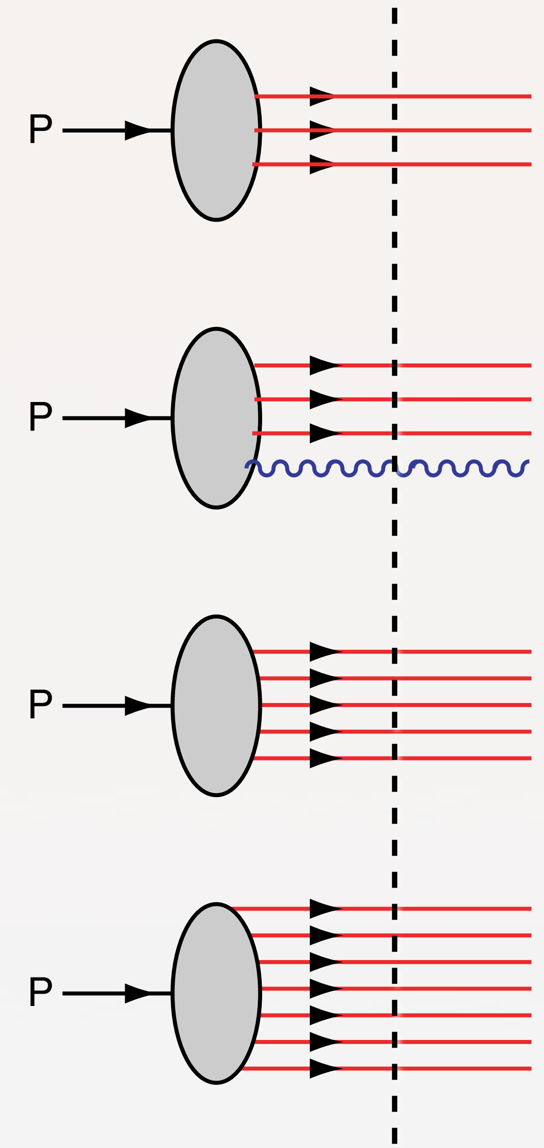
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states

BFKL Pomeron

Hidden Color!

Light-Front QCD

Heisenberg Matrix Formulation

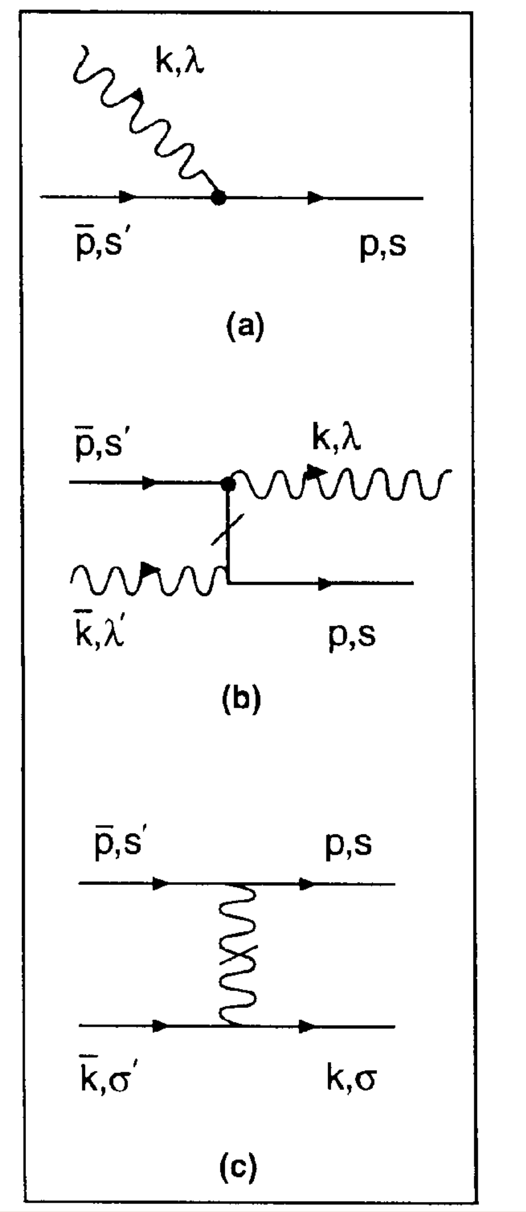
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

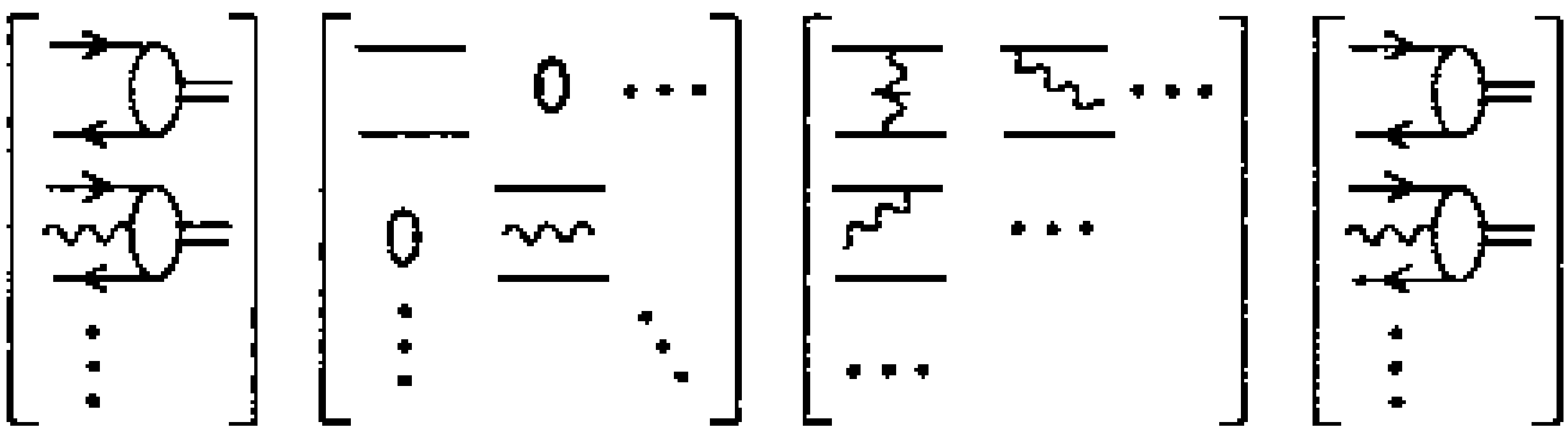
$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$



Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions!

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right)\begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}\begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



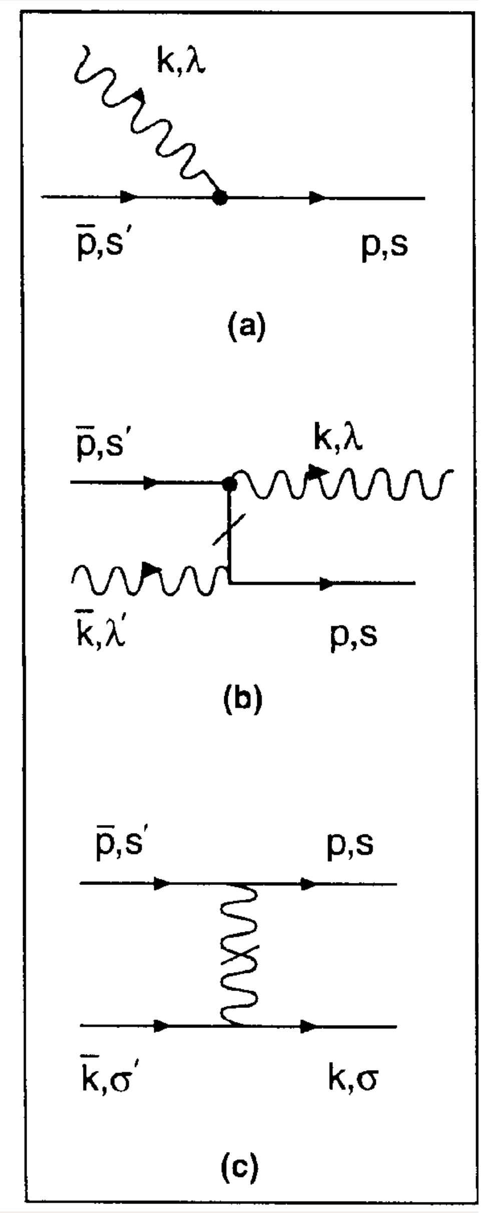
G.P. Lepage, sjb

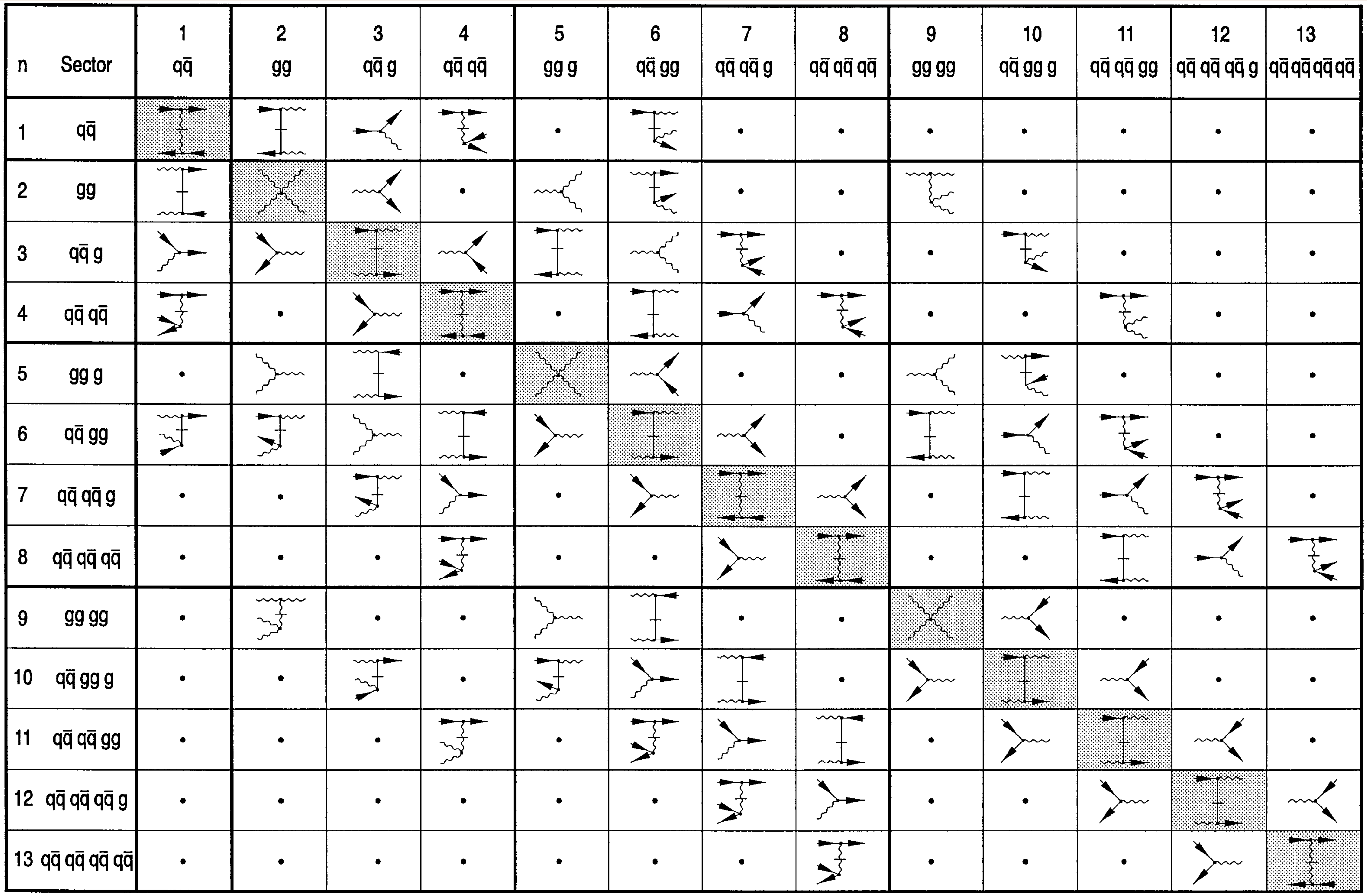
$A^+ = 0$

Light-Front QCD

Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$





Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

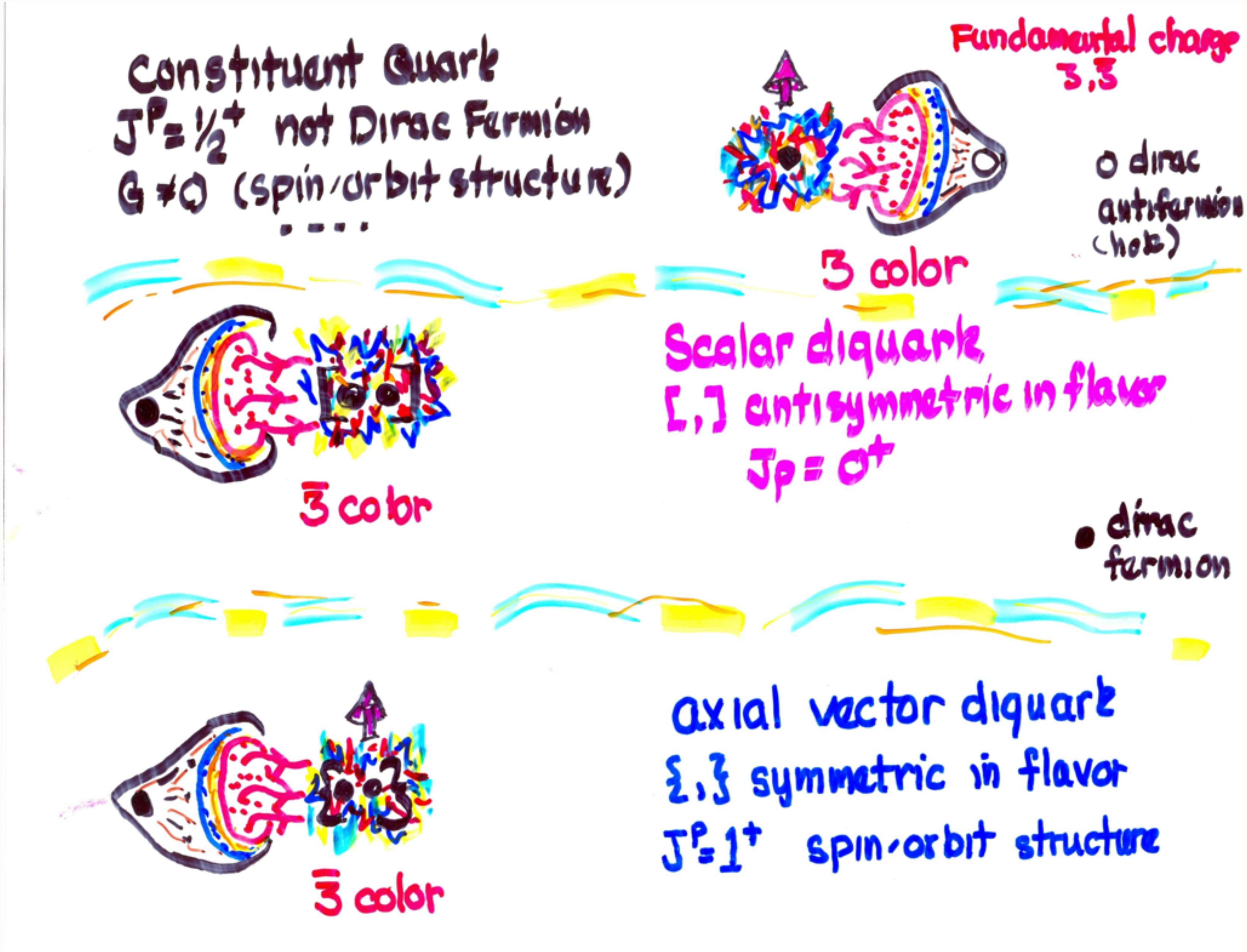
$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

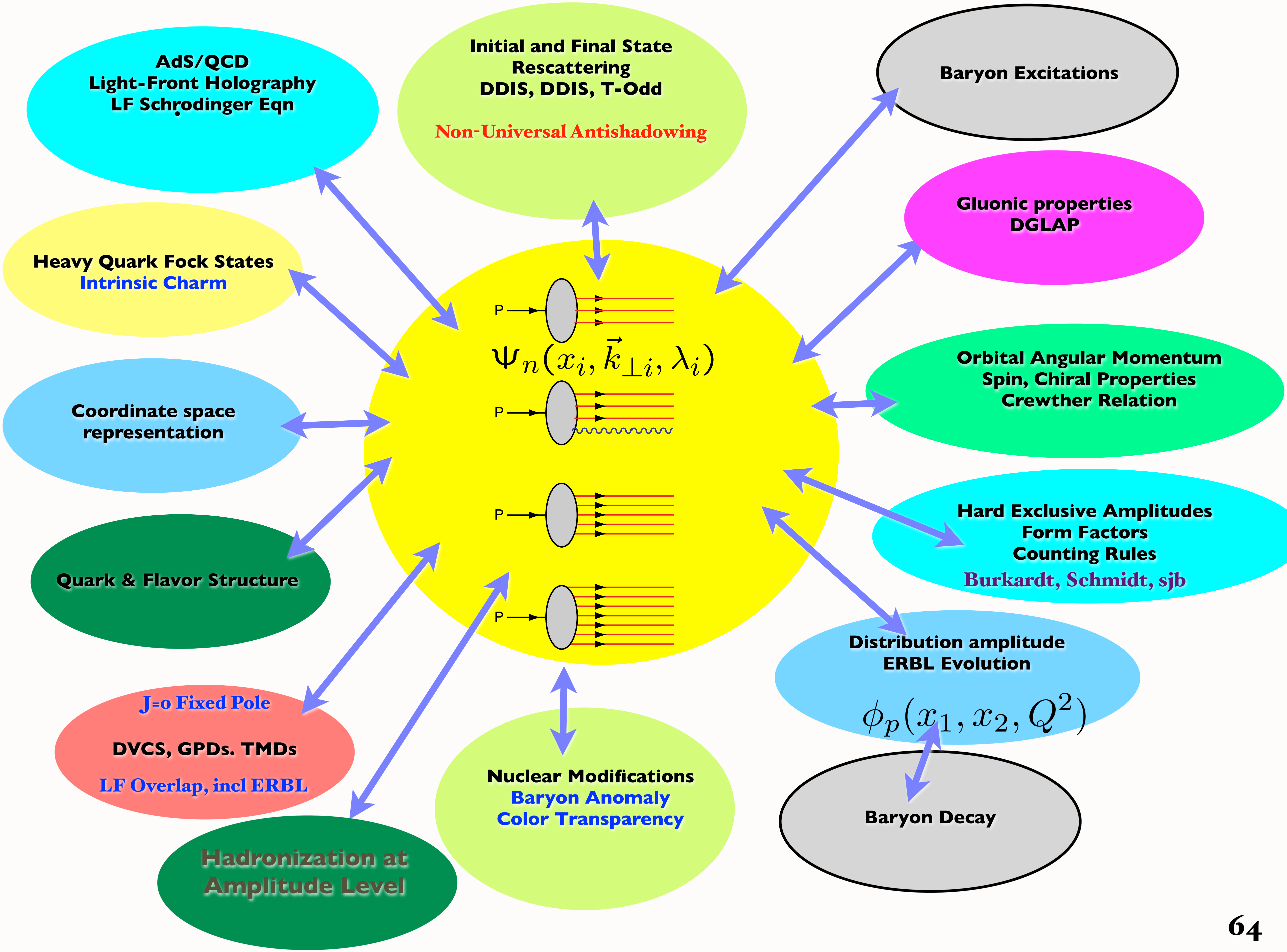
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



Constituent Quark
J^P = ½⁺ not Dirac Fermion
G ≠ 0 (spin/orbit structure)
Fundamental charge
3,3̄
3 color
o dirac antifermion (hole)
Scalar diquark
[,] antisymmetric in flavor
Jp = 0⁺
3̄ color
• dirac fermion
axial vector diquark
{,} symmetric in flavor
J^P = 1⁺ spin-orbit structure
3̄ color

QCD and the LF Hadron Wavefunctions



Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved by every interaction

LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

Special Features of LF Spin

- LF Helicity and chirality refer to z direction, not the particle's 3-momentum p
- LF spinors are eigenstates of $S^z = \pm\frac{1}{2}$
- Gluon polarization vectors are eigenstates with $S^z = \pm 1$

$$\epsilon^\mu = (\epsilon^+, \epsilon^-, \vec{\epsilon}_\perp) = (0, 2\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp)$$

$$\vec{\epsilon}^{\,\pm}_\perp = \mp\frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \qquad k^\mu \epsilon_\mu = 0$$

G. P. Lepage and sjb

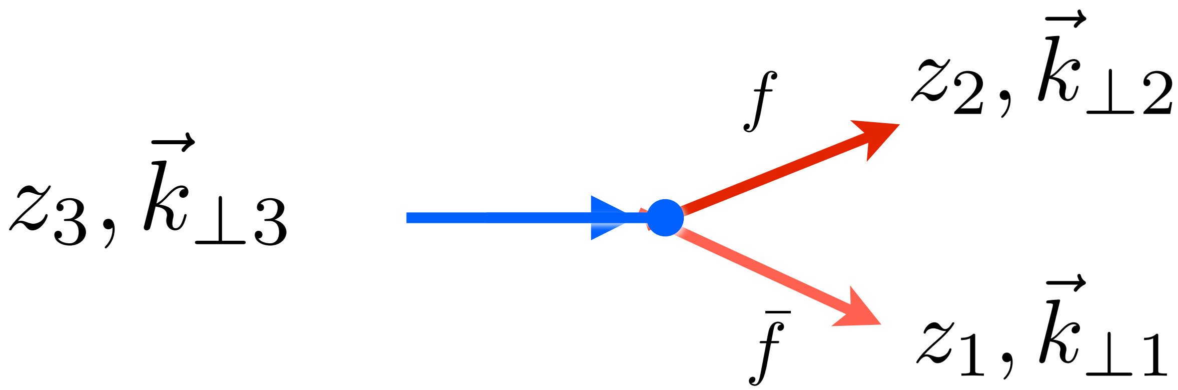
$$\left.\begin{matrix} u_\uparrow(p) \\ u_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}}\left(p^+ + \beta m + \alpha_\perp \cdot \mathbf{p}_\perp\right) \times \left\{\begin{matrix} \chi(\uparrow) \\ \chi(\downarrow) \end{matrix}\right. ,$$

$$\left.\begin{matrix} v_\uparrow(p) \\ v_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}}\left(p^+ - \beta m + \vec{\alpha}_\perp \cdot \vec{\mathbf{p}}_\perp\right) \times \left\{\begin{matrix} \chi(\downarrow) \\ \chi(\uparrow) \end{matrix}\right.$$

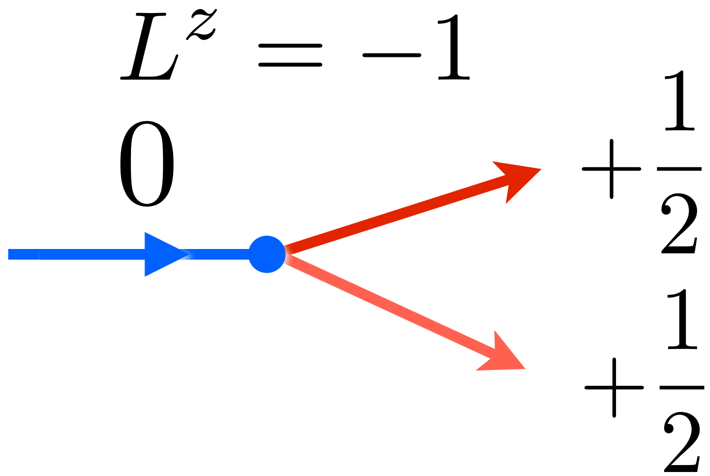
$$\chi(\uparrow) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},$$

Melosh not needed

Angular Momentum on the Light-Front

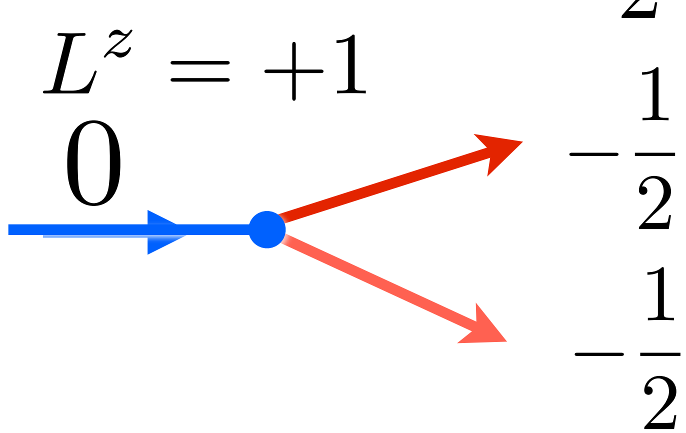


P-Wave Decay
Spin-0 coupling
to fermion pair



spinor overlap

$$< ij > = < i - | j + > = -\sqrt{2 z_i z_j} \vec{\epsilon}_{\perp}^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$



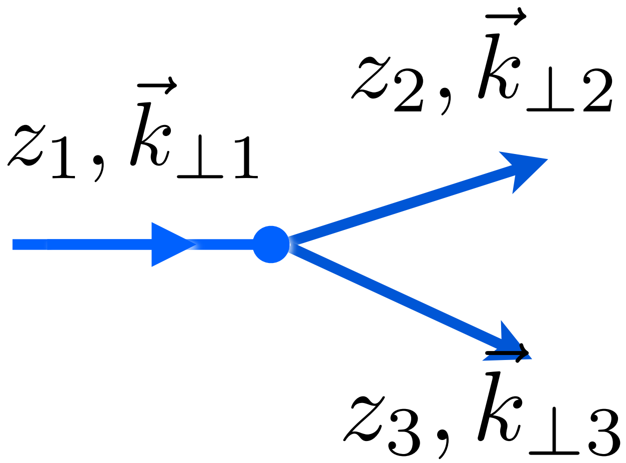
$$[ij] = < i^+ | j - > = \sqrt{2 z_i z_j} \vec{\epsilon}^{\,(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

$$< ij > [ij] = z_i z_j \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)^2 = \mathcal{M}_{ij}^2$$

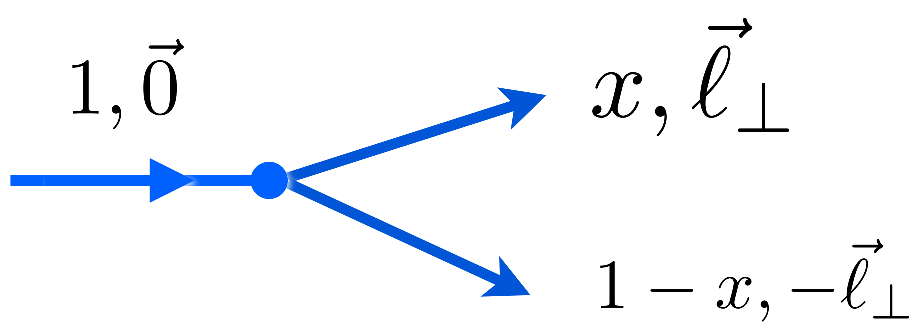
Identity

Angular Momentum on the Light-Front

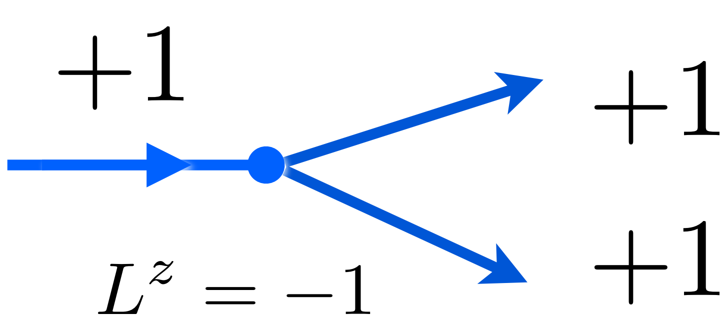
Triple-Gluon Coupling



$$g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \vec{v}_{23} = g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp 2}}{z_2} - \frac{\vec{k}_{\perp 3}}{z_3}\right)$$



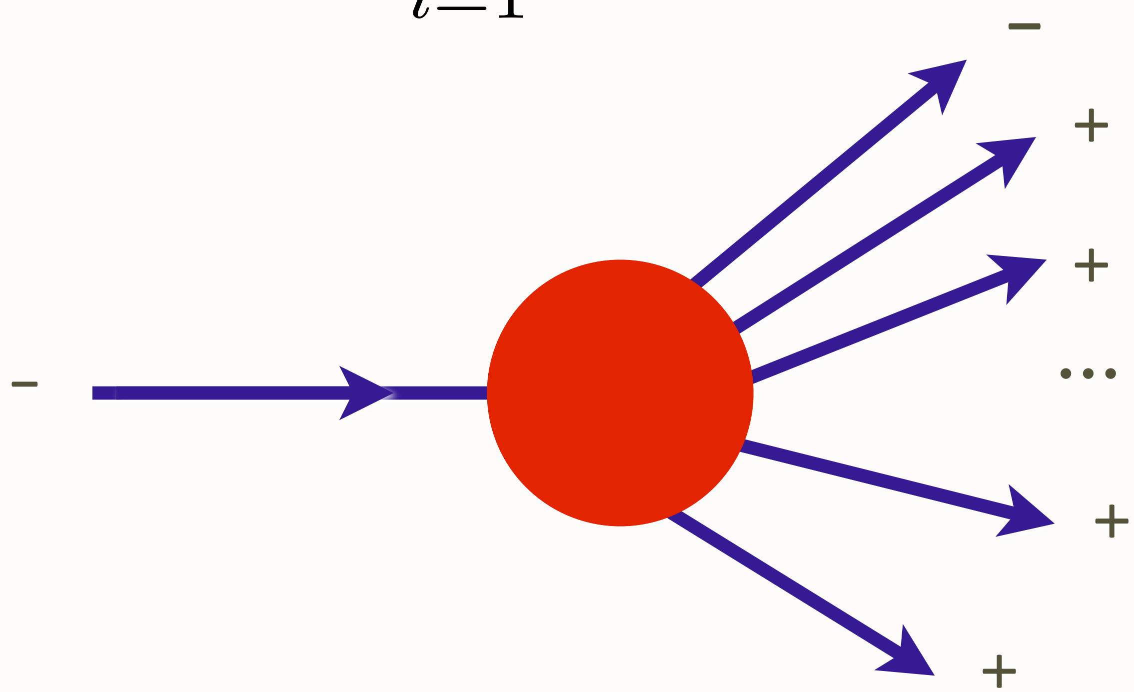
$$g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \vec{v}_{23} = g \vec{\epsilon}_\perp^{\,+} \cdot \frac{\vec{\ell}_\perp}{x(1-x)}$$



$$< ij >= -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

$$M(-1 \to -1 + 1 + 1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-2) + L^z$$

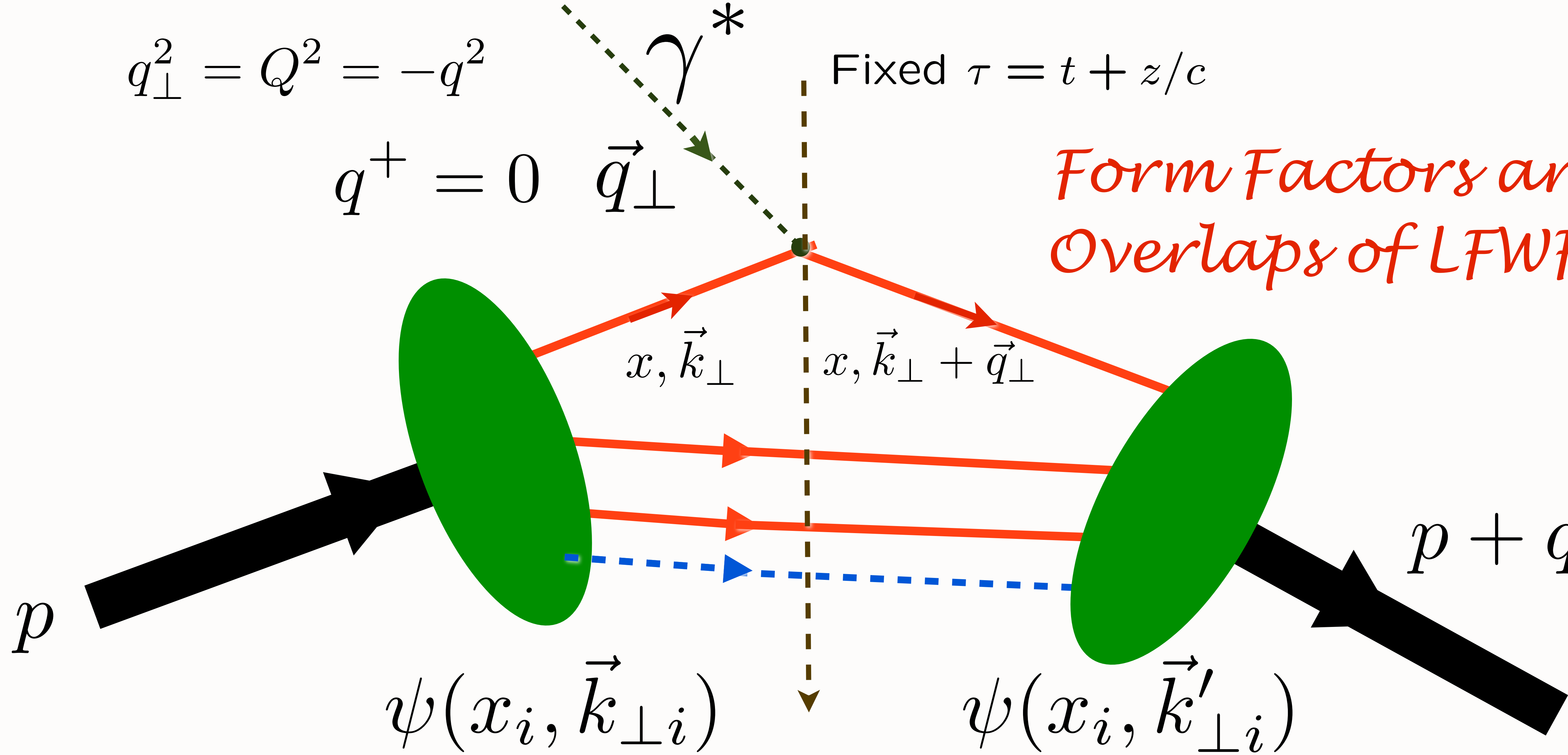


Vanishes Because Maximum $|L^z| = n - 2$

Light Front Analog of MHV rules

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West

Exact LF Formula for Pauli Form Factor

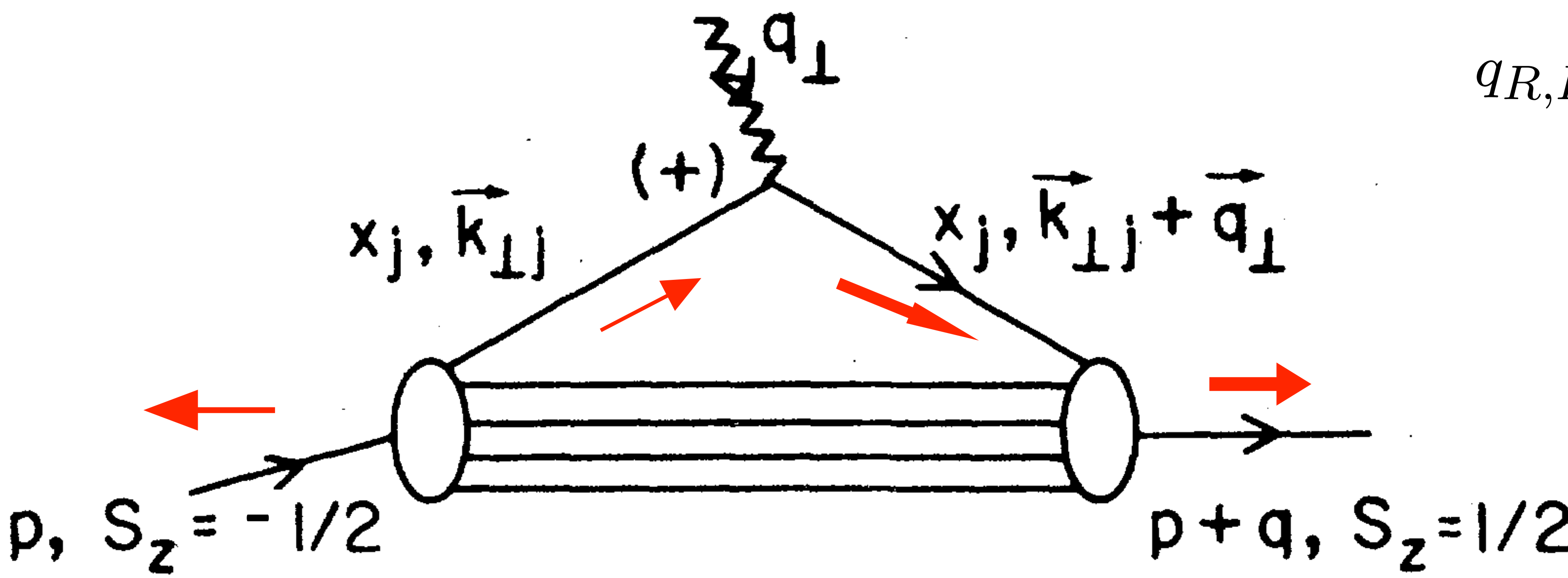
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

PHYSICAL REVIEW D **75,** 073008 (2007)

Connection between the Sivers function and the anomalous magnetic moment

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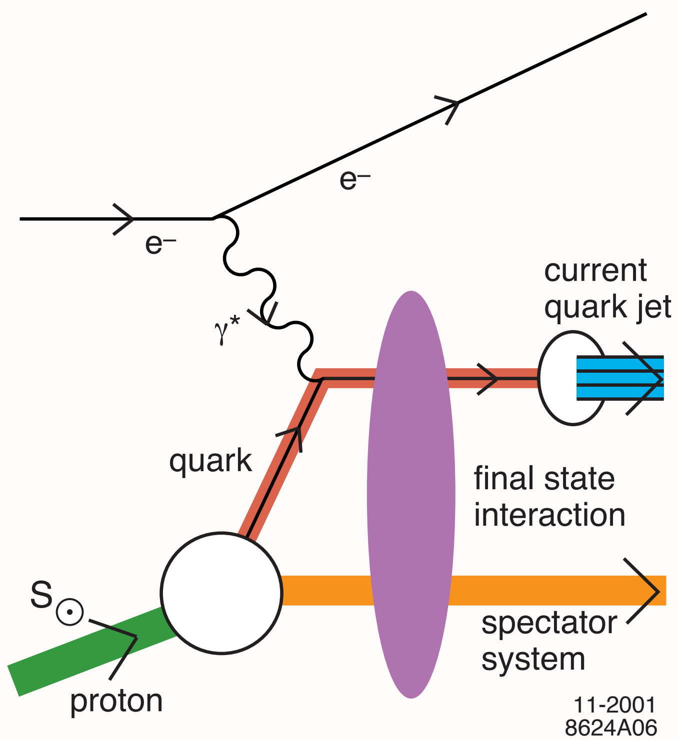
(Received 8 January 2007; revised manuscript received 14 February 2007; published 9 April 2007)

The same light-front wave functions of the proton are involved in both the anomalous magnetic moment of the nucleon and the Sivers function. Using the diquark model, we derive a simple relation between the anomalous magnetic moment and the Sivers function, which should hold in general with good approximation. This relation can be used to provide constraints on the Sivers single spin asymmetries from the data on anomalous magnetic moments. Moreover, the relation can be viewed as a direct connection between the quark orbital angular momentum and the Sivers function.



$$\frac{A_{UT}^{\mathrm{Siv}}(\pi^+)}{A_{UT}^{\mathrm{Siv}}(\pi^-)} \approx \frac{2e_u^2 f_{1T}^{\perp u} D_1^{\pi^+/u}}{e_d^2 f_{1T}^{\perp d} D_1^{\pi^-/d}} \approx \frac{2e_u^2 \kappa_u}{e_d^2 \kappa_d} = -3.3.$$

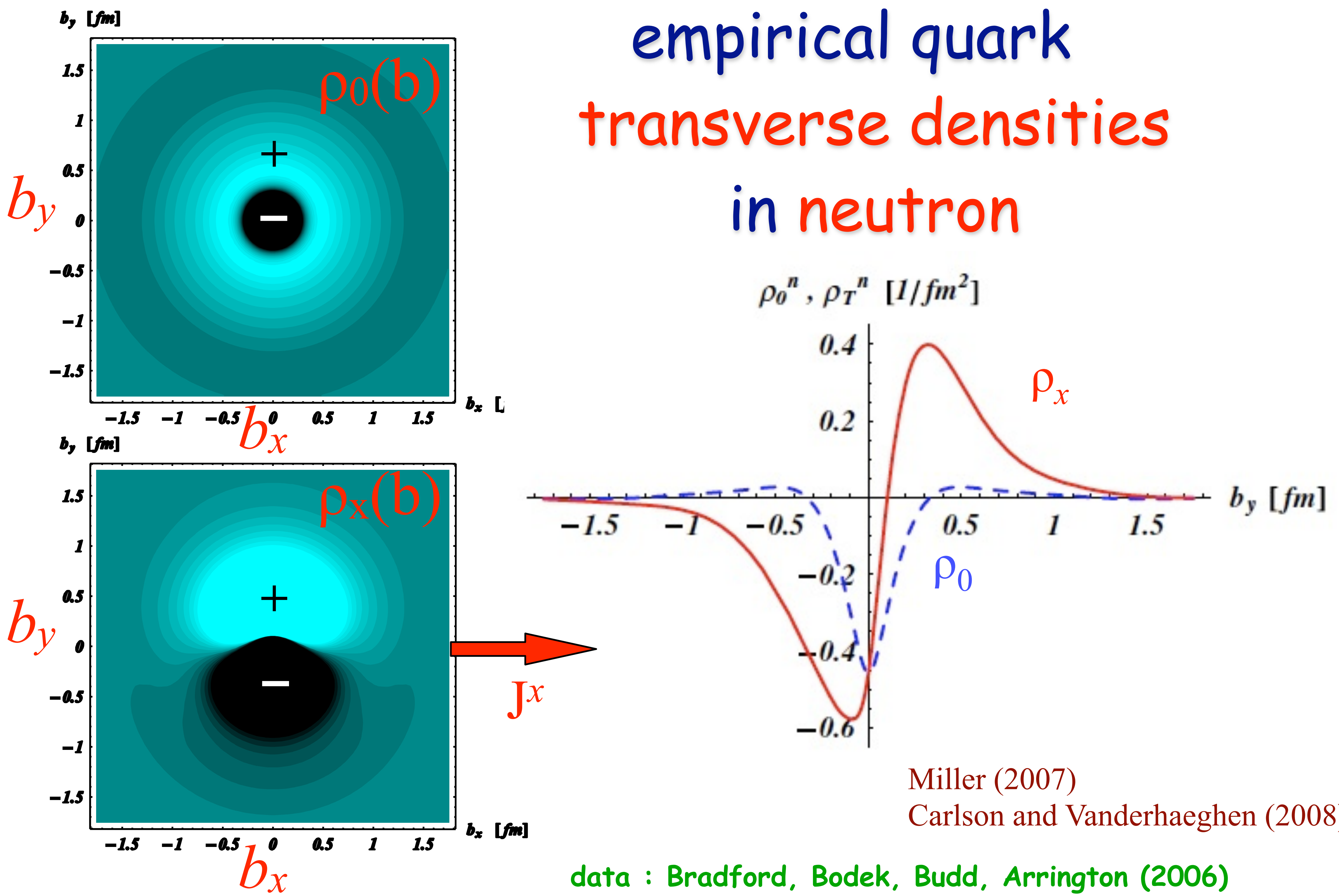
$$\frac{A_{UT}^{\mathrm{Siv}}(\pi^0)}{A_{UT}^{\mathrm{Siv}}(\pi^-)} \approx \frac{2e_u^2 f_{1T}^{\perp u} D_1^{\pi^0/u} + e_d^2 f_{1T}^{\perp d} D_1^{\pi^0/d}}{e_d^2 f_{1T}^{\perp d} D_1^{\pi^-/d}} \approx \frac{2e_u^2 \kappa_u + e_d^2 \kappa_d}{2e_d^2 \kappa_d} = -1.15,$$

$$\kappa_p = (2)(2/3)\kappa_{u/p} + (-1/3)\kappa_{d/p},$$

$$\kappa_n = (2)(-1/3)\kappa_{u/p} + (2/3)\kappa_{d/p}.$$

$$\frac{A_{UT}^{\mathrm{Siv}}(K^+)}{A_{UT}^{\mathrm{Siv}}(K^0)} \approx \frac{2e_u^2 f_{1T}^{\perp u} D_1^{K^+/u}}{e_d^2 f_{1T}^{\perp d} D_1^{K^0/d}} \approx \frac{4e_u^2 \kappa_u}{e_d^2 \kappa_d} = -6.6.$$

Using measured form factors, find the empirical quark transverse densities in neutron



Miller (2007)
Carlson and Vanderhaeghen (2008)

data : Bradford, Bodek, Budd, Arrington (2006)

Paul Hoyer Losjin 2 September 2011

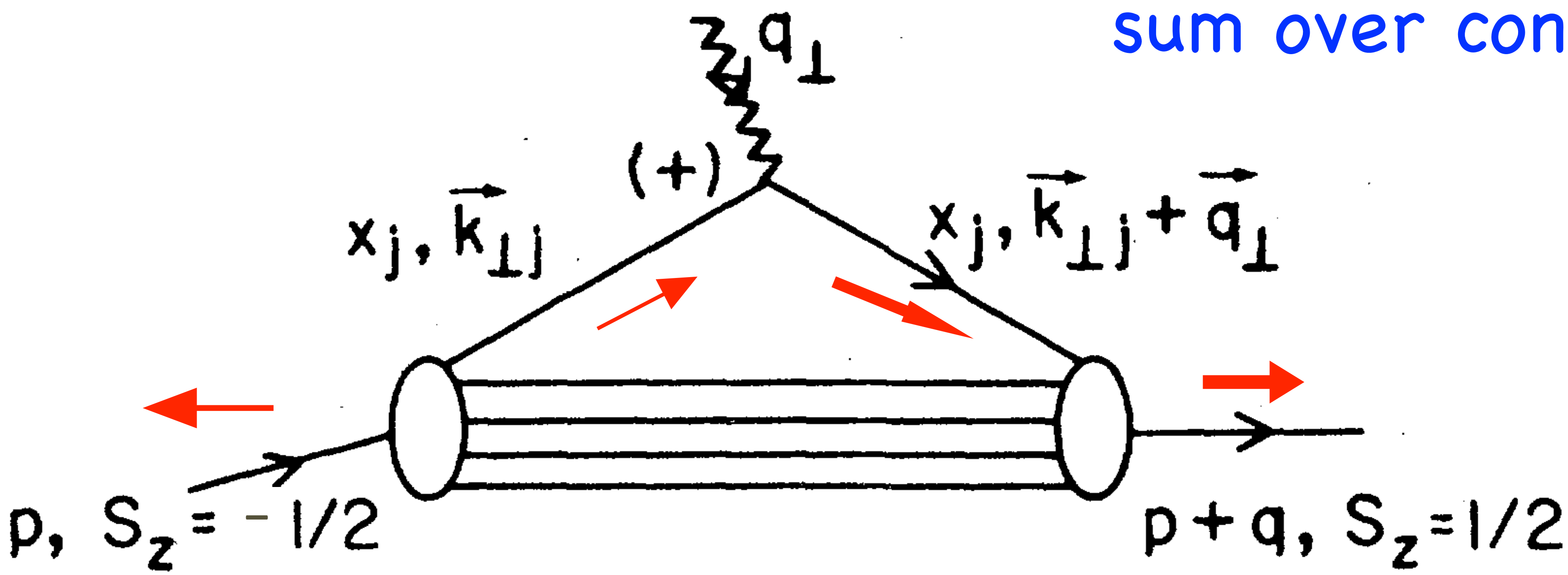
A Transversity Theorem!

