Measuring Systemic Risks in Insurance - Reinsurance Networks

- Stanford University 2012 -

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November, 2012

Outline

1. Modeling Focus, Framework and Question of Interests
   - Focus of the Talk: Insurance Risk
   - The Model: A High-level Overview
   - Insurance: Basics
   - Modeling Framework
   - Questions of Interest
2. Counter-party Risk and Settlement Mechanism
   - A Stylized Contractual Model
   - Default Settlement Mechanism
3. Qualitative Risk Analysis
   - A Tractable Stylized Risk Factor Model
   - A First Qualitative Analysis
   - The Role of Risk Mitigators
4. Enhancing Qualitative Analysis with Efficient Simulation Tools
5. Examples and Conclusions

The Impact of Catastrophic Events

- **Atlantic City, NJ. About 10 days ago...**

Focus of the Model

- Stochastic Risk Networks:

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  1) Multidimensional stochastic processes describing evolution of risk exposures

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  2) Features: counterparty risk, default contagion effects, factor dependence, interconnections, combinatorial nature, etc.

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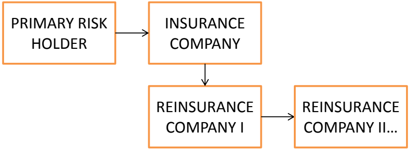
  1) Multidimensional stochastic processes describing evolution of risk exposures *subject to resource (capital) constraints*

  2) Features: counterparty risk, default contagion effects, factor dependence, interconnections, combinatorial nature, etc.

  3) Features $\longrightarrow$ Challenges

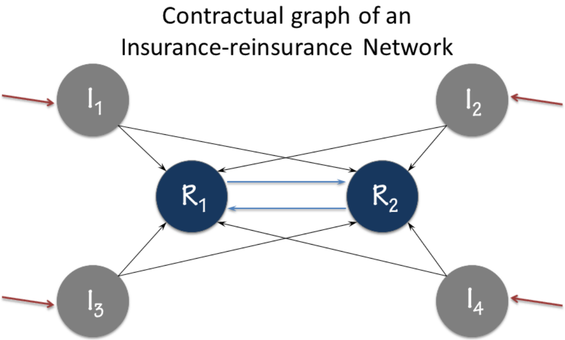
Insurance: Basics

- **Insurance flow...**



Insurance-Reinsurance Risk Networks: Framework

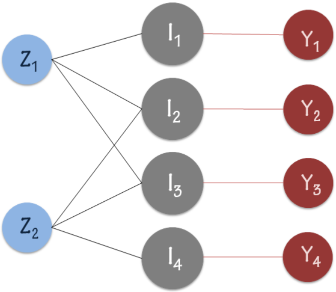
- **Contractual diagram:**



Insurance-Reinsurance Risk Networks: Framework (Con'd)

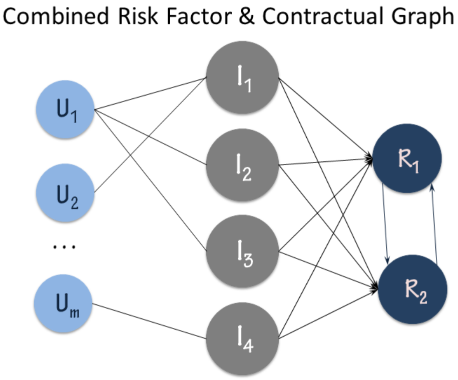
- **Risk (Factor) Exposure Map:**

Risk Factor Influence Graph



Insurance-Reinsurance Risk Networks: Framework (Con'd)

- **Combined Factor Exposure & Contractual Directed Graph**



Insurance-Reinsurance Risk Analysis: Questions of Interest

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3) Role of risk mitigators:
   - *How does the presence of reinsurers alter the systemic landscape of the system?*
   - *How to enhance the role of reinsurers?*

Insurance-Reinsurance Risk Framework: Goal

Goal

Design models that allow to study these questions systematically both in qualitative and quantitative ways...

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A Stylized Contractual Model

- **Insurance-reinsurance network:**

A Stylized Contractual Model

- **Insurance-reinsurance network:**

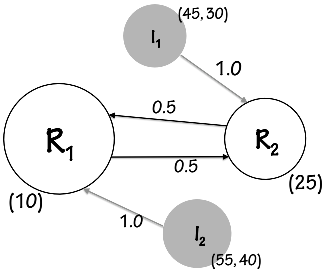
  1) Insurance companies (set $\mathcal{I}$) & reinsurance companies (set $\mathcal{R}$), $\mathcal{R} \cap \mathcal{I} = \varnothing$.

