Matching and Price Competition: Comment

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The theory of two-sided matching markets has interested researchers for its theoretical appeal and relevance to real-life applications. The matching of medical residents and hospitals in the United States has been studied extensively by Alvin E. Roth (1984) and others. The National Resident Matching Program (NRMP), the matching authority in the US hospital-resident matching market, runs a centralized matching mechanism that is a variant of the deferred acceptance algorithm of David Gale and Lloyd S. Shapley (1962), redesigned by Roth and Elliott Peranson (1999).

A recent antitrust case against the NRMP charged that the centralized matching mechanism suppressed wages of residents. Although the lawsuit itself was dismissed, it sparked discussion about the effect of centralized matching on wages and efficiency.[1] Jeremy Bulow and Jonathan Levin (2006, BL henceforth) investigate a matching market with price competition where each firm can hire only one worker and show that (a) the average wage is lower, (b) profit of each firm is higher, (c) wages are more compressed, and (d) the resulting matching is slightly less efficient in the presence of the matching mechanism than in any competitive equilibrium.[2] Although BL declare "we have chosen our assumptions for analytical simplicity and transparency, not as the most realistic possible model of the residency match" (654), these results were often interpreted as an argument against the NRMP and led to discussion about potential changes of the matching mechanism. Vincent P. Crawford (forthcoming), for instance, proposes the "Flexible-Salary Match," in which hospitals are allowed to indicate several levels of possible wages, and medical students are allowed to express preferences over pairs of hospitals and wages.[3]

We show that conclusions (a) and (b) above do not necessarily hold when firms may hire more than one worker and the number of workers in different firms are different.[4] More specifically, we present an example with multiple positions in which the average worker wage is higher in the equilibrium with the matching mechanism than in a competitive equilibrium; and profit of each firm is lower in the equilibrium with the matching mechanism than in a competitive equilibrium. Our findings may explain why some of the results of BL disagree with empirical findings of Muriel Niederle and Roth (2003, 2004), who find little or no effect of the centralized matching on wages in some medical matching markets in the United States. Note that different firms hire different numbers of workers in many labor markets like the NRMP.[5]

There are other reasons why the conclusions and policy implications of BL may not be applicable to markets like the NRMP. For example, a competitive equilibrium may not be a

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[1] As footnote 2 of BL notes, the antitrust case was dismissed in 2004 after Congress intervened on the side of the defendants. The appellate court upheld the dismissal in June 2006. A brief account of the antitrust case is found at http://kuznets.fas.harvard.edu/~aroth/alroth.html#Antitrust.

[2] Regarding point (d), more precisely Proposition 5 of BL shows that inefficiency per firm is of the order of the reciprocal of the number of firms in their model.

[3] Georgy Artemov (forthcoming) discusses implications of introducing such a mechanism with flexible wages for wage volatility.

[4] Our example does not imply that other conclusions of BL do not generalize. See discussion presented later.

[5] Program descriptions are posted in the Fellowship and Residency Electronic Interactive Database (FREIDA), http://www.ama-assn.org/ama/pub/category/2007.html. For example, among the total of 13 residency programs in internal medicine in Massachusetts that posted program sizes on that Web site (the data were collected on October 17, 2006), program sizes (sum of the numbers of first-, second-, and third-year resident positions) ranged between 20 and 164. Median was 43, mean was 62.8, and the standard deviation was 47.6. Variation in program sizes is observed in other specialties and regions as well.

good model of the decentralized market without the matching mechanism, for such reasons as unraveling and congestion (Roth 1984; Roth and Xiaolin Xing 1994). Motivated by this idea, Hideo Konishi and Margarita Sapozhnikov (2006) and Siva Anantham and Jennifer Stack (2006) show that worker wages may be lower than those in a competitive equilibrium in their models of decentralized markets without the matching mechanism. Niederle (forthcoming) notes that the NRMP has a feature she calls ordered contracts: hospital programs can offer more than one term of contract. She finds that there is an equilibrium with the matching mechanism that coincides with the firm-optimal competitive equilibrium when hospitals can offer ordered contracts.

I. An Example with Multiple Positions

Consider the following market. There are two firms, 1 and 2. Firms hire different numbers of workers: firm 1 has one position to fill with a worker, whereas firm 2 has two positions. There are three workers 1, 2, and 3. Let Δ_m be worker m's parameter of quality. Surplus of firm 1 from hiring worker m is $v(1; m) = \Delta_m$. Firm 2's surplus from hiring workers m and m' is $v(2; m, m') = 2(\Delta_m + \Delta_{m'})$. A vacant position is identified as a worker of quality zero hired at wage zero. Let $\Delta_1 = 1$, $\Delta_3 = 3$, and $\Delta_2 \in (1, 3)$.[6] The payoff to a firm is the surplus minus the wages it pays to the worker(s) it hires, and the payoff of a worker is the wage she obtains.