Anomalous gravitomagnetic moment B(0)

Terayev, Okun: *B(0) Must vanish because of Equivalence Theorem*

graviton

sum over constituents



Hwang, Schmidt, sjb; Donoghue, Holstein

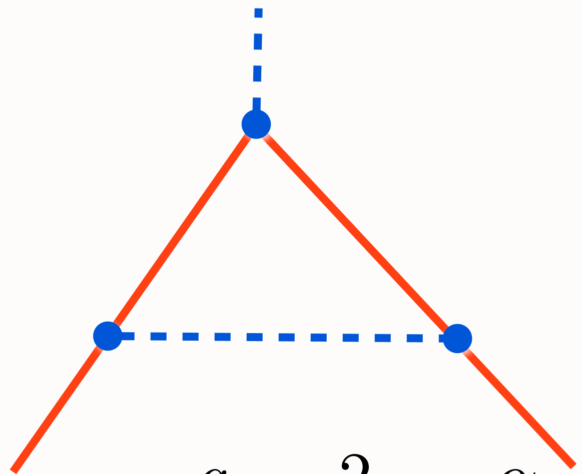
$$B(0) = 0$$

Each Fock State

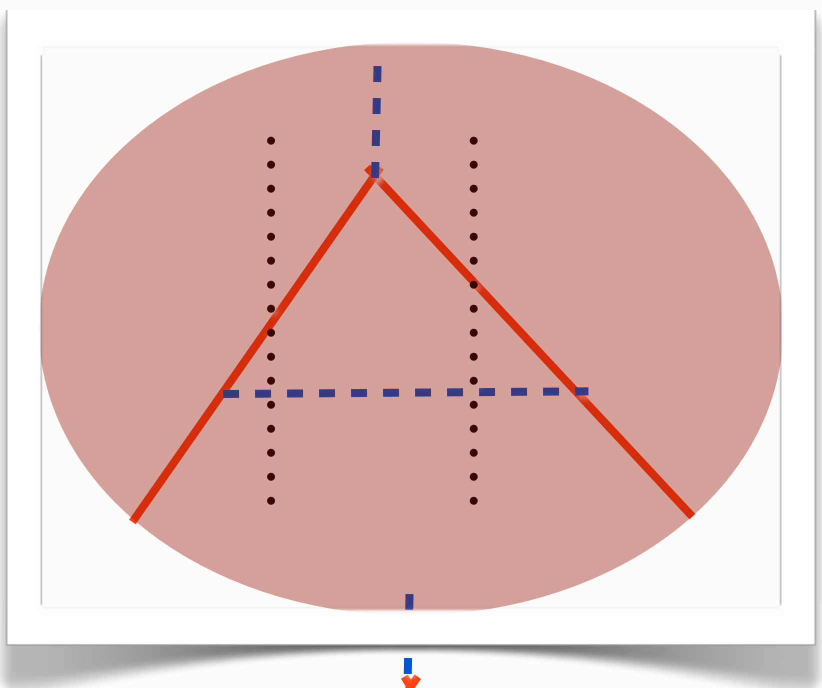
LF formalism essential

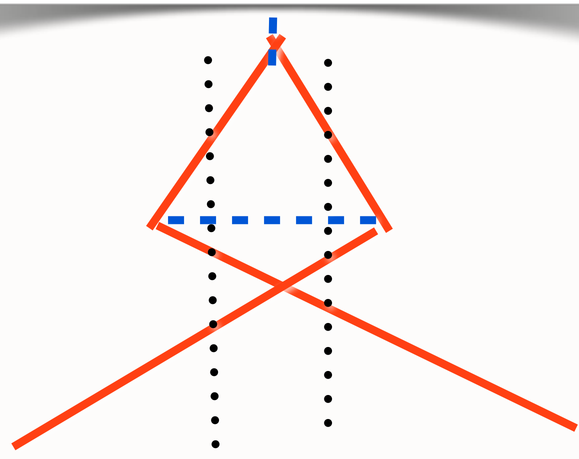
Wick Theorem

Feynman diagram =
single front-form time-ordered diagram!

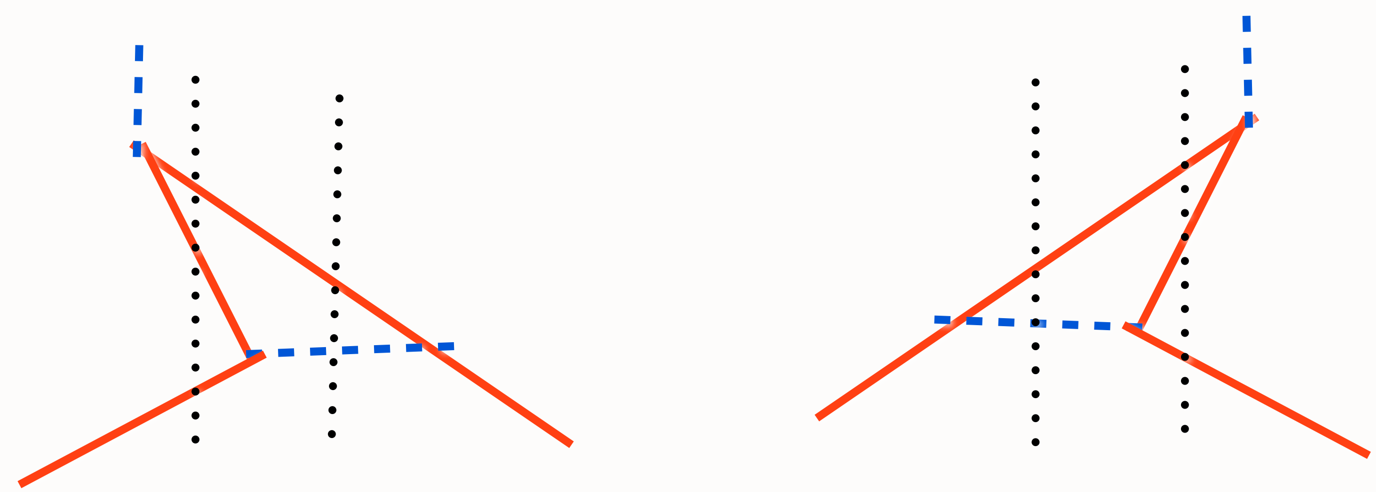


$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

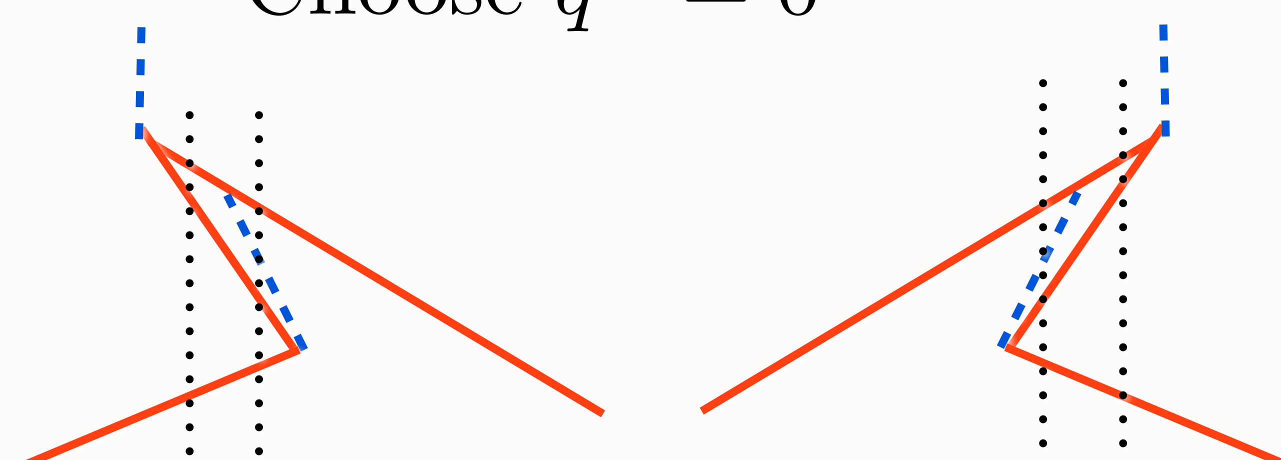




Also $P \to \infty$ observer frame (Weinberg)

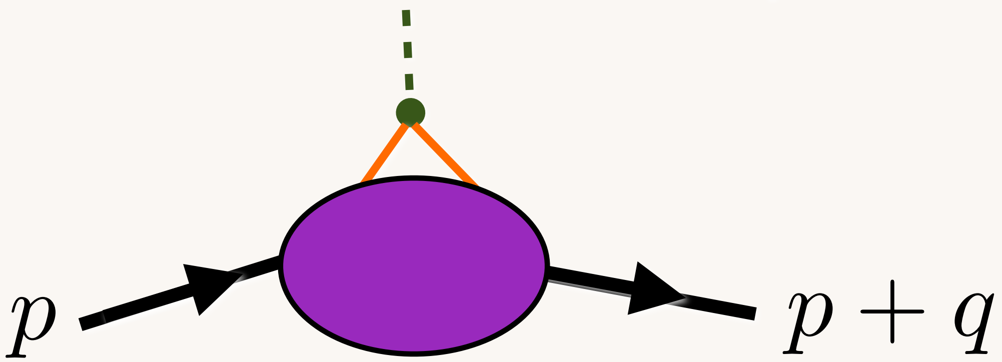


Choose $q^+ = 0$



Calculation of proton form factor in Instant Form

$$< p+q|J^{\mu}(0)|p >$$



- **Need to boost proton wavefunction from p to p+q: Extremely complicated dynamical problem; particle number changes**

- **Need to couple to all currents arising from vacuum**

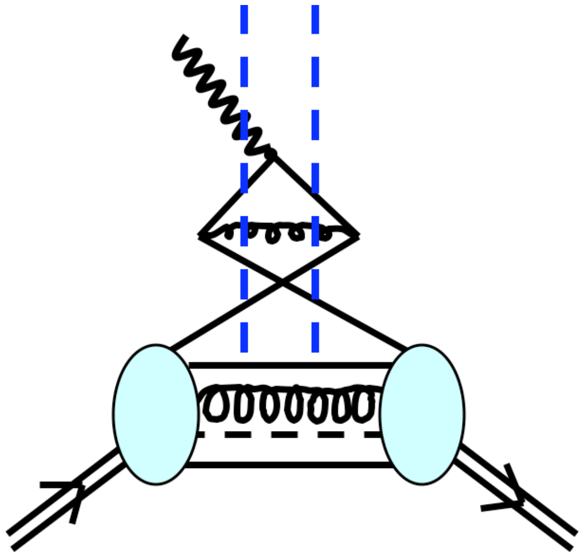
- **Each time-ordered contribution is frame-dependent**

- **Divide by disconnected vacuum diagrams**

Calculation of Hadron Form Factors
Instant Form

- **Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum**

- **Pair creation from vacuum occurs at any time before probe acts -- acausal**



- **Knowledge of hadron wavefunction insufficient to compute current matrix elements**

- **Requires dynamical boost of hadron wavefunction -- unknown except at weak binding**
  Hoyer, Vantinnen, Primack, sjb

- **Complex vacuum even for QED**

- **None of these complications occur for quantization at fixed LF time (front form)**

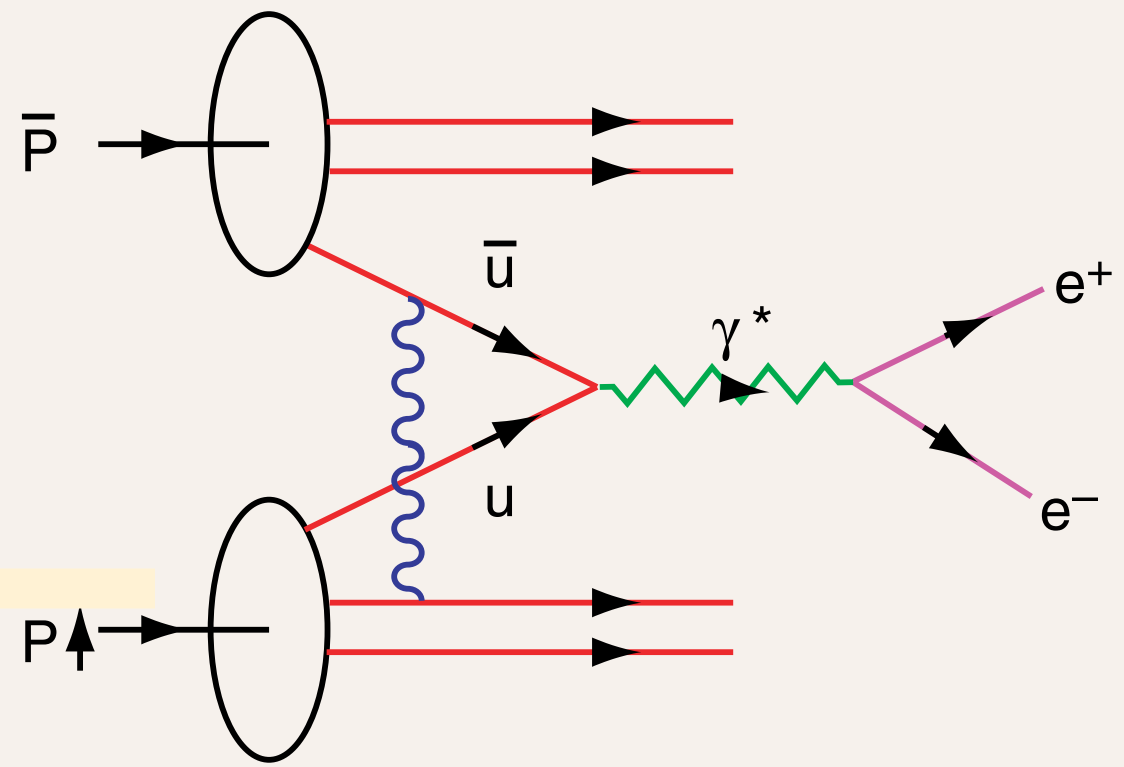
Key QCD Experiment

Collins;
Hwang, Schmidt.
sjb

Measure single-spin asymmetry A_N in Drell-Yan reactions

Leading-twist Bjorken-scaling A_N from S, P-wave initial-state gluonic interactions

Predict: $A_N(DY) = -A_N(DIS)$
Opposite in sign!

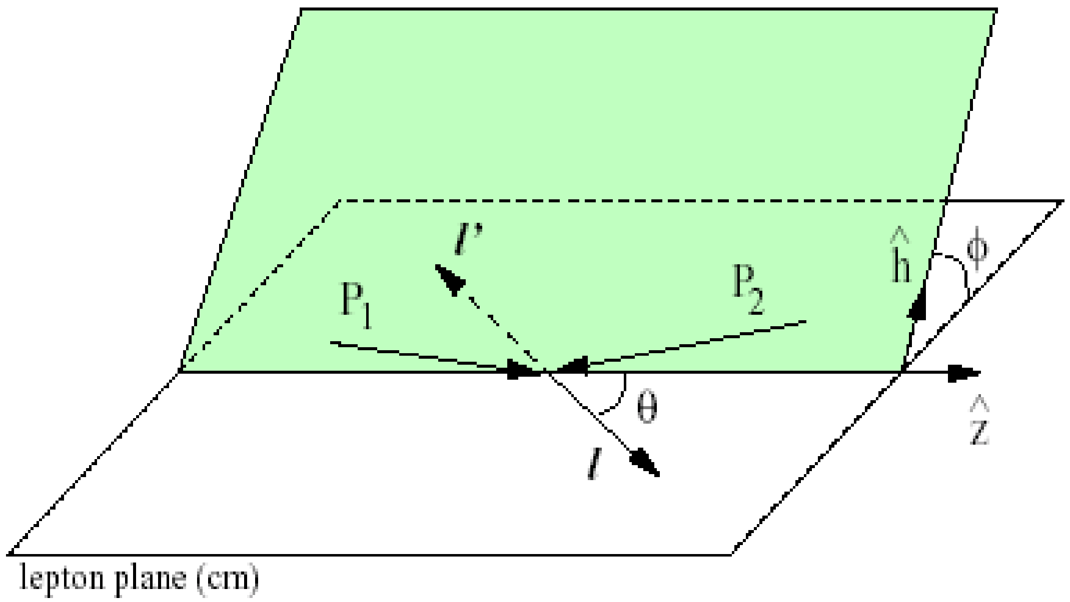


$$\bar{p}p_{\uparrow} \to \ell^+\ell^- X$$

$\vec{S} \cdot \vec{q} \times \vec{p}$ correlation

Drell-Yan angular distribution

Unpolarized DY



- Experimentally, a violation of the Lam-Tung sum rule is observed by sizeable cos2Φ moments
- Several model explanations
  - higher twist
  - spin correlation due to non-triva QCD vacuum
  - Non-zero Boer Mulders function

$\mathrm{Lam-Tung\ SR:}\ 1-\lambda=2\nu$

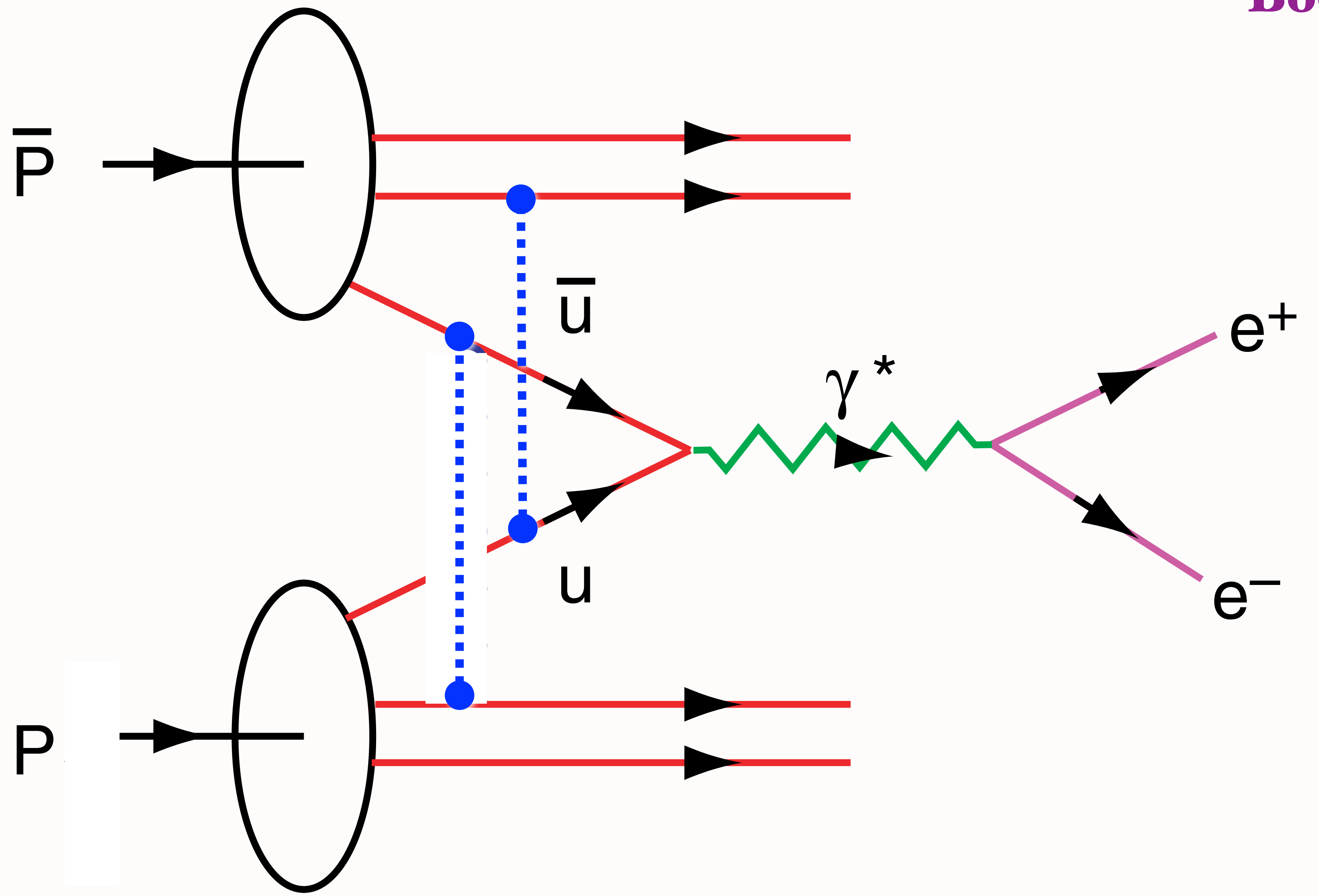
$\mathrm{NLO\ pQCD:}\ \lambda\approx 1\ \mu\approx 0\ \nu\approx 0$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega}=\frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

$$\mathrm{Experiment:}\ \nu\simeq 0.6$$

B. Seitz

Boer, Hwang, sjb



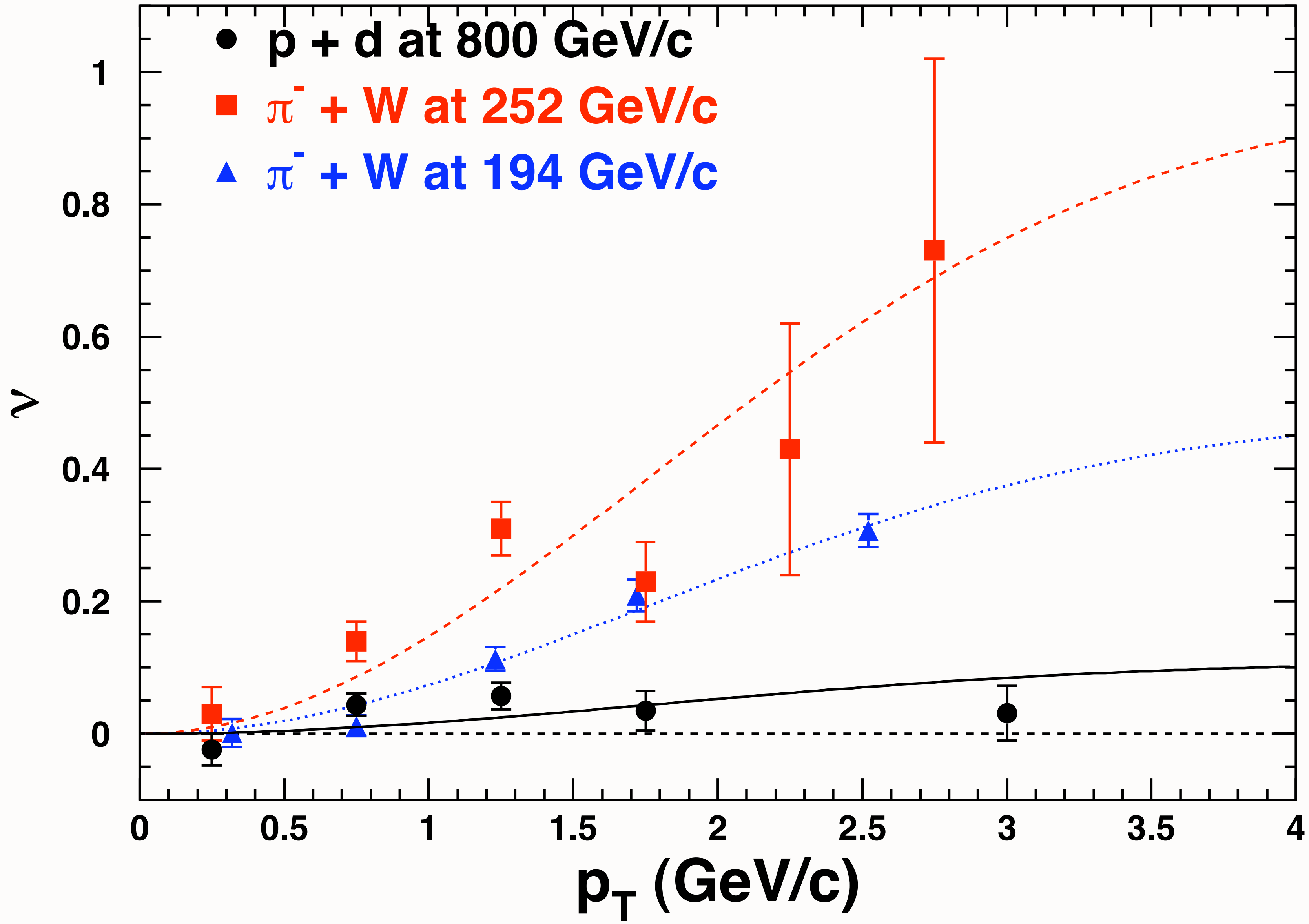
DY $\cos 2\phi$ correlation at leading twist from double ISI

Product of Boer - Mulders Functions

$$h_1^{\perp}(x_1, \boldsymbol{p}_{\perp}^2) \times \bar{h}_1^{\perp}(x_2, \boldsymbol{k}_{\perp}^2)$$

Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)



Huge Effect in $\pi W \to \mu^+ \mu^- X$

Negligible Effect $pd \to \mu^+ \mu^- X$

Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

Double Initial-State Interactions generate anomalous $\cos 2\phi$

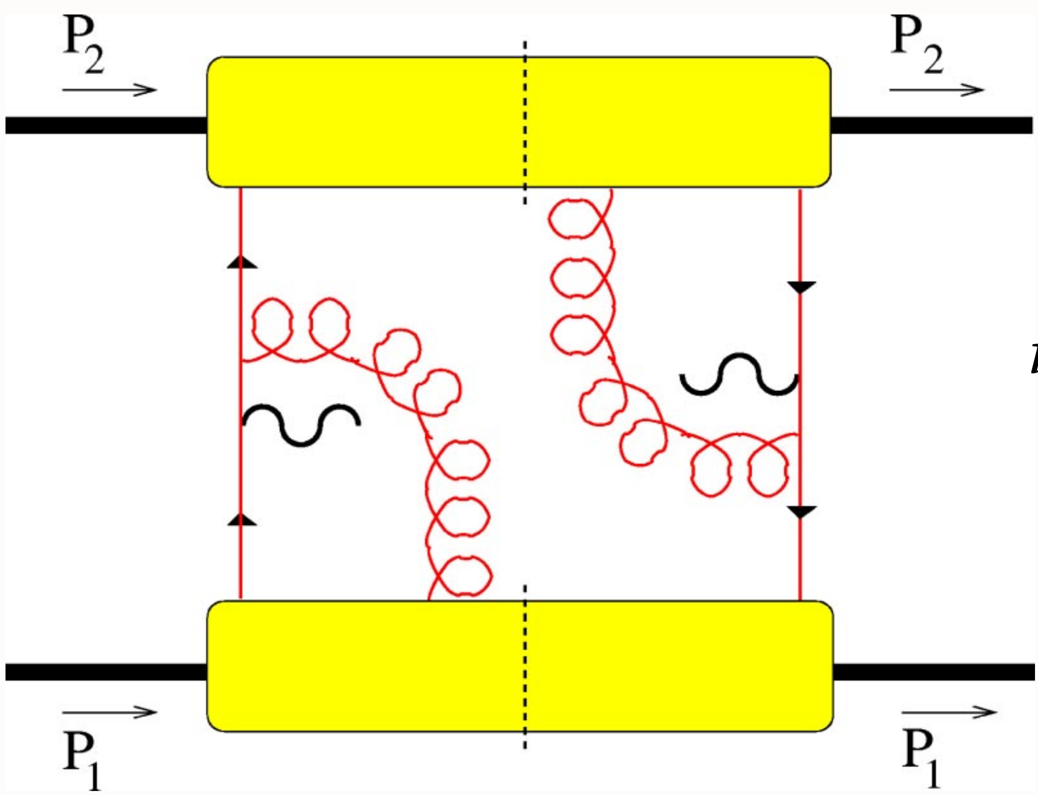
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$

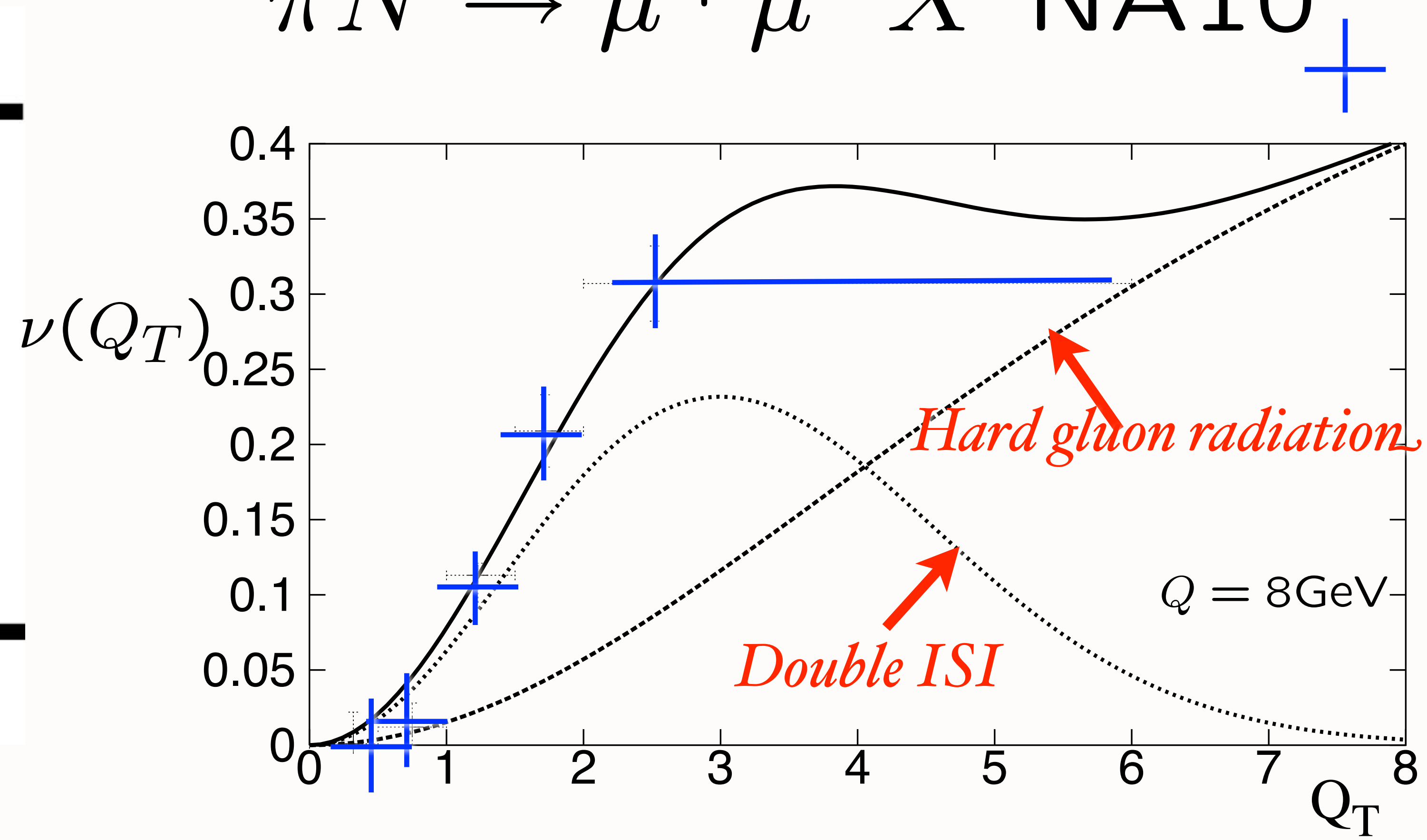
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$



Violates Lam-Tung relation!

$\pi N \to \mu^+ \mu^- X$ NA10

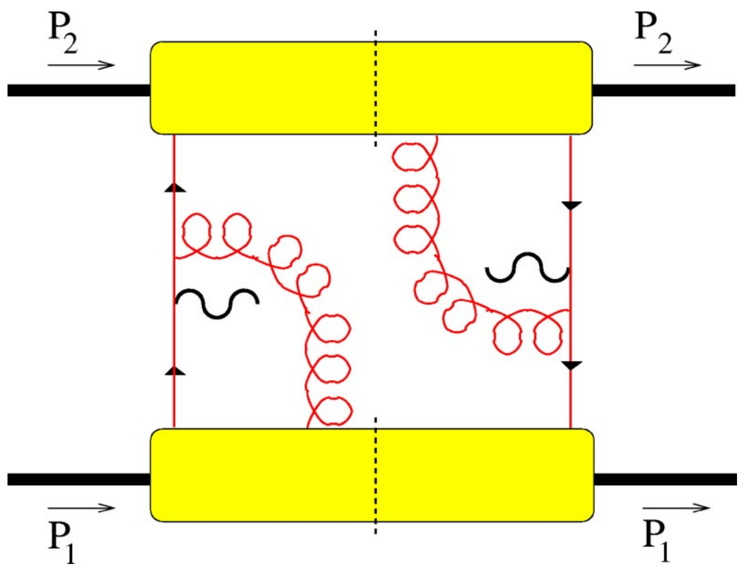


Model: Boer,

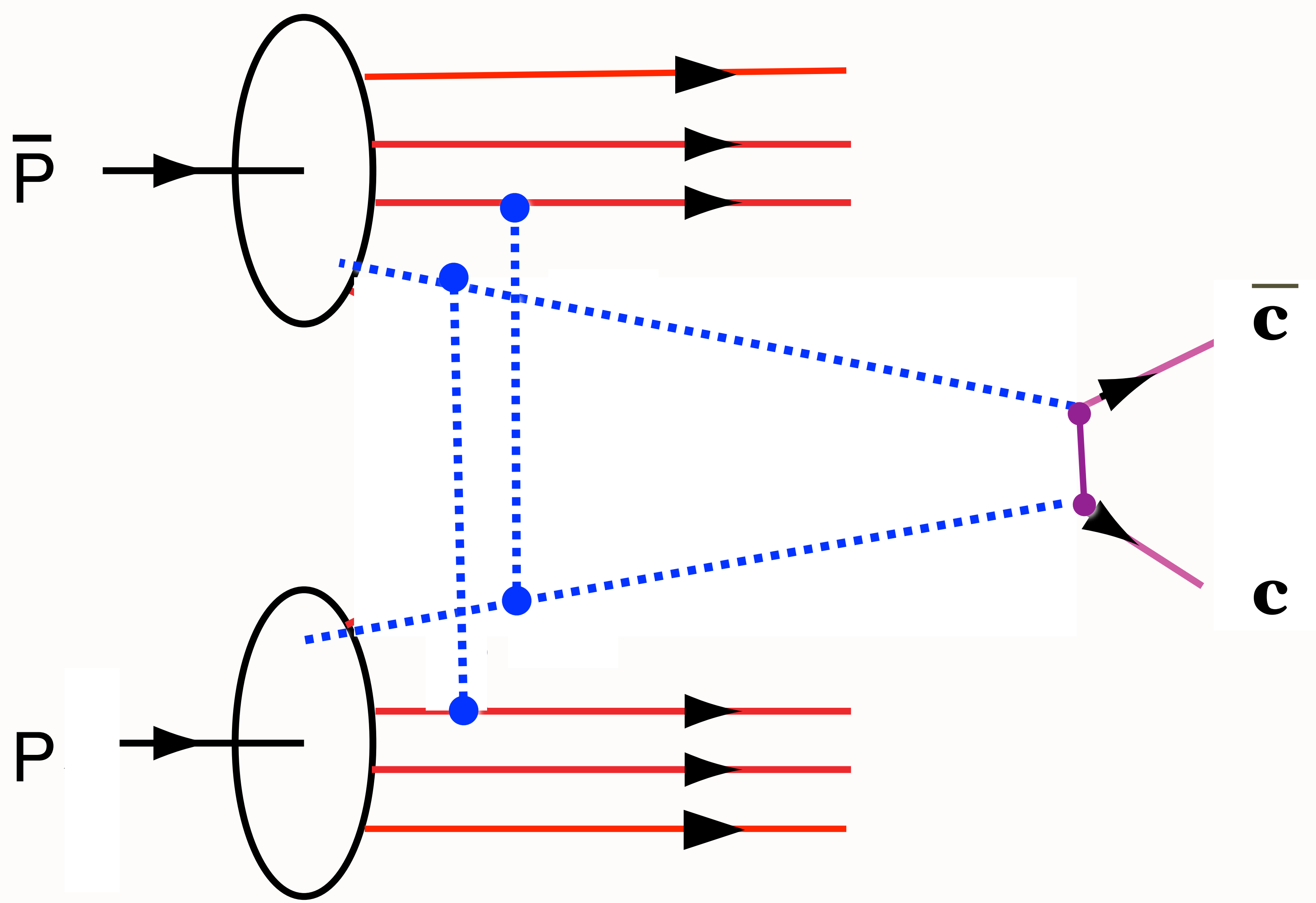
Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation



- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization



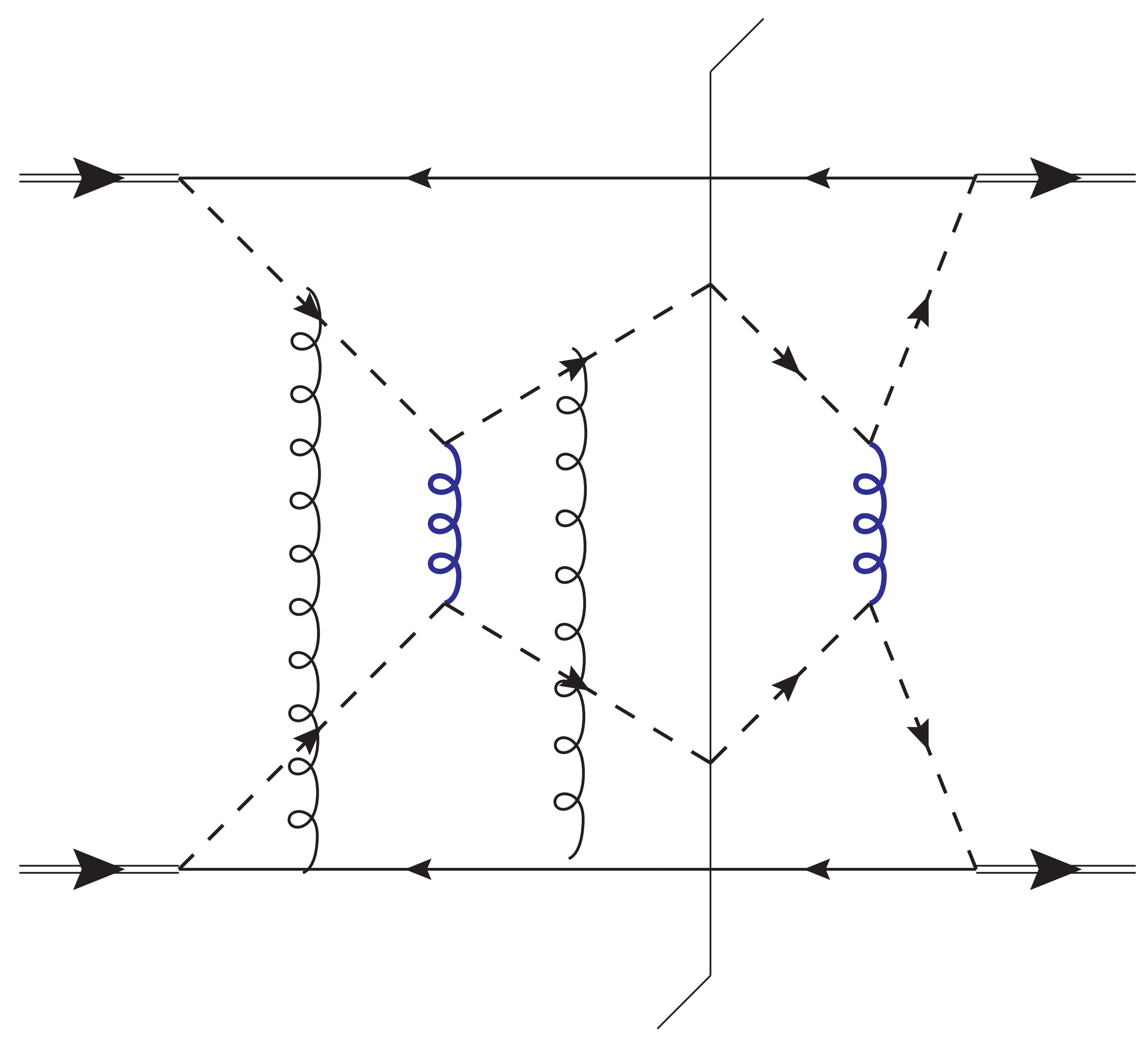
$\cos 2\phi$ **correlation for quarkonium production at leading twist from double ISI**

Enhanced by gluon color charge

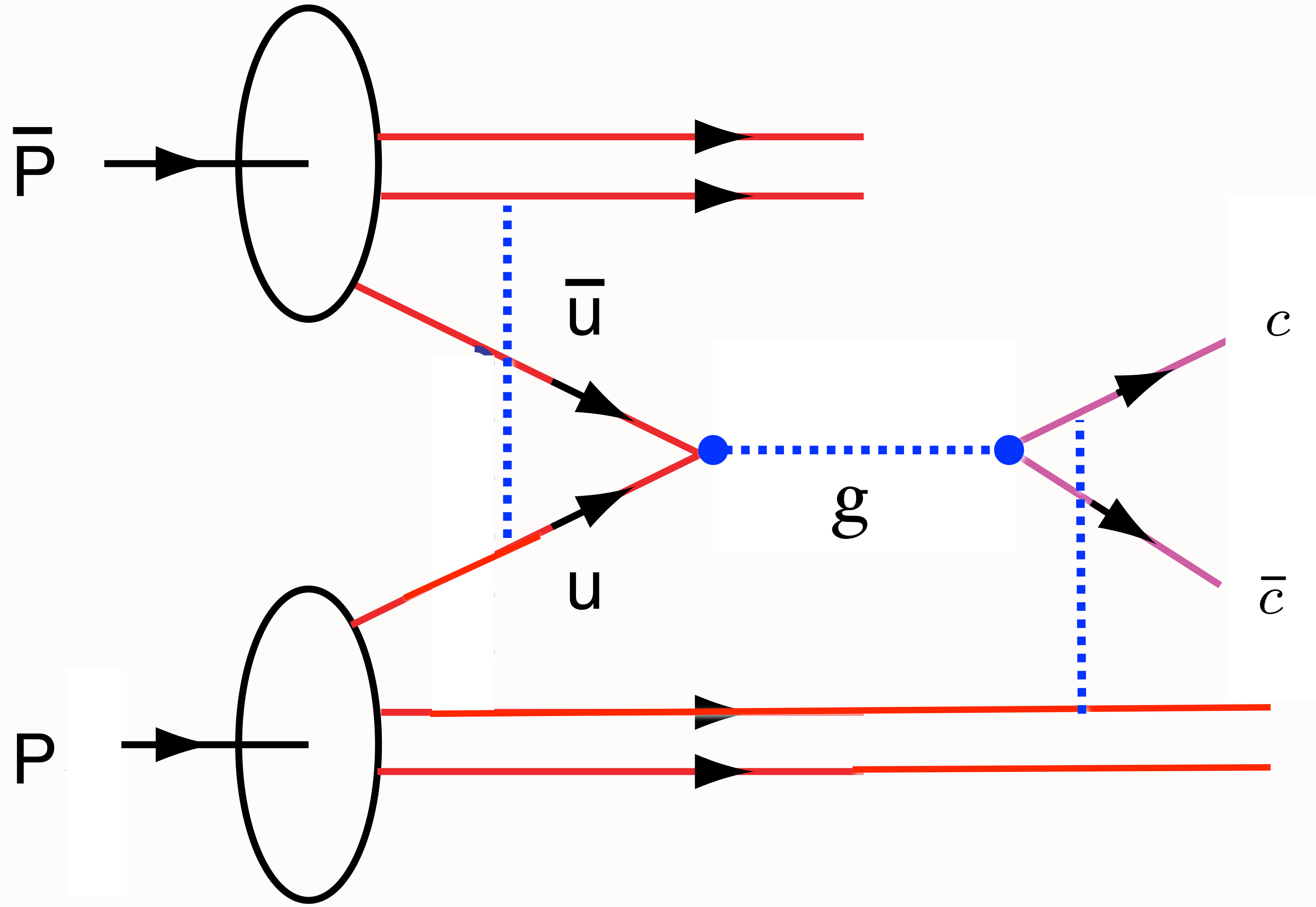
Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

e-Print: **arXiv:0705.2141** [hep-ph]

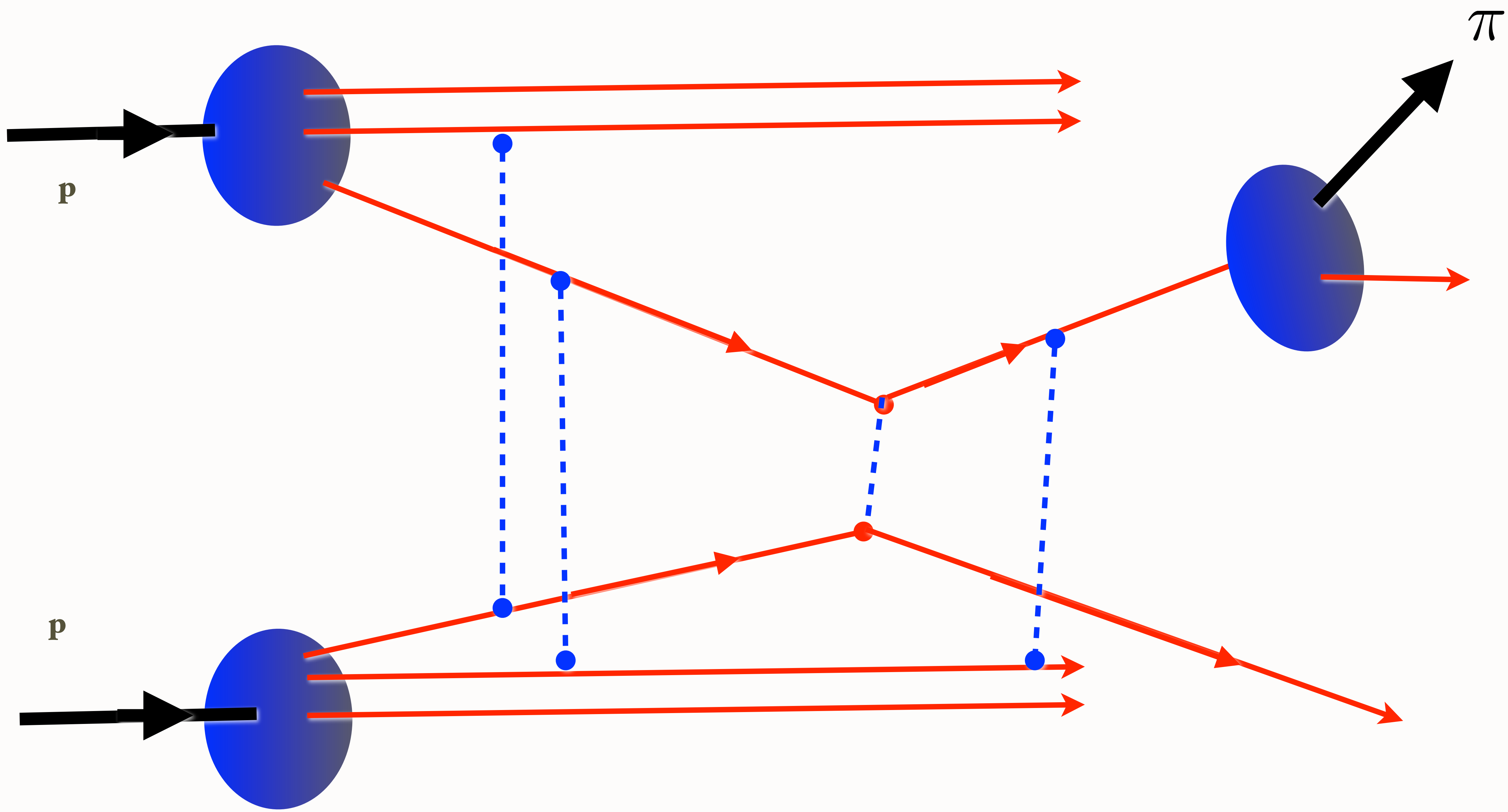


The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.



