MATHEMATICS

Courses offered by the Department of Mathematics are listed under the subject code MATH on the Stanford Bulletin's ExploreCourses web site.

The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics, and also participates in the program leading to the B.S. in Mathematical and Computational Science, and the M.S. and Ph.D. degree programs offered through the Institute for Computational & Mathematical Engineering.

Mission of the Undergraduate Program in Mathematics

The mission of the undergraduate program in Mathematics is to provide students with a broad understanding of mathematics encompassing logical reasoning, generalization, abstraction, and formal proof. Courses in the program teach students to create, analyze, and interpret mathematical models and to communicate sound arguments based on mathematical reasoning and careful data analysis. The mathematics degree prepares students for careers in the corporate sector and government agencies, and for graduate programs in mathematics.

Learning Outcomes (Undergraduate)

The department expects undergraduate majors in the program to be able to demonstrate the following learning outcomes. These learning outcomes are used in evaluating students and the department's undergraduate program. Students are expected to demonstrate:

1. problem solving skills,
2. the ability to formulate proofs and to structure mathematical arguments,
3. the ability to communicate mathematical ideas via extended written presentation.

Advanced Placement in Mathematics for Freshmen

Students with strong ability in mathematics often take one or more semesters of college-equivalent courses in mathematics while they are still in high school. Under certain circumstances, it is possible for such students to secure both advanced placement and credit toward the bachelor's degree. A decision as to placement and credit is made by the department after consideration of the student's performance on the Advanced Placement Examination in Mathematics (forms AB or BC) of the College Entrance Examination Board, and also after consideration of transfer credit in mathematics from other colleges and universities.

The department does not give its own advanced placement examination. Students can receive either 5 or 10 units of advanced placement credit, depending on their scores on the CEEB Advanced Placement Examination or the IB Exam. Entering students who have credit for two quarters of single variable calculus (10 units) are encouraged to enroll in:

		Units
Select one of the following series:		
Standard Series		
MATH 51	Linear Algebra and Differential Calculus of Several Variables	5
MATH 52	Integral Calculus of Several Variables	5
MATH 53	Ordinary Differential Equations with Linear Algebra	5
Honors Series		
MATH 51H	Honors Multivariable Mathematics	5
MATH 52H	Honors Multivariable Mathematics	5
MATH 53H	Honors Multivariable Mathematics	5

These three-course sequences, which can be completed during the freshman year, supply the necessary mathematics background for most majors in science and engineering. They also serve as excellent background for the major or minor in Mathematics, or in Mathematical and Computational Science.

Students who have credit for one quarter of single variable calculus (5 units) should take:

		Units
MATH 42	Calculus (Autumn Quarter)	5
MATH 51	Linear Algebra and Differential Calculus of Several Variables (Winter Quarter)	5
Options available in Spring Quarter include:		
MATH 52	Integral Calculus of Several Variables	5
or MATH 53	Ordinary Differential Equations with Linear Algebra	

For proper placement, contact the Department of Mathematics.

Bachelor of Science in Mathematical and Computational Science

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the "Mathematical and Computational Science (http://exploredegrees.stanford.edu/schoolofhumanitiesandsciences/mathematicalandcomputationalscience)" section of this bulletin.

Introductory and Undergraduate Courses

The department offers two sequences of introductory courses in single variable calculus.

1. MATH 41and MATH 42 present single-variable calculus at a fast pace. Differential calculus is covered in the first quarter, integral calculus in the second.
2. MATH 19, MATH 20, and MATH 21 cover the material in MATH 41, MATH 42 in three quarters instead of two.

There are options for studying multivariable mathematics:

1. MATH 51 Linear Algebra and Differential Calculus of Several Variables, MATH 52 Integral Calculus of Several Variables, and MATH 53 Ordinary Differential Equations with Linear Algebra cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize applications. MATH 51 covers differential calculus in several variables and introduces matrix theory and linear algebra; MATH 52 covers integral calculus in several variables and vector analysis; MATH 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is strongly recommended for incoming freshmen with 10 units of advanced placement credit.
2. MATH 51H Honors Multivariable Mathematics, MATH 52H Honors Multivariable Mathematics, and MATH 53H Honors Multivariable Mathematics cover the same material as MATH 51, MATH 52, MATH 53, and much more with more emphasis on theory and rigor.

