Voter Preferences, Polarization, and Electoral Policies

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Abstract

A probabilistic voting model with voter utility functions that are not necessarily concave is examined. When voters are polarized, there is a convexity threshold of their utility function below which policy convergence is a unique equilibrium, and above which policy divergence is a unique equilibrium. Divergent equilibrium is more likely when voters become more polarized. Social welfare is maximized in each divergent equilibrium, but not necessarily in every convergent equilibrium. When there is more than one policy issue, the candidates' equilibrium policies diverge on issues for which utility functions are convex and converge on issues for which they are concave.

1 Introduction

The celebrated "median-voter theorem" (Hotelling (1929), Downs (1957) and Black (1958)) predicts that electoral candidates who try to maximize the vote share end up taking the same policy position. In practice, however, their policies are not necessarily the same: While candidates may take similar positions on some issues such as economic policy, they differ sharply on other issues, especially on those with religious content. For example, the

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2008 Democratic National Platform declared that they "support the full inclusion of all families, including same-sex couples, in the life of our nation, and support equal responsibility, benefits, and protections," and "oppose the Defense of Marriage Act."[1] The Republican platform in the same year described the Democrats' opposition to this act as unbelievable, and wrote that they "call for a constitutional amendment that fully protects marriage as a union of a man and a woman."[2]

These observations raise a number of questions. Why do political candidates take similar positions on some issues and not on others? More generally, what determines candidates' policy positions? What are the welfare implications of policy convergence and divergence?

This paper addresses these questions using a probabilistic voting model without aggregate preference shocks. While the setup closely follows that of standard models, we employ one novel feature. Specifically, in our model voters' utility function is not necessarily concave.

In our model, when voters are more polarized than in the uniform distribution, there is a convexity threshold of voters' utility function (i) below which policy convergence is a unique equilibrium, and (ii) above which policy divergence is a unique equilibrium. As the voters become more polarized, divergent policies prevail in equilibrium for a wider range of voter utility functions. Moreover, we find that social welfare is maximized in each divergent equilibrium, but not necessarily in every convergent equilibrium.

We also introduce a model with more than one policy issue, for example, tax policy and same-sex marriage. In that model, the candidates' equilibrium policies diverge on "convex issues," i.e., issues for which voters' utility function is convex, while they converge on "concave issues," as analogously defined. If voters' utility function is convex on religious or moral issues and concave on economic issues (as we discuss below), then our model predicts policy convergence in economic policies and divergence on moral issues, as observed in American politics (see Glaeser et al. (2005)).

Because most of the existing works on spatial models of elections assume that voters' utility functions are concave, convex utility functions might seem at first glance to be unrealistic. Justification for concave utility functions,

[1]The following is a main excerpt from the act. "No State, territory, or possession of the United States, or Indian tribe, shall be required to give effect to any public act, record, or judicial proceeding of any other State, territory, possession, or tribe respecting a relationship between persons of the same sex that is treated as a marriage under the laws of such other State, territory, possession, or tribe, or a right or claim arising from such relationship."

[2]For more evidence and discussion on issues other than gay marriage, see Glaeser et al. (2005).

however, seems unclear. A skeptical view has been eloquently expressed by Osborne (1995) as follows:

> The assumption of concavity is often adopted, first because it is associated with 'risk aversion' and second because it makes it easier to show that an equilibrium exists. However, I am uncomfortable with the implication of concavity that extremists are highly sensitive to differences between moderate candidates (a view that seems to be shared by Downs 1957, 119-20). Perhaps the Republican and Democratic parties in the United States are run by people whose opinions are extreme relative to those of the average voter for these parties (Tim Feddersen has made this point to me), but does Tony Benn really perceive a huge difference between Margaret Thatcher and Enoch Powell? (David laidler suggested this specific example.) Further, it is not clear that evidence that people are risk averse in economic decision-making has any relevance here. I conclude that in the absence of any convincing empirical evidence, it is not clear which of the assumptions is more appropriate.

Indeed, non-concave utility functions are used extensively in the empirical literature. Poole and Rosenthal (1997), for instance, argue that concave utility functions do not fit the data well. In the theoretical literature, although concavity is often assumed, Shepsle (1972) and Aragones and Postlewaite (2002) allow for convex utility functions, relating them to the "intensity" of voter preferences. We also believe utility functions that are not concave are sometimes plausible and useful in the voting context and pursue the implications different properties of voters' preferences have for the candidates' policy positions.

Economic policy is arguably a concave issue, given the evidence that individuals are risk averse in financial decisions. By contrast, voters may have convex utility functions on moral or religious issues such as same-sex marriage. For example, when a civil union law was introduced in Connecticut in 2005, Anne Stanback, president of a gay right advocacy organization Love Makes a Family, commented "It's bittersweet" as it was a move in the right direction but did not go far enough (*Boston Globe*, April 21, 2005). The attitude of gay people to State Supreme Court was in a sharp contrast, when it ruled to legalize same-sex marriages in 2008: "The opinion in Connecticut was hailed by jubilant gay couples and their advocates" (*New York Times*, October 11, 2008). This difference is remarkable, given that laws defining civil unions provide most of the legal benefits of marriage in all but name "marriage." Indeed, the majority opinion in the State Supreme Court declares "marriage and civil unions do embody the same legal rights under our law." Even so, the Court's majority opinion seems to have recognized the large utility difference for gay couples between a civil union and a marriage, as it continues to write "they [marriage and civil unions] are by no means equal" and rules in favor of gay couples. The strong dissatisfaction of gay couples about civil union and the contrasting happiness they feel about the traditional marriage suggest a convex utility function, meaning that voters have strong feelings regarding policy changes around their bliss points.

Abortion gives another intuition for convex utility functions. A pro-life activist may equate with murder and find it (almost) equally abhorrent, even if it is conducted at an early stage of pregnancy.

While the anecdotes above are suggestive at best, they present new possibility of understanding electoral competitions. Recall that our theory predicts that politicians tend to converge on concave issues and diverge on convex issues. This prediction, together with the intuition that economic policy and moral policy are concave and convex issues, respectively, is consistent with the observed pattern in American politics, where political platforms often resemble each other in economic issues but differ sharply where moral issues are concerned.

In addition to convex utility functions, polarization of voter preferences plays an important role in our model. Popular media has been reporting polarization of Americans in recent years.[3] Meanwhile, there seems to be less consensus among researchers. McCarty et al. (2006) present evidence that suggests the existence of voter polarization and its recent increase in terms of income. DiMaggio et al. (1996) and Evans (2003) find that voters have not polarized on most issues but they have done so on abortion. Other researchers, such as Fiorina (2006), argue that polarization does not exist, or at least has not increased among the general public over the past few decades, but that politicians have become polarized. We do not take a strong stance on empirical evidence and instead provide theoretical predictions on electoral outcomes given voter distributions, an approach that enables us to understand the implications of voter polarization.

Finally, we mention a line of literature that explains policy divergence, and discuss the difference from our paper. Palfrey (1984) considers the possibility of a third party candidate; Alesina (1988) studies repeated interactions of policy-motivated candidates; Roemer (1994) investigates policy-

[3]For example, Gelman (2008, Figure 3.2) finds that newspapers and magazines have recently increased their use of political catchphrases such as "polarizing, polarized," "red state," and "blue state."

motivated candidates in a setting with uncertainty about the position of the median voter; Osborne and Slivinski (1996) and Besley and Coate (1997) consider citizen-candidate models; Aragones and Palfrey (2000) and Kartik and McAfee (2007) allow for differences in the personal qualities of candidates (such as charisma); and Glaeser et al. (2005) consider the abilities of politicians to target political messages toward their core constituents, among others. Compared to these works, the departure of our model from the Hotelling-Downs framework is kept minimal. For example, we obtain divergent equilibria even without policy motivation of candidates or the possibility of entry by a third party. In addition, our explanation of policy convergence and divergence based on voters' utility function is novel and enables us to obtain insights on the relationship between equilibrium policies and social welfare.

The plan of the paper is as follows. Section 2 introduces the model. Section 3 studies uni-dimensional policy space. In Subsection 3.1, we consider a special case in which the voter distribution is perfectly polarized. The intuition behind the results in this subsection helps us understand the intuition for our main results in the next subsection. In Subsection 3.2, we consider general voter distributions. We formally analyze how the degrees of voter polarization and convexity of voters' utility function influence policy positions in equilibrium. We also consider the welfare implication of the convexity of voters' utility function. In Section 4, we consider the case of a multi-dimensional policy space. Section 5 concludes. All the proofs are relegated to the Appendix.