Problem for factorization when both ISI and FSI occur

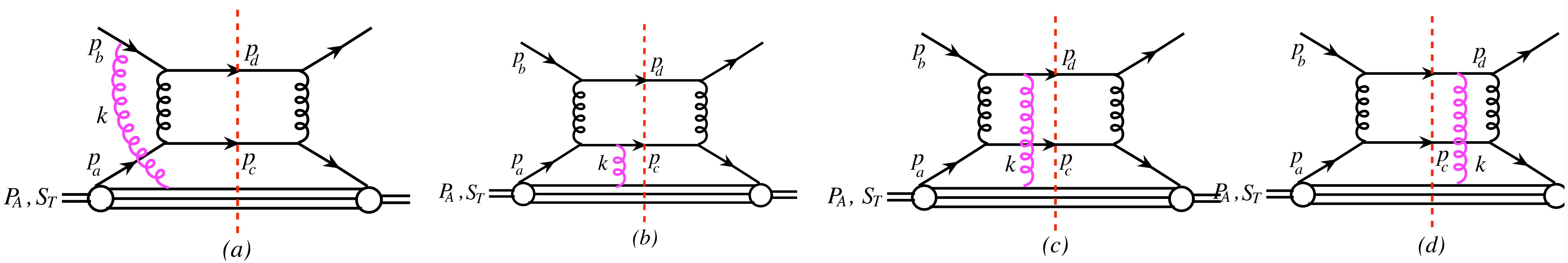
Important Corrections from Initial and Final State Corrections



Sivers & Collins Odd-T Spin Effects, Co-planarity Correlations

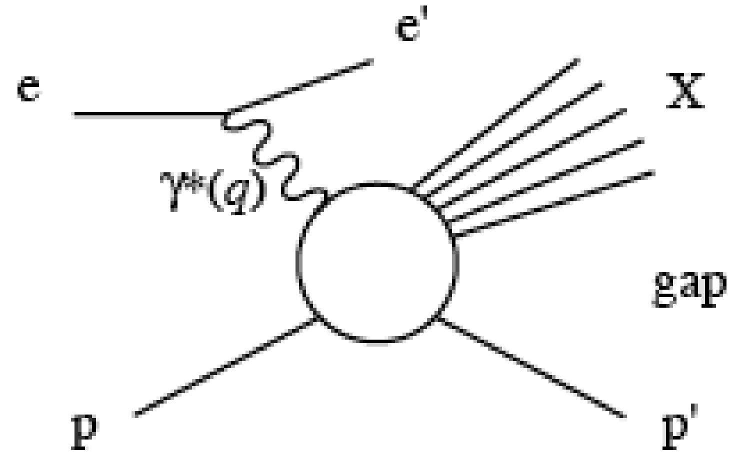
Observation

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- Color factors entirely due to color structure of the partonic subprocess
- consider channel $qq' \to qq'$



Gamberg

DDIS



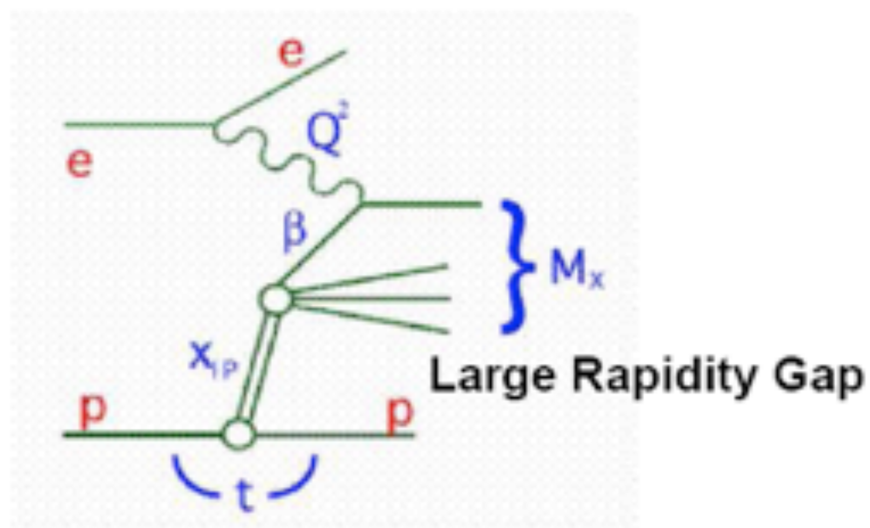
FNAL: Goulianas

- In a large fraction ($\sim$ 10–15%) of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum
- This leaves a large *rapidity gap* between the proton and the produced particles
- The t-channel exchange must be *color singlet* $\rightarrow$ a pomeron??

Diffractive Deep Inelastic Lepton-Proton Scattering de Roeck

10% to 15% of DIS events are diffractive !

Diffractive Structure Function F_2^D



Diffractive inclusive cross section

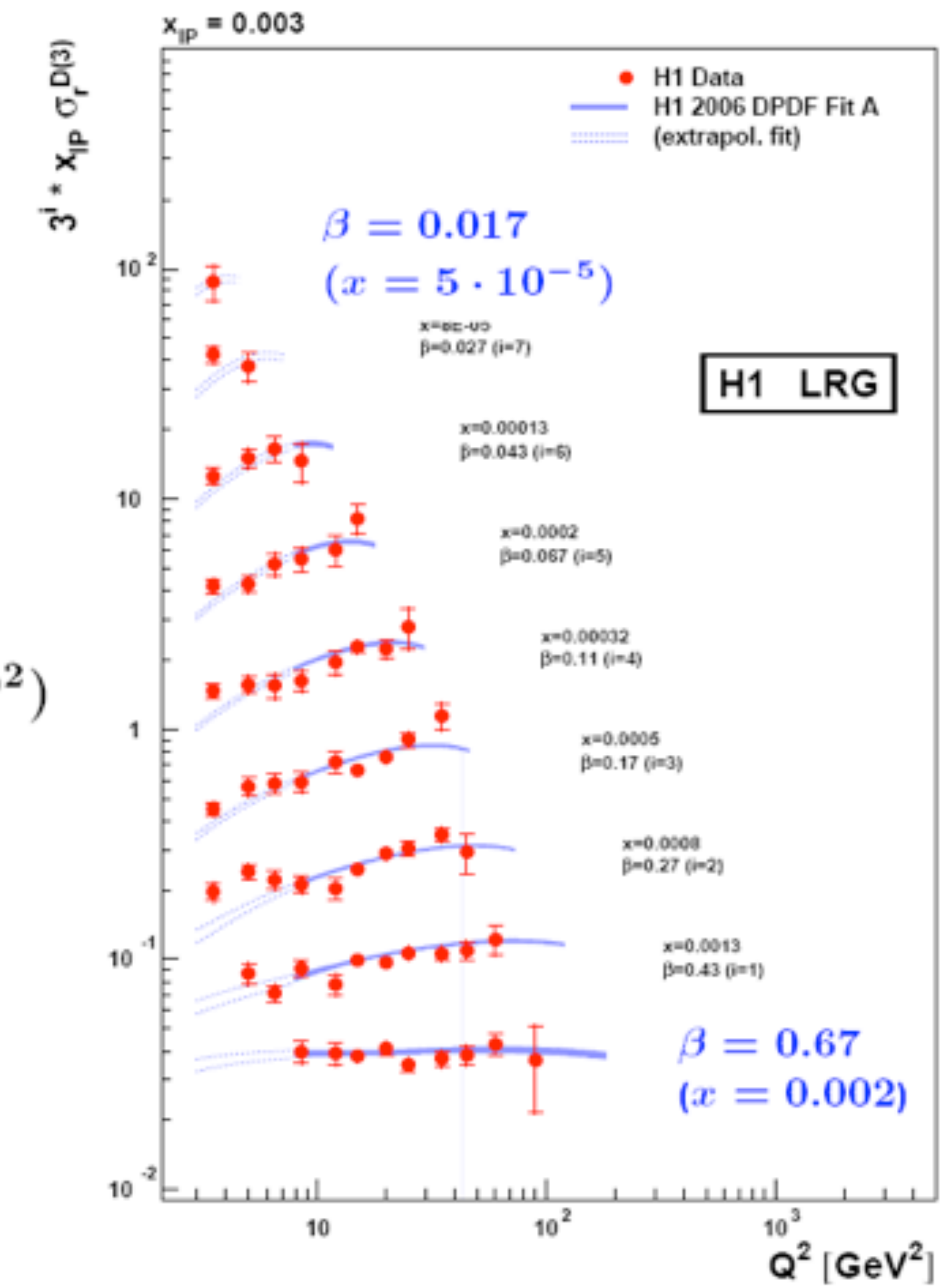
$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\,d\beta\,dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P},\beta,Q^2)$$

$$F_2^D(x_{I\!P},\beta,Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta,Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

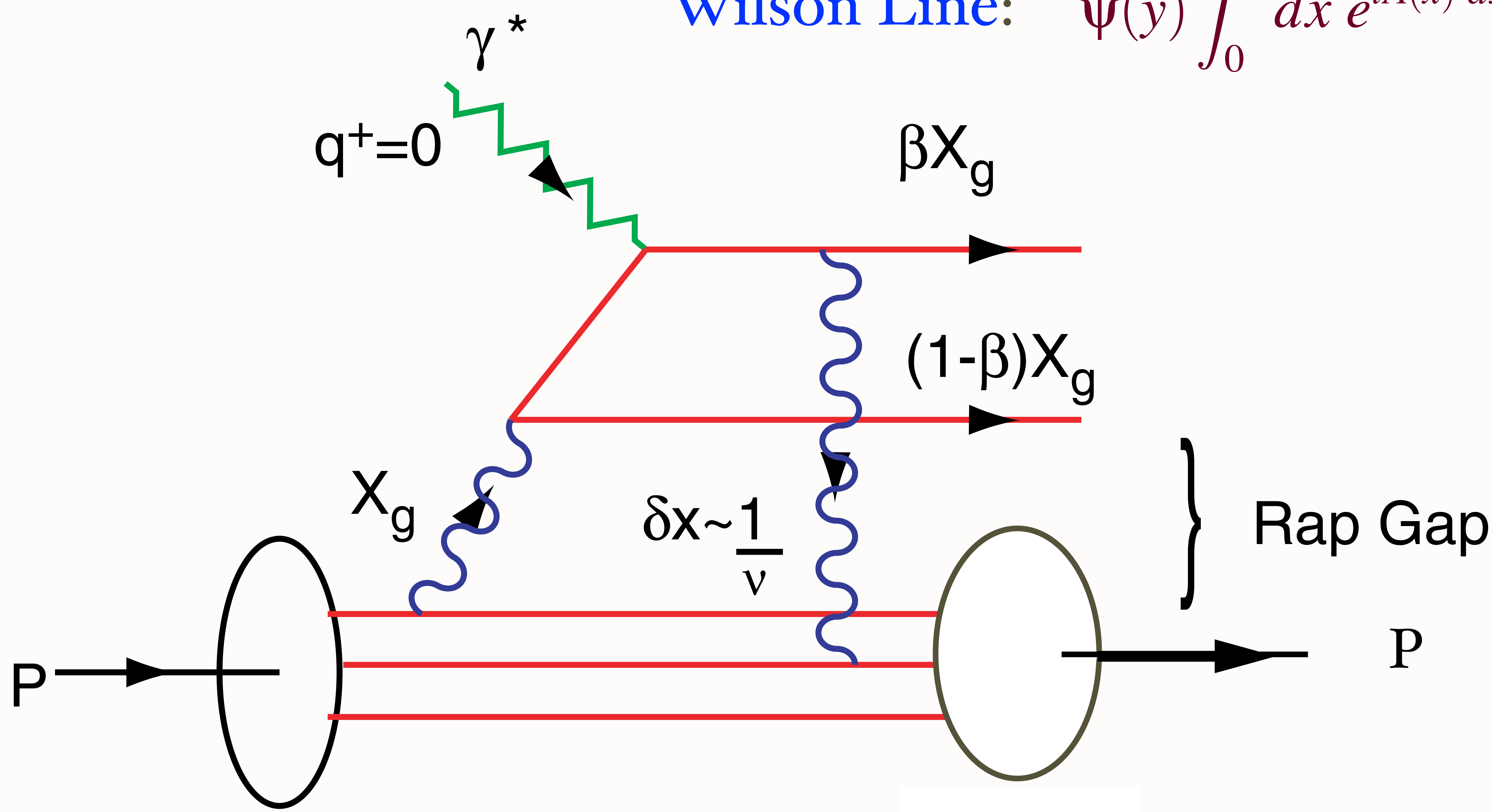
Precise measurements *sys 5%, stat 5–20%*



Hoyer, Marchal, Peigne, Sannino, sjb

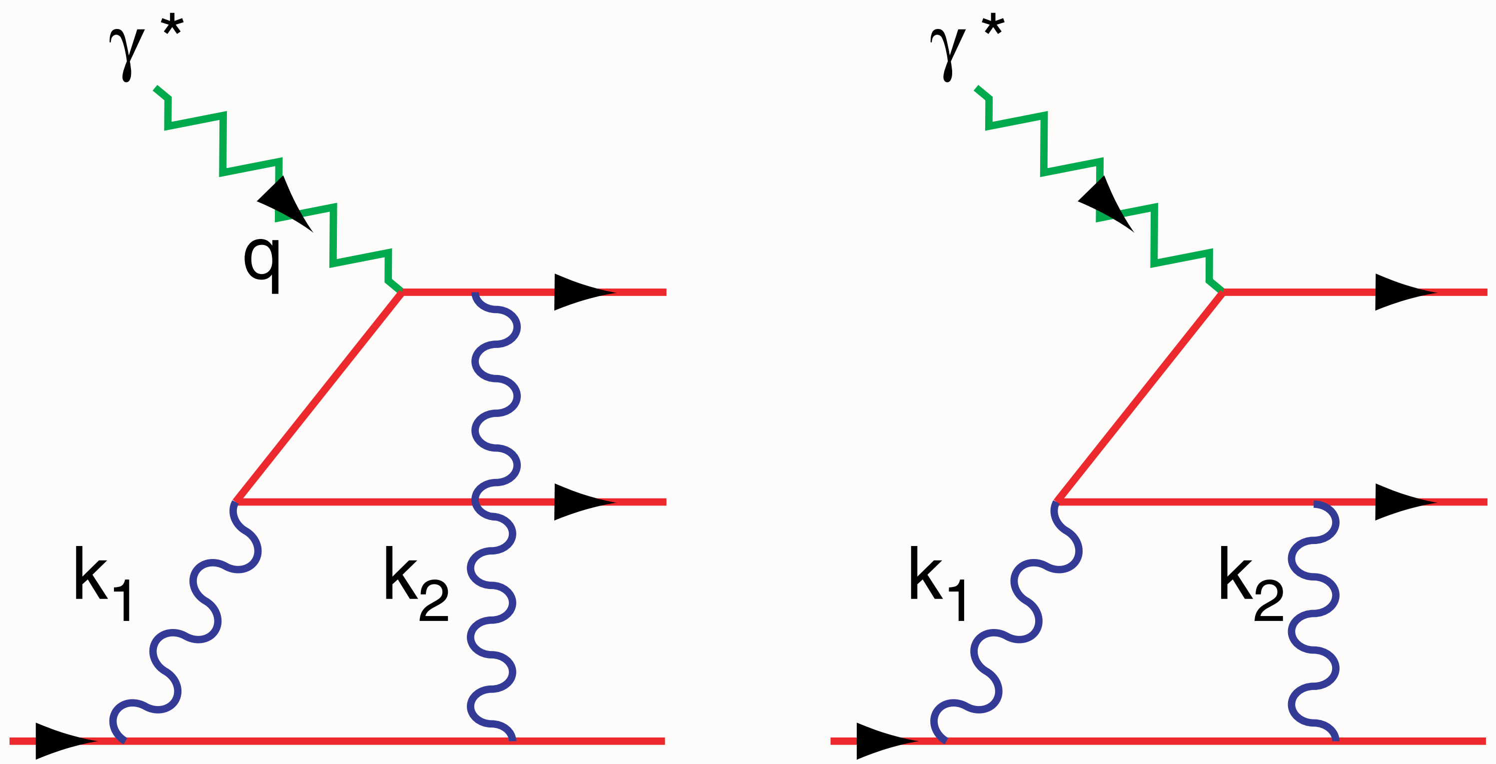
QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



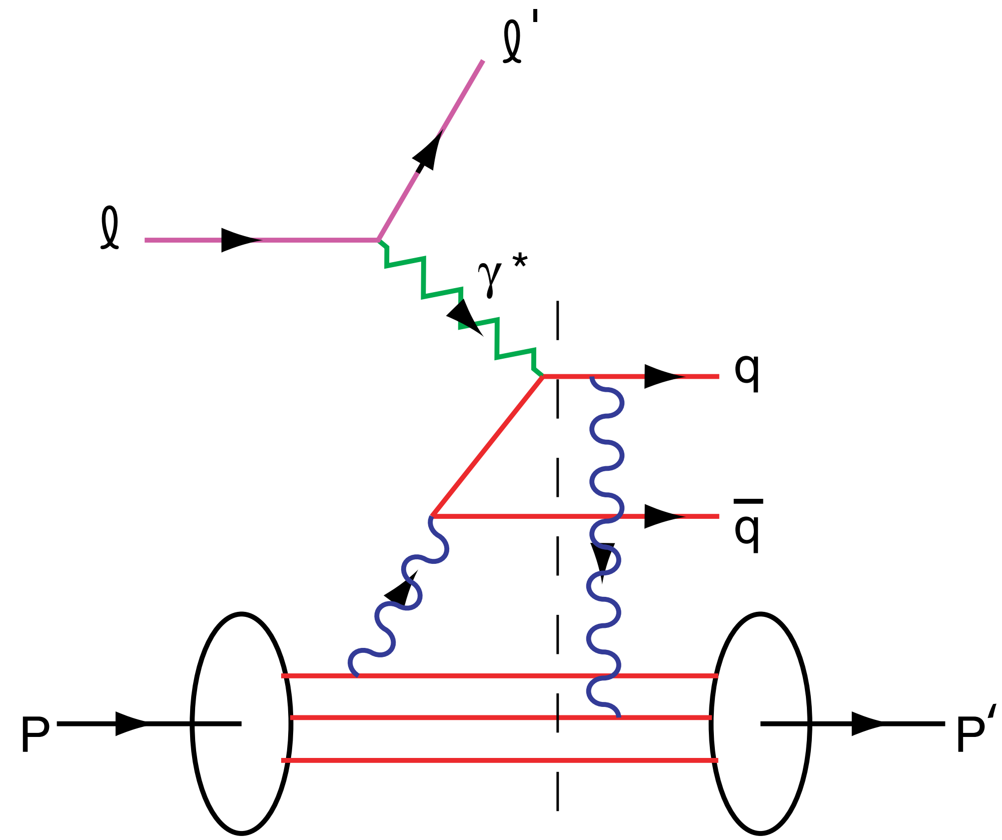
Reproduces lab-frame color dipole approach

Final State Interactions in QCD



Feynman Gauge Light-Cone Gauge

Result is Gauge Independent

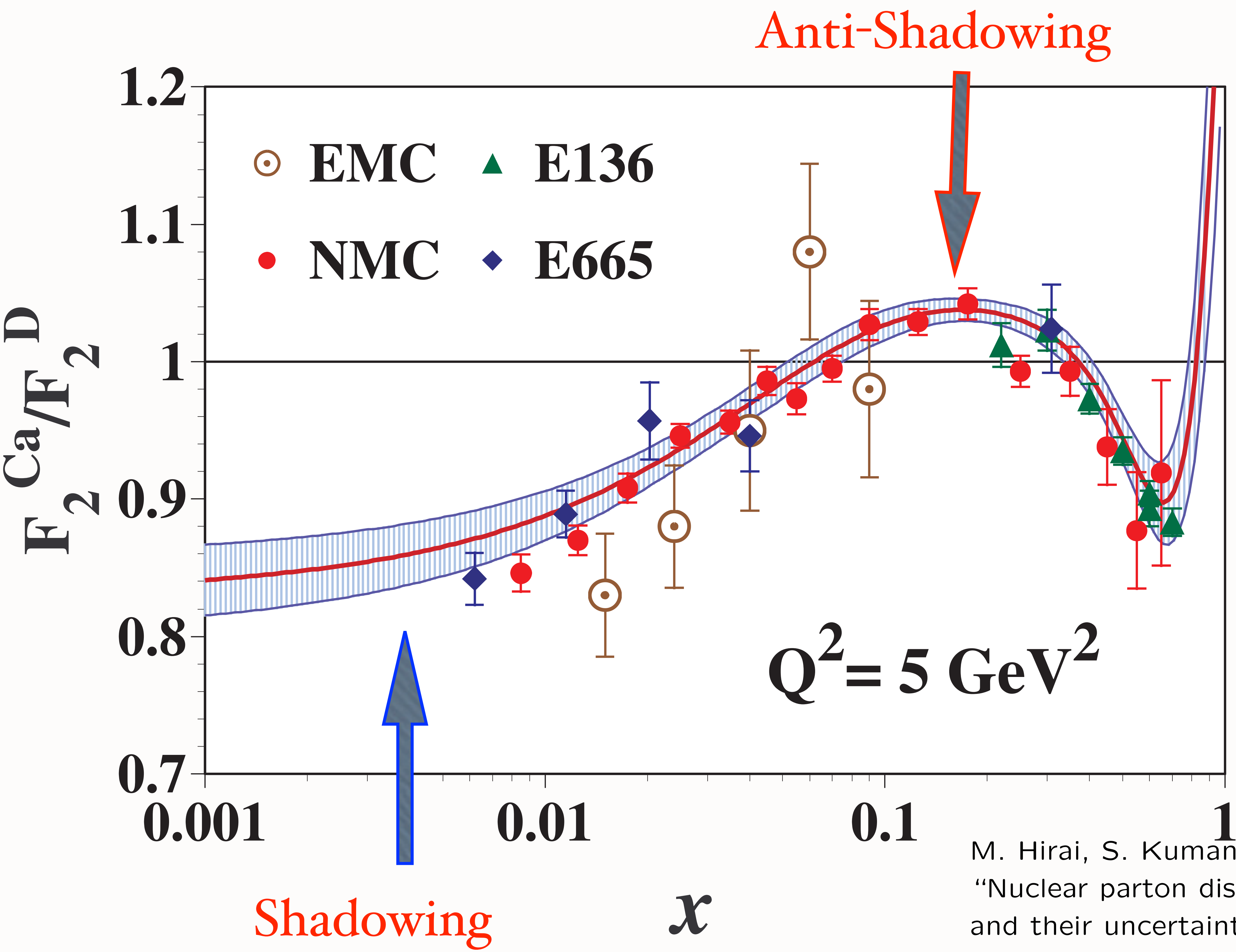


Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron and DDIS

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

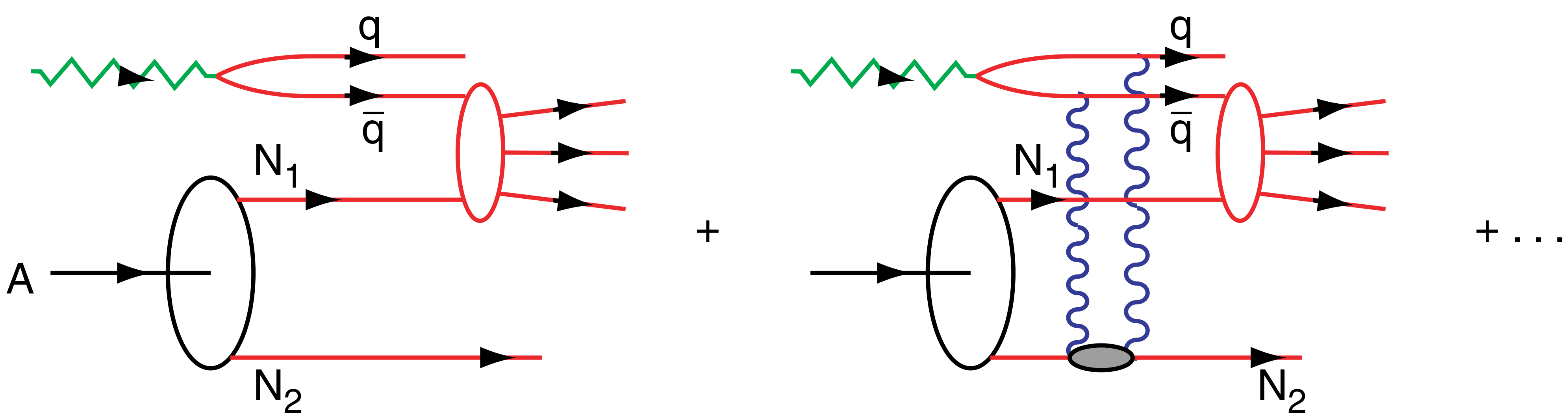
Physics of FSI not in Wavefunction of Target!



M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

Stodolsky
Pumplin, sjb
Gribov

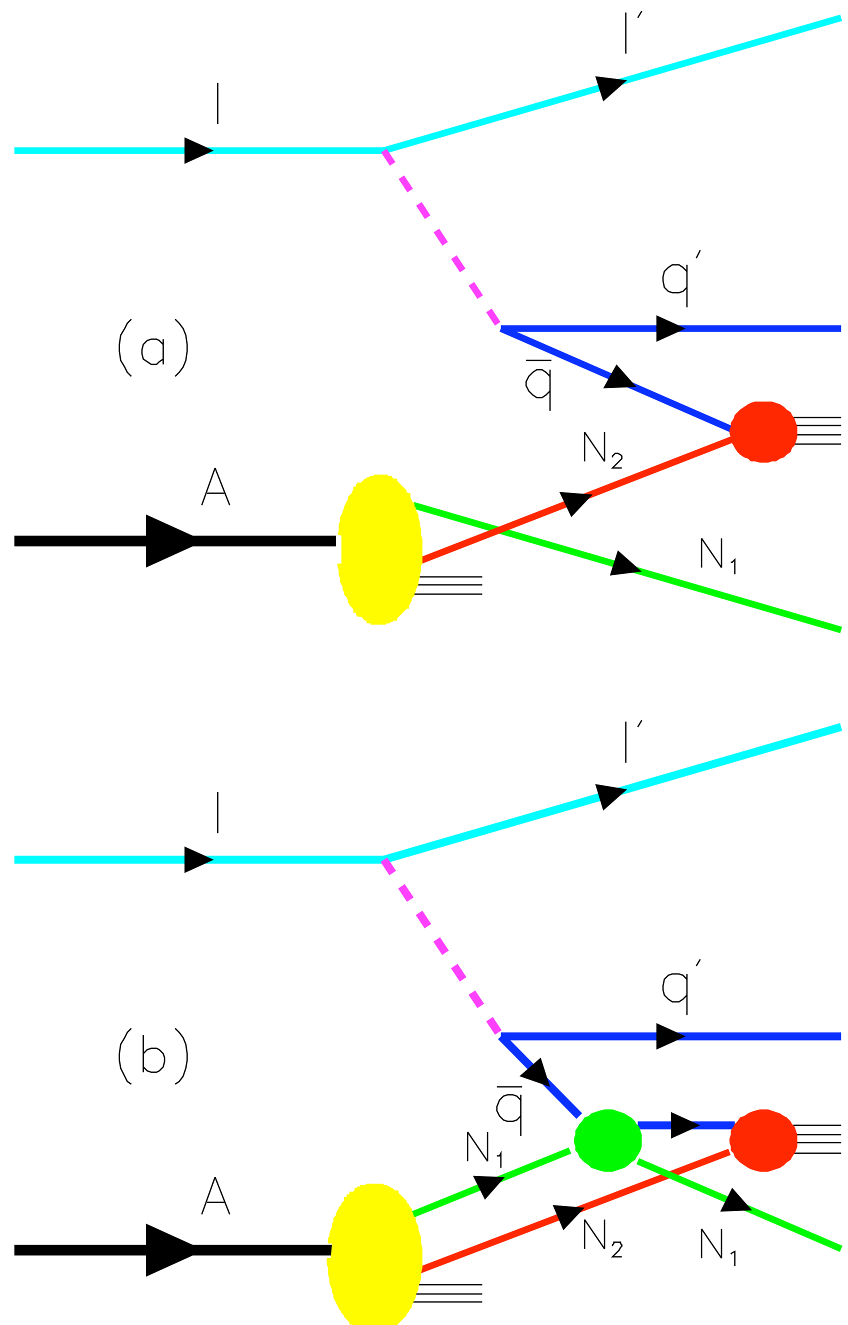
Nuclear Shadowing in QCD



Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus



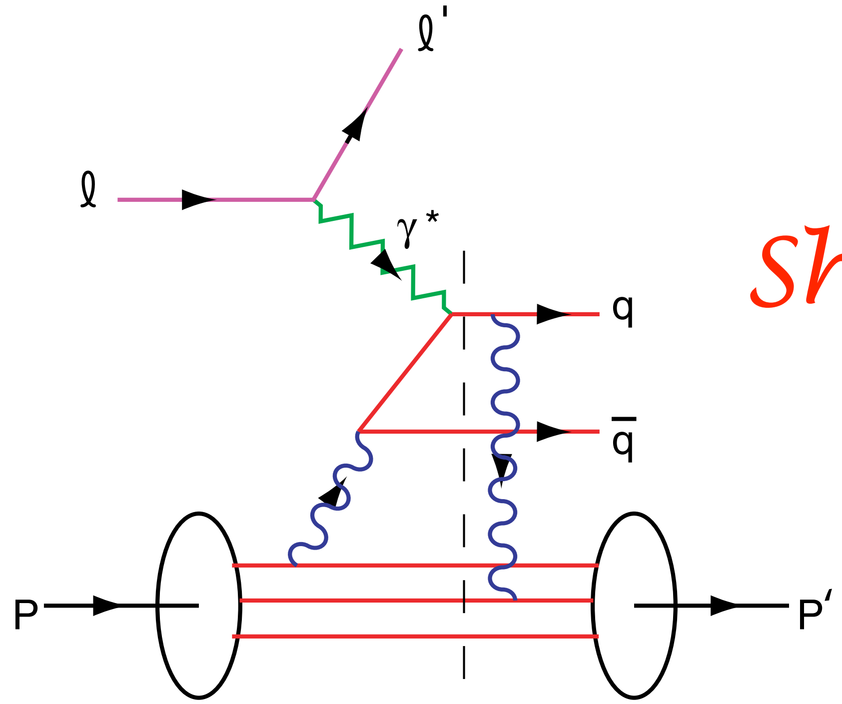
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B : $1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

$\rightarrow$ Shadowing of the DIS nuclear structure functions.

Observed HERA DDIS produces nuclear shadowing



Shadowing depends on leading-twist DDIS

Integration over on-shell domain produces phase i

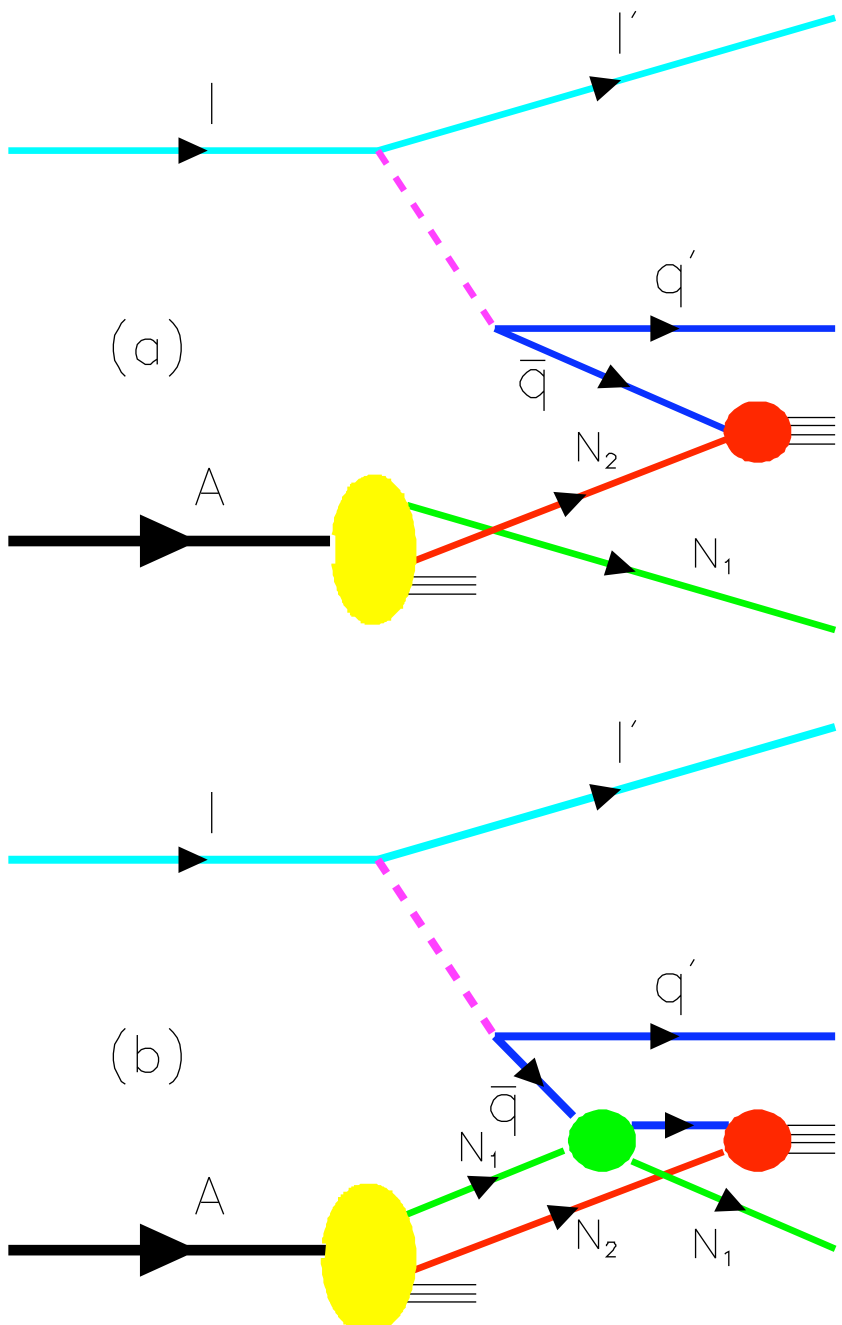
Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb



The one-step and two-step processes in DIS on a nucleus.

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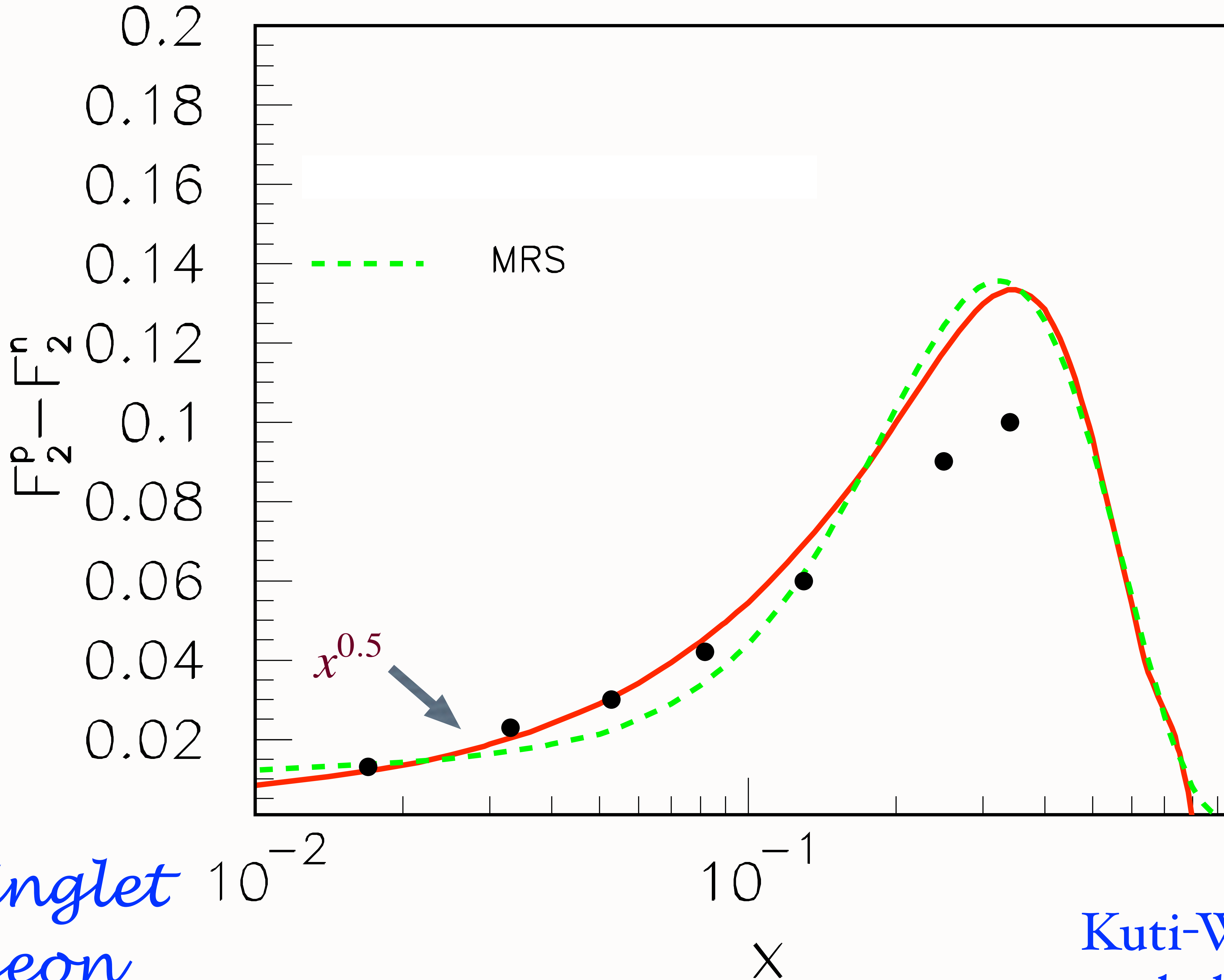
If the scattering on nucleon N_1 is via ~~pomeron~~ *Reggeon* exchange, the one-step and two-step amplitudes are ~~opposite~~ in phase, thus ~~diminishing~~ *increasing* the $\bar{q}$ flux reaching N_2.

→ ***Anti-*** Shadowing of the DIS nuclear structure functions.

Schmidt, Yang, sjb



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i) \times i = \frac{1}{\sqrt{2}}(i+1)$$

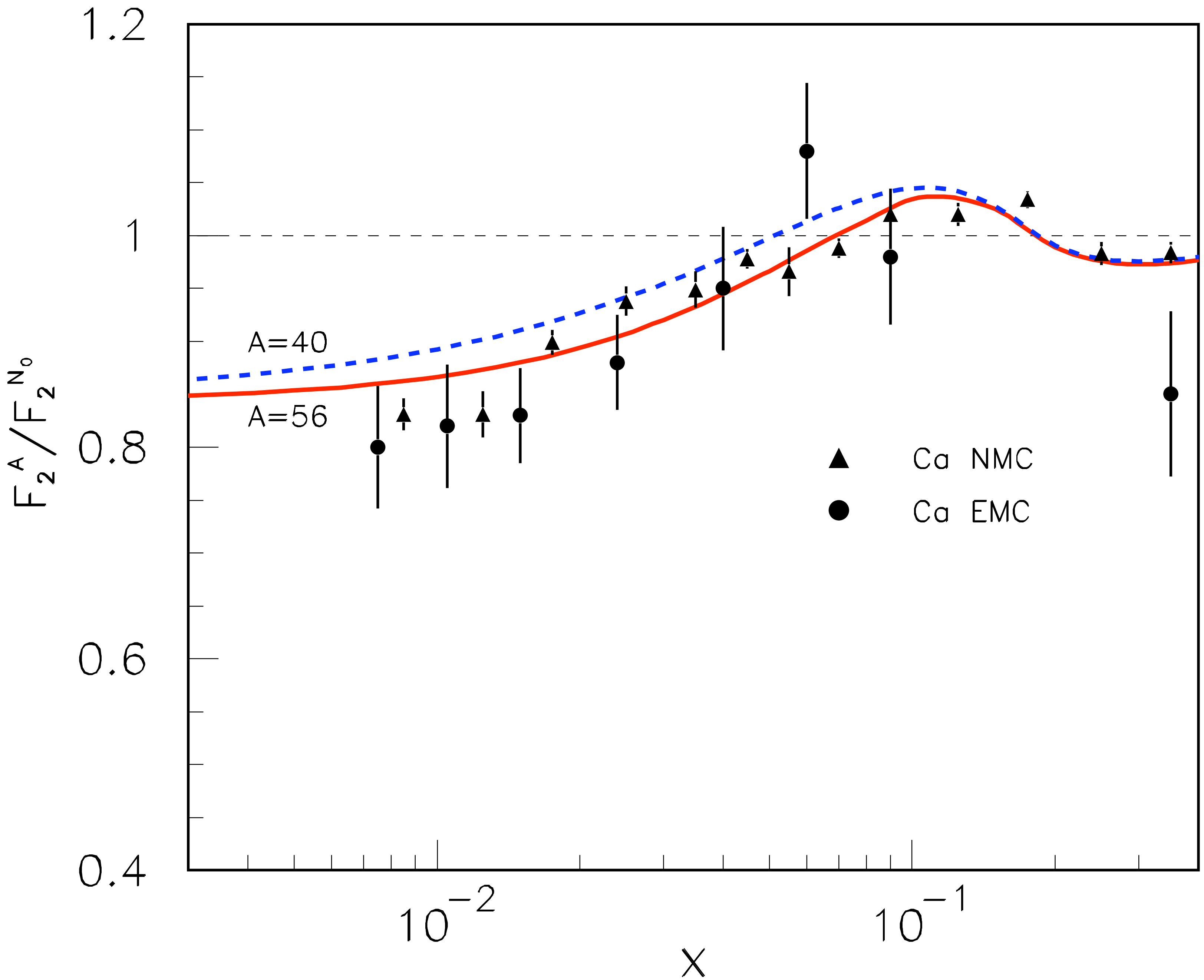
Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

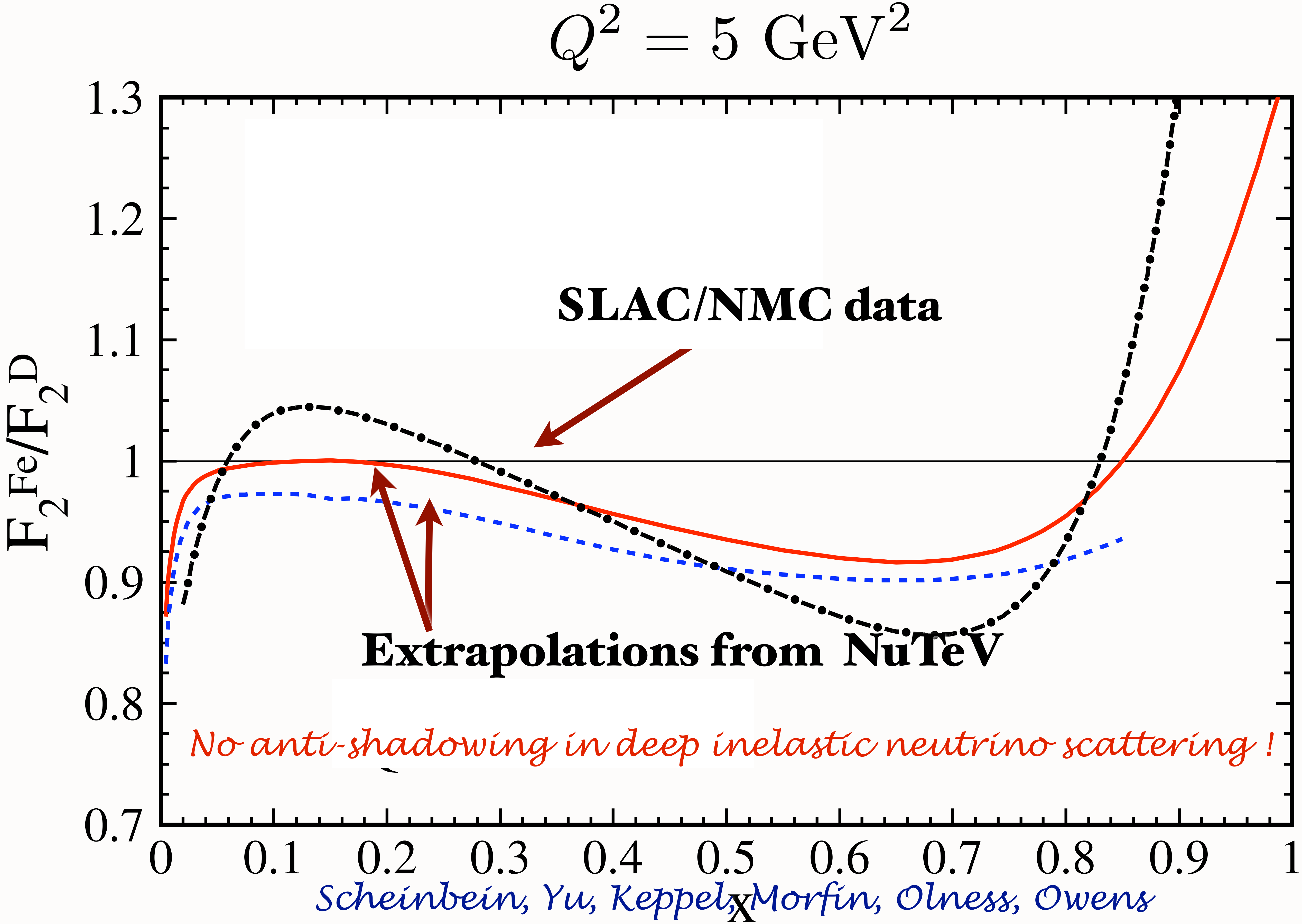
Different for couplings of $\gamma^*, Z^0, W^\pm$

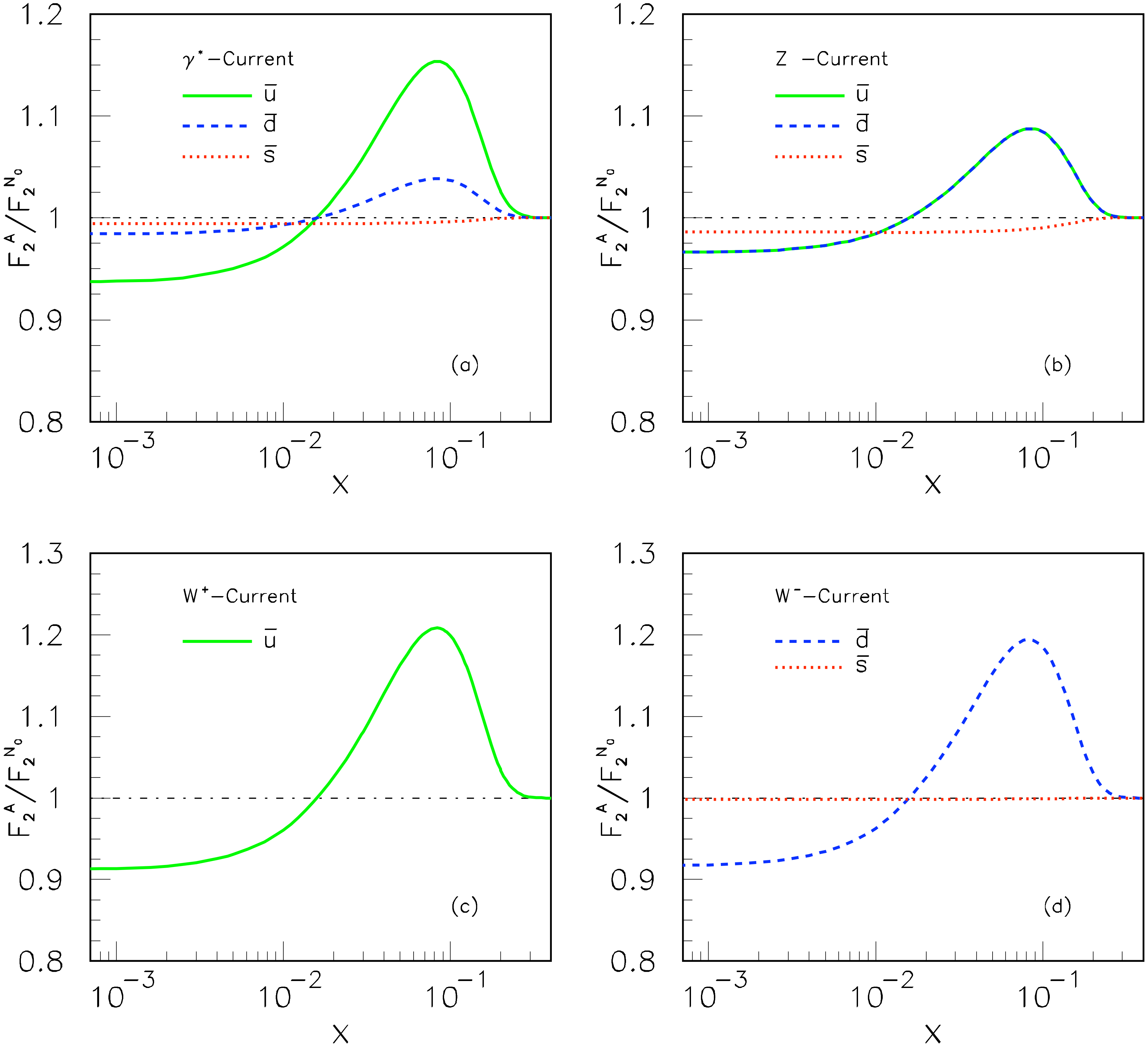
Critical test: Tagged Drell-Yan



Predicted nuclear shadowing and and antishadowing at $Q^2 = 1\ \text{GeV}^2$

S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

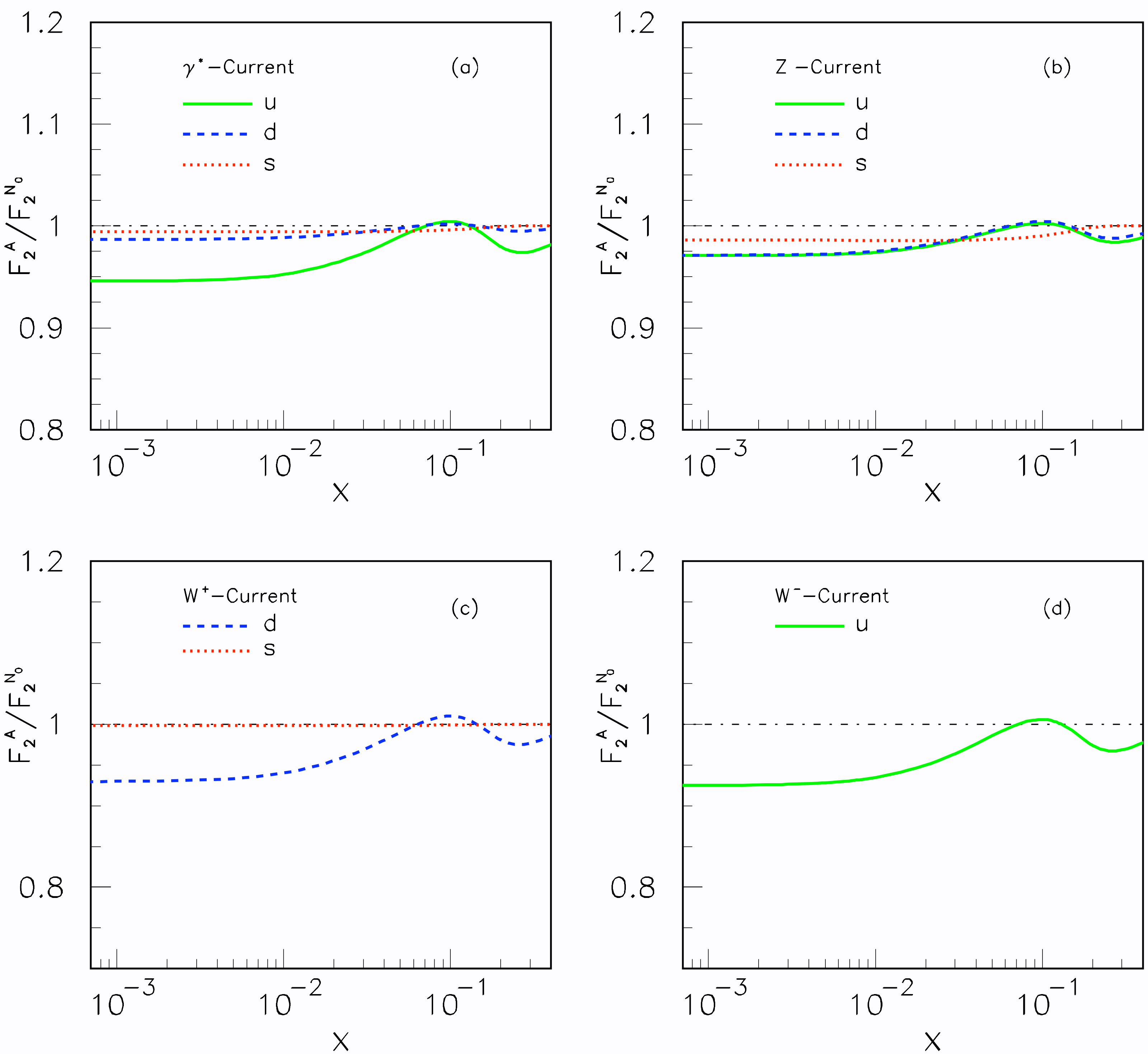

$Q^2 = 5\ \mathrm{GeV}^2$
1.3
1.2
1.1
1
0.9
0.8
0.7
F_2^{Fe}/F_2^D
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
Scheinbein, Yu, Keppel, Morfin, Olness, Owens



Schmidt, Yang; sjb

Nuclear Antishadowing not universal !