The department offers three classes on linear algebra:

		Units
MATH 51	Linear Algebra and Differential Calculus of Several Variables	5
or MATH 51H	Honors Multivariable Mathematics	
MATH 104	Applied Matrix Theory	3
MATH 113	Linear Algebra and Matrix Theory	3

Learning Outcomes (Graduate)

The Master's Degree is conferred upon candidates who have developed advanced knowledge and skills in Mathematics. This is achieved through completion of courses, in the primary field as well as related areas, and experience with independent work and specialization.

The Ph.D. is conferred upon candidates who have demonstrated substantial scholarship and the ability to conduct independent research and analysis in Mathematics. Through completion of advanced coursework and rigorous skills training, the doctoral program prepares students to make original contributions to the knowledge of Mathematics and to interpret and present the results of such research.

Bachelor of Science in Mathematics

The following department requirements are in addition to the University's basic requirements for the bachelor's degree:

Students wishing to major in Mathematics must satisfy the following requirements:

1. Department of Mathematics courses totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, STATS 116 Theory of Probability, PHIL 151 Metalogic, and PHIL 152 Computability and Logic count as Department of Mathematics courses.
2. Additional courses taken from Department of Mathematics courses numbered 101 and above or from approved courses in other disciplines with significant mathematical content, totaling at least 15 units credit. At least 9 of these units must be taken for a letter grade.
3. A Department of Mathematics adviser must be selected, and the courses selected under items '1' and '2' above must be approved by the department's director of undergraduate studies, acting under guidelines laid down by the department's Committee for Undergraduate Affairs. The Department of Mathematics adviser can be any member of the department's faculty.
4. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation.

Students are normally expected to complete either the sequence MATH 19 Calculus, MATH 20 Calculus, MATH 21 Calculus or the sequence MATH 41 Calculus, MATH 42 Calculus (but not both). Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may receive 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses numbered 51 and above. Students with an Advanced Placement score of at least 3 in BC math or at least 4 in AB math may receive 5 units credit and fulfill requirement '1' by taking at least 44 units of Department of Mathematics courses numbered 42 and above.

Sophomore seminar courses may be counted among the choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed. For example, students transferring from other universities may be allowed credit for some courses completed before their arrival at Stanford. However, at least 24 units of the 49 units under item '1' above and 9 of the units under item '2' above must be taken at Stanford. In all cases, approval for variations in the degree requirements must be obtained from the department's Committee for Undergraduate Affairs. Application for such approval should be made through the department's director of undergraduate studies. The policy of the Mathematics Department is that no courses other than the MATH 50 series and below may be double-counted toward any other University major or minor.

It is to be emphasized that the above regulations are minimum requirements for the major; students contemplating graduate work in mathematics are strongly encouraged to include the courses MATH 116 Complex Analysis, MATH 120 Groups and Rings, MATH 121 Galois Theory, MATH 147 Differential Topology or MATH 148 Algebraic Topology, and MATH 171 Fundamental Concepts of Analysis in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program, discussed below.

To help develop a sense of the type of course selection (under items '1' and '2' above) that would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses that could be taken in fulfillment of the Mathematics major requirements:

Example 1

A general program (a balanced program of both pure and applied components, without any particular emphasis on any one field of mathematics or applications) as follows:

		Units
Select one of the following series or Advanced Placement credit (see the Overview tab for details):		10
MATH 19 & MATH 20 & MATH 21	Calculus and Calculus and Calculus	
MATH 41 & MATH 42	Calculus and Calculus	
Math 50 Series:		15
MATH 51 & MATH 52 & MATH 53	Linear Algebra and Differential Calculus of Several Variables and Integral Calculus of Several Variables and Ordinary Differential Equations with Linear Algebra	
MATH 104 or MATH 113	Applied Matrix Theory Linear Algebra and Matrix Theory	3
MATH 106	Functions of a Complex Variable	3
MATH 109	Applied Group Theory	3
MATH 110	Applied Number Theory and Field Theory	3
MATH 115	Functions of a Real Variable	3
Plus any selection of at least eight of the following courses, including three Department of Mathematics courses:		24
MATH 108	Introduction to Combinatorics and Its Applications	
MATH 131P	Partial Differential Equations I	
MATH 132	Partial Differential Equations II	
MATH 143	Differential Geometry	
MATH 146	Analysis on Manifolds	
MATH 147	Differential Topology	
MATH 148	Algebraic Topology	
MATH 152	Elementary Theory of Numbers	
MATH 161	Set Theory	
CME 108	Introduction to Scientific Computing	
ECON 50	Economic Analysis I	
PHYSICS 41	Mechanics	
PHYSICS 43	Electricity and Magnetism	
PHYSICS 45	Light and Heat	
STATS 116	Theory of Probability	
Total Units		64

The courses from other departments are only meant as examples; there are many suitable courses in several departments that can be taken to fulfill part or all of requirement '2'.