2 Basic Model

There is a one-dimensional policy space, $\mathcal{P} := \{0, \frac{1}{2}, 1\}$. A continuum of voters are distributed according to a probability mass function $f : \mathcal{P} \to [0, 1]$ with $f\left(\frac{1}{2}\right) = c$ and $f(0) = f(1) = \frac{1-c}{2}$ for $c \in [0, \frac{1}{3})$.[4] c represents the degree of centralization of the voter distribution. We say that the voter distribution is **polarized** if $c = 0$, and will study the polarized distribution in detail later. Two candidates A and B simultaneously determine their positions, x_A and x_B. Candidate $i = A, B$ obtains a share of votes

$$P(x_i, x_{-i}) = \sum_{x \in \mathcal{P}} \frac{\mathrm{e}^{\lambda u(|x - x_i|)}}{\mathrm{e}^{\lambda u(|x - x_i|)} + \mathrm{e}^{\lambda u(|x - x_{-i}|)}} f(x), \tag{1}$$

[4] As we mentioned in the Introduction, we focus on the voter distributions that entail some degree of polarization, by assuming $c < \frac{1}{3}$.

where $\lambda > 0$ is fixed, x_{-i} is the policy position taken by i's opponent, and $u : \mathcal{P} \to \mathbb{R}$ is a decreasing function, i.e. $u(x) < u(x')$ if $x > x'$.[5]

The above specification is called the logit model. One "microfoundation" of this logit model is as follows.[6] u is the voters' deterministic utility function which is assumed to be homogeneous across them. The voters are subject to independently and identically distributed random shocks that affect their overall utility. Specifically, the overall utility of a voter with bliss point x when the elected candidate is i is written as $u(|x - x_i|) + \xi_i$, where the first term $u(|x - x_i|)$ is the utility from the policy implemented, and ξ_i is a random shock on utility for that voter about candidate i. The logit specification assumes that the random shock term ξ_i follows the extreme value distribution independently and identically across voters, and it is used in the literature extensively.[7] Each voter is assumed to vote for the candidate who generates more overall utility to her if she is elected, knowing her own realized values of the random term.[8,9] The parameter $\lambda > 0$ represents how strongly voters respond to difference in policy positions.[10] Intuitively, the larger λ is, the more the voters care about the policy positions. Our formulation implicitly assumes that no voters abstain, so that $P(x_A, x_B) + P(x_B, x_A) = 1$ for every possible pair of x_A and x_B.

Without loss of generality, we normalize the utility function by setting $u(0) = 1$ and $u(1) = 0$, and set $u\left(\frac{1}{2}\right) = \frac{1}{2} - v$ where $v \in (-\frac{1}{2}, \frac{1}{2})$. Parameter v denotes the degree of convexity of the utility function. Hence, we say that u is **convex** if $v \geq 0$ and u is **concave** if $v \leq 0$. We say that u is strictly convex if $v > 0$ and u is strictly concave if $v < 0$.

There are only three points in the policy space in our model. While it may be tempting to assume that voters are distributed over a continuous policy space, our three-point model has at least two advantages over such

[5]For any $i \in \{A, B\}$, $-i$ denotes the candidate different from i.

[6]Other interpretations are given in the literature (see, for example, Persson and Tabellini (2000)). For example, voters may not be optimizing and voting randomly, or the randomness reflects subjective beliefs on the part of candidates. The interpretation we give here seems to be the most consistent with the standard rational choice framework.

[7]See, for example, Anderson et al. (1992) and Yang (1995).

[8]When both candidates generate the same overall utility to a voter, each candidate is chosen by this voter with some arbitrary probability. The specification does not affect the analysis since such an event happens with probability zero.

[9]Since there are only two candidates, voting for the candidate with the higher overall utility is a weakly dominant strategy for each voter. See Myerson and Weber (1993) and Fey (1997) for issues related to voters' strategic behavior when there are more than two candidates.

[10]In the current context, one could interpret λ as the "salience" or relative importance of the political issue in consideration, compared to the idiosyncratic utility for each voter.

a model. First, our model is simple and very tractable. With a continuous policy space, by contrast, even the existence of Nash equilibrium is not guaranteed. Second, our model allows us to unambiguously order all the possible voter utility functions with respect to their convexity/concavity by a single parameter. With a continuous policy space, however, utility functions cannot necessarily be ordered by their convexity/concavity. Other studies such as Aragones and Postlewaite (2002) and Carrillo and Castanheira (2008) also use three-point distributions.

Given a profile of positions (x_A, x_B) chosen by the candidates, let $w(x_A, x_B)$ be the "winner" of the election: Formally, let $w(x_A, x_B)$ be i if $P(x_i, x_{-i}) > \frac{1}{2}$, and A and B each with probability $\frac{1}{2}$ if $P(x_A, x_B) = P(x_B, x_A) = \frac{1}{2}$. Each candidate $i = A, B$ has a payoff function:

$$U_i(x_A, x_B) = a_i \cdot \mathbb{I}_{\{i=w(x_A,x_B)\}} + \epsilon b_i(|x_{w(x_A,x_B)} - \bar{x}_i|), \tag{2}$$

where a_i is a positive constant, ϵ is a nonnegative constant, and $b_i(\cdot)$ is a decreasing function whose argument is the distance between the realized policy and i's bliss point, denoted by $\bar{x}_i$. We assume that $\bar{x}_A = 0$ and $\bar{x}_B = 1$. We also assume that, for each i, $\epsilon\left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)\right) < a_i$. This assumption holds if b_i is concave (i.e. $b_i(\frac{1}{2}) \leq \frac{b_i(0)+b_i(1)}{2}$) or if $\epsilon > 0$ is sufficiently small. As we formally state in Proposition 4, this assumption implies that candidates primarily care about the vote share, and if two policy choices give candidate A (resp. candidate B) the same probability of winning the election, she prefers the left (resp. the right) policy. In the remainder of the paper, we assume $\epsilon > 0$ unless stated otherwise.

A profile of mixed strategies is a Nash equilibrium if the strategy of each candidate maximizes her expected utility given the strategy of the other candidate.

Social welfare of a policy x is

$$W(x) = \sum_{x' \in \mathcal{P}} u(|x' - x|) f(x').$$

We say that a (mixed) strategy profile is **welfare maximizing** if for all (x_A, x_B) that realizes with positive probability under that strategy profile, $P(x_i, x_{-i}) \geq \frac{1}{2}$ implies $W(x_i) \geq W(x')$ for all $x' \in \mathcal{P}$. That is, every policy position that wins the election with positive probability maximizes social welfare.

3 Policy Divergence and Policy Convergence

3.1 Illustrative Example: Polarized Distribution

This subsection is devoted to the analysis in a particularly simple environment. Recall we say that a distribution of voters is **polarized** if $f\left(\frac{1}{2}\right) = c = 0$. That is, f is a polarized distribution if it has point masses on 0 and 1, each of which has a weight of $\frac{1}{2}$. A polarized distribution emerges in a political situation where one half of the voters share one bliss point, and the other half share the other bliss point.

Proposition 1. *Suppose the voter distribution is polarized.*

1. *If voters' utility function is convex, then* $(0, 1)$ *is a unique Nash equilibrium.*
2. *Otherwise,* $(\frac{1}{2}, \frac{1}{2})$ *is a unique Nash equilibrium.*

Note that the two parts of the proposition show the uniqueness of Nash equilibrium for any utility function of voters.

We offer intuition of Proposition 1. Suppose that candidates A and B are at 0 and 1, respectively, and consider the incentive of candidate A. Candidate A experiences gain and loss by moving from 0 to $\frac{1}{2}$: she receives more votes from the voters at 1, while she loses votes from the voters at 0. If voters have convex utility function, they care more about policy changes when the proposed policy is close to their bliss points than when it is far. If candidate A moves toward the middle ($\frac{1}{2}$), then she loses more votes from the voters that are close to her (i.e. the voters at 0) than she wins votes from voters who are far away (i.e. the voters at 1). A symmetric argument holds for candidate B. Thus divergence is an equilibrium when voters have convex preferences. On the contrary, voters with concave utility functions care more about policy changes when the policy is far from their bliss points. Thus candidates have incentives to position at the middle, so that they can win reasonably many votes from both sides of the distribution of voters.

The exact relationship between convexity and divergence depends on our logit specification of the probability with which candidates get votes. Note however that the prediction does not rely on the value of λ, so the result is robust with respect to this parameter.

Voters' utility function is often assumed to be concave in the literature, and policy convergence has been shown under that assumption (see Banks and Duggan (2005)). Proposition 1 demonstrates the importance of the concavity assumption for such results by showing that both policy convergence and divergence can occur depending on the utility functions. Note that Proposition 1 provides a necessary and sufficient condition for policy convergence, which is uncommon in the literature.

Proposition 2. *Suppose the voter distribution is polarized. Then the (unique) Nash equilibrium is welfare maximizing.*

As seen in Proposition 1, we may or may not observe policy divergence in equilibrium, depending on voters' utility function. However, Proposition 2 demonstrates that social optimum is attained in equilibrium, whether or not the divergence occurs.