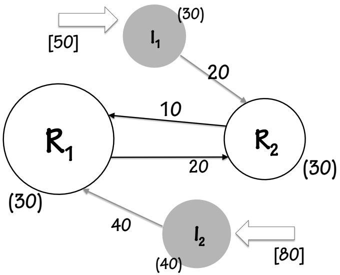
  2) Each insurer I_i, $i \in \mathcal{I}$ enters reinsurance with multiple reinsurers with some given proportions.

  3) Stop-loss reinsurance contracts and proportional retrocession.

  4) Spill-over proportions at the time of default denoted by $\rho_{r,i}$, $\tilde{\rho}_{r,r'}$.

Need for Well-defined Default Contagion Mechanism





Initial configurations:

- (45, 30): (initial reserve, reinsurance deductible)
- (10), (25): initial reserves
- 0.5, 1.0: risk transfer ratio

Before settlements:

- [50], [80]: claim sizes
- (30), (40): effective claim sizes
- 10, 20, 40: transferred amount

Default Mechanism: An Equilibrium Formulation

Formulation

$$
\begin{aligned}
\min \sum_{i\in\mathcal{I}^+} \pi_i^- &+ \sum_{r\in\mathcal{R}^+} \psi_r^- \\
s.t. \pi_i^+ - \pi_i^- &= e_i + C_i - L_i - \sum_{r\in\mathcal{R}^+} \psi_r^- \cdot \rho_{r,i},\ \forall i \in \mathcal{I}^+ \\
\psi_r^+ - \psi_r^- &= e_r + C_r - L_r - \sum_{r'\in\mathcal{R}^+, r'\neq r} \left(\psi_{r'}^- \cdot \tilde{\rho}_{r',r} - \kappa \psi_r^- \cdot \tilde{\rho}_{r,r'}\right),\ \forall r \in \mathcal{R}^+ \\
\pi_i^+, \pi_i^-, \psi_r^+, \psi_r^- &\geq 0.
\end{aligned}
$$

Theorem (Blanchet & Shi (2012b))

The LP has a unique optimal solution. The solution is independent of the objective function as long as this one is strictly increasing in π_i^- and ψ_r^-. Moreover, in equilibrium π_i^-, ψ_r^- are losses and π_i^+, ψ_r^+ surpluses (i.e. you can't have both $\pi_i^+ > 0$ and $\pi_i^- > 0$).

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Our formulation:

- Netting of default
- Solution can be obtained from LOCAL interactions (no need for central planner).

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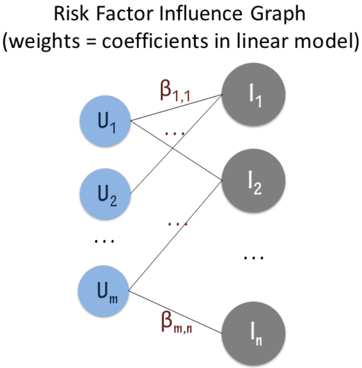
Dependence Structure in Risk Factor Influence Graph

- **Claims:** *Linear model* of m independent *power-law-tail* risk factors

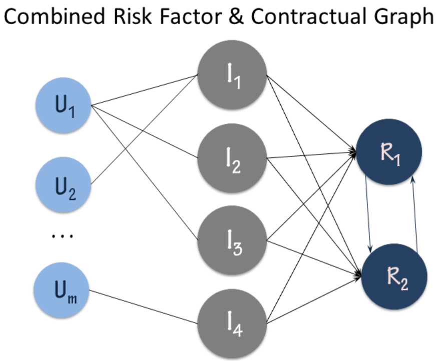
$$P\left(U_j > b\right) \approx b^{-\alpha_j}, \quad j = 1, \ldots, m,$$

as $b \longrightarrow \infty$.



Summary of Dependence: How Risk Factors Influence Companies?

- Write $G_{i,j} = 1$ if there is directed path from j-th factor to company i, otherwise $G_{i,j} = 0$ (e.g. $G_{R_1,m} = 1$, $G_{I_3,m} = 0$)



Some Qualitative Analysis

Theorem (Blanchet & Shi (2012b))

Let $\mathcal{A}$ be a given set of companies, assume that $e_i, e_r = \Theta(b)$ as $b \to \infty$.

$$P(\text{ruin of set } \mathcal{A}) \approx b^{-\lambda(\mathcal{A})},$$

where λ is the optimal solution to the following (multi-dimensional) Knapsack problem

$$\lambda(\mathcal{A}) = \min \sum_{j=1}^{m} \alpha_j x_j$$

$$\text{s.t.} \sum_{j=1}^{m} G_{i,j} x_j \geq 1, \ \forall i \in \mathcal{A}$$

$$x_j \in \{0, 1\}.$$

Jump to numerical results...

Qualitative Risk Analysis: Some Observations

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  5. What is the role of reinsurance?
  6. Quantitatively coarse.