Shadowing and Antishadowing of DIS Structure Functions



S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of

$$\sin^2\theta_W$$

Test in flavor-tagged lepton-nucleus collisions

Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: Destructive Interference of Two-Step and One-Step Processes *Pomeron Exchange*

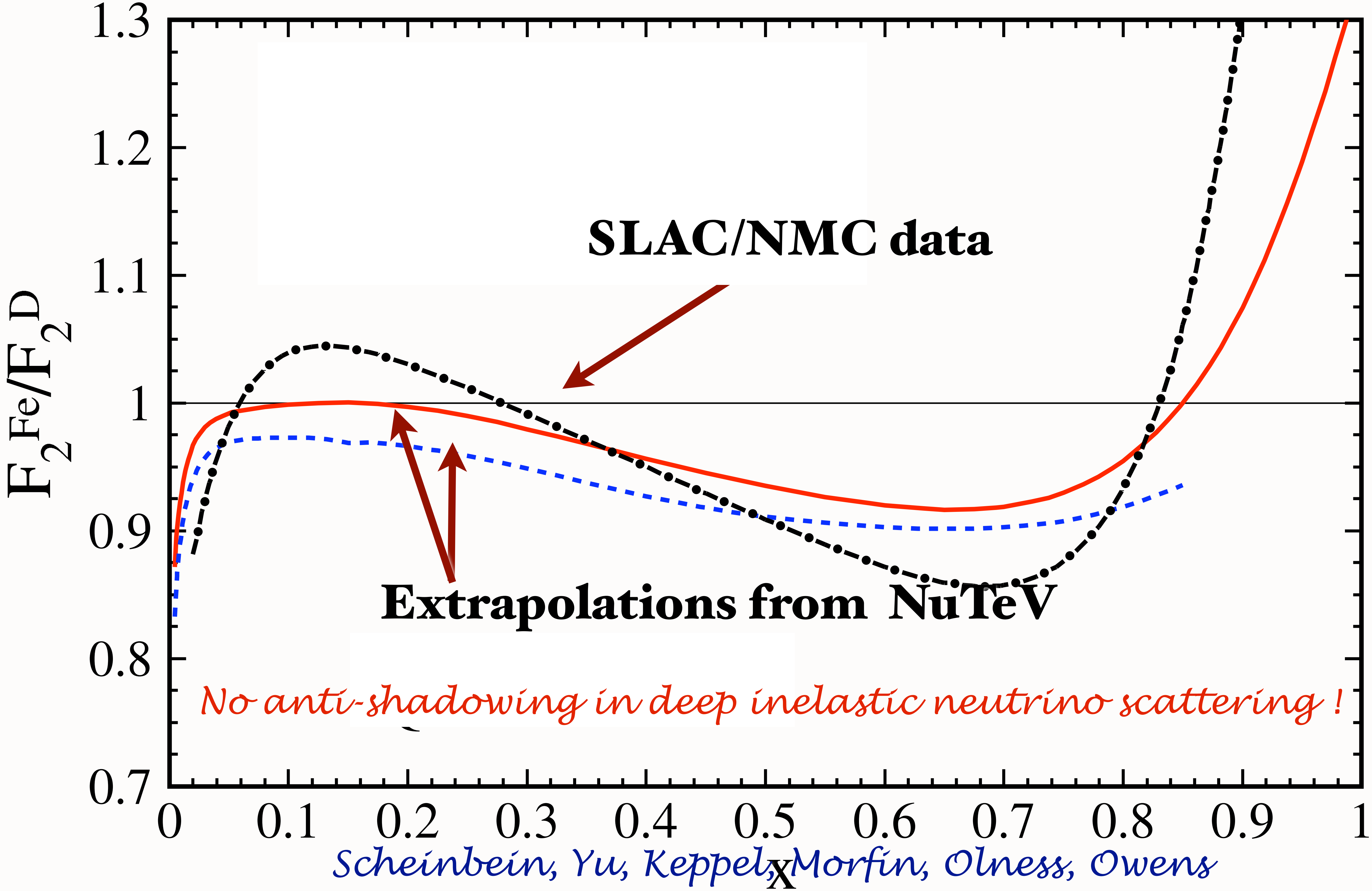
- Antishadowing: Constructive Interference of Two-Step and One-Step Processes! *Reggeon and Odderon Exchange*

- Antishadowing is Not Universal! Electromagnetic and weak currents: different nuclear effects !

Jian-Jun Yang
Ivan Schmidt
Hung Jung Lu
sjb

Can explain NuTeV result!

$Q^2 = 5\ \mathrm{GeV}^2$



Scheinbein, Yu, Keppel, Morfin, Olness, Owens

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

Hwang, Schmidt, sjb,

Mulders, Boer

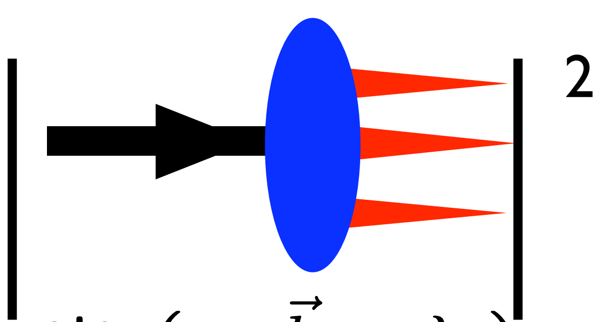
Qiu, Sterman

Collins, Qiu

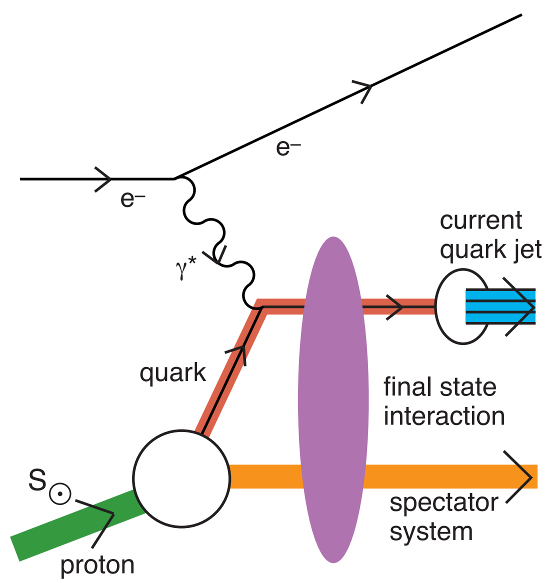
Pasquini, Xiao, Yuan, sjb

Burkardt

Hoyer



$$\left|\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$

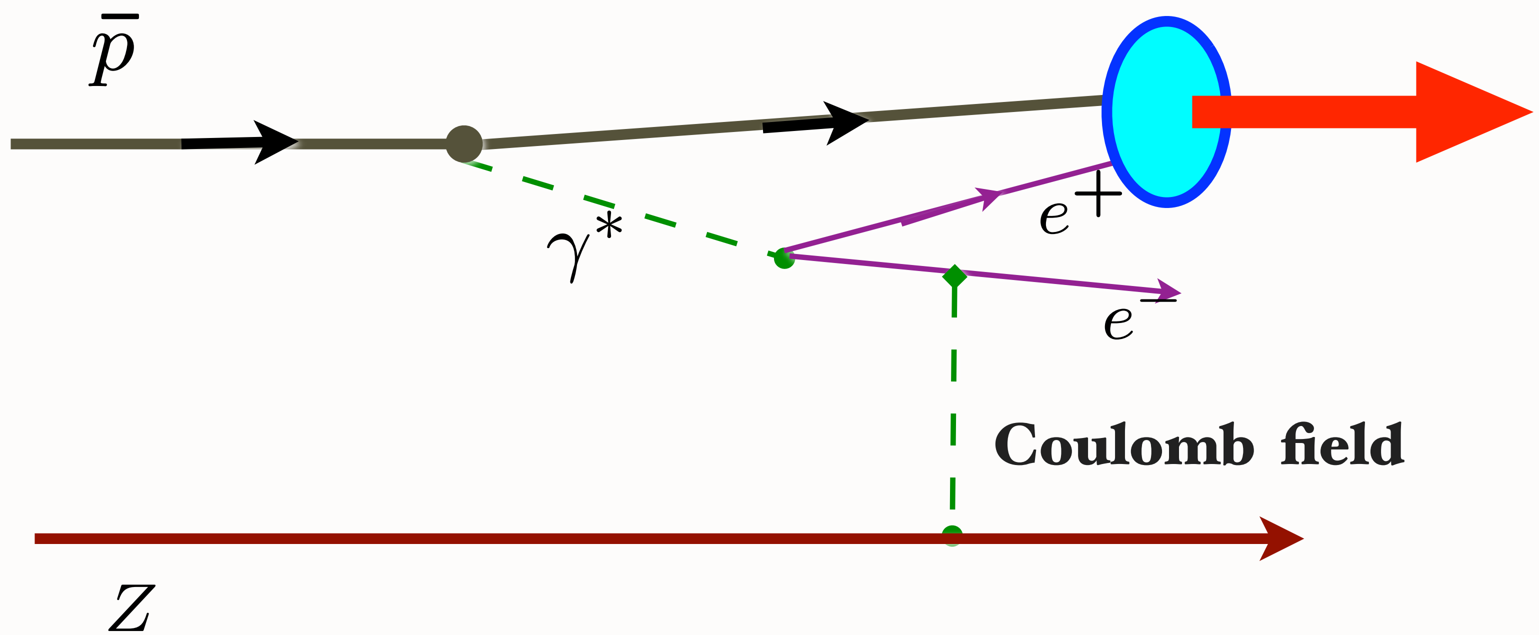


Lensing

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$\bar{H}(\bar{p}e^+)$

$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

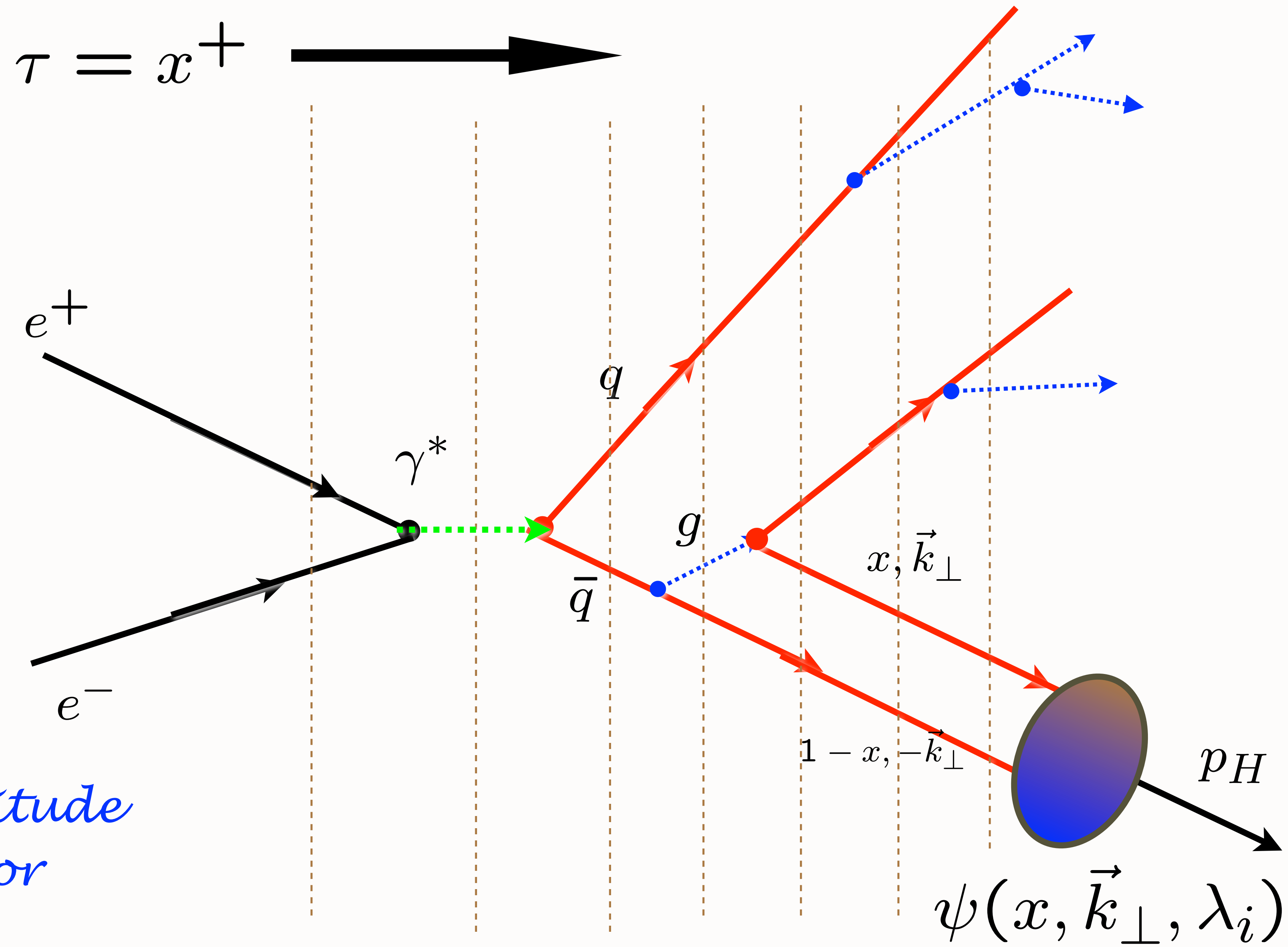
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of** off-shell **co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

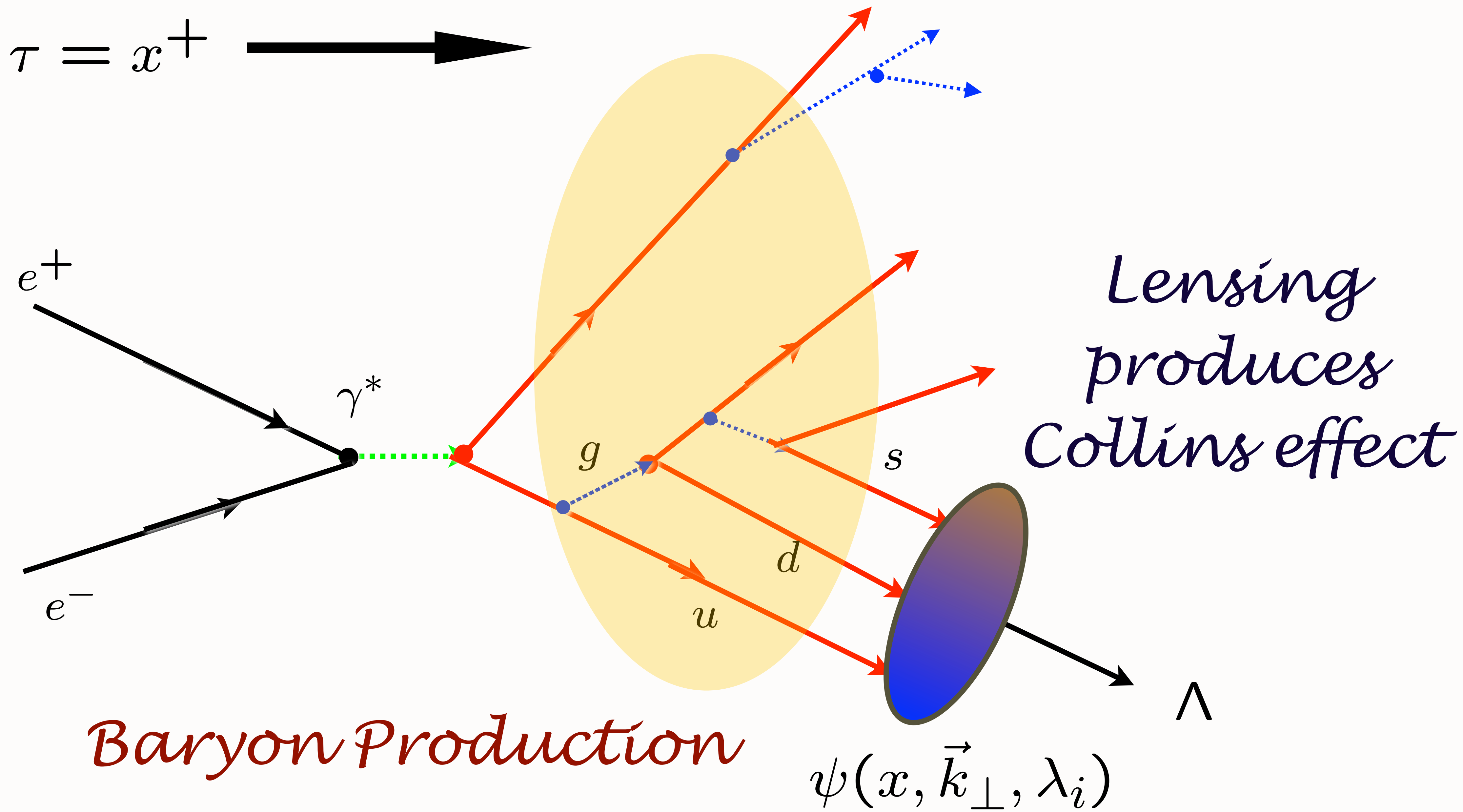
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



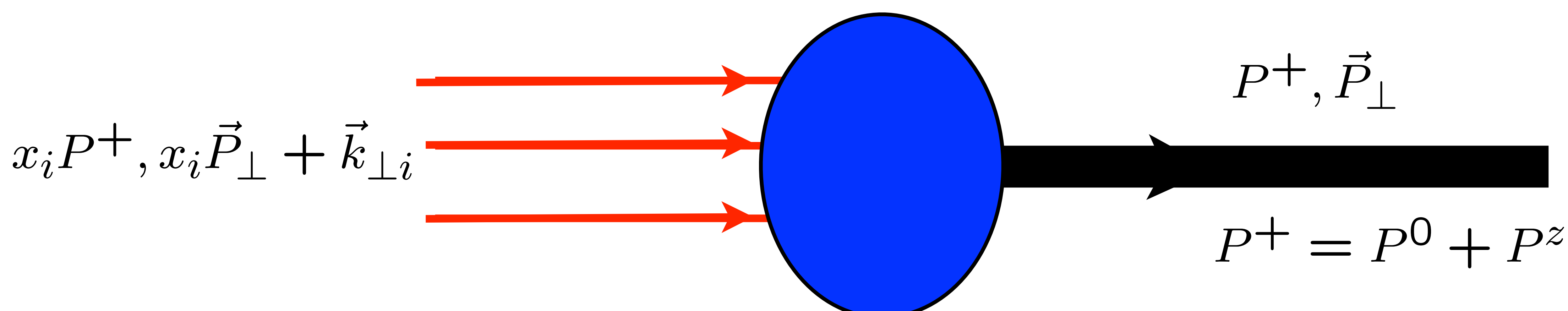
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Features of LF T-Matrix Formalism

"Event Amplitude Generator"

Hadronization at the Amplitude Level!

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin
- Color Transparency
- Lensing



Bjorken, Kogut, Soper; Blankenbecler, Gunion, sjb; Blankenbecler,Schmidt

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

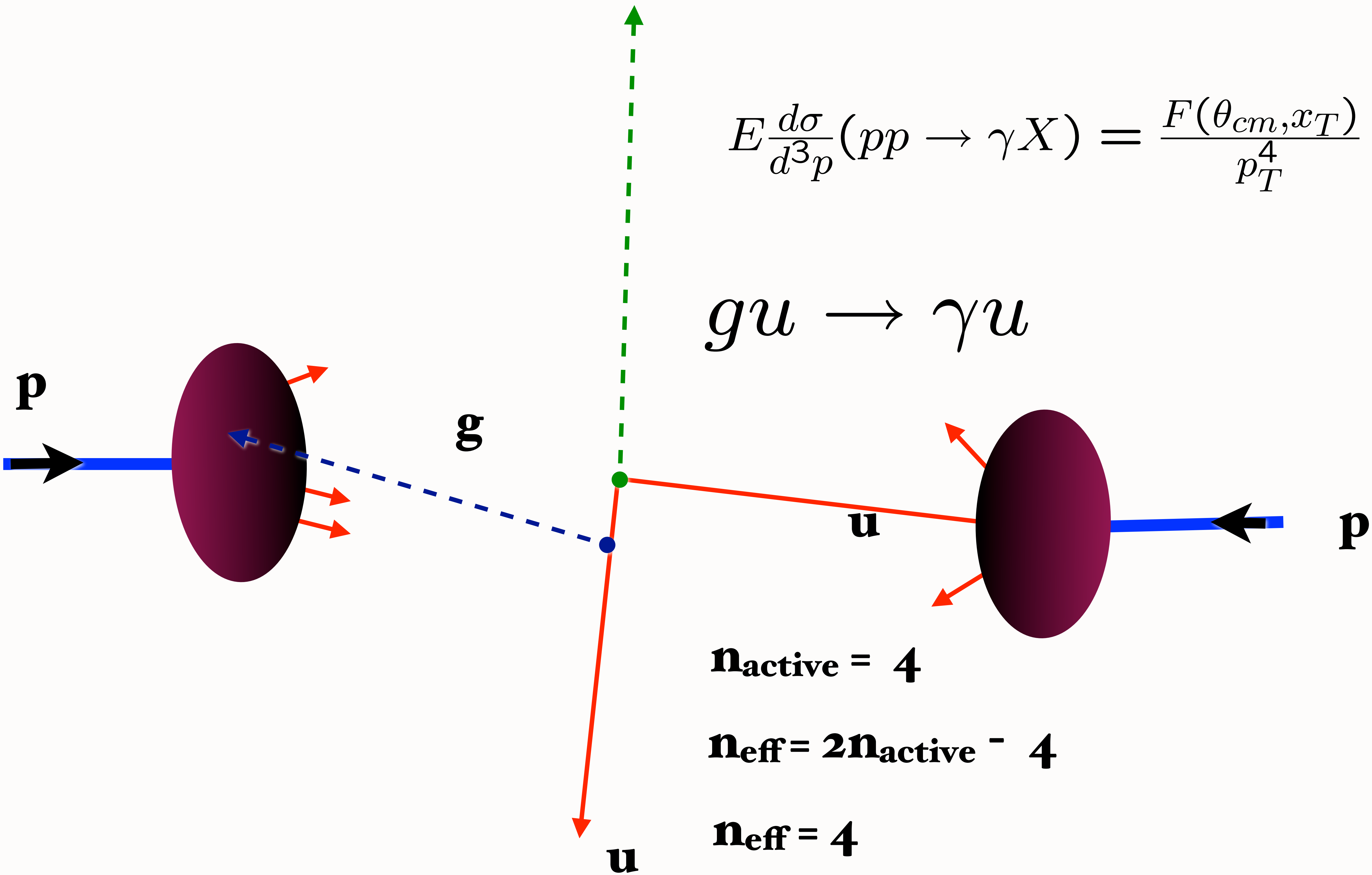
$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}} \qquad x_T = \frac{2p_T}{\sqrt{s}}$$

Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

scaling law: $n_{eff} = 2\, n_{active} - 4$

$$pp \to \gamma X$$



$$E\frac{d\sigma}{d^3p}(pp \to \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \to \gamma u$$

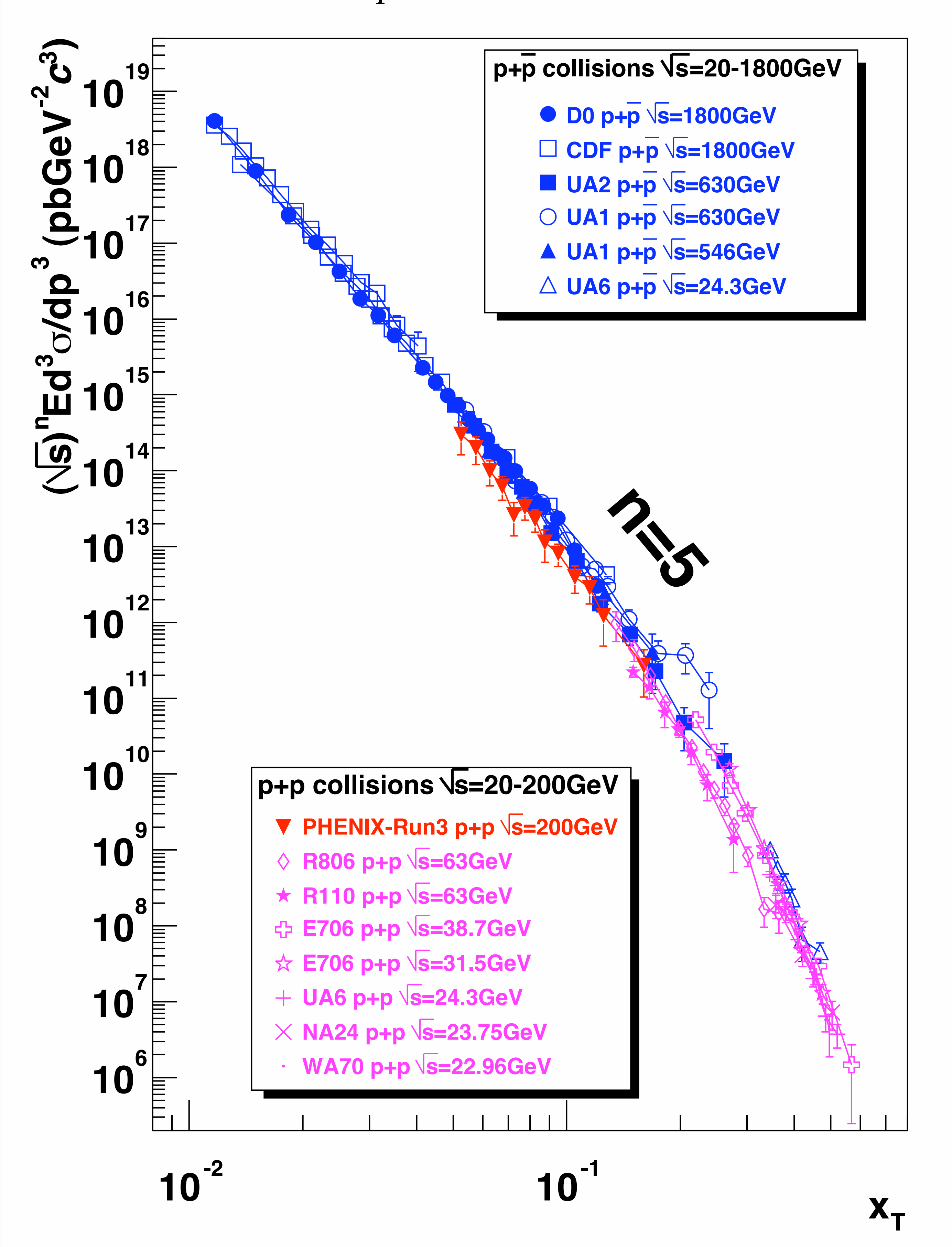
$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

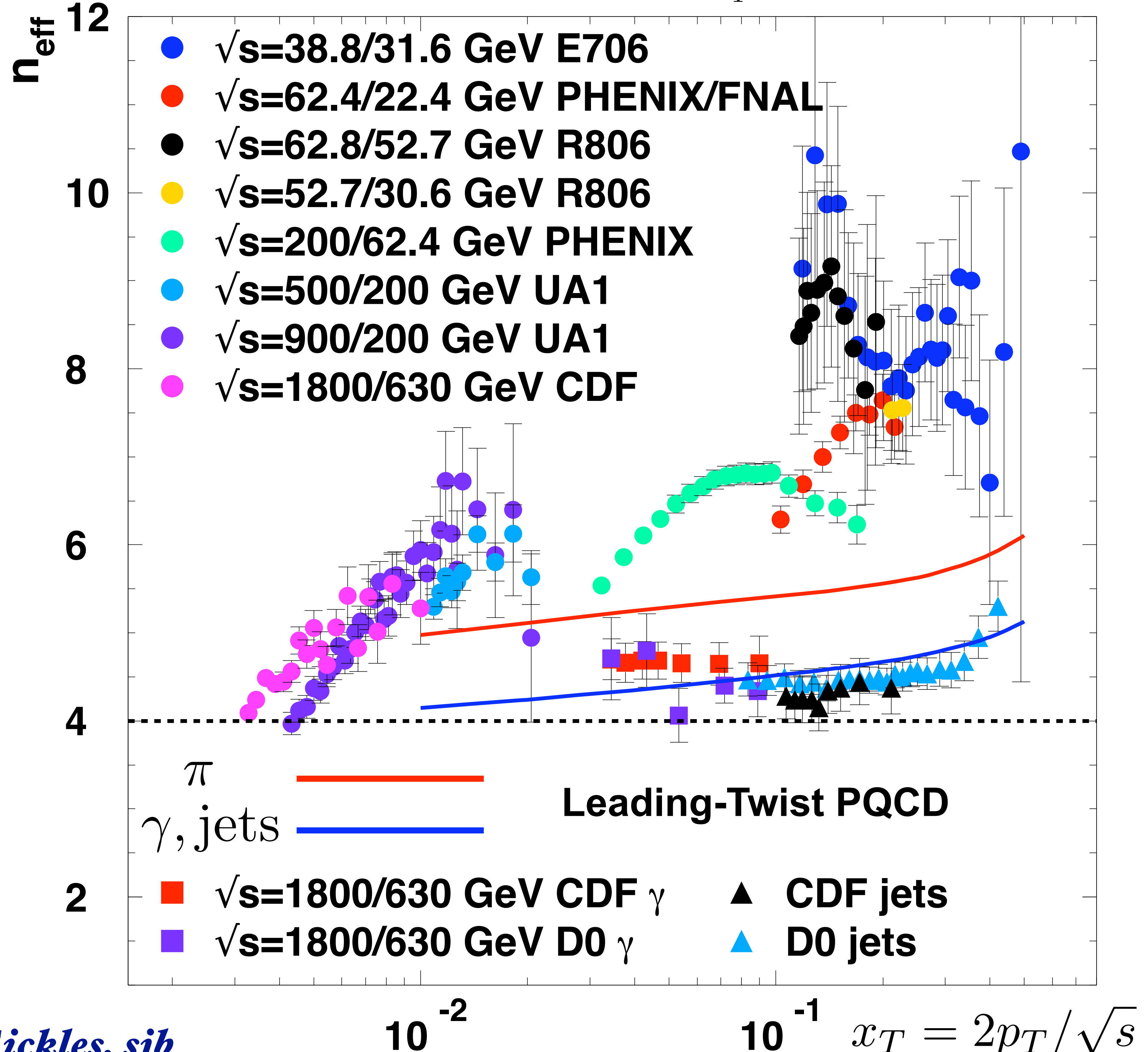
$\sqrt{s}^n E\frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed x_T

Tannenbaum



x_T-scaling of direct photon production: consistent with PQCD

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\rm eff}}}$$



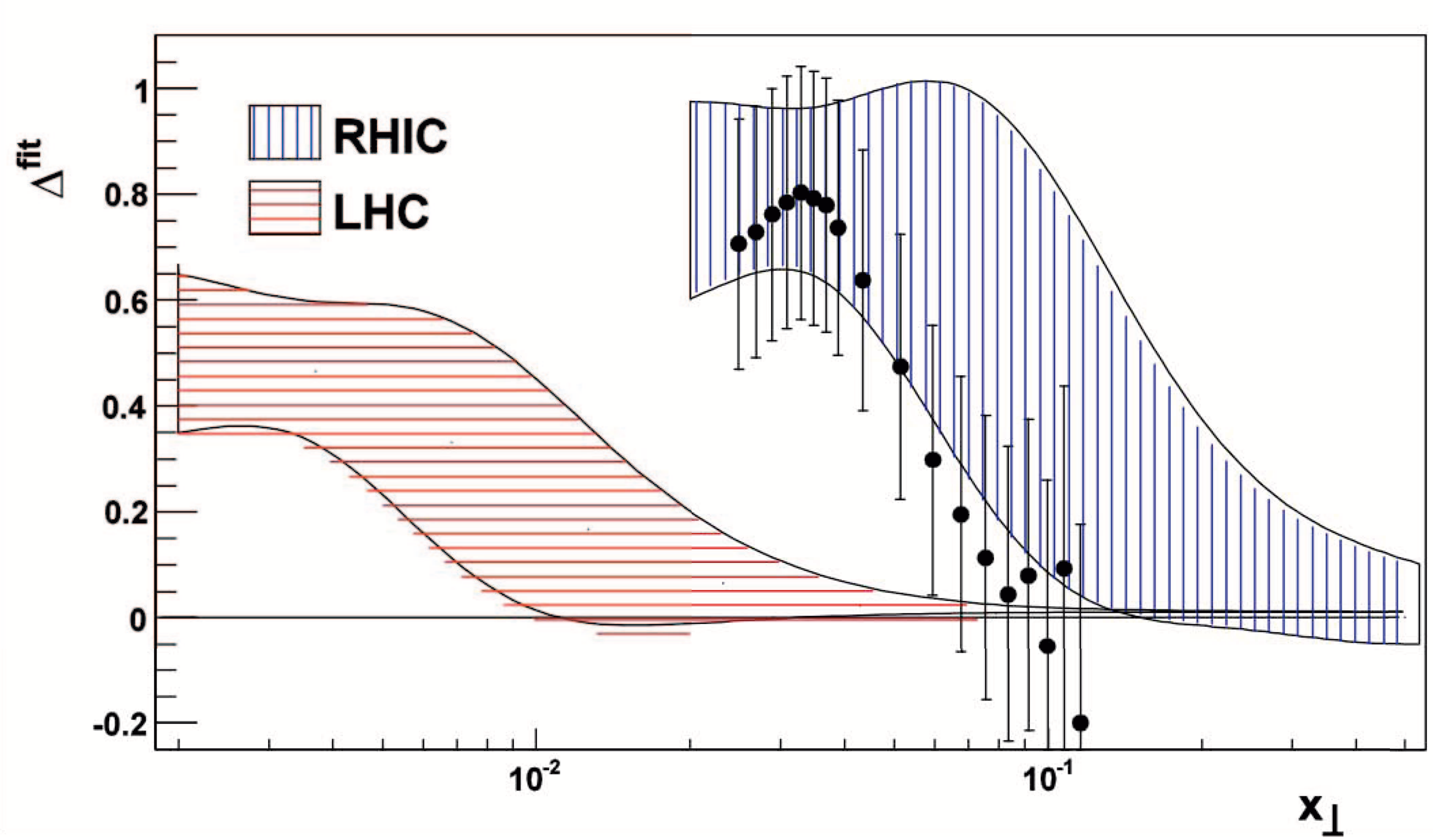
Arleo, Hwang, Sickles, sjb

RHIC/LHC predictions

PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

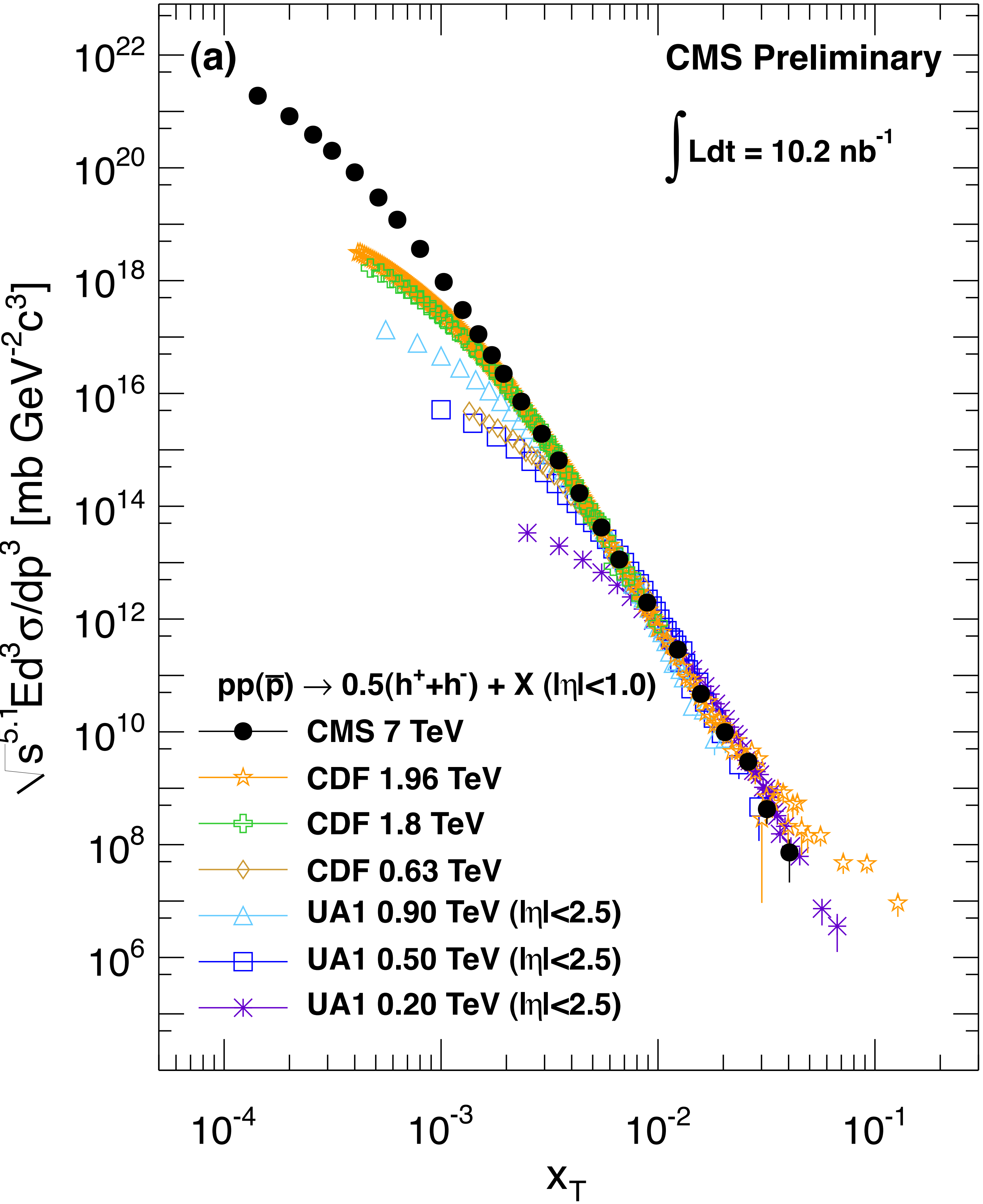
[A. Bezilevsky, APS Meeting]



- Magnitude of Δ and its $x_\perp$-dependence consistent with predictions

Jet-triggered charged particle transverse momentum spectra in pp collisions at 7 TeV

The CMS Collaboration



x_T scaling fails

at the LHC

Inclusive invariant cross sections, scaled by $\sqrt{s}^{5.1}$

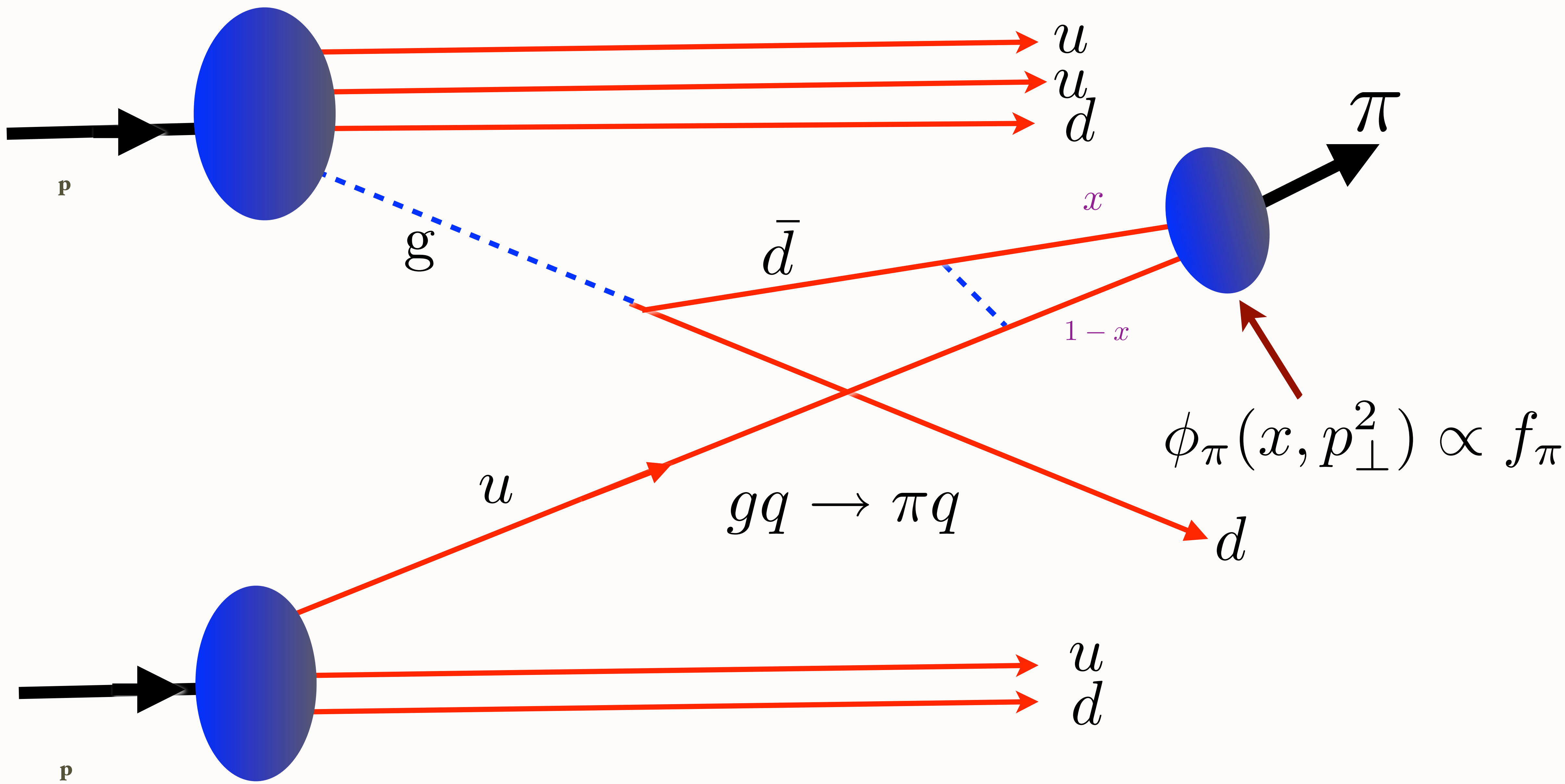
Direct Higher Twist Processes

- QCD predicts that hadrons can interact directly within hard subprocesses
- Exclusive and quasi-exclusive reactions
- Form factors, deeply virtual meson scattering
- Controlled by the hadron distribution amplitude

$$\phi_H(x_i, Q)$$

- Satisfies ERBL evolution

Direct Contribution to Hadron Production

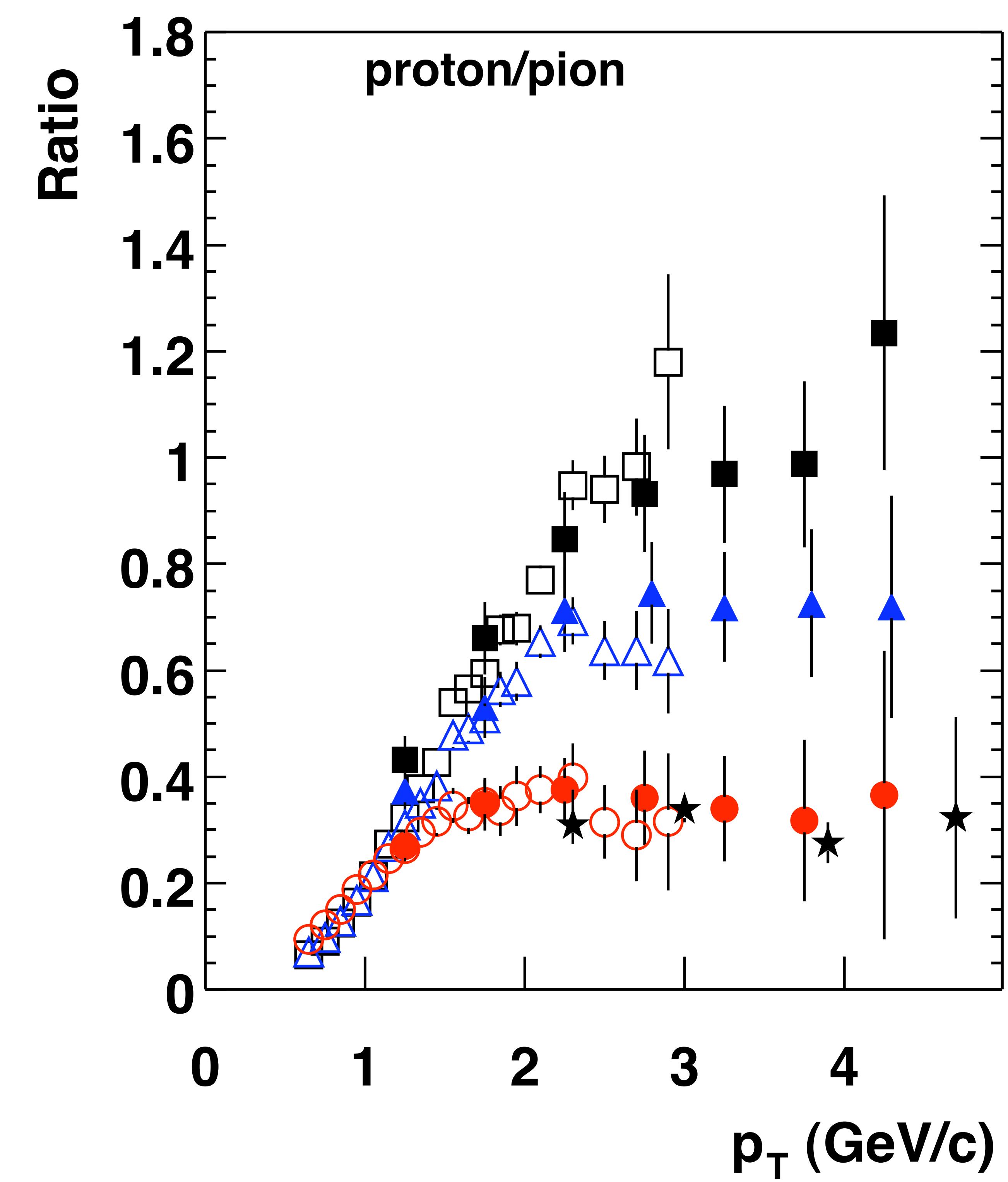


$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

No Fragmentation Function

S. S. Adler *et al.* PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Particle ratio changes with centrality!



Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process

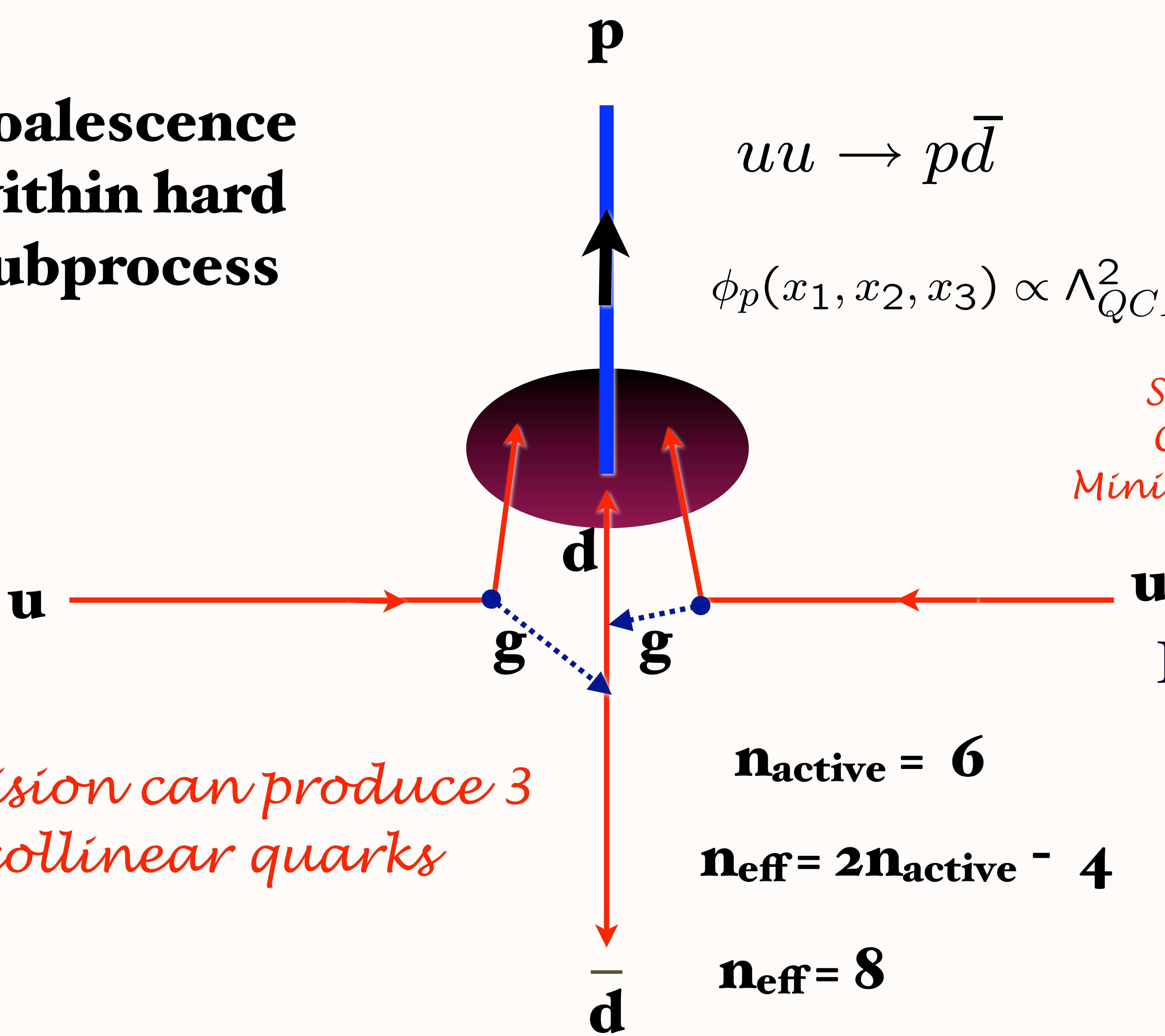
← **Central**

← **Peripheral**

Tannenbaum: "Baryon Anomaly"

Baryon can be made directly within hard subprocess!

