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Designing an Optimal 'Tech Fix' Path to Global Climate Stability:

Integrated Dynamic Requirements Analysis for the 'Tech Fix'

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ABSTRACT

This paper analyzes the requirements for a social welfare-optimized transition path toward a carbon-free economy, focusing particularly on the deployment of low-carbon technologies, and the roles of engineering upgrading of extant facilities, and directed R&D to enhancing their productivity. The goal in each case is to achieve timely supply-side transformations in the global production regime that will avert catastrophic climate instability, and do so in a manner that minimizes the social welfare costs of stabilizing the level of the atmospheric concentration of greenhouse gases (GHG). This "planning-model" approach departs from conventional IAM exercises by dispensing with the need to make (generally dubious) assumptions about the macro-level consequences of behaviors of economic and political actors in response to market incentives and specific public policy instruments, such as a carbon tax. It shifts attention instead to the need for empirical research on critical technical parameters, and problems of inter-temporal coordination of investment and capacity utilization that will be required to achieve a timely, welfare-optimizing transition. A suite of heuristic integrated models is described, in which global macroeconomic growth is constrained by geophysical system with climate feedbacks, including extreme weather damages from global warming driven by greenhouse gas emissions, and the threshold level GHG concentration beyond which the climate system will be "tipped into" catastrophic runaway warming. A variety of technological options are identified, each comprising an array of specific techniques that share a distinctive instrumental role in controlling the concentration level of atmospheric CO_2. The development of low-carbon technologies through investment in R&D, and their deployment embodied in new physical capital formation, is explicitly modeled; as is the implementation of known engineering techniques to "upgrade" existing fossil-fueled production facilities. The social-welfare efficient exercise of the available technological options is shown to involve sequencing different investment and production activities in separate temporal "phases" that together form a transition path to a sustainable low-carbon economy—one in which gross CO2-emissions do not exceed the Earth's "natural" abatement capacity. Parametric variations of the "tipping point" constraint in these models will permit exploration of the corresponding modification in the required sequencing and durations of investment and production in the phases that form the optimal transition path. The preliminary solutions (using mufti-phase optimal control methods) expose important dynamic complementarities among technological options that are often presented as substitutes by current climate policy discussions.

JEL codes: Q540, Q550, O310, O320, O330

Keywords: global warming, tipping points, catastrophic climate instability, technology fix options, R&D investments, capital-embodied innovations, optimal sequencing, IAM and DIRAM policy design approaches, multi-phase optimal control, sustainable endogenous growth

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1. Overview and Organization of the Paper

Our main purpose in these pages is to stimulate fresh thinking about nothing less than the design of a coherent global program that would mobilize the technical and economic resources that will be needed to implement effective and timely actions stabilize the Earth's climate. More specifically, the concern of the paper focuses on analysing the requirements for a social welfare-optimized transition path toward a low-carbon production regime that would be both viable and sufficient to avert catastrophic climate instability, and thereby preserve the future possibilities of sustainable development and continuing economic growth.

To fix ideas and begin to make analytical headway, we describe a suite of heuristic "integrated" model of long-run macroeconomic growth, i.e., models in which endogenous growth is constrained by feedbacks from the geophysical system in which the global economic sub-system is embedded. Two among the feedback effects that directly constrain the economic growth are modeled explicitly: firstly, there is the potential for continuing emissions of greenhouse gases (GHG) to raise the level of their atmospheric concentration (measured in CO_2-equivalent ppmv), increasing radiative forcing and correspondingly elevated mean temperature of the Earth's surface, elevated moisture content of the atmosphere and, consequently more frequent rates of extreme weather damages to infrastructures and losses in productive capacity. The second, and still more serious constraint arises from recognition of the existence of one or more climate "tipping points" or thresholds that once crossed would initiate a self-reinforcing, "runaway warming" process that would bring in its train catastrophic damages to human welfare and well-being.

A policy commitment that embraces the Precautionary Principle and therefore seeks to avert a global climate disaster will be seen to be tantamount to imposing a hard "carbon-budget" constraint on the optimal transition path to a stabilized climate. This follows from defining a catastrophe tipping point in terms of a critical threshold level of CO_2–*e ppmv* in the atmosphere, which (given the latter measure's initial level) serves to fix the cumulative net additions volume of GHG emissions, and hence in the CO2-equivalent atmospheric concentration of those gases) that does not cross the stipulated threshold leading to catastrophe. Thus, any conjectured catastrophe tipping point sets a "carbon-budget," and a corresponding climate-stabilizing transition path that can be optimized – if it is feasible, given the other constraints on the dynamical reallocation of global resources.

A second obtrusive respect in which the approach to be described here deviates from the style of integrated climate policy assessment modeling (IAM) that has become familiar in the field of environmental and energy economics applied to climate policy issues,[1] is to be seen in its explicit attention to the technological transformations that will need to be effected in order for a program of climate stabilization to succeed. Rather than positing an unchanging aggregate production function for the global economy, we propose to consider a portfolio of technological options comprising several distinguishable classes of linear techniques, each characterized by a common instrumental role in controlling the concentration level of atmospheric CO_2. The development of alternative, low-carbon technologies through investment in directed R&D, and their deployment embodied in new physical capital formation, are explicitly modeled, as are investments in implementing known engineering techniques to "upgrade" existing fossil-fueled production facilities—lowering their average flow rate of CO_2 per unit of output and thereby rendering existing capital stock "greener."

Explicit modeling of distinct elements in the array of available and latent technological options, and the assumption that the techniques in question must be embodied in physical capital (whether infrastructural or directly productive assets) represent an important respect in which the present approach departs from the leading integrated (policy) assessment models' treatment of technological changes in the global production regime, whether through the further diffusion of available techniques (and the scrapping physical capital embodying earlier vintages), or the development and deployment of recent technological innovations.[2]

The social-welfare efficient deployment of the available technological options is shown to involve sequencing different investment and production activities in distinct temporal "phases" each of which satisfy the first-order conditions for an optimal control solution, and taken together form a transition path to a sustainable low-carbon economy—one in which gross CO_2-emissions do not exceed the Earth's "natural" abatement capacity. Following the minimax-regret criterion for decisions about policy actions, welfare optimization of the tech fix program must satisfy the condition that the design achieves stabilization of the atmospheric concentration of $CO_{2\text{-eqiv}}$ ppmv at a level

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[1] See further discussion in Sect. 4 (below), especially the IAM models following and extending the paradigmatic DICE model of Nordhaus (1993, 2002, 2007) and other contributions to this literature reviewed in Kelly and Kolstad (1999), and more recently in Ackerman et al. (2009).

[2] The "technological portfolio" approach presented here (based on functional distinctions among "techniques") is consistent in spirit with previous efforts (e.g., Bosetti et al., 2006, and other contributions to the same special issue of *Energy Journal*) to combine detailed "bottom up" representations of engineering realities in the energy sector with "top down" modelling of the global macroeconomic and climate systems. In order to explicitly examine the implications of switching between technologies that must be embodied in physical capital, we have sacrificed engineering details for greater computational tractability of the resulting multi-stage optimal control problem.

just below the specified "catastrophe tipping point." Given the current atmospheric concentration of GHG, this serves to impose a conditional net "carbon emissions budget" constraint upon the optimal transition path corresponding to the specified "catastrophe tipping point". By solving the model for an array of alternative conjectured tipping-point, it is then possible to explore the dynamics of the altered technological and investment requirements that will (just) expend the budget by taking the indicated transition paths to the corresponding stable climate.

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Interpreting application of the Precautionary Principle as "minimax regret" strategy, would provide a formal justification for investigating the requirements of welfare-optimal transition paths that would suffice to halt net additions to the atmospheric concentration of GHGs just before the system crossing a conjectured catastrophic tipping-point.[3] , is that doing so would eschew relying upon expected utility maximizing solutions. To obtain the latter would call for quantitative assessments of the balance between future societal welfare benefits and costs. But, in the present incomplete state of scientific and economic understanding of the dynamic processes that would be involved, those assessments entail accepting highly dubious quantitative guesses regarding the time distribution and magnitudes of material damages to populations and productive capacity that would entail catastrophic losses in utility terms. Furthermore, if the possibility of abrupt and catastrophic climate changes is to be explicitly acknowledged, it also would be necessary to accept uncertain and highly subjective estimates of the probability distribution of the GHG concentration levels corresponding to the threshold values of various "tipping elements" in the Earth's climate system.

There is, however, and available attractive alternative to the classic "minimax regret" strategy for rational choice decisions under uncertainty, which typically is invoked as a formalization of the Precautionary Principle whose application could account for human actors observed departures from the predictions implied by Savage's (1951) postulated "sure-thing principle.[4] As formulated by Loomes and Sugden (1982),

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[3] The minimax-regret criterion introduced by Savage (1951)calls for choosing the action(s) from the available set that will minimize the magnitude of 'regret' associated with the worst case social welfare outcomes. When the outcome variables are utilities (as is usual in welfare analysis), the minimax-regret criterion is based on differences in expected outcomes, and the presence of uncertainty requires assigning (subjective) probabilities to alternative plausible states of the world. The worst case outcome of not taking a given program that would stabilize the climate before reaching level k of MGT is the castastrophic onset of irreversible, self-reinforcing global warming – which would *ex hypothesis* ensue from exceeding level k. Under an alternative state there is no such tipping point before or at MGT level k, and the outcome of taking the action is the welfare loss of the unnecessary stabilization effort to stop there. If the welfare indexes are linear, and the states of the world are independent of the action in question, then this approach is consistent with expected utility theory, where subjective probabilities may be assigned to the states of the world, and the decision criterion is the difference between the expected payoff for not taking and taking the stabilization action. This approach has been implemented by Cai, Judd and Lontzek (2012b), Lemoine and Trager (2012). But, as will be seen, one can justify asserting the Precautionary Principle without having to assign subjective probabilities to the tipping point's conjectured locations. .

[4] See Loomes and Sugden (1982), Slovic and Tversky (1974), Kahenman and Tversky (1979).

this approach holds that a rational actor choses between actions (including a particular action and inaction) so as the maximize the mathematical expectation of *modified utility*. Supposing that expected modified utility is the objective function maximized by rational actors is justified on the grounds of simplicity and consistency with the empirical evidence of most of the well documented behavioral departures from the conventional framework of maximizing expected utility among available choices. [5]

Under the assumption that the current generation places a value on future generations' welfare (conceiving of the latter as a subjective state of felicity), decisions to be taken in the face of an uncertain existential threat -- such as that of entering into an irreversible catastrophic state that will entail extreme deprivations for future generations -- may weight current sacrifices of utility against the prospect of maintaining more hopeful conditions for future generations. Costly actions of this kind would, in effect, sacrifice the subjective felicity of current material consumption in order to avoid the subjective infelicity of contemplating the situation that they have made inadequate sacrifices to avert the "hopeless" situation in which future generations would be left by materialization of the catastrophic alternative.[6]

This would seem to commend itself as the rational application of the Precautionary Principle in regard to decisions on actions aimed at stabilizing the Earth's climate. That, of course, is premised on accepting the propositions that (i) there exist one or more "tipping points" in the atmosphere concentration level of GHG), and (ii) at least one of these thresholds would be crossed within the lifetimes of present members of the world's population, unless major cuts in the volume of GHG emissions are not begun immediately. Uncertainties about the extent of the material sacrifices of current economic welfare that an effective program of that kind will entail – due to uncertainties and ambiguities in the location of the tipping-point (s), in the state dependent dynamics

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[5] This approach proceeds from the view that psychological experiences of regret (and rejoicing) cannot be properly conceptualized as "rational" or "irrational" – because they, like sorrow and joy are subjective states; the class of 'objects' that may be taken to produce utility is thus explicitly held to include subjective states (feelings about a material state of the world) as well as the utility directly derived from that world state. Rather than following Kahenman and Tversky (1979, 1981) by super-imposing a theory of systematic behavioral violations upon the expected utility theory deriving from von Neumann and Morgenstern (1947), Loomes and Sugden (1982) formulate a "modified utility function" for the decision agent, in which increments or decrements in utility (felicity) corresponding to the experienced sensations (joy vs. regret) that are associated with contemplation of the factual and counter-factual outcomes of conditions that actually have or might have been achieved. In other words, this formulation assumes that the degree of regret (or joy) experienced depends upon comparison of the utility of "what is" with "what might have been."

[6] Belief in the possibility of "a better future", or at least a future more viable than the present undergirds deferral of gratification, and individual behaviors (saving, investing, acquiring knowledge) that play critical roles in the accumulation of productive tangible assets, as well as the formation and maintenance of formal institutions and social organizations that structure cooperation in the collective creation of public goods. Therefore, it is reasonable to view the widespread prospective "loss of hope" for a viable material future among members of the human population as posing a grave "existential threat".

of the of Earth's climate under conditions of continued warming, and in the effectiveness with which both public and private resource reallocation plans can be implemented, and corrected in mid-course -- imply a non-zero probability that some of the sacrifice of the social welfare of future generations will be perceived to be "excessive" or even unnecessary.[7]

Our approach in exploring the requirements for optimally designed transitions to a stabilized climate therefore proceeds from the position that in the comparison between the two contemplated sources of expected regret, the preponderance in the balance is overwhelmingly on the side of deciding upon actions to *seek to avoid inadequately strong stabilization measures* and to accept the higher risks of regrets associated with having taken actions that will turn out to have been unnecessarily costly. In other words, this formulation assumes that the degree of regret (or joy) experienced depends upon comparison of the utility of "what is" with "what might have been." Decisions that must be made in the face of the existential threat of entering an irreversible catastrophic state – recognition of which would deprive future generations (that share the decision-taker's notions of rationality) of "hope" for the possibility of all things conditional on the sustainable existence of a recognizable form of civil society. Preservation of that hopeful state of warrants actions entailing degrees of material sacrifice that will not compromise the viability of the future state in which hopefulness can be maintained. Costly actions of this kind will, in effect, sacrifice the material conditions of the viable world left to future generations in order to avert the hopelessness of the catastrophic alternative.

It should also be remarked here that "excessive precaution" is subject to correction in some measure once it has become recognized; and there would remain scope for arranging subsequent further compensation for those in future generations that had been most damaged by errors in policy planning and implementation failures, rending such remedial actions an option for resource reallocation and redistributive decisions by their contemporaries in the stabilized and economically sustainable ("green") global regime of production**.** The same remedial options, however, will be available where feasible precautionary options remain inadequately exploited. Our approach in exploring the requirements for optimally designed transitions to a stabilized climate therefore proceeds from the view that in comparisons between the two contemplated sources of expected regret, the preponderance in the balance is overwhelmingly on the side of deciding upon actions calculated *to avoid inadequately*

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[7] Such "excessive precaution," however, would be subject to correction in some measure once it has been initiated; as well as to subsequent further adjustment by future generations that may better recognize the scale of catastrophe that has been averted. This course of action would also provide further options for resource reallocation and redistributive decisions in the stabilized and economically sustainable ("green") global regime of production**.** The same remedial options, however, are not available in the case where feasible precautionary options are insufficiently exploited.

strong stabilization measures and to accept the entailed higher risks of regrets associated with having taken precautions that will turn out to have been unnecessary.[8]

To study the requirements of a timely (catastrophe-averting) transition, we formulate a sequence of optimal control sub-problems linked together by transversality conditions, the solution of which determines the optimum allocation of resources and sequencing of the several phases implied by the options under consideration. Parametric variations of the tipping point constraint in these models will permit exploration of the corresponding modification in the required sequencing and durations of investment and production in the phases that form the optimal transition path. The preliminary solutions using multi-phase optimal control methods expose the important dynamic complementarities among technological options that are often presented as substitutes by current climate policy discussions. Further, the “planning-model” approach departs from conventional IAM exercises also by eschewing dubious assumptions about the behaviors of economic and political actors in response to market incentives and specific public policy instruments; it shifts attention instead to the need for empirical research on critical technical parameters. and problems of inter-temporal coordination of investiment and capacity utilization that will be required to achieve a timely, welfare-optimizing transition.

2. Background and Motivation

Climate change is now convincingly linked to increasing atmospheric concentrations of GHG (greenhouse gases).[9] Among those who have examined the relevant scientific data there has been a growing consensus that this poses an inter-related host of worrisome problems. Moreover, rather than conveniently going away, or being gradually being accommodated by adaptations on the part of the world’s peoples, these problems are likely to grow worse. While on many specific points of climate science uncertainties, doubts and disagreement persist, the underlying physical and chemical processes responsible for anthropogenic “greenhouse effects” warming the Earth’ surface are firmly grounded, as is the accumulating mass of empirical observations attesting to the rise in mean global temperature that has taken place

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[8] The germ of the following argument for embracing the Precautionary Principle, formulated as the case for adopting a *minimax regret* strategy, was advanced in a climate policy brief published on the eve of the 2009 UN Copenhagen Conference (see David, et al. (2009: pp. 2-3). In the interim, the emergence consensus among climate and environmental scientists on the likely existence of “tipping points” initiating abrupt and catastrophic alterations of Earth’s climate (further discussed in section 3.1, below), and the absence of any significant globally concerted actions on rapid reduction of GHG emissions, has rendered only more compelling the case for designing (costly) policies that could rapidly stabilize the atmospheric concentration of GHG.

[9] See IPCC (2007),”Summary for Policy Makers,” and the Stern Review (Stern, 2006) for an extensive overview of the causes and consequences of climate change.

during the past two centuries.[10] Together, these findings firmly undergird the scientifically informed warnings about the consequences of continuing "business as usual" based on burning fossil fuels.