Note that Proposition 2 enables analysts to evaluate social welfare *without* reference to primitive of the model except voter distributions. This is potentially useful, as analysts can make welfare judgment without much information, such as realized policy positions or utility functions.

In the next subsection, we consider more general voter distributions than the polarized distribution. It will turn out that some, but not all, of the insights in this section carry over to those general cases.

3.2 Main Results

In this subsection, we investigate how equilibrium policies are affected by voters' utility function, randomness added to it, and polarization of the voter distribution.

Before presenting the main results, we offer two basic results that prove useful in subsequent analysis. First, the following result allows us to focus on pure strategy equilibria without loss of generality.

Proposition 3. *Each candidate uses a pure strategy in any Nash equilibrium.*

Before presenting the second result, recall that we assume $\epsilon(b_i(0)-2b_i(\frac{1}{2})+b_i(1)) < a_i$ for each i. As we mentioned, this condition holds whenever $\epsilon > 0$ is sufficiently small. This distinguishes our model with most existing ones with policy-motivated candidates, in that effectively we only assume lexicographically weaker preferences for policies than politicians' primary interests in winning votes. In that sense the departure of our setup from the standard Hotelling model is kept minimal. Indeed, it is clear from the definition that any strict Nash equilibrium in the game with $\epsilon = 0$ is a Nash equilibrium of the game with sufficiently small $\epsilon > 0$, so our requirement of candidates' policy preferences is mild. Even so, interestingly this policy preference term does rule out some equilibria and enables us to obtain a unique prediction under certain circumstances as we will see shortly. In fact, the following proposition shows that a Nash equilibria with $\epsilon > 0$ are equivalent to a certain refinement of Nash equilibria of the game with $\epsilon = 0$.

Proposition 4. *(x_A^*, x_B^*) is a Nash equilibrium of the game with $\epsilon > 0$ if and only if it is a Nash equilibrium of the game with $\epsilon = 0$ with the additional property that, for each i, there exists no x_i' such that $P(x_i', x_{-i}^*) = P(x_i^*, x_{-i}^*)$ and $|x_i' - \bar{x}_i| < |x_i^* - \bar{x}_i|$.*

The proposition shows that analyzing a game with $\epsilon > 0$ is equivalent to analyzing a game with $\epsilon = 0$ as long as we analyze a (slight) refinement of Nash equilibrium, in the sense that we focus on Nash equilibria in which each candidate i announces the closest policy to $\bar{x}_i$ among the set of policies that gives the highest vote share to her, without loss of generality. In this sense, the model with policy preferences ($\epsilon > 0$) departs only minimally from the standard model of purely office-motivated candidates ($\epsilon = 0$). Indeed, all our results except for uniqueness of equilibrium holds also with $\epsilon = 0$. Thus, even if we stick to the model with politicians who maximize vote share, we still obtain policy divergence as a Nash equilibrium under a wide range of environments.

Recall that c denotes the degree of centralization of the voter distribution. For any $\lambda > 0$, let

$$\bar{c}(v, \lambda) = \left(2 + \frac{\mathbf{e}^{\frac{1}{2}\lambda} - \mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v}} \right)^{-1}$$

for any $v > 0$, and $\bar{c}(0, \lambda) = 0$.[11]

Theorem 1. *1. If voters' utility function is convex and $c \le \bar{c}(v, \lambda)$, then $(0, 1)$ is a unique Nash equilibrium.*

2. Otherwise, $(\frac{1}{2}, \frac{1}{2})$ is a unique Nash equilibrium.

The basic intuition of this theorem can be explained in an analogous manner as we did for Proposition 1. Suppose that candidates A and B are at 0 and 1, respectively, and consider the incentive of candidate A. Candidate A experiences gain and loss by moving from 0 to $\frac{1}{2}$: the gain from more votes from the voters at $\frac{1}{2}$ and 1, and the loss from less votes from the voters at 0. If voters' utility function is sufficiently convex, they care more about policy changes when the proposed policy is close to their bliss points than when it is far. If candidate A moves toward the middle ($\frac{1}{2}$), then she loses more votes from the voters that are close to her (i.e. the voters at 0) than she wins votes from the voters who are far away (i.e. the voters at $\frac{1}{2}$ and 1). A symmetric argument holds for candidate B. Thus divergence is an equilibrium when voters' utility function is sufficiently convex. The other cases can be similarly explained.

[11] $\bar{c}(0, \lambda) = \lim_{v \searrow 0} \bar{c}(v, \lambda)$ by Proposition 5 below.

Further intuition for the case of $v > 0$ can be explained as follows[12]: Suppose that $(0, 1)$ is a Nash equilibrium for some $c = c'$. Then, for any $c < c'$, we should expect it to be a Nash equilibium. Intuitively speaking, this is because, since more voters are at the "extreme positions" (i.e. 0 or 1) when $c < c'$ than when $c = c'$, the convexity of voters' utility function implies (with the logic explained in the previous subsection) more incentive to situate themselves at extreme positions.[13] Conversely, if $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium for some $c = c''$, then for any $c > c''$ we should expect it to be a Nash equilibrium. This is because, since more voters are at the "middle points" when $c > c''$ than when $c = c''$, the incentive to take "extreme points" (implied by the convexity of u) decreases, so candidates still want to take the middle position. In other words, there exist thresholds, $c_*(v, \lambda)$ and $c^*(v, \lambda)$, for divergence and convergence, respectively, to be equilibria.

While we have explained the intuition for Theorem 1, note that there are more contents in this theorem: (i) there exists a unique Nash equilibrium for each c, (ii) thresholds $c_*(v, \lambda)$ and $c^*(v, \lambda)$ are identical, and (iii) we can analytically solve for threshold $\bar{c}(v, \lambda)$. (iii) is useful since it enables us to conduct simple comparative statics, as shown in the following proposition.

Proposition 5. *1. $\bar{c}(v, \lambda)$ is strictly increasing in v, $\lim_{v \searrow 0} \bar{c}(v, \lambda) = 0$, and $\lim_{v \nearrow \frac{1}{2}} \bar{c}(v, \lambda) = \frac{1}{3}$ for any λ.*

2. $\bar{c}(v, \lambda)$ is strictly decreasing in λ for any $v > 0$, and $\lim_{\lambda \searrow 0} \bar{c}(v, \lambda) = \frac{2v}{4v+1}$ and $\lim_{\lambda \to \infty} \bar{c}(v, \lambda) = 0$ for any v.

Part 1 of the Proposition shows that a higher degree of convexity of voters' utility function makes a divergent equilibrium possible even when the voter distribution is more concentrated in the median. Moreover, as voters' utility function approaches a linear function (i.e., $v \to 0$), the threshold $\bar{c}(v, \lambda)$ needed for a divergent equilibrium approaches zero, and when the utility function becomes linear, the divergent equilibrium persists only under the polarized distribution, $c = 0$. As convexity becomes very large ($v \to \frac{1}{2}$), the divergent equilibrium becomes prevalent and the threshold approaches $c = 1/3$, corresponding to the uniform distribution.

Part 2 of the Proposition shows that the more strongly voters care about policies relative to idiosyncratic random preferences, the more polarization of voters are needed for divergent equilibria. Furthermore, as the degree of

[12]The cases for $v \leq 0$ is standard in the literature. See, for example, Banks and Duggan (2005).

[13]We point out that, given a convex voter utility function, the existence of small enough a value c such that policy divergence is an equilibrium under c is not surprising given Proposition 1. Nontrivial contents of this theorem are explained shortly.

randomness becomes much more significant than voters' deterministic preference, the threshold needed for a divergent equilibrium approaches a limit that depends on the degree of convexity of the preference (v). As the randomness becomes negligible relative to the deterministic preference of the voters, the divergence becomes difficult and the threshold approaches zero.

Note that, for any degree of randomness λ, policy divergence occurs under voter distributions that have centralizations close to (but less than) $\frac{1}{3}$ as long as the utility function is sufficiently convex, while even very large degree of randomness does not necessarily imply policy divergence under such a voter distribution. This analysis shows that sufficient convexity, rather than sufficient randomness, is essential to have divergence in equilibrium: The only randomness we need is just a slightest degree of it. This fact might appear contradictory to the "median voter theorem" as predicted by the standard Hotelling model, where there is no randomness in voters' utility. However there is no inconsistency. As we have seen in Part 2, once we fix the value of $v < \frac{1}{2}$, $\bar{c}(v, \lambda)$ approaches 0 as the randomness vanishes ($\lambda \to \infty$).

We plot in Figure 1 the value of $\bar{c}(v, \lambda)$ with respect to v, for four values of λ: 0.0001, 2, 5, 10, and 15. The graph illustrates that more convexity and randomness allow policy divergence under distributions with more centralization. Notice that, for any fixed v, $\bar{c}(v, \lambda)$ approaches zero as λ increases.