Coalescence within hard subprocess



$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

Baryon anomaly

$$qq \to B\bar{q}$$

Collision can produce 3 collinear quarks

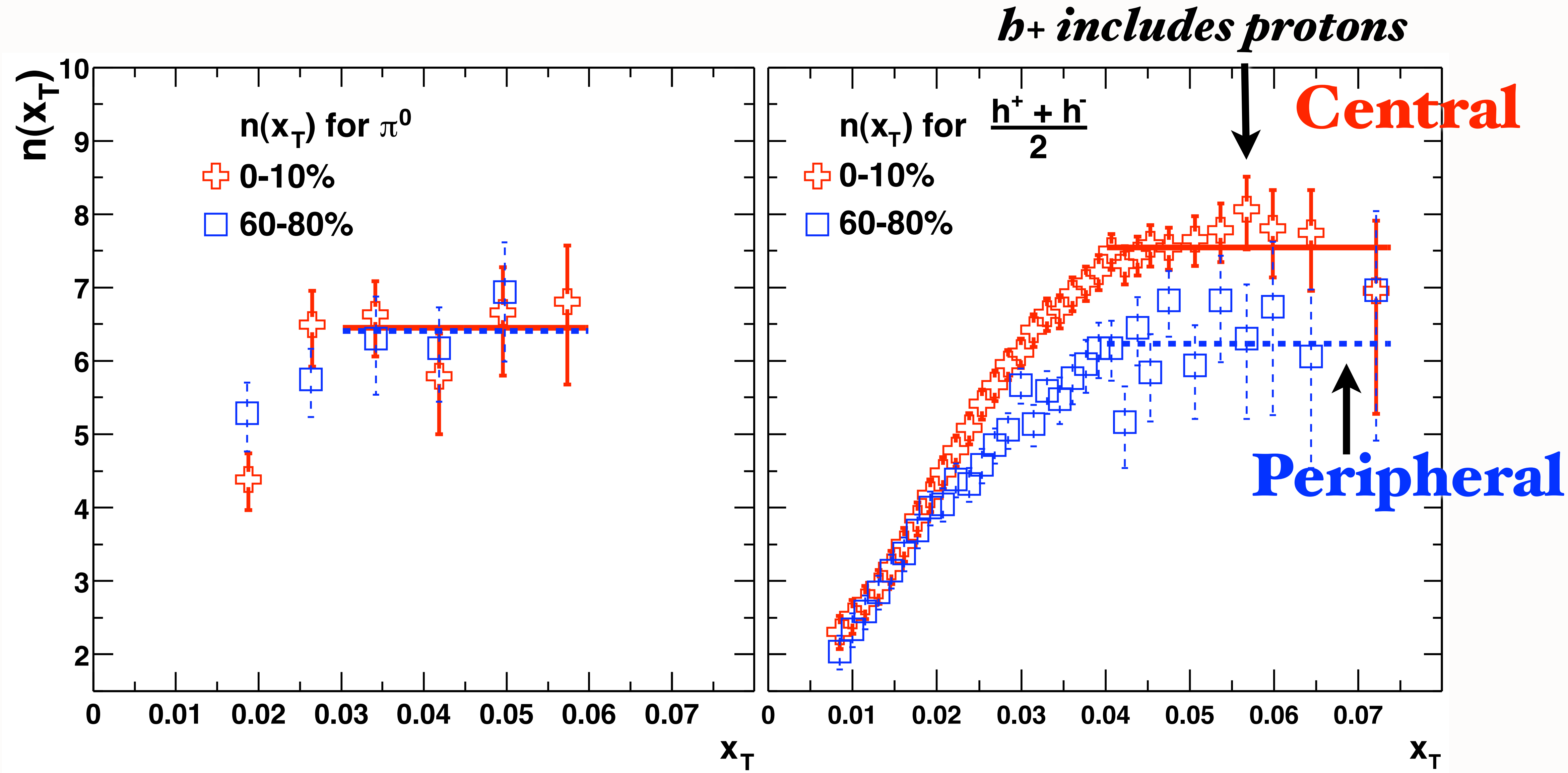
$\mathbf{n_{active} = 6}$

$\mathbf{n_{eff} = 2n_{active} - 4}$

$\mathbf{n_{eff} = 8}$

Power-law exponent $n(x_T)$ for π^0 and h spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV

S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev.* C **69**, 034910 (2004) [nucl-ex/0308006].

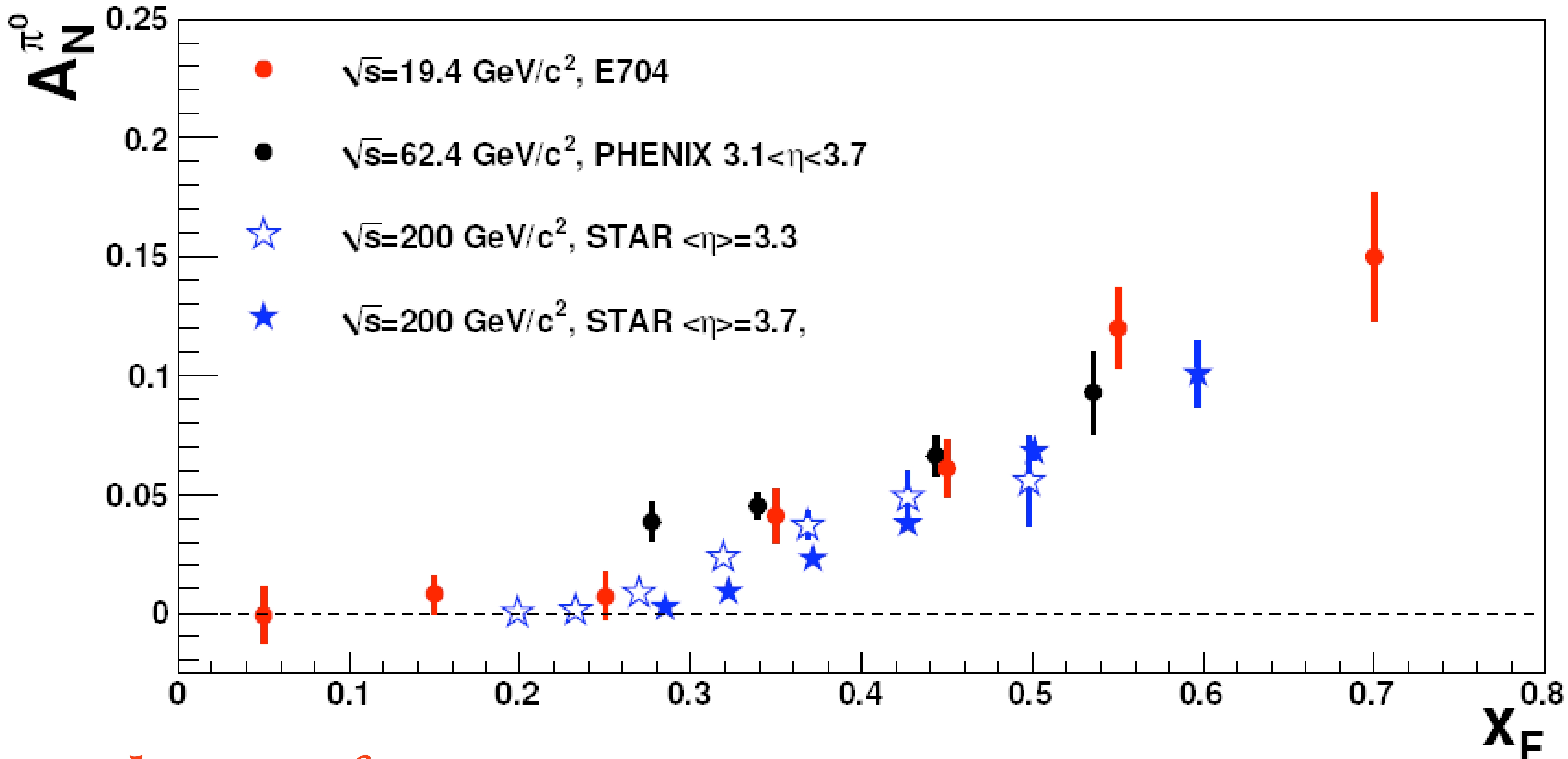


Proton power changes with centrality !

Proton production dominated by
color-transparent direct high n_{eff} subprocesses

A_N in $p^\uparrow p \to \pi X$, the big challenge

$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\uparrow}{d\sigma^\uparrow + d\sigma^\uparrow}$$



Contributions from Direct Processes? Reggeon Exchange?

John Koster talk

$$\pi^- N \to \mu^+ \mu^- X \text{ at } 80\ \mathrm{GeV}/c$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi .$$

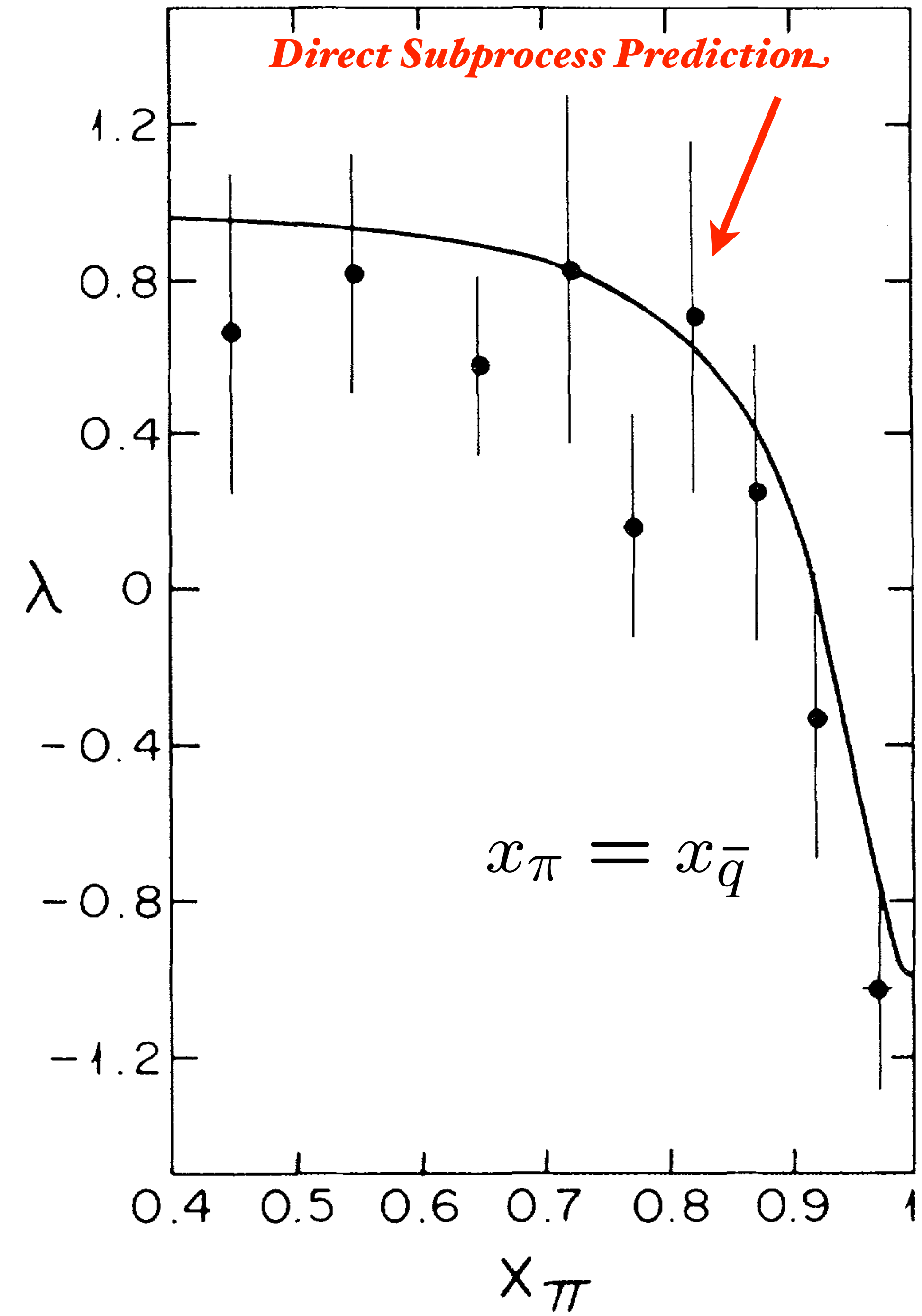
$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16\ \mathrm{GeV}^2/c^2$$

$$Q^2 = M^2$$

Dramatic change in angular distribution at large x

$$x_\pi = x_{\bar{q}}$$



Chicago-Princeton Collaboration

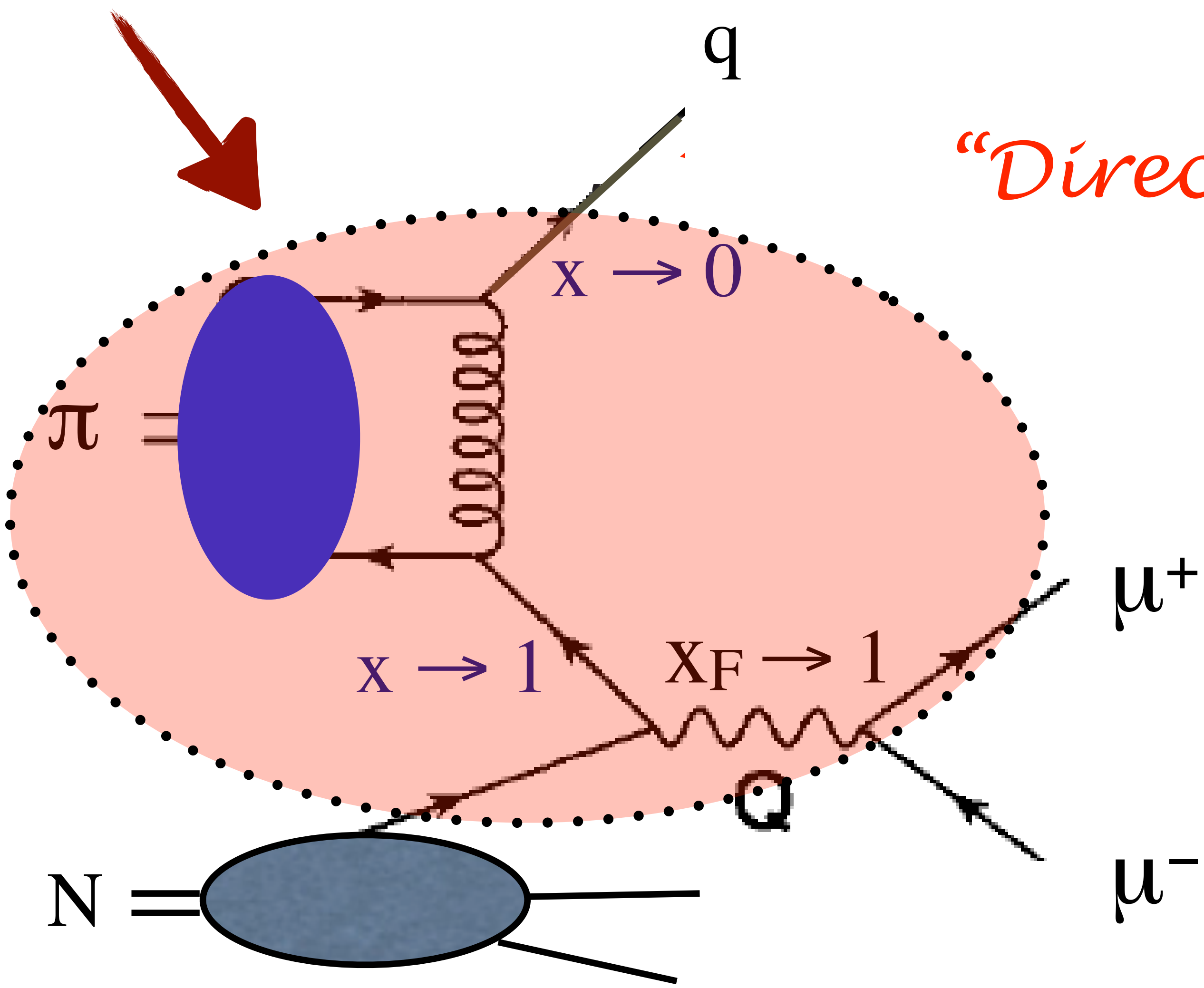
Phys.Rev.Lett.55:2649,1985

Example of a higher-twist direct subprocess

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

Distribution amplitude from AdS/CFT

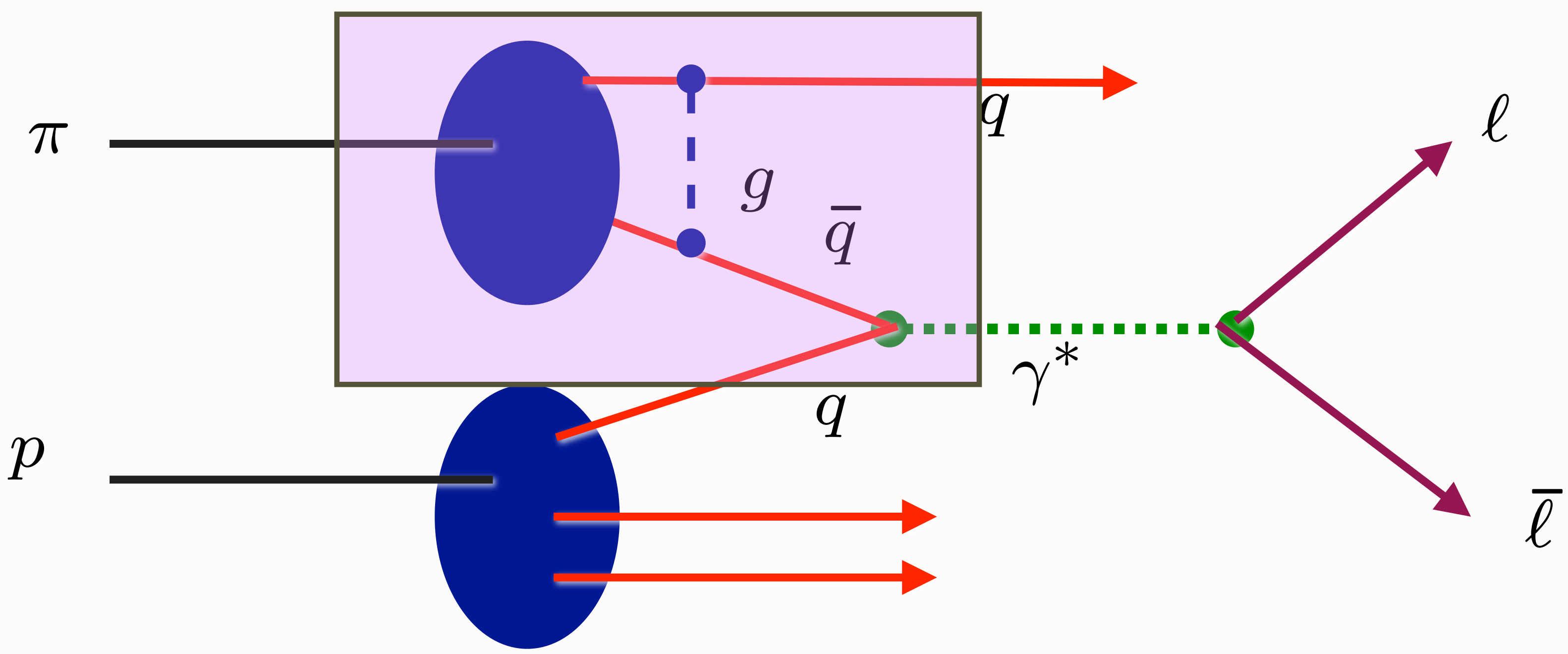


Berger, sjb
Khoze, Brandenburg, Muller, sjb

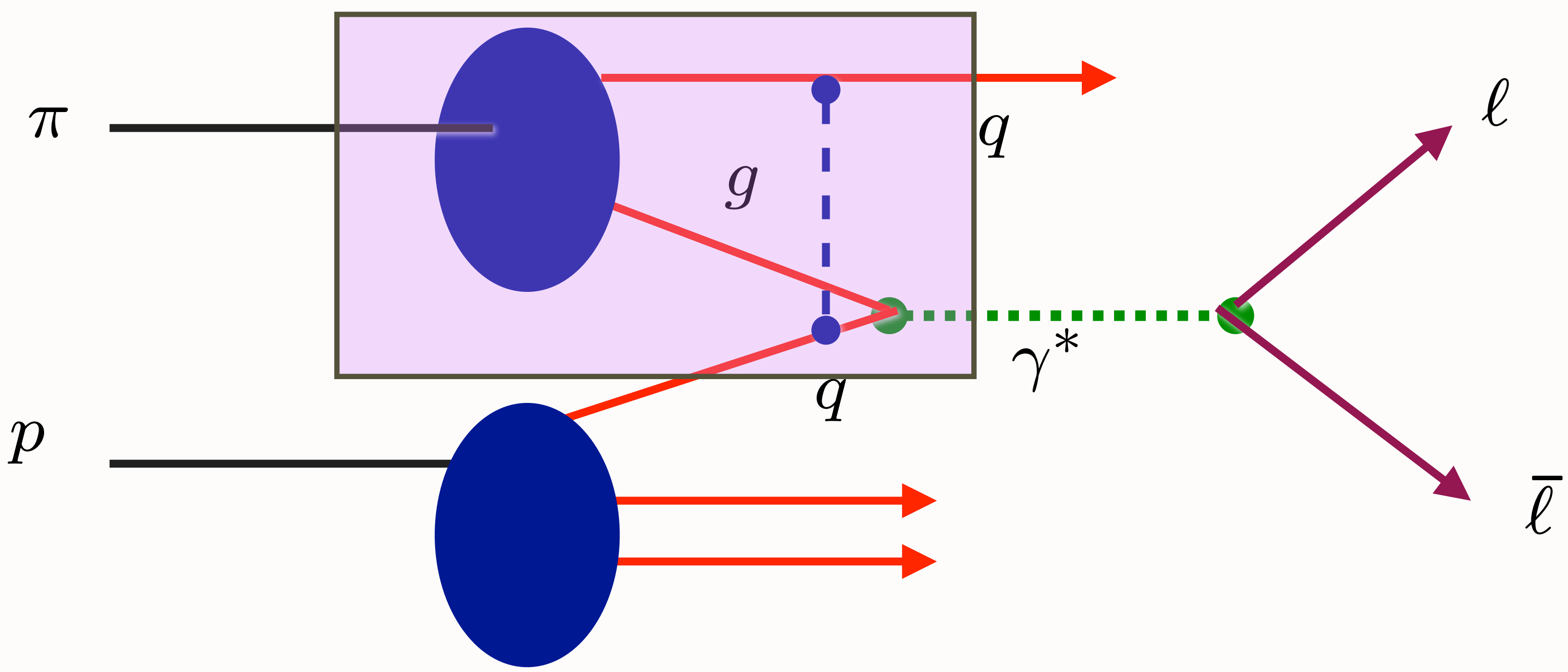
Similar higher twist terms in jet hadronization at large z

Hoyer Vanttinen

Berger, Lepage, sjb



$$\pi q \rightarrow \gamma^* q$$



Initial State Interaction

Pion appears directly in subprocess at large x_F

All of the pion's momentum is transferred to the lepton pair

Lepton Pair is produced longitudinally polarized

$\pi^- N \to \mu^+ \mu^- X$ at 80 GeV/c

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

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Dramatic change in angular distribution at large x_F

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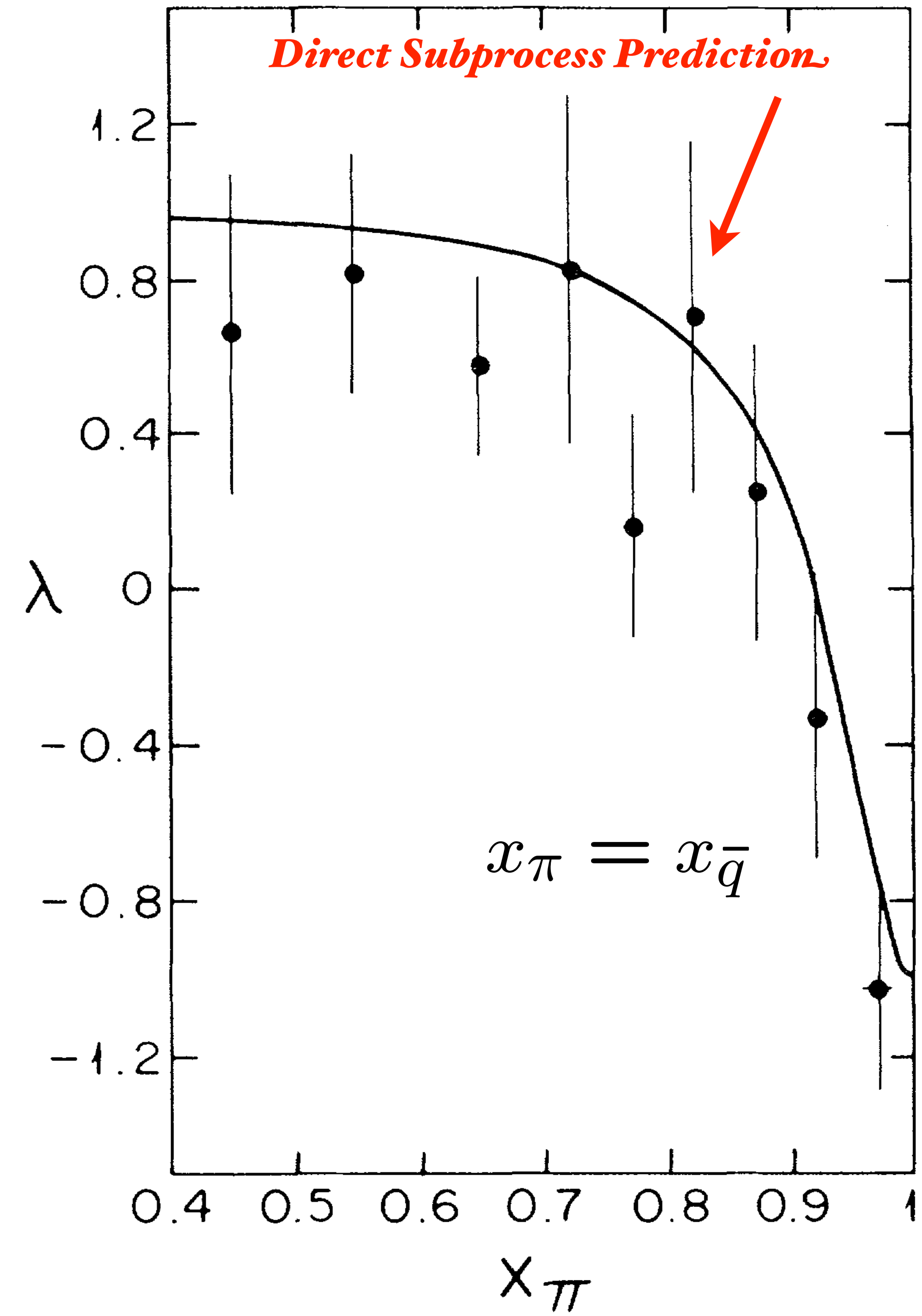


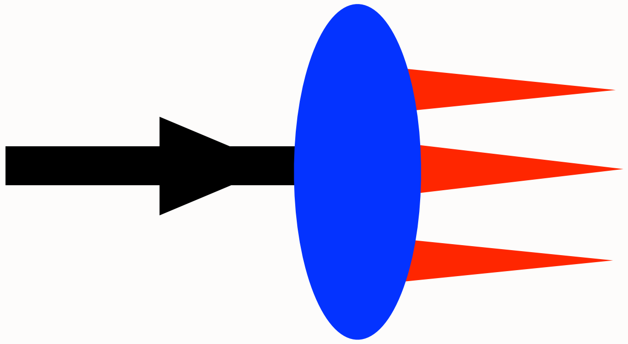
Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

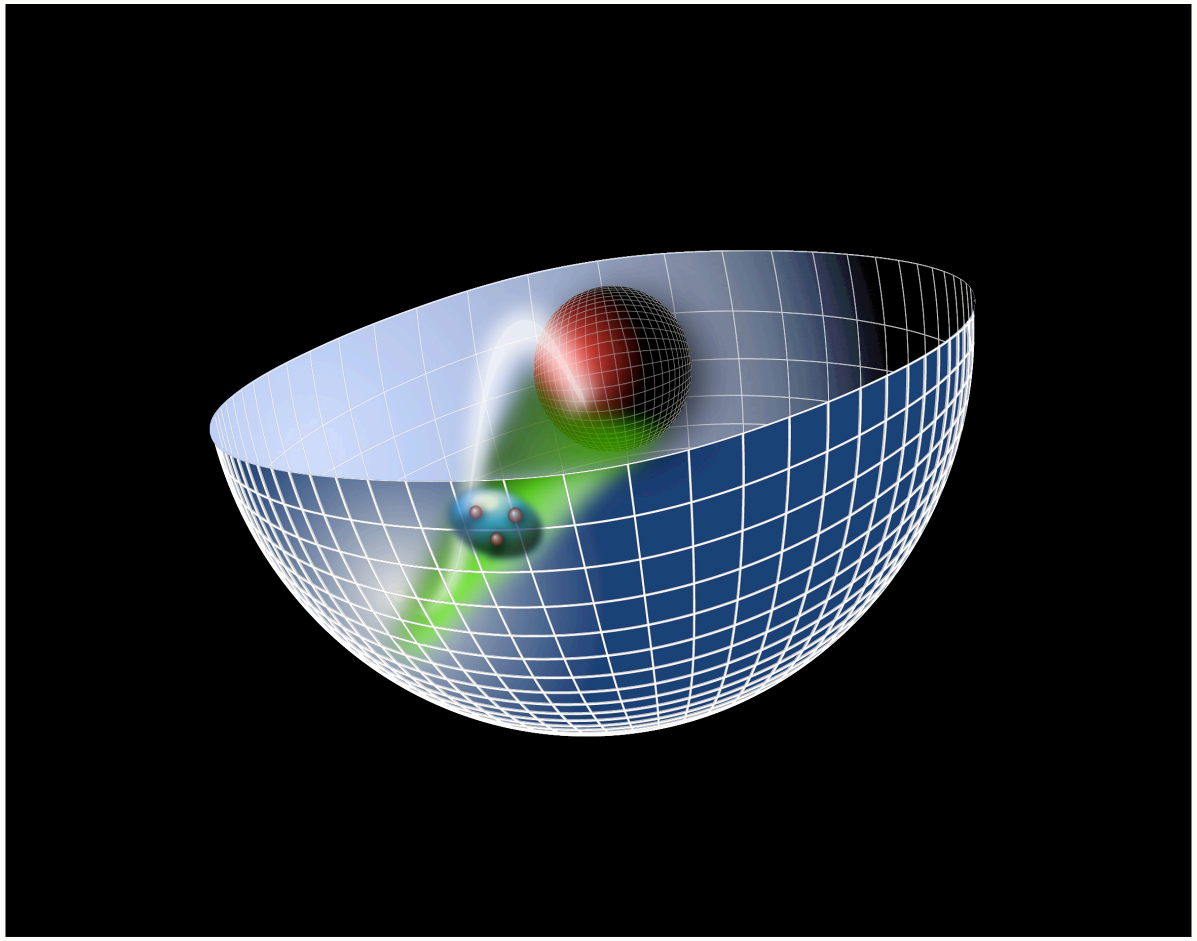
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions,
Running coupling in IR



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond

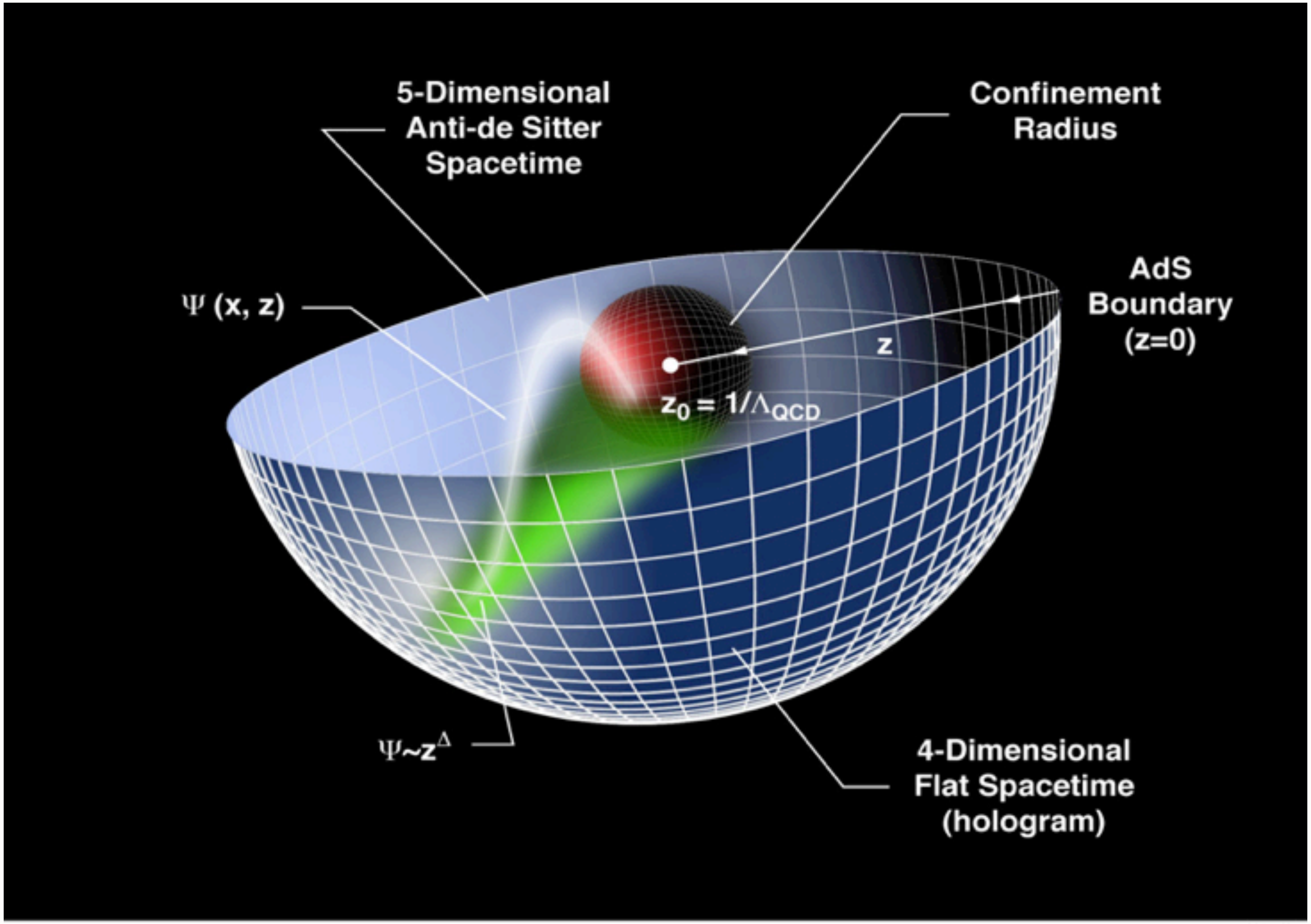
Central problem for strongly-coupled gauge theories

Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Transversity**
- **Insight into QCD Condensates**
- **Systematically improvable**

de Teramond, sjb

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

- de Teramond, sjb

Positive-sign dilaton

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\phi(z) = \mathcal{M}^2\phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action Dilaton-Modified AdS_5

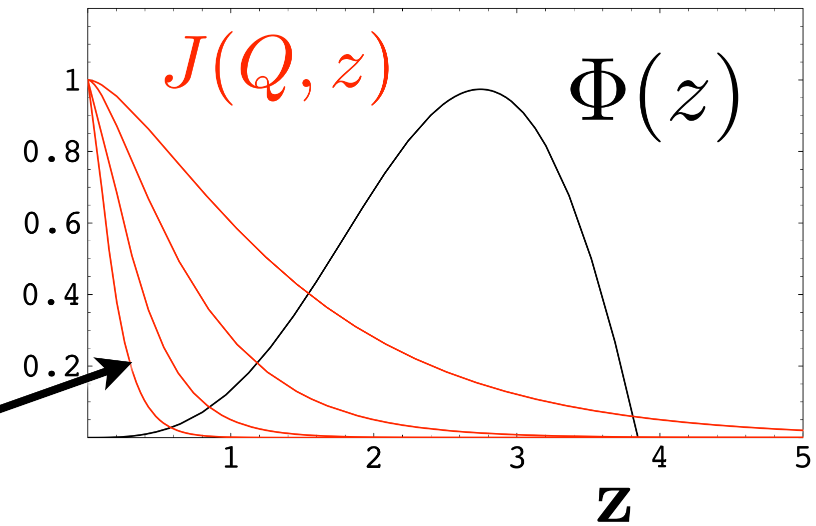
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2 from small $z \sim 1/Q$



Polchinski, Strassler de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

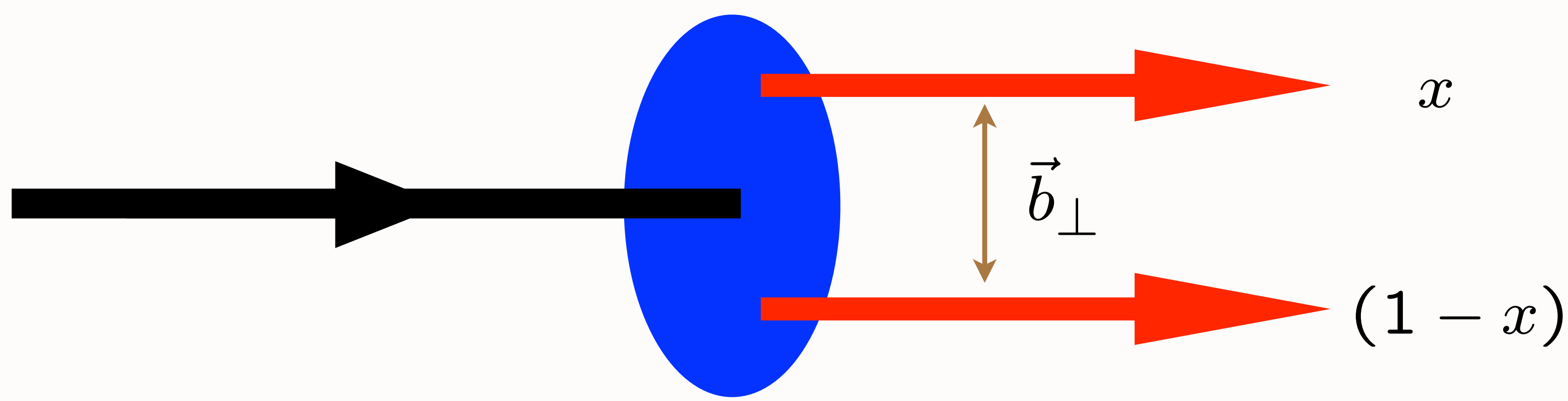
$$\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},$$

Identical to LF Holography obtained from electromagnetic current de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables $(\vec{\zeta},\varphi), \ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

QCD Meson Spectrum

$$H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x,\vec{k}_\perp) = M^2\ \psi_{LF}(x,\vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta,S,L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta,S,L) = \kappa^4\zeta^2 + \kappa^2(L+S-1/2)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

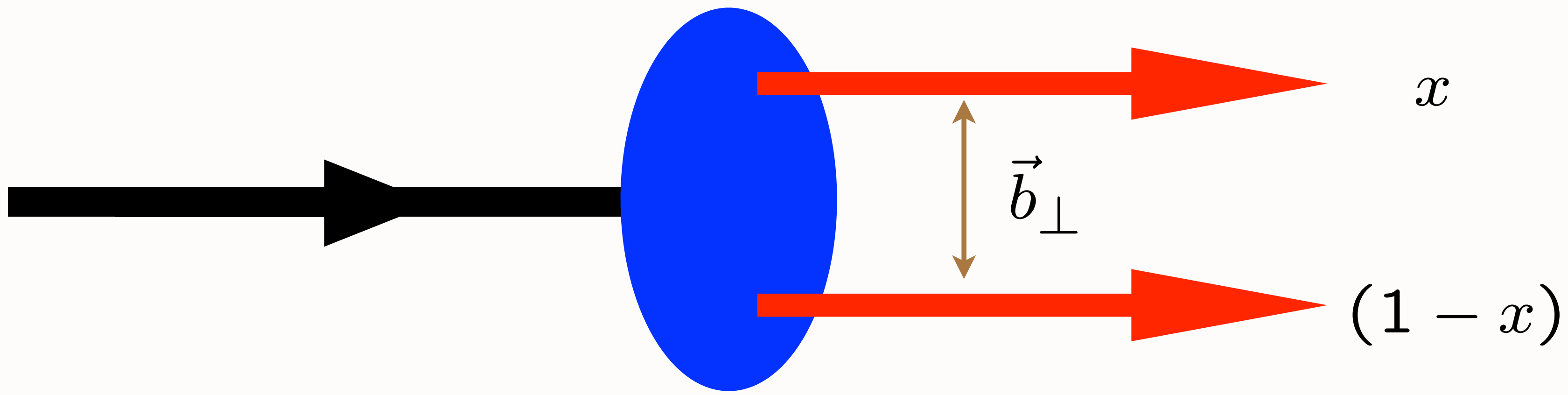
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