The rising levels of moisture that will accompany the warming of the Earth's surface in the temperate latitudes can be expected to drive more frequent and more severe weather cycles, bringing heavier precipitation, stronger winds and weather-related damages to physical property and losses of human life. Moreover, it is more than conceivable that at some point in the not very distant future continuing the current rate of GHG emissions and unimpeded global warming will usher in an age of catastrophically abrupt climate changes, characterized by chaotic instabilities in Earth's climates.[11] This would drastically curtail the fraction of the world's current population that would be able to achieve a state of "adaptive survival" preserving substantial resemblances to present state of civilization.

Mitigating environmental damages from CO_2, methane and dangerous particulate emissions by more extensively deploying known technologies, and replacing carbon-based production systems with those that utilize new and economically efficient "carbon-free" technologies, are two obvious courses of action that may be able to avert these grim future prospects. Together with still more radical adaptations that possibly may be achieved by geo-engineering to capture and sequester existing atmospheric carbon and methane, these form the core of the technological options whose development and deployment are the focus of this exploratory examination of the dynamics of a feasible and timely transition to a sustainable low-carbon global economy.

A program of public and private R&D investments yielding directed technical changes of this kind should be viewed as a "supply-side strategy" in the campaign to stabilize global GHG concentrations at a viable level. The intention of this paper is give "technology fix" program of that kind its warranted central place in economic research on the design of policy to mitigate CO_2 emissions. That would constitute a quite radical reorientation of the discussion, for, on the occasions during the past decade when technological change figured explicitly in the economics literature devoted to climate change policy, it usually has been presented as an ancillary and optional complement to implementing "carbon pricing." Carbon taxes, or the "cap and trade" schemes that have

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[10] See Solomon et al., eds. (2007) on the physical basis and scientific evidence for IPCC conclusions regarding global warming and its sources in human activity. "Climate skepticism" and "global warming denial" should not be confused with the generally acknowledged scientific uncertainties that still surround the values some key parameters in quantitative models of the processes involved in anthropogenic global warming -- e.g., the magnitude of the "climate sensitivity" (linking changes in mean global surface temperature to relative gain in atmospheric GHG concentration), and the critical level of temperature gain that would trigger irreversible runaway warming and catastrophic climate instability. See further discussion below and details in footnotes 6-7.

[11] See e.g., Alley (2000); Alley, Marotzke, Nordhaus, Overpeck et al., (2003); Hall and Behl (2006), and further discussion in Sect. 2.1 (below).

been advocated as the means of creating a global market for carbon-emissions permits and derivative instruments, would work to mitigate CO_2 emissions primarily by raising the current and expected future relative prices of fossil fuels as a source of energy. Were they to have that effect, the volume of demand for goods and services (e.g., electricity supply, transportation) that were especially carbon-intensive in their methods of production would be reduced by the private optimizing adjustments made by producers and consumers response to the altered structure of prices of energy sources and carbon-intensive final goods and services.[12]

That strategy has gained broad support among economists as the approach that more directly attacks the root cause of the global warming externality--namely, unregulated and un-priced releases of CO_2 –while leaving the choice of mitigation tactics to agents in the private sector. An unfortunate consequence of the preoccupation of economists with assessing the effectiveness of carbon pricing and trying to gauge the welfare-optimal schedule of carbon taxes led to the development of computational models that gloss over the complicated resource allocation dynamics of the transition to a low-carbon global regime of production; and, in so doing, have deflected attention from the potentially greater role of directed technology policy initiatives by the public sector that would combine technical standard-setting with direct and indirect funding subsidies, and the possibilities institutional changes that would remove existing barriers to rapid global diffusion of effective "green technology innovations."

Consequently. it is hard to point to a body of previous integrated analysis devoted to assessing global climate policy designs that consider an array of technological options involving investment in deploying available engineering techniques that will lower the carbon-intensity of existing production facilities, directed R&D expenditures to enhance the cost competiveness and reliability of carbon-free energy sources (vis-à-vis fossil fuels), and adoption subsides to speed diffusion of innovations and the realization of further incremental improvements from learning under field conditions. But it is encouraging to note that academic papers that discussed the pertinence of endogenous technological change in the context of climate policy had begun to emerge even before the debacle of the attempt at the 2009 Copenhagen Conference strengthen and expand to Kyoto Treaty protocols.[13]

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[12] This aspect of the environmental and energy economics literature is well represented in the extensive review article by Aldy, Krupnick, Newell et al. (2010), which, in the concise words of its abstract "provides an exhaustive review of critical issues in the design of climate mitigation policy by pulling together key findings and controversies from diverse literatures on mitigation costs, damage valuation, policy instrument choice, technological innovation, and international climate policy." The authors focus first on the broad issue of how high policy assessments suggest the near- and medium-term price set on greenhouse gas emissions would need to be, both under cost-effective stabilization of global climate and under net benefit maximization or Pigouvian emissions pricing. They then turn to discussion of the appropriate scope of regulation, questions regarding policy instrument choice, complementary technology policy, and international policy architectures."

[13] See, e.g., Carraro, van der Zwaan and Gerlagh (2003). Jaffe, Newell and Stavins (2006) is salient among these publications, as much for the cogency of its insights and arguments as for the sparseness of other

Neither these early forays into this area, nor other works to our knowledge, ventured to examine the required dynamic sequencing of *directed* R&D and other technology policies affecting public and private sector investments required for a timely transition to a low-GHG emitting production regime, the policy issue that is the focus of the research reported here. [14]

It has been suggested that CO_2 emissions mitigation achieved by the introduction of carbon-taxes or "cap-and trade" schemes for "pricing" transferable emissions permits, could indirectly achieve what would otherwise have to be accomplished by means of social expenditures for public research and development projects and tax subsidies for private investment in "green" R&D, and possibly implementing the resultant innovations in carbon free physical capital formation.[15] The argument is that the effect of raising the cost of carbon-intensive processes and products to producers and consumers will be to stimulate producers' demands for offsetting, carbon-free technologies and induce incremental private funding of the search of those innovative methods.

Although that is a possibility, so too is the opposite outcome. If raising carbon taxes does effectively curtail demand for highly carbon-intensive products, the result

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contributions in this genre. They point out (*Ibid*., p.169) that because the U.S. and other developed countries had not been actively pursuing significant environmental policy intervention, "there is little environmental policy-induced incentive to develop technologies that reduce greenhouse gas emissions. In this second-best setting, policy to foster greenhouse gas-reducing technology may be one of the main policy levers available and can be justified on economic grounds so long as it has positive net benefits." The same passage, however, concludes with the caveat that using "technology policy as a substitute for, rather than a complement to environmental policy" could be a costly approach -- implying that the payoff to R&D is more risky than counting on the responses to increases in the relative price of fossil fuels, and that the pure "technology push" approach sacrifices the potential benefits of induced innovation responses from the private sector.

[14] In the wake of the Stern Report (2007) and the Copenhagen Conference economic arguments for greater public and private investment in R&D, and subsidies for the diffusion of advanced technologies directed to lowering CO_2 emissions began appearing with greater more frequently – although these typically remained unspecific. Early statements proposing a "technology fix" approach include Arrow, Cohen, David et al. (2008), Nelson and Sarewitz (2008), Klemperer (2007/2009), David (2009), David, Huang, Soete and van Zon (2009), and Hendry (2010). None, to our knowledge, venture to examine the *required* dynamic path of *directed R&D* and *capital formation embodying innovations* that would lower the global rate of GHG emissions per unit of real output, which is the focus of the present work. Such theoretical and computational modelling of endogenous and induced technical change that has been undertaken to date has largely following the pioneering work of Goulder and Schneider (1999), which is noted below.

[15] Among the pioneering efforts to examine the proposition that raising the price of carbon would induce beneficial private investments in technological innovation, see the working papers by Nordhaus (1997), and Goulder and Mathai (1998, published in 2002). Goulder and Schneider (1999) carried this line of inquiry farther by using a computable multi-sector general equilibrium model to quantitatively assess the extent to which real GDP costs of imposing CO_2 abatement taxes would be altered by the effects of endogenous technological improvements resulting from R&D investment induced by the effect of the tax on the relative prices of carbon vs. non-carbon inputs. This modelling exercise did not, however, undertake an evaluation of the efficacy of the hypothesized induced technical changes in terms of the magnitude of the absolute abatement of the economy's aggregate emissions rate. More recent exercises in integrated climate policy assessment have undertaken to do so, as is noticed below (in section 2).

may well be generally adverse expectations about market growth and a consequent weakening of incentives for R&D investment in all production techniques for in those lines of business.[16] Moreover, commitment to a future schedule of rising unit taxes on fossil fuels might well have the perverse first-order effect of immediately depressing the future value of fossil fuel deposits relative to their present value -- if the scheduled rate of increase in the "carbon tax" was higher than the interest rate. Under that condition, private owners of the resource deposits would have an immediate incentive to raise the resource extraction rate in order to bringing forward in time the exploitation of those resource deposits, with the consequence (*ceteris paribus*) that near-term prices in the markets for carbon fuels would be depressed.[17] Even though, following that one-time drop, it would be expected that the present value of carbon deposits in the ground and the relative market price of carbon fuels would resume rising at the rate of interest --as called for by Hotelling's (1931) analysis, the relative market price of carbon fuel might not regain its pre-tax regime level for some considerable period of time. During that interval, at least, the intended mitigation of CO_2 emissions would be reversed, as also would be the direction of such impetus as would formerly have been imparted towards induced technical invention and innovation aimed at saving carbon-energy inputs.

Whether or not the long-term effect of a high and rising carbon tax eventually would be sufficient to halt the withdrawal of fossil fuels before from the known reserves were totally exhausted (the outcome that has come to be labelled as the "strong" form of Green Paradox, to distinguish it from "weak" form in which there is a transient anti-conservative effect on the pace of resource extraction), is a more complicated and uncertain matter, in which the direction of the overall environmental effect would turn on upon still other conditions.[18] But in a world in which one should not count on fiscal

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[16] The question cannot be settled on theoretical grounds, but see Gans (2009) for analysis of the conditions under which carbon-taxes would have a perverse effect of discouraging carbon emissions-reducing technology research and innovation. Acemoglu, Aghion, Bursztynx, and Hemous (2010/2012) explore the conditions for carbon taxes in a growth model with environmental (carbon emissions) constraints and endogenous directed technical change driven by private sector R&D investments induced by the effects of an essentially transient carbon tax. They conclude that with a sufficiently high elasticity of substitution between highly carbon-intensity and low carbon-intensity inputs in production, carbon taxes can switch directed R&D to "clean" (low carbon intensity) technological research and need not be continued indefinitely; there is a similar reinforcing role for "temporary R&D subsidies". Hourcade, Pottier and Espagne (2011), however, shown that the elasticity of substitution required to produce these results in the "illustrative" integrated climate model presented by Acemoglu et al. (2010/ 2012) are implausibly high, both from the engineering viewpoint and from their implications for the price elasticity of demand for carbon energy sources, which are inconsistent with the available empirical evidence from those markets. In addition the specification of the climate system in the "illustrative" computational model presented in the latter part of this paper departs from the standard climate science models in a way that leads to serious underestimation of the costs of the required CO_2 mitigation strategy—so much so that the corrected computations vitiate the authors' policy conclusions.

[17] Discussion of the latter problem was broached first by Sinn (2008, 2009: pp.10-13) who provided a label --"The Green Paradox" –under which a theoretical literature soon developed. See, e.g., Edenhofer and Kalkuhl (2010), Hoel (2010), Withagen (2010), van der Ploeg and Withagen (2012).

[18] Among conditions that would rule out the "strong" form of the Green Paradox (see, esp. van der Ploeg (2010) and Edenhofer and Kalkuhl (2010)) include: (i) a high level of the unit tax on carbon extraction, (ii) marginal costs of resource extraction that increased as an inverse function of the size of the (unexhausted)

and regulatory instruments being optimally designed and consistently used by political authorities, let alone by assemblies or sovereign states, the possibilities of unintended "Green Paradox" outcomes should caution against reliance on carbon-pricing schemes as the "best policy response" to the threats posed by global warming.

It seems pertinent here also to notice that the distinction drawn between "weak" and "strong" Green Paradoxes in the theoretical literature loses practical relevance when there is no assurance that global warming will afford humanity sufficient time to even come close to exhausting the Earth's known deposits of fossil fuel, as is pre-supposed by all the models that have been used to find conditions under which the "strong" form of Green Paradox could or could not be realized.

As will be seen shortly (from section 2.1, below), climate scientists now regard it to be quite possible that we already have arrived at a state where a quite modest further gain in the planet's mean surface temperature could push the geo-physical system past a critical climate "tipping point," initiating a self-reinforcing cascade of tipping elements that would irreversibly usher in an epoch of catastrophic climate instability. There is thus little comfort in the knowledge that the unintended immediate perverse boost in the rates of extracting and burning fossil fuel – brought about by resource owners' wealth-preserving reactions to the prospect of high and rising future carbon taxes -- might be only a transient effect. Before that initial impulse boosting the volume of CO_2 had dissipated, the trajectory of the climate could have been irretrievably altered. It is in that context that the precautionary principle paradoxically serves to inveigh against advice to rely exclusively upon so uncertain and potentially dangerous a strategy of combatting goal warming by promising a future "ramping up" of the carbon tax rate.

Beyond this, the foregoing review of the worrisome potential drawbacks of exclusive reliance on carbon taxes to "fix the GHG emissions externality," there is still greater grounds for doubts about the efficacy of the alternative proposal that gained widespread popularity among environmental and energy economists – namely, downstream cap-and-trade schemes, which price the emissions from burning carbon, but not the carbon material that is the energy source.

It is not at all obvious that putting a price on the emissions from the burning of hydrocarbons necessarily would suffice to raise the combined price of the material and its combustion, so the possibility of a perverse relative price change cannot be

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resource deposits, (iii) a unit tax that was not scheduled to increase at a rate exceeding the resource owners' time-discount rate, and (iv) was *ad hoc* in not being set in accord with an optimized inter-temporal program of resource use. Further, although not considered in the theoretical just cited, the practical value of the analytical distinction between the "strong" and "weak" form of the paradox becomes dubious, when one recognizes that positive feed-back effects from the rise in global temperature caused by "transiently" accelerated CO_2 emissions could alter the dynamics of the climate system sufficiently to initiate a self-reinforcing ("runaway") process of warming. See section 2.1 (below) for further discussion.

definitively ruled out.[19] By contrast, the proposal of an "up-stream" implementation of the cap and trade mechanism (see Repetto 2011) is administratively attractive because effective restriction of the volume of fossil fuels available from "first sellers" it would necessarily raise the cost of carbon-based energy sources. Further monitoring and enforcement of the "caps" would be greatly simplified by the limited number of "first sellers" and the ease of identifying them – especially in comparison with the myriad downstream users that would require CO_2 emission permits. Theoretical analysts of the problematic possibility of a perverse "Green Paradox" effect of rising carbon are agreed that the anti-conservation reaction on the part of owners of fossil fuel deposits would not materialize were a high enough ad valorem tax rate to be imposed sooner rather than promised for the future.

But, as efficient and administratively feasible as it might be for central government authorities in the largest economics to quickly impose coordinated tight regulatory caps on first vendors, such actions would be tantamount to directly restricting extraction and importation of fossil fuels. The political economy of domestic legislation and international treaty negotiations, however, seem likely to militate strongly against practical implementation of this approach to fixing the greenhouse gas externality – both presently and for some time to come. That would seem to deprive proposed up-stream cap-and-trade mechanisms of any practical advantage vis-à-vis the more familiar "downstream" variety. [20]

In these circumstances it should be remarked that the present and likely future social and political resistances to stringent and enforceable carbon-pricing policies

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[19] This source of ambiguity regarding the effects of using the "cap and trade" mechanism to set a positive price on carbon emissions, it should be emphasized, is quite distinct from the concerns about the possible disincentive effects of carbon taxes' negative impacts on demands for carbon-intensive goods and services – e.g., those relying heavily on production processes using energy sources such as coal, oil and natural gas. The theory of exhaustible natural resource pricing, following the classic work of Hotelling (1931) tells us that the resource valuation must rise at the rate of interest, and with marginal extraction costs constant, the price of the flow of materials traded in the market must rise *pari passus* with that of the resource deposit. Subsequent theoretical contributions to exhaustible resource economics have pointed out that downward pressure exerted by anticipations of advances in "backstop" technologies permitting substitution of carbon-free energy sources (and, more generally, more energy-efficient production process innovations) would lower the levels from which the market prices of coal, oil and natural gas would be trending upwards (see Heal (1976), Dasgupta, Heal and Majumdar (1978), Dasgupta and Heal (1980)). This effect would work to slow, if not temporarily halt the diffusion of those innovations. Whether the magnitude of the first-order offset on prices in carbon markets would be large enough to perversely neutralize the intended impact of a global cap-and-trade scheme, or so large as to temporarily *lower* the after-tax unit cost of burning carbon fuels, is an empirical question (obviously of policy-relevance) that deserves more attention than it has received. On first consideration, it seems that the answer must turn on the extent of the anticipated demand displacement in relation to the size of the resource reserves that could be accessed from the markets in question. We are grateful to Lawrence Goulder observation that the latter considerations make the magnitude of the offset-effects greater in the case of high quality oil reserves than it would be for the world's vast coal deposits.