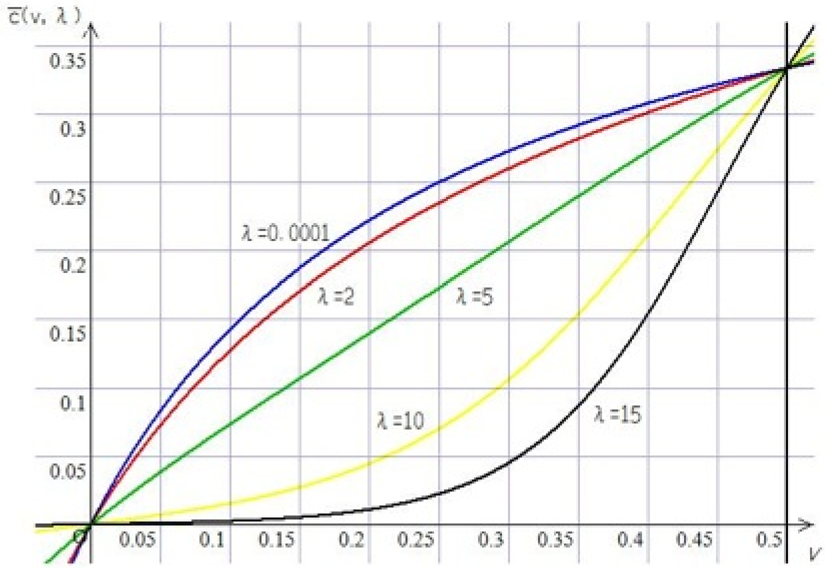

Figure 1: Relationship between v and $\bar{c}(v, \lambda)$ for different values of λ.

The next result shows that the divergent equilibrium is welfare maximizing.

Theorem 2. *If* $(0,1)$ *is a Nash equilibrium, then it is welfare maximizing.*

The converse of this result is not true: Even if $(0,1)$ is welfare maximizing, it is not necessarily a Nash equilibrium. Also, in such a case the only equilibrium is $(\frac{1}{2},\frac{1}{2})$. Hence, in our model with convex utility function, *the convergent equilibrium is not necessarily welfare maximizing.*[14] This conclusion contrasts with the standard results in the literature (see, for example, Banks and Duggan (2005)) that the convergent equilibrium is welfare maximizing.

4 Multi-Dimensional Policy Space

The primary interest of this paper is to see how positions of political candidates are determined in a strategic situation and how positions are related to the nature of the political issues. In this section we propose a model in which there are more than one policy issue. We will see that political candidates diverge on some issues and converge in others in equilibrium, and issues with divergent policy positions are precisely those on which voters have a convex utility function.

A continuum of voters are distributed on $\mathcal{P} := \{0,\frac{1}{2},1\}^n$ according to a probability mass function f on $\mathcal{P}$. The interpretation is that each dimension of the policy space corresponds to one policy issue. Two candidates A and B with bliss points $(\bar{x}_A,\bar{x}_B) = ((0,\dots,0),(1,\dots,1))$ simultaneously determine their positions, $x\in\mathcal{P}$ and $y\in\mathcal{P}$. The payoff of each candidate i is given by equation (2) as before, except that the vote share function (1) is replaced by its multi-dimensional generalization,

$$P_i(x,y) = \sum_{x'\in\mathcal{P}} \frac{\mathbf{e}^{\lambda\sum_{k=1}^n \delta_k u_k(|x'_k - x_k|)}}{\mathbf{e}^{\lambda\sum_{k=1}^n \delta_k u_k(|x'_k - x_k|)} + \mathbf{e}^{\lambda\sum_{k=1}^n \delta_k u_k(|x'_k - y_k|)}} f(x'),$$

where $\lambda > 0$ is fixed and $\sum_{k=1}^n \delta_k u_k(\cdot)$ represents voters' utility function. Implicit in the definition is the assumption that voters' utility function is additive across different policy issues. For each k, we assume that u_k is a decreasing function satisfying $u_k(0) = 1$ and $u_k(1) = 0$, and $\delta_k > 0$.

[14] An example is as follows: Let $\lambda = 1$, $c = 0.248$, and $v = \frac{1}{4}$. Then, it can be shown that $c = 0.248 > 0.246\cdots = \bar{c}(\frac{1}{4},1)$, so that $(\frac{1}{2},\frac{1}{2})$ is the unique Nash equilibrium, while it is not welfare maximizing, since $W(0) = W(1) = 0.438 > 0.436 = W(\frac{1}{2})$.

Parameter δ_k represents the relative importance of the k'th policy issue for voters.

To obtain a sharp prediction, we assume that the distribution of voters $f : \mathcal{P} \to [0, 1]$ is **polarized**, that is, $\text{supp}(f) \subseteq \{0, 1\}^n$ and $f(x) = f(x')$ for all $x, x' \in \mathcal{P}$ with $x'_k = 1 - x_k$ for all $k = 1, \dots, n$.

The concept of a polarized distribution is a generalization of the corresponding notion in Subsection 3.1. The class of polarized distributions subsumes as a special case a distribution that puts a $\frac{1}{2}$ mass on $(0, \dots, 0)$ and the other $\frac{1}{2}$ mass on $(1, \dots, 1)$. Also included in this class is a distribution where each vertex of the n-dimensional unit cube $\{0, 1\}^n$ has an identical weight and all other points have weight zero. However, the notion of polarized distribution is more general: For example, in the 2-dimensional policy space, the distribution in which fraction $\frac{1}{3}$ of the voters are situated at $(0, 0)$ and $(1, 1)$ each, and $\frac{1}{6}$ at $(0, 1)$ and $(1, 0)$ each, is a polarized distribution.

We first provide a characterization of a Nash equilibrium, and then gives a welfare analysis.

Theorem 3. *Suppose the voter distribution is polarized. There exists a unique Nash equilibrium. The unique Nash equilibrium* $(x^*_A, x^*_B) = ((x^*_{A1}, \dots, x^*_{An}), (x^*_{B1}, \dots, x^*_{Bn}))$ *is given by*

$$(x^*_{Ak}, x^*_{Bk}) = \begin{cases} (0, 1) & \textit{if } u_k \textit{ is convex,} \\ \left(\frac{1}{2}, \frac{1}{2}\right) & \textit{otherwise.} \end{cases}$$

The result shows that candidates' positions have clear dichotomy. More specifically, candidates diverge on "convex issues," that is, issues for which voters' utility function is convex, while they converge on "concave issues." Also note that the equilibrium is shown to be unique and its expression is explicitly given in the theorem. The case where $n = 1$ corresponds to Proposition 1.

As in Subsection 3.1, we can show that a Nash equilibrium is welfare maximizing whether or not it is convergent. We follow essentially the same analysis as before.

Extending the definition before, social welfare of a policy x is

$$W(x) = \sum_{x' \in \mathcal{P}} \sum_{k=1}^{n} \delta_k u_k(|x'_k - x_k|) f(x'),$$

where $x = (x_1, \dots, x_n)$ and $x' = (x'_1, \dots, x'_n)$.

Given this definition of $W(x)$, we say that a (mixed) strategy profile is welfare maximizing if for all (x_A, x_B) that realizes with positive probability under that strategy profile, $P(x_i, x_{-i}) \geq \frac{1}{2}$ implies $W(x_i) \geq W(x')$ for all $x' \in \mathcal{P}$. That is, every policy position that wins the election with positive probability maximizes social welfare.

Proposition 6. *Suppose the voter distribution is polarized. Then the (unique) Nash equilibrium is welfare maximizing.*

The proof is omitted, as it is a relatively easy adaptation of the proof of Proposition 2.

We have seen in Proposition 2 that the unique Nash equilibrium is welfare maximizing in the model with a uni-dimensional policy space. The above proposition generalizes this result: The unique Nash equilibrium is welfare maximizing even in a multi-dimensional policy space.

5 Conclusion

We considered a probabilistic voting model with utility functions that are not necessarily concave. In our model, when voters are more polarized than in the uniform distribution, there is a convexity threshold of voters' utility function (i) below which policy convergence is a unique equilibrium, and (ii) above which policy divergence is a unique equilibrium. As voters become more polarized, divergent policies prevail in equilibrium for a wider range of voter utility functions. Moreover, social welfare is maximized in each divergent equilibrium, but not necessarily in every convergent equilibrium. When there are more than one policy issue and voter distribution is polarized, the candidates' equilibrium policies diverge on issues for which utility functions are convex and converge on issues for which utility functions are concave.

We conclude the paper with possible directions for future research. First, the paper suggests that it may be interesting to empirically identify on which policy issues voters have convex and concave utility functions, respectively. Second, investigating the implications of convexity would be a fruitful approach. We are not aware of such studies, but they could prove useful for understanding the various policy positions taken by electoral candidates in reality.

A Appendix

Propositions 1 and 2 are proven in Sections A.3 and A.5, respectively. We begin by proving Proposition 3.