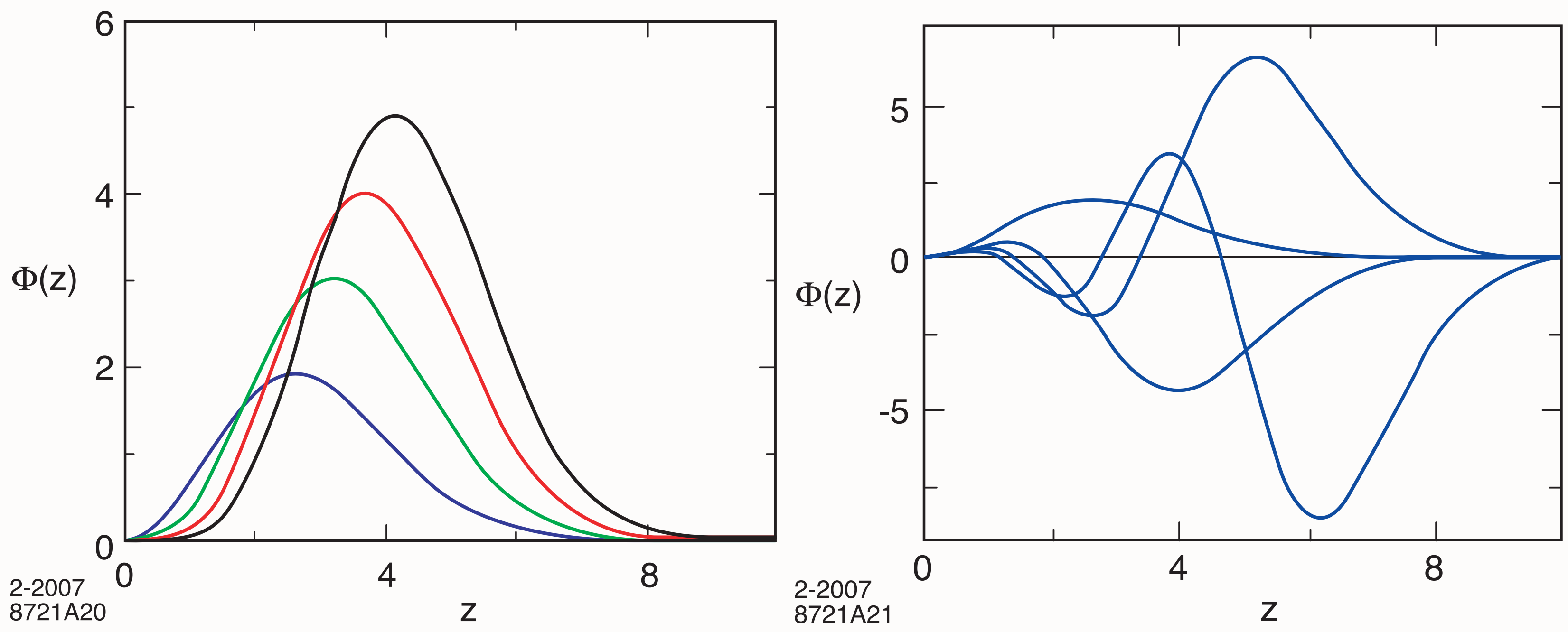
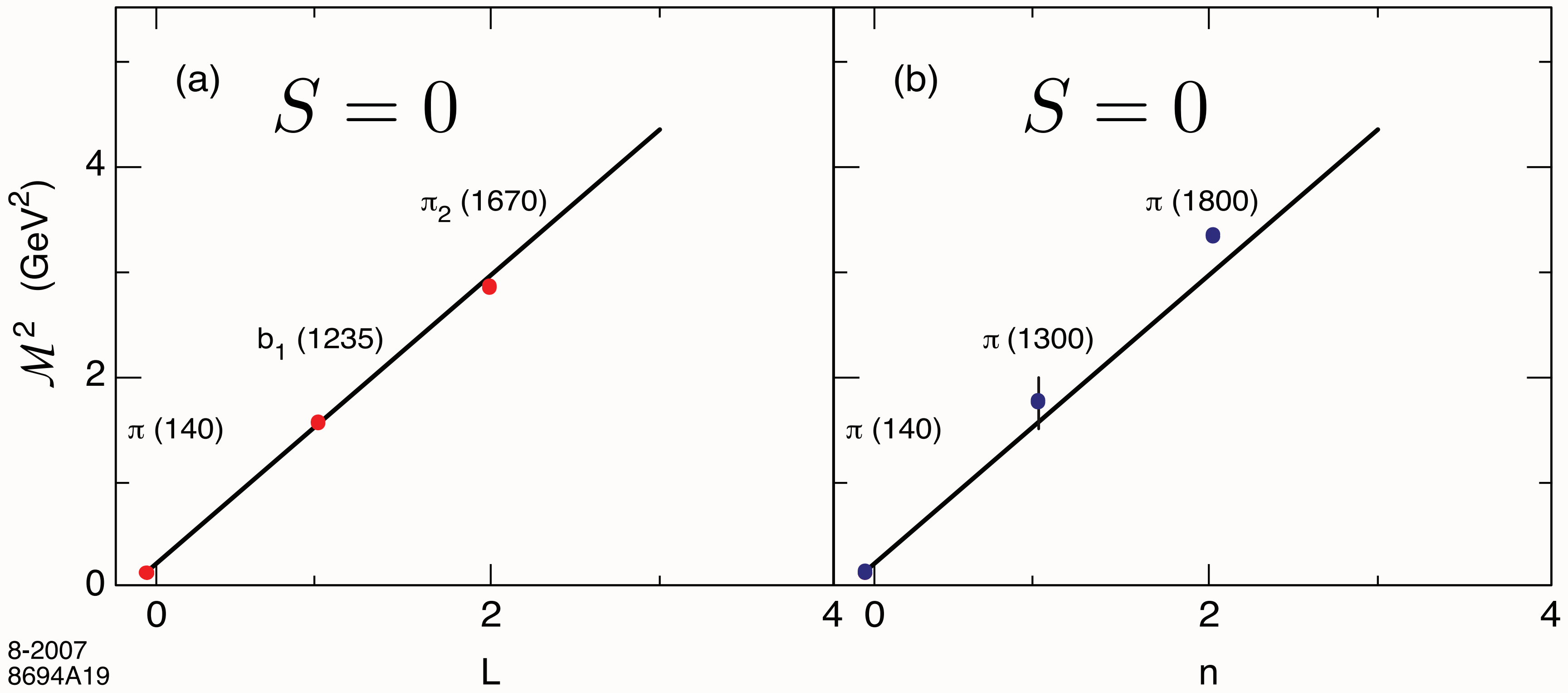


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

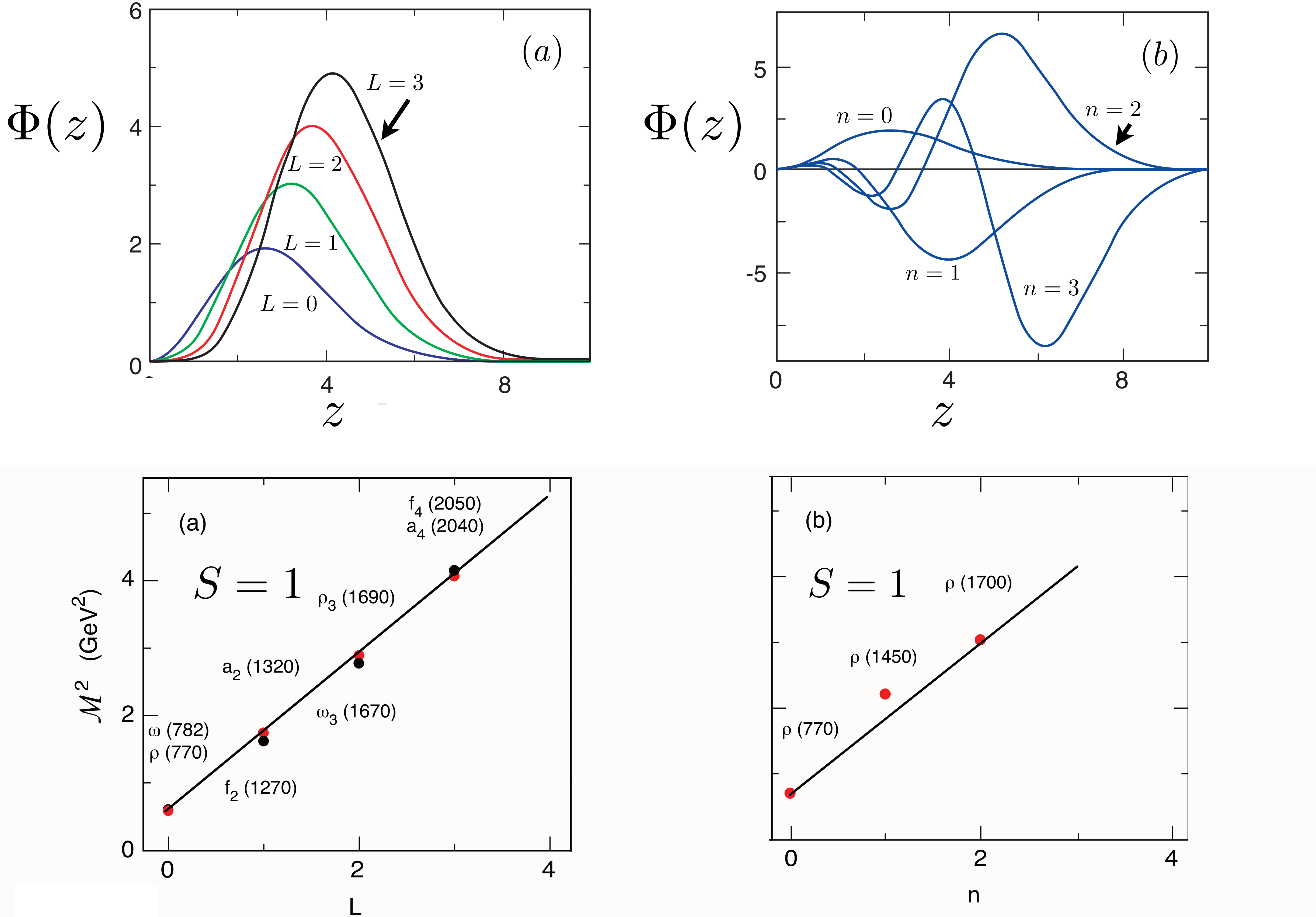
- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$



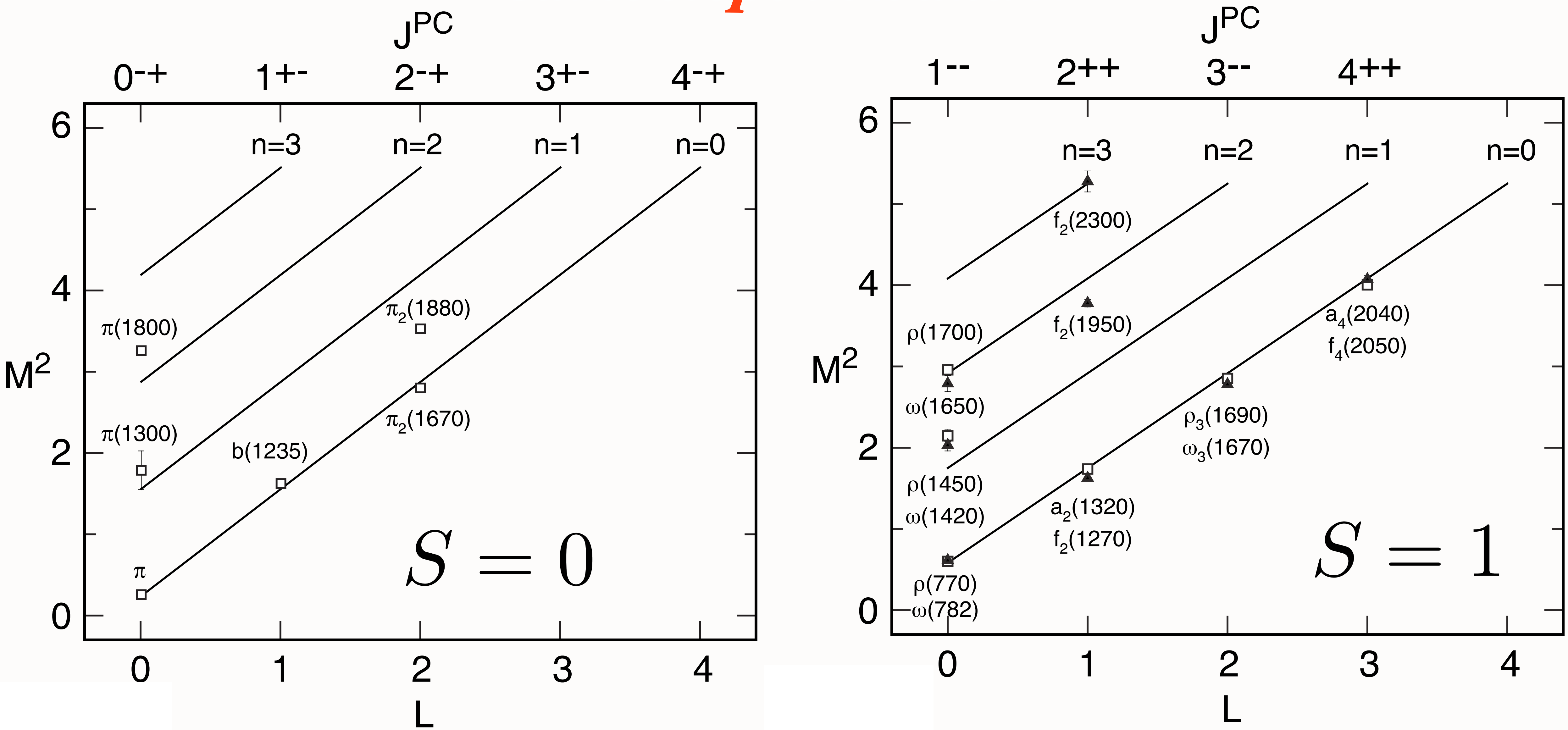
(a)
Φ(z)
L = 3
L = 2
L = 1
L = 0
z
(b)
Φ(z)
n = 0
n = 1
n = 2
n = 3
z
(a)
S = 1
f4 (2050)
a4 (2040)
ρ3 (1690)
a2 (1320)
ω3 (1670)
ω (782)
ρ (770)
f2 (1270)
M² (GeV²)
L
(b)
S = 1
ρ (1700)
ρ (1450)
ρ (770)
n

Bosonic Modes and Meson Spectrum

$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \to 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

String Theory

Mapping of Poincare' and Conformal SO (4,2) symmetries of 3+1 space to AdS5 space

AdS/CFT

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

Conformal behavior at short distances + Confinement at large distance

AdS/QCD

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

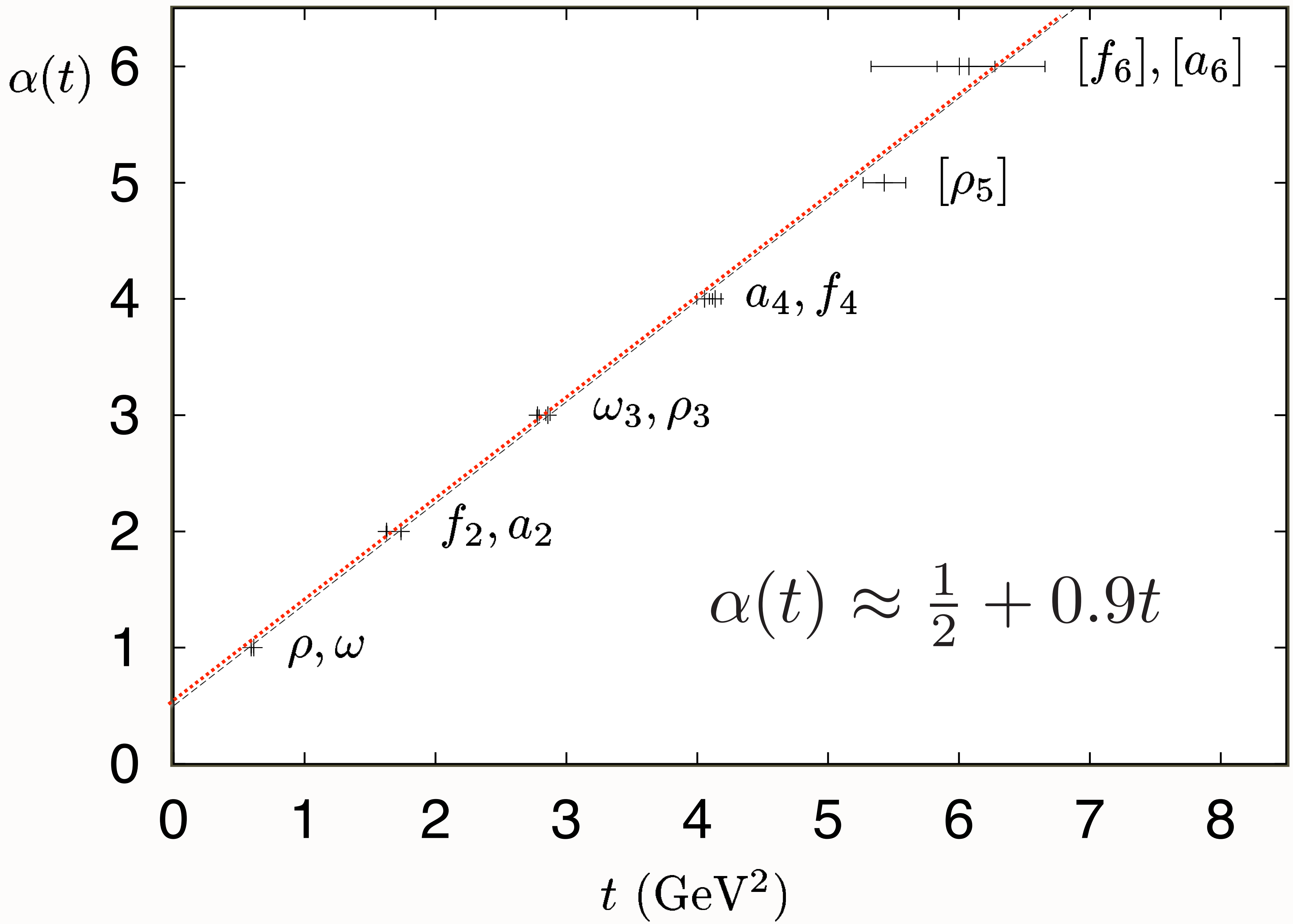
J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

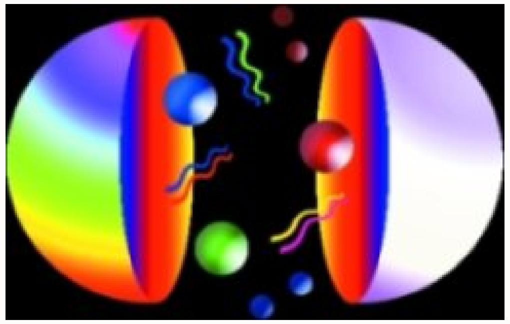


AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in Ads/CFT



From Nick Evans

- Action for massive fermionic modes on AdS_5:

$$S[\overline{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z)$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

Hard Wall

- Solution $(\mu R = \nu + 1/2)$

$$\Psi(z) = C z^{5/2} \left[J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm \left(z = 1/\Lambda_{QCD} \right) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \, \Lambda_{QCD}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \, \Lambda_{QCD}$$

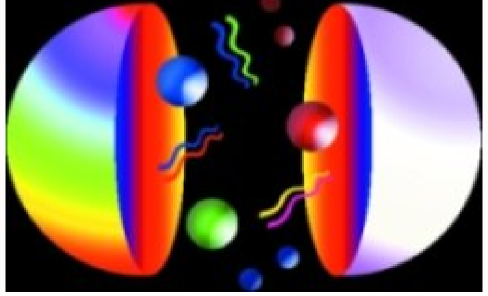
  with scale independent mass ratio

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1}\sqrt{g}e^{\varphi}(z)\left(i\overline{\Psi}e^M_A\Gamma^A D_M\Psi + h.c + \varphi(z)\overline{\Psi}\Psi - \mu\overline{\Psi}\Psi\right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x}\Psi(z)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + 2\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu}e^{-\kappa^2 z^2/2}L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu}e^{-\kappa^2 z^2/2}L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$ positive parity

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\,\Lambda_{\rm QCD}$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

Soft Wall

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

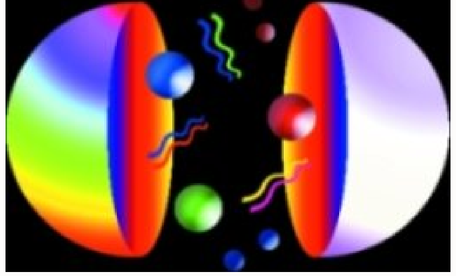
- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta)$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

• **Δ spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)**

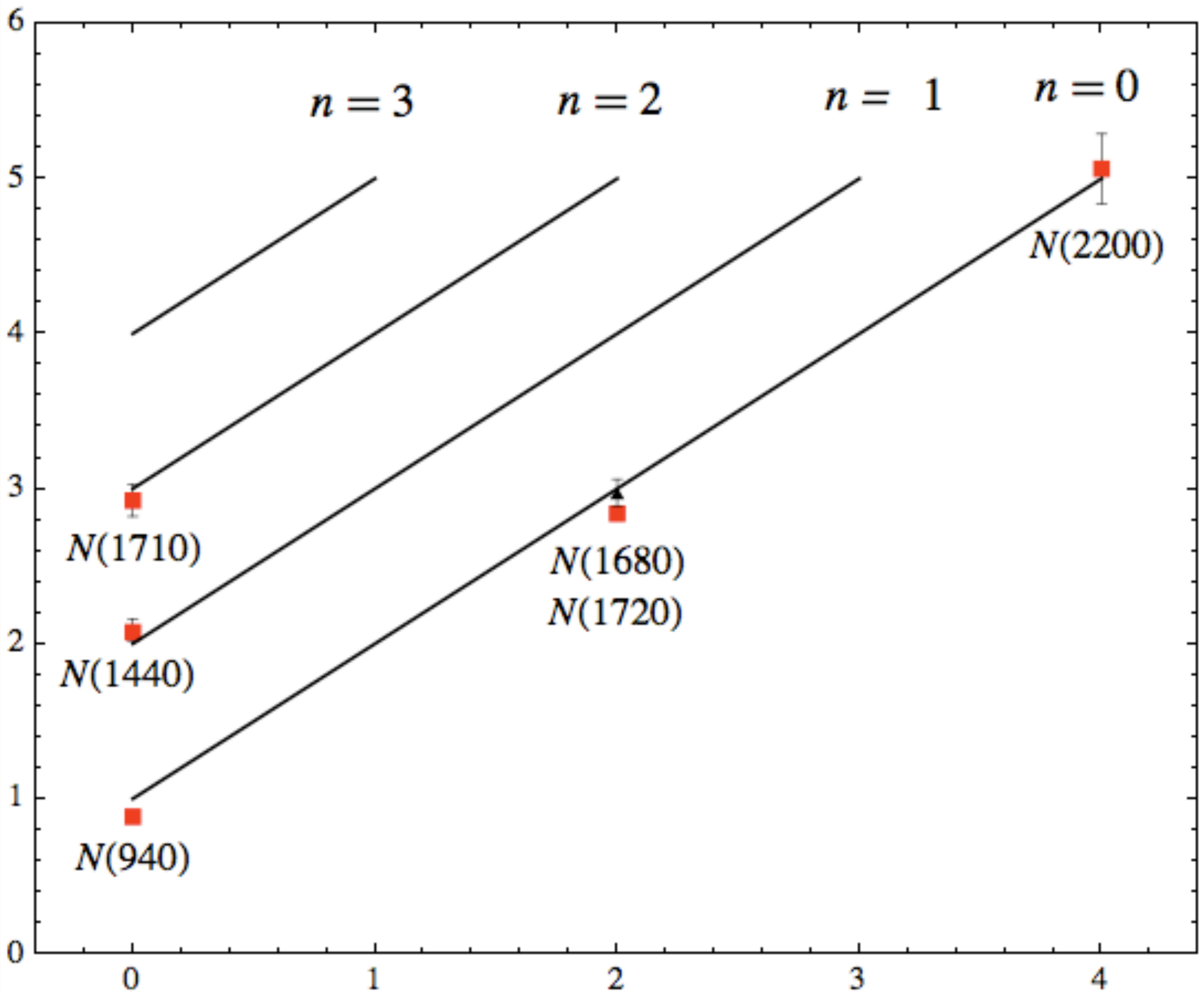
$4\kappa^2$ for $\Delta n = 1$

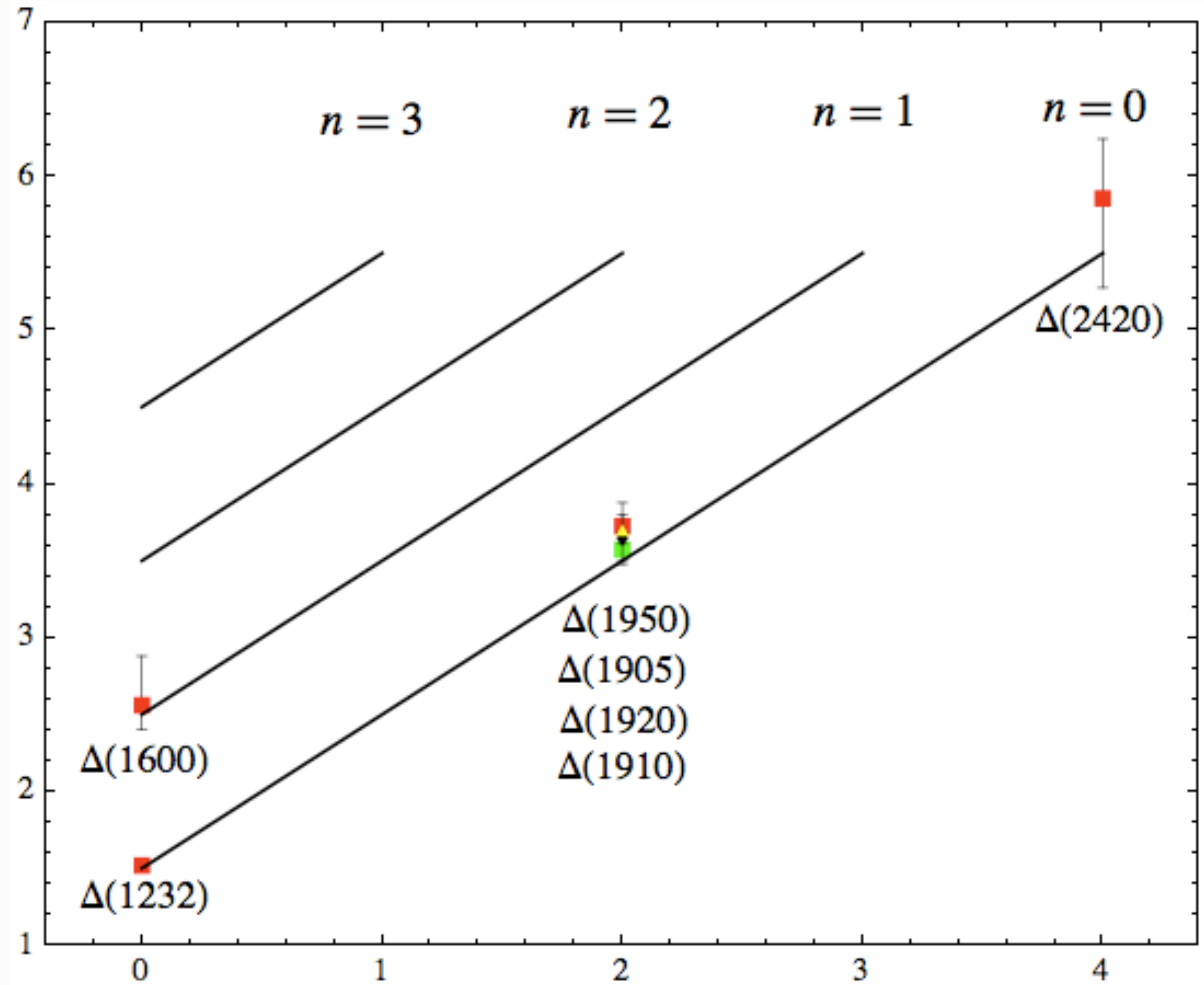
$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$

Same multiplicity of states for mesons and baryons!

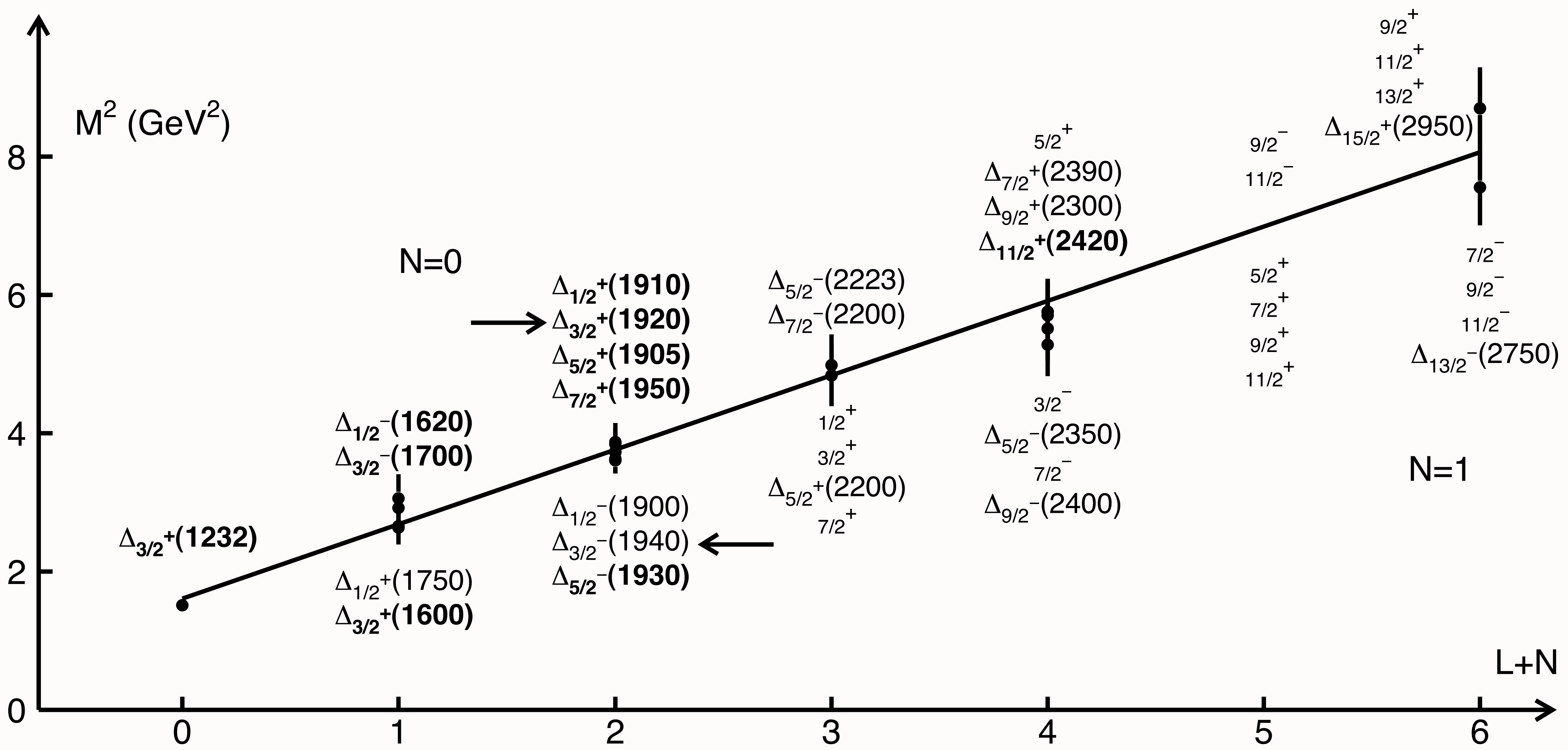
$\mathcal{M}^2$





L

Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

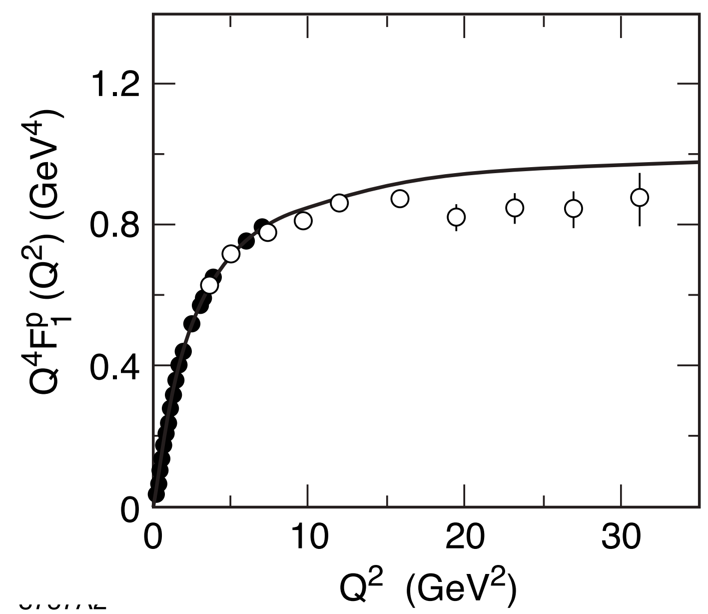


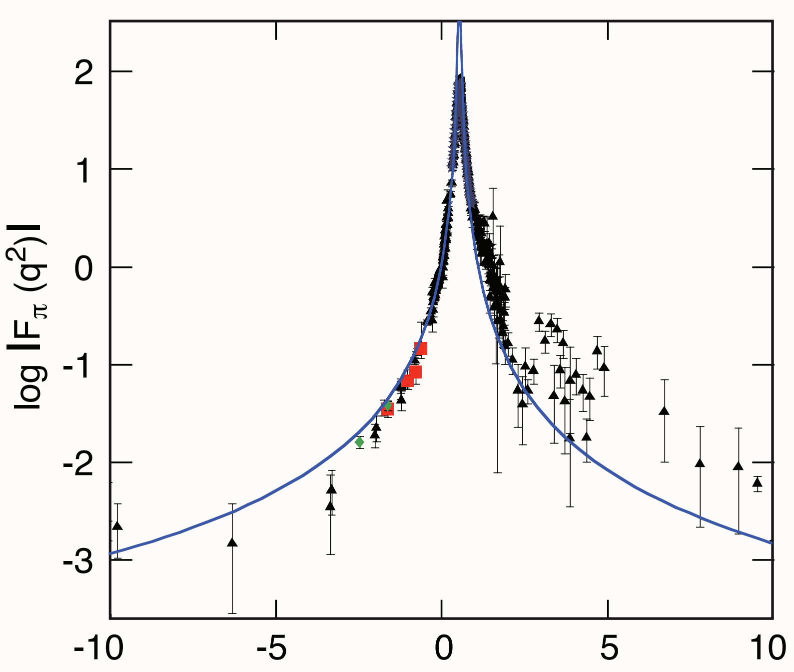
E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Other Applications of Light-Front Holography

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form factors of composite hadrons
- n-parton holographic mapping
- Heavy flavor mesons





hep-th/0501022

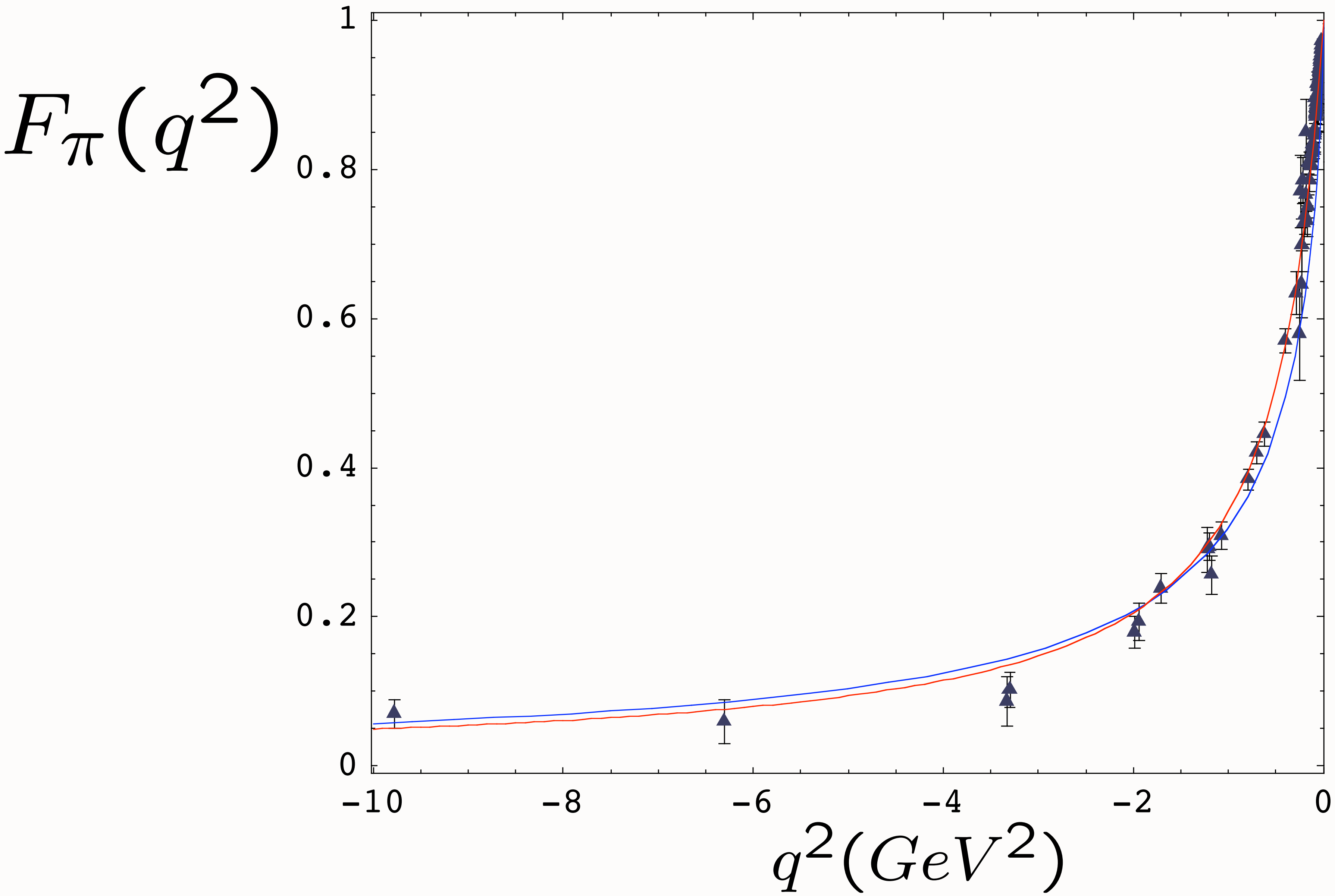
hep-ph/0602252

arXiv:0707.3859

arXiv:0802.0514

arXiv:0804.0452

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

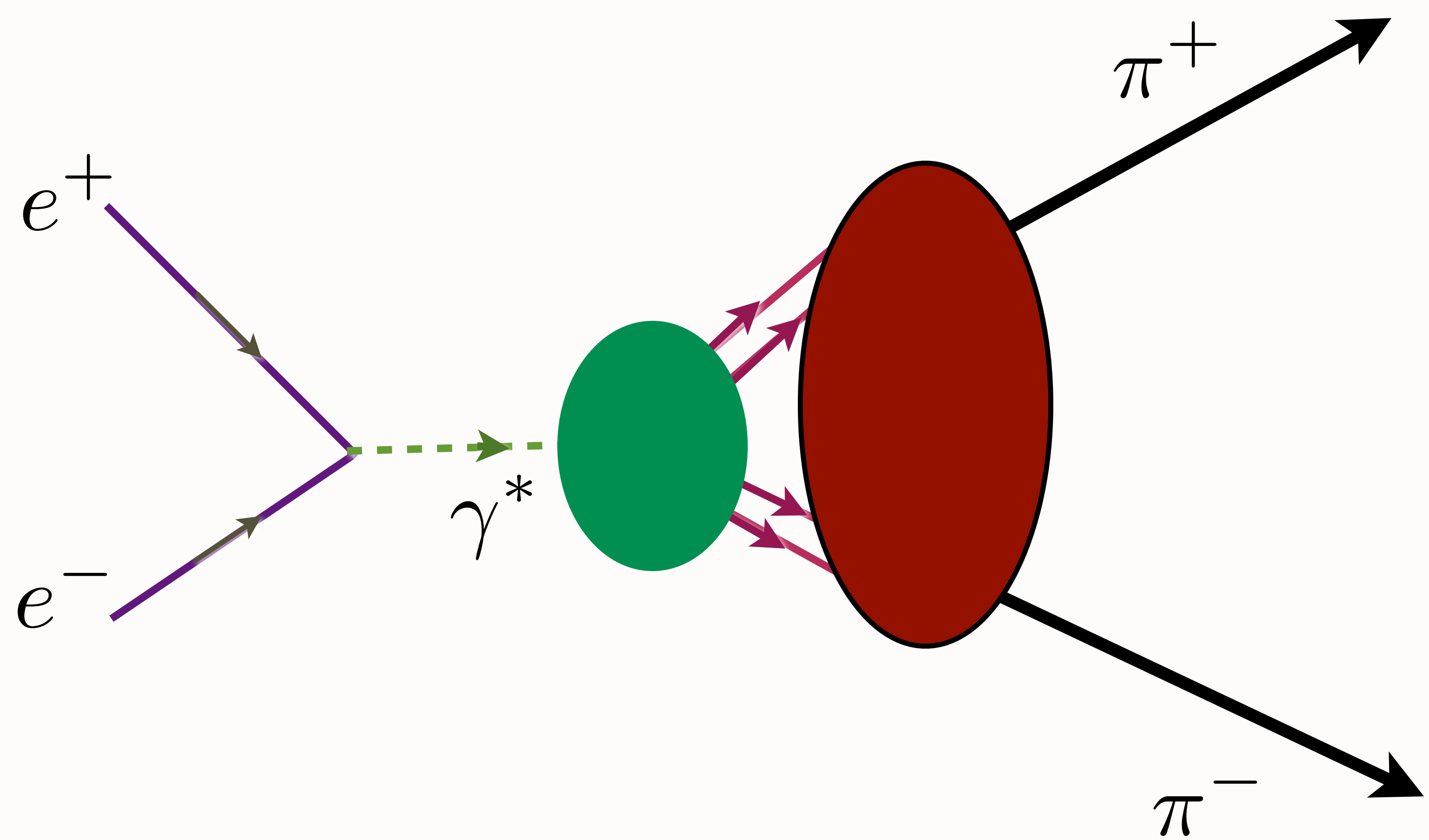
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

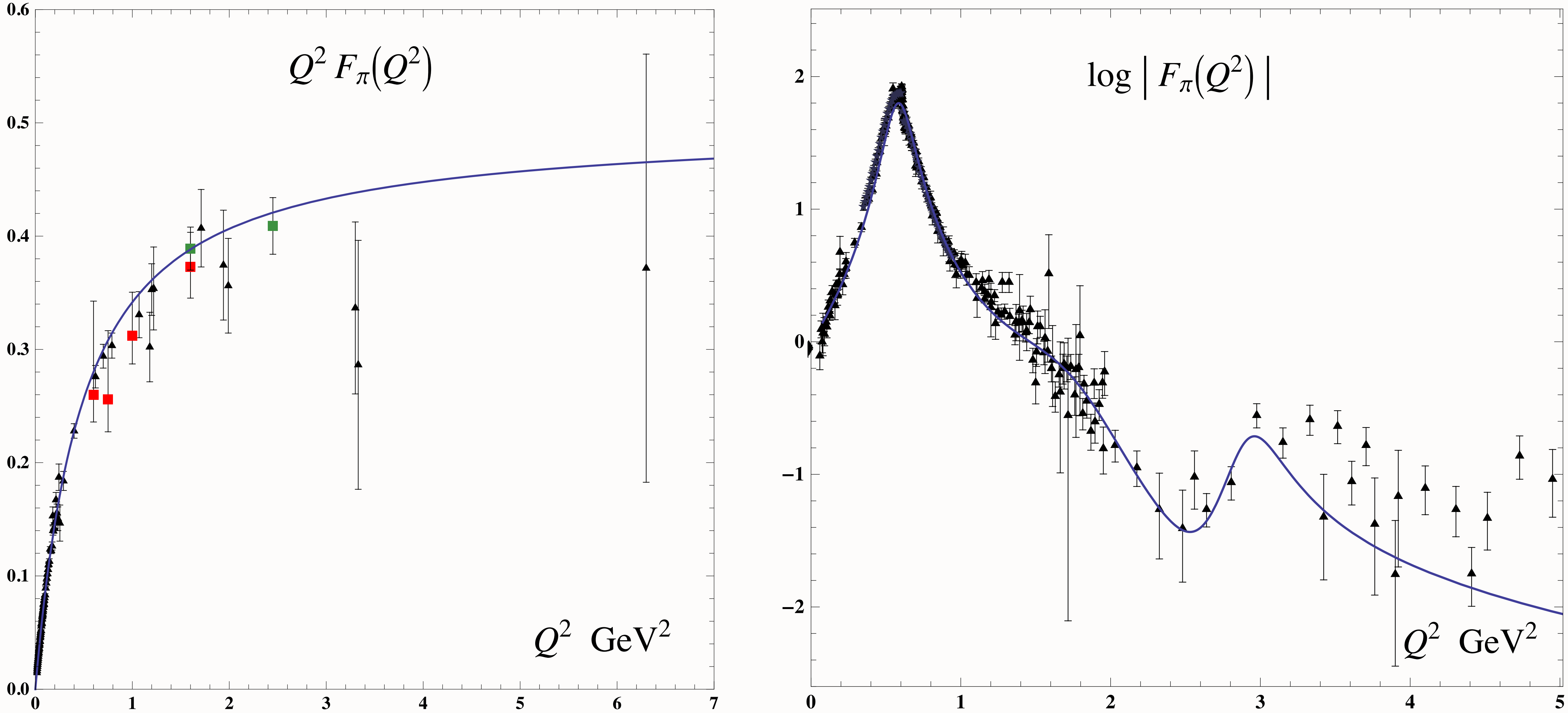
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current brings in higher Fock states and more vector meson poles



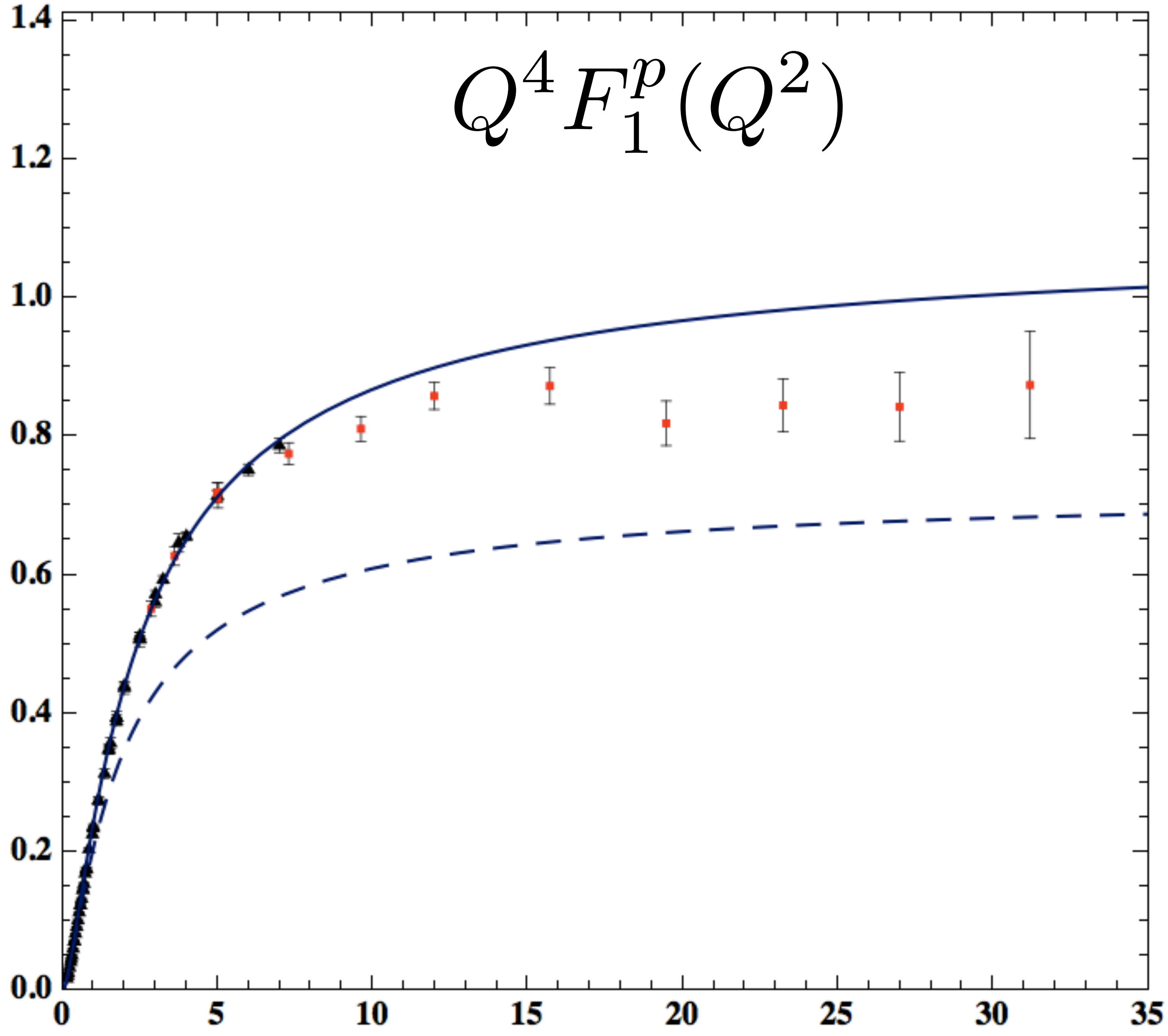


Spacelike and Timelike Pion Form Factor

Structure of the space- and time-like pion form factor in light-front holography for a truncation of the pion wave function up to twist four. Triangles are the data compilation from Baldini *et al.*, [42] red squares are JLAB 1 [43] and green squares are JLAB 2. [44]

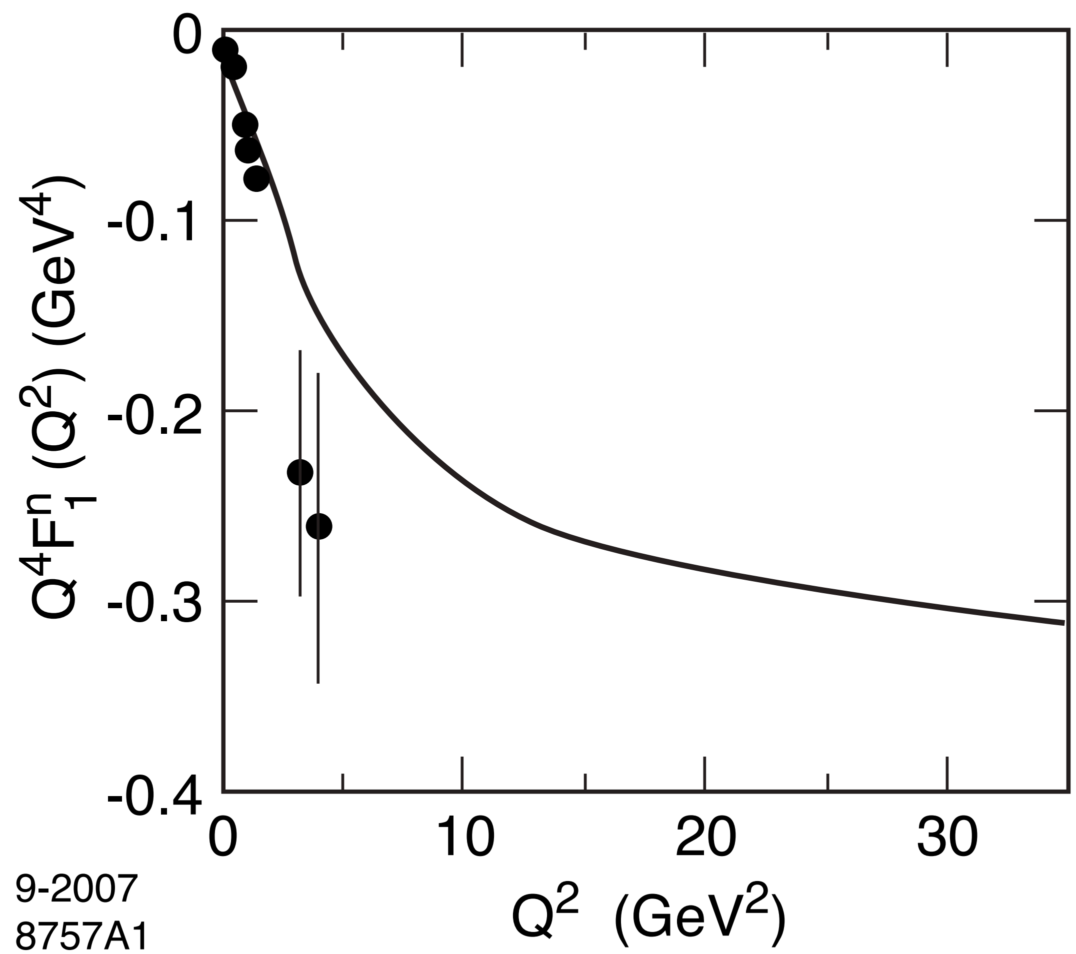
$$|\pi>=\psi_{\bar{q}q/\pi}|\bar{q}q> +\psi_{\bar{q}q\bar{q}q/\pi}|q\bar{q}\bar{q}q>$$

AdS/QCD $\kappa = 0.54 \text{ GeV}$



$Q^4 F_p^1(Q^2)$ in a negative (dashed line, $\kappa = 0.3877$ GeV) and positive dilaton backgrounds (continuous line, $\kappa = 0.5484$ GeV). The data compilation is from Diehl.