[20] Indeed, it seem more than likely that government regulations banning unlicensed first sales of fossil carbon sources of energy would be characterized as tantamount to state expropriation and control of those forms of natural resource wealth, and met with strong political opposition from powerful economic interests in the West as nothing less than "de facto socialization" of their privately owned mineral wealth.

might be weakened were there credible grounds for anticipating that the economic welfare costs entailed in switching to alternative energy sources would be significant reduced by a concerted multi-national program of directed technological innovation. Seen from this angle, the attention that economists have paid to the potentiality that raising the relative price of fossil fuels would induce (private) investments to reduce the carbon-intensity of production processes and final goods and services may have put the policy cart before the horse.[21] While the dynamics of this techno-politico nexus will need to be thought through carefully, the argument for systematic economic analysis of the potential role technology policies in effecting a timely, climate stabilizing transition to a viable "low carbon" global economy therefore can rest in part on the possible temporal asymmetry of complementary (or super-modular) relationships between the two major policy approaches.

When viewed in that expanded and explicitly dynamic perspective, a program that starts with "tech fix" may make that policy initiative complementary to (and not a substitute for) fiscal and regulatory instruments designed to get market prices of carbon fuel to reflect their marginal social costs. This consideration provides a rationale for according greater attention to the requirements of a "technology push" strategy than it has received in previous and contemporary economic research on climate policy design conceived primarily in determining the level and time-profile of an optimal global "carbon tax" --or its equivalent implementation by means of a credible future schedule for the stock of globally available tradable permits to emit CO_2.

The latter considerations warrant close attention to the technological portfolio problem, especially in view of the uncertain nature of the technical constraints on the potential performance improvements of the multiple fossil-fuel based techniques of energy generation and utilization. The relevant constrains have to do not only with physical limitations on the scope both for enhancing their respective productivities (by lowering those processes' carbon- intensity), and for directly curtailing the rates at which they release CO_2 and other GHGs into the atmosphere. Equally relevant are the constraints upon the distributions of the magnitudes and speeds with which improvements in specific technologies or families of related technologies can be antcitipated to flow from prior directed R&D expenditures. Uncertainties of the same sort surround the technical constraints on raising the productivity of "alternative" (non-fossil fuelled) technologies for generating energy.

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[21] The force of this point is not vitiated by policy arguments that recognize that publicly subsidized R&D may well be needed to supplement the induced response of private innovation (see, e.g., Acemoglu, Aghion, Bursztynx, and Hemous, 2012), because "free-riding" on the anticipated externalities (in the form of informational "spill-overs") resulting from R&D outlays is likely to reduce the aggregate volume of the latter investments to a socially sub-optimal, while racing for patent protection and first mover advantages would tend to produce excess R&D commitment to invention in the case of some technical areas.

Analysis of the requirements of an explicit "tech-fix" approach to stabilizing the global climate therefore demands more than identifying a subset of the array of available technologies upon which CO_2-emissions mitigating measure should be focused, however broad the resulting portfolio of selected "core techniques" might turn out to be. To be useful, the portfolio has to be specified dynamically. Not only must it be constructed so as to render feasible at least one path for the timely transition to a climate-stabilizing global production regime -- i.e., characterized by zero net additions to the atmospheric concentration level of GHGs in CO_2-equivalent ppmv. Subject to that global constraint, the "tech-fix" path should be designed to be "social welfare optimizing", in the sense that it will minimize the welfare burden of the transition process, taking into account the damages incurred by the consequences of the rising temperature(s) of Earth's land and ocean surfaces until stabilization has been achieved. Timing does matter: both in regard to the sequencing of investments in research and development activities directed to enhancing the performance of specific techniques prior to their deployment, and to the time distribution of tangible investments that will be needed in order to deploy those technologies that must be embodied in fixed capital structures and equipment, including the necessary supporting infrastructures.

3. Preliminaries for modelling a "Tech Fix" strategy's dynamic requirements

All of the previously mentioned supply-side options, however, require paying explicit attention to major changes in production systems and/or changes in life-style and consumption patterns. Furthermore, to explore the dynamics of transitions towards a less carbon-intensive production regime and the role that research policy would have to play, it is necessary to consider the temporal duration of R&D activities directed to altering the performance characteristics of various classes of technology, as well as the duration of tangible capital formation processes required by technologies that must be "embodied" in new physical plant and equipment. Such questions can be most usefully explored in this paper within the framework of a heuristic model of endogenous global economic growth, which is the research approach pursued here. .

It is no less essential to begin by explicitly taking into account the dynamic behavior of the geophysical system within which the global economy is set. That larger physical context quite obviously imposes both natural resource and environmental constraints upon the production regime, as well as impinging directly upon human welfare – in this instance through the continuing anthropogenic emissions of greenhouse gases and their destabilizing impacts upon global climate and the accompanying alteration of weather patterns.

3.1 Climate science realities and the geophysical system constraints

There are now firm scientific foundations for the growing concerns that to allow GHG emissions to continue at anything close to their present rate may soon set in motion irreversible runaway global warming, with potentially catastrophic consequences for global ecosystems and for human life as we know it. This conclusion has survived the persisting doubts voiced by a dwindling fringe of academic "climate change sceptics" and outright deniers of the seriousness of the problems that continued global warming will pose.[22]

The mass of evidence provided by chemical analysis of the gases in the deep ice cores that have been extracted from Arctic and mountain glaciers during the past decade-and-a-half has allowed climate scientists to document a history of abrupt alterations of the Earth's climate(s) during pre-Holocene epoch. The record points to the possibility that even modest global warming driven by continued anthropogenic emissions of GHG could trigger the onset of irreversible global warming, but a form of climate instability that during the most recent transition between glacial and stadial epochs featured *a* prolonged era of "climate flickering."[23] This latter term refers to a climate regime characterized by high frequency switching (with periodicities as short as 3 to 5 years) between markedly warmer and cooler average temperatures -- the latter differing by as much as half the 12^{o} C. range between the lowest and highest extremes recorded during Earth's glacial and stadial episodes. The possibility of catastrophic state change of that sort undermines the sanguine suppositions that the continued emissions of CO_2 would simply result in a smooth transition to a warmer global climate to which human societies would be able to successfully adapt.

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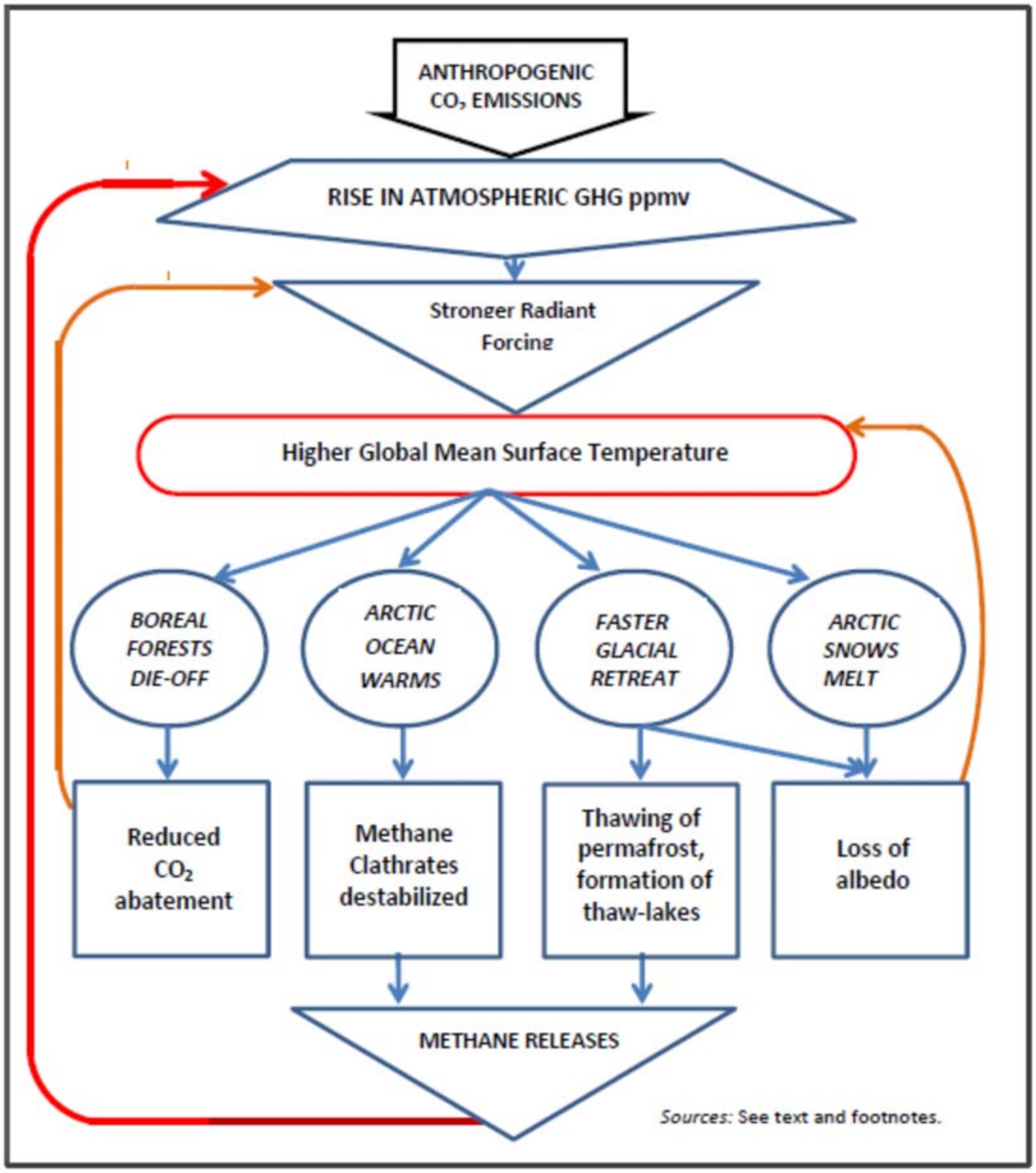
[22] Despite the evidence presented by Solomon et al. (2007), doubts have been raised about conclusion that the warming trend observable during the second half of the 20th century resulted from radiative forcing *due to human activities*. These doubts are rooted in criticism of IPCC models of general climate circulation models (GCMs) on account of their poor performance in predicting the high frequency and high relative amplitude variations in global climate during the past 50-60 years. See, e.g., Scarfetta (2011) and references therein for studies showing that the latter variations can be more accurately predicted by statistically fitting harmonic (Fourier series) regression models that combine indicators of multiple astronomical cycles with differing frequencies. Such research, however, sheds scant light on the dynamics of the physical forcing processes that would account casually for the observed statistical correlation, nor does the existence of unexplained fluctuations around a trend vitiate the positive statistical significance of the trend itself. Recent theoretical efforts, such as that by Stockwell (2011), offer general physical basis for the skeptics' contention that periodic radiative forcing from astronomical sources, not accounted for in the IPCC's GCMs, could be responsible for the latter's failures in predicting recent high frequency variations observed in the Earth's temperature. The trouble with this line of speculative criticism seems obvious enough: variations in solar radiation are not thought to be a concurrently emerging feature in the history of our planet's astronomical environment. Consequently it is hard to understand how that phenomenon could account for the "hockey-stick" up-turn observed in the last half-millennium's worth of data indicating the long-term trend in the Earth's temperature.

[23] Hall and Behl (2006) review the findings from recent advances in climate science, and point out that its implications thoroughly undermine the supposition long maintained by mainstream contributors to the literature on energy and environmental economics, namely, that "climate change" driven by rising GHG concentration levels could be satisfactorily modelled as a smooth transition to a higher equilibrium level of the global mean surface temperature – as has been assumed by the integrated assessment models (IAMs) that have figured prominently and remain salient in the energy and environmental economics field. See Hall and Behl (2006: pp.461-462) for a detailed discussion of this and related features of the geophysical sub-system in Nordhaus and Boyer's (2000) updating of the original (Nordhaus 1994) DICE model, The assumption that radiative forcing due to the accumulation of atmospheric CO_2 would drive a smooth transition to a higher equilibrium temperature of the Earth's surface is retained by Nordhaus (2010) in the latest update of DICE (2010), as well as in the annualized version of the model that Cai Judd and Lontzek (2012) create as a basis for DSICE, a stochastic version of the model.

The positive feedbacks that would drive runaway global warming dynamics and the manifold entailed damages to human welfare are those generated by the amplification of the linkage between elevated GHG concentration levels of anthropogenic origins and stronger radiative forcing that produces faster warming of Earth's surface. The warming at various points triggers self-reinforcing alterations in the behaviour of the geophysical system. These feedback processes would operate to increase the strength of the radiative forcing resulting from a given atmospheric GHG concentration level; boost the surface absorption of heat, and hence the rate of warming produced by a given level of radiative forcing; supplement direct anthropogenic emissions of CO2 by triggering releases of methane (a much more powerful GHG) – which, at lower temperatures, would have remained naturally sequestered in the permafrost beneath glacial ice and in the methane hydrates lying on the shallow seabed at the northern edge of the arctic ocean.

Some of the intervening steps of such sequences (depicted schematically in Figure 1) have a cascade-like sub-structure that serve to extend and further accelerate the positive feedback process. This creates a potential for the rising concentration of

Figure 1: Geophysical system feedbacks drive climate destabilization



"tipping point" in the Earth's climate system that abruptly triggers the onset of a catastrophic runaway warming process.[24]

[24] On *abrupt climate changes* see Alley, Marotzke, Nordhaus, Overpeck et al.(2003); Stern (2007), pp. 11-14, for a short overview of positive feedbacks from global warming, including reduction of albedo through reduced ice-coverage of the arctic regions, thawing of permafrost and induced releases of methane. see Lenton, Held, Kriegler et al., (2008), for specifics of identified major "tipping elements" in the Earth's climate system; Barnosky, Hadly, Bascompte et al. (2012) on evidence indicative of an approaching global ecosystem tipping point. O'Riordan and Lenton (2011) provide a brief non-technical presentation of the concepts of *tipping points* and *tipping elements* and their present policy relevance.

Consider, for example, that by lowering *albedo* (the fraction of sunlight reflected by the earth's surface) glacial retreat leads immediately to greater local heat absorption in the affected regions. This promotes thawing of the exposed permafrost and the creation thaw-lakes from which methane generated by the anaerobic decay of underlying organic material will bubble to the surface, as it does from continually flooded rice paddies. Further, warming of the large peat deposits in the arctic Siberian shelf would augment the release of methane from that natural source; and even a moderate rise in the upper ocean's temperature, particularly in the shallower Arctic waters can destabilize the methane clathrate compounds that had formed on the seabed there during an earlier epoch. Methane matters, because it is a powerful greenhouse gas whose liberation from natural sequestration on the land and under the oceans' surface is an important potential source of *self-reinforcing* dynamics. As the (red) positive feedback loops in Figure 1 depict, in principle those effects can be powerful enough to drive continuing a warming trend even when anthropogenic CO2 emissions have been suppressed to rates that can be matched by the Earth's natural abatement capacities.

Methane (CH_4) is an unstable molecule that is subject to immediate degradation by exposure in the atmosphere to the OH radical , which sets in motion a chemical chain reaction that within several days will have begun converting the newly added methane molecules to molecules of water and CO_2.[25] Although the life of a molecule of methane is a very transient one, the gas has a global warming potential (GWP) -- the metric expressing the ratio of its absolute global warming potential relative to that of CO_2 – averaging 72 over the 20 year horizon following its release, and declining to an annual average GWP of 26 over the 100 year horizon. The GWP values reflect both very strong short-lived direct effects of on radiative forcing of a "shot" of CH4 throughout the 12 years of its perturbation (impulse decay) lifetime, and the persisting indirect effects of the CO_2 molecules that the decaying "methane shot" adds to the atmosphere.[26]

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[25] See Archer (2011:Ch5) for a lucid exposition of the methane cycle, drawing largely on Jacob (1999). The OH radical (denoted OH●) is produced naturally by the effect of sunlight on water, which causes the later to lose a hydrogen atom, and to immediately steal one from an available methane molecule --yielding H_2O and the methyl radical (CH_3●), which a chain of simple chemical reactions ends up producing 2 stable molecules of water and one of carbon dioxide.