Example 2

A theoretical program recommended for those contemplating possible later graduate work providing an introduction to the main areas of mathematics both broader and deeper than the general program outlined above:

		Units
Select one of the following series or Advanced Placement credit (see the Overview tab for details):		10
MATH 19 & MATH 20 & MATH 21	Calculus and Calculus and Calculus	
MATH 41 & MATH 42	Calculus and Calculus	
Math 50 Series:		15
MATH 51	Linear Algebra and Differential Calculus of Several Variables	
or MATH 51H	Honors Multivariable Mathematics	
MATH 52	Integral Calculus of Several Variables	
or MATH 52H	Honors Multivariable Mathematics	
MATH 53	Ordinary Differential Equations with Linear Algebra	
or MATH 53H	Honors Multivariable Mathematics	
In addition to the series, the following courses are recommended:		
MATH 106	Functions of a Complex Variable	3
or MATH 116	Complex Analysis	
MATH 113	Linear Algebra and Matrix Theory	3
MATH 120	Groups and Rings	3
MATH 171	Fundamental Concepts of Analysis	3
Plus nine or more 3-unit math courses numbered 121 or higher (the logic courses PHIL 151 and PHIL 152 are considered to be such courses), including at least one algebra course, one analysis course, and one geometry/topology course. (See the description of the honors program below)		27
Total Units		64

In addition, those contemplating eventual graduate work in Mathematics should consider including at least one graduate-level math course such as MATH 205A Real Analysis, MATH 210A Modern Algebra I, or MATH 215A Complex Analysis, Geometry, and Topology or MATH 215B Complex Analysis, Geometry, and Topology. Such students should also consider the possibility of entering the honors program.

Example 3

An applied mathematics* program:

		Units
Select one of the following series or Advanced Placement credit (see the Overview tab for details):		10
MATH 19 & MATH 20 & MATH 21	Calculus and Calculus and Calculus	
MATH 41 & MATH 42	Calculus and Calculus	
Math 50 Series:		15
MATH 51 & MATH 52 & MATH 53	Linear Algebra and Differential Calculus of Several Variables and Integral Calculus of Several Variables and Ordinary Differential Equations with Linear Algebra	
MATH 104	Applied Matrix Theory	3
MATH 106	Functions of a Complex Variable	3
MATH 108	Introduction to Combinatorics and Its Applications	3
MATH 109	Applied Group Theory	3
MATH 110	Applied Number Theory and Field Theory	3
MATH 115	Functions of a Real Variable	3
MATH 131P	Partial Differential Equations I	3
MATH 132	Partial Differential Equations II	3
STATS 116	Theory of Probability	3-5
Plus at least 12 units of additional courses in applied mathematics, including, for example, suitable courses from the departments of Physics, Computer Science, Economics, Engineering, and Statistics.		12
Total Units		64-66

* Students interested in applied mathematics, but desiring a broader-based program than the type of program suggested in Example 3, including significant computational and/or financial and/or statistical components, are encouraged to also consider the Mathematics and Computational Science program.

Honors Program

The honors program is intended for students who have strong theoretical interests and abilities in mathematics. The goal of the program is to give students a thorough introduction to the main branches of mathematics, especially analysis, algebra, and geometry. Through the honors thesis, students may be introduced to a current or recent research topic, although occasionally more classical projects are encouraged. The program provides an excellent background with which to enter a Master's or Ph.D. program in Mathematics. Students completing the program are awarded a B.S. in Mathematics with Honors.