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

$$\cdots$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}, \quad N,$$

Positive Dilaton Background $\exp\left(+\kappa^2 z^2\right)$

$$\mathcal{M}_n^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$$

$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}$$

$$Q^2 \to \infty$$

Constituent Counting

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\big(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\big)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\Big(1+\frac{Q^2}{\mathcal{M}_\rho^2}\Big)\Big(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\Big)\Big(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\Big)}$$

with $\mathcal{M}_{\rho_n}^{\,2} \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Elastic and Transition Form Factors

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)}, \qquad F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)\left(1+\frac{Q^2}{M_{\rho''}^2}\right)},$$

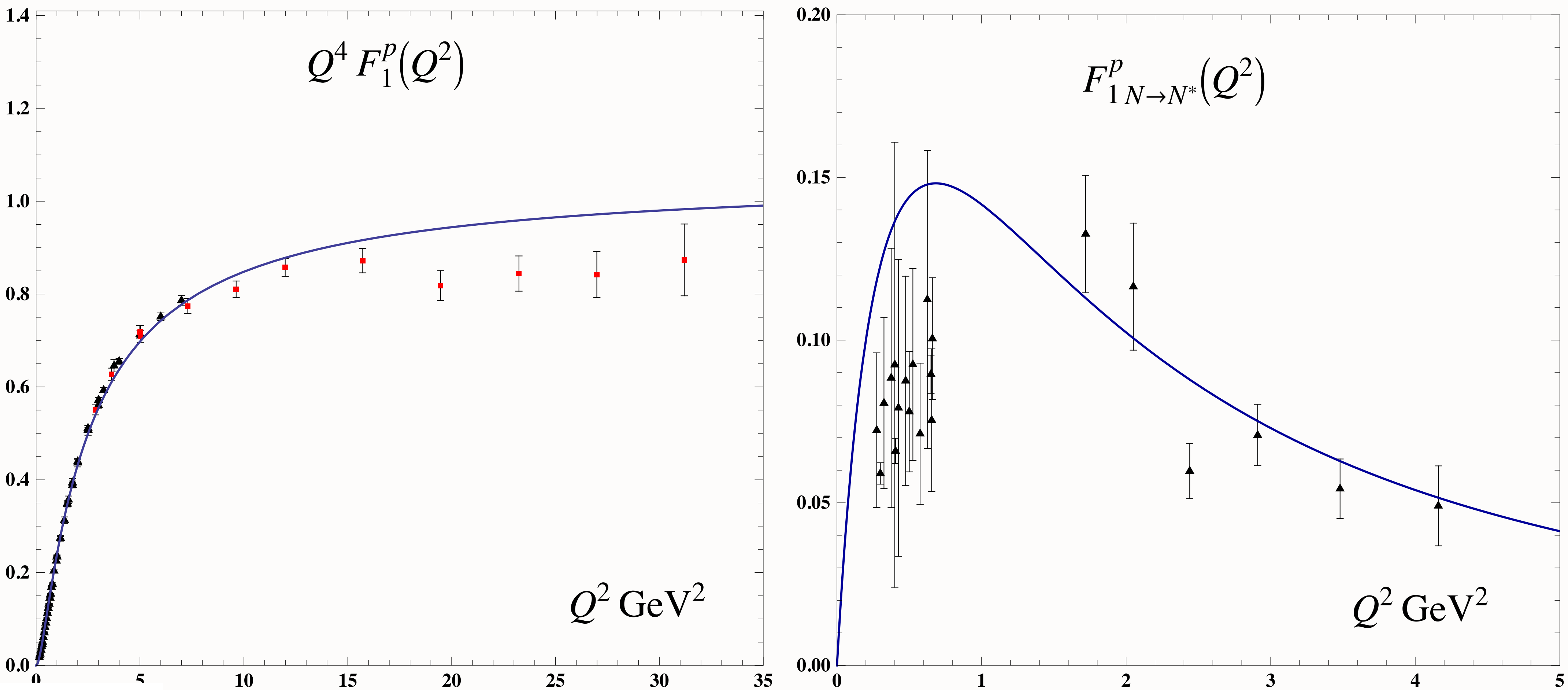


Figure 2: Dirac proton form factors in light-front holographic QCD. Left: scaling of proton elastic form factor $Q^4F_1^p(Q^2)$. Right: proton transition form factor $F^p_{1\,N\to N^*}(Q^2)$ to the first radial excited state. Data compilation from Diehl [32] (left) and JLAB [33] (right).

Guy de Teramond, sjb

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chira**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Higher Fock States

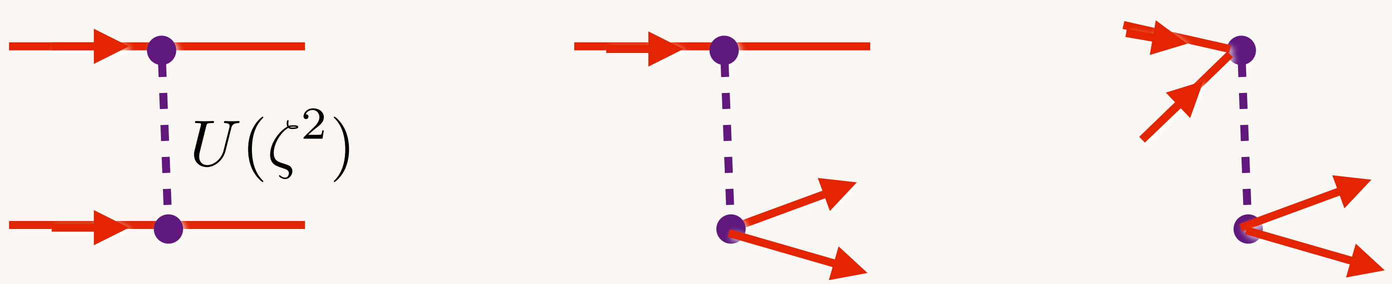
- Exposed by timelike form factor through dressed current.

- Created by confining interaction

$$P^-_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- Similar to QCD(1+1) in lcg

- No explicit gluons - quark interchange dominates exlusive reactions



de Teramond, sjb

AdS/QCD and Light-Front Holography

- Hadrons are composites of quark and anti-quark constituents
- Explicit gluons absent!
- Higher Fock states with extra quark/anti-quark pairs created by confining potential
- Dominance of Quark Interchange in Hard Exclusive Reactions
- Short-distance behavior matches twist of interpolating operator at short distance -- guarantees dimensional counting rules --

PHYSICAL REVIEW D VOLUME 49, NUMBER 1 1 JANUARY 1994

Comparison of 20 exclusive reactions at large t

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K. Johns,[4,§] M. Kmit,[1,||] D. I. Lowenstein,[1] X. Ma,[3] Y. I. Makdisi,[1]
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(Received 28 May 1993)

We report a study of 20 exclusive reactions measured at the AGS at 5.9 GeV/c incident momentum, 90° center of mass. This experiment confirms the strong quark flow dependence of two-body hadron-hadron scattering at large angle. At 9.9 GeV/c an upper limit had been set for the ratio of cross sections for $(\bar{p}p \to \bar{p}p)/(pp \to pp)$ at 90° c.m., with the ratio less than 4%. The present experiment was performed at lower energy to gain sensitivity, but was still within the fixed angle scaling region. A ratio $R(\bar{p}p/pp) \approx 1/40$ was measured at 5.9 GeV/c, 90° c.m. in comparison to a ratio near 1.7 for small angle scattering. In addition, many other reactions were measured, often for the first time at 90° c.m. in the scaling region, using beams of $\pi^\pm$, $K^\pm$, p, and $\bar{p}$ on a hydrogen target. There are similar large differences in cross sections for other reactions: $R(K^-p \to \pi^+\Sigma^-/K^-p \to \pi^-\Sigma^+) \approx 1/12$, for example. The relative magnitudes of the different cross sections are consistent with the dominance of quark interchange in these 90° reactions, and indicate that pure gluon exchange and quark-antiquark annihilation diagrams are much less important. The angular dependence of several elastic cross sections and the energy dependence at a fixed angle of many of the reactions are also presented.

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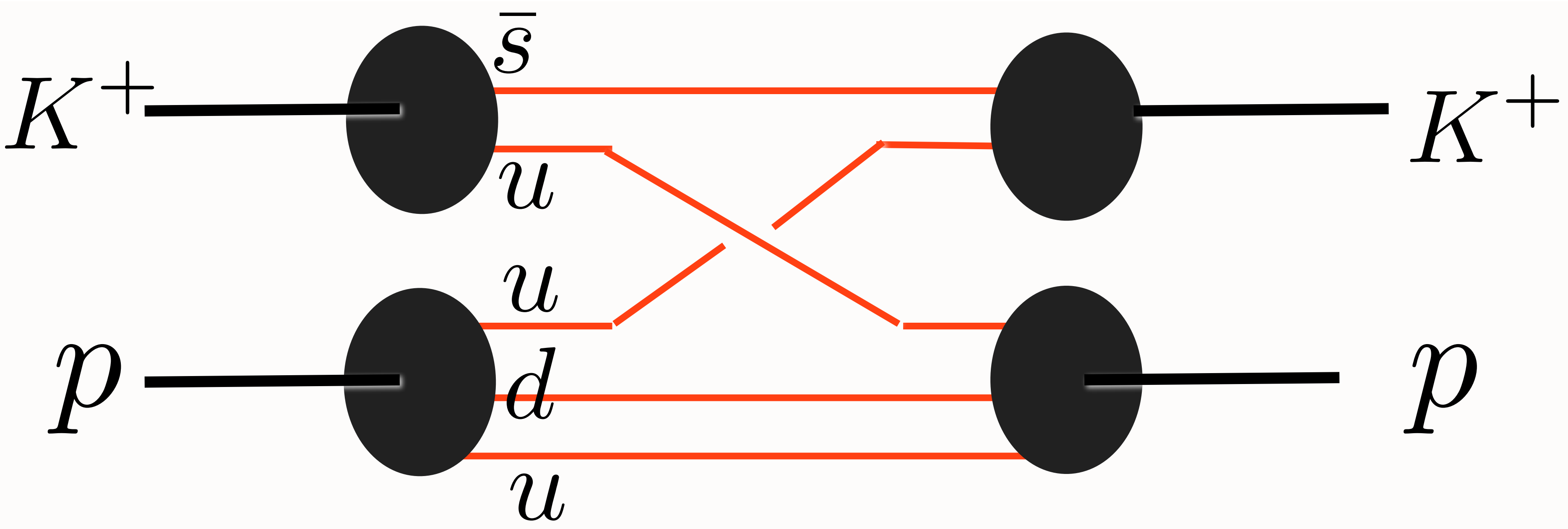
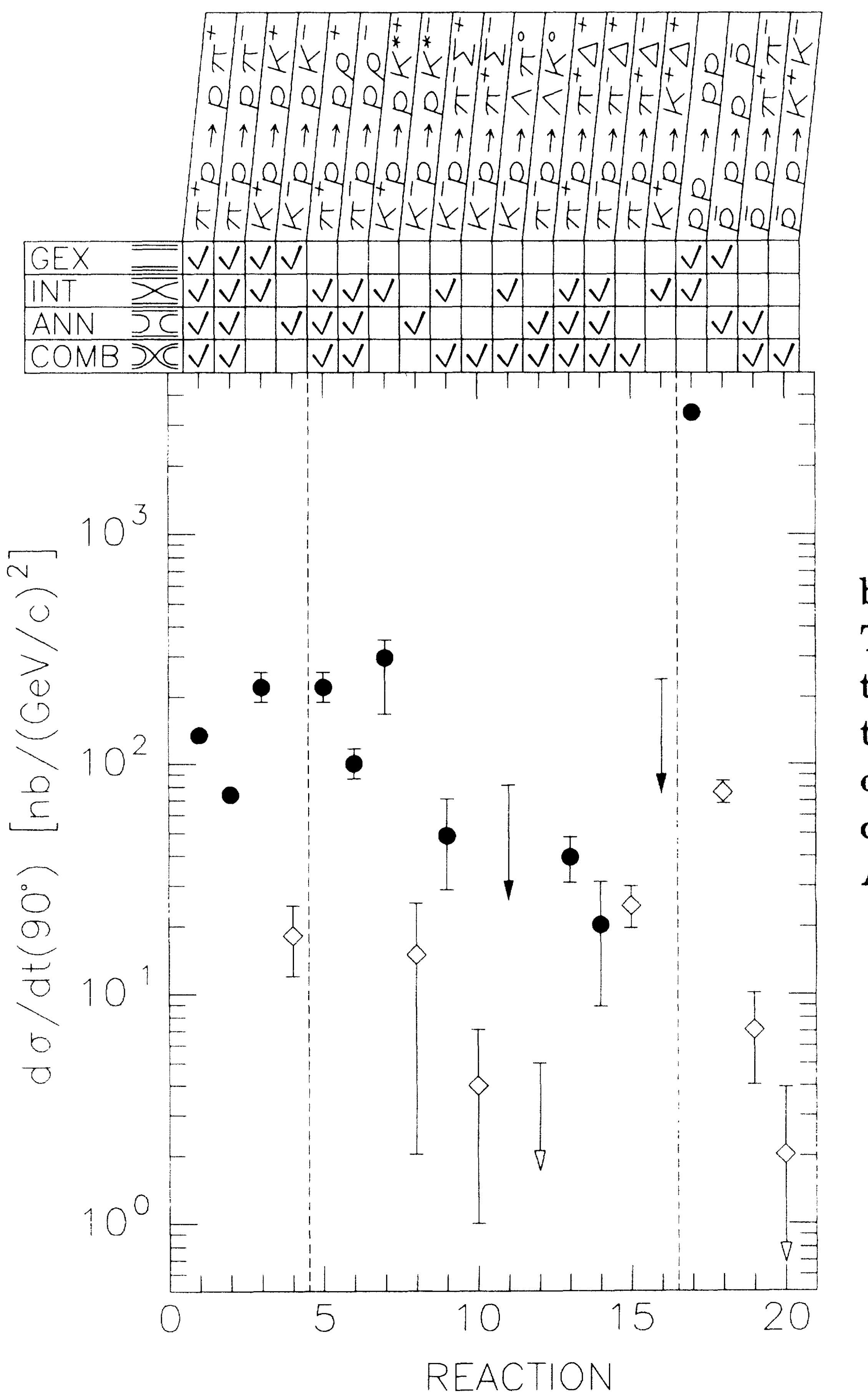




Differential cross sections for the 16 meson-baryon and 4 baryon-baryon measured in this experiment. The cross sections are at, or extrapolated from, near 90° center of mass. The four quark flow diagrams which contribute to each of the 20 reactions are given in the chart at the top of the figure. Those reactions which have a contribution from quark interchange(INT) are given by the solid black points. As can be seen, these are the largest cross sections.

Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2 P_{q\bar q}}\, \kappa\, z^2 e^{-\kappa^2 z^2/2}, \quad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar q}$
- Find $\left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\bar q}}\, \kappa / \sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar q} Q^2 (1-x)/4\pi^2 f_\pi^2\, x}\right]$$

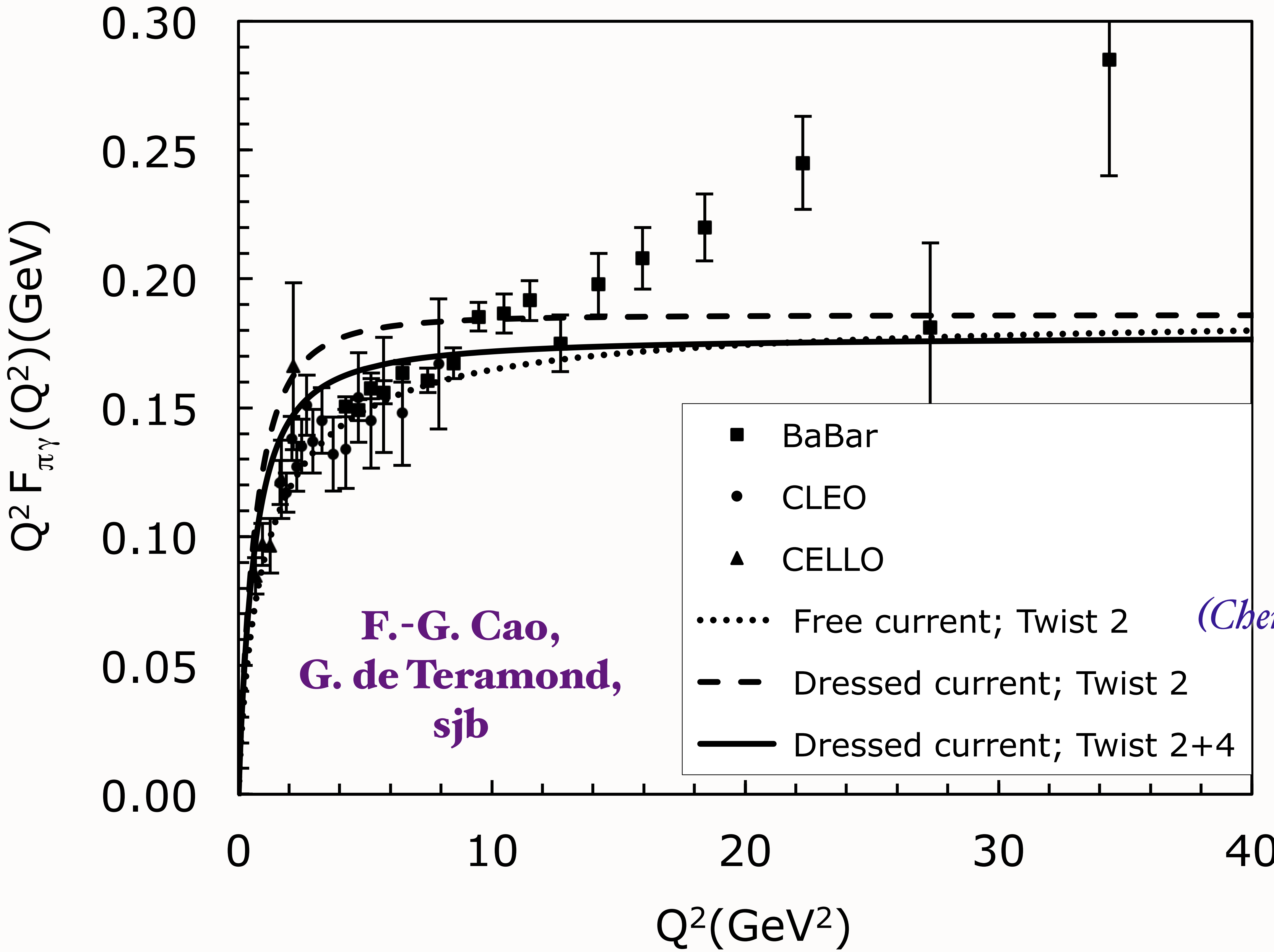
normalized to the asymptotic DA $[P_{q\bar q} = 1 \to$ Musatov and Radyushkin (1997)]

G.P. Lepage, sjb

- Large Q^2 TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

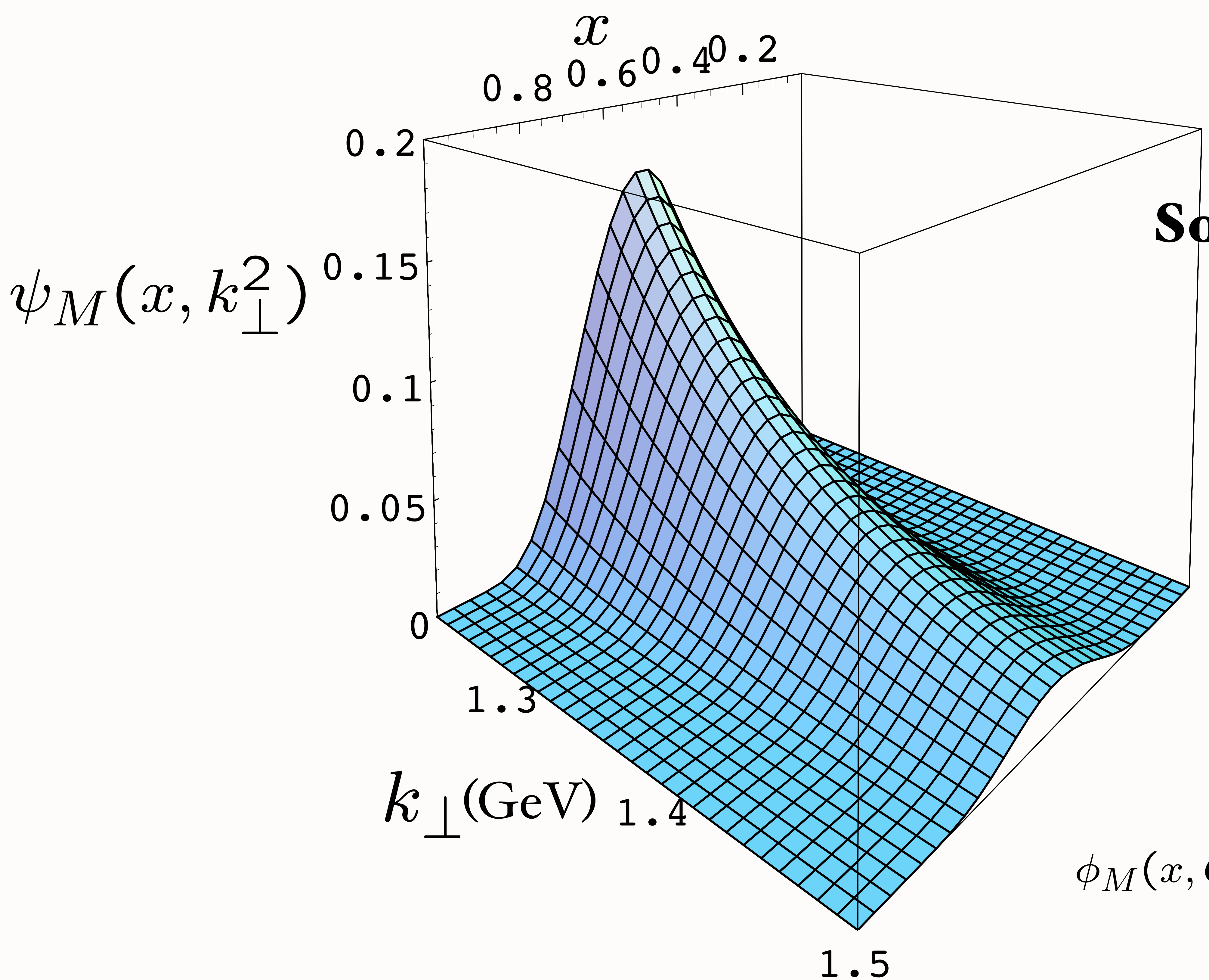
Photon-to-pion transition form factor

Lepage, sjb $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$



(Chern-Simons)

Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

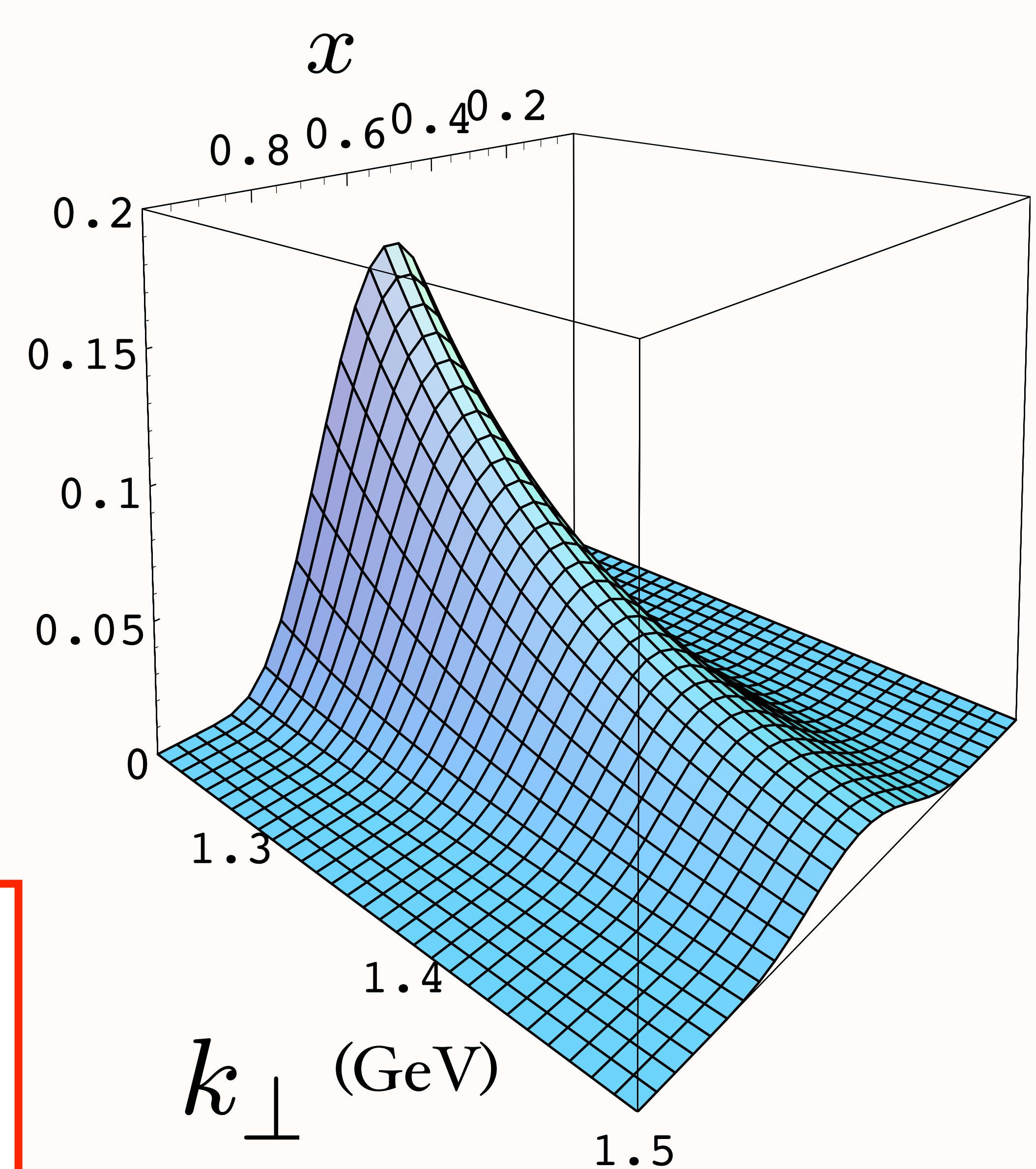
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

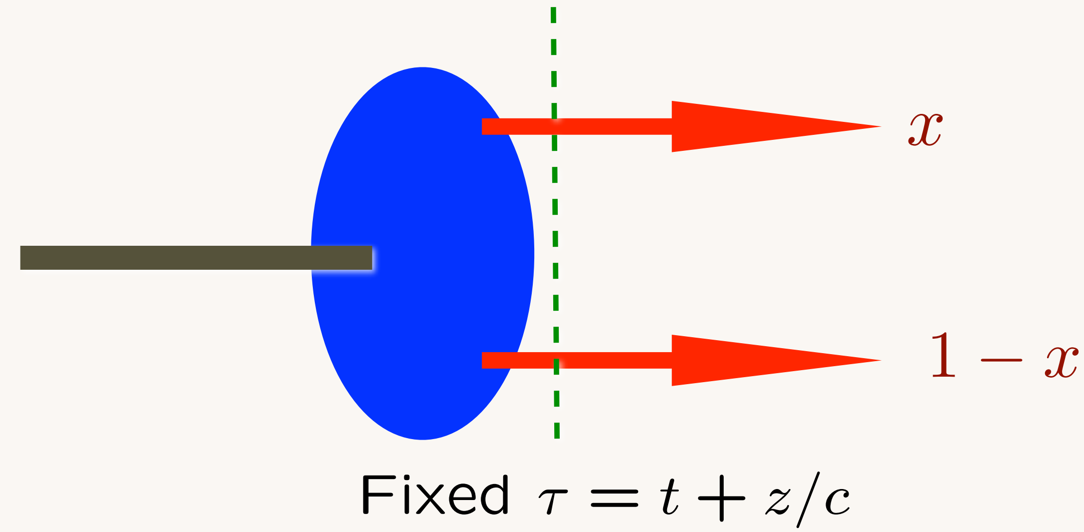
Connection of Confinement to TMDs

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

Generalized parton distributions in AdS/QCD

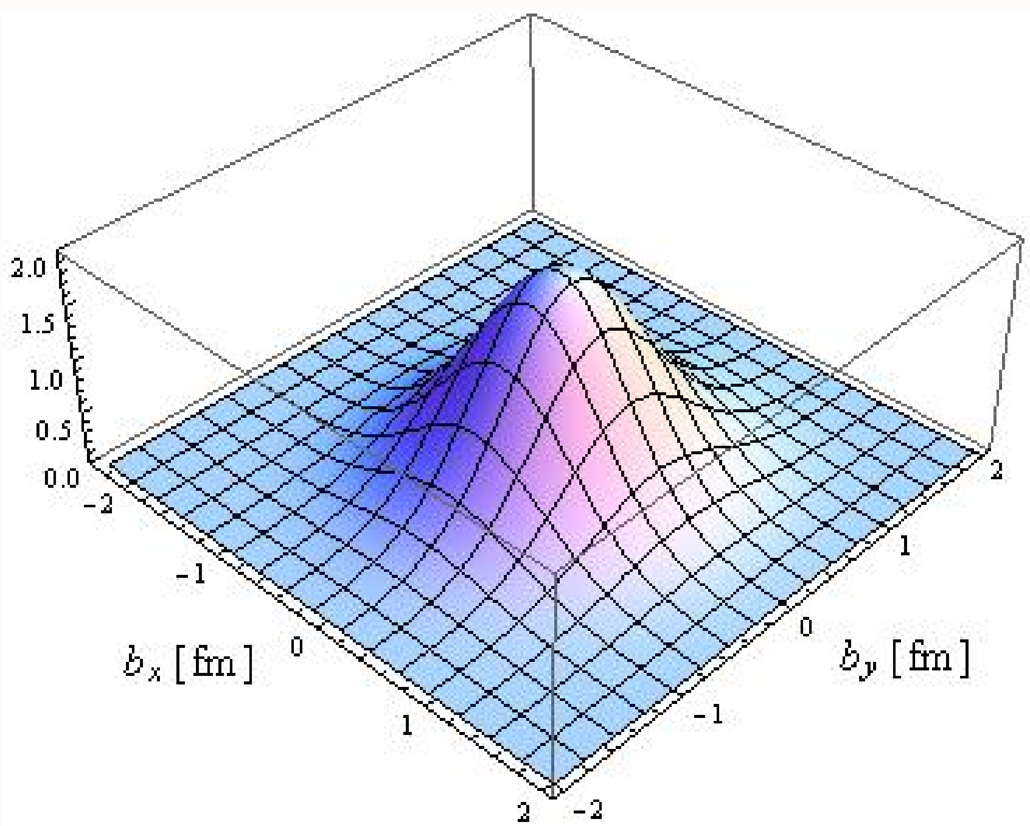
Alfredo Vega[1], Ivan Schmidt[1], Thomas Gutsche[2], Valery E. Lyubovitskij[2*]

[1] *Departamento de Física y Centro Científico y Tecnológico de Valparaíso, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

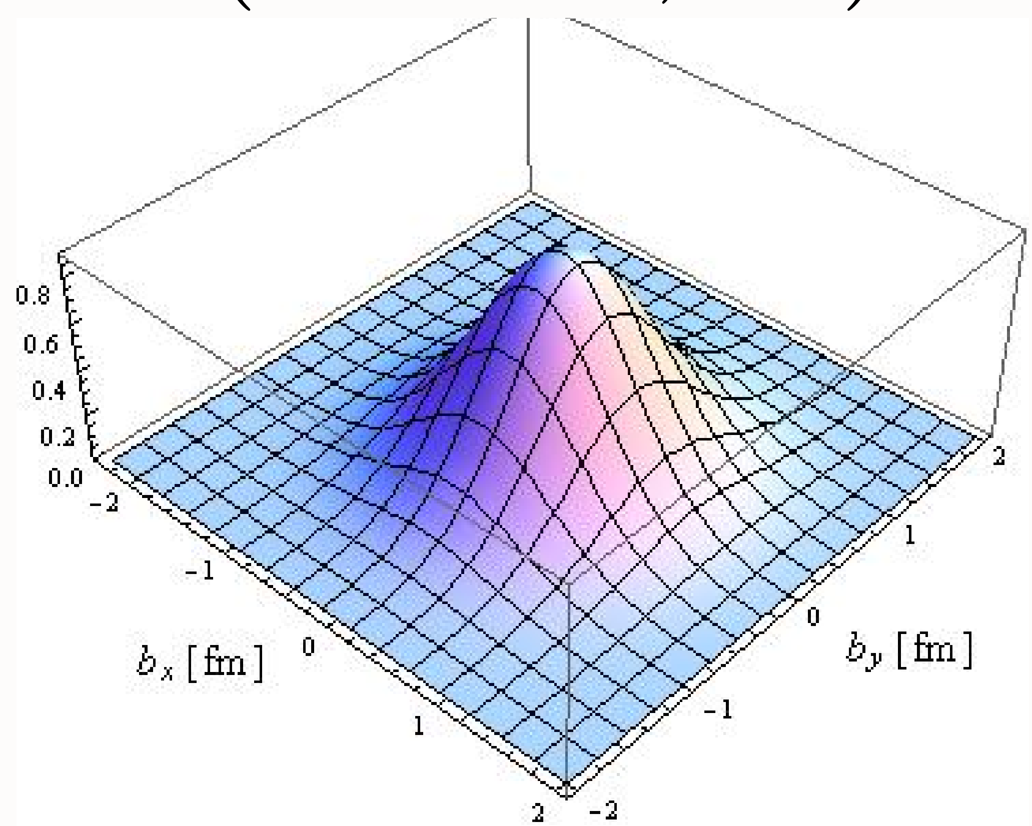
[2] *Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D–72076 Tübingen, Germany*

(Dated: January 19, 2011)

$$u(x = 0.1, \vec{b}_\perp)$$



$$d(x = 0.1, \vec{b}_\perp)$$



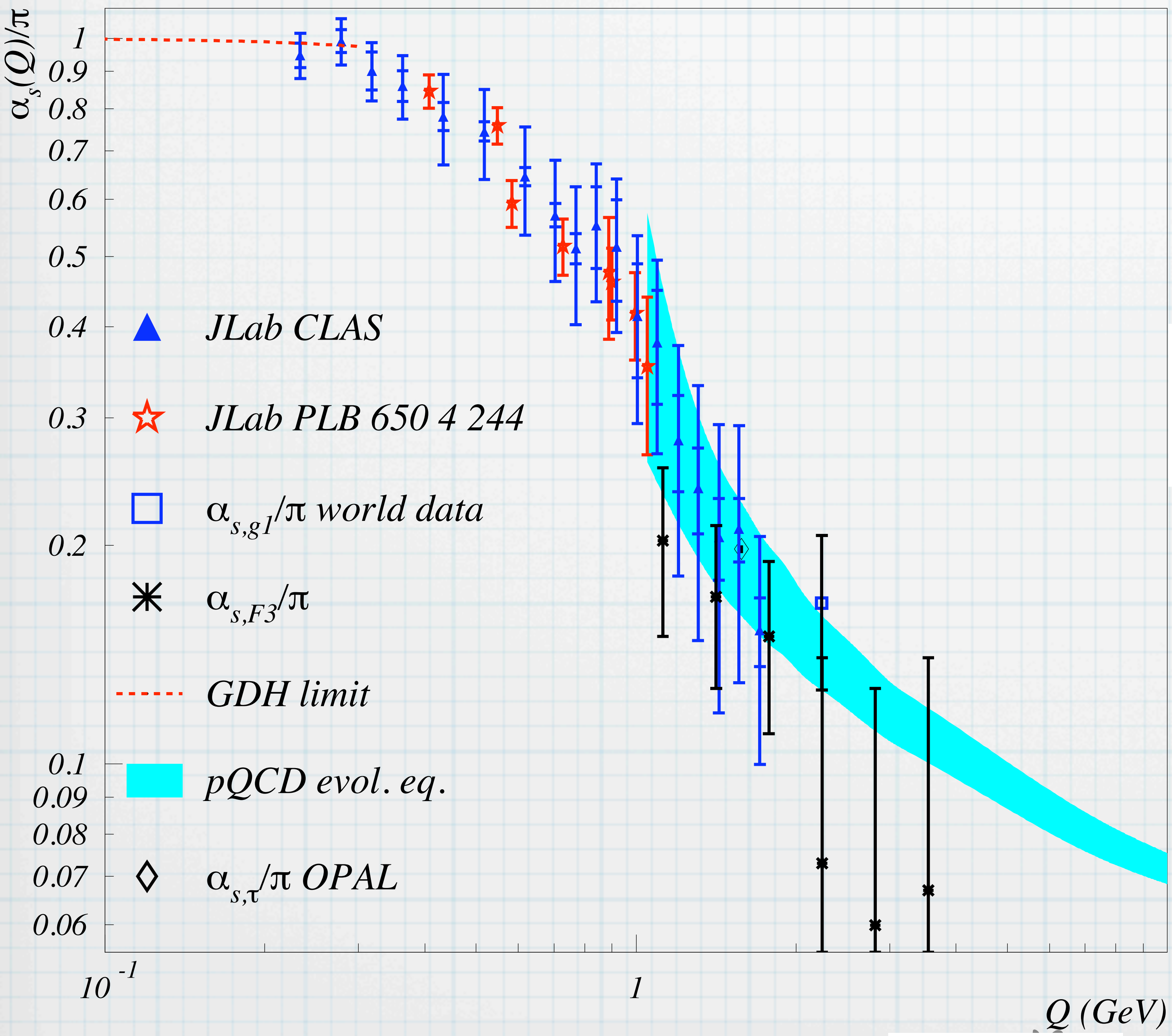
Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS5**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

Nearly conformal QCD?

Define α_s from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left(g_1^p(x,Q^2) - g_1^n(x,Q^2) \right) = \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g_1}}{\pi} \right)$$



g1 = spin dependent structure functio

Recent JLab data from EG1(2008), CLAS, and Hall A

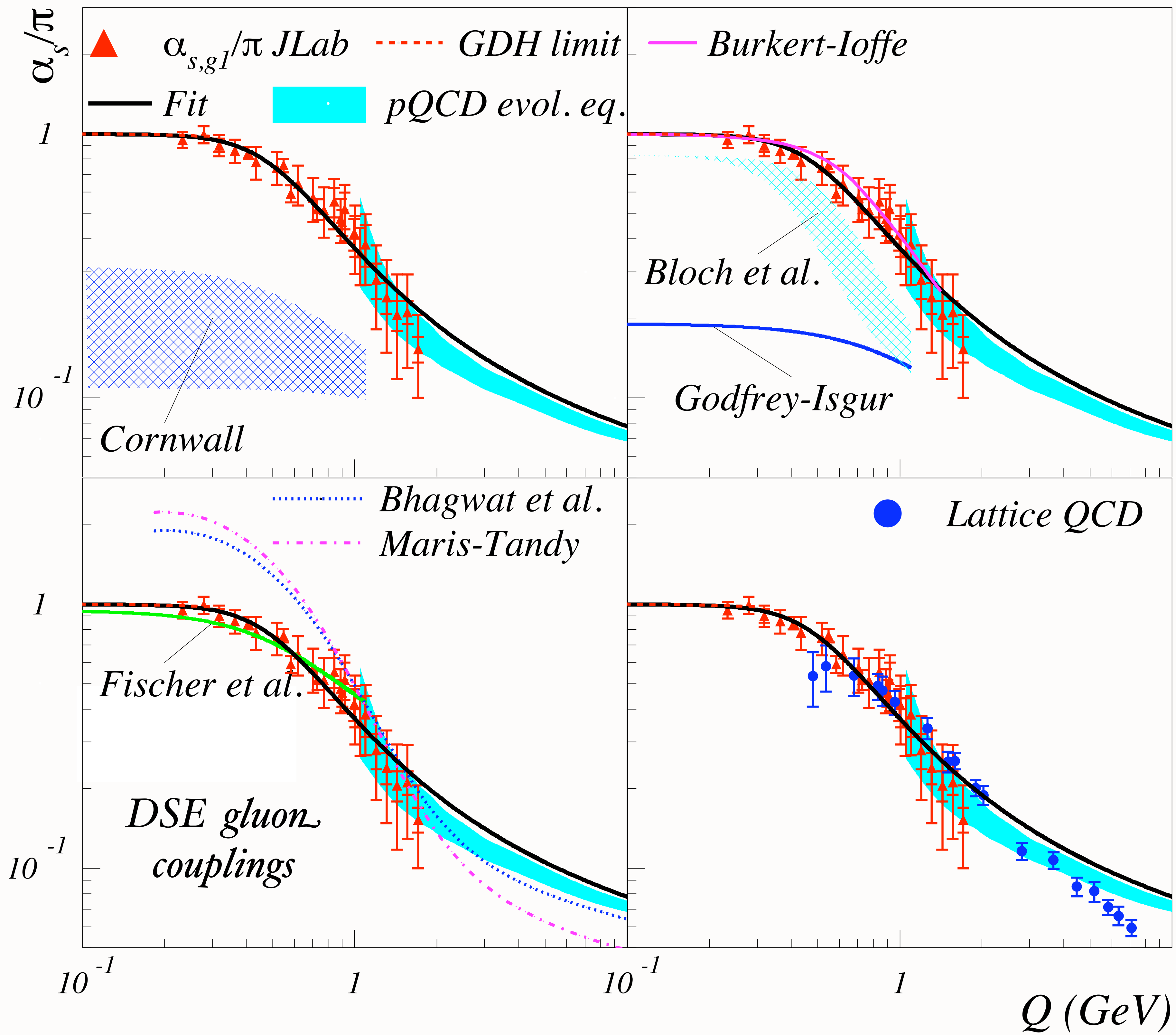
α_s runs only modestly at small Q^2

Gribov

Fig. from 0803.4119, Duer et al.

Deur, de Teramond, sjb

Deur, Korsch, et al.



Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\,dz\,\sqrt{g}\,e^{\varphi(z)}\,\frac{1}{g_5^2}\,G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

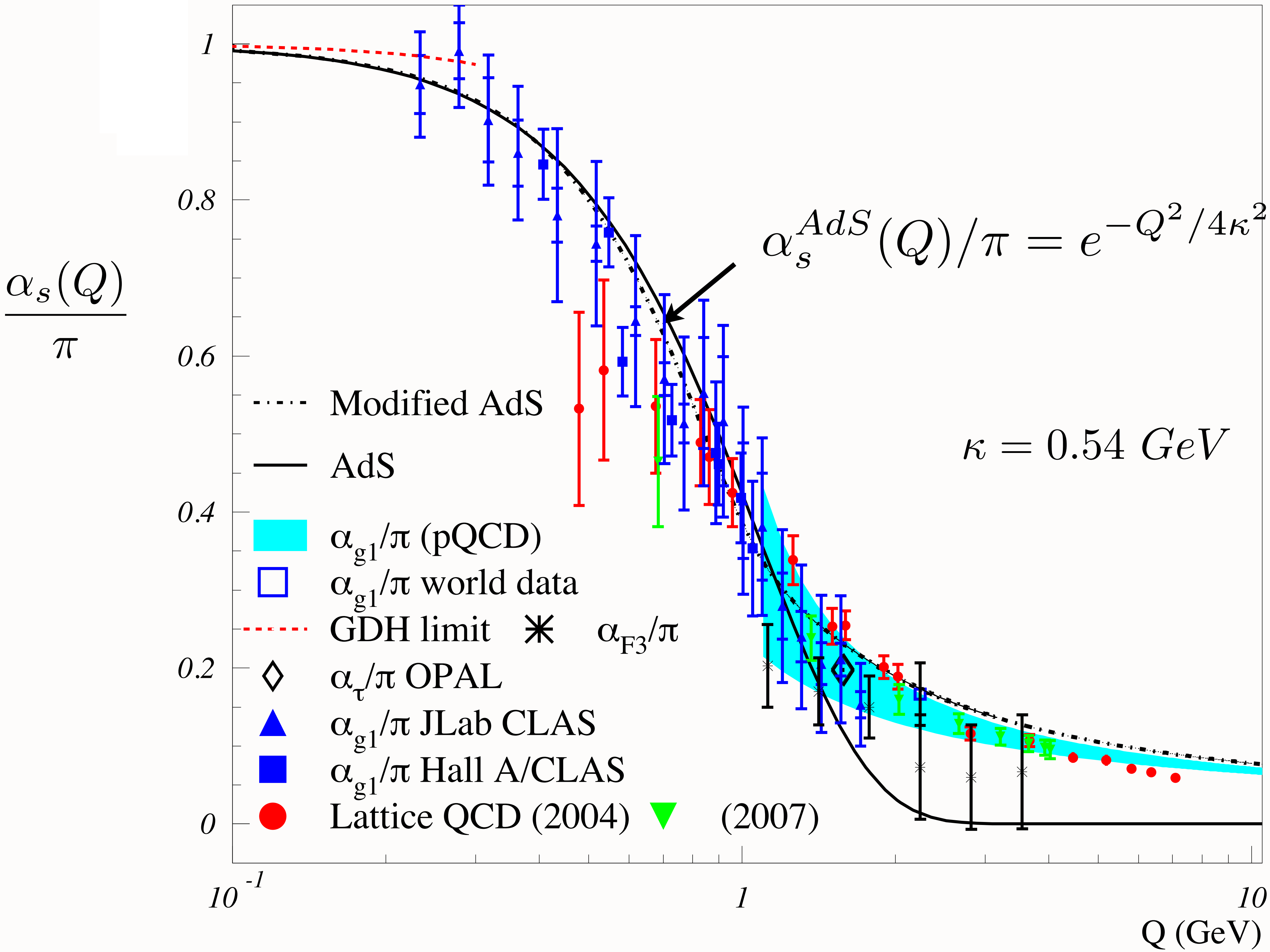
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

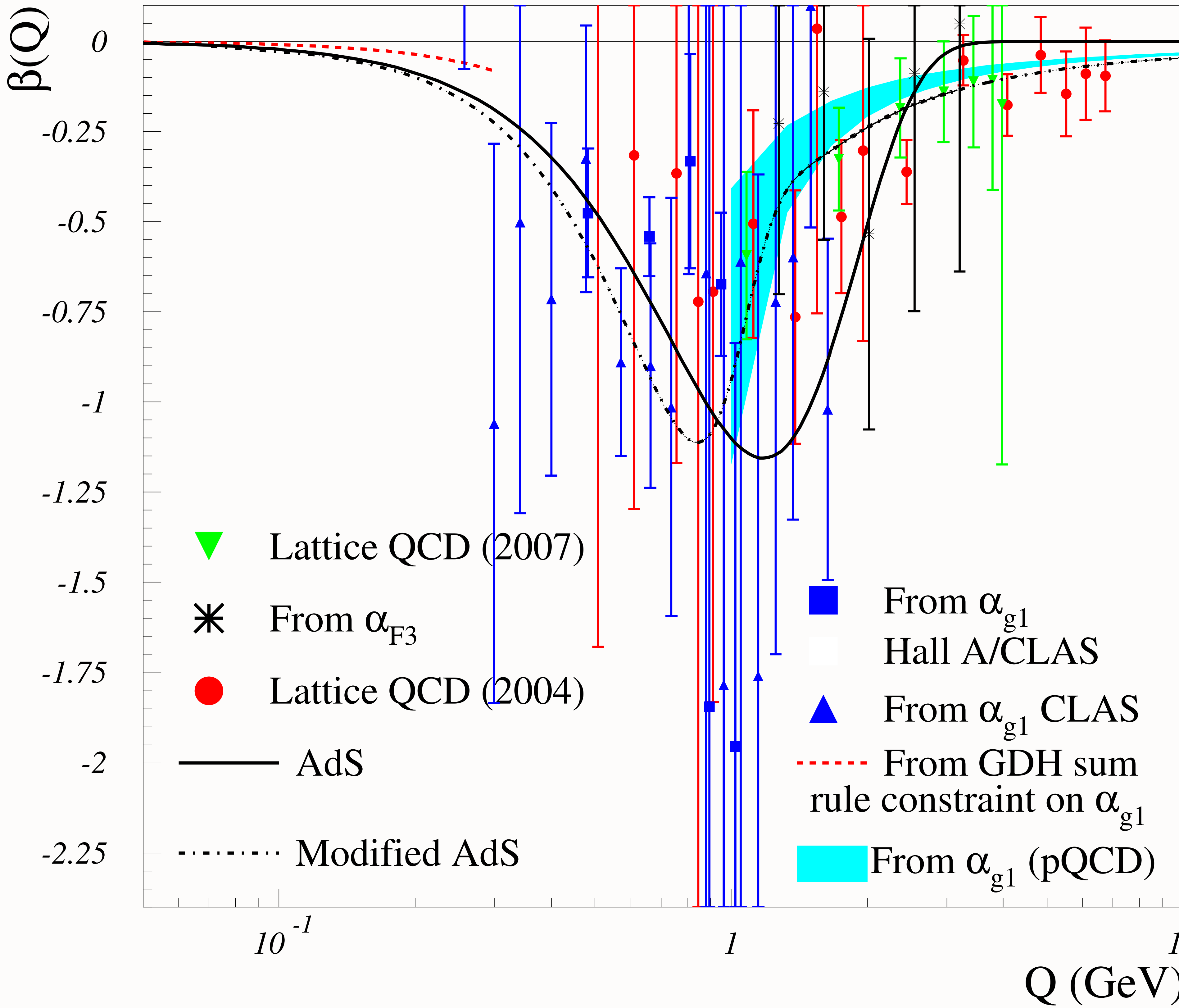
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Deur, de Teramond, sjb

$$\beta^{AdS}(Q^2) = \frac{d}{d\log Q^2}\alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2}e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb

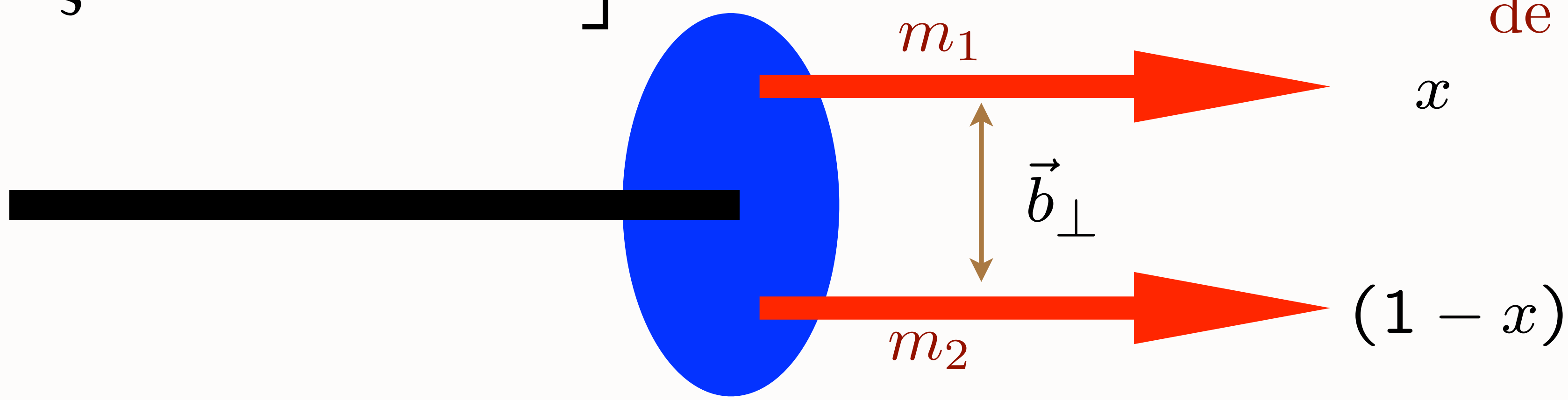
Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

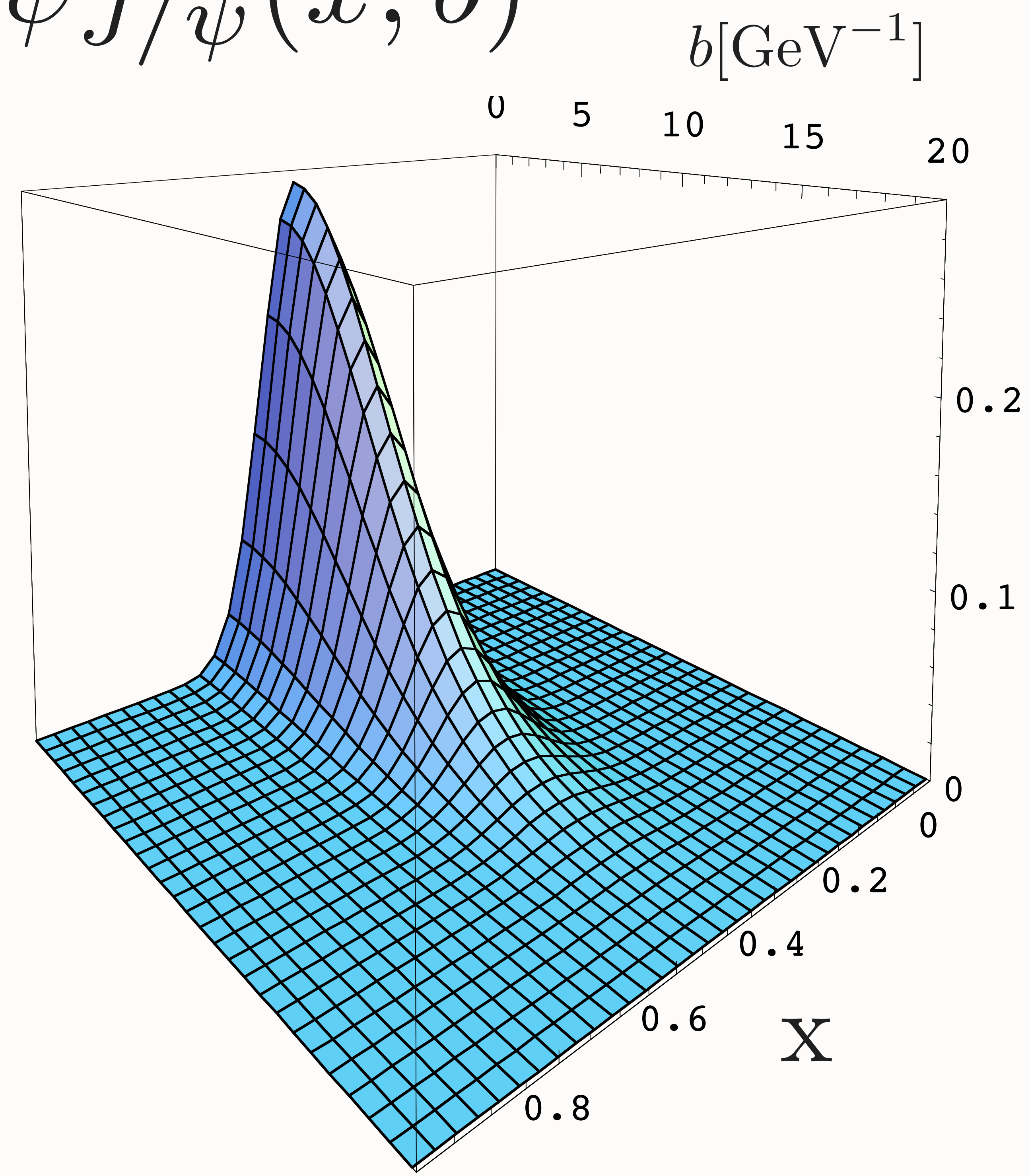
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$\kappa = 0.375$ GeV

$\psi_{J/\psi}(x, b)$



$m_a = m_b = 1.25$ GeV

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$

$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

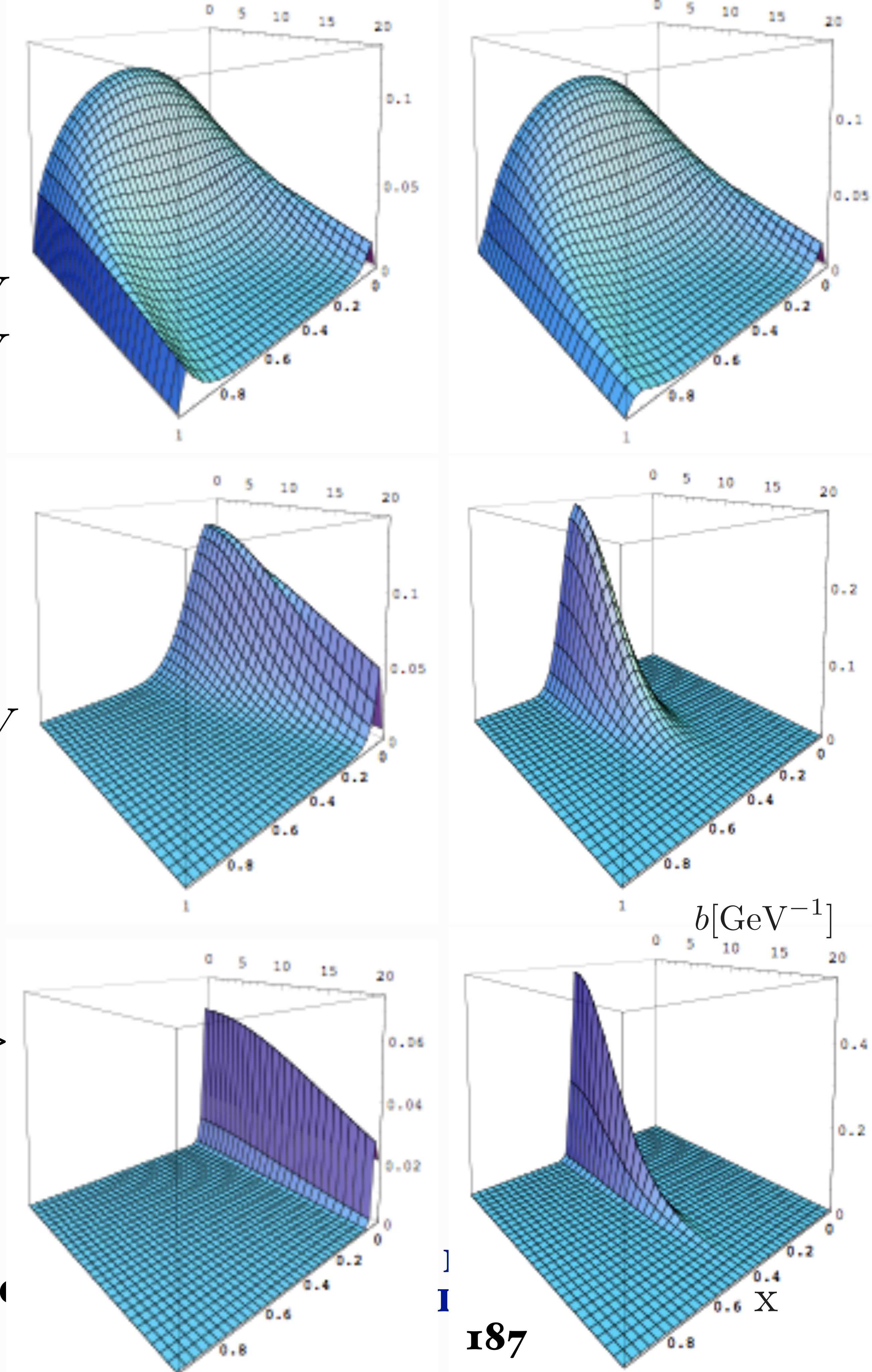
$m_c = 1.25\ GeV$

$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$

$|\eta_b >= |b\bar{b} >$



$\kappa = 375\ MeV$

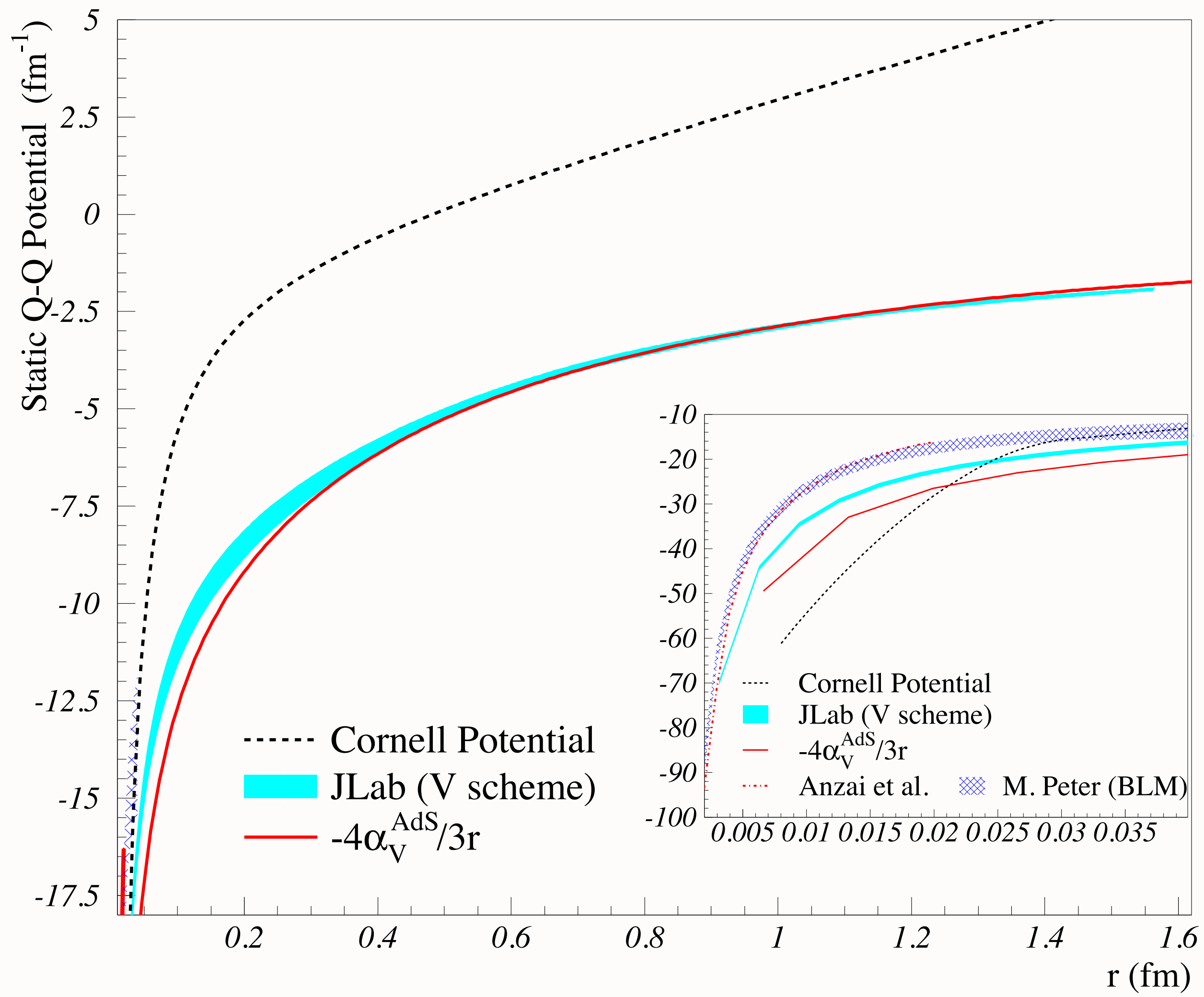
Static $\overline{Q}Q$ Potential

- For heavy quarks LF holographic equations reduce to NR Schrödinger equation in configuration space

$$V(r) = -\frac{4}{3}\frac{\alpha_V(r)}{r} + V_{conf}(r)$$

de Teramond, sjb

where $V_{conf} \simeq \frac{1}{2} m_{red}\omega^2 r^2$, $m_{red} = m_Q m_{\overline{Q}}/(m_Q + m_{\overline{Q}})$ and $\omega = \kappa^2/(m_Q + m_{\overline{Q}})$



Comparison of Coulomb $\overline{Q}Q$ potential with Cornell potential

Features of Soft-Wall AdS/QCD

- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion ($m_q = 0$)**
- **Regge Trajectories: universal slope in n and L**
- **Valid for all integer J & S.**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large Nc limit required**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize H_{LF} on AdS basis**

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Pauli, Hornbostel, Hiller, McCartor, Chabysheva, sjb

- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
  G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb Proc.Nat.Acad.Sci. 108 (2011) 45-50 "Condensates in Quantum Chromodynamics and the Cosmological Constant"

C. Roberts, R. Shrock, P. Tandy, sjb Phys.Rev. C82 (2010) 022201 "New Perspectives on the Quark Condensate"

Gell-Mann Oakes Renner Formula in QCD

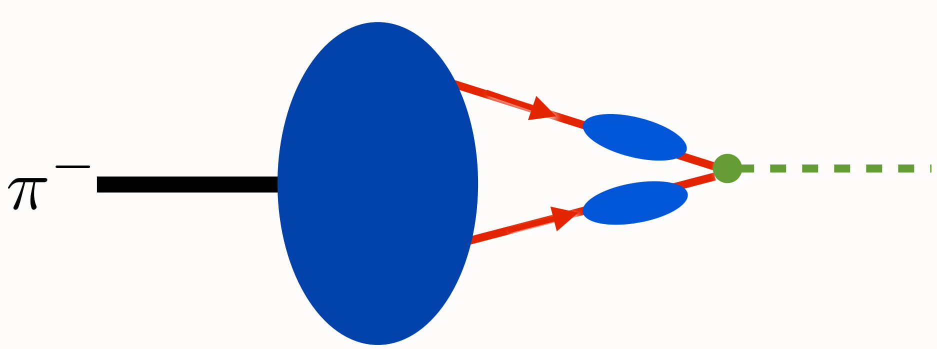
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

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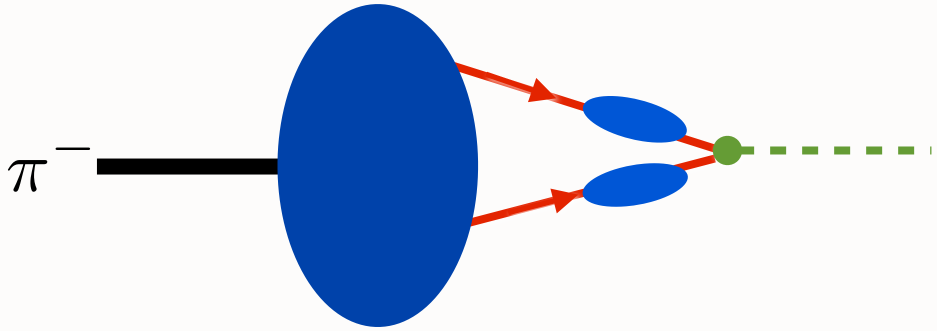


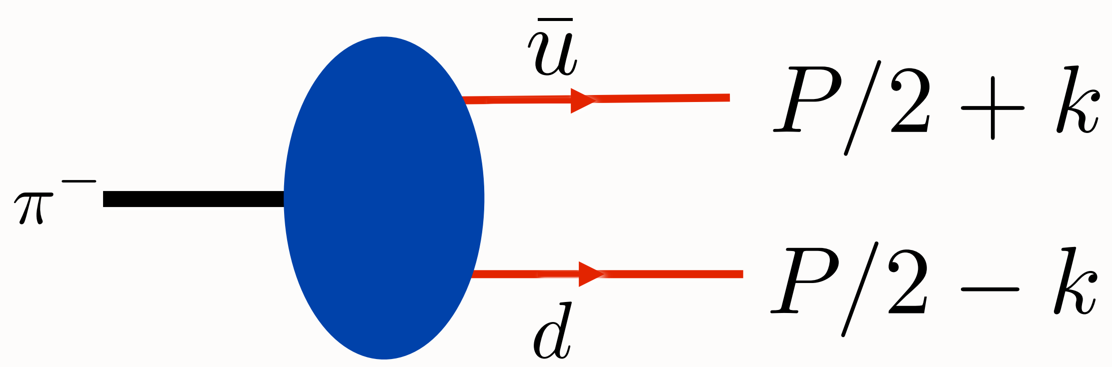
$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

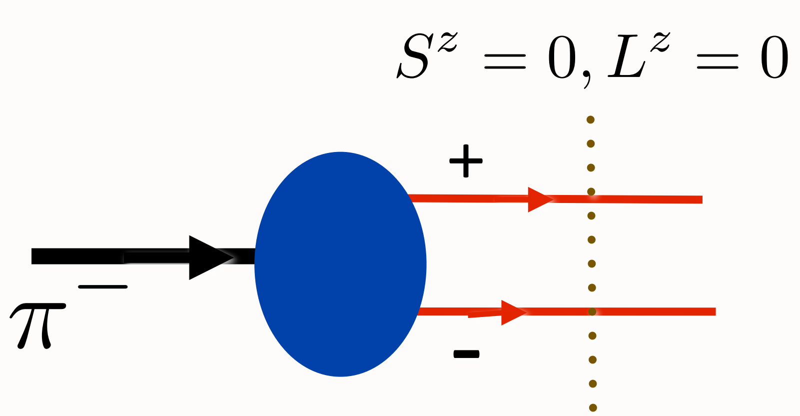
General Form of Bethe-Salpeter Wavefunction

$$\Gamma_\pi(k;P) = i\gamma_5 E_\pi(k,P) + \gamma_5 \gamma \cdot P F_\pi(k;P)$$
$$+\gamma_5 \gamma \cdot k G_\pi(k;P) - \gamma_5 \sigma_{\mu\nu} k^\mu P^\nu H_\pi(k;P)$$

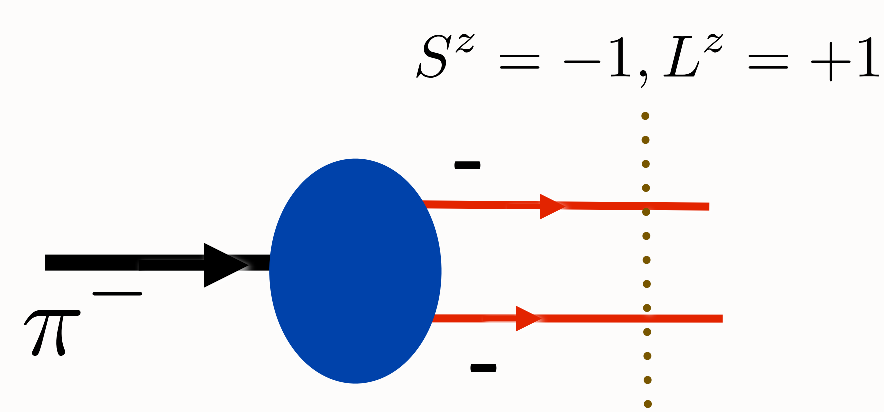
$\Gamma_\pi(k;P)$



Allows both $< 0|\bar{q}\gamma_5\gamma_\mu q|\pi >$ and $< 0|\bar{q}\gamma_5 q|\pi >$

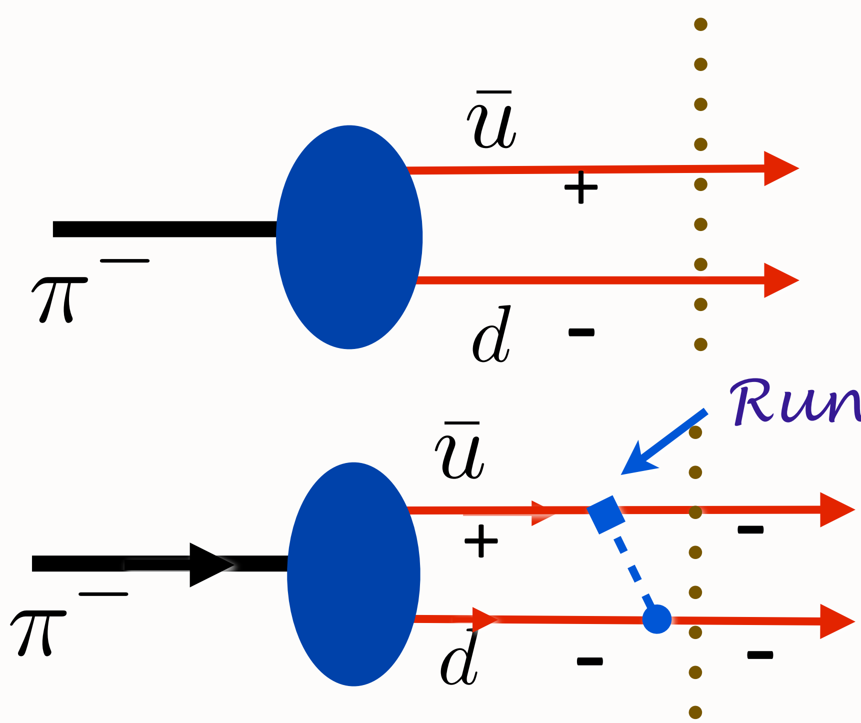


LFWFs



Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$



Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar\gamma^\mu q \gamma_5 q|0 > \sim f_\pi$$

Running constituent mass at vertex

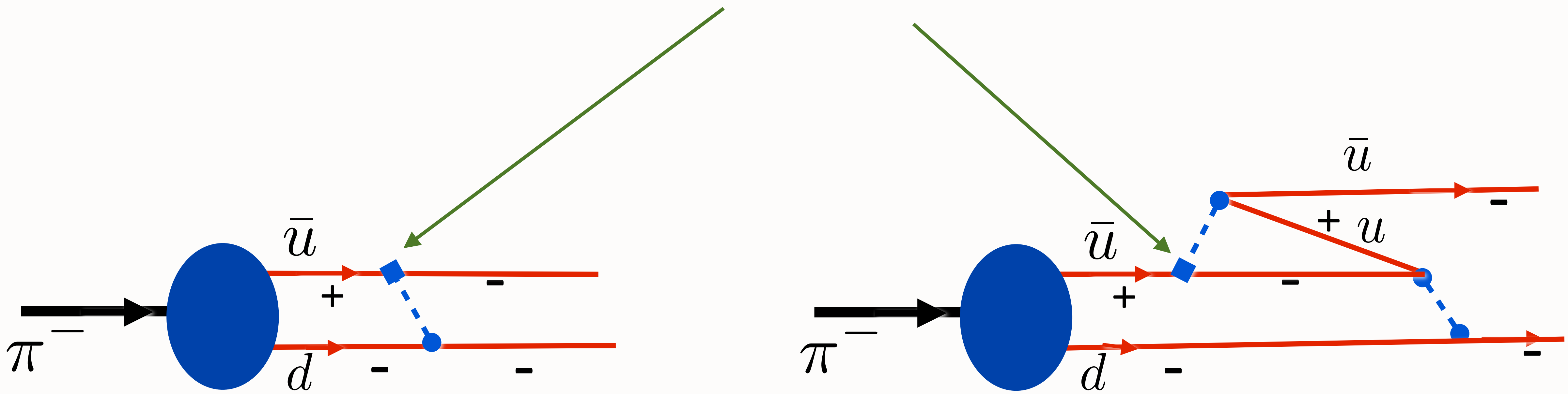
$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0 > \sim \rho_\pi$$

$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

Angular Momentum Conservation

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

Running constituent mass at vertex



$$L^z = +1, S^z = -1$$

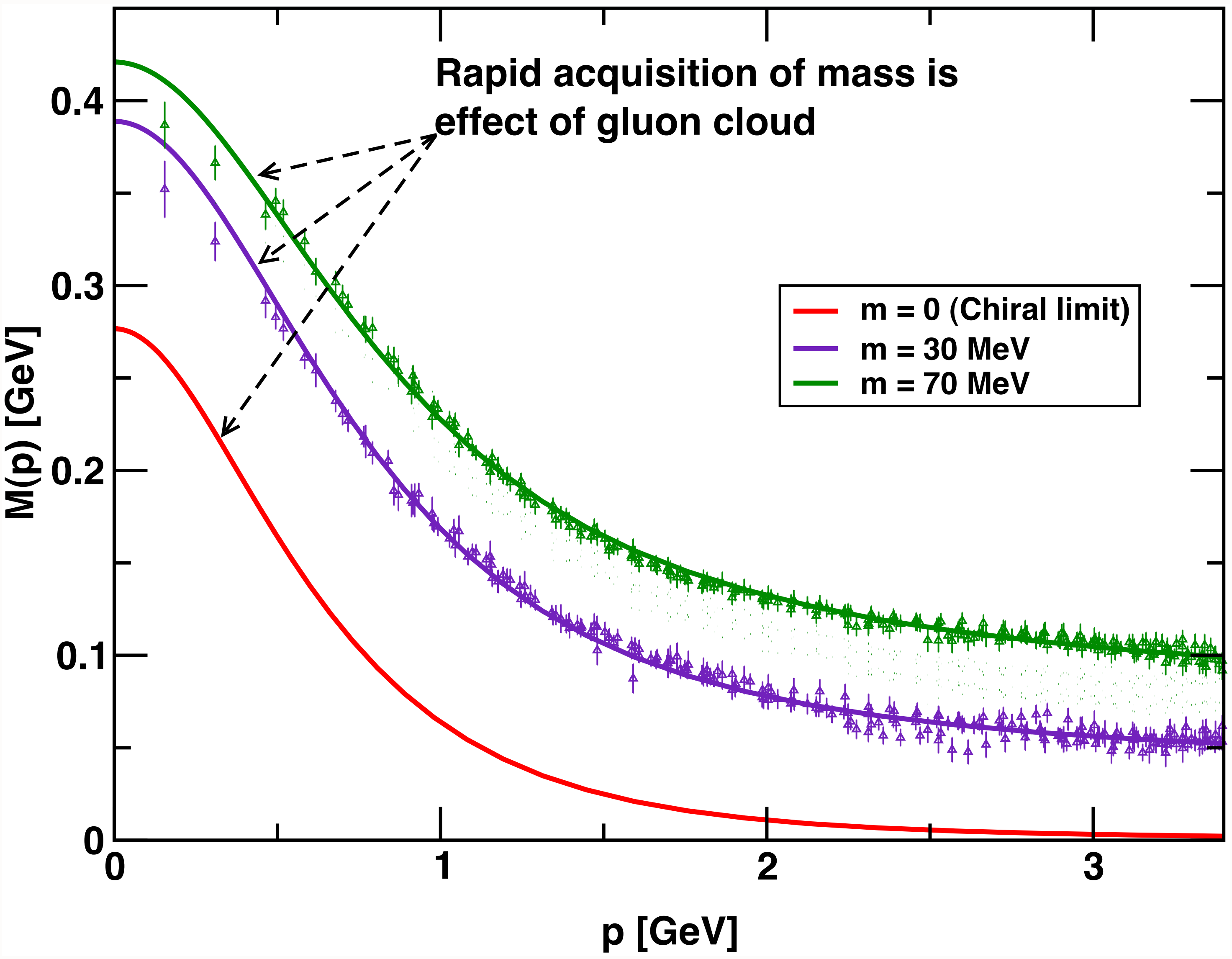
$L^z = 0, S^z = 0$ LF wavefunction couples to $< \pi|\bar{\gamma}^\mu q\gamma_5 q|0 >$

$L^z = +1, S^z = -1$ LF wavefunction couples to $< \pi|\bar{q}\gamma_5 q|0 >$

$m(\ell^2;\zeta) = B(\ell^2;\zeta)/A(\ell^2;\zeta)$ *running quark mass*

Running quark mass in QCD

$$S^{-1}(p) = i\gamma \cdot p \; A(p^2) + B(p^2) \qquad m(p^2) = \frac{B(p^2)}{A(p^2)}$$



Dyson-Schwinger

Chang, Cloet, El-Bennich Klahn, Roberts

Consistent with EW input at high p^2

Survives even at m=0!

Spontaneous Chiral Symmetry Breaking!

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

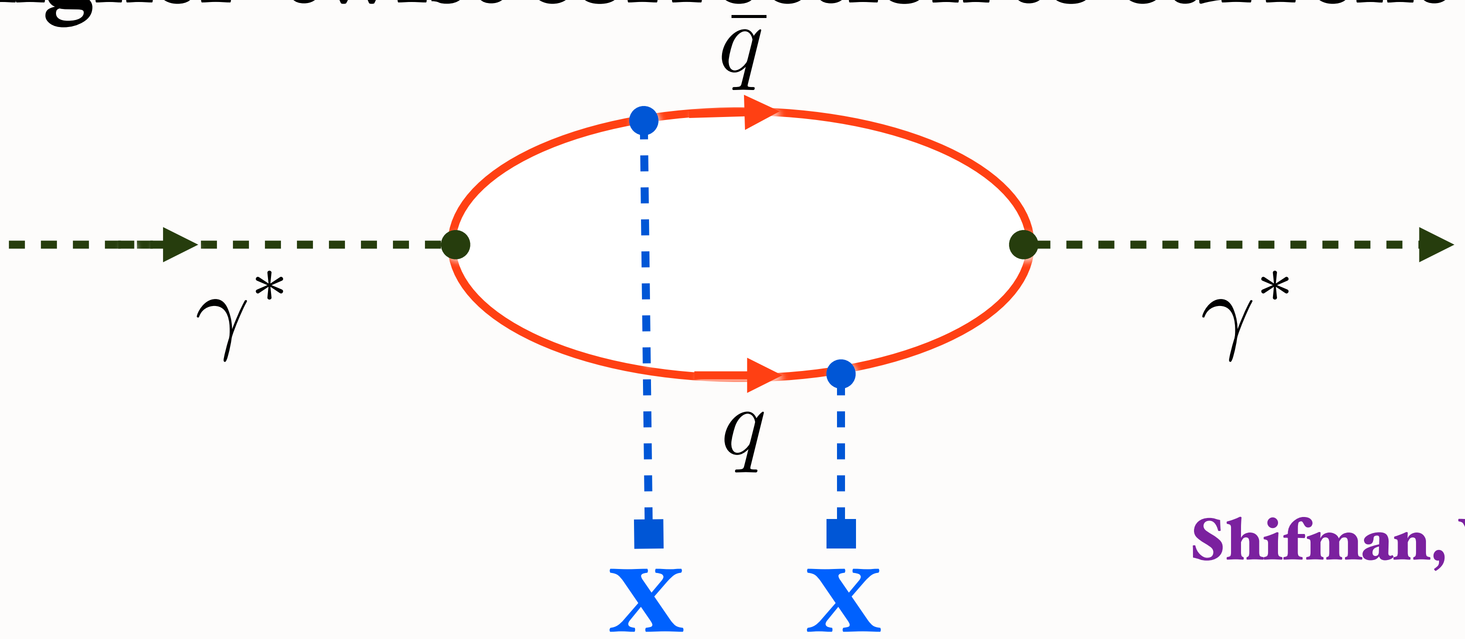
On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.

Light-Front Formalism

Is there evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$$e^+e^- \to X, \ \tau \text{ decay}, \ Q\bar{Q} \text{ phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi} \frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

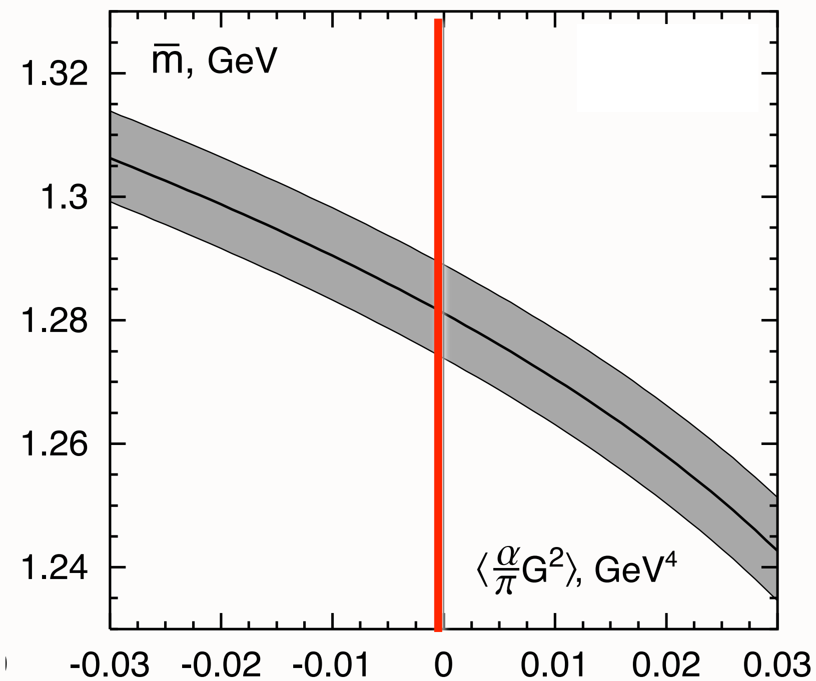
Determinations of the vacuum Gluon Condensate

$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

$$-0.005 \pm 0.003 \text{ from } \tau \text{ decay.}$$ Davier et al.

$$+0.006 \pm 0.012 \text{ from } \tau \text{ decay.}$$ Geshkenbein, Ioffe, Zyablyuk

$$+0.009 \pm 0.007 \text{ from charmonium sum rules}$$ Ioffe, Zyablyuk

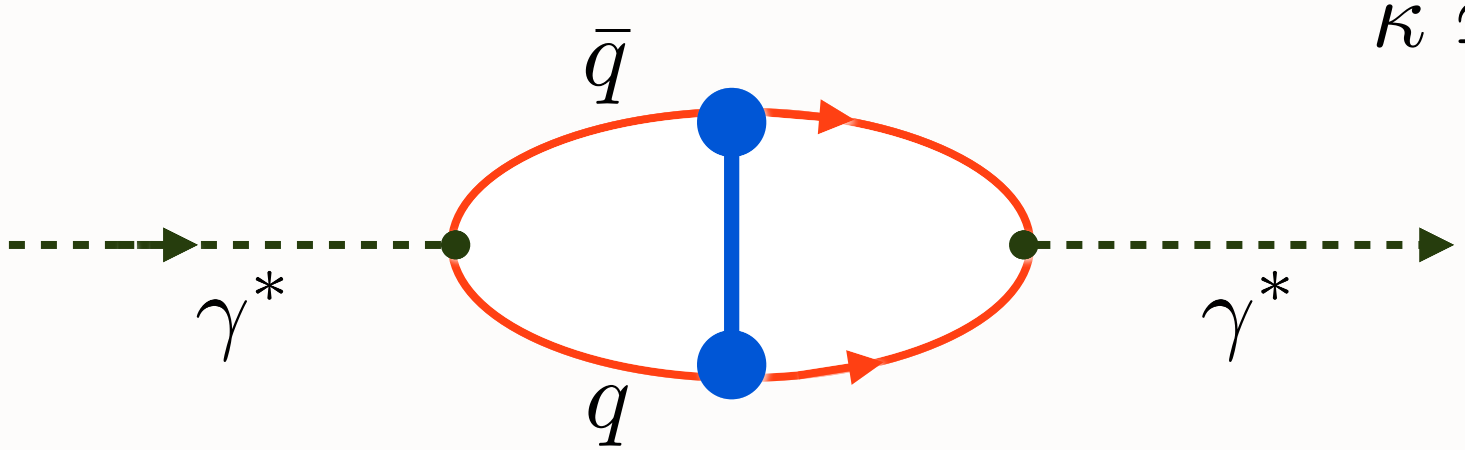


Consistent with zero vacuum condensate

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

mimics dimension-4 gluon condensate $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena
- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"
- Find: $$\frac{<0|\bar{q}q|0>}{f_\pi} \rightarrow -<0|i\bar{q}\gamma_5 q|\pi>= \rho_\pi$$

$$<0|\bar{q}i\gamma_5 q|\pi> \text{ similar to } <0|\bar{q}\gamma^\mu\gamma_5 q|\pi>$$

- Zero contribution to cosmological constant! Included in hadron mass
- ρ_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

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[4] *Department of Physics, Peking University, Beijing 100871, China*
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We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind   Maris, Roberts, Tandy   Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

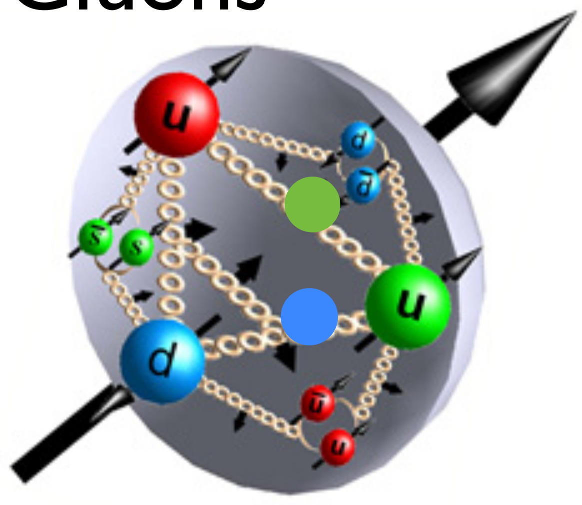
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent for spacelike observables
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

Transversity

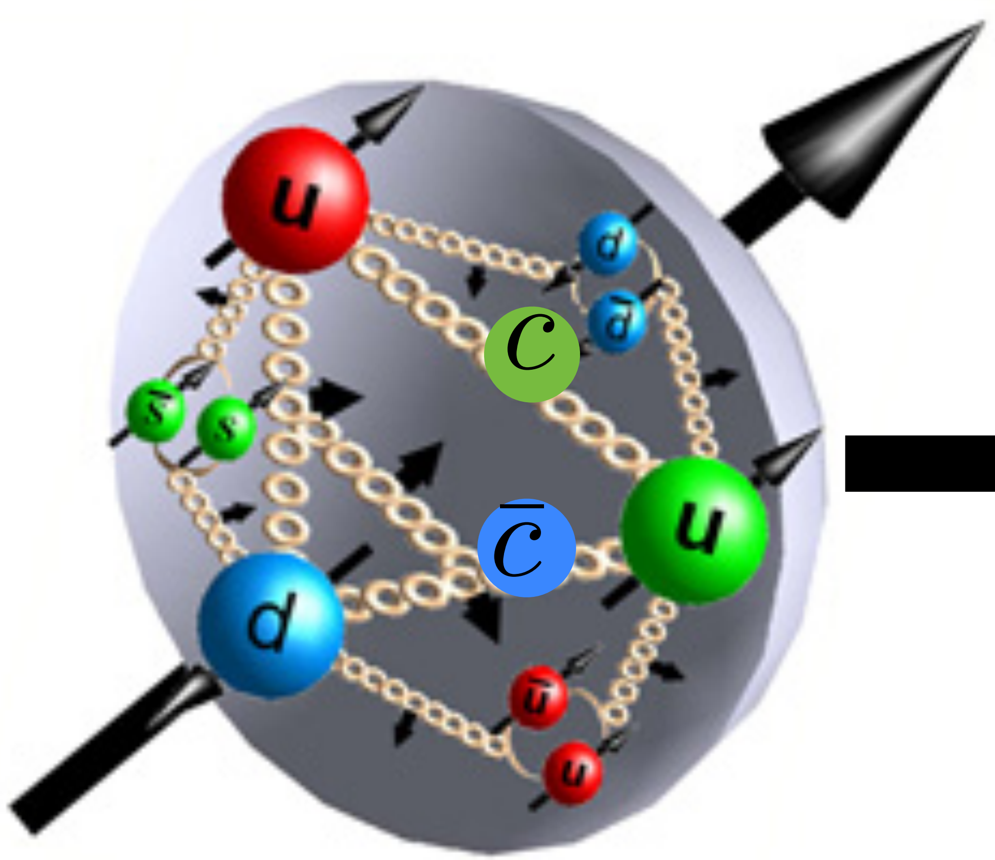
Angular Momentum Structure, and the Spin Dynamics of Hadrons

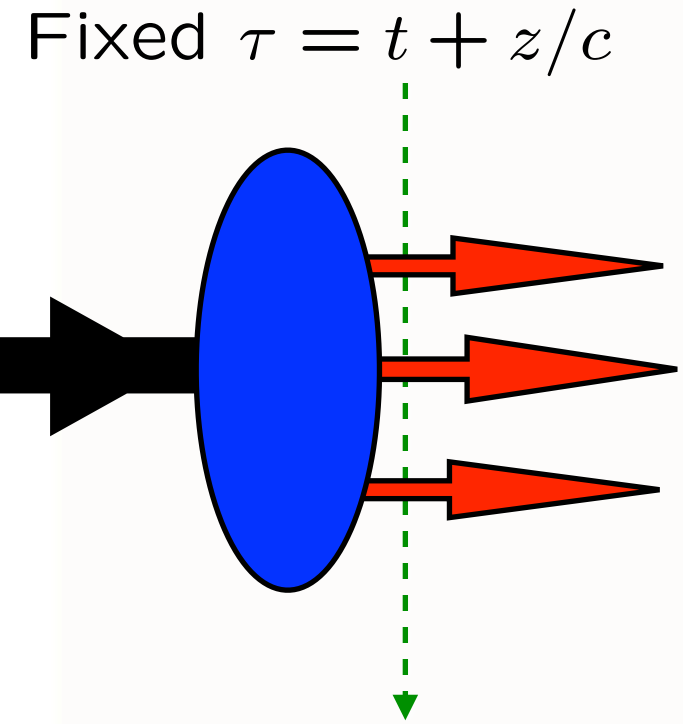
- Test Fundamentals of Gauge Structure of QCD
- Fundamental Measures of Hadron Structure
- Angular Momentum of Confined Quarks and Gluons
- Breakdown of Conventional Wisdom
- Breakdown of Factorization Ideas
- Crucial Experiment Tests, Measurements

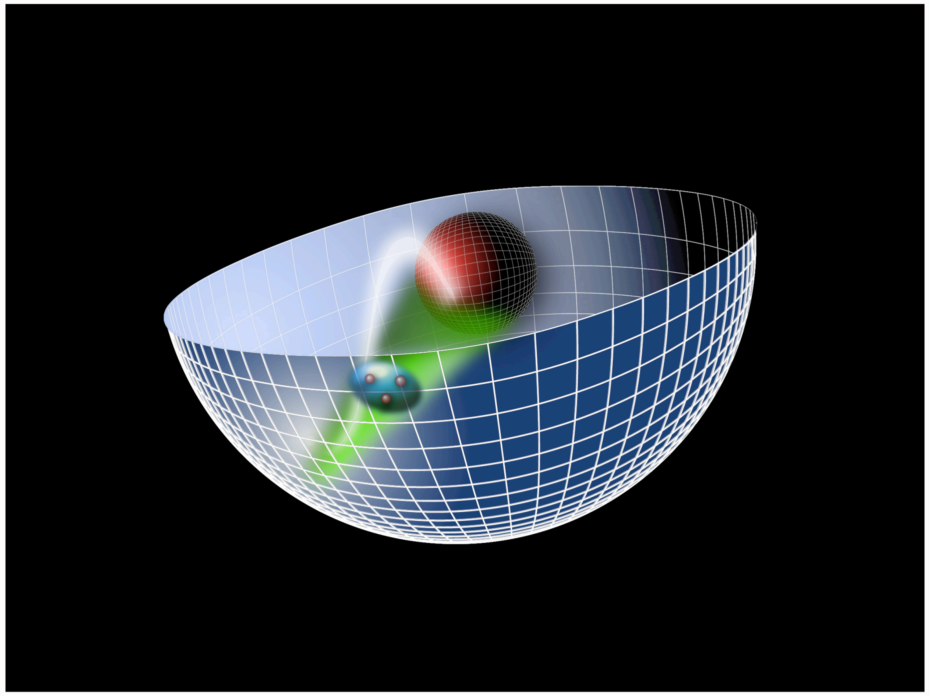


Remarkable array of theory and experimental talks

Light-Front Holography and Proton Transversity







Thanks for an outstanding meeting!

Franco Bradamante (chair) / Trieste

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