A.1 Proof of Proposition 3

Proof. Let $\Delta(\mathcal{P})$ be the set of a candidate's mixed strategies, and for candidate i and any $\alpha_i, \alpha_{-i} \in \Delta(\mathcal{P})$, let

$$U_i(\alpha_i, \alpha_{-i}) := \sum_{(x_i, x_{-i}) \in \mathcal{P}^2} \alpha_i(x_i) \cdot \alpha_{-i}(x_{-i}) \cdot U_i(x_i, x_{-i}).$$

A profile of mixed strategies (α_A, α_B) is a Nash equilibrium if $\alpha_i \in \arg\max_{\alpha'_i \in \Delta(\mathcal{P})} U_i(\alpha'_i, \alpha_{-i})$ for each $i = A, B$.

Consider a Nash equilibrium (α_A, α_B). We first show that $\alpha_A(1) = 0$. First note that, by symmetry, A's winning probability is the same when she plays a pure strategy $x_A = 0$ as when she plays a pure strategy $x_A = 1$. Suppose first that the probability that A wins the election at $x_A = 1$ is zero. By symmetry, this occurs only when $\alpha_B(\frac{1}{2}) = 1$ and $P(1, \frac{1}{2}) < \frac{1}{2}$. In this case, by playing $x_A = 1$, A's winning probability is 0, while the realized policy is $\frac{1}{2}$ with probability 1. But by playing $x_A = \frac{1}{2}$, A's winning probability is $\frac{1}{2}(> 0)$, while the realized policy is, again, $\frac{1}{2}$ with probability 1. Thus in this case A cannot put a positive probability on $x_A = 1$ in a Nash equilibrium.

Consider next the other case, i.e. the cases in which the probability that A wins the election at $x_A = 1$ is positive. Notice again that, by symmetry, the winning probability is the same when A plays $x_A = 0$ as when she plays $x_A = 1$. Since whenever she wins the realized policy is closer to her bliss point when $x_A = 0$ than when $x_A = 1$, pure strategy $x_A = 0$ gives a strictly higher payoff to candidate A than pure strategy $x_A = 1$ does. Thus, again in this case, candidate A puts probability zero on $x_A = 1$ in any Nash equilibrium.

Thus we conclude that $\alpha_A(1) = 0$. A symmetric argument shows that $\alpha_B(0) = 0$.

In order to prove the proposition, now consider two cases. Suppose first that $P(0, \frac{1}{2}) \geq \frac{1}{2}$. Given any realized action of B, A's winning probability is strictly positive and weakly larger when $x_A = 0$ than when $x_A = \frac{1}{2}$. Also, whenever A wins the election, the realized policy is strictly closer to A's bliss point when $x_A = 0$ than when $x_A = \frac{1}{2}$, and whenever A loses, it is the same when $x_A = 0$ as when $x_A = \frac{1}{2}$. Hence under this assumption candidate A takes a pure strategy, $x_A = 0$ in the best response.

Next, suppose that $P(0, \frac{1}{2}) < \frac{1}{2}$. Then, candidate A's expected payoff from $x_A = 0$ is:

$$\frac{\alpha_B(1)}{2} a_i + \epsilon \left(\frac{\alpha_B(1)}{2} b_i(0) + (1 - \alpha_B(1)) b_i(\frac{1}{2}) + \frac{\alpha_B(1)}{2} b_i(1) \right).$$

On the other hand, her expected payoff from $x_A = \frac{1}{2}$ is:

$$\frac{1 + \alpha_B(1)}{2} a_i + \epsilon b_i(\frac{1}{2}).$$

We show that the latter is strictly larger than the former. To see this, subtract the former from the latter:

$$\left(\frac{1+\alpha_B(1)}{2}a_i + \epsilon b_i(\frac{1}{2})\right) - \left(\frac{\alpha_B(1)}{2}a_i + \epsilon\left(\frac{\alpha_B(1)}{2}b_i(0) + (1-\alpha_B(1))b_i(\frac{1}{2}) + \frac{\alpha_B(1)}{2}b_i(1)\right)\right)$$
$$= \frac{1}{2}\left(a_i - \alpha_B(1)\cdot\epsilon\left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)\right)\right). \quad (3)$$

Expression (3) is obviously strictly positive if $b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)$ is negative. Otherwise, (3) is the smallest if $\alpha_B(1) = 1$, in which case it is strictly positive because of the assumption that $\epsilon\left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)\right) < a_i$. This shows that the payoff at $x_A = \frac{1}{2}$ is the unique best response. Hence candidate A takes a pure strategy in a Nash equilibrium.

A symmetric argument shows that candidate B uses a pure strategy in any best response, completing the proof. □

A.2 Proof of Proposition 4

Proof. We show the "Only if" direction first and then the "If" direction.

"Only if" direction. Let (x_A^*, x_B^*) be a pure strategy Nash equilibrium of the game with $\epsilon > 0$ (by Proposition 3, we can focus on pure strategy Nash equilibria without loss of generality). We begin by showing that each candidate obtains the vote share of $\frac{1}{2}$. Suppose the contrary, i.e. that some candidate i gets a vote share strictly smaller than $\frac{1}{2}$. Then, by deviating from x_i^* to $x_i = x_{-i}^*$, candidate i can strictly increase the winning probability, while this deviation does not change the realized policy (namely x_{-i}^*). This contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium. Hence in (x_A^*, x_B^*), each candidate gets the vote share of $\frac{1}{2}$. Thus, in particular, the winninng probability in (x_A^*, x_B^*) is $\frac{1}{2}$.

From the proof of Proposition 3, $x_A^* \neq 1$ and $x_B^* \neq 0$. Hence there are four cases: (i) $(x_A^*, x_B^*) = (0, \frac{1}{2})$, (ii) $(x_A^*, x_B^*) = (0, 1)$, (iii) $(x_A^*, x_B^*) = (\frac{1}{2}, \frac{1}{2})$, and (iv) $(x_A^*, x_B^*) = (\frac{1}{2}, 1)$.

In each case, we will suppose that candidate A has an incentive to deviate from (x_A^*, x_B^*) in the game with $\epsilon = 0$, and derive contradictions. By symmetry this is sufficient to show that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon = 0$.

First, consider case (i). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = \frac{1}{2}$ has to give a strictly higher winning probability than x_A^*. But it would give A the winning probability of $\frac{1}{2}$ by symmetry, which is the same as when she takes x_A^*, a contradiction.

Second, consider case (ii). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = \frac{1}{2}$ has to give a strictly higher winning probability than x_A^* (1 instead of $\frac{1}{2}$). But then by assumption $\epsilon\left(b_i(0) - 2b_i(\frac{1}{2}) + b_i(1)\right) < a_i$, we have $a_i + \epsilon b_i(\frac{1}{2}) > \frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(0) + b_i(1)\right)$, which implies that in the game with $\epsilon > 0$, $x_A = \frac{1}{2}$ gives a higher payoff to candidate A than x_A^* does. This contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

Third, consider case (iii). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = 0$ has to give a strictly higher winning probability than x_A^*. But if it were the case, then in the game with $\epsilon > 0$, the deviation from x_A^* to 0 gives candidate A a higher winning probability (1 instead of $\frac{1}{2}$) as well as the realized policy closer to her bliss point (0 instead of $\frac{1}{2}$). Thus candidate A would be better off by taking 0 instead of x_A^*, which contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

Finally, consider case (iv). If A has an incentive to deviate in the game with $\epsilon = 0$, $x_A = 0$ has to give a strictly higher winning probability than x_A^*. But it would give A the winning probability of $\frac{1}{2}$ by symmetry, which is the same as when she takes x_A^*, a contradiction.

To conclude, we have shown that whenever (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$, it is also a Nash equilibrium in the game with $\epsilon = 0$.

Now we verify that in (x_A^*, x_B^*) candidate A does not have another choice x_A' such that $|x_A' - \bar{x}_A| < |x_A^* - \bar{x}_A|$ that gives her the same vote share. If such x_i' exists, then it is immediate that the payoff from x_A^* is strictly less than the one from x_A' in the game with $\epsilon > 0$, since x_A' is closer to A's bliss point than x_A^*. This contradicts the assumption that (x_A^*, x_B^*) is a Nash equilibrium in the game with $\epsilon > 0$.

"If" direction. Let (x_A^*, x_B^*) be a Nash equilibrium of the game with $\epsilon = 0$ with the property that, for each $i = A, B$, candidate i has no other choice of policy x_i' such that $P(x_i', x_{-i}^*) = P(x_i^*, x_{-i}^*)$ and $|x_i' - \bar{x}_i| < |x_i^* - \bar{x}_i|$. This implies that the vote share at x_i^* is $\frac{1}{2}$, and there is no other choice x_i' that gives strictly higher vote share. Suppose that (x_A^*, x_B^*) is not a Nash equilibrium of the game with $\epsilon > 0$. In the game with $\epsilon > 0$, the expected payoff at x_i^* is $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$. The expected payoff at another choice x_i' is $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$ if x_i' gives the vote share of $\frac{1}{2}$, and it is $\epsilon b_i(|x_{-i}^* - \bar{x}_i|)$ if x_i' gives the vote share strictly less than $\frac{1}{2}$.