It is recommended that the sequence MATH 51H Honors Multivariable Mathematics, MATH 52H Honors Multivariable Mathematics, and MATH 53H Honors Multivariable Mathematics be taken in the freshman year. To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

1. The selection of courses under items '1' and '2' above must contain:

		Units
MATH 106	Functions of a Complex Variable	3
or MATH 116	Complex Analysis	
MATH 120	Groups and Rings	3
MATH 171	Fundamental Concepts of Analysis	3

And must also include seven additional 3-unit Math courses numbered 121 or higher. (The logic courses PHIL 151 Metalogic and PHIL 152 Computability and Logic can also be used.) These seven courses must include at least:

		Units
One Algebra Course:		
MATH 121	Galois Theory	3
MATH 122	Modules and Group Representations	3
MATH 152	Elementary Theory of Numbers	3
MATH 154	Algebraic Number Theory	3
One Analysis Course:		
MATH 131P	Partial Differential Equations I	3
MATH 132	Partial Differential Equations II	3
MATH 136	Stochastic Processes	3
MATH 151	Introduction to Probability Theory	3
MATH 155	Analytic Number Theory	3
MATH 172	Lebesgue Integration and Fourier Analysis	3
MATH 173	Theory of Partial Differential Equations	3

MATH 175	Elementary Functional Analysis	3
One Geometry/Topology Course:		
MATH 143	Differential Geometry	3
MATH 144		3
MATH 145	Algebraic Geometry	3
MATH 146	Analysis on Manifolds	3
MATH 147	Differential Topology	3
MATH 148	Algebraic Topology	3
MATH 149		3

2. All courses counting towards the honors requirements (MATH 106/MATH 116, MATH 120, MATH 171, all 7 additional Math courses used to fulfill the major requirement, and MATH 197) must be taken for a letter grade.
3. Students must have an average GPA of at least a 3.0 across all math classes counting towards the major at the time of applying for honors to be eligible for acceptance into the honors program, as well as upon graduation to graduate with honors.
4. Majors interested in honors can apply in winter quarter of their junior year at the earliest, and no later than the last day of classes in the spring quarter of junior year.
5. Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty and map out a concentrated reading program under the direction and guidance of the adviser. This will occur when the student applies for honors. During the senior year, the student must enroll in MATH 197 Senior Honors Thesis with his/her adviser for a total of 6 units (recommended to be spread over three quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for MATH 197 Senior Honors Thesis are required in addition to the 64 units required of the major. (See the major requirements at the top of the page.)
6. The deadline for the senior thesis final draft is the Monday of week 8 of the student's graduation quarter.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). MATH 205A Real Analysis, MATH 210A Modern Algebra I, and MATH 215A Complex Analysis, Geometry, and Topology or MATH 215B Complex Analysis, Geometry, and Topology are especially recommended in this context.

Students with questions about the honors program should see the department's director of undergraduate studies.

Minor in Mathematics

To qualify for the minor in Mathematics, a student should complete, for a letter grade, at least six Department of Mathematics courses numbered 51 or higher, totaling a minimum of 24 units. For the purposes of this requirement, STATS 116 Theory of Probability, PHIL 151 Metalogic, and PHIL 152 Computability and Logic count as Department of Mathematics courses. No other courses from outside the Department of Mathematics may be used towards the minor in Mathematics.

It is recommended that these courses include:

Math Minor		Units
MATH 51	Linear Algebra and Differential Calculus of Several Variables	5
or MATH 51H	Honors Multivariable Mathematics	
MATH 52	Integral Calculus of Several Variables	5
or MATH 52H	Honors Multivariable Mathematics	
MATH 53	Ordinary Differential Equations with Linear Algebra	5
or MATH 53H	Honors Multivariable Mathematics	
Plus three additional MATH courses		9
Total Units		24

At least 12 of the units applied toward the minor in Mathematics must be taken at Stanford. The policy of the Mathematics Department is that no courses other than the MATH 50 series and below may be double-counted toward any other University major or minor.

Master of Science in Mathematics

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees)" section of this bulletin. Students should pay particular attention to the University's course requirements for graduate degrees.

University Coterminal Requirements

Coterminal master's degree candidates are expected to complete all master's degree requirements as described in this bulletin. University requirements for the coterminal master's degree are described in the "Coterminal Master's Program (http://exploredegrees.stanford.edu/cotermdegrees)" section. University requirements for the master's degree are described in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees/#masterstext)" section of this bulletin.

After accepting admission to this coterminal master's degree program, students may request transfer of courses from the undergraduate to the graduate career to satisfy requirements for the master's degree. Transfer of courses to the graduate career requires review and approval of both the undergraduate and graduate programs on a case by case basis.