A competitive equilibrium is a wage for each worker and an allocation of workers to firms such that (a) wages are individually rational, that is, each firm and each worker gets a nonnegative payoff; and (b) at the going worker wages, no firm has a strict incentive to change the set of workers it hires. There are a range of competitive equilibria with the same matching of workers and firms at different wages. There exists a competitive equilibrium, called the *firm-optimal competitive equilibrium,* which is preferred to any competitive equilibrium by every firm. Following BL, we focus on the firm-preferred competitive equilibrium. In the firm-optimal competitive equilibrium, firm 1 hires worker 1 at the lowest individually rational wage $p_1^F = 0$. Given that, firm 2 hires workers 2 and 3 at wages $p_2^F = \Delta_2 - 1$ and $p_3^F = 2$.

Next we consider a simple game of matching with impersonal wages. Each firm simultaneously makes a nonnegative wage offer which applies to any worker(s) it hires. The offers are observed by the workers. Then matching follows. Firms rank workers from the highest-quality worker to the lowest-quality worker: 3, 2, and 1 in this order. Workers rank firms from the one with the higher wage. So assortative matching occurs and wages are paid as announced: worker 3 is matched to the firm with the highest wage offer, worker 2 is always matched to firm 2, and worker 1 is matched to the firm with the lowest wage.

Now we compute the Nash equilibrium, called the *pricing equilibrium,* of the matching game. By a logic analogous to cases with one position per firm in BL, two firms take mixed strategies over some common interval $[0, \bar{p}]$ with $\bar{p} \geq 0$. Moreover, there is no atom in the distributions of wage offers except possibly for one firm at $p = 0$.[7] Let $g_n(p)$ be the probability density that firm n makes an offer of wage p. In order for firm 1 to randomize over the support $[0, \bar{p}]$ of the mixed strategy, the following should hold at $p \in [0, \bar{p}]$:

$$g_2(p) \times (3 - 1) = 1.$$

The left-hand side is the expected benefit of raising the wage offer by a unit amount, thus hiring worker 3 instead of 1. The equation requires that this benefit be equal to the cost (the additional wage it pays to a worker) that appears on the right-hand side. Similarly, firm 2's incentive condition requires

$$g_1(p) \times 2 \times (3 - 1) = 2.$$

The benefit of raising the wage offer by a unit amount is twice the expected improvement of the worker quality, since firm 2 is of higher

[6] Note that we consider a case where surplus functions are multiplicative and firm 2's surplus is additive in worker quality in order to be as close to BL as possible. In particular, these surplus functions are increasing in worker quality and have increasing differences in firm quality and worker quality, which are crucial for BL's result, Proposition 4.

[7] It turns out that neither firm has an atom in this particular example.

quality, 2. On the other hand, the cost appearing on the right-hand side is 2 instead of 1, since firm 2 must pay the same wages to each of the two workers it hires.

The equations above imply $g_1(p) = g_2(p) = \frac{1}{2}$ for every $p \in [0, \bar{p}]$. Therefore, $\bar{p} = 2$, and both firms randomize uniformly between 0 and 2.

Now we compare average wages in the firm-optimal competitive equilibrium and the pricing equilibrium. The average wage in the firm-optimal competitive equilibrium is $[0 + (\Delta_2 - 1) + 2]/3 = (\Delta_2 + 1)/3$. The average wage in the pricing equilibrium is 1, since both firms uniformly randomize between 0 and 2 and these firms hire the entire set of workers. Therefore, if $\Delta_2 < 2$, the average wage in the firm-optimal competitive equilibrium is lower than in the pricing equilibrium.

Next we consider firm profits. The payoff of firm 1 is 1 in the firm-optimal competitive equilibrium, as it hires worker 1 at wage zero. Firm 1's profit is 1 in the pricing equilibrium as well, since paying zero is one of firm 1's best responses in the pricing equilibrium, and it results in hiring worker 1 with certainty. Firm 2's profit in the firm-optimal competitive equilibrium is

$$2 \times (\Delta_2 + 3) - [(\Delta_2 - 1) + 2] = \Delta_2 + 5.$$

In the pricing equilibrium, paying zero is one of firm 2's best responses. Hence, profit of firm 2 in the pricing equilibrium is

$$2 \times (1 + \Delta_2) = 2\Delta_2 + 2.$$

Therefore, firm 2's profit is lower in the pricing equilibrium than in the competitive equilibrium for every $\Delta_2 < 3$.