[26] The GWP average values for methane of 72,4, 26.3, and 7.6 over the 20-yr., 100-yr. and 500-yr horizons are calculated by Boucer et. al. (2009: Table 1). These agree closely with the values presented by IPCC (2007: AR4-Working Group I, Ch.2, p. 212). Because they include the indirect global warming effects of the CO2 produced by the oxidation of CH4, the values in the text (above) are higher than the corresponding direct effects reported by the 2001 IPCC Third Assessment Report, IPCC 2001: Sect. 6.12.1,Table 6.7) gives (direct) Global Warming Potential (GPW) values for methane of 62, 23 and 7over the 20-yr., 100-yr. and 500 yr. time horizons — also calculated as annual averages on the basis of a 12 year impulse decay lifetime for Ch_4 , and a CO_2 response function that assumes mixing ratio with other trace gasses that is constant over a 500 year period. The GPW calculation for CH_4 differs from that for the other trace gases, in that it uses value for the methane's perturbation lifetime that takes into account the feedback effects of its degradation on the atmospheric concentration moisture (H_2O) and the availability

Clouds of CH_4 bubbling up to the surface of the arctic ocean's shallows and rapidly diffusing into atmospheric circulation, would thus cause the CO_2-equivalent concentration level and the strength of radiant forcing to spike upwards. Consequently, the strong global warming effect a sudden jump in atmospheric methane concentration (unlike that of gains in CO_2 ppmv) will dissipate rapidly if the higher rate of emissions is not maintained, leaving behind only the persisting climate effects of the elevated moisture level and the CO2 concentration produced by the chemical reactions that degrade it. A surge in warming resulting from a prolonged rise in the rate of methane flux, however, could be sufficient to induce further positive feedback effects on mean global temperature and have a potential to trigger a self-reinforcing trend rise in the atmospheric concentration of CO_2.[27]

Methane gas clouds could be abruptly released by the warming of shallower ocean waters along the Artic Ocean's edges, since it is estimated that a rise of only 2° C. in the water's temperature there would be enough to destabilize the methane clathrate deposits that hold CH_4 trapped in their cage-like molecular structure. This, the so-called "Clathrate Gun" effect now is thought to be a primary mechanism of the pre-historic episodes of pronounced high frequency temperate fluctuations that have left a record in the deep artic ice-cores.[28]

Yet another perverse feedback loop involves forest die-back. This worries scientists studying the boreal forest stretching across the northern latitudes of the north American continent and Russia -- a zone whose trees represent between 25 and 30 percent of the world's forest cover, and which is reported to have experienced the most pronounced temperature increases observed anywhere on the planet during the last quarter of twentieth century. In the northern latitude of Siberia the predominant needle-shedding larch forests are being replaced by evergreen conifers that grow more rapidly

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of OH radicals (the principal natural sink for CH_4). See the previous footnote on the chemistry of the methane cycle.

[27] While it appears that there is little likelihood of this happening spontaneously through natural events -- such as disruptions of the deep ocean seabed by earthquakes that opened undersea vents for methane -- that would occur on a large enough scale to have catastrophic climate consequences, the size and instability of the Earth's methane hydrate reserves and the uncertainties surrounding the present state of scientific knowledge lead a sober climate scientist like Archer (2011) to characterize those potentialities as "frightening".

[28] Even quite moderate warming of the ocean waters along the continental shelves is thought to be sufficient to destabilize the methane clathrate compounds that have formed on the seabed in northern latitudes. On the "Clathrate Gun" hypothesis and it's relevance as an explanation of abrupt climate change and the phenomenon of "climate flickering" at the end of the last ice age, see Kennett et al (2003), Maslin et al. (2004), Hall and Behl (2006), Reagan and Moridis (2007). The high frequency of the temperature fluctuations recorded in the ice-cores are held to militate against the earlier theory that this abrupt and catastrophic alteration of the climate system could have been produced by a "thermohaline collapse", which would result climate cycles of much lower frequencies.

in the summer warmth, but the evergreen trees absorb more sunlight and their expansion is thus contributing to the global warming trend.[29]

The loss of predictability of local environmental changes accompanying profound ecological damage and losses of human life and welfare, and the socio-economic repercussions that would exacerbate the process thereby set in motion, can be left to the imagination at this point. Suffice it to say that this prospect, however uncertain its timing and magnitude are at present, strengthens the force of arguments that the precautionary principle should be embraced firmly and become the touchstone of national and international climate stabilization policy measures designed to avert the disaster of crossing a tipping point into regime of runaway warming and the possible onset of catastrophe climate flickering.

3.2 The way forward

Obviously that is much easier said than done, on many counts. Aiming for a timely attainment of some stable level of atmospheric CO2-equiv. concentration with an ex ante optimum set of policy instruments would have to allow for the likelihood of unforeseen events that cause the program to falling short of attaining the appropriate (moving) CO2 emissions mitigation targets. Getting it "back on track," however, would only have become technically more difficult and entailing still larger sacrifices of economic welfare and well-being for the world's people (not to mention other species). In view of the potentially insurmountable adaptation costs and welfare damages involved in passing from a viable quasi-stable global climate mode (characterized by the slow and continuous warming trend) to one of irreversibly accelerating and eventually chaotic regime of global warming the most sensible course of action is to is to firmly embrace the implications of the precautionary principle.

This judgment of the present situation calls for a commitment to design and seek to implement climate stabilization policies required by application of the Precautionary Principle grounded on regret theory. The latter's aim is to minimize the subjective probability of the worst-case outcomes being realized, i.e. determining the steps required to avert those (catastrophic) future outcomes that would occasion the maximum regret were they perceived to be immanent. A feasible initial step in that direction is to investigate the nature and extent of global mobilization and allocation of resources required in order to implement the technical means of mitigating the emission of GHGs so as to stabilize atmospheric concentration of the CO2-equiv ppmv at just below a conjectured likely level of the catastrophic tipping point.

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[29] Whether this is compensated by their greater capacity for absorbing CO_2 is not clear. If such is the case, it is being offset in the southern drier boreal zone, where water-stressed trees in the summer grow less rapidly and the forests are being replaced by grasslands and pasture. http://en.wikipedia.org/wiki/Boreal_forest#Climate_change

Given the uncertainties that surround both the scientific and technological conditions which will constrain any such an undertaking, the latter conjecture necessarily will be subjective. Nevertheless, for the purposes of the analysis it can be treated as an exogenous parameter that imposes a key constraint upon the design of a welfare optimizing "technological fix". By setting the level of this parameter, given the presently existing atmospheric concentration level of CO2-equiv ppmv, will in effect set a hard "carbon-budget constraint" on the optimal mitigation program; in effect it fixes the cumulative net volume of CO2 emissions that can be expended during the just-in-time transition to a stabilized climate that would avert the catastrophe of overshooting the (conjectured) tipping point.

Parametric variation of the hypothesized tipping point can reveal the sensitivity of the technical and economic investment requirements of the (constrained) optimal transition path. {Such explorations will not have much practical purpose if the lower end of the range of variations is not truncated at a tipping point level above those that already have been surpassed, because whether or not the actually of that state is perceived, ex hypothesis the conjectured catastrophe cannot be averted. If a low a priori probability were to be attached to that conjectured location of the tipping point, this would have the same effect, in sufficing to lower the amount of attention devoted to the requirements of retrieving something worthwhile within the discouragingly tight constraints of the conjectured state of the system. The one thing this approach seeks to avoid is inaction intended to avoid the expenditure of economic resources on mitigation efforts by waiting to accumulate information that could increase the precision of the a priori subjective probabilities assigned to the range of tipping point locations. This follows from the plausibility of the present scientific consensus that the economic-climate system present situation and likely future path without major migitation efforts makes it unlikely that a catastrophic tipping point will not be reached until far in the future.

Our modelling approach in this paper builds upon the pioneering work of economists on climate policy analysis represented in so-called *integrated assessment models* (IAMs) that provide a simplified characterization of the essential features of the geo-physical system.[30] These quantity the linkage between the flow of GHG emissions from economic activity and the augmented radiative forcing that drives higher mean global temperatures at the Earth's surface (the "green-house" effect). Modelling the carbon cycle takes account of the natural sequestration of CO_2 in the forests and oceans, and the consequent lagged effect in the adjustment of the accumulated atmospheric stock of GHG (equivalent CO_2 ppmv); the "climate sensitivity" parameter describes the equilibrium mean surface temperature's response to a doubling of the CO_2-*e*

[30] For surveys and reviews of the evolving field of research on IAMs, after Nordhaus (1993a and 1993b), see Dowlatabadi (1995), Kelly and Kolstad (1999), Parker, Letcher, Jakeman et al (2003), Ackerman, DeCanio, Howarth and Sheeran (2009).

concentration in the atmosphere. [31] Within that characterization of the physical framework, our model adds specifications of the production sector of an endogenous growth model in which directed R&D expenditures can raise the productivity of newly formed capital goods embodying novel "carbon-free" technologies, and hence enhance the economic efficiency of those additions to aggregate (sustainable) production capacity.[32] This is the nexus through which public "research policy" interventions that boost investments in science and R&D can positively affect social welfare by lowering the costs of switching to production systems characterized by low, or in the limit "GHG emissions-free" technologies.

Nevertheless, the research approach pursued here gives priority to understanding the inter-temporal resource allocation requirements of a program of technological changes that could halt global warming by completing the transition to a "green" (zero net CO2- emission) production regime *within the possibly brief finite interval that remains before Earth's climate is driven beyond a catastrophic tipping point*. This paper formulates a deterministic multi-phase model of a just-in-time transition that permit analysis of the requirements of the transition paths that can avert that terrible outcome in a welfare-optimal ways, each being conditional on the conjectured tipping point concentration level of CO2 and the array of technological options that can be exercised to stop short of that point.

In proceeding in this way we are cognizant of the controversial proposition advanced by Weitzman (2009b), that the scientific uncertainty surrounding the behavior of the geo-physical system, such as the distribution of the "climate sensitivity" parameter, raises the possibility of catastrophically large outcomes that cannot be ignored by supposing their materialization would be governed by the vanishingly small likelihoods found at the extrema of 'thin-tailed" probability distributions. Indeed, it seems quite sensible to view setting low targets for the permissible accumulation of

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[31] For purposes of our model described below, atmospheric GHG (ppmv) concentration levels are given by an initial baseline level plus a scalar function of the integral of (CO_2 -*e*) emissions from the baseline date, the latter being proportional to the mean rate of CO_2 emitted per unit of output produced (proportional to utilized capacity) and not being absorbed by the forests and the upper and lower ocean. This simplification ignores the lag effects of emissions on changes in atmospheric CO_2 that result from the carbon-cycle diffusion of the gas between the atmosphere and the upper ocean, between the upper ocean and the lower ocean, and the upper ocean and the atmosphere. In addition it linearizes the relationship between the atmospheric concentration of carbon relative to its level at a base date (E_t/E_0, in our notation) and the absolute gain in radiative forcing from the atmosphere, $F_t - F_0$. Widely used IAMs -- e.g., versions of DICE due to Nordhaus (2002, 2007) – typically represent the change in atmospheric radiative forcing as taking the form $\Delta F_t = \eta[\log (E_t/E_0) - \log(2)]$ with adjustments for the direct and indirect effects of radiative forcing from the upper and lower oceans. Ignoring the latter, and the lags in the relationship between changes in the radiative forcing and those in Earth's surface temperature, the parameter η is the approximate "climate sensitivity": the expected long-run gain in T relative to its base level resulting from doubling the atmospheric CO_2 concentration. Our simplified dynamics of the geophysical subsystem is more acceptable in modeling the transition paths to climate stabilization than it would be in simulating the course of carbon emissions and temperature changes over the 600year long time span of optimally "moderated" CO_2 emissions envisaged by DICE (2007) and its precursor IAMs.

[32] Presently, R&D outcomes are completely deterministic in our model, but we envisage the introduction of stochasticity in several relationships, including this one.

GHGs in the Earth's atmosphere to be a form of "insurance against catastrophic climate damages" especially when their expected magnitude cannot be reliably gauged, as De Canio (2003) and Weitzman (2010) have suggested. [33]

At the same time, however, we are persuaded that heeding the findings of climate scientists militates strongly against continuing to work within the accepted framework of the IAM research literature for reasons that are distinct from, and far more compelling than troublesome thoughts that the robustness of inter-temporal cost-benefit analysis may be imperiled by the existence of "fat-tailed" probability distributions, or that inconceivably large damages to humanity might render its application especially misleading in the context of designing climate policy. The particular findings about the phenomenon of a potential climate catastrophe are those concerning its form, which also has implications for the magnitude of the entailed damages. Abrupt climate changes in the form of "climate flickering" introduce major discontinuities, so that to represent the dynamics of the climate system properly in an integrated policy assessment model would vitiate hopes of being able to identify the social welfare efficient program of intervention in response to global warming by consistent application of optimal control analysis. Discontinuities introduce inextricable non-convexities in optimization function, and convexity is necessary to assure that and optimal solution exists.[34] Ignoring the

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[33] As disturbing as the "dismal theorem" advanced by Weitzman (2009a, 2009b) at first appeared to be, especially for economists reluctant to discard cost-benefit analysis -- a widely applied and generally useful tool for quantitative policy analysis, the "fat-tails problem" grounded in reality, and not an artifact of remediable technical features of Weitzman's model, See Millner (2012) for a lucid and insightful review of the controversy ignited by Weitzman's "dismal theorem," which points out that the emergence of a "fat-tailed" distribution in Weitzman's mathematical model results from the introduction of Bayesian statistics; not from the uncertainties that doubtless remain in scientific understanding about the determinants of such key matters as "the climate sensitivity" or the whereabouts of catastrophic "tipping points". Furthermore, as Millner has shown, the apparently dismal conclusions for economic policy analysis (due to the non-existence of solutions to cost-benefit calculations involving possibly enormous damages) are an artifact of the constant relative risk aversion (CRRA) property implicit in the traditional specification of the form of the social utility function. This, too, shown to be technically remediable; the "harmonic absolute risk aversion" (HARA) utility function--a generalization of the traditional (CRRA) utility function that is widely used in the finance literature--notonly yields finite welfare measures when risks are fat-tailed; under plausible parameter conditions it also makes policy evaluations relatively insensitive to the tails of the consumption distribution.

[34] This is the critical point made by Hall and Behl (2006: esp. pp. 460-463). Given its importance, their clear articulation of the defects arising from the failure of the field of economic research to pay adequate attention to the phenomena revealed by the advances in paleoclimatology remains inadequately recognized, and although it merits repeating *in extenso,* the follow summary extract will have to suffice here: "Abrupt climate change, both warming and cooling, would result in physical and economic destruction of the capital stock …and the rate of return on investment would be further altered in a discontinuous manner with each climate flicker and the expectation of additional climate flickering….The destruction of capital from climate flickering should change the expected return on capital investments. The cost of adapting to climate change should increase with flickering taken into account. The value of technological change to adapt to sudden decreases in temperature, precipitation and ambient CO2, for example, should be lost with subsequent sudden increases in temperature, precipitation, and ambient CO2. There is a fundamental problem with economic optimization models like DICE for economic analysis of climate instability: a solution does not exist. Economic optimization requires convexity, and climate instability results in non-convex optimization functions."(p.460)

non-convexities can save the resulting model's "solve-ability." But Galileo taught us that to "save the phenomena" would be better – even with the trouble of having to discard the misleading yet conveniently familiar model.

The timing, as well as the magnitude of remedial actions in response to the challenges posed by global warming is priority topic of concern. The Intergovernmental Panel on Climate Change (IPCC) has concurred in the conclusion that there is a critical threshold at 2 degrees C. of global warming from the earth's preindustrial mean temperature beyond which the probability of being able to avert catastrophic runaway global warming begins to drop below 50-50. Due to the cumulative volume of persisting GHG emissions up until this point in time, however, the world already is committed to a future rise in mean global temperature (relative to the present) on the order of 0.5 to 1 degrees Kelvin– even if carbon emissions were completely stopped. Even if the" tipping-point" were set at a temperature gain of 3 degrees, this leaves relatively little room for further warming without catastrophic consequences, as the Stern Report pointed out (Stern, 2006: p.15). The implication is that under a "business as usual" rate of GHG emissions there may not be very much time left before the critical threshold will be crossed and the problem facing the world's population will be transformed to one of learning to adapt as best we can to the almost unimaginable mounting damage and societal disruptions entailed in coping with a destabilized climate system.

Against this worrisome background, it is a particularly harsh fact of present economic life that to make major changes in production systems also takes considerable time (and resources). This is because the transition from carbon-based production to carbon-free production involves first the (further) development of carbon-free alternatives and secondly the subsequent implementation of these alternatives through investment in tangible capital formation projects, including infrastructure modifications that will have long gestation periods. Furthermore, the development of the new technologies required will entail the commitment of resources to R&D projects of uncertain and possible extended durations. The claims of these programs of tangible and intangible investment therefore will press upon consumption levels for some period without yielding resource savings or significantly lowering the rate of GHG emissions.

Thus, the reality of transitioning towards a carbon-free economy in time to avoid crossing the critical temperature threshold into climate instability will be, at best, an uncomfortable, unremitting and perilous journey for humanity. The particular analytical challenge that is taken up in this paper is how best to schedule R&D expenditures and related tangible capital formation at the macro-level so as to maximise the societal welfare (or more realistically minimize the damage to social welfare) associated with the entire transition path towards a carbon-free production regime in a GHG-stabilized environment, taking into account the diversion of resources away from consumption that those investments entail. Towards this end we have constructed a multi-phase, multi-technology endogenous growth model that allows for the expenditure of R&D resources on the creation and improvement of "carbon-free technology" and acknowledges that this form of investment diverts real resources from consumption and hence adversely affects societal welfare in the short run (doing 'nothing' would have negative welfare effects in the long run, however).

Taken together, the economic and geo-physical sub-systems pose a timing problem, since old carbon-based technologies generate CO_2 emissions, and postponing the implementation of new, carbon-free technologies reduces the time available for building up new capacity while cumulative emissions of CO_2 (and other GHG) remain below the associated critical threshold concentration (and associated mean global surface temperature) that can trigger the onset of climate instability. Starting soon enough to undertake the R&D (which takes both time and resources) can make available a better performing family of low-carbon and carbon-free technologies in time to embody that knowledge in new production facilities and avoid crossing that "tipping point", but how soon is soon enough will depend upon the rate of R&D investments and their impact on the productivity of carbon-free production facilities, as well as the economy's capacity to replace "dirty" carbon-using capacity with a carbon-free capital stock.