In this master's program, courses taken three quarters prior to the first graduate quarter, or later, are eligible for consideration for transfer to the graduate career. No courses taken prior to the first quarter of the sophomore year may be used to meet master's degree requirements.

Course transfers are not possible after the bachelor's degree has been conferred.

The University requires that the graduate adviser be assigned in the student's first graduate quarter even though the undergraduate career may still be open. The University also requires that the Master's Degree Program Proposal be completed by the student and approved by the department by the end of the student's first graduate quarter.

The following are specific departmental requirements:

Candidates must complete an approved course program of 45 units of courses beyond the department requirements for the B.S. degree, of which at least 36 units must be Mathematics Department courses, taken for a letter grade. The Mathematics Department courses must include at least 18 units numbered 200 or above. The candidate must have a grade point average (GPA) of 3.0 (B) over all course work taken in Mathematics, and a GPA of 3.0 (B) in the 200-level courses considered separately. Course work for the M.S. degree must be approved during the first quarter of enrollment in the program by the department's Director of Graduate Studies.

The Financial Mathematics M.S. degree program is no longer offered through the School of Humanities and Sciences. The Institute for Computational and Mathematical Engineering (ICME (https://icme.stanford.edu)) now offers a master's degree track in Mathematical and Computational Finance (http://exploredegrees.stanford.edu/schoolofengineering/instituteforcomputationalandmathematicalengineering/#masterstext).

Doctor of Philosophy in Mathematics

The University's basic requirements for the doctorate (residence, dissertation, examinations, etc.) are discussed in the "Graduate Degrees (http://exploredegrees.stanford.edu/graduatedegrees)" section of this bulletin. The following are specific departmental requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the Department area examination and the University oral examination.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For further information concerning degree programs, fellowships, and assistantships, inquire of the department's student services office.

Ph.D. Minor in Mathematics

Requirements for the Ph.D. Minor in Mathematics are:

		Units
Complete both of the following Sequences: [1, 2]		
Sequence 1		
MATH 106	Functions of a Complex Variable	3
or MATH 116	Complex Analysis	
MATH 131P	Partial Differential Equations I	3
MATH 132	Partial Differential Equations II	3
Sequence 2		
MATH 113	Linear Algebra and Matrix Theory	3
MATH 120	Groups and Rings	3
or MATH 152	Elementary Theory of Numbers	
Additional Courses		
21 units of 200-level MATH courses [3]		21
Total Units		36

1. The 100-level courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.
2. A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for Sequence 1 or 2.
3. The 200-level courses must be taken at Stanford and approved by the Department of Mathematics Ph.D. minor adviser.

Emeriti: Gregory Brumfiel, Gunnar Carlsson, Solomon Feferman, Robert Finn, Yitzhak Katznelson, Joseph Keller, Georg Kreisel, Harold Levine, Tai-Ping Liu, R. James Milgram, Donald Ornstein, Richard Schoen, Leon Simon

Chair: Brian White

Professors: Simon Brendle (on leave), Daniel Bump, Emmanuel Candes, Sourav Chatterjee, Ralph L. Cohen, Brian Conrad, Amir Dembo (on leave Winter), Persi Diaconis, Yakov Eliashberg, Jacob Fox, Soren Galatius, Eleny Ionel (on leave), Steven Kerckhoff, Jun Li, Rafe Mazzeo, Maryam Mirzakhani (on leave Autumn), George Papanicolaou, Lenya Ryzhik, Kannan Soundararajan, Ravi Vakil, Andras Vasy, Akshay Venkatesh, Brian White, Lexing Ying, Zhiwei Yun

Assistant Professors: Thomas Church, Jack Poulson, John Pardon

Szegö Assistant Professors: Riddhipratim Basu, Alexei Entin, Yu Gu, Hilaf Hasson, Yu-Shen Lin, Davi Maximo, Jenny Wilson, Tian Yang, Tianyi Zheng, Xuwen Zhu

Lecturers: Marion Campisi, Susie Kimport, Mark Lucianovic, George Schaeffer, Wojciech Wieczorek

Courtesy Professors: Renata Kallosh

Consulting Professors: Brian Conrey, David Hoffman

Clay Fellow: Alex Wright

Distinguished Poncaire Professor: Tadashi Tokieda