In the former case, to compare $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$ and $\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right)$, note that x_i^* and x_i' give the same vote share. This implies, by assumption, that x_i^* is weakly closer to $\bar{x}_i$ than x_i' is.

This implies that $b_i(|x_i^* - \bar{x}_i|) \geq b_i(|x_i' - \bar{x}_i|)$. Hence

$$\frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i^* - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right) \geq \frac{a_i}{2} + \frac{\epsilon}{2}\left(b_i(|x_i' - \bar{x}_i|) + b_i(|x_{-i}^* - \bar{x}_i|)\right). \quad (4)$$

In the latter case, first note that $x_A^* \neq 1, x_B^* \neq 0$ by the proof of Proposition 3, so $|x_i^* - \bar{x}_i| \leq |x_{-i}^* - \bar{x}_i|$. Therefore

$$\begin{aligned}\epsilon b_i(|x_{-i}^* - \bar{x}_i|) &\leq \frac{\epsilon}{2}(b_i(|x_A^* - \bar{x}_i|) + b_i(|x_B^* - \bar{x}_i|) \\ &< \frac{a_i}{2} + \frac{\epsilon}{2}(b_i(|x_A^* - \bar{x}_i|) + b_i(|x_B^* - \bar{x}_i|). \quad (5)\end{aligned}$$

Conclusions (4) and (5) of these two cases show that candidate i does not have an incentive to deviate from (x_A^*, x_B^*) in the game with $\epsilon > 0$. This completes the proof.

□

A.3 Proof of Proposition 1 and Theorem 1

Since Proposition 1 is a special case of Theorem 1 when $c = 0$, we prove Theorem 1.

Proof. First, we show that if there exists a Nash equilibrium, then it must be unique. To see this, let (x_A^*, x_B^*) and (x_A^{**}, x_B^{**}) be Nash equilibria. Since each candidate obtains the vote share of $\frac{1}{2}$ in any equilibrium as shown in the proof of Proposition 4, we have

$$\frac{1}{2} = P(x_A^*, x_B^*) \geq P(x_A^{**}, x_B^*) = 1 - P(x_B^*, x_A^{**}) \geq 1 - P(x_B^{**}, x_A^{**}) = P(x_A^{**}, x_B^{**}) = \frac{1}{2}.$$

Hence, it must be the case that $P(x_A^*, x_B^*) = P(x_A^{**}, x_B^*)$. This equation together with the condition that (x_A^*, x_B^*) is a Nash equilibrium implies that $|x_A^* - \bar{x}_A| \leq |x_A^{**} - \bar{x}_A|$. By a symmetric argument, we obtain $|x_A^* - \bar{x}_A| \geq |x_A^{**} - \bar{x}_A|$ Hence, we have $|x_A^* - \bar{x}_A| = |x_A^{**} - \bar{x}_A|$. Since $\bar{x}_A = 0$, we have $x_A^* = x_A^{**}$. We can apply an analogous argument to show that $x_B^* = x_B^{**}$. Therefore, we conclude that if there exists a Nash equilibrium, then it must be unique.

Part 1. We will show that $(x_A^*, x_B^*) = (0, 1)$ is a Nash equilibrium (and hence the unique Nash equilibrium) if $v \geq 0$ and $c \leq \bar{c}(v, \lambda)$.

By Proposition 4, the assumption $(\bar{x}_A, \bar{x}_B) = (0, 1)$, and symmetry, $(0, 1)$ is a Nash equilibrium if and only if the vote share of candidate B is at most

$\frac{1}{2}$ when she chooses position $\frac{1}{2}$ while candidate A chooses 0. Note that

$$\begin{aligned} P(x,0) &= \frac{1-c}{2} \cdot \frac{\mathbf{e}^{\lambda u(x)}}{\mathbf{e}^{\lambda u(x)} + \mathbf{e}^{\lambda u(0)}} + c \cdot \frac{\mathbf{e}^{\lambda u(|\frac{1}{2}-x|)}}{\mathbf{e}^{\lambda u(|\frac{1}{2}-x|)} + \mathbf{e}^{\lambda u(\frac{1}{2})}} + \frac{1-c}{2} \cdot \frac{\mathbf{e}^{\lambda u(1-x)}}{\mathbf{e}^{\lambda u(1-x)} + \mathbf{e}^{\lambda u(1)}} \\ &= \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda[u(0)-u(x)]}} + c \cdot \frac{1}{1+\mathbf{e}^{\lambda\left[u(\frac{1}{2})-u(|\frac{1}{2}-x|)\right]}} + \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda[u(1)-u(1-x)]}} \\ &= \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda[1-u(x)]}} + c \cdot \frac{1}{1+\mathbf{e}^{\lambda\left[u(\frac{1}{2})-u(|\frac{1}{2}-x|)\right]}} + \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda[-u(1-x)]}}. \end{aligned}$$

Therefore the condition that the vote share of candidate B with position $x = \frac{1}{2}$ against position 0 of the opponent be no more than $\frac{1}{2}$ is:

$$\begin{aligned} & P\left(\frac{1}{2},0\right) = \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}} + c \cdot \frac{1}{1+\mathbf{e}^{\lambda\left[u(\frac{1}{2})-u(0)\right]}} + \frac{1-c}{2} \cdot \frac{1}{1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}} \leq \frac{1}{2} \\ \Longleftrightarrow\ & c\left(-\frac{1}{1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}} + 2 \cdot \frac{\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}}{1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}} - \frac{1}{1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}}\right) \\ & \leq 1 - \frac{1}{1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}} - \frac{1}{1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}} \\ \Longleftrightarrow\ & c\left(-\left(1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}\right) + 2\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}\left(1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}\right) - \left(1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}\right)\right) \\ & \leq \left(1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}\right)\left(1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}\right) - \left(1+\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}\right) - \left(1+\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}\right) \\ \Longleftrightarrow\ & c\left(-2-\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]} + \mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]} + 2\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]}\mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}\right) \leq \mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]} \cdot \mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]} - 1 \\ \Longleftrightarrow\ & c \leq \frac{\mathbf{e}^{\lambda\left[1-2u(\frac{1}{2})\right]} - 1}{\mathbf{e}^{\lambda\left[1-u(\frac{1}{2})\right]} - \mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]} + 2\left(\mathbf{e}^{\lambda\left[1-2u(\frac{1}{2})\right]} - 1\right)} \\ \Longleftrightarrow\ & c \leq \frac{\mathbf{e}^{2\lambda v} - 1}{\mathbf{e}^{\lambda\left(\frac{1}{2}+v\right)} - \mathbf{e}^{\lambda\left(v-\frac{1}{2}\right)} + 2\left(\mathbf{e}^{2\lambda v} - 1\right)}. \end{aligned} \tag{6}$$

If $v = 0$, then the right hand side of inequality (6) is equal to zero, thus showing the statement for $v = 0$. If $v > 0$, then inequality (6) is equivalent to

$$c \leq \left(2 + \frac{\mathbf{e}^{\lambda\left(\frac{1}{2}+v\right)} - \mathbf{e}^{\lambda\left(v-\frac{1}{2}\right)}}{\mathbf{e}^{2\lambda v} - 1}\right)^{-1} = \left(2 + \frac{\mathbf{e}^{\frac{1}{2}\lambda} - \mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v}}\right)^{-1} = \bar{c}(v,\lambda), \tag{7}$$

showing the statement for $v > 0$.

Part 2. We consider two cases. First, assume $v \geq 0$ and $c > \bar{c}(v, \lambda)$. Then, by inequality (7), we have $P(0, \frac{1}{2}) = 1 - P(\frac{1}{2}, 0) < \frac{1}{2}$. By symmetry, $P(1, \frac{1}{2}) < \frac{1}{2}$. Meanwhile, it is clear that $P(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$. Hence, $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium of the game with $\epsilon = 0$. Moreover, it trivially satisfies "the additional property" in the statement of Proposition 4 since $P(x_i', \frac{1}{2}) < P(\frac{1}{2}, \frac{1}{2})$ for all $x_i' \neq \frac{1}{2}$. Therefore, by Proposition 4, we conclude that $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium for $\epsilon > 0$.