The following section of the paper demonstrates how this and related questions can be answered by formulating and solving a sequence of optimal control sub-problems for each distinctive phase in the transition from a "business-as- usual" carbon based economy to a sustainable carbon free economy, and then tying optimized phases together in "stacked Hamiltonians" by the use of transversality conditions that guarantee the optimality of the transition path as a whole. The sub-sections of 2 apply this approach in developing increasingly more complicated models, all constructed on the basis of the most elementary growth model. Section 3 comments on a number of insights that the three partial models provide about questions of timing in exercising various technology-fix options. It also reports preliminary results from the use of sensitivity analysis to assess the effects on optimal investment paths of variations in the assumed location of climate "tipping points". The paper concludes in Section 4 with a brief review of what has been learned, what remains to be studied, and the proximate next steps in pursuing this line of research into the design and implementation requirement for a successful technology fix for global climate instability.

4. Modeling the phases of a timely transition to an essentially "carbon-free" production system

4.0 An incremental model-building agenda

Rather than undertaking from the outset to specify the structure of an equivalent deterministic dynamical system that incorporates numerous features of each of the various possible technological strategies could be deployed to stabilize global climate, we proceed towards that goal in a step-by-step manner, taking three discrete partial model-building steps, and investigating what can be learned about the dynamics of an optimal transition path from each of them, considered separately.

We start from a basic model in which there are two available technological options, a mature carbon-intensive technology that is embodied in existing fixed production facilities and a carbon-free technology that has yet to be deployed by appropriate capital formation. The tableau in Figure 1 (below) provides a summary overview of activities that distinguish the three phases of the transition that transforms the economy of our Basic Model from a carbon-burning "business as usual" regime to one in which the switch to producing exclusively with carbon-free capital facilities has stabilized the global climate. The tableau's horizontal columns indicate the two different production technologies that can be used by the generic economy, whereas the different kind of tangible (and intangible are compatible with efficient resource allocation in each phase of the transition path investments) that may be undertaken are shown in the rows. The shading (in red, for the Basic Model) shows the combinations of concurrent investment and production activities that.

Figure 1: Basic Model (Red)

Investment	***Production (Capacity in Operation)***		
	Old Carbon High emission rate	**Greened Carbon** Lower emission rate	**Carbon Free** Zero emission rate
Carbon-using capital K^A	**production business as usual**		
Carbon-free capital K^B	**joint production**		**joint production**
			carbon free production
R&D on carbon--free technologies			
R&D on geo-engineering			

Next, we introduce into the Basic Model set-up the possibility of undertaking R&D expenditures during the "business as usual phase". These investments in "directed technological change" are aimed to reduce the unit capital costs of production facilities embodying the carbon-free technology, raising the average productivity of K^B enough to match that of capacity based on the carbon-using technology. In this elementary model of a three-phase "endogenous R&D-driven transition", only when that technological goal is attained does R&D expenditures (and further tangible investment in carbon-intensive production capacity) come to a halt, and tangible capital formation begins deploying the carbon-free technology. This is indicated in Figure 2 by the tableau's blue shaded areas.

Figure 2: Basic Model with R&D (Blue)

Investment	***Production (Capacity in Operation)***		
	Old Carbon High emission rate	**Greened Carbon** Lower emission rate	**Carbon Free** Zero emission rate
Carbon-using capital K^A	**production business as usual**		
Carbon-free capital K^B	**joint production**		**joint production**
			carbon free production
R&D on carbon-free technology	**production business as usual**		
R&D on geo-engineering			

The third of the partial models to be examined extends the structure of the basic model by allowing for the exploitation of a third class of technological opportunities. These do not require significant R&D investments, since they make use of known "core" engineering techniques to provide an "intermediate", less carbon-intensive set of production technologies that in effect serve to "green" existing carbon-using direct production facilities and infrastructure. Because they are characterized by lower rates of CO_2 emissions per unit of capital than the old carbon-based technologies (even if a higher capital cost per unit of output), they provide a reduced emissions-output ratio. Thus they can "buy time" to make the tangible investments needed to switch to a carbon-free production regime. Figures 3.1 and 3.2 display in tableau form the pair of alternative 5-phase trajectories that arise in the case of the buy-time model. Each version can be solve for an optimal transition path, but to find the optimum optimorum it is necessary to evaluate and compare the implied present value of the social welfare index associated with each of them.[35]

[35] Note that the production constraints in the model, including those derived from the technological parameter values, may determine which, between alternative trajectories can satisfy the optimality

Figure 3.1: Buy Time Model Trajectory 1 (Dark Green)

Investment	*Production (Capacity in Operation)*		
	Old Carbon High emission rate	**Greened Carbon** Lower emission rate	**Carbon Free** Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-economizing K^D (retrofitted K^A)	joint old & green carbon production	joint old & green carbon production	
		Greened carbon production	
Carbon-free capital K^B		Joint greened carbon and carbon-free production	Joint greened carbon and carbon-free production
			Carbon-free regime

Figure 3.2: Buy time model Trajectory 2 (Light Green)

Investment	*Production (Capacity in Operation)*		
	Old Carbon High emission rate	**Greened Carbon** Lower emission rate	**Carbon Free** Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-economizing K^D (retrofitted K^A)	Joint old and green carbon production	Joint old and green carbon production	
Carbon-free capital K^B	Joint old and green carbon, carbon-free production	Joint old and green carbon, carbon-free production	Joint old and green carbon, carbon-free production
		Green carbon and carbon-free production	Green carbon and carbon-free production
			Carbon-free production

In the optimal control solutions each of the three foregoing "partial" models of a climate stabilizing transition, the internally optimized phases of the model are tied together by transversality conditions. That makes it possible to obtain the "requirements" of the entire optimal transition path, in terms of the optimum durations of its phases, and the optimized within-phase levels of activity for production, consumption, and investment rates (both the intangible R&D and the alternative tangible forms of technology-embodying capital formation) – as well as the implied flows of CO_2 emissions up to the point at which the global economy's production regime

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requirements, so that answer to the question of which to choose become an empirical matter. See section 2.3 for further discussion.

reaches the goal of a carbon-free state that averts the onset of irreversible climate instability. Even in the simplest of possible growth-model setting that has been specified, the solutions of each of these successively more complicated models to find their respective overall optimal transition paths must be obtained by numerical analysis of the resulting systems of "stacked Hamiltonians" – the details of which following section discusses *ad seriatim*.

This modelling framework differs from the standard AK growth model in its specification of distinct phases that form the transition path, phases during which production capacity embodying new (comparatively "clean") technologies is built up capacity embodying old ("dirtier') techniques is run down and eventually discarded. A central task for the analysis of each of the models, therefore, must be to find the optimal configuration of specific investment and production activities in conjunction with the their optimal sequencing on the transition path to a viable stabilized climate.

The questions addressed by this modelling exercise are not predictive. Rather than venturing to throw light on what will happen, they ask what has to be done, and when must it be done in order to get from a carbon-based production system to a carbon free production system that would avert runaway global warming, while maintaining the highest present value of social welfare that is consistent with attaining that goal.

Our starting point in the analysis is to set up a Hamiltonian system with a standard CIES inter-temporal utility function in which consumption (per capita) is its argument, since population is implicitly assumed to be constant and therefore can be suppressed.[36] Similarly, labor service inputs in production activities are assumed constant, which is consistent with our having selected the simplest possible overall dynamic setting, namely, the AK-model of endogenous economic growth due to Rebelo (1991), and Barro and Sala-i-Martin (2004, ch. 4) as the platform on which to develop the Basic Model of technology switching. We have extended classic AK-framework, however, by replacing its specification of a single, linear capital-using production technology by the assumption that there are multiple discrete (linear) technologies that may be used either simultaneously or sequentially, including one that can be enhanced by directed investments in R&D activities.

The reduced unit cost of capital embodying the carbon-free technology as a result of directed R&D expenditures, however, is only realized through the technology's subsequent deployment in gross tangible capital formation. To put a somewhat sharper point on this, it is not enough to think about R&D policies and programs. Mechanisms of diffusion into use, as distinct from the dissemination of information about technological

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[36] Readers interested in the details of the structural equations and solutions of the models discussed here may consult the "Technical Annex: Modeling Optimal Multiphase Transition Paths to Sustainable Growth" (July 2011), which is available at http://siepr.stanford.edu/system/files/shared/OptimalMulti.pdf, or can be downloaded as a supplement to an earlier version of this paper (presented on 12/14 at the SIEPR-GEEG Social Science and Technology Seminar) at: http://siepr.stanford.edu/programs/SST_Seminars/index.html.

innovations also must be figured explicitly among the requirements of a "technology fix." Similarly, in the "buy time model," the use of existing core engineering methods to reduce the ratio of CO2-emissions to real output in carbon-based production facilities entails incremental tangible investments in retrofitting parts of the existing (K^A) capital stock.

This feature of the approach pursued here rests on the premise that *embodiment in physical capital* goods is necessary to implement changes in the technologies that would lower the global production regime's carbon-intensity, and in those that would reduce the CO2 emissions-intensity per unit of output from the carbon-fuelled production facilities production. The "embodiment" assumption is especially appropriate when considering the impact of technical innovations in energy supply systems. [37] Explicit recognition of this constitutes another important respect in which the present framework of analysis (and its account of the dynamics of the optimal climate-stabilizing transition path) departs from the ways in which the effects of endogenous technological change have been treated in previous economic contributions to integrated modeling and assessment of climate policy measures.[38]

Although the AK-setting is extremely simple, because we allow for different phases in the transition from carbon based to carbon free production, the resulting model is able to generate a set of time paths for the transition to a stabilized climate (and stationary level of atmospheric GHG concentration) that, in practice, can be determined only by the use of numerical methods.[39] A future, more complicated

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[37] The development and use of energy technologies is viewed as an integrated system comprising research discoveries and inventions, the creation of commercial products and processes, their initial deployment and adoption into commercial operations, and subsequent wider diffusion – the view embraced recently by the Report of the President's Council of Advisors on Science and Technology (PCAST, 2012). Accordingly, Ernest Moniz (2012: p. 82), former Undersecretary of the U.S. Department of Energy and a PCAST member, emphasizes the importance of tangible fixed capital formation in considering policies designed to stimulate "energy technology innovation": "Adoption and diffusion are the stages at which materiality of [novel] products and processes are realized (or not). *Innovation, as I use it here, refers to the end-to-end system including market diffusion, not front-end R&D alone*." By contrast

[38] One may compare the implicit assumption – common to each of the following salient contributions on the subject -- that technological change (whether exogenous or endogenous) is disembodied, and therefore not affected by the rate of investment in tangible capital formation: Goulder and Schneider (1999), Nordhaus (2002), Bounanno, Carraro and Galeotti (2003), Edenhofer, Carrir and Galeotti (2004), Popp (2004), Lessman, Kemfert et al. (2006), Sue Wing (2006). Rather strikingly, the useful survey provided by Gillingham, Newell and Pizer (2008) of the different approaches used in modeling endogenous technological change in the context of climate policy assessment models, does not refer to the distinction between embodied and disembodied technological change, and omits mention of adoption and diffusion from its comments on the ways in which economists represent R&D, and learning-by-doing as determinants of general or energy sector-specific change in productivity or GHG-emission intensity.

[39] Given the fact that we distinguish explicitly between different sub-phases during the transition, we end up with a 'stack' of Hamiltonian problems that are linked together through a set of transversality conditions (TVC's), See Leonard and Van Long (1992: esp. Ch. 7). The first-order conditions, in combination with the TVC's and the given initial values of the state variables (and the given terminal value of cumulative emissions), give rise to a set of strongly non-linear constraints on the remaining initial values of the model's implied time paths for co-state and control variables. Using these initial values, the simplicity of the AK-setting allows closed form analytical expressions to be obtained direct integration for

extension of this set-up is envisaged, in which the cumulative increase in GHG concentration, or the mean global surface temperature gain associated with it, enters the utility function negatively.[40]

The transition phases mentioned earlier are linked together through transversality conditions that implicitlydefine the phase changes in terms of rather general optimality conditions that often give rise to conditions on the co-state variables that have a clear-cut economic interpretation. For example, it can be shown that it is never optimal to simultaneously invest in two existing technologies that are different with respect to their capital productivity and their emission characteristics.

Thus, in the current model setting, the existence of a cumulative emission tipping-point will generate notional emission costs associated with the use of carbon-based capital.[41] Investment in the latter type of capital will stop and that in carbon free capital will start the moment the shadow price of carbon based capital falls below that of carbon free capital. As the cost of investment is represented by the welfare loss associated with consumption foregone, and as one unit of investment generates one unit of consumption foregone in both cases, this transversality condition implies the requirement that investment will take place in the technology that generates the highest marginal net welfare per unit of investment. Likewise, the discarding of existing capacity is typically an activity that signals the end of a particular phase and the beginning of the next one.

The optimum timing of such a phase change, and hence the phases' durations, will be governed by a general transversality condition, namely that the shadow price of an incremental unit of production capacity embodying the particular type of technology should be zero at the exact moment that it is taken out of production; otherwise, if it had a positive productive use, why discard it? This transversality condition turns out to be

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the time paths of all variables—except for the cumulative stock of GHG emissions. The resulting paths themselves are in part highly non-linear and largely intractable (except for their underlying roots/structural equations) necessitating recourse to numerical exercises in order to investigate the properties of the model, some of which are not intuitively transparent *a priori*. Moreover, the path for cumulative emissions cannot be obtained by analytical means and requires numerical integration starting from some initial guess of the length of the BAU phase. The optimum duration of the BAU phase then is implied by the requirement that total cumulative emissions over the entire transition (all phases) should just match a pre-specified "threshold" value. Evidently a different approach will be required by a stochastic formulation of the transition process – of which the model discussed here represents the equivalent deterministic system.

[40] See Arrow (2009) and Weitzman (2009a, b) on the significance of specifying an additive temperature effect that is negative, interpreting this as a direct "environmental" effect upon social welfare. An alternative formulation is available -- in which the negative term in the social utility function is the inverse of the difference the critical "threshold," or "tipping point" temperature (beyond which the warming process is expected to become self-reinforcing) and Earth's prevailing global mean surface temperature. This formulation would have essentially the same consequences for inter-temporal resource allocation, but it would admit the adverse psychic effect of approaching the expected point at which humanity will, for all intents and purposes have lost its ability to stabilize the warming trend and avert the catastrophic onset of climate instability.

[41] We use the term "notional" here to denote the existence of a real cost, that is, however, generally not, or at least not fully, paid for in practice.

equivalent to the requirement that a unit of existing carbon-based capital should be discarded at the moment that the marginal welfare benefits of using that unit of carbon-based capacity drop below the corresponding marginal emission costs.[42]

4.1 The Basic Model –Technology Switching

The first and foundational version of our suite of multi-phase transition models is called (appropriately enough) the Basic Model. In this there are two already available production technologies, one that is carbon-based and already in use, and a carbon-free technology that at the outset is yet to be deployed.[43] This set-up creates the possibility of there being three distinct phases. The features of this Basic ("technology switching") model that have been already described are summarized by the system of equations presented in Table 1 (below).

Table 1
Phase-Specific Equations of the Basic 3-Phase Transition Model

$Y^A = AK^A$	(carbon-based production, BAU, JPR)
$\dot{K^A} = Y^A - C - \delta^A K^A$	(net investment, BAU)
$\dot{K^A} = -\delta^A K^A$	(net investment, JPR)
$\dot{E}^A = \varepsilon^A Y^A$	(Net CO2 emissions flow;, BAU,JPR)
$Y^B = BK^B$	(carbon-free production JPR,CFR)
$\dot{K^B} = Y^B - C - \delta^B . K^B$	(net investment, CFR)
$\dot{K^B} = Y^A + Y^B - C - \delta^B . K^B$	(net investment, JPR)
$\bar{E} = E_0 + \int_0^{TJPR} \dot{E}^A dt + \int_{TJPR}^{TCFR} \dot{E}^A dt$	(Cumulative net CO2 3missions)
$U = \int_0^{\infty} \{e^{-\rho . t}\}\{C^{1-\theta}/(1-\theta)\}\, dt$	(intertemp. social welfare, ALL phases)
$H_{TJPR}^{BAU} = H_{TJPR}^{JPR} \Leftrightarrow \lambda^A = \lambda^B$	(TVC defining TJPR (equality Hamiltonians))
$H_{TCFR}^{JPR} = H_{TCFR}^{CFR} \Leftrightarrow \lambda^B A = -\varepsilon\, \lambda^S$	(TVC defining TCFR: deactivation of K^A)
λ_{TCFR}^A=0	(TVC value of deactivated capital = 0)
$lim_{t\to\infty}\ \lambda^B . K^B = 0$	(TVC, standard AK TVC CFR phase)

[42] This condition closely resembles the negative quasi-rent condition that governs the economic lifetime of clay-clay vintages in a perfect competition setting (see Malcomson (1975), for example).

[43] For purposes of this analysis it is supposed that the period under consideration commences when a carbon-free (B) technology becomes available, although with average productivity less than that of the capital (K^A) embodying the carbon-using technology; equivalently, the cost of the production capacity (K^B) per unit of output initially exceeds that of K^A .