The intuition for the example above is as follows. In the pricing equilibrium, worker 2 is hired by firm 2 whether firm 2 offers a higher wage than firm 1 or not. On the other hand, firm 2 has to offer the same wages to both workers it hires. The expected wage for worker 2 is thus made higher because of the competition for worker 3. This positive effect on worker 2 is so large that the average worker wage is higher in the pricing equilibrium than in the firm-optimal competitive equilibrium. On the other hand, firm 2 is made worse off in the pricing equilibrium than in the firm-optimal competitive equilibrium since it should pay high wages not only to worker 3 but also to worker 2 in order to compete for worker 3, even though firm 2 ends up hiring worker 2 irrespective of the wage it offers.

We summarize our finding as follows. The "nondiscrimination" feature of wage offers plays an important role for wage suppression and increase in firm profits when one firm hires one worker. The same feature, however, may raise worker wages and reduce firm profits through a different channel when a firm may hire more than one worker.

II. Discussion and Conclusion

Our analysis relied crucially on the assumption that a firm offers the same wage to all workers it hires. While in principle wages can be different across workers within a firm, this assumption seems to be satisfied in many labor markets, such as the NRMP. In the NRMP, most hospitals indicate one term of contract such as wage level for all the potential workers prior to the execution of matching.[8] The survey study by Niederle, Deborah D. Proctor, and Roth (2006) also suggests that each hospital program often offers the same wages and terms of contracts to all its workers. More specifically, in the market of gastroenterology fellows, all but 4 out of 63 programs offered the same wage to all their fellows; all but 4 offered the same hours on call; all but 18 offered the same amount of time for research. These pieces of evidence suggest that, as BL eloquently put it, "[a hospital] cannot offer $5x$ for the obstetrical Barry Bonds, but only x for the obstetrical Mario Mendoza" (653).

Another notable feature of our example is that different firms hire different numbers of workers. In that example, the less efficient firm 1 makes an aggressive offer because it is also small, which makes the cost of a high wage offer relatively small in comparison to the larger firm 2. To see this point clearly, consider the market in which all N firms have identical $k \geq 2$ positions and there are kN workers. Then a

[8] Hospitals in the NRMP can in principle offer more than one wage level by using ordered contracts (Niederle 2006). However, only about 7 percent of hospitals use ordered contracts.

firm with the highest wage offers is matched to the k best workers, the firm with the second highest wage offer to the $(k + 1)$st, ... , $(2k)$th best workers, and so forth. Therefore the problem is equivalent to the one in which each firm hires only one worker and there are N workers, where the quality of the mth most efficient worker is the average of those of the $[k(m - 1) + 1]$st, ... , (km)th most efficient workers in the original problem. Hence, the analysis of pricing equilibria of BL can be applied without modification if all the firms hire multiple but equal numbers of workers.

Before finishing the paper, we discuss possible directions of future research. In addition to wage depression and rise in firm profits, BL discover wage compression and slight inefficiency of a centralized matching (conclusions (c) and (d) mentioned above). Whether these results hold with multiple positions is an open question, as our example does not contradict them.[9] Analyzing these issues would be interesting, since some reform proposals of the NRMP mechanism such as the Flexible-Salary Match (Crawford forthcoming) may not only raise the average wage, but also alleviate wage compression and especially improve efficiency of the matching.

Another issue of interest is whether the firm-optimal competitive equilibrium is a natural model of a decentralized market. While the question is raised by several authors (cited in an introductory paragraph), BL give some justification in their original environment. More specifically, they observe that, when each firm hires one worker, the firm-optimal competitive equilibrium results in the same allocation as in a form of Vickrey auction, which may be another natural benchmark. This equivalence does not hold, however, when firms are allowed to hire more than one worker, and the Vickrey outcome may be preferred by firms to the firm-optimal competitive equilibrium. It would be interesting to investigate relationships among competitive equilibria, pricing equilibria, and other models such as Vickrey auctions when firms hire multiple workers.

Our findings also await empirical study. While the effect identified in this paper may partially explain why wages do not appear to depend much on whether centralized matching is present (Niederle and Roth 2003, 2004), other explanations are proposed in the literature.[10] Finally, multiple positions may have other important consequences in the labor market. For example, if a firm raises wages of its new workers, there may be pressure to raise wages of existing workers. Given such pressure, a firm may want to keep wage offers to its new workers low.[11] Analyzing a richer model of this kind would be an interesting research topic.

[9] In our example workers 1 and 2 are made better off and worker 3 worse off in the pricing equilibrium than in the firm-optimal competitive equilibrium. In particular, wage compression occurs in this market. The issue of efficiency in large markets as formulated by BL is beyond the scope of our analysis, as we focus on a simple example of a small market.

[10] See papers cited in the introductory paragraphs, for example.

[11] We are grateful to an anonymous referee for suggesting this possibility.

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