Next assume $v < 0$. First, note that

$$\begin{aligned}
&\frac{1}{1 + \mathbf{e}^{\lambda[u(1) - u(\frac{1}{2})]}} + \frac{1}{1 + \mathbf{e}^{\lambda[u(0) - u(1 - \frac{1}{2})]}} > 1 \\
\iff & 1 + \mathbf{e}^{\lambda[u(0) - u(1 - \frac{1}{2})]} + 1 + \mathbf{e}^{\lambda[u(1) - u(\frac{1}{2})]} > (1 + \mathbf{e}^{\lambda[u(0) - u(1 - \frac{1}{2})]})(1 + \mathbf{e}^{\lambda[u(1) - u(\frac{1}{2})]}) \\
\iff & \mathbf{e}^{\lambda[u(0) - u(1 - \frac{1}{2})]}\mathbf{e}^{\lambda[u(1) - u(\frac{1}{2})]} < 1 \\
\iff & u(0) - u(1 - \frac{1}{2}) + u(1) - u(\frac{1}{2}) < 0 \\
\iff & u(0) + u(1) < u(\frac{1}{2}) + u(1 - \frac{1}{2}) \\
\iff & v < 0,
\end{aligned}$$

so we have

$$\begin{aligned}
P\left(\frac{1}{2}, 0\right) &= \frac{1 - c}{2} \cdot \frac{1}{1 + \mathbf{e}^{\lambda\left[1 - u(\frac{1}{2})\right]}} + c \cdot \frac{1}{1 + \mathbf{e}^{\lambda\left[u(\frac{1}{2}) - u(0)\right]}} + \frac{1 - c}{2} \cdot \frac{1}{1 + \mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}} \\
&> \frac{1 - c}{2} \cdot \left(\frac{1}{1 + \mathbf{e}^{\lambda\left[1 - u(\frac{1}{2})\right]}} + \frac{1}{1 + \mathbf{e}^{\lambda\left[-u(\frac{1}{2})\right]}}\right) + \frac{c}{2} \\
&> \frac{1}{2}.
\end{aligned}$$

Therefore $P(0, \frac{1}{2}) = 1 - P(\frac{1}{2}, 0) < \frac{1}{2}$, while $P(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$ by symmetry. Again, by Proposition 4, $(\frac{1}{2}, \frac{1}{2})$ is a Nash equilibrium. □

A.4 Proof of Proposition 5

Proof. We show the first part and second part of the proposition in sequence.

Part 1. $\mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v}$ is strictly increasing in v, implying that $\bar{c}(v, \lambda)$ is strictly increasing in v for any fixed $\lambda > 0$. It is straightforward to see that $\bar{c}(v, \lambda)$ approaches 0 as v goes to 0 and approaches $\frac{1}{3}$ as v approaches $\frac{1}{2}$, for any fixed $\lambda > 0$.

Part 2. Consider the derivative,

$$\frac{\partial\left(\frac{\mathbf{e}^{\frac{1}{2}\lambda}-\mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v}-\mathbf{e}^{-\lambda v}}\right)}{\partial\lambda} = \frac{\partial(\mathbf{e}^{\lambda})}{\partial\lambda}\cdot\frac{\partial\left(\frac{t^{\frac{1}{2}}-t^{-\frac{1}{2}}}{t^{v}-t^{-v}}\right)}{\partial t}\Bigg|_{t=\mathbf{e}^{\lambda}}$$

$$= \mathbf{e}^{\lambda}\cdot\left(\frac{\frac{1}{2}\left(t^{\frac{1}{2}-1}+t^{-\frac{1}{2}-1}\right)(t^{v}-t^{-v})-v\left(t^{\frac{1}{2}}-t^{-\frac{1}{2}}\right)(t^{v-1}+t^{-v-1})}{(t^{v}-t^{-v})^{2}}\right)\Bigg|_{t=\mathbf{e}^{\lambda}}$$

$$= \frac{G(\mathbf{e}^{\lambda},v)}{(\mathbf{e}^{\lambda v}-\mathbf{e}^{-\lambda v})^{2}}, \tag{8}$$

where we define

$$G(t,v)=\frac{1}{2}(t^{\frac{1}{2}}+t^{-\frac{1}{2}})(t^{v}-t^{-v})-v(t^{\frac{1}{2}}-t^{-\frac{1}{2}})(t^{v}+t^{-v}).$$

By definition,

$$G(1,v)=0 \text{ for any } v. \tag{9}$$

Differentiating $G(t,v)$ with respect to t,

$$\begin{aligned}\frac{\partial G(t,v)}{\partial t} &= \frac{1}{2}\cdot\frac{1}{2}\cdot t^{-1}(t^{\frac{1}{2}}-t^{-\frac{1}{2}})(t^{v}-t^{-v})+\frac{1}{2}\cdot v\cdot t^{-1}(t^{\frac{1}{2}}+t^{-\frac{1}{2}})(t^{v}+t^{-v})\\ &\quad -v\cdot\frac{1}{2}\cdot t^{-1}(t^{\frac{1}{2}}+t^{-\frac{1}{2}})(t^{v}+t^{-v})-v\cdot v\cdot t^{-1}(t^{\frac{1}{2}}-t^{-\frac{1}{2}})(t^{v}-t^{-v})\\ &= (\frac{1}{2}-v)(\frac{1}{2}+v)t^{-1}(t^{\frac{1}{2}}-t^{-\frac{1}{2}})(t^{v}-t^{-v}). \end{aligned} \tag{10}$$

Expression (10) is strictly positive for any $v\in(0,\frac{1}{2})$ and $t>1$. This and equation (9) imply that $G(t,v)$ is strictly positive for any $t>1$ and $v\in(0,\frac{1}{2})$. Since $\mathbf{e}^{\lambda}>1$ for any $\lambda>0$, this implies that the expression (8) is strictly positive for any $v\in(0,\frac{1}{2})$. Hence, $\bar{c}(v,\lambda)$ is strictly decreasing in λ for any $v\in(0,\frac{1}{2})$.

To obtain the limit values of $\bar{c}(v,\lambda)$, first note that the statement holds trivially for $v=0$ by definition: $\bar{c}(0,\lambda)=0$ for all λ. For $v>0$, by L'Hopital's rule, we have

$$\lim_{\lambda\searrow 0}\frac{\mathbf{e}^{\frac{1}{2}\lambda}-\mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v}-\mathbf{e}^{-\lambda v}}=\lim_{\lambda\searrow 0}\frac{\frac{1}{2}\left(\mathbf{e}^{\frac{1}{2}\lambda}+\mathbf{e}^{-\frac{1}{2}\lambda}\right)}{v\left(\mathbf{e}^{\lambda v}+\mathbf{e}^{-\lambda v}\right)}=\frac{\frac{1}{2}\cdot 2}{v\cdot 2}=\frac{1}{2v}.$$

Hence, $\lim_{\lambda\searrow 0}\bar{c}(v,\lambda)=\left(2+\frac{1}{2v}\right)^{-1}=\frac{2v}{4v+1}$. It is straightforward to see that $\lim_{\lambda\to\infty}\bar{c}(v,\lambda)=0$.

□

A.5 Proofs of Proposition 2 and Theorem 2

Before showing the results, first note that $W(0) = W(1)$ by symmetry, and

$$
\begin{aligned}
& W(0) \geq W(\frac{1}{2}) \\
\iff & \frac{1-c}{2} \cdot u(0) + c \cdot u(\frac{1}{2}) + \frac{1-c}{2} \cdot u(1) \geq \frac{1-c}{2} \cdot u(\frac{1}{2}) + c \cdot u(0) + \frac{1-c}{2} \cdot u(\frac{1}{2}) \\
\iff & \frac{1-c}{2} \cdot 1 + c \cdot \left(\frac{1}{2} - v\right) + \frac{1-c}{2} \cdot 0 \geq \frac{1-c}{2} \cdot \left(\frac{1}{2} - v\right) + c \cdot 1 + \frac{1-c}{2} \cdot \left(\frac{1}{2} - v\right) \\
\iff & 2(1-c) + 2c\,(1-2v) \geq (1-c)\,(1-2v) + 4c + (1-c)\,(1-2v) \\
\iff & c\,(-2 + 2(1-2v) + (1-2v) - 4 + (1-2v)) \geq 2(1-2v) - 2 \\
\iff & c\,(4(1-2v) - 6) \geq -4v \\
\iff & c(1+4v) \leq 2v. \qquad (11)
\end{aligned}
$$