A second phase opens with the start of positive investment in facilities embodying the CFR technology, and the cessation of BAU capital formation, therefore marks the opening of the Basic model's second phase. From this point forward there is negative net investment in carbon-using facilities, since the physical depreciation of the existing stock is no longer being offset by positive gross investment in capital of that type. This phase of the Basic model is referred to as 'the joint production' (JPR) phase, as capital goods embodying both classes of technology being used to generate output. The transition to a carbon-free production regime and climate stabilization is complete with the shut-down of the remaining carbon-using facilities and, hence, stoppage of the flow of CO_2 emissions just as the cumulative stock of emissions (and the temperature) approach their respective critical threshold levels. That marks the beginning of the third and final phase of system's transition to sus tainable, carbon-free growth.[44]

The intensity of the flows during each phase, as well as the moments in time at which the various phase changes are scheduled, all follow from the first-order conditions (FOCS), the transversality conditions and from the given initial and terminal values for the state-variables that are part of the optimum control problem. Events and phase-changes in the context of the Basic Model are summarised in Figure 4 (below),

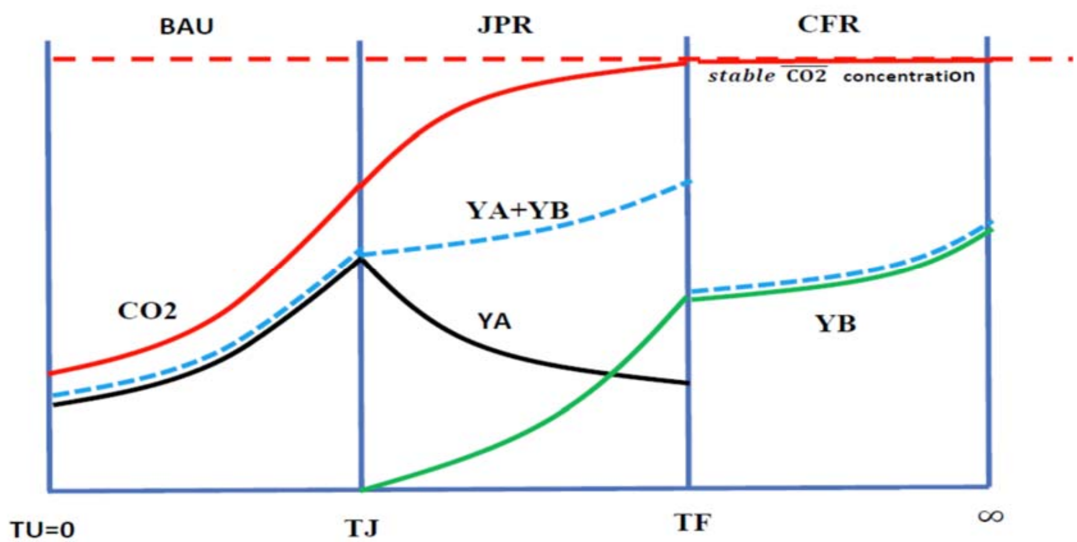

Figure 4. The 3 - Phase "Basic" (Technology Switching) Model:

This transition path to a Carbon Free Economy requires an eventual cut In production just before the "trigger-point" atmospheric concentration of CO_2-equivalent ppmv is reached, but output growth (YB) can resume thereafter.

[44] As the BAU technology has higher initial capital productivity than CFR technology, welfare would not be maximised if the flow of CO2 emissions ceased before the cumulative emission limit was reached.

which shows the development over time of the stock of carbon-based capital K^A, the stock of carbon-free capital K^B and cumulative emissions, ΣCO_2

The points marked on the time-axis of the Figure (TU, TJ, TF) indicate the optimal moments at which the respective phases (BAU, JPR, CFR) begin. It may be noted that although the cumulative stock of emissions continues to rise during the JPR phase, is does so at a decreasing rate as the stock of carbon-based capital K^A is run down and replaced by its carbon-free substitute. The final, carbon-free production phase starts when cumulative emissions atmospheric concentration (ΣCO_2) almost reaches the expectedthr climate "tipping-point, and the curve marked ***CO_2*** becomes flat to the right of TF.

Evidently, the various phases in the model are qualitatively different. The first phase, i.e. the BAU phase, uses a high growth technology which unfortunately quickly raises the stock of cumulative CO_2 emissions that is bounded from above by the cumulative threshold. Before that level is reached, investment in carbon free technology must have taken place during phase JPR to bring carbon free production capacity up to a level where the switch from using carbon-based capacity to using carbon free capacity would not force changes in consumption levels that are too disruptive, since consumers dislike consumption shocks, as is implied by our use of a social welfare function that depends upon the present (discounted) discounted levels of per capita consumption, and is of the (CIES) form allowing for relative risk aversion.

or t>TF the world is "green", and, given the more expensive carbon-free production facilities that are required to stabilize the atmospheric CO_2-equivalent concentration level, the output (per capita) will have to grow for some while at the relatively slow pace at which the K^B is accumulating with the gross carbon-free capital formation rate constrained not to cut too heavily into consumption. Eventually, however, the build-up of the carbon-free capital stock is sufficient to support both rising consumption and a higher rate of investment in K^B, which thereafter accumulates at a quickened pace.[45]

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[45] The structure of the Basic Model has some commonalities with the analysis in Valente (2003) of a two-phase endogenous growth model in which production based on essential (energy) inputs obtained from exhaustible resources switches to a "backstop technology" that provides a constant supply of sustained (e.g., solar) energy. R&D investments permit growth through productivity improvements that occur at the same rate with either technologies (although the levels of productivity can differ), and the optimal timing of technology switching is determined by welfare maximization in which utility depends upon discounted per capita consumption. Valente similarly obtains optimal control solutions for each stage and ties these paths together with transversality conditions. But, unlike in the present analysis, the point at which the renewable technology can be efficiently embodied in tangible capital used in production is not endogenously determined by directed R&D expenditures and the diversion of output to building the "renewables-base" capital stock. Nor is a positive shadow price explicitly attached to using the exhaustible resource in Valente's analysis, because it abstracts from the geophysical climate-system constraints that affect the optimal transition path. As will be seen, the research approach here examines much more complicated, multi-stage transitions.

4.2 A Basic Model with Endogenous (R&D-driven) Technological Change

In addition to the Basic Model, we have specified a version in which R&D can be undertaken to improve the productivity of the CFR technology before its actual implementation through gross investment in the CFR technology.[46] R&D requires resources to be invested now in return for a future intangible asset in the form of knowledge of how to embody CFR production techniques in production facilities whose capital cost per unit of output will be lower than is the case at present. Surprisingly, this view of the role to be played by R&D in controlling future GHG emissions is something of a novelty in the small economic literature that has employed integrated assessment modelling of climate policy options, because insofar as the contribution of investments in R&D to "directed" technology change and innovation has been considered at all, the "direction" has been taken to be the lowering of CO_2 emissions per unit total output in the economy.[47] To sharpen the contrast with the latter approach, Section 2.3 (below) examines the option of achieving greater efficiencies in the use of carbon-based energy can be achieved *without R&D expenditures*, but by investments in the engineering implementation of existing technological knowledge that would result in higher unit (tangible) capital costs of carbon-based production facilities.[48]

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[46] van Zon and David (2013b) presents the complete analysis of a calibrated version of the Basic model with endogenous R&D (as described here). The latter paper extensively revises van Zon and David (2012) and adds a model that incorporates endogenously timed upward shift in the average annual rate of (unscheduled) losses of production capacity due to the accumulation of CO_2 in the atmosphere. The qualitative results display the main substantive findings of the solutions and sensitivity analysis obtained with the preliminary calibration of the Basic + R&D model.

[47] To the best of our knowledge, the earliest previous investigation of the effects of allowing for e*ndogenous* technological change in a computational climate policy assessment model appears in the related papers by Goulder and Mathai (2000), and Goulder and Schneider (1999), which specify the effect of R&D expenditures on changes in an economy-wide total factor productivity coefficient. In these pioneering investigations of the impact of induced technological change on the attractiveness of CO_2 abatement policies, absolute changes in productivity (or real marginal cost) levels, rather than the proportionate rate of change in productivity was specified as a positive increasing (decreasing) function of R&D expenditures. In Goulder and Schneider (1999: pp.216, 223) this functional relationship not further restricted in their mathematical analysis of a 2-period model; whereas the linear specification, $\dot{B} = \varsigma \cdot R$ was used in their dynamic multi-period general equilibrium simulation studies. Nordhaus (2002) takes a different approach, in effect assuming that induced research effort focuses solely on reducing the CO_2 emissions intensity of production. Nordhaus posits a separate production function for carbon-based energy inputs which are used in fixed proportion to the non-energy inputs in an economy-wide production function. He then specifies that the rate of change of the carbon-intensity coefficient in energy production is given by $(d\sigma/\sigma) = - [\psi R^{\beta} - \Omega]$, where $\Psi>0$ is a constant and $\Omega>0$ is a (constant) rate of obsolence of the technical knowledge gained through R&D and reflected in σ. The empirical findings of Griliches (1973), Hall (1995) and others on the link between commercial R&D and industrial productivity growth are cited by Nordhaus in support of this specification, although in general the time-spans to which the data on industry-level and firm-level data relate are far shorter, and hardly of the global sembocope typically contemplated in IAM models.

[48] Of course, this distinction is not necessary because one can model the situation in which R&D expenditures are allocated between the two "directions" of technological change, lowering raising the ratio of gross output per unit of CO2 emissions, and raising the productivity of carbon-free tangible capital used in production. But because the present investigation is confined to modeling the paths to successful climate stabilization, and the time scale for that is far shorter than the hundreds of years contemplated in the existing variants of the Nordhaus (1993a, 1993b, 1999, 2002, 2007) DICE model, it is not

Furthermore, in specifying the impact of such R&D investments on the unit cost of CFR capital goods (K_B) we deliberately depart from the "R&D production function" formulation that is familiar in endogenous growth models following Lucas (1988), Romer (1990) and Aghion and Howitt (1991). Rather than taking the instantaneous rate of improvement in the productivity of CFB-embodying capital, $\dot{B}/B$, to be proportional to the current absolute flow of R&D resource inputs, *R*, as in

$$\dot{B} = \varsigma \cdot R \cdot B \qquad 0 < \varsigma < 1 \text{ ,}$$

an alternative specification is proposed here:

$$\dot{B} = \varsigma \cdot R^{\beta} \cdot (\bar{B} - B), \;\; 0 < \beta < 1 \text{ ,}$$

where ς is a constant productivity parameter, and $\bar{B}$ represents the maximum value that *B* can attain.[49]

There are several features of this specification that argue for its' use in the present applications context. Firstly, it has the advantage of introducing decreasing returns to R&D in a setting that, unlike the conventional endogenous growth models, excludes the possibility of a specific technology being rendered infinitely productive (and so resulting in infinitely rapid growth) merely by the application of more and more massive R&D expenditures at any particular moment in time.[50] Allowing decreasing marginal returns in R&D recognizes that at a given stage in the advance of knowledge the state of fundamental scientific understanding of the physical processes involved may still be inadequate to permit the effective application of more and more resources to the solution of a particular practical problem -- such as the further improvement of the productivity of a particular class of technology-embodying capital facilities.

Secondly, this formulation of the effects of investment in R&D activities may be thought to reflect a Platonic world in which a finite number of solution possibilities for technical transformations are present from the start of time, but these as a rule will not

unreasonable to suppose that the presently existing stock of implementable techniques for enhancing the productivity of carbon-energy inputs could suffice for a considerable number of decades without requiring "refreshment" by focused investment of R&D efforts. See further discussion of the "buy time" option in section 2.3, below.

[49] With $\bar{B}, \zeta, \beta$ as constant positive parameters, *B* and *R* therefore become additional state- and control variables in our dynamic system.

[50] The equilibrium (steady-state) growth rate in a standard AK-model rises linearly with the rate growth in the productivity of capital (cf. Barro and Sala-i-Martin, 2004), and therefore with the flow rate of R&D expenditures. The existence of technology-specific intrinsic productivity limits set – in the limit -- by the physical properties of the materials, chemical and electrical processes entailed in production makes this so implausible that even its assertion as a metaphor is of dubious usefulness. Popp (2002) presents evidence the R&D investment in the energy sector is subject to diminishing returns, and the WITCH model of Bosetti et al. (2006) represents this by setting the elasticity of the "flow of new ideas" to be b< 1-c, where c is that flow's elasticity w. r.t. the stock of ideas and (c+b) < 1.

reveal themselves spontaneously. They can be uncovered, however, and formulated for practical application through costly research and development procedures based upon the existing state of fundamental scientific knowledge, rather than being created *de novo* and without limit by the expenditure of resources in the performance of R&D activities.[51] That more restrictive view of the transformative power of investment in R&D is appropriate not only for the foregoing general reasons, but also because the concern in this context is not with the undirected global expansion of the technological opportunity set typically envisaged in theoretical growth models. Rather, the aim of the "directed R&D" in the present model is to enhance the economic properties of particular kinds of process inventions, with new product inventions only insofar as alterations in product characteristics are consequential for the raising the efficiency of capital inputs into carbon-free production processes.[52]

Within the framework created by introducing this (or any other) R&D production function into the Basic model, there are again three distinct phases of the transition to a stabilized climate. Table 2 compares the phases in the original basic model with the version introducing R&D:

Table 2. Comparison of the Basic and Endogenous R&D Models

Basic model			
Phase	**BAU1** → Business as usual	**JPR1** → Joint production	**CFR1** Carbon-free
Investment in	Carbon-based capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	Carbon-free capital only
Endogenous (R&D-driven) Technological Change Model			
Phase	**BAU2** → Business as usual	**JPR2** → Joint production	**CFR2** Carbon-free
Investment in	Carbon-based capital	Carbon-free capital	Carbon-free capital
R&D investment in	Carbon-free technology	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and carbon-free capital	Carbon-free capital only

[51] It also has the advantage of being jointly concave in B and R, which is a necessary condition for the welfare maximization problem to have a solution.

[52] Following this interpretation, adding endogenous technological change to the Basic Model allows us to characterize the optimal path of *global R&D that is directed to increasing the productivity of CFR capital*. Correspondingly the impact of R&D investment on economic welfare is modeled as being felt indirectly, rather than directly in the form of pure product quality enhancements. In other words, the welfare gains come through reduction of the sacrifice of consumption utility required in the transition, and for the subsequent sustained growth of (per capita) consumption under stabilized climatic conditions.

From Table 2 it may be seen that while the R&D model resembles the Basic Model in having three phases and two optimum switching moments, it differs in having three state variables and two control variables. This is the case because it turns out to be optimal to initiate R&D directed to improving a particular technology from the very first moment that information becomes available about the existence of a potentially workable CFR-technology. On the assumption that the CFR technology is discovered at time zero but is less productive than the mature carbon-using technology when embodied in equally costly production facilities, R&D activity then should be undertaken (only) during the phase in which tangible capital formation and production adhere to the carbon-dependent features of the Basic model's BAU phase. Following that, a JPR phase will commence with gross investment directed to deploying the improved CFR technology in KB-type production facilities.

4.3 A "Buy-Time Model" – Greening Carbon-based Capital in the Basic Model

We now turn to the third of the partial models, in which the structure of the basic model is enriched by introducing a third category of technologies, namely a known "intermediate" class of carbon-using production techniques that are characterized by lower rates of CO_2 emissions per unit of capital but capital productivity per unit of output that is lower than of the mature carbon-based technologies, but not as much lower than that of the initial (pre-R&D) versions of the carbon-free techniques of production.[53]

This conceptualization corresponds broadly to the variety of well-grounded engineering methods may be used to upgrade carbon-using production facilities, whether by improving the energy efficiency of residential, business and government office buildings by better insulation, heating and cooling systems, and reducing the

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[53] On the latter, see the recent noticed example of the annual consumption of over $3 billion work of electric power in the U.S. by HD TV "set-top" boxes supplied by cable companies, due to the choice of low cost designs that that do not actually stop draining power when they are switched off (see, "Atop TV Sets, a Power Drain that Runs Nonstop," *The New York Times*, June 26, 2011: p.1.). The McKinsey Global Institute devoted considerable attention in the years before the financial crisis to studies of current and near-term options for energy efficiency routes to CO2 emissions-reductions and private costs saving through upgrading of production and distribution systems. See, e.g., Farrel and Rennes (2008); Grove and R. Burgelman (2008). In the U.S., numerous proposals for this kind of "retrofitting" have had difficulty gaining policy-traction due to the prevailing policy bias towards research subsidizes to support "innovation" in renewable energy technologies." More recently, the case for expanded exploitation of natural gas and greater investment in other opportunities to improve the productivity of carbon-based technologies that (similarly) offer higher output per volume of GHG emitted has been cogently and vigorously advanced by Burton Richter (2010). During the crisis and recession years the attention of the European Commission shifted away from the expensive and longer-term research envisaged by its ambitious Strategic Energy Technologies Plan [SET, COM (2007) 723]; it focused instead on a variety of shorter-term tactics aimed at stimulating aggregate demand in ways that would implement already available technologies for "green" purposes, notably : retro-fitting buildings for greater energy efficiency, supporting the automotive industry to increase production of low-CO_2 vehicles using electric batteries and second generation bio-fuels (see discussion in David, *op.cit.* (2009). In the U.S., numerous proposals for this kind of "retrofitting" have had difficulty gaining policy-traction due to the prevailing policy bias towards research subsidizes to support "innovation" in renewable energy technologies.

passive consumption electricity by electrical and electronic appliances by configuring them to switch off completely when not in use, and so forth. But it also subsumes proposals for the expanded exploitation of natural gas as a reduced GHG-emissions source of energy, including the acknowledgement that incremental costs per BTU would be required to curtail the environmental damage currently associated with "fracking" methods.