Enhancing the Role of Reinsurance Companies

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Formulation

$$\lambda(\rho,\mathcal{A}) = \min \sum_{j=1}^{m} \rho\alpha_j x_j + \sum_{j=1}^{m} \alpha_j y_j$$

$$\text{s.t.} \quad \sum_{j=1}^{m} G_{l,j} x_j \geq 1, \ \forall l \in \mathcal{R}(\mathcal{A}) \qquad [\textit{Default of counterparties of } \mathcal{A}]$$

$$\sum_{j=1}^{m} G_{i,j}(x_j + y_j) \geq 1, \ \forall i \in \mathcal{A} \qquad [\textit{Default of } \mathcal{A}]$$

$$x_j, y_j \in \{0,1\}.$$

Guidance for Regulatory Policies

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- What is the minimum extra capital needed to inject/hold for a given set of $k \geq 1$ companies?

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How to Improve Upon Asymptotics via Efficient Monte Carlo

- Let $\mathcal{A}$ be a set of companies & $e_i, e_r = \Theta(b)$, want to compute

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Goal

Design an *efficient* Monte Carlo estimator $Z_{\mathcal{A}}(b)$ such that

$$\frac{E[Z_{\mathcal{A}}^2(b)]}{P(\text{Ruin of set } \mathcal{A})^2} = O(1)$$

as $b \to \infty$.

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- In addition, we want cost-per-replication of $Z_{\mathcal{A}}(b)$ to be $O(1)$ as $b \to \infty$.
- Want conditional expectations given ruin of $\mathcal{A}$, also with complexity $O(1)$.

Constructing an Efficient Monte Carlo Estimator

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- Also $\delta > 0$ such that for all $b > 0$

$$\delta P\left(C_b\right) \leq P(\text{Ruin of set } \mathcal{A} \text{ by time } M)$$

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- Using the structure of the linear program one can explicitly **find** $\gamma > 0$ such that

$$\{\text{Ruin of set } A\} \subset \cup_{(x_1,...,x_m)} \{\cap_{i \in A} \cup_{j: G_{i,j} x_j \geq 1} \{U_j > \gamma b\}\},$$

where $(x_1, ..., x_m)$ is feasible for the Knapsack problem.

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$$P\left(\cap_{i\in A} \cup_{j:G_{i,j}x_j\geq 1} \{U_j > \gamma b\}\right)$$

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- *Output estimator*

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Proposition

$$Z_{\mathcal{A}} \leq \frac{\max_{\mathbf{x}} P\left(\cap_{i \in \mathcal{A}} \cup_{j: G_{i,j} x_j \geq 1} \{U_j > \gamma b\}\right)}{\min_{\mathbf{x}} w(\mathbf{x})}.$$

In particular, the estimator is strongly efficient.

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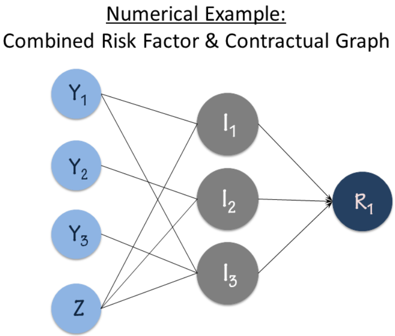
Numerical Examples

Scenario (power indices)

	1	2	3
α_1	4.9	3.9	2.1
α_2	5.2	2.2	2.8
α_3	6.3	3.3	2.3
α_Z	2.1	6.1	3.4
$\lambda(\rho, \mathcal{A})$	2.1	5.5	4.9



- Most likely cause of ruin of companies $\mathcal{A} = \{2, 3\}$

Back to Knapsack formulation...

Numerical Examples

- 10^6 Replications
- CSD=Cond. system loss given default of $\mathcal{A}$

Numerical results for 3 scenarios, $\mathcal{A} = \{2, 3\}$

Scenario # 1.	$b = 10^7$
$\hat{p}(s.e./\hat{p}(\%))$	1.03×10^{-8} (2.961%)
95% $C.I.$	$(0.97, 1.09) \times 10^{-8}$
$\widehat{CSD}$	1.857×10^7
Scenario # 2.	$b = 10^5$
$\hat{p}(s.e./\hat{p}(\%))$	9.78×10^{-11} (2.90%)
95% $C.I.$	$(0.92, 1.03) \times 10^{-10}$
$\widehat{CSD}$	1.092×10^5
Scenario # 3.	$b = 10^6$
$\hat{p}(s.e./\hat{p}(\%))$	6.64×10^{-11} (5.272%)
95% $C.I.$	$(5.96, 7.33) \times 10^{-11}$
$\widehat{CSD}$	8.337×10^5

Conclusions

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- Thank you!
- Draft version: Stochastic Risk Networks: Modeling, Analysis and Efficient Monte Carlo downloadable on SSRN