Proof of Theorem 2. By Theorem 1, the assumption that $(0, 1)$ is a Nash equilibrium implies $v \geq 0$. When $v \geq 0$, inequality (11) is equivalent to $c \leq \frac{2v}{1+4v} := \hat{c}(v, \lambda)$. Thus it suffices to show $\bar{c}(v, \lambda) \leq \hat{c}(v, \lambda)$ for all $v \geq 0$ and $\lambda > 0$. This inequality $\bar{c}(v, \lambda) \leq \hat{c}(v, \lambda)$ holds for $v = 0$ since $\bar{c}(0, \lambda) = \hat{c}(0, \lambda) = 0$. If $v > 0$, by algebraic manipulation we obtain

$$
\begin{aligned}
\bar{c}(v, \lambda) \leq \hat{c}(v, \lambda) \iff & \left(2 + \frac{\mathbf{e}^{\frac{1}{2}\lambda} - \mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v}}\right)^{-1} \leq \frac{2v}{1+4v} \\
\iff & 1 + 4v \leq 2v \left(2 + \frac{\mathbf{e}^{\frac{1}{2}\lambda} - \mathbf{e}^{-\frac{1}{2}\lambda}}{\mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v}}\right) \\
\iff & \mathbf{e}^{\lambda v} - \mathbf{e}^{-\lambda v} \leq 2v(\mathbf{e}^{\frac{1}{2}\lambda} - \mathbf{e}^{-\frac{1}{2}\lambda}).
\end{aligned}
$$

For all $\lambda > 0$, the left hand side of the last inequality is convex with respect to v and its right hand side is linear with respect to v. Also, an equality holds between these two sides both at $v = 0$ and at $v = \frac{1}{2}$. Hence, the last inequality holds for all $v \in [0, \frac{1}{2}]$ and $\lambda > 0$, implying that $\bar{c}(v, \lambda) \leq \hat{c}(v, \lambda)$ for all $v \in [0, \frac{1}{2})$ and $\lambda > 0$. This completes the proof. □

Proof of Proposition 2. If $v \geq 0$, then the statement of the Proposition is a special case of Theorem 2 since $c = 0 \leq \bar{c}(v, \lambda)$ for the polarized distribution. So assume $v < 0$. Recall that $c = 0$ holds for the polarized distribution. This implies that inequality (11) is violated, since its left hand side is zero and its right hand side is strictly negative. Therefore $W(\frac{1}{2}) > W(0) = W(1)$. By Proposition 1, the unique Nash equilibrium is $(\frac{1}{2}, \frac{1}{2})$, thus completing the proof. □

A.6 Proof of Theorem 3

Proof. Let (x_A^*, x_B^*) be the strategy profile as defined in the statement of the Theorem.

For any k such that u_k is convex, by definition we have

$$u_k(0) + u_k(1) \geq 2u_k(\frac{1}{2}).$$

For any other k, u_k is strictly concave and we have

$$u_k(0) + u_k(1) < 2u_k(\frac{1}{2}).$$

Therefore, for any $i \in \{A, B\}$ and $j \neq i, x, y \in \{0,1\}^n$ with $y_k = 1 - x_k$ for all k and any $x_i' = (x_{i1}', \ldots, x_{in}') \in \mathcal{P}$,

$$\sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - x_k|) + \sum_{k=1}^{n} \delta_k u_k(|x_{ik}' - y_k|) \leq \sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - x_k|) + \sum_{k=1}^{n} \delta_k u_k(|x_{jk}^* - y_k|),$$

with strict inequality if there exists k such that $x_{ik}' \neq \frac{1}{2}$ and u_k is strictly concave. Rearranging terms, this inequality is equivalent to

$$\begin{aligned}
& 0 \leq \textstyle\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|) \\
& \quad + \textstyle\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - y_k|) \\
\iff \quad & 1 \leq \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]} \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - y_k|)]} \\
\iff \quad & (1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]}) + (1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - y_k|)]}) \\
& \leq (1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]})(1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - y_k|)]}) \\
\iff \quad & \frac{1}{1+\mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]}} + \frac{1}{1+\mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - y_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - y_k|)]}} \leq 1,
\end{aligned}$$

with strict inequality if there exists k such that $x_{ik}' \neq \frac{1}{2}$ and u_k is strictly concave. Therefore,

$$\begin{aligned}
P(x_i', x_j^*) &= \sum_{x \in \mathcal{P}} \frac{1}{1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]}} f(x) \\
&= \frac{1}{2}[\sum_{x \in \mathcal{P}} (\frac{1}{1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - x_k|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - x_k|)]}} \\
&+ \frac{1}{1 + \mathbf{e}^{[\sum_{k=1}^n \delta_k u_k(|x_{jk}^* - (1-x_k)|) - \sum_{k=1}^n \delta_k u_k(|x_{ik}' - (1-x_k)|)]}}) f(x)] \\
&\leq \frac{1}{2} \\
&= P(x_i^*, x_j^*),
\end{aligned}$$

with strict inequality if there exists k such that $x'_{ik} \neq \frac{1}{2}$ and u_k is strictly concave. Therefore, the only possible deviation x'_i by i from x^*_i that does not reduce her winning probability is one where $x'_{ik} \neq \bar{x}_{ik}$ for some k's such that u_k's are convex, while $x'_{ik} = x^*_{ik}$ for all other k's. But this deviation would keep the winning probability for i unchanged at $\frac{1}{2}$ at best while strictly reducing her payoff when she wins, thus it is not a profitable deviation.

Uniqueness of the Nash equilibrium holds by an analogous argument as in the proof of Proposition 1 and Theorem 1 and hence is omitted. □

References

Alesina, Alberto, "Credibility and Policy Convergence in a Two-Party System with Rational Voters," *American Economic Review*, 1988, *78*, 796–805.

Anderson, Simon P, Andr de Palma, and Jacques-Francois Thisse, *Discrete Choice Theory of Product Differentiation*, Cambridge: MIT Press, 1992.

Aragones, Enriqueta and Andrew Postlewaite, "Ambiguity in election games," *Review of Economic Design*, 2002, *7*, 233–255.

_ **and Thomas Palfrey**, "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory*, 2000, *103*, 131–161.

Banks, Jeffrey S. and John Duggan, "Probabilistic Voting in the Spatial Models of Elections: The Theory of Office-Motivated Candidates," in David Austen-Smith and John Duggan, eds., *Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks*, Heidelberg: Springer, 2005.

Besley, T. and S. Coate, "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 1997, *112*, 85–114.

Black, Duncan, *The Theory of Committees and Elections*, Cambridge: Cambridge University Press, 1958.

Carrillo, Juan and Micael Castanheira, "Information and Strategic Political Polarization," 2008.

DiMaggio, Paul, John Evans, and Bethany Bryson, "Have American's Social Attitudes Become More Polarized?," *American Journal of Sociology*, 1996, *102*, 690–755.

Downs, Anthony, *An Economic Theory of Democracy*, New York: Harper and Row, 1957.

Evans, John, "Have American's Social Attitudes Become More Polarized? An Update," *Social Science Quarterly*, 2003, *84*, 71–90.

Fey, Mark, "Stability and Coordination in Duverger's Law: A Formal Model of Preelection Polls and Strategic Voting," *American Political Science Review*, 1997, *91*, 135–147.

Fiorina, Morris P., *Culture War? The Myth of a Polarized America*, Pearson Longman, 2006.

Gelman, Andrew, *Red State, Blue State, Rich State, Poor State*, Princeton: Princeton University Press, 2008.

Glaeser, Edward L., Giacomo A. M. Ponzetto, and Jesse M. Shapiro, "Strategic Extremism: Why Republicans and Democrats Divide on Religious Values," *Quarterly Journal of Economics*, 2005, *120*, 1283–1330.

Hotelling, Harold, "Stability in Competition," *Economic Journal*, 1929, *39*, 41–57.

Kartik, Navin and R. Preston McAfee, "Signaling Character in Electoral Competition," *American Economic Review*, 2007, *97*, 852–870.

McCarty, Nolan, Keith T. Poole, and Howard Rosenthal, *Polarized America*, Cambridge: The MIT Press, 2006.

Myerson, Roger B. and Robert Weber, "A Theory of Voting Equilibria," *American Political Science Review*, 1993, *87*, 102–114.

Osborne, Martin J., "Spatial Models of Political Competition Under Plurality Rule: A Survey of Some Explanations of The Number of Candidates and The Positions They Take," *Canadiann Journal of Economics*, 1995, *2*, 261–301.

_ **and A. Slivinski**, "A model of political competition with citizen-candidates," *Quarterly Journal of Economics*, 1996, *111*, 65–96.

Palfrey, Thomas R., "Spatial equilibrium with entry," *Review of Economic Studies*, 1984, *51*, 139–156.

Persson, Torsten and Guido Tabellini, *Political Economics: Explaining Economic Policy*, Cambridge: MIT Press, 2000.

Poole, Keith T. and Howard Rosenthal, *Congress: a political-economic history of roll call voting*, New York: Oxford University Press, 1997.

Roemer, John, "A Theory of Policy Differentiation in Single Issue Electoral Competition," *Social Choice and Welfare*, 1994, *11*, 355–380.

Shepsle, Kenneth, "The Strategy of Ambiguity: Uncertainty and Electoral Competition," *American Political Science Review*, 1972, *66*, 555–568.

Yang, C. C., "Endogenous Tariff Formation under Representative Democracy: A Probabilistic Voting Model," *American Economic Review*, 1995, *85*, 956–963.