Break-through research and novel engineering principles, however, are not required to exploit this class of carbon-based energy sources and production technologies here, but specific applications of core engineering knowledge adaptation of existing designs to local contexts will raise the average unit capital costs carbon-using plant equipment that has undergone this kind of "green-upgrading," as well as that of a "less dirty" energy source such as natural gas, and safer nuclear power-plants with provisions for long-term sequestration of their toxic waste.[54] The gain to be had by seizing this "low-hanging fruit" comes in the reduced rate of CO_2, which means undertaking the entailed incremental capital expenditures constitutes a way to "buying time" early in the transition in order to be able to subsequently proceed more slowly in replacing the whole carbon-based production regime with one based on new, carbon-free production facilities.

Not surprisingly, therefore, the extent to which the "buy time option" will be attractive to exploit in the context of our Basic model, by building up production capacity (K^D) in this intermediate "greened" form rather than capital that embodies the mature, more carbon-intensive technology, depends not only on the associated capital productivity but also on the reduction gained in emissions per unit of output. It turns out that introducing the possibility of utilizing this third class of technologies gives rise to a model in which there are 5 distinct phases that can be arranged in either of two alternative transition trajectories, as indicated by Figures 3.1 and 3.2 and detailed in Table 2.[55]

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[54] It may be noted that the extraction and processing of fossil fuels, including the catalytic cracking of petroleum, and hydraulic fracking of natural gas also results in emissions of methane, a short-lived GHG especially high GWP (vis-à-vis CO2) during the 20 years following its release into the atmosphere (see above, sect 3.1). In order to include technological measures for methane mitigation in an integrated analysis of climate stabilization, however, it will be necessary to construct a considerably more complex representation of the climate sub-system than the one that is approximated here. In addition to modeling the carbon cycle exchanges of CO2 between the atmosphere and the upper and lower ocean, the fast dynamics of methane's interaction with OH radicals and consequent degradation into CO2, and its high GWP must be modeled explicitly.

[55] Note that the act of 'buying time' involves using a technology with a relatively high output/emission ratio. Hence, logically speaking, buying time is associated with production using that technology rather than with investment in that technology. Therefore, in this case, the BTM phase is subdivided into three sub-phases, in which we have positive output from the BTM technology (here indexed 'D'), and the CFR phase starts when the BTM-technology is deactivated. Typically, the j-th BTM sub-phase of trajectory k is labeled BTM.

Moreover, which trajectory will be the one that is optimal for this simple economy to follow depends upon the parameter configuration that sets the BTM technology's output/emission ratio. Given these 'technical data', the choice between the alternative trajectories that will be followed in exploiting the "buy time option" is prescribed by comparing the welfare valuations of the two solutions of the two optimal control programs.[56] Trajectory 1, as described by the following Table, is found to be the welfare dominant member of the pair when the that ratio is high, whereas when the ratio is low it is better to follow Trajectory 2, which calls for a shorter period of investment in building stocks of K^D and an earlier switch (in the third phase rather than the fourth) to gross capital formation embodying the carbon-free technologies.

In Table 3 (below), the header lines contain the labels for the sub-phases in each trajectory.[57] The entries below each phase label belonging to a trajectory contain a

Table 3. Alternative trajectories involving the buy time (BTM) technology

			Trajectory 1		
Phase	**BAU1** → Business as usual	**BTM11** → Buying time	**BTM21** → Buying time	**BTM31** → Buying time	**CFR1** Carbon-free
Investment in	Carbon-based capital	Reduced emission capital	Reduced emission capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	Reduced emission capital only	Carbon-free capital and reduced emission capital	Carbon-free capital only
			Trajectory 2		
Phase	**BAU2** → Business as usual	**BTM12** → Buying time	**BTM22** → Buying time	**BTM32** → Buying time	**CFR2** Carbon-free
Investment in	Carbon-based capital	Low emissions capital	Carbon-free capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	All three types of capital	Carbon-free capital and reduced emission capital only	Carbon-free capital only

[56] That the model itself directs attention in this way to the relevance of empirical information about certain parameters is worth notice, because it may help in prioritizing areas warranting more concrete and detailed empirical research in engineering and applied economics.

[57] Typically, the j-th BTM sub-phase of trajectory k is labeled BTMj,k.

shorthand description of the activities taking place during or at the beginning of the sub-phases. Notice that the main difference between both trajectories is the time at which the carbon-using A-technology is de-activated. As a consequence, output during sub-phase BTM22 of trajectory 2 comes from three different sources, so the equation describing the accumulation of BTM capital is more complicated than that for trajectory 1.[58]

By solving the stacked optimum control problems associated with the model versions outlined above, we arrive at a complete and intertemporally consistent description of the nature and timing of the various phases, as well as the shape of the time-paths of tangible and intangible capital investment, levels of production and consumption, as well as cumulative emissions as a function of the structural parameters of the model. These parameters include the probable location of the cumulative emissions threshold, the productivity of the R&D process as well as the productivity differences between carbon-based and carbon-free technologies, and the 'standard' preference parameters for the consumption component of the social utility function, i.e. the rate of time discount and the intertemporal elasticity of substitution.

5. Preliminary results, experiments with physical system interactions

To this point in the present research program, the several models described in the foregoing pages have not been consistently calibrated on data for the global economy. Nor have we completed the the integration of the BTM model with the endogenous R&D model.[59] The latter remains a matter of particular interest because being able to extend the length of time during which R&D expenditures are maintained is (as far as we have been able to see) is likely to be the most productive use of the added time "bought" by CO_2 emission-rate reductions effected by exercising the BTM option. Using known techniques to upgrading carbon-using infrastructures and directly productive capital facilities certainly would be a socially more productive purpose than hastening the inevitable descent into climate instability just for the sake of prolonging current enjoyment of the higher consumption levels associated with "business as usual."

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[58] Although this makes the entire model considerably more difficult to solve, it was solvable for *Mathematica.*

[59] The Basic Model, and the Basic + R&D Model have been so calibrated and the results of the optimal solutions obtained are discussed and compared in van Zon and David (2013). Solutions have been obtained for a revised specification for the Basic + BTM Model discussed here (see David and van Zon (2013, which the working paper refers to as the "Greening upgrade" model). The structure of the latter is t comparable to that of the other two models in the suite, but it has not been comparably re-calibrated. Once that has been done and solutions have been obtained, it would be appropriate to undertake solution of an integrated model by adding "Greening upgrade" to the Basic + R&D Model, and try to find a solution for the multistage optimal transition path.

Comparisons of the results obtained with the different partial models, however, yields a number of useful insights about the role of directed R&D investment in the transition to climate stabilization. One striking and intuitively understandable finding that emerges from the optimized solution of the endogenous R&D model is highlighted by its juxtaposition with the features of the transition path found for the basic model. In the latter case, business-as-usual investment in carbon-using capital (K^A) comes to a halt when carbon-free technology can be embodied in production facilities, regardless of their greater unit capital cost; whereas in the former model the BAU phase ends only when R&D succeeds in rendering the unit costs of production facilities embodying CFR techniques competitive with that of K^A.[60]

Allowing for the possibility of investment in R&D directed towards lowering the unit capital costs of carbon-free production processes has the effect of raising the tangible investment in carbon-using production facilities, shortening the absolute and relative duration of the BAU phase -- leaving the length of the subsequent joint production phase essentially the same. In other words, being able to invest in the research required for a "technical fix" has the expected result of speeding the beginning of CFR production, the cessation of investment in carbon-using production facilities, and correspondingly, the retirement of the old carbon-based capital stock.

The underlying economic logic here is that the anticipation of being able to build more productive CFR capacity in the future generates a heightened *derived* demand for BAU capacity, and hence a higher rate of tangible capital formation in carbon-based facilities -- since the desired stock of CFR capital is a produced means of production. The resulting higher volume of K^A generates a larger output flow, providing a greater pool of resources that can be spent on R&D investment during the BAU phase, and subsequently for CFR-embodying capital formation in the JPR phase. It should be noted, however, that this would be a dangerous plan to have pursued were the technical improvements in carbon-free production systems that had been expected to result from R&D expenditure to fall short of expectations. In such circumstances – hardly unrealistic in view of the uncertainties surrounding the performance of R&D -- it would be necessary to eventually make the required switch to much more costly CFR production capacity, attended by consequently greater losses of consumption and welfare.

[60] Strictly speaking, the last part of this statement is not quite true in the model on which this discussion is based, because the upper limit on the capital productivity of the CFR technology, B-bar is set below the that for the carbon-using technology. R&D removes as much as possible of the productivity deficit of K^B relative to K^A before implementing the CFR technology tangible capital. B. But this is in no way essential, and there are no a priori reasons for not entertaining the possibility that CRF technologies can (eventually) have lower unit capital costs than carbon-based technologies, so that if the exponent and constant in the productivity change-R&D input function are quite small, the model is the model is not inconsistent with the observation that the world we live in is (still) a dependent upon a carbon-using regime of production.

This suggests the practical relevance of combining the endogenous R&D model and the BTM model, because, by adding the " buy time option" of reducing the CO_2 emissions rates on the carbon-using production capacity, there would be more time left to build-up the necessary carbon-free capital stock. Since that capital formation process would weigh all the more heavily of consumption levels, exercising the "buy time option" can serve as a partial insurance against the adverse welfare consequences of disappointed expectations regarding the effectiveness of R&D investment in enhancing the productivity production facilities embodying carbon-free technologies. Although the two options often seem to be discussed as alternative, substitute "climate policies" they are more properly seen to be complements in an expanded "technology fix" portfolio.

It is a fairly straightforward matter to conduct sensitivity experiments with these partial models, in order to get a qualitative impression of the impact of parameter variations that, given the deterministic form the model can begin to suggest the range of distributions in the dynamics the system exhibit were the model component reformulated more realistically in stochastic terms. Moreover, such sensitivity experiments also shed light on the way in which the requirements for a successful climate-stabilizing transition will be affected by -- and therefore need to make *ex ante* allowance for -- alterations in the perceived and actual dynamic feedbacks arising from interactions between the economic and the geo-physical subsystems. [61]

Since at this stage of our work the results of the solution of the model considering both the buy-time option and R&D directed to innovations in carbon free production technology are still being analysed, we report the results of using sensitivity experiments with the "external" physical specifications of our partial economic models as a preliminary means of exploring the impacts of their interactions. Among the variety of computational experiments that can be readily executed, the following pair has yielded results that are of principal interest: (i) lowering the cumulative GHG emissions tipping-point, and (ii) increasing the annual rate of physical depreciation of carbon-based production capacity.

The first of these offers a simple way to assess the gross effects of making precautionary allowances for the ambiguity surrounding the exact level of the atmospheric GHG concentration level that will be a "tipping point" into the domain of irreversible climate instability.[62] The robust result we find for all variants of lowering

[61] For example, a more cautionary stance towards the dangers of surpassing the cumulative emissions threshold may involve a lowering of the 'model'- threshold below its 'real world' expected level.

[62] The implications of allowing for the ambiguity in the location of the "tipping point" beyond which warming becomes a self-reinforcing process even where GHG emissions of immediate anthropogenic orgins have ceased completely will need to be assessed within the context of a fully integrated stochastic version of our model. Lemoine and Traeger (2011) introduce probabilistic functions for crossing a climate regime tipping point into a recursive formulation of the DICE integrated assessment model for establishing optimal climate policies. The tipping point's probability and timing thus become endogenously determined by the chosen emission policy. Recognizing that the probability distribution for the "temperature Threshold" corresponding to the "tipping point" is not known with confidence, policy

the critical threshold for temperature gain (or equivalently, the cumulative emissions tipping point) is that the required length of the transition to a stabilized climate would be shortened by abridgement of the BAU phase, leaving the duration of the joint production phase essentially unaltered. Initially, output is reduced but consumption is raised, so that investment in tangible capital formation is postponed; although consumption subsequently will be reduced, there also is less investment in the new CFR technology, and, indeed and lower levels of activity across the board – compared to those found for the (basic) reference model when the temperature-gain threshold is set at a higher, less constraining level.

The general finding, then, is that with less scope for CO_2 emissions before the expected critical threshold will be reached, production with carbon-using capacity must be curtailed and the length of the transition abridged; with less time to build up carbon-free capital goods, the eventual growth of output and consumption has to be deferred until climate stabilization with a carbon-free production regime has been achieved. A further, consistent result applies in the specific cases of the endogenous R&D and BMT models: because a lower critical threshold leaves less time to do R&D or to "buy time". In in order to counter that effect to some extent, the distribution of R&D activity over time must be shifted to in favour of R&D now and against R&D later.

Thus, the planning messages to be taken from this is that setting a lower GHG ppmv target --in keeping with greater aversion to risk ambiguity and consequent adherence to the minimize regret strategy dictated by the "precautionary principle," calls for an initially high and rapidly ramped up the rate of R&D expenditures when sufficient research advances have made it attractive to devote greater resources to development and engineering implementations. Similarly, it warrants boosting at the outset the rate of capital formation in production facilities with upgraded output/emissions performance, so as to smooth the adverse impact on per capital consumption of having to quickly deploy low and zero carbon-burning production capacity. Later, once the build-up of carbon free production capacity is well under way both types of investment will have to decline, first relative to output and then absolutely,.

Our second set of experiments provide a simplified way of emulating the higher frequency of damages that can be expected to result from extreme weather events associated with changing climate(s), and to assess the impact on the optimized transition plan of anticipating the consequences of warming that already is "in the pipeline," due to the past history of GHG emissions. An increased rate of unscheduled capital losses (damages from increased climate instability) has the effect of postponing the arrival of the CFR phase, primarily because production using the carbon based

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decisions can be made that reflect "ambiguity aversion" by selecting strategies that reduce the downside variance around the expected "threshold temperature. Lemoine and Traeger's simulations show that under reasonable parameterization of the DICE model, allowance for tipping points in this way can raise the near-term social cost of carbon emissions by as much as 50%.

technology is negatively affected, leading to a lower flow of CO_2 emissions in the process. This is "the good news", but, unfortunately it is not the whole story.

Since it seems reasonable for purposes of this analysis to assume that these negative climate effects will impact old carbon-based facilities most severely, if not exclusively,[63] the rate of return on investment in carbon-using facilities will be lower than is the case in the absence of allowance for this feedback of climate destabilization (i.e., in the base model). The "bad news" is that this results in lower initial rates of carbon-using capital formation (gross investment in K^A and K^D) and higher rates of consumption in an extended BAU phase. Consequently the constraint placed on the growth of productive capacity during this early phase of the transition curtails the rate of R&D investment, and the global economy's subsequent ability to rapidly accumulate the necessary stock of capital embodying carbon-free technologies.

One implication of the foregoing findings would seem to be that expenditures aimed at averting "unscheduled losses" in production capacity due to climate instability-related damages, may be quite a "good initial investment". It should be noted that under the assumption of increased damages, the level of R&D activity also is affected negatively; but that in the context of a fully endogenous integrated model the climate-related capital losses will be driven by strong non-convexity in the effects of rising GHG concentration levels and consequently higher mean surface temperatures. They are thus likely to take a heavier toll on existing production capacity at dates farther in the future, rather than from the outset of the transition, and the insurance motivation will push the shadow-price of GHG farther upwards. Here the details of heat exchange dynamics between atmosphere, the upper oceans and the lower ocean will matter, and accordingly, climate science research progress specifically directed to reduce the present uncertainties regarding those questions, as well as the endogenous processes affecting the climate sensitivity, could significantly reshape the optimal course of policy implementations.

These sources of non-stationarity in the climate feedback from alterations in the level of CO_2 emissions can be emulated (albeit roughly) by a further simulation experiment in which the physical depreciation rate on carbon-using capital moves to the higher level only after the tangible investment in that type of capital has stopped. We may suppose that the anticipation of those future capital losses, as before, will exert a depressing effect (albeit less heavily) upon the expected net rate of return to investment in KA during the BAU phase. Consequently, although the induced near-term reduction in tangible (carbon-technology using) capital facilities, and the rise of consumption per

[63] The consideration underlying this assumption is the a large portion of the carbon-based infrastructure and directly productivity capital stock will be legacies from the BAU regime and the historically antecedent state, it will be located in temperate latitudes and built in fashions that will leave it more vulnerable to the effects of coastal and riverine flooding, and serve wind-damage from hurricanes and monsoons, than the recently constructed carbon-free facilities. The latter, ideally, will have been designed and sited with those risks in mind.

capita will occur during the BAU phase, that would not happen from the phase's outset. Instead, it would be concentrated in the period just before the carbon-using stock attains it maximum, when (under the assumed timing of the increased rate of unscheduled capital losses) the weather-created damages start to occur with greater frequency.[64] The overall effect of these alterations in the timing of intangible (R&D) investments upon on the terminal value of the productivity of capital embodying CFR technologies is bounded, however, because the BAU phase during which R&D is taking place will have been stretched out. That change compensates for the cumulative effect of a decrease in the flow rate of R&D inputs, and the so ameliorate the latter's negative impact upon the extent of productivity improvements in K^B.

A major message that emerges from the foregoing modelling exercise and parameterization exercises is one that might well have been anticipated at its outset. What we learn from heuristic model building of the present kind is how to think about the problem, and not necessary what to conclude about the best course of action in meeting the challenge it poses. Even the highly simplified dynamical system that we are studying has sufficient interconnected and mutually interdependent "moving parts" to tax one's unassisted intuitions as to the ways that variations in the parameters of the geophysical and economic subsystems will alter the optimized transition paths to a stabilized global climate.

Moreover, while the simplicity of the heuristic growth model makes more transparent the logic of changes in the directions of investment and production activity in their various forms, the sequenced phase structure of the transition is not so immediately obvious. Nor can the impacts of parametric variations on the optimal phases' relative durations be ascertained without undertaking explicitly quantitative analyses. The latter can serve to prioritize areas for empirical research, by identifying technical parameters (and in models with richer specifications of agents' behaviors, critical behavioural parameters)--the magnitudes of which are found to strongly impact the welfare properties and shape the resource allocation requirements in the early stages of the optimal transition path.

Building such models, and investigating the implications of more sophisticated, integrated systems of economic-climate interactions, should not be seen as a pursuit intended to produce a substitute for the exercise of intuitive judgements about matters of economic policy design. In the end, the latter will have to weigh many important practical considerations regarding human behaviors, culture and politics that will resist accurate capture in tractable quantitative models. Instead, we regard the exercise of

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[64] This difference in timing has the effect of releasing resources that allow for a faster build-up of carbon-free capital (by assumption, designed so as to be not susceptible to the elevated severity of the weather). In addition, due to the specification adopted for the R&D production function in our model, the release of those resources at a later point in the future, after considerable R&D expenditures have taken place, is less costly (in terms of CFR productivity gains foregone) than is the case when the volume of R&D is lowered from the outset.

experience-based intuitions and quantitative analyses to be complementary ingredients in the policy design process; and believe they will work more effectively and reliably when each is allowed to inform, sharpen and qualify the conclusions to which one is led by their joint employment.

6. What will be learned from the next stages of this research program?

The idealized "optimal planning" framework for endogenous macroeconomic growth has been found to be well suited to incorporating representations of the technical aspects of the array of existing and potential technological options and their respective resource requirements, as well as those required to operationalize a welfare-optimal transition path. Application of multi-phase optimal control analysis provides DIRAM solutions that describe the optimal flows of tangible and intangible capital formation, along with the production flows using carbon-based or alternative technologies in each of the phases, as well as the sequencing and respective durations of the latter which completely describe the completed transition path.

This very concrete way of setting out what has to be accomplished technologically in each of the phases, in our view, provides useful starting point for thinking about how to design and coordinate the multiplicity of diverse tasks that will need to be undertaken in order to adequately respond to the daunting existential challenges posed by global warming and climate instability. Beyond those already noticed in the preceding pages, other, still more intricate and computationally demanding modeling and analyses of temporally extended multi-phase transition paths will be necessary to shed fuller light on the complexities entailed in working out the proper dynamic sequencing for the integrated development and exploitation of the variety of complementary and competing technology policy options. Among the myriad "additional options", the following handful deserve priority positions on the agenda for continued future DIRAM research – by virtue of their varied functional attributes:

(i) Expanded investments in both engineering research and physical equipment required to greatly extend and integrate existing electric power grids, upgrading these to create interoperable "smart grid" platforms that will combine information-intensive load-smoothing pricing mechanisms with thus use of energy storage techniques (e.g., batteries, flywheels, pumped water reservoirs) to raise the utilization rates of intermittent renewable sources of electricity generation, thereby lowering the latter's unit fixed costs and raising both the private and social rate of return to that form of capital formation;[65]

[65] Tabors, Parker and Caramanis (2013) point to the importance (in addition to the attention focused on technical engineering aspects of smart grid development), of defining and implementing the use of standard metrics for intermittency as a quality-dimension of electricity supply, as well as a "platform of platforms" approach to interoperability and efficient market performance in the pricing of diverse geographically distributed sources of electric power.

(ii) R&D expenditures on risky exploratory research programs having longer time horizons than the norm for applied projects because they seek low-frequency "break-through" discoveries and inventions to enable commercial exploitation of little-used alternative renewable energy sources (e.g., thermal pumps), and energy storage technologies by would drastically lowering their unit capital costs.

(iii) Long-term resource support for experimental geo-engineering projects that work in parallel on the development and field-testing of safely scalable "back-stop" technologies for atmospheric carbon capture and sequestration (ACCS), and locally deployable solar radiation management (SRM) techniques;[66]

(iv) Capital formation for reforestation with fast-growing leafy trees in order to efficiently raise the natural capacity CO_2 abatement capacity of the Earth's present forest cover as far as possible. Although it is highly unlikely that this measure could compensate for the degraded abatement capacity of the oceans that would result from continuing warming, pursuing this option should be seen not as "a fix" but as a "buy time" strategy in its effects on the net volume of CO_2 that is added to the atmosphere by burning carbon. Policy measures aimed at slowing or actually halting the clear-cutting of forests would work in the same direction, and might also involve compensatory capital formation to raise agricultural yields and the livestock carrying capacity of already cleared lands, thereby tending to reduce the economic pressures that are driving deforestation due to human agency.

(v) "defensive" expenditures for engineering design research and capital formation to implement physical reinforcements and additions to existing infrastructure that would reduce the latter's own vulnerability to damage caused by extreme weather events and extensive flooding. This would serve to mitigate direct and indirect losses of productive capacity as well as providing a measure of protection from loss of lives and livelihoods in some forms of natural disasters that are likely to become more destructive due to regional climate changes.

Quite obviously, it will be no mean task for future research to develop informative heuristic representations of the foregoing dynamic processes, and to explore the way(s) to sequence the exercise of the enlarged set of policy options in an integrated

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[66] Under the assumptions of our deterministic optimal control models of the climate-stablizing transition path, those techniques never would need to deployed in a "back-stop" role, even were to some among them to be economically as well as technically feasible and environmentally safe to implement. Yet, in the latter circumstances the investment in finding those method could nevertheless have a substantial social pay-off, especially when the transition to a low carbon global production regime had to be completed in reasonably short order. In that case it is most likely that a considerable stock of operational carbon-based capital would have to be de-activated before the start of the sustainable economic growth phase. By deploying atmospheric carbon capture (ACC) techniques that were (*ex hypothesis*) technically effective and economically practical, the concentration level of CO_2 could be gradually lowered far enough to allowing the "moth-balled" carbon-based production facilities to be brought back into production, thereby yielding a finite flow of social quasi-rents. A fast transition also would imply that the latter's present value would not have been so severely reduced by discounting. Of course, in a stochastic control setting, "back-stop" geo-engineering investment would have positive insurance value even were the research results to turn out never to be needed, or deemed too risky in their potential environmental side-effects to be deployed only to capture the private quasi-rents.

multiphase model.[67] But that task will be rendered more feasible by tackling it within the simplifying framework of a "social planning model," as the latter setting dispenses with a large number of complicated assumptions -- concerning the market behaviors of private economic agents, and the ability of political authorities to coordinate and implement coherent regulatory measures and institutional enforcement mechanisms on a global scale -- that otherwise would have to be specified and empirically justified.

The present paper focuses on understanding the deterministic system of the modelling framework we have constructed, and has indicated the directions in which we can proceed to complete the integration of the partial models within a complete endogenous economic and geo-physical system. In the next major stage of this research program it will be important to begin by replacing the simplistic representation of the carbon cycle lags in the effects of current CO2 on the atmospheric concentration level of that GHG, and to explicitly specify the resulting gain in radiative forcing and the resulting rise in the mean global surface temperature. It will then be possible to further extend the Basic Model by introducing a continuous endogenous "damage process" – which impacts welfare indirectly, through the losses of productive capacity caused by the positive temperature-dependent effects of the frequency/severity of "extreme weather" events that disable a fraction of productive capacity. Using the resulting "Basic" platform, the solution of a calibrated model that integrates the partial models for the two "tech fix" options discussed here, while allowing computation of the way that endogenous R&D (directed to raise productivity of carbon free technologies) and the "Buy Time" re-engineering of existing and incremental additions to carbon-using production facilities jointly affect the duration of the optimized transition's phases, and the welfare indices associated with the entire path. From there it will be straightforward to go on to examine the robustness of those findings to variations in parameter specifications. Of particular interest, indeed concern, will be the alterations in the phase structure and allocation of resources varying that result from varying the conjectured location of the climate-system's tipping point.

Further broadening of the array of 'tech fix' options that are modelled can consider the question of whether and when to begin concerted exploratory R&D on alternative geo-engineering approaches to creating effective and environmentally manageably "backstop" technologies –whether in the form of solar radiation management (SRM), or air capture of carbon and its sequestration (ACCS). In this case it will be appropriate to consider when, if ever, such research should be discontinued on the ground that the deployment of carbon free technologies had progressed far enough

[67] The natural starting point for the incremental expansion of the portfolio of tangible and intangible investments is to add the option of multiple vintages "of buy time" retrofitting of carbon-based production capacity to the calibrated 3-phase growth model of an optimal transition to a stabilized global climate, the solution of which is presented in van Zon and David (2013). The latter model features directed R&D in renewable (carbon-free) technologies that must be embodied in new physical plant, which is designed and situated to be far less vulnerable than the existing CO2-based capital stock to unscheduled output losses arising from severe weather damage driven rising atmospheric CO2 concentration levels.

to make its timely completion feasible at a welfare sacrifice that would be no greater than that of continued research on an untried approach to stabilizing the climate would call for field experimentation at scales that carried hard-to-assess risks of causing serious environmental disruption.

To tackle the latter issue properly, however, would call for abandoning the deterministic framework in which our DIRAM research program has been developed. There is a good bit to be said for shifting as soon to a stochastic control reformulation, retaining the "planning model" approach in order to minimize reliance on assumptions about the behaviours private market actors under uncertainly. This shift would entail specifying the probability distributions governing the "climate sensitivity" of the physical system,[68] as well as explicitly acknowledging the stochastic nature of the output of R&D expenditure inputs, and the realized payoffs of technological advances permitting higher productivity in the available stock of capital embodying carbon-free technologies.

Simple intuitions about the altered qualitative insights that would emerge from these and other reformulations of the DIRAM framework suggest that the greater importance of "buying insurance" against future down-side risks would force greater early sacrifices of consumption in order raise (risky) investment R&D on carbon-free technologies and concurrently raising the volume and pace of "buy time" retrofitting expenditures that lowered the ratio of CO2 emissions per unit of output produced with the existing capital stock. But the point of the exercise is to gain insights regarding the quantitative alterations that allowance for uncertainties would require in the magnitudes of resource commitments and durations of the optimal transition phases.

Once the DIRAM approach is carried into the domain of stochastic optimization it would be feasible to use these models to explore questions about the many ways in which an optimal multi-stage plan may fail to be realized, and what might be done –with attendant welfare costs –to ameliorate the resulting damage.

In the nearer-term future, however, there still are many interesting things to learn, and questions to answer by exploring the equivalent deterministic version of our model. For different parameter constellations we are able to show how the timing of the phases will change, and how the evolution over time of welfare per capita will be affected, but also how the required R&D expenditures and the volume of gross capital formation will change, both in terms of their distribution and intensity over time. Further, we are able to use parameter sensitivity experiments to compute how

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[68] The nature of the distribution of the feedback parameter describing the physical system's response (in terms of global mean surface temperature to a doubling of the atmospheric GHG concentration level is surrounded by very substantial uncertainties arising from the complexities of the interactions among the physical and chemical processes that underlie the feedback sequences discussed in section 2.1 (above). In the present context there is an important issue as to whether it is reasonable to assume that the effects of changes in the mean atmospheric GHG concentration on the distribution around the mean "climate sensitivity" are variance-preserving.

uncertainty regarding the position of the climate tipping point, taken in combination with different degrees of relative risk averse "precautionary behaviour" – on the part of the representative consumer (or her social planning agent) affects the optimum timing and intensities of the various activities distinguished in the model.

The flip side of the latter computational analysis is our ability to explore the welfare damage of "policy implementation failures". Starting from the optimal program for the transition to a viable and stable GHG concentration level, it is feasible to show the effects of time inconsistencies in the implementation of multi-period private sector plans, and of budgetary or political constraints that result in specifically timed postponements of public investment in R&D and energy infrastructure programs. Alternatively, this approach could assess the impacts of uncompensated private cuts in the upgrading of carbon-based production facilities, or in the roll-out of the carbon-free technology when, under normal business conditions it would be advantageous to undertake installation of the required capital investments.

How much the timing of action matters in this system, and the penalties for deviation from the optimal path can be investigated under the assumption that no effort is made to re-optimize from the position that has resulted in the aftermath of such "shocks", and also under the assumption of trying to "catch up" after a particular range of delays by means of re-optimizing the transition path. Thus, it is possible within this heuristic modelling framework to computationally describe the relationship between the lengths of implementation delays during specific phases of a successful transition to climate stability, and the consequent costs in terms of net-welfare foregone.

7. Centering "Tech Fix" in the climate policy mix: a beginning, not a conclusion:

This initial stage in our projected explorations of the "requirements" for a socially optimal transition to sustainable global economic growth in a viable stabilized climate has been motivated primarily by the conviction that it is vitally important to give specific "technology fix" options a central place in structured analysis of policies that can respond effectively to the challenge of global warming. Environmental and energy economists have been quick to emphasize the allocational efficiency of various fiscal and regulatory means of raising the market prices of carbon fuels, and some have suggested that the efficacy of that policy approach would be enhanced by their effects in inducing "carbon-saving" technical innovation from private sector.

As has been pointed out, however, the latter proposition presupposes that raising the cost of fossil fuel sources of energy would not generate negative income effects that dampened private R&D expenditures by energy-intensive sectors of the economy. Furthermore, the integrated assessment models that economists have developed as quantitative tools to guide policies aiming to raise the price of carbon typically fail to show the conditions under which they could be expected to elicit a sufficient flow of innovations that were directed specifically to curtailing and eventually displacing fossil fuels as a dominant global source of energy, or, indeed, to have the desired effect of mitigating emissions of CO2. the first orddesire. In

To address this "policy assessment gap" would entail giving greater attention to specifying the relevant characteristics of the array of energy-using technologies, and the corresponding "endogenous technological progress" sub-system, and to connect them with the other components of the aggregate economic growth models that will govern the pace and extent of deployment of improved production techniques.

The dynamic integrated requirements analysis modeling (DIRAM) approach that has been introduced here may be viewed as a preliminary step towards a more illuminating representation within the context of familiar IAMs of the potential role of technological measures in the transition from the present dominance of carbon-based energy to a low carbon global regime of production. It surely will occur to some readers to question according priority to elaborating the technological specifics of existing IAMs, on the ground that many other features of these highly stylized models also warrant further elaboration. Moreover, greater policy relevance calls for a spatially disaggregated approach that would take account of geographical and ecological variations affecting both the global economy and the climate system. Indeed, there already have been many efforts along just such lines, impelled by a policy interest in assessing the differential regional and national incidence of the costs of curtailing global GHG emissions.[69] Others might argue for the importance of recognizing the endogeneity of changes in the size, age structure and spatial distribution of global population, and more sophisticated welfare framework to account for demographic as well as economic changes in the transition to a viable stabilized climate.

Even were the latter qualification to be waved away, there should still be room to consider the likelihood that arriving at domestic and international agreements to impose future taxes on fossil fuels would be politically less arduous where there good prospects of the availability of affordable technological innovations that could significantly reduce the adverse impacts of such commitments on future real profit rates and consumption levels. Just such expectations could be created by prior commitment on the part of the already developed and scientifically and technically advance countries to major coordinated technological programs -- such as those aimed at significantly increasing

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[69] See, Brock, Engstrom and Xepapadaes (2012), for an integrated global model in which the spatial variations of mean temperature (in the northern hemisphere) are endogenously generated along with the evolution of the corresponding regional economies. The RICE model (Nordhaus 2007, 2010) and other previous contributions to the literature provided spatial distributions of estimated *damages* due to rising MGT for zones around the equator, showing the regional incidence of welfare losses along the dynamic path of the global economy that imposed a global carbon tax that reflected only the aggregate magnitude of those damages. .

the efficiency of energy distribution and use, and at lowering the real unit costs of non-fossil fuel sources of energy.

Rising prices for carbon-based energy would encourage any form cost-saving innovation activities and, by that token, it would raise the perceived marginal social payoff from expanding public support for science and technology research – if only to create a knowledge infrastructure that would lower the costs of directed innovation in the affected sectors and lines of business. Nevertheless, the augmented public R&D funding would have to be forthcoming, and additional, differential subsidy measures would need to be introduced to direct private R&D (and subsequent deploy the results) toward lowering the unit costs of carbon-based and alternative energy sources.

Alternatively, a well thought out supply-side climate policy that started with the goal of expanding a variety of applications-oriented R&D programs would tend to create credible expectations of substantial future resource savings with carbon-free production facilities, and a concomitantly smaller sacrifice of material welfare entailed in restricting GHG emissions. That could contribute to weakening economically motivated resistance to a schedule of gradually rising carbon taxes and therefore impart wider and stronger commitments to national and international agreements on coordinated and verifiable actions that would curtail GHG emissions. Bundling proposed commitments from lower income developing economies with reciprocal loan subsidies and cost concessions in the transfer to new “greener” and carbon-free technologies would constitute a credible package for negotiations that would grow in its attractiveness as the R&D programs